Not a Typical Firm: The Joint Dynamics of Firms, Labor Shares, and Capital-Labor Substitution

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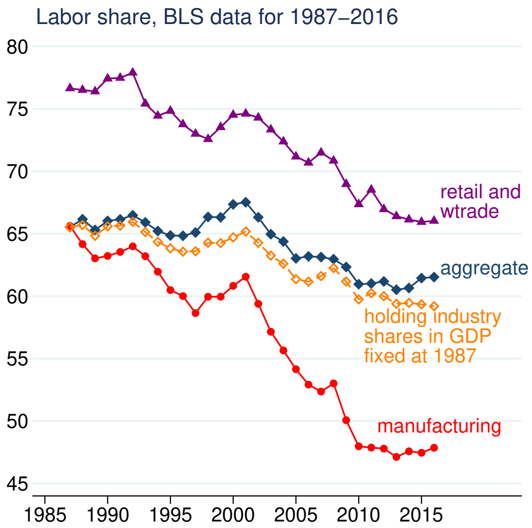
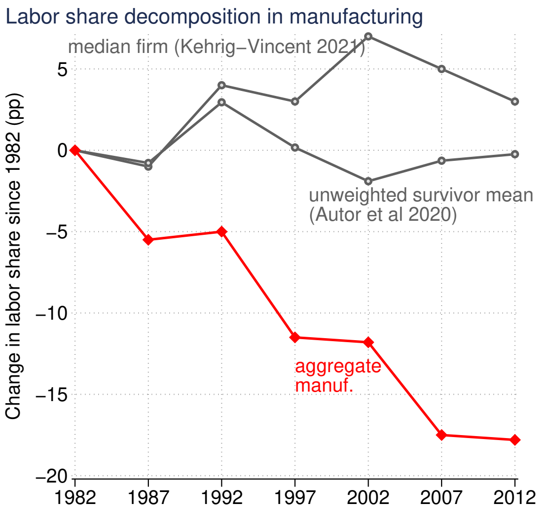
The Decline in the US Labor Share



Two broad explanations:

- Technology: capital–labor substitution, automation
  (Karabarbounis–Neiman 2014; Eden–Gagl 2018; Hubmer 2020; Acemoglu–Restrepo, 2018)
- Concentration: reallocation to high markup firms
  (Barkai 2020; De Loecker–Eeckhout–Unger 2020; Autor–Dorn–Katz–Patterson–V.Reenen 2020)

The Role of Firms in the Decline of the Labor Share



- Decline not uniform across firms: not visible for typical firm (Autor et al. 2020; Kehrig–Vincent 2021)
- Challenges technology view:
  - rules out simple story where all firms face same factor prices and have access to same technologies
  - suggests key role for reallocation rather than capital-labor substitution

This Paper

1. Firm dynamics model with costly K–L substitution to assess technology + micro facts.
   - Key element: fixed cost of automating additional tasks. Matches studies on adoption of new capital-intensive techs (Acemoglu et al. 2020; US Census 2020) details
   - Capital prices ↓: for large firms, labor share ↓ (K–L substitutes); for typical firm, labor share ↑ (K–L complements)
2. Allow for variable markups and quantitatively decompose labor share change into technology and concentration.
3. Direct evidence from firm-level markup estimates supports our findings.
   - Important to allow for differences in technology by firm size.

Model: Firm Production Function

- Task-based production $\Rightarrow$ as if CES production function in K,L:

$$y = z \cdot \left(\alpha^{\frac{1}{\eta}} K^{\frac{\eta-1}{\eta}} + g(\alpha)^{\frac{1}{\eta}} L^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \quad , \quad g'(\cdot) < 0$$

- Firm's share of automated tasks $\alpha \in [0, 1]$: endogenous, fixed cost of adjusting.
- Firm productivity z: exogenous, follows Markov process.

micro foundation

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- Firm's share of automated tasks $\alpha \in [0, 1]$: endogenous, fixed cost of adjusting.
- Firm productivity z: exogenous, follows Markov process.
- Given factor prices, $\alpha^* \equiv$ cost-minimizing $\alpha \Rightarrow$ different notions of K–L elasticities:

K–L elasticity (fixed α) $= \eta$

where $\eta > 0$ is the task substitution elasticity.

K–L elasticity (adjust to α^*) $\eta^* = \eta + \gamma$

where $\gamma > 0$ parametrizes task realloca-tion across production factors.

micro foundation

Model: Non-CES Demand Side

- Kimball aggregator $H(\cdot)$

$$\int_\theta \lambda \cdot H\left(\frac{y(\theta)}{\lambda \cdot Y}\right) m(\theta) d\theta = 1, \quad \theta = (z, \alpha)$$

  where λ is (exogenous) proxy for "market size"

- Normalizing price of final good to 1 yields demand function

$$y(\theta) = Y \cdot \lambda \cdot D\left(\frac{p(\theta)}{\rho}\right), \quad \rho = \text{comp. price index} \neq 1, \quad (H' = D^{-1})$$

- **Key assumptions:** Marshall's second laws: details

$$-\frac{D'(x)}{D(x)} x \text{ greater than 1 and increasing in } x; \qquad x + \frac{D(x)}{D'(x)} \text{ positive and log-concave}$$

  (markups higher for prod firms) (passthroughs lower for prod firms)

Model: Dynamics and Equilibrium

- Value function of incumbent with technology (z, α):

$$V(z,\alpha) = \pi(z,\alpha) + \int \max\left\{0, -c_f + \max_{\alpha' \in [\alpha,1]} \left\{-c_a \cdot (\alpha' - \alpha) + \beta \mathbb{E}\left[V(z', \alpha')|z\right]\right\}\right\} dG(c_f)$$

- Endogenous entry and exit
  - Diffusion of automation through imitation: entrants start at $\alpha_0 = \bar{\alpha}$ (e.g. Perla, Tonetti and Waugh, 2021). Not essential.

- Supply of capital and labor
  - capital supply fully elastic, produced from final good at rate q
  - labor in fixed supply and fully mobile across firms

Model: Transitional Dynamics

- Start economy in steady state ('82) and study two driving forces of labor share over 1982–2012:

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  2. Concentration: increase in λ (rising effective market size) $\rightarrow$ affects primarily markup μ_t

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Labor share (of value added)

of a firm can be written as

$$s_{\ell,t} = \frac{\varepsilon_{\ell,t}}{\mu_t}$$

Same holds for the aggregate economy when taking appropriate averages over firms.

Result: Response of Labor Shares to Falling Capital Price (q-shock)

Proposition

Assume CES demand (fixed markup). Following a uniform increase in q, the economy converges to a new steady state, where the aggregate share of labor in cost changes by

$$d \ln \varepsilon_\ell = \frac{1 - \varepsilon_\ell}{\varepsilon_\ell} \cdot (1 - \eta^*) \cdot d \ln q.$$

At the same time, for an incumbent firm with low realizations of z along the transition,

$$d \ln \varepsilon_\ell(\theta) = \frac{1 - \varepsilon_\ell(\theta)}{\varepsilon_\ell(\theta)} \cdot (1 - \eta) \cdot d \ln q.$$

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$\Rightarrow$ If $\eta < 1 < \eta^*$, then in aggregate $\varepsilon_\ell \downarrow$ but $\varepsilon_\ell(\theta) \uparrow$ for typical firm version with $\eta^* < 1$

$\Rightarrow$ Distribution of firms, each of them exhibits some elasticity $\in [\eta, \eta^*)$

Result: Response of Labor Shares to Increase in Market Size (λ-shock)

Proposition

Assume fixed α (fixed technology). An increase in λ affects stationary equilibrium as follows:

1. *all firms reduce their markups $\mu(z)$ ($\Rightarrow s_\ell \uparrow$)*
2. *for any two firms with $z > z'$, the relative market share of z increases $\frac{\omega(z)}{\omega(z')} \uparrow$ ($\Rightarrow s_\ell \downarrow$)*
3. *latter effect dominates ($s_\ell \downarrow$ on net) iff log-convex z-distribution*

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$\Rightarrow$ Pareto distribution is log-linear benchmark with zero net effect

Calibration: Technology

By sector. First: manufacturing.

- Task substitution elasticity $\eta = 0.5$. (Humlum 2019)
- Induced $K - L$ elasticity $\eta^* = 1.35$ (Hubmer 2020; Karabarbounis & Neiman, 2014).
- Automation cost c_a: match differential adoption of new capital tech by firm size over transition: $\frac{\mathbb{E}[\Delta\alpha_f | \text{firm f in employment P99+}]}{\mathbb{E}[\Delta\alpha_f | \text{firm f in employment P0-75}]} = 1.96$ details on data: Annual Business Survey model vs. data

  $\Rightarrow$ Top firms twice as likely to adopt new capital tech (relative to typical firm)

Calibration: Non-CES Demand system

- $H(\cdot)$: Klenow-Willis aggregator $\Rightarrow$ $\text{demand elasticity} := \sigma \cdot \text{rel. quantity}^{-\frac{\nu}{\sigma}}$
  - $\sigma = 6.1$: matches aggregate markup of 15%
  - $\nu/\sigma = 0.22$: matches difference between median and aggregate labor share in 1982
- Productivity process given by

$$z = \exp\left(F^{-1}_{\text{Weibull}(\zeta,n)}(\Phi(\tilde{z}))\right), \qquad \tilde{z}' = \rho_z \tilde{z} + \epsilon'_z$$

which ensures that

$$\ln z \sim \text{Weibull}(\zeta, n) \Rightarrow \mathbb{P}(\ln z \geq x) = \exp\left(-\left(\frac{x}{\zeta}\right)^n\right)$$

  - $n = 0.78, \zeta = 0.086$ to match top sales shares (CR4 and CR20)
  - more log convex than Pareto ($n = 1$); but not too much!

details

Quantitative Findings: Manufacturing 1982–2012

Automation and rise in comp. calibrated to match

- decline in labor share
- observed rise in concentration

Inferred shocks

- $d \ln q = 1.50$
- $d \ln \lambda = 0.03$

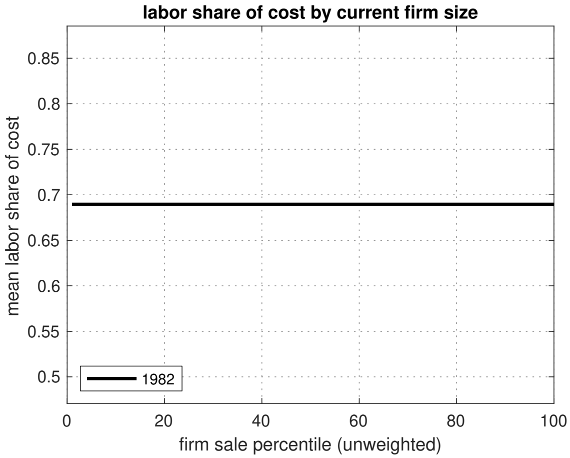
	Data	Model: Full	Model: q-shock	Model: λ-shock
Δ aggregate s_ℓ [a]	−17.8	−17.4	−16.6	−0.0
Δ concentration (CR4) [b]	6.0	5.9	4.3	1.4
Δ median $s_{\ell,f}$ [a]	3.0	4.3	3.5	0.1
Δ ln agg. markup	.	1.3	1.3	0.1

In percentage points. [a] Kehrig–Vincent (2021). [b] Autor et al (2020): Average manuf. industry sales concentration.

inferred price of capital decline and automation fixed cost of plausible magnitude
diffusion

Firm-level Labor Share Dynamics in Response to q-shock

- In 1982 st. state, uniform technology.



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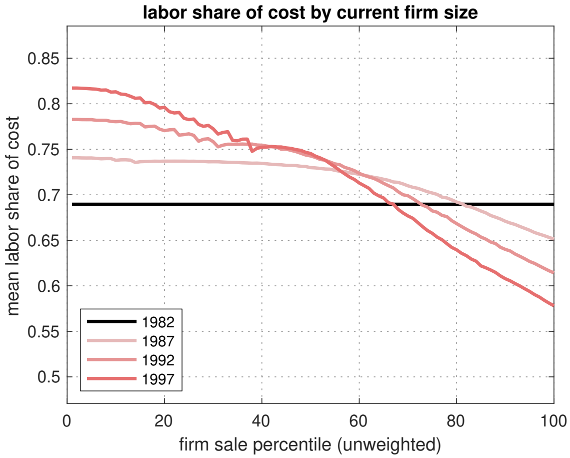

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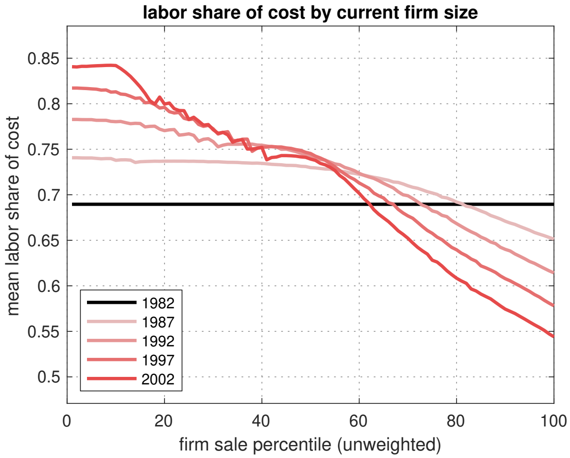

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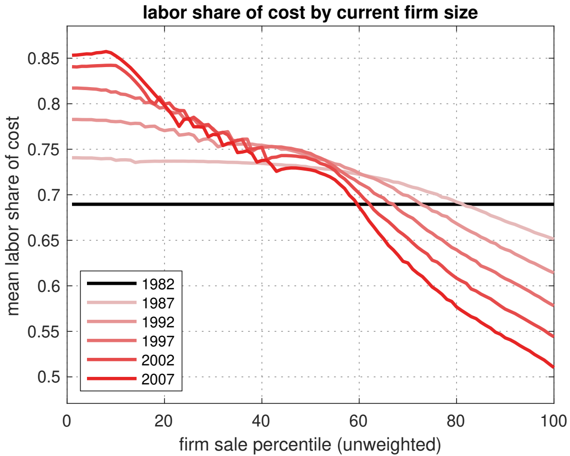

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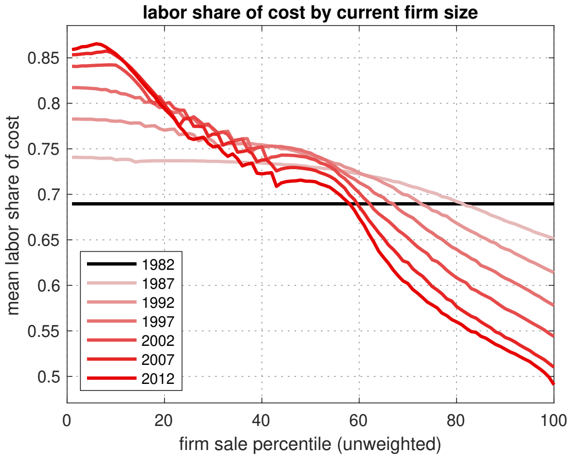

Zooming in on Effect of Decline in Price of Capital (q-shock)

Untargeted Moments:

1. Generates endogenous increases in productivity dispersion productivity dispersion
2. ... and, therefore, sales concentration in line with data sales concentration
3. Explains empirical firm-level labor share decompositions in Autor et al 2020 Autor et al
4. ... and dynamics in Kehrig-Vincent 2021 Kehrig and Vincent

similar quantitative findings with simple CES demand side (i.e., uniform markups)

In Retail Sector, Concentration More Important

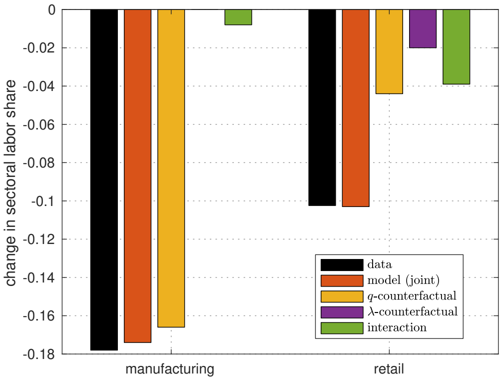

Figure: Data: Kehrig & Vincent (2021) for manufacturing, BLS MFP Tables for retail.

- In retail, $\Delta s_\ell^{data} = -10.2pp$.
- Model: $\sim$ 40% due to q-shock, $\sim$ 20% due to λ-shock, $\sim$ 40% due to interaction details all sectors

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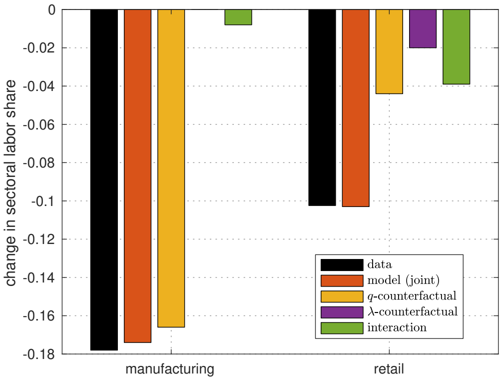

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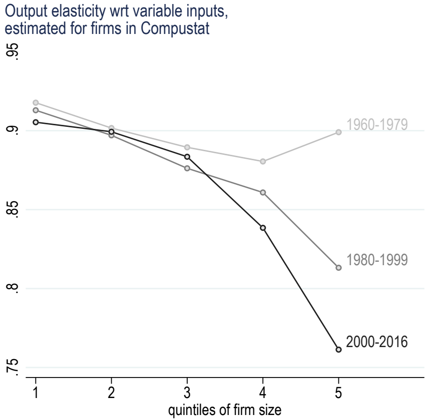
- Model: $\sim$ 40% due to q-shock, $\sim$ 20% due to λ-shock, $\sim$ 40% due to interaction details all sectors
- Why different inference to manuf.?
  1. Higher concentration level ($\rightarrow$ more log-convex z-dist $\rightarrow$ λ-shock more potent)
  2. Stronger concentration increase ($\rightarrow$ infer larger $d\ln\lambda =$ ~~0.03~~ 0.41)
  3. s_ℓ^{data} fell less ($\rightarrow$ infer smaller $d\ln q =$ ~~1.50~~ 0.75)

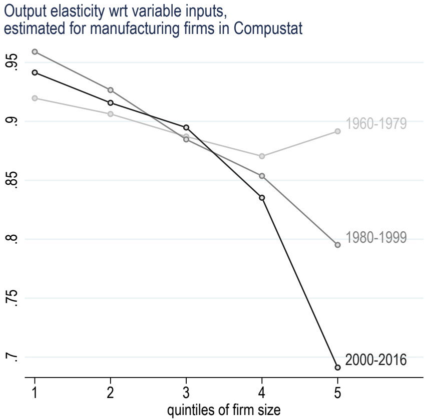
Direct Evidence from Markup Estimates

- Now provide direct evidence using Compustat data. Goals:
  1. Complementary accounting decomposition: more general markup variation.
  2. Validate model predictions: Two drivers (q- and λ-shocks) have distinct implications for markups (μ) and output elasticities (ε).
- We follow the literature (DeLoecker–Eeckhout–Unger, 2020) in using the production function approach to recover $\varepsilon_{v,f,t} \Rightarrow$ compute $\mu_{f,t} = \frac{\varepsilon_{v,f,t}}{s_{v,f,t}}$ (only $s_{v,f,t}$ directly observed).
  - We allow technology ($\varepsilon_{v,f,t}$) to vary by time period, industry, and firm size.

assumptions

Finding 1: Clockwise Rotation in Elasticities

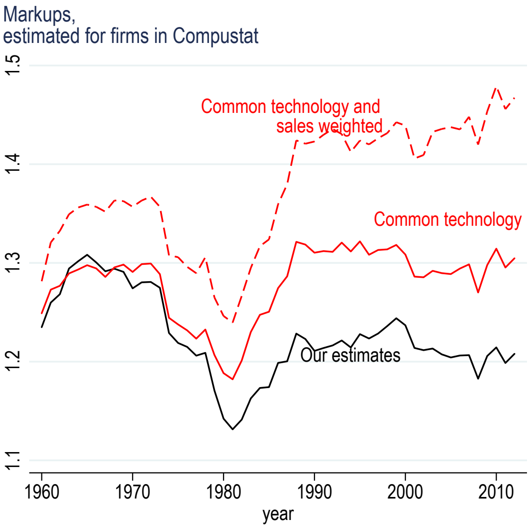




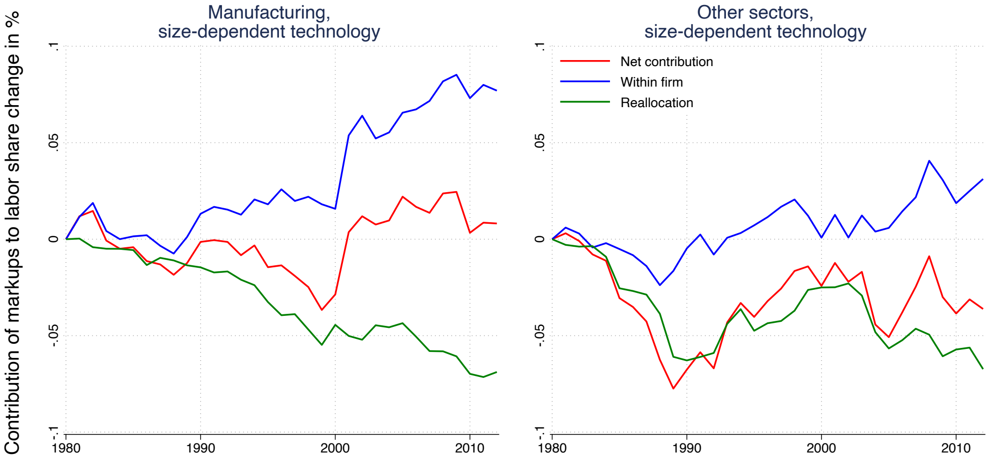
As in model (q-shock), clockwise rotation in $\varepsilon_{v,f,t} \Rightarrow$ top firms switch to cap-intensive techs.

Finding 2: Only Minor Increase in Aggregate Markup

- Common technology, sales weighting (DLEU headline): replicate strong increase in aggregate markup
- Common technology, cost-weighting (Edmond, Midrigan, Xu, 2021; Baqaee, Farhi 2020): mild increase
- Our estimate with **heterogenous technology** and cost-weighting: **no trend/minor increase**



Finding 3: Within firm markups fall ($s_{\ell,f}\uparrow$), reallocation to high-markup firms ($s_\ell\downarrow$). ~ 0 net effect in manuf., < 0 other sectors.



Within/between components as in model (λ-shock); similar net markup contribution

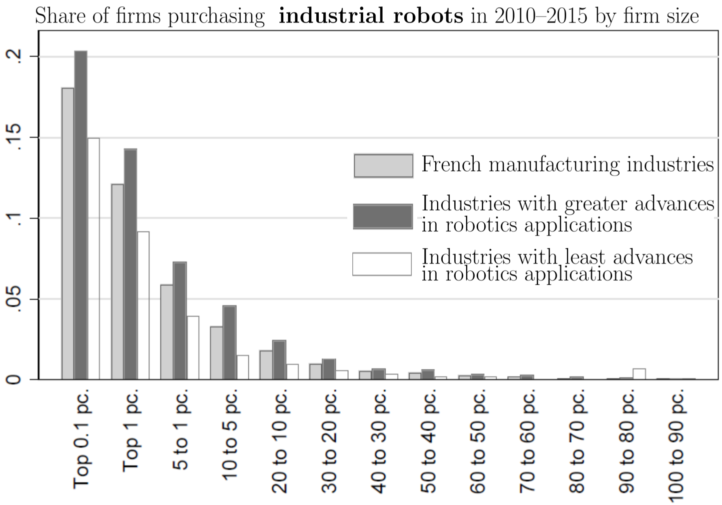
Concluding Remarks

- Model of K–L substitution with a fixed cost per task matches firm-level facts on the decline of labor share well in manufacturing
- In other sectors, in particular retail, rising competition and reallocation to more productive high-markup firms also play an important role
- Direct evidence from firm-level markup estimates supports our findings

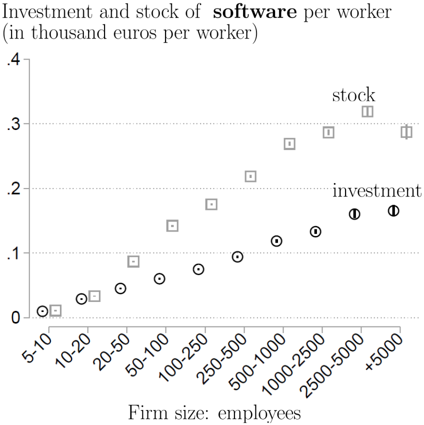
$\Rightarrow$ Highlights importance of allowing for differences in technology across firms, both to assess role of technology, and also to assess role of markups/concentration!

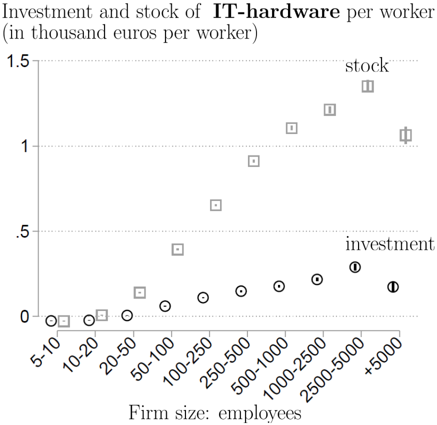
Appendix Slides

Skewed Adoption of Capital Intensive Technologies
Acemoglu–Lelarge–Restrepo 2020: Industrial robots in France return



Skewed Adoption of Capital Intensive Technologies
Lashkari–Bauer–Boussard 2019: IT in France return





Capital tech adoption rates (2018): Annual Business Survey (US) return

	AI			Industrial robots		
age \employment	P0-P75	P99+	Ratio	P0-P75	P99+	Ratio
0-5 years	4%	6%	1.50	2%	6%	3.00
6-10 years	3%	7%	2.33	2%	6%	3.00
11-20 years	3%	10%	3.33	1%	10%	10.00
21+ years	2%	7%	3.50	1%	13%	13.00
	Specialized equipment			**Specialized software**		
	P0-P75	P99+	Ratio	P0-P75	P99+	Ratio
0-5 years	18%	30%	1.67	38%	71%	1.87
6-10 years	17%	20%	1.18	37%	65%	1.76
11-20 years	17%	31%	1.82	36%	70%	1.94
21+ years	18%	34%	1.89	34%	71%	2.09

$\Longrightarrow$ Target adoption differential (weighted average of ratios): 1.96

Micro Foundation of Production Function 1/2

- Production requires a continuum of tasks

$$y = z \cdot \left(\int_0^1 \mathcal{Y}(x)^{\frac{\eta-1}{\eta}} dx \right)^{\frac{\eta}{\eta-1}}$$

- Tasks $\in [0, \alpha]$ are automated and can (will) be produced by capital

$$\mathcal{Y}(x) = \begin{cases} \psi_k(x)k(x) + \psi_\ell(x)\ell(x) & \text{if } x \leq \alpha \\ \psi_\ell(x)\ell(x) & \text{if } x > \alpha \end{cases} \qquad \frac{\psi_\ell(x)}{\psi_k(x)} \text{ increasing in } x$$

- Normalizing $\psi_k(x) = 1$, unit cost (if all tasks in $[0, \alpha]$ produced by K) "as if" CES

$$c(z, \alpha) = \frac{1}{z} \left[\alpha \left(\frac{1}{q} \right)^{1-\eta} + g(\alpha) W^{1-\eta} \right]^{\frac{1}{1-\eta}},$$

with endogenous share parameters: $g(\alpha) = \int_\alpha^1 \psi_\ell(x)^{\eta-1} dx$ return

Micro Foundation of Production Function 2/2

- Parametrizing the labor productivity schedule as

$$\psi_\ell(x) = \left(x^{\frac{1-\eta-\gamma}{\gamma}} - 1\right)^{\frac{1}{1-\eta-\gamma}},$$

  we get that

$$g(\alpha) = \int_\alpha^1 \psi_\ell(x)^{\eta-1} dx = \left(1 - \alpha^{\frac{\eta+\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\eta+\gamma-1}}$$

- In particular, with this parameterization the induced K–L elasticity η^* (adjusting α to α^*) is equal to the constant $\eta + \gamma$.

- In general, have that

$$\eta^* = \eta + \underbrace{\text{task reallocation}}_{\text{function of } \frac{\psi_\ell}{\psi_k}(\cdot) \text{ steepness}} > \eta$$

return

Result: Response of Labor Shares to q-shock at Marginal Tasks

Proposition

Assume CES demand (fixed markup). Following an increase in $q_0(x)$ for all tasks $x > \alpha^*$ by $d \ln q > 0$, the economy converges to a new steady state with $Y \uparrow$, $w \uparrow$, and $\alpha^* \uparrow$. The aggregate share of labor in costs changes by

$$d \ln \varepsilon_\ell = -(1 - \varepsilon_\ell) \cdot (\eta^* - \eta) \cdot d \ln q + (1 - \varepsilon_\ell) \cdot (1 - \eta^*) \cdot d \ln w$$

At the same time, for an incumbent firm with low realizations of z along the transition, the share of labor in costs changes by

$$d \ln \varepsilon_\ell(\theta) = (1 - \varepsilon_\ell(\theta)) \cdot (1 - \eta) \cdot d \ln w.$$

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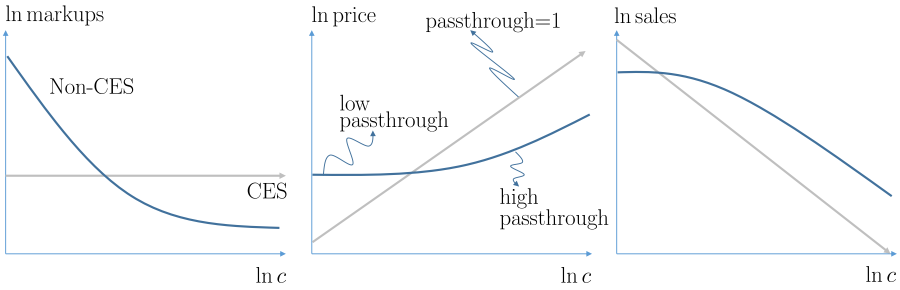
$$d\ln \varepsilon_\ell(\theta) = (1-\varepsilon_\ell(\theta))\cdot(1-\eta)\cdot d\ln w.$$

$\Rightarrow$ With non-uniform capital price change across tasks, can have that in aggregate $\varepsilon_\ell\downarrow$ but $\varepsilon_\ell(\theta)\uparrow$ for typical firm even if $\eta^* < 1$ return

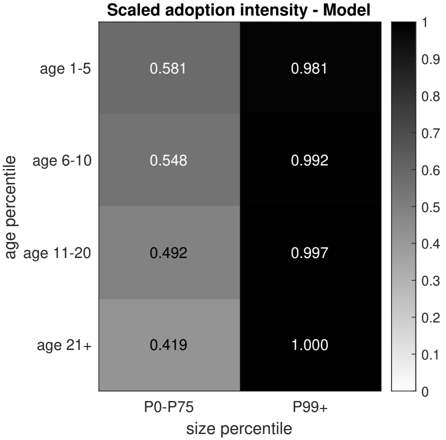
Non-CES Demand: Markups, Pricing and Sales

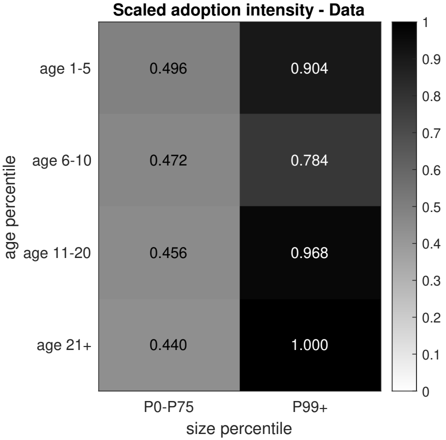


- markups decreasing in unit cost following from Marshall's weak second law
- pass-throughs increasing in unit cost following from Marshall's strong second law
- mapping productivity → sales more concave than under CES demand

return

Capital Tech Adoption Rates in Model and Data (ABS, 2018)

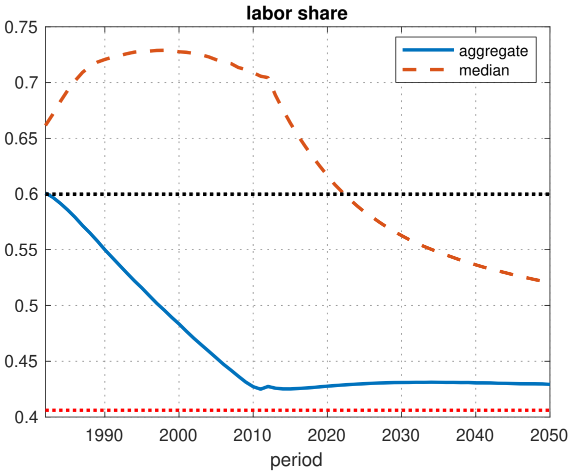




$\Rightarrow$ Size gradient targeted (c_a); age gradient balances diffusion on entry vs. older firms more likely to have cycled through high-z states and automated return

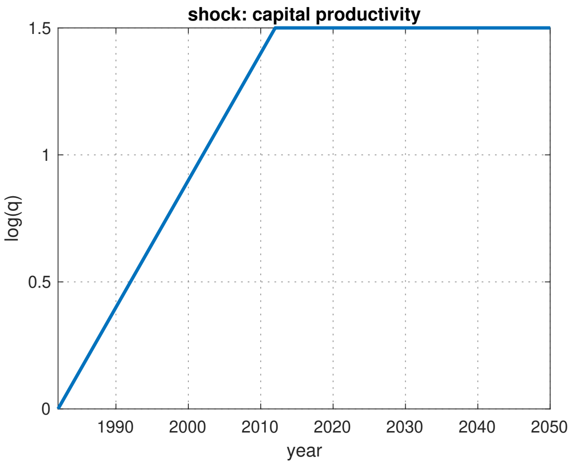
Transitional Dynamics: Aggregate Labor Share

- Economy starts in steady state in 1982 $\rightarrow$ all firms have the same technology (but not markup)
- In terms of aggregate factor shares, fast transition.
- In terms of median firm, slow transition even with diffusion.
  - W/o diffusion, very similar results until now, no convergence going forward.



shock   example firm history

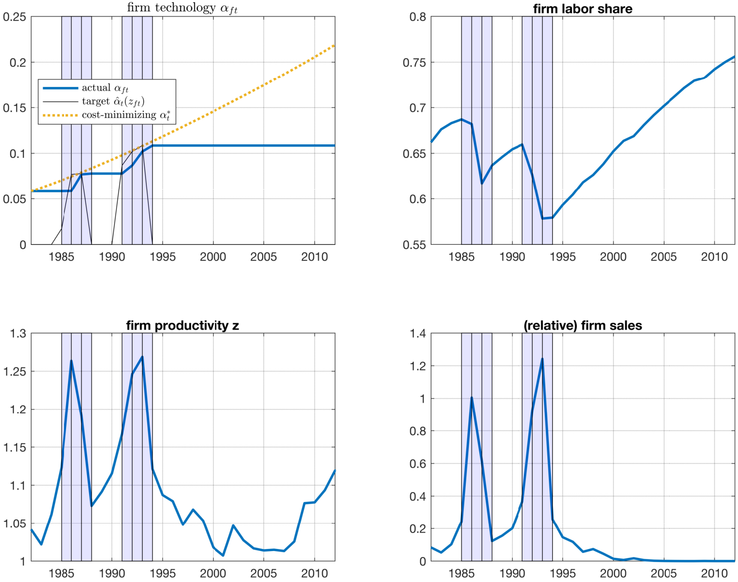
Model: Time series of shock (investment-specific technical change q)



return

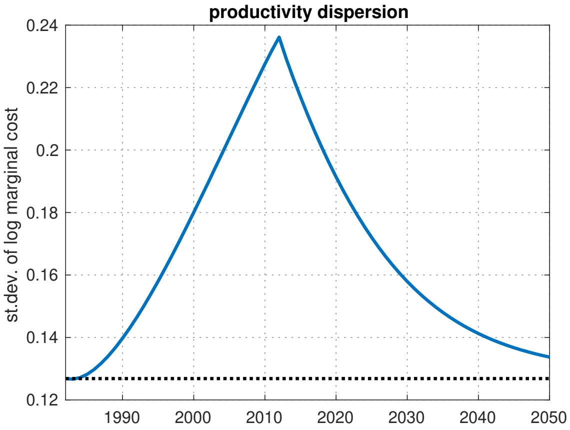
Model: Example timeline one particular firm

return



Productivity dispersion back

- endogenous automation choice $\rightarrow$ endogenous (temporary) increase in productivity dispersion
- broadly in line with data: Decker, Haltiwanger, Jarmin, Miranda (2020) find 5 log points increase 1980s to 2000s (TFP, U.S. manufacturing)
    - model with q-shock only: $+7.1$ log points



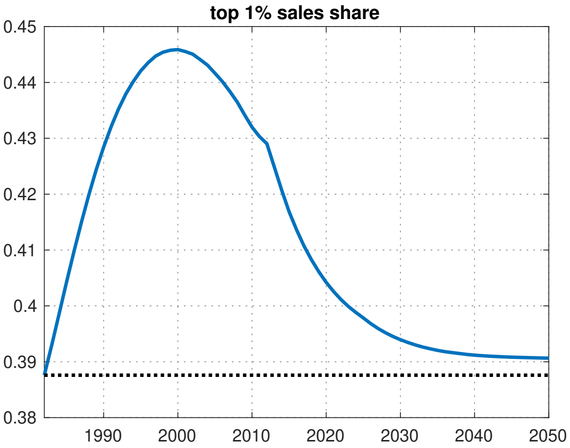
Sales concentration back

- endogenous (temporary) increase in productivity dispersion $\rightarrow$ endogenous (temporary) increase in sales concentration

- broadly in line with data:

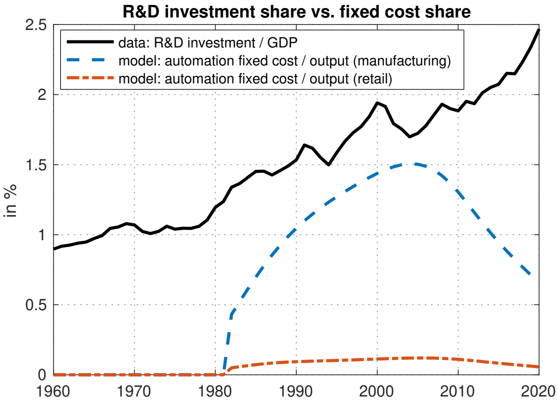
	Data	Model (q-shock only)
Δ CR4	0.060	0.043
Δ CR20	0.052	0.081

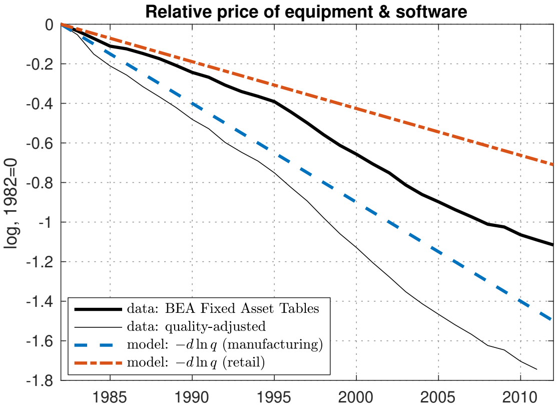
Autor et al (2020) data, manufacturing, 1982–2012. In p.p.



Comparing inferred parameters and shocks to data

back





Decomposition of Labor Share in Manufacturing 1982–2012 back

- The labor share in an industry is

$$s_\ell := \sum_f \omega_f \times s_{\ell f}$$

$s_{\ell f}$ =labor share firm f,

ω_f =share firm f in value added

- Melitz–Polanec decomposition in Autor et al. (2020)

$$\begin{aligned}\Delta s_\ell &= \Delta \bar{s}_\ell && \text{(unw. mean)} \\ &+ \Delta \text{cov}(\omega_f, s_{\ell f}) && \text{(covariance)} \\ &+ \text{entry} + \text{exit}\end{aligned}$$

	Data	Model (q-shock)
Δ Unweighted survivors' mean	-0.2	5.0
Δ Covariance	-18.7	-21.5
Entry	5.9	1.2
Exit	-5.5	-1.3
Δ Aggregate	-18.5	-16.6

Data: Autor et al (2020), manufacturing, compensation share of value added. In p.p.

Unpacking the Changing Covariance Between Size and Labor Share back

- Covariance can be further decomposed

$$
\begin{aligned}
&\Delta \mathrm{cov}(\omega_f, s_{\ell f}) \\
&= \mathrm{cov}(\Delta\omega_f, s_{\ell f}) \qquad \text{(market share dynamics)} \\
&+ \mathrm{cov}(\omega_f, \Delta s_{\ell f}) \qquad \text{(labor share by size dynamics)} \\
&+ \mathrm{cov}(\Delta\omega_f, \Delta s_{\ell f}) \qquad \text{(cross dynamics)}.
\end{aligned}
$$

- Kehrig and Vincent (2021): cross dynamics important in manufacturing

	Data	Model (q-shock)
Market share dynamics	4.7	6.6
Labor share dynamics*	-4.3	2.8
Cross dynamics	-23.2	-25.7

Note: Kehrig and Vincent (2021) data from balanced panel of manufacturing establishments. In p.p.

Table: Steady state calibration of the non-CES demand model: Manufacturing

	Parameter		Moment	Data	Model
	I. Parameters governing steady state in 1982				
$\ln q_0$	Inverse capital price	−6.55	Aggregate labor share	60.1%	60.2%
σ	Demand elasticity	6.10	Aggregate markup	1.150	1.150
ν/σ	Demand supra-elasticity	0.22	Median labor share ratio	1.169	1.101
ζ	Weibull scale	0.086	Top 20 firms' sales share	69.7%	69.7%
n	Weibull shape	0.78	Top 4 firms' sales share	40.0%	40.0%
	II. Parameters governing firm dynamics				
$\underline{c}_f$	Minimum fixed cost	$4.6 \cdot 10^{-6}$	Entry (=exit) rate	0.062	0.063
ξ	Dispersion fixed cost	0.310	Size of exiters	0.490	0.488
μ_e	Entrant productivity	0.876	Size of entrants	0.600	0.601

Notes: Aggregate and median LS correspond time averages in manufacturing sector 1967–1982 (Kehrig and Vincent, 2020); median displayed as ratio over aggregate. Aggregate markup from Barkai (2020). Concentration measures are from Autor et al. (2020): manufacturing sector in 1982. Model equivalents refer to top 1.1% and top 5.5% of firms ranked by sales (on average 364 firms per 4-digit manufacturing industry). Data moments in Panel II follow the model with CES demand. All eight parameters jointly calibrated to match the eight corresponding moments.

return

Calibrating the Non-CES Demand system: Retail 1982–2012

- $H(x)$: Klenow-Willis aggregator $\Rightarrow$ demand elasticity $:= \sigma \cdot \text{rel. quantity}^{-\frac{\nu}{\sigma}}$
  - $\sigma = 6.1 \to \sigma = 9.0$
  - $\nu/\sigma = 0.22 \to \nu/\sigma = 0.20$

- Productivity process given by

$$z = \exp\left(F^{-1}_{\text{Weibull}(\zeta,n)}(\Phi(\tilde{z}))\right), \qquad \tilde{z}' = \rho_z \tilde{z} + \epsilon'_z$$

  which ensures that

$$\ln z \sim \text{Weibull}(\zeta, n) \Rightarrow \mathbb{P}(\ln z \geq x) = \exp\left(-\left(\frac{x}{\zeta}\right)^n\right)$$

  - $n = 0.78, \zeta = 0.086 \to n = 0.54, \zeta = 0.023$
  - more log convex than in manufacturing

table return

Transitional Dynamics: Retail 1982–2012 return

Automation and rise in comp. calibrated to match

- decline in labor share
- observed rise in concentration

Inferred shocks

- $d \ln q = 0.71$ (1.50 in manuf)
- $d \ln \lambda = 0.41$ (0.03 in manuf)

		Model		
	Data	Full	q-shock	λ-shock
Δ aggregate LS[a]	−10.2	−10.3	−4.4	−2.0
Δ concentration: CR4[b]	14.0	11.5	0.1	10.9
Δ CR20[b]	16.3	20.4	0.4	18.8
Δ uw. mean LS[b]	4.4	3.0	−2.4	1.4
$\Delta \ln$ agg. markup	.	3.3	0.1	2.9

In percentage points. [a] BLS Multifactor Productivity Tables. [b] Autor et al (2020).

Table: Steady state calibration of the non-CES demand model: Retail

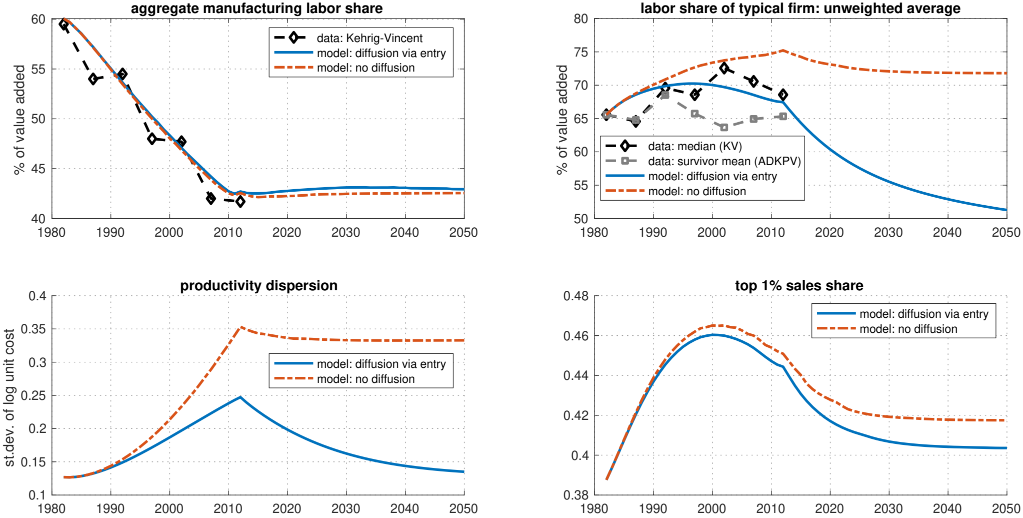
	Parameter		Moment	Data	Model
	I. Parameters governing steady state in 1982				
$\ln q_0$	Inverse capital price	-7.35	Aggregate labor share	70.4%	70.5%
σ	Demand elasticity	9.0	Aggregate markup	1.150	1.150
ν/σ	Demand supra-elasticity	0.20	Median labor share ratio	1.169	1.106
ζ	Weibull scale	0.023	Top 20 firms' sales share	29.9%	29.9%
n	Weibull shape	0.54	Top 4 firms' sales share	15.1%	15.1%
	II. Parameters governing firm dynamics				
$\underline{c}_f$	Minimum fixed cost	$5.2 \cdot 10^{-7}$	Entry (=exit) rate	0.062	0.062
ξ	Dispersion fixed cost	0.250	Size of exiters	0.490	0.488
μ_e	Entrant productivity	0.868	Size of entrants	0.600	0.599

Notes: Aggregate LS corresponds to the BLS MFP estimate for the retail sector. The ratio median-to-aggregate is from Kehrig and Vincent (2020); refers to manufacturing, since in retail the data does not allow to compute the labor share of value added. Aggregate markup from Barkai (2020). Two concentration measures are from Autor et al. (2020), correspond to retail in 1982. Model equivalents refer to top 0.023% and 0.116% of firms ranked by sales (on average 17,259 firms per 4-digit retail industry). All eight parameters jointly calibrated to match eight corresponding moments.

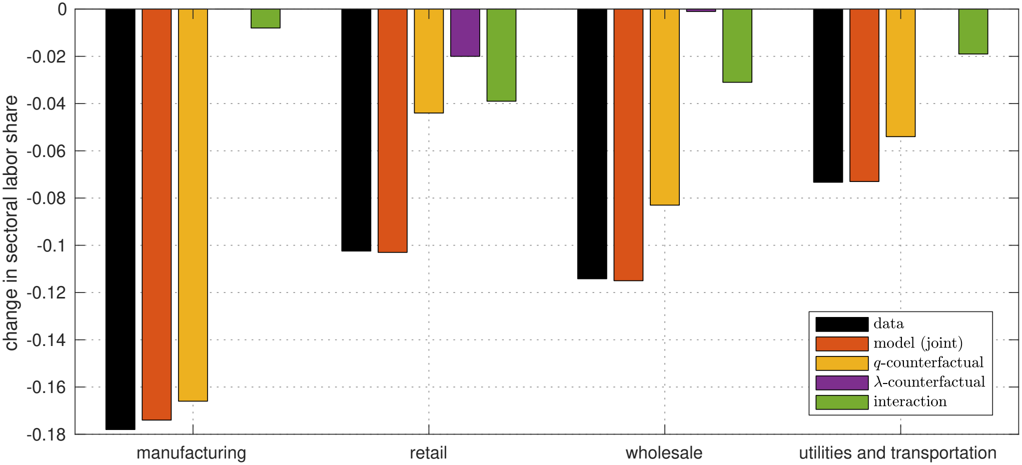
return

Model with (baseline) and without diffusion

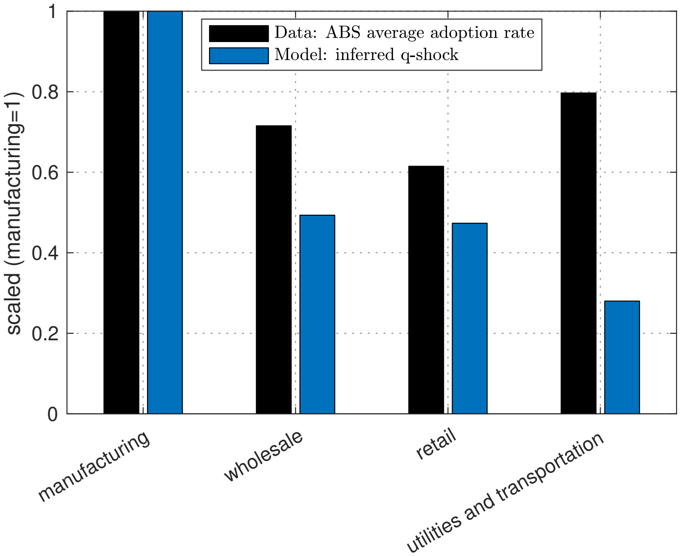
return



Quantitative Findings: All Sectors return



All Sectors: Inferred q-shock vs. adoption rates in data (ABS) return



Quantitative Findings (Manufacturing): Robustness, CES Demand 1/n

Baseline model with non-CES demand ⇒ Re-calibrate under CES demand

Inferred shocks

- $d\ln q$ = ~~1.50~~ 1.66
- ~~$d\ln\lambda = 0.03$~~

Inferred parameters

- c_a = ~~0.36~~ 0.51

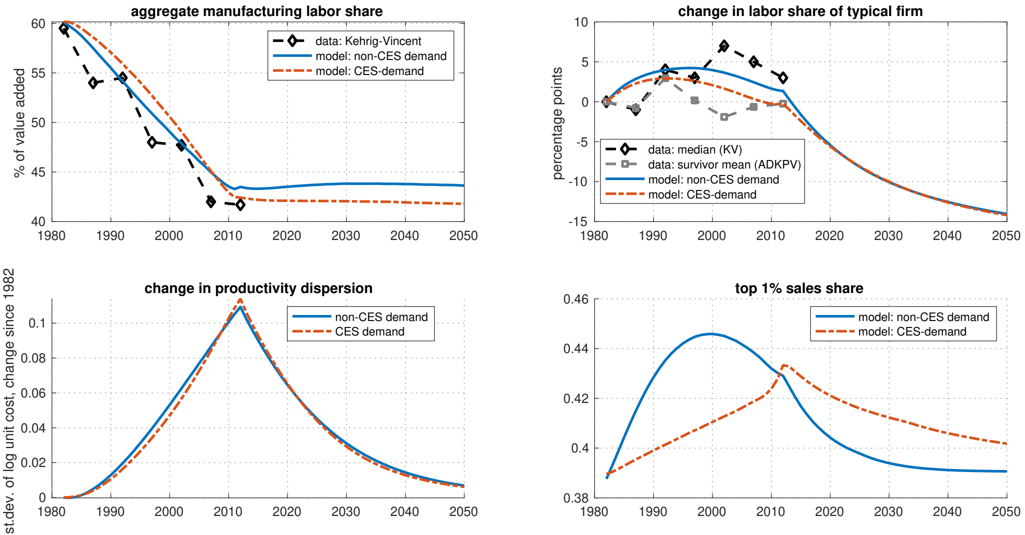
Demand side:	DATA	MODEL: NON-CES FULL	MODEL: NON-CES q-SHOCK	MODEL: CES q-SHOCK
Δ aggregate s_ℓ [a]	−17.8	−17.4	−16.6	−17.7
Δ concentration: CR4 [b]	6.0	5.9	4.3	4.9
Δ concentration: CR20 [b]	5.2	9.9	8.1	7.3
Δ median $s_{\ell,f}$ [a]	3.0	4.3	3.5	1.9
Δ ln agg. markup	.	1.3	1.3	.

In percentage points. [a] Kehrig–Vincent (2021). [b] Autor et al (2020): Average manuf. industry sales concentration.

return

Quantitative Findings: Robustness, CES Demand 2/n

return



Decomposition in Manufacturing 1982–2012, Robustness CES 3/n return

- The labor share in an industry is

$$s_\ell := \sum_f \omega_f \times s_{\ell f}$$

$s_{\ell f}$ =labor share firm f,

ω_f =share firm f in value added

- Melitz–Polanec decomposition in Autor et al. (2020)

$$\begin{aligned}\Delta s_\ell &= \Delta \bar{s}_\ell && \text{(unw. mean)} \\ &+ \Delta \text{cov}(\omega_f, s_{\ell f}) && \text{(covariance)} \\ &+ \text{entry} + \text{exit}\end{aligned}$$

	Data	Model	
		non-CES	CES
Δ Unweighted survivors' mean	−0.2	5.0	2.5
Δ Covariance	−18.7	−21.5	−20.9
Entry	5.9	1.2	0.9
Exit	−5.5	−1.3	−0.5
Δ Aggregate	−18.5	−16.6	−17.7

Data: Autor et al (2020), manufacturing, compensation share of value added. In p.p.

Unpacking the Changing Covariance, Robustness CES 4/n

return

- Covariance can be further decomposed

$$\Delta \text{cov}(\omega_f, s_{\ell f})$$
$$= \text{cov}(\Delta\omega_f, s_{\ell f}) \quad \text{(market share dynamics)}$$
$$+ \text{cov}(\omega_f, \Delta s_{\ell f}) \quad \text{(labor share by size dynamics)}$$
$$+ \text{cov}(\Delta\omega_f, \Delta s_{\ell f}) \quad \text{(cross dynamics)}.$$

- Kehrig and Vincent (2021): cross dynamics important in manufacturing

	Data	Model	
		non-CES	CES
Market share dynamics	4.7	6.6	0.0
Labor share dynamics*	-4.3	2.8	-3.8
Cross dynamics	-23.2	-25.7	-14.7

Note: Kehrig and Vincent (2021) data from balanced panel of manufacturing establishments. In p.p.

Markup estimation: Assumptions return

A1 differences in the price of variable inputs reflect quality,

A2 revenue is given by a revenue production function of the form

$$\ln y_{ft} = z_{ft} + \varepsilon^{R}_{vc(f)t} \cdot \ln v_{ft} + \varepsilon^{R}_{kc(f)t} \cdot \ln k_{ft} + \epsilon_{ft},$$

where $c(f)$ denotes groups of firms with a common technology and same process for their revenue productivity, and ϵ_{ft} is an i.i.d ex-post shock orthogonal to k_{ft} and v_{ft}

A3 unobserved productivity z_{ft} evolves according to a Markov process of the form

$$zf, t = g(z_{ft-1}) + \zeta_{ft},$$

where ζ_{ft} is orthogonal to k_{ft} and v_{ft-1}, and

A4 the gross output PF exhibits CRTS in capital and variable input $\Rightarrow$ quantity elasticities

$$\varepsilon_{vft} = \varepsilon^{R}_{vc(f)t} \Big/ \left(\varepsilon^{R}_{vc(f)t} + \varepsilon^{R}_{kc(f)t}\right)$$

Markup estimation: Method

1. First-stage regression to purge measurement error:

$$\ln \tilde{y}_{ft} = \mathbb{E}[\ln y_{ft} | \ln x_{ft}, \ln k_{ft}, \ln v_{ft}, t, c(f)] = h(\ln x_{ft}, \ln k_{ft}, \ln v_{ft}; \theta_{c(f)t}).$$

2. Second stage: Given any pair of revenue elasticities $\varepsilon^R_{vc(f)t}$ and $\varepsilon^R_{vc(f)t}$ compute

$$\tilde{z}_{ft} = \ln \tilde{y}_{ft} - \varepsilon^R_{vc(f)t} \cdot \ln v_{ft} - \varepsilon^R_{kc(f)t} \cdot \ln k_{ft},$$

estimate the flexible model

$$\tilde{z}_{ft} = g(\tilde{z}_{ft-1}; \theta_{c(f)t}) + \tilde{\zeta}_{ft},$$

and form the following moment conditions that identify the revenue elasticities:

$$\mathbb{E}\left[\zeta_{ft} \otimes (\ln k_{ft}, \ln v_{ft-1})\right] = 0.$$

return

Contribution of Markups/Reallocation to Labor Share Decline

- We can write the labor share of an industry i as

$$s_{\ell t} := \frac{\varepsilon_{\ell t}}{\mu_t},$$

- Holding fixed common technology $\varepsilon_{\ell,t}$ (model and data pre-1980), compute counterfactual labor share that reflects only changes in markups:

$$d \ln s_{\ell t}^{cf} := -d \ln \mu_t \quad \text{where} \quad \frac{1}{\mu_t} := \sum_f \omega_{f,t} \frac{1}{\mu_{f,t}}.$$

- We further want to distinguish changes in this counterfactual labor share due to within firm and reallocation component of markups.
    - Expect that at firm level, markups fall $\rightarrow$ positive within component.
    - ... and reallocation to high-markup firms $\rightarrow$ negative reallocation component.