Dynamic Banking and the Value of Deposits

Patrick Bolton 
Ye Li 
Neng Wang 
Jinqiang Yang

Columbia & Imperial College 
OSU 
Columbia 
SUFIE
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- Depositors accept a low rate for payment convenience ("money premium")
- Banks face uncertainty in deposit flows

A dynamic model of depository institution with endogenous risk-taking, deposit-taking, short-term borrowing, payout policy, equity issuance.
A Model of Depository Institutions

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US banks

Cash-rich US banks move to reduce corporate deposits

JPMorgan Chase and Citigroup take unusual step to avoid additional capital requirement

Imani Moise in New York MAY 4 2021

Banks including JPMorgan Chase and Citigroup have held conversations with some large corporate clients about putting cash into money market funds rather than in deposits, according to people briefed on the talks.

Deposits held at the three largest US banks by assets — JPMorgan, Bank of America and Citi — climbed $243bn in the first three months of the year, on top of a record $1tn inflow last year. In 2019 they rose by $92bn.
Introduction

Model

Results
Model: The Setup

- $A_t$ loans: return $\frac{dA_t}{A_t} = (r + \alpha_A) \, dt + \sigma_A \, d\mathcal{W}_t^A$

- $B_t$ short-term bonds: interest expenses $r \, dt$
  - The bank issues bonds when $B_t > 0$ and holds risk-free asset when $B_t < 0$
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- $X_t$ deposits: \[ \frac{dX_t}{X_t} = - (\delta_X \, dt - \sigma_X d\mathcal{W}_t^X) + n (i_t) \, dt, \text{ with } n' (i_t) > 0 \]
  - Net withdrawal rate: $\delta_X \, dt - \sigma_X d\mathcal{W}_t^X$, corr. $(d\mathcal{W}_t^X, d\mathcal{W}_t^A) = \phi \, dt$
  - Effectively deposits have long duration (Drechsler, Savov, Schnabl, 2021)
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  - The costs of running deposit franchise: $C(n(i_t), X_t) dt$
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- The law of motion of equity capital $K_t$: \[ dK_t = A_t \left[ (r + \alpha_A) \, dt + \sigma_A d\mathcal{W}_t^A \right] - B_t r dt - X_t i_t dt - C \left( n \left( i_t \right), X_t \right) dt - dU_t + dF_t \]
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  $$dK_t = A_t \left[ (r + \alpha_A) \ dt + \sigma_A dW^A_t \right] - B_t r dt - X_t i_t dt - C(n(i_t), X_t) \ dt - dU_t + dF_t$$
  - $U_t$ is the cumulative payout and $F_t$ is the cumulative issuances
  - Under equity issuance costs $H_t$: max $\mathbb{E} \left[ \int_{t=0}^{\infty} e^{-\rho t} (dU_t - dF_t - dH_t) \right]$
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Results: Endogenous Risk Aversion, Payout Policy and Equity Issuance

- 2 state variables: equity capital $K_t$ and deposit stock $X_t$
  - A transformation from $(K_t, X_t)$ to $(k_t, X_t)$ where $k_t = K_t / X_t$
  - The value function $V_t = V(K_t, X_t) = v(k_t)X_t$

- Equity-to-deposit ratio, $k_t \equiv K_t / X_t \in [k, k_0]$ drives the choice variables
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  - Equity capital marginal q: $V_K(K_t, X_t) = v'(k)$
  - Deposit marginal q: $V_X(K_t, X_t) = v(k) - v'(k)k$

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- Equity-to-deposit ratio, $k_t \equiv K_t / X_t \in [k, \bar{k}]$, drives the choice variables
  - 5 control variables: risky asset $A_t$ (*liquid → no coordination failure/run*), short-term borrowing $B_t$, deposit rate $i_t$, payout $dU_t$, equity issuance $dF_t$
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  - \( V_K (K_t, X_t) = v'(k_t) = 1 \) at dividend payout boundary \( k_t = k \)
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  - $V_K(K_t, X_t) = v'(k_t) = 1$ at dividend payout boundary $k_t = \bar{k}$
  - $V_K(K_t, X_t) = v'(k_t) > 1$ for $k_t \in [k, \bar{k})$, highest at issuance boundary $k$
Results: Endogenous Risk Aversion, Payout Policy and Equity Issuance

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  - $V_{kK}(K_t, X_t) < 0$, risk-averse towards $K_t$ fluctuation under issuance costs (Brunnermeier, Sannikov, 2014; Klimenko, Pfeil, Rochet, Nicolo, 2016; Phelan, 2016)
Results: Optimal Deposit Rate

\[ i(k) = \frac{V_X(X,K)}{V_K(X,K)} \frac{1}{\theta \omega} = \frac{v(k) - v'(k)k}{v'(k)} \frac{1}{\theta \omega} \]

- \( \omega \): semi-elasticity of deposit demand (Drechsler, Savov, Schnabl, 2017)
  - The rate-dependent component of deposit flow: \( n(i)dt = \omega idt \) calibration

- \( \theta \): the convex cost of running deposit franchise \( C(n(i), X) = \frac{\theta n(i)^2}{2} X \)
Results: Optimal Deposit Rate

\[ i(k) = \frac{V_X(X,K)}{V_K(X,K)} - \frac{1}{\omega} = \frac{\nu(k) - \nu'(k)k}{\nu'(k)} - \frac{1}{\omega} \]

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- Hayashi “investment” policy, investing in sticky depositor/customer base
  - Adjusted \( Q \), \( \frac{V_X(X,K)}{V_K(X,K)} \): building the deposit base vs. earnings
Results: Optimal Deposit Rate

\[ i(k) = \frac{V_X(X,K)}{V_K(X,K)} - \frac{1}{\omega} \]  
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- Deposit-rate lower bound: \( i(k) \geq 0 \) (Heider, Saidi, Schepens, 2019)
Results: Deposit Marginal $\phi$ and Optimal Deposit Rate

A: Deposit Marginal $\phi$

B: Deposit Rate
Results: The Mechanism of Dynamic Deposit Marginal q

\[ dK_t = A_t \left[ (r + \alpha A) dt + \sigma A dW^A_t \right] - B_t r dt - X_t i_t dt - C (n(i_t), X_t) dt - dU_t + dF_t \]

\[ C(n(i_t), X_t) = \frac{\theta}{2} n(i_t)^2 X_t \] and balance-sheet identity \( X_t + K_t = A_t - B_t \) \( \Rightarrow \)

\[ dK_t = K_t r dt + A_t (\alpha A dt + \sigma A dW^A_t) + X_t \left[ r - i_t - \frac{\theta}{2} n(i_t)^2 \right] dt - dU_t + dF_t \]

\( \text{net deposit spread} > 0 \)
Results: The Mechanism of Dynamic Deposit Marginal q

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\[ dX_t = X_t \left( n (i_t) - \delta_X \right) dt + X_t \sigma_X dW_t^X \]

Deposit inflows are cheap sources of funds, so the bank earns the deposit spread. Add risk to the future trajectory of equity capital:

\[ dW \] net deposit spread > 0

The risk concern dominates when the bank is undercapitalized. The bank is endogenously risk averse under equity issuance costs, so the deposit marginal \( q, V_{X} (X_t, K_t) \), turns negative as \( k \) falls to \( k_{7/10} \).
Results: The Mechanism of Dynamic Deposit Marginal $q$

\[ dK_t = A_t \left[ (r + \alpha_A) dt + \sigma_A dW_t^A \right] - B_t r dt - X_t i_t dt - C \left( n(i_t), X_t \right) dt - dU_t + dF_t \]

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- Deposit inflows ...
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  - add risk to the future trajectory of equity capital: \( dW_t^X < 0 \rightarrow E_t[dK_t] \downarrow \)
Results: The Mechanism of Dynamic Deposit Marginal $q$

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  \[ \rightarrow \text{Deposit marginal } q, V_X(X, K), \text{ turns negative as } k \text{ falls to } k \]
Results: Banking in a Low Interest Rate Environment

- The level of $r$ determines the bank’s flexibility in adjusting its deposit rate $i$.
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- The level of $r$ determines the bank’s flexibility in adjusting its deposit rate $i$
  - Away from $k$ (costly equity issuance), the bank tunes up $i$ so that when $k$ falls, it can tune down $i$ to reduce deposits and de-risk
The level of $r$ determines the bank’s flexibility in adjusting its deposit rate $i$

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- A high $r$ means the bank can adjust $i$ to a high level to build up flexibility when $k$ is high and still earn a positive deposit spread $r - i$
**Results: Banking in a Low Interest Rate Environment**

- The level of $r$ determines the bank’s flexibility in adjusting its deposit rate $i$
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- The distance between $r$ and 0 measures the flexibility in managing deposits
  - Low $r$: deposit risk management is more difficult and bank value declines
Results: Risk-Taking

\[ \frac{A}{K} = \frac{K + X + B}{K} = \frac{\alpha_A}{\gamma(k)\sigma_A^2} + \frac{\sigma_X}{\sigma_A} \phi \]

- Merton’s portfolio choice, wealth \( K \) (equity) and risky asset \( A \) (loans)
- \( \gamma(k) \equiv \frac{-V_{KK}(X,K)K}{V_K(X,K)} = -\frac{v''(k)k}{v'(k)} \) decreases in \( k = K/X \)
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  - \( \frac{A}{K} \) increases in \( k \), so capital requirement limits procyclicality in risk-taking

- Evidence from Copeland, Duffie, and Yang (2021): intraday payment risk

\[ \sigma_X \rightarrow \gamma(k) \rightarrow \text{bank demands safe assets, i.e., } B < 0 \]

- A large literature on the synergy between deposit-taking and lending

- Dynamic Optimization under Parametric Choices

- \( A/K \) Graphs (State Space)

- \( A/K \) Graphs (Long Run)

- \( B \) Graphs

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Results: Risk-Taking

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  - \(\frac{A}{K}\) increases in \(k\), so capital requirement limits procyclicality in risk-taking
  - A high-\(k\) bank uses \(B > 0\) to amplify leverage

Evidence from Copeland, Duffie, and Yang (2021): intraday payment risk \(\sigma_X\) \(\gamma(k)\) \(\phi\): Hedging the “background risk” in deposit-flow uncertainty

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  - A high-\( k \) bank uses \( B > 0 \) to amplify leverage
  
  - A low-\( k \) bank uses \( B < 0 \) (i.e., hold bonds) to de-risk asset side of B/S
Results: Risk-Taking

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\frac{A}{K} = \frac{K + X + B}{K} = \frac{\alpha_A}{\gamma(k)\sigma^2_A} + \frac{\sigma_X}{\sigma_A}\phi
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- \( \gamma(k) \equiv -\frac{V_{KK}(X,K)K}{V_K(X,K)} = -\frac{\nu''(k)k}{\nu'(k)} \) decreases in \( k = \frac{K}{X} \)
  
  - \( A/K \) increases in \( k \), so capital requirement limits procyclicality in risk-taking
  
  - A high-\( k \) bank uses \( B > 0 \) to amplify leverage
  
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Results: Risk-Taking

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- Merton’s portfolio choice, wealth \( K \) (equity) and risky asset \( A \) (loans)

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  - A high-\( k \) bank uses \( B > 0 \) to amplify leverage
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- A dynamic model of depository institution with practical applications:
  - (1) procyclical risk-taking; (2) procyclical short-term debt; (3) procyclical
    dividend payout; (4) countercyclical equity issuance; (5) jump risk
Equity Capital Marginal $q$ and Risk-Taking

A: Marginal Value of Equity Capital

B: Loans/Capital

Back to Risk-Taking
Stationary Distribution of Capital-Deposit Ratio

A: Stationary Probability Density

B: Cumulative Distribution Function
Capital requirement does not always bind (Gropp, Heider, 2010; Begenau, Bigio, Majerovitz, Vieyra, 2019)
Short-Term Debts

A: Bonds/Deposits

\[ \frac{B}{X} \]

Capital/Deposits \( k \)

B: Bonds/Deposits (Distribution)

\[ \frac{B}{X} \]

c.d.f.\( (k) \)

Back to Risk-Taking
Introducing Jump Risk in Loan Returns

A: Deposit Marginal q

B: Deposit Rate

C: Loans/Capital

D: Bonds/Deposits
What Happened during Covid-19 – Deposit Influx

$865 Billion in April 2020

Source: FDIC
What Happened during Covid-19 – Weakened Capital

Quarterly loan-loss provisions

Source: FDIC
What Happened during Covid-19 – Depressed Valuation

S&P 500

KBW Bank Index

Back to Summary
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  - Banks have deposit market power (Drechsler, Savov, and Schnabl, 2017) but deposits are short-term debts
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→ Marginal value of deposits is positive and banks only worry about outflows

- Deposit marginal \( q \) can be negative, and inflow implies future risk
## Calibration

**Table: Parameter Values**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>risk-free rate</td>
<td>$r$</td>
<td>1%</td>
<td>FRED: Fed Fund Rate</td>
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<tr>
<td>discount rate</td>
<td>$\rho$</td>
<td>4.5%</td>
<td>Literature</td>
</tr>
<tr>
<td>bank excess return</td>
<td>$\alpha_A$</td>
<td>0.2%</td>
<td>FRED: Bank ROA</td>
</tr>
<tr>
<td>asset return volatility</td>
<td>$\sigma_A$</td>
<td>10%</td>
<td>Literature</td>
</tr>
<tr>
<td>deposit flow (mean)</td>
<td>$\delta_X$</td>
<td>0</td>
<td>Literature</td>
</tr>
<tr>
<td>deposit flow (volatility)</td>
<td>$\sigma_X$</td>
<td>5%</td>
<td>Literature</td>
</tr>
<tr>
<td>deposit maintenance cost</td>
<td>$\theta$</td>
<td>0.5</td>
<td>Deposits/Total Liabilities</td>
</tr>
<tr>
<td>deposit demand semi-elasticity</td>
<td>$\omega$</td>
<td>5.3</td>
<td>Literature</td>
</tr>
<tr>
<td>corr. between deposit and asset shocks</td>
<td>$\phi$</td>
<td>0.8</td>
<td>Prob.(Capital Requirement Binds)</td>
</tr>
<tr>
<td>equity issuance fixed cost</td>
<td>$\psi_0$</td>
<td>0.1%</td>
<td>Issuance-to-Equity Ratio</td>
</tr>
<tr>
<td>equity issuance propositional cost</td>
<td>$\psi_1$</td>
<td>5.0%</td>
<td>Literature</td>
</tr>
<tr>
<td>SLR requirement parameter</td>
<td>$\xi_L$</td>
<td>20</td>
<td>Regulation</td>
</tr>
<tr>
<td>capital requirement parameter</td>
<td>$\xi_K$</td>
<td>14.3</td>
<td>Regulation</td>
</tr>
</tbody>
</table>
Model: Optimization – HJB Equation

- Payout, \( dU_t \), and issuance, \( dF_t \), set boundaries for bank capital \( K_t \)
  - In the interior region, the value function satisfies the HJB equation

\[
\rho V(X, K) = \max_{\pi^A, i} \left( V_X(X, K) \left[ -X \delta_X + n(i) X \right] + \frac{1}{2} V_{XX}(X, K) X^2 \sigma_X^2 \right.
\]
\[
+ \left. V_K(X, K) (X + K) \left( r + \pi^A \alpha_A \right) - V_K(X, K) \left[ iX + C(n(i), X) \right] \right)
\]
\[
+ \frac{1}{2} V_{KK}(X, K) (X + K)^2 \left( \pi^A \sigma_A \right)^2 + V_{XK}(X, K) (X + K) \pi^A \sigma_A X \sigma_X \phi .
\]

- The bank controls \( \pi^A = A / (X + K) \) and \( i \)
  - Given states \( X \) and \( K \), B/S identity, \( A = X + B + K \), implies \( B \)
The bank raises equity only if

\[ V(X, K + dF_t) - V(X, K) \geq dF_t + dH_t = \psi_0 X + (1 + \psi_1) M_t. \]

Capital raised: \( dF_t = M_t \), given by \( V_K(X, K + M_t) = 1 + \psi_1 \)

Issuance costs: \( dH_t = \psi_0 X + \psi_1 M_t \)
- Fix cost scaled by \( X \) for value function be homogeneous in \( X \)

The bank pays out dividend only if

\[ V(X, K) - V(X, K - dU_t) \leq dU_t \text{ i.e., } V_K(X, K) \leq 1. \]
- Optimality and smooth-pasting condition: \( V_{KK}(X, K) = 0 \)
Model: Optimal Risk-Taking

\[
\frac{A}{K} = \min \left\{ \frac{\alpha_A + \epsilon (X, K) \sigma_A \sigma_X \phi}{\gamma (X, K) \sigma_A^2}, \, \zeta_K \right\}
\]

**Endogenous Risk Aversion:** \( \gamma (X, K) \equiv -\frac{V_{KK} (X, K) K}{V_K (X, K)} \)

**Hedging Motive:** \( \epsilon (X, K) \equiv \frac{V_{XK} (X, K) X}{V_K (X, K)} \)

**\( \gamma \):** Concavity, \( V_{KK} (X, K) < 0 \), from the equity issuance costs

**\( \epsilon \):** Hedging motive from background risk in the randomness of deposit flow
Model: Optimal Deposit Rate

\[ V_X (X, K) \cdot n' (i) = V_K (X, K) \left[ 1 + C_n (n (i), X) \cdot n' (i) \right] . \]

**LHS**: Marginal benefit of adding deposits

**RHS**: Marginal cost of paying deposit rates and deposit maintenance costs (from adding more deposits)

Back to Optimization
Model: Solution under Homogeneity

- State space transformation: \((K_t, X_t) \rightarrow (k_t, X_t)\) where
  \[k_t = \frac{K_t}{X_t}\]

- Value function: \(V(X, K) = v(k)X\) and HJB equation (ODE)
  \[
  \rho v(k) = \max_{\pi^A, i} \left[ v(k) - v'(k)k \right] (-\delta X + \omega i) + \frac{1}{2} v''(k) k^2 \sigma_X^2 \\
  + v'(k) (1 + k) \left( r + \pi^A \alpha_A \right) + \frac{1}{2} v''(k) (1 + k)^2 \left( \pi^A \sigma_A \right)^2 \\
  - v'(k) \left[ i + \frac{\theta}{2} (\omega i)^2 \right] - v''(k) k (1 + k) \pi^A \sigma_A \sigma_X \phi.
  \]

- The bank controls \(\pi^A = A / (X + K)\) and \(i\)
  - Given states \(X\) and \(K\), B/S identity, \(A = X + B + K\), implies \(B\)
The bank pays out dividend at $k_t = \bar{k}$ with the ODE boundary

$$v'(\bar{k}) = 1,$$

- Optimality and smooth-pasting condition: $v''(\bar{k}) = 0$ to pin down $\bar{k}$

The bank raises equity at $k_t = \underline{k}$ with the ODE boundary

$$v'(\underline{k} + m) = 1 + \psi_1,$$

- Capital raised: $dm = M_t / X_t$, given by $v(\underline{k} + m) - v(\underline{k}) = \psi_0 + (1 + \psi_1) m$
- Determining $\underline{k}$: $v(\underline{k})$ is globally concave so $\underline{k} = 0$
- SLR: the bank raises equity to stay in compliance $(X + K) / K = \frac{1}{\underline{k}} + 1 \leq \zeta_L$