Monetary Policy and Wealth Effects: The Role of Risk and Heterogeneity *

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Abstract

We study the role of wealth effects, i.e. the revaluation of stocks, bonds, and human wealth, in the monetary policy transmission mechanism. The analysis of wealth effects requires to incorporate realistic asset-pricing dynamics and heterogeneous households' portfolios. Thus, we build an analytical heterogeneous-agents model with two main ingredients: i) rare disasters and ii) positive private debt. The model captures time-varying risk premia and precautionary savings in a linearized setting that nests the textbook New Keynesian model. Quantitatively, the model matches the empirical response of asset prices as well as the heterogeneous impact on borrowers and savers. We find that wealth effects induced by time-varying risk and private debt account for the bulk of the output response to monetary policy.

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1 Introduction

A long tradition in monetary economics emphasizes the role of wealth effects, i.e. the revaluation of real and financial assets, in the economy's response to changes in monetary policy. Its importance can be traced back to both classical and Keynesian economists, such as Pigou, Patinkin, Metzler and Tobin.[1] Keynes himself described the effects of interest rate changes as follows:

> There are not many people who will alter their way of living because the rate of interest has fallen from 5 to 4 per cent, if their aggregate income is the same as before. [...] Perhaps the most important influence, operating through changes in the rate of interest, on the readiness to spend out of a given income, depends on the effect of these changes on the appreciation or depreciation in the price of securities and other assets.
> - John Maynard Keynes, The General Theory of Employment, Interest, and Money.

Recently, wealth effects have regained relevance. In an influential paper, Kaplan et al. (2018) build a quantitative heterogeneous-agents New Keynesian (HANK) model and find only a small role for the standard intertemporal-substitution channel, leading the way to a more important role for wealth effects. Much of the literature has focused on the role of heterogenous marginal propensities to consume (MPCs) in settings with idiosyncratic income risk. In contrast, relatively little work has been done on the role of aggregate risk and risk premia, which are central to capture the wealth effects induced by monetary policy, despite ample evidence of their relevance.[2] A reason for this is that incorporating risk premia represents a challenge for the standard New Keynesian framework, as models with rich asset-pricing dynamics require the use of complex global or high-order perturbation methods. However, these models lack the insights on the role of these channels provided by analytically tractable models.

Our paper fills this gap. We provide a tractable unifying framework to study the role of risk and household heterogeneity in the monetary transmission mechanism. We obtain time-varying risk premia without the need for higher-order perturbation techniques, which allows us to provide a complete analytical characterization of the channels involved. Moreover, we capture key features of HANK models, such as precautionary savings and heterogeneous marginal propensities to consume (MPCs), in a setting with positive private debt, a combination that has been elusive in the analytical HANK literature. The model quantitatively captures key features of the monetary transmission mechanism, including important asset-pricing moments, such as the term premium, the equity premium, and corporate spreads, as well as the differential responses of borrowers and savers to monetary shocks observed in the data. Despite still being stylized, the

[1] The revaluation of government liabilities was central to Pigou (1943) and Patinkin (1965), while Metzler (1951) considered stocks and money. Tobin (1969) focused on how monetary policy interacted with the value of real assets.

[2] The effect of monetary policy on stock prices is considered by e.g. Bernanke and Kuttner (2005) and Kekre and Lenel (2020), while the effect on bonds is studied by e.g. Gertler and Karadi (2015) and Hanson and Stein (2015). The role of heterogeneous portfolios and the associated redistribution channel was originally considered by Auclert (2017). Cieslak and Vissing-Jorgensen (2020) show that policymakers track the behavior of stock markets because of its consumption wealth effect, while Chodorow-Reich et al. (2019) empirically establish the importance of this channel on the dynamics of consumption.

ability of the model to match these moments suggests that it can provide a useful assessment of the quantitative importance of the channels through which monetary policy affects households' consumption.[3] In particular, we find that time-varying risk and private debt account for more than 80% of the economy's response to a monetary shock. We conclude that risk and household heterogeneity combined are, therefore, major drivers of the economy's response to monetary policy.

We build an analytical HANK model with two main ingredients: i) rare disasters and ii) positive private debt. Rare disasters allow us to capture both a precautionary savings motive and realistic risk premia. Private debt is an important component of households' portfolios, representing 75% of GDP, and, as recently shown by Cloyne et al. (2020), borrowers account for the bulk of the response of aggregate consumption to changes in interest rates. Thus, by incorporating private debt, we are able to capture the role of revaluations in both gross and net asset positions.

We begin our analysis by considering an economy populated by two types of households, borrowers and savers, where borrowers are relatively impatient. Households are subject to borrowing constraints, and, in equilibrium, borrowers will be constrained at all times. By allowing households to borrow positive (but limited) amounts, we depart from most of the analytical HANK literature that focuses on the case of zero private liquidity. The zero liquidity assumption allowed the analytical literature to capture two key features of quantitative HANK models: a precautionary savings motive and heterogeneous MPCs. We capture these same two features in an economy with positive private debt by introducing an *aggregate* disaster risk, where the productivity of the economy is permanently reduced after a shock hits, as in the work of Barro (2006, 2009).[4] Moreover, this formalization allows us to effectively discipline the magnitude of the precautionary savings motive with asset-pricing data.

We then study the impact of monetary shocks by perturbing the economy around a stationary equilibrium with *positive aggregate risk* instead of adopting the more common approach of approximating around a non-stochastic steady state. By perturbing around the stochastic stationary equilibrium, we are able to obtain time variation in precautionary motives and risk premia using a first-order approximation, while the standard approach would require a third-order approximation (see e.g. Andreasen 2012). Moreover, by linearizing around an economy with zero monetary risk, we are able to solve for the stochastic stationary equilibrium in closed form, avoiding the need to compute the risky steady state numerically, as in Coeurdacier et al. (2011). This hybrid approach allows us to capture the effect of aggregate risk on asset prices in a linearized model.

Our first result states that output satisfies an *aggregate Euler equation*, where its sensitivity to interest rates depends on the disaster risk and on the level of private debt. With zero private liq-

[3]In order to obtain tractability, our model does not incorporate the rich heterogeneity in households' MPCs emphasized in the literature. However, our focus is on borrowers and savers, and we use the empirical findings in Cloyne et al. (2020) to discipline the model.

[4]Rare disasters have been widely used to explain a range of a asset-pricing facts; see e.g. Rietz (1988), Barro (2006), Gabaix (2008), Wachter (2013), Farhi and Gabaix (2016), and Barro and Liao (2020).

uidity and constant disaster probability, our economy features a *discounted* Euler equation, where output is less sensitive to future interest rate changes due to a precautionary motive, as in the incomplete-markets model of McKay et al. (2017). The presence of private debt acts in the opposite direction, as it pushes the economy towards *compounding* in the Euler equation, even with acyclical income inequality. We find that the second effect dominates in our calibration, so the aggregate Euler equation features compounding, even though, at the micro level, savers' Euler equation always features discounting.

We turn next to the channels through which monetary policy affects the economy. We show that equilibrium output can be characterized as the sum of four terms: the *intertemporal-substitution effect* (ISE); the *inside wealth effect*, i.e. the change in valuation of assets in zero net supply; the *outside wealth effect*, i.e. changes in the valuation of assets in positive net supply; and a *time-varying risk effect*.[5]

The ISE corresponds to the output response that operates through changes in the *timing* of output but not its overall (present value) level. While this channel is quantitatively important in the textbook New Keynesian model, we find that it has a marginal impact in the presence of heterogeneous agents and risk.

Most of the response in the economy can be explained by wealth effects and the associated time-varying risk effect. The inside wealth effect corresponds to a channel that is present only with heterogeneous MPCs and positive private debt.[6] It captures the aggregate implications of the differential response of borrowers and savers to changes in interest payments. An increase in nominal interest rates creates a positive wealth effect on savers, as they receive a higher income from private lending, and a corresponding loss to borrowers. Given the higher MPC for borrowers, this generates a negative aggregate response of output on impact.

Time-varying risk has a significant impact on how output responds to monetary shocks. When the probability of disaster is constant, the model is able to capture important unconditional asset-pricing moments, such as the level of the equity premium and an upward-sloping yield curve, but it fails to generate the observed response of risk premia to monetary shocks. This failure has important real consequences, as aggregate risk has then only a minor impact on the response of output and inflation. With time-varying disaster risk, the model is able to simultaneously match how long-term bonds, corporate spreads, and equities respond to monetary shocks in the data, and the impact on output increases almost threefold. This highlights the importance of matching the empirical response of asset prices to properly assess the role of risk in determining how monetary policy affects the economy.

Finally, the outside wealth effect is the sum of the change in wealth for all households in the economy. This includes the change in the value of stocks, government bonds, and human wealth,

[5]The notion of inside/outside wealth is reminiscent of inside/outside money as used by Gurley and Shaw (1960), and, more recently, inside/outside liquidity by Holmstrom and Tirole (2011).

[6]Note that previous analytical HANK models focused on either the case of heterogeneous MPC but no private debt, as in Bilbiie (2018), or positive private debt and no heterogeneity in MPC, as in Acharya and Dogra (2020).

net of the impact of discount rates on the present discounted value of consumption. We find that the outside wealth effect interacts with the presence of private debt in interesting ways. As we mentioned above, private debt introduces a force towards compounding in the Euler equation. This compounding *amplifies* the effect of changes in the value of households' wealth on equilibrium output, analogously to the *debt-deflation* effect. Lower asset prices reduce aggregate demand, which lowers output and inflation. Lower inflation increases the real burden on borrowers, generating an extra effect on aggregate demand.

An important result of our analysis is that the outside wealth effect is tightly connected to the response of fiscal policy to monetary shocks. In particular, we show that the outside wealth effect is proportional to the revaluation of public debt and the fiscal backing, that is, the change in taxes and transfers in response to monetary shocks. Intuitively, in a closed economy, the government is the only trading counterpart to the household sector as a whole, so the outside wealth effect can be inferred from the impact of monetary policy on government finances. More importantly, this result implies that we can use standard VAR techniques to identify the fiscal response to a monetary shock and discipline the ability of the model to generate quantitatively meaningful wealth effects. These findings have important implications for the quantitative assessment of monetary models. We find that when constrained to match the estimated fiscal response, the standard RANK model generates a substantially weaker output response to monetary shocks than when fiscal backing is determined by the standard Taylor rule that restricts monetary shocks to an AR(1) process. Equivalently, these results imply that the standard Taylor equilibrium requires a (passive) fiscal response that is counterfactually large. Our results can be made consistent with a Taylor equilibrium by allowing a more general specification of the monetary shock. In this case, we can use both monetary and fiscal data to discipline the parameters of the interest rate rule. It is in this context that the presence of heterogeneity and risk becomes particularly relevant, as these forces can compensate for the missing fiscal response.

To quantify the importance of the channels that are present in the model, we decompose the response of output by sequentially adding time-varying risk and private debt to the standard RANK model. We find that adding time-varying risk accounts for more than 50% of the overall output response, while private debt accounts for roughly 30%. Moreover, we find that time-varying risk has a larger impact on the economy in the presence of private debt and vice versa, showing the importance of considering risk and heterogeneity simultaneously.

Literature review. Wealth effects have a long tradition in monetary economics. Pigou (1943) relied on a wealth effect to argue that full employment could be reached even in a liquidity trap. Kalecki (1944) argued that these effects apply only to government liabilities, as inside assets cancel out in the aggregate, while Tobin highlighted the role of private assets and high-MPC borrowers.[7]

[7] Tobin (1982) describes the role of inside assets: "The gross amount of these 'inside' assets was and is orders of magnitude larger than the net amount of the base. Aggregation would not matter if we could be sure that the marginal propensities to spend from wealth were the same for creditors and debtors. But if the spending propensity were

Our work is closely related to two strands of literature. First, it relates to the analytical HANK literature, such as Werning (2015), Debortoli and Galí (2017), and Bilbiie (2018, 2019). While this literature focuses mostly on how the cyclicality of income interacts with differences in MPCs, we focus instead on how heterogeneous asset positions interact with differences in MPCs. We see these two channels as mostly complementary: even though Cloyne et al. (2020) do not find significant differences in income sensitivity across borrowers and savers, Patterson (2019) found a positive covariance between MPCs and the sensitivity of earnings to GDP across different demographic groups, suggesting that the income-sensitivity channel is operative for a different cut of the data. We share with Eggertsson and Krugman (2012) and Benigno et al. (2020) the emphasis on private debt, but they abstract from a precautionary motive and focus instead on the implications of deleveraging. Iacoviello (2005) also considers a monetary economy with private debt, but focuses instead on the role of housing as collateral. Our work is also related to Auclert (2017), which studies the redistribution channel of monetary policy arising from portfolio heterogeneity. Our paper emphasizes the redistribution channel in the context of a general equilibrium setting with aggregate risk.

Second, our paper is also closely related to work on how monetary policy affects the economy through changes in asset prices, including models with sticky prices, such as Caballero and Simsek (2020), and models with financial frictions, such as Brunnermeier and Sannikov (2016), Drechsler et al. (2018), and Di Tella (2019).[8] In recent contributions, Kekre and Lenel (2020) consider the role of the marginal propensity to take risk in determining the risk premium and shaping the response of the economy to monetary policy, and Campbell et al. (2020) use a habit model to study the role of monetary policy in determining bond and equity premia. Our model highlights instead the role of heterogeneous MPCs, positive private liquidity, and disaster risk in an analytical framework that preserves the tractability of standard New Keynesian models.

A recent literature studies rare disasters and business cycles. Gabaix (2011) and Gourio (2012) consider a real business cycle model with rare disasters, while Andreasen (2012) and Isoré and Szczerbowicz (2017) allow for sticky prices. They focus on the effect of changes in disaster probability, while we study monetary shocks in an analytical HANK model with rare disasters.

Our result regarding how asset revaluations depend on fiscal variables is related to work on fiscal policy and asset prices. Croce et al. (2012) and Gomes et al. (2013) study how fiscal policy affects asset prices in neoclassical economies, while Jiang (2019) and Corhay et al. (2018) study exchange rates and bond returns, respectively, in a fiscally active regime.

Outline. The paper is organized as follows. Section 2 presents the model used in the analysis. It shows how heterogeneity, positive private liquidity, and risk feed into the the aggregate Euler equation. In Section 3 we study the equilibrium dynamics, focusing on the determination of inside

systematically greater for debtors, even by a small amount, the Pigou effect would be swamped by this Fisher effect."

[8] A related literature focuses instead on the revaluation of housing, as in Berger et al. (2018) and Guren et al. (2018).

and outside wealth effects. We turn to the role of risk in Section 4. We conclude in Section 5.

2 D-HANK: An Analytical Rare Disasters HANK Model

In this section, we consider an analytical HANK model with two main ingredients: the possibility of rare disasters and positive private liquidity. By introducing aggregate disaster risk instead of the commonly adopted idiosyncratic income risk, we are able to capture a precautionary savings motive and an explicit role for liquidity in a setting with heterogeneous MPCs without having to keep track of a non-degenerate distribution of wealth.

2.1 The Model

Environment. Time is continuous and denoted by $t \in \mathbb{R}_+$. The economy is populated by households, firms, and a government. There are two types of households, *borrowers* and *savers*, who differ in their discount rates. A mass $0 \leq \mu_b < 1$ of households are borrowers and a mass $\mu_s = 1 - \mu_b$ are savers. Households can borrow or lend at a riskless rate, but they are subject to a borrowing constraint.

Firms can produce final or intermediate goods. Final-goods producers operate competitively and combine intermediate goods using a CES aggregator with elasticity $\epsilon > 1$. Intermediate-goods producers use labor as their only input and face Rotemberg pricing adjustment costs.[9] Intermediate-goods producers are subject to an aggregate productivity shock: with Poisson intensity $\lambda_t \geq 0$, they receive a shock that permanently reduces their productivity. This shock is meant to capture the possibility of rare disasters: low-probability, large drops in productivity and output, as in the work of Barro (2006, 2009). We say that periods that predate the realization of the shock are in the *no-disaster state*, and periods that follow the shock are in the *disaster state*. The disaster state is an absorbing state, and there are no further shocks after the disaster is realized. Assuming an absorbing disaster state simplifies the presentation, but it can be easily relaxed, as shown in Appendix A.2.[10]

The government sets fiscal policy, comprising a sales tax on intermediate-goods producers and transfers to borrowers and savers, and monetary policy, specified by an interest rate rule subject to a sequence of monetary shocks. We assume that the government issues long-term nominal bonds which pay exponentially decaying coupons, as in Woodford (2001), where the coupon in period t is given by $e^{-\psi_d t}$. The rate of decay ψ_d is inversely related to the bond's duration, where a perpetuity corresponds to $\psi_d = 0$ and the limit $\psi_d \to \infty$ corresponds to the case of short-term bonds. We denote by $Q_{L,t}$ the nominal price of the bond in the no-disaster state and by $Q^*_{L,t}$ the

[9] Rotemberg costs simplify the derivations in the disaster state, but they are not essential for our results.

[10] Allowing for partial recovery after a disaster, as in Barro et al. (2013) and Gourio (2012), introduces dynamics in the disaster state, but it does not change the main implications for the no-disaster state, which is our focus.

price of the bond in the disaster state, where the star superscript is used throughout the paper to denote variables in the disaster state.

Households' problem. Households face a portfolio problem where they choose how much to invest in short-term and long-term bonds. For simplicity, we assume that borrowers issue only short-term bonds and the government issues only long-term bonds. The nominal return on the long-term bond is given by

$$dR_t^L = \left[\frac{1}{Q_{L,t}} + \frac{\dot{Q}_{L,t}}{Q_{L,t}} - \psi_d\right] dt + \frac{Q_{L,t}^* - Q_{L,t}}{Q_{L,t}} d\mathcal{N}_t,$$

where $\mathcal{N}_t$ is a Poisson process with arrival rate λ_t.

Let $B_{j,t} = B_{j,t}^S + B_{j,t}^L$ denote the total value of bonds (in real terms) held by a type-j household, $j \in \{b, s\}$, that is, the sum of short-term ($B_{j,t}^S$) and long-term ($B_{j,t}^L$) bonds. The problem of a household of type j is to choose consumption $C_{j,t}$, labor supply $N_{j,t}$, and long-term bonds $B_{j,t}^L$, given an initial real value of bonds $B_{j,t}$, to solve the following problem:

$$V_{j,t}(B_{j,t}) = \max_{[C_{j,z}, N_{j,z}, B_{j,z}^L]_{z \geq t}} \mathbb{E}_t \left[\int_t^{t^*} e^{-\rho_j(z-t)} \left(\frac{C_{j,z}^{1-\sigma}}{1-\sigma} - \frac{N_{j,z}^{1+\phi}}{1+\phi}\right) dz + e^{-\rho_j(t^*-t)} V_{j,t^*}^*(B_{j,t^*}^*)\right],$$

subject to the flow budget constraint

$$dB_{j,t} = \left[(i_t - \pi_t)B_{j,t} + r_{L,t}B_{j,t}^L + \frac{W_t}{P_t}N_{j,t} + \Pi_{j,t} + \widetilde{T}_{j,t} - C_{j,t}\right] dt + B_{j,t}^L \frac{Q_{L,t}^* - Q_{L,t}}{Q_{L,t}} d\mathcal{N}_t,$$

and the borrowing constraints

$$B_{j,t} \geq -\overline{D}_p \quad \text{and} \quad B_{j,t}^L \geq 0,$$

where $\rho_b > \rho_s > 0$, W_t is the nominal wage, P_t is the price level, $\Pi_{j,t}$ denotes real profits from corporate holdings, $\widetilde{T}_{j,t}$ denotes government transfers, and $r_{L,t} \equiv \frac{1}{Q_{L,t}} + \frac{\dot{Q}_{L,t}}{Q_{L,t}} - \psi_d - i_t$ is the excess return on long-term bonds conditional on there being no disasters. The random (stopping) time t^* represents the period in which the aggregate shock hits the economy. $V_{j,t^*}^*(\cdot)$ and B_{j,t^*}^* denote, respectively, the value function and the real value of bonds in the disaster state. The non-negativity constraint on $B_{j,t}^L$ captures the assumption that only the government can issue long-term bonds.

We assume that $B_{s,0} > 0$ and $B_{b,0} = -\overline{D}_p$. For sufficiently large ρ_b, borrowers are constrained in all periods. We also assume that $\Pi_{b,t} = 0$, that is, firms are entirely owned by savers.[11]

In Appendix A, we show that the labor supply is determined by the standard condition

$$\frac{W_t}{P_t} = N_{j,t}^{\phi} C_{j,t}^{\sigma}.$$

[11] Alternatively, we could have assumed that households can trade shares of the firms. In steady-state, borrowers would choose to sell their shares and firms would be entirely held by savers.

The Euler equation for short-term bonds, if $B_{j,t} > -\overline{D}_p$, is given by

$$\frac{\dot{C}_{j,t}}{C_{j,t}} = \sigma^{-1}(i_t - \pi_t - \rho_j) + \frac{\lambda_t}{\sigma}\left[\left(\frac{C_{j,t}}{C^*_{j,t}}\right)^{\sigma} - 1\right], \tag{1}$$

where $C^*_{j,t}$ is the consumption of household j in the disaster state.[12] The first term captures the usual intertemporal-substitution force present in RANK models. The second term captures the *precautionary savings motive* generated by the disaster risk, and it is analogous to the precautionary motive that emerges in HANK models with idiosyncratic risk.

The Euler equation for long-term bonds, if $B^L_{j,t} > 0$, is given by

$$r_{L,t} = \underbrace{\lambda_t\left(\frac{C_{s,t}}{C^*_{s,t}}\right)^{\sigma}}_{\substack{\text{price of}\\ \text{disaster risk}}} \underbrace{\frac{Q_{L,t} - Q^*_{L,t}}{Q_{L,t}}}_{\substack{\text{quantity of}\\ \text{risk}}}. \tag{2}$$

This expression captures a risk premium on long-term bonds, which pins down the level of long-term interest rates in equilibrium. The premium on long-term bonds is given by the product of the *price of disaster risk*, the compensation for a unit exposure to the risk factor, and the *quantity of risk*, the loss the asset suffers conditional on switching to the disaster state.

Firms' problem. Intermediate-goods producers are indexed by $i \in [0,1]$ and operate the linear technology $Y_{i,t} = A_t N_{i,t}$. Productivity in the no-disaster state is given by $A_t = A$ and productivity in the disaster state is given by $A_t = A^*$, where $0 < A^* < A$. Intermediate-goods producers choose the *rate-of-change* of prices $\pi_{i,t} = \dot{P}_{i,t}/P_{i,t}$, given the initial price $P_{i,0}$, to maximize the expected discounted value of real (after-tax) profits subject to Rotemberg quadratic adjustment costs:

$$Q_{i,t}(P_{i,t}) = \max_{[\pi_{i,z}]_{z\geq t}} \mathbb{E}_t\left[\int_t^{t^*} \frac{\eta_z}{\eta_t}\left((1-\tau)\frac{P_{i,z}}{P_z}Y_{i,z} - \frac{W_z}{P_z}\frac{Y_{i,z}}{A} - \frac{\varphi}{2}\pi^2_{i,t}\right)dz + \frac{\eta_{t^*}}{\eta_t}Q^*_{i,t^*}(P_{i,t^*})\right], \tag{3}$$

subject to the demand $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$ and $\dot{P}_{i,t} = \pi_{i,t}P_{i,t}$, where η_t denotes the stochastic discount factor (SDF) that is relevant to firms and $Q^*_{i,t}(P_i)$ denotes the firms' value function in the disaster state. Note that the price $P_{i,t}$ is a state variable in the firms' problem and $\pi_{i,t}$ is a control variable. The parameter φ controls the magnitude of the pricing adjustment costs. We assume that these costs are rebated to households, so they do not represent real resource costs. Moreover, as firms are owned by savers, we assume that firms discount profits using the SDF $\eta_t = e^{-\rho_s t}C^{-\sigma}_{s,t}$.

Combining the first-order condition and the envelope condition for problem (3), we obtain the

[12] In discrete time, we obtain $C^{-\sigma}_{j,t} = (1-\rho_j\Delta t)(1+r_t\Delta t)\left[(1-\lambda_t\Delta t)C^{-\sigma}_{j,t+\Delta t} + \lambda_t\Delta t(C^*_{j,t+\Delta t})^{-\sigma}\right]$. After some rearrangement, we get $\frac{C^{-\sigma}_{j,t+\Delta}-C^{-\sigma}_{j,t}}{\Delta t} = -(r_t-\rho_j)C^{-\sigma}_{j,t+\Delta t} - \lambda_t((C^*_{j,t+\Delta t})^{-\sigma} - C^{-\sigma}_{j,t+\Delta t}) + o(\Delta t)$, which gives equation (1) as $\Delta t \to 0$.

non-linear New Keynesian Phillips curve:

$$\dot{\pi}_t = \left(i_t - \pi_t + \lambda_t \frac{\eta_t^*}{\eta_t}\right)\pi_t - \varphi^{-1}(\epsilon - 1)\left(\frac{\epsilon}{\epsilon - 1}\frac{W_t}{P_t}\frac{1}{A} - (1 - \tau)\right)Y_t, \tag{4}$$

assuming a symmetric initial condition $P_{i,0} = P_0$, for all $i \in [0, 1]$.

Time-varying risk. Motivated by the literature on time-varying risk premia and rare disasters, we allow for a time-varying disaster probability.[13] To capture the effect of monetary policy on the market price of risk, as documented e.g. by Gertler and Karadi (2015) and Hanson and Stein (2015), we assume that monetary shocks affect directly the probability of disasters: $\lambda_t = \lambda(i_t - r_n)$, for a given function $\lambda(\cdot)$. Importantly, we assume that $\lambda(\cdot)$ is an increasing function, such that a contractionary monetary shock raises the probability of a disaster.[14] This reduced-form assumption allows us to capture the quantitative effect of monetary policy on risk premia in a parsimonious way while keeping the analysis tractable.

Government. The government's flow budget constraint in the no-disaster state is given by

$$\dot{D}_{g,t} = (i_t - \pi_t + r_{L,t})D_{g,t} + \sum_{j \in \{b,s\}} \mu_j \widetilde{T}_{j,t} - \tau Y_t,$$

and the No-Ponzi condition $\lim_{t\to\infty} \mathbb{E}_0[\eta_t D_{g,t}] \leq 0$, $D_{g,t}$ denotes the real value of government debt and $D_{g,0} = \overline{D}_g$ is given. We assume that government transfers to borrowers are determined by the policy rule $\widetilde{T}_{b,t} = \widetilde{T}_b(Y_t)$, where transfers depend on aggregate output and the elasticity of $\widetilde{T}_b(\cdot)$ determines the cyclicality of government transfers to borrowers and, ultimately, the cyclicality of borrowers' consumption.

In the no-disaster state, monetary policy is determined by the policy rule

$$i_t = r_n + \phi_\pi \pi_t + u_t, \tag{5}$$

where $\phi_\pi > 1$, u_t represents monetary shocks, and r_n denotes the real rate when $\pi_t = u_t = 0$ at all periods. In the disaster state, we assume that there are no monetary shocks, that is, $i_t^* = r_n^* + \phi_\pi \pi_t^*$. By abstracting from the policy response after a disaster, we isolate the impact of changes in monetary policy during "normal times."

Market clearing. The market-clearing conditions for goods, labor, and bonds are given by

$$\sum_{j \in \{b,s\}} \mu_j C_{j,t} = Y_t, \quad \sum_{j \in \{b,s\}} \mu_j N_{j,t} = N_t, \quad \sum_{j \in \{b,s\}} \mu_j B_{j,t}^L = D_{g,t}, \quad \sum_{j \in \{b,s\}} \mu_j B_{j,t}^S = 0.$$

[13]See Tsai and Wachter (2015) for a review of this literature.

[14]For direct evidence on this channel, see Schularick et al. (2021), who find that contractionary monetary policy shocks lead to a substantial increase in the probability of future crises.

2.2 Equilibrium dynamics

Stationary equilibrium. We define a stationary equilibrium as an equilibrium in which all variables are constant in each aggregate state. In particular, the economy will be in a stationary equilibrium in the absence of monetary shocks, that is, $u_t = 0$ for all $t \geq 0$. Since variables are constant in each state, we drop time subscripts and write, for instance, $C_{j,t} = C_j$ and $C^*_{j,t} = C^*_j$. For ease of exposition, we follow Bilbiie (2019) and focus on a *symmetric* stationary equilibrium, where $\widetilde{T}_b$ implements the same consumption level for each household, and discuss the general case $C_b \neq C_s$ in the appendix.

The natural interest rate, the real rate in the stationary equilibrium, is given by

$$r_n = \rho_s - \lambda \left[\left(\frac{C_s}{C^*_s} \right)^{\sigma} - 1 \right],$$

where $0 < C^*_s < C_s$ and, with a slight abuse of notation, $\lambda = \lambda(0) > 0$ is the disaster intensity when $i_t = r_n$. The presence of a precautionary motive depresses the natural interest rate relative to the one that would prevail in a non-stochastic economy. Moreover, we show in Appendix A.2.1 that the precautionary motive depends on the extent to which savers can self-insure. In particular, holding everything else constant, a higher level of private debt $\overline{D}_p$ implies a weaker precautionary motive and a higher natural interest rate.

From Equation (2), we obtain the term spread, the difference between the yield on the long-term bond and the short-term rate,[15]

$$i_L - r_n = \lambda \left(\frac{C_s}{C^*_s} \right)^{\sigma} \frac{Q_L - Q^*_L}{Q_L},$$

where i_L is the yield on the long-term bond in the stationary equilibrium. We show in Appendix A.2.2 that the term spread $i_L - r_n$ is strictly positive. Thus, our model generates an upward-sloping yield curve, where the yield on the long-term bond exceeds the natural (short-term) rate, consistent with the data.[16]

Log-linear dynamics. Following the practice in the literature on monetary policy, we focus on a log-linear approximation of the equilibrium conditions. However, instead of linearizing around the non-stochastic steady state, we linearize the equilibrium conditions around the (stochastic) symmetric stationary equilibrium described above. Formally, we perturb the allocation around the economy where $u_t = 0$, while the standard approach would perturb around the economy

[15] The result follows from noting that the yield on the bond is given by $i_{L,t} = Q^{-1}_{L,t} - \psi_d$ and, in a stationary equilibrium, the expected excess return conditional on no disaster r_L equals the term spread $i_L - r_n$.

[16] The mechanism behind the upward-sloping yield curve is related to the lack of precautionary savings in the disaster state. We would obtain similar results by introducing expropriation and inflation in a disaster, as in Barro (2006).

where $u_t = \lambda_t = 0$.[17] This will enable us to capture the effects of (time-varying) precautionary savings and risk premia in a linear setting.

Let lower-case variables denote log-deviations from the stationary equilibrium, e.g., $c_{j,t} \equiv \log C_{j,t}/C_j$ and $n_{j,t} \equiv \log N_{j,t}/N_j$. Borrowers' consumption is given by

$$c_{b,t} = (1-\alpha)(w_t - p_t + n_{b,t}) + T_{b,t} - (i_t - \pi_t - r_n)\overline{d}_p,$$

where $1-\alpha \equiv \frac{WN}{PY}$ is the labor share in the stationary equilibrium, $T_{j,t} \equiv \frac{\widetilde{T}_{j,t} - \widetilde{T}_j}{Y}$, and $\overline{d}_p \equiv \frac{\overline{D}_p}{Y}$. Using the fact that transfers satisfy $T_{b,t} = T_b'(Y)Yy_t$, and solving for the real wage, we obtain

$$c_{b,t} = \chi_y y_t - \chi_r(i_t - \pi_t - r_n), \tag{6}$$

where

$$\chi_y \equiv \frac{T_b'(Y)Y + (1-\alpha)(1+\phi)(1+\phi^{-1}\sigma)}{1+(1-\alpha)\phi^{-1}\sigma}, \qquad \chi_r \equiv \frac{\overline{d}_p}{1+(1-\alpha)\phi^{-1}\sigma}.$$

The coefficient χ_y controls the cyclicality of income inequality and has been extensively studied by the literature on analytical HANK models. We focus throughout the paper on the case in which $0 < \chi_y < \mu_b^{-1}$, such that the consumption of both agents increases with y_t.[18] The second term is not present in the commonly studied case of zero private liquidity, $\overline{d}_p = 0$, and it captures the impact of monetary policy on the consumption of constrained agents that is not directly mediated by aggregate output y_t. The coefficient χ_r plays an important role in the analysis that follows.

Next, consider the savers' problem. Recall that we assume that the disaster probability depends on the interest rate, $\lambda_t = \lambda(i_t - r_n)$. In our linearized setting, the only relevant parameter is the semi-elasticity of the disaster probability with respect to monetary shocks, $\epsilon_\lambda \equiv \lambda'(0)/\lambda(0)$. We focus on the case in which $\epsilon_\lambda \geq 0$, with $\epsilon_\lambda = 0$ corresponding to a constant probability benchmark. Then, the savers' Euler equation is given by

$$\dot{c}_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma c_{s,t} + \chi_p \epsilon_\lambda (i_t - r_n), \tag{7}$$

where $\chi_p \equiv \frac{\lambda}{\sigma}\left[\left(\frac{C_s}{C_s^*}\right)^\sigma - 1\right]$ is a parameter capturing the strength of the precautionary motive in the stationary equilibrium. Importantly, time-varying disaster risk introduces a new precautionary savings channel for savers, which ultimately shapes the impact of changes in nominal rates on households' consumption.

Combining condition (6) for borrowers' consumption, equation (7) for savers' Euler equation, and the market-clearing condition for goods, we can derive the evolution of aggregate output.

[17] This method then differs from the perturbation procedure considered by Fernández-Villaverde and Levintal (2018), and it is also distinct from Coeurdacier et al. (2011), as we linearize around a stochastic steady state of an economy with no monetary shocks, instead of the stochastic steady state of the economy with both shocks.

[18] The role of χ_y, including the case where $\chi_y > \mu_b^{-1}$, was originally considered by Bilbiie (2008). The cyclicality of income inequality also plays an important role in the aggregation results in Werning (2015) and Bilbiie (2018).

Proposition 1 characterizes the dynamics of aggregate output and inflation.

Proposition 1 (Aggregate dynamics)**.** *The dynamics of output and inflation is described by the conditions:*

i. ***Aggregate Euler equation:***

$$\dot{y}_t = \tilde{\sigma}^{-1}(i_t - \pi_t - r_n) + \delta y_t + v_t, \tag{8}$$

where $\tilde{\sigma}^{-1}$, δ, and v_t are given by

$$\tilde{\sigma}^{-1} \equiv \frac{(1-\mu_b)\sigma^{-1} - \mu_b \chi_r \rho}{1-\mu_b \chi_y}, \qquad \delta \equiv \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} - \frac{\mu_b \chi_r \kappa}{1-\mu_b \chi_y},.$$

and

$$v_t \equiv \frac{\mu_b \chi_r}{1-\mu_b \chi_y}(\rho(i_t - r_n) - \dot{i}_t) + \frac{1-\mu_b}{1-\mu_b \chi_y}\chi_p \epsilon_\lambda (i_t - r_n)$$

ii. ***New Keynesian Phillips curve:***

$$\dot{\pi}_t = \rho \pi_t - \kappa y_t, \tag{9}$$

where $\rho \equiv \rho_s + \lambda$ and $\kappa \equiv \varphi^{-1}(\epsilon - 1)(1-\tau)(\phi + \sigma)$.

Proof. See Appendix B.1. □

Condition (8) represents the *aggregate Euler equation* for this economy. The aggregate Euler equation has three terms. The first term, the product of the (aggregate) elasticity of intertemporal substitution (EIS) and the real interest rate, corresponds to the one present in RANK models. The dependence of the aggregate EIS on the cyclicality of inequality is well-known in the literature, while the result that private liquidity may reduce $\tilde{\sigma}^{-1}$ is, to the best of our knowledge, new.[19]

The second term, δy_t, captures how the impact of real interest rate changes can be compounded or discounted in equilibrium. The sign of δ and, therefore, whether the economy exhibits compounding or discounting, depends on two forces.[20] With disaster risk but in the absence of private debt, so that $\lambda > 0$ and $\chi_r = 0$, we obtain $\delta > 0$. This corresponds to the discounted Euler equation of McKay et al. (2017), where aggregate disaster risk plays the role of idiosyncratic income risk. In contrast, if $\lambda = 0$ but $\chi_r > 0$, we get compounding, that is, $\delta < 0$. As a contractionary monetary shock depresses the economy and reduces inflation in all periods, it increases the real burden of debt for borrowers, amplifying the effect of the monetary shock. This amplification translates into a compounded response of output to future interest rate changes. More generally, the aggregate Euler equation (8) can feature compounding or discounting. In particular, if λ is sufficiently small, we can have that savers' consumption satisfies a discounted Euler equation while the aggregate Euler equation features compounding.

[19] In the calibration for the numerical exercises, we obtain $\tilde{\sigma}^{-1} > 0$. However, most of our results do not rely on this.

[20] Note that, assuming $v_t = 0$, output is given by $y_t = -\int_t^\infty e^{-\delta(s-t)}(i_s - \pi_s - r_n)ds$, so the effect of future changes in the real interest rates on output is dampened (or discounted) if $\delta > 0$ and it is compounded if $\delta < 0$.

The third term in the aggregate Euler equation, v_t, captures a direct effect of monetary policy on households, one which is not mediated by changes in aggregate demand. First, in an economy with positive private liquidity, monetary policy directly affects borrowers' disposable income, the high-MPC agents in this economy. Second, the time-varying component of the disaster risk directly impacts the savers' precautionary savings motives. Therefore, monetary policy has real effects even in the complete absence of intertemporal-substitution forces.

Finally, Proposition 1 defines the New Keynesian Phillips curve in this economy. The linearized Phillips curve coincides with the one obtained from models with Calvo pricing. As in a textbook New Keynesian model, inflation is given by the present discounted value of future output gaps

$$\pi_t = \kappa \int_t^\infty e^{-\rho(s-t)} y_s ds.$$

One distinction relative to the standard formulation is that future output gaps are not discounted by the natural rate r_n, but by a higher rate $\rho > r_n$. This is a consequence of the riskiness of the firm's value, so the appropriate discount rate incorporates an adjustment for risk.

Asset prices. The response of asset prices to monetary policy depends in an important way on the behavior of the price of disaster risk. In its log-linear form, the price of disaster risk is given by

$$p_{d,t} \equiv \sigma c_{s,t} + \epsilon_\lambda (i_t - r_n). \tag{10}$$

Note that the expression has two terms. The first term captures the change in the savers' consumption drop if the disaster shock is realized. The second term represents the change in the disaster probability after a monetary shock.

We show in Appendix A.3 that the (linearized) price of the long-term bond in $t = 0$ is given by

$$q_{L,0} = -\underbrace{\int_0^\infty e^{-(\rho+\psi_d)t}(i_t - r_n)dt}_{\text{path of nominal interest rates}} - \underbrace{\int_0^\infty e^{-(\rho+\psi_d)t} r_L p_{d,t} dt}_{\text{term premium}}. \tag{11}$$

The yield on the long-term bond, expressed as deviations from the stationary equilibrium, is given by $-Q_L^{-1} q_{L,0}$, which can be decomposed into two terms: the path of nominal interest rates, as in the expectations hypothesis, and a *term premium*, capturing variations in the compensation for holding long-term bonds. Because the term premium responds to monetary shocks, the expectation hypothesis does not hold in this economy. This is important since the term premium accounts for the bulk of the response of long rate to monetary policy in the data.

We have a pricing condition for stocks which is analogous to the one for bonds, where the equity premium depends on the price of disaster risk and the quantity of risk:[21]

$$\frac{\Pi_t}{Q_t} + \frac{\dot{Q}_t}{Q_t} - (i_t - \pi_t) = \lambda_t \left(\frac{C_{s,t}}{C^*_{s,t}}\right)^{\sigma} \frac{Q_t - Q^*_t}{Q_t},$$

where Q_t is the value of a claim on firms' profits. The price in period 0 is given by

$$q_0 \;=\; \underbrace{\frac{Y}{Q}\int_0^{\infty} e^{-\rho t}\left[(1-\tau)y_t - (1-\alpha)(w_t - p_t + n_t)\right]dt}_{\text{dividends}} - \underbrace{\int_0^{\infty} e^{-\rho t}\left[i_t - \pi_t - r_n + r_S p_{d,t}\right]dt}_{\text{discount rate}}, \quad (12)$$

where $r_S \equiv \lambda\left(\frac{C_s}{C^*_s}\right)^{\sigma}\frac{Q-Q^*}{Q}$ is the (conditional) equity premium in the stationary equilibrium. This expression shows that the valuation of assets responds to changes in monetary policy through two channels: a *dividend channel*, capturing changes in firms' profits, and a *discount rate channel*, capturing changes in real interest rates and risk premia.

2.3 Intertemporal budget constraint and discount-rate neutrality

Individual intertemporal budget constraint. An intertemporal budget constraint, computed with the SDF η_t, holds with equality for both types of households:

$$\mathbb{E}_0\left[\int_0^{\infty}\frac{\eta_t}{\eta_0}C_{j,t}dt\right] = B_{j,0} + \mathbb{E}_0\left[\int_0^{\infty}\frac{\eta_t}{\eta_0}\Pi_{j,t}dt\right] + \mathbb{E}_0\left[\int_0^{\infty}\frac{\eta_t}{\eta_0}\left(\frac{W_t}{P_t}N_{j,t} + \widetilde{T}_{j,t}\right)dt\right],$$

where the intertemporal budget constraint holds for savers because of the transversality condition and holds for borrowers because $\lim_{t\to\infty}\mathbb{E}_t[\eta_t] = 0$ and $B_{b,t}$ is constant. Thus, the present discounted value of consumption equals the value of households' assets: the total value of short-term and long-term bonds, the value of stocks (the discounted value of profits), and the value of human wealth (the discounted value of labor income inclusive of transfers).

Let $Y_{j,t} \equiv \Pi_{j,t} + \frac{W_t}{P_t}N_{j,t} + \widetilde{T}_{j,t}$ denote household j's income. Then, we can write the intertemporal budget constraint as $Q_{C_j,0} = B_{j,0} + Q_{Y_j,0}$, where $Q_{C_j,t}$ is the value of a claim on consumption and $Q_{Y_j,t}$ is the value of a claim on income for household j. In log-linear form, we obtain:

$$\int_0^{\infty} e^{-\rho t}c_{j,t}dt = \overline{b}^L_j q_{L,0} + \int_0^{\infty} e^{-\rho t}\left[y_{j,t} - (q_{y_j} - q_{c_j})(i_t - \pi_t - r_n) - \lambda\left(\frac{C_s}{C^*_s}\right)^{\sigma}\left(q_{y_j} - q^*_{y_j} - (q_{c_j} - q^*_{c_j})\right)p_{d,t}\right]dt.$$

where $y_{j,t} \equiv \frac{Y_{j,t}-Y_j}{Y_j}$, $q_{c_j} \equiv \frac{Q_{C_j}}{Y}$, and $q_{y_j} \equiv \frac{Q_{Y_j}}{Y}$. This expression illustrates how consumption on average reacts to changes in income and discount rates. Importantly, the effect of changes in discount rates depends on the *mismatch* on households' balance sheets. Consider first the case of borrowers, where $\overline{b}^L_b = 0$, $q_{y_b} > q_{c_b}$, and $q_{y_b} - q^*_{y_b} = q_{c_b} - q^*_{c_b}$. An increase in the real interest rate has a nega-

[21] Note that this represents the equity premium conditional on no disasters. The unconditional equity premium is given by the closely related expression: $\lambda_t\left[\left(\frac{C_{s,t}}{C^*_{s,t}}\right)^{\sigma} - 1\right]\frac{Q_t - Q^*_t}{Q_t}$.

tive impact on the average consumption of borrowers, everything else constant. In the language of Auclert (2017), they have an unhedged interest rate exposure. In contrast to borrowers, savers are also exposed to changes in risk premia. Movements in risk premia affect average consumption to the extent that there are differences in the riskiness of income and consumption which are not hedged by the bonds. Therefore, changes in risk premia depends on the *unhedged risk exposure* of household j.[22]

Aggregate intertemporal budget constraint. By combining the intertemporal budget constraint for each household, we obtain the economy's aggregate intertemporal budget constraint:

$$\int_0^\infty e^{-\rho t} \left(\mu_b c_{b,t} + (1-\mu_b) c_{s,t}\right) dt = \Omega_0, \tag{13}$$

where Ω_0 denotes the outside wealth effect, and it is given by

$$\Omega_0 \equiv \overline{d}_g q_{L,0} + \int_0^\infty e^{-\rho t} \left[(1-\tau) y_t + T_t + \overline{d}_g (i_t - \pi_t - r_n + r_L p_{d,t})\right] dt, \tag{14}$$

where $\overline{d}_g \equiv \frac{D_g}{Y}$ is the public debt-to-GDP ratio.

Condition (13) presents the (linearized) aggregate intertemporal budget constraint. Note that a simple rearrangement of (13) and (14) gives the government's intertemporal budget constraint (or valuation equation). By writing it this way, we make explicit the role of the *outside wealth effect* Ω_0, which captures the revaluation of assets in positive net supply: stocks, human wealth, and government bonds. This is in contrast with the effect of the revaluation of assets in zero net supply, such as private debt, which we refer to as the *inside wealth effect*. We define the wealth effect net of the impact of changes in discount rates in the present discounted value of consumption, which can be thought of as a form of households' liabilities. This definition implies that a negative wealth effect means that the household's original consumption bundle is not affordable after the shock.

An important implication of Equation (14) is that an increase in discount rates do not create a negative (outside) wealth effect in the absence of government debt, everything else constant. This may sound surprising at first, as an increase in interest rates or risk premia would reduce the value of stocks even absent changes in dividends. However, this does not take into account the impact on the value of households' planned consumption. Without government debt, aggregate consumption equals dividends plus (after-tax) labor income, so there is no unhedged interest rate or risk exposure. The initial consumption bundle is still affordable. In the presence of short-term government debt or when government debt is a perpetuity, one can show that Ω_0 does not depend directly on $p_{d,t}$. In this case, the unhedged risk exposure is zero, so changes in risk premia do not affect Ω_0, everything else constant.

[22]Formally, the unhedged risk exposure at date t corresponds to $q_{y_j} - q^*_{y_j} - (q_{c_j} - q^*_{c_j}) + e^{-\psi_d t}(\overline{b}^L_s - \overline{b}^{L,*}_s)$. If the unhedged risk exposure is zero at all dates, then variations in the price of risk cancel out of the intertemporal budget constraint.

3 Monetary Policy and Wealth Effects

In this section, we study how households' balance sheets determine the impact of monetary policy on the dynamics of the economy. The main result of this section presents a decomposition that identifies the contribution of the different forces of the model to the aggregate dynamics of the economy. In particular, we isolate the role of intertemporal substitution, precautionary savings, and wealth effects in the transmission of monetary shocks to the economy. To derive this decomposition, we proceed in two steps. First, we express the evolution of output and inflation in terms of equilibrium policy variables, that is, the path of nominal interest rates $\{i_t\}$ and the corresponding fiscal backing $\{T_t\}$. Second, we derive an *implementability result* which shows how to map the path of policy variables to an underlying monetary shock u_t in the interest rate rule (5).

3.1 The dynamic system

We can express output and inflation in terms of policy variables by solving the system of differential equations described in Proposition 1:

$$\begin{bmatrix} \dot{y}_t \\ \dot{\pi}_t \end{bmatrix} = \begin{bmatrix} \delta & -\tilde{\sigma}^{-1} \\ -\kappa & \rho \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \tilde{v}_t \\ 0 \end{bmatrix}, \tag{15}$$

where $\tilde{v}_t \equiv \tilde{\sigma}^{-1}(i_t - r_n) + v_t$ depends only on the path of the nominal interest rate. The eigenvalues of the system are given by

$$\overline{\omega} = \frac{\rho + \delta + \sqrt{(\rho+\delta)^2 + 4(\tilde{\sigma}^{-1}\kappa - \rho\delta)}}{2}, \qquad \underline{\omega} = \frac{\rho + \delta - \sqrt{(\rho+\delta)^2 + 4(\tilde{\sigma}^{-1}\kappa - \rho\delta)}}{2}.$$

The following assumption, which we will assume holds for all subsequent analysis, guarantees that the eigenvalues are real-valued and that they have opposite signs, that is, $\overline{\omega} > 0$ and $\underline{\omega} < 0$.

Assumption 1. *The following condition holds: $\rho\delta < \tilde{\sigma}^{-1}\kappa$.*

Assumption 1 implies that the system lacks exactly one boundary condition. This is consistent with the results in Acharya and Dogra (2020), who find that indeterminacy under an interest rate peg requires a discounting parameter that is not overly large. Next, we show that the missing boundary condition can be provided by an intertemporal budget constraint.

From equation (13), we have that the aggregate intertemporal budget constraint is a necessary equilibrium condition. The next lemma establishes the sufficiency of the aggregate intertemporal budget constraint for pinning down the equilibrium. That is, it shows that if $[y_t, \pi_t]_0^\infty$ satisfies system (15) and the aggregate intertemporal budget constraint (in its log-linear form), then we can determine the value of consumption and labor supply for each household, wages, and prices such that all equilibrium conditions are satisfied.

Lemma 1. *Suppose that, given a path for the nominal interest rate* $[i_t]_0^\infty$, $[y_t, \pi_t]_0^\infty$ *satisfy system* (15) *and the aggregate intertemporal budget constraint* (13)*. Then,* $[y_t, \pi_t]_0^\infty$ *can be supported as part of a competitive equilibrium.*

Proof. See Appendix B.2. □

Therefore, the equilibrium dynamics can be characterized as the solution to the dynamic system (15), subject to the boundary condition (13).

3.2 Intertemporal substitution, risk and wealth effects

The next proposition characterizes the output response to a sequence of monetary policy shocks, for a given value of the outside wealth effect Ω_0. We provide a full characterization of Ω_0 in Section 3.3. For ease of exposition, we focus on the case of exponentially decaying nominal interest rates; that is, we assume $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, where ψ_m determines the persistence of the path of interest rates.

Proposition 2 (Aggregate output in D-HANK)**.** *Suppose that* $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$*. The path of aggregate output is then given by*

$$y_t = \underbrace{\sigma^{-1}\hat{y}_t}_{\text{ISE}} + \underbrace{\chi_p \epsilon_\lambda \hat{y}_t}_{\text{time-varying risk}} + \underbrace{\frac{\mu_b \chi_r}{1-\mu_b}\psi_m \hat{y}_t}_{\text{inside wealth effect}} + \underbrace{(\rho - \underline{\omega})e^{\underline{\omega} t}\Omega_0}_{\substack{\text{GE multiplier}\times \\ \text{outside wealth effect}}}, \tag{16}$$

where $\hat{y}_t$ *is given by*

$$\hat{y}_t = \frac{1-\mu_b}{1-\mu_b\chi_y}\frac{(\rho - \underline{\omega})\, e^{\underline{\omega} t} - (\rho + \psi_m)\, e^{-\psi_m t}}{(\overline{\omega} + \psi_m)\,(\underline{\omega} + \psi_m)}(i_0 - r_n), \tag{17}$$

and satisfies

$$\int_0^\infty e^{-\rho t}\hat{y}_t dt = 0, \quad \frac{\partial \hat{y}_0}{\partial i_0} < 0.$$

Proof. See Appendix B.3. □

Proposition 2 shows that output can be decomposed into four terms: an intertemporal-substitution effect (ISE), a time-varying risk channel, a revaluation of inside assets, and a revaluation of outside assets. An important feature of the decomposition is that each term can be included or excluded by choosing the value of the parameters that determine the channel's strength. Moreover, given a path of nominal interest rates, the ISE, the time-varying risk, and the inside wealth effect are uniquely determined.

The ISE captures the equilibrium implications of the intertemporal-substitution channel. In the absence of outside wealth effects, monetary policy affects only the *timing* of output, as the present value of economic activity is determined entirely by Ω_0 (that is, we have that $\int_0^\infty e^{-\rho t}\hat{y}_t dt = 0$ and

$\int_0^\infty e^{-\rho t} y_t dt = \Omega_0$). Similar in logic to the substitution effect in introductory microeconomics, an increase in nominal interest rates reduces consumption today, while it increases future consumption.[23] In this sense, the intertemporal-substitution channel of monetary policy operates simply by shifting demand over time, and it is ineffective in the absence of an intertemporal-substitution motive; that is, we obtain $\sigma^{-1}\hat{y}_t = 0$ in all periods if $\sigma^{-1} = 0$. Note that $\hat{y}_t$ is multiplied by a factor that is increasing in the parameter that controls the (counter-) cyclicality of inequality χ_y. This amplification lies at the heart of the mechanism in analytical HANK models with zero private liquidity.

The second term captures the role of time-varying risk. Given Ω_0, time-varying risk amplifies the response of output in a way that is similar to that of the ISE, and the magnitude of the amplification depends crucially on the strength of the precautionary motive, as captured by $\chi_p = \frac{\lambda}{\sigma}\left[\left(\frac{C_s}{C_s^*}\right)^\sigma - 1\right]$, and the degree of time-varying risk, as captured by ϵ_λ. Consider a contractionary monetary shock. If $\epsilon_\lambda > 0$, the increase in the nominal interest rate increases the probability of a disaster shock, which increases savers' precautionary motive. Thus, aggregate demand decreases. As the nominal interest rate reverts back to its long-run level, the probability of disaster decreases, and savers' demand increases above its long-run target. Thus, we have that the present discounted value of the time-varying risk term is zero, i.e. $\int_0^\infty e^{-\rho t}\chi_p \epsilon_\lambda \hat{y}_t dt = 0$.

The third term corresponds to the inside wealth effect, and it is present only in economies with positive private debt and heterogeneous MPCs. The inside wealth effect is analogous to the ISE and the time-varying risk in many respects, as it operates by shifting demand over time, and it satisfies $\int_0^\infty e^{-\rho t}\chi_r \psi_m \hat{y}_t dt = 0$. A key distinction is that the strength of the inside wealth effect depends on the persistence of the monetary shock, and it is equal to zero when the monetary shock is permanent, $\psi_m = 0$.

An important implication of this result is that the effectiveness of monetary policy depends on the persistence of monetary shocks. For instance, by promising to keep interest rates low for a very long period of time, the monetary authority increases the persistence of the shock and, therefore, reduces the importance of inside wealth effects and the overall output response. To understand this result, note that an increase in interest rates has a negative impact on borrowers and a positive impact on savers. When the shock is temporary, the impact of the change in interest rates is initially larger on borrowers, as savers respond less strongly to the change in wealth to smooth consumption. If the shock is permanent, however, there is no reason to smooth the shock. In this case, the savers' response coincides with the borrowers' response, and the inside wealth effect is exactly zero. Thus, it is the *variability* of interest rates rather than the average level that matters for the inside wealth effect.

The last term in expression (16) plays a crucial role, as the outside wealth effect determines the average level of output. Holding everything else constant, the impact of a wealth effect Ω_0 on

[23] Our definition of ISE coincides with the textbook substitution effect in the limit case $\lambda = \kappa = 0$, where there is no distinction between changes in nominal and real rates, and there is no precautionary motive.

consumption would be simply $\rho\Omega_0$, as households attempt to smooth the impact of the change in wealth over time. However, the response of initial consumption is amplified in general equilibrium, as a positive wealth effect generates inflation, which reduces real interest rates and shifts consumption to the present. As we show in the next section, this effect can be quantitatively significant. Moreover, an important factor determining the size of the GE multiplier is the discounting/compounding parameter δ. It can be shown that the GE multiplier in period 0 is decreasing in δ. Therefore, the precautionary savings motive *dampens* the effect of the outside wealth effect, while positive private debt reduces the value of δ, which *increases* the effect of the outside wealth effect.

Inflation. The next proposition characterizes the response of inflation to monetary policy shocks in the context of our heterogeneous-agent economy.

Proposition 3 (Inflation in D-HANK)**.** *Suppose* $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$*. The path of inflation is given by*

$$\pi_t = \sigma^{-1}\hat{\pi}_t + \chi_p \epsilon_\lambda \hat{\pi}_t + \frac{\mu_b \chi_r}{1-\mu_b}\psi_m \hat{\pi}_t + \kappa e^{\underline{\omega} t}\Omega_0, \tag{18}$$

where $\hat{\pi}_t$ *is given by*

$$\hat{\pi}_t = \frac{1-\mu_b}{1-\mu_b\chi_y}\frac{\kappa(e^{\underline{\omega} t} - e^{-\psi_m t})}{(\underline{\omega}+\psi_m)(\overline{\omega}+\psi_m)}(i_0 - r_n), \tag{19}$$

and satisfies $\frac{\partial \hat{\pi}_t}{\partial i_0} \geq 0$.

Proof. See Appendix B.3. □

Inflation can be analogously decomposed into four terms. The first three terms capture the impact of the ISE, the inside wealth effect and the time-varying risk, while the last term captures the impact of the outside wealth effect. Because $\hat{\pi}_0 = 0$, the first three terms are initially zero. This implies that initial inflation is determined entirely by the outside wealth effect, a consequence of the forward-looking nature of the New Keynesian Phillips curve. Moreover, $\hat{\pi}_t$ is *increasing* in the nominal interest rate. That is, this economy features a *Neo-Fisherian* behavior in the absence of the outside wealth effect, as an increase in interest rates leads to an increase in inflation. This sheds new light on how monetary policy controls inflation: monetary policy is able to reduce inflation by increasing interest rates *only* if it creates a negative net revaluation of households' assets.

3.3 Outside Wealth Effects

We consider next the determination of the outside wealth effect Ω_0. The outside wealth effect depends on the path of output and inflation:

$$\Omega_0 = \int_0^\infty e^{-\rho t} \left[(1-\tau)y_t + T_t + \overline{d}_g(i_t - \pi_t - r_n + r_L p_{d,t})\right] dt - \overline{d}_g \int_0^\infty e^{-(\rho+\psi_d)t}[i_t - r_n + r_L p_{d,t}]dt. \tag{20}$$

But output, inflation and the price of risk in turn depend on the outside wealth effect,

$$y_t = \chi\hat{y}_t + (\overline{\omega} - \delta)e^{\underline{\omega}t}\Omega_0, \qquad \pi_t = \chi\hat{\pi}_t + \kappa e^{\underline{\omega}t}\Omega_0, \tag{21}$$

$$p_{d,t} = \hat{p}_{d,t} + \sigma\left(\frac{1-\mu_b\chi_y}{1-\mu_b}(\rho - \underline{\omega}) - \frac{\mu_b\chi_r}{1-\mu_b}\kappa\right)e^{\underline{\omega}t}\Omega_0, \tag{22}$$

where $\chi \equiv \sigma^{-1} + \chi_p\epsilon_\lambda + \frac{\mu_b\chi_r}{1-\mu_b}\psi_m$, and $\hat{p}_{d,t}$ collect the terms that are a function only of $[i_t]_0^\infty$.

This simultaneity reflects the fact that asset prices react to the level of aggregate demand, as shown by equation (20), and that spending decisions depend on the level of asset prices, as shown by (21). By combining these expressions, we can express Ω_0 in terms of policy variables, that is, the path of nominal interest rates, i_t, and the fiscal backing to the monetary shock, T_t. In particular, we can express Ω_0 as follows:

$$\Omega_0 = \underbrace{(1-\epsilon_\Omega)\Omega_0}_{\text{aggregate demand effect}} + \underbrace{\int_0^\infty e^{-\rho t}\left[T_t + \overline{d}_g(i_t - \chi\hat{\pi}_t - r_n + r_L\hat{p}_{d,t})\right]dt - \overline{d}_g\int_0^\infty e^{-(\rho+\psi_d)t}[i_t - r_n + r_L\hat{p}_{d,t}]dt}_{\text{direct effect}},$$

where ϵ_Ω is a constant defined in the appendix. The first term captures the impact of aggregate demand on the valuation of stocks, bonds, and human wealth, while the second term captures the impact of changes in monetary and fiscal variables that are not mediated by aggregate demand. Assumption 2 guarantees that outside wealth reacts less than one-to-one to aggregate demand.[24]

Assumption 2. *The parameters of the model are such that* $\epsilon_\Omega \in (0, 1)$.

The next proposition shows that the outside wealth effect can be expressed as the product of a *multiplier* and an *autonomous term*, that is, a term that does not depend directly on Ω_0.

[24] Assumption 2 implies that either the primary surplus or the cost of servicing the debt increases with economic activity, as captured by Ω_0. It essentially implies that monetary policy has fiscal consequences.

Proposition 4. *Suppose Assumption 2 holds. The outside wealth effect is then given by*

$$\Omega_0 \;=\; \frac{1}{\epsilon_\Omega}\left[\int_0^\infty e^{-\rho t}\left[T_t + \bar{d}_g(i_t - \chi\hat{\pi}_t - r_n + r_L\hat{p}_{d,t})\right]dt - \bar{d}_g\int_0^\infty e^{-(\rho+\psi_d)t}[i_t - r_n + r_L\hat{p}_{d,t}]dt\right]. \tag{23}$$

Proof. See Appendix B.4. □

Proposition 4 introduces an important relationship between the model-implied revaluation of assets in positive net supply, Ω_0, and the equilibrium path of policy variables. For example, expression (23) shows that, in the absence of any fiscal backing ($T_t = 0$) or government debt ($\bar{d}_g = 0$), the outside wealth effect is zero. Monetary policy still has an effect on the value of stocks and human wealth, as can be seen in (12), but the reduction in the value of households' assets is exactly offset by the reduction in the value of households' liabilities (in the form of consumption), as discussed in Section 2.3. Under Assumption 2, the aggregate demand effect cannot sustain a positive value of Ω_0 in the absence of a direct effect of policy variables.

By incorporating fiscal data into the analysis, this relationship provides a way to discipline the model's economic forces. One can estimate the fiscal response to a monetary shock in the data and introduce the estimated values into expression (23) to obtain the model's prediction for Ω_0.

3.4 Implementability condition

The results in (16) and (18) express output and inflation in terms of the path of nominal interest rates and of the outside wealth effect Ω_0, while equation (23) gives Ω_0 in terms of the underlying fiscal backing T_t. In combination, these results demonstrate how the policy variables (i_t, T_t) affect output and inflation. However, both the nominal interest rate and the associated fiscal backing are endogeneous variables and depend on the monetary policy rule (5). The next proposition shows how the monetary authority can implement a desired equilibrium path of nominal interest rates and fiscal backing by appropriately choosing the exogenous process for the monetary shock u_t.

Proposition 5 (Implementability)**.** *Let y_t be given by (16) and π_t be given by (18), for a given path of nominal interest rates $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, where $\psi_m \neq -\underline{\omega}$, and the associated fiscal backing T_t. Let $[\bar{i}_t, \bar{y}_t, \bar{\pi}_t]_0^\infty$ denote the (bounded) solution to the system comprising the Taylor rule (5), the aggregate Euler equation (8), and the New Keynesian Phillips curve (9), and suppose the monetary shock u_t is given by*

$$u_t = \nu e^{-\psi_m t}(i_0 - r_n) + \theta e^{\underline{\omega} t}. \tag{24}$$

Then, there exists parameters ν and θ such that $\bar{i}_t = i_t$, $\bar{y}_t = y_t$, and $\bar{\pi}_t = \pi_t$.

Proof. See Appendix B.5. □

Proposition 5 shows that the process for the monetary shock uniquely pins down (i_t, T_t), so one can equivalently express the solution either in terms of equilibrium policy variable or in terms of the underlying process for the u_t. The formulation in equation (24) generalizes the process for monetary shocks frequently used in the literature, where the parameter θ is usually set to zero. While ν simply scales the shock such that the initial nominal interest rate equals a given i_0, θ pins down the outside wealth effect Ω_0 and the underlying fiscal backing. An important feature of specification (24) is that the path of the nominal interest rate is the same for any value of θ, so the parameter θ affects only Ω_0.[25]

The extra degree of freedom given by the parameter θ will be important to discipline the outside wealth effect empirically. If we impose $\theta = 0$, we obtain the standard process $u_t = e^{-\psi_m t} u_0$ for some innovation u_0. In this case, the corresponding fiscal backing is given by

$$\int_0^\infty e^{-\rho t} T_t dt = -\frac{\tau}{(\overline{\omega} + \psi_m)(\underline{\omega} + \psi_m)} \frac{1-\mu_b}{1-\mu_b \chi_y} \left(\sigma^{-1} + \chi_p \epsilon_\lambda + \frac{\mu_b \chi_r}{1-\mu_b} \psi_m \right) (i_0 - r_n),$$

where we set $\overline{d}_g = 0$ for simplicity. This particular value may be inconsistent with its empirical counterpart, which implies that the outside wealth effect implied by the model will also be counterfactual. By considering the generalized process (24), the model will be able to simultaneously match the persistence of the equilibrium interest rate and the corresponding fiscal backing.

4 The Quantitative Importance of Wealth Effects

In this section, we study the quantitative importance of wealth effects in the transmission of monetary shocks. We calibrate the model to match key unconditional and conditional moments, including of asset-pricing dynamics and the fiscal response to a monetary shock. We find that household heterogeneity and time-varying risk (rather than its steady-state level) are the predominant channels of transmission of monetary policy. Notably, time-varying risk and household heterogeneity interact, both in amplifying the response of output to a monetary shock and in improving the microeconomic predictions of the model.

4.1 Calibration

The parameter values are chosen as follows. The discount rate of savers is chosen to match a natural interest rate of $r_n = 1\%$. We assume a Frisch elasticity of one, $\phi = 1$, and set the elasticity of substitution between intermediate goods to $\epsilon = 6$, common values adopted in the literature. The fraction of borrowers is set to $\mu_b = 30\%$, and the parameter $\overline{d}_p$ is chosen to match a household debt-to-disposable income ratio of 1 (consistent with the U.S. Financial Accounts). The parameter $\overline{d}_g$ is chosen to match a public debt-to-GDP ratio of 66%, and we assume a duration of five years, consistent with the historical average for the United States. The tax rate is set to $\tau = 0.27$ and the

[25]This can be seen by noting that the sign of the effect of a monetary shock on nominal interest rates depends on the persistence of the shock (see chapter 3 of Galí 2015). If $\psi_m < |\underline{\omega}|$, a contractionary shock increases nominal rates, while it reduces nominal rates if $\psi_m > |\underline{\omega}|$. If $\psi_m = |\underline{\omega}|$, then the nominal interest rate do not react to a monetary shock.

parameter $T_b'(Y)$ is chosen such that $\chi_y = 1$, which requires countercyclical transfers to balance the procyclical wage income. A value of $\chi_y = 1$ is consistent with the evidence in Cloyne et al. (2020) that the net income of mortgagors and non-mortgagors reacts similarly to monetary shocks. The pricing cost parameter φ is chosen such that κ coincides with its corresponding value under Calvo pricing and an average period between price adjustments of three quarters. The half-life of the monetary shock is set to three and a half months to roughly match what we estimate in the data.

We calibrate the disaster risk parameters in two steps. For the stationary equilibrium, we adopt a calibration mostly based on the parameters adopted by Barro (2009). We choose λ (the steady-state disaster intensity) to match an annual disaster probability of 1.7%, and A^* to match a drop in output of $1 - \frac{Y}{Y^*} = 0.39$.[26] The risk-aversion coefficient is set to $\sigma = 4$, a value within the range of reasonable values according to Mehra and Prescott (1985), but is substantially larger than $\sigma = 1$, a value often adopted in macroeconomic models. Note that the equity premium in the stationary equilibrium is

$$\frac{\Pi}{Q} + \frac{\mathbb{E}_t[dQ]}{Qdt} - r_n = \lambda \left[\left(\frac{C_s}{C_s^*} \right)^{\sigma} - 1 \right] \frac{Q - Q^*}{Q},$$

where Q and Q^* are the value of intermediate-good firms in the no-disaster and disaster states, respectively. Our calibration implies an equity premium in the stationary equilibrium of 6.1%, in line with the observed equity premium of 6.5%. This suggests that the model is able to match movements in marginal utility caused by the rare disaster when $\sigma = 4$. Moreover, by setting $\sigma = 4$ we obtain a micro EIS of $\sigma^{-1} = 0.25$, in the ballpark of an EIS of 0.1 as recently estimated by Best et al. (2020). We discuss the calibration of ϵ_λ, which determines the elasticity of asset prices to monetary shocks, in the next subsection.

For the policy variables, we estimate a standard VAR augmented to incorporate fiscal variables, and compute empirical IRFs applying the recursiveness assumption of Christiano et al. (1999). From the estimation we obtain the path of monetary and fiscal variables: the path of the nominal interest rate, the change in the initial value of government bonds, and the path of fiscal transfers. We provide the details of the estimation in Appendix C. Figure 1 shows the dynamics of fiscal variables in the estimated VAR in response to a contractionary monetary shock. Government revenues fall in response to the contractionary shock, while government expenditures fall on impact and then turn positive, likely driven by the automatic stabilizer mechanisms embedded in the government accounts. The present value of interest payments increases by 69 bps and the initial valye of government debt drops by 50 bps.[27] In contrast, the present value of transfers T_t,

[26] As discussed in Barro (2006), it is not appropriate to calibrate A^*/A to the average magnitude of a disaster, given that empirically the size of a disaster is stochastic. We instead calibrate A^*/A to match $\mathbb{E}[(C_s/C_s^*)^\sigma]$ using the empirical distribution of disasters reported in Barro (2009).

[27] The present discounted value of interest payments is calculated as $\sum_{t=0}^{\mathcal{T}} \left(\frac{1-\lambda}{1+\rho_s}\right)^{\frac{t}{4}} \left[\overline{d}_t^g (\hat{i}_{L,t} - \hat{\pi}_t)\right]$, where $\mathcal{T}$ is the truncation period, $\hat{i}_{L,t}$ is the IRF of the 5-year rate estimated in the data, and $\hat{\pi}_t$ is the IRF of inflation. We choose $\mathcal{T} = 60$ quarters, when the main macroeconomic variables, including government debt, are back to their pre-shock values. Other present value calculations follow a similar logic.

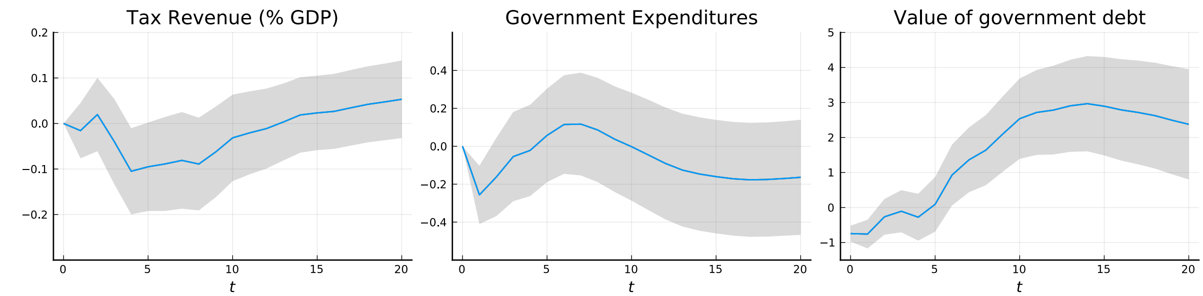

Figure 1: Estimated fiscal response to a monetary policy shock

Note: IRFs computed from a VAR identified by a recursiveness assumption, as in Christiano et al. (1999). Variables included: real GDP per capita, CPI inflation, real consumption per capita, real investment per capita, capacity utilization, hours worked per capita, real wages, tax revenues over GDP, government expenditures per capita, federal funds rate, 5-year constant maturity rate and the real value of government debt per capita. We estimate a four-lag VAR using quarterly data for the period 1962:1-2007:3. The real value of government debt and the 5-year rate are ordered last, and the fed funds rate is ordered third to last. Gray areas are bootstrapped 95% confidence bands. See Appendix C for the details.

drops by 12 bps.[28] Moreover, we cannot, at the 95% confidence level, reject the possibility that the present discounted value of the primary surplus does not change in response to monetary shocks and that the increase in interest payments is entirely compensated by the initial reaction in the value of government bonds.

4.2 Asset-pricing implications of time-varying risk

Recall that the price of a long-term government bond is given by

$$q_{L,0} = -\int_0^\infty e^{-(\rho+\psi_d)t}(i_t - r_n)dt - \int_0^\infty e^{-(\rho+\psi_d)t} r_L(\sigma c_{s,t} + \epsilon_\lambda(i_t - r_n))dt,$$

where $p_{d,t} = \sigma c_{s,t} + \epsilon_\lambda(i_t - r_n)$ is the price of the disaster risk. We use this expression and calibrate ϵ_λ to match the initial response of the 5-year yield on government bonds. Consistent with Gertler and Karadi (2015) and our own estimates reported in Appendix C, we find that a 100 bps increase in the nominal interest rate leads to an increase in the 5-year yield of roughly 20 bps. This procedure leads to a calibration of ϵ_λ of 2.25, which implies an annual increase in the probability of disaster of roughly 0.55% after a 100 bps increase in the nominal interest rate. Figure 2 shows the response of the yield on the long bond and the contributions of the path of future interest rates and the term premium. We find that the bulk of the reaction of the 5-year yield reflects movements in the term premium, a finding that is consistent with the evidence.

The model is also able to capture the responses of other asset prices, which are not directly targeted in the calibration. We consider first the response of the *corporate spread*, the difference

[28]In the data, expenditures also include the response of government consumption and investment. When run separately, however, we cannot reject the possibility that the sum of these two components is equal to zero in response to monetary shocks.

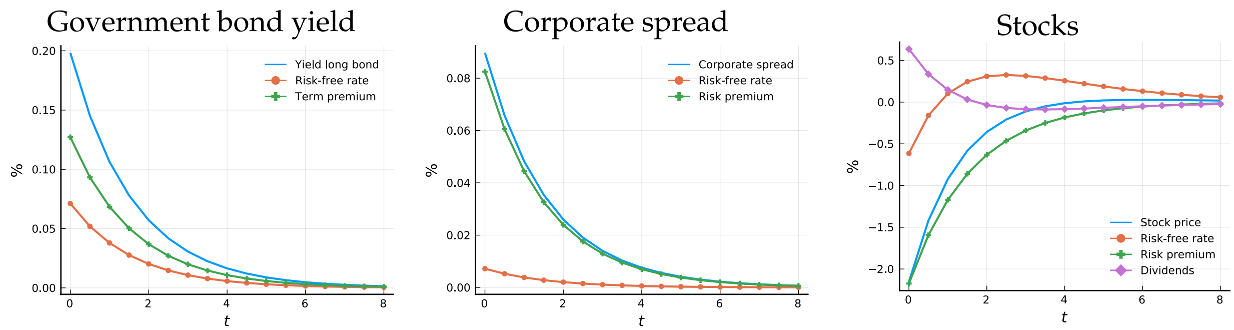

Figure 2: Asset-pricing response to monetary shocks with time-varying risk.

between the yield on a corporate bond and the yield on a government bond (without risk of default) with the same promised cash flow. This corresponds to the way in which the GZ spread is computed in the data by Gilchrist and Zakrajšek (2012). Let $e^{-\psi_c t}$ denote the coupon paid by the corporate bond. We assume that the monetary shock is too small to trigger a corporate default, but the corporate bond defaults if a disaster occurs, where lenders recover the amount $1 - \zeta_c$ in case of default. We calibrated ψ_c and ζ_c to match a duration of 6.5 years and a credit spread of 200 bps in the stationary equilibrium, which is consistent with the estimates reported by Gilchrist and Zakrajšek (2012). Note that the calibration targets the *unconditional* level of the credit spread. We evaluate the model on its ability to generate an empirically plausible *conditional* response to monetary shocks.

The price of the corporate bond can be computed analogously to the computation of the long-term government bond

$$q_{C,0} = -\int_0^\infty e^{-(\rho+\psi_c)t}(i_t - r_n)dt - \int_0^\infty e^{-(\rho+\psi_c)t}\left[\lambda\left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_C - Q_C^*}{Q_C}(\sigma c_{s,t} + \epsilon_\lambda(i_t - r_n))\right]dt,$$

where Q_C and Q_C^* denote the price of the corporate bond in the stationary equilibrium in the no-disaster and disaster states, respectively. Given the price of the corporate bond, we can compute the corporate spread. Figure 2 shows that the corporate spread responds to monetary shocks by 8.9 bps. We introduce the excess bond premium (EBP) in our VAR and find an increase in the EBP of 6.5 bps and an upper bound of the confidence interval of 10.9 bps, consistent with the prediction of the model. Thus, even though this was not a targeted moment, time-varying risk is able to produce quantitatively plausible movements in the corporate spread.

Another moment that is not targeted by the calibration is the response of stocks to monetary shocks. We find a substantial response of stocks to changes in interest rates, which is explained mostly by movements in the risk premium. In contrast to the empirical evidence, we find a *positive* response of dividends to a contractionary monetary shock. This is the result of the well-known feature of sticky-prices models that profits are strongly countercyclical. This counterfactual prediction could be easily solved by introducing some form of wage stickiness. Despite the positive

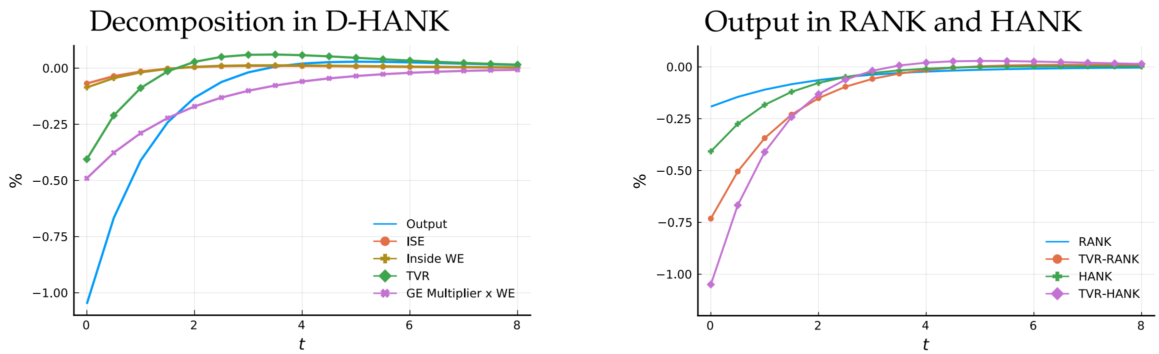

Figure 3: Output in RANK and HANK.

Note: In both plots, the path of the nominal interest rate is given by $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, where $i_0 - r_n$ equals 100 bps, and the fiscal backing corresponds to the value estimated in Section 4.1.

response of dividends, the model generates a decline in stocks of 2.15% in response to a 100 bps increase in interest rates, which is smaller than the point estimate of Bernanke and Kuttner (2005), but is still within their confidence interval.[29] Fixing the degree of countercyclicality of profits would likely bring the response of stocks closer to their point estimate.

4.3 Wealth effects in the monetary policy transmission mechanism

Figure 3 (left) presents the response of output and its components to a monetary shock in the New Keynesian model with heterogeneous agents and time-varying risk. We find that output reacts by -1.05% to a 100 bp increase in the nominal interest rate, which is consistent with the empirical estimates of e.g. Miranda-Agrippino and Ricco (2020). In terms of its components, time-varying risk (TVR) and the outside wealth effect are the two main components determining the output dynamics, representing 39% and 47% of the output response, respectively. In contrast, the ISE accounts for only 6.5% of the output response, indicating that intertemporal substitution plays only a minor role in the monetary transmission mechanism.

These findings stand in sharp contrast to the dynamics in the absence of heterogeneity and time-varying risk. Figure 3 (right) plots the response of output for different combinations of heterogeneity ($\mu_b > 0$ and $\mu_b = 0$) and time-varying risk ($\epsilon_\lambda > 0$ and $\epsilon_\lambda = 0$). By shutting down the two channels, denoted by "RANK" in the figure, the initial response of output would be only -0.14%, a more than sevenfold reduction in the impact of monetary policy. There are two reasons for this result. First, our calibration of $\sigma = 4$ implies an EIS that is one fourth of the standard calibration. This significantly reduces the quantitative importance of the ISE, even if the intertemporal substitution channel represents a large fraction of the output response in the RANK model. Second, our estimate of the fiscal response is substantially lower than the one implied by a stan-

[29] We follow common practice in the asset-pricing literature and report the response of a levered claim on firms' profits, using a debt-to-equity ratio of 0.5, as in Barro (2006).

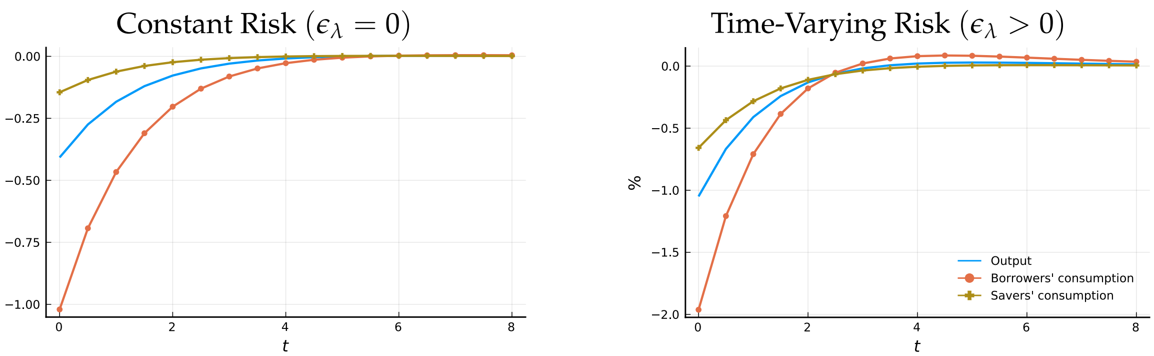

Figure 4: Consumption of borrowers and savers with constant risk and time-varying risk.

Note: In both plots, the path of the nominal interest rate is given by $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, where $i_0 - r_n$ equals 100 bps, and the fiscal backing corresponds to the value estimated in Section 4.1.

dard Taylor equilibrium that imposes an AR(1) process for the monetary shock. We discuss the role of fiscal backing and the implications for the New Keynesian model in Section 4.5 below.

Figure 3 (right) also plots the response of output when there is household heterogeneity but not time-varying risk ("HANK" in the figure), and the response of output when there is time-varying risk but not household heterogeneity ("TVR-RANK" in the figure). We find that heterogeneity increases the response of output by 22 bps while time-varying risk increases it by 54 bps. Notably, by combining both features we get an increase in the response of output of 86 bps, which is 10 bps larger than the sum of the individual effects. Thus, heterogeneity and time varying risk reinforce each other. In terms of the fraction of the response of output that can be attributed to each channel, we find that 20.5% can be attributed to household heterogeneity, 51.5% corresponds to time-varying risk, and 9.7% is the amplification effect of heterogeneity together with time-varying risk, while the remaining contribution represents the channels in the RANK model.

Finally, time-varying risk is important for properly capturing the heterogeneous response to monetary policy. Figure 4 shows that borrowers are disproportionately affected by monetary shocks. The magnitude of the relative response of borrowers and savers, however, is too large in the economy without time-varying risk. The drop in borrowers' consumption is 7 times greater than the decline in savers' consumption with a constant disaster probability, while it is 3 times greater in the economy with time-varying risk. Cloyne et al. (2020) estimate a relative peak response of mortgagors and home-owners of roughly 3.6. Therefore, allowing for time-varying risk is also important if we want to capture the heterogeneous impact of monetary policy.

4.4 The limitations of the constant disaster risk model

We have established that time-varying risk can significantly amplify the effect of monetary policy on output. Here, we show that it is the time-varying component of the disaster risk rather than its level in the stationary equilibrium that matters for the effects of monetary policy. In other words, it

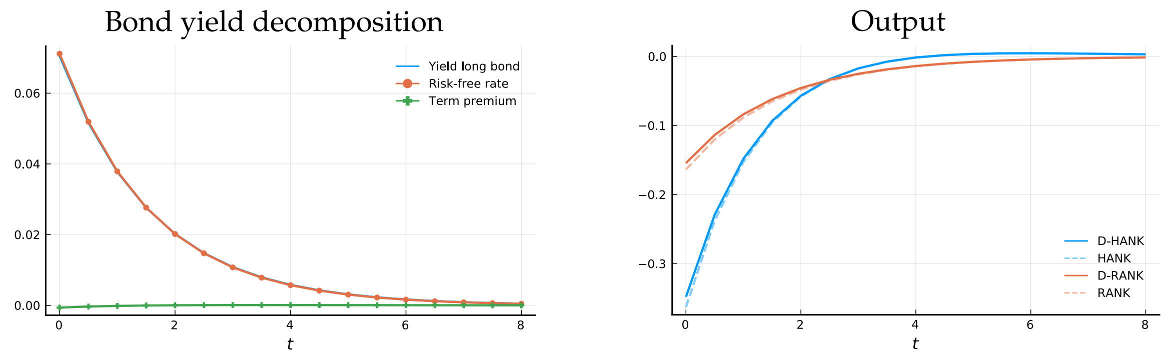

Figure 5: Long-term bond yields and output for economies with and without risk.

is the *conditional* asset-pricing moments rather than the *unconditional* ones that affect the monetary transmission mecanism.

Consider the response of asset prices to monetary shocks. Figure 5 shows that the yield on the long bond increases by 6.5 bps, which implies a decline of the value of the bond of 32 bps (given a 5-year duration), less than half of the response estimated by the VAR in Section 4.1. Moreover, movements in the yield of the long bond are almost entirely explained by the path of nominal interest rates, while the term premium is nearly indistinguishable from zero. This stands in sharp contrast to the evidence reported in Gertler and Karadi (2015) and Hanson and Stein (2015). Similarly, it can be shown that most of the response of stocks in the model is explained by movements in interest rates instead of changes in risk premia, a finding that is inconsistent with the evidence documented in e.g. Bernanke and Kuttner (2005).

Figure 5 enables us to compare the response of output to monetary shocks for the heterogeneous-agent economy with aggregate risk (D-HANK) and without aggregate risk (HANK), and similarly for the representative-agent economy. We find that risk has only a minor impact on the response of output. Aggregate risk increases the value of the discounting parameter δ, which reduces the GE multiplier and dampens the initial impact of the monetary shock. Given that the term premium barely moves, disaster risk plays only a small role in determining the outside wealth effect. In contrast, the role of heterogeneity can be clearly seen by comparing the response of the D-HANK and D-RANK economies.

Therefore, while introducing a constant disaster probability allows the model to capture important *unconditional* asset-pricing moments, such as the (average) risk premium or the upward-sloping yield curve, the model is unable to match key *conditional* moments, in particular, the response of asset prices to monetary policy. The limitations of the model with constant disaster probability in matching conditional asset-pricing moments were recognized early on in the literature, leading to an assessment of the implications of *time-varying disaster risk*, as in Gabaix (2012) and Wachter (2013). This justifies our focus on time-varying disaster risk and how it affects the asset-pricing response to monetary shocks and, ultimately, its impact on real economic variables.

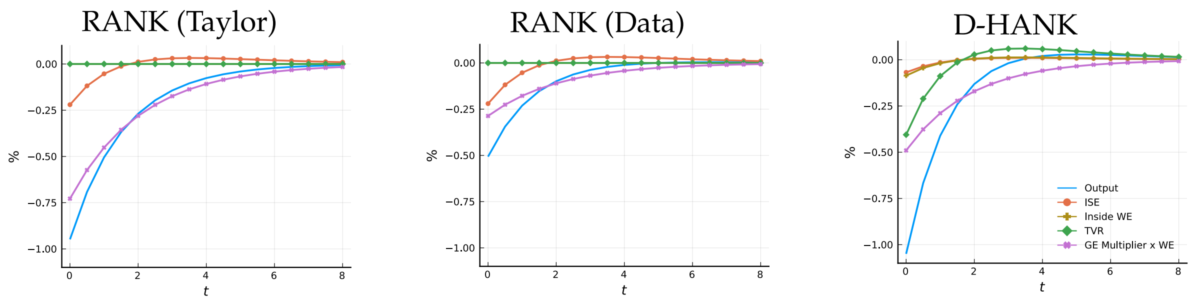

Figure 6: Output in RANK vs D-HANK with time-varying risk.

Note: The first two panels show output in RANK ($\mu_b = \lambda = 0$) with unit EIS ($\sigma^{-1} = 1$). In the left panel, fiscal backing is determined by a Taylor rule, while in the middle panel fiscal backing corresponds to the value estimated in the data. The right panel corresponds to the D-HANK economy with time-varying risk and the estimated fiscal backing.

4.5 The role of fiscal backing

We have found that time-varying risk and heterogeneity substantially amplify the impact of monetary policy on the economy. To properly assess the importance of these two channels, however, it was crucial to control for the implicit fiscal backing, as discussed in Section 3.3. Figure 6 illustrates this point. In the three panels, we show the impact of a monetary shock that leads to an increase in nominal interest rates on impact of 100 bps. In the left panel, we consider a RANK economy ($\mu_b = \lambda = 0$) with the standard value for the EIS ($\sigma^{-1} = 1$) and fiscal backing implicitly determined by a Taylor rule with a monetary shock that follows a standard AR(1) process. In the middle panel, we consider the same economy but the fiscal backing corresponds to the value estimated in the data, which corresponds to a Taylor equilibrium with a monetary shock that follows the more general specification. The right panel shows our D-HANK model with time-varying risk and the calibrated value of the EIS, $\sigma^{-1} = 0.25$. The response of the textbook economy is only slightly smaller than that of our D-HANK economy despite the lack of time-varying risk or heterogeneous agents. The main difference is the value of the implicit fiscal backing, which is almost ten times larger in the textbook economy compared with the one we estimated in the data. The response of output drops by almost half when the fiscal backing is the same as in the data.

Note that the value of the EIS also plays an important role. Even with fiscal backing directly from the data, the response of output is still significant, only slightly less than that in our D-RANK with time-varying risk. But this same response comes from very different channels. In the RANK economy, the ISE accounts for roughly 40% of the output response, while in our D-RANK the ISE accounts for less than 7% of that response. Therefore, the strong response of the ISE in RANK relies to a great extent on having a value of the EIS that is much larger than the empirical estimates, as in Best et al. (2020).

These results suggest that the quantitative success of the RANK model (see, for example, Christiano et al. (2005)) are likely to be the result an implausibly large fiscal backing in response to monetary shocks and a strong intertemporal-substitution channel, which compensate for missing heterogeneous agents and risk channels. Once we discipline the fiscal backing with data and calibrate the EIS to the estimates obtained from microdata, our model suggests that heterogeneous agents and, in particular, time-varying risk are crucial for generating quantitatively plausible output dynamics. It is important to note, however, that our model made several simplifications to incorporate indebted agents and time-varying aggregate risk without sacrificing the tractability of standard macro models. A natural extension would be to incorporate these channel into a medium-sized DSGE model to obtain a better assessment of the quantitative properties of the New Keynesian model.

5 Conclusion

In this paper we provide a novel unified framework in which we analyze the role of heterogeneity and risk in a tractable linearized New Keynesian model. We are able to study the implications of positive private liquidity and heterogeneous MPCs, a combination that has been proven elusive to the analytical HANK literature. Moreover, we capture both important unconditional asset-pricing moments, such as the equity premium and an upward-sloping yield curve, and conditional moments, such as the response of government bonds, corporate bonds, and equities, in response to monetary shocks. Despite its richness, the model can be fully characterized in closed-form.

We show how monetary policy affects the economy through the intertemporal-substitution channel as well as inside and outside wealth effects, and time-varying risk premia. We find that wealth effects play an important role in the transmission of monetary shocks. Quantitatively, we find that time-varying risk explains roughly 50% of the output response, while the presence of private debt accounts for 30% of the response.

The methods introduced in this paper can be applied beyond the current model. For instance, it can be applied to a full quantitative HANK model with idiosyncratic risk, extending the results of Ahn et al. (2018) to allow for time-varying risk premia. Alternatively, one could introduce risky household debt, or a richer capital structure for firms, and study the pass-through of monetary policy to households and firms. These methods may enable us to bridge the gap between the extensive existing work on heterogeneous agents and monetary policy and the emerging literature on the role of asset prices in the transmission of monetary shocks.

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Appendix: For Online Publication

A Derivations for Sections 2 and 3

In this appendix, we provide the derivations of the expressions provided in Section 2 and 3. We consider first the optimality conditions for the non-linear model, then the derivation of the stationary equilibrium, and, finally, the log-linear equilibrium conditions.

A.1 The non-linear model

Households' problem. The household problem is given by

$$V_{j,t}(B_j) = \max_{[C_{j,z},N_{j,z},B^L_{j,z}]_{z\geq t}} \mathbb{E}_t \left[\int_t^{t^*} e^{-\rho_j(z-t)} \left(\frac{C_{j,z}^{1-\sigma}}{1-\sigma} - \frac{N_{j,z}^{1+\phi}}{1+\phi} \right) dz + e^{-\rho_j(t^*-t)} V^*_{j,t^*}(B^*_{j,t^*}) \right], \tag{A.1}$$

subject to the flow budget constraint

$$dB_{j,t} = \left[(i_t - \pi_t) B_{j,t} + r_{L,t} B^L_{j,t} + \frac{W_{j,t}}{P_t} N_{j,t} + \frac{\Pi_{j,t}}{P_t} + \tilde{T}_{j,t} - C_{j,t} \right] dt + B^L_{j,t} \frac{Q^*_{L,t} - Q_{L,t}}{Q_{L,t}} d\mathcal{N}_t, \tag{A.2}$$

and the borrowing constraint and no-negativity constraint for long-term bonds

$$B_{j,t} \geq -\overline{D}_p, \qquad B^L_{j,t} \geq 0, \tag{A.3}$$

given the initial condition $B_{j,t} = B_j \geq -\overline{D}_p$, where $r_{L,t} \equiv \frac{1}{Q_{L,t}} + \frac{\dot{Q}_{L,t}}{Q_{L,t}} - \psi_d - i_t$ is the excess return on long-term bonds conditional on no disasters.[30]

The HJB equation is given by

$$\rho_j V_{j,t} = \max_{C_j,N_j,B^L_j} \left\{ \frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{N_{j,t}^{1+\phi}}{1+\phi} + \frac{\partial V_{j,t}}{\partial B} \left[(i_t - \pi_t) B_{j,t} + r_{L,t} B^L_{j,t} + \frac{W_{j,t}}{P_t} N_{j,t} + \frac{\Pi_{j,t}}{P_t} + \tilde{T}_{j,t} - C_{j,t} \right] + \dot{V}_{j,t} + \lambda_t (V^*_{j,t} - V_{j,t}) \right\}, \tag{A.4}$$

where $V^*_{j,t}$ is evaluated at $B^*_{j,t} = B_{j,t} + B^L_{j,t} \frac{Q^*_{L,t} - Q_{L,t}}{Q_{L,t}}$, the value of household j's wealth after the disaster.

The first-order conditions are given by

$$C_{j,t}^{-\sigma} = \frac{\partial V_{j,t}}{\partial B}, \qquad N_{j,t}^{\phi} = \frac{\partial V_{j,t}}{\partial B} \frac{W_{j,t}}{P_t}, \qquad \frac{\partial V_{j,t}}{\partial B} r_{L,t} \leq \lambda_t \frac{\partial V^*_{j,t}}{\partial B} \frac{Q_{L,t} - Q^*_{L,t}}{Q_{L,t}}. \tag{A.5}$$

The solution is also subject to the state-constraint boundary condition[31]

$$\frac{\partial V_{j,t}(-\overline{D}_p)}{\partial B} \geq \left(-(i_t - \pi_t)\overline{D}_p + \frac{W_{j,t}}{P_t} N_{j,t} + \frac{\Pi_{j,t}}{P_t} + \tilde{T}_{j,t} \right)^{-\sigma}, \tag{A.6}$$

which implies that $\dot{B}_{j,t} \geq 0$ at $B_{j,t} = -\overline{D}_p$, which guarantee that the borrowing constraint is not violated.

[30] Formally, households also face the constraint $B^*_{j,t} \geq -\overline{D}_p$, which effectively imposes an upper bound on $B^L_{j,t}$. Given that this constraint will not be binding in equilibrium, we presented the relaxed problem without this constraint.

[31] See Achdou et al. (2017) for a discussion of state-constraint boundary conditions in the context of continuous-time savings problems with borrowing constraints.

Combining the first-order conditions for consumption and labor, we obtain the labor-supply condition

$$\frac{W_t}{P_t} = N_{j,t}^{\phi} C_{j,t}^{\sigma}, \tag{A.7}$$

which coincides with the expression provided in the main text.

The first-order condition for long-term bonds, expressed as an equality, can be written as

$$r_{L,t} = \lambda_t \frac{(C_t^*)^{-\sigma}}{C_t^{-\sigma}} \frac{Q_{L,t} - Q_{L,t}^*}{Q_{L,t}}, \tag{A.8}$$

using the first-order condition for consumption to eliminate the partial derivatives of the value function.

The envelope condition with respect to $B_{j,t}$ for an unconstrained household is given by

$$\rho_j \frac{\partial V_{j,t}}{\partial B} = (i_t - \pi_t) \frac{\partial V_{j,t}}{\partial B} + \frac{\mathbb{E}_t [d \left(\frac{\partial V_{j,t}}{\partial B} \right)]}{dt}. \tag{A.9}$$

Combining the expression above with the first-order condition for consumption, we obtain

$$\frac{\mathbb{E}_t [dC_{j,t}^{-\sigma}]}{dt} = -(i_t - \pi_t - \rho_j) C_{j,t}^{-\sigma}. \tag{A.10}$$

Expanding the expectation of the marginal utility, we obtain the Euler equation for savers

$$\frac{\dot{C}_{s,t}}{C_{s,t}} = \sigma^{-1}(i_t - \pi_t - \rho_j) + \frac{\lambda_t}{\sigma} \left[\left(\frac{C_{s,t}}{C_{s,t}^*} \right)^{\sigma} - 1 \right], \tag{A.11}$$

which coincides with expression (7) provided in the main text.

Firms' problem. Final goods are produced according to the production function

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\epsilon}{\epsilon - 1}} \right)^{\frac{\epsilon - 1}{\epsilon}}. \tag{A.12}$$

The problem of the final-goods producer is given by

$$\max_{[Y_{i,t}]_{i=0}^1} P_t \left(\int_0^1 Y_{i,t}^{\frac{\epsilon - 1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon - 1}} - \int_0^1 P_{i,t} Y_{i,t} di. \tag{A.13}$$

The demand for variety i and the price level are given by

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t, \qquad P_t = \left(\int_0^1 P_{i,t}^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}. \tag{A.14}$$

The intermediate-goods producers' problem is given by

$$Q_{i,t}(P_i) = \max_{[\pi_{i,s}]_{s \geq t}} \mathbb{E}_t \left[\int_t^{t^*} \frac{\eta_s}{\eta_t} \left((1 - \tau) \frac{P_{i,s}}{P_s} Y_{i,s} - \frac{W_s}{P_s} \frac{Y_{i,s}}{A_s} - \frac{\varphi}{2} \pi_s^2(j) \right) ds + \frac{\eta_{t^*}}{\eta_t} Q_{i,t^*}^*(P_{i,t^*}) \right], \tag{A.15}$$

subject to $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$ and $\dot{P}_{i,t} = \pi_{i,t} P_{i,t}$, given $P_{i,t} = P_i$.

The HJB equation for this problem is

$$0 = \max_{\pi_{i,t}} \eta_t \left((1-\tau) \frac{P_{i,t}}{P_t} Y_{i,t} - \frac{W_t}{P_t} \frac{Y_{i,t}}{A} - \frac{\varphi}{2} \pi_{i,t}^2 \right) dt + \mathbb{E}_t[d(\eta_t Q_{i,t})]. \tag{A.16}$$

Applying Ito's lemma for jump processes, we obtain

$$\mathbb{E}_t[d(\eta_t Q_{i,t})] = \mathbb{E}_t[dQ_{i,t}] + \mathbb{E}_t[d\eta_t] + \lambda_t (\eta_t^* - \eta_t) \left(Q_{i,t}^* - Q_{i,t}\right) dt. \tag{A.17}$$

The drift of $Q_{i,t}$ and η_t are given by

$$\mathbb{E}_t[dQ_{i,t}] = \left[\frac{\partial Q_{i,t}}{\partial t} + \frac{\partial Q_{i,t}}{\partial P_i} \pi_{i,t} P_{i,t} + \lambda_t (Q_{i,t}^* - Q_{i,t}) \right] dt, \qquad \mathbb{E}_t[d\eta_t] = -(i_t - \pi_t)\eta_t dt, \tag{A.18}$$

where we used the fact that the value of firm i is a function of calendar time and the price level, so the drift of $Q_{i,t}$ can be computed using Ito's lemma, and we used the household's Euler equation to obtain the drift of the SDF, $\eta_t = e^{-\rho t} C_{s,t}^{-\sigma}$.

Combining Equations (A.17) and (A.18), the drift of $\eta_t Q_{i,t}$ can be written as

$$\begin{aligned} \frac{\mathbb{E}_t[d(\eta_t Q_{i,t})]}{\eta_t dt} &= \frac{\partial Q_{i,t}}{\partial t} + \frac{\partial Q_{i,t}}{\partial P_i} \pi_{i,t} P_{i,t} + \lambda_t (Q_{i,t}^* - Q_{i,t}) - (i_t - \pi_t)\, Q_{i,t} + \lambda_t \left(\frac{\eta_t^* - \eta_t}{\eta_t} \right) (Q_{i,t}^* - Q_{i,t}) \\ &= \frac{\partial Q_{i,t}}{\partial t} + \frac{\partial Q_{i,t}}{\partial P_i} \pi_{i,t} P_{i,t} - (i_t - \pi_t) Q_{i,t} + \lambda_t \left(\frac{C_{s,t}}{C_s^*} \right)^{\sigma} \left(Q_{i,t}^* - Q_{i,t}\right). \end{aligned} \tag{A.19}$$

Plugging the result above into Equation (A.16), the firms' HJB equation can be written as

$$0 = \max_{\pi_{i,t}} \left((1-\tau) \frac{P_{i,t}}{P_t} - \frac{W_t}{P_t} \frac{1}{A} \right) \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t - \frac{\varphi}{2} \pi_{i,t}^2 + \frac{\partial Q_{i,t}}{\partial t} + \frac{\partial Q_{i,t}}{\partial P_i} \pi_{i,t} P_{i,t} - (i_t - \pi_t) Q_{i,t} + \lambda_t \frac{\eta_t^*}{\eta_t} \left(Q_{i,t}^* - Q_{i,t}\right), \tag{A.20}$$

where $\eta_t^* \equiv e^{-\rho t} (C_{s,t}^*)^{-\sigma}$.

The first-order condition is given by

$$\frac{\partial Q_{i,t}}{\partial P_i} P_{i,t} = \varphi \pi_{i,t}. \tag{A.21}$$

The change in π_t conditional on no disaster is then given by

$$\left(\frac{\partial^2 Q_{i,t}}{\partial t \partial P_i} + \frac{\partial^2 Q_{i,t}}{\partial P_i^2} \pi_{i,t} P_{i,t} \right) P_{i,t} + \frac{\partial Q_{i,t}}{\partial P_i} \pi_{i,t} P_{i,t} = \varphi \dot{\pi}_{i,t}. \tag{A.22}$$

The envelope condition with respect to $P_{i,t}$ is given by

$$\begin{aligned} 0 = &\left((1-\epsilon)(1-\tau) \frac{P_{i,t}}{P_t} + \epsilon \frac{W_t}{P_t A} \right) \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_{i,t}} + \frac{\partial^2 Q_{i,t}}{\partial t \partial P_i} + \frac{\partial^2 Q_{i,t}}{\partial P_i^2} \pi_{i,t} P_{i,t} + \frac{\partial Q_{j,t}}{\partial P_i} \pi_{i,t} - (i_t - \pi_t) \frac{\partial Q_{i,t}}{\partial P_i} \\ &+ \lambda_t \frac{\eta_t^*}{\eta_t} \left(\frac{\partial Q_{i,t}^*}{\partial P_i} - \frac{\partial Q_{i,t}}{\partial P_i} \right). \end{aligned} \tag{A.23}$$

Multiplying the expression by $P_{i,t}$ and using Equations (A.22) and (A.23), we obtain

$$0 = \left((1-\epsilon)(1-\tau)\frac{P_{i,t}}{P_t} + \epsilon\frac{W_t}{P_t A}\right)\left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t + \varphi\dot{\pi}_t - (i_t - \pi_t)\varphi\pi_{i,t} + \lambda_t\frac{\eta_t^*}{\eta_t}\varphi(\pi_{i,t}^* - \pi_{i,t}). \tag{A.24}$$

Rearranging the expression above, we obtain the non-linear New Keynesian Phillips curve

$$\dot{\pi}_t = \left(i_t - \pi_t + \lambda_t\frac{\eta^*}{\eta_t}\right)\pi_t - (\epsilon-1)\varphi^{-1}\left(\frac{\epsilon}{\epsilon-1}\frac{W_t}{P_t}\frac{1}{A} - (1-\tau)\right)Y_t, \tag{A.25}$$

where we have assumed that $P_{i,t} = P_t$ for all $i \in [0,1]$ and that $\pi_t^* = 0$.

A.2 Stationary equilibrium

Introducing recurrent shocks. Suppose aggregate productivity follows the following process:[32]

$$\frac{dA_t}{A_t} = -\zeta d\mathcal{N}_t, \tag{A.26}$$

given $A_0 = A$ and $0 < \zeta < 1$, where $\mathcal{N}_t$ is a Poisson process with arrival rate $\lambda_t = \lambda(i_t - \rho; A_t)$.

This formulation generalizes the one considered in the text. Note that the above process implies that aggregate productivity after a Poisson event is given by $A(1-\zeta) \equiv A^*$. The setting discussed in the main text corresponds to the case where $\lambda(i_t - \rho; A) > 0$ and $\lambda(i_t - \rho; A_t) = 0$ for $A_t < A$. This formulation also captures the case of recurrent shocks assumed by Barro (2009), where λ_t is constant and positive for all t.

A.2.1 Consumption and natural interest rate

Consumption determination in a stationary equilibrium. In a stationary equilibrium, all variables are independent of calendar time and depend only on aggregate productivity A_t, so they are constant between the realization of disasters. We can then write, for instance, consumption and output as $C_{j,t} = \mathbf{C}_j(A_t)$ and $Y_t = \mathbf{Y}(A_t)$, where $\mathbf{C}_j(\cdot)$ and $\mathbf{Y}(\cdot)$ are functions we need to solve for.

First, note that, imposing $\dot{C}_{s,t} = 0$ in the Euler equation (1), we obtain the natural rate $\mathbf{r}_n(A)$

$$\mathbf{r}_n(A) = \rho_s - \lambda(A)\left[\left(\frac{\mathbf{C}_s(A)}{\mathbf{C}_s(A(1-\zeta))}\right)^{\sigma} - 1\right], \tag{A.27}$$

where we abuse notation and write $\lambda(A)$ instead $\lambda(0; A)$ in a stationary equilibrium.

The consumption of borrowers satisfies the condition

$$\mathbf{C}_b(A) = \left[A(1-\tau)(1-\epsilon^{-1})\right]^{\frac{1+\phi}{\phi}}\mathbf{C}_b^{-\frac{\sigma}{\phi}}(A) + \mathbf{T}_b(A) - \mathbf{r}_n(A)\overline{D}_p, \tag{A.28}$$

where $\mathbf{T}_b(A)$ represents the level of transfers as a function of productivity, and we used the labor supply condition to solve for N_b and the fact that the real wage is given by

$$\frac{W_t}{P_t} = A_t(1-\tau)(1-\epsilon^{-1}), \tag{A.29}$$

[32] The process can be easily generalized to allow for trend growth, $dA_t = gA_t dt - \zeta A_t d\mathcal{N}_t$. The expressions in the text apply to this case as well after all variables are properly detrended.

obtained by imposing $\pi_t = 0$ in the non-linear New Keynesian Phillips curve (4).

The consumption of savers satisfies the condition

$$\mathbf{C}_s(A) = \left[A(1-\tau)(1-\epsilon^{-1})\right]^{\frac{1+\phi}{\phi}} \mathbf{C}_s^{-\frac{\sigma}{\phi}}(A) + \frac{1-(1-\tau)(1-\epsilon^{-1})}{1-\mu_b}\mathbf{Y}(A) - \frac{\mu_b \mathbf{T}_b(A)}{1-\mu_b} + \frac{\mathbf{r}_n(A)\mu_b}{1-\mu_b}\overline{D}_p, \quad \text{(A.30)}$$

where used the government's flow budget constraint to eliminate $\tilde{T}_{s,t}$, the market clearing condition for bonds to write $B_s = \frac{\overline{D}_g + \mu_b \overline{D}_p}{1-\mu_b}$ and the fact that profits received by savers can be written as

$$\Pi_{s,t} = (1-\tau)\frac{1-(1-\epsilon^{-1})}{1-\mu_b}. \quad \text{(A.31)}$$

Given that $\mathbf{Y}(A) = \sum_{j\in\{b,s\}} \mu_j \mathbf{C}_j(A)$, the above conditions provide a pair of functional equations that determine $\mathbf{C}_j(A)$. To ease notation, we write $C_j \equiv \mathbf{C}_j(A)$ and $C_j^* \equiv \mathbf{C}_j(A(1-\zeta))$ to denote variables in the no-disaster and disaster states, respectively.

Symmetric stationary equilibrium. Note that, using $\mu_b = 0$ and $\mathbf{C}_s = \mathbf{Y}$ in the expression for $\mathbf{C}_s$, we can solve for output in a representative-agent economy:

$$\overline{Y}(A) = A^{\frac{1+\phi}{\sigma+\phi}} \left[(1-\tau)(1-\epsilon^{-1})\right]^{\frac{1}{\sigma+\phi}}. \quad \text{(A.32)}$$

We obtain the output level $\overline{Y}(A)$ in the economy with $\mu_b > 0$ if $\mathbf{T}_b(A)$ satisfies

$$\mathbf{T}_b(A) = \left[1-(1-\tau)(1-\epsilon^{-1})\right]\overline{Y}(A) + \left(\rho_s - \lambda(A)\left[\left(\frac{\overline{Y}(A)}{\overline{Y}(A(1-\zeta))}\right)^{\sigma} - 1\right]\right)\overline{D}_p. \quad \text{(A.33)}$$

Plugging the value of $\mathbf{T}_b(A)$ into the expression for $\mathbf{C}_j(A)$, we obtain

$$\begin{aligned}\mathbf{C}_b(A) = {} & \left[A(1-\tau)(1-\epsilon^{-1})\right]^{\frac{1+\phi}{\phi}} \mathbf{C}_b^{-\frac{\sigma}{\phi}}(A) + [1-(1-\tau)(1-\epsilon^{-1})]\sum_{j\in\{b,s\}} \mu_j \mathbf{C}_j(A) \\ & + \lambda(A)\overline{D}_p\left[\left(\frac{\mathbf{C}_s(A)}{\mathbf{C}_s(A(1-\zeta))}\right)^{\sigma} - \left(\frac{\overline{Y}(A)}{\overline{Y}(A(1-\zeta))}\right)^{\sigma}\right], \quad \text{(A.34)} \\ \mathbf{C}_s(A) = {} & \left[A(1-\tau)(1-\epsilon^{-1})\right]^{\frac{1+\phi}{\phi}} \mathbf{C}_s^{-\frac{\sigma}{\phi}}(A) + [1-(1-\tau)(1-\epsilon^{-1})]\sum_{j\in\{b,s\}} \mu_j \mathbf{C}_j(A) \\ & - \lambda(A)\frac{\mu_b \overline{D}_p}{1-\mu_b}\left[\left(\frac{\mathbf{C}_s(A)}{\mathbf{C}_s(A(1-\zeta))}\right)^{\sigma} - \left(\frac{\overline{Y}(A)}{\overline{Y}(A(1-\zeta))}\right)^{\sigma}\right]. \quad \text{(A.35)}\end{aligned}$$

A solution to the above system is $\mathbf{C}_j(A) = \overline{Y}(A)$. In this case, we obtain a symmetric stationary equilibrium, where consumption in both households is the same. Moreover, consumption is a power function of productivity, which implies that C_s^* is given by

$$C_s^* = (1-\zeta)^{\frac{1+\phi}{\sigma+\phi}} C_s. \quad \text{(A.36)}$$

Therefore, in a symmetric stationary equilibrium, the real interest rate is given by

$$\mathbf{r}_n(A) = \rho_s - \lambda(A)\left[(1-\zeta)^{-\sigma\frac{1+\phi}{\sigma+\phi}} - 1\right]. \tag{A.37}$$

Note that this result holds for both the case of non-recurrent shocks, where $\lambda(A) > 0$ and $\lambda(A^*) = 0$, and the case of recurrent shocks, where $\lambda(A)$ is independent of productivity level A.

Role of private debt. In a symmetric stationary equilibrium, the government effectively taxes all of the income savers receive by lending to borrowers, so the natural interest rate is independent of the level of private debt. To allow a role for private debt in determining the natural rate, we assume that transfers to borrowers satisfy the condition

$$\mathbf{T}_b(A^*) = \left[1-(1-\tau)(1-\epsilon^{-1})\right]\overline{Y}(A^*), \tag{A.38}$$

and transfers to borrowers are given by (A.33) if $A_t \neq A^*$. This provides the minimal deviation from the symmetric stationary equilibrium that allows a role for private debt in determining the natural rate. For simplicity, we focus on the case of non-recurrent shocks.

Proposition 6. *Suppose $\boldsymbol{T}_b(A^*)$ is given by (A.38) and the shocks are non-recurrent. Let $C_s = \boldsymbol{C}_s(A)$ and $C_s^* = \boldsymbol{C}_s(A^*)$, where $A^* = A(1-\zeta)$. Then, the natural interest rate $r_n = r_n(A)$ is given by*

$$r_n = \rho_s - \lambda\left[\left(\frac{C_s}{C_s^*}\right)^{\sigma} - 1\right], \tag{A.39}$$

and it is strictly increasing in $\overline{D}_p$.

Proof. Consider first the determination of (C_s^*, C_b^*). The condition for C_b^* is given by

$$0 = \left[A^*(1-\tau)(1-\epsilon^{-1})\right]^{\frac{1+\phi}{\phi}}(C_b^*)^{-\frac{\sigma}{\phi}} + \left[1-(1-\tau)(1-\epsilon^{-1})\right]\overline{Y}(A^*) - r_n^*\overline{D}_p - C_b^*, \tag{A.40}$$

where $r_n^* = \rho_s$ in the case of non-recurrent shocks. .

The right-hand side of the above expression is positive for sufficiently small C_b^*, it is negative for sufficiently large C_b^*, and it is strictly decreasing in C_b^*. Therefore, there is a unique solution to the above non-linear equation, given the value of $\overline{D}_p$, which we denote by $C_b^*(\overline{D}_p)$. Applying the implicit function theorem to the above expression, we can show that $C_b^*(\overline{D}_p)$ is strictly decreasing in $\overline{D}_p$:

$$\frac{\partial C_b^*(\overline{D}_p)}{\partial \overline{D}_p} = -\frac{r_n^*}{1+\frac{\sigma}{\phi}\left[A^*(1-\tau)(1-\epsilon^{-1})\right]^{\frac{1+\phi}{\phi}}(C_b^*)^{-\frac{\sigma+\phi}{\phi}}} < 0. \tag{A.41}$$

Savers' consumption is determined by $g(C_s^*) = 0$, where $g(C_s^*)$ is given by

$$g(C_s^*) = \left[A^*(1-\tau)(1-\epsilon^{-1})\right]^{\frac{1+\phi}{\phi}}(C_s^*)^{-\frac{\sigma}{\phi}} - (1-\tau)(1-\epsilon^{-1})C_s^* + \mu_b\frac{(1-(1-\tau)(1-\epsilon^{-1}))(C_b^*(\overline{D}_p)-\overline{Y}(A^*)) + r_n^*\overline{D}_p}{1-\mu_b}. \tag{A.42}$$

Note that $g(\cdot)$ is strictly decreasing, and it approaches infinity as $C_s^* \to 0$ and approaches $-\infty$ as $C_s^* \to \infty$. The continuity of $g(\cdot)$ then implies that a solution $C_s^*(\overline{D}_p)$ exists and is unique.

Applying the implicit function theorem, we obtain

$$\frac{\partial C_s^*}{\partial \overline{D}_p} = \frac{1}{\Delta_C} \frac{\mu_b r_n^*}{1-\mu_b} \left[1 - \frac{1-(1-\tau)(1-\epsilon^{-1})}{1+\frac{\sigma}{\phi}\left[A^*(1-\tau)(1-\epsilon^{-1})\right]^{\frac{1+\phi}{\phi}} (C_b^*)^{-\frac{\sigma+\phi}{\phi}}} \right] > 0, \tag{A.43}$$

where $\Delta_C \equiv (1-\tau)(1-\epsilon^{-1}) + \frac{\sigma}{\phi}\left[A^*(1-\tau)(1-\epsilon^{-1})\right]^{\frac{1+\phi}{\phi}} (C_b^*)^{-\frac{\sigma+\phi}{\phi}}$, where we used Equation (A.41).

Given that the solution coincides with $\overline{Y}(A^*)$ if $\overline{D}_p = 0$, we conclude that $C_s^*(\overline{D}_p) > \overline{Y}(A^*)$ for $\overline{D}_p > 0$. Since $C_s = \overline{Y}(A)$, r_n is increasing in $\overline{D}_p$ and it is larger than in the symmetric stationary equilibrium. □

A.2.2 Risk premia

Equity premium. The value of the intermediate-goods firms satisfies the standard pricing condition

$$Q_t = \mathbb{E}_t \left[\int_t^\infty \frac{\eta_s}{\eta_t} \Pi_s ds \right], \tag{A.44}$$

where $\Pi_t = (1-\tau)Y_t - \frac{W_t}{P_t} N_t$, which represents the sum of the profit of the intermediate-goods producer and the rebate to households of the cost of adjusting prices.

We can write the condition above in recursive form:

$$\frac{\Pi_t}{Q_t} dt + \frac{\mathbb{E}_t[d(\eta_t Q_t)]}{\eta_t Q_t} = 0. \tag{A.45}$$

Applying Ito's lemma for jumping processes, we get an expression for the expected return on the firm:

$$\frac{\Pi_t}{Q_t} dt + \frac{\mathbb{E}_t[dQ_t]}{Q_t} = \left[\underbrace{(i_t - \pi_t)}_{\text{interest rate}} + \underbrace{\lambda_t \frac{(C_{s,t}^*)^{-\sigma} - C_{s,t}^{-\sigma}}{C_{s,t}^{-\sigma}} \frac{Q_t - Q_t^*}{Q_t}}_{\text{risk premium}} \right] dt. \tag{A.46}$$

In a stationary equilibrium, profits are given by $\Pi_t = \epsilon^{-1}(1-\tau)Y_t$ and the value of the firm is given by $Q_t = \mathbf{Q}(A_t)$. We can write the dividend-yield as follows:

$$\frac{\Pi_t}{Q_t} = r_{n,t} + \lambda_t \left(\frac{C_{s,t}}{C_{s,t}^*} \right)^\sigma \left(1 - \frac{\Pi_t}{Q_t} \frac{Q_t^*}{\Pi_t^*} \frac{\Pi_t^*}{\Pi_t} \right). \tag{A.47}$$

Rearranging the above expression, we obtain

$$\frac{\Pi_t}{Q_t} = \frac{\rho_s + \lambda_t}{1 + \lambda_t \frac{C_{s,t}}{C_{s,t}^*}^\sigma \frac{Q_t^*}{\Pi_t^*} \frac{\Pi_t^*}{\Pi_t}}, \tag{A.48}$$

using the fact that $r_{n,t} + \lambda_t \left(\frac{C_{s,t}}{C_{s,t}^*} \right)^\sigma = \rho_s + \lambda_t$

We will consider the value of Π_t / Q_t under both non-recurrent and recurrent shocks. Consider first the case where $\lambda_t > 0$ in the no-disaster state and $\lambda_t = 0$ in the disaster state. Therefore, $\Pi_t^* = \rho_s Q_t^*$ and Π_t / Q_t

can be written as

$$\frac{\Pi_t}{Q_t} = \frac{\rho_s + \lambda_t}{1 + \lambda_t \frac{C_{s,t}}{C^*_{s,t}}^{\sigma} \frac{(1-\zeta_Y)}{\rho_s}}, \tag{A.49}$$

where $1 - \zeta_Y \equiv \frac{Y^*}{Y}$.

The (unlevered) equity premium can then be written as

$$\frac{\Pi}{Q}dt + \lambda \frac{Q^* - Q}{Q} - r_n = \lambda \left((1-\zeta_Y)^{-\sigma} - 1\right) \left(1 - \frac{(\rho_s + \lambda_t)(1-\zeta_Y)}{\rho_s + \lambda_t \frac{C_{s,t}}{C^*_{s,t}}^{\sigma}}\right), \tag{A.50}$$

using the fact that $\Pi^*_t / \Pi_t = 1 - \zeta_Y$.

Consider the case of recurrent shocks, such that $\lambda_t = \lambda$ for all $t \geq 0$. The dividend-yield is then given by

$$\frac{\Pi_t}{Q_t} = r_n + \lambda(1-\zeta_y)^{-\sigma}\zeta_Y, \tag{A.51}$$

using the fact that $\frac{\Pi}{Q} = \frac{\Pi^*}{Q^*}$.

In this case, the equity premium is given by

$$\frac{\Pi}{Q}dt + \lambda \frac{Q^* - Q}{Q} - r_n = \lambda \left((1-\zeta_Y)^{-\sigma} - 1\right)\zeta_Y, \tag{A.52}$$

which coincides with the expression for the risk premium in Barro (2009).

Term spread. From the first-order condition (A.8), the excess return on the long-term bonds satisfies

$$\frac{1}{Q_{L,t}} + \frac{\dot{Q}_{L,t}}{Q_{L,t}} - \psi_d - i_t = \lambda_t \left(\frac{C_t}{C^*_t}\right)^{\sigma} \frac{Q_{L,t} - Q^*_{L,t}}{Q_{L,t}}. \tag{A.53}$$

Let $i_{L,t}$ denote the yield on the long-term bond, then $i_{L,t}$ satisfies

$$Q_{L,t} = \int_t^{\infty} e^{-(i_L + \psi_d)(s-t)} ds \Rightarrow \frac{1}{Q_{L,t}} = i_{L,t} + \psi_d. \tag{A.54}$$

We consider next a stationary equilibrium with non-recurrent shocks. Combining the previous two expressions, we obtain that the term spread, the difference between the long and short interest rate, is given by

$$i_L - r_n = \lambda \left(\frac{C_s}{C^*_s}\right)^{\sigma} \frac{Q_L - Q^*_L}{Q_L}. \tag{A.55}$$

The price of the long-term bond in the disaster state is given by

$$Q^*_L = \frac{1}{i^*_L + \psi_d}, \tag{A.56}$$

where $i^*_L = r^*_n$ is the yield on the long-term bond.

We can then express the term spread as follows

$$i_L - r_n = \lambda \left(\frac{C_s}{C^*_s}\right)^{\sigma} \frac{i^*_L - i_L}{i^*_L + \psi_d}. \tag{A.57}$$

Rearranging the above expression, we obtain

$$i_L - r_n = \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{r_n^* - r_n}{r_n^* + \psi_d + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma}}.$$

Note that $i_L > r_n$ and the difference is decreasing in ψ_d, so the yield increases with the bond duration. The term spread coincides with the expected excess return on the bond conditional on no disaster. The (unconditional) expected excess return on the long-term pond is given by

$$\frac{1}{Q_L} - \psi_d + \lambda \frac{Q_L^* - Q_L}{Q_L} - r_n = i_L - r_n - \lambda \frac{i_L^* - i_L}{i_L^* + \psi_d} = \left[1 - \left(\frac{C_s^*}{C_s}\right)^{\sigma}\right] (i_L - r_n), \tag{A.58}$$

which is proportional to the term spread.

Yield curve. We show next that the yield curve is upward-sloping in this model. Let $Q_{L,t}(\tau)$ denote the price of a (real) zero-coupon bond maturing τ periods ahead. The value of the bond is given by

$$Q_{L,t}(\tau) = \mathbb{E}_t \left[e^{-\rho_s \tau} \left(\frac{C_{s,t+\tau}}{C_{s,t}}\right)^{-\sigma}\right] = e^{-(\rho_s+\lambda)\tau} \left(\frac{C_{s,t+\tau}}{C_{s,t}}\right)^{-\sigma} + (1 - e^{-\lambda\tau}) e^{-\rho_s \tau} \left(\frac{C_{s,t+\tau}^*}{C_{s,t}}\right)^{-\sigma}. \tag{A.59}$$

In a stationary equilibrium, we obtain

$$Q_L(\tau) = e^{-(\rho_s+\lambda)\tau} + (1 - e^{-\lambda\tau}) e^{-\rho_s \tau} \left(\frac{C_s^*}{C_s}\right)^{-\sigma}. \tag{A.60}$$

The yield on the bond is given by $r_L(\tau) \equiv -\log Q_L(\tau)/\tau$ and can be written as

$$r_L(\tau) = \rho_s + \lambda - \frac{1}{\tau} \log \left[1 + (e^{\lambda\tau} - 1) \left(\frac{C_s}{C_s^*}\right)^{\sigma}\right]. \tag{A.61}$$

Approximating the above expression, we obtain

$$r_L(\tau) = r_n + \lambda \left(\left(\frac{C_s}{C_s^*}\right)^{\sigma} - 1\right) \tau + \mathcal{O}(\tau^2), \tag{A.62}$$

which is increasing in τ.

Corporate bond premium. We can also price a *corporate bond*, which is a defautable bond. Let $Q_{C,t}$ denote the value of a bond that pays off coupons $e^{-\psi_C t}$ in nominal terms in the absence of default. We assume that monetary shocks are too small to trigger default, so there is no default in the no-disaster state, and that the bond suffers a loss $1 - \zeta_c$ conditional on a disaster.

The value of the corporate bond in the disaster state is given by

$$Q_C^* = \frac{1 - \zeta_c}{r_n^* + \psi_c}. \tag{A.63}$$

A derivation analogous to the one for government bonds shows that the yield on the corporate bond, which is given by $i_{C,t} = Q_{C,t}^{-1} - \psi_C$, can be expressed as follows in a stationary equilibrium

$$i_C - r_n = \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_C - Q_C^*}{Q_C}. \tag{A.64}$$

In the stationary equilibrium, the value of the corporate bond in the no-disaster state is given by

$$Q_C = \frac{1 + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_C^*}{\psi_C + \rho_s + \lambda}. \tag{A.65}$$

The yield on the corporate bond is given by $i_{C,t} = Q_{C,t}^{-1} - \psi_C$. The *corporate spread*, the difference between the yield on the corporate bond and a government bond with the same coupons, is given by in the stationary equilibrium

$$r_C = \frac{\psi_C + \rho_s + \lambda}{1 + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_C^*} - \frac{\psi_C + \rho_s + \lambda}{1 + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_C^*/(1 - \zeta_c)}. \tag{A.66}$$

A.3 Log-linear dynamics

We use lower-case variables to denote log-deviations from the stationary equilibrium, e.g. $c_{j,t} \equiv \log C_{j,t}/C_j$ and $n_{j,t} = \log N_{j,t}/N_j$. We derive the equilibrium conditions for the general case where C_b may differ from C_s, and then specialize to the $C_b = C_s$ case considered in Section 2.

A.3.1 Consumption of borrowers and savers

Labor supply and market clearing. The labor supply condition can be written as

$$w_t - p_t = \phi n_{j,t} + \sigma c_{j,t}. \tag{A.67}$$

Log-linearizing the market-clearing conditions for consumption and labor, we obtain

$$\mu_b^c c_{b,t} + (1 - \mu_b^c) c_{s,t} = y_t, \qquad \mu_b^n n_{b,t} + (1 - \mu_b^n) n_{s,t} = n_t, \tag{A.68}$$

where $\mu_b^c \equiv \frac{\mu_b C_b}{Y}$ and $\mu_b^n \equiv \frac{\mu_b N_b}{N}$.

Given $c_{b,t}$ and y_t, we can use the above equations to solve for the real wage, savers' consumption, and labor supply for both agents. Equating the labor-supply condition for both agents, we obtain

$$\begin{aligned} n_{s,t} &= n_{b,t} + \phi^{-1}\sigma(c_{b,t} - c_{s,t}) \\ &= n_{b,t} + \phi^{-1}\sigma(1 - \mu_b^c)^{-1}(c_{b,t} - y_t), \end{aligned} \tag{A.69}$$

using the market-clearing condition for goods to eliminate $c_{s,t}$.

Plugging the above expression into the market-clearing condition for labor, we obtain

$$n_{b,t} = (1 + \phi^{-1}\sigma) y_t - \phi^{-1}\sigma c_{b,t} + \phi^{-1}\sigma \frac{\mu_b^c - \mu_b^n}{1 - \mu_b^c}(y_t - c_{b,t}). \tag{A.70}$$

The real wage is given by

$$w_t - p_t = (\phi + \sigma) y_t + \sigma \frac{\mu_b^c - \mu_b^n}{1 - \mu_b^c} (y_t - c_{b,t}). \tag{A.71}$$

Borrowers' consumption. Linearizing the borrowers' budget constraint, we obtain

$$c_{b,t} = \frac{WN_b}{PC_b} (w_t - p_t + n_{b,t}) + T_{b,t} - (i_t - \pi_t - r_n) \overline{d}_P. \tag{A.72}$$

where $T_{b,t} \equiv \frac{\tilde{T}_{b,t} - \tilde{T}_b}{C_b}$, and $\overline{d}_p \equiv \frac{\overline{D}_p}{C_b}$.

Plugging the expressions for the real wage and labor supply into the above expression, we obtain

$$\begin{aligned} c_{b,t} = & \frac{WN_b}{PC_b} \left[(1 + \phi^{-1})(\phi + \sigma) y_t - \phi^{-1} \sigma c_{b,t} + (1 + \phi^{-1}) \sigma \frac{\mu_b^c - \mu_b^n}{1 - \mu_b^c} (y_t - c_{b,t}) \right] \\ & + T_{b,t} - (i_t - \pi_t - r_n) \overline{d}_P. \end{aligned} \tag{A.73}$$

Transfers to borrowers are given by

$$T_{b,t} = T_b'(\overline{Y}) y_t. \tag{A.74}$$

Combining the previous two conditions, we obtain

$$c_{b,t} = \chi_y y_t - \chi_r (i_t - \pi_t - r_n), \tag{A.75}$$

where

$$\chi_y \equiv \frac{T_b'(\overline{Y}) + \frac{WN_b}{PC_b} \left[(1 + \phi^{-1})(\phi + \sigma) + (1 + \phi^{-1}) \sigma \frac{\mu_b^c - \mu_b^n}{1 - \mu_b^c} \right]}{1 + \frac{WN_b}{PC_b} \left[\phi^{-1} \sigma + (1 + \phi^{-1}) \sigma \frac{\mu_b^c - \mu_b^n}{1 - \mu_b^c} \right]} \tag{A.76}$$

$$\chi_r \equiv \frac{\overline{d}_p}{1 + \frac{WN_b}{PC_b} \left[\phi^{-1} \sigma + (1 + \phi^{-1}) \sigma \frac{\mu_b^c - \mu_b^n}{1 - \mu_b^c} \right]}. \tag{A.77}$$

The expression in the text is obtained by imposing $C_b = C_s = Y$, so $\mu_b^c = \mu_b^n = \mu_b$ and $1 - \alpha \equiv \frac{WN}{PY}$.

Savers' consumption. From the borrowers' consumption and market clearing, we obtain

$$c_{s,t} = \frac{1 - \mu_b \chi_y}{1 - \mu_b} y_t + \frac{\mu_b \chi_r}{1 - \mu_b} (i_t - \pi_t - r_n). \tag{A.78}$$

A.3.2 Asset pricing

Stocks and human wealth. The value of the firm satisfies the condition

$$\frac{\Pi_t}{Q_t} + \frac{\dot{Q}_t}{Q_t} = i_t - \pi_t + \lambda_t \left(\frac{C_{s,t}}{C_{s,t}^*} \right)^{\sigma} \left(1 - \frac{Q^*}{Q_t} \right). \tag{A.79}$$

After some rearrangement, the above expression can be written as

$$\dot{Q}_t = \left[i_t - \pi_t + \lambda_t \left(\frac{C_{s,t}}{C^*_{s,t}}\right)^{\sigma}\right] Q_t - \lambda_t \left(\frac{C_{s,t}}{C^*_{s,t}}\right)^{\sigma} Q^* - \Pi_t. \tag{A.80}$$

Log-linearizing the above expression, we obtain

$$\dot{q}_t = \rho q_t + (i_t - \pi_t - r_n) + \lambda \left(\frac{C_s}{C^*_s}\right)^{\sigma} \frac{Q - Q^*}{Q} (\sigma c_{s,t} + \epsilon_\lambda (i_t - r_n)) - \frac{Y}{Q} \left((1-\tau) y_t - (1-\alpha)(w_t - p_t + n_t)\right), \tag{A.81}$$

where we defined $q_t \equiv (Q_t - Q)/Q$.

Solving the above equation forward, we obtain

$$\begin{aligned} q_0 = {} & \frac{Y}{Q} \int_0^\infty e^{-\rho t} \left[(1-\tau) y_t - (1-\alpha)(w_t - p_t + n_t)\right] dt \\ & - \int_0^\infty e^{-\rho t} \left[i_t - \pi_t - r_n + \lambda \left(\frac{C_s}{C^*_s}\right)^{\sigma} \frac{Q - Q^*}{Q} (\sigma c_{s,t} + \epsilon_\lambda (i_t - r_n))\right] dt. \end{aligned} \tag{A.82}$$

The change in the value of stocks depends on the change in dividends, firms' profits, and on changes in the discount rate, captured by changes in the real interest rate and the risk premium. Note that the present discount value of savers' consumption is given by

$$\int_0^\infty e^{-\rho t} c_{s,t} dt = \frac{\mu_b \chi_r}{1-\mu_b} \int_0^\infty e^{-\rho t} (i_t - \pi_t - r_n) dt + \frac{1 - \mu_b \chi_y}{1 - \mu_b} \Omega_0, \tag{A.83}$$

using $c_{s,t} = \frac{1-\mu_b \chi_y}{1-\mu_b} y_t + \frac{\mu_b \chi_r}{1-\mu_b} (i_t - \pi_t - r_n)$.

For a given value of Ω_0, an increase in real rates will raise the risk premium on average. Therefore, the response of the risk premium will amplify the response of real rates when $\Omega_0 = 0$.

Similarly, if we define human wealth as the present discount value of (after-tax) labor income $H_t = \mathbb{E}_t \left[\int_t^\infty \frac{\eta_z}{\eta_t} \left(\frac{W_z}{P_z} N_z + T_z\right) dz\right]$, we can then write the linearized value of human wealth as follows:

$$h_0 = \frac{Y}{H} \int_0^\infty e^{-\rho t} \left[(1-\alpha)(w_t - p_t + n_t) + T_t\right] dt \tag{A.84}$$

$$- \int_0^\infty e^{-\rho t} \left[i_t - \pi_t - r_n + \lambda \left(\frac{C_s}{C^*_s}\right)^{\sigma} \frac{H - H^*}{H} (\sigma c_{s,t} + \epsilon_\lambda (i_t - r_n))\right] dt. \tag{A.85}$$

Long-term bonds. The pricing condition for bonds can be written as

$$\frac{1}{Q_{L,t}} + \frac{\dot{Q}_{L,t}}{Q_{L,t}} - \psi_d - i_t + \lambda_t \left(\frac{C_{s,t}}{C^*_{s,t}}\right)^{\sigma} \frac{Q^*_{L,t} - Q_{L,t}}{Q_{L,t}} = 0. \tag{A.86}$$

Rearranging the above expression, we obtain

$$\dot{Q}_{L,t} = (i_t + \psi_d) Q_{L,t} - 1 + \lambda_t \left(\frac{C_{s,t}}{C^*_{s,t}}\right)^{\sigma} (Q_{L,t} - Q^*_{L,t}). \tag{A.87}$$

Linearizing the above expression, we obtain

$$\dot{q}_{L,t} = (\rho + \psi_d)q_{L,t} + i_t - r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_L - Q_L^*}{Q_L}(\sigma c_{s,t} + \epsilon_\lambda(i_t - r_n)). \tag{A.88}$$

Solving the above equation forward, we obtain

$$q_{L,0} = -\int_0^\infty e^{-(\rho+\psi_d)t} \left[i_t - r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_L - Q_L^*}{Q_L}(\sigma c_{s,t} + \epsilon_\lambda(i_t - r_n))\right] dt. \tag{A.89}$$

The yield on the long-term bond can then be written as

$$i_{L,0} - i_L = (i_L + \psi_d) \int_0^\infty e^{-(\rho+\psi_d)t} \left[i_t - r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_L - Q_L^*}{Q_L}(\sigma c_{s,t} + \epsilon_\lambda(i_t - r_n))\right] dt, \tag{A.90}$$

using the fact that $Q_L^{-1} = i_L + \psi_d$.

Corporate bonds. The linearized price of the corporate bond is given by an expression analogous to the one for government bonds:

$$q_{C,0} = -\int_0^\infty e^{-(\rho+\psi_c)t} \left[i_t - r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_C - Q_C^*}{Q_C}(\sigma c_{s,t} + \epsilon_\lambda(i_t - r_n))\right] dt. \tag{A.91}$$

The yield on the corporate bond is $i_{C,0} - i_C = -Q_C^{-1} q_{C,0}$, which can be written as

$$i_{C,0} - i_C = (i_C + \psi_c) \int_0^\infty e^{-(\rho+\psi_c)t} \left[i_t - r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_C - Q_C^*}{Q_C}(\sigma c_{s,t} + \epsilon_\lambda(i_t - r_n))\right] dt, \tag{A.92}$$

using the fact that $Q_C^{-1} = i_C + \psi_c$.

The corporate spread is $r_{C,0} = i_{C,0} - \bar{i}_{C,0}$, where $\bar{i}_{C,0}$ is the yield of a government bond with the same coupons as the corporate bond.

$$r_{C,0} = r_C \int_0^\infty e^{-(\rho+\psi_c)t}(i_t - r_n)dt + \left[(i_C + \psi_c)(i_C - i) - (\bar{i}_C + \psi_c)(\bar{i}_C - i)\right] \int_0^\infty e^{-(\rho+\psi_C)t}(\sigma c_{s,t} + \epsilon_\lambda(i_t - r_n))dt,$$

where $i_C - i = \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_C - Q_C^*}{Q_C}$ is the difference between the corporate bond yield and the short-term nominal rate in the stationary equilibrium and $\bar{i}_C - i$ is the corresponding object for a bond without default risk.

A.3.3 Flow budget constraints.

The flow budget constraint for savers can be written as

$$\begin{aligned}\bar{b}_s \dot{b}_{s,t} = {} & (i_t - \pi_t - r_n)\bar{b}_s + \hat{r}_{L,t}\bar{b}_s^L + r_n \bar{b}_s b_{s,t} + r_L \bar{b}_s^L b_{s,t}^L + \frac{WN_s}{PC_s}(w_t - p_t + n_{s,t}) \\ & + \frac{(1-\tau)y_t - (1-\alpha)(w_t - p_t + y_t)}{1-\mu_b^c} + T_{s,t} - c_{s,t},\end{aligned} \tag{A.93}$$

where $1-\alpha \equiv \frac{WN}{PY}$ is the labor share, $T_{s,t} \equiv \frac{\tilde{T}_{s,t}-\tilde{T}_s}{C_s}$, $\overline{b}_s \equiv \frac{\overline{B}_s}{C_s}$, $\overline{b}_s^L \equiv \frac{\overline{B}_s^L}{C_s}$, and

$$\hat{r}_t^L = \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_L - Q_L^*}{Q_L} \left[\sigma c_{s,t} + \epsilon_\lambda (i_t - \rho)\right] + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_L^*}{Q_L} q_{L,t}. \tag{A.94}$$

The government's budget constraint is given by

$$\overline{d}_g \dot{d}_{g,t} = (i_t - \pi_t + \hat{r}_L - r_n)\overline{d}_g + (r_n + r_L)\overline{d}_g d_{g,t} + \sum_{j\in\{b,s\}} \mu_j^c T_{j,t} - \tau y_t. \tag{A.95}$$

B Proofs

B.1 Proof of Proposition 1

Proof. Consider first the New Keynesian Phillips curve

$$\dot{\pi}_t = \left(i_t - \pi_t + \lambda_t \frac{\eta_t^*}{\eta_t}\right)\pi_t - (\epsilon - 1)\varphi^{-1}\left(\frac{\epsilon}{\epsilon - 1}\frac{W}{PA}e^{w_t - p_t} - (1-\tau)\right) Y e^{y_t}. \tag{B.1}$$

Linearizing the above expression, and using $\frac{\epsilon}{\epsilon-1}\frac{W}{P}\frac{1}{A} = (1-\tau)$, we obtain

$$\dot{\pi}_t = \left(r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma}\right)\pi_t - \varphi^{-1}(\epsilon - 1)(1-\tau)(w_t - p_t). \tag{B.2}$$

Using the expression for $w_t - p_t$, we obtain

$$\dot{\pi}_t = (\rho_s + \lambda)\,\pi_t - \kappa y_t, \tag{B.3}$$

where $\kappa \equiv \varphi^{-1}(\epsilon-1)(1-\tau)(\phi+\sigma)$ and we used the fact that $r_n + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} = \rho_s + \lambda$.

Consider next the generalized Euler equation. From the market-clearing condition for goods and borrowers' consumption, we obtain

$$c_{s,t} = \frac{1-\mu_b \chi_y}{1-\mu_b} y_t + \frac{\mu_b \chi_r}{1-\mu_b}(i_t - \pi_t - r_n). \tag{B.4}$$

The Euler equation for savers is given by

$$\dot{c}_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} c_{s,t} + \sigma^{-1} p_d \epsilon_\lambda (i_t - r_n), \tag{B.5}$$

where $p_d \equiv \lambda\left[\left(\frac{C_s}{C_s^*}\right)^{\sigma} - 1\right]$ is the price of disaster risk and $\epsilon_\lambda \equiv \lambda'(0)/\lambda(0)$ is the semi-elasticity of $\lambda_t = \lambda(i_t - r_n)$ with respect to i_t.

Combining the previous two conditions, we obtain

$$\begin{aligned}\dot{y}_t &= \frac{1-\mu_b}{1-\mu_b\chi_y}\sigma^{-1}(i_t-\pi_t-r_n) + \frac{1-\mu_b}{1-\mu_b\chi_y}\left[\lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} c_{s,t} + \sigma^{-1}p_d\epsilon_\lambda(i_t-r_n)\right] - \frac{\mu_b\chi_r}{1-\mu_b\chi_y}(\dot{i}_t-\dot{\pi}_t) \\ &= \left[\frac{1-\mu_b}{1-\mu_b\chi_y}\sigma^{-1} - \frac{\mu_b\chi_r}{1-\mu_b\chi_y}\rho\right](i_t-\pi_t-r_n) + \left[\lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} - \frac{\mu_b\chi_r}{1-\mu_b\chi_y}\kappa\right]y_t - \frac{\mu_b\chi_r}{1-\mu_b\chi_y}(\dot{i}_t-\rho(i_t-r_n)) \\ &+ \frac{1-\mu_b}{1-\mu_b\chi_y}\sigma^{-1}p_d\epsilon_{\lambda,i}(i_t-r_n), \end{aligned} \tag{B.6}$$

where we used (B.3) and (B.4) to replace for $\dot{\pi}_t$ and $c_{s,t}$ in the second equality.

We can then write the aggregate Euler equation:

$$\dot{y}_t = \tilde{\sigma}^{-1}(i_t-\pi-r_n) + \delta y_t + v_t, \tag{B.7}$$

where the constants $\tilde{\sigma}^{-1}$, δ, and v_t are defined in the proposition.

□

B.2 Proof of Lemma 1

Proof. We first derive the (non-linear) intertemporal budget constraint, then derive its log-linearized version and show the sufficiency of system (15) and the intertemporal budget constraint.

Non-linear intertemporal budget constraint. The dynamics of the SDF can be written as

$$\frac{d\eta_t}{\eta_t} = -(i_t-\pi_t)dt + \frac{(C_{s,t}^*)^{-\sigma} - C_{s,t}^{-\sigma}}{C_{s,t}^{-\sigma}}(d\mathcal{N}_t - \lambda_t dt). \tag{B.8}$$

Applying Ito's lemma to $d(\eta_t B_{j,t})$, we obtain

$$\begin{aligned}\mathbb{E}[d(\eta_t B_{j,t})] &= \eta_t\mathbb{E}_t[dB_{j,t}] + B_{j,t}\mathbb{E}_t[d\eta_t] + \lambda_t(\eta_t^*-\eta_t)(B_{j,t}^*-B_{j,t})dt \\ &= \eta_t\left[(i_t-\pi_t)B_{j,t} + \left(r_{L,t}+\lambda_t\frac{Q_{L,t}^*-Q_{L,t}}{Q_{L,t}}\right)B_{j,t}^L + \frac{W_{j,t}}{P_t}N_{j,t} + \frac{\Pi_{j,t}}{P_t} + \tilde{T}_{j,t} - C_{j,t}\right]dt \\ &- (i_t-\pi_t)\eta_t B_{j,t}dt + \lambda_t(\eta_t^*-\eta_t)B_{j,t}^L\frac{Q_{L,t}^*-Q_{L,t}}{Q_{L,t}}dt. \end{aligned} \tag{B.9}$$

Using expression (2) to replace $r_{L,t}$ if $B_{j,t}^L > 0$, we obtain

$$\mathbb{E}[d(\eta_t B_{j,t})] = \eta_t\left[\frac{W_{j,t}}{P_t}N_{j,t} + \frac{\Pi_{j,t}}{P_t} + \tilde{T}_{j,t} - C_{j,t}\right]dt. \tag{B.10}$$

Integrating $d(\eta_t B_{j,t})$ and taking expectations, we obtain

$$\mathbb{E}_0[\eta_t B_{j,t}] - \eta_0 B_{j,0} = \mathbb{E}_0\left[\int_0^t \eta_z\left(\frac{W_z}{P_z}N_{j,z} + \Pi_{j,z} + \tilde{T}_{j,z} - C_{j,z}\right)dz\right]. \tag{B.11}$$

Note that $\lim_{t\to\infty}\mathbb{E}_0[\eta_t B_{b,t}] = 0$, as $B_{b,t}$ is constant in equilibrium and $\lim_{t\to\infty}\mathbb{E}_0[\eta_t] = 0$. The borrowing constraint implies that $\lim_{t\to\infty}\mathbb{E}_0[\eta_t B_{s,t}] \geq 0$, which, combined with the No-Ponzi condition for the government, market clearing for bonds, and the previous result for borrowers, implies that $\lim_{t\to\infty} \mathbb{E}_0[\eta_t B_{s,t}] = 0$. Therefore, we conclude that $\lim_{t\to\infty} \mathbb{E}_0[\eta_t B_{j,t}] = 0$ for both types of households.

Taking the limit as $t \to \infty$ of the previous expression, using the fact that $\lim_{t\to\infty} \mathbb{E}_0[\eta_t B_{j,t}] = 0$, and aggregating across households, we obtain

$$\underbrace{\mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0}\left(\mu_b C_{b,t} + (1-\mu_b)C_{s,t}\right) dt\right]}_{\equiv Q_{C,0}} = D_{g,0} + \underbrace{\mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0}\left((1-\tau)Y_t + \tilde{T}_t\right) dt\right]}_{\equiv Q_{Y,0}}, \tag{B.12}$$

using the fact that $\frac{W_t}{P_t}N_t + (1-\mu_b)\Pi_{s,t} = (1-\tau)Y_t$.

The above expression can then be written as

$$Q_{C,0} = D_{g,0} + Q_{Y,0} \tag{B.13}$$

where $Q_{C,0}$ is the initial value of the consumption claim and $Q_{Y,0}$ is the initial value of a claim on after-tax profits, wages, and transfers.

Log-linearized intertemporal budget constraint. The linearized budget constraint can be written as

$$Q_C q_{C,0} = \overline{D}_g q_{L,0} + Q_Y q_{Y,0}, \tag{B.14}$$

where $q_{C,0} \equiv \log \frac{Q_{C,0}}{Q_C}$, $q_{Y,0} \equiv \log \frac{Q_{Y,0}}{Q_Y}$, and Q_C and Q_Y denote the value of the consumption claim and the (after-tax) income claim, respectively, in the no-disaster state of the stationary equilibrium.

Following a derivation analogous to the one for the value of stocks in Section A.3, we obtain

$$q_{C,0} = \frac{Y}{Q_C}\int_0^\infty e^{-\rho t}\left[\mu_b c_{b,t} + (1-\mu_b)c_{s,t}\right] dt - \int_0^\infty e^{-\rho t}\left[i_t - \pi_t - r_n + \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_C - Q_C^*}{Q_C} p_{d,t}\right] dt \tag{B.15}$$

$$q_{Y,0} = \frac{Y}{Q_Y}\int_0^\infty e^{-\rho t}\left[(1-\tau)y_t + T_t\right] dt - \int_0^\infty e^{-\rho t}\left[i_t - \pi_t - r_n + \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_Y - Q_Y^*}{Q_Y} p_{d,t}\right] dt, \tag{B.16}$$

where $T_t = \mu_b T_{b,t} + (1-\mu_b)T_{s,t}$, $p_{d,t} \equiv \sigma c_{s,t} + \epsilon_\lambda(i_t - r_n)$ is the (log-linear) price of disaster risk.

The intertemporal budget constraint can then be written as

$$\int_0^\infty e^{-\rho t}\left[\mu_b c_{b,t} + (1-\mu_b)c_{s,t}\right] dt = \overline{d}_g q_{L,0} + \int_0^\infty e^{-\rho t}\left[(1-\tau)y_t + T_t\right] dt + \\ \int_0^\infty e^{-\rho t}\frac{Q_C - Q_Y}{Y}(i_t - \pi_t - r_n) + \int_0^\infty e^{-\rho t}\left[\lambda\left(\frac{C_s}{C_s^*}\right)^\sigma\left(\frac{Q_C - Q_Y}{Y} - \frac{Q_C^* - Q_Y^*}{Y}\right) p_{d,t}\right] dt. \tag{B.17}$$

The left-hand side represents the dividend effect on the consumption claim. The right-hand side is given by the sum of the revaluation of government bonds, the dividend effect on the claim on profits, wages, and transfers, and the last two terms capture the net discount rate effect. Note that discount rate effects only appear to the extent there is some mismatch between assets and liabilities, that is, if $Q_C \neq Q_Y$.

In a stationary equilibrium, we infer that $Q_C = \overline{D}_g + Q_Y$ and $Q_C^* = \overline{D}_g + \overline{D}_g \frac{Q_L^* - Q_L}{Q_L} + Q_Y^*$, which allows us to write

$$\Omega_0 = \int_0^\infty e^{-\rho t}\left[(1-\tau)y_t + T_t + \overline{d}_g(i_t - \pi_t - r_n + r_L p_{d,t})\right] dt - \overline{d}_g \int_0^\infty e^{-(\rho+\psi_d)t}\left[i_t - r_n + r_L p_{d,t}\right] dt, \tag{B.18}$$

where $r_L \equiv \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_L - Q_L^*}{Q_L}$ and we used the expression for $q_{L,0}$ derived in Section A.3.2.

Sufficiency of the intertemporal budget constraint. Suppose $[y_t, \pi_t]_0^\infty$ satisfy system (15) and the intertemporal budget constraint (13) in the no-disaster state. We will show that $[y_t, \pi_t]_0^\infty$ can be supported as an equilibrium. Consider first the disaster state. We set $T_{s,t} = \rho_s \overline{b}_s b_{s,t^*}$, where t^* denotes the time the economy switches to the disaster state, and $i_t^* = \rho_s$. All the remaining variables are equal to zero in the disaster state. The equation for the real wage, $w_t^* - p_t^* = (\phi + \sigma) y_t^* = 0$, and the labor-supply condition for each household, $w_t^* - p_t^* = \sigma c_{j,t}^* + \phi n_{j,t}^*$, are also satisfied. The same applies to the savers' Euler equation and the market clearing condition for goods, labor, and bonds. Borrowers' and savers' budget constraint are satisfied,

$$c_{b,t}^* = (1-\alpha)(w_t^* - p_t^* + n_{b,t}^*) + T_{b,t}^* - (i^* - \pi_t^* - \rho_s)\overline{d}_p \tag{B.19}$$

$$\overline{b}_s \dot{b}_{s,t} = \overline{b}_s \rho_s b_{s,t} + (1-\alpha)(w_t^* - p_t^* + n_{s,t}^*) + \frac{(1-\tau)y_t^* - (1-\alpha)(w_t^* - p_t^* + n_t^*)}{1-\mu_b} + T_{s,t}^* + (i^* - \pi_t^* - \rho_s)\overline{b}_s - c_{s,t}^*, \tag{B.20}$$

which implies that $b_{s,t}$ is constant in the disaster state at the b_{s,t^*} level.

Consider now the no-disaster state. The real wage is given by $w_t - p_t = (\phi + \sigma) y_t$. Borrowers' and savers' consumption are given by

$$c_{b,t} = \chi_y y_t - \chi_r (i_t - \pi_t - r - n), \qquad c_{s,t} = \frac{1 - \mu_b \chi_y}{1 - \mu_b} y_t + \frac{\mu_b \chi_r}{1 - \mu_b}(i_t - \pi_t - r_n), \tag{B.21}$$

and the labor supply is given by $n_{j,t} = \phi^{-1}(w_t - p_t) - \phi^{-1}\sigma c_{j,t}$.

By construction, the market-clearing condition for goods and labor and the labor supply for each household are all satisfied. Because y_t satisfies the aggregate Euler equation, the savers' Euler equation is also satisfied. Because π_t satisfies the New Keynesian Phillips curve, the optimality condition for firms is satisfied. Bond holdings by savers and government debt evolve according to

$$\overline{b}_s \dot{b}_{s,t} = r_n \overline{b}_s b_{s,t} + (1-\alpha)(w_t - p_t + n_{s,t}) + T_{s,t} + \frac{(1-\tau)y_t - (1-\alpha)(w_t - p_t + n_t)}{1-\mu_b}$$
$$+ (i - \pi_t - r_n)\overline{b}_s + \hat{r}_{L,t}\overline{b}_s^L + r_L \overline{b}_s^L b_{s,t}^L - c_{s,t}, \tag{B.22}$$

$$\overline{d}_g \dot{d}_{g,t} = \overline{d}_g (r_n + r_L) d_{g,t} + T_t - \tau y_t + (i_t - \pi_t - r_n + \hat{r}_{L,t})\overline{d}_g, \tag{B.23}$$

where $b_{s,0} = \frac{\overline{b}_s^L}{\overline{b}_s} q_{L,0}$ and $d_{g,0} = \overline{d}_g q_{L,0}$.

The value of $c_{b,t}$ is such that the flow budget constraint for borrowers also hold:

$$0 = (1-\alpha)(w_t - p_t + n_{b,t}) + T_{b,t} - (i_t - \pi_t - r_n)\overline{d}_p - c_{b,t}. \tag{B.24}$$

Aggregating the budget constraint of borrowers and savers and using the market clearing condition for goods and labor, we obtain

$$(1-\mu_b)\overline{b}_s \dot{b}_{s,t} = r_n (1-\mu_b)\overline{b}_s b_{s,t} + T_t - \tau y_t + (i - \pi_t - r_n)\left[(1-\mu_b)\overline{b}_s - \mu_b \overline{d}_p\right] + (\hat{r}_{L,t} + r_L b_{s,t}^L)(1-\mu_b)\overline{b}_s^L \tag{B.25}$$

Note that $\overline{b}_s b_{s,l} = \overline{b}_s^S b_{s,t}^S + \overline{b}_s^L b_{s,t}^L$. We set $b_{s,t}^S = 0$, so the market for short-term bonds clear at all periods. It remains to show that the market for long-term bonds also clears. Subtracting the government's flow budget constraint from the condition above, we obtain

$$(1-\mu_b)\overline{b}_s^L \dot{b}_{s,t}^L - \overline{d}_g \dot{d}_{g,t} = (r_n + r_L)((1-\mu_b)\overline{b}_s^L b_{s,t}^L - \overline{d}_g d_{g,t}), \tag{B.26}$$

using $\overline{b}_s b_{s,t} = \overline{b}_{s,t}^L b_{s,t}^L$ and $(1-\mu_b)\overline{b}_s - \mu_b \overline{d}_p = (1-\mu_b)\overline{b}_s^L = \overline{d}_g$.

Integrating the expression above, we obtain

$$(1-\mu_b)\overline{b}_s^L b_{s,t}^L - \overline{d}_g d_{g,t} = e^{(r_n+r_L)t}\left[(1-\mu_b)\overline{b}_s^L b_{s,0}^L - \overline{d}_g d_{g,0}\right] = 0, \tag{B.27}$$

where the second equality uses the initial conditions for $b_{s,0}^L$ and $\overline{d}_g$.

Therefore, the market clearing condition for long-term bonds is satisfied in all periods. The only condition that remains to be checked is the No-Ponzi condition for the government or, equivalently, the aggregate intertemporal budget constraint. Because condition (13) is satisfied, the No-Ponzi condition for the government is also satisfied. □

B.3 Proof of Propositions 2 and 3

Proof. We can write dynamic system (15) in matrix form as follows:

$$\dot{Z}_t = AZ_t + B\nu_t, \tag{B.28}$$

where $B = [1, 0]'$.

Applying the spectral decomposition to matrix A, we obtain

$$A = V\Omega V^{-1}, \tag{B.29}$$

where

$$V = \begin{bmatrix} \frac{\rho-\overline{\omega}}{\kappa} & \frac{\rho-\underline{\omega}}{\kappa} \\ 1 & 1 \end{bmatrix}; \qquad V^{-1} = \frac{\kappa}{\overline{\omega}-\underline{\omega}}\begin{bmatrix} -1 & \frac{\rho-\underline{\omega}}{\kappa} \\ 1 & -\frac{\rho-\overline{\omega}}{\kappa} \end{bmatrix}; \qquad \Omega = \begin{bmatrix} \overline{\omega} & 0 \\ 0 & \underline{\omega} \end{bmatrix}. \tag{B.30}$$

Decoupling the system, we obtain

$$\dot{z}_t = \Omega z_t + b\nu_t, \tag{B.31}$$

where $z_t = V^{-1}Z_t$ and $b = V^{-1}B$.

Solving the equation with a positive eigenvalue forward and the one with a negative eigenvalue backward, we obtain

$$z_{1,t} = -b_1 \int_t^\infty e^{-\overline{\omega}(z-t)} \nu_z dz \tag{B.32}$$

$$z_{2,t} = e^{\underline{\omega}t} z_{2,0} + b_2 \int_0^t e^{\underline{\omega}(t-z)} \nu_z dz. \tag{B.33}$$

Rotating the system back to the original coordinates, we obtain output and inflation

$$y_t = V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right)e^{\underline{\omega}t} - V_{11}V^{11}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + V_{12}V^{21}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz \tag{B.34}$$

$$\pi_t = V_{22}\left(V^{21}y_0 + V^{22}\pi_0\right)e^{\underline{\omega}t} - V_{21}V^{11}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + V_{22}V^{21}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz, \tag{B.35}$$

where $V^{i,j}$ is the (i,j) entry of matrix V^{-1}.

Integrating $e^{-\rho t}y_t$ and using the intertemporal budget constraint,

$$\Omega_0 = V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right)\frac{1}{\rho-\underline{\omega}} - \frac{1}{\rho-\overline{\omega}}V_{11}V^{11}\int_0^\infty \left(e^{-\overline{\omega}t} - e^{-\rho t}\right)\nu_t dt + \frac{1}{\rho-\underline{\omega}}V_{12}V^{21}\int_0^\infty e^{-\rho t}\nu_t dt. \tag{B.36}$$

Rearranging the above expression, we obtain

$$V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right) = (\rho-\underline{\omega})\Omega_0 + \frac{\rho-\underline{\omega}}{\rho-\overline{\omega}}V_{11}V^{11}\int_0^\infty \left(e^{-\overline{\omega}t} - e^{-\rho t}\right)\nu_t dt - V_{12}V^{21}\int_0^\infty e^{-\rho t}\nu_t dt. \tag{B.37}$$

Consumption is then given by

$$y_t = \tilde{y}_t + (\rho-\underline{\omega})e^{\underline{\omega}t}\Omega_0, \tag{B.38}$$

where

$$\tilde{y}_t = -\frac{\overline{\omega}-\rho}{\overline{\omega}-\underline{\omega}}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + \frac{\overline{\omega}-\delta}{\overline{\omega}-\underline{\omega}}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz - \frac{\rho-\underline{\omega}}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t}\int_0^\infty e^{-\overline{\omega}z}\nu_z dz. \tag{B.39}$$

Inflation is given by

$$\pi_t = \tilde{\pi}_t + \kappa e^{\underline{\omega}t}\Omega_0, \tag{B.40}$$

where

$$\tilde{\pi}_t = \frac{\kappa}{\overline{\omega}-\underline{\omega}}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + \frac{\kappa}{\overline{\omega}-\underline{\omega}}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz - \frac{\kappa}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t}\int_0^\infty e^{-\overline{\omega}z}\nu_z dz. \tag{B.41}$$

We can write $\tilde{y}_t$ and $\tilde{\pi}_t$ as follows:

$$\tilde{y}_t = -\epsilon^F_{y,t}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + \epsilon^B_{y,t}\int_0^t \left(e^{-\underline{\omega}z} - e^{-\overline{\omega}z}\right)\nu_z dz \tag{B.42}$$

$$\tilde{\pi}_t = \epsilon^F_{\pi,t}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + \epsilon^B_{\pi,t}\int_0^t \left(e^{-\underline{\omega}z} - e^{-\overline{\omega}z}\right)\nu_z dz, \tag{B.43}$$

where

$$\epsilon^F_{y,t} = \frac{(\overline{\omega}-\rho) + (\overline{\omega}-\delta)e^{-(\overline{\omega}-\underline{\omega})t}}{\overline{\omega}-\underline{\omega}}, \qquad \epsilon^B_{y,t} = \frac{\rho-\underline{\omega}}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t} \tag{B.44}$$

$$\epsilon^F_{\pi,t} = \frac{\kappa}{\overline{\omega}-\underline{\omega}}\left(1 - e^{-(\overline{\omega}-\underline{\omega})t}\right), \qquad \epsilon^B_{\pi,t} = \frac{\kappa}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t}, \tag{B.45}$$

and

$$\nu_t = \frac{1-\mu_b}{1-\mu_b\chi_y}\sigma^{-1}(i_t - r_n) + \frac{1-\mu_b}{1-\mu_b\chi_y}\chi_p\epsilon_\lambda(i_t - r_n) - \frac{\mu_b\chi_r}{1-\mu_b\chi_y}\dot{i}_t. \tag{B.46}$$

If $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, then $\dot{i}_t = -\psi_m(i_t - r_n)$. This allows us to write

$$\tilde{y}_t = \sigma^{-1}\hat{y}_t + \chi_p\epsilon_\lambda\hat{y}_t + \frac{\mu_b\chi_r}{1-\mu_b}\psi_m\hat{y}_t, \qquad \tilde{\pi}_t = \sigma^{-1}\hat{\pi}_t + \chi_p\epsilon_\lambda\hat{\pi}_t + \frac{\mu_b\chi_r}{1-\mu_b}\psi_m\hat{\pi}_t \tag{B.47}$$

where

$$\hat{y}_t = \frac{1-\mu_b}{1-\mu_b\chi_y}\left[-\frac{(\overline{\omega}-\rho)e^{\overline{\omega}t}+(\overline{\omega}-\delta)e^{\underline{\omega}t}}{(\overline{\omega}-\underline{\omega})(\psi_m+\overline{\omega})}e^{-(\psi_m+\overline{\omega})t}+\frac{\rho-\underline{\omega}}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t}\left(\frac{1-e^{-(\psi_m+\underline{\omega})t}}{\psi_m+\underline{\omega}}-\frac{1-e^{-(\psi_m+\overline{\omega})t}}{\psi_m+\overline{\omega}}\right)\right](i_0-r_n),$$
$$= \frac{1-\mu_b}{1-\mu_b\chi_y}\left[-\frac{\psi_m+\rho}{(\psi_m+\underline{\omega})(\psi_m+\overline{\omega})}e^{-\psi_m t}+\frac{\rho-\underline{\omega}}{(\psi_m+\overline{\omega})(\psi_m+\underline{\omega})}e^{\underline{\omega}t}\right](i_0-r_n), \tag{B.48}$$

and

$$\hat{\pi}_t = \frac{1-\mu_b}{1-\mu_b\chi_y}\left[\kappa\frac{e^{\overline{\omega}t}-e^{\underline{\omega}t}}{(\overline{\omega}-\underline{\omega})(\psi_m+\overline{\omega})}e^{-(\psi_m+\overline{\omega})t}+\kappa\frac{e^{\underline{\omega}t}}{\overline{\omega}-\underline{\omega}}\left(\frac{1-e^{-(\underline{\omega}+\psi_m)t}}{\underline{\omega}+\psi_m}-\frac{1-e^{-(\overline{\omega}+\psi_m)t}}{\overline{\omega}+\psi_m}\right)\right](i_0-r_n)$$
$$= \frac{1-\mu_b}{1-\mu_b\chi_y}\frac{\kappa(e^{\underline{\omega}t}-e^{-\psi_m t})}{(\underline{\omega}+\psi_m)(\overline{\omega}+\psi_m)}(i_0-r_n), \tag{B.49}$$

consistent with the results in Propositions 2 and 3.

Note that the present discounted value of $\hat{y}_t$ is given by

$$\int_0^\infty e^{-\rho t}\hat{y}_t dt = \frac{1-\mu_b}{1-\mu_b\chi_y}\left[-\frac{\psi_m+\rho}{(\psi_m+\underline{\omega})(\psi_m+\overline{\omega})}\frac{1}{\rho+\psi_m}+\frac{\rho-\underline{\omega}}{(\psi_m+\overline{\omega})(\psi_m+\underline{\omega})}\frac{1}{\rho-\underline{\omega}}\right](i_0-r_n)=0, \tag{B.50}$$

and the initial value $\hat{y}_0$ satisfies

$$\frac{\partial\hat{y}_0}{\partial i_0} = -\frac{1-\mu_b}{1-\mu_b\chi_y}\frac{1}{\psi_m+\overline{\omega}} < 0 \tag{B.51}$$

□

B.4 Proof of Proposition 4

Proof. From conditions (11) and (13), we can write Ω_0 as follows:

$$\Omega_0 = \int_0^\infty e^{-\rho t}\left[(1-\tau)y_t + T_t + \overline{d}_g(i_t-\pi_t-r_n+r_L p_{d,t})\right]dt - \overline{d}_g\int_0^\infty e^{-(\rho+\psi_d)t}\left[i_t-r_n+r_L p_{d,t}\right]dt. \tag{B.52}$$

We can write the price of disaster risk as follows

$$\begin{aligned} p_{d,t} &= \sigma c_{s,t} + \epsilon_\lambda(i_t-r_n) \\ &= \sigma\frac{1-\mu_b\chi_y}{1-\mu_b}y_t + \sigma\frac{\mu_b\chi_r}{1-\mu_b}(i_t-\pi_t-r_n)+\epsilon_\lambda(i_t-r_n) \\ &= \sigma\frac{1-\mu_b\chi_y}{1-\mu_b}\hat{y}_t + \sigma\frac{\mu_b\chi_r}{1-\mu_b}(i_t-\chi\hat{\pi}_t-r_n)+\epsilon_\lambda(i_t-r_n)+\epsilon_{p_d,\Omega}e^{\underline{\omega}t}\Omega_0 \\ &= \hat{p}_{d,t}+\epsilon_{p_d,\Omega}e^{\underline{\omega}t}\Omega_0, \end{aligned} \tag{B.53}$$

where $\epsilon_{p_d,\Omega} \equiv \sigma\left(\frac{1-\mu_b\chi_y}{1-\mu_b}(\overline{\omega}-\delta)-\frac{\mu_b\chi_r}{1-\mu_b}\kappa\right)$.

Using the fact that $y_t = \chi\hat{y}_t + (\overline{\omega}-\delta)e^{\underline{\omega}t}\Omega_0$, $\int_0^\infty e^{-\rho t}y_t dt = \Omega_0$, and $\pi_t = \chi\hat{\pi}_t + \kappa e^{\underline{\omega}t}\Omega_0$, we obtain

$$\Omega_0 = (1-\epsilon_\Omega)\Omega_0 + \int_0^\infty e^{-\rho t}\left[T_t + \overline{d}_g(i_t-\chi\hat{\pi}_t-r_n+r_L\hat{p}_{d,t})\right]dt - \overline{d}_g\int_0^\infty e^{-(\rho+\psi_d)t}(i_t-r_n+r_L\hat{p}_{d,t})\,dt, \tag{B.54}$$

where

$$\epsilon_\Omega \equiv \tau + \frac{\kappa \overline{d}_g}{\overline{\omega} - \delta} - \frac{\psi_d r_L \overline{d}_g}{\rho + \psi_d - \underline{\omega}} \epsilon_{p_d,\Omega}. \tag{B.55}$$

Rearranging the expression for Ω_0, we obtain

$$\Omega_0 = \frac{1}{\epsilon_\Omega} \left[\int_0^\infty e^{-\rho t} \left[T_t + \overline{d}_g (i_t - \chi \hat{\pi}_t - r_n + r_L \hat{p}_{d,t}) \right] dt - \overline{d}_g \int_0^\infty e^{-(\rho+\psi_d)t} \left(i_t - r_n + r_L \hat{p}_{d,t} \right) dt \right]. \tag{B.56}$$

□

B.5 Proof of Proposition 5

Proof. We divide this proof in three steps. First, we derive the condition for local uniqueness of the solution under the policy rule (5). Second, we derive the path of $[y_t, \pi_t, i_t]_0^\infty$ for a given path of monetary shocks. Third, we show how to implement a given path of nominal interest rates $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ and a given value of Ω_0, which maps to a given value of fiscal backing $\int_0^\infty e^{-\rho t} T_t dt$.

Equilibrium determinacy First, note that we can write ν_t as follows,

$$\begin{aligned}
\nu_t &= \frac{1-\mu_b}{1-\mu_b \chi_y} \sigma^{-1}(i_t - r_n) - \frac{\mu_b \chi_r}{1-\mu_b \chi_y} \dot{i}_t + \frac{1-\mu_b}{1-\mu_b \chi_y} \chi_p \epsilon_\lambda (i_t - r_n) \\
&= \left[\frac{1-\mu_b}{1-\mu_b \chi_y} \sigma^{-1} + \frac{1-\mu_b}{1-\mu_b \chi_y} \chi_p \epsilon_\lambda \right] (\phi_\pi \pi_t + u_t) - \frac{\mu_b \chi_r}{1-\mu_b \chi_y} (\phi_\pi \dot{\pi}_t + \dot{u}_t) \\
&= \tilde{\sigma}^{-1} \left[1 + \frac{1-\mu_b}{1-\mu_b \chi_y} \tilde{\sigma} \chi_p \epsilon_\lambda \right] \phi_\pi \pi_t + \frac{\mu_b \chi_r}{1-\mu_b \chi_y} \phi_\pi \kappa y_t + \tilde{\nu}_t,
\end{aligned} \tag{B.57}$$

where $\tilde{\nu}_t \equiv \left[\frac{1-\mu_b}{1-\mu_b \chi_y} \sigma^{-1} + \frac{1-\mu_b}{1-\mu_b \chi_y} \chi_p \epsilon_\lambda \right] u_t - \frac{\mu_b \chi_r}{1-\mu_b \chi_y} \dot{u}_t$.

The dynamic system for y_t and π_t can now be written as

$$\begin{bmatrix} \dot{y}_t \\ \dot{\pi}_t \end{bmatrix} = \begin{bmatrix} \tilde{\delta} & -\tilde{\sigma}^{-1}(1 - \tilde{\phi}_\pi) \\ -\kappa & \rho \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tilde{\nu}_t, \tag{B.58}$$

where $\tilde{\delta} \equiv \delta + \frac{\mu_b \chi_r}{1-\mu_b \chi_y} \phi_\pi \kappa$ and $\tilde{\phi}_\pi \equiv \left[1 + \frac{1-\mu_b}{1-\mu_b \chi_y} \tilde{\sigma} \chi_p \epsilon_\lambda \right] \phi_\pi$.

The eigenvalues of the system above are given by

$$\overline{\omega}_T = \frac{\rho + \tilde{\delta} + \sqrt{(\rho + \tilde{\delta})^2 + 4(\tilde{\sigma}^{-1}(1-\tilde{\phi}_\pi)\kappa - \rho\tilde{\delta})}}{2}, \qquad \underline{\omega}_T = \frac{\rho + \tilde{\delta} - \sqrt{(\rho + \tilde{\delta})^2 + 4(\tilde{\sigma}^{-1}(1-\tilde{\phi}_\pi)\kappa - \rho\tilde{\delta})}}{2}. \tag{B.59}$$

The system has a unique bounded solution if both eigenvalues have positive real parts. A necessary condition for the eigenvalues to have positive real parts is

$$\rho + \delta + \frac{\mu_b \chi_r}{1-\mu_b \chi_y} \phi_\pi \kappa > 0 \iff \phi_\pi > -(\rho + \delta) \left(\frac{\mu_b \chi_r \kappa}{1-\mu_b \chi_y} \right)^{-1} = 1 - \left(\rho + \lambda \left(\frac{C_s}{C_s^*} \right)^\sigma \right) \frac{1-\mu_b \chi_y}{\mu_b \chi_r \kappa}. \tag{B.60}$$

If the condition above is violated, then the real part of $\underline{\omega}_T$ is negative. Another necessary condition for the eigenvalues to have positive real parts is

$$\tilde{\sigma}^{-1}(1-\tilde{\phi}_\pi)\kappa < \rho\left[\delta + \frac{\mu_b\chi_r}{1-\mu_b\chi_y}\phi_\pi\kappa\right], \tag{B.61}$$

which after some rearrangement gives us

$$\phi_\pi > \frac{\tilde{\sigma}^{-1}-\rho\delta/\kappa}{\tilde{\sigma}^{-1}+\frac{\mu_b\chi_r\rho}{1-\mu_b\chi_y}+\frac{(1-\mu_b)\chi_p\epsilon_\lambda}{1-\mu_b\chi_y}} = 1 - \frac{\chi_p\epsilon_\lambda + \frac{\rho\lambda}{\kappa}\frac{1-\mu_b\chi_y}{1-\mu_b}\left(\frac{C_s}{C_s^*}\right)^\sigma}{\chi_p\epsilon_\lambda+\sigma^{-1}}. \tag{B.62}$$

If the above condition is violated, then the eigenvalues are real-valued and $\underline{\omega}_T < 0$. This establishes the necessity of the condition

$$\phi_\pi > \max\left\{1 - \frac{\chi_p\epsilon_\lambda + \frac{\rho\lambda}{\kappa}\frac{1-\mu_b\chi_y}{1-\mu_b}\left(\frac{C_s}{C_s^*}\right)^\sigma}{\chi_p\epsilon_\lambda+\sigma^{-1}}, 1-\left(\rho+\lambda\left(\frac{C_s}{C_s^*}\right)^\sigma\right)\frac{1-\mu_b\chi_y}{\mu_b\chi_r\kappa}\right\}. \tag{B.63}$$

Suppose that the above condition is satisfied. If the eigenvalues are complex-valued, then the above condition guarantees that $\rho+\tilde{\delta} > 0$, so the eigenvalues' real part is positive. If the eigenvalues are real-valued, the above condition guarantees that $\sqrt{(\rho+\tilde{\delta})^2 + 4(\tilde{\sigma}^{-1}(1-\phi_\pi)\kappa - \rho\tilde{\delta})} < \rho+\tilde{\delta}$, so $\underline{\omega}_T > 0$. Therefore, the condition is necessary and sufficient for both eigenvalues to have a positive real part. Note that, if $\mu_b\chi_y < 1$, then $\phi_\pi > 1$ is sufficient to guarantee the local uniqueness of the solution.

Solution to the dynamic system. The dynamic system for $[y_t, \pi_t]_{t=0}^\infty$ is given by

$$\begin{bmatrix} \dot{y}_t \\ \dot{\pi}_t \end{bmatrix} = \begin{bmatrix} \tilde{\delta} & -\tilde{\sigma}^{-1}(1-\tilde{\phi}_\pi) \\ -\kappa & \rho \end{bmatrix}\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\tilde{v}_t. \tag{B.64}$$

In matrix form, the system is given by

$$\dot{\tilde{Z}}_t = \tilde{A}\tilde{Z}_t + B\tilde{v}_t,$$

where $B = [1,0]'$.

Applying the spectral decomposition to matrix $\tilde{A}$, we obtain

$$\tilde{A} = \tilde{V}\Omega_T\tilde{V}^{-1},$$

where

$$\tilde{V} = \begin{bmatrix} \frac{\rho-\overline{\omega}_T}{\kappa} & \frac{\rho-\underline{\omega}_T}{\kappa} \\ 1 & 1 \end{bmatrix}; \qquad \tilde{V}^{-1} = \frac{\kappa}{\overline{\omega}_T - \underline{\omega}_T}\begin{bmatrix} -1 & \frac{\rho-\underline{\omega}_T}{\kappa} \\ 1 & -\frac{\rho-\overline{\omega}_T}{\kappa} \end{bmatrix}; \qquad \Omega_T = \begin{bmatrix} \overline{\omega}_T & 0 \\ 0 & \underline{\omega}_T \end{bmatrix}.$$

Decoupling the system, we obtain

$$\dot{\tilde{z}}_t = \Omega_T\tilde{z}_t + \tilde{b}\tilde{v}_t,$$

where $\tilde{z}_t = \tilde{V}^{-1}\tilde{Z}_t$ and $\tilde{b} = \tilde{V}^{-1}B$.

Solving the system forward, we obtain

$$z_{1,t} = -\tilde{V}^{11} \int_t^\infty e^{-\overline{\omega}_T(z-t)} \tilde{v}_z dz$$
$$z_{2,t} = -\tilde{V}^{21} \int_t^\infty e^{-\underline{\omega}_T(z-t)} \tilde{v}_z dz,$$

where $\tilde{V}^{i,j}$ is the (i, j) entry of the matrix $\tilde{V}^{-1}$.

Rotating the system back to the original coordinates, we obtain output and inflation

$$y_t = -\tilde{V}_{11}\tilde{V}^{11} \int_t^\infty e^{-\overline{\omega}_T(z-t)} \tilde{v}_z dz - \tilde{V}_{12}\tilde{V}^{21} \int_t^\infty e^{-\underline{\omega}_T(z-t)} \tilde{v}_z dz$$
$$\pi_t = -\tilde{V}_{21}\tilde{V}^{11} \int_t^\infty e^{-\overline{\omega}_T(z-t)} \tilde{v}_z dz - \tilde{V}_{22}\tilde{V}^{21} \int_t^\infty e^{-\underline{\omega}_T(z-t)} \tilde{v}_z dz.$$

We rewrite the above expression as follows,

$$y_t = -\frac{\overline{\omega}_T - \rho}{\overline{\omega}_T - \underline{\omega}_T} \int_t^\infty e^{-\overline{\omega}_T(z-t)} \tilde{v}_z dz + \frac{\underline{\omega}_T - \rho}{\overline{\omega}_T - \underline{\omega}_T} \int_t^\infty e^{-\underline{\omega}_T(z-t)} \tilde{v}_z dz$$
$$\pi_t = -\frac{\kappa}{\overline{\omega}_T - \underline{\omega}_T} \int_t^\infty \left(e^{-\underline{\omega}_T(z-t)} - e^{-\overline{\omega}_T(z-t)} \right) \tilde{v}_z dz,$$

where $\tilde{v}_t \equiv \left[\frac{1-\mu_b}{1-\mu_b\chi_y}\sigma^{-1} + \frac{1-\mu_b}{1-\mu_b\chi_y}\chi_p\epsilon_\lambda \right] u_t - \frac{\mu_b\chi_r}{1-\mu_b\chi_y}\dot{u}_t$.

Using the fact that $u_t = e^{-\psi_m t}u_0$, we obtain

$$y_t = -\frac{\rho + \psi_m}{(\overline{\omega}_T + \psi_m)(\underline{\omega}_T + \psi_m)} \frac{1-\mu_b}{1-\mu_b\chi_y} \left(\sigma^{-1} + \chi_p\epsilon_\lambda + \frac{\mu_b\chi_r}{1-\mu_b}\psi_m \right) u_t$$
$$\pi_t = -\frac{\kappa}{(\overline{\omega}_T + \psi_m)(\underline{\omega}_T + \psi_m)} \frac{1-\mu_b}{1-\mu_b\chi_y} \left(\sigma^{-1} + \chi_p\epsilon_\lambda + \frac{\mu_b\chi_r}{1-\mu_b}\psi_m \right) u_t,$$

where $(\overline{\omega}_T + \psi_m)(\underline{\omega}_T + \psi_m) = \tilde{\sigma}^{-1}\kappa(\tilde{\phi}_\pi - 1) + (\tilde{\delta} + \psi_m)(\rho + \psi_m) > 0$.

The wealth effect is given by

$$\Omega_0 = -\frac{1}{(\overline{\omega}_T + \psi_m)(\underline{\omega}_T + \psi_m)} \frac{1-\mu_b}{1-\mu_b\chi_y} \left(\sigma^{-1} + \chi_p\epsilon_\lambda + \frac{\mu_b\chi_r}{1-\mu_b}\psi_m \right) u_0. \tag{B.65}$$

The nominal interest rate is given by

$$i_t = r_n + \left[1 - \frac{\kappa\phi_\pi}{(\overline{\omega}_T + \psi_m)(\underline{\omega}_T + \psi_m)} \frac{1-\mu_b}{1-\mu_b\chi_y} \left(\sigma^{-1} + \chi_p\epsilon_\lambda + \frac{\mu_b\chi_r}{1-\mu_b}\psi_m \right) u_t \right] u_t$$
$$= r_n + \frac{(\delta + \psi_m)(\rho + \psi_m) - \tilde{\sigma}^{-1}\kappa}{(\tilde{\delta} + \psi_m)(\rho + \psi_m) + \tilde{\sigma}^{-1}\kappa(\tilde{\phi}_\pi - 1)} u_t.$$

Note that if $\psi_m = -\underline{\omega} > 0$, then the nominal interest is given by

$$
\begin{aligned}
i_t - r_n &= \frac{(\delta - \underline{\omega})(\rho - \underline{\omega}) - \tilde{\sigma}^{-1}\kappa}{(\tilde{\delta} - \underline{\omega})(\rho - \underline{\omega}) + \tilde{\sigma}^{-1}\kappa(\tilde{\phi}_\pi - 1)} u_t \\
&= \frac{\rho\delta - \tilde{\sigma}^{-1}\kappa - \underline{\omega}(\rho + \delta) + \underline{\omega}^2}{(\tilde{\delta} - \underline{\omega})(\rho - \underline{\omega}) + \tilde{\sigma}^{-1}\kappa(\tilde{\phi}_\pi - 1)} u_t \\
&= \frac{\overline{\omega}\underline{\omega} - \underline{\omega}(\overline{\omega} + \underline{\omega}) + \underline{\omega}^2}{(\tilde{\delta} - \underline{\omega})(\rho - \underline{\omega}) + \tilde{\sigma}^{-1}\kappa(\tilde{\phi}_\pi - 1)} u_t = 0, \qquad \text{(B.66)}
\end{aligned}
$$

using the fact that $\overline{\omega}\underline{\omega} = \rho\delta - \tilde{\sigma}^{-1}\kappa$ and $\overline{\omega} + \underline{\omega} = \rho + \delta$.

Despite the zero interest rate, the impact on output and inflation is non-zero. In particular, the outside wealth effect is given by

$$
\Omega_0 = -\frac{1}{(\overline{\omega}_T - \underline{\omega})(\underline{\omega}_T - \underline{\omega})} \left(\frac{1 - \mu_b}{1 - \mu_b \chi_y} (\sigma^{-1} + \chi_p \epsilon_\lambda) - \frac{\mu_b \chi_r}{1 - \mu_b \chi_y} \underline{\omega} \right) u_0. \qquad \text{(B.67)}
$$

Implementability condition. Suppose $u_t = \nu e^{-\psi_m t}(i_0 - r_n) + \theta e^{\underline{\omega} t}$ and denote by $(\bar{i}_t, \overline{y}_t, \overline{\pi}_t)$ the value of the nominal interest rate, output, and inflation under the Taylor rule. Given the linearity of the system, the solution will be sum of the solutions for $u_{1,t} = \nu e^{-\psi_m t}(i_0 - r_n)$ and $u_{2,t} = \theta e^{\underline{\omega} t}$. The nominal interest rate is then given by

$$
\begin{aligned}
\bar{i}_t - r_n &= \frac{(\delta + \psi_m)(\rho + \psi_m) - \tilde{\sigma}^{-1}\kappa}{(\tilde{\delta} + \psi_m)(\rho + \psi_m) + \tilde{\sigma}^{-1}\kappa(\tilde{\phi}_\pi - 1)} \nu e^{-\psi_m t}(i_0 - r_n) + \frac{(\delta - \underline{\omega})(\rho - \underline{\omega}) - \tilde{\sigma}^{-1}\kappa}{(\tilde{\delta} - \underline{\omega})(\rho - \underline{\omega}) + \tilde{\sigma}^{-1}\kappa(\tilde{\phi}_\pi - 1)} \nu e^{\underline{\omega} t} \\
&= e^{-\psi_m t}(i_0 - r_n), \qquad \text{(B.68)}
\end{aligned}
$$

using the fact that the nominal interest rate is zero under $u_{2,t}$ and

$$
\nu = \frac{(\tilde{\delta} + \psi_m)(\rho + \psi_m) + \tilde{\sigma}^{-1}\kappa(\tilde{\phi}_\pi - 1)}{(\delta + \psi_m)(\rho + \psi_m) - \tilde{\sigma}^{-1}\kappa} = \frac{(\overline{\omega}_T + \psi_m)(\underline{\omega}_T + \psi_m)}{(\overline{\omega} + \psi_m)(\underline{\omega} + \psi_m)}. \qquad \text{(B.69)}
$$

The outsider wealth effect $\overline{\Omega}_0 = \int_0^\infty e^{-\rho t} \overline{y}_t dt$ is given by

$$
\overline{\Omega}_0 = -\frac{\frac{1-\mu_b}{1-\mu_b\chi_y}(\sigma^{-1} + \chi_p \epsilon_\lambda) + \frac{\mu_b \chi_r}{1-\mu_b\chi_y}\psi_m}{(\overline{\omega}_T + \psi_m)(\underline{\omega}_T + \psi_m)} \nu(i_0 - r_n) - \frac{\frac{1-\mu_b}{1-\mu_b\chi_y}\sigma^{-1} - \frac{\mu_b\chi_r}{1-\mu_b\chi_y}\underline{\omega}}{(\overline{\omega}_T - \underline{\omega})(\underline{\omega}_T - \underline{\omega})}\theta \qquad \text{(B.70)}
$$

To implement a $\overline{\Omega}_0 = \Omega_0$, we must choose θ as follows

$$
\theta = \frac{(\overline{\omega}_T + |\underline{\omega}|)(\underline{\omega}_T + |\underline{\omega}|)}{\frac{1-\mu_b}{1-\mu_b\chi_y}(\sigma^{-1} + \chi_p\epsilon_\lambda) + \frac{\mu_b\chi_r}{1-\mu_b\chi_y}|\underline{\omega}|} (\Omega_0^{AR(1)} - \Omega_0), \qquad \text{(B.71)}
$$

where $\Omega_0^{AR(1)} = -\frac{1}{(\overline{\omega}_T + \psi_m)(\underline{\omega}_T + \psi_m)} \left(\frac{1-\mu_b}{1-\mu_b\chi_y}\sigma^{-1} + \frac{\mu_b\chi_r}{1-\mu_b\chi_y}\psi_m \right) \nu(i_0 - r_n)$.

Given the process for u_t and the values of ν and θ, output can be written as

$$\begin{aligned}\overline{y}_t &= -\frac{\rho+\psi_m}{(\overline{\omega}+\psi_m)(\underline{\omega}+\psi_m)}\frac{1-\mu_b}{1-\mu_b\chi_y}\left(\sigma^{-1}+\chi_p\epsilon_\lambda+\frac{\mu_b\chi_r}{1-\mu_b}\psi_m\right)e^{-\psi_m t}(i_0-r_n)-(\rho-\underline{\omega})\left(\Omega_0^{AR(1)}-\Omega_0\right)e^{\underline{\omega}t} \\ &= \frac{(\rho-\underline{\omega})e^{\underline{\omega}t}-(\rho+\psi_m)e^{-\psi_m t}}{(\overline{\omega}+\psi_m)(\underline{\omega}+\psi_m)}\frac{1-\mu_b}{1-\mu_b\chi_y}\left(\sigma^{-1}+\chi_p\epsilon_\lambda+\frac{\mu_b\chi_r}{1-\mu_b}\psi_m\right)(i_0-r_n)+(\overline{\omega}-\delta)e^{\underline{\omega}t}\Omega_0 \\ &= \sigma^{-1}\hat{y}_t+\chi_p\epsilon_\lambda\hat{y}_t+\frac{\mu_b\chi_r}{1-\mu_b}\psi_m\hat{y}_t+(\overline{\omega}-\delta)e^{\underline{\omega}t}\Omega_0 \qquad \text{(B.72)}\end{aligned}$$

which coincides with (16), where we used $\rho-\underline{\omega}=\overline{\omega}-\delta$. This result also implies that $\overline{\pi}_t=\pi_t$, as $\overline{\pi}_t = \kappa\int_0^\infty e^{-\rho t}\overline{y}_t dt=\kappa\int_0^\infty e^{-\rho t}y_t dt=\pi_t$.

Fiscal transfers. To determine the value of fiscal transfers to savers, we need first to solve for the real interest rate and for the wealth effect. The real interest rate is given by

$$\begin{aligned}i_t-\pi_t-r_n &= \left[1-\frac{\kappa(\phi_\pi-1)}{(\overline{\omega}_T+\psi_m)(\underline{\omega}_T+\psi_m)}\left(\frac{1-\mu_b}{1-\mu_b\chi_y}\sigma^{-1}+\frac{\mu_b\chi_r}{1-\mu_b\chi_y}\psi_m\right)\right]u_t \\ &= \frac{(\rho+\psi_m)\left(\delta+\frac{\mu_b\chi_r\kappa}{1-\mu_b\chi_y}+\psi_m\right)}{\tilde{\sigma}^{-1}\kappa(\phi_\pi-1)+(\tilde{\delta}+\psi_m)(\rho+\psi_m)}u_t.\end{aligned}$$

The wealth effect is given by

$$\Omega_0=-\frac{1}{(\overline{\omega}_T+\psi_m)(\underline{\omega}_T+\psi_m)}\left(\frac{1-\mu_b}{1-\mu_b\chi_y}\sigma^{-1}+\frac{\mu_b\chi_r}{1-\mu_b\chi_y}\psi_m\right)u_0.$$

Note that an increase in u_0 always creates a negative wealth effect regardless of the sign of the response of interest rates. The present discounted value of transfers satisfies

$$\begin{aligned}\int_0^\infty e^{-\rho t}\mu_s T_{s,t}dt &= \int_0^\infty e^{-\rho t}\left[(\tau-\mu_b T_b'(Y))y_t-\overline{d}_g(i_t-\pi_t-r_n)\right]dt \\ &= (\tau-\mu_b T_b'(Y))\Omega_0-\overline{d}_g\frac{\epsilon_{r,u}u_0}{\rho+\psi_m}.\end{aligned}$$

□

C Empirical Evidence on the Fiscal Response to Monetary Shocks

We estimate the empirical IRFs using a VAR identified by a recursiveness assumption, as in Christiano et al. (1999), extended to include fiscal variables.

The variables included in the VAR are: real GDP per capita, CPI inflation, real consumption per capita, real investment per capita, capacity utilization, hours worked per capita, real wages, tax revenues over GDP, government expenditures per capita, the federal funds rate, the 5-year constant maturity rate, and the real value of government debt per capita. We estimate a four-lag VAR using quarterly data for the period 1962:1-2007:3. The identification assumption of the monetary shock is as follows: the only variables that are allowed to react contemporaneously to the monetary policy shock are the federal funds rate, the 5-year rate and the value of government debt. All other variables, including government tax revenues and expenditures, are allowed to react with a lag of one quarter. This assumption is the natural extension of Christiano et al. (1999): while agents' decisions (with agents, in our case, including households and the government) cannot react to the shock contemporaneously, financial variables (in our case, the federal funds rate, the 5-year rate, and the value of government debt) immediately incorporate the information of the shock.

Data sources All the variables are obtained from standard sources (see below), except for the real value of debt, which we construct from the series provided by Hall et al. (2018).[33] These data provide the market value of government debt held by private investors at a monthly frequency from 1776 through 2018. We transform the series into quarterly frequency by keeping the market value of debt in the first month of the quarter. This choice is meant to avoid capturing changes in the market value of debt arising from changes in the *quantity* of debt after a monetary shock instead of changes in *prices*.

The data sources are:

Nominal GDP: BEA Table 1.1.5 Line 1
Real GDP: BEA Table 1.1.3 Line 1
Consumption Durable: BEA Table 1.1.3 Line 4
Consumption Non Durable: BEA Table 1.1.3 Line 5
Consumption Services: BEA Table 1.1.3 Line 6
Private Investment: BEA Table 1.1.3 Line 7
GDP Deflator: BEA Table 1.1.9 Line 1
Capacity Utilization: FRED CUMFNS
Hours Worked: FRED HOANBS
Nominal Hourly Compensation: FRED COMPNFB
Civilian Labor Force: FRED CNP16OV
Nominal Revenues: BEA Table 3.1 Line 1
Nominal Expenditures: BEA Table 3.1 Line 21
Nominal Transfers: BEA Table 3.1 Line 22
Nominal Gov't Investment: BEA Table 3.1 Line 39
Nominal Consumption of Net Capital: BEA Table 3.1 Line 42
Effective Federal Funds Rate (FF): FRED FEDFUNDS
5-Year Treasury Constant Maturity Rate: FRED DGS5

[33] For recent work using a similar data construction, see e.g., Cochrane (2019) and Jiang et al. (2019).

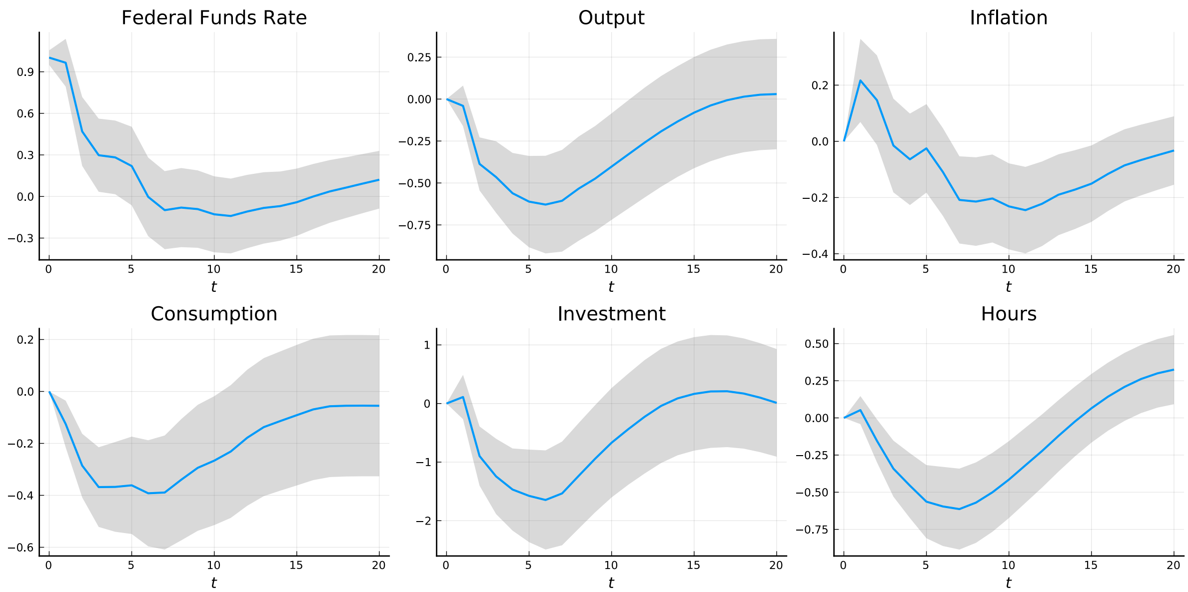

Figure C.1: Estimated IRFs.

Market Value of Government Debt: Hall et al. (2018)

VAR estimation. Figure C.1 shows the results. As is standard in the literature, we find that a contractionary monetary shock increases the federal funds rate and reduces output and inflation on impact. Moreover, the contractionary monetary shock reduces consumption, investment, and hours worked.

The Government's Intertemporal Budget Constraint The fiscal response in the model corresponds to the present discounted value of fiscal transfers over an infinite horizon, that is, $\sum_{t=0}^{\infty} \tilde{\beta}^t T_t$, where $\tilde{\beta} = \frac{1-\lambda}{1+\rho_s}$. We next consider the empirical counterpart of this quantity. First, we calculate a truncated intertemporal budget constraint from period zero to $\mathcal{T}$:

$$\underbrace{b_y b_0}_{\substack{\text{debt}\\ \text{revaluation}}} = \sum_{t=0}^{\mathcal{T}} \tilde{\beta}^t \left[\underbrace{\tau y_t + \tau_t}_{\text{tax revenue}} - \underbrace{\tilde{\beta}^{-1} b_y (i_{t-1}^m - \pi_t - r^n)}_{\text{interest payments}} \right] - \underbrace{T_{0,\mathcal{T}} + \tilde{\beta}^{\mathcal{T}} b_y b_{\mathcal{T}}}_{\substack{\text{other transfers/expenditures}\\ \text{\& final debt}}} \tag{C.1}$$

The right-hand side of (C.1) is the present value of the impact of a monetary shock on fiscal accounts. The first term represents the change in revenues that results from the real effects of monetary shocks. If a contractionary monetary shock generates a recession, government revenues will naturally decrease as a consequence, both because output decreases and because the average tax decreases if the tax system is progressive. The second term represents the change in interest payments on government debt that results from change in nominal rates. For example, a contractionary monetary shock increases nominal payments on government debt. The last two terms are adjustments in transfers and other government expenditures, and the final debt position at period $\mathcal{T}$, respectively. In particular, $T_{0,\mathcal{T}}$ represents the present discounted value of transfers from period 0 through $\mathcal{T}$. Provided that $\mathcal{T}$ is large enough, such that (y_t, τ_t, i_t) have

	(1) Revenues	(2) Interest Payments	(3) Transfers & Expenditures	(4) Debt in T	(5) Initial Debt	(1) - (2) - (3) + (4) - (5) Residual
Data	-26 [-72.89,20.89]	68.88 [30.01,107.75]	-12.09 [-48.74,24.56]	2.91 [-12.79,18.62]	-49.74 [-68.03,-31.46]	30.13 [-4.74,65]

Table C.1: The impact on fiscal variables of a monetary policy shock

Note: Calculations correspond to a a 100 bps unanticipated interest rate increase. Confidence interval at 95% confidence level.

essentially converged to the steady state, then the value of debt at the terminal date, $b_{\mathcal{T}}$, equals (minus) the present discounted value of transfers and other expenditures from period $\mathcal{T}$ onward. Hence, the last two terms combined can be interpreted as the present discounted value of fiscal transfers from zero to infinity.

The left-hand side represents the revaluation effect of the *initial* stock of government debt. In the presence of long-term bonds, a contractionary monetary shock reduces the initial value of government bonds. Hence, part of the adjustment in response to the shock comes from a reduction in the value of debt, instead of coming entirely from raising present or future taxes.

Table C.1 shows the impact on the fiscal accounts of a monetary policy shock, both in the data and in the estimated model. We start by testing whether our estimate of the fiscal response to a monetary shock is consistent with the government's intertemporal budget constraint. To test this, we apply equation (C.1) to the data and check whether the difference between the left-hand side and the right-hand side is different from zero. We decompose the fiscal response in the data into six groups: the present value (PV) of revenues, the PV of interest payments, the PV of transfers and expenditures, the final value of debt, the initial value of debt, and a residual. The residual is calculated as

$$\text{Residual} = \text{Revenues - Interest Payments - Transfers + Debt in } \mathcal{T} \text{ - Initial Debt}$$

We truncate the calculations to quarter 60, that is, $\mathcal{T} = 60$ (15 years) in equation (C.1). The results reported in Table C.1 imply that we cannot reject the possibility that the residual is zero and, therefore, we cannot reject the possibility that the intertemporal budget constraint of the government is satisfied in our estimation.

The adjustment of the fiscal accounts in the data corresponds to the patterns we observed in Figure 1. The contractionary monetary policy shock leads to an increase in the present value of interest payments and of transfers and expenditures. The present value of revenues drops in response to the shock, mostly as a result of the recession generated by the monetary shock. The response of initial debt is quantitatively important, and it accounts for the bulk of the adjustment in the fiscal accounts.

EBP. To estimate the response of the corporate spread in the data, we add the EBP measure of Gilchrist and Zakrajšek (2012) into our VAR (ordered after the fed funds rate). Since the EBP is only available starting in 1973, we reduce our sample period to 1973:1-2007:7. The estimated IRFs are in line with those obtained for the longer sample. We find a significant increase of the EBP on impact, of 6.5 bps, in line with the estimates reported in Gertler and Karadi (2015).

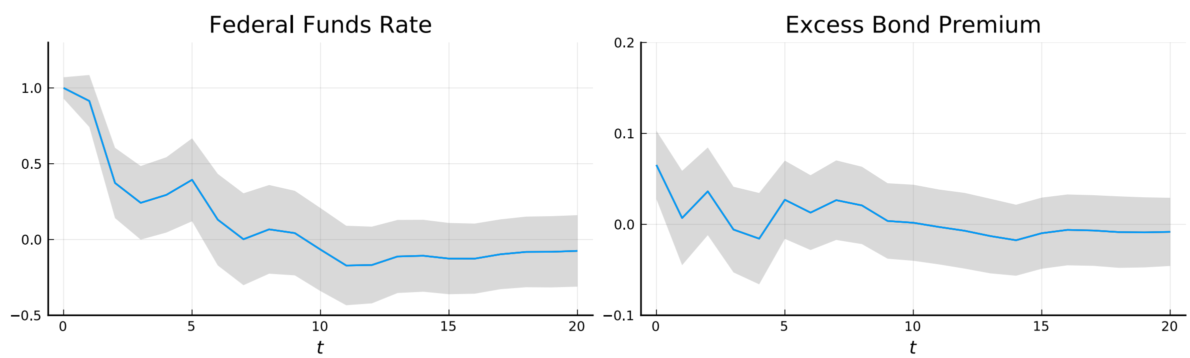

Figure C.2: IRFs for the federal funds rate and excess bond premium.