RESTRUCTURING VS. BANKRUPTCY*

Jason Roderick Donaldson† Edward R. Morrison‡ Giorgia Piacentino§ Xiaobo Yu¶

July 15, 2022

Abstract

We develop a model of a firm in financial distress. Distress can be mitigated by filing for bankruptcy (which is costly) or preempted by restructuring (which is impeded by a collective action problem). We find that bankruptcy and restructuring are complements, not substitutes: Reducing bankruptcy costs facilitates restructuring, rather than crowding it out. And so does making bankruptcy more debtor-friendly, under a sufficient condition that seems likely to hold now in the United States. The model gives new perspectives on current relief policies (e.g., subsidies to firms in bankruptcy) and on long-standing legal debates (e.g., the efficiency of the absolute priority rule).

*For helpful comments, we thank Douglas Baird, Vincent Buccola, Anthony Casey, Winston Dou, Edie Hotchkiss, Rich Hynes, Melissa Jacoby, Josh Mitts, Martin Oehmke, Mark Roe, David Schoenherr, Jan Starmans, Suresh Sundaresan, David Thesmar, Wenyu Wang, Michelle White, Yao Zeng, and seminar participants at the Bank of Italy, Chicago Booth, Columbia, the Corporate Restructuring and Insolvency Seminar, FIRS, the FT Webinar, HKUST, the Law and Macro conference, Maryland, NBER CF Spring 2020 Meeting, New York Fed, NYU, the Princeton-Stanford Conference on Corporate Finance and the Macroeconomy under COVID-19, Temple, UBC, USC, the Virtual Finance Theory Seminar, and Washington University in St. Louis. Larisa Antonisse, John Clayton, Fengming Dong, Thomas Horton, and Tianyu Wu provided excellent research assistance. This project received generous support from the Richard Paul Richman Center for Business, Law, and Public Policy at Columbia University.

†Washington University in St. Louis and CEPR.
‡Columbia University and NBER.
§Columbia University, CEPR, and NBER.
¶Columbia University.
1 Introduction

Policies targeting distressed firms often aim to avert liquidation and its associated costs, such as supply-chain and labor-market disruptions, and thereby avoid recessions. But these policies can help only insofar as private solutions fail. When a firm enters financial distress, there are generally two private solutions that could allow it to avoid liquidation. One is bankruptcy reorganization; the other is an out-of-court restructuring agreement with creditors. They are about equally common, each constituting about forty percent of corporate defaults (Moody’s (2020)). Both reduce leverage by exchanging existing debt for new securities (debt or equity). Restructurings have the advantage of avoiding the costs of bankruptcy (e.g., legal fees and court delays), but they are inhibited by a collective action (“hold-out”) problem among dispersed creditors (Bernardo and Talley (1996) and Gertner and Scharfstein (1991)). In practice, restructurings typically take a specific form to circumvent this very problem: a “distressed exchange” of old debt for higher-priority new debt. But even successful restructurings do not preclude a future bankruptcy case—more than a third are followed by a bankruptcy within the next three years (Altman and Kuehne (2020)).

Although restructuring and bankruptcy are well-understood individually, much of the literature conflates them or treats them as substitutes. In this paper, we study a distressed firm but recognize that it could choose to file for bankruptcy even after a successful restructuring.

---

1The remaining defaults are missed payments.

2Similarly, in their study of financially distressed firms during the period 1978-87, Gilson, John, and Lang (1990) find that about half resolved distress through an out-of-court restructuring.

3See, e.g., Latham & Watkins Capital Market Practice (2020) (“Bondholders who do not participate in the exchange offer are known as ‘holdouts,’ and the holdout issue is often the Achilles’ heel of exchange offers....”), Moody’s (2017) (“Even though distressed exchanges have many obvious advantages, they are not perfect solutions for all distressed companies. One of the challenges in distressed exchanges is the holdout problem, which occurs when one or more creditors have an incentive to reject a deal that collectively benefits all creditors.”), and Antonoff (2013) (“Among the principal issues that arise in out-of-court restructurings are the problems of holdouts and free riders.”).


5See, e.g., Asquith, Gertner, and Scharfstein (1994), Becker and Josephson (2016), Favara, Schroth, and Valta (2012), Franks and Torous (1994), Gertner and Scharfstein (1991), and Gilson, John, and Lang (1990). In some papers, such as Fan and Sundaresan (2000) and Hart and Moore (1994, 1998), bankruptcy serves as the outside option for renegotiation. These papers, however, typically do not model the bankruptcy choice, which is instead synonymous with liquidation.
This allows us to explore how restructuring and bankruptcy interact and to address the following questions: How do key parameters of the bankruptcy environment—its deadweight costs and the extent to which it is “creditor friendly”—affect the likelihood of an out-of-court restructuring? During a crisis, what are the optimal policies for mitigating corporate distress? In particular, how do policies that subsidize bankruptcy\(^6\) (which allocate scarce resources to the firms in deepest distress) compare to policies that subsidize debt restructuring\(^7\) (which prevent distress from arising in the first place)?

The model we develop casts doubt on conventional policy wisdom. We find that an efficient bankruptcy system increases the likelihood of out-of-court restructurings. Restructuring and bankruptcy are complements, not substitutes. Conversely, a creditor-friendly bankruptcy system can impede out-of-court restructuring, under a condition that we argue is likely to hold in the U.S. now. Turning to crisis policies, we find that subsidies to bankrupt firms can backfire, increasing the costs of bankruptcy in equilibrium. By contrast, subsidies to secured creditors—including subsidies to secured creditors of firms in bankruptcy—can be socially optimal (though their benefits may ultimately be enjoyed entirely by equity holders).

**Model preview.** We model a firm in financial distress. It has risky assets \(v\) and debt \(D_0\) held by dispersed creditors. There are two dates. At date 0, before \(v\) is realized, the firm can propose a restructuring of its debt. At date 1, \(v\) is realized, and the firm has a choice: repay the debt or file for bankruptcy. We assume bankruptcy is costly in the sense that it generates deadweight costs \((1 - \lambda)v\). In bankruptcy, creditors bargain with equity holders to capture a fraction \(\theta\) of the value available for distribution \((\lambda v)\). This fraction \(\theta\) measures the “creditor friendliness” of bankruptcy, which depends on the legal environment. In this setup, a restructuring that reduces the firm’s debt and thereby avoids bankruptcy’s deadweight costs has the potential to make everyone better off, including creditors who have their debt written-down.

Two key assumptions underlie our results. The first is that when each creditor decides whether to accept a restructuring offer, it takes the decisions of other creditors as given. This is what we mean when we say creditors are “dispersed”: They cannot act collectively. The second assumption is that the firm *chooses* whether to file for bankruptcy, and it can act strategically: The firm can choose to file in order to benefit from a debtor-friendly code \((\theta < 1)\) even if it could repay its debt in full. These assumptions reflect practice: Dispersed creditors seem to be the main impediment to exchange offers (as discussed in footnote 3), and 98 percent of corporate bankruptcies are initiated by debtors, not forced by creditors (Hynes and Walt (2020)), including numerous well-known bankruptcies (e.g., Texaco) filed “strategically” by solvent corporations (Cole (2002); Moody’s (2006)).

---

\(^{6}\)See, e.g., DeMarzo, Krishnamurthy, and Rauh (2020).
\(^{7}\)See, e.g., Blanchard, Philippon, and Pisani-Ferry (2020) and Greenwood and Thesmar (2020).
Results preview. In our model, the hold-out problem dooms any restructuring involving an exchange of debt for equity or for equal-priority debt: An individual creditor knows that if others accept the offer, the firm will avoid distress and likely be able to pay its debt in full. Because creditors are dispersed, each has incentive to hold out. However, as in Bernardo and Talley (1996) and Gertner and Scharfstein (1991), the firm can restructure if it offers to exchange existing debt for higher-priority debt. Creditors accept a write-down in the face value of the debt (which decreases their payment if the firm does not file) in exchange for an increase in their priority (which increases their payment if it does file).

Each creditor accepts an exchange offer if priority in bankruptcy is valuable, conditional on others accepting. Priority is valuable if the creditor believes (i) bankruptcy is a meaningful possibility (otherwise priority is moot, because all debt is likely to be paid in full anyway) and (ii) its recovery in bankruptcy will be substantial (otherwise priority is useless, because recoveries will be small anyway). That is our first insight. Behind it is a new take on “bargaining in the shadow of the law” (Mnookin and Kornhauser (1979)). The bankruptcy (the law) serves not as an outside option in the event that restructuring (bargaining) fails, but rather as an “inside option” in the event that restructuring succeeds.

Our analysis hinges on a second insight as well: The probability of bankruptcy, and hence the value of priority, is determined by a strategic decision of the firm—whether to file. As is typical in the hold-out literature—in which bankruptcy is generally not a strategic decision—the firm in our model files for bankruptcy when its asset value $v$ falls below a threshold, which we denote by $\hat{v}$. But, unlike in that literature, $\hat{v}$ depends on the parameters of the bankruptcy environment: Bankruptcy is more attractive to the firm when bankruptcy costs are low ($\lambda$ is high) and when the Code is debtor-friendly ($\theta$ is low). Hence, the bankruptcy filing threshold, $\hat{v}$, is increasing in $\lambda$ and decreasing in $\theta$, all else equal. But all else is not equal, as these parameters also have an effect on creditors’ decision to accept a restructuring offer—an effect that turns out not to be obvious.

This leads to our first main result: A decline in bankruptcy costs (an increase in $\lambda$) facilitates restructuring. To see why, recall that restructuring is feasible only insofar as creditors are willing to accept write-downs in exchange for priority. And recall that the value of priority has two components (i) the probability of bankruptcy and (ii) the recovery value in bankruptcy. Both increase as $\lambda$ increases: An increase in $\lambda$ (i) makes it more attractive to the firm to file (an indirect effect) and (ii) increases recovery values for senior debt (a direct effect).

Our second main result is that an excessively creditor-friendly bankruptcy law (high $\theta$) can deter out-of-court restructuring. Like $\lambda$, the optimal $\theta$ should maximize the value of seniority. Unlike $\lambda$, $\theta$ must balance two effects. One is the direct effect we just saw: Increasing $\theta$ increases
recovery values for senior debt. But now there is a countervailing indirect effect: Increasing $\theta$ makes filing for bankruptcy less attractive to the firm because more value is diverted to creditors at the expense of equity holders. As the likelihood of bankruptcy declines, the value of seniority in bankruptcy declines as well.

We derive a condition to test whether the creditor friendliness of bankruptcy is inefficiently high in the sense that a small decrease in $\theta$ would make restructuring easier. Because the condition depends only on a few “sufficient statistics,” not the whole distribution of $v$, we can apply it to off-the-shelf estimates in the literature. Our calculation suggests this condition is likely satisfied in the U.S. today: An increase in creditor friendliness is likely to have a minor effect on creditor recovery values, but a decrease could have a significant effect on the filing probability. The net effect is that restructurings, which avoid the deadweight costs of bankruptcy, would be more prevalent in the U.S. if bankruptcy laws were made less creditor friendly. This finding is consistent with intuition, expressed by leading bankruptcy attorneys, that the creditor-friendliness of U.S. law impedes out-of-court restructurings.\(^8\)

We then use our model to evaluate how a utilitarian social planner should allocate its marginal dollar among a general set of subsidies, targeting each layer of the capital structure (equity, secured debt, and unsecured debt) inside or outside bankruptcy. This leads to our third main result: Subsidies to equity outside bankruptcy are equivalent to subsidies to secured debt (inside or outside it), and both are optimal. It does not matter whether the government creates incentives (i) for the firm not to file for bankruptcy ex post (effectively subsidizing it for repaying its debt) or (ii) for creditors to restructure debt ex ante (effectively subsidizing them for debt write-downs).

Other subsidies backfire. Subsidies to equity in bankruptcy induce excessive filing ex post; subsidies to unsecured debt (inside or outside bankruptcy) induce hold-outs ex ante. Overall, our result underscores that policy interventions must complement private solutions for resolving distress—viz., restructurings—rather than substitute for them.\(^9,^{10}\)

The subsidies that we study nest many policies, including several implemented/proposed in response to the COVID-19 pandemic. We show how our result can be applied, off-the-shelf, to

\(^8\)See, e.g., Miller and Marcus (1989) ("It is only the leverage afforded by the possibility of resort to protection under the Code that, in certain cases, serves as a catalyst to the accomplishment of so-called ‘out of court’ restructurings. As resort to the formal reorganization process under chapter 11 of the Code becomes less of a deterrent to creditors, out of court restructurings are likely to become more infrequent or conclude upon terms which are more onerous to debtors—making subsequent bankruptcies and a greater number of liquidations more likely.")

\(^9\)These include, e.g., the Debtor-in-Possession Financing Facility (DIPFF) proposed by DeMarzo, Krishnamurthy, and Rauh (2020).

\(^{10}\)A caveat to this policy analysis, which takes the firm’s initial debt $D_0$ as given, is that anticipated policy interventions could affect how much the firm borrows in the first place. An ex post analysis seems especially appropriate for unanticipated crises like the COVID-19 pandemic. See, however, Appendix D in which we study how the firm’s ex ante borrowing decision is affected by the bankruptcy environment.
show that grants and forgivable loans (which benefit unsecured debt) can impede restructuring.\footnote{These include, e.g., the loan guarantees proposed by Blanchard, Philippon, and Pisani-Ferry (2020) and the Paycheck Protection Program (PPP). (E.g., United Airlines will receive a total of $5 billion through the PPP. Of the $5 billion the airline expects to receive, approximately $3.5 billion will be a direct grant and approximately $1.5 billion will be a low interest rate loan.)} Better are policies that either directly facilitate restructuring agreements\footnote{These include, e.g., the government support of restructuring proposed by Blanchard, Philippon, and Pisani-Ferry (2020) and Greenwood and Thesmar (2020) and some that have been implemented in the past directly via the tax code. (In 2012, for example, IRS Regulation TD9599 reduced the taxes that creditors owe upon restructuring. Campello, Ladika, and Matta (2018) show that this policy led bankruptcy risk to fall by nearly 20 percent and restructurings to double.)} or that benefit secured debt in bankruptcy.

**Extensions.** We explore several extensions. (i) We show how secured creditors exercising control in bankruptcy process can either facilitate or deter restructuring, depending on how control is exercised. We also explore deviations from the absolute priority rule (APR), finding that those between senior and junior are never optimal, even though those between debt and equity can be. (ii) We include court congestion and show that this can generate financial instability in the form of multiple equilibria. We argue that bankruptcy policy thus matters for financial stability. (iii) We allow for ex ante costs of financial distress, arising from debt overhang or risk-shifting, as well as ex post costs arising from, e.g., judicial errors or bargaining frictions. We find that, although these costs unambiguously increase the benefits of restructuring, their effect on the likelihood of restructuring is complex. (iv) Finally, we allow creditors to be concentrated as well as dispersed. We find that restructurings will include debt-for-equity swaps when creditors are sufficiently concentrated but only debt-for-debt swaps (swapping junior unsecured debt for senior secured debt) when they are dispersed.

**Literature.** Our paper bridges two strands of the bankruptcy literature. One focuses on the hold-out problem as an impediment to restructuring.\footnote{Our paper complements papers studying other restructuring frictions, such as asymmetric information (Bulow and Shoven (1978), Giammarino (1989), and White (1980, 1983)). Our work departs from papers in which such frictions are absent and, as a result, Coasean bargaining among investors leads to efficiency (e.g., Baird (1986), Haugen and Senbet (1978), Jensen (1986), and Roe (1983)).} Roe (1987) was among the first to focus on this problem in the context of bondholders, whose inability to coordinate (exacerbated by federal law) can prevent efficient restructuring and render bankruptcy necessary.\footnote{In corporate finance, this idea is also central to Grossman and Hart’s (1980) model in which free-riding shareholders refuse efficient takeovers.} Gertner and Scharfstein (1991) study the problem more formally, showing that a debtor can induce claimants to agree to a restructuring via an “exchange offer” that offers seniority to consenting creditors (and thereby denies non-consenting creditors).\footnote{Roe and Tung (2016) also study exchange offers and show that a successful exchange can nonetheless be followed by a bankruptcy filing.} Bernardo and Talley (1996) show that the
ability to make such exchange offers can distort a firm’s investment incentives.\footnote{Haugen and Senbet (1988) discuss ways to solve the coordination problem contractually (though some of the solutions could run afoul of the Trust Indenture Act). For example, the indenture could permit the firm to repurchase the bonds at any time at a specified price (e.g., the price quoted in the most recent trade).} In these papers, however, bankruptcy is not a choice; it is an automatic consequence of the firm’s inability to pay its debts.

A separate strand of the literature focuses on the bankruptcy decision and explores the effects of bankruptcy rules, such as the APR, on this decision. Baird (1991) and Picker (1992), for example, assess whether these rules induce firms to enter Chapter 11 when doing so maximizes recoveries to dispersed unsecured creditors. Picker (1992) concludes that, because the filing decision is held by shareholders, optimal rules might permit violations of the APR in order to induce filings that maximize ex post recoveries. These papers, however, do not consider how rules affecting the bankruptcy filing decision also affect the likelihood of a successful restructuring ex ante.\footnote{Another strand of the literature is exemplified by Mooradian (1994), Povel (1999), and White (1994), who view bankruptcy as a screening device that can induce liquidation of inefficient firms and the reorganization or restructuring of efficient firms.}

Our paper is also related to several other lines of research. A large literature studies the effects of creditor priority on bankruptcy outcomes and ex ante investment decisions (examples include Adler (1995) and Bebchuk (2002)). Recent work has focused on the optimal “creditor friendliness” of bankruptcy laws, showing that the optimal level depends on judicial ability in bankruptcy and the quality of contract enforcement outside of bankruptcy (see Ayotte and Yun (2009)) as well as on the extent to which default imposes personal costs on owners and managers (see Schoenherr and Starmans (2020)).\footnote{Sautner and Vladimirov (2017) also study optimal creditor friendliness, showing that greater creditor friendliness can facilitate ex ante restructuring when the firm has a single creditor who is unsure about firm cash flows during restructuring but sure about them in bankruptcy.} Our work contributes to this literature because we show how creditor friendliness in bankruptcy (ex post) affects the restructuring decision ex ante.

Our work also contributes to research on the determinants of debt structure (recently surveyed by Colla, Ippolito, and Li (2020)) and the drivers of debt renegotiation (e.g., Roberts and Sufi (2009)).

**Layout.** Section 2 presents the model. In Section 3, we characterize the equilibrium and show that restructuring is feasible only insofar as priority is valuable. Section 4 studies the effects of the bankruptcy environment on restructuring and derives the first two main results. In Section 5, we analyze policies for alleviating financial distress. Section 6 explores extensions. In Section 7, we conclude with a discussion of the model’s broader implications. All proofs and omitted derivations appear in the Appendix along with several additional microfoundations and robustness exercises.
2 Model

We study a two-date model of a single firm. It has assets with random positive value \( v \sim F \) and initial debt \( D_0 \) owed to a continuum of identical, risk-neutral creditors.\(^{19}\) The firm is controlled by risk-neutral equity holders (or managers and directors acting in their interest).

2.1 Restructuring

The firm could avoid distress by deleveraging ("restructuring") to \( D < D_0 \) at date 0.\(^{20}\) To do so, it makes a take-it-or-leave-it offer to exchange each creditor’s debt for new claims.\(^{21}\) We focus on the most common claims in real-world restructurings: equity and debt (Gilson, John, and Lang (1990); see, however, Appendix G on more general claims).\(^{22}\)

The main friction in the model is that there is a collective action problem among creditors. Each decides whether to accept the offer taking others’ decisions as given. Although we study a firm with dispersed creditors, this hold-out problem exists even with a small number of creditors (even just two) because no creditor internalizes fully the benefit of its write-down on the default probability. Distress costs are another friction, which we define next.

2.2 Financial Distress: Bankruptcy

We capture financial distress by the costs of bankruptcy that arise when the firm does not repay its debt \( D \) at date 1. Bankruptcy leads to deadweight costs \((1 - \lambda)v\), which may derive from professional fees; inefficient judicial decisions; separations from suppliers, trade creditors, ...

\(^{19}\) See Appendix D on the determination of \( D_0 \).  
\(^{20}\) Restricting attention to restructuring at date 0 turns out to be w.l.o.g.; see Appendix B, in which we study restructuring at date 1.  
\(^{21}\) We are assuming that the restructuring takes the form of an exchange offer, as is typical for corporate bonds in the U.S., where any restructuring is subject to the Trust Indenture Act (TIA). The TIA prohibits modifications to the face, coupon, or maturity of existing bonds without unanimous consent, something generally deemed infeasible (see, e.g., Hart (1995), Ch. 5 on why). Similar prohibitions commonly appear in syndicated bank loans, as discussed in Sufi (2007). In practice, however, some exchange offers are conditioned on acceptance by a minimum percentage of creditors; without that acceptance, the deal is off. These provisions make no difference to our baseline analysis with a continuum of creditors, but they could with a finite number of creditors (cf. Bagnoli and Lipman (1988) and Section 6.4).  
\(^{22}\) We abstract from the possibility that outstanding debt has covenants that could impede new senior debt issuance, such as so-called “negative pledge covenants.” This is a reasonable first approximation because, unlike core bond terms, such covenants typically can be removed via “exit consents” as long as a simple majority of bond holders accept (Kahan and Tuckman (1993)). Moreover, such covenants offer only weak protection against dilution via new secured debt anyway (Bjerre (1999)), notwithstanding that they sometimes can deter issuance (Donaldson, Gromb, and Piacentino (2020a)).
or customers; and other factors (e.g., Titman (1984)).\textsuperscript{23} Here, $D$ denotes the firm’s debt at the end of date 1, i.e., the outcome of a restructuring (if one has taken place) or the initial debt $D_0$ (if not).

If the firm pays $D$ in full, creditors get $D$ and equity holders get the residual $v - D$. But the firm need not repay; it can file for bankruptcy instead. In this case, creditors get a fraction $\theta$ of the bankruptcy value $\lambda v$. We refer to $\theta$ as the “creditor friendliness” of the bankruptcy system, which captures creditors’ bargaining power in bankruptcy (as modeled explicitly in Appendix A). If $\theta < 1$, then equity receives something in bankruptcy even if creditors are not paid in full.\textsuperscript{24} Thus, total payoffs to equity holders and creditors are:

\begin{equation}
\text{equity payoff} = \begin{cases} 
v - D & \text{if repayment,} \\
(1 - \theta)\lambda v & \text{if bankruptcy,} 
\end{cases}
\end{equation}

and

\begin{equation}
\text{debt payoff} = \begin{cases} 
D & \text{if repayment,} \\
\theta \lambda v & \text{if bankruptcy.} 
\end{cases}
\end{equation}

Observe that we focus on asset values, not cash flows. The reason is that, for the type of firms the model captures—those with dispersed debt holdings—solvent problems (low asset values) are likely a necessary condition for financial distress. Liquidity problems (low cash flows) are insufficient because such firms are likely to be able to raise capital to meet liquidity problems for at least three reasons: (i) They are likely to be owned by deep-pocketed equity holders who will inject capital to preserve going-concern value if asset values are high (as in, e.g., Black and Cox (1976) and Leland (1994)); (ii) they are likely to have access to capital markets, and creditors will lend against collateral if asset values are high (see, e.g., Chaney, Sraer, and Thesmar (2012)); and (iii) they are likely to be able to sell capital, and buyers will pay high prices if asset values are high (see, e.g., Asquith, Gertner, and Scharfstein (1994)).

\textsuperscript{23}We focus on ex post/direct costs of distress in our baseline model; we extend it to include ex ante/indirect costs in Section 6.3. See, e.g., Davydenko, Strebulaev, and Zhao (2012) and Dou et al. (2020) for estimates of such costs.

\textsuperscript{24}Such deviations from the APR in favor of equity over debt are not uncommon (see Eberhart, Moore, and Roenfeldt (1990), Franks and Torous (1989), and Weiss (1990)).
2.3 Timeline

In summary, the timing is as follows: At date 0, debt can be restructured or not. At date 1, the asset value $v$ is realized and, then, the firm repays its debt or files for bankruptcy.

3 Equilibrium Characterization and the Value of Priority

Here, we use backward induction to characterize the symmetric pure-strategy subgame-perfect equilibrium of the model. The key results in this section are that (i) restructuring is subject to a hold-out problem that is resolved by exchanging old debt for new debt with higher priority and (ii) the feasible write-down in a restructuring is increasing in the value of priority in bankruptcy.

3.1 Default and the Bankruptcy Filing Decision

Solving backwards, we start with the firm’s choice between repayment and filing for bankruptcy, given assets $v$ and debt $D$ at date 1. Given the equity payoffs in equation (1), the firm files when the payoff from filing, $(1 - \theta)\lambda v$, is higher than the payoff from repaying, $v - D$, or the asset value $v$ is below a threshold, which we denote by $\hat{v}(D)$:

$$v < \hat{v}(D) := \frac{D}{1 - (1 - \theta)\lambda}. \tag{3}$$

Notice that, if the deadweight costs of bankruptcy destroy all value ($\lambda = 0$) or the bankruptcy system is perfectly creditor friendly ($\theta = 1$), firms will file for bankruptcy only when the value of the firm’s assets $v$ is less than its debt $D$ (i.e., when the firm is “insolvent”). But if bankruptcy preserves at least some value ($\lambda > 0$) and yields some payoff to equity ($\theta < 1$), a firm may file even when it is solvent ($v > D$). The more debtor friendly the law is, the more likely the firm is to file when it is solvent.\(^{25}\)

3.2 Restructuring

Restructuring can reduce debt and therefore avoid the deadweight costs of bankruptcy. However, as we show next, a hold-out problem prevents efficient restructurings (debt-to-equity exchanges). Other, typically less-efficient, restructurings (debt-to-debt exchanges) are feasible, but only when old debt is exchanged for new debt with higher priority.

\(^{25}\)Consistent with this observation, Adler, Capkun, and Weiss (2012) find that as the law has become more creditor friendly, the asset quality of filing firms has deteriorated (relative to the face value of their debt).
3.2.1 The Hold-out Problem: Restructuring to Equity or Pari Passu Debt Is Infeasible

The inefficiency in the model is debt-induced financial distress, which restructuring to equity would eliminate:

**Lemma 1.** For any $\alpha$ such that

\[
\frac{\mathbb{E}[v - 1_{\{v \geq \hat{v}\}D_0 - 1_{\{v < \hat{v}\}}\lambda \theta v}]}{\mathbb{E}[v]} > \alpha > \frac{\mathbb{E}[v - 1_{\{v \geq \hat{v}\}D_0 - 1_{\{v < \hat{v}\}}\lambda \theta v} - (1 - \lambda)\mathbb{E}[1_{\{v < \hat{v}\}}v]}{\mathbb{E}[v]},
\]

restructuring debt to equity worth a fraction $1 - \alpha$ of the assets makes the firm and creditors both strictly better off.

This result recalls the Coase Theorem: Inefficiencies can be avoided by an appropriate assignment of property rights. However, the collective action problem can make it hard to agree on an assignment. Even though a restructuring to equity can eliminate all inefficiencies (distress costs), creditors might not accept it: They accept what makes them better off individually, which may not coincide with what makes them better off collectively.

**Lemma 2.** There is no ex ante restructuring of debt to equity that (uncoordinated) creditors are willing to accept and the firm is willing to offer.

Intuitively, a hold-out problem makes equity restructuring too expensive for the equity holders. Because creditors are dispersed and cannot coordinate, each makes its restructuring decision independently, taking the decisions of other creditors as given.

To see why, consider the vantage point of a single creditor. If all other creditors consent to a restructuring, this creditor knows the firm will become solvent and able to pay this creditor’s debt in full. So the creditor holds out, withholding consent to the restructuring, unless the restructuring offers an equity stake that is at least as valuable as payment in full. Because all creditors reason identically, all will hold out and the only restructuring that will succeed is one that leaves equity holders no better off than if the firm did not restructure at all.

The same problem afflicts a restructuring that exchanges existing debt for new debt with lower face value $D < D_0$ but the same priority as the original debt (“pari passu”): Deleveraging would decrease distress costs for all creditors but each creditor takes the decisions of other creditors as given, conditions its decision on a successful deleveraging, and therefore has incentive to hold out. As a result, no restructuring to equal-priority debt is feasible:

\[\text{It may be useful to illustrate how a marginal decrease in debt can make all creditors better off: It can be better to receive less with a higher probability than more with a lower one. Creditors with debt } D_0 \text{ are better off}\]
Lemma 3. There is no ex ante restructuring of debt to equal-priority debt that creditors are willing to accept and the firm is willing to offer.

3.2.2 Solving the Hold-out Problem: Restructuring to Senior Debt

The firm has another option: Offer to exchange existing debt for senior debt, which must be paid ahead of the existing debt in bankruptcy. Because such an exchange punishes hold-outs by diluting (“priming”) the existing debt, it makes restructuring to \( D < D_0 \) feasible.\(^{27}\)

An individual creditor will accept this restructuring if the value of senior debt with face value \( D \) is greater than the value of junior debt with face value \( D_0 \), conditional on the restructuring being successful (so the filing probability, unaffected by a single infinitesimal creditor’s decision, is \( F(\hat{v}(D)) \)):

\[
\left( 1 - F(\hat{v}(D)) \right) D + F(\hat{v}(D)) E \left[ \theta \lambda v \mid v < \hat{v}(D) \right] \geq \left( 1 - F(\hat{v}(D)) \right) D_0. \tag{5}
\]

The left-hand side is a creditor’s expected payoff from accepting the offer: If there is no future bankruptcy, it gets \( D \); if there is one, it gets a unit share of the recovery (each creditor has a claim with the same face value). The right-hand side is its expected payoff from holding out: If there is no future bankruptcy, it gets \( D_0 \); if there is, it gets zero. The reason the bankruptcy payoff is zero is that the firm cannot pay all debt in full (for all \( v \leq \hat{v}, D > \theta \lambda v \) by equation (3)) and all other debt is senior (by virtue of the restructuring), making the payoff to junior debt zero.\(^{28}\)

Rearranged, inequality (5) describes the feasibility of a restructuring that reduces debt by decreasing debt if

\[
\left. \frac{\partial}{\partial D} \right|_{D=D_0} \left( 1 - F(\hat{v}(D)) \right) D + F(\hat{v}(D)) E \left[ \theta \lambda v \mid v < \hat{v}(D) \right] < 0
\]

or, computing,

\[
\frac{1 - F(\hat{v}(D_0))}{f(\hat{v}(D_0))\hat{v}(D_0)} < \frac{1 - \lambda}{1 - (1 - \theta) \lambda}.
\]

Let us make two observations. (i) The condition can be satisfied only if \( \lambda \) is sufficiently small: If \( \lambda = 1 \), there are no bankruptcy costs to avoid by reducing debt, so creditors are always better off with more debt. (ii) It can be satisfied more easily when \( f(\hat{v}(D_0)) \) is large—that is, when a small reduction in debt from \( D_0 \) has a significant impact on the probability of default. At any rate, restructurings always increase total surplus in our model, even if they do not implement Pareto improvements.


\(^{28}\)We are assuming that senior debt is always paid ahead of junior debt. That is, there are no deviations from the APR that favor junior creditors at the expense of senior creditors. Although this assumption appears to be a good approximation of reality (Bris, Welch, and Zhu (2006)), we relax it in Section 6.1. Moreover, we show that deviations favoring junior creditors at the expense of senior debt are suboptimal from a welfare point of view.
Proposition 1. Restructuring to senior debt: For any $D$ such that

$$D_0 - D \leq \frac{F(\hat{v}(D))}{1 - F(\hat{v}(D))} \mathbb{E} \left[ \theta \lambda v \mid v < \hat{v}(D) \right],$$

restructuring the initial debt $D_0$ to senior debt with face value $D < D_0$ is accepted by creditors and makes the firm strictly better off.

The right-hand side of inequality (6) illustrates how the write-down $D_0 - D$ increases with both (i) the likelihood of a bankruptcy filing $F(\hat{v})$ and (ii) creditors’ recovery value in default $\mathbb{E} \left[ \theta \lambda v \mid v \leq \hat{v}(D) \right]$. These two ingredients, which underlie the rest of our analysis, reflect the value of priority afforded by senior debt: (i) If a firm never goes bankrupt, priority has no value (even the most junior creditor is paid in full) and (ii) if total recovery value is sufficiently low, priority has little value (even the senior debt is paid little).

A restructuring increases total efficiency by decreasing leverage and thereby decreasing distress costs. It implements a Pareto improvement if distress is sufficiently costly, in which case creditors benefit more from avoiding it than they suffer from write-downs (per footnote 26). Otherwise, it constitutes a so-called “coercive exchange” in which creditors accept a restructuring that makes them worse off because they want to avoid being diluted by new senior debt.\(^{29}\) Such coercive exchange need not be a concern for policy makers: It has no deadweight costs and creditors can demand compensation ex ante for the foreseeable risk of such an exchange. (To be sure, the risk could distort the flow of credit to firms. We show, however, that incorporating this margin into our model does not materially alter our conclusions; see Appendix D.)

3.3 Write-downs and Secured Credit Spreads

The feasible write-down $D_0 - D$ (inequality (6)) can be expressed in terms of observable yields. Doing so helps show that our model accords with reality.

To do this, we rewrite the condition for a feasible write-down (inequality (5)) in terms of continuously compounded yields-to-maturity, $y^s$ and $y^u$, conditional on the write-down:

$$De^{-y^s} \geq D_0 e^{-y^u}. \quad (7)$$

\(^{29}\)In corporate restructurings, this so-called “hold-in” problem could be more of a theoretical possibility than a practical reality, as it seems restructurings tend not to harm creditors (Chatterjee, Dhillon, and Ramírez (1995)). It seems to be the opposite in sovereign restructurings, in which bankruptcy is not an option Donaldson, Kremens, and Piacentino (2021).
Here, \( y^s \) is the yield on senior (secured) debt (which creditors receive in a restructuring); \( y^u \) is the yield on junior (unsecured) bonds (which they exchange). Noting that the inequality will bind in equilibrium (the firm chooses the lowest feasible debt level \( D \)), rearranging, and approximating, we obtain: \(^{30}\)

\[
\% \text{ write-down} \approx \text{secured credit spread}.
\] (8)

This illustrates how priority helps solve the hold-out problem: Creditors are willing to accept write-downs only to the extent that seniority is valuable (as measured by the secured credit spread).

As the quantities on each side of equation (8) are observable, we can use it to compare our model to the real world.\(^{31}\) Estimates in Benmelech, Kumar, and Rajan (2020) suggest that, for distressed (low-rated) firms, the (unannualized) secured credit spread \( y^u - y^s \) is about 42 percent (about six percent, annualized, for bonds with maturity of about seven years). Equation (8) says this should equal the percentage write-down in a restructuring, which seems to accord with the data: Studying distressed exchanges of unsecured for secured debt, Mooradian and Ryan (2005) find a mean write-down of 44 percent.

4 Analysis of the Bankruptcy Environment

Here we present our main insights, which follow from comparative statics on the condition for an individual creditor to accept a restructuring—inequality (6). We focus on the key parameters of the bankruptcy environment: deadweight costs \((1 - \lambda)\) and creditor friendliness \((\theta)\).

4.1 How the Costs of Bankruptcy Affect Restructuring

Define an individual creditor’s gain from accepting the restructuring relative to holding out, given others accept, as \( \Delta \). Using inequality (5), we can write it as:

\[
\Delta := \left(1 - F(\hat{v}(D))\right)(D - D_0) + F(\hat{v}(D)) \mathbb{E} [\theta \lambda v \mid v < \hat{v}(D)].
\] (9)

\(^{30}\) The details are as follows: First suppose inequality (7) binds and rearrange to write \( \log(D_0/D) \leq y^u - y^s \). The left-hand side is approximately the proportion of debt that can be written down \((D_0 - D)/D_0\). The right-hand side is the spread between secured and unsecured credit. Using the approximation \( \log(1 - x) \approx -x \), re-write the left-hand side:

\[
\log \left( \frac{D_0}{D} \right) = - \log \left(1 - \frac{D_0 - D}{D_0} \right) \approx \frac{D_0 - D}{D_0}.
\]

\(^{31}\) Although, to be precise, the spread must be conditional on successful restructuring. Additionally, to measure the spread in practice, the firm must have some other debt that is not restructured.
The equilibrium write-down, or lowest face value $D^\ast$, corresponds to $\Delta = 0$, i.e., to creditors’ incentive constraint (IC) binding (inequality (5)). Differentiating $D^\ast$ with respect to $\lambda$, we obtain the next result:

**Proposition 2. Bankruptcy costs:** Reducing bankruptcy costs (increasing $\lambda$) facilitates restructuring in the sense that the maximum write-down $D_0 - D^\ast$ is increasing in $\lambda$.

This is a central result of our paper: Restructuring and bankruptcy are complements, not substitutes. This is true for two reasons, which echo the ingredients that make priority valuable (see Proposition 1):

(i) The more efficient bankruptcy is, the more likely the firm is to file, and priority in bankruptcy is more valuable when it is more likely.

(ii) The more efficient bankruptcy is, the more creditors recover in bankruptcy, and priority in bankruptcy is more valuable when recovery values are higher.\(^{32}\)

In other words, as bankruptcy costs fall, priority in bankruptcy becomes more valuable, which increases the likelihood that creditors will accept write-downs in exchange for priority. Hence, contrary to common intuition, policies that reduce bankruptcy costs actually facilitate out-of-court restructuring. This adds support to Brunnermeier and Krishnamurthy’s (2020) conclusion that “reducing the cost of bankruptcy is unambiguously beneficial to society” (p. 6).

Schoenherr and Starmans (2020) provide some evidence consistent with our finding that a reduction in bankruptcy costs will facilitate restructuring. They find that the number of out-of-court restructurings, as well as the share that are successful, increased after a reform that reduced the deadweight cost of bankruptcy.

Finally, Proposition 2 is consistent with several stylized facts about bankruptcy practice. As Bratton and Levitin (2018) explain, bankruptcy costs have declined during the past two decades as firms have increasingly filed “prepackaged” cases in which a majority of creditors have already consented to a reorganization plan. Other innovations during the past two decades, such as restructuring support agreements, may also have reduced bankruptcy costs, as Casey, Tung, and Waldock (2020) show. During the same decades that bankruptcy costs have declined, distressed exchanges have become a more common solution to corporate default (see Moody’s (2017)). This is what our theory predicts.

\(^{32}\)To see why, recall, from the IC in inequality (5), that creditors accept a restructuring only if the payoff in bankruptcy from accepting senior debt is high relative to the payoff in bankruptcy from holding out, which, conditional on others accepting, is equal to zero. Thus, as senior creditors’ payoff in bankruptcy increases, so does the write-down creditors are willing to accept.
4.2 How the Creditor Friendliness of Bankruptcy Affects Restructuring

The feasibility of a restructuring also depends on the creditor friendliness of the bankruptcy system. Here, we differentiate the equilibrium face value \( D^* \) with respect to \( \theta \) to obtain our next result:

**Proposition 3. Creditor friendliness:** An increase in creditor friendliness (\( \theta \)) facilitates restructuring, in the sense that the maximum feasible write-down \( D_0 - D^* \) increases, if and only if \( \partial \Delta / \partial \theta \) is positive (see equation (74) in the Appendix). Moreover, if

\[
\frac{1 - F(D_{\theta=1}^*)}{D_{\theta=1}^* f(D_{\theta=1}^*)} < \lambda, \tag{10}
\]

where \( D_{\theta=1}^* \) denotes the solution to \( \Delta = 0 \) with \( \theta = 1 \), then there is an interior level of creditor friendliness \( \theta^* \in (0, 1) \) that maximizes the feasible write-down.

The ambiguity in this result stems from the fact that, although restructuring is facilitated when priority in bankruptcy becomes more valuable, creditor friendliness has two effects on the value of priority, which again echo the ingredients that make priority valuable (see Proposition 1):

(i) By increasing what creditors receive in the event of bankruptcy, creditor friendliness makes priority *more* valuable.

(ii) By reducing the payoff to equity holders, creditor friendliness reduces their incentive to file for bankruptcy, which makes priority *less* valuable.

Condition (10) tells us that, when creditor friendliness is high (\( \theta \) is near 1), further increases in \( \theta \) reduce the size of the write-down in a successful restructuring (as illustrated in Appendix C via an explicitly solved example). This implies that the optimal level of creditor friendliness is less than 1 (\( \theta^* < 1 \)). In other words, *the optimal bankruptcy system does not maximize creditor recoveries.*

This means that violations of debt-equity priority can be optimal. We show below, however, that the opposite is true for violations of secured-unsecured priority: They make restructuring harder (Section 6.1).

This finding hinges on our assumption that firms *choose* to file for bankruptcy. If bankruptcy were automatic, as is typical in the hold-out literature but at odds with practice, effect (ii) listed above—the effect of the filing decision on the value of priority—would be absent. Only effect

---

33Bisin and Rampini (2005) uncover a related downside of creditor friendliness: By discouraging filings, low bankruptcy payoffs to equity can make it hard for a bank to enforce exclusive contracts.
— the effect of the recovery value — would be relevant and, as a result, an increase in creditor friendliness would always facilitate restructurings.

4.3 Is the U.S. Bankruptcy Code Too Creditor Friendly?

We can use our model to assess whether existing laws are too creditor friendly in the sense that a marginal reduction in creditor friendliness would increase the feasibility of restructurings.

To do this, we make two additional assumptions, both of which seem relatively weak. The first is that $D^*$ is a continuous function of $\theta$ with a unique local minimum.\(^\text{34}\) Under this assumption, a bankruptcy system is excessively creditor friendly if an increase in creditor friendliness $\theta$ decreases the size of the attainable write-down:

$$\frac{\partial D^*}{\partial \theta} > 0.$$  \hspace{1cm} (11)

The second assumption is that $vf(v)$ is increasing in the bankruptcy region $[0, \hat{v}]$.\(^\text{35}\) Under this assumption, as we show in Appendix I, a sufficient condition for the system to be too creditor friendly (inequality (11)) is

$$\frac{D^*}{D_0} \leq \frac{1 - \lambda(1 - \theta)}{2 - \lambda(2 - \theta)}.$$  \hspace{1cm} (12)

Condition (12) captures the horse race between the two effects of an increase in $\theta$ on the value of priority — making it (i) more valuable by increasing the recovery value in bankruptcy $\theta \lambda v$ and (ii) less valuable by decreasing the likelihood of a bankruptcy filing $F(\hat{v})$. The right-hand side is increasing in $\theta$, revealing that effect (i) is dominant when $\theta$ is low, but (ii) is dominant when $\theta$ is high. The reason is that recovery values matter only insofar as bankruptcy happens, so they matter for low $\theta$, when the firm is likely to file, but not for high $\theta$, when it is unlikely to.

We can use (12) to assess whether U.S. bankruptcy law is too creditor friendly. The empirical literature offers approximations for each term in the condition:

- $\lambda$: This term captures the direct costs of bankruptcy. In studies of corporate reorganizations (mostly involving large corporations), the literature consistently estimates $\lambda > 90$ percent. (See Hotchkiss et al. (2008), Table 1, for a summary of twelve studies.)

- $\theta$: A number of papers investigate the value retained by equity holders in bankruptcy. They suggest that $\theta > 85$ percent is a conservative lower bound. In most cases, creditors are paid in full before equity is paid anything. (See Hotchkiss et al. (2008), Section 5.1, for a summary of estimates.)

\(^{34}\)This holds for the commonly-used distributions we have analyzed, such as the uniform; see Appendix C.

\(^{35}\)As long as $f$ is unimodal, it suffices that the bankruptcy threshold $\hat{v}$ is below its mode, or, e.g., that bankruptcy is a tail event.
• $D^*/D_0$: As mentioned above, Mooradian and Ryan (2005) find a mean write-down of 44 percent.

Taking these numbers at face value, condition (12) becomes

$$\frac{56\%}{2 - 90\% \times (1 - 85\%)} \approx 90\%,$$

which is satisfied. Note that this condition is sufficient, but far from necessary, and that the estimates from the literature are conservative. This leads us to believe that current law is likely too creditor friendly.

Giambona, Lopez-de Silanes, and Matta (2019) provide evidence supporting this conclusion. They find that an exogenous increase in creditor protection led to an increase in bankruptcy filings. This finding could be surprising because nearly all bankruptcies are initiated by debtors (Why should they file bankruptcy more often when they expect less in bankruptcy?). The finding is, however, consistent with our calculations, which show that creditor-friendly bankruptcy rules can impede restructuring, resulting in more bankruptcies.

5 Relief Policy

We now turn to the policy implications of our model. We take the vantage point of a social planner choosing how to allocate a marginal dollar (“subsidy”) to maximize (utilitarian) welfare. We first analyze the effects of subsidies to each layer of the capital structure, inside and outside of bankruptcy, taking into account their “direct effects” on financial distress and “indirect effects” on restructuring. We then apply the results to specific policies, including some initiated in response to the COVID-19 pandemic.

5.1 Planner’s Problem for a Marginal Dollar

We consider a planner with a budget $\varepsilon$ that can be spent on subsidies, denoted by vector $s \geq 0$, with associated (expected) costs $q$. We assume the planner has no other instruments at its disposal and therefore must respect creditors’ IC, shown in inequality (5) (equivalently, $\Delta(s) = 0$). The planner therefore minimizes the expected deadweight costs of bankruptcy (the only
inefficiency in the model) subject to creditors’ IC and its own budget constraint:

\[
\begin{cases}
\min & F(\hat{v}(s)) \mathbb{E} \left[ (1 - \lambda)\hat{v}(s) \mid v \leq \hat{v}(s) \right] \\
\text{s.t.} & \Delta(s) = 0 \\
& \mathbf{q} \cdot \mathbf{s} = \varepsilon
\end{cases}
\] (14)

over feasible subsidies \( \mathbf{s} \).

We can simplify the planner’s problem in two ways. First, because policies \( \mathbf{s} \) affect the objective only through the filing threshold \( \hat{v} \), the planner’s objective is equivalent to minimizing \( \hat{v} \). Second, because we assume the planner’s budget is small, \( \varepsilon \to 0 \), only the marginal effect matters. Thus, the planner minimizes

\[
\left. \frac{d\hat{v}}{d\varepsilon} \right|_{\varepsilon=0} = \sum_i \left. \frac{\partial \hat{v}}{\partial s_i} \frac{ds_i}{d\varepsilon} \right|_{\varepsilon=0} = \sum_i -\frac{\partial \Delta/\partial s_i}{\partial \hat{v}/\partial \varepsilon} \frac{1}{q_i} \Bigg|_{\varepsilon=0},
\]

having used the chain rule to differentiate along each constraint.\(^{36}\) The terms in the sum have an economic interpretation: Each is the product “policy efficacy” \( \times \) “bang for the buck.”

## 5.2 General Policies

We start with a general set of subsidies: We allow the planner to subsidize each layer of the capital structure—equity (\( E \)), unsecured debt (\( U \)), and secured debt (\( S \))—conditional on the firm being either inside bankruptcy (\( B \)) or outside it (\( O \)). With these indices, \( \mathbf{s} = (s_B^E, s_U^E, s_S^E, s_B^U, s_U^U, s_S^U) \).

With these policies, the creditors’ IC changes. Creditors prefer restructuring only if secured debt with face value \( D \) (plus subsidy \( s_B^O \) outside bankruptcy and subsidy \( s_U^B \) inside it) is worth more than unsecured debt with face value \( D_0 \) (plus subsidy \( s_U^O \) outside bankruptcy and \( s_U^B \) inside it):

\[
(1 - F(\hat{v}(s))(D + s_B^O) + F(\hat{v}(s))\mathbb{E} [\theta \lambda v + s_S^R] \geq (1 - F(\hat{v}(s))(D_0 + s_U^O) + F(\hat{v}(s))s_U^B. \quad (16)
\]

We assess the policies using the objective in equation (15). Note that each policy’s cost \( q_i \) is the probability it is paid—a dollar subsidy inside bankruptcy has expected cost \( q_i = F(\hat{v}) \) and one outside bankruptcy \( q_i = 1 - F(\hat{v}) \). This leads to our next result.

**Proposition 4.** The welfare-effect of subsidies is summarized in Table 1. Subsidies to equity

---

\(^{36}\)The expression for \( \partial \hat{v}/\partial s_i \) comes from the IC and for \( ds_i/d\varepsilon \) from the budget constraint.
outside bankruptcy and to secured debt inside or outside bankruptcy (and combinations thereof) are welfare-equivalent and optimal. All other subsidies backfire, decreasing welfare.

The moral is that as long as the whole subsidy is used to deter bankruptcy, it does not matter how the subsidy is allocated: It can be used to discourage the firm from filing ex post (by bribing equity holders with $s^O_E$ not to file) or to incentivize creditors to restructure ex ante (by bribing creditors with $s^B_S$ or $s^O_S$ to participate in a distressed exchange).\footnote{The optimal policies induce the same bankruptcy threshold. They differ only in the allocation of value between the firm and its creditors. But it would be wrong to think that subsidies to equity do not benefit debt (they can by reducing the filing probability) or that subsidies to debt do not benefit equity (they can by facilitating debt reduction).} But policies that subsidize equity in bankruptcy backfire: They incentivize excessive filing ex post.\footnote{Such a subsidy to equity in bankruptcy also facilitates restructuring ex ante—incentivizing filing increases the value of priority. But it also induces bankruptcy costs, something other policies that facilitate restructuring—i.e., subsidies to secured debt—do not.} Subsidies to unsecured debt also backfire: They disincentivize restructuring ex ante.

<table>
<thead>
<tr>
<th>$s_i$</th>
<th>$s^B_E$</th>
<th>$s^B_S$</th>
<th>$s^B_U$</th>
<th>$s^O_E$</th>
<th>$s^O_S$</th>
<th>$s^O_U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{d\hat{v}}{d\varepsilon} \frac{\partial \Delta}{\partial \hat{v}}$</td>
<td>$\frac{1 - F(\hat{v})}{F(\hat{v})}$</td>
<td>$-1$</td>
<td>$1$</td>
<td>$-1$</td>
<td>$-1$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

Table 1: Welfare effect of general subsidies. (Lower values correspond to higher welfare; see equation (15), but note that we multiply by the denominator $\frac{\partial \Delta}{\partial \hat{v}}$ for brevity, as it is the same for all policies.)

### 5.3 Specific Policies

Some of these general subsidies correspond to real-world policies and recent proposals. Subsidies to equity in bankruptcy ($s^B_E$) include policies that permit shareholders to retain ownership interests during a bankruptcy reorganization. For example, the U.S. Congress recently amended small-business bankruptcy laws to permit reorganization plans that allow owners to retain their interests, as discussed in Morrison and Saavedra (2020). Proposition 4 suggests that, in our model, policies like this are inferior to a policy that subsidizes secured creditors in bankruptcy, e.g., by extending them new credit at below-market rates, as in DeMarzo, Krishnamurthy, and Rauh’s (2020) proposed DIPFF.

But many other policies are combinations of the general subsidies considered above. We can apply Proposition 4 to them as well.

1. **Asset subsidies.** A policymaker can inject cash to increase the firm’s asset value. The incidence of the subsidy depends on whether the firm files for bankruptcy. If it does, the
subsidy is split: A fraction $1 - \theta$ goes to equity and a fraction $\theta$ to secured debt. If it does not, all debt is repaid and the subsidy goes to equity. A dollar subsidy ($q \cdot s = 1$) therefore corresponds to $s_E^B = 1 - \theta$, $s_S^B = \theta$, and $s_E^O = 1$ (and $s_i = 0$ otherwise).

2. **Asset subsidies to firms in bankruptcy.** If the asset subsidy only benefits firms in bankruptcy, a fraction $1 - \theta$ will go to equity in bankruptcy and a fraction $\theta$ to secured debt. A dollar subsidy therefore corresponds to $s_E^B = (1 - \theta)/F(\hat{v})$ and $s_S^B = \theta/F(\hat{v})$ (and $s_i = 0$ otherwise).

3. **Restructuring subsidies.** A policy maker can “bribe” creditors to accept a restructuring, paying them conditional on participating in the exchange offer. One way to do this is to alter the tax consequences of restructurings, as discussed in Campello, Ladika, and Matta (2018). Another is for the government to announce that it will effectively subsidize lenders who write-down their loans, as discussed in Blanchard, Philippon, and Pisani-Ferry (2020). In our model, this corresponds to a subsidy to secured debt both in and out of bankruptcy. Thus, a dollar subsidy corresponds to $s_S^B = 1$ and $s_S^O = 1$ (and $s_i = 0$ otherwise).

4. **Debt purchases (and forgiveness).** A policymaker can purchase debt in the market and write it off, effectively paying a fair price to reduce the firm’s debt. This bears some resemblance to quantitative easing programs in which central banks purchase corporate debt, with the twist that the central bank does not enforce repayment on the purchased debt. In our model, such a decrease in unsecured debt, which is paid nothing in bankruptcy, is equivalent to a subsidy to secured debt outside bankruptcy, so a dollar subsidy corresponds to $s_S^O = 1/(1 - F(\hat{v}))$ (and $s_i = 0$ otherwise).\(^{39}\)

This analysis, summarized in Table 2, reveals that asset subsidies are suboptimal, because they include subsidies to equity in bankruptcy, which distort the filing decision. Restructuring subsidies and debt purchases are optimal policies.

<table>
<thead>
<tr>
<th>$s$</th>
<th>Blanket assets</th>
<th>Assets in bankruptcy</th>
<th>Restructuring</th>
<th>Debt purchases</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d\hat{v} \partial \Delta \bigg</td>
<td>_{\varepsilon = 0}$</td>
<td>$-\theta$</td>
<td>$-1 + \frac{1 - \theta}{F(\hat{v})}$</td>
<td>$-1$</td>
</tr>
</tbody>
</table>

Table 2: The welfare effects of specific subsidies: The entries correspond to $\nabla \hat{v} \cdot s$ for each policy $s$ described in Section 5.3.\(^{30}\)

\(^{39}\)To see this, re-write the IC in equation (16), with only the secured-debt-out-of-bankruptcy subsidy positive ($s_S^O > 0$ and $s_i = 0$ otherwise), and rearrange to get: $F(\hat{v})\mathbb{E}[\theta \lambda v] \geq (1 - F(\hat{v}) (D_0 - s_S^O - D)$, which is tantamount to a reduction in $D_0$.\(^{30}\)
6 Extensions

We now relax several of our baseline assumptions, namely that (i) APR was respected between classes of debt, (ii) filing had no externalities on other firms, (iii) costs of debt arose solely in bankruptcy, and (iv) creditors were dispersed.

6.1 Secured Creditor Power and Priority Rules

We have assumed thus far that senior debt is paid strictly before junior debt in bankruptcy. In other words, there are no APR deviations that favor unsecured creditors at the expense of secured creditors. Although this is a good first approximation, in practice the division of surplus between secured and unsecured creditors depends on post-filing decisions, such as the decision to liquidate or reorganize. Liquidation is likely to favor secured creditors who seek quick payouts, whereas reorganization is likely to favor unsecured creditors who want to gamble on the going concern.

Here, we extend the model to capture different levels of secured creditor power, which we denote by $\rho$. Specifically, as in the baseline model, we assume that there are two classes of debt—senior (secured) and junior (unsecured). Unlike the baseline model, however, we also assume that senior creditors are more likely to be paid first as their power ($\rho$) increases. Specifically, we assume that senior debt is paid first with probability $\rho$, but shares pro-rata with junior debt with probability $1 - \rho$ (i.e., they are treated as if they are equal in priority). We still assume that equity gets a fraction $1 - \theta$ of the value in bankruptcy. What is changing here is how the fraction $\theta$ is divided among creditors.$^{41}$

To explore how $\rho$ affects restructuring, we start with the creditors’ IC for accepting a write-down from $D_0$ to $D$:

$$
\left(1 - F(\hat{v}(D))\right)D + F(\hat{v}(D))\mathbb{E}\left[\theta \lambda v \mid v < \hat{v}(D)\right]
\geq \left(1 - F(\hat{v}(D))\right)D_0 + (1 - \rho)F(\hat{v}(D))\mathbb{E}\left[\theta \lambda v \mid v < \hat{v}(D)\right] \frac{D_0}{D}. \tag{17}
$$

The difference between the above and the condition in inequality (5) is that with probability $1 - \rho$, a hold-out creditor’s junior debt receives a positive recovery value in bankruptcy (an accepting

---

$^{40}$The entries come from computing $\frac{d\hat{v}}{ds} = \nabla \hat{v} \cdot s$ for each policy $s$, noting that the components $\frac{\partial \hat{v}}{\partial s_i}$ of the gradient $\nabla \hat{v}$ can be obtained from Table 1 (see equation (15)).

$^{41}$Although we interpret creditor power mainly as a policy parameter describing the bankruptcy code or judicial preferences, it could also reflect market forces. Notably, Jiang, Li, and Wang (2012) find that when firms’ unsecured debt is held by hedge funds, total payoffs to creditors tend to increase in bankruptcy (our $\theta$ is higher) and so do payoffs to unsecured creditors (our $\rho$ is lower). Thus, our analysis suggests a possible downside of hedge fund participation in debt markets: It can make restructuring harder (see Section 4.3).
creditor’s payoff is unchanged because it takes as given that others accept). Rearranging, we see that a write-down $D_0 - D$ is feasible if:

$$D_0 - D \leq \frac{\theta \lambda F(\hat{v}(D))}{1 - F(\hat{v}(D))} \mathbb{E} \left[ v \mid v \leq \hat{v}(D) \right] \left( 1 - (1 - \rho) \frac{D_0}{D} \right).$$

(18)

This is identical to the original feasibility condition in equation (6) except for the final expression in brackets on the right-hand side. Indeed, when the APR is enforced strictly ($\rho = 1$), inequality (18) reduces to the original feasibility condition. Because the right-hand side is increasing in $\rho$, we have the next result:

**Proposition 5.** Strict enforcement of the priority of senior over junior debt, i.e., $\rho = 1$, facilitates restructuring, in the sense that it maximizes the feasible write-down in inequality (18).

This suggests a counterpoint to arguments in favor of relaxing the APR between secured and unsecured debt (e.g., Bebchuk and Fried (1996)), which often emphasize that the APR gives secured debt power to dilute unsecured debt. Our model captures this, but suggests that dilution is not necessarily inefficient; it can facilitate restructuring and thereby helps circumvent financial distress. It thus helps rationalize observed practice: Equity-debt violations are more common than secured-unsecured violations (Bris, Welch, and Zhu (2006)).

Our analysis so far has assumed that secured creditor control amounts only to a transfer away from unsecured creditors. But it could instead reduce total surplus. For example, secured creditors could force quick sales, potentially at fire sale prices, at the expense of other claimants, as Ayotte and Ellias (2020), Antill (2020), and Ayotte and Morrison (2009) document. Hence, in Appendix E, we allow secured creditors to exercise control over the bankruptcy process and show that such control can facilitate or deter restructuring, depending on how control is exercised. If secured creditors manipulate the bankruptcy process to divert value from unsecured creditors without reducing payoffs to equity (as in Ayotte and Ellias (2020)), a marginal increase in creditor control can facilitate the likelihood of restructuring. Intuitively, reducing unsecured creditors’ payoff in bankruptcy reduces their payoff from holding out, inducing them to accept write-downs. But if secured creditors induce excessive liquidations that reduce payoffs to all investors, including equity (as in Ayotte and Morrison (2009) and Antill (2020)), a marginal increase in secured creditor control reduces the likelihood of restructuring. Intuitively, reducing equity holders’ payoff in bankruptcy reduces their payoff from filing, inducing them to file less and thus reducing the value of priority.

**Tort claimants.** Priority rules appear in another place in policy debates: Should tort claimants—“accidental” or “involuntary” creditors—be treated on par with or ahead of other creditors in bankruptcy? Our model allows us to evaluate the effects of alternative priority rules...
on the likelihood of restructuring before bankruptcy. We do so in Appendix F and we find that, to facilitate restructuring, tort claimants should be prioritized above junior debt but below senior debt. That way, the law increases the difference in payoffs between junior and senior debt. It thus makes priority more valuable, facilitating restructuring.

6.2 Court Congestion

We have assumed thus far that the costs of bankruptcy \((1 - \lambda)v\) do not depend on whether restructuring occurs. This is reasonable for an individual firm because a single restructuring is unlikely to affect the efficiency of courts. However, taken in aggregate, restructurings can affect the costs of bankruptcy—if there are more out-of-court restructurings, fewer firms will file for bankruptcy, and courts are likely to be less congested. In other words, the costs of bankruptcy could be increasing in the aggregate number of firms that file. Here we show that this effect can create a feedback loop, amplifying the effects of bankruptcy costs on the hold-out problem.

We assume that there is a unit of ex ante identical firms with independent asset values and we assume that the costs of bankruptcy increase with the number of firms that file (which equals the probability that any individual firm files \(F(\hat{v})\), by the law of large numbers). We assume that courts can process a maximum number of filings \((\kappa)\) before experiencing “congestion costs” (see, e.g., Iverson (2018)). Specifically, we assume that

\[
\text{bankruptcy costs} = 1 - \lambda_H - 1_{\{F(\hat{v}) > \kappa\}}(\lambda_L - \lambda_H),
\]

which says that bankruptcy costs are equal to \(1 - \lambda_H\) if the number of bankruptcies \(F(\hat{v})\) is below the threshold “court capacity” \(\kappa\) and increase to \(1 - \lambda_L\) if they are above it.

From Proposition 2, we know that high bankruptcy costs impede restructuring, making it hard to reduce debt, and hence making bankruptcy itself more likely. Now, with congestion costs, this can create an amplification spiral. If bankruptcy filings exceed the court’s threshold \((F(\hat{v}) > \kappa)\), bankruptcy costs increase, by assumption. This reduces the feasibility of restructuring (by Proposition 2) and increases filings, closing the loop.

The spiral has the potential to generate financial instability in the form of multiple equilibria:

**Proposition 6.** Suppose

\[
\hat{v}(D_\lambda^{*} = \lambda_H) < F^{-1}(\kappa) < \hat{v}(D_\lambda^{*} = \lambda_L),
\]

where \(D^{*}\) is the face value that makes creditors’ IC (inequality (5)) bind in the baseline model for the indicated value of \(\lambda\). There are two equilibria:

- There is a “good” equilibrium in which the probability of filing is low, courts are not con-
gested, and the costs of bankruptcy are low; and

• there is a “bad” equilibrium in which the probability of filing is high, courts are congested, and the costs of bankruptcy are high.

This result suggests that bankruptcy policy cannot be separated from financial stability regulation: Congestion itself can create panic-like coordination failures. Bankruptcy policy is not just about mitigating the costs of filings at the margin, but also about preventing mass filings altogether. Indeed, increasing court capacity \( \kappa \)—so that the second inequality in condition (20) is violated—can eliminate the “bad” equilibrium. This adds support to the argument that avoiding court congestion should be a policy priority in response to COVID-19 (see Iverson, Ellias, and Roe (2020)).

6.3 Endogenous Asset Values and Debt Overhang

We have assumed thus far that, although the asset value \( v \) is uncertain ex ante, its ex post distribution is exogenous. We have therefore focused on so-called “direct” costs of liquidation and bankruptcy, ignoring the “indirect” costs that can arise due to, e.g., debt overhang (Myers (1977)). Here, we incorporate endogenous asset values and show conditions under which debt overhang amplifies or attenuates our results.

We assume that the firm can make an investment before the asset value \( v \) is realized, exerting effort \( \eta \) to improve the distribution of \( v \). Specifically, we assume that increasing \( \eta \) to \( \eta' > \eta \) improves the distribution of \( v \) from \( F^{\eta} \) to \( F^{\eta'} \succ F^{\eta} \), where “\( \succ \)” indicates first-order stochastic dominance. We also assume that the firm will exert less effort when it has high levels of debt because the costs of effort are borne solely by equity holders, whereas the benefits are shared with creditors. (Rather than model the firm’s decision directly, however, we simply assume the firm exerts more effort when it has less debt.)

To formulate a comparative-statics result, we define the parameter \( \lambda^{\eta} \) as follows: the smallest \( \lambda \) for which it is feasible to write down debt sufficiently to reduce the bankruptcy filing threshold, \( \hat{v} \), to a given level.\(^{42} \)\(^{43} \) The next result describes how \( \lambda^{\eta} \) depends on \( \eta \).

**Proposition 7.** If the probability of default \( F(\hat{v}) \) does not depend on \( \eta \), then \( \lambda^{\eta} \) is decreasing in \( \eta \). On the other hand, if the tail conditional expectation \( \mathbb{E}[v \mid v \leq \hat{v}(D)] \) does not depend on \( \eta \), then \( \lambda^{\eta} \) is increasing in \( \eta \).

\(^{42} \)We work with \( \hat{v} \) (inequality (3)) instead of the debt \( D \) directly only because it makes the math easier.

\(^{43} \)Proposition 2, which shows that high \( \lambda \) facilitates restructuring, implies such a \( \lambda^{\eta} \) is well-defined.
This result reveals that the effect of debt overhang on restructuring depends on how the overhang affects the distribution of values ex post. High effort can have two effects, one present under each of the conditions in the proposition.

- If effort does not affect the probability of default $F(\hat{v})$, then increasing $\eta$ makes restructuring easier. In this situation, higher effort increases the probability of high asset values, increasing creditor recoveries in bankruptcy and making priority more valuable. In this case, the more there is to gain from restructuring, the more likely it is to occur—creditors restructure to access these gains.

- If, on the other hand, effort affects the probability of default, $F(\hat{v})$, but does not affect the value of the firm’s assets in bankruptcy $E[v|v \leq \hat{v}]$, then increasing $\eta$ makes restructuring harder. In this situation, higher effort reduces the probability of bankruptcy, reducing the value of seniority and making priority less valuable. In this case, the more there is to gain from restructuring, the less likely it is to occur—creditors hold out to free ride on these gains.

Overall, this result adds nuance to Brunnermeier and Krishnamurthy’s (2020) argument that an efficient bankruptcy system helps resolve debt-overhang problems.

### 6.4 Concentrated Debt Holdings and Debt-Equity Exchanges

So far, we have focused on debt held by dispersed and infinitesimally small creditors. In this setting, we find that the feasible restructurings involve swapping junior debt for senior debt. Although this is a reasonable approximation of reality, we do observe some exchange offers that swap debt for equity (see, e.g., Asquith, Gertner, and Scharfstein (1994)). In this section, we relax the assumption that debt is dispersed and show that debt-for-equity exchanges occur if and only if debt holdings are sufficiently concentrated.

To show this, we introduce a measure of creditor concentration, $\xi$, defined as the probability that the firm’s debt is held by a single large creditor or a group of creditors acting in concert (otherwise, it is held by a unit of dispersed creditors, as in the baseline). The firm does not know the distribution of creditors when it makes its exchange offer, but creditors know the distribution when they accept or reject the offer.\(^{44}\)

To explore how $\xi$ affects restructuring, we need to separately analyze the creditors’ IC when debt is dispersed and when it is concentrated. With dispersed debt, the hold-out condition is

\(^{44}\)A firm’s debt can become more or less concentrated as it approaches default and bankruptcy; the measure $\xi$ can be interpreted as reflecting the firm’s expectations regarding future concentration.
the same as in our baseline model (inequality (5)). As we saw there, a debt-for-equity exchange
is infeasible; the firm just offers a minimum amount of senior debt. With concentrated debt, by
contrast, a feasible restructuring may include swapping debt for equity. In this case, the creditor
will accept a combination of new debt $D$ and new equity $1 - \alpha$ if its payoff exceeds its outside
option:

$$
F(\hat{v}(D)) \mathbb{E} \left[ (\theta + (1 - \theta)(1 - \alpha)) \lambda v \mid v \leq \hat{v}(D) \right] \\
+ (1 - F(\hat{v}(D))) \mathbb{E} \left[ D + (1 - \alpha)(v - D) \mid v \geq \hat{v}(D) \right] \\
\geq F(\hat{v}(D_0)) \mathbb{E} \left[ \lambda v \mid v \leq \hat{v}(D_0) \right] + (1 - F(\hat{v}(D_0))) D_0.
$$

(21)

The left-hand side resembles expressions we have seen before: It is the combined payoff of debt
with face value $D$ and a fraction $1 - \alpha$ of the equity. If the creditor accepts the restructuring,
the firm may subsequently file for bankruptcy (first term) or avoid that outcome (second term).
Either way, the creditor receives a payoff on account of both its new debt claim and its new
equity interest. The right-hand side of inequality (21) differs from what we have seen before: It
is the payoff to debt if no write-down takes place. Unlike dispersed creditors, the concentrated
creditor internalizes the fact that, if it does not accept, the restructuring will fail.

The firm knows these IC conditions, but does not know whether its creditors are dispersed
or concentrated when it makes a restructuring offer. Because it can offer a mix of (i) senior debt
and (ii) equity, it has three options:

1. It can offer (i) the smallest amount of senior debt such that dispersed creditors accept but
   (ii) no equity. In this case, the large creditor may or may not accept. Either way, this will
   be an attractive option when $\xi$ is low (creditors are likely dispersed).

2. It can offer (i) no debt but (ii) the smallest amount of equity such that the large creditor
   accepts. In this case, the dispersed creditors will not accept, but this will nonetheless be
   an attractive option when $\xi$ is high (creditors are likely concentrated).

3. It can offer (i) the smallest amount of senior debt that makes dispersed creditors accept
   and (ii) enough equity to make the large creditor accept too. It turns out that this entails
   leaving some rent to the dispersed creditors, but could be optimal to ensure the offer is
   accepted.

The firm’s choice among these options depends on a trade-off. If it makes an offer that is accepted
all of the time (case 3), it minimizes its exposure to the deadweight costs of bankruptcy but the
offer may be overly generous, allowing creditors to capture rents. If the firm makes a less generous
offer (cases 1 and 2), it can reduce the rents paid to creditors, but it will expose itself to the deadweight costs of bankruptcy if the offer is rejected.

Comparing the firms’ payoffs from these options gives the following result:

**Proposition 8.** Suppose \( D_0 \) is sufficiently large. There are thresholds, \( \bar{D} \), \( \xi \), and \( \bar{\xi} \) (given explicitly in equations (94), (116) and (117) in the proof) such that:

- For \( D^* \geq \bar{D} \) and \( \xi < \bar{\xi} \), there are three regions:
  - If \( \xi \leq \xi \), the firm offers senior debt only (and only dispersed creditors accept).
  - If \( \xi < \xi \leq \bar{\xi} \), the firm offers a mix of senior debt and equity (and all creditors accept).
  - If \( \xi > \bar{\xi} \), it offers equity only (and only the concentrated creditor accepts).

- Otherwise, there are two regions: Below a threshold, the firm offers senior debt (and all creditors accept) and, above the threshold, it offers equity only (and only the concentrated creditor accepts).

Overall, this result says that our baseline analysis is robust to some creditor concentration, but that higher levels of concentration induce the firm to use equity as well. Thus, our framework can explain the use of equity in exchange offers observed in practice. Our result is consistent with James’s (1995) finding that banks take equity in restructuring because they, unlike dispersed bondholders, internalize the effect of write-downs (see also Jostarndt and Sautner (2009)).

### 7 Discussion and Conclusion

We develop a model built on two observations about debt restructuring with dispersed creditors. The first is that restructuring is difficult, if not impossible, unless the firm can dilute existing debt with high-priority (senior) debt in an exchange offer. The second is that priority is only as valuable as bankruptcy is likely: Creditors do not care about priority if the firm will never enter bankruptcy, but they do care when bankruptcy is likely, and the value they place on priority increases with the probability of a bankruptcy filing. Although intuitive, this observation yields a counterintuitive implication: Bankruptcy and restructuring are complements. Policies that make bankruptcy attractive to equity holders also facilitate restructuring. This implication drives most of the results in our paper, which show how key parameters of the bankruptcy environment, such as its deadweight costs and creditor friendliness, affect restructurings.

Our analysis could matter for policymakers. It provides a simple way to evaluate whether current bankruptcy law is excessively creditor friendly (or debtor friendly). Our calculations
suggest that U.S. law is too creditor friendly: A reduction in \( \theta \) would likely increase the frequency of restructurings.

Further, as policymakers consider policies to aid struggling businesses during crises such as the COVID-19 pandemic, our model shows that they should prioritize policies that facilitate restructuring (e.g., those that reward creditors for restructuring debts) and policies that increase the payoff to senior lenders in bankruptcy (e.g., a government backstop to DIP loans extended by senior lenders).

Our analysis also has broad implications for the design of bankruptcy policy outside of a crisis. For example, the APR priority can aggravate or mitigate financial distress, depending on the claims involved. It can facilitate restructuring to the extent that it helps secured creditors maintain seniority over other debt claims, but can undermine restructuring if it prevents equity from being paid in bankruptcy.

The model can shed light on other recent controversies in bankruptcy policy. Scholars and practitioners have expressed concern about loans (“DIP loans”) extended to firms in bankruptcy. The vast majority of these loans are extended by pre-existing senior lenders, the rates of return on these loans are thought to be highly (perhaps excessively) profitable (as argued by Eckbo et al. (2019)), and the terms of the DIP loans allow senior lenders to exercise control over speed and outcomes of the bankruptcy process (as discussed by Ayotte and Morrison (2009), among others). Our model suggests a different perspective on DIP loans. These loans increase the payoff to senior lenders in bankruptcy and protect senior lenders from dilution (because they allow the lenders to exercise control over the process). Seen this way, the criticized features of DIP loans can actually facilitate restructuring, thereby avoiding the deadweight costs of bankruptcy.

Finally, our findings may shed light on the evolution of the U.S. bankruptcy law and practice: During the late 19th century, the U.S. lacked a stable bankruptcy law. In that void, lawyers developed techniques for reorganizing companies using non-bankruptcy devices, such as the “equity receivership,” which was a proto-Chapter 11 procedure but was often criticized because many companies (especially railroads) used it as a device to (i) maximize the returns to secured creditors, (ii) give a payoff to equity, and (iii) squeeze out unsecured creditors (Miller and Berkovich (2006)). In other words, private parties developed a technique that seems to resemble what our model recommends.
Appendix

A Out-of-court Liquidation and Bargaining Foundation for $\theta$

In the baseline model, we assume that the firm either repays its debt or files for bankruptcy. In practice, it can also default without filing. In this case, creditors have the right to liquidate assets. Here we model these two options explicitly.

1. **Liquidation.** If the firm defaults and does not file for bankruptcy, creditors can seize the firm’s assets. We assume that their liquidation (or redeployment) value is less than the value to incumbent equity holders, leading to deadweight costs $(1 - \mu)v$. All of the remaining value $\mu v$ goes to creditors; equity holders get nothing. Creditors also have the option to liquidate if bargaining with equity holders breaks down after a bankruptcy filing, as we describe next.

2. **Bankruptcy.** To avoid liquidation, the firm can file for bankruptcy. We assume the same bankruptcy costs as in the baseline model. The value of the firm net of bankruptcy costs, $\lambda v$, is distributed through a structured bargaining process. In particular, creditors can insist on a payoff no less than their recovery in a bankruptcy liquidation, $\mu \lambda v$ (this is called the “best interests test” and represents an outside option during the bargaining). The surplus created by avoiding liquidation, $\lambda v - \mu \lambda v$, is split between the parties. The split is a function of various bankruptcy rules that allocate bargaining power to creditors in some cases (e.g., rules governing adequate protection and lift-stay motions) and equity holders in others (e.g., deviations from absolute priority, third-party releases, and, more broadly, management retention agreements). We model this bargaining environment using

---

45 One thing we abstract from here is that creditors can file an “involuntary” bankruptcy case against a firm. Under U.S. law, they must prove that the firm is “generally not paying such debtor’s debts as such debts become due” 11 U.S.C. §303(h)(1). Courts have not given a precise or consistent definition of “generally not paying,” but it appears to describe a situation where the firm has defaulted on multiple debts that account for a substantial fraction of total debt (Levin and Sommer (2020)). This is a situation close to insolvency, that is, $v \leq D$. Equation (3), however, shows that the firm will choose to file when $v \leq \hat{v}(D)$. As discussed above, $\hat{v}(D)$ will exceed $D$ whenever $\theta$ and $\lambda$ are greater than 0. This suggests that creditor power to start a case is relevant only in the (unusual) situation where the bankruptcy law offers no payout to equity or has no deadweight costs. In practice, involuntary filings account for about two percent of corporate bankruptcy filings (Hynes and Walt (2020)).

46 For the microfoundations of this wedge in value, see, e.g., Aghion and Bolton (1992), Hart (1995), and Shleifer and Vishny (1992). For evidence on the deadweight costs of liquidation, relative to reorganization, see Bernstein, Colonelli, and Iverson (2019).

47 See Bisin and Rampini (2005) and von Thadden, Berglöf, and Roland (2010) for models rationalizing the institution of bankruptcy. See Waldock (2020) for a comprehensive empirical study of bankruptcy filings by large corporations in the U.S.
the generalized Nash bargaining protocol: Creditors get their liquidation value $\mu \lambda v$ plus a fraction $\hat{\theta}$ of the surplus created by avoiding liquidation, where $\hat{\theta}$ is their bargaining power.

When a firm reorganizes in bankruptcy, creditors bargain collectively and are guaranteed (via the “best interests test”) a payoff no lower than what they would receive in a liquidation ($\mu \lambda v$). The extent to which their payoff exceeds $\mu \lambda v$ depends on the value available for distribution in a reorganization ($\lambda v$) and their bargaining power ($\hat{\theta}$). Thus,

$$\text{creditors' payoff} = \text{liquidation value} + \hat{\theta} \times \text{surplus from reorganization}$$

$$= \mu \lambda v + \hat{\theta} (\lambda v - \mu \lambda v)$$

$$= (\mu + (1 - \mu)\hat{\theta}) \lambda v$$

$$\equiv \theta \lambda v,$$

where $\theta := \mu + (1 - \mu)\hat{\theta}$ is the “creditor friendliness” of the baseline model (Section 2.2), now expressed as a combination of the value of creditors’ outside option ($\mu$) and their direct bargaining power in bankruptcy court ($\hat{\theta}$).

B Ex Post Restructuring

In our baseline model, we allow for restructuring only at Date 0, abstracting from it at Date 1. Here, we show this assumption is without loss of generality because restructuring is generally infeasible ex post, when there is no uncertainty about firm value:

**Lemma 4.** There is no ex post restructuring that (uncoordinated) creditors are willing to accept and that the firm is willing to offer.\(^{48}\)

**Proof.** We consider equity and debt restructuring in turn.

**Equity restructuring.** There are two cases corresponding to $v \leq \hat{v}(D)$. In each case, each individual creditor must be better off accepting fraction $1 - \alpha$ of the equity than keeping their debt and equity holders must be better off offering it than not.

In either case, there is no bankruptcy following a successful restructuring, so an individual creditor, taking others’ acceptance as given, must be better off accepting than getting repaid in full:

$$ (1 - \alpha)v \geq D \quad \text{(26)}$$

\(^{48}\)To be precise, change in debt that does not affect anything else—it leaves payoffs and actions unchanged—could be possible. For simplicity, we ignore such immaterial debt changes.

30
Case 1: \( v \geq \hat{v}(D) \). In this case, there is no bankruptcy even absent a restructuring, so equity must be better off offering it than repaying in full:

\[
\alpha v > v - D. \tag{27}
\]

This is mutually incompatible with creditors’ accepting (condition (26)).

Case 2: \( v < \hat{v}(D) \). In this case, the firm would file for bankruptcy absent a restructuring. So equity holders must prefer their residual claim on the firm \( \alpha \) after a restructuring to what they get in bankruptcy absent one, \((1-\theta)\lambda\):

\[
\alpha v \geq (1-\theta)\lambda v. \tag{28}
\]

This is mutual incompatible with creditors’ accepting (condition (26)) and the assumption that \( v < \hat{v}(D) \).

**Debt restructuring.** Denote the total amount of debt after a potential successful restructuring to \( D' < D \). Again, there are two cases, now corresponding to \( v \leq \hat{v}(D') \).

Case 1: \( v \geq \hat{v}(D') \). In this case, an individual creditor accepts the restructuring whenever its payoff \( D' \) from accepting exceeds its payoff \( D \) from holding out, or \( D' \geq D \), i.e., it never accepts a write-down.

Case 2: \( v < \hat{v}(D') \). In this case, the firm files for bankruptcy even if a restructuring is successful. Thus, it is payoff equivalent to not restructuring: the firm gets \((1-\theta)\lambda v\) and creditors get \(\theta \lambda v\).\(^{49}\)

Ex post restructuring is subject to such a severe hold-out problem that no debt reduction is feasible. The reason is that an effective restructuring reduces debt enough that the firm repays its debt for sure—all uncertainty being resolved, the probability that the firm files for bankruptcy is zero. Thus, a hold-out creditor anticipates being repaid in full with certainty. And no creditor is willing to accept a write-down on its debt if its debt is valued at par. Hence, any ex post restructuring is doomed to fail.

\(^{49}\)Note: If we allowed for mixed equilibria, one could exist in this case, albeit an unrealistic one. If the firm could commit to file randomly, say with probability \( p \), then it could induce the creditors to accept. Such random filing is time consistent only if \( \hat{v}(D') = v \), i.e., if the firm is indifferent between filing and not conditional on the offer \( D' \) being accepted. This strategy is implemented with \( D' = (1-\lambda(1-\theta))v \) (which makes the firm indifferent to filing if the offer is accepted) and \( p = \frac{D_0-D'}{D_0-(1-\lambda)\theta v} \) (which makes the creditors indifferent to accepting it).
C Example: Optimal Creditor Friendliness if \( v \) Is Uniform

Here, we illustrate Proposition 3 for \( v \) uniform, \( F(v) \equiv v/\bar{v} \) on \([0, \bar{v}]\). In this case, creditors’ binding IC (\( \Delta = 0 \) in equation (9)) becomes:

\[
\frac{\lambda \theta}{2 \bar{v}} \left( \hat{v}(D^*) \right)^2 = (D_0 - D^*) \left( 1 - \frac{\hat{v}(D^*)}{\bar{v}} \right).
\]

Substituting for \( \hat{v} \) from equation (3) and solving gives:\(^{50}\)

\[
D^* = \frac{(1 - (1 - \theta) \lambda) \bar{v} + D_0 - \sqrt{D_0 - (1 - (1 - \theta) \lambda) \bar{v}}} {2 - \frac{\lambda \theta}{1 - (1 - \theta) \lambda}}. \tag{30}
\]

This expression illustrates that the write-down \( D_0 - D^* \) is increasing in \( \lambda \), as per Proposition 2, and is hump-shaped in \( \theta \), as per Proposition 3; see Figure 1.

![Figure 1: The percentage write-down for uniform \( v \) on \([0, \bar{v}]\), with \( \bar{v} = 100 \), \( D_0 = 50 \), and \( \lambda = 1/2 \).](image)

---

^{50}The other root of the quadratic is larger than \( D_0 \) and is omitted.
So far, we have taken the initial debt level $D_0$ as exogenous. Our results are thus exposed to a Lucas-type critique: By studying the effect of the bankruptcy environment on how the debt is restructured, we neglect to study its effect on how the debt comes to be in the first place. This approach has strengths: With $D_0$ as a starting point, not an outcome, our analysis does not depend on its source. By contrast, to endogenize $D_0$, we need to make specific assumptions about the firm’s motives for borrowing, such as exploiting tax shields as in the trade-off theory, financing investment as in $q$ theory, or disciplining management as in agency theory. Our model also retains tractability while allowing for general asset distributions ($F$). The models with endogenous debt, by and large, do not. Nonetheless, in this appendix, we present one model of endogenous debt to show that our main results with respect to the bankruptcy environment—how it affects the write-down $D_0 - D$ (Proposition 2 and Proposition 3)—are robust to endogenizing $D_0$, at least for numerical examples.

We suppose that the firm takes debt $D_0$ to borrow a fixed amount $I$ from a continuum of competitive creditors, presumably to finance an investment. Net of the investment, the firm value is $v$, which we suppose here is uniform: $v \sim [0, \bar{v}]$. The model proceeds as described in Section 2.

The face value of debt $D$ is thus determined by creditors’ break-even condition:

$$(1 - F(\hat{v}(D)))D + F(\hat{v}(D))\mathbb{E}[\theta \lambda v | v \leq \hat{v}(D)] = I. \quad (31)$$

This depends on $D_0$ via the face value $D$, which, in turn, is the outcome of a restructuring per equation (5). Substituting $F(v) = v/\bar{v}$ and $D = (1 - (1 - \theta)\lambda)\hat{v}$, this can be re-written as

$$\left(1 - \frac{\hat{v}}{\bar{v}}\right)\left(1 - (1 - \theta)\lambda\right)\hat{v} + \frac{\theta \lambda \hat{v}^2}{2\bar{v}} = I. \quad (32)$$

This is a quadratic equation in $\hat{v}$ with a unique positive solution

$$\hat{v} = \frac{(1 - \theta)\lambda - 1 + \sqrt{(1 - (1 - \theta)\lambda)^2 - \frac{4I}{\bar{v}}\left(\frac{\theta \lambda}{2} - (1 - (1 - \theta)\lambda)\right)}}{\frac{2}{\bar{v}}(\frac{\theta \lambda}{2} - (1 - (1 - \theta)\lambda))}. \quad (33)$$

We can combine this expression for $\hat{v}$ from the break-even condition with the expression for the write-down (Proposition 1) to analyze how the write-down depends on $\lambda$ and $\theta$. 

33
The restructuring condition gives an expression for the write-down \( D_0 - D \) in terms of \( \hat{v} \):

\[
D_0 - D = F(\hat{v}(D)) \frac{1}{1 - F(\hat{v}(D))} \mathbb{E}[\theta \lambda v \mid v \leq \hat{v}(D)] = \frac{\lambda \theta \hat{v}^2_D}{2(\bar{v} - \hat{v}_D)},
\]

having made use of the assumption that \( v \) is uniform. Substituting the expression for \( \hat{v} \) from the break-even condition (3), we can plot the write-down as a function of \( \lambda \) and \( \theta \).

Figure 2 illustrates our main results in this model with endogenous \( D_0 \): The equilibrium write-down is generally increasing in bankruptcy efficiency \( \lambda \) (Proposition 2) and can be increasing, decreasing, or hump-shaped in creditor friendliness \( \theta \) (Proposition 3).

\[ \begin{array}{c}
\text{bankruptcy efficiency } \lambda \\
\text{creditor friendliness } \theta \\
\end{array} \]

Figure 2: The plots above show how the equilibrium write-down \( D_0 - D \) depends on \( \lambda \) (left panel) and \( \theta \) (right panel) with \( D_0 \) endogenous. The parameters used are as follows: both panels: \( F(v) = v/100 \) and \( I = 10 \); left panel: \( \theta = 0.4 \) (solid blue), 0.6 (dotted red), 0.8 (dashed violet); right panel: \( \lambda = 0.4 \) (solid blue), 0.6 (dotted red), 0.8 (dashed violet).

### E Further Results on Secured Creditor Power

Here we use the framework of Section 6.1 to ask how secured creditor control interacts with creditor friendliness and inefficient liquidation in bankruptcy.

#### E.1 Interaction of Secured Creditor Power and Creditor Friendliness

We now ask how secured creditor power affects the write-down-maximizing level of creditor friendliness: If secured creditors recover more relative to unsecured creditors (\( \rho \) is higher), should
creditors as a whole get more or less relative to equity (a higher or lower \( \theta \)) in order to maximize the write-down \( D_0 - D \)? We find that they should get more:

**Proposition 9.** Suppose that the write-down is maximized at a unique interior level of creditor friendliness \( \theta^* \) that is not an inflection point (as in, e.g., the uniform case in Figure 1). Increasing the secured creditor power \( \rho \) increases the optimal level of creditor friendliness. That is, \( \frac{d\theta^*}{d\rho} > 0 \).

To see the intuition for this result, recall that \( \theta^* \) is chosen to maximize the value of priority, balancing the increase in creditor recovery value against the decrease in the filing probability. Because high secured creditor power \( \rho \) increases recovery value without affecting the filing probability, \( \theta^* \) increases to balance the two effects.

**Proof.** The IC in inequality (44) can be re-written as

\[
\Delta = \left(1 - F(\hat{v})\right)(D - D_0) + \left(1 - (1 - \rho)\frac{D_0}{D}\right) \lambda \theta \int_0^{\hat{v}} vdF(v) \geq 0. \tag{35}
\]

The binding IC, \( \Delta = 0 \), defines the written-down debt level \( D^* \); minimizing \( D^* \) over \( \theta \) defines the optimal level of creditor friendliness. Thus, by the chain rule, the effect of \( \rho \) on \( \theta^* \) is given by:

\[
\frac{d\theta^*}{d\rho} = -\frac{\frac{\partial}{\partial \rho} \left( \frac{\partial \Delta}{\partial \theta} \right)}{\frac{\partial^2 \Delta}{\partial \theta^2}}. \tag{36}
\]

Note that the denominator is negative at \( \theta^* \) given that we have assumed that \( \theta^* \) is an interior local minimum and not an inflection point.\(^{51}\)

We compute the denominator directly, step by step:

\(^{51}\)To see why, differentiate the defining condition for \( \theta^* \), i.e., \( \frac{dD^*}{d\theta} \bigg|_{\theta = \theta^*} = -\frac{\partial u_\rho}{\partial D} \bigg|_{\theta = \theta^*} = 0 \), to get

\[
\frac{d^2 D^*}{d\theta^2} = -\left( \frac{\partial \Delta}{\partial D} \right)^{-2} \left[ \frac{\partial \Delta}{\partial D} \frac{d}{d\theta} \left( \frac{\partial \Delta}{\partial \theta} \right) - \frac{d}{d\theta} \left( \frac{\partial \Delta}{\partial D} \right) \frac{\partial \Delta}{\partial \theta} \right]
= -\left( \frac{\partial \Delta}{\partial D} \right)^{-1} \left( \frac{\partial^2 \Delta}{\partial \theta^2} \right)_{\theta^*, D^*} > 0,
\]

having simplified using the conditions of optimality for \( \theta^* \), i.e., \( \frac{dD^*}{d\theta} \bigg|_{\theta = \theta^*} = 0 \) and \( \frac{\partial u_\rho}{\partial \theta} \bigg|_{\theta = \theta^*} = 0 \). The result follows from the fact that \( \frac{\partial \Delta}{\partial D} > 0 \) (see, e.g., equation (71)).

35
• First,

\[
\frac{\partial \Delta}{\partial \theta} = \left[ f(\hat{v})(D_0 - D) + \left(1 - (1 - \rho) \frac{D_0}{D}\right) \lambda \hat{v} f(\hat{v})\right] \frac{\partial \hat{v}}{\partial \theta} 
\]
\[
+ \left(1 - (1 - \rho) \frac{D_0}{D}\right) \lambda \int_{\hat{v}}^0 v dF(v)
\]
\[
= -\lambda \hat{v} f(\hat{v})(\hat{v}_0 - \hat{v}) + \left(1 - (1 - \rho) \frac{D_0}{D}\right) \lambda \left(\int_{\hat{v}}^0 v dF(v) - \frac{\lambda \theta \hat{v} f(\hat{v})}{1 - \lambda(1 - \theta)} \hat{v}\right),
\]

having used that \(D = (1 - \lambda(1 - \theta)) \hat{v}, D_0 = (1 - \lambda(1 - \theta)) \hat{v}_0\), and

\[
\frac{\partial \hat{v}}{\partial \theta} = -\frac{\lambda \hat{v}}{1 - \lambda(1 - \theta)}.
\]

• Second,

\[
\frac{\partial^2 \Delta}{\partial \rho \partial \theta} = \frac{D_0}{D} \lambda \left(\int_{\hat{v}}^0 v dF(v) - \frac{\lambda \theta \hat{v} f(\hat{v})}{1 - \lambda(1 - \theta)} \hat{v}\right).
\]

To determine its sign, it turns out that we can use two facts:

– Using \(D < D_0\) in \(\Delta = 0\) above, we get that

\[
1 - (1 - \rho) \frac{D_0}{D} > 0.
\]

– Given \(\theta^*\) is optimal, \(\partial \Delta / \partial \theta|_{\theta = \theta^*} = 0\). This, together with the last inequality, implies that

\[
\int_{\hat{v}}^0 v dF(v) > \frac{\lambda \theta \hat{v} f(\hat{v})}{1 - \lambda(1 - \theta)} \hat{v}.
\]

This implies that \(\partial^2 \Delta / \partial \rho \partial \theta\) in equation (40) is positive and, thus, given the above, that

\[
\frac{d\theta^*}{d\rho} > 0.
\]

\[\square\]

E.2 Inefficiencies of Secured Creditor Control

Here we relax the assumption that liquidation costs do not depend on the division of surplus. We assume instead that secured creditor power can lead to inefficient liquidation. These costs could be born by creditors or equity holders. We consider each case in turn.
E.2.1 Creditors Bear the Costs of Secured Creditor Power

First, we assume that unsecured debt receives only a fraction $\zeta$ of what is left after secured debt and equity are paid, so $1 - \zeta$ captures the inefficiency of secured creditor power. With this modification, the creditors’ IC in equation (17) becomes

$$
\left(1 - F(\hat{v}(D))\right)D + F(\hat{v}(D))\mathbb{E}[\theta \lambda v \mid v < \hat{v}(D)] \\
\geq \left(1 - F(\hat{v}(D))\right)D_0 + \zeta(1 - \rho)F(\hat{v}(D))\mathbb{E}[\theta \lambda v \mid v < \hat{v}(D)] \frac{D_0}{D}. \tag{44}
$$

Observe that $\zeta$ above plays the same role that $1 - \rho$ does (cf. equation (18)). Therefore, Proposition 5 and Proposition 9 imply that an increase in $\zeta$ makes restructuring harder and reduces the optimal level of creditor friendliness $\theta^*.$

E.2.2 Equity Holders Bear the Cost of Secured Creditor Power

Now we assume that for creditors to gain $1$ in bankruptcy, equity holders must forgo more than $1.$ Specifically, for every $(1 - \gamma/2)\theta \lambda v$ that creditors get, equity gives up $(1 - \theta)\lambda v.$ Thus, $\gamma$ measures the inefficiencies they induce ex post. If $\gamma = 0,$ the model is the same as the baseline. Increasing $\gamma$ decreases the total surplus.

To explore how the inefficiencies of creditor power could affect restructuring, we explore how the optimal level of creditor friendliness depends on the inefficiencies induced by secured creditor power $\gamma$ (cf. Section 4.2 and Section 4.3). This gives the next result:

**Proposition 10.** Suppose that the write-down is maximized at a unique interior level of creditor friendliness $\theta^*$ that is not an inflection point (as in, e.g., the uniform case in Figure 1). Increasing the inefficiency of secured creditor power $\gamma$ decreases the optimal level of creditor friendliness. That is, $d\theta^*/d\gamma < 0.$

Intuitively, if giving creditors power destroys value in bankruptcy, then giving them more power is likely to make them even less willing to accept a restructuring. Hence, the larger is $\gamma,$ the larger is the region of $\theta$ for which making the code more creditor friendly makes restructuring harder.

**Proof.** We begin from the creditors’ IC:

$$
\Delta = (1 - F(\hat{v})) (D - D_0) + \lambda \theta \left(1 - \frac{\gamma \theta}{2}\right) \int_0^{\hat{v}} v dF(v), \tag{45}
$$
which is just the creditors’ IC in equation (9) modified to include the inefficiencies captured by \( \gamma \). The binding IC, \( \Delta = 0 \), defines the written-down debt level \( D^* \) and minimizing \( D^* \) over \( \theta \) defines the optimal level of creditor friendliness \( \theta^* \). Thus, by the chain rule, the effect of \( \gamma \) on \( \theta^* \) is given by:

\[
\frac{d\theta^*}{d\gamma} = -\frac{\partial}{\partial \gamma} \left( \frac{\partial \Delta}{\partial \theta} \right) = -\frac{\partial^2 \Delta}{\partial \gamma \partial \theta}.
\] (46)

Note that the denominator is negative given that we have assumed that \( \theta^* \) is an interior local minimum and not an inflection point (see footnote 51).

We compute the numerator \( \frac{\partial^2 \Delta}{\partial \gamma \partial \theta} \) directly, step by step:

- First,

\[
\frac{\partial \Delta}{\partial \theta} = \lambda (1 - \gamma \theta) \int_0^\hat{v} v dF(v) + \left[ D_0 - D + \lambda \theta \left( 1 - \frac{\gamma \theta}{2} \right) \right] f(\hat{v}) \frac{\partial \hat{v}}{\partial \theta} \] (47)

\[
= \lambda (1 - \gamma \theta) \int_0^\hat{v} v dF(v) - \lambda \left[ \hat{v}_0 - \hat{v} + \frac{\lambda \theta (2 - \gamma \theta)}{2(1 - \lambda (1 - \theta))} \right] \hat{v} f(\hat{v}),
\] (48)

having used that \( D = (1 - \lambda (1 - \theta)) \hat{v} \), \( D_0 = (1 - \lambda (1 - \theta)) \hat{v}_0 \), and

\[
\frac{\partial \hat{v}}{\partial \theta} = \frac{-\lambda \hat{v}}{1 - \lambda (1 - \theta)}.
\] (49)

- Second,

\[
\frac{\partial^2 \Delta}{\partial \gamma \partial \theta} = -\lambda \theta \left( \int_0^\hat{v} v dF(v) - \frac{\lambda \theta}{2(1 - \lambda (1 - \theta))} \hat{v}^2 f(\hat{v}) \right).
\] (50)

To determine the sign, it turns out that we can use the fact that \( \theta^* \) is optimal, so \( \partial \Delta / \partial \theta \big|_{\theta=\theta^*} = 0 \). That is, the derivative in equation (48) is zero. Manipulating, we get:

\[
\lambda (1 - \gamma \theta) \left( \int_0^\hat{v} v dF(v) - \frac{\lambda \theta}{2(1 - \lambda (1 - \theta))} \hat{v}^2 f(\hat{v}) \right) - \lambda \left[ \hat{v}_0 - \hat{v} + \frac{\lambda \theta}{2(1 - \lambda (1 - \theta))} \right] \hat{v} f(\hat{v}) = 0
\] (51)

and, thus, that:

\[
- \int_0^\hat{v} v dF(v) + \frac{\lambda \theta}{2(1 - \lambda (1 - \theta))} \hat{v}^2 f(\hat{v}) = -(1 - \gamma \theta)^{-1} \left[ \hat{v}_0 - \hat{v} + \frac{\lambda \theta}{2(1 - \lambda (1 - \theta))} \right] \hat{v} f(\hat{v}),
\] (52)

which is the numerator in equation (50). It is negative given that \( \hat{v}_0 > \hat{v} \).

Substituting into equation (46), we see that \( d\theta^*/d\gamma \) is negative.
F Tort Claimants

Here, we study the role of “accidental”/tort creditors. To do so, we suppose the firm has outstanding tort claims equal to $T$. If $T$ is not paid in full prior to a bankruptcy filing, different priority rules correspond to different types of taxes in bankruptcy: If tort claims are treated on-par with secured claims, they are equivalent to tax $\tau_s$ on senior debt. If they are treated on-par with unsecured debt, they represent a tax $\tau_j$ on junior debt. If they are junior to unsecured debt, there is no tax on creditors (and they will often go unpaid).

Within this set-up, we now return to creditors’ incentive to accept a restructuring. Their IC becomes:

$$(1 - F(\hat{v}(D + T)))D + F(\hat{v}(D + T))\mathbb{E} \left[ (1 - \tau_s)\theta \lambda v \mid v < \hat{v}(D + T) \right] \geq (1 - F(\hat{v}(D + T)))D_0 + (1 - \rho)F(\hat{v}(D + T))\mathbb{E} \left[ (1 - \tau_j)\theta \lambda v \mid v < \hat{v}(D + T) \right] \frac{D_0}{D}. \quad (53)$$

This condition is easiest to satisfy if $\tau_s$ is small and $\tau_j$ is large. This suggests that, to facilitate restructuring, tort claimants should be paid behind secured debt, but ahead of unsecured debt. That ordering makes priority valuable by (i) increasing the value of secured debt and (ii) decreasing the value of unsecured debt.

G Mixed Offers

In our baseline model, we study exchange offers that include a single security, equity or debt. In Section 6.4, we show that offers that include a mix and debt and equity could arise with a concentrated creditor. Here, we show they never arise in our baseline model with dispersed creditors.

Suppose that the firm offers creditors a mix of a proportion of equity $1 - \alpha$ and senior debt with face value $D$ in exchange for their junior debt $D_0$.

Noting that the bankruptcy decision condition in equation (3) is unchanged by new equity (i.e., that the firm will file whenever $v \leq \hat{v}(D)$), we can write a creditor’s payoffs as follows:
• If it accepts, it gets:

\[
\text{payoff}_{\text{acc.}} = (1 - F(\hat{v}))D + \int_{0}^{\hat{v}} (\theta + (1 - \alpha)(1 - \theta))\lambda v dF(v)
\]

\[+ \int_{\hat{v}}^{\infty} (1 - \alpha)(v - D)dF(v). \tag{54}\]

• If it rejects, it gets:

\[
\text{payoff}_{\text{rej.}} = (1 - F(\hat{v}))D_0. \tag{55}\]

So the firm chooses \(\alpha\) and \(D\) to

maximize \(\int_{0}^{\hat{v}} \alpha \lambda (1 - \theta)v f(v) dv + \int_{\hat{v}}^{\infty} \alpha (v - D)f(v) dv \tag{56}\)

subject to creditors' IC that

\[
\text{payoff}_{\text{acc.}} \geq \text{payoff}_{\text{rej.}}. \tag{57}\]

Supposing that the constraint binds and substituting it in the objective, we can re-write the problem as:

maximize \(\int_{0}^{\hat{v}} \lambda v dF(v) + \int_{\hat{v}}^{\infty} v dF(v) - (1 - F(\hat{v}(D)))D_0. \tag{58}\)

Now observe that this does not depend on \(\alpha\) and that \(D\) appears only in \(\hat{v}\). Hence, if there is an interior optimum, we can maximize the objective with respect to \(\hat{v}\) directly to get:

\[-\lambda f(\hat{v})\hat{v} + f(\hat{v})\hat{v} + f(\hat{v})D_0 = 0. \tag{59}\]

Substituting in for \(\hat{v}\) from equation (3) and solving, we find that:

\[D = \frac{1 - \lambda(1 - \theta)}{1 - \lambda}D_0 > D_0. \tag{60}\]

Still supposing an interior optimum, substitute \(D_0\) into the binding constraint in (57) to get that:

\[(1 - \alpha) \left( \int_{0}^{\hat{v}} + (1 - \theta)\lambda v dF(v) + \int_{\hat{v}}^{\infty} (v - D)dF(v) \right) =

\[= -(1 - F(\hat{v})))\theta\lambda \hat{v}(D) - \int_{0}^{\hat{v}} \theta \lambda v dF(v). \tag{61}\]

But this implies that new equity is negative. So we conclude that there is not an interior optimum,
but, rather, that $1 - \alpha = 0$. Substituting into the constraint (57), we get:

$$(1 - F(\hat{v}))D + \int_{0}^{\hat{v}} \theta \lambda v f(v) dv \geq (1 - F(\hat{v}))D_0,$$

which is the usual constraint.

### H Proofs

**H.1 Proof of Lemma 1**

Creditors are better off accepting the equity share $1 - \alpha$ if its value is greater than their expected payoff in bankruptcy:

$$(1 - \alpha)E[v] \geq E[1_{\{v \geq \hat{v}\}}D_0 + 1_{\{v < \hat{v}\}}\theta \lambda v].$$

Similarly, equity holders are better off if their residual claim ($\alpha$ of the assets) is worth more than what they expect in bankruptcy:

$$\alpha E[v] > E[1_{\{v \geq \hat{v}\}}(v - D_0) + 1_{\{v < \hat{v}\}}(1 - \theta)\lambda v] \equiv E[\max\{v - D_0, (1 - \theta)\lambda v\}].$$

These inequalities can be rewritten and combined as inequality (4) in the statement of the result. Since the left-most term is always strictly greater than the right-most term, an appropriate debt-to-equity restructuring can implement a strict Pareto improvement (and avoid all costs of financial distress).

**H.2 Proof of Lemma 2**

First, recall that each creditor accepts a restructuring offer only if it makes the creditor better off, given that other creditors accept. That is, an individual creditor must prefer getting a fraction $1 - \alpha$ of the assets to holding its original debt with face value $D_0$. If all other creditors agree to the restructuring, the firm is effectively all equity (assuming the individual creditor is infinitesimally small). A creditor therefore accepts if:

$$(1 - \alpha)E[v] \geq D_0.$$
can be re-written and combined as:

\[
E[v] - D_0 \geq \alpha E[v] \geq E[v] - D_0 + \mathbb{E}\left[1_{v<\hat{v}}\left\{(1 - \theta)\lambda v - (v - D_0)\right\}\right].
\] (66)

The last expectation is positive, because the term in braces is positive for \( v < \hat{v} \) by the definition of \( \hat{v} \) (equation (3)). Hence, the right-most term is greater than the left-most term; no restructuring of debt to equity is feasible.

\[\square\]

**H.3 Proof of Lemma 3**

An individual creditor will accept a restructuring to pari passu debt if

\[
\left(1 - F(\hat{v}(D))\right)D + F(\hat{v}(D))\mathbb{E}\left[\theta \lambda v \mid v < \hat{v}(D)\right] \geq \left(1 - F(\hat{v}(D))\right)D_0 + F(\hat{v}(D))\frac{D_0}{D}\mathbb{E}\left[\theta \lambda v \mid v < \hat{v}(D)\right].
\] (67)

The left-hand side of the inequality above is the expected payoff if the creditor accepts: If there is no future bankruptcy, the creditor gets \( D \); if there is one, creditor gets a unit share of the recovery (given it has the same face value \( D \) as the unit of other identical creditors). The right-hand side is the expected payoff if the creditor holds out: If there is no future bankruptcy, it gets \( D_0 \); if there is, it gets a share \( D_0/D \) of the recovery value (given it has face value \( D_0 \) and each of the unit of other identical creditors have face value \( D \)). Observe that, because the creditor is small, its decision does not affect the firm’s filing decision; hence, the bankruptcy threshold \( \hat{v} \) depends on the new level of total debt \( D \) even if the creditor holds out. Denoting the price of (unsecured) debt with face value one by \( e^{-y^u} \) (\( y^u \) is the continuously compounded yield to maturity), this condition can be re-written as

\[
D e^{-y^u} \geq D_0 e^{-y^u}.
\] (68)

This is never satisfied, implying immediately that no restructuring to equal priority debt is feasible.

**H.4 Proof of Proposition 1**

The proof is in the text. \[\square\]
H.5 Proof of Proposition 2

First, we re-write $\Delta$ in equation (9) as:

$$\Delta := \left(1 - F(\hat{v}(D))\right)(D - D_0) + \lambda \theta \int_0^{\hat{v}(D)} v dF(v). \quad (69)$$

The maximum write-down $D_0 - D^*$ corresponds to the minimum face value $D^*$, or to $\Delta = 0$. To see how $D^*$ depends on $\lambda$, we use the chain rule to write:

$$\frac{\partial D^*}{\partial \lambda} = -\frac{\partial \Delta}{\partial \lambda} \frac{\partial \Delta}{\partial D}. \quad (70)$$

Computing, we see that the denominator is positive:

$$\frac{\partial \Delta}{\partial D} = f(\hat{v}) \left(\frac{\partial \hat{v}}{\partial D} D_0 - \hat{v}\right) + (1 - F(\hat{v})) + \lambda \theta \hat{v} f(\hat{v}) \frac{\partial \hat{v}}{\partial D} > 0, \quad (71)$$

given that all terms are positive.\footnote{To see why, note that $\frac{\partial \hat{v}}{\partial D}(D_0) = \hat{v}(D_0)$ and $D_0 > D^*$.} The numerator is positive too:

$$\frac{\partial \Delta}{\partial \lambda} = \theta \int_0^{\hat{v}(D^*)} v dF(v) + \left(\lambda \theta \hat{v} + D_0 - D^*\right) f(\hat{v}) \frac{\partial \hat{v}}{\partial \lambda} > 0. \quad (72)$$

This proves the result in the text. \hfill $\Box$

H.6 Proof of Proposition 3

To prove the first part of the result—how $D^*$ depends on $\theta$—we use the chain rule to write:

$$\frac{\partial D^*}{\partial \theta} = -\frac{\partial \Delta}{\partial \theta} \frac{\partial \Delta}{\partial D}. \quad (73)$$

The denominator is as in equation (71) above. Recall that it is positive. The numerator is:

$$\frac{\partial \Delta}{\partial \theta} = \lambda \int_0^{\hat{v}} v dF(v) + (\lambda \theta \hat{v} + D_0 - D^*) f(\hat{v}) \frac{\partial \hat{v}}{\partial \theta}. \quad (74)$$
Given that the last term is negative, that is,

\[
\frac{\partial \hat{v}}{\partial \theta} = -\frac{\lambda D}{(1 - (1 - \theta)\lambda)^2} < 0, \tag{75}
\]

this expression can change sign depending on parameters. This proves the first part of the result.

To prove the second part of the result on the existence of an interior \( \theta^* \), we show that it follows from continuity: We show that \( \theta^* \) is always decreasing in \( \theta \) at \( \theta = 0 \) and, under the condition in the result, is increasing in \( \theta \) at \( \theta = 1 \), so \( D^* \) must be minimized for an interior value \( \theta^* \in (0, 1) \).

From equations (71) and (73), we know that \( \partial D^*/\partial \theta \) has the opposite sign of \( \partial \Delta/\partial \theta \), which is given in equation (74). We compute:

- At \( \theta = 0 \), we have \( \Delta = (1 - F(\hat{v}))(D - D_0) \); hence, \( D^* = D_0 \). Now, substituting into equation (74),

\[
\frac{\partial \Delta}{\partial \theta} \bigg|_{\theta=0} = \lambda \int_0^{\hat{v}} vdF(v) + (D_0 - D^*) f(\hat{v}) \frac{\partial \hat{v}}{\partial \theta} \tag{76}
\]

\[
= \lambda \int_0^{\hat{v}} vdF(v) > 0. \tag{77}
\]

- At \( \theta = 1 \), we have \( \hat{v} = D \) and \( \Delta = (1 - F(D))(D - D_0) + \lambda \int_0^D vdF(v) \); hence, \( \lambda \int_0^{D^*} vdF(v) = (1 - F(D^*))(D_0 - D^*) \). Now, substituting into equation (74),

\[
\frac{\partial \Delta}{\partial \theta} \bigg|_{\theta=1} = \lambda \int_0^{D^*} vdF(v) - (D_0 - D^*)\lambda D^* f(D^*) \tag{78}
\]

\[
= (D_0 - D^*) \left[ 1 - F(D^*) - \lambda D^* f(D^*) \right]. \tag{79}
\]

Given \( D_0 > D^* \), a sufficient condition for this to be negative (and hence for the existence of an interior minimum) is for the term in square brackets to be negative (at \( \theta = 1 \)), which is the condition in the statement of the result.

**H.7 Proof of Proposition 4**

We have to work with equation (15). Given that the denominator \( \partial \Delta/\partial \hat{v} \) is the same for each subsidy and we know the bang for the buck \( q_i \), we need only to calculate the numerator \( -\partial \Delta/\partial s_i \).
for each subsidy in turn. Writing $\Delta$ as

$$\Delta = \int_0^{\hat{v}} \left( \lambda \theta \hat{v} + s^B_S - s^B_U \right) dF(\hat{v}) + \int_{\hat{v}}^{\infty} \left( (1 - \lambda(1 - \theta))\hat{v} + s^O_E - s^O_B - D_0 + s^O_S - s^O_U \right) dF(\hat{v}), \quad (80)$$

we derive the sensitivities $-\partial \Delta / \partial s_i$ for each $s_i$ as follows:

1. Equity inside bankruptcy: $-\partial \Delta / \partial s^B_E = 1 - F(\hat{v})$.
2. Unsecured debt inside bankruptcy: $-\partial \Delta / \partial s^B_U = F(\hat{v})$.
3. Secured debt inside bankruptcy: $-\partial \Delta / \partial s^B_S = -F(\hat{v})$.
4. Equity outside bankruptcy: $-\partial \Delta / \partial s^O_E = -\left(1 - F(\hat{v})\right)$.
5. Unsecured debt outside bankruptcy: $-\partial \Delta / \partial s^O_U = 1 - F(\hat{v})$.
6. Secured debt outside bankruptcy: $-\partial \Delta / \partial s^O_S = -(1 - F(\hat{v}))$.

Multiplying these sensitivities by the “bang for the buck” $1/q_i$, with $q_i = F(\hat{v})$ for subsidies inside bankruptcy and $1 - F(\hat{v})$ for outside, equation (15) gives the expressions in the table (Table 1).

Finally, to ensure that we want to choose the smallest entry in Table 1, i.e., that minimizing $d\hat{v}/d\varepsilon$ is equivalent to minimizing $d\hat{v}/d\varepsilon$, we need to check that $\partial \Delta / \partial \hat{v} > 0$ at $s = 0$ in equation (15). This holds:

$$\left. \frac{\partial \Delta}{\partial \hat{v}} \right|_{\hat{v}=0} = (\lambda \theta \hat{v} + s^B_S - s^B_U - (1 - \lambda(1 - \theta))\hat{v}$$

$$- s^O_E + s^B_E + D_0 - s^O_S - s^O_U) f(\hat{v}) + (1 - \lambda(1 - \theta))(1 - F(\hat{v})) \right|_{\hat{v}=0}$$

$$= (D_0 - (1 - \lambda)\hat{v}) f(\hat{v}) + (1 - \lambda(1 - \theta))(1 - F(\hat{v})) > 0. \quad (82)$$

\[\Box\]

**H.8 Proof of Proposition 5**

Note that the result is not quite as immediate as it might seem from equation (18), because we have to take into account that the equilibrium debt level $D^*$ and, hence, the default threshold $\hat{v} = \hat{v}(D^*)$ depend on $\rho$. That said, the immediate intuition does hold: Increasing $\rho$ increases the write-down. To prove it, we use implicit differentiation.
First, define the difference in a creditor’s payoffs from accepting versus rejecting an offer, given that other creditors accept:

\[
\Delta = (1 - F(\hat{v}))(D - D_0) + F(\hat{v})\mathbb{E}[\lambda \theta v | v < \hat{v}] \left(1 - (1 - \rho)\frac{D_0}{D}\right) \tag{83}
\]

\[
= (1 - F(\hat{v}))(D - D_0) + \left(1 - (1 - \rho)\frac{D_0}{D}\right) \int_0^{\hat{v}} \lambda \theta v dF(v). \tag{84}
\]

Now compute the derivative \(dD^*/d\rho\) in two steps:

\[
\frac{\partial \Delta}{\partial D} = (1 - F(\hat{v})) + (1 - \rho)D_0D^{-2} \int_0^{\hat{v}} \lambda \theta v dF(v) + \\
+ \left(D_0 - D + \left(1 - (1 - \rho)\frac{D_0}{D}\right) \lambda \theta \hat{v}\right) f(\hat{v}) \frac{\partial \hat{v}}{\partial D} > 0
\]

and

\[
\frac{\partial \Delta}{\partial \rho} = \frac{D_0}{D} \int_0^{\hat{v}} \lambda \theta v dF(v) > 0. \tag{86}
\]

So,

\[
\frac{dD^*}{d\rho} = -\frac{\partial \Delta}{\partial \rho} < 0 \tag{87}
\]

and \(\rho = 1\) maximizes the write-down.

\[\square\]

**H.9 Proof of Proposition 6**

The proof is in the text. \[\square\]

**H.10 Proof of Proposition 7**

We first re-write the creditors’ biding IC in inequality (5) as a function of \(\eta\) as:

\[
\Delta = \int_{\hat{v}(D)}^{\infty} \left((1 - \lambda^n(\hat{v})(1 - \theta))\hat{v} - D_0\right) dF^n(v) + \int_{0}^{\hat{v}(D)} \theta \lambda^n(\hat{v}) v dF^n(v) = 0, \tag{88}
\]
having substituted for \( D = (1 - \lambda^n(\hat{v})(1 - \theta))\hat{v} \) from equation (3). Solving, we can write the minimum \( \lambda^n(\hat{v}) \) needed to restructure to the debt level \( D \) as:

\[
\lambda^n(\hat{v}) = \frac{\int_{\hat{v}}^\infty (D_0 - \hat{v})dF^n(v)}{\int_0^{\hat{v}} \theta \hat{v}dF^n(v) - \int_{\hat{v}}^\infty (1 - \theta)\hat{v}dF^n(v)}
\]

\[
= \frac{(D_0 - \hat{v})(1 - F^n(\hat{v}))}{\theta F^n(\hat{v}) \mathbb{E}^n[v|v < \hat{v}] - (1 - \theta)(1 - F^n(\hat{v}))\hat{v}},
\]

where \( \mathbb{E}^n \) denotes the expectation given the distribution \( F^n \).

Now we show each part of the result in turn.

**Tail expectation** \( \mathbb{E}^n[v|v < \hat{v}] \) does not depend on \( \eta \). Given that, by assumption \( F^n(\hat{v}) \geq F^n(\hat{v}) \), we have that \( F^n(\hat{v}) < F^n(\hat{v}) \). This implies that the numerator in equation (90) is increasing in \( \eta \) and the denominator is decreasing in \( \eta \). Thus, \( \lambda^n(\hat{v}) > \lambda^n(\hat{v}) \).

**Default probability** \( F^n(\hat{v}) \) does not depend on \( \eta \). That is, \( F^n(\hat{v}) = F^n(\hat{v}) \). Given that, by assumption \( F^n(\hat{v}) \geq F^n(\hat{v}) \), we have that \( F^n(\hat{v}) \mathbb{E}^n[v|v < \hat{v}] > F^n(\hat{v}) \mathbb{E}^n[v|v < \hat{v}] \). This implies that the denominator of equation (90) is increasing in \( \eta \). And, since \( F^n(\hat{v}) = F^n(\hat{v}) \), the numerator is the same under \( \eta \) and \( \eta' \). Thus, \( \lambda^n(\hat{v}) < \lambda^n(\hat{v}) \).

\[\square\]

**H.11 Proof of Proposition 8**

Here, we solve for the restructuring offer that equity holders make if they face a large, concentrated creditor (denoted by \( L \) below) with probability \( \xi \) and small, dispersed creditors (denoted by \( S \)) with probability \( 1 - \xi \).

\[\text{53} \text{Given } F^n(\hat{v}) \text{ does not depend on } \eta, \text{ this is akin to the fact that } F^n(\hat{v}) \geq F^n(\hat{v}) \text{ implies } \mathbb{E}^n[v|v > \hat{v}] > \mathbb{E}^n[v|v < \hat{v}] \text{ and can be proved via a change of variables, } v := (F^n)^{-1}(F^n(\hat{v}));
\]

\[
F^n(\hat{v}) \mathbb{E}^n[v|v < \hat{v}] \equiv \int_0^{\hat{v}} vdF^n(v) = \int_0^{\hat{v}} (F^n)^{-1}(F^n(\hat{v}))dF^n((F^n)^{-1}(F^n(\hat{v})))
\]

\[
= \int_0^{\hat{v}} (F^n)^{-1}(F^n(\hat{v}))dF^n(\hat{v})
\]

\[
\leq \int_0^{\hat{v}} \hat{v}dF^n(\hat{v}) \equiv F^n(\hat{v}) \mathbb{E}^n[v|v < \hat{v}],
\]

where the last inequality follows from the definition of stochastic dominance. That is, \( F^n(\hat{v}) \geq F^n(\hat{v}) \) or, equivalently, \( \hat{v} \geq (F^n)^{-1}(F^n(\hat{v})) \). Note, critically, that the assumption that \( F^n(\hat{v}) \) does not depend on \( \eta \) allowed us to change variables without changing the bounds of integration (the result would not obtain without that assumption).
Ultimately, there are three possibilities:

1. The firm makes an offer that \( L \) accepts and \( S \) reject.
2. The firm makes an offer the \( L \) rejects and \( S \) accept.
3. The firm makes an offer that both \( L \) and \( S \) accept.

The proof involves simply (i) computing current equity’s optimal restructuring given each of these possibilities and (ii) comparing its payoffs (denoted by \( u \) below) from each possibility given these optimal restructurings. But this involves some steps:

Step 1: Write \( L \)’s and \( S \)’s ICs to accept a restructuring offer.

Step 2: Write current equity’s payoffs \( u \) in benchmarks in which \( \xi = 1 \) and \( \xi = 0 \). That is, creditors are concentrated for sure or dispersed for sure (these expressions are useful in the subsequent comparisons).

Step 3: Calculate current equity’s payoffs \( u \) for each possibility 1–3 above.

Step 4: Compare these payoffs to determine the optimal restructuring offer.

**Step 1: IC constraints.** Here, we define the conditions for each type of creditor to accept the firm’s offer of a restructuring to debt \( D \) and equity \( 1 - \alpha \).

- **Large creditor’s IC.** Define the difference in its payoffs from accepting and rejecting the offer (inequality (21) with the expectations expanded as integrals) as \( \Delta_L \):

\[
\Delta_L := \int_0^{\hat{v}(D)} (\theta + (1 - \theta)(1 - \alpha))\lambda v f(v)dv + \int_{\hat{v}(D)}^{\infty} (D + (1 - \alpha)(v - D))f(v)dv \\
- \int_0^{\hat{v}(D_0)} \theta \lambda v f(v)dv - \int_{\hat{v}(D_0)}^{\infty} D_0 f(v)dv. \tag{91}
\]

Its IC is thus \( \Delta_L \geq 0 \).

- **Small creditors’ IC.** Define their difference in payoffs from accepting and rejecting the offer as \( \Delta_S \):

\[
\Delta_S := \int_0^{\hat{v}(D)} (\theta + (1 - \theta)(1 - \alpha))\lambda v f(v)dv \\
+ \int_{\hat{v}(D)}^{\infty} (D + (1 - \alpha)(v - D))f(v)dv - \int_{\hat{v}(D)}^{\infty} D_0 f(v)dv. \tag{92}
\]

Their IC is thus \( \Delta_S \geq 0 \).
Note the difference between these conditions: Whereas \( L \) takes into account the effect of restructuring on the default probability, \( S \) do not (this creates the hold-out problem in the baseline model).

When equity holders make a restructuring offer, they can choose \( D \) and \( 1 - \alpha \) that will be accepted if there is a large creditor, if there are small creditors, or in both cases. Thus, they have to take into account which IC is tighter.

Comparing \( \Delta_L \) and \( \Delta_S \) reveals that \( L \)'s IC is tighter than \( S \)'s if
\[
\Delta_S - \Delta_L = \int_0^{\hat{v}(D_0)} \lambda \theta v f(v) dv - \int_{\hat{v}(D)}^{\hat{v}(D_0)} D_0 f(v) dv \geq 0. \tag{93}
\]
Because this inequality does not depend on \( \alpha \), it is satisfied whenever \( D \) is above a threshold, \( \tilde{D} \), which solves
\[
\Delta_L - \Delta_S \bigg|_{D=\tilde{D}} = 0. \tag{94}
\]

**Step 2:** Current equity’s payoff \( u \) for \( \xi = 1 \) and \( \xi = 0 \). To find the firm’s payoff, we define its payoff in the following benchmark cases:

- \( u_L \) is current equity holders’ payoff if there is a large concentrated creditor (i.e., if \( \xi = 1 \)):
  \[
u_L := \int_0^{\hat{v}(D)} \lambda v f(v) dv + \int_{\hat{v}(D)}^\infty v f(v) dv - \left( \int_0^{\hat{v}(D_0)} \lambda \theta v f(v) dv + \int_{\hat{v}(D_0)}^\infty D_0 f(v) dv \right). \tag{95}\]

- \( u_S \) is current equity’s payoff if there are small dispersed creditors (i.e., if \( \xi = 0 \) as in the baseline model):
  \[
  u_S := \int_0^{\hat{v}(D)} \lambda v f(v) dv + \int_{\hat{v}(D)}^\infty v f(v) dv - \int_{\hat{v}(D_0)}^\infty D_0 f(v) dv. \tag{96}
  \]

- \( u_\emptyset \) is current equity’s payoff if there is no restructuring (i.e., if \( D = D_0 \)):
  \[
  u_\emptyset := \int_0^{\hat{v}(D_0)} \lambda (1 - \theta) v f(v) dv + \int_{\hat{v}(D_0)}^\infty (v - D_0) f(v) dv. \tag{97}
  \]

These expressions are useful, because equity holders’ payoff for \( \xi \in (0,1) \) will be a weighted average of them.

**Step 3:** Current equity payoff calculation. There are three possible cases, depending on which IC binds, which we consider in turn.
Case 1: $\Delta_L \geq 0 > \Delta_S$. In this case, only $L$ accepts the restructuring but $S$ do not. So the restructuring succeed with probability $\xi$ and equity holders’ problem is to

$$\text{maximize } u = \xi u_L + (1 - \xi) u_\varnothing$$

over $\alpha$ and $D$. Given $u_\varnothing$ does not depend on $\alpha$ or $D$, this is the same as maximizing $u_L$. Differentiating with respect to $D$, we find:

$$-(1 - \lambda) \hat{\varrho} f(\hat{v}) < 0,$$

for all $D$. So the solution is $D = 0$. We can find $1 - \alpha$ from the binding IC ($\Delta_L = 0$):

$$(1 - \alpha) \mathbb{E}[v] = \int_0^{\hat{v}(D_0)} \lambda \vartheta v f(v) dv + \int_{\hat{v}(D_0)}^{\infty} D_0 f(v) dv.$$

Defining $u_{L}^{\max}$ as the maximum of $u_L$ in this case, we have:

$$u_{L}^{\max} = \int_0^{\infty} v f(v) dv - \left( \int_0^{\hat{v}(D_0)} \lambda \vartheta v f(v) dv + \int_{\hat{v}(D_0)}^{\infty} D_0 f(v) dv \right).$$

So the expected payoff is:

$$u = \xi u_{L}^{\max} + (1 - \xi) u_\varnothing.$$

Case 2: $\Delta_S \geq 0 > \Delta_L$. In this case, only the small creditors’ IC is satisfied. So the restructuring succeed with probability $1 - \xi$ and and equity holders’ problem is to

$$\text{maximize } u = \xi u_\varnothing + (1 - \xi) u_S$$

over $\alpha$ and $D$. Given $u_\varnothing$ does not depend on $\alpha$ or $D$, this is the same as maximizing $u_S$. This is the baseline case of dispersed creditors studied in Section G: $1 - \alpha = 0$ and $D = D^\ast$.

Defining $u_{S}^{\max}$ as the maximum of $u_S$ in this case, we have:

$$u_{S}^{\max} = \int_0^{\hat{v}(D)} \varrho f(v) dv + \int_{\hat{v}(D)}^{\infty} v f(v) dv - \int_{\hat{v}(D)}^{\infty} D_0 f(v) dv.$$

So the expected payoff is

$$u = \xi u_\varnothing + (1 - \xi) u_{S}^{\max}.$$

Case 3: $\Delta_S \geq 0$ and $\Delta_L \geq 0$. In this case, both ICs are satisfied and equity holders’ problem
is to
\[ \text{maximize } u = \xi u_L + (1 - \xi) u_S \quad (106) \]
over \( \alpha \) and \( D \).

Here, there are three sub-cases:

- **Case 3(i):** \( \Delta_L > \Delta_S = 0 \) \( (\iff D < \hat{D}) \). In this case, the firm chooses \( \alpha \) and \( D \) so that \( S \)'s IC binds and \( L \) accepts the restructuring on \( S \)'s terms. Hence, the equity holders problem is to
\[ \text{maximize } u = \xi u_S + (1 - \xi) u_S = u_S \quad (107) \]
over \( \alpha \) and \( D \). From case 2, we know that to maximize \( u_S \), equity holders set \( 1 - \alpha = 0 \) and \( D = D^* \). So the expected payoff is:
\[ u = u_S^{\text{max}}. \quad (108) \]

- **Case 3(ii):** \( \Delta_S > \Delta_L = 0 \) \( (\iff D > \hat{D}) \). This cannot arise in equilibrium: Given \( S \)'s IC is slack \( (\Delta_S > 0) \), equity holders can increase their surplus by decreasing \( D \) (and increasing \( 1 - \alpha \) so as not to violate \( L \)'s IC).

- **Case 3(iii):** \( \Delta_S = \Delta_L = 0 \) \( (\iff D = \hat{D}) \). Given both ICs are binding, \( D = \hat{D} \) by definition. Here, the expected payoff is:
\[ u = u_S(\hat{D}) = u_L(\hat{D}). \quad (109) \]

In this case, the offer includes both debt \( \hat{D} \) and an equity stake \( 1 - \alpha \), which we can solve for from the binding ICs:
\[ 1 - \alpha = \frac{\int_{v(D_0)}^{\hat{D}} \lambda \theta v f(v)dv + \int_{v(\hat{D})}^{\infty} P_{L}(v\cdot f(v)dv - \int_{v(\hat{D})}^{\infty} \hat{D} f(v)dv}{\int_{v(\hat{D})}^{\infty} (1 - \theta) \lambda v f(v)dv + \int_{v(\hat{D})}^{\infty} (v - \hat{D}) f(v)dv}. \quad (110) \]

Combining these sub-cases, we have that equity’s expected payoff is:
\[ u = \begin{cases} 
  u_S^{\text{max}} & \text{if } D^* < \hat{D}, \\
  u_S(\hat{D}) & \text{if } D^* \geq \hat{D}. 
\end{cases} \quad (111) \]

**Step 4: Payoff comparison.** Combining the cases above, we have that equity’s expected
payoff is:

\[
  u = \begin{cases}
    \max \left\{ \xi u_L^{\max} + (1 - \xi) u_\emptyset, \xi u_\emptyset + (1 - \xi) u_S^{\max}, u_S^{\max} \right\} & \text{if } D^* < \bar{D}, \\
    \max \left\{ \xi u_L^{\max} + (1 - \xi) u_\emptyset, \xi u_\emptyset + (1 - \xi) u_S^{\max}, u_S(\bar{D}) \right\} & \text{if } D^* \geq \bar{D}.
  \end{cases}
\]

To simplify the thresholds above, first observe that, from the definitions of \( u_L^{\max}, u_S^{\max}, \) and \( u_S(\bar{D}) \) and the assumption that \( D_0 \) is sufficiently large (and hence \( u_\emptyset \) sufficiently small), we have:

\[
  u_L^{\max} \geq u_L(\bar{D}) = u_S(\bar{D}) > u_\emptyset
\]

and

\[
  u_S^{\max} \geq u_L(\bar{D}) = u_S(\bar{D}) > u_\emptyset.
\]

With this, we can re-write \( u \) as:

\[
  u = \begin{cases}
    \max \left\{ \xi u_L^{\max} + (1 - \xi) u_\emptyset, u_S^{\max} \right\} & \text{if } D^* < \bar{D} \\
    \max \left\{ \xi u_L^{\max} + (1 - \xi) u_\emptyset, \xi u_\emptyset + (1 - \xi) u_S^{\max}, u_S(\bar{D}) \right\} & \text{if } D^* \geq \bar{D}.
  \end{cases}
\]

To give the formulation in the statement of the result, we divide the expression above into cases for \( D^* \leq \bar{D} \) and \( \xi \leq \bar{\xi} \), where:

\[
  \xi := \frac{u_S^{\max} - u_S(\bar{D})}{u_S^{\max} - u_\emptyset},
\]

\[
  \bar{\xi} := \frac{u_S(\bar{D}) - u_\emptyset}{u_L^{\max} - u_\emptyset}.
\]

• If \( D^* \geq \bar{D} \) and \( \xi < \bar{\xi} \), then:

\[
  u = \begin{cases}
    \xi u_\emptyset + (1 - \xi) u_S^{\max} & \text{if } \xi \leq \xi, \\
    u_S(\bar{D}) & \text{if } \xi \in (\xi, \bar{\xi}], \\
    \xi u_L^{\max} + (1 - \xi) u_\emptyset & \text{if } \xi > \bar{\xi}
  \end{cases}
\]

The analysis above implies this corresponds to debt for \( \xi \leq \xi \), a mix of debt and equity for \( \xi \in (\xi, \bar{\xi}] \), and equity for \( \xi > \bar{\xi} \).
• If $D^* \geq \tilde{D}$ and $\xi \geq \tilde{\xi}$, then:

$$u = \begin{cases} 
\xi u_\varnothing + (1 - \xi) u_{S_{\max}} & \text{if } \xi \leq \frac{u_{S_{\max}} - u_\varnothing}{u_L^{\max} + u_S^{\max} - 2u_\varnothing}, \\
\xi u_L^{\max} + (1 - \xi) u_\varnothing & \text{if } \xi > \frac{u_{S_{\max}} - u_\varnothing}{u_L^{\max} + u_S^{\max} - 2u_\varnothing}.
\end{cases}$$  \hspace{1cm} (119)

The analysis above implies this corresponds to debt for $\xi$ below the threshold $(u_{S_{\max}} - u_\varnothing)/(u_L^{\max} + u_S^{\max} - 2u_\varnothing)$ and equity above it.

• If $D^* < \tilde{D}$, then:

$$u = \begin{cases} 
u_{S_{\max}} & \text{if } \xi \leq \frac{u_{S_{\max}} - u_\varnothing}{u_L^{\max} - u_\varnothing} \\
\xi u_L^{\max} + (1 - \xi) u_\varnothing & \text{if } \xi > \frac{u_{S_{\max}} - u_\varnothing}{u_L^{\max} - u_\varnothing}.
\end{cases}$$  \hspace{1cm} (120)

The analysis above implies this corresponds to debt for $\xi$ below the threshold $(u_{S_{\max}} - u_\varnothing)/(u_L^{\max} - u_\varnothing)$ and equity above it.

\[ \square \]

I Omitted derivation from Section 4.3

Given equations (73) and (74), inequality (11) is equivalent to:

\[ 0 > \frac{\partial \Delta}{\partial \theta} = \lambda \int_0^{\hat{v}} v dF(v) - (\lambda \hat{v} \hat{v} + D_0 - D^*) f(\hat{v}) \frac{\partial \hat{v}}{\partial \theta} \]
\[ = \lambda \int_0^{\hat{v}} v dF(v) - (\lambda \hat{v} \hat{v} + D_0 - D^*) f(\hat{v}) \frac{\lambda \hat{v}}{1 - (1 - \theta) \lambda}. \hspace{1cm} (121) \]

Now use the assumption that $vf(v)$ is increasing on $[0, \hat{v}]$ to see that

\[ \int_0^{\hat{v}} v dF(v) = \int_0^{\hat{v}} vf(v) dv \leq \int_0^{\max_{v \in [0,\hat{v}]} vf(v) dv = \max_{v \in [0,\hat{v}]} vf(v) \int_0^{\hat{v}} dv = \hat{v}^2 f(\hat{v}). \hspace{1cm} (123) \]

Using the above, we can get a sufficient condition for (123):

$$\hat{v}^2 f(\hat{v}) < (\lambda \hat{v} \hat{v} + D_0 - D^*) f(\hat{v}) \hat{v} \frac{1}{1 - (1 - \theta) \lambda}. \hspace{1cm} (124)$$

Rearranging gives condition (12) in the text.

\[ \square \]
References


