Cognitive Imprecision and Strategic Behavior*

Cary Frydman† Salvatore Nunnari‡

July 13, 2021

Abstract

We propose and experimentally test a theory of strategic behavior in which players are *cognitively imprecise* and perceive the state of the world with noise. We focus on 2×2 *regime change* games. When players observe the state precisely and this is common knowledge, there are multiple equilibria. However, when players are cognitively imprecise, each player holds a slightly different perception of the state and this transforms the game into one of incomplete information. Relying on arguments from the global games literature, we show that small perceptual noise can select a unique equilibrium. When combined with a further assumption of efficient coding, this prediction delivers our main testable hypothesis: the amount of noise with which players implement the unique equilibrium strategy is increasing in the *ex-ante* volatility of the state of the world. We find strong empirical support for this prediction in a pre-registered experiment. For a given state, the distribution of observed actions depends systematically on the environment to which players' perceptual systems have adapted. Our results suggest that the theory of global games can potentially be applied even in the presence of precise public information (e.g., market prices) because of players' cognitive imprecision. More broadly, our framework offers a cognitive foundation for models of imperfect best response (e.g., QRE).

Keywords: Perception, Efficient Coding, Coordination, Global Games

JEL Codes: C72, C92, D91, E71

*We are grateful to Benjamin Enke, Nicola Gennaioli, Lawrence Jin, Vijai Krishna, Nicola Pavoni, and audiences at Florida State University, Princeton University, University of Glasgow, Indiana University, University of Insubria, Bocconi University, MiddExLab Virtual Seminar, and the HBS Workshop on Cognitive Noise for helpful comments and interesting discussions. Luca Congiu provided excellent research assistantship. Frydman acknowledges financial support from the NSF. Nunnari acknowledges financial support from the European Research Council (POPULIZATION Grant No. 852526).

†University of Southern California, Marshall School of Business, cfrydman@marshall.usc.edu.

‡Bocconi University, Department of Economics, salvatore.nunnari@unibocconi.it.

1 Introduction

Over the past decade, economists have begun investigating the effects of *cognitive imprecision* on individual decision-making (see Woodford 2020 for a review). This agenda proposes that the decision-maker's perception of the economic environment does not coincide with the objective environment, due to information processing constraints in the brain. To date, the emphasis of cognitive imprecision in economics has largely been in the domains of choice under risk and intertemporal choice. For example, even when the decision-maker has linear utility and perfectly patient time preferences, theoretical models with cognitive imprecision can generate as-if risk aversion (Khaw, Li and Woodford, Forthcoming) and as-if time discounting (Gabaix and Laibson, 2017). Initial experimental tests in both the domains of risky choice and intertemporal choice have produced encouraging results (Enke and Graeber, 2019; Gershman and Bhui, 2020; Khaw, Li and Woodford, Forthcoming; Frydman and Jin, Forthcoming).

Motivated by this growing evidence from the domain of individual decision-making, it is natural to ask whether cognitive imprecision also affects strategic behavior. This line of inquiry is important not only to test whether cognitive imprecision extends into other economic domains, but also because imposing psychologically-grounded restrictions on perception can affect equilibrium predictions. Indeed, in many *regime change* games that are used to model bank runs, currency attacks, and revolutions, there can be multiple equilibria — especially when each player perceives the state of the world with perfect precision[1] However, as pointed out by Woodford (2020), if one adds a small amount of cognitive imprecision to the model, so that each player has a slightly different perception of the state, then coordination becomes more difficult and multiplicity breaks down. Thus, perceptual noise may prove useful in selecting a unique equilibrium. This line of reasoning follows directly from the vast literature on *global games*, with the important distinction that, here, we interpret the noise as arising

[1]For examples of regime change models in these domains, see Diamond and Dybvig (1983), Obstfeld (1996), Morris and Shin (1998), Atkeson (2000), Goldstein and Pauzner (2005), and Edmond (2013).

from perceptual errors, rather than from more traditional sources of asymmetric information (Carlsson and Van Damme, 1993; Morris and Shin, 2003; Angeletos and Lian, 2016).

In this paper, we theoretically develop and experimentally test the hypothesis that perceptual noise systematically affects behavior in games. The hypothesis enables us to apply standard results from the global games literature, without explicitly introducing private information. Specifically, we analyze a 2×2 simultaneous move game of regime change where players can choose to invest or not in an asset. If both players invest, the regime changes. Each player's payoff depends on the value of a fundamental and on the action of the other player. While theory predicts multiple equilibria for a range of fundamental values in the complete information version of the game, a small amount of perceptual noise implies the existence of a unique equilibrium: each player invests if and only if the fundamental crosses a threshold. Our experimental design is optimized to test this prediction.

Several previous experimental studies have found empirical support for the global games prediction that the probability of investing is monotonic in the fundamental (Heinemann, Nagel and Ockenfels, 2004, 2009; Cabrales, Nagel and Armenter, 2007; Szkup and Trevino, 2020; Goryunov and Rigos, 2020). However, our interpretation of noise in the global games model as stemming from perceptual constraints generates a novel testable hypothesis, which, if validated, should make the results from the global games literature more broadly applicable. Specifically, we assume a particular type of cognitive imprecision in our model, called *efficient coding*. This assumption delivers sharp predictions about context-dependent perception, and hence behavior. Thus, if our experimental results are consistent with the stronger assumption of efficient coding, then this provides evidence for (the weaker assumption of) cognitive imprecision, which is the key ingredient for generating a unique equilibrium.

Roughly speaking, efficient coding posits that the distribution of perceptual noise is optimally adapted to the decision-maker's environment. This principle implies that, when a fundamental parameter is drawn from a symmetric and unimodal distribution, a player's perceptual resources are optimally allocated towards perceiving more accurately those fundamentals close to the mean of the distribution, since a bulk of the density is concentrated near the mean. But when the distribution becomes wider — so that fundamentals are more volatile — perceptual resources are then optimally readjusted away from the center of the distribution; thus, at least for values near the mean of the distribution, there is an increase in cognitive imprecision.[2]

To test this hypothesis, we conduct a pre-registered experiment in which subjects are randomly matched into pairs and play a regime change game, in each of three hundred rounds. The game is characterized by the value of a fundamental parameter, which is clearly displayed to both subjects on each round as a two-digit Arabic numeral, such as "48". We rely on subjects' inherent cognitive imprecision to transform this "public" signal into a private signal, owing to idiosyncratic perceptual errors. The perceptual error, of course, induces uncertainty over the fundamental; but, more importantly, it also induces strategic uncertainty over the other player's action, which is key to breaking the multiplicity of equilibria. In order to provide a targeted test of cognitive imprecision as the source of private information, we manipulate the volatility of the fundamental across a high volatility and a low volatility condition. Under efficient coding, this manipulation should endogenously change the amount of noise in a subject's perceived value of the fundamental.

Our data are consistent with the hypothesis that subjects perceive the fundamental parameter with imprecision. We also find clear evidence supporting the stronger hypothesis of efficient coding in the regime change game, which manifests as a context-dependent probability of investing. Specifically, we observe that the probability of investing is monotone in the fundamental, and we find that this monotonic relationship is significantly stronger in the low volatility condition than in the high volatility condition. Importantly, this result holds when restricting to the same set of games, i.e., those games characterized by a fundamental in a fixed range. In light of our model, we interpret the observed treatment effect as a conse-

[2]Such an assumption has been validated in many papers on sensory perception (Girshick et al., 2011; Wei and Stocker, 2015; Payzan-LeNestour and Woodford, Forthcoming) and in economic decision making (Polania, Woodford and Ruff, 2019; Frydman and Jin, Forthcoming).

quence of more accurate perception of fundamentals in the low volatility condition. Further evidence comes from the distribution of response times, which indicates that subjects make decisions significantly more quickly when they are adapted to the low volatility distribution of fundamentals.

We emphasize that, in both conditions, the strong monotonic relationship that we observe between fundamentals and investing is not predicted under the complete information version of the game. As such, the data suggest that even when subjects receive no explicit private signals from the experimenter, private information is inherent in the game because subjects encode the fundamental with idiosyncratic perceptual noise. Our results offer an explanation for earlier experimental papers on regime change games which, surprisingly, find little difference in behavior when manipulating the provision of private information (Heinemann, Nagel and Ockenfels, 2004; Van Huyck, Viriyavipart and Brown, 2018).[3] Such a result can be explained when one takes a broader view of the potential sources of private information to also include perceptual errors.

Because our experimental tests are based on equilibrium predictions, one important and implicit assumption we make is that subjects are aware of their own perceptual noise and that of their opponent. Although this assumption is much weaker than the assumption of common knowledge of the noisy signal technology that is usually invoked in global games, it is important for the equilibrium arguments to follow.[4]

To investigate the validity of this assumption, we conduct a second experiment, where subjects are asked to classify whether a two-digit number is greater than a reference level

[3]Moreover, when discussing an experiment where there is no explicit private information about payoffs, Heinemann, Nagel and Ockenfels (2009) argue that "Of course, players know the true payoff. Their uncertainty about others' behavior makes them behave *as if* they are uncertain about payoffs" (p. 203). Our results indicate that it may well be the case that subjects do not know the true payoff. Heinemann, Nagel and Ockenfels (2009) also estimate the standard deviation of *as if* private signals in their experimental data and find that the model implies a fairly large standard deviation; one interpretation of their result is that a portion of the large estimate of noise in private signals is driven by errors in perceiving the fundamental.

[4]Morris, Shin and Yildiz (2016) show that a weaker assumption of "common certainty of uniform rank beliefs" is sufficient to deliver the standard global games result. In our case of a 2×2 symmetric game, this assumption states that each player assigns probability $1/2$ to the other player's signal being greater than his own.

of 55 (which is chosen to be the same as the threshold in the unique equilibrium of the regime change game from our first experiment). We then incentivize subjects to report their beliefs about their own accuracy and the accuracy of the average of all other subjects in the experiment. We find that subjects do report that they are aware of their own errors and of the errors of others in the classification task. Moreover, subjects are aware that discriminating between a number close to the threshold, say "54", is harder than discriminating between a number far from the threshold, say "47." This property has been shown theoretically to have important implications for equilibrium selection (Morris and Yang, 2019), and has recently been formalized in the rational inattention literature using a so-called "neighborhood cost function" (Hébert and Woodford, Forthcoming). The data from our second experiment therefore provide novel evidence supporting the assumption that subjects are aware that nearby states are systematically harder to distinguish compared to far-away states.

In our regime change game experiment, we emphasize that the fundamental is represented by a two-digit Arabic numeral that is clearly displayed to both subjects. Thus, any cognitive imprecision that we observe in the experiment is likely to be a lower bound on the imprecision in perceiving more complex, but publicly available, stimuli in the field (e.g., market prices). This is important because previous theoretical work has shown that the amount of noise in public signals, relative to that in private signals, determines whether there exists a unique equilibrium (Morris and Shin, 2003; Angeletos and Werning, 2006; Hellwig, Mukherji and Tsyvinski, 2006).

The remainder of the paper proceeds as follows: Section 2 presents the model and derives the theoretical predictions for our experimental manipulation. Sections 3 and 4 describe the experimental design and report the experimental results for Experiment 1 (the regime change game) and Experiment 2 (the number classification task), respectively. Section 5 discusses some assumptions of our theoretical framework and its connection with existing behavioral game theory models. Section 6 concludes.

2 Model

Here we provide a theory of strategic interaction in which a game payoff is perceived imprecisely but also efficiently, according to the principle of efficient coding (Barlow, 1961; Wei and Stocker, 2015; Khaw, Li and Woodford, Forthcoming; Frydman and Jin, Forthcoming). The bulk of our theoretical results depend only on the assumption of cognitive imprecision; as we discuss in more detail below, the assumption of efficient coding provides us with an identification strategy in the experiment to test the source of noisy behavior. Here we illustrate the implications of the theory in a 2×2 simultaneous-move game which, for some parameters, captures the essential features of a coordination game. Consider the following game

	Not Invest	Invest
Not Invest	θ, θ	θ, a
Invest	a, θ	b, b

Table 1: The Game

where $b > a$. In what follows, we always assume that a and b are perceived precisely by both players, and we are interested in the effects of imprecise coding of θ.[5] In Section 2.1, we assume that θ is perceived precisely by both players. In Sections 2.2 and 2.3, we introduce imprecise coding of θ and investigate the consequences for strategic behavior and equilibrium outcomes.

2.1 Precise Coding

First, we consider the game where θ is perceived precisely by both players. This is a game of complete information, which has the following Nash equilibria:

[5]Our assumption that a and b are perceived without noise can be justified, for example, through a learning mechanism. In our experiment, we keep a and b constant across all rounds, so the amount of noise in perceiving a and b is arguably minimal.

- If $\theta > b$, then Invest is a strictly dominated action for each player, and (Not Invest, Not Invest) is the unique Nash (and dominant strategy) equilibrium.

- If $\theta < a$, then Not Invest is a strictly dominated action for each player, and (Invest, Invest) is the unique Nash (and dominant strategy) equilibrium.

- If $a \leq \theta \leq b$, then there are two Nash equilibria in pure strategies, (Not Invest, Not Invest) and (Invest, Invest), and a Nash equilibrium in mixed strategies.

When $a \leq \theta \leq b$, one of the two pure Nash equilibria is *risk dominant*. In particular, according to Harsanyi and Selten (1988), (Not Invest, Not Invest) risk-dominates (Invest, Invest) if Not Invest is associated with the largest product of deviation losses (and vice versa). It follows that (Not Invest, Not Invest) risk dominates (Invest, Invest) if and only if $(\theta - a)^2 > (b - \theta)^2$. To illustrate the predictions of the complete information game using a particular example, we assign values to a and b that we also use in our experiment:

Example 1 *Assume* $a = 47$, $b = 63$. *Then, if* $\theta > 63$, *(Not Invest, Not Invest) is the unique Nash equilbrium; if* $\theta < 47$, *(Invest, Invest) is the unique Nash equilibrium; and if* $47 \leq \theta \leq 63$, *there are two Nash equilibria in pure strategies, (Not Invest, Not Invest) and (Invest, Invest). When* $\theta \in [47, 55)$, *the risk dominant Nash equilibrium is (Invest, Invest) and when* $\theta \in (55, 63]$, *the risk dominant Nash equilibrium is (Not Invest, Not Invest).*

In summary, when players are endowed with the ability to perfectly perceive all payoffs, and thus there is common knowledge of θ, theory predicts multiple equilibria when θ takes on values in the intermediate range, $47 \leq \theta \leq 63$.

2.2 Imprecise Coding

Suppose now that players perceive θ with noise. This assumption is backed up by a large literature in numerical cognition, which finds that people encode numerical quantities with noise, even when the quantities are presented symbolically (see Dehaene 2011 for a review). Suppose further that the distribution of noisy signals, conditional on θ, is common knowledge.

Assumption 1 (Cognitive Imprecision) *Each player has a common prior that θ is distributed normally: $\theta \sim \mathcal{N}(\mu_\theta, \sigma_\theta^2)$. Moreover, each player i, $i = \{1, 2\}$, observes a noisy internal representation of θ, $S_i = \theta + \epsilon_i$, where each ϵ_i is independently and normally distributed: $\epsilon_i \sim \mathcal{N}(0, \sigma_S^2)$. This is common knowledge.*

The specific way in which we model the variance of the noisy internal representation has two implications that are worth noting. First, the literature on numerical cognition typically imposes an assumption that larger numbers are perceived with more noise (so that, for example, σ_S^2 scales linearly with θ). For analytical tractability, we assume σ_S^2 is constant in θ. Second, Assumption 1 implies that σ_S^2 does not vary with the distribution of θ. In Section 2.3, we relax this restriction in order to allow θ to be efficiently coded, as a function of the distribution of θ.

Given Assumption 1, we follow the global games literature and restrict our analyses to monotone equilibria of this incomplete information game; that is, equilibria in which actions are monotonic in the internal representation, S_i. In a *monotone equilibrium*, players' mutual best response is to choose Invest if and only if their internal representation is below a threshold $k^\star$. Adapting results from the global games literature (Carlsson and Van Damme, 1993; Morris and Shin, 2003; Morris, 2010) to the game in Table 1, with the further assumption that $\mu_\theta = (a + b)/2$ (as in our experiment), we can establish there exists a monotone equilibrium such that player i invests if and only if $S_i \leq \mu_\theta$, for any value of σ_θ and σ_S. Furthermore, if the noise in the internal representation is sufficiently small, then this is the unique monotone equilibrium.

Proposition 1 (Equilibrium Existence and Uniqueness) *Suppose Assumption 1 and $\mu_\theta = (a + b)/2$. There exists an equilibrium of the game where each player invests if and* *only if $S_i \leq \mu_\theta$ (or, equivalently, $E[\theta|S_i] \leq \mu_\theta$). Moreover, if $\frac{\sigma_\theta^2\sqrt{2\sigma_\theta^2\sigma_S^2+\sigma_S^4}}{(b-a)\sigma_S^2\sqrt{\sigma_\theta^2+\sigma_S^2}} > \frac{1}{\sqrt{2\pi}}$, this is the unique monotone equilibrium of the game.*

Proposition 1 implies a particular set of comparative statics with respect to θ. Specifically, the probability of investing is pinned down by the distribution of the internal representation: $Pr[\text{Invest}|\theta] = Pr\left[S_i \leq \mu_\theta|\theta\right] = \Phi\left(\frac{\mu_\theta-\theta}{\sigma_S}\right)$, where $\Phi(\cdot)$ is the cumulative density function of the standard normal. This result indicates that, in the unique monotone equilibrium, the probability of investing is monotonically decreasing in θ. Moreover, once the noise in internal representation is sufficiently small relative to σ_θ (so that the conditions in Proposition 1 are satisfied), the probability of investing does not depend on the prior volatility, σ_θ.

Corollary 1 (Comparative Statics on θ and σ_θ) *Assume the conditions in Proposition 1 are satisfied. In the unique monotone equilibrium, the probability that each player invests for a given value of θ is $Pr[Invest|\theta] = Pr\left[S_i \leq \mu_\theta|\theta\right] = \Phi\left(\frac{\mu_\theta-\theta}{\sigma_S}\right)$. This probability is decreasing in θ and does not change with the variance of the distribution of θ.*

Note that, in deriving Proposition 1, we assumed common knowledge of the distribution of internal representations. However, precise knowledge of the underlying information structure is not necessary for this equilibrium to arise. As we show in Appendix B, it is enough to assume that (i) $\mu_\theta = (a+b)/2$, (ii) $E[\epsilon_i] = 0$, (iii) the distribution of ϵ_i is symmetric, quasiconcave and independent of the realized value of θ, and (iv) the distribution of θ is symmetric and continuous on $\mathbb{R}$. Figure 1 summarizes the behavioral prediction of our theory of cognitive imprecision.

2.3 Imprecise and Efficient Coding

In this section, we continue to assume that players have imprecise perception, but we now allow the noisy internal representation to adapt to the player's environment. There is substantial empirical evidence, mainly from the literature on sensory perception, which demonstrates that the distribution of noisy internal representations is *optimally* adapted to the

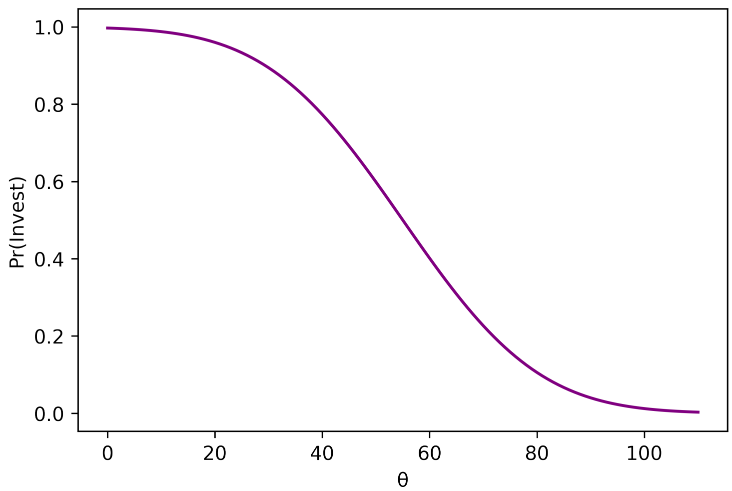


Figure 1: Predicted Probability of Investing as a Function of θ with Imprecise Coding. Note: $a = 47$, $b = 63$, $\theta \sim N(55, \sigma_\theta)$, $\sigma_S = 20$; the prediction is the same for any value of $\sigma_\theta > 0$.

statistical regularities of the environment. This principle is called *efficient coding*, and recent work has empriically documented effects of efficient coding in economic choices (Polania, Woodford and Ruff, 2019; Frydman and Jin, Forthcoming). Here, we make the assumption that efficient coding operates in a strategic setting.

When modeling efficient coding, one needs to take a stand on (i) the constraints imposed by the perceptual system and (ii) the objective function of the decision-maker. We follow the model of efficient coding from Khaw, Li and Woodford (Forthcoming), which assumes that the perceptual system is constrained in encoding the value θ, and must use a noisy internal representation, $S_i = m(\theta) + \epsilon_i$, where $m(\theta)$ is an encoding function. Furthermore, the encoding function is assumed to be linear, $m(\theta) = \xi + \psi\theta$, and there is a channel capacity constraint, $E[m^2] \leq \Omega^2$.

Turning to the player's performance objective, we assume the player minimizes the mean squared error between θ and its estimate.[6] With the additional assumption of a lognormal prior, Khaw, Li and Woodford (Forthcoming) show that, as the volatility of the prior distribution increases, so does the amount of noise in the internal representation. In Appendix C,

[6] In this environment, our interpretation is that the minimization process, and the generation of noisy signals, is performed unconsciously by the perceptual system.

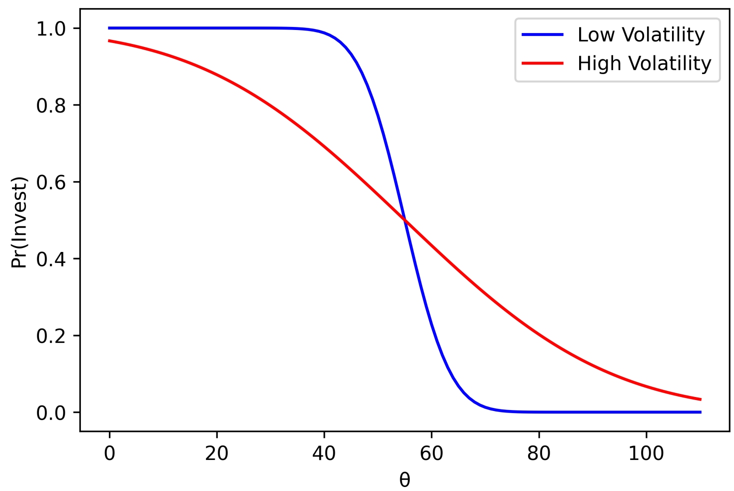


Figure 2: Predicted Probability of Investing as a Function of θ with Efficient Coding. Note: The blue line denotes the prediction for a low volatility distribution with $\theta \sim N(55, 20)$; the red line denotes the prediction for a high volatility distribution with $\theta \sim N(55, 400)$; we set the following parameter values: $a = 47$, $b = 63$, and $\omega = 1.5$.

we confirm this result holds for the normal prior distribution that players hold in the game we analyze. For the purpose of analyzing strategic behavior, the important elements of our efficient coding assumptions are summarized by:

Assumption 2 (Efficient Coding) *The precision of the internal representation depends on the distribution of θ. In particular, the standard deviation of the internal representation is proportional to the standard deviation of θ, that is, $\sigma_S = \omega\sigma_\theta$, where $\omega > 0$.*

At an intuitive level, efficient coding implies that perceptual resources are allocated so as to better discriminate between different values of θ that are expected to occur more frequently under the player's prior beliefs. Specifically, as the volatility of the prior decreases, perceptual resources are reallocated towards a narrower range. A key testable prediction of this theory is that manipulating σ_θ affects the optimal encoding rule and, thus, the noise in implementing the equilibrium monotone strategy of the game analyzed above. This prediction is summarized in the following proposition:

Proposition 2 (Comparative Statics on θ and σ_θ with Efficient Coding) *Suppose Assumption 1, Assumption 2, $\mu_\theta = (a + b)/2$, and $\sqrt{\omega(1+\omega)} < \frac{\sqrt{6\pi}}{(b-a)}\sigma_\theta^2$. In the unique mono-*

tone equilibrium of the game, we have that $Pr[Invest|\theta] = \Phi\left(\frac{55-\theta}{\omega\sigma_\theta}\right)$*. Thus, increasing the precision of the distribution of* θ *(that is,* $1/\sigma_\theta$*) increases the sensitivity of choices to* θ *(that is, the rate at which* $Pr[Invest|\theta]$ *decreases with* θ*) for values of* θ *close to* μ_θ*.*

Under our assumption of efficient coding, theory predicts that, in equilibrium, the probability of investing will depend not only on θ, but also on the prior distribution from which θ is drawn. Thus, if we experimentally manipulate the volatility of the prior, we should see that, in the unique equilibrium of the game obtained when ω is sufficiently small, the probability of investing is more sensitive to θ when the prior volatility is smaller. This prediction is shown in Figure 2, and motivates our experimental design.

3 Experiment 1: Simultaneous-Move Game

3.1 Experimental Design

In this experiment, we test the model by incentivizing subjects to play a simultaneous-move game, and we manipulate the distribution that generates the fundamental payoff, θ. We pre-register the experiment and recruit 300 subjects from the online data collection platform, Prolific.[7] We restrict our sample to subjects who (i) were UK nationals and residents, (ii) did not have any previous "rejected" submissions on Prolific, and (iii) answered all comprehension quiz question correctly. Subjects were paid 2 GBP ($\sim$ 2.8 USD) for completing the experiment, and they had the opportunity to receive additional earnings based on their choices and the choices of other participants.

The experiment consists of 300 rounds, and each subject participates in all rounds. In each round, a subject is randomly matched with another subject and, together, they play the simultaneous-move game in Table 1. In all rounds, we set the payoff parameters $a = 47$ and $b = 63$. The only feature of the game that varies across rounds is the value θ, which is an i.i.d. draw from the condition-specific distribution $f(\theta)$. We do not provide subjects with any

[7] The pre-registration document is available in Appendix E.

feedback on their opponent's decision in each round, nor do we provide any feedback about their payoff in a given round. We chose not to provide interim feedback for two reasons: (i) to shut down learning about a subject's opponent and (ii) to analyze behavior in each round as a one-shot game.[8] At the end of the experiment, one round was selected at random, and subjects were paid according to the number of points they earned in that round, which in turn, depended on their action, their opponent's action, and the (round-specific) value of θ. Points were converted to GBPs using the rate 20:1. The average duration of the experiment was $\sim$ 25 minutes and average earnings, including the participation fee, were $\sim$ 5.5 GBP ($\sim$ 7.7 USD).

Subjects were randomized into one of two experimental conditions: a *high volatility* condition or a *low volatility* condition, which differ only based on the distribution of θ. In the high volatility condition, $f(\theta)$ is normally distributed with mean 55 and variance 400. In the low volatility condition, $f(\theta)$ is normally distributed with mean 55 and variance 20. In both conditions, after drawing θ from its respective distribution, we round θ to the nearest integer, and we re-draw θ if the rounded value is less than 11 or greater than 99. We implemented these modifications to the normal distribution in order to control complexity and ensure that θ is a two-digit number in each round. We do not give subjects any explicit information about $f(\theta)$ in the instructions, as our intention is to test whether a subject's perceptual system can adapt to the statistical properties of the environment without explicit top-down information. Indeed, subjects read the same instructions and are asked the same questions in the comprehension quiz in both conditions.[9]

Recall that, in the complete information version of the game that subjects face, there are multiple equilibria when θ is in the range, [47, 63]. We therefore focus our analyses on games

[8]While subjects do not receive explicit feedback about their opponent's choice, it is still possible that they learn about the strategic environment through repeated exposure to the game, as in Weber (2003) and Rick and Weber (2010). Indeed, we find evidence that response times decrease over the course of the experiment, a signature pattern of learning. Part of these learning effects are likely due to the general strategic environment, but we also find that some learning takes place over the distribution of $f(\theta)$. We discuss this further in Section 3.2. Table 3 in Appendix D shows that results are robust to focusing on subsamples where subjects have the same experience with the same game (i.e., θ) in both conditions.

[9]The experimental instructions are available in Appendix E.

Option A	**Option B**
45	**47** if other participant chooses A **63** if other participant chooses B

Figure 3: Sample Screenshot Shown to Participants in Experiment 1. Note: In this round, the realized value of θ is 45, which is clearly and explicitly displayed to both subjects. Subjects choose either "Option A" or "Option B" by pressing one of two different keys on the keyboard.

for which θ lies in this range, which occurs on 93% of rounds in the low volatility condition and on 31% of rounds in the high volatility condition. We pre-register that our main analyses are restricted to those rounds for which θ is in [47, 63], which we call "common rounds." This is a crucial feature of our design, because it allows us to compare behavior across conditions, using the exact same set of games, and varying only the context — which is characterized by the distribution of possible games.[10] In choosing the variance of $f(\theta)$ for each condition, we thus struck a balance between generating a substantial number of common rounds to analyze, and creating a large difference in prior variance across conditions; the latter is necessary to generate different predictions across conditions. As outlined in our pre-registration, we also exclude the first 30 rounds from our analyses, in order to allow subjects time to adapt to the distribution of θ.

Figure 3 provides a screenshot of a single round shown to subjects. In order to avoid framing effects, we label the two options "Option A" and "Option B", and the left-right location of each option is randomized across rounds. The number "45" is the realized value

[10]In the original Carlsson and Van Damme (1993) paper on global games, the authors conclude that their uniqueness result is "driven by the fact that, in a global game, the uncertainty forces the players to take account of the entire class of a priori possible games. . . " Here, we experimentally manipulate the distribution of possible games, which we think of as varying the context of the game.

of θ on the specific round shown in Figure 3. We emphasize that while the number is clearly displayed to all subjects, and thus would traditionally be interpreted as public information, here we rely on cognitive imprecision to transform the fundamental value into private information. In other words, we assume that the constraints on a subject's perceptual system make it impossible to perfectly perceive the fundamental value. Under the hypothesis of efficient coding outlined in the model section, we expect the amount of imprecision to vary endogenously with the distribution of θ in each condition. Thus, we use efficient coding to identify cognitive imprecision, which is the core mechanism that generates the unique equilibrium structure in the game.

3.2 Experimental Results

Choice Behavior

Following our pre-registration, we restrict our analysis to common rounds in which subjects made a decision with a response time greater than 0.5 seconds. After applying this restriction, we are left with 50,129 decisions, of which 73.0% are in the low volatility condition and 27.0% are in the high volatility condition. Across both conditions, subjects choose to invest on 58.9% of rounds, and exhibit an average response time of 1.64 seconds.

In Figure 4, we plot the probability of investing as a function of the fundamental, separately for the two experimental conditions. One can see that, in both conditions, the aggregate data are consistent with the hypothesis that subjects implement strategies that are monotone in θ. This finding is consistent with previous experimental results on coordination games (Heinemann, Nagel and Ockenfels, 2004, 2009; Szkup and Trevino, 2020). Importantly, the aggregate data are inconsistent with a discrete jump in the probability of investing at any level of the fundamental (assuming all subjects use the same threshold); rather there is a smooth and decreasing relationship between the probability of investing and the fundamental. This is important because the noisy perception of the fundamental is what generates the prediction of a (noisy) threshold strategy at the subject level, and the amount

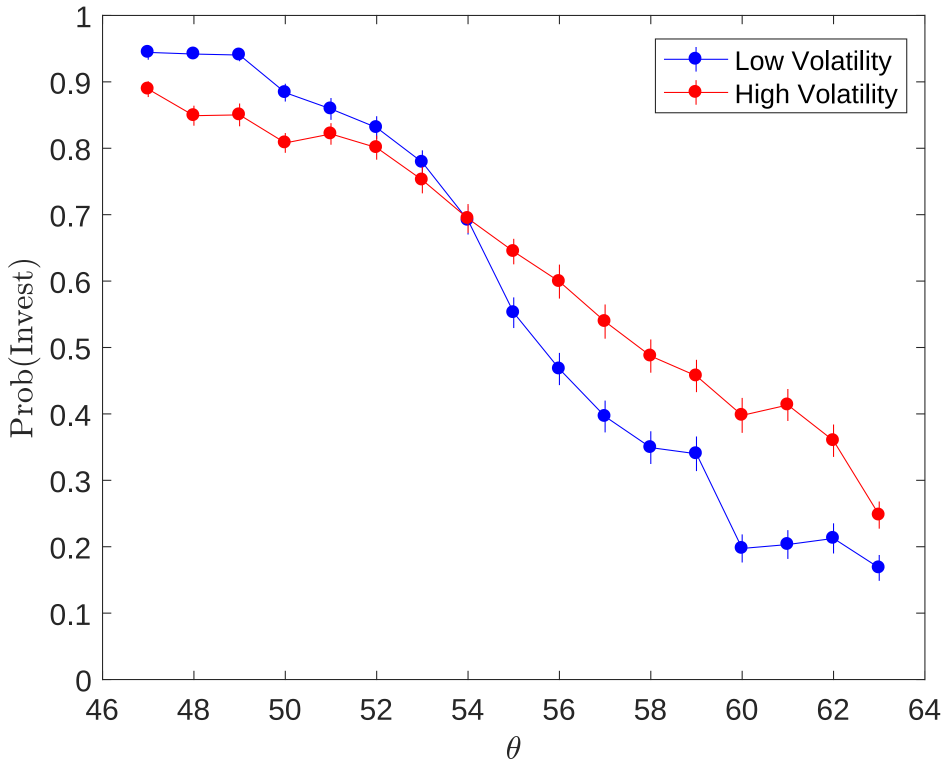


Figure 4: Observed Probability of Investing as a Function θ. Note: For each value of θ between 47 and 63, we plot the proportion of rounds in which a subject chooses to invest, separately for each of the two experimental conditions. Data are pooled across subjects and are shown for rounds 31-300, after an initial 30-round adaptation period. Vertical bars across each dot denote two standard errors of the mean. Standard errors are clustered by subject.

of noise is reflected in the shape of the psychometric curve. Thus, in both conditions, the aggregate data are consistent with cognitive imprecision.

In order to provide a more targeted test of cognitive imprecision, we exploit the variation in the distribution of θ across our two experimental conditions. Specifically, efficient coding predicts systematically different behavior across conditions, and any evidence of efficient coding necessarily implies some degree of cognitive imprecision in subjects' perception. Consistent with the hypothesis of efficient coding, we see from Figure 4 that the probability of investing is more sensitive to the fundamental in the low volatility condition, compared to the high volatility condition.

To formally test the difference in slope, we estimate a series of mixed effects logistic

Dependent Variable: Pr(Invest)	(1)	(2)	(3)	(4)
$(\theta - 55)$	-0.458***	-0.467***	-0.577***	-0.481***
	(0.033)	(0.039)	(0.051)	(0.037)
$(\theta - 55)$ x Low	-0.499***	-0.351***	-0.487***	-0.374***
	(0.063)	(0.059)	(0.076)	(0.061)
Low	-0.182	-0.335	-0.170	-0.275
	(0.386)	(0.343)	(0.423)	(0.360)
Late				-0.022
				(0.121)
$(\theta - 55)$ x Late				0.013
				(0.021)
Low x Late				0.092
				(0.154)
Low x $(\theta - 55)$ x Late				-0.065*
				(0.034)
Constant	1.351***	1.316***	1.465***	1.292***
	(0.221)	(0.224)	(0.222)	(0.229)
Observations	50,129	13,196	12,861	25,864
Rounds	31-300	31-100	231-300	31-100
				+ 231-300

Table 2: Mixed Effects Logistic Regression Results. Note: The dependent variable takes value 1 if the subject chooses to Invest, and 0 otherwise. The variable *Low* takes the value 1 if the round belongs to the low volatility condition and 0 otherwise. The variable *Late* takes the value 1 if the round is 231 or beyond, and 0 otherwise. Only data from rounds where $46 < \theta < 64$ are included. There are random effects on $(\theta - 55)$ and the intercept. Standard errors of the fixed effect estimates are clustered at the subject level. ***, **, * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

regressions. Column (1) of Table 2 confirms our main result: the coefficient on the interaction term $(\theta - 55)$ x *Low* is significantly negative, indicating that the probability of investing decreases in the fundamental more rapidly when a subject is adapted to the low volatility condition. Columns (2) and (3) show that this result holds within early (first 70 trials after adaptation) and late (last 70 rounds of the session) subsamples. Column (4) indicates that the treatment effect becomes moderately stronger over the course of the experiment. The strengthening of the treatment effect over the course of the experiment suggests that subjects have not fully adapted to the distribution by round 100, and additional rounds of play provide the opportunity for further adaptation.

Response Times

We can also use response times as another source of data to analyze adaptation over the course of the experiment. Figure 5 shows that, in both conditions, response times (averaged across subjects) decrease dramatically over the first 50 trials.[11] The rapid decrease in response times in each condition likely reflects learning about both the strategic environment and the distribution of θ. Evidence for learning about the distribution of θ comes from the fact that there is a clear separation of the two time series across conditions: response times are, on average, longer in the high volatility condition. Thus, not only is the probability of investing more sensitive to the fundamental in the low volatility condition, but decisions are also executed significantly more quickly over the course of rounds $31 - 300$ (1.51 seconds vs. 2.01 seconds, $p < 0.001$).

We can also analyze cross-sectional variation in response times, which illuminates how response time varies with the fundamental. Figure 6 shows that, in the high volatility condition, the peak is at 55, whereas in the low volatility condition, the peak is at 54. If subjects are implementing the unique equilibrium threshold strategy, which involves discriminating whether the fundamental is above or below 55, then models of sequential sampling from

[11] We again restrict to common rounds and, thus, there is less data in the high volatility time series, which is why it appears noisier than the low volatility time series.

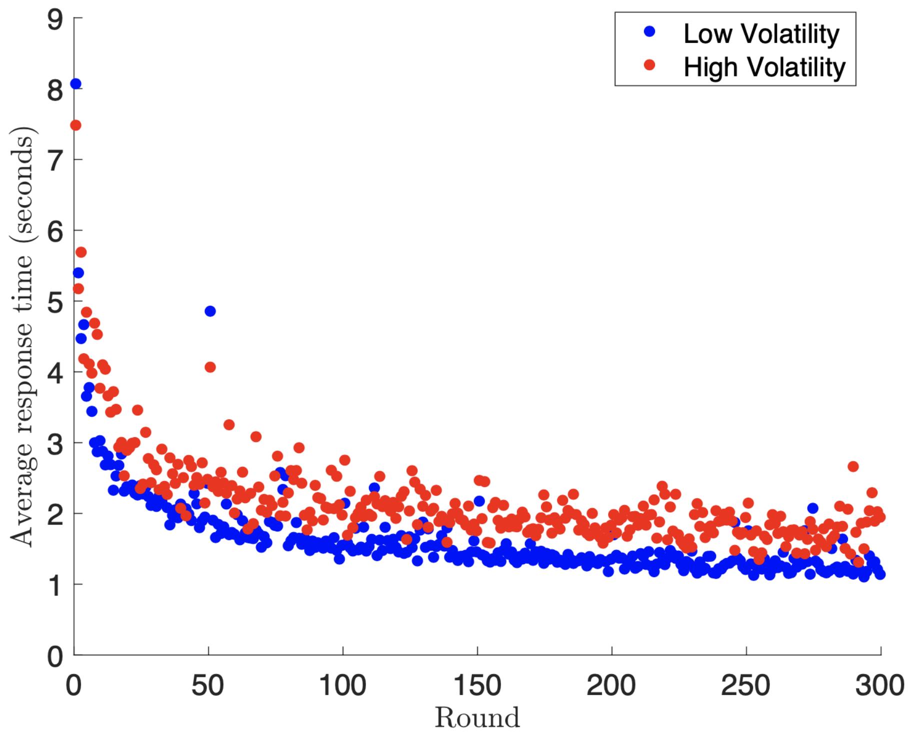


Figure 5: Time Series of Average Response Time. Note: For each condition and round number, the figure plots the response time averaged across subjects. Data are restricted to rounds for which $46 < \theta < 64$

the mathematical psychology literature (Ratcliff, 1978; Krajbich, Armel and Rangel, 2010) would predict that response times should peak at the predicted threshold of 55, since these are the most "difficult" discrimination problems. The response time data provide some support for this prediction. We also see that, in both conditions, there is an uptick in response times as θ approaches 63. This could, in part, reflect the fact that not all subjects play the same threshold strategy.

4 Experiment 2: Number Classification Task

4.1 Experimental Design

Because we are relying on cognitive imprecision to endogenously transform the complete information game into an incomplete information game, our theoretical analysis also assumes

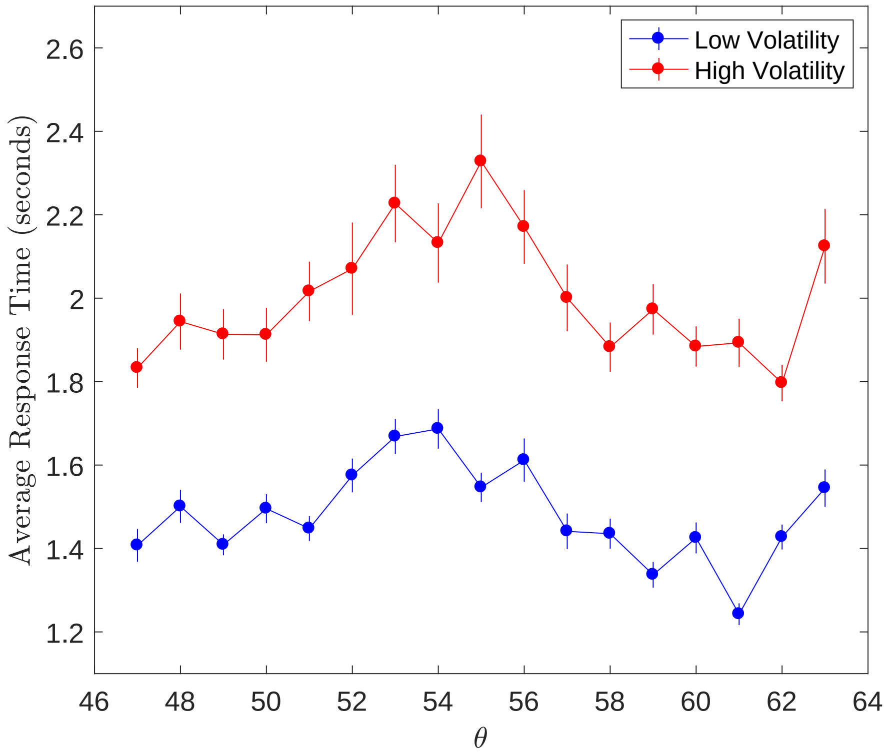


Figure 6: Average Response Time as a Function of θ. Note: Response times are averaged across subjects and across rounds. Vertical bars denote two standard errors of the mean. Standard errors are clustered by subject.

common knowledge of cognitive imprecision. While we cannot directly test the common knowledge assumption, here we report results from a second experiment that is designed to investigate whether subjects are at least aware of their own imprecision and the imprecision of their opponent. If subjects are not aware of cognitive imprecision (either their opponent's or their own), this naivete would cast doubt on the mechanism which generates a unique equilibrium.

Our method for studying awareness of imprecision is to create a simplified version of the previous experiment, but one that retains the core individual decision-making prediction that subjects should play a threshold strategy. Specifically, we employ a task from the numerical cognition literature where subjects are incentivized to quickly and accurately classify whether a two-digit number is larger or smaller than the number 55. Note that this strategy is identical to the theoretical prediction in the unique equilibrium from the previous experiment; here we exogenously impose the strategy on subjects without any strategic considerations or equilibrium requirements. We then incentivize subjects to report beliefs about errors in their own classification and in the classification of others.

For this second experiment, we recruit 300 subjects from Prolific who did not participate in Experiment 1. We pay subjects 1 GBP for completing the study, in addition to earnings from three phases of the experiment. In Phase 1, on each of 150 rounds, subjects are incentivized to quickly and accurately classify whether a two-digit Arabic numeral on the experimental display screen is larger or smaller than 55. In particular, subjects earned $(1.5 \times \text{accuracy} - 1 \times \text{speed})$ GBPs, where 'accuracy' is the percentage of trials where the subject classified the number correctly, and 'speed' is the average response time in seconds. As in Experiment 1, there are two conditions, and the only difference across conditions is the distribution from which the two-digit Arabic numeral is drawn. Furthermore, we use the same two distributions as in Experiment 1: in the high volatility condition θ is normally distributed with mean 55 and variance 400; in the low volatility condition, θ is normally distributed with mean 55 and variance 20. We also round each value of θ to the nearest integer and re-draw if the rounded integer is less than 11 or greater than 99 (again, to ensure that each number contains exactly two digits).

In Phase 2, subjects are incentivized to report beliefs about others' performance in the task. Not only are we interested in eliciting awareness of imprecision, we also collect data to test whether subjects believe that others are more imprecise when the number on screen is closer to the reference level of 55. We implement this latter test because the specific noise structure in perception plays an important role in generating the unique equilibrium we explored in the previous experiment; in particular, recent theoretical work has shown that an important feature of the noise structure in generating a unique equilibrium is that nearby states are harder to discriminate compared to far away states (Morris and Yang, 2019; Hébert and Woodford, Forthcoming). To investigate this property of the noise structure, we ask subjects to consider the 149 other participants in their experimental condition of the study, who also just completed Phase 1. We then ask subjects the following two questions:

1. Consider only trials where the number on screen was equal to 47. In what percentage of these trials do you think the other participants gave a correct answer, that is, they correctly classified whether the number was smaller or larger than 55?

2. Consider only trials where the number on screen was equal to 54. In what percentage of these trials do you think the other participants gave a correct answer, that is, they correctly classified whether the number was smaller or larger than 55?

For each of the two questions, we pay the subject 0.5 GBP if their forecast is within 1% of the true percentage.[12] Question 1 elicits beliefs about others' imprecision when the distance between the number is far from the threshold (47 vs. 55), whereas Question 2 elicits beliefs about others' imprecision when the distance is close (54 vs. 55). While we could have asked subjects about their beliefs about others' imprecision for a range of numbers — rather than the single numbers 47 and 55 — this would have introduced a confound, since the distribution of numbers is different across conditions.

In Phase 3, we ask subjects about their own performance on the number classification task (that they completed in Phase 1). This question is not trivial because we do not provide subjects with feedback after any round in Phase 1 (nor after the end of Phase 1). Here, we are also interested in subjects' awareness of their own imprecision for numbers that are close and far from the threshold. Specifically, we ask subjects the following two questions:

1. Consider only trials where the number on screen was between 52 and 58. In what percentage of these trials do you think you correctly classified whether the number was smaller or larger than 55?

[12] Following Hartzmark, Hirshman and Imas (Forthcoming), we chose to use this elictation procedure as opposed to more complex mechanisms such as the Binarized Schoring Rule (BSR) due to recent evidence showing that the BSR can systematically bias truthful reporting (Danz, Vesterlund and Wilson, 2020).

2. Consider only trials where the number on screen was less than 52 or greater than 58. In what percentage of these trials do you think you correctly classified whether the number was smaller or larger than 55?

For each of these two questions, we again reward subjects with 0.50 GBP if they provide an answer that is within 1% of their true accuracy. All subjects first go through Phase 1, and the order of Phase 2 and Phase 3 is randomized across subjects. One concern is that when asking subjects about their performance in Phase 1, we are testing memory, not *ex-ante* beliefs. This is a reasonable concern, and an alternative is to have subjects forecast their performance before undertaking the classification task. However, in this case, subjects' classification performance would be endogenous to their beliefs, and would invalidate the incentive compatibility of our belief elicitation procedure. For this reason, we opted to implement Phase 1 first for all subjects.

4.2 Experimental Results

We first report results from subjects' behavior in Phase 1, which are shown in Figure 7. In the left panel, we replicate the classic results from previous experiments on number discrimination, whereby subjects exhibit errors, and these errors increase as the number on screen approaches the threshold (Dehaene, Dupoux and Mehler, 1990). Moreover, we see that, for numbers between 47 and 63, errors are systematically higher in the high volatility condition (Frydman and Jin, Forthcoming). Similar patterns are reflected in the response times shown in the right panel of Figure 7: response times increase as the number approaches the threshold of 55, and response times are systematically longer in the high volatility condition.

The purpose of Phase 1 is to create a dataset about performance, over which we can ask subjects about their beliefs in Phases 2 and 3. In the left panel of Figure 8, we see that subjects believe their performance in the classification task exhibits imprecision (that is, beliefs about accuracy are less than 100%). Moreover, we see that subjects are aware that mistakes for numbers closer to the threshold (that is, greater than 52 and less than 58)

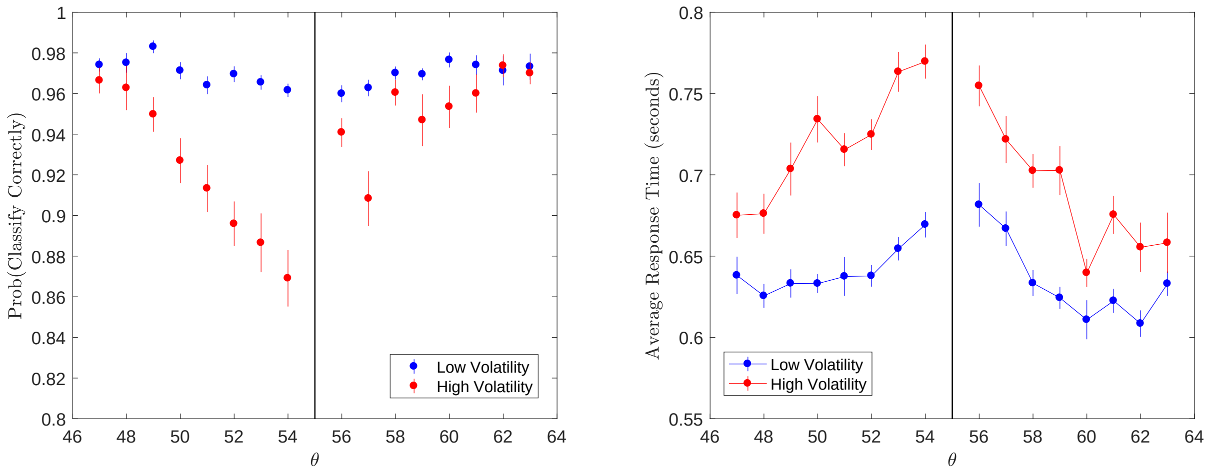


Figure 7: Accuracy and Response Times in the Classification Task in Experiment 2. Note: Left panel shows the proportion of rounds on which subjects correctly classified the stimulus as greater than or less than the reference level of 55. Right panel shows the average response time on rounds where subjects correctly classified the stimulus. In both panels, the vertical bars denote two standard errors of the mean. Standard errors are clustered by subject.

are more likely than on numbers that are farther from the threshold (that is, less than 52 or greater than 58; $p < 0.001$). In the middle panel, we see that subjects report similar beliefs about other subjects' imprecision; beliefs about others' errors when discriminating 54 vs. 55 are greater than when discriminating 47 vs. 55 ($p < 0.001$).

Our data also enable us to test one other feature of beliefs about others' imprecision. As outlined in our pre-registration, we test whether beliefs about others' accuracy on rounds when $\theta = 54$ is higher for those subjects who experience the low volatility distribution in Phase 1.[13] Such a test investigates the hypothesis that subjects are aware that others' perception of a given number varies as a function of the experienced distribution. Indeed, the right panel of Figure 8 shows that, for $\theta = 54$, subjects who experience the high volatility distribution in Phase 1 report that others make more errors, compared to those subjects who experience the low volatility distribution in Phase 1 ($p = 0.048$).

[13]Pre-registration documents are available in Appendix E.

Figure 8: Beliefs about Own and Others' Accuracy in the Classification Task. Note: Panel (a) shows the average belief about own accuracy for values of θ that are far ($\theta < 52$ or $\theta > 58$) and close ($51 < \theta < 59$) to the threshold 55. Panel (b) shows the average belief about others' accuracy for values of θ that are far ($\theta = 47$) and close ($\theta = 54$) to the threshold 55. Panel (c) shows the average belief about others' accuracy when $\theta = 54$, split by experimental condition. In all panels, vertical bars denote two standard errors of the mean.

5 Discussion

As we discussed in Section 2.3, the efficient coding rule we use to model perception is only one of several plausible specifications (Ma and Woodford, 2020). In particular, there are other possible objective functions that players may have, besides minimizing the mean squared error of the estimate of θ. For example, a prominent alternative efficient coding objective from the literature on sensory perception is to maximize the mutual information between the state and its noisy internal representation. In Appendix C, we confirm that the coding rule we use in our model is robust to this alternative objective.

Yet another alternative objective that has been examined in the economics literature is to maximize expected financial gain. Because we are analyzing perception and decision-making in a strategic environment, the coding rule that maximizes expected financial gain would need to incorporate beliefs about the opponent's actions, and hence, the opponent's efficient coding objective. Such a model of efficient coding over the opponent's perceived value of fundamentals would entail a variety of additional and substantial assumptions, for which we currently have little empirical guidance. For this reason, we have opted to maintain the assumption that each player's coding scheme is efficient in the sense of minimizing the perceptual error of the fundamental.

Throughout the paper, we have highlighted the role that perceptual noise can play in satisfying the assumptions that generate the classic global games result. However, with the additional assumption of efficient coding, we have shown both theoretically and experimentally how cognitive imprecision can generate *context-dependent* behavior in games. Our framework can thus be seen as offering a potential cognitive foundation for stochastic and context-dependent choice in games. As such, our results are related to the well-known theory of Quantal Response Equilibrium (McKelvey and Palfrey, 1995, 1998), in which players stochastically best respond to each other. Interestingly, in the original QRE paper, McKelvey and Palfrey propose that *"to the extent that we can find observable independent variables that a priori one would expect to be correlated with the precision of these [expected payoff] estimates, one can make predictions about the effects of different experimental treatments that systematically vary these independent variables."* Efficient coding provides one such independent variable, which is the volatility of the payoff distribution. In related work, Friedman (2020) proposes a model that endogenizes the precision parameter in QRE, though it is the set of payoffs in the *current game* that determine the precision parameter — rather than the distribution of payoffs within a class of games, as in our model.

Our results also relate to another behavioral theory of games called Level-k Thinking (Stahl and Wilson, 1994, 1995; Nagel, 1995). In one version of this theory, there are different types of players, and each type best responds to another type who exhibits one less degree of strategic sophistication. For example, a Level-0 type would be characterized by no strategic sophistication and, thus, would exhibit purely random behavior. A Level-1 type would then best respond to a Level-0 player, and a Level-2 player would best respond to a Level-1 player, and so on. What are the predictions of Level-k Thinking for our game? Given that Level-0 players randomize, the expected utility of a Level-1 player from Invest is

$$EU_{L1}(\text{Invest}) = \frac{1}{2}a + \frac{1}{2}b$$

Thus, $EU_{L1}(\text{Invest}) > EU(\text{Not Invest})$ if and only if $\theta < (a+b)/2$. Next, under the assumption that Level-2 players believe they are facing a Level-1 opponent, the expected utility from Invest for a Level-2 player is

$$EU_{L2}(\text{Invest}) = \begin{cases} b \text{ if } \theta < (a+b)/2 \\ a \text{ if } \theta > (a+b)/2 \end{cases}$$

When $\theta < (a+b)/2$, then $EU_{L2}(\text{Invest}) = b > \theta$. Conversely, when $\theta > (a+b)/2$, then $EU_{L2}(\text{Invest}) = a < \theta$. Thus, Level-2 players choose Invest if and only if $\theta < (a+b)/2$. Using the same logic, we obtain the same prediction for all upper levels.

In sum, the fraction of subjects who choose Invest is:

$$\Pr[\text{Invest}] = \begin{cases} \Pr[L_0]\frac{1}{2} + (1 - \Pr[L_0]) & \text{if } \theta < (a+b)/2 \\ \Pr[L_0]\frac{1}{2} & \text{if } \theta > (a+b)/2 \end{cases}$$

where $\Pr[L_0]$ is the fraction of Level-0 players in the population. The theory therefore predicts that, in the aggregate, the probability of investing exhibits a sharp decrease at $\theta = (a+b)/2$. More importantly, Level-k Thinking does not predict any difference across our experimental treatments; thus the theory would need to be augmented with some extra feature in order to explain the clear context-dependence we observe in our data.

6 Conclusion

We have provided and experimentally validated a framework for analyzing strategic behavior, when players have imprecise perception of the state of the world. Our results are in line with previous experiments on global games, which find evidence consistent with an equilibrium where all players invest once the fundamental crosses a threshold (Heinemann, Nagel and Ockenfels, 2004, 2009; Cabrales, Nagel and Armenter, 2007; Szkup and Trevino, 2020; Goryunov and Rigos, 2020). At the same time, our experimental data suggest that the predictions from the global games literature may be more applicable than previously thought. Even when there is no explicit private information given to players, imprecise perception can serve as a source of private information. Interestingly, the particular manner in which we model imprecise perception is closely connected to the noise structure used in the global games literature to select a unique equilibrium.

We also find empirical evidence of *context-dependent* strategic behavior, which is consistent with efficient coding. We argue that the unstable strategic behavior that we observe across experimental conditions is a consequence of the efficient use of cognitive resources. In our setting of a 2×2 regime change game, efficient coding provides a mechanism that modulates the probability that two players coordinate and play the same action. As such, the framework we present here may prove useful in serving as a cognitive foundation for other imperfect best response models in game theory, such as quantal response equilibrium.

References

Angeletos, George-Marios and Chen Lian, "Incomplete Information in Macroeconomics: Accommodating Frictions in Coordination," in "Handbook of Macroeconomics," Vol. 2, Elsevier, 2016, pp. 1065–1240.

_ **and Iván Werning**, "Crises and Prices: Information Aggregation, Multiplicity, and Volatility," *American Economic Review*, 2006, *96* (5), 1720–1736.

Atkeson, Andrew, "[Rethinking Multiple Equilibria in Macroeconomic Modeling]: Comment," *NBER macroeconomics annual*, 2000, *15*, 162–171.

Barlow, Horace B, "Possible Principles Underlying the Transformation of Sensory Messages," *Sensory Communication*, 1961, *1* (01).

Cabrales, Antonio, Rosemarie Nagel, and Roc Armenter, "Equilibrium Selection through Incomplete Information in Coordination Games: An Experimental Study," *Experimental Economics*, 2007, *10* (3), 221–234.

Carlsson, Hans and Eric Van Damme, "Global Games and Equilibrium Selection," *Econometrica*, 1993, pp. 989–1018.

Chambers, Christopher P and Paul J Healy, "Updating Toward the Signal," *Economic Theory*, 2012, *50* (3), 765–786.

Danz, David, Lise Vesterlund, and Alistair J Wilson, "Belief Elicitation: Limiting Truth Telling with Information on Incentives," 2020. NBER Working Paper No. 27327.

Dehaene, Stanislas, *The Number Sense: How the Mind Creates Mathematics*, Oxford University Press, 2011.

_ , Emmanuel Dupoux, and Jacques Mehler, "Is Numerical Comparison Digital? Analogical and Symbolic Effects in Two-Digit Number Comparison," *Journal of Experimental Psychology: Human Perception and Performance*, 1990, *16* (3), 626.

Diamond, Douglas W and Philip H Dybvig, "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 1983, *91* (3), 401–419.

Edmond, Chris, "Information Manipulation, Coordination, and Regime Change," *Review of Economic Studies*, 2013, *80* (4), 1422–1458.

Enke, Benjamin and Thomas Graeber, "Cognitive Uncertainty," 2019. NBER Working Paper No. 26518.

Friedman, Evan, "Endogenous Quantal Response Equilibrium," *Games and Economic Behavior*, 2020, *124*, 620–643.

Frydman, Cary and Lawrence J Jin, "Efficient Coding and Risky choice," *Quarterly Journal of Economics*, Forthcoming.

Gabaix, Xavier and David Laibson, "Myopia and Discounting," 2017. NBER Working Paper No. 23254.

Gershman, Samuel J and Rahul Bhui, "Rationally Inattentive Intertemporal Choice," *Nature Communications*, 2020, *11* (1), 1–8.

Girshick, Ahna R, Michael S Landy, and Eero P Simoncelli, "Cardinal Rules: Visual Orientation Perception Reflects Knowledge of Environmental Statistics," *Nature Neuroscience*, 2011, *14* (7), 926–932.

Goldstein, Itay and Ady Pauzner, "Demand–Deposit Contracts and the Probability of Bank Runs," *The Journal of Finance*, 2005, *60* (3), 1293–1327.

Goryunov, Maxim and Alexandros Rigos, "Discontinuous and Continuous Stochastic Choice and Coordination in the Lab," 2020. Lund University, Department of Economics.

Harsanyi, John C and Reinhard Selten, *A General Theory of Equilibrium Selection in Games*, The MIT Press, 1988.

Hartzmark, Samuel M, Samuel Hirshman, and Alex Imas, "Ownership, Learning, and Beliefs," *Quarterly Journal of Economics*, Forthcoming.

Hébert, Benjamin M and Michael Woodford, "Neighborhood-Based Information Costs," *American Economic Review*, Forthcoming.

Heinemann, Frank, Rosemarie Nagel, and Peter Ockenfels, "The Theory of Global Games on Test: Experimental Analysis of Coordination Games with Public and Private Information," *Econometrica*, 2004, *72* (5), 1583–1599.

_ , _ , and _ , "Measuring Strategic Uncertainty in Coordination Games," *Review of Economic Studies*, 2009, *76* (1), 181–221.

Hellwig, Christian, Arijit Mukherji, and Aleh Tsyvinski, "Self-Fulfilling Currency Crises: The Role of Interest Rates," *American Economic Review*, 2006, *96* (5), 1769–1787.

Huyck, John Van, Ajalavat Viriyavipart, and Alexander L Brown, "When Less Information is Good Enough: Experiments with Global Stag Hunt Games," *Experimental Economics*, 2018, *21* (3), 527–548.

Khaw, Mel Win, Ziang Li, and Michael Woodford, "Cognitive Imprecision and Small-Stakes Risk Aversion," *Review of Economic Studies*, Forthcoming.

Krajbich, Ian, Carrie Armel, and Antonio Rangel, "Visual Fixations and the Computation and Comparison of Value in Simple Choice," *Nature Neuroscience*, 2010, *13* (10), 1292–1298.

Ma, Wei Ji and Michael Woodford, "Multiple Conceptions of Resource Rationality," *Behavioral and Brain Sciences*, 2020, *43.*

McKelvey, Richard D and Thomas R Palfrey, "Quantal Response Equilibria for Normal Form Games," *Games and Economic Behavior*, 1995, *10* (1), 6–38.

_ **and** _ , "Quantal Response Equilibria for Extensive Form Games," *Experimental economics*, 1998, *1* (1), 9–41.

Morris, Stephen, "Global Games," in Steven Durlauf and Lawrence E. Blume, eds., *Game Theory*, Plagrave Macmillan, 2010.

_ **and Hyun Song Shin**, "Unique equilibrium in a model of self-fulfilling currency attacks," *American Economic Review*, 1998, pp. 587–597.

_ **and** _ , "Global Games: Theory and Applications," in Mathias Dewatripont, Lars Peter Hansen, and Stephen J. Turnovsky, eds., *Advances in Economics and Econometrics (Proceedings of the Eighth World Congress of the Econometric Society)*, Cambridge University Press, 2003.

_ **and Ming Yang**, "Coordination and Continuous Stochastic Choice," 2019.

_ **, Hyun Song Shin, and Muhamet Yildiz**, "Common Belief Foundations of Global Games," *Journal of Economic Theory*, 2016, *163*, 826–848.

Nagel, Rosemarie, "Unraveling in Guessing Games: An Experimental Study," *The American Economic Review*, 1995, *85* (5), 1313–1326.

Obstfeld, Maurice, "Models of Currency Crises with Self-Fulfilling Features," *European Economic Review*, 1996, *40* (3-5), 1037–1047.

Payzan-LeNestour, Elise and Michael Woodford, "Outlier Blindness: A Neurobiological Foundation for Neglect of Financial Risk," *Journal of Financial Economics*, Forthcoming.

Polania, Rafael, Michael Woodford, and Christian C Ruff, "Efficient Coding of Subjective Value," *Nature neuroscience*, 2019, *22* (1), 134–142.

Ratcliff, Roger, "A Theory of Memory Retrieval.," *Psychological Review*, 1978, *85* (2), 59.

Rick, Scott and Roberto A Weber, "Meaningful Learning and Transfer of Learning in Games Played Repeatedly without Feedback," *Games and Economic Behavior*, 2010, *68* (2), 716–730.

Stahl, Dale O and Paul W Wilson, "Experimental Evidence on Players' Models of Other Players," *Journal of Economic Behavior and Organization*, 1994, *25* (3), 309–327.

_ and _ , "On Players' Models of Other Players: Theory and Experimental Evidence," *Games and Economic Behavior*, 1995, *10* (1), 218–254.

Szkup, Michal and Isabel Trevino, "Sentiments, Strategic Uncertainty, and Information Structures in Coordination Games," *Games and Economic Behavior*, 2020, *124*, 534–553.

Weber, Roberto A, "Learning with No Feedback in a Competitive Guessing Game," *Games and Economic Behavior*, 2003, *44* (1), 134–144.

Wei, Xue-Xin and Alan A Stocker, "A Bayesian Observer Model Constrained by Efficient Coding can Explain 'anti-Bayesian' Percepts," *Nature Neuroscience*, 2015, *18* (10), 1509.

Woodford, Michael, "Modeling Imprecision in Perception, Valuation, and Choice," *Annual Review of Economics*, 2020, *12*, 579–601.

Appendix A: Proofs

Proof of Proposition 1

First, we show that, when the conditions in the statement of the Proposition are satisfied, there exists a unique monotone equilibrium of the game.

Since the distribution of S_i conditional on θ (that is, the likelihood function) is conjugate to the prior distribution of θ, we have a closed form solution for the distribution of player i's posterior beliefs over θ. In particular, player 1's posterior distribution of θ given S_1 is

$$\theta|S_1 \sim \mathcal{N}\left(\frac{\sigma_S^2}{\sigma_\theta^2+\sigma_S^2}\mu_\theta + \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_S^2}S_1, \frac{\sigma_\theta^2\sigma_S^2}{\sigma_\theta^2+\sigma_S^2}\right) \tag{1}$$

Thus, we have:

$$EU[\text{Not Invest}|S_1] = E[\theta|S_1] = \frac{\sigma_S^2\mu_\theta + \sigma_\theta^2 S_1}{\sigma_\theta^2+\sigma_S^2} \tag{2}$$

On the other hand, player 1's expected utility from investing is

$$EU[\text{Invest}|S_1] = a + (b-a)\Pr[\text{Opponent Invests}|S_1] \tag{3}$$

Assume player 1 believes his opponent uses a monotone strategy with threshold k. In this case, player 1's expectation that the opponent invests is $\Pr[S_2 \leq k|S_1]$. Player 1's distribution of S_2 given S_1 is:

$$S_2|S_1 \sim \mathcal{N}\left(E[\theta|S_1] = \frac{\sigma_S^2}{\sigma_\theta^2+\sigma_S^2}\mu_\theta + \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_S^2}S_1, \frac{2\sigma_\theta^2\sigma_S^2+\sigma_S^4}{\sigma_\theta^2+\sigma_S^2}\right)$$

Thus, we have:

$$\Pr[S_2 \leq k|S_1] = \Phi\left(\frac{k\sqrt{\sigma_\theta^2+\sigma_S^2}}{\sqrt{2\sigma_\theta^2\sigma_S^2+\sigma_S^4}} - \frac{\sigma_S^2\mu_\theta+\sigma_\theta^2 S_1}{\sqrt{\sigma_\theta^2+\sigma_S^2}\sqrt{2\sigma_\theta^2\sigma_S^2+\sigma_S^4}}\right) = \Phi\left(\frac{(\sigma_\theta^2+\sigma_S^2)\,k - \sigma_S^2\mu_\theta - \sigma_\theta^2 S_1}{\sqrt{2\sigma_\theta^2\sigma_S^2+\sigma_S^4}\sqrt{\sigma_\theta^2+\sigma_S^2}}\right)$$

where $\Phi(\cdot)$ is the cumulative distribution of the standard normal.

Player 1's best response is to invest if and only if

$$\begin{aligned} EU[\text{Not Invest}|S_1] &\leq EU[\text{Invest}|S_1, k] \\ \frac{\sigma_S^2\mu_\theta + \sigma_\theta^2 S_1}{\sigma_\theta^2 + \sigma_S^2} &\leq a + (b-a)\Phi\left(\frac{\left(\sigma_\theta^2 + \sigma_S^2\right)k - \sigma_S^2\mu_\theta - \sigma_\theta^2 S_1}{\sqrt{2\sigma_\theta^2\sigma_S^2 + \sigma_S^4}\sqrt{\sigma_\theta^2 + \sigma_S^2}}\right) \end{aligned} \tag{4}$$

If we write $\overline{S}(k)$ for the unique value of S_1 such that player 1 is indifferent between investing and not investing (this is well defined since player 1's expected payoff from not investing is strictly increasing in S_1 and player 1's expected payoff from investing is strictly decreasing in S_1), the best response of player 1 is to follow a monotone strategy with threshold equal to $\overline{S}(k)$, that is, to invest if and only if $S_1 \leq \overline{S}(k)$.

Observe that as $k \to -\infty$ (player 2 never invests), $EU[\text{Invest}|S_1, k]$ tends to a, so $\overline{S}(k)$ tends to $\frac{\left(\sigma_\theta^2+\sigma_S^2\right)a-\sigma_S^2\mu_\theta}{\sigma_\theta^2}$. As $k \to \infty$ (player 2 always invests), $EU[\text{Invest}|S_1]$ tends to b, so $\overline{S}(k)$ tends to $\frac{\left(\sigma_\theta^2+\sigma_S^2\right)b-\sigma_S^2\mu_\theta}{\sigma_\theta^2}$. A fixed point of $\overline{S}(k)$ — that is a value $k^\star$ such that $\overline{S}(k^\star) = k^\star$ — is a monotone equilibrium of the game where each player invests if and only if his signal is below $k^\star$. Since $\overline{S}(k)$ is a mapping from $\mathbb{R}$ to itself and is continuous in k, there exists $k \in \left[\frac{\left(\sigma_\theta^2+\sigma_S^2\right)a-\sigma_S^2\mu_\theta}{\sigma_\theta^2}, \frac{\left(\sigma_\theta^2+\sigma_S^2\right)b-\sigma_S^2\mu_\theta}{\sigma_\theta^2}\right]$, such that $\overline{S}(k) = k$ and a threshold equilibrium of this game exists.

When is there a unique equilibrium? Define $W(\overline{S}(k), k)$ as

$$W(\overline{S}(k), k) = \frac{\sigma_S^2\mu_\theta + \sigma_\theta^2\overline{S}(k)}{\sigma_\theta^2 + \sigma_S^2} - a - (b-a)\Phi\left(\frac{\left(\sigma_\theta^2 + \sigma_S^2\right)k - \sigma_S^2\mu_\theta - \sigma_\theta^2\overline{S}(k)}{\sqrt{2\sigma_\theta^2\sigma_S^2 + \sigma_S^4}\sqrt{\sigma_\theta^2 + \sigma_S^2}}\right)$$

At a fixed point, $\overline{S}(k^\star) = k^\star$. Thus, we have:

$$W(k^\star) = \frac{\sigma_S^2\mu_\theta + \sigma_\theta^2 k^\star}{\sigma_\theta^2 + \sigma_S^2} - a - (b-a)\Phi\left(\frac{\sigma_S^2}{\sqrt{2\sigma_\theta^2\sigma_S^2 + \sigma_S^4}\sqrt{\sigma_\theta^2 + \sigma_S^2}}\left(k^\star - \mu_\theta\right)\right)$$

Then,

$$\frac{\partial W(k^\star)}{\partial k^\star} = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_S^2} - \phi\left(\frac{\sigma_S^2}{\sqrt{2\sigma_\theta^2\sigma_S^2+\sigma_S^4}\sqrt{\sigma_\theta^2+\sigma_S^2}}\left(k^\star-\mu_\theta\right)\right)\frac{\sigma_S^2(b-a)}{\sqrt{2\sigma_\theta^2\sigma_S^2+\sigma_S^4}\sqrt{\sigma_\theta^2+\sigma_S^2}}$$

And there is a unique fixed point if and only if $\frac{\partial W(k^\star)}{\partial k^\star} > 0$ at the fixed point. When $\frac{\partial W(k^\star)}{\partial k^\star} < 0$, there are at least three fixed points.

Since $\phi(y) \leq \frac{1}{\sqrt{2\pi}}$, this is a sufficient condition for $\frac{\partial W(k^\star)}{\partial k^\star} > 0$:

$$\begin{aligned}
\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_S^2} &> \frac{1}{\sqrt{2\pi}}\frac{\sigma_S^2(b-a)}{\sqrt{2\sigma_\theta^2\sigma_S^2+\sigma_S^4}\sqrt{\sigma_\theta^2+\sigma_S^2}} && (5)\\
\frac{\sigma_\theta^2\sqrt{2\sigma_\theta^2\sigma_S^2+\sigma_S^4}}{(b-a)\sigma_S^2\sqrt{\sigma_\theta^2+\sigma_S^2}} &> \frac{1}{\sqrt{2\pi}} && (6)\\
\sqrt{2\pi} &> \frac{(b-a)\sigma_S^2\sqrt{\sigma_\theta^2+\sigma_S^2}}{\sigma_\theta^2\sqrt{2\sigma_\theta^2\sigma_S^2+\sigma_S^4}} && (7)
\end{aligned}$$

We can find the same sufficient condition if we use implicit differentiation to derive a condition which guarantees that $\frac{\partial\overline{S}(k)}{\partial k} \in (0,1)$ and, thus, $\overline{S}(k)$ is a contraction.

$$\frac{\partial\overline{S}(k)}{\partial k} = -\frac{\partial W/\partial k}{\partial W/\partial\overline{S}(k)} = \frac{\phi\left(\frac{\left(\sigma_\theta^2+\sigma_S^2\right)k-\sigma_S^2\mu_\theta-\sigma_\theta^2\overline{S}(k)}{\sqrt{2\sigma_\theta^2\sigma_S^2+\sigma_S^4}\sqrt{\sigma_\theta^2+\sigma_S^2}}\right)\frac{(b-a)\sigma_\theta^2+(b-a)\sigma_S^2}{\sqrt{2\sigma_\theta^2\sigma_S^2+\sigma_S^4}\sqrt{\sigma_\theta^2+\sigma_S^2}}}{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_S^2}+\phi\left(\frac{\left(\sigma_\theta^2+\sigma_S^2\right)k-\sigma_S^2\mu_\theta-\sigma_\theta^2\overline{S}(k)}{\sqrt{2\sigma_\theta^2\sigma_S^2+\sigma_S^4}\sqrt{\sigma_\theta^2+\sigma_S^2}}\right)\frac{(b-a)\sigma_\theta^2}{\sqrt{2\sigma_\theta^2\sigma_S^2+\sigma_S^4}\sqrt{\sigma_\theta^2+\sigma_S^2}}}$$

Note that, since $\phi(\cdot) \in (0,1)$, $\frac{\partial\overline{S}(k)}{\partial k}$ is always positive. It is also less than 1 if the following condition is satisfied:

$$\begin{aligned}
\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_S^2} &> \phi\left(\frac{\left(\sigma_\theta^2+\sigma_S^2\right)k-\sigma_S^2\mu_\theta-\sigma_\theta^2\overline{S}(k)}{\sqrt{2\sigma_\theta^2\sigma_S^2+\sigma_S^4}\sqrt{\sigma_\theta^2+\sigma_S^2}}\right)\frac{(b-a)\sigma_S^2}{\sqrt{2\sigma_\theta^2\sigma_S^2+\sigma_S^4}\sqrt{\sigma_\theta^2+\sigma_S^2}}\\
\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_S^2}\frac{\sqrt{2\sigma_\theta^2\sigma_S^2+\sigma_S^4}\sqrt{\sigma_\theta^2+\sigma_S^2}}{(b-a)\sigma_S^2} &> \phi\left(\frac{\left(\sigma_\theta^2+\sigma_S^2\right)k-\sigma_S^2\mu_\theta-\sigma_\theta^2\overline{S}(k)}{\sqrt{2\sigma_\theta^2\sigma_S^2+\sigma_S^4}\sqrt{\sigma_\theta^2+\sigma_S^2}}\right)\\
\frac{\sigma_\theta^2\sqrt{2\sigma_\theta^2\sigma_S^2+\sigma_S^4}}{(b-a)\sigma_S^2\sqrt{\sigma_\theta^2+\sigma_S^2}} &> \frac{1}{\sqrt{2\pi}} && (8)
\end{aligned}$$

When the condition in equation (8) is satisfied, there exists a constant $G < 1$ such that

$|\partial\overline{S}(k)/\partial k| \leq G < 1$ for all $k \in \mathbb{R}$. This means that, when the condition in equation (8) is satisfied, $\overline{S}(k)$ is a contraction and, thus, by the Banach fixed point (or contraction mapping) theorem, $\overline{S}(k)$ has exactly one solution.

This shows that, when the conditions in the statement of the Proposition are satisfied, there exists a unique monotone equilibrium of the game.

Second, we show that when Assumption 1 is satisfied and $\mu_\theta = \frac{(a+b)}{2}$, there exists a monotone equilibrium of the game where $k^\star = \mu_\theta$ for any value of σ_θ and σ_S. When the third condition from Proposition 1 is satisfied, this is the unique monotone equilibrium of the game.

Assume player 2 uses a threshold strategy where he invests if and only if $S_2 \leq k = \mu_\theta$. Is this an equilibrium, that is, is $\overline{S}(\mu_\theta) = \mu_\theta$? $\overline{S}(\mu_\theta)$ is the value of S_1 such that the following equation is satisfied with equality:

$$\frac{\sigma_S^2\mu_\theta + \sigma_\theta^2 S_1}{\sigma_\theta^2 + \sigma_S^2} = a + (b-a)\Phi\left(\frac{\left(\sigma_\theta^2 + \sigma_S^2\right)k - \sigma_S^2\mu_\theta - \sigma_\theta^2 S_1}{\sqrt{2\sigma_\theta^2\sigma_S^2 + \sigma_S^4}\sqrt{\sigma_\theta^2 + \sigma_S^2}}\right)$$
$$\frac{\sigma_S^2\mu_\theta + \sigma_\theta^2 S_1}{\sigma_\theta^2 + \sigma_S^2} = a + (b-a)\Phi\left(\frac{\sigma_\theta^2\mu_\theta - \sigma_\theta^2 S_1}{\sqrt{2\sigma_\theta^2\sigma_S^2 + \sigma_S^4}\sqrt{\sigma_\theta^2 + \sigma_S^2}}\right)$$

If we set $\overline{S}(\mu_\theta) = \mu_\theta$, we get:

$$\mu_\theta = a + (b-a)\Phi(0)$$
$$\mu_\theta = \frac{(a+b)}{2}$$

Proof of Proposition 2

The condition $\sqrt{\omega(1+\omega)} < \frac{\sqrt{6\pi}}{(b-a)}\sigma_\theta^2$ ensures uniqueness of the equilibrium. It is obtained by replacing $\sigma_S = \omega\sigma_\theta$ (Assumption 2) in the condition for a unique monotone equilibrium from Proposition 1 and re-arranging terms. From Proposition 1 and the condition above, we know that there exists a unique monotone equilibrium of the game with imprecise and efficient cognition where each player invests if and only if the signal he receives is smaller than μ_θ. In this equilibrium, $Pr[Invest|\theta] = Pr\left[S_i \leq \mu_\theta|\theta\right] = \Phi\left(\frac{\mu_\theta - \theta}{\omega\sigma_\theta}\right)$ and $\frac{\partial Pr[Invest|\theta]}{\partial\theta} = -\phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{1}{\omega\sigma_\theta}\right)$. Thus, $Pr[Invest|\theta]$ grows with σ_θ if $\theta < \mu_\theta$ and it decreases with σ_θ is $\theta > \mu_\theta$. Moreover, the sensitivity of choices to θ decreses with σ_θ for values of θ around the cutoff.

Indeed, we have

$$\begin{aligned}
\frac{\partial Pr\left[Invest|\theta\right]}{\partial\theta\partial\sigma_\theta} &= \phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{1}{\omega\sigma_\theta^2}\right) + \phi'\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta^2}\right)\left(\frac{1}{\omega\sigma_\theta}\right) \\
&= \phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{1}{\omega\sigma_\theta^2}\right) - \left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta^2}\right)\left(\frac{1}{\omega\sigma_\theta}\right) \\
&= \phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{1}{\omega\sigma_\theta^2}\right) - \phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{(\mu_\theta-\theta)^2}{\omega^3\sigma_\theta^4}\right) \\
&= \phi\left(\frac{\mu_\theta-\theta}{\omega\sigma_\theta}\right)\left(\frac{\omega^2\sigma_\theta^2-(\mu_\theta-\theta)^2}{\omega^3\sigma_\theta^4}\right)
\end{aligned}$$

which is positive if and only if $(\mu_\theta - \theta)^2 < \omega^2\sigma_\theta^2$.

(In the second line, we used the fact that $\phi'(x) = -x\phi(x)$.)

Appendix B: Robustness of Equilibrium with $k^\star = \mu_\theta$

Let us introduce the following definitions from Chambers and Healy (2012):

Definition 1 *A random variable with cumulative density function F and mean μ is **symmetric** if, for every $a \geq 0$, $F(\mu + a) = 1 - \lim_{x \to a^-} F(\mu - a)$.*

Definition 2 *A random variable is **quasiconcave** (or unimodal) if it has a density function f such that for all $x, x' \in \mathbb{R}$ and $\lambda \in (0,1)$, $f(\lambda x + (1-\lambda)x') \geq \min\{f(x), f(x')\}$.*

Definition 3 *The error term ϵ_i satisfies **symmetric dependence** with respect to the random variable θ if, for each realization of θ, $\epsilon_i|\theta$ has a continuous density function $f_{\epsilon_i|\theta}$ satisfying $f_{\epsilon_i|\theta}(\epsilon_i|\mu_\theta + a) = f_{\epsilon_i|\theta}(\epsilon_i|\mu_\theta - a)$ for almost every ϵ_i and a in $\mathbb{R}$. (Note that error terms that are independent of θ satisfy this definition).*

Consider the following assumptions:

(A1) $S_i = \theta + \epsilon_i$

(A2) $E[\theta] = \mu_\theta < \infty$

(A3) θ is a symmetric random variable and its density is continuous on $\mathbb{R}$

(A4) $E[\epsilon_i|\theta] = 0$ for each θ

(A5) ϵ_i is a symmetric and quasiconcave random variable

(A6) ϵ_i satisfies symmetric dependence with respect to θ

Lemma 1 (Chambers and Healy 2012, Proposition 2) *Assume A1-A6. A Bayesian agent updates his beliefs over θ in the direction of the signal, that is, for almost every $S_i \in \mathbb{R}$, there exists some $\alpha \geq 0$ such that $E[\theta|S_i] = \alpha S_i + (1-\alpha)\mu_\theta$.*

Proposition 3 *Assume A1-A6 and $\mu_\theta = (a+b)/2$. There exists a monotone equilibrium of the game where $k^\star = \mu_\theta$.*

Proof of Proposition 3 The proof can be carried out with general values for a and b (such that $b > a$). For ease of exposition, we focus on the experimental parameters: $a = 47$, $b = 63$, $\mu_\theta = 55$. Assume that player j uses threshold $k_j = 55$, that is, he invests if and only if $S_j < 55$. We want to show that player i's best response is to use the same threshold, $k_i = 55$. Player i prefers to invest if and only if $EU[\text{Not Invest}|S_i] < EU[\text{Invest}|S_i, k_j]$. Thus, we want to show that (1) when $S_i = 55$, $EU[\text{Not Invest}|S_i] = EU[\text{Invest}|S_i, k_j = 55]$; (2) when $S_i < 55$, $EU[\text{Not Invest}|S_i] < EU[\text{Invest}|S_i, k_j = 55]$; and (3) when $S_i > 55$, $EU[\text{Not Invest}|S_i] > EU[\text{Invest}|S_i, k_j = 55]$.

By Lemma 1, $EU[\text{Not Invest}|S_i] = E[\theta|S_i] = \alpha S_i + (1-\alpha)\mu_\theta$ where $\alpha \geq 0$. Note also that $EU[\text{Invest}|S_i, k_j = 55] = 47 + (63-47)Pr[S_j < k_j = 55|S_i]$. First, we prove (1). Assume $S_i = 55$. We want to show that $EU[\text{Not Invest}|S_i] = EU[\text{Invest}|S_i, k_j = 55]$. By Lemma 1, $EU[\text{Not Invest}|S_i = 55] = \alpha S_i + (1-\alpha)\mu_\theta = \alpha(55) + (1-\alpha)(55) = 55$. Thus, the equality we want to show becomes $55 = 47 + (63-47)Pr[S_j < k_j = 55|S_i = 55]$. This equality is satisfied if and only if $Pr[S_j < k_j = 55|S_i = 55] = 1/2$. By A1 and A4 (and linearity of expectation), $E[S_j|S_i] = E[\theta|S_i] = 55$. By A5, the density of of $S_j|S_i$ is symmetric. Thus, the probability S_j takes a value below its posterior mean (55) is 1/2. This proves (1).

Second, we prove (2). Assume $S_i < 55$. By Lemma 1, $EU[\text{Not Invest}|S_i] = \alpha S_i + (1-\alpha)55$. This is smaller than 55 for any positive α. This also means that, by A1 and A4, $E[S_j|S_i] = E[\theta|S_i] < 55$. The probability that the opponent invests is the posterior probability that his signal is below 55 (given S_i). Since the conditional distribution of the opponent's signal is symmetric around its mean (by A5), the median is equal to the mean. This means that the conditional CDF of the opponent signal equals 1/2 at the posterior mean, is greater than 1/2 for values of S_j above the mean and is lower than 1/2 for values of S_j below the mean. Since the posterior mean of the opponent's signal is lower than 55, the probability that player j's signal is lower than 55 (conditional on $S_i < 55$) is greater than 1/2. Thus, $EU[\text{Invest}|S_i, k_j = 55] = 47 + (63-47)Pr[S_j < k_j = 55|S_i] > 55$. This proves that $EU[\text{Invest}|S_i, k_j = 55] > 55 > EU[\text{Not Invest}|S_i]$. (3) can be proven analogously.

Appendix C: Derivation of Efficient Coding Assumption

Here we adapt the theoretical derivation of efficient coding from Khaw, Li and Woodford (Forthcoming) to our framework where the distribution of θ is normal rather than lognormal. Suppose that the internal representation S of θ is drawn from

$$S|\theta \sim N(m(\theta), \sigma_S^2) \tag{9}$$

where the *encoding rule*, $m(\theta)$, is a linear transformation of θ, $m(\theta) = \xi + \psi\theta$. Parameters ξ and ψ are endogenous while the precision parameter σ_S is exogenous. We assume that the cognitive process producing the internal representation is subject to a "power constraint"

$$E[m^2] \leq \Omega^2 \leq \infty \tag{10}$$

The efficient coding hypothesis requires that the encoding rule $m(\theta)$ is chosen (among all linear functions satisfying the constraint) so as to maximize the system's objective function, for a given prior distribution of θ. As in Khaw, Li and Woodford (Forthcoming), we assume that the system produces an estimate of θ on the basis of S, $\tilde{\theta}(S)$, and that the goal of the design problem is to have a system that achieves as low as possible a mean squared error of this estimate. Given a noisy internal representation, the estimate which minimizes the mean squared error is the mean of the posterior distribution of θ, that is, $\tilde{\theta}(S) = E[\theta|S]$ for all S. The goal of the design problem is, thus, to minimize the variance of the posterior distribution of θ.

Consider the transformed internal representation, $\tilde{S} \equiv (S - \xi)/\psi$. The distribution of the transformed internal representation conditional on θ is $\tilde{S}|\theta \sim N(\theta, \sigma_S^2/\psi^2)$. Thus, the distribution of θ given the (transformed) internal representation is

$$\theta|\tilde{S} \sim N\left(\mu_\theta + \frac{\sigma_\theta^2}{\sigma_\theta^2 + (\sigma_S^2/\psi^2)}(\tilde{S} - \mu_\theta), \frac{\sigma_\theta^2(\sigma_S^2/\psi^2)}{\sigma_\theta^2 + (\sigma_S^2/\psi^2)}\right) \tag{11}$$

The variance of the posterior distribution of θ is strictly increasing in the variance of $\tilde{S}$, σ_S^2/ψ^2. Thus, it is desirable to make ψ as large as possible (in order to make the mean squared error of the estimate as small as possible) consistent with the power constraint. When the distribution of θ is normal, we have

$$E[m^2] = \xi^2 + \psi^2 E[\theta^2] + 2\xi\psi E[\theta] = (\xi + \psi\mu_\theta)^2 + \psi^2\sigma_\theta^2 \leq \Omega \tag{12}$$

The largest value of ψ consistent with this constraint is achieved when

$$\xi = -\psi\mu_\theta \;,\; \psi = \frac{\Omega}{\sigma_\theta} \tag{13}$$

Thus, $m^\star(\theta) = -\frac{\Omega}{\sigma_\theta}\mu_\theta + \frac{\Omega}{\sigma_\theta}\theta$ and

$$\tilde{S}|\theta \quad \sim \quad N\left(\theta, \frac{\sigma_S^2}{\Omega^2}\sigma_\theta^2\right) \tag{14}$$

Defining $\omega \equiv \frac{\sigma_S^2}{\Omega^2}$, we recover the noisy internal representation posited in Assumption 2.

The same optimal coding rule obtains under an alternative goal of the system. Consider the more conventional hypothesis from sensory perception literature, whereby the encoding rule is assumed to maximize the Shannon mutual information between the objective state θ and its subjective representation S. Denote with ρ_θ the precision of θ and with ρ_S the precision of S. We have $\theta \sim N\left(\mu_x, \frac{1}{\rho_\theta}\right)$, $S|\theta \sim N\left(\xi + \psi\theta, \frac{1}{\rho_S}\right)$, $\tilde{S}|\theta \sim \left(\theta, \frac{1}{\rho_{\tilde{S}}}\right)$, and $\theta|\tilde{S} \sim N\left(\frac{\rho_\theta\mu_\theta + \rho_{\tilde{S}}\tilde{S}}{\rho_\theta + \rho_{\tilde{S}}}, \frac{1}{\rho_\theta + \rho_{\tilde{S}}}\right)$, where $\tilde{S} = \frac{S-\xi}{\psi}$ and $\rho_{\tilde{S}} = \psi^2/\sigma_S^2$. The Shannon mutual information between θ and $\tilde{S}$ is

$$I(\theta, \tilde{S}) = \frac{1}{2}\log_2\left(\frac{\sigma_\theta^2}{\sigma_{\theta|\tilde{S}}^2}\right) = \frac{1}{2}\log_2\left(1 + \frac{\rho_{\tilde{S}}}{\rho_\theta}\right) \tag{15}$$

which is strictly increasing in $\rho_{\tilde{S}}$ and, thus, strictly decreasing in $\sigma_{\tilde{S}}^2$. This means that, as for the previous goal, it is desirable to make ψ as large as possible (consistent with the power constraint).

Appendix D: Additional Experimental Results

Dependent Variable: Pr(Invest)	(1)	(2)	(3)	(4)
$(\theta - 55)$	-0.384***	-0.389***	-0.364***	-0.447***
	(0.039)	(0.041)	(0.036)	(0.047)
$(\theta - 55)$ x Low	-0.266***	-0.275***	-0.285***	-0.299***
	(0.051)	(0.054)	(0.052)	(0.063)
Low	-0.317	-0.356	-0.129	-0.207
	(0.276)	(0.285)	(0.283)	(0.317)
Constant	1.067***	1.202***	0.993***	1.174***
	(0.181)	(0.199)	(0.192)	(0.217)
Observations	4,263	4,053	3,677	3,255
Rounds of Experience with Game (θ)	3	4	5	6

Table 3: Mixed Effects Logistic Regressions Estimates. Note: The dependent variable takes value 1 if the subject chooses to Invest, and 0 otherwise. The variable *Low* takes value 1 if the round belongs to the low volatility condition and 0 otherwise. Only data from rounds where $46 < \theta < 64$ are included. There are random effects on $(\theta - 55)$ and the intercept. Standard errors of the fixed effect estimates are clustered at the subject level. ***, **, * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

Appendix E: Experimental Instructions and Pre-Registrations

Experiment 1

Welcome!

You will earn £2 for completing this study and will have the opportunity to **earn more money** depending on your decisions during the study.

Specifically, at the end of the study, the computer will randomly select one question. You will receive points from the randomly selected question and the number of points depends on your decision and the decision of another participant. Points will be converted to pounds using the rate 20 points = £1. For example, if you earned 60 points for the selected question, you would then earn 60/20 = £3 (in addition to the completion fee).

All questions are equally likely to be selected so **make all choices carefully**.

The next pages give detailed instructions. Following the instructions, you will take a quiz on them. You will be allowed to continue and will be entitled to payment **only if you answer all questions on the quiz correctly**.

Instructions (1/2)

The study is separated into 6 parts of 50 rounds each.

In each round, you are randomly matched with another participant, who we call your **opponent**.

In each round, both you and your opponent will be asked to choose between **two options**:

"Option A" or "Option B"

Here is how to earn points:

- If you choose Option A, the number of points you receive does not depend on whether your opponent chooses Option A or B. The amount of points you receive for choosing Option A can be different in different rounds and will be displayed on your screen.
- If you choose Option B, the number of points you receive depends on your opponent's decision: if your opponent chooses Option A, you will receive 47 points; if your opponent also chooses Option B you will receive 63 points.

Importantly, your opponent is reading these same exaxt instructions. This means that:

- If your opponent chooses Option A, his/her payoff does not depend on your decision and the number of points he/she earns are those given by Option A.
- If your opponent chooses Option B, the number of points he/she receives depends on your decision: if you choose Option A, your opponent will receive 47 points; if you also choose Option B, your opponent will receive 63 points.

Instructions (2/2)

Below is an example screen from the study:

Option A	**Option B**
53	**47** if other participant chooses A **63** if other participant chooses B

In this example, Option A is on the LEFT side of the screen and Option B is on the RIGHT.

In each round, you will choose one of the two options by pressing either the "A" key on your keyboard for the LEFT option or the "L" key on your keyboard for the RIGHT option. On some rounds, Option A will be on the LEFT, and in other rounds it will be on the RIGHT.

In the example above:

- Option A pays you 53 points regardless of your opponent's decision, while Option B pays you 47 points if your opponent chooses Option A and 63 points if your opponent chooses Option B.
- Note also that, if your opponent chooses Option A, he/she earns 53 points regardless of your decision. If your opponent, instead, chooses Option B, he/she earns 47 points if you choose Option A and 63 points if you choose Option B.

Cognitive Imprecision and Strategic Behavior - PROLIFIC (#58473)

Created: 02/16/2021 09:11 AM (PT)
Shared: 06/30/2021 11:57 AM (PT)

1) Have any data been collected for this study already?
No, no data have been collected for this study yet.

2) What's the main question being asked or hypothesis being tested in this study?
We will test a theory of strategic behavior in which a subject's perception of a game payoff is based on a noisy internal representation of the payoff (due to information processing constraints in the brain). Specifically, we focus on 2x2 simultaneous-move games and we model perception using the principle of efficient coding, which implies that perception becomes more precise as the dispersion of encountered stimuli (i.e., payoffs) is reduced. Assuming that a game payoff is perceived noisily and that players are aware of the noisy perception means that the game is one of incomplete information. Equilibrium analysis of this incomplete information game delivers our main hypothesis: manipulating the prior distribution from which a game payoff is drawn during an experimental session will affect the distribution of choices in a given game (that is, keeping the payoffs of the game fixed). In particular, we hypothesize that increasing the volatility of the prior distribution of a game payoff will make observed choices less responsive to that payoff.

3) Describe the key dependent variable(s) specifying how they will be measured.
In each round of a simultaneous-move game, a subject can choose one of two actions: "Option A" or "Option B". This binary action is the key dependent variable of the study. We will also collect response times for each round.

4) How many and which conditions will participants be assigned to?
The game played on each round is symmetric, and the only difference across rounds is the payoff X. Specifically, if a subject chooses "Option A," they receive X points, regardless of their opponent's action. If a subject chooses "Option B", they receive 47 points if their opponent chooses Option A, and 63 points if their opponent chooses Option B. There are two conditions: a high volatility condition and a low volatility condition; subjects are randomly assigned to one of the two conditions. In each condition there are 300 rounds. In the high volatility condition, we draw X from a normal distribution with mean 55 and variance 400. In the low volatility condition, we draw X from a normal distribution with mean 55 and variance 20. We then implement the following two steps:

1) X is rounded to the nearest integer
2) We re-draw X if the nearest integer is less than 11 or greater than 99, which ensures that X contains exactly 2-digits

5) Specify exactly which analyses you will conduct to examine the main question/hypothesis.
We will test whether the degree of noise in subjects' action selection is greater in the high volatility condition compared to the low volatility condition, for values of X in the range [47, 63] in all rounds 31-300. Specifically, we will conduct regressions where the dependent variable is a dummy variable equal to 1 if the choice is "Option B" (and 0 otherwise), and the main independent variable is X. Our main test of interest is whether the probability that subjects choose "Option B" decreases more rapidly as a function of X in the low volatility condition compared to the high volatility condition.

6) Describe exactly how outliers will be defined and handled, and your precise rule(s) for excluding observations.
After reading the experimental instructions and before the first round of the game, subjects will take a comprehension quiz, which consists of 3 questions. Any subject who does not answer all 3 questions correctly will be routed out of the experiment and excluded from the dataset. Of those subjects who pass the comprehension quiz, we will exclude observations for which response times are excessively fast, as defined by a response time that is less than 500 milliseconds.

7) How many observations will be collected or what will determine sample size? No need to justify decision, but be precise about exactly how the number will be determined.
We will collect data until we have N=300 subjects (150 subjects in each of the two conditions) who have correctly passed the comprehension quiz (defined as answering all 3 questions correctly).

8) Anything else you would like to pre-register? (e.g., secondary analyses, variables collected for exploratory purposes, unusual analyses planned?)
As a secondary analysis, we will test whether response times are faster in the low volatility condition compared to the high volatility condition (conditional on X). We will also explore whether the peak of the response time distribution is associated with the equilibrium threshold predicted by a theory in which a game payoff is based on a subject's noisy internal representation. These secondary analyses will be restricted to rounds that satisfy both of the following criteria: 1) X is in the range [47, 63]; 2) the round number is greater than 30

Experiment 2

Thank you for participating in this study!

Before we begin, please close all other applications on your computer and put away your cell phone. This study will last approximately **10 minutes**. During this time, we ask your complete and undistracted attention. You will earn £1 for completing the study and you will have the opportunity to **earn more money** depending on your answers during the study.

This study consists of **two phases**. The instructions for Phase 1 are given in the next page. After you go through Phase 1, you will be given a new set of instructions for Phase 2.

When you are ready to continue, press ENTER.

In **Phase 1**, you will see a series of numbers and will be asked to classify whether each number is **larger or smaller than 55**. If the number displayed is smaller than 55, press the "A" key on your keyboard. If the number displayed is larger than 55, press the "L" key.

Your bonus payment will depend on the speed and accuracy of your classification. Specifically:

Bonus Payment = £ (1.5 x accuracy – 1 x speed)

where "accuracy" is the percentage of trials where you correctly classified the number as larger or smaller than 55, and "speed" is the average amount of time it takes you to classify the number on all trials throughout the study, in seconds.

Thus, you make the most money by answering as **quickly** and as **accurately** as possible.

For example, if you correctly classified the number on all trials and it took you 0.3 seconds to respond to each question, you would earn £(1.5 x 100% - 10 x 0.3) = £1.20. If instead you only classified 70% of the numbers correctly and took 0.8 seconds to respond to each question, you would earn £(1.5 x 70% - 10 x 0.8) = £0.25.

Phase 1 will be separated into 3 parts of 50 trials each. In between, you can take a short break.

Before starting with the classification task, you will be asked a question to check your understanding of the instructions. You will be allowed to continue **only if you answer this question correctly**.

When you are ready to continue with the comprehension question, press ENTER.

This is **Phase 2** of the study.

Phase 2 consists of four questions, two on this page and two on the next one.

There are 99 other participants in this study.

Consider the task completed by **the other participants** in Phase 1.

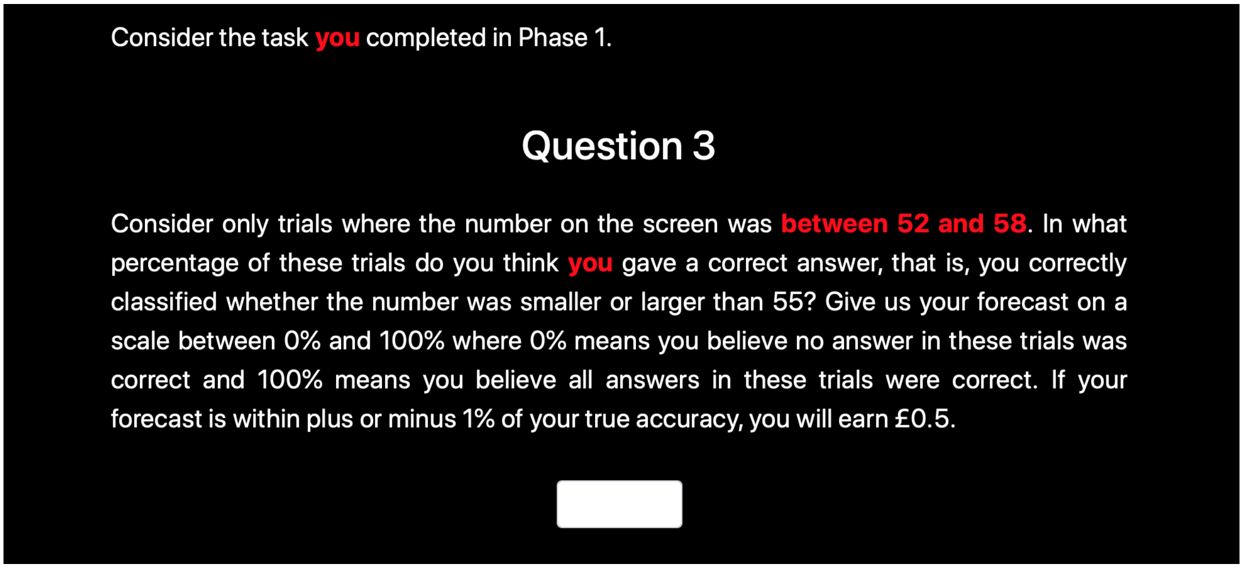
Question 1

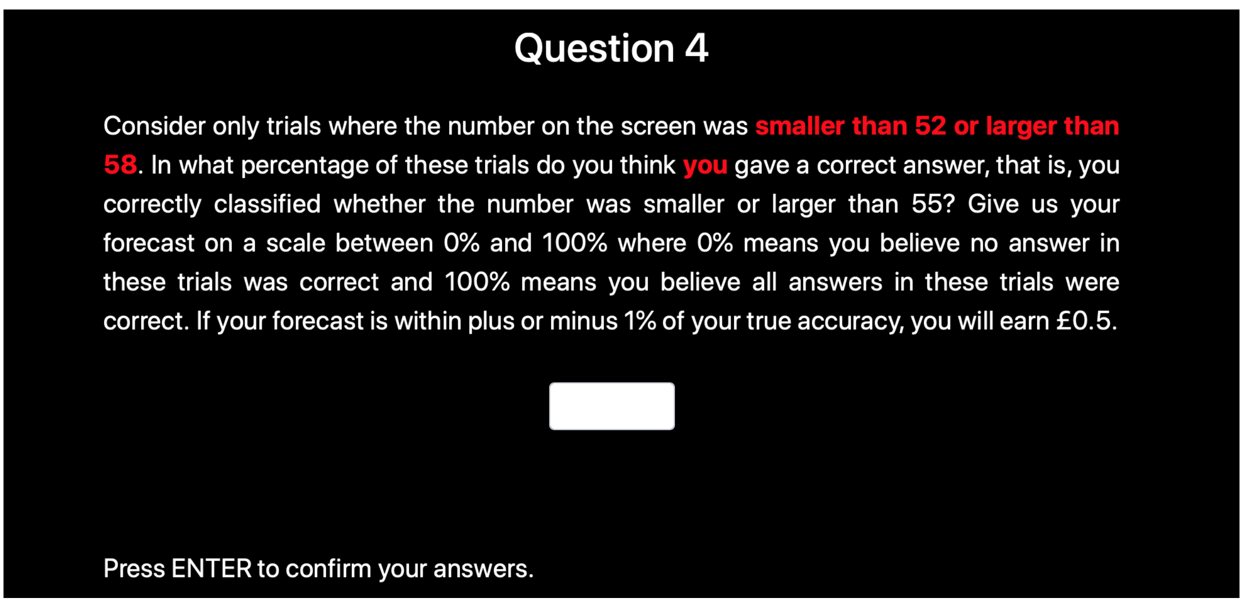
Consider only trials where the number on the screen was **equal to 47**. In what percentage of these trials do you think **the other participants** gave a correct answer, that is, they correctly classified whether the number was smaller or larger than 55? Give us your forecast on a scale between 0% and 100%, where 0% means you believe no answer in these trials was correct and 100% means you believe all answers in these trials were correct. If your forecast is within plus or minus 1% of the true percentage, you will earn £0.5.

Question 2

Consider only trials where the number on the screen was **equal to 54**. In what percentage of these trials do you think **the other participants** gave a correct answer, that is, they correctly classified whether the number was smaller or larger than 55? Give us your forecast on a scale between 0% and 100%, where 0% means you believe no answer in these trials was correct and 100% means you believe all answers in these trials were correct. If your forecast is within plus or minus 1% of the true percentage, you will earn £0.5.

Press ENTER to confirm your answers.


Consider the task you completed in Phase 1.
Question 3
Consider only trials where the number on the screen was between 52 and 58. In what percentage of these trials do you think you gave a correct answer, that is, you correctly classified whether the number was smaller or larger than 55? Give us your forecast on a scale between 0% and 100% where 0% means you believe no answer in these trials was correct and 100% means you believe all answers in these trials were correct. If your forecast is within plus or minus 1% of your true accuracy, you will earn £0.5.


Question 4
Consider only trials where the number on the screen was smaller than 52 or larger than 58. In what percentage of these trials do you think you gave a correct answer, that is, you correctly classified whether the number was smaller or larger than 55? Give us your forecast on a scale between 0% and 100% where 0% means you believe no answer in these trials was correct and 100% means you believe all answers in these trials were correct. If your forecast is within plus or minus 1% of your true accuracy, you will earn £0.5.
Press ENTER to confirm your answers.

Awareness of Own and Others' Cognitive Imprecision - PROLIFIC (#60101)

Created: 03/05/2021 07:16 AM (PT)
Shared: 06/30/2021 11:58 AM (PT)

1) Have any data been collected for this study already?
No, no data have been collected for this study yet.

2) What's the main question being asked or hypothesis being tested in this study?
We will test whether subjects are aware of their own and others' cognitive imprecision. We will study this question in the setting of a classic number discrimination task where a subject is incentivized to classify whether a two-digit Arabic numeral is greater than or smaller than a reference number (e.g., Dehaene, Dupoux and Mehler 1990). The innovation of this experiment is to also incentivize subjects to provide their beliefs about their own cognitive imprecision and the cognitive imprecision of other participants. In particular, we hypothesize that subjects are aware that (i) they make mistakes in such a number discrimination task, (ii) other subjects make mistakes in such a number discrimination task, and (iii) both own and others' mistakes are more frequent as the stimulus number becomes closer to the reference number.

3) Describe the key dependent variable(s) specifying how they will be measured.
There are three parts to the experiment:

Part 1: Subjects will go through a number discrimination task in which they are incentivized to accurately and quickly discriminate whether a 2-digit number that we denote as X is bigger or smaller than the reference number of "55". We will collect accuracy and response time data for each of the one hundred fifty trials in this part of the task.

Part 2: We will ask the subject to give beliefs about their own accuracy in Part 1, over the domains (X<52 or X>58) and (51< X< 59). Specifically, for each of the two domains, we will incentivize subjects to report the percentage of trials on which they believe they made accurate choices.

Part 3: We will ask the subject to give beliefs about other subjects' accuracy in Part 1, for the values of X=54 and X=47 (in the same experimental condition, see below). Specifically, for each of the two values of X, we will incentivize the subject to report the percentage of trials on which he believes all subjects (who went through the same condition as him) made accurate choices.

4) How many and which conditions will participants be assigned to?
There are two conditions, and the only difference across conditions is in Part 1 (though the order of Parts 2 and 3 will be randomized at the subject level). Subjects are randomized into the two conditions. The two conditions are "high volatility" and "low volatility" and they refer to the different distributions from which numbers are drawn in Part 1. In each condition there are 150 rounds. In the high volatility condition, we draw X from a normal distribution with mean 55 and variance 400. In the low volatility condition, we draw X from a normal distribution with mean 55 and variance 20. We then implement the following two steps:

1) X is rounded to the nearest integer
2) We re-draw X if the nearest integer is less than 11 or greater than 99, which ensures that X contains exactly 2-digits

5) Specify exactly which analyses you will conduct to examine the main question/hypothesis.
We will test whether (i) the stated beliefs about own and others' accuracy from Parts 2 and 3 are statistically different from 100%, (ii) the stated belief about own accuracy in "far comparisons" (that is, for X in the range [11, 51] or in the range [59, 99]) is greater than the stated belief about own accuracy in "close comparisons" (that is, for X in the range [52, 58]), and (iii)) the stated belief about others' accuracy for X=47 is greater than the stated belief about others' accuracy for X=54.

6) Describe exactly how outliers will be defined and handled, and your precise rule(s) for excluding observations.
Subjects will take a comprehension quiz after reading the experimental instructions for Part 1, which consists of 1 question. Any subject who does not answer the question correctly will be routed out of the experiment and excluded from the dataset.

7) How many observations will be collected or what will determine sample size? No need to justify decision, but be precise about exactly how the number will be determined.
We will collect data until we have N=300 subjects (150 subjects in each of the two conditions) who have correctly passed the comprehension quiz (defined as answering the single quiz question correctly).

8) Anything else you would like to pre-register? (e.g., secondary analyses, variables collected for exploratory purposes, unusual analyses planned?)

As a secondary analysis, we will assess whether (i) actual accuracy in the number classification task in the range [47, 63], on trials 31-150, is higher in the low volatility condition compared to the high volatility condition, conditional on X, (ii) response times in the number classification task are faster in the low volatility condition compared to the high volatility condition in the range [47, 63], on trials 31-150, and conditional on X, and (iii) perceived others' accuracy is higher for X=54 in the low volatility condition compared to the high volatility condition.