Public Debt Bubbles in Heterogeneous Agent Models with Tail Risk

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INTRODUCTION

Observations from the 21st Century

- Many developed economies have, and are expected to continue to have, **large public debts**.
  - Japan, US, UK ...

- These countries also have, and are expected to continue to have, **large primary deficits**.

- **Real interest rates** have been, and are expected to continue to be, well **below economic growth rates**.

Public Debt Bubble?

- This evidence suggests that:
  - Public debt $>$ present value of future primary surpluses (negative infinity).

- People expect governments to raise revenue to repay debt through new debt issue, not through taxation.

- That is, there may be a (rational) **public debt bubble** (Brunnermeier, et al. (2020)) that could open a role for welfare-improving changes in government policy (Blanchard (2019)).

Bubbles as Curiosa?

- Despite the data, most academic (and non-academic) analyses focus on the non-bubble case.

- One key reason: public debt bubbles lie outside the domain of widely used/taught modern macro models.

- Well-known: public debt bubbles *can* exist in two-period-lived overlapping generations models (Samuelson (1958), Diamond (1965)).

  - But these aren't widely used in modern ("serious"?) macro.

- Public debt bubbles cannot exist in much more widely used immortal representative agent models.

This Paper: Setup

- I take a standard class of modern macro models (Aiyagari-Bewley-Huggett or ABH) and perturb it slightly.

- In the models, agents can trade (elastically supplied) one-period public debt with fixed real return $r < g = 0$.

- Government uses a fraction α of the revenue from bond sales (net of interest payments) for a uniform lump-sum transfer.

This Paper: Findings

- The deficit (in any period) is an increasing function of r and grows without bound as α nears 1.

- The long-run public debt grows without bound as r grows to zero.

- Long-run expected utility from private and public consumption are both strictly increasing in r.

Punchlines

- Public debt bubbles - as we seem to see in the data - aren't exotic oddities from the point of view of modern macro.

- Rather: (Arbitrarily) large public debt bubbles are consistent with (arbitrarily) small perturbations of heterogeneous agent macro models.

- In these perturbed models: bubbles, no matter their size, are always too small.

What is the "Perturbed" ABH Model?

- Take any ABH model with idiosyncratic Markov endowment shocks.

- Add a new "urgent-to-consume" state in which agents have linear utility.

  - probability of transition to new state is p.

  - marginal utility in new state is ν/p

- Focus on p near zero ("small" perturbation)

  - Usual LLN argument: When p is close to zero, few transitions take place in the data.

  - But agents still demand public debt as a form of precautionary saving against (severe) downside risk.

Related Literature

- Large literature on rational bubbles in macro (Martin and Ventura (2018)).

- Recent related papers:

  - Reis (2020) - policy implications of $r < g < MPK$.

  - Aguiar, Amador, and Arellano (2021) - constructing Pareto improvements in ABH models when r is low.

  - Brumm, et al. (2021) - example models showing that $r < g$ doesn't mean that more debt is better.

Outline

1. Class of Perturbed ABH Models

2. Solution to the Individual Problem

3. Aggregates

4. Conclusions and Extensions

PERTURBED ABH MODELS

Stochastics

- Unit measure of households.

- Household states evolve according to identical stochastically independent Markov chains.

- The Markov chain has state space $\{1, 2, .., J\}$ and aperiodic and irreducible transition matrix $\widehat{\Gamma}$.

- Its unique stationary density is $\widehat{\mu}$.

Preferences and Endowments

- A household in state i has endowment y_i of nondurable/nonstorable consumption.

- A household in state i has momentary utility function u_i, which satisfies $u_i', -u_i'' > 0$ and Inada conditions.

- All households have common discount factor $\beta \in (0, 1)$.

Adding a Low Probability State

- Add a state 0, in which households have endowment y_0 and momentary utility function:

$$\nu c/p, \nu > 0 \text{ and } 0 < p < 1.$$

- The $(J+1) \times (J+1)$ transition matrix is $\Gamma(p)$, where:

$$\begin{aligned}
\Gamma_{i0}(p) &= p, 1 \leq i \leq J \\
\Gamma_{ij}(p) &= (1-p)\widehat{\Gamma}_{ij}, 1 \leq i, j \leq J \\
\Gamma_{0j}(p) &= (1-\rho)\widehat{\mu}_j, 1 \leq j \leq J \\
\Gamma_{00}(p) &= \rho
\end{aligned}$$

- The stationary density is:

$$\mu_0(p) = \frac{p}{1+p-\rho}; \mu_i(p) = (1-\mu_0(p))\widehat{\mu}_i, 1 \leq i \leq j$$

Low p

- I focus on what happens when p is near zero.
- The various probabilities are arbitrarily close to the original model.
- BUT: marginal utility of consumption in state 0 is close to infinity.
- State 0 thus represents a low-probability (and rarely observed) state with a high urgency to consume.

Bubble Condition

- In what follows, I assume that (*ex-ante* state 0 marginal utility) ν is sufficiently high that:

$$u_j'(y_j) < \beta\nu/(1-\beta).$$

- There is an open interval of such ν.

- This **bubble condition** ensures that households demand a positive amount of bonds in all non-zero states ... even when their real return is negative.

SOLUTION TO THE INDIVIDUAL PROBLEM

Bond Sales/Purchases

- In each date, the government sells one-period bonds.

- The bonds pay off 1 unit of consumption and have a constant price $q > 1$ that is chosen by the government.

- Households each begin life with $\bar{B}_1 \geq 0$ units of bonds (that mature in period 1).

- The government makes a positive transfer τ_t to all households in each period t.

- Households cannot short sell bonds (relaxed in paper).

Low p and q

- The constant bond price $q > 1$ implies that the real interest rate is negative (less than the growth rate).

- Recall: p is probability of transiting to auxiliary state 0.

- I restrict (p, q) to be in the interval $\Lambda = (0, \bar{p}) \times (1, \bar{q})$ where:

$$u_j'(y_j) < \frac{\beta\nu}{\bar{q} - \beta(1 - \bar{p})}, j = 1, ..., J$$
$$0 < \bar{p} < \rho \text{ (persistence of state 0).}$$

- The existence of $(\bar{p}, \bar{q})$ is implied by the earlier bubble condition imposed on the marginal utility parameter ν.

A Simple Solution

- Suppose (p, q) is in Λ and define:

$$c_i^*(p,q) = u_i'^{-1}\left(\frac{\beta\nu}{q-\beta(1-p)}\right), i = 1, .., J$$

- Then, in history $(s_1, ..., s_t)$, where $s_t > 0$, it is optimal for households to choose:

$$\begin{aligned} c_t(s^t) &= c_{s_t}^*(p,q), s_t > 0 \\ qb_{t+1}(s^t) &= (y_{s_t} + \tau_t - c_{s_t}^*(p,q)) + b_t(s^t). \end{aligned}$$

- If $s_t = 0$, then:

$$\begin{aligned} c_t(s^t) &= b_t(s^t) + y_{s_t} + \tau_t \\ b_{t+1}(s^t) &= 0 \end{aligned}$$

Why?

- In state $j, j = 1, .., J$, households are marginally indifferent between buying more or less bonds

$$qu_j'(c_j^*(p,q)) - \beta \sum_{i=1}^{J} \Gamma_{ji} u_i'(c_i^*(p,q)) - \beta p(\nu/p)$$
$$= q\frac{\beta\nu}{q-\beta(1-p)} - \beta(1-p)\frac{\beta\nu}{q-\beta(1-p)} - \beta\nu = 0.$$

- In state 0, households are borrowing-constrained b/c:

$$q\nu/p - \beta(1-\rho)\frac{\beta\nu}{q-\beta(1-p)} - \beta\rho\nu/p$$

is strictly increasing in q and is zero when $q = \beta$.

- Transversality condition is satisfied, because households hit short-sales constraint infinitely often along almost every sample path.

An Interpretation

- At each date, households in state $j > 0$ give a gift to the government:

$$(y_j + \tau_t - c_j^*(p, q))$$

- Their bondholdings are then the cumulation of past gifts, discounted at $(1/q) < 1$.

- When a household enters date 0, it immediately cashes in its gift account (because of its urgent need for consumption).

- It begins rebuilding the "gift account" after it re-enters a non-zero state.

- The per-capita "gift account" represents per capita public debt.

AGGREGATES

Definitions

- Let $\bar{B}_t(p, q, \alpha)$ represent per-capita holdings of public debt in period t.

- Here, α represents the transfer from the government, as it sets:

$$\tau_t(p, q, \alpha) = \alpha(q\bar{B}_{t+1}(p, q, \alpha) - \bar{B}_t(p, q, \alpha)), 0 \leq \alpha < 1$$

- The government uses the remaining revenue for government purchases.

Public Debt Formula: Building Blocks

- Suppose (p, q) is in Λ.

- Assume households' initial states are distributed according to stationary density $\mu(p)$.

- Let:

$$\Delta_j^*(p,q) = y_j - u_j'^{-1}\left(\frac{\beta\nu}{q - \beta(1-p)}\right)$$

be the household's "gift" to the government in state j (net of the returned/reinvested transfer).

Public Debt Evolution Over Time

- Then, when p is near zero, the public debt at the end of period t is :

$$\bar{B}_{t+1}(p,q,\alpha) \approx \frac{\left(\sum_{i=1}^{J} \widehat{\mu}_i \boldsymbol{\Delta}_i^*(0,q)\right)}{(1-\alpha)} \frac{(1-1/q^t)}{q-1} + \bar{B}_1/q^t$$

- Recall: $\widehat{\mu}$ is the stationary density in the economy WITHOUT the auxiliary state.

Why q? Why α?

- The q part of the formula is simply geometric discounting at work.

- The α part of the formula captures the households (in states $j > 0$) re-investing their transfers.

Main Results 1: Properties of Public Debt

- Given t, $\bar{B}_t(p, q, \alpha)$ is an increasing function of $(1/q, \alpha)$.

- When p is near zero, period t debt is unbounded as a function of transfers (α):

$$lim_{\alpha \to 1} lim_{p \to 0} \bar{B}_t(p, q, \alpha) = \infty.$$

- When p is near zero, long-run debt is unbounded as a function of q:

$$lim_{q \to 1} lim_{t \to \infty} lim_{p \to 0} \bar{B}_t(p, q, \alpha) = \infty.$$

Primary Deficits

- The primary deficit is endogenously determined as:

$$D_t(p,q,\alpha) = q\bar{B}_{t+1}(p,q,\alpha) - \bar{B}_t(p,q,\alpha)$$

(that is, the amount borrowed beyond what's used to pay interest + principal).

- For p near zero, the primary deficit is well-approximated by:

$$D_t(p,q,\alpha) \approx \frac{\left(\sum_{i=1}^{J} \widehat{\mu}_i \Delta_i^*(0,q)\right)}{(1-\alpha)}.$$

Main Results 2: Properties of Deficits

- When p is low:

- The primary deficit is strictly decreasing (but bounded) as a function of q.

- The primary deficit is, at any date, increasing in α and is unbounded as a function of α:

$$lim_{\alpha \to 1} lim_{p \to 0} D_t(p, q, \alpha) = \infty.$$

Welfare

- I calculate ex-ante expected utility from private consumption in each period t as a function $W_t(p, q, \alpha)$.
  - all agents begin life with same bondholdings.
  - agents begin life behind "veil of ignorance" in terms of the draw of their initial state from $\mu(p)$.

- As before, I focus on the case in which p is near zero.

- But - unlike before - the zero state matters in welfare calculations, because ex-ante marginal utility remains at ν even when p is near zero.

Main Results 3: Welfare

- Define $W_t^0(q,\alpha) = lim_{p\to 0} W_t(p,q,\alpha)$.
- For any (q,t), W_t^0 is strictly increasing in α.
- For any α, and for t sufficiently large (in particular, larger than $\frac{(2-\beta)}{(1-\beta)}$), then W_t^0 is strictly decreasing in q.

What About Government Purchases?

- When p is near zero, then government purchases are approximately:

$$G_t(p,q,\alpha) = (1-\alpha)D_t(p,q,\alpha) = \sum_{i=1}^{J} \widehat{\mu}_i \Delta_i^*(0,q).$$

- Government purchases are independent of α when p is near zero.

- Note that $lim_{p\to 0} G_t(p,q,\alpha)$ is strictly decreasing in q.

CONCLUSIONS AND EXTENSIONS

Bubbles as Products of Standard Models

- I consider small (in a probabilistic sense) perturbations of a class of standard ABH models.

- In these slightly perturbed models, public debt bubbles ($r < g = 0$) emerge as equilibria.

- In these equilibria:

  - Government policy choices can give rise to **arbitrarily large** debt and deficit levels.

  - long-run welfare is strictly increasing in debt levels.

Extensions: Private Credit

- In this presentation, I've not allowed households to borrow.

- But, in the paper, I show that the above results generalize to the case in which agents have a fixed but positive borrowing limit.

- One caveat: with borrowing, the long-run welfare results apply only when q is sufficiently near 1.

Extensions: Capital

- In the paper, I add capital (as in Aiyagari (1994)). I abstract from redistribution (set $\alpha = 0$).

- I focus on steady-states, in which capital is constant over time.

- When p is near zero, lower values of q (higher r) induce steady states with:
  - lower capital
  - **higher** output and public consumption
  - lower private consumption AND higher expected utility from private consumption.

Extensions: Strictly Concave Utility in State Zero

- In the paper, I provide a numerical example in which utility is log in state zero (but there is still a large MU shock in that state).

- For $q = 1.02$, long-run debt/GDP ratio is (approximately) 2.65 when p is about 0.025 (and ρ is near zero).

- **Lesson: Very large debt bubbles (by historical standards) can be sustainable as equilibria** even without linear utility.