Dynamic Banking and the Value of Deposits

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 - A dynamic model of depository institution with endogenous risk-taking, deposit-taking, short-term borrowing, payout policy, equity issuance



US banks

Cash-rich US banks move to reduce corporate deposits

JPMorgan Chase and Citigroup take unusual step to avoid additional capital requirement

Imani Moise in New York MAY 4 2021



Banks including JPMorgan Chase and Citigroup have held conversations with some large corporate clients about putting cash into money market funds rather than in deposits, according to people briefed on the talks.

Deposits held at the three largest US banks by assets — JPMorgan, Bank of America and Citi climbed \$243bn in the first three months of the year, on top of a record \$1tn inflow last year. In 2019 they rose by \$92bn.

Introduction

Model

Results

- ullet A_t loans: return $rac{dA_t}{A_t}=(r+lpha_A)\,dt+\sigma_A d\mathcal{W}_t^A$
- B_t short-term bonds: interest expenses r dt
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- X_{t} deposits: $\frac{dX_{t}}{X_{t}} = -\left(\delta_{X}dt \sigma_{X}d\mathcal{W}_{t}^{X}\right) + n\left(i_{t}\right)dt$, with $n'\left(i_{t}\right) > 0$
 - Net withdrawal rate: $\delta_X dt \sigma_X d\mathcal{W}_t^X$, $\mathit{corr.}\left(d\mathcal{W}_t^X,\ d\mathcal{W}_t^A\right) = \phi dt$
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- U_t is the cumulative payout and F_t is the cumulative issuances
- Under equity issuance costs H_t : max $\mathbb{E}\left[\int_{t=0}^{\infty}e^{-\rho t}\left(dU_t-dF_t-dH_t\right)\right]$

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- 2 state variables: equity capital K_t and deposit stock X_t
 - A transformation from (K_t, X_t) to (k_t, X_t) where $k_t = K_t/X_t$
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- Equity-to-deposit ratio, $k_t \equiv K_t/X_t \in [\underline{k}, \overline{k}]$, drives the choice variables
 - 5 control variables: risky asset A_t (liquid \rightarrow no coordination failure/run), short-term borrowing B_t , deposit rate i_t , payout dU_t , equity issuance dF_t

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 - $V_{KK}\left(K_t,X_t\right)<0$, risk-averse towards K_t fluctuation under issuance costs (Brunnermeier, Sannikov,2014; Klimenko, Pfeil, Rochet, Nicolo,2016; Phelan,2016)

Dynamic Optimization

Dynamic Optimization under Parametric Choices

Results: Optimal Deposit Rate

$$i(k) = \frac{\frac{V_X(X,K)}{V_K(X,K)} - \frac{1}{\omega}}{\theta\omega} = \frac{\frac{v(k) - v'(k)k}{v'(k)} - \frac{1}{\omega}}{\theta\omega}$$

- \bullet ω : semi-elasticity of deposit demand (Drechsler, Savov, Schnabl, 2017)
 - The rate-dependent component of deposit flow: $n(i)dt = \omega idt$ calibration



• θ : the convex cost of running deposit franchise $C\left(n\left(i\right),X\right)=\frac{\theta n\left(i\right)^{2}}{2}X$

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 - Adjusted Q, $\frac{V_X(X,K)}{V_K(X,K)}$: building the deposit base vs. earnings

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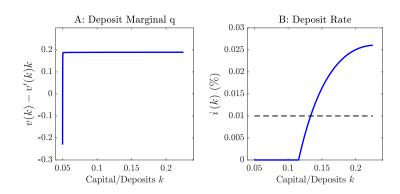
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- Deposit-rate lower bound: $i(k) \ge 0$ (Heider, Saidi, Schepens, 2019)

Results: Deposit Marginal q and Optimal Deposit Rate



$$\begin{split} d\mathcal{K}_t &= A_t \left[(r + \alpha_A) \ dt + \sigma_A d\mathcal{W}_t^A \right] - B_t r dt - X_t i_t dt - C \left(n \left(i_t \right), X_t \right) dt - dU_t + dF_t \\ &\quad C \left(n \left(i_t \right), X_t \right) = \frac{\theta}{2} n (i_t)^2 X_t \text{ and balance-sheet identity } X_t + K_t = A_t - B_t \quad \Rightarrow \\ dK_t &= K_t r dt + A_t \left(\alpha_A dt + \sigma_A d\mathcal{W}_t^A \right) + X_t \underbrace{\left[r - i_t - \frac{\theta}{2} n (i_t)^2 \right]}_{\text{net deposit spread} > 0} dt - dU_t + dF_t \end{split}$$

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 - \rightarrow Deposit marginal q, $V_X(X, K)$, turns negative as k falls to \underline{k}

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- The distance between r and 0 measures the flexibility in managing deposits
 - Low r: deposit risk management is more difficult and bank value declines

Results: Risk-Taking

$$\frac{A}{K} = \frac{K + X + B}{K} = \frac{\alpha_A}{\gamma(k)\sigma_A^2} + \frac{\sigma_X}{\sigma_A}\phi$$

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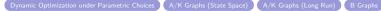
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- $\frac{\sigma_X}{\sigma_A}\phi$: Hedging the "background risk" in deposit-flow uncertainty
 - A large literature on the synergy between deposit-taking and lending

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- $\frac{\sigma_X}{\sigma_A}\phi$: Hedging the "background risk" in deposit-flow uncertainty
 - A large literature on the synergy between deposit-taking and lending









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- SLR was restored in 2021 (to prevent banks being "lazy", holding bonds?)
 - Banks will take risk, and the outcome depends on what kind of risk

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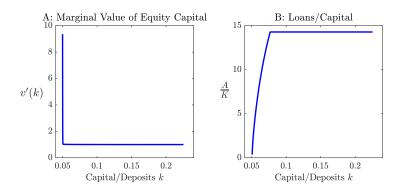


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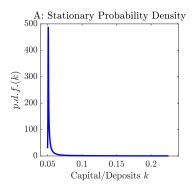
- A dynamic model of depository institution with practical applications:
 - (1) procyclical risk-taking; (2) procyclical short-term debt; (3) procyclical dividend payout; (4) countercyclical equity issuance; (5) jump risk graphs

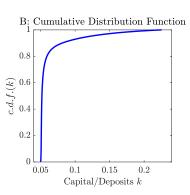
Equity Capital Marginal q and Risk-Taking



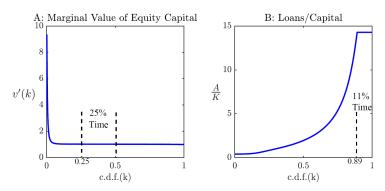
1/16

Stationary Distribution of Capital-Deposit Ratio





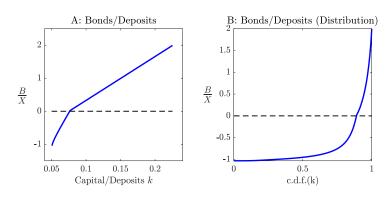
Equity Capital Marginal q and Risk-Taking over the Long Run



 Capital requirement does not always bind (Gropp, Heider, 2010; Begenau, Bigio, Majerovitz, Vieyra, 2019)

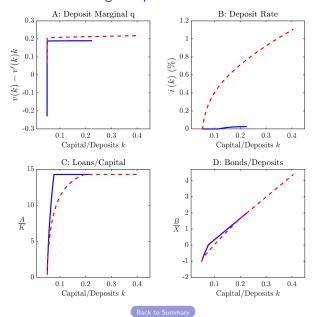
Back to Risk-Taking

Short-Term Debts

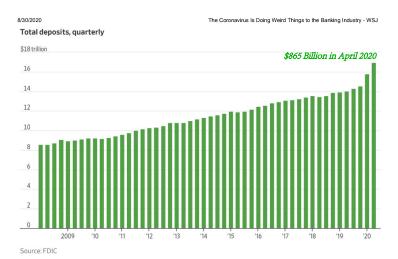


Back to Risk-Taking

Introducing Jump Risk in Loan Returns

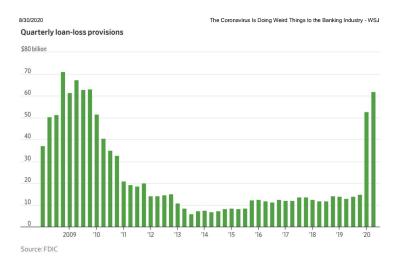


What Happened during Covid-19 - Deposit Influx



Back to Summary

What Happened during Covid-19 - Weakened Capital



Back to Summary

What Happened during Covid-19 - Depressed Valuation



- Deposits pay an interest rate below the prevailing risk-free rate
 - Banks have deposit market power (Drechsler, Savov, and Schnabl, 2017)
 but deposits are short-term debts
 - Deposits as means of payment: short-term debts with convenience yield

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 - ullet Deposit marginal q can be negative, and inflow implies future risk



Calibration

Table: PARAMETER VALUES

Parameters	Symbol	Value	Target
risk-free rate	r	1%	FRED: Fed Fund Rate
discount rate	ρ	4.5%	Literature
bank excess return	α_A	0.2%	FRED: Bank ROA
asset return volatility	$\sigma_{\mathcal{A}}$	10%	Literature
deposit flow (mean)	δ_X	0	Literature
deposit flow (volatility)	σ_{χ}	5%	Literature
deposit maintenance cost	θ	0.5	Deposits/Total Liabilities
deposit demand semi-elasticity	ω	5.3	Literature
corr. between deposit and asset shocks	φ	8.0	Prob.(Capital Requirement Binds)
equity issuance fixed cost	ψ_0	0.1%	Issuance-to-Equity Ratio
equity issuance propositional cost	ψ_1	5.0%	Literature
SLR requirement parameter	ξ̃L	20	Regulation
capital requirement parameter	ξκ	14.3	Regulation

Back to Deposit Rate

Model: Optimization – HJB Equation

- Payout, dU_t , and issuance, dF_t , set boundaries for bank capital K_t
 - In the interior region, the value function satisfies the HJB equation

$$\begin{split} \rho V\left(X,K\right) &= \max_{\pi^{A},\,i} \,\, V_{X}\left(X,K\right) \left[-X\delta_{X} + n\left(i\right)X\right] + \frac{1}{2}V_{XX}\left(X,K\right)X^{2}\sigma_{X}^{2} \\ &+ V_{K}\left(X,K\right)\left(X+K\right)\left(r+\pi^{A}\alpha_{A}\right) - V_{K}\left(X,K\right)\left[iX+C\left(n\left(i\right),X\right)\right] \\ &+ \frac{1}{2}V_{KK}\left(X,K\right)\left(X+K\right)^{2}\left(\pi^{A}\sigma_{A}\right)^{2} + V_{XK}\left(X,K\right)\left(X+K\right)\pi^{A}\sigma_{A}X\sigma_{X}\phi \,. \end{split}$$

- The bank controls $\pi^A = A/(X+K)$ and i
 - Given states X and K, B/S identity, A = X + B + K, implies B



Model: Equity Issuance and Payout

• The bank raises equity only if

$$V\left(X,K+dF_{t}
ight)-V\left(X,K
ight)\geq dF_{t}+dH_{t}=\psi_{0}X+\left(1+\psi_{1}\right)M_{t}$$
 .

- Capital raised: $dF_t = M_t$, given by $V_K(X, K + M_t) = 1 + \psi_1$
- Issuance costs: $dH_t = \psi_0 X + \psi_1 M_t$
 - Fix cost scaled by X for value function be homogeneous in X
- The bank pays out dividend only if

$$V\left(X,K
ight)-V\left(X,K-dU_{t}
ight)\leq dU_{t}$$
 i.e., $V_{K}\left(X,K
ight)\leq1$.

- Optimality and smooth-pasting condition: $V_{KK}(X, K) = 0$



Model: Optimal Risk-Taking

$$\frac{A}{K} = \min \left\{ \frac{\alpha_{A} + \epsilon \left(X, K \right) \sigma_{A} \sigma_{X} \phi}{\gamma \left(X, K \right) \sigma_{A}^{2}} , \ \xi_{K} \right\}$$

Endogenous Risk Aversion:
$$\gamma(X, K) \equiv \frac{-V_{KK}(X, K) K}{V_{K}(X, K)}$$

Hedging Motive:
$$\epsilon(X, K) \equiv \frac{V_{XK}(X, K) X}{V_{K}(X, K)}$$

- γ : Concavity, $V_{KK}(X, K) < 0$, from the equity issuance costs
- ϵ : Hedging motive from background risk in the randomness of deposit flow



Model: Optimal Deposit Rate

$$V_{X}\left(X,K\right)n'\left(i\right) = V_{K}\left(X,K\right)\left[1 + C_{n}\left(n\left(i\right),X\right)n'\left(i\right)\right] \ .$$

LHS: Marginal benefit of adding deposits

RHS: Marginal cost of paying deposit rates and deposit maintenance costs (from adding more deposits)



Model: Solution under Homogeneity

ullet State space transformation: $(K_t, X_t) o (k_t, X_t)$ where

$$k_t = \frac{K_t}{X_t}$$

• Value function: V(X, K) = v(k)X and HJB equation (ODE)

$$\begin{split} \rho v\left(k\right) &= \max_{\pi^{A},i} \; \left[v\left(k\right) - v'\left(k\right) \, k \right] \left(-\delta_{X} + \omega i \right) + \frac{1}{2} v''\left(k\right) \, k^{2} \sigma_{X}^{2} \\ &+ v'\left(k\right) \left(1 + k\right) \left(r + \pi^{A} \alpha_{A}\right) + \frac{1}{2} v''\left(k\right) \left(1 + k\right)^{2} \left(\pi^{A} \sigma_{A}\right)^{2} \\ &- v'\left(k\right) \left[i + \frac{\theta}{2} \left(\omega i\right)^{2} \right] - v''\left(k\right) \, k \left(1 + k\right) \pi^{A} \sigma_{A} \sigma_{X} \phi \,. \end{split}$$

- The bank controls $\pi^A = A/(X+K)$ and i
 - Given states X and K, B/S identity, A = X + B + K, implies B

Back to Solution Back to Risk-Taking Back to Deposit Rate Optimal Payout and Issuance Optimal π^A Optimal i

Model: Equity Issuance and Payout under Homogeneity

ullet The bank pays out dividend at $k_t=\overline{k}$ with the ODE boundary

$$v'(\overline{k})=1$$
,

- Optimality and smooth-pasting condition: $v''\left(\overline{k}\right)=0$ to pin down \overline{k}
- The bank raises equity at $k_t = \underline{k}$ with the ODE boundary

$$v'\left(\underline{k}+m
ight)=1+\psi_1$$
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- Capital raised: $dm=M_t/X_t$, given by $v\left(\underline{k}+m\right)-v\left(\underline{k}\right)=\psi_0+(1+\psi_1)\,m$
- Determining \underline{k} : v(k) is globally concave so $\underline{k} = 0$
- SLR: the bank raises equity to stay in compliance (X+K) / $K=rac{1}{k}+1\leq \xi_L$

