# Financial and Total Wealth Inequality with Declining Interest Rates*

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**Abstract**

Financial wealth inequality and long-term real interest rates track each other closely over the post-war period. Faced with unanticipated lower real rates, households which rely more on financial wealth must see large capital gains to afford the consumption that they planned before the decline in rates. Lower rates beget higher financial wealth inequality. Inequality in total wealth, the sum of financial and human wealth and the relevant concept for household welfare, rises much less than financial wealth inequality and even declines at the top of the wealth distribution. A standard incomplete markets model reproduces the observed increase in financial wealth inequality in response to a decline in real interest rates because high financial-wealth households have a financial portfolio with high duration.

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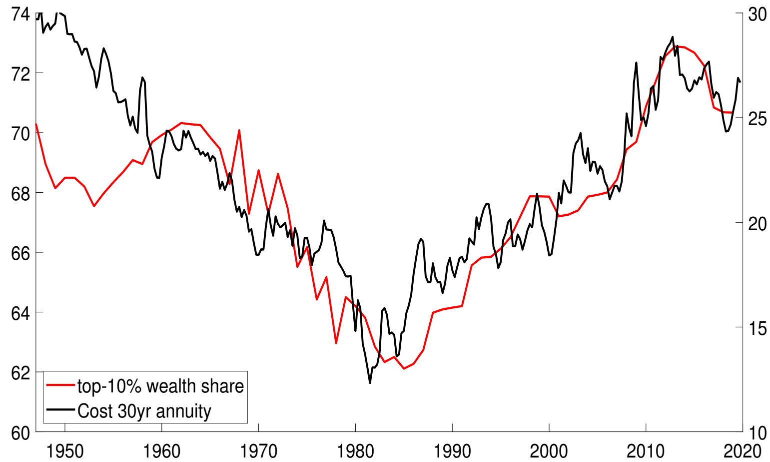
# 1 Introduction

Financial wealth inequality has increased substantially over the past several decades in the U.S. and many other countries. According to the World Inequality Database, the fraction of U.S. financial wealth held by the top-10% wealthiest households has increased from 63.0% in the 1980s to 71.9% in the 2010s, an increase of 8.9% points. The share of financial wealth held by the top-1% increased from 24.6% to 35.1% over the same period, a 10.5% point increase. Over the same period, long term nominal rates have declined dramatically. The 10-year U.S. nominal Treasury yield declined from 10.6% in the 1980s to 2.4% in the 2010s. Given that 10-year average expected inflation fell by much less than 8 percentage points over this period, and not at all since 2003, real rates declined as well. The 10-year real bond yield averaged 2.1% in 2003 before falling to 0.3% in 2016 and -0.60% in 2020. We argue that these two changes are related. Large changes in the distribution of financial wealth are to be expected if households want to finance the same consumption stream in the low rate environment as the one that they had planned prior to the onset of lower rates.

Consider a simple example to fix ideas. A 50-year old in 1982 who wants to spend $10,000 per year for the next 30 years had to set aside $125,000. In 2012, a 50-year old with the same desire needs $291,000, or 2.5 times as much financial wealth. Indeed, Figure 1 plots the cost of a savings instrument that provides $1 of consumption in each of the next thirty years. The price of this 30-year real annuity is about $30 in the early 1950s. The price then falls to a low of $12.5 in 1981.Q3 when long-term real interest rates peak. As interest rates fall over the next three decades, the price of the consumption annuity more than doubles to $29.1 in 2012.Q4. In contrast, a 30-year old with many more years left in the labor market is partially hedged against this real interest rate change. The market value of the 30-year old's human wealth in 2019 is much larger than that of the 30-year old in 1982 as the valuation reflects the lower interest rates. She may not need to adjust financial savings to the same extent as the 50-year old to afford the same consumption plan. This example also illustrates how inequality in total wealth, the sum of financial and human wealth, may behave very differently from that in financial wealth in the face of declining interest rates. The 30-year old has little financial wealth while the 50-year old has little human wealth. Figure 1 shows that the top wealth share and the cost of the real annuity comove strongly in the U.S. We find the same pattern in the U.K and in France.

This evidence suggests that large changes in the distribution of financial wealth are to be expected, and even desirable, in the wake of large changes in real rates. To make this point rigorously, we analyze a Bewley-style incomplete markets equilibrium economy with heterogeneous agents. The decline in long-term real rates in the model arises from a slowdown in the long-run growth rate of the economy. This slowdown is isomorphic to a decrease in the rate of time preference of all households in a stationary version of this economy. Since there is no preference heterogeneity and all households have merely become equally more patient, it is natural to ask

Figure 1: Top Inequality and the Cost of Real Annuity




*Note:* The figure plots the top-10% financial wealth share for the United States (red line). The data is annual from 1947 until 2019 from the World Inequality Database. It also plots (black line) the price of a 30-year inflation-indexed annuity which pays $1 in real terms for the next 30 years. A dynamic affine term structure model, estimated on quarterly data from 1947.Q1-2019.Q4 and spelled out in Appendix E delivers the term structure of real bond yields. The price of the annuity equals the sum of the prices of the real zero-coupon bonds of maturities 1 through 30.

whether we can implement the same consumption allocation in the economy with low rates as the one that prevailed when rates were high. We show that the equilibrium consumption allocation in the model with high rates remains an equilibrium in the model with low rates, provided that agents' initial financial wealth is adjusted. Conversely, if the financial wealth distribution does not change in response to changes in long rates, the new equilibrium will result in large changes in the consumption distribution.

First, we use a version of the model with ex ante identical households to analyze the normative implications of a decline in rates for the wealth distribution. When the households's consumption is fully hedged against rate shocks, the model predicts an increase in financial wealth inequality in case of a positive cross-sectional correlation between financial wealth and the duration of the household's excess consumption plan. When households are ex-ante identical and labor income shocks are persistent, low-wealth households are households who experienced a recent history of bad idiosyncratic income shocks. These households tend to have a high duration of human wealth, reflecting the expected long-run mean reversion of their labor income. Their consumption plan has a low duration because of consumption smoothing. Their duration of excess consumption, consumption minus income, is low. These households have low excess consumption duration and low financial wealth. The converse is true for high-wealth households, households who have been able to accumulate financial wealth thanks to a sequence of fortunate labor income shocks. With this positive cross-sectional correlation, the wealth-weighted duration of financial wealth exceeds the equal-weighted duration, and the right tail of the financial wealth distribution grows larger when rates decline. The rise in financial top-wealth inequality is what the model predicts should happen when households' consumption is fully hedged against a decline in real interest rates. Since everyone is still able to afford the same consumption plan, nobody is worse off.[1] We refer to this as the *compensated* financial wealth distribution.

Second, to analyze the quantitative relevance of this mechanism, we calibrate a Bewley model with ex-ante heterogeneity across households. Households differ by age and by the duration of their financial wealth portfolio. They face income risk over the life-cycle, calibrated using Panel Study of Income Dynamics data. We add a superstar income state to enable the model to match the financial wealth Gini of the 1980s. We calibrate the heterogeneity in the duration of financial wealth using data on the composition of households' financial portfolios from the Survey of Consumer Finances. We combine portfolio shares from the SCF with durations of major asset classes obtained from an auxiliary asset pricing model. We find that U.S. households have an equally-weighted average duration of financial wealth of 15.43, which is below the value-weighted (or aggregate) duration of financial wealth of 25.72. We observe substantial heterogeneity in financial durations by wealth level and by age. Low-wealth households have low financial durations, driven by their higher share of deposit-like assets, the presence of consumer debt, and lower shares of housing, private business, and stock market wealth. The reverse is true for high-wealth households. Conditional on wealth, financial durations are declining in age. This heterogeneity in financial duration is a new empirical finding, and crucial for the response of financial inequality to interest rates.

We compute the model at a long-term real interest rate of 4.82%, the level that prevailed in the 1980s, and at a 0.34% long-term real rate, the level that prevailed in the 2010s. The interest rate change is due to an unanticipated decline in the expected growth rate of the economy.

In a first step, we ask the positive question: what actually happens to financial wealth in the calibrated model after rates decline unexpectedly? When financial wealth durations are heterogeneous as in the data, we find that the model can account for the entire rise in financial wealth inequality between the 1980s and 2010s. The *repriced* financial wealth distribution exactly matches the increases in the observed financial wealth Gini. It features increases in the top-10% and top-1% wealth shares that are close to the data.

Human wealth inequality is much lower, and rises by much less when rates decline. Young agents have both high levels of human wealth and high human wealth durations, explaining the increase in human wealth inequality when rates decline. In contrast to top financial wealth shares, top human wealth shares fall modestly. Total wealth inequality, which is the welfare-relevant concept, shows only a modest increase in Gini and a small decline in top wealth shares. The decline in rates has not led to large increases in total wealth inequality.

[1] As we explain in section 5, households are fully hedged against rate shocks when households can consume the same consumption shares as a fraction of aggregate consumption.

In a second step, we ask the normative question: how much additional financial wealth each household would need to be able to afford the old consumption plan, as a fraction of aggregate consumption, under the new, lower interest rate? This *compensated* financial wealth distribution is a rightward shift of the original wealth distribution. While all households require more financial wealth to finance the old consumption allocation, young households require the largest compensation. Since they must save for retirement for many years, the loss in compound interest hits them particularly hard. Young households have a high duration of their excess consumption plan in spite of the high duration of their human wealth, because, after retirement, they still consume but have no labor income. So, their human wealth provides an incomplete hedge against rate shocks.

While the wealthy see a large increase in financial wealth under the compensated distribution (as much of 40.9% of the increase in aggregate wealth goes to the top-1%), the top-1% and top-10% financial wealth shares and the Gini nevertheless fall since the required increase in financial wealth for the young is greater still. In other words, the large human wealth of the young does not provide a large enough hedge against interest rate declines. This shows that the life-cycle aspect is a crucial addition, adding meaningfully to the intuition coming from the Bewley model with ex ante identical, infinitely-lived households.

The rest of the paper is organized as follows. The next section discusses the related literature. Section 3 shows that the share of the top percentiles tracks the cost of an indexed annuity quite closely in the U.S., U.K., and France. Section 4 sets up an incomplete markets economy with aggregate uncertainty and infinitely-lived households who face idiosyncratic income risk. A first insight from this model is that one needs to use the same discount rate for the household's future labor income, financial income, and consumption to arrive at a measure of household wealth that properly aggregates. The second and main result in this section is that financial wealth becomes more concentrated in response to decline in the interest rate if financial wealth and the duration of financial wealth covary positively. This covariance condition is naturally satisfied in an incomplete markets economy with persistent labor income shocks. Section 5 quantifies the effect of an interest rate change by adding a life-cycle component to the model as well as heterogeneity across demographic groups. Section 6 concludes. Appendix A contains details on data sources and construction. Appendix B contains the proofs of the propositions. Appendix C shows that the connection between low expected returns and high financial wealth inequality arises under minimal assumptions. Appendix D contains some details of the calibrated model. Appendix E provides an auxiliary asset pricing model used to infer real interest rates and durations of the components of financial wealth.

## 2 Related Literature

A large strand of recent literature documents the evolution of income inequality as well as financial wealth inequality over the past century (Piketty and Saez, 2003; Piketty, 2015; Alvaredo, Chancel, Piketty, Saez and Zucman, 2018b). Most of the evidence suggests that financial wealth inequality has increased in many countries over the past decades. Zucman (2019) reviews the empirical literature on the topic. Benhabib and Bisin (2018) survey economic theories of wealth inequality.

Much of the literature on wealth inequality adopts a backward-looking approach and explores the connection between past returns and current wealth. This literature has argued that high past rates of return and heterogeneity therein helps account for the increase in financial wealth inequality (Piketty and Zucman, 2015; Fagereng, Guiso, Malacrino and Pistaferri, 2020; Bach, Calvet and Sodini, 2020; Hubmer, Krusell and Smith, 2020; Cox, 2020).

But wealth is also the current value of the household's future consumption stream. Human wealth is the value of future labor income and financial wealth is the value of future consumption minus income. We bring an asset pricing perspective to the discussion on inequality. We impute a valuation by discounting future cash flows. When rates declines, households need more wealth to finance the same consumption stream. Households that have mostly human wealth are likely to be better hedged. Households with mostly financial wealth need enough duration in their portfolio in order to finance future consumption. To keep consumption shares unchanged, a decline in real rates needs to entail a reallocation of financial wealth towards those households who rely mostly on their (current and future) financial wealth to finance future consumption.

Discount rates matter. In a simple partial equilibrium model, Moll (2020) explains that small discount rate-induced changes in the wealth distribution may have smaller welfare effects than cash flow-induced changes. We make a related point in a version of the Bewley-style general equilibrium model with aggregate and idiosyncratic risk. Recently, Catherine, Miller and Sarin (2020) show that discounting social security transfers at time-varying discount rates has quantitatively important implications for wealth inequality.

Greenwald, Lettau and Ludvigson (2019) point to increases in the share of output accruing to profits as a key source of the rise in equity values since 1989. While we motivate our main experiment using a drop in the real risk-free rate, the decline in expected returns applies more broadly to other financial assets. This decline could arise either from a highly persistent change in the real risk free rate or to a decrease in risk premia. To the extent that economic forces have varied these quantities across time and across different financial assets, our methodology could be extended to capture these more detailed patterns. The auxiliary asset pricing model in Appendix E indeed shows declines in expected real returns not only on bonds but also on stocks and housing.

Our paper is related to recent work by Auclert (2019), who explores the effect of cross-sectional variation in the duration of households' financial assets for the effectiveness of monetary policy. We consider a setting with aggregate risk, we develop measures of household duration based on a no-arbitrage dynamic asset pricing model and household financial portfolios, and we assess quantitatively the extent to which households have hedged their consumption plan against interest rate innovations. In earlier work, Doepke and Schneider (2006) focus on the distributional consequences of inflation. Our work instead focuses on the distributional effects of changes in long-term real rates. Gomez and Gouin-Bonenfant (2020) study the effects of lower interest rate on the cost of raising new capital for entrepreneurs, linking the decline in interest rates to the rise in wealth inequality through a different channel.

There are important normative implications for fiscal policy. The compensated distribution that allows all households to implement their old consumption plans features less top total wealth inequality than both the old distribution and the actual repriced distribution, but a similar total wealth Gini. This suggests that a tax on top-wealth households may be able to improve on the repriced consumption distribution. In our life-cycle model, we find that young households are hurt most by a reduction in rates. In that respect, our model speaks to the inter-generational distribution of the burden of taxation. A large literature studies optimal labor and capital income taxation in Bewley models with idiosyncratic risk, endogenous labor supply, and capital formation (Aiyagari, 1995; Panousi and Reis, 2017; Heathcote, Storesletten and Violante, 2017; Krueger and Ludwig, 2018; Boar and Midrigan, 2020). We take labor income as given and do not model capital formation, but instead focus on the distributional implications of lower interest rates.

As an aside, we resolve an outstanding issue in the literature on how to compute an individual's human wealth. A common approach in the literature is to use the individual's own SDF to compute human wealth. Instead, Lustig, Van Nieuwerburgh and Verdelhan (2013) propose using the same stochastic discount factor (SDF) that prices traded assets to discount an individual's labor income stream. In this paper we show that using individual SDFs results in a wealth measure that does not aggregate. For wealth accounting, the aggregate SDF is more convenient, because the aggregate value of individual wealth is consistent with market valuations.

By emphasizing total wealth (inequality), of which human wealth (inequality) forms a very significant component, our work contributes to the literature on measuring wealth (inequality). Our paper provides new and detailed statistics on the duration of financial wealth for U.S. households. Related, Kuhn, Schularick and Steins (2020) study how housing and equity portfolio shares differ across the wealth distribution and result in differing financial wealth dynamics for the middle class and the top of the financial wealth distribution. Recent work discusses the measurement of private business income and wealth (Kopczuk, 2017; Saez and Zucman, 2016; Piketty, Saez and Zucman, 2018; Smith, Yagan, Zidar and Zwick, Working Papers; Kopczuk and Zwick, 2020). In our theoretical work, we sidestep this issue by recognizing that financial wealth is the present discounted value of the future stream of consumption minus labor income. In our empirical work, we infer the duration of private business wealth from that of small stocks.

The literature has proposed a long list of candidates for such a growth slowdown: demographics (Summers, 2014; Eggertsson and Mehrotra, 2014; Eichengreen, 2015), a productivity slowdown due to a plateau in educational attainment or diminishing technological progress (Gordon, 2017), a global saving glut and/or shortage of safe assets (Bernanke et al., 2005; Caballero, Farhi and Gourinchas, 2008), government spending that leads to depressed future aggregate demand (Mian, Straub and Sufi, 2020), a decline in competition (Gutiérrez and Philippon, 2017), a decline in desired investment due to lower relative prices of capital goods (Rachel and Smith, 2017), among others. Lower tax progressivity could lead to more saving by the rich, more aggregate wealth, and lower rates (Hubmer et al., 2020). However, Heathcothe, Storesletten and Violante (2020) argue that once transfers are considered, the U.S. tax system has not become less progressive. Alternatively, a rise in income inequality could be the origin of lower interest rates. Mian et al. (2020) argue that the rich have a higher propensity to save than the poor; Fagereng, Blomhoff Holm, Moll and Natvik (2019) provide empirical evidence consistent with this from Norway. This reduces aggregate demand and the real rate of interest in the wake of an exogenous increase in income inequality, for example, due to skill-biased technological change. In our work, we consider a decline in real rates driven by a decline in the expected growth rate of the economy. While the interest rate is endogenous in the Bewley model of Section 4, our model features standard homothetic preferences. The model in Section 5 keeps labor income inequality constant, in order to isolate the effect of a decline in the long-run growth rate of the economy.[2] Our conclusions regarding the differing behavior of financial and total wealth inequality are not sensitive to the source of the decline in interest rates.

# 3 Empirical Evidence

In this section we document a strong time-series correlation between the evolution of long-term real interest rates and wealth inequality. While our focus is on the U.S. in most of the paper, this section documents that this correlation is present also in the United Kingdom and in France. This evidence suggests that households are partially hedged against changes in long real rates.

Figure 2 shows the wealth share of the top-10% of the population in the left panels and the wealth share of the top-1% of the population in the right panels. Wealth shares from the World Inequality Database. For the U.S., we also plot the wealth shares constructed from the SCF+ (Survey of Consumer Finances). For the U.K. (France), we have have added post-2012 (post-2014) wealth shares from the Credit Suisse (CS) Global Wealth report to complement the WID data. Each panel

[2] Hubmer et al. (2020) show that a rise in earnings risk actually lowers wealth inequality as it strengthens precautionary savings motives meaningfully for all but the richest households. A rise in top-income inequality, in contrast, can increase wealth inequality.

also plots the price of a thirty-year real annuity, computed either from nominal yields and inflation or alternatively from an affine asset pricing model. Construction details are in Appendix A.1. The top row is for the U.S., the middle row for the U.K., and the bottom row for France. The sample is 1947-2019.[3]

For both inequality measures, there is a strong positive correlation between financial wealth inequality and the annuity price. Put differently, there is a strong negative association between financial wealth inequality and long-term real interest rates. Between 1947 and 1982, the top-10% (top-1%) wealth share falls from 70% (29%) to 63% (24%) in the U.S. as the annuity becomes cheaper. From 1982 until 2015, the top-10% (top-1%) wealth share rises from 63% (24%) to 73% (36%). During this period, the cost of the annuity more than doubles. There is a small decline in top wealth shares from 2015 until 2019, which is expected to have reversed again in 2020.

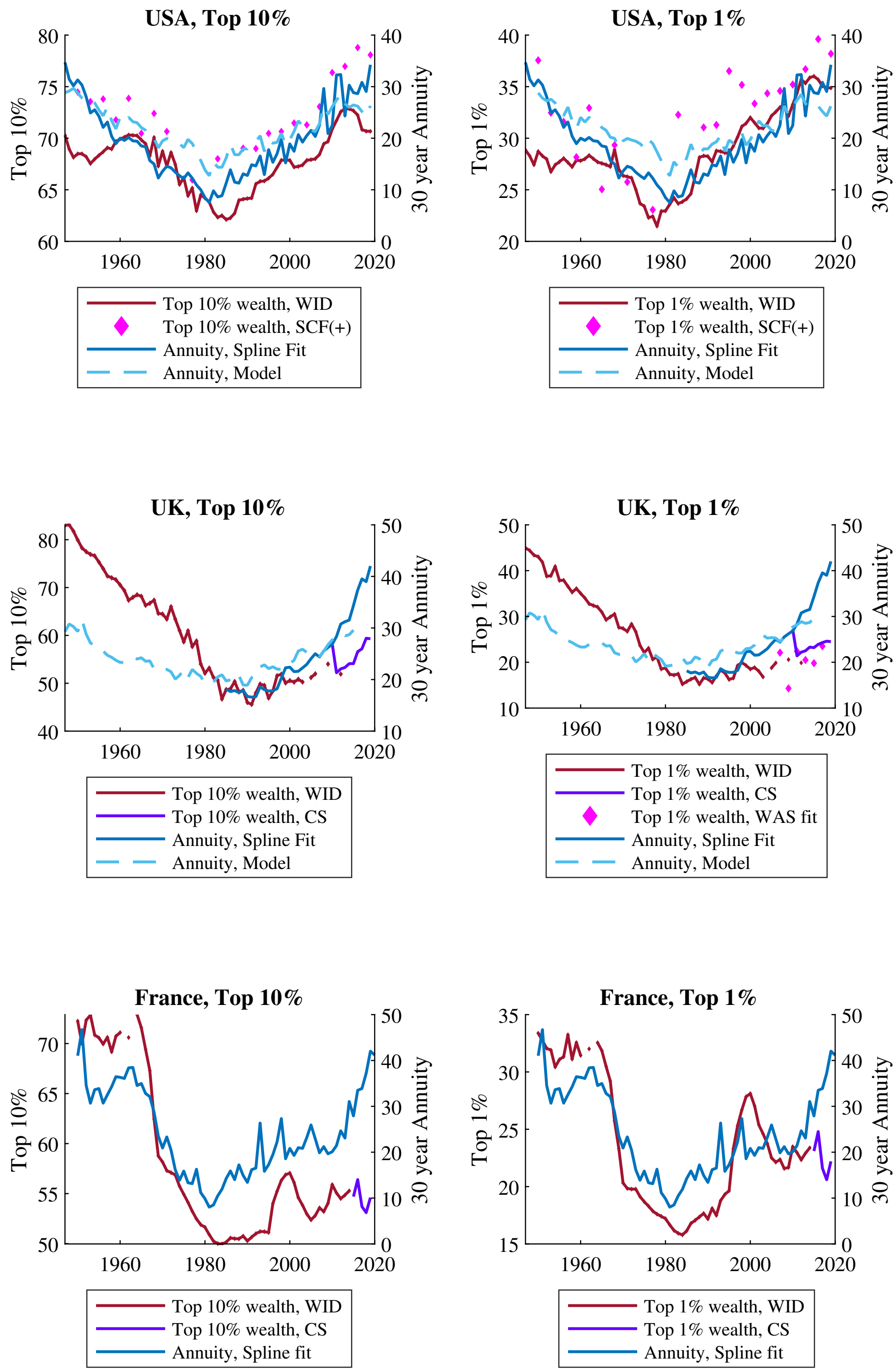
The patterns in both wealth inequality and the evolution of the cost of the annuity are similar in the UK and in France. Rachel and Smith (2017) show that the decline in the real rate has occurred across a broad set of developed and emerging market countries. While many other factors no doubt differ across countries, this shared trend in rates should result in a global rise in financial wealth inequality.

Wealth measures are valuation metrics. From the household budget constraint, it follows that wealth is the present value of future household consumption, and human wealth is the present value of household labor income. Financial wealth is the difference between these two wealth measures. As a result, there is a tight connection between wealth inequality and long rates. In section C of the Appendix, we use a Campbell-Shiller decomposition of household wealth to make this connection under minimal assumptions. When long-term real rates decline and aggregate valuation ratios increase, we expect measures of inequality to increase because wealth is being marked-to-market and different households have different exposure to real rates, even in the absence of news about the distribution of future consumption shares. Wealth inequality measures are not immune to discount rate variation. This is consistent with the evidence in Figure 2.

Next, we analyze this relationship in a fully articulated, dynamic general equilibrium model where consumption is optimally determined and interest rates are set in equilibrium.

[3]For France we start our sample in 1950 since inflation was very high coming out of the WW-II, resulting in implausible real bond yield estimates.

Figure 2: Top Financial Wealth Inequality and Cost of Real Annuity




*Note:* Each panel plots a financial wealth inequality measure against a measure of the cost of a 30-year real annuity. The inequality measure in the left panels is the share of financial wealth going to the top-10% of the population. The right panels plot the share of the top-1% of the population. The wealth shares are from the World Inequality Database and, the SCF+ (U.S.), the Credit Suisse Global Wealth report (U.K., post 2012; France, post 2014), the U.K. Wealth and Assets Survey (WAS) (U.K., post 2012). Details on annuities and wealth shares in Appendix A.1.

# 4 Incomplete Markets Model

To analyze the effects of changes in discount rates on the distribution of wealth, we use a standard Bewley (1986) endowment economy in which ex-ante identical agents face idiosyncratic and aggregate risk. We use an endowment economy to isolate the valuation effects. We first show how to solve this model by transforming the problem into a stationary model without aggregate risk. Next, we use the model to arrive at a method of valuing individual human wealth that is consistent with aggregation. Third, we let the economy undergo a decline in the interest rate, arising from a slowdown in expected economic growth, and show that this increases the inequality in financial wealth.

## 4.1 Endowments

Time is discrete, infinite, and indexed by $t \in [0, 1, 2, ...)$. The aggregate endowment $e$ follows the stochastic process:

$$e_t(z^t) = e_{t-1}(z^{t-1})\lambda_t(z_t)$$

where $\lambda(z_t)$ denotes the stochastic growth rate of the aggregate endowment and $z_t$ the aggregate state. The history of aggregate shocks is denoted by $z^t = \{z_t, z_{t-1}, \cdots\}$. A share $\alpha_t(z_t)$ of the aggregate endowment is financial income, the remaining $1 - \alpha_t(z_t)$ share represents aggregate labor income.

Household labor income $y$ follows the stochastic process:

$$y_t(s^t) = \widehat{y}_t(z^t, \eta^t)(1 - \alpha_t(z_t))e_t(z^t),$$

Households are subject to idiosyncratic income shocks, whose history is denoted by $\eta^t$. The ratio of individual to aggregate labor income, which we refer to as the labor income share, is given by $\widehat{y}_t(z^t, \eta^t)$. The $\eta_t$ shocks are i.i.d. across households and persistent over time. The idiosyncratic shock process is assumed to be independent from the aggregate shock process. We use $s^t = (z^t, \eta^t)$ to summarize the history of aggregate and idiosyncratic shocks, and $\pi(s^t) = \pi(z^t, \eta^t)$ to denote the unconditional probability that state $s^t$ will be realized. If the aggregate and idiosyncratic states are independently distributed, then we can decompose state transition probabilities into an aggregate and idiosyncratic component:

$$\pi(z_{t+1}, \eta_{t+1}|z^t, \eta^t) = \phi(z_{t+1}|z^t)\varphi(\eta_{t+1}|\eta^t).$$

## 4.2 Preferences

Households maximize discounted expected utility:

$$U(c) = \sum_{t=1}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \frac{c(s^t)^{1-\gamma}}{1-\gamma},$$

where the coefficient of relative risk aversion $\gamma > 1$, and the subjective time discount factor $0 < \beta < 1$.

## 4.3 Technology

Households trade state-contingent bonds $a_t(s^t, z_{t+1})$ at prices $q_t(z^t, z_{t+1})$ and shares in the Lucas tree $\sigma_t(s^t)$ at price $\nu_t(z^t)$ satisfying the budget constraint:

$$c_t(s^t) + \sum_{z_{t+1}} a_t(s^t, z_{t+1}) q_t(z^t, z_{t+1}) + \sigma_t(s^t)\nu_t(z^t) \leq W_t(s^t).$$

Household cash on hand $W$ evolves according to:

$$W_{t+1}(s^{t+1}) = a_t(s^t, z_{t+1}) + \widehat{y}_{t+1}(\eta^{t+1}, z^{t+1})(1 - \alpha(z_{t+1}))e_{t+1}(z^{t+1}) + \left(\alpha(z_{t+1})e_{t+1}(z^{t+1}) + \nu_{t+1}(z^{t+1})\right)\sigma_t(s^t).$$

Households are subject to state-uncontingent and state-contingent solvency constraints:

$$\sum_{z_{t+1}} a_t(s^t, z_{t+1}) q_t(z^t, z_{t+1}) + \sigma_t(s^t)\nu_t(z^t) \geq K_t(s^t)$$

$$a_t(s^t, z_{t+1}) + \left(\alpha(z_{t+1})e_{t+1}(z^{t+1}) + \nu_{t+1}(z^{t+1})\right)\sigma_t(s^t) \geq M_t(s^t, z_{t+1})$$

where $K$ and $M$ denote generic borrowing limits. Incomplete risk sharing arises from two sources: the lack of an asset whose payoff depends on the idiosyncratic income shock $\eta^t$ and the borrowing constraints.

## 4.4 Transformation into Stationary Economy

We can transform the stochastically growing economy into a stationary economy with a constant aggregate endowment following Alvarez and Jermann (2001); Krueger and Lustig (2010). To that end, define the deflated consumption allocations:

$$\widehat{c}_t(s^t) = \frac{c_t(s^t)}{e_t(z^t)}, \forall s^t,$$

the deflated transition probabilities and the deflated subjective time discount factor:

$$\begin{aligned}\widehat{\pi}(s_{t+1}|s^t) &= \frac{\pi(s_{t+1}|s^t)\lambda_{t+1}(z_{t+1})^{1-\gamma}}{\sum_{s_{t+1}}\pi(s_{t+1}|s^t)\lambda_{t+1}(z_{t+1})^{1-\gamma}}, \\ \widehat{\beta}(s^t) &= \beta\sum_{s_{t+1}}\pi_t(s_{t+1}|s^t)\lambda_{t+1}(z_{t+1})^{1-\gamma}.\end{aligned}$$

Agents in the deflated economy with these preferences:

$$U(\widehat{c})(s^t) = \frac{\widehat{c}(s^t)^{1-\gamma}}{1-\gamma} + \sum_{s_{t+1}}\widehat{\beta}(s_{t+1}, s^t)\widehat{\pi}(s_{t+1}|s^t)U(\widehat{c})(s_{t+1}, s^t) \tag{1}$$

rank consumption plans identically as in the original economy. Under the maintained assumption of independence of aggregate and idiosyncratic risk, the deflated aggregate transition probabilities and the deflated time discount factor are:

$$\begin{aligned}\widehat{\phi}(z_{t+1}|z^t) &= \frac{\phi(z_{t+1}|z^t)\lambda_{t+1}(z_{t+1})^{1-\gamma}}{\sum_{z_{t+1}}\phi(z_{t+1}|z^t)\lambda_{t+1}(z_{t+1})^{1-\gamma}}, \\ \widehat{\beta}(z^t) &= \beta\sum_{z_{t+1}}\phi(z_{t+1}|z^t)\lambda_{t+1}(z_{t+1})^{1-\gamma}.\end{aligned}$$

These are risk-neutral probabilities. When there is predictability in aggregate consumption growth, shocks to expected growth manifest themselves as taste shocks in the deflated economy. If aggregate growth shocks are i.i.d. over time, then the deflated time discount factor is constant and given by:

$$\widehat{\beta} = \beta\sum_{z_{t+1}}\phi(z_{t+1})\lambda_{t+1}(z_{t+1})^{1-\gamma}. \tag{2}$$

This i.i.d. assumption on aggregate growth shocks is the assumption we will make, noting that it can easily be relaxed. In what follows, we also assume that aggregate factor shares are constant: $\alpha_t(z_t) = \alpha,\ \forall t$. By definition, labor income shares average to one across households:

$$\sum_{\eta^t}\varphi(\eta^t|\eta_0)\widehat{y}_t(\eta^t) = 1$$

### 4.5 Equilibrium in the Stationary Economy

Agents trade a single risk-free bond and a stock. The stock yields a dividend $\alpha$ in each period. Given initial financial wealth $\theta_0$, interest rates $\widehat{R}_t$ and stock prices $\widehat{v}_t$, households choose consumption $\{\widehat{c}_t(\theta_0, \eta^t)\}$, bond positions $\{\widehat{a}_t(\theta_0, \eta^t)\}$, and stock positions $\{\widehat{\sigma}_t(\theta_0, \eta^t)\}$ to maximize expected utility (1) subject to the budget constraint:

$$\widehat{c}_t(\eta^t) + \frac{\widehat{a}_t(\eta^t)}{\widehat{R}_t} + \widehat{\sigma}_t(\eta^t)\widehat{v}_t = (1-\alpha)\widehat{y}_t(\eta^t) + \widehat{a}_{t-1}(\eta^{t-1}) + \widehat{\sigma}_{t-1}(\eta^{t-1})(\widehat{v}_t + \alpha),$$

and subject to borrowing constraints:

$$\frac{\widehat{a}_t(\eta^t)}{\widehat{R}_t} + \widehat{\sigma}_t(\eta^t)\widehat{\nu}_t \geq \widehat{K}_t(\eta^t), \quad \forall \eta^t$$

$$\widehat{a}_t(\eta^t) + \widehat{\sigma}_t(\eta^t)(\widehat{\nu}_{t+1} + \alpha) \geq \widehat{M}_t(\eta^t), \quad \forall \eta^t.$$

**Definition 1.** For a given initial distribution of wealth $\Theta_0$, a Bewley equilibrium is a list of consumption choices $\{\widehat{c}_t(\theta_0, \eta^t)\}$, bond positions $\{\widehat{a}_t(\theta_0, \eta^t)\}$, and stock positions $\{\widehat{\sigma}_t(\theta_0, \eta^t)\}$ as well as stock prices $\widehat{\nu}_t$, and interest rates $\widehat{R}_t$ such that each household maximizes its expected utility, and asset markets and goods markets clear.

$$\int \sum_{\eta^t} \varphi(\eta^t|\eta_0)\widehat{a}_t(\theta_0, \eta^t)d\Theta_0 = 0,$$

$$\int \sum_{\eta^t} \varphi(\eta^t|\eta_0)\widehat{\sigma}_t(\theta_0, \eta^t)d\Theta_0 = 1.$$

$$\int \sum_{\eta^t} \varphi(\eta^t|\eta_0)\widehat{c}_t(\theta_0, \eta^t)d\Theta_0 = 1.$$

In the deflated economy, the return on the aggregate stock equals the risk-free rate:

$$\widehat{R}_t = \frac{\widehat{\nu}_{t+1} + \alpha}{\widehat{\nu}_t}. \tag{3}$$

The equilibrium stock price equals the present discounted value of the dividends:

$$\widehat{\nu}_t = \sum_{\tau=0}^{\infty} \widehat{R}^{-1}_{t\to t+\tau}\alpha,$$

discounted at the cumulative gross risk-free rate, defined as: $\widehat{R}_{t\to t+T} = \Pi_{k=0}^{T}\widehat{R}_{t+k}$. Note that $\widehat{R}_{t\to t} = \widehat{R}_t$ and define $\widehat{R}_{t\to t-1} = 1$.

### 4.6 Equilibrium in the Growing Economy

We can map the equilibrium in the detrended economy into an equilibrium in the stochastically growing economy.

**Proposition 4.1.** If $\{\widehat{c}_t(\theta_0, \eta^t), \widehat{a}_t(\theta_0, \eta^t), \widehat{\sigma}_t(\theta_0, \eta^t)\}$ and $\{\widehat{\nu}_t, \widehat{R}_t\}$ are a Bewley equilibrium, then $\{c_t(\theta_0, s^t), a_t(\theta_0, s^t, z_{t+1}), \sigma_t(\theta_0, s^t)\}$ as well as asset prices $\{\nu_t(z^t), q_t(z^t, z_{t+1})\}$ are an equilibrium of the stochastically growing economy with:

$$\begin{aligned} c_t(\theta_0, s^t) &= \widehat{c}_t(\theta_0, \eta^t) e_t(z^t) \\ a_t(\theta_0, s^t, z_{t+1}) &= \widehat{a}_t(\theta_0, \eta^t) e_t(z^t) \\ \sigma_t(\theta_0, s^t) &= \widehat{\sigma}_t(\theta_0, \eta^t) \\ \nu_t(z^t) &= \widehat{\nu}_t e_t(z^t) \\ q_t(z^t, z_{t+1}) &= \frac{\widehat{\phi}(z_{t+1})}{\lambda(z_{t+1})} \frac{1}{\widehat{R}_t}. \end{aligned}$$

The proof is provided in Krueger and Lustig (2010). The last equation implies the following relationship between the interest rate in the growing economy and the stationary economy:

$$R_t = \left( \sum_{z_{t+1}} q_t(z^t, z_{t+1}) \right)^{-1} = \left( \sum_{z_{t+1}} \frac{\widehat{\phi}(z_{t+1})}{\lambda(z_{t+1})} \right)^{-1} \widehat{R}_t. \tag{4}$$

### 4.7 Wealth Accounting

What is the right discount rate when measuring household wealth? If we want a wealth measure that can be aggregated, we have to use the same discount rate for all claims.

**Proposition 4.2.** At time 0, the financial wealth of each household equals the present discounted value of future consumption minus future labor income.

$$\theta_0 = \sum_{\tau=0}^{\infty} \sum_{\eta^\tau} \frac{\varphi(\eta^\tau)}{\widehat{R}_{0\to\tau-1}} \left( \widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau) \right)$$

As the proof in the appendix shows, the proposition follows easily from iterating forward on the one-period budget constraint. In this iteration, we take expectations over financial wealth in all future states using the objective probabilities of the idiosyncratic events $\varphi(\eta^\tau)$, and discount by the cumulative risk-free rate $\widehat{R}_{0\to\tau-1}$.

Aggregate financial wealth in the economy in period 0 is given by:

$$\int \theta_0 d\Theta_0 = \int (\widehat{a}_{-1}(\theta_0) + \widehat{\sigma}_{-1}(\theta_0)\widehat{\nu}_0) d\Theta_0 = 0 + 1\widehat{\nu}_0,$$

where we have used market clearing in the bond and stock markets at time 0.

Aggregating the cost of the excess consumption plan across all households, using the fact that labor income shares average to 1, and imposing goods market clearing at time 0, we get:

$$\int \sum_{\tau=0}^{\infty} \widehat{R}^{-1}_{0\to\tau-1} \sum_{\eta^\tau} \varphi(\eta^\tau) \left( \widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau) \right) d\Theta_0 = \sum_{\tau=0}^{\infty} \widehat{R}^{-1}_{0\to\tau-1} \alpha = \widehat{\nu}_0.$$

The aggregate cost of households' excess consumption plan, or households' aggregate financial wealth, exactly equals the stock market value $\widehat{v}_0$, the only source of net financial wealth in the economy. This result relies on market clearing:

$$\int \sum_{\eta^\tau} \varphi(\eta^\tau)\left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right) d\Theta_0 = \alpha,$$

at each time $t$, because $\int \sum_{\eta^\tau} \varphi(\eta^\tau)\widehat{c}_\tau(\eta^\tau) d\Theta_0 = 1$ from market clearing, and the labor income shares sum to one as well.

The choice of the actual probability measure $\varphi(\cdot)$ and rate $\widehat{R}$ to compute an individual's human capital, the expected present discounted value of her labor income stream, may seem arbitrary. After all, claims to labor income are not traded in this model and markets are incomplete. The key insight is that, using any other pricing kernel to discount individual labor income and consumption streams may result in a value of aggregate financial wealth different from the value of the Lucas tree. To see this, consider using a distorted measure $\psi(\eta^\tau)\varphi(\eta^\tau)$ different from the actual measure $\varphi(\eta^\tau)$, where the household-specific wedges satisfy $\mathbb{E}_0[\psi_t] = 1, \forall t$. Under this different measure, the goods markets do not clear and the labor shares do not sum to one, unless the household-specific wedges do not covary with consumption and income shares:

**Proposition 4.3.** Wealth measures aggregate if and only if the following orthogonality conditions holds for the househehold-specific wedges and household consumption and income:

$$Cov_0\left(\psi_t, \widehat{c}_t\right) = 0, \quad Cov_0\left(\psi_t, \widehat{y}_t\right) = 0.$$

For all other wedge processes $\psi_t(\eta^\tau)$, the resource constraint is violated:

$$\int \sum_{\eta^\tau} \psi(\eta^\tau)\varphi(\eta^\tau)\left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right) d\Theta_0 \neq \alpha,$$

It is common in the literature to use the household's own IMRS to compute human capital (e.g., Huggett and Kaplan, 2016). The household's IMRS is a natural choice because it ties the valuation of human wealth directly to welfare. However, this approach does not lend itself to aggregation. The wedges

$$\psi(\eta^{t+1}) = \frac{u'(\widehat{c}(\eta_{t+1}, \eta^t))}{u'(\widehat{c}_t(\eta_0))},$$

do not satisfy the zero covariance restrictions of the proposition. Imperfect consumption insurance implies that:

$$Cov_0(\psi_t, \widehat{c}_t) \leq 0, \quad Cov_0(\psi_t, \widehat{y}_t) \leq 0.$$

**Proposition 4.4.** If the cross-sectional covariance between the household-specific wedges and consumption is negative ($Cov_0(\psi_t, \widehat{c}_t) \leq 0$), then the aggregate valuation of individual wealth is less than the market's valuation of total wealth.

When aggregating, this pricing functional undervalues human wealth and therefore also total wealth.[4] In sum, while pricing claims to consumption and labor income using the household's IMRS is sensible from a welfare perspective, this approach does not lend itself to wealth accounting and aggregation.

### 4.8 Interest Rate Decline

We now analyze the main exercise of the paper, which is to let the economy undergo an unexpected and permanent decrease in the interest rate ("MIT shock"). Since interest rates are endogenously determined, we generate this decrease through a decrease in the expected growth rate of the economy:

$$\mathbb{E}[\lambda] = \sum_{z_{t+1}} \phi(z_{t+1})\lambda(z_{t+1}) \rightarrow \mathbb{E}[\widetilde{\lambda}] = \sum_{z_{t+1}} \phi(z_{t+1})\widetilde{\lambda}(z_{t+1})$$

where $\mathbb{E}[\widetilde{\lambda}] < \mathbb{E}[\lambda]$. A lower expected growth rate manifests itself as a higher subjective time discount factor in the stationary economy, provided that the coefficient of relative risk aversion $\gamma > 1$, or, equivalently, the elasticity of intertemporal substitution is smaller than one:

$$\widetilde{\beta} = \beta \sum_{z_{t+1}} \phi(z_{t+1})\widetilde{\lambda}_{t+1}(z_{t+1})^{1-\gamma} > \widehat{\beta}.$$

In the transformed incomplete markets economy, the size of the decline in the rate of time preference is governed by the EIS ($1/\gamma$). Just like in a representative agent economy, the larger the EIS, the smaller the effect of a decline in the expected growth rate of aggregate consumption on the risk-free rate.

In the simple case of log-normally distributed aggregate consumption growth, we obtain the following expression for the rate of time preference in the stationary economy:

$$\log \widehat{\beta} = \log \beta - \gamma \mathbb{E}[\log \lambda] - \frac{1}{2}\gamma(1-\gamma)Var[\log \lambda]. \tag{5}$$

Hence, the change in the transformed rate of time preference in response to the growth shock is given by: $\frac{d \log \widehat{\beta}}{d\mathbb{E}[\log \lambda]} = -\gamma$.

It is natural to ask whether we can still implement the equilibrium consumption allocation $\{\widehat{c}_t(\theta_0, \eta^t)\}$ from the economy with high rates in the economy with low rates. Given that the time discount factor of all agents increased by the same amount, there should be no motive to trade away from these allocations. The following proposition shows that the old consumption

[4] Since the factor shares are constant, the consumption claim is in the span of traded assets. Financial wealth is the value of the Lucas tree, which equals $\alpha$ times the value of a claim to total consumption.

allocation is indeed still an equilibrium in the low interest rate economy, provided that initial financial wealth is scaled up for every household.

**Proposition 4.5.** If the allocations and asset market positions $\{\widehat{c}_t(\theta_0,\eta^t),\widehat{a}_t(\theta_0,\eta^t),\widehat{\sigma}_t(\theta_0,\eta^t)\}$ and asset prices $\{\widehat{v}_t,\widehat{R}_t\}$ are a Bewley equilibrium in the economy with $\widehat{\beta}$ and natural borrowing limits $\{\widehat{K}_t(\eta^t)\}$,

$$\widehat{K}_t(\eta^t) = \sum_{\tau=t}^{\infty} \widehat{R}_{t\to\tau-1}^{-1} \sum_{\eta^\tau|\eta^t} \varphi(\eta^\tau|\eta^t)(1-\alpha)\widehat{y}_\tau(\eta^\tau),$$

then the allocations and asset market positions $\{\widehat{c}_t(\widetilde{\theta}_0,\eta^t),\widehat{a}_t(\widetilde{\theta}_0,\eta^t),\widehat{\sigma}_t(\widetilde{\theta}_0,\eta^t)\}$ and asset prices $\{\widetilde{v}_t,\widetilde{R}_t\}$ will be an equilibrium of the economy with $\widetilde{\beta}$ and natural borrowing limits $\{\widetilde{K}_t(\eta^t)\}$,

$$\widetilde{K}_t(\eta^t) = \sum_{\tau=t}^{\infty} \widetilde{R}_{t\to\tau-1}^{-1} \sum_{\eta^\tau|\eta^t} \varphi(\eta^\tau|\eta^t)(1-\alpha)\widehat{y}_\tau(\eta^\tau),$$

asset prices are given by

$$\widetilde{\beta}\widetilde{R}_t = \widehat{\beta}\widehat{R}_t, \text{ and } \widetilde{v}_t = \sum_{\tau=0}^{\infty} \widetilde{R}_{t\to t+\tau}^{-1}\alpha,$$

and every household's initial wealth is adjusted as follows:

$$\widetilde{\theta}_0 = \theta_0 \frac{\sum_{\tau=0}^{\infty} \widetilde{R}_{0\to\tau}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau)\left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right)}{\sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau)\left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right)}.$$

The proof is in the appendix. Aggregate financial wealth undergoes an adjustment equal to the ratio of the price of two perpetuities:

$$\frac{\sum_{\tau=0}^{\infty} \widetilde{R}_{0\to\tau}^{-1}}{\sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau}^{-1}} = \frac{\widetilde{v}_0}{\widehat{v}_0}.$$

Intuitively, with lower interest rates, all asset prices are higher than in the high-rate economy. The Lucas tree becomes more valuable. A fraction $1-\alpha$ of this tree reflects aggregate human wealth, the remaining fraction is aggregate financial wealth. Each individual's financial wealth adjustment differs, and depends on the expected discounted value of the same future excess consumption plan discounted at different rates. The higher one's expected future excess consumption, the larger the initial financial wealth adjustment needed to implement the old equilibrium allocation.

To a first-order approximation, i.e., for a small change in the interest rate, the adjustment in initial financial wealth needed for agents to keep their initial consumption plan is given by the duration of their planned consumption in excess of labor income. This is the duration households will need in their net financial assets in order to be fully hedged against interest rate risk.

**Characterizing Interest Rate Sensitivity Using Duration of Excess Consumption** Define the duration of a household's excess consumption plan at time 0, following the realization of the idiosyncratic labor income shock $\eta_0$, as follows:

$$D^{c-y}(\theta_0, \eta_0) = \frac{\sum_{\tau=0}^{\infty} \sum_{\eta^\tau|\eta_0} \tau \widehat{R}_{0\to\tau}^{-1} \varphi(\eta^t|\eta_0) \left(\widehat{c}_\tau(\eta^\tau|\eta_0) - (1-\alpha)\widehat{y}(\eta^\tau|\eta_0)\right)}{\sum_{\tau=0}^{\infty} \sum_{\eta^\tau|\eta_0} \varphi(\eta^t|\eta_0) \widehat{R}_{0\to\tau}^{-1} \left(\widehat{c}_\tau(\eta^\tau|\eta_0) - (1-\alpha)\widehat{y}(\eta^\tau|\eta_0)\right)}$$

The duration measures the sensitivity of the cost of its excess consumption plan to a change in the interest rate. In our endowment economy, aggregate consumption is fixed. We are interested in the valuation effects of interest rate changes.[5] The duration of the excess consumption claim equals the value-weighted difference of the duration of the consumption claim and that of the labor income claim:

$$D^{c-y} = \frac{P_0^c}{P_0^{c-y}} D^c - \frac{P_0^y}{P_0^{c-y}} D^y.$$

where $P_0^{c-y} = \theta_0$ is household financial wealth, $P_0^y$ is human wealth, and $P_0^c$ is total household wealth, the sum of financial and human wealth. Households with a high positive duration of excess consumption face a large increase in the cost of their consumption plan when interest rates go down, insofar that this increased cost is not offset fully by the increase in their human wealth.

The duration of the aggregate excess consumption claim, the aggregate duration for short, equals:

$$D^a = \frac{\sum_{\tau=0}^{\infty} \tau \widehat{R}_{0\to\tau}^{-1}}{\sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau}^{-1}}$$

This is the duration of a claim to aggregate consumption minus aggregate labor income, or equivalently to aggregate financial income. It is the duration of a perpetuity in the stationary economy. Recall that $\widehat{\nu}_0 = \nu_0 = \alpha \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau}^{-1}$ denotes aggregate financial wealth.

**Proposition 4.6.** The aggregate duration equals the wealth-weighted average duration of households' excess consumption claims:

$$D^a = \int D^{c-y}(\theta_0, \eta_0) \frac{\theta_0}{\nu_0} d\Theta_0.$$

The proof follows directly from the definition of the household specific duration measure and market clearing.

The next proposition is the main result. It shows that, when households that are richer than

[5] Households in the detrended economy's equilibrium face a deterministic interest rate, and do not anticipate interest rate changes. Auclert (2019) was the first to conduct this type of duration analysis in a model with endogenous labor supply to gauge the effects of monetary policy on consumption.

average tend to have excess consumption plans of higher duration, then the (equally-weighted) average household's excess consumption plan duration is smaller than the aggregate duration.

**Proposition 4.7.** If $cov(\theta_0, D^{c-y}(\theta_0)) > 0$ then $\int D^{c-y}(\theta_0, \eta_0) d\Theta_0 \leq D^a$ and lower interest rates increase financial wealth inequality.

The proof follows from recognizing the following relationship between (cross-sectional) expectations and covariances:

$$D^a = \mathbb{E}\left[\frac{\theta_0}{\nu_0^a} D^{c-y}(\theta_0, \eta_0)\right] = \mathbb{E}\left[D^{c-y}(\theta_0, \eta_0)\right] + cov\left[\frac{\theta_0}{\nu_0}, D^{c-y}(\theta_0, \eta_0)\right].$$

The proposition says that under the covariance condition, if all households are perfectly hedged in their portfolio, then wealth inequality should increase when rates decline.

In this class of Bewley models, agents with low financial wealth have encountered a bad history of labor income shocks. If labor income is highly persistent, their labor income is low today and in the near future relative to labor income in the distant future (because of mean-reversion). This pattern makes the duration of their labor income stream high. But since the household is smoothing consumption inter-temporally, $D^c < D^y$. As a result, low-wealth agents tend to have low duration of their excess consumption plan. Conversely, rich agents have high labor income and high excess consumption duration. Consumption smoothing is the force that makes the covariance assumption satisfied in a Bewley model where the only source of heterogeneity is income shock realizations. It follows immediately that, under the stated covariance restriction, the increase in the cost of the excess consumption plan for the average household is smaller than the aggregate (per capita) wealth increase. Put differently, financial wealth inequality should increase when rates go down if households want to afford their old consumption plans.

Low-financial wealth households in a Bewley model have high-duration human wealth, which provides a natural interest rate hedge. High financial-wealth households have low-duration human wealth and need to increase financial wealth by more when rates decline to be able to afford the old consumption plan.

The insights of this normative proposition apply more broadly. The covariance condition can be verified in a richer model with ex-ante heterogeneity across households, like the one discussed in the next section. It can also be tested in the data, with the additional observation that households' financial portfolios may not have the same duration as their excess consumption plans. In other words, real-world households may not be fully hedged, unlike the households in the Bewley model.

Next, we measure the actual duration of the household's financial assets in the data, denoted $D^{fin}$, which can differ from the duration of the excess consumption claim $D^{c-y}$. If they differ, the household is not fully hedged. We use a calibrated life-cycle version of the Bewley model with overlapping generations to assess how well these households are really hedged against interest rate risk.

## 5 Calibrated Model with Ex-ante Heterogeneity

The previous section showed that in a Bewley model a rise in financial wealth inequality is required when interest rates decline when agents are fully hedged. In this section, we aim to quantify this effect in a model with realistic heterogeneity among households. The model introduces overlapping generations of finitely-lived agents, generating heterogeneity by age. We feed in the actual heterogeneity in financial wealth duration, and we investigate how the financial, human, and total wealth distributions change with low versus high interest rates.

### 5.1 Calibration

We conduct our analysis in the stationary version of this economy, using the mapping described in Proposition 4.1 to go from objects in the stochastically growing economy to objects in the stationary economy.

**Aggregate Output Growth Process** We assume that aggregate output growth $\lambda$ follows an i.i.d. log-normal process $\log \lambda \sim N(g, \sigma_\lambda^2)$, where $g = 0.01893$ and $\sigma_\lambda = 0.02319$ are the average annualized growth and volatility of log real per-capita GDP in the U.S. data.

**Preferences and Stationarity** Households have CRRA preferences with risk aversion $\gamma$ equal to 2. Substituting into (4), and using the initial risk-free rate 4.82% from the data (see below) implies that the initial risk-free rate in the stationary economy is $\hat{R} = 1.0294$. We set $\widehat{\beta} = 1/\widehat{R}$, implying $\hat{\beta} = 0.9715$. For easier interpretation, converting back to the growing economy using (2) implies that the true preference parameter $\beta$ is equal to 0.9898.

**Size of Decline in Real Yields** In the model, the expected growth rate experiences an unanticipated decline, giving rise to decline in the real rates.

According to the auxiliary asset pricing model in Appendix E, the ten-year real bond yield averaged 4.82% in the 40 quarters of the 1980s decade and 0.34% in the 2010s decade.[6] The asset pricing model shows similarly large declines in expected real returns on the aggregate stock market and on housing wealth, as shown in Table 1. Other stock indices such as value and infrastructure stocks show larger declines, while growth and small stocks show smaller declines. Expected returns on total wealth, measured as a claim to GDP or to aggregate consumption, show

[6] The asset pricing model matches the available data on Treasury Inflation-Indexed Securities over the period for which they are available. The model-implied yield changes are similar for real bonds of different maturities.

large declines around 12-13% points. In other words, the decline in expected returns was broad-based.

Table 1: Expected Real Returns Decade Averages

| Asset | 1980s | 2010s | Decline |
|---|---|---|---|
| Ten-year real bond yield | 4.82% | 0.34% | 4.48% |
| Aggregate stock market | 7.98% | 2.00% | 5.98% |
| Growth stocks | 5.21% | 3.53% | 1.68% |
| Value stocks | 18.50% | 7.19% | 11.31% |
| Infrastructure stocks | 11.75% | 2.35% | 9.40% |
| Small stocks | 3.57% | 3.18% | 0.39% |
| Housing wealth | 8.24% | 4.89% | 3.35% |
| GDP claim | 15.90% | 2.80% | 13.10% |
| Consumption claim | 15.27% | 2.84% | 12.43% |

*Note:* The table reports model-implied real expected real returns and average them over the 40 quarters in the 1980s and the 40 quarters of the 2010s. The model is described in Appendix E.

This change in $R$ is the result of an unexpected and permanent decline in the expected aggregate growth rate of the economy (an MIT shock). We calibrate the model to a decline in real rates of 4.48%. In the stationary model, interest rates must be adjusted for growth. Using the formula

$$\hat{R}_t = R_t \exp\left\{-g + \left(\gamma - \frac{1}{2}\right)\sigma_\lambda^2\right\}$$

obtained from (4) in the lognormal case, the adjusted rates $\widehat{R}$ decline from 2.83% to $\widetilde{R}$ -1.57%. Using our value of $\gamma = 2$, (5) implies that a decline in rates of 4.48% can be generated using a decline in expected growth $\mathbb{E}[\log \lambda]$ of 2.24%. Following Proposition 4.5, we also adjust the discount factor to preserve the relation $\tilde{\beta}\tilde{R} = \hat{\beta}\hat{R} = 1$.

**Regular Income Component** The income process consists of a regular component and a superstar component. The regular income process for household $i$ of age $a$ at time $t$ that is not currently in the superstar state takes the form standard in the literature, given by:

$$\log\left(y_{t,a}^i\right) = m_t + \chi' X_t^i + z_t^i, \tag{6}$$

$$z_{t+1}^i = \alpha_i + \varepsilon_{t+1}^i + \nu_{t+1}^i, \tag{7}$$

$$\varepsilon_{t+1}^i = \rho\varepsilon_t^i + \eta_{t+1}^i, \tag{8}$$

where $m_t$ is a year-fixed effect and $X_t^i$ is a vector of household characteristics that includes a cubic function of age.[7] When calibrating the model, we normalize the age profile $\chi' X_t^i$ so that its mean is equal to unity during working life.

The stochastic income component $z_t^i$ contains a household-fixed effect $\alpha^i$, a persistent component $\varepsilon_{t+1}^i$, and an i.i.d. component $\nu_{t+1}^i$. We have: $\mathbb{E}[z^i] = \mathbb{E}[\alpha^i] = E[\nu^i] = \mathbb{E}[\varepsilon^i] = 0$ and $Var[\nu^i] = \sigma_\nu^2$, $Var[\eta^i] = \sigma_\eta^2$, $Var[\alpha^i] = \sigma_\alpha^2$, and $Var[\varepsilon_0^i] = \sigma_{\varepsilon,0}^2$. Note that the income risk parameters are common across groups. The parameters are estimated by GMM using PSID data from 1970 until 2017, as detailed in Appendix A.2. Figure A5 in that appendix plots the deterministic life-cycle income profile.

The literature typically estimates (6)-(8) on labor income for white males between ages 25 and 55. We deviate from this practice in three ways, all of which are important for our purposes. First, we consider a broader income concept. Second, we consider the entire life-cycle from age 18 to 80. Third, we focus on households rather than individuals.

First, from the model's perspective, the relevant notion of income includes transfers. It is the risk in this income that the household is hedging by trading in financial markets (borrowing and saving). To that end, we measure income in the data as income from wages and salaries, the labor income component of proprietor's income, and government transfers (unemployment benefits, social security, other government transfers), and private defined-benefit pension income. Obtaining consistent data on the various components of transfers is involved because successive waves of the PSID use different variable codes for the same concepts. Appendix A.2 provides the details. Catherine et al. (2020) also focuses on after-transfer income.

Second, we are interested in the entire life-cycle. We start at age 18 and go until age 80. Because our income concept includes transfers such as unemployment benefits and retirement income from public or private defined-benefit pension plans, we do not have to model labor force participation decisions or retirement decisions. Our approach captures the average decisions made in the data. For example, we do not need to make the assumption that retirement starts at age 65, that income in retirement is some constant fraction of pre-retirement income, or that income risk disappears in retirement. We can let the data speak on these issues. Since our income concept includes income from part-time work, it captures income earned by students, for example. We assign to students the educational achievement they will attain even before they have completed their education, so that they are classified in the correct group.

Third, we focus on households, aggregating income across its adult members. This absolves us from having to model demographic changes such as getting married, getting divorced, getting widowed. We simply follow households identified by the head of household as designated in the

[7] We have verified that our results are similar if we estimate the year fixed effect and the age profile separately for groups of households that depend on education (college completion or not), race (white or non-white), gender (male or non-male), giving rise to 8 groups in total. Since it makes little difference, we only consider one group here.

data.

**Superstar Income Component** To help the model match the level of wealth inequality in the high-interest rate regime, we enrich the income process in (6)-(8) with a superstar income state. This state has a high income level $Y^{sup}$. Households enter in this state with probability $p_{12}^{sup}$ when they are in the normal income state, and return to the normal state with probability $p_{21}^{sup}$ when they currently are in the superstar income state. The income level $Y^{sup}$ is chosen to match the wealth Gini in the 1980s exactly, which requires a value equal to 75 times average income. The transition probability parameters $p_{12}^{sup} = 0.0002$ and $p_{21}^{sup} = 0.975$ are taken from Boar and Midrigan (2020). There is about a 1% probability of entering in the superstar income state over one's life-time. Conditional on entering, the state has an expected duration of 40 years.

In the computations, we discretize the stochastic income process $z$, with the extra superstar state, as a markov chain.

**Mortality Risk** For simplicity, we assume that households in each age-gender group share mortality risk within their cohort.[8]

## 5.2 Financial Duration

In the stationary economy, agents trade a single risk-free asset with heterogeneous duration. As the previous section explained, having only safe assets is without much loss of generality since a model with aggregate risk in total income maps into a stationary economy without aggregate risk as long as the idiosyncratic and aggregate risk are uncorrelated. The presence of aggregate risk in the growing economy affects the time discount factor and hence the equilibrium risk-free rate in the stationary economy.

The risk-free asset is long-lived, modeled as a zero-coupon bond. Therefore, its duration equals its maturity. Agents start life at age 18 with zero financial assets. Households in the stationary economy anticipate a constant risk-free rate, and hence are ex ante indifferent with respect to the duration of their portfolio. The source of the unexpected decline in interest rates is an unanticipated decline in the expected growth rate of the aggregate endowment (GDP).

We feed in the actual duration of household portfolios into the simulation. The real world's counter-part to the model's financial asset is a portfolio of various financial and real assets that households own. As Table 2 shows, household assets consist of (i) cash, deposits, and money market instruments, (ii) stocks held directly and indirectly in mutual funds and pension accounts, (iii) real estate, (iv) private business wealth, and (v) fixed income assets (directly and indirectly

[8]This is implemented as a tontine system, where all agents of a certain age pool resources to eliminate mortality risk. Our results are not sensitive to this assumption. Future extensions could add an operative bequest motive. They would make the life-cycle model closer to the infinite-horizon model of the previous section.

held). Household liabilities consist of mortgage, student, and consumer debt. The duration of a financial portfolio is the weighted average duration of the components of the financial portfolio, where the weights are the portfolio weights $\omega(k)$ of the various financial assets $k$:

$$D_{t,i}^{fin} = \sum_k \omega_t^i(k) D_t(k). \tag{9}$$

To measure the duration of each of the components of financial wealth, $D_t(k)$, we build a rich asset pricing model, detailed in Appendix E. It prices bonds of various maturities, both nominal and real, the aggregate stock market, several cross-sectional stock market factors including small, growth, value, and infrastructure stocks, and household housing wealth. For these assets, the model provides a McCauley duration in each quarter from 1947.Q1 until 2019.Q4. We use the durations for the 1980s, averaged across the 40 quarters in that decade.

We use the model-implied duration of the aggregate stock market to proxy for the duration of households' directly- and indirectly-held stock market wealth. We use the duration of small stocks to proxy for the duration of household business wealth.[9] We use the duration of owner-occupied housing wealth to measure the duration of households' real estate assets. For cash and deposits, we assume a duration of 0.25 years. For fixed income, we assume a duration of 4 years.[10]

For student debt, we assume a duration of 4.5 years. Student loans are typically 10 year annuities. At an interest rate of 5.8%, the average rate on outstanding student loans in 2017, the duration is 4.56. At higher the interest rates that prevailed in the 1980s, the duration would be slightly smaller. For consumer debt, we assume a duration of 1 year. Much of this debt is revolving debt, while some of it is 24-month personal loans. The personal loans are amortizing.[11] For mortgage debt, we obtain data for the Bloomberg-Barclays Aggregate MBS Index. It is a representative portfolio of all outstanding U.S. pass-through mortgage-backed securities. The average McCauley duration of this representative mortgage portfolio in 1989 and 1990 was 5.2 years. Most mortgage debt in the U.S. is 30-year fixed-rate mortgages. The reasons for this much lower du-

[9] To understand the high value for the duration of small stocks, consider a back-of-the-envelope calculation based on the Gordon Growth Model where the McCauley duration is $(1+r)/(r-g)$. Under perfect foresight, we can use the average realized real return and average realized real dividend growth rate from 1985–2020 to proxy for the expected real return and expected real dividend growth rate in the 1980s. For the smallest market capitalization decile, we find $r = 8.01\%$ and $g = 6.55\%$. This delivers a duration of 75.8, close to the number we obtain using our more sophisticated SDF model and the bottom quintile of market caps. For comparison, for stocks in the largest decile of market capitalization, we obtain $r = 7.94\%$ and $g = 0.81\%$, resulting in a duration of 15.2. The expected returns on large and small stocks are nearly identical. Thus, the high duration for small stocks arises from its high cash flow growth rate.

[10] For reference, the maturity of outstanding U.S. Treasury marketable securities averages 62 months between 2000 and 2020. The duration is strictly smaller than the maturity since bonds pay coupons. For example, if the coupon rate is 4.65% and the bond pays semi-annual coupons, then the duration is 4.5 years. Other corporate and international bonds and loans held by U.S. households tends to have somewhat lower duration than U.S. Treasuries because there are fewer long-term bonds and coupons are higher.

[11] We exclude auto debt since we also exclude vehicles from assets. The reason is that our consumption measure includes durable consumption.

Table 2: Duration of the Household Financial Wealth Portfolio 1980s

| | Duration | Portfolio Shares |
|---|---|---|
| **Assets** | | |
| Cash and Deposits | 0.25 | 11.60 |
| Equities | 28.78 | 11.61 |
| Real Estate | 14.89 | 48.75 |
| Private Business Wealth | 61.25 | 24.56 |
| Fixed Income | 4.00 | 17.75 |
| **Liabilities** | | |
| Mortgage Debt | 5.20 | 12.12 |
| Student Debt | 4.50 | 0.27 |
| Other Debt | 1.00 | 1.88 |
| **Aggregate Duration** | | **25.72** |
| **Average Duration** | | **15.43** |

*Note:* The column "Duration" reports the duration of the asset, again averaged over all quarters in the 1980s. For Equities, Private Business Wealth, and Real Estate, the durations are computed form the asset pricing model in Appendix E, averaging across the 40 quarters in the 1980s. The column "Portfolio Shares" reports the wealth-weighted average or aggregate portfolio weights. Liabilities receive negative portfolio weights. These weights are based on the 1989 SCF.

ration than 30 are several: amortization, high interest rates, and prepayment.[12] The resulting durations are reported in the first column of Table 2.

Next, we collect data from the Survey of Consumer Finances (SCF) on household portfolio shares, the $\omega_t^i(k)$ in (9). The wealth-weighted portfolio weights are reported in the remaining columns of Table 2. The details are in Appendix A.3.

We calculate the duration for each household separately, combining the household-level portfolio weights with the asset-specific durations listed in the first column. The last two rows of Table 2 report the wealth-weighted (aggregate) and equally-weighted (average) financial duration among households. When pooling all households, the average financial duration is 15.43. This value is much lower than the wealth-weighted duration of 25.72.

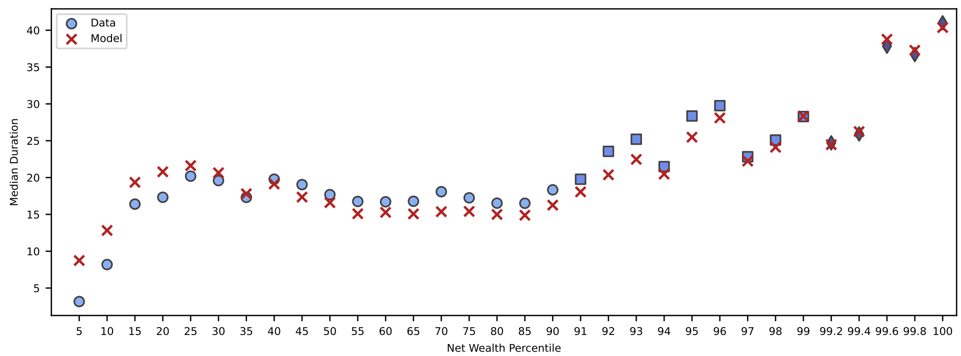
As shown in Proposition 4.7, if the aggregate duration exceeds the average duration of financial wealth, then a reduction in interest rates leads to a rise in financial wealth inequality. This condition, which results from a positive cross-sectional covariance between duration of financial wealth and financial wealth itself, is clearly satisfied in the U.S. data. It occurs because richer households tend to hold more private business wealth, equities, and housing wealth, which are long-duration assets, hold fewer short-duration assets (cash), and hold less debt (negative duration). The goal of this section is to analyze how much financial wealth inequality the model can generate given the observed decline in real interest rates.

In the data, financial duration is strongly correlated with the level of financial wealth. Using

[12] The average maturity of the outstanding MBS portfolio in 1989-1990 was 9.8 years and the average coupon rate was 9.35%.

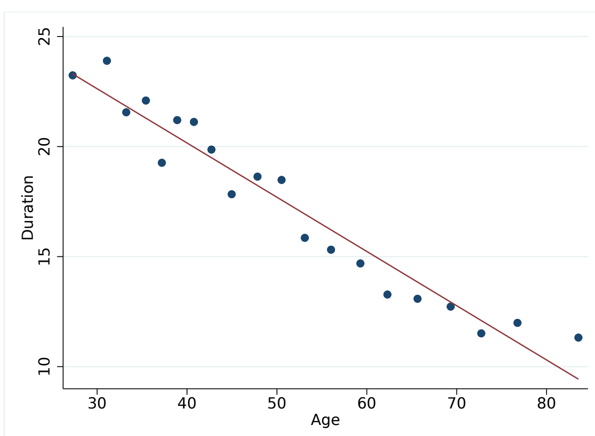
the SCF data, the dots in Figure 3 plots the average duration by wealth bin. Since higher-wealth agents are more important for aggregate wealth outcomes, Figure 3 displays 5% bins up to the 90th percentile, then 1% bins up to the 99th percentile, and 0.2% bins for the top 1%. The figure shows that wealthier households hold longer-duration financial portfolios.

Figure 3: Financial Duration by Net Worth Wealth Percentiles




The second key data pattern is variation in financial duration by age. Figure 4 displays a binscatter of measured duration in our SCF data by age, after controlling for net wealth using dummies for each of the bins constructed in Figure 3. Figure 4 shows that there is a strongly negative relationship between age and duration.

Figure 4: Financial Duration by Net Worth Wealth Percentiles




Our empirical results in Appendix A.3 show that adding other covariates yields little additional power to explain variation in financial duration across households. Therefore, we approximate financial duration using the regression:

$$D_i^{fin} = \alpha + \beta Age_i + \sum_j \gamma_j NetWealthBin_{i,j} + \varepsilon_i, \tag{10}$$

where $NetWealthBin_{i,j}$ is a dummy for whether household $i$ falls in financial wealth bin $j$, as defined for Figure 3 above. We then calibrate financial durations in the model to be equal to the fitted value:

$$\widehat{D}_i^{fin} = \hat{\alpha} + \hat{\beta} Age_i + \sum_j \hat{\gamma}_j NetWealthBin_{i,j} \tag{11}$$

applied household by household, where hats denote sample estimates. This procedure delivers the close fit between model and data observed in Figure 3, where the small discrepancies are due to slight differences in the relationship between age and net wealth percentile in model and data. The model delivers an equal-weighted duration of 16.9 and a value-weighted duration of 28.5, both close to their empirical counterparts in Table 2.

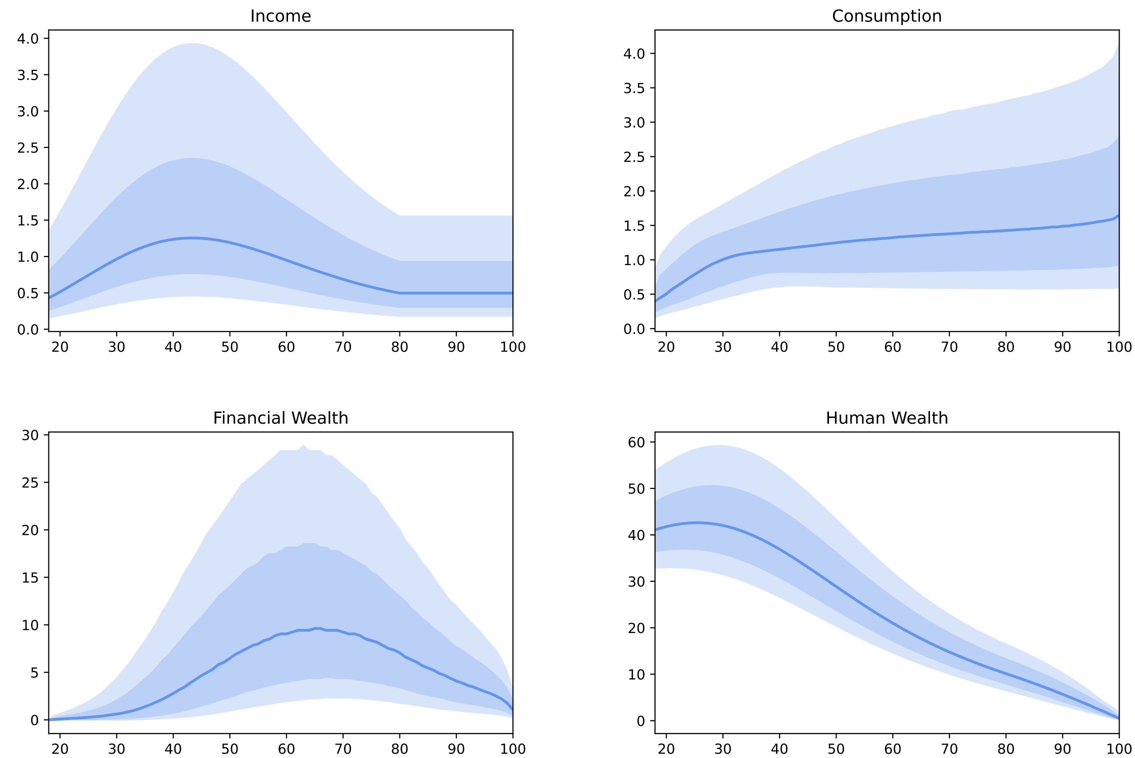
### 5.3 High Interest-Rate Regime

We begin by describing the properties of the model in its stationary distribution under the high interest rate regime. Figure 5 displays the life cycle profiles of income, consumption, financial wealth, and human wealth. The axes are normalized such that 1 represents the typical income during working life. Income inequality is increasing over the life cycle because of the accumulation of income shocks and because of the increase in average income over the life cycle profile. The income inequality drops after retirement but is still non-negligible since agents have heterogeneous retirement income and still face some income risk.

The top-right panel shows that both the level and dispersion of consumption are rising over the life cycle, with dispersion falling in retirement when income risk reduces. This is consistent with the data which show that consumption inherits the hump-shaped profile from income (e.g., Krueger and Perri, 2006).

Turning to wealth, financial wealth in the bottom left panel increases in preparation for retirement, and is subsequently run down during retirement. Financial wealth inequality rises and falls over the life cycle. Human wealth in the bottom right panel is decreasing in age. There are two effects at play. Human wealth rises as the households' highest-earning periods are brought closer to the present. Human wealth falls due to the overall decrease in the remaining periods of work. The latter effect dominates. Total wealth consists almost exclusively of human wealth when young. As households age and accumulate financial wealth, a larger share of total wealth becomes financial wealth. However, human wealth remains a large component of total wealth throughout

Figure 5: Life Cycle Profiles



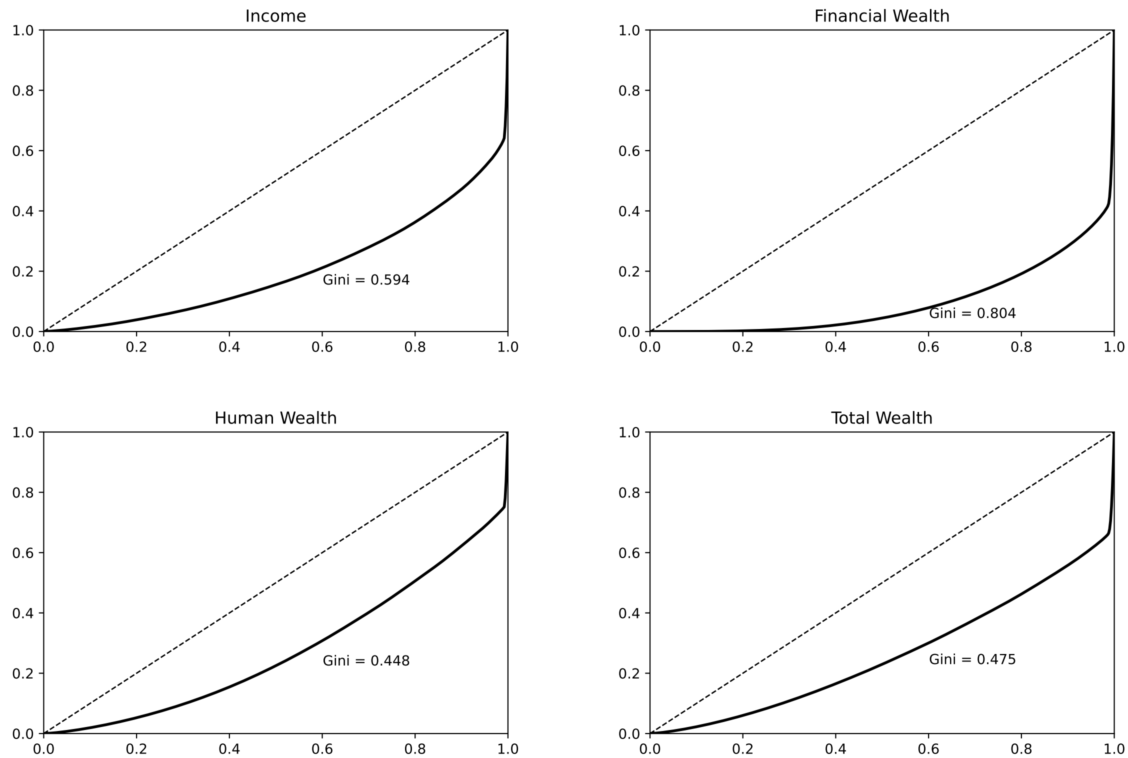

*Note:* This figure plots the life cycle profiles by age for the all agents of all groups combined. The axes are normalized so that the average income across all agents of all ages is equal to unity. The center line displays the median, while the dark and light bands represent 66.7% and 95% percentile bands. Although agents in the model have a maximum age of 100, we truncate the plot at age 90 due the relatively small sample of agents surviving past this age.

the life-cycle.

Figure 6 displays the Lorenz curves for consumption and wealth for all households (in all groups), and reports the Gini coefficients. The model generates a Gini coefficient for (after-transfer) household income of 0.594. Consumption inequality (not plotted) closely tracks income inequality and has a Gini coefficient of 0.558. Financial wealth is much more unequally distributed than human wealth or total wealth. The Gini coefficients of human and total wealth are 0.448 and 0.475, compared to the Gini of financial wealth of 0.804. The low total wealth inequality arises from (i) the importance of human wealth in total wealth, and (ii) the negative cross-sectional correlation between financial and human wealth.

Figure 7 displays the duration of human and total wealth by age. Human wealth represents a claim on lifetime income whereas total wealth represents a claim on lifetime consumption. Both of these durations are similar because of the importance of human wealth in total wealth. These durations are high when young, around 30, and drop rapidly as age increases, since there are

Figure 6: Lorenz Curves




*Note:* This figure plots the Lorenz curve for each variable, obtained from a long simulation of the model.

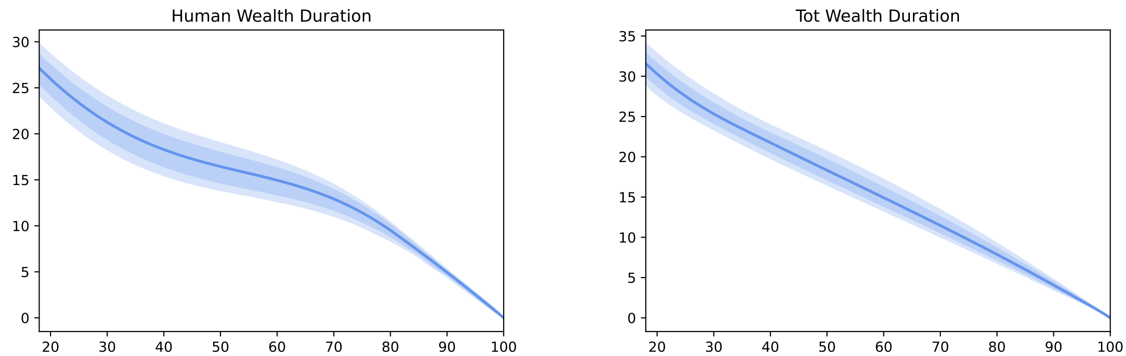
fewer years of life remaining to earn labor/pension income.

### 5.4 Change to Low Interest Rates

In this section we apply the main experiment of an unanticipated, permanent decline in the real interest rate from 4.82% to 0.34% in the growing economy, corresponding to a decline from 2.83% to -1.57% in the stationary economy. Before turning to the response of households' actual wealth portfolios, we first note that agents' prior consumption plans may no longer be budget feasible. Thus, even if financial wealth were unchanged, the change in interest rates could have large effects on lifetime consumption and welfare.

To study the impact of the change in interest rates, we first simulate the model to generate an initial draw from the model's stationary distribution. We then change the interest rate, re-solve the model at the new interest rate, and simulate forward 50 periods (years). To isolate the effect of the rate change, we subtract out the results of the simulation with the same idiosyncratic shock realizations under the old interest rate. We do not clear the bond market in this exercise. As a result, when interest rates decline, the economy produces excess savings. We rebate those savings

Figure 7: Wealth Durations




*Note:* This figure plots the durations of labor income (human) wealth (left panel) and consumption (right panel). The plots display durations computed for many agents simulated from the stationary equilibrium of the model. The economy is normalized so that the average income is equal to unity. The center line displays the median, while the dark and light bands represent 66.7% and 95% percentile bands.

to households so as to keep the total resources of the economy unchanged before and after the interest rate change. Appendix D explains the details.

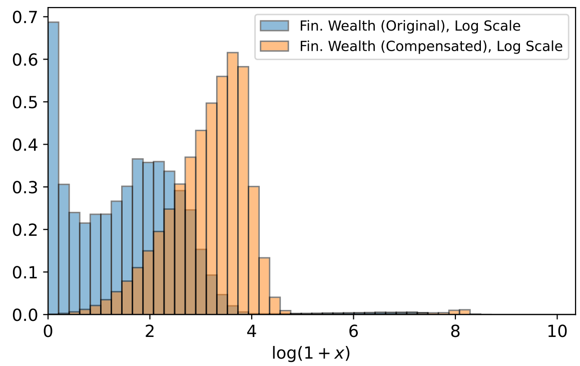
#### 5.4.1 The Compensated Wealth Distribution

To establish an intuitive baseline that is consistent with the theoretical analysis in Proposition 4.5 of Section 4.8, we compute the change in financial wealth that would be required to maintain the prior consumption allocation in the high interest-rate economy. We refer to the counterfactual wealth allocation in which "fully hedged" households receive this financial wealth as the *compensated* financial wealth distribution (defined as $\widetilde{\theta}$ in the theory above).

The resulting distribution of financial wealth, alongside the original (pre-shock) distribution, is displayed in Figure 8. To ensure that the full distribution is visible, we display transformed variables $\log(1+x)$ on the x-axis.[13] This comparison shows two major differences between the pre-shock and compensated distribution. First, the compensated distribution is shifted substantially to the right. Households in this economy mostly save ($c_t < y_t$) earlier in life before dissaving ($c_t > y_t$) in old age. When rates are much lower, households lose much of the effect of compound interest on their retirement savings. As a result, the aggregate amount of financial wealth in the compensated distribution exceeds the pre-shock total by 166.6%. As can be seen from the plot, this rightward shift extends up to the very top, implying that even the wealthiest individuals must be compensated with additional financial assets to attain their old consumption plans. Indeed, more than one third (40.9%) of new financial wealth accrues to top-1% financial wealth holders under

[13]Because many agents have zero financial wealth, a standard log transform would be inappropriate in this context.

Figure 8: Histogram, Compensated vs. Original Financial Wealth Distribution




*Note:* This plot displays the distribution of financial wealth under the stationary distribution and under the compensated distribution drawn from the stationary distribution of the economy. The x-axis displays a transformation $\log(1+x)$ of the original data. Each distribution is top coded at the top 0.1% of the pre-shock wealth distribution.

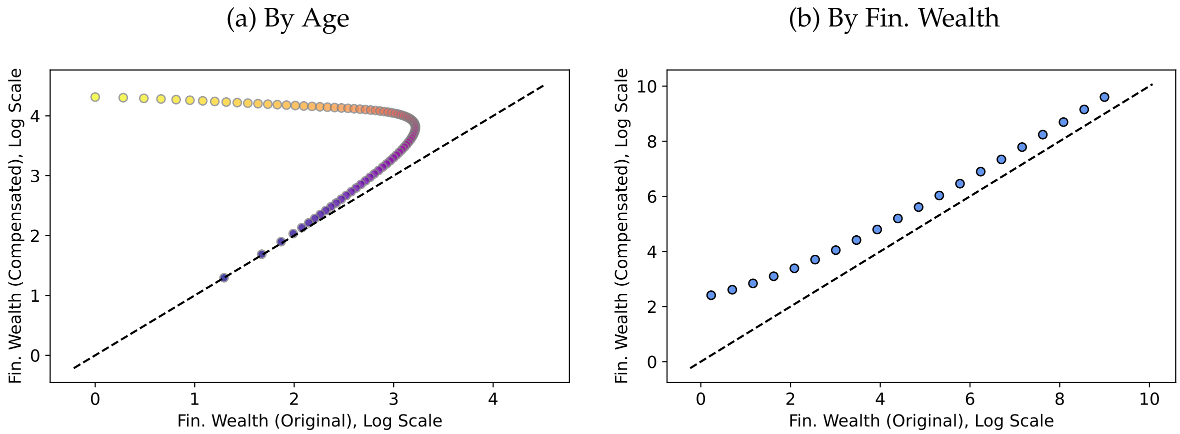
the compensated distribution.

Second, although the wealthiest gain under this compensated distribution, the financial wealth Gini falls substantially in the compensated distribution, as the less wealthy gain proportionally more. Visually, while the original high interest-rate distribution of financial wealth is heavily right-skewed, the compensated distribution is actually left-skewed. Quantitatively, the share of financial wealth held by the top-1% decreases from 55.3% in the baseline economy to 44.8% in the compensated economy.

To see why inequality falls in the compensated distribution, we can turn to Figure 9. Panel (a) compares the original (horizontal axis) and compensated financial wealth distributions (vertical axis) by age. The youngest agents (light/yellow) in the top left have close to zero financial wealth in the original distribution, but require the most financial wealth in the compensated distribution. As households age, their actual wealth initially increases, but their compensated wealth falls. Finally, late in life, both actual and compensated wealth fall rapidly toward zero, with the actual and compensated distributions close to coinciding for these older households.

This result is perhaps surprising, since the young have virtually their entire asset portfolio invested in human wealth. Because human wealth has a very long duration (left panel of Figure 7), it is well-hedged against interest rate changes. The key challenge the young face in a low interest rate environment, however, is not from their current portfolio, but their future portfolios. Due to the life cycle profile of income, the young plan to save during middle age, then dissave during retirement. Under a low interest rate, the young will be unable to accumulate enough interest on their future savings, making their original consumption plans unattainable without large infusions of financial wealth today. In contrast, older agents have already benefited from

Figure 9: Scatterplots, Compensated vs. Original Financial Wealth Distribution




*Note:* Panel (a) plots the distribution of original financial wealth against the distribution of compensated financial wealth by age. Each dot represents one year of age, with the lightest (yellow) dots representing the youngest agents and the darkest (purple) dots representing the oldest agents. Both variables are plotted using the transform $\log(1 + x)$. The dashed line represents equality between the original and compensated distributions. Panel (b) plots the same distribution by bins of original financial wealth in place of age.

the higher rate of return in accumulating their retirement assets, while the oldest are dissaving, consuming principal rather than interest. These households are less affected by the loss of high-return investment opportunities, and require little compensation.

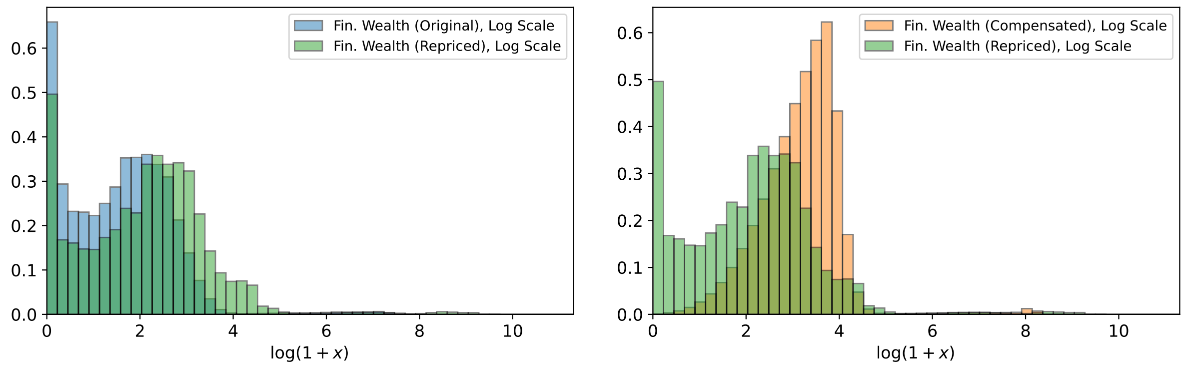
Panel (b) aggregates over ages to present the total compensation required for various levels of pre-shock financial wealth. The lowest levels of financial wealth mix young agents who have not begun saving with old agents who are spending down assets late in life. As a result, this group mixes over agents requiring the largest and smallest amounts of compensation. Quantitatively, the young make up a disproportionate share of this group and dominate the aggregate result, so that the least wealthy agents in this economy require the most compensation, measured as the vertical distance from the dot to the dashed 45-degree line. As wealth increases, we move toward the middle-aged individuals in the economy, who require a non-zero level of compensation, but less than those at the bottom of the wealth distribution. Finally, the wealthiest agents in the top bin, whose wealth is more driven by their income realizations than by demographics, also require a strictly positive level of compensation, but less than that of the least wealthy.

### 5.4.2 The Repriced Wealth Distribution

Having computed the financial wealth distribution required to keep consumption plans constant, we can compare it to the financial wealth distributions that actually results under low interest rates. We refer to this distribution as the *repriced* distribution. Unlike the compensated distribution, it depends on the duration of financial wealth, which we approximate using (11). We

Figure 10: Histograms, Repriced Financial Wealth Distribution

(a) Repriced vs. Original (b) Repriced vs. Compensated



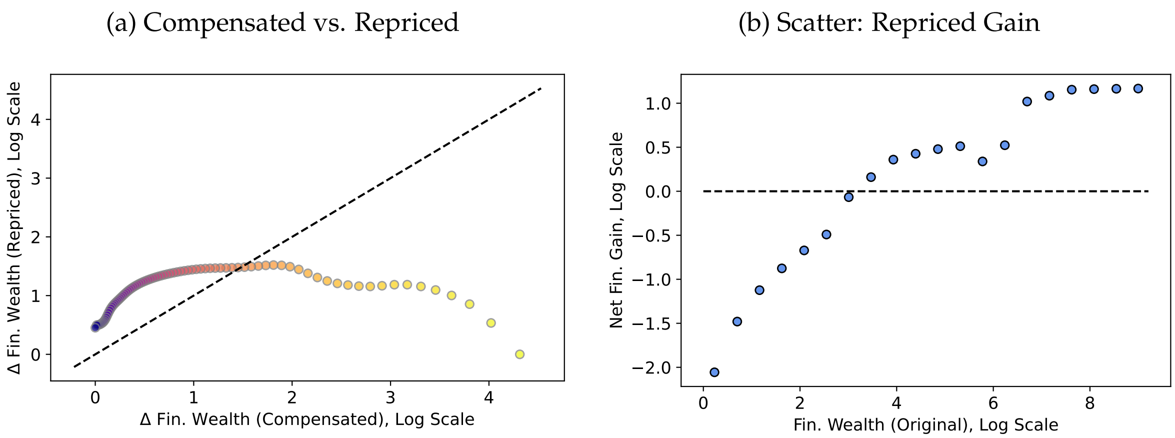

*Note:* This plot displays the distribution of financial wealth under the repriced distribution, compared to the original distribution and compensated distribution. All distributions are drawn from the stationary distribution of the economy. The x-axis displays a transformation $\log(1 + x)$ of the original data. Each distribution is top coded at the top 0.1% of the pre-shock wealth distribution.

implement assets with this duration as zero coupon bonds with maturity equal to their duration, and compute the actual change in financial asset values following our assumed change in interest rates.[14] The repriced distribution is displayed in Figure 10. Panel (a) shows that repricing shifts the financial wealth distribution to the right. Lower interest rates increase aggregate financial wealth by 185.4%. This increase in wealth is in fact larger than required under the compensated distribution (166.6%), showing that sufficient gains exist to compensate all agents. However, the distributions display strikingly different shapes, with the repriced distribution leaving many more agents at very low wealth levels.

Figure 11 compares changes in the repriced vs. compensated distributions by age in Panel (a) and by wealth in Panel (b). Panel (a) shows that repricing delivers virtually no additional financial wealth to the young, despite their large need for compensating transfers. In contrast, the old are, if anything, slightly over-hedged, receiving more wealth under repricing than needed to afford their former consumption plan. These are the points above the 45-degree line. Panel (b) displays the net gain from repricing, defined as the change in repriced wealth net of the change in compensated wealth. The figure reinforces this finding, showing that only the wealthiest agents gain on net from repricing, while the least wealthy experience a large net loss from the interest rate change, as repricing fails to appropriately compensate these households.

[14]While the duration is sufficient to compute the change in portfolio value as the shock size approaches zero, for large shocks this local approximation breaks down, making the exact timing of the cash flows relevant for the change in portfolio value. Using zero-coupon assets eliminates this complexity.

Figure 11: Scatterplots, Repriced Financial Wealth Distribution




*Note:* This plot displays the distribution of financial wealth under the repriced distribution, compared to the compensated distribution. Panel (a) displays the change in financial wealth relative to the original distribution for the compensated (x-axis) and repriced (y-axis) distributions. Both axes display a transformation $\log(1 + x)$ of the original data. Each dot represents one year of age, with the lightest (yellow) dots representing the youngest agents and the darkest (purple) dots representing the oldest agents. Panel (b) displays original financial wealth on the x-axis and the net financial gain (repriced minus compensated wealth) on the y-axis. The x-axis displays the transform $\log(1 + x)$, while the y-axis displays the difference in transformed values. Each dot represents one bin from the original wealth distribution. All distributions are drawn from the stationary distribution of the economy.

### 5.4.3 Financial and Total Wealth Inequality

Our model's combined implications for inequality following a fall in interest rates are summarized in Table 3. Each row of the table displays a different statistic measuring inequality. The first two columns display the statistics from the data. We take the 1980s to be the period preceding the interest rate decline, and the 2010s to be the period following the interest rate decline. These measures are computed from the SCF+, as detailed in Appendix A.3.9. The next four columns display the results from the model. They report results for the initial pre-shock, high-interest rate distribution, and results for the various distributions after the interest rate decline: the compensated, the repriced with homogenous duration, and the repriced with heterogeneous duration distributions.

The top panel of Table 3 shows that our model is able to produce a realistic level of financial wealth inequality for the pre-shock period, matching the 0.804 Gini and coming close to matching the top-10% wealth share. It overstates the top-1% financial wealth share.

Turning to the last column, corresponding to the repriced distribution under heterogeneous durations, we see that this model can explain more than 100% of the 0.069 increase in the financial wealth Gini over the intervening period. The model also produces an increase in the top-10% financial wealth share of 13.8% points, more than the observed increase in the data of 8.8% points. The model produces a large increase in the top-1% financial wealth share of 17.5% points. This overstates the 5.5% point increase in the SCF+ measure but is closer to the 10.5% points increase in

Table 3: Inequality, Model Comparison

| | Data | | Model | | |
|---|---|---|---|---|---|
| | Before | After | Initial | Repriced | Comp |
| Gini FW | 0.804 | 0.873 | 0.804 | 0.891 | 0.643 |
| Top-10% share FW | 68.6% | 77.4% | 71.5% | 85.2% | 57.7% |
| Top-1% share FW | 31.7% | 37.2% | 55.3% | 72.8% | 44.8% |
| Gini HW | – | – | 0.448 | 0.488 | 0.488 |
| Top-10% share HW | – | – | 37.7% | 35.4% | 35.4% |
| Top-1% share HW | – | – | 25.2% | 19.3% | 19.3% |
| Gini TW | – | – | 0.475 | 0.563 | 0.525 |
| Top-10% share TW | – | – | 44.2% | 48.8% | 41.9% |
| Top-1% share TW | – | – | 32.2% | 36.5% | 27.2% |

*Note:* Top 1%, Top 10% financial wealth shares as well as financial wealth Gini coefficients are estimated using SCF surveys. For the Before period we use the average values in 1983 and 1989. For the After period we use the average values in 2010, 2013 and 2016. More details on the computations are provided in Appendix A.3.9. For model results, the columns represent the pre-shock wealth distribution ("Initial"), the compensated distribution ("Comp"), and the repriced distribution ("Repriced").

the WID measure listed in the opening paragraph of the paper.[15] In short, lower expected returns on financial assets and heterogeneity in financial durations are quantitatively strong enough to explain (more than) all of the rise in financial wealth inequality in the data.

At the same time, the top panel of Table 3 shows that a compensated distribution, allowing agents to afford their prior consumption plans, would have corresponded to a major *decrease* in inequality, with top wealth shares and the Gini coefficient all falling substantially compared to their pre-shock levels. This suggests that the actual allocations in the data failed to fully compensate younger and less wealthy individuals, leaving them less well off than they were prior to the rate shock.

Turning to the center panel of Table 3, we observe that all three human wealth inequality indicators are much lower than their financial wealth inequality counterparts in the initial distribution. Lower interest rates modestly increase the human wealth Gini from 0.448 to 0.488. Younger households own most of the human wealth, and have a high duration of human wealth. The interest rate decline generates the largest increase for the highest-human wealth households, explaining the rise in human wealth inequality. However, the 0.040 increase in the human wealth Gini is less than half as large as the 0.087 increase in the financial wealth Gini implied by the

[15] As noted in Section 3, the rise in the top-1% financial wealth share in the United States was even larger, at 12% points, when measured between 1982 and 2015 according to the World Inequality Database. This is nearly identical to what the model produces. The SCF+ generates an increase in the top-1% financial wealth share of 7.2% between the 1983 and 2016 surveys. The WID generates a 8.9% point increase in the top-10% share between the 1980s and the 2010s, which is nearly identical to the 8.8% point increase in the top-10% share in the SCF+ over the same period. Hence, the disagreement between data sources is concentrated in the top-1% only.

model. The model predicts a substantial decline in the top-1% human wealth share, which spills over to a modest decline in the top-10% share. The top percentile of human wealth contains many households who currently are in the superstar income state. Since that state arrives at random times in the life-cycle, ends with 2.5% probability each period, the human wealth duration of the superstars is lower than that of typical young households.[16] Hence, a decline in interest rates lowers the top-1% human wealth share.

The bottom panel of Table 3 reports on total wealth inequality, where total wealth is the sum of financial and human wealth. Since human wealth is by far the largest component of total wealth for most households, the total wealth Gini (0.475) is close to the human wealth Gini (0.448) and much lower than the financial wealth Gini (0.804). When interest rates decline, the total wealth Gini rises by 0.088, a magnitude similar to the rise in the financial wealth Gini. However, the differences are not as stark at the top of the wealth distribution. The top-10% total wealth share increases by only 4.6% points, far less than the 13.8% point rise in the corresponding financial wealth share. The top-1% total wealth share rises by a similar 4.3% points, far below the 17.5% point increase in the top-1% financial wealth share.

The behavior of the top total wealth percentile in response to an interest rate decline can be thought of as the composition of the responses of two types of households in the top 1%. The first group consists of older households who hold most of their wealth in financial wealth. These households have typically saved for a long time, and likely entered the superstar state sometime in the past, but have since transitioned out of it. The second cluster are households who currently are in the superstar state. They are younger on average and have much lower ratios of financial to total wealth. The wealth dynamics of the former cluster are governed by the dynamics of the top-1% financial wealth sharh, which increases sharply, while the wealth dynamics of the second cluster are governed by the dynamics of the top-1% human wealth share, which falls sharply. The effect of the first cluster dominates, and on net, there is a modest increase in the total wealth share of the top-1%. The main take-away is that top total wealth inequality does not rise nearly as much as top financial wealth inequality when rates decline. Since consumption is ultimately what matters to the households in the model, and total wealth is the present value of consumption, the most relevant measure of wealth inequality has changed less than inequality in the more easily measured financial wealth data.

Finally, we note that the repriced distribution for total wealth features more inequality than the compensated distribution. Abstracting from incentive effects—which may well be very important—progressive (total) wealth taxation would help move the economy under the repriced distribution closer to that under the compensated distribution.

[16]The average human wealth duration of households in the superstar state is 12.0 compared to 17.5 for those not in the superstar state. Intuitively, the exit rate acts as an additional discount rate which lowers the duration. Moreover, when younger agents enter the superstar state, it pulls forward their income profile, again lowering its duration.

#### 5.4.4 Robustness to Private Business Durations.

Because we do not measure private business durations in the data, we approximate them using the duration of a portfolio of small public equities, which our empirical asset pricing model estimates to be 61.25. To explore robustness to this, we recompute our results using two alternative assumptions. First, Table A6 assumes that the duration of private business wealth is equal to the estimated duration of all public equities (28.78). Since the share of private business wealth in the household's portfolio is increasing in financial wealth, this lower financial duration dampens the positive link between financial wealth and financial duration, reducing the resulting rise in inequality from a fall in rates. Under this calibration, the financial wealth Gini, top-10% share, and top-1% share, by 0.026, 4.9%, and 6.3%, compared to 0.069, 8.8%, and 5.5% (10.5%), respectively, in the SCF (WID) data. Next, we consider an intermediate specification using a duration of 40 for private business wealth. These results, displayed in Table A7, fall between those in Table A6 and our benchmark model in Table 3, with repricing raising the financial wealth Gini, top-10% share, and top-1% share, by 0.049, 8.2%, and 10.4%, closely approximating the (WID) data. These results show that while our quantitative findings do vary with the estimated duration of private business wealth, the ultimate conclusion that our duration mechanism explains a large share of the rise in financial wealth inequality observed since the 1980s is robust.

## 6 Conclusion

A persistent decline in real interest rates, like the one experienced in much of the world between the 1980s and the 2010s, naturally leads to a rise in financial wealth inequality. Households whose wealth is predominantly made up of financial rather than human wealth, and particularly those with short-maturity assets, must increase savings to be able to afford the same consumption plan. We show how a standard incomplete markets Bewley model predicts that a decline in rates increases financial wealth inequality. We establish that households display large heterogeneity in the duration of their financial wealth portfolio. Once the observed positive correlation between financial wealth and financial wealth duration is taken into account, the model that feeds in the observed decline in interest rates explains all of the rise in financial wealth inequality. Human wealth inequality is much lower than financial wealth inequality, and increases by much less when rates decline. Since human wealth represents a majority of total wealth, the effect of lower rates on top total wealth shares is modest. While most households have been made worse off by the decline in interest rates, due to imperfectly hedged portfolios of human and financial wealth, the costs have fallen disproportionately on young and low-wealth households.

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# A Data Appendix

## A.1 Inequality Data

The top wealth shares presented in 2 are from the World Inequality Database. The data for the U.S. are available until 2019, for the U.K. until 2012, and for France until 2014.

Our primary source of data for the top wealth shares presented in 2 is the World Inequality Database maintained by WID team. As the WID time series for the UK and France have a limited window of observation, we augment the WID estimates with additional measures of top wealth shares obtained from survey data and the Credit Suisse Global Wealth Report (Shorrocks, Davies and Lluberas, 2020), where available and necessary. This serves to increase the size of the observation window and provides additional robustness to our results.

For the United States the WID time series provides complete coverage. In addition, we report survey estimates of top wealth shares from the Survey of Consumer Finances (SCF) and the SCF+, the database developed by Kuhn et al. (2020), from 1950 to 1983. We slightly modify the definition of total financial net-wealth by subtracting vehicles (for both the SCF and SCF+ data).[17]

For the United Kingdom the time series of top wealth shares from the WID ends in 2012. From 2012 onwards, we rely on top wealth share estimates from the Credit Suisse Global Wealth Report (Shorrocks et al., 2020). In addition, we construct estimates of the top 1% wealth share by augmenting survey microdata from the U.K. Wealth and Assets Survey (WAS) with observations from the Sunday Times Rich List to estimate the top 1% wealth share implied by fitting a Pareto tail to the wealth distribution, following Vermeulen (2018). We choose this method of estimating the top wealth share because in the periods of overlap between the WAS and WID the estimates of top wealth inequality in the raw survey data do not align well with the estimates from the WID which are based on the work of Alvaredo, Atkinson and Morelli (2018a) using administrative estate tax records. The most likely cause of this misalignment is undersampling of the rich in the WAS, which can be remediated by the Pareto-tail fitting exercise using rich list observations proposed by Vermeulen (2018).

For France the time series of top wealth shares from the WID ends in 2014. As for the U.K., we rely on top wealth share estimates from the Credit Suisse Global Wealth Report (Shorrocks et al., 2020) for the time period from 2014 onwards.

We construct the price of a real 30 year annuity by estimating the historical real yield curve for each country. Letting $y^r_{t,m}$ denote the real yield at maturity $m$ at time $t$ the cost of the annuity is calculated as

$$a_t = \sum_{m=1}^{30} \frac{1}{(1 + y^r_{t,m})^m}$$

[17] Note that the SCF+ database uses a definition of total financial net-wealth that is consistent with the SCF.

Due to varying availability of data we use three different approaches to estimate the real yield curve that lead to broadly consistent estimates. Firstly, for the UK post 1985 we use historical time series of real yields from  to fit a spline through these points and construct the real yield curve directly. Secondly, for the U.S. and France we use the time series of historical nominal yields and inflation provided by Global Financial Data, augmented with data from the Macrohistory database constructed by Jordà, Schularick and Taylor (2017), to annually estimate real yields at different maturities and then fit a spline through the estimated real yields to construct the real yield curve. We construct real yields for each year by estimating an AR(1) process for inflation on a sample of 50 years prior and then subtracting forecasted inflation from nominal yields at all available maturities (3-month treasury yields and 10-year government bond yields for all periods, as well as 30-year government bond yields for later periods). Thirdly, for the U.K. and U.S. we also use model estimates of the real yield curve from .

We construct the price of a real 30 year annuity by estimating the historical real yield curve for each country. Letting $y_t^r(h)$ denote the real yield at maturity $h$ at time $t$ the cost of the annuity is calculated as:

$$\sum_{h=1}^{30} \frac{1}{(1+y_t^r(h))^h}$$

Due to varying availability of data and for robustness, we use three different approaches to estimate the real yield curve that lead to broadly consistent estimates.

First, for the UK post 1985 we use historical time series of real yields of various maturities available from the Bank of England. We fit a spline through these points and construct the real yield curve directly.

Second, for the U.S. and France we use the time series of historical nominal yields and inflation provided by Global Financial Data, augmented with data from the Macrohistory database constructed by Jordà et al. (2017), to estimate real yields at different maturities and then fit a spline through the estimated real yields to construct the real yield curve. We construct real yields for each year by estimating an AR(1) process for inflation on a rolling sample of 50 years of past data, and then subtracting forecasted inflation from nominal yields at all available maturities. Those are 3-month treasury yields and 10-year government bond yields for all periods, as well as 30-year government bond yields for later years.

Third, for the U.K. and U.S. we also use model estimates of the real yield curve. The U.S. estimates are from the model in Section E. The U.K. estimates are from a similar model estimated for the U.K. in Jiang, Lustig, Van Nieuwerburgh and Xiaolan (2021).

## A.2 Income Data

### A.2.1 Data Source: PSID

The Panel Study of Income Dynamics (PSID) is a household panel survey that began in 1968. The PSID was originally designed to study the dynamics of income and poverty. Thus, the original 1968 PSID sample was drawn from two independent samples: an over-sample of 1,872 low income families from the Survey of Economic Opportunity (the "SEO sample") and a nationally representative sample of 2,930 families designed by the Survey Research Center at the University of Michigan (the "SRC sample"). A total of approximately 500 post-1968 immigrant families were added in 1997/1999 to update the PSID by adding a representative sample of recent immigrants to the United States: this sample is called the 1997 PSID Immigrant Refresher Sample. A total of 615 post-1997 immigrant families were added in 2017 to update the PSID by adding a representative sample of recent immigrants to the United States: this sample is called the 2017 PSID Immigrant Refresher Sample. We use data from the SRC sample starting in 1970 and ending with the 2017 wave.

### A.2.2 PSID Income variables

We construct the following income variables: *labinc2f* is labor income excluding transfers but including the labor part of business and farm income for both head and eventual spouse, *transf* which are total households transfer (including Social Security Income and other transfers). These two variables are then summed to *labinc3f* which is our measure of total household income for both head and eventual spouse. Here we detail the construction of these variables. [18]

**labinc2f**

- 1970 - 1993.
  - Total labor income of head, including wages and salaries, labor part of business income and farm income (1993:V23323).
  - Spouse's total labor income, including labor part of business income and farm income (1993:V23324)
- 1993 - 2017
  - Reference Person's total labor (including wages and other labor) excluding Farm and Unincorporated Business Income, (2017:ER71293)
  - Labor Part of Business Income from Unincorporated Businesses (2017:ER71274)

[18] Note that PSID variables tickers changed in each survey so here in order to indicate a specific ticker we define it as follow (YYYY:Ticker).

– Reference Person's and Spouse's/Partner's Income from Farming (2017:ER71272)
– Wife's Labor Income, Excluding Farm and Unincorporated Business Income (2017:ER71321)
– Wife's Labor Part of Business Income from Unincorporated Businesses (2017:ER71302)

Note that farm's income includes both labor and asset portions of income.

**transf**

- 1970-1993
  - Total Transfer Income of Head and Wife/"Wife" (1993:V22366)
  - Total Transfer Income of Others (1993:V22397)
- 1994-2003
  - Head's and Wife's Total Transfer Income, Except Social Security (2017:ER71391)
  - Other Total Transfer Income, Except Social Security (2017:ER71419)
  - Total Family Income from Social Security (1994:ER4152)
- 2004-2017
  - Head's and Wife's Total Transfer Income, Except Social Security (2017:ER71391)
  - Other Total Transfer Income, Except Social Security (2017:ER71419)
  - Reference Person's Income from Social Security (2017:ER71420)
  - Spouse's/Partner's Income from Social Security (2017:ER71422)
  - Others Income from Social Security (2017:ER71424)

**labinc3f** We then construct *labinc3f* by summing *labinc2f* and total family transfers *transf*.

Figure A1 plots the three variables described above averaged across all households. All variables are deflated to 2016 dollars using the CPI index.

We then split the sample in different cohorts: those age from 20 to 40 (Young), 40 to 60 (Middle) and 60 to 80 (Elderly). Figure A2 plots the same set of variables for these cohorts.

### A.2.3 Aggregation: NIPA vs PSID

We compare the PSID aggregates to the NIPA table aggregates from NIPA Table 2.1. We use NIPA *Wages and salaries* and compare to *labinc2f*. We then use the Census data on US number of households to compute *Wages and salaries* per households (note that our PSID measures are at the household level). In Figure A3, the left-hand side plot exhibits the nominal amount (thousands

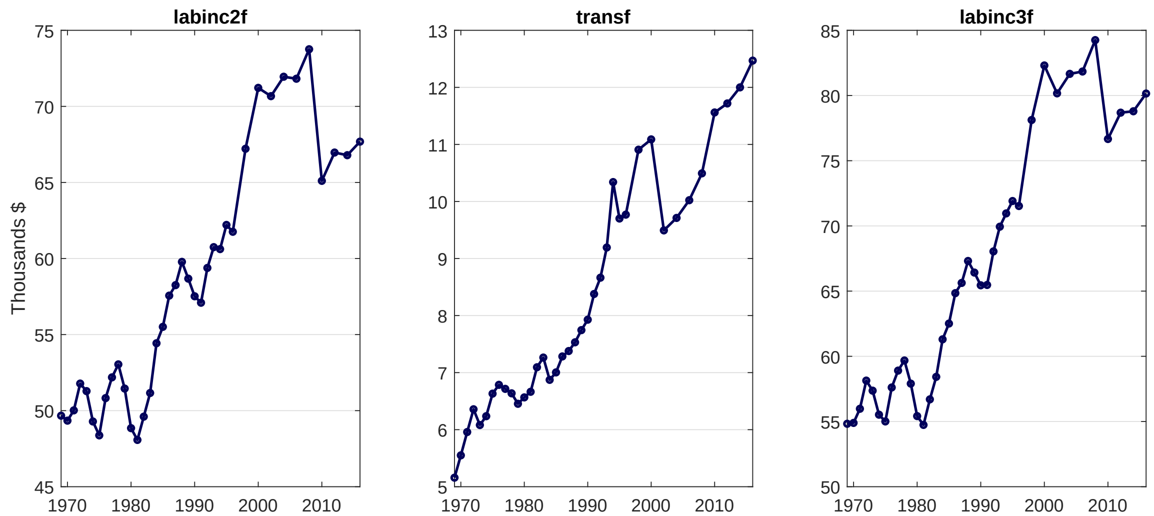



Figure A1: PSID average

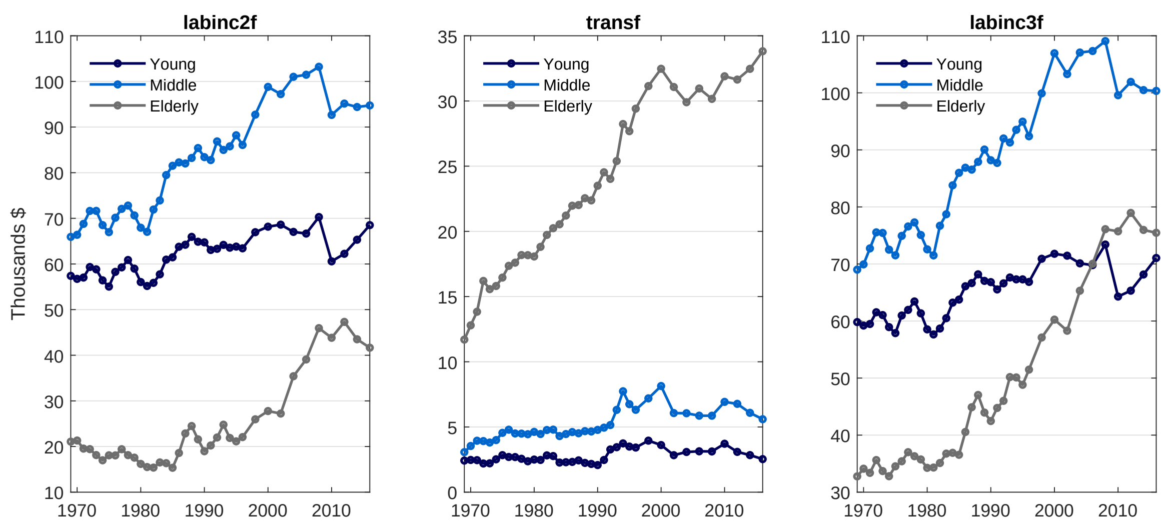



Figure A2: PSID by Cohorts

$ per households) for the NIPA aggregates, the PSID simple average and PSID weighted average using the longitudinal weights. The right-hand side plot exhibits the real variables (deflated using the CPI index). The variables are indexed such that the index are equal to 100 in 2000. While the two definitions of income (NIPA *Wages and salaries* vis-a-vis our measure *labinc2f*) are not strictly identical, we find that they evolve quite closely over our sample.

### A.2.4 Group Definitions

Our groups are defined based on gender, race and education. Here we detail the variables used from the PSID. **Sex.** We use the sex of the head of the household (2017:ER66018).

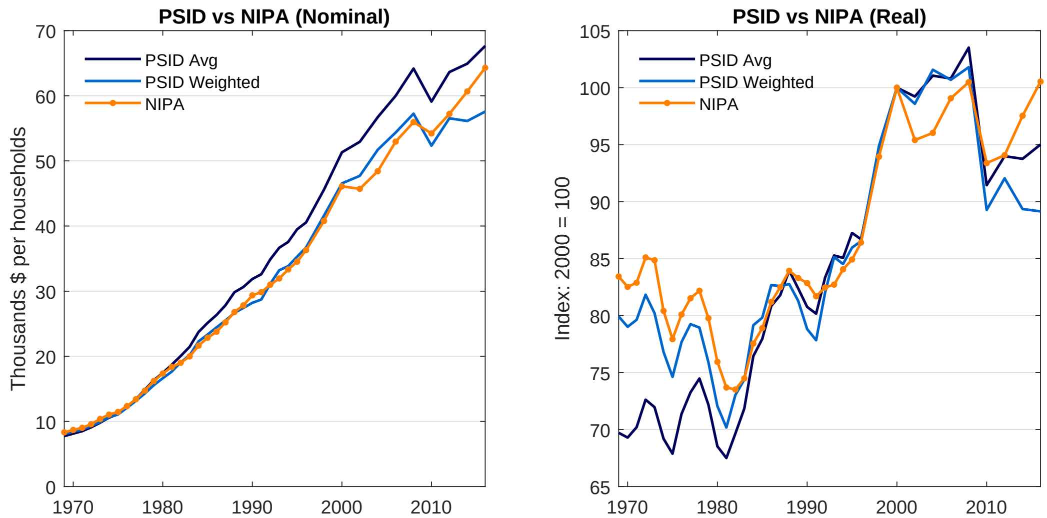



Figure A3: PSID vs NIPA Income

**Race.** We use the variable race (2017:ER70882). We only have an indicator function if the head is white and zero for all other races.

**Education.** We use a measure of years completed of education (2017:ER34548). The question in the survey is: "What is the highest grade or year of school that (you/he/she) has completed?". We make the following assumption: Education is based on highest level of educational achievement with perfect foresight. So, income of an 18 year old who goes to college later should be part of the college income profile. We define an individual to be college educated if they have 16 years of schooling or more. This definition is consistent with Heathcote, Perri and Violante (2010). Before 1975, we use the variable (1975:V4198).

Based on the above variables we measure the labor income for different groups. Figure A4 plots the variable *labinc3f* averaged across all households in each group.

### A.2.5 Estimating Income Process

We estimate the income profile for different groups following Meghir and Pistaferri (2004). The income process for household $i$ in group $g$ of age $a$ at time $t$ is given by (6)-(8). The estimation proceeds in two steps. In the first step, we estimate the year-fixed effects and the coefficients on the deterministic income profile $\chi$ from (6). In the second stage, we estimate the risk parameters using the residuals $z_{it}$ from the first step. This estimation is done by GMM as detailed below.

Figure A5 plots the deterministic income profile of the different groups, evaluated at the 2016 year-fixed effects. The graph plots the expected income profile for the average person in each group who is 18 years old in 2016, expressed in thousands of 2016 dollars.

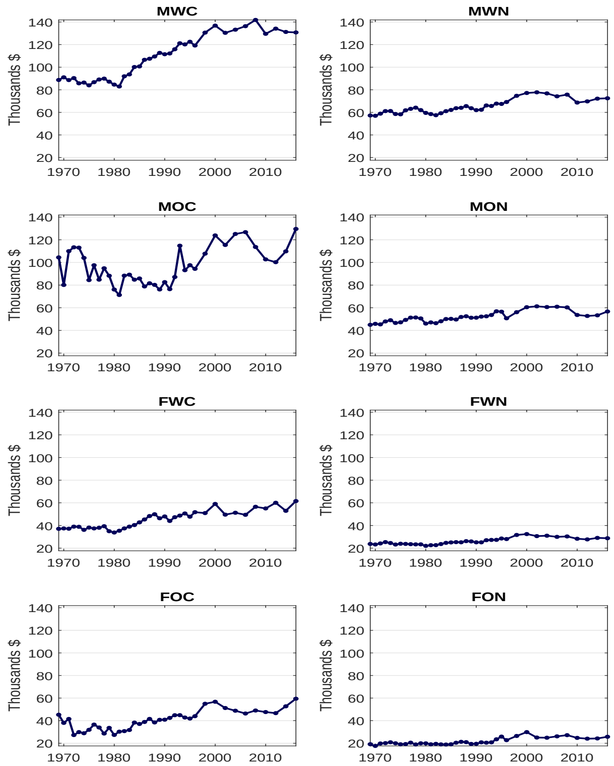



Figure A4: Labor Income by Group

*Note:* Average total labor income (*labinc2f*) of each group. Variables are in 2016 thousands dollars. M stands for male, F for female; W stands for white, O for all other races; C stands for college, N for non-college.

Figure A6 plots the income at different age, expressed relative to the level at age 18.

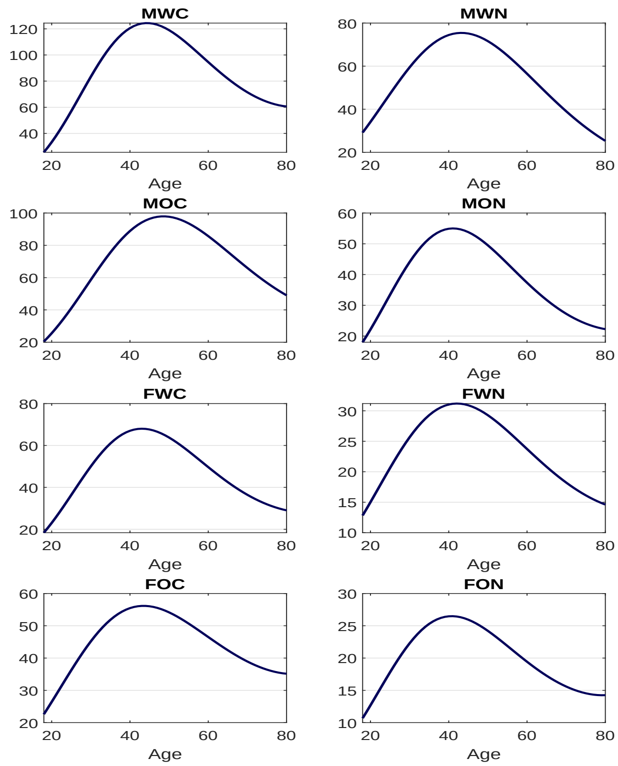



Figure A5: Income Profile by Group

*Note:* This figure displays the life cycle income profile of households within different groups. M stands for male, F for female; W stands for white, O for all other races; C stands for college, N for non-college. We use the 2016 year fixed effects. The figure is in thousands of 2016 dollars. The model is estimated according to Equation (6)-(8) on PSID data from 1970 to 2017.

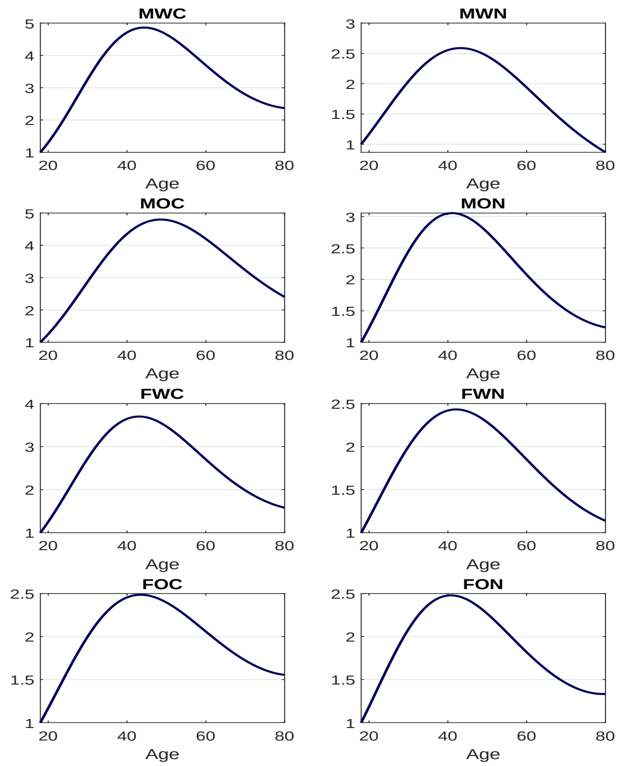



Figure A6: Income Profile by Group

Using Equation (6)-(8), and define $j$ as equal to the age of the households minus the minimum age (18), we find that:

$$E[z^i_j, z^i_{j+h}] = \sigma^2_\alpha + E[{\varepsilon^i_j}^2] + \sigma^2_\nu \qquad \text{if } h = 0 \tag{12}$$

$$E[z^i_j, z^i_{j+h}] = \sigma^2_\alpha + \rho^h E[{\varepsilon^i_j}^2] \qquad \text{if } h > 0 \tag{13}$$

$$E[{\varepsilon^i_j}^2] = \rho^{2j}\sigma^2_{\varepsilon_0} + \sum_{k=1}^{j} \rho^{2(j-k)}\sigma^2_\eta \tag{14}$$

We then use a GMM estimation to estimate $\theta = (\rho, \sigma_\nu, \sigma_\eta, \sigma_\alpha, \sigma_{\varepsilon_0})$. We use a Minimum Distance Estimator, where the weighting matrix is the identity matrix.

**Sample Selection.** We use PSID data from 1970 to 2017. We only include households whose head is 18 to 80 years old. We only include households which were in the survey for three or more periods. We exclude households with zero or negative income. In each year, we trim the top 2.5% of households by their income.

We pool all households together, after removing group-specific year-fixed effects and cubic age-profiles, and estimate the idiosyncratic risk parameters $\theta$. The point estimates are displayed in Panel A in Table A1. These are the parameters used in the main text.

Table A1: Idiosyncratic Risk Parameter Estimates

| | $\rho$ | $\sigma^2_\eta$ | $\sigma^2_\nu$ | $\sigma^2_\alpha$ | $\sigma^2_{\varepsilon_0}$ | N. Obs. |
|---|---|---|---|---|---|---|
| All | 0.950 | 0.023 | 0.195 | 0.066 | 0.194 | 10638 |

*Note:* $\rho$, $\sigma^2_\eta$, $\sigma^2_\nu$, $\sigma^2_\alpha$, $\sigma^2_{\varepsilon_0}$ are estimated using Equation Equation (6)-(8). Data runs from 1970 to 2017.

## A.3 Portfolio Shares

The Survey of Consumer Finances (SCF) is a statistical survey of the balance sheet, pension, income and other demographic characteristics of families in the United States. We use data from the Summary Extract Data – that is, the extract data set of summary variables used in the Federal Reserve Bulletin. It includes data from the triennial surveys beginning in 1989.[19]

### A.3.1 Variables

We collect the following variable:

**Total Financial Assets**. This includes:

1. All types of transaction account (liquid assets)
2. Certificates of deposit
3. Directly held pooled investment funds (exc. money mkt funds)
4. Savings bonds
5. Directly held stocks
6. Directly held bonds (excl. bond funds savings bonds)
7. Cash value of whole life insurance
8. Other managed assets
9. Quasi-liquid retirement accounts
10. Other misc. financial assets

**Cash & Deposits** This includes all types of transaction account (liquid assets) and certificated of deposits. The list of variables are:

[19]The SCF Flow Chart provides information on how variables are constructed https://www.federalreserve.gov/econres/files/networth%20flowchart.pdf. The code on how different variables in the Summary Extract Data are constructed can be found here: https://www.federalreserve.gov/econres/files/bulletin.macro.txt

1. Money market accounts
2. Checking accounts (excl. money market)
3. Savings accounts
4. Call accounts
5. Prepaid cards
6. Certificates of deposit

**Equities (direct & indirect).** Total value of financial assets held by household that are invested in stock. That includes:

1. directly-held stock
2. Stock mutual funds: full value if described as stock mutual fund, 1/2 value of combination mutual funds
3. RAs/Keoghs invested in stock: full value if mostly invested in stock, 1/2 value if split between stocks/bonds or stocks/money market, 1/3 value if split between stocks/bonds/money market
4. Other managed assets with equity interest (annuities, trusts, MIAs): full value if mostly invested in stock, 1/2 value if split between stocks/MFs & bonds/CDs, or ”mixed/diversified”, 1/3 value if ”other”
5. Thrift-type retirement accounts invested in stock: full value if mostly invested in stock,1/2 value if split between stocks and interest earning assets

The allocation rules for mixed investments in 3), 4), and 5) do not apply to 2004 since new questions in 2004 directly ask the share of stock in those assets.

**Real Estate.** The real estate variable includes:

1. Primary residence
2. Residential property excluding primary residence (e.g., vacation homes)

**Private Business Wealth.** Businesses (with either an active or nonactive interest). Businesses include both actively and nonacitvely-managed business(es). Value of active business(es) calculated as net equity if business(es) were sold today, plus loans from the household to the business(es), minus loans from the business(es) to the household not previously reported, plus value of personal assets used as collateral for business(es) loans that were reported earlier. Value of nonactive business(es) is calculated as the market value of the business(es).

**Fixed Income.** Fixed income is calculated as the residual of Total financial assets minus Cash & Deposits and Equity (direct & indirect).

**Mortgage Debt.** This includes:

1. Debt secured by prim. resid. (mortgages, home equity loans, HELOCs)
2. Debt secured by other residential property

**Student Debt.** Total value of education loans held by household. This includes education loans that are currently in deferment and loans in scheduled repayment period. We exclude installment loans: these are mostly student loans (which we accounts for separately), vehicle loans (which we do not account as debt as vehicles are part of consumption).

**Consumer and Other Debt.** This includes:

1. Other lines of credit (not secured by resid. real estate)
2. Credit card balances after last payment
3. Other installments other than vehicles debt and student debt

**Net Wealth.** We calculate net wealth for each household as the difference between total assets (Cash & Deposits, Equities (direct & indirect), Real Estate, Private Business Wealth and Fixed Income) and total liabilities (Mortgage Debt, Student Debt and Consumer and Other Debt).

#### A.3.2 SCF 1983

The 1983 SCF is not included in the original SCF bulletin database. We try to match the SCF bulletin data as close as possible. We do not have the equity variable so we use the methodology described in Section A.3.3 to measure the indirect holdings of equity and fixed-income held through mutual funds and pension funds. The list of variables derived from the 1983 are:

**Income**:

+ B3205 Income in wages and salary.
+ B3206 Income from a professional practice, business, or farm
+ B3212 workers or unemployment compensation income
+ B3213 Child support, alimony, inheritance, gifts, financial support.
+ B3214 Adc, afdc, food stamps, ssi, welfare, other public assistance
+ B3215 Retirement, annuity, pension, disability, survivor benefits.
+ B3216 Other Income. This incldues:

- settlements from lawsuits, divorce, insurance (8 cases)
- gambling winnings (5 cases)
- educational scholarships or grants, GI bill, fellowship (9 cases)
- other source (14 cases)

**Real estate**:

+ B3708 Current value of home

+ B3801 Aggregate gross value of other properties

**Cash and Deposits**:

+ B3401 Total dollar in unrestricted checking accounts

+ B3418 Total dollar amount in all money market and call accounts

+ B3434 Total dollar amount in all savings or share accounts

+ B3453 Total dollar amount of certificates of deposits

**Stocks**:

+ B3465 Total dollar amount of stock held in investment clubs.

+ B3466 Total dollar amount of publicly traded stock in own company

+ B3467 Total dollar amount of other publicly traded stock.

**Equity**

+ Stocks

+ Indirect equity holdings through mutual funds

+ Indirect equity holdings through pension

**Mutual funds**

+ B3462 Total dollar amount of stocks and mutual funds.

+ B3470 Total dollar amount in trust accounts.

- Stocks

**Pension**

+ B3446 Total dollar amount in ira or keogh accounts [IRAKH]

+ B3306 Total thrift-type pension account assets [THRIFT]

+ B4929 (H) + B5029 (S) Dollar Amount in Defined Contribution Account [FUTPEN]

+ CURRPEN is set to 0. Note that in the SCF from 1989 to 2001, CURRPEN is always set to 0

**Fixed Income**:

+ B3457 Total face amount of u.s. government savings bonds.

+ B3458 Total face amount of bonds

+ B3477 Total dollar amount of loans owed to household and gas leases.

+ B3475 Dollar cash value of whole life insurance.

+ B3601 Aggregate gross value of land contracts and notes.

+ Indirect fixed income holdings through mutual funds

+ Indirect fixed income holdings through pension

**Private Business Wealth**

+ B3501 Net value of business with no management interest

+ B3502 Total net value of business with a management interest

**Mortgage**

+ B3318 Total real estate debt

**Other Debt**

+ B3319 Total consumer Debt

We set student debt to 0 as it is not available.

### A.3.3 Look-through

We follow the methodology of Leombroni, Piazzesi, Schneider and Rogers (2020).

In order to look-through the mutual funds and pension holdings we use data from the US financial account. We compute the mutual funds equity holdings using Corporate Equities (LM653064100)[20]. We compute mutual funds bond holdings using Debt securities (LM654022005). We compute the shares dividing equity holdings and bond holdings by total mutual funds assets (LM654090000).

We compute the DC pension total equity holdings as the sum of Corporate equities (LM573064133) and indirect holdings through mutual funds. We compute the indirect holdings as mutual fund shares (LM573064255) $\times$ mutual funds equity shares. We compute the DC pension total bond holdings as the sum of Debt securities (LM574022035) and indirect holdings through mutual funds. We compute the indirect holdings as mutual fund shares (LM573064255) $\times$ mutual funds bonds shares. We then divide the total equity holdings by total DC pension assets (FL574090055) to estimate the shares.

For each household we then multiply the nominal holdings of mutual funds and pension by the calculated indirect portfolio shares. We then apportion the indirect equity and fixed income holdings to the direct holdings.

### A.3.4 Groups

From the SCF data, we extract the sex of the reference person, the education attainment and the race. Using the education attainment we divide the sample into households with college degree and households without college degree. We only include households older than 25 years old. Table A2 provides information on the different groups.

Table A2: Summary Statistics by Group

| Groups | Population Share (%) | Median Age | Median NW | Average NW | Std NW | Negative NW | Zero NW |
|---|---|---|---|---|---|---|---|
| MWC | 16.82 | 42 | 286.80 | 824.77 | 3427.81 | 5.46 | 0.00 |
| MWN | 41.20 | 46 | 112.41 | 308.22 | 1180.69 | 7.54 | 2.16 |
| MOC | 2.61 | 41 | 84.03 | 452.45 | 1770.41 | 10.84 | 1.02 |
| MON | 12.26 | 42 | 28.01 | 90.83 | 279.27 | 13.37 | 10.46 |
| FWC | 3.62 | 51 | 167.92 | 320.96 | 644.89 | 8.83 | 0.00 |
| FWN | 14.12 | 63 | 53.59 | 137.82 | 341.55 | 9.65 | 3.99 |
| FOC | 0.79 | 40 | 20.54 | 85.51 | 158.10 | 38.73 | 5.89 |
| FON | 8.57 | 49 | 0.37 | 35.32 | 112.57 | 16.88 | 28.88 |
| All | 100.00 | 46 | 85.24 | 323.47 | 1654.48 | 9.38 | 5.28 |

*Note:* Groups Information. SCF 1989. Column 1 is in percentage. Column 3, 4 and 5 in 2016 thousands dollars, Column 6 and 7 are in percentage. M stands for male, F for female; W stands for white, O for all other races; C stands for college, N for non-college.

[20] The code refers to the Financial Account codes

### A.3.5 Holdings

We compute the holdings for each of the assets and liabilities for each household. Table A3 shows summary statistics for the distribution of asset holdings. Note that Private Business Wealth is a measure net of loans from the business to the households and hence may also be a negative number.

Table A3: Portfolio Holdings

| | Mean | std | Min | 25% | 50% | 75% | 90% | 95% | Max |
|---|---|---|---|---|---|---|---|---|---|
| Cash and Deposits | 37.51 | 241.25 | 0.00 | 0.75 | 4.11 | 22.09 | 82.90 | 158.71 | 67789 |
| Equities | 37.55 | 327.24 | 0.00 | 0.00 | 0.00 | 5.00 | 52.28 | 142.84 | 103864 |
| Real Estate | 157.70 | 292.76 | 0.00 | 0.00 | 84.02 | 199.79 | 382.78 | 560.16 | 65347 |
| Private Business Wealth | 79.45 | 1226.41 | -1335.06 | 0.00 | 0.00 | 0.00 | 9.34 | 186.72 | 258180 |
| Fixed Income | 57.43 | 459.32 | -0.00 | 0.00 | 3.73 | 27.63 | 103.07 | 223.51 | 171201 |
| Mortgage Debt | 39.22 | 96.43 | 0.00 | 0.00 | 0.00 | 50.41 | 123.24 | 192.32 | 30006.12 |
| Student Debt | 0.87 | 5.57 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 3.73 | 166.18 |
| Other Debt | 6.08 | 49.85 | -0.00 | 0.00 | 0.26 | 3.73 | 11.02 | 20.54 | 4201.23 |
| Net Wealth | 323.47 | 1654.48 | -3368.92 | 7.66 | 85.24 | 269.44 | 644.58 | 1208.09 | 290288.20 |

*Note:* Data are based on SCF 1989 and are reported in 2016 thousands dollars. Note that Private Business Wealth is a measure net of loans from the business to the households. For this reason some observations are negative.

### A.3.6 Financial Duration

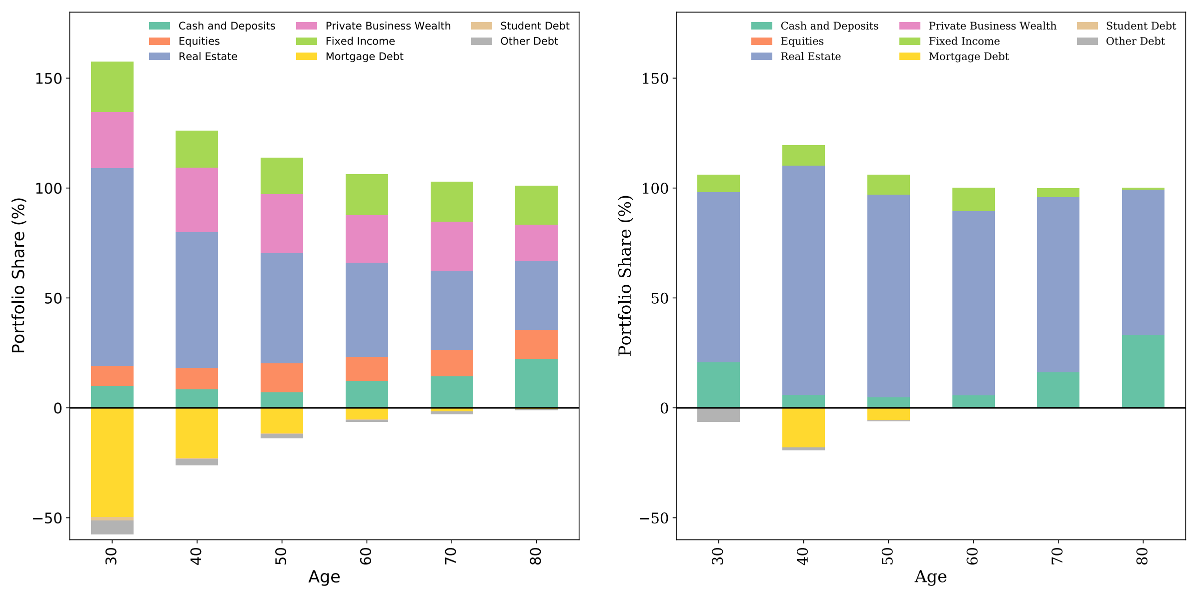
For the purpose of our duration calculation, we exclude households with zero net-wealth but positive assets. We then compute household's portfolio share in each asset by dividing the dollar holdings in the asset by the households net wealth. Using the portfolio shares, we compute the durations of the household's financial portfolio by multiplying the asset duration of an asset (assets durations are reported in the first column of Table 2) by the portfolio share of that asset, and summing over all assets in the portfolio. We trim household financial durations by excluding the top and bottom 2.5% of observations. In the last row of Table 2, we report the average duration, by averaging over all households (using the SCF sampling weights). Similarly, we compute average durations by group by averaging durations among the households in a group (using the SCF sampling weights).

Table 2 also reports value-weighted portfolio shares for each asset. They are obtained by summing dollar holdings of an asset among all households (households in a group) by the total dollar holdings of all assets among all households (households in a group). Aggregate durations are then obtained by multiplying the value-weighted portfolio weights for each asset by the duration of that asset, and summing over assets. They are reported in the last but one row of Table 2.

Figure A7 shows portfolio shares by age in the 1989 SCF. We bundle households into different cohort groups: 25-35, 35-45, 45-55, 55-65, 65-75, 75-85. Figure A7a uses the value-weighted portfolio shares. Figure A7b plots the median portoflio share in each asset category, and then rescales the resulting shares so that they sum to 100%.

Figure A7: Portfolio Shares by Cohorts

(a) Wealth-Weighted Portfolio Shares (b) Median Portfolio Shares




*Note:* Portfolio shares by age in the 1989 SCF. We bundle households into different cohort groups: 25-35, 35-45, 45-55, 55-65, 65-75, 75-85. The top panel uses the value-weighted portfolio shares. The bottom panel uses the median portfolio share in each asset category, and then rescales the resulting shares so that they sum to 100%. We exclude households with zero net wealth as the portfolio shares are undefined.

Figure A8 provides further information on the distribution of durations across households. Figure A8a plots the average duration by cohort. We bundle households into cohort groups and estimate the average duration. Figure A8b bundles households in wealth-weighted percentile and estimate the average duration of households in each bin.[21] Figure A8c and A8d rank households according to their wealth and income percentile, respectively, and estimate the average duration of each group.

Figure A8e and Figure A8f also rank households according to their wealth and income. Then plot the average duration of each group against the average net-wealth or income.

We also evaluate more formally the correlation between financial duration and some covariates of interest. First, we regress household financial duration on household position in the Lorenz Curve. To calculate households' positions, we rank households by their net-wealth, then calculate the cumulative sum of net-wealth and divide by the aggregate net-wealth. We then add a dummy for each group, a quadratic function of age and the log of household income. We exclude

[21] Households are ranked according to their net-wealth and allocated to different bins. Each bin is designed such that the share of total wealth held by the households in each bin is the same across different bins.

households with zero net-wealth and positive assets (as the duration is indeterminate) and trim the bottom/top 2.5% of households ranked by their duration. The regression estimate take into account survey weights. Table A4 reports the estimation results.

#### A.3.7 Financial Duration Over Time

Figure A9 uses information from each SCF survey from 1983 till 2019. Figure A9a compute the aggregate (wealth-weighted) while A9b computes the average average (equally-weighted) duration over time. We use two different specifications for the duration of assets. Full sample computes the duration of the asset using the information over the whole sample; the duration of each asset is kept constant over time. The time varying specification computes time varying duration measures for equity, private business wealth and real estate. We then use these time varying measures to compute the portfolio duration.

#### A.3.8 Financial Duration - Robustness

To make sure our results are robust we replicate Figure 3 using different samples and different duration estimates for business wealth. Figure A10a uses a measure of business duration equal to 40, Figure A10b uses as duration of business wealth, the same duration used for equity (28.7), A10c plots the median duration instead of the average duration, Figure A10d uses a different trimming: trim the bottom/top 1%.

Table A5 reproduces the last two rows of Table 2 but using a duration of business wealth equal to 40.

We also provide a robustness check for Figure **??**. In Figure A11 we plot both average duration (Figure A11a) as well as the median duration (Figure A11b).

#### A.3.9 Wealth Shares, Income Shares, and Gini Coefficients

We estimate the net-wealth shares held by the top-10% and top-1%. We also estimate gini coefficients. We use the SCF+ database developed by Kuhn et al. (2020) in order to have a longer time series of wealth and income. We slightly modify their definition of total financial net-wealth by subtracting vehicles and other non-financial wealth.

Figure A12 plots the top shares and the gini coefficient for financial (net) wealth. Table A8 computes averages for these moments, computed over all surveys in the 1980s and all surveys in the 2010s, for both financial wealth and income. The income moments in this table are from the SCF. We define household income as SCF total household income minus capital income.

Table A4: Determinants of Household-level Financial Duration

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|---|
| Age | 0.034 | -0.36*** | -0.33*** | | | | -0.18*** | -0.20*** |
| | (0.51) | (-5.80) | (-5.12) | | | | (-2.76) | (-3.11) |
| Age Squard | -0.0023*** | 0.00100* | 0.00097* | | | | -0.00044 | -0.00013 |
| | (-3.96) | (1.84) | (1.76) | | | | (-0.78) | (-0.23) |
| Net-Wealth | 0.0000010*** | | | | | | | |
| | (8.18) | | | | | | | |
| Income | 0.0000015*** | | | | | | | |
| | (3.06) | | | | | | | |
| Log-Net-Wealth | | 2.43*** | | | | | | |
| | | (27.40) | | | | | | |
| Log-Income | | 0.065 | | | | | | 0.60*** |
| | | (0.42) | | | | | | (3.33) |
| Net-Wealth Pctl | | | 0.11*** | | | | | |
| | | | (16.04) | | | | | |
| Income Pctl | | | 0.038*** | | | | | |
| | | | (5.88) | | | | | |
| Lorenz | | | | 0.17*** | | 0.15*** | 0.20*** | 0.19*** |
| | | | | (18.97) | | (14.48) | (20.40) | (16.74) |
| MWC | | | | | 7.87*** | 4.63*** | 1.67** | 0.99 |
| | | | | | (12.09) | (6.65) | (2.29) | (1.34) |
| MWN | | | | | 5.39*** | 3.89*** | 2.46*** | 2.03*** |
| | | | | | (9.04) | (6.52) | (3.92) | (3.18) |
| MOC | | | | | 6.74*** | 4.97*** | 2.19* | 1.73 |
| | | | | | (5.07) | (3.81) | (1.68) | (1.34) |
| MON | | | | | 4.56*** | 4.16*** | 2.50*** | 2.15*** |
| | | | | | (6.11) | (5.63) | (3.29) | (2.81) |
| FWC | | | | | 0.19 | -1.52 | -2.44** | -2.86*** |
| | | | | | (0.19) | (-1.62) | (-2.50) | (-2.93) |
| FWN | | | | | -0.76 | -1.47** | -0.040 | -0.25 |
| | | | | | (-1.19) | (-2.33) | (-0.06) | (-0.39) |
| FOC | | | | | 3.06*** | 2.13* | 1.15 | 0.69 |
| | | | | | (2.61) | (1.84) | (1.11) | (0.68) |
| Constant | 21.8*** | 4.56** | 23.8*** | 15.1*** | 12.9*** | 12.6*** | 23.6*** | 17.9*** |
| | (11.85) | (2.10) | (13.46) | (77.03) | (23.23) | (22.72) | (12.34) | (6.76) |
| Observations | 13145 | 13145 | 13145 | 13145 | 13145 | 13145 | 13145 | 13145 |
| $R^2$ | 0.098 | 0.201 | 0.159 | 0.053 | 0.051 | 0.084 | 0.157 | 0.159 |
| Adjusted $R^2$ | | | | | | | | |

*Note:* Data based on SCF 1989. T-stats in parentheses ($^{*}$ $p < 0.10$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$)

Table A5: Robust Aggregate and Average Duration

| | All | MWC | MWN | MOC | MON | FWC | FWN | FOC | FON |
|---|---|---|---|---|---|---|---|---|---|
| Aggregate Duration | 20.50 | 21.65 | 20.49 | 24.95 | 19.39 | 15.28 | 14.15 | 18.51 | 16.64 |
| Average Duration | 14.33 | 18.68 | 15.89 | 16.26 | 13.36 | 11.86 | 10.07 | 11.98 | 7.30 |

*Note:* Data are based on SCF 1989. This table reproduces the last two rows of Table 2 but using a duration of business wealth equal to 40. M stands for male, F for female; W stands for white, O for all other races; C stands for college, N for non-college.

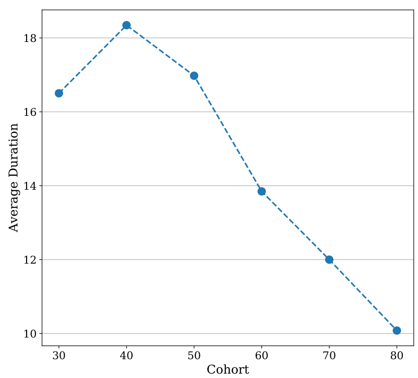
Table A6: Inequality, Model Comparison, Private Business Duration = Public Equity Duration

| | Data | | Model | | |
|---|---|---|---|---|---|
| | Before | After | Initial | Repriced | Comp |
| Gini FW | 0.804 | 0.873 | 0.804 | 0.830 | 0.643 |
| Top-10% share FW | 68.6% | 77.4% | 71.5% | 76.4% | 57.7% |
| Top-1% share FW | 31.7% | 37.2% | 55.3% | 61.6% | 44.8% |
| Gini HW | – | – | 0.448 | 0.488 | 0.488 |
| Top-10% share HW | – | – | 37.7% | 35.4% | 35.4% |
| Top-1% share HW | – | – | 25.2% | 19.3% | 19.3% |
| Gini TW | – | – | 0.475 | 0.505 | 0.525 |
| Top-10% share TW | – | – | 44.2% | 41.9% | 41.9% |
| Top-1% share TW | – | – | 32.2% | 28.3% | 27.2% |

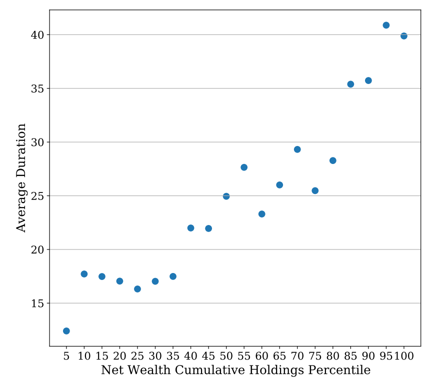
*Note:* Top 1%, Top 10% financial wealth shares as well as financial wealth Gini coefficients are estimated using SCF surveys. For the Before period we use the average values in 1983 and 1989. For the After period we use the average values in 2010, 2013 and 2016. More details on the computations are provided in Appendix A.3.9. For model results, the columns represent the pre-shock wealth distribution ("Initial"), the compensated distribution ("Comp"), and the repriced distribution ("Repriced").

Figure A8: Distribution of Durations

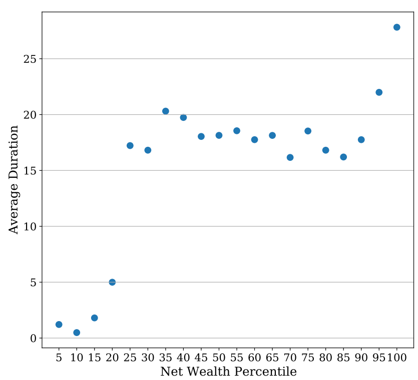
(a) Average Duration by Cohort



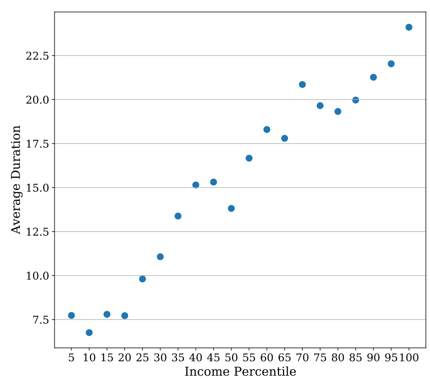

(b) Average Duration by Wealth-Weighted Percentiles



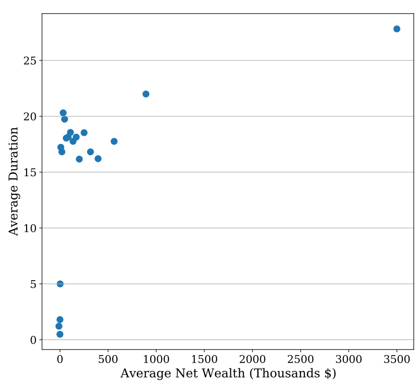

(c) Average Duration by Net-Wealth Percentile



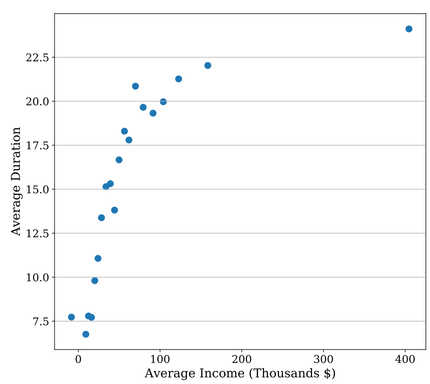

(d) Average Duration by Income Percentile




(e) Duration vs Net-Wealth

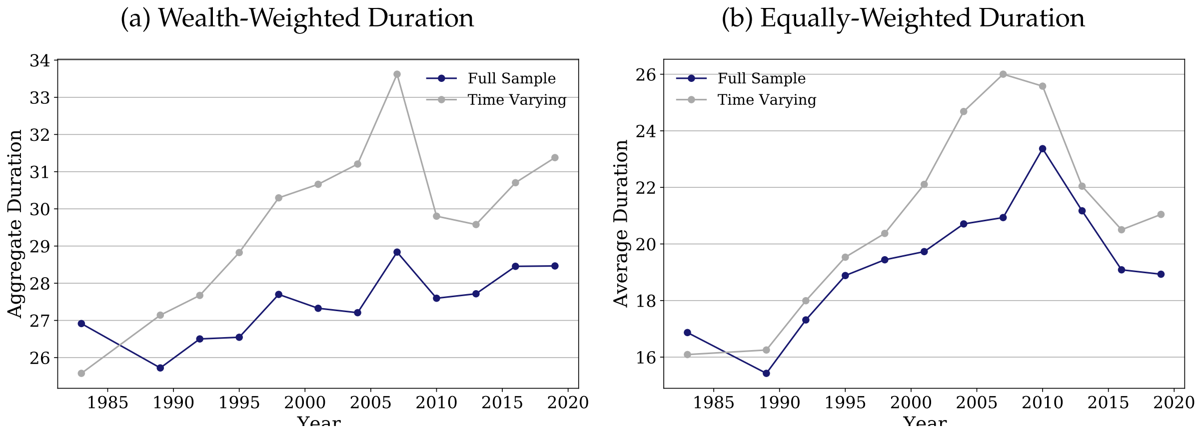



(f) Duration vs Income




*Note:* Data are based on SCF 1989. We exclude households with zero net wealth and positive assets (as their portfolio shares would be indeterminate) and we trim the data based on households' overall duration: we exclude the top/bottom 2.5%. Panel (a) plots the average duration by cohort. We bundle households into cohort groups and estimate the average duration. Panel (b) bundles households in wealth-weighted percentile and estimate the average duration of households in each bin. Panel (c) and Panel (d) rank households according to their wealth and income percentile, respectively, and estimate the average duration of each group. Then plot the average duration of each group against the average net-wealth (Panel (e)) or income (Panel (f)).

Figure A9: Financial Duration Over Time




*Note:* A9 uses information from each SCF survey from 1983 till 2019. Figure A9a compute the aggregate (wealth-weighted) while A9b computes the average average (equally-weighted) duration over time. We use two different specifications for the duration of assets. Full sample computes the duration of the asset using the information over the whole sample; the duration of each asset is kept constant over time. The time varying specification computes time varying duration measures for equity, private business wealth and housing.

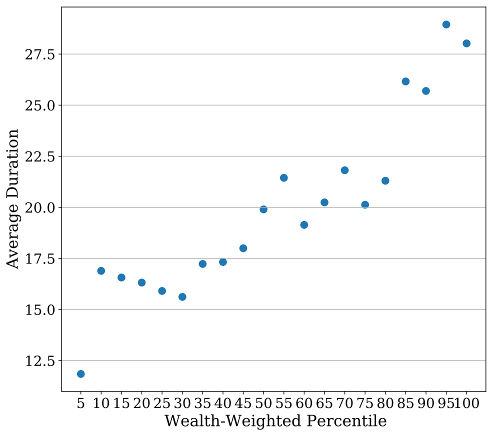
Table A7: Inequality, Model Comparison, Private Business Duration = 40

| | Data | | Model | | |
|---|---|---|---|---|---|
| | Before | After | Initial | Repriced | Comp |
| Gini FW | 0.804 | 0.873 | 0.804 | 0.853 | 0.643 |
| Top-10% share FW | 68.6% | 77.4% | 71.5% | 79.7% | 57.7% |
| Top-1% share FW | 31.7% | 37.2% | 55.3% | 65.7% | 44.8% |
| Gini HW | – | – | 0.448 | 0.488 | 0.488 |
| Top-10% share HW | – | – | 37.7% | 35.4% | 35.4% |
| Top-1% share HW | – | – | 25.2% | 19.3% | 19.3% |
| Gini TW | – | – | 0.475 | 0.522 | 0.525 |
| Top-10% share TW | – | – | 44.2% | 43.9% | 41.9% |
| Top-1% share TW | – | – | 32.2% | 30.7% | 27.2% |

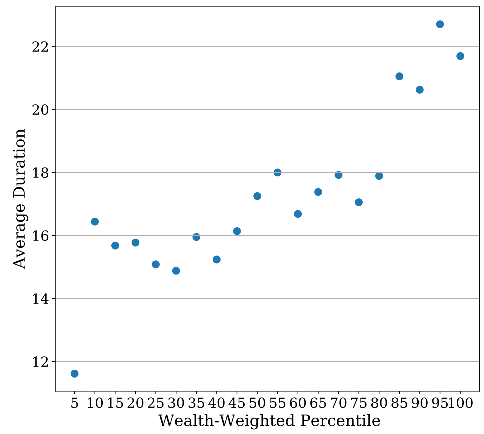
*Note:* Top 1%, Top 10% financial wealth shares as well as financial wealth Gini coefficients are estimated using SCF surveys. For the Before period we use the average values in 1983 and 1989. For the After period we use the average values in 2010, 2013 and 2016. More details on the computations are provided in Appendix A.3.9. For model results, the columns represent the pre-shock wealth distribution ("Initial"), the compensated distribution ("Comp"), and the repriced distribution ("Repriced").

Figure A10: Duration by Net Worth, Robustness

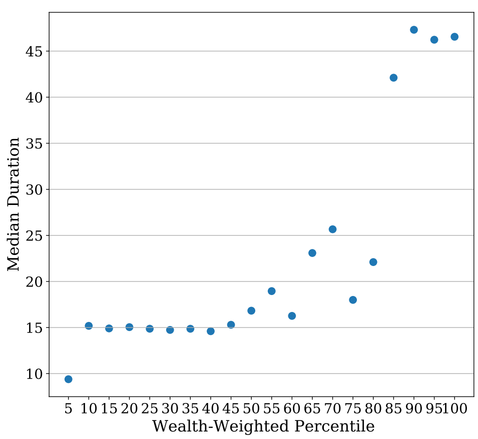
(a) Business Wealth Duration = 40



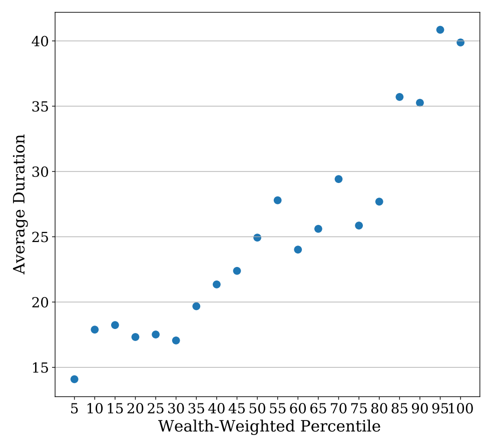

(b) Business Wealth Duration = Equity Duration




(c) Median Duration

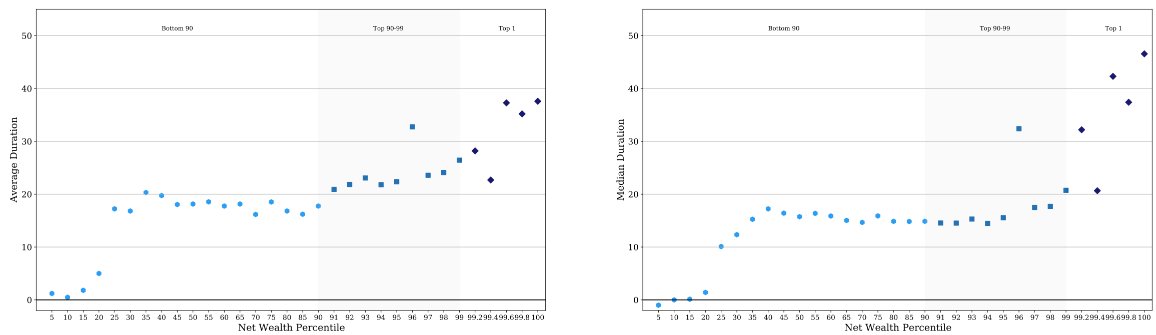



(d) Trim top / bottom 1%




*Note:* Data are based on SCF 1989. We always exclude households with zero net wealth and positive assets (as their portfolio shares would be indeterminate). Figure A10a uses a measure of business duration equal to 40, Figure A10b uses as duration of business wealth, the same duration used for equity (28.7), A10c plots the median duration instead of the average duration, Figure A10d uses a different trimming: trim the bottom/top 1%.

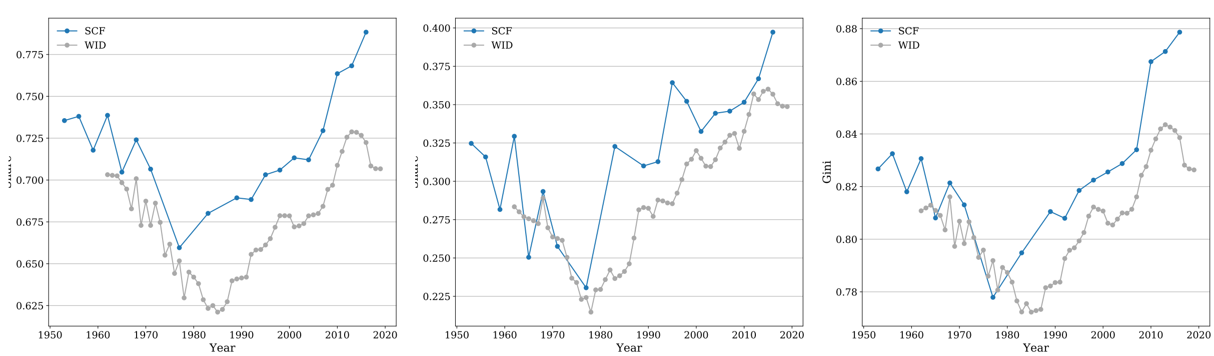
Figure A11: Financial Duration by Net Worth Population Percentiles, Robustness



(a) Average Duration (b) Median Duration

*Note:* This figure provides a robustness check for Figure **??**. We plot both average duration (Figure A11a) as well as the median duration (Figure A11b). Data are based on SCF 1989. We exclude households with zero wealth and positive assets holdings. We trim the top/bottom 2.5% of households ranked by the duration of their portfolio.

Figure A12: Financial Wealth Inequality in the SCF+



(a) Top 10% Share (b) Top 1% Share (c) gini

*Note:* Data are based on SCF+ database developed by Kuhn et al. (2020) and WID database.

Table A8: Summary Statistics Wealth and Income Inequality in SCF

| | SCF | | WID | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1980s | 2010s | 1980s | 2010s | | | | |
| Wealth: Top 1 Share (%) | 28.8 | 37.2 | 25.3 | 35.1 | | | | |
| Wealth: Top 10 Share (%) | 67.6 | 77.3 | 63.2 | 71.8 | | | | |
| Wealth: gini (×100) | 79.4 | 87.2 | 77.8 | 83.6 | | | | |
| | SCF | | WID | | PSID | | PSID (ex transf.) | |
| | 1980s | 2010s | 1980s | 2010s | 1980s | 2010s | 1980s | 2010s |
| Income: Top 1 Share (%) | 11.5 | 18.3 | 12.2 | 18.6 | 6.4 | 9.5 | 8.1 | 11.8 |
| Income: Top 10 Share (%) | 36.3 | 45.5 | 36.3 | 45.1 | 29.2 | 34.3 | 35.4 | 41.7 |
| Income: gini (×100) | 48.2 | 56.1 | 48.7 | 57.9 | 42.8 | 47.8 | 56.9 | 62.7 |

*Note:* Shares and Gini coefficients estimated using the SCF+ developed by Kuhn et al. (2020), the WID database and the PSID. We use our income variable *labinc3f* from the PSID as well the income variable excluding transfers (*labinc2f*). From the SCF+ we use the total income variable excluding capital gain. SCF+ 1980s average over the surveys in 1977, 1983 and 1989.

# B Proofs

## B.1 Proof of proposition 4.2

*Proof.* The one-period budget constraint:

$$\widehat{c}_t(\eta^t) + \frac{\widehat{a}_t(\eta^t)}{\widehat{R}_t} + \widehat{\sigma}_t(\eta^t)\widehat{v}_t = (1-\alpha)\widehat{y}_t(\eta^t) + \widehat{a}_{t-1}(\eta^{t-1}) + \widehat{\sigma}_{t-1}(\eta^{t-1})(\widehat{v}_t + \alpha),$$

can be restated, using equation (3), as:

$$\widehat{c}_t(\eta^t) - (1-\alpha)\widehat{y}_t(\eta^t) + \frac{\widehat{a}_t(\eta^t) + \widehat{\sigma}_t(\eta^t)(\widehat{v}_{t+1} + \alpha)}{\widehat{R}_t} = \widehat{a}_{t-1}(\eta^{t-1}) + \widehat{\sigma}_{t-1}(\eta^{t-1})(\widehat{v}_t + \alpha). \tag{15}$$

Rewriting (15) one period later:

$$\widehat{c}_{t+1}(\eta^{t+1}) - (1-\alpha)\widehat{y}_{t+1}(\eta^{t+1}) + \frac{\widehat{a}_{t+1}(\eta^{t+1}) + \widehat{\sigma}_t(\eta^{t+1})(\widehat{v}_{t+2} + \alpha)}{\widehat{R}_{t+1}} = \widehat{a}_t(\eta^t) + \widehat{\sigma}_t(\eta^t)(\widehat{v}_{t+1} + \alpha).$$

Multiply this equation by $\varphi(\eta_{t+1}|\eta^t)$ and sum across all states $\eta_{t+1}$ to obtain:

$$\begin{aligned} & \sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta^t) \left( \widehat{c}_{t+1}(\eta^{t+1}) - (1-\alpha)\widehat{y}_{t+1}(\eta^{t+1}) + \frac{\widehat{a}_{t+1}(\eta^{t+1}) + \widehat{\sigma}_t(\eta^{t+1})(\widehat{v}_{t+2} + \alpha)}{\widehat{R}_{t+1}} \right) \\ = \; & \widehat{a}_t(\eta^t) + \widehat{\sigma}_t(\eta^t)(\widehat{v}_{t+1} + \alpha), \end{aligned}$$

where we used the fact that $\sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta^t) = 1$ on the right-hand side. Next, substitute this expression back into (15) to obtain:

$$\begin{aligned} & \widehat{c}_t(\eta^t) - (1-\alpha)\widehat{y}_t(\eta^t) + \widehat{R}_t^{-1} \sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta^t) \left( \widehat{c}_{t+1}(\eta^{t+1}) - (1-\alpha)\widehat{y}_{t+1}(\eta^{t+1}) \right) \\ & + \widehat{R}_{t\to t+1}^{-1} \sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta^t) \left( \widehat{a}_{t+1}(\eta^{t+1}) + \widehat{\sigma}_t(\eta^{t+1})(\widehat{v}_{t+2} + \alpha) \right) = \widehat{a}_{t-1}(\eta^{t-1}) + \widehat{\sigma}_{t-1}(\eta^{t-1})(\widehat{v}_t + \alpha). \end{aligned}$$

Define financial wealth, scaled by the aggregate endowment, as:

$$\widehat{\theta}_t = \widehat{a}_{t-1}(\eta^{t-1}) + \widehat{\sigma}_{t-1}(\eta^{t-1})(\widehat{v}_t + \alpha).$$

Continuing the forward substitution, we end up with the following expression:

$$\widehat{\theta}_t \;=\; \sum_{\tau=t}^{\infty} \widehat{R}_{t\to\tau-1}^{-1} \sum_{\eta^\tau|\eta^t} \varphi(\eta^\tau|\eta^t) \left( \widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau) \right).$$

where $\varphi(\eta^t|\eta^t) = 1$. Financial wealth must equal the cost of the household's excess consumption plan, where excess refers to the part not paid for with labor income. Noting that $e_0 = 1$ so that

$\widehat{\theta}_0 = \theta_0$, writing this expression at time zero:

$$\theta_0 = \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau) \left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right)$$

recovers the statement of the proposition. □

### B.2 Proof of Proposition 4.3

*Proof.* We note that the cross-sectional expectation of the product can be decomposed in the standard way:

$$\int \sum_{\eta^\tau} \varphi(\eta^\tau)\psi(\eta^\tau) \left(\widehat{c}_\tau(\eta^\tau)\right) d\Theta_0 = \mathbb{E}_0[\psi_\tau c_\tau] = \mathbb{C}ov_0[\psi_\tau, c_\tau] + \mathbb{E}_0\left[\psi_\tau\right] \mathbb{E}_0\left[c_\tau\right].$$

If the orthogonality condition is satisfied, then the following result obtains:

$$\int \sum_{\eta^\tau} \varphi(\eta^\tau)\psi(\eta^\tau) \left(\widehat{c}_\tau(\eta^\tau)\right) d\Theta_0 = \mathbb{E}_0[\psi_\tau c_\tau] = \mathbb{E}_0[\psi_\tau]\mathbb{E}_0[c_\tau] = \mathbb{E}_0[c_\tau] = 1,$$

because $\mathbb{E}_0[\psi_t] = 1$. □

### B.3 Proof of Proposition 4.4

*Proof.* This inequality $0 \geq Cov(\psi_t, \widehat{c}_t)$ directly implies that the following inequalities obtain:

$$\int \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau)\psi(\eta^\tau)\widehat{c}_\tau(\eta^\tau) d\Theta_0 \leq \int \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau)\widehat{c}_\tau(\eta^\tau) d\Theta_0 = \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1},$$
$$\int \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau)\psi(\eta^\tau)\widehat{y}_\tau(\eta^\tau) d\Theta_0 \leq \int \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau)\widehat{y}_\tau(\eta^\tau) d\Theta_0 = \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1}.$$

As a result, this new measure implies an aggregate value of individual wealth that falls short of total wealth, $\sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1}$. Note that even though this claim to total consumption is itself not traded, the Lucas tree is a claim to $\alpha$ of the same cash flow stream. The market value of the Lucas tree is $\alpha \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1}$, and hence the value of total wealth has to be $\sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1}$. □

### B.4 Proof of proposition 4.5

*Proof.* An unconstrained household's Euler equation in the high-growth economy is given by:

$$1 = \widehat{\beta}\widehat{R}_t \sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta^t) \frac{u'(\widehat{c}(\eta_{t+1}, \eta^t))}{u'(\widehat{c}_t(\eta^t))}.$$

This Euler equation is satisfied because the allocations and prices constitute a Bewley equilibrium in the high-growth economy. This household's Euler equation in the new economy with lower interest rates is still satisfied at the old consumption allocation. This can be seen by plugging in the new equilibrium interest rates:

$$\widetilde{R}_t \widetilde{\beta} = \widehat{\beta} \widehat{R}_t,$$

to recover the unconstrained household's Euler equation in the low-growth economy:

$$1 = \widetilde{\beta} \widetilde{R}_t \sum_{\eta_{t+1}} \phi(\eta_{t+1} | \eta_t) \frac{u'(\widehat{c}(\eta^t, \eta_{t+1})}{u'(\widehat{c}_t(\eta^t))}.$$

We allocate the following amount of financial wealth at time 0 to ensure the household can afford the same consumption plan:

$$\widetilde{\theta}_0(\theta_0, \eta_0) \quad = \quad \sum_{\tau=0}^{\infty} \widetilde{R}_{0\to\tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^t) \left( \widehat{c}_\tau(\eta^\tau) - (1-\alpha) \widehat{y}_\tau(\eta^\tau) \right).$$

Aggregating this initial financial wealth across households:

$$\int \widetilde{\theta}_0 d\Theta_0 \quad = \quad \alpha \sum_{\tau=0}^{\infty} \widetilde{R}_{0\to\tau}^{-1} = \tilde{v}_0,$$

where we have used the goods market clearing condition and the definition of labor income shares. The last equation shows that the new allocation of initial financial wealth uses up all aggregate financial wealth in the economy. Finally, note that the natural borrowing constraints are not binding in the high-growth economy. They remain non-binding in the low-growth economy because consumption is nonnegative. Hence, the allocations are feasible, and they satisfy the sufficient conditions for optimality. □

# C Wealth Inequality and Expected Returns: A Model-free Approach

To develop an initial understanding of the relationship between financial wealth inequality and interest rates under minimal assumptions, we derive closed-form expressions using a log-linearization of the household budget constraint.

We work in a stationary version of the economy in which the aggregate endowment is constant. The hatted variables denote the stationary economy. Section 4 shows how to map these hatted variables into the corresponding variables of the stochastically growing economy. In the stationary economy, investors are computing expectations under the risk-neutral measure. Shocks to expected growth of the aggregate endowment show up as shocks to the risk-free rate.

Let $\widehat{wc}^a_{t+1}$ denote the aggregate log wealth-consumption ratio. Given the constant aggregate endowment in the de-trended economy, $\widehat{wc}^a_{t+1}$ is also the log price-dividend ratio of a perpetuity. The Campbell and Shiller (1988) log-linearization of the aggregate budget constraint around the mean aggregate log wealth-consumption ratio delivers the following expression for the real return on total wealth:

$$\widehat{r}^a_{t+1} = \rho_a \widehat{wc}^a_{t+1} + k^a - \widehat{wc}^a_t.$$

The usual aggregate consumption growth term is zero because the aggregate endowment is constant. The linearization coefficient $\rho^a$ depends only on the mean of the log aggregate wealth-consumption ratio $\widehat{wc}^a$:

$$\rho^a \equiv \frac{e^{\widehat{wc}^a}}{e^{\widehat{wc}^a}+1}, \qquad k^a \equiv \log(1+exp(\widehat{wc}^a)) - \widehat{wc}^a \rho^a.$$

The linearization constant $\rho^a$ captures the McCauley duration of the aggregate consumption claim.
By iterating forward on the linearized return equation and imposing a TVC condition, $\lim_{j\to\infty}(\rho^a)^j \widehat{wc}^a_{t+j} = 0$, and taking expectations, we obtain the standard expression for the aggregate log wealth-consumption ratio as the PDV of future returns:

$$\widehat{wc}^a_t = \frac{k^a}{1-\rho^a} - \mathbb{E}_t \sum_{j=1}^{\infty} (\rho^a)^{j-1} \widehat{r}^a_{t+j}.$$

The wealth-consumption ratio can be linked to the yield on a perpetuity $\widehat{yp}^a_t$:

$$\widehat{yp}^a_t = -(1-\rho^a)\widehat{wc}^a_t + k^a = (1-\rho^a)\mathbb{E}_t \sum_{j=1}^{\infty} (\rho^a)^{j-1} \widehat{r}^f_{t+j-1},$$

since $\widehat{r}^f_t = \widehat{r}^a_t$ in the detrended economy. The unconditional average yield on the perpetuity is given by: $\widehat{yp}^a = \mathbb{E}[\widehat{r}^f]$.[22] For simplicity, we assume that the risk-free rate follows and AR(1) process

[22] As shown by Krueger and Lustig (2010), the expectations hypothesis holds in the stochastically detrended economy,

with persistence $\phi$. Given the AR(1) process for the risk-free rate, the yield on the perpetuity can be expressed as:

$$\widehat{yp}^a_t = \widehat{yp}^a + \frac{1-\rho^a}{1-\rho^a\phi}\left(\widehat{r}^f_t - \mathbb{E}[\widehat{r}^f_t]\right).$$

The yield on the perpetuity governs the dynamics of the aggregate wealth-consumption ratio. When the yield increases, the wealth-consumption ratio decreases and vice versa. Lustig et al. (2013) estimate the wealth-consumption ratio and show that it tracks the inverse of long-term real bond yields closely. [23]

Aggregate consumption equals aggregate labor income plus aggregate financial income. We assume that the factor shares are constant. As a result, the aggregate human wealth-labor income ratio $\widehat{hy}^a_t$ is identical to the aggregate wealth consumption ratio $\widehat{wc}^a_t$.

Next, we turn to the dynamics of the household's wealth-consumption ratio $\widehat{wc}^i_t$. To focus on the valuation effects, we assume that consumption shares $\Delta\widehat{c}^i_t$ (labor income shares) follow a random walk with drift $\mu^c_i$ ($\mu^y_i$). We assume that this is the only source of heterogeneity other than the initial shares $\widehat{c}^i_0$.[24] The household's log human wealth-income ratio is denoted by $\widehat{hy}^i_t$. The household's log wealth-consumption ratio equals the PDV of future consumption share growth and risk-free rates:

$$\widehat{wc}^i_t = \frac{k^c_i + \mu^c_i}{1-\rho^c_i} + \mathbb{E}_t\sum_{j=1}^{\infty}(\rho^c_i)^{j-1}\Delta\widehat{c}^i_{t+j} - \mathbb{E}_t\sum_{j=1}^{\infty}(\rho^c_i)^{j-1}\widehat{r}^f_{t+j-1}.$$

where the linearization constants $\rho^c_i$ and $k^c_i$ are defined analogously to their aggregate counterparts $rho^a$ and $k^a$.

**Corollary C.1.** The log wealth-consumption ratio (human wealth-income ratio) of household $i$ relative to the aggregate ratio is given by:

$$\begin{aligned}
\widehat{wc}^i_t - \widehat{wc}^a_t &= \frac{k^c_i+\mu^c_i}{1-\rho^c_i} - \frac{k^a}{1-\rho^a} + \frac{\mathbb{E}[\widehat{r}^f](\rho^a-\rho^c_i)}{(1-\rho^c_i)(1-\rho^a)} + \frac{\phi(\rho^c_i-\rho^a)}{(1-\rho^c_i\phi)}(\widehat{wc}^a_t - \widehat{wc}^a) \\
\widehat{hy}^i_t - \widehat{wc}^a_t &= \frac{k^h_i+\mu^h_i}{1-\rho^h_i} - \frac{k^a}{1-\rho^a} + \frac{\mathbb{E}[\widehat{r}^f](\rho^a-\rho^h_i)}{(1-\rho^h_i)(1-\rho^a)} + \frac{\phi(\rho^h_i-\rho^a)}{(1-\rho^h_i\phi)}(\widehat{wc}^a_t - \widehat{wc}^a), \\
\widehat{wc}^i_t &= \frac{k^c_i - \mathbb{E}[\widehat{r}^f]+\mu^c_i}{1-\rho^c_i} - \frac{1-\rho^a\phi}{1-\rho^c_i\phi}\frac{(\widehat{yp}^a_t-\widehat{yp}^a)}{(1-\rho^a)}, \quad \widehat{wc}^a_t = \frac{k^a-\mathbb{E}[\widehat{r}^f]}{1-\rho^a} - \frac{(\widehat{yp}^a_t-\widehat{yp}^a)}{(1-\rho^a)}.
\end{aligned}$$

To develop some intuition, consider a simple cross-section with only "workers" and "capital-

and the yield on the perpetuity will only reflect future risk-free rates.

[23]If we allow for bond risk premia that do not vary over time, the risk premium would add an additional constant to the expression for the yield on the perpetuity.

[24]This assumption is not essential, but it makes the analysis that follows more tractable. The next section considers households that choose the optimal consumption path in a dynamic general equilibrium incomplete markets economy. Here, we want to focus on valuation effects.

ists." The capitalists have lower duration of their human wealth: $\rho^h_w > \rho^a > \rho^h_{cap}$. Assume that the duration of their consumption claim does not differ: $\rho^c_w = \rho^a = \rho^c_{cap}$. Their initial consumption and total wealth are also identical. Now consider a decrease in long rates that pushes up the aggregate wealth-consumption ratio. The distribution of total wealth does not change in response to $wc^a_t$ by virtue of the equal consumption durations: $wc^w_t - wc^{cap}_t = 0$. However, the human wealth of the workers goes up by more than the human wealth of the capitalists:

$$\widehat{hy}^w_t - \widehat{hy}^{cap}_t = const + \left( \frac{\phi(\rho^h_w - \rho^a)}{1-\rho^h_w\phi} - \frac{\phi(\rho^h_{cap} - \rho^a)}{1-\rho^h_{cap}\phi} \right) (\widehat{wc}^a_t - \widehat{wc}^a),$$

because $\rho^h_w > \rho^h > \rho^h_{cap}$.

The financial wealth (FW) of the capitalists, which is given by the difference between their total wealth and human wealth $\exp(\widehat{c}^{cap}_t + \widehat{wc}^{cap}_t) - \exp(\widehat{y}^{cap}_t + \widehat{hy}^{cap}_t)$, increases when rates decline ($\widehat{wc}^a_t - \widehat{wc}^a > 0$):

$$FW^{cap}_t = \exp(\widehat{wc}^a_t) \left( \exp(\widehat{c}^{cap}_t) - \exp(\widehat{y}^{cap}_t + \frac{\mathbb{E}[\widehat{r}^f](\rho^a - \rho^h_{cap})}{(1-\rho^h_{cap})(1-\rho^a)} + \frac{\phi(\rho^h_{cap} - \rho^a)}{(1-\rho^h_{cap}\phi)} (\widehat{wc}^a_t - \widehat{wc}^a) \right)$$

The capitalists suffer a relative decline in human wealth that is offset by an increase in their financial wealth, leaving total wealth unchanged.

Next, we characterize the cross-sectional variance of total wealth assuming that all households have the same initial consumption. We use subscript $xs$ to denote cross-sectional moments. The main result in this section is that there is a negative relationship between long rates and the cross-sectional dispersion of total wealth.

**Proposition C.2.** If there is no initial consumption dispersion and wealth is log-normally distributed, the cross-sectional coefficient of variation of total wealth (TW) is bounded below by:

$$\frac{Std_{xs}(TW)}{\mathbb{E}_{xs}(TW)} \approx std_{xs}(w) = \left( Var_{xs} \left[ \widehat{wc}^i_t + \widehat{c}^i_t \right] \right)^{1/2} \geq \left( Var_{xs} \left[ \widehat{wc}^i \right] + Var_{xs} \left[ \frac{\phi(\rho^c_i - \rho^a)}{(1-\rho^c_i\phi)} \right] (\widehat{wc}^a_t) \right)^{1/2}.$$

The cross-sectional coefficient of variation of human wealth (HW) is bounded below by:

$$\frac{Std_{xs}(HW)}{\mathbb{E}_{xs}(HW)} \approx std_{xs}(h) = \left( Var_{xs} \left[ \widehat{hy}^i_t + \widehat{y}^i_t \right] \right)^{1/2} \geq \left( Var_{xs} \left[ \widehat{hy}^i \right] + Var_{xs} \left[ \frac{\phi(\rho^h_i - \rho^a)}{(1-\rho^h_i\phi)} \right] (\widehat{wc}^a_t) \right)^{1/2}.$$

It immediately follows from the proposition that when long rates decline and the aggregate wealth-consumption ratio increases, the cross-sectional dispersion of total wealth and human wealth increase (at least weakly). The size of the increase in wealth inequality increases in the cross-sectional variance of the duration of total wealth:

$$Var_{xs}\left[\frac{\phi(\rho_i^c - \rho^a)}{(1-\rho_i^c\phi)}\right].$$

To derive results for financial wealth, we need to make an additional assumption, namely that financial and human wealth vary negatively. We use $\alpha$ to denote the capital income share.

**Proposition C.3.** If there is no initial consumption dispersion, human and total wealth are log-normally distributed, and human wealth and financial wealth covary negatively, then the cross-sectional coefficient of variation of financial wealth (FW) is bounded below by:

$$\frac{Std_{xs}(FW)}{\mathbb{E}_{xs}(FW)} \geq \sqrt{\frac{1}{(1-\alpha)^2}\left(Var_{xs}\left[\widehat{wc}^i\right] + Var_{xs}\left[\frac{\phi(\rho_i^c-\rho^a)}{(1-\rho_i^c\phi)}\right](\widehat{wc}_t^a)^2\right) + \frac{\alpha^2-2\alpha}{(1-\alpha)^2}\left(Var_{xs}\left[\widehat{hy}^i\right] + Var_{xs}\left[\frac{\phi(\rho_i^h-\rho^a)}{(1-\rho_i^h\phi)}\right](\widehat{wc}_t^a)^2}$$

If the cross-sectional variance of total wealth is smaller than the variance of human wealth, which seems plausible when financial and human wealth are negatively correlated and given that households seek to smooth consumption, we also obtain the following bound:

$$\frac{Std_{xs}(FW)}{\mathbb{E}_{xs}(FW)} \geq std_x(a) = \left(Var_{xs}\left[\widehat{wc}_t^i + \widehat{c}_t^i\right]\right)^{1/2} \geq \left(Var_{xs}\left[\widehat{wc}^i\right] + Var_{xs}\left[\frac{\phi(\rho_i^c - \rho^a)}{(1-\rho_i^c\phi)}\right](\widehat{wc}_t^a)\right)^{1/2}.$$

When rates go down and $\widehat{wc}_t^a$ goes up, financial wealth inequality rises.

## C.1 Proof of Corollary C.1

*Proof.* Next, we analyze the household's wealth-consumption ratio. A Campbell and Shiller (1988) log-linearization of the return equation around the mean log wealth-consumption ratio $wc^i$ for household $i$ delivers the following expression for the log returns on a claim to household $i$'s consumption stream:

$$\widehat{r}_{t+1}^i = \Delta\widehat{c}_{t+1}^i + \rho_i^c\widehat{wc}_{t+1}^i + k_i^c - \widehat{wc}_t^i, \tag{16}$$

where the linearization coefficient $\rho_i^c$ depends only on the mean of the log wealth-consumption ratio $wc^i$: $\rho_i^c \equiv \frac{e^{\widehat{wc}^i}}{e^{\widehat{wc}^i}+1}$. By taking unconditional averages of the return in Equation 16, we obtain:

$$(1-\rho_i^c)\widehat{wc}^i = \mu_i^c - \mathbb{E}[\widehat{r}_{t+1}^i] + k_i^c,$$

where $\mu_i^c$ denotes the household's average consumption growth rate. We obtain the following expression for the average wealth-consumption ratio:

$$\widehat{wc}^i = \frac{\mu_i^c - \mathbb{E}[\widehat{r}_{t+1}^i]}{1-\rho_i^c} + \frac{k_i^c}{1-\rho_i^c}.$$

By iterating forward on the linearized return equation and imposing a no-bubble condition,

$$\lim_{j\to\infty}(\rho_i^c)^j\widehat{wc}_{t+j}^i = 0,$$

we obtain an expression for the log wealth-consumption ratio:

$$\widehat{wc}_t^i = \frac{k_i^c}{1-\rho_i^c} + \left[\sum_{j=1}^{\infty}(\rho_i^c)^{j-1}\Delta\widehat{c}_{t+j}^i\right] - \left[\sum_{j=1}^{\infty}(\rho_i^c)^{j-1}\widehat{r}_{t+j}^i\right].$$

We take expectations at time $t$ to obtain:

$$\widehat{wc}_t^i = \frac{k_i^c}{1-\rho_i^c} + \mathbb{E}_t\left[\sum_{j=1}^{\infty}(\rho_i^c)^{j-1}\Delta\widehat{c}_{t+j}^i\right] - \mathbb{E}_t\left[\sum_{j=1}^{\infty}(\rho_i^c)^{j-1}\widehat{r}_{t+j}^a\right].$$

In our baseline model, the expected one period returns on the consumption-claim have to be equal to the return on the aggregate consumption claim. As shown in section 4 , in this incomplete markets economy, to compute measures of wealth that can be aggregated, we discount all consumption claims using the discount rate on the aggregate consumption claim. If the expectations hypothesis holds, then the household's log wealth-consumption ratio is determined by the PDV of future consumption growth and risk-free rates:

$$\widehat{wc}_t^i = \frac{k_i^c}{1-\rho_i^c} + \mathbb{E}_t\sum_{j=1}^{\infty}(\rho_i^c)^{j-1}\Delta\widehat{c}_{t+j}^i - \mathbb{E}_t\sum_{j=1}^{\infty}(\rho_i^c)^{j-1}\widehat{r}_{t+j-1}^f.$$

As a result, we obtain the following expression for :

$$\widehat{wc}_t^i = \frac{k_i^c+\mu_i^c}{1-\rho_i^c} - \mathbb{E}_t\sum_{j=1}^{\infty}(\rho_i^c)^{j-1}\widehat{r}_{t+j-1}^f.$$

Using the autoregressive process for the risk-free rate, we obtain the following expression for the log wealth-consumption ratio:

$$\widehat{wc}_t^i = \frac{k_i^c - \mathbb{E}[\widehat{r}^f] + \mu_i^c}{1-\rho_i^c} + \frac{\widehat{r}_t^f - \mathbb{E}[\widehat{r}^f]}{1-\rho_i^c\phi}.$$

Hence, we obtain the following expression for the household's consumption wealth ratio in deviation from the average:

$$\widehat{wc}_t^i - \widehat{wc}^i = \frac{1-\rho^a\phi}{1-\rho_i^c\phi}(\widehat{wc}_t^a - \widehat{wc}^a).$$

As a result, we have that the log household wealth-consumption ratio can be stated as:

$$\widehat{wc}_t^i \quad = \quad \frac{k_i^c - E[\widehat{r}^f] + \mu_i^c}{1-\rho_i^c} - \frac{1-\rho^a\phi}{1-\rho_i^c\phi}\frac{(\widehat{yp}_t^a - \widehat{yp}^a)}{(1-\rho^a)}$$

By the same token, the aggregate wealth consumption ratio can be stated as:

$$\widehat{wc}_t^a \quad = \quad \frac{k^a - E[\widehat{r}^f]}{1-\rho^a} - \frac{(\widehat{yp}_t^a - \widehat{yp}^a)}{(1-\rho^a)}$$

We can apply the same approach to derive an expression for the log human wealth-income ratio. The household's log human wealth-income ratio is determined by the PDV of future income growth and risk-free rates:

$$\widehat{hy}_t^i = \frac{k_i^c}{1-\rho_i^h} + \mathbb{E}_t \sum_{j=1}^{\infty} (\rho_i^h)^{j-1} \Delta \widehat{y}_{t+j}^i - \mathbb{E}_t \sum_{j=1}^{\infty} (\rho_i^h)^{j-1} \widehat{r}_{t+j-1}^f.$$

Given the AR(1) process for the risk-free rate, when log household income $i$ follows a random walk with drift $\mu_i^h$, the household's log human wealth-income ratio can be expressed as:

$$\widehat{hy}_t^i - \widehat{wc}_t^a = \frac{k_i^h + \mu_i^h}{1-\rho_i^h} - \frac{k^a}{1-\rho^a} + \frac{\mathbb{E}[\widehat{r}^f](\rho^a - \rho_i^h)}{(1-\rho_i^h)(1-\rho^a)} + \frac{\phi(\rho_i^h - \rho^a)}{(1-\rho_i^h\phi)}(\widehat{wc}_t^a - \widehat{wc}^a).$$

□

## C.2 Proof of Proposition C.2

*Proof.* In the economy without consumption dispersion, the cross-sectional standard deviation of log household wealth is bounded below by:

$$\sigma\left[\widehat{wc}_t^i + \widehat{c}_t^i\right] \geq \quad \left(Var_x\left[\widehat{wc}^i\right] + Var_x\left[\tfrac{\phi(\rho_i^c - \rho^a)}{(1-\rho_i^c\phi)}\right](\widehat{wc}_t^a)\right)^{1/2}.$$

As the wealth/consumption ratio increases, the lower bound on the cross-sectional standard deviation of wealth increases. As interest rates declines and $\widehat{wc}_t^a$ increases, the cross-sectional variance of the log wealth distribution increases. We know that the household's log wealth-consumption ratio can be stated as:

$$\widehat{wc}_t^i - \widehat{wc}_t^a \quad = \quad \widehat{wc}^i - \widehat{wc}^a + \frac{\mathbb{E}[\widehat{r}^f](\rho^a - \rho_i^c)}{(1-\rho_i^c)(1-\rho^a)} + \frac{\phi(\rho_i^c - \rho^a)}{(1-\rho_i^c\phi)}(\widehat{wc}_t^a - \widehat{wc}^a).$$

Hence, the cross-sectional variance of log wealth is given by the cross-sectional variance of consumption, plus the cross-sectional variance of wealth-consumption and the cross-sectional covariance:

$$\begin{aligned}
Var_x\left[\widehat{wc}_t^i+\widehat{c}_t^i\right] &= Var_x\left[\widehat{c}_t^i\right]+Var_x\left[\widehat{wc}_t^i\right]+2Cov_x\left[\widehat{wc}_t^i,\widehat{c}_t^i\right],\\
&= Var_x\left[\widehat{c}_t^i\right]+Var_x\left[\widehat{wc}^i\right]+2Cov_x\left[\widehat{wc}^i,\widehat{c}_t^i\right]\\
&+ 2Cov_x\left[\widehat{wc}^i,\widehat{c}_t^i\right]+2\widehat{wc}_t^a Cov_x\left[\frac{\phi(\rho_i^c-\rho^a)}{(1-\rho_i^c\phi)},\widehat{wc}^i\right]\\
&+ Var_x\left[\frac{\phi(\rho_i^c-\rho^a)}{(1-\rho_i^c\phi)}\right](\widehat{wc}_t^a)^2.
\end{aligned}$$

Assume that there is no dispersion in initial consumption shares. Then the cross-sectional variance of log wealth is given by:

$$Var_x\left[\widehat{wc}_t^i+\widehat{c}_t^i\right] = Var_x\left[\widehat{wc}^i\right]+2\widehat{wc}_t^a Cov_x\left[\frac{\phi(\rho_i^c-\rho^a)}{(1-\rho_i^c\phi)},\widehat{wc}^i\right]+Var_x\left[\frac{\phi(\rho_i^c-\rho^a)}{(1-\rho_i^c\phi)}\right](\widehat{wc}_t^a)^2.$$

Since $\mu_i^c$ is the only source of heterogeneity, this implies that $Cov_x\left[\frac{\phi(\rho_i^c-\rho^a)}{(1-\rho_i^c\phi)},\widehat{wc}^i\right]\geq 0$, because $\rho_i^c$ depends on $\mu_i^c$.

We can use the cumulant-generating function to back out the moments of wealth in levels. Let $w$ denote the log of wealth. The cross-sectional coefficient of variation of wealth is given by:

$$\left(\frac{\mathbb{E}_x[\exp 2w]-(\mathbb{E}_x[\exp w])^2}{(\mathbb{E}_x[\exp w])^2}\right)^{1/2} = \sqrt{\left[\exp\left(\sum_{j=2}^{\infty}2^{j-1}\kappa_{j,t+1}(w_{t+1})/j!\right)-1\right]},$$

where $\kappa_{j,t+1}(w_{t+1})$ denotes the $j$-th cumulant.

We can use the cumulant-generating function to back out the moments of wealth in levels. Let $w$ denote the log of wealth. The first moment of wealth in levels is given by:

$$\mathbb{E}_x[\exp w] = \exp(\kappa_{1,t+1}(w_{t+1})+\sum_{j=2}^{\infty}\kappa_{j,t+1}(w_{t+1})/j!),$$

where $\kappa_{j,t+1}$ denotes the $j-th$ cumulant of the log wealth distribution. Similarly, the second moment of wealth in levels is given by:

$$\mathbb{E}_x[\exp 2w] = \exp(2\kappa_{1,t+1}(w_{t+1})+\sum_{j=2}^{\infty}2^j\kappa_{j,t+1}(w_{t+1})/j!).$$

As a result, the cross-sectional variance of wealth is

$$\mathbb{E}_x[\exp 2w]-(\mathbb{E}_x[\exp w])^2 = \exp\left(2\kappa_{1,t+1}(w_{t+1})+\sum_{j=2}^{\infty}2^j\kappa_{j,t+1}(w_{t+1})/j!\right)$$

$$- \exp\left(2\kappa_{1,t+1}(w_{t+1}) + 2\sum_{j=2}^{\infty}\kappa_{j,t+1}(w_{t+1})/j!\right).$$

As a result, the cross-sectional variance of wealth is given by:

$$\mathbb{E}_x[\exp 2w] - (\mathbb{E}_x[\exp w])^2 = \exp\left(2\kappa_{1,t+1}(w_{t+1}) + 2\sum_{j=2}^{\infty}\kappa_{j,t+1}(w_{t+1})/j!\right) \left[\exp\left(\sum_{j=2}^{\infty}2^{j-1}\kappa_{j,t+1}(w_{t+1})/j!\right) - 1\right].$$

Hence, if we scale the cross-sectional variance of wealth by the cross-sectional mean we obtain:

$$\left(\frac{\mathbb{E}_x[\exp 2w] - (\mathbb{E}_x[\exp w])^2}{(\mathbb{E}_x[\exp w])^2}\right)^{1/2} = \sqrt{\left[\exp\left(\sum_{j=2}^{\infty}2^{j-1}\kappa_{j,t+1}(w_{t+1})/j!\right) - 1\right]}$$

In the case of log-normal wealth distribution, all of the higher-order cumulants drop out ($\kappa_{j,t} = 0, k > 2$). □

## C.3 Proof of proposition C.3

*Proof.* The cross-sectional variance of financial wealth $A = W - H$ can be stated as:

$$\frac{Var_x(A)}{\mathbb{E}_x(A)^2} = \frac{\mathbb{E}_x(W)^2}{\mathbb{E}_x(A)^2}\frac{Var_x(W)}{\mathbb{E}_x(W)^2} + \frac{\mathbb{E}_x(H)^2}{\mathbb{E}_x(A)^2}\frac{Var_x(H)}{\mathbb{E}_x(H)^2} - 2\frac{\mathbb{E}_x(H)\mathbb{E}_x(W)}{\mathbb{E}_x(A)^2}Cov_x(W,H).$$

$1-\alpha$ denotes the capital share. The previous equation can be restated as follows:

$$\frac{Var_x(A)}{\mathbb{E}_x(A)^2} = \frac{1}{(1-\alpha)^2}\frac{Var_x(W)}{\mathbb{E}_x(W)^2} + \frac{\alpha^2}{(1-\alpha)^2}\frac{Var_x(H)}{\mathbb{E}_x(H)^2} - 2\frac{\alpha}{(1-\alpha)^2}Cov_x(W,H).$$

Note that $Cov_x(W,H) = Cov_x(A,H) + Var_x(H)$. We assume that $Cov_x(A,H) < 0$. As a result, using $Cov_x(W,H) \geq Var_x(H)$, we obtain:

$$\frac{Var_x(A)}{\mathbb{E}_x(A)^2} \geq \frac{1}{(1-\alpha)^2}\frac{Var_x(W)}{\mathbb{E}_x(W)^2} + \frac{\alpha^2}{(1-\alpha)^2}\frac{Var_x(H)}{\mathbb{E}_x(H)^2} - 2\frac{\alpha}{(1-\alpha)^2}Var_x(H).$$

If human wealth and financial wealth covary negatively, the cross-sectional variance of financial wealth is bounded below (approximately) by the following expression

$$\frac{Var_x(A)}{\mathbb{E}_x(A)^2} \geq \frac{1}{(1-\alpha)^2}Var_x(w) + \frac{\alpha^2 - 2\alpha}{(1-\alpha)^2}Var_x(h).$$

provided that $Cov_x(A,H) < 0$. If $Var_x(w_t) \leq Var_x(h_t)$ for all $t$, which seems plausible given that households seek to smooth consumption, we also obtain:

$$\frac{Var_x(A)}{\mathbb{E}_x(A)^2} \geq Var_x(w).$$

□

## D Life-Cycle Model Details

Each agent in the life-cycle model with age $j$, portfolio of financial assets $\{a_{k,t}\}$, and idiosyncratic labor income state $z$ solves the Bellman equation:

$$V_j(a_t; z_t) = \max_{a_{t+1}} \frac{c_t^{1-\gamma}}{1-\gamma} + \beta s_j \mathbb{E}_t \Big[ V_{j+1}(a_{t+1}; z_{t+1}) \Big] \tag{17}$$

subject to the budget constraint:

$$c_t \leq y_t + \sum_{k=0}^{K} (q_k + \delta_k) s_j^{-1} a_{k,t} - q_k a_{k,t+1} \tag{18}$$

where $y$ is after-tax income as specified in equations (6) and (7), $s_j$ is the probability of surviving to age $j+1$, $q_k$ and $\delta_k$ are the prices and cash flows, respectively, of the set of risk free financial assets available to the household. The term $s_j^{-1}$ in the budget constraint (18) represents that households enter an annuity or tontine system in which surviving households receive the assets of households in their age cohort who died, proportional to their asset holdings. This assumption ensures a sufficiently strong savings motive for older households in the absence of a bequest motive.

We can generalize the problem through some convenient variable substitutions. First, we can simplify the asset structure. In a stationary equilibrium, without aggregate shocks or changes to the interest rate, the specific form of the financial assets is arbitrary, although it will be relevant for repricing assets following an interest rate shock. As a result, we can define $x$ to be the start-of-period value of the entire portfolio, including both its cash flow and continuation value:

$$\theta_t = \sum_{k=0}^{K} (q_k + \delta_k) a_{k,t}.$$

By no arbitrage, we have

$$\frac{q_k + \delta_k}{q_k} = R$$

for all $k$, which implies

$$\sum_{k=0}^{K} q_k a_{k,t+1} = \sum_{k=0}^{K} (q_k + \delta_k) R^{-1} a_{k,t+1} = R^{-1} \theta_{t+1}.$$

Substituting now yields the simplified the budget constraint

$$c_t \leq y_t + \theta_t - R^{-1} \theta_{t+1}. \tag{19}$$

Under a constant interest rate, the problem can therefore be solved as if the agents held one-period debt with face value $\theta$ in each period, allowing us to use a single solution to characterize economies with portfolios over many possible assets.

**Compensated Distribution.** To compute the compensated distribution under a change from interest rate $R$ to $\tilde{R}$, we first compute total wealth under the original and new interest rates:

$$\Omega_t = \sum_{\tau=0}^{\infty} R^{-\tau} c_{t+\tau}$$
$$\tilde{\Omega}_t = \sum_{\tau=0}^{\infty} \tilde{R}^{-\tau} c_{t+\tau}.$$

We next compute human wealth under the original and new interest rates:

$$\Upsilon_t = \sum_{\tau=0}^{\infty} R^{-\tau} y_{t+\tau}$$
$$\tilde{\Upsilon}_t = \sum_{\tau=0}^{\infty} \tilde{R}^{-\tau} y_{t+\tau}.$$

The implied amount of financial wealth that makes the original consumption plan affordable is therefore

$$\begin{aligned} \theta_t^{comp} &= \tilde{\Omega}_t - \tilde{\Upsilon}_t \\ &= \theta_t + (\tilde{\Omega}_t - \Omega_t) + (\tilde{\Upsilon}_t - \Upsilon_t) \end{aligned}$$

where $\theta_t$ is pre-shock financial wealth.

**Repriced Distribution.** To compute the repriced distribution following a change from interest rate $R$ to $\tilde{R}$, we will need to specify the specific asset structure. We assume that agents hold zero coupon bonds with maturity $m$, which implies $q_m = R^{-m}$. At the moment of the interest rate change, the repriced (post-shock) financial wealth $\theta_t^{repriced}$ is related to pre-shock financial wealth

$\theta_t$ according to the formula

$$\theta_t^{repriced} = \left(\frac{\tilde{q}_m}{q_m}\right)\theta_t = \left(\frac{\tilde{R}}{R}\right)^{-m}\theta_t \tag{20}$$

for a household with bonds of maturity (duration) $m$. For our computations, we set $m$ equal to financial wealth duration, and apply (20).

# E Affine Asset Pricing Model

This appendix develops a reduced-form asset pricing model. The asset pricing model is used for three main purposes. First, to compute long-term real bonds yields, the cost of a 30-year real annuity, and expected returns on stocks and housing wealth. Second, to compute the McCauley duration of the aggregate stock market, small stocks, and real estate wealth in a manner that is consistent with the history of bond and stock prices. Third, the model delivers the price and duration of a claim to aggregate consumption and to aggregate labor income.

The asset pricing model in the class of exponentially-affine SDF models. A virtue of the reduced-form model is that it can accommodate a substantial number of aggregate risk factors. We argue that it is important to go beyond the aggregate stock and bond markets to capture the risk embedded in households' financial asset portfolios as well as the aggregate risk in consumption and labor income claims. Similar models are estimated in Lustig et al. (2013); Jiang, Lustig, Van Nieuwerburgh and Xiaolan (2019); Gupta and Van Nieuwerburgh (2021).

## E.1 Setup

### E.1.1 State Variable Dynamics

Time is denoted in quarters. We assume that the $N \times 1$ vector of state variables follows a Gaussian first-order VAR:

$$\boldsymbol{z}_t = \boldsymbol{\Psi} \boldsymbol{z}_{t-1} + \boldsymbol{\Sigma}^{\frac{1}{2}} \boldsymbol{\varepsilon}_t, \tag{21}$$

with shocks $\boldsymbol{\varepsilon}_t \sim i.i.d. \mathcal{N}(0, I)$ whose variance is the identity matrix. The companion matrix $\Psi$ is a $N \times N$ matrix. The vector $z$ is demeaned. The covariance matrix of the innovations to the state variables is $\Sigma$; the model is homoscedastic. We use a Cholesky decomposition of the covariance matrix, $\Sigma = \Sigma^{\frac{1}{2}} \Sigma^{\frac{1}{2}\prime}$, which has non-zero elements only on and below the diagonal. The Cholesky decomposition of the residual covariance matrix allows us to interpret the shock to each state variable as the shock that is orthogonal to the shocks of all state variables that precede it in the VAR. We discuss the elements of the state vector and their ordering below. The (demeaned) one-quarter bond nominal yield is one of the elements of the state vector: $y^{\$}_{t,1} = y^{\$}_{0,1} + e'_{yn} z_t$, where $y^{\$}_{0,1}$ is the unconditional average 1-quarter nominal bond yield and $e_{yn}$ is a vector that selects the element of the state vector corresponding to the one-quarter yield. Similarly, the (demeaned) inflation rate is part of the state vector: $\pi_t = \pi_0 + e'_{\pi} z_t$ is the (log) inflation rate between $t-1$ and $t$. Lowercase letters denote logs.

### E.1.2 Stochastic Discount Factor

The nominal SDF $M^{\$}_{t+1} = \exp(m^{\$}_{t+1})$ is conditionally log-normal:

$$m^{\$}_{t+1} \quad = \quad -y^{\$}_{t,1} - \frac{1}{2}\mathbf{\Lambda}'_t\mathbf{\Lambda}_t - \mathbf{\Lambda}'_t\varepsilon_{t+1}. \tag{22}$$

Note that $y^{\$}_{t,1} = -\mathbb{E}_t[m^{\$}_{t+1}] - 0.5\text{Var}_t[m^{\$}_{t+1}]$. The real log SDF $m_{t+1} = m^{\$}_{t+1} + \pi_{t+1}$ is also conditionally Gaussian. The innovations in the vector $\varepsilon_{t+1}$ are associated with a $N \times 1$ market price of risk vector $\mathbf{\Lambda}_t$ of the affine form:

$$\mathbf{\Lambda}_t = \mathbf{\Lambda}_0 + \mathbf{\Lambda}_1 z_t. \tag{23}$$

The $N \times 1$ vector $\mathbf{\Lambda}_0$ collects the average prices of risk while the $N \times N$ matrix $\mathbf{\Lambda}_1$ governs the time variation in risk premia. Asset pricing amounts to estimating the market prices of risk $(\mathbf{\Lambda}_0, \mathbf{\Lambda}_1)$. We specify the moment conditions used to identify the market prices of risk below.

### E.1.3 State Vector Elements

The state vector contains the following $N = 22$ variables, in order of appearance: (1) real GDP growth, (2) GDP price inflation, (3) the nominal short rate (3-month nominal Treasury bill rate), (4) the spread between the yield on a five-year Treasury note and a three-month Treasury bill, (5) the log price-dividend ratio on the CRSP value-weighted stock market, (6) the log real dividend growth rate on the CRSP stock market. Elements 7, 9, 11, and 13 are the log price-dividend ratios on the first size quintile of stocks (small), the first book-to-market quintile of stocks (growth), the fifth book-to-market quintile of stocks (value), and a listed infrastructure index (infra). Elements 8, 10, 12, and 14 are the corresponding log real dividend growth rates. Element 15 is the log price-dividend ratio on housing wealth, element 16 is log real dividend growth on housing wealth. Finally, the state vector contains the log change in the consumption/GDP ratio $\Delta cx$ in 17th, the log change in the log labor income/GDP ratio $\Delta lx$ in 18th, the log level of the consumption/GDP ratio $cx$ in 19th, and the log level of the labor income/GDP ratio $lx$ in 20th position.

$$\begin{aligned} z_t = \quad & \Big[\pi_t, x_t, y^{\$}_{t,1}, y^{\$}_{t,20} - y^{\$}_{t,1}, pd^m_t, \Delta d^m_t, pd^{small}_t, \Delta d^{small}_t, \\ & pd^{growth}_t, \Delta d^{growth}_t, pd^{value}_t, \Delta d^{value}_t, pd^{infra}_t, \Delta d^{infra}_t \\ & pd^{hw}_t, \Delta d^{hw}_t, \Delta cx_{t+1}, \Delta lx_{t+1}, cx_{t+1}, lx_{t+1}\Big]'. \end{aligned} \tag{24}$$

This state vector is observed at quarterly frequency from 1947.Q1 until 2019.Q4 (292 observations). This is the longest available time series for which all variables are available. Inflation is the log change in the GDP price deflator. For the yields, we use the average of daily Constant

Maturity Treasury yields within the quarter. All dividend series are deseasonalized by summing dividends across the current month and past 11 months. Small stocks are the bottom 20% of the market capitalization distribution, growth stocks the bottom 20% of the book-to-market distribution, and value stocks the top 20% of the book-to-market distribution. The infrastructure stock index is measured as the value-weighted average of the eight relevant Fama-French industries (Aero, Ships, Mines, Coal, Oil, Util, Telcm, Trans). We subtract inflation from all nominal dividend growth rates to obtain real dividend growth rates.

Dividend growth on housing wealth is measured as housing services consumption growth from the Bureau of Economic analysis Table 2.3.5. The price-dividend ratio is the ratio of owner-occupied housing wealth from the Financial Accounts of the United States Table B.101.h divided by housing services consumption. The resulting price-dividend ratio on housing wealth averages 16.1 (for annualized dividends) between 1947 and 2019. We subtract inflation from dividend growth on housing wealth and we also subtract 0.6% per quarter to reflect the fact that the size of the housing stock is growing and we are only interested in the rental price change, not the change in the quantity of housing. The resulting real rental growth rate is 1.82% per year, which is in line with (and still on the higher end of the numbers reported in) the literature.

Aggregate consumption is measured as non-durables plus services plus durable services consumption. Durable services consumption is constructed as the depreciation rate (20%) multiplied by the stock of durables. The stock of durables itself is computed using the perpetual inventory method. This series is divided by nominal GDP and logs are taken.

Aggregate labor income is measured as wages and salaries plus business income (proprietors' income with inventory valuation and capital consumption adjustments) plus transfer income (personal current transfer receipts) minus taxes (Personal current taxes and Contributions for government social insurance, domestic). This series is divided by nominal GDP and logs are taken. Real consumption growth can then be written as the sum of real GDP growth plus the change in the consumption/GDP ratio:

$$\Delta c^a_{t+1} = x_{t+1} + \Delta cx_{t+1}$$

and similar for labor income growth.

All state variables are demeaned with the observed full-sample mean. The first 18 equations of the VAR are estimated by OLS equation by equation. We recursively zero out all elements of the companion matrix $\Psi$ whose t-statistic is below 2.2. The resulting point estimates for $\Psi$ and $\Sigma^{\frac{1}{2}}$ are reported below.

The dynamics of $cx$ are pinned down by the dynamics of $\Delta cx$:

$$cx_{t+1} = cx_t + \Delta cx_{t+1} = \left(e_{cx} + e_{cxgr}\Psi\right)' z_t + e_{cxgr}\gamma^{\frac{1}{2}}\varepsilon_{t+1}$$

Therefore the 19st row of $\Psi$ is identical to the 17th row, except that $\Psi(19,19) = \Psi(17,19) + 1$.

Similarly, the 20th row of $\Psi$ is identical to the 18th row, except that $\Psi(20,20) = \Psi(18,20) + 1$. The innovations to the 19th and 20th row are not independent innovations but determined by the innovations that precede it. The level variables $cx$ and $lx$ are only added to the VAR to enforce cointegration between consumption and GDP and between labor income and GDP. As a result of this cointegration, the aggregate consumption and labor income claims will have the same aggregate risk as the GDP claim.

## E.2 Estimation

### E.2.1 Bond Pricing

In this setting, nominal bond yields of maturity $\tau$ are affine in the state variables:

$$y^{\$}_{t,\tau} = -\frac{1}{\tau}A^{\$}_{\tau} - \frac{1}{\tau}\left(B^{\$}_{\tau}\right)' z_t.$$

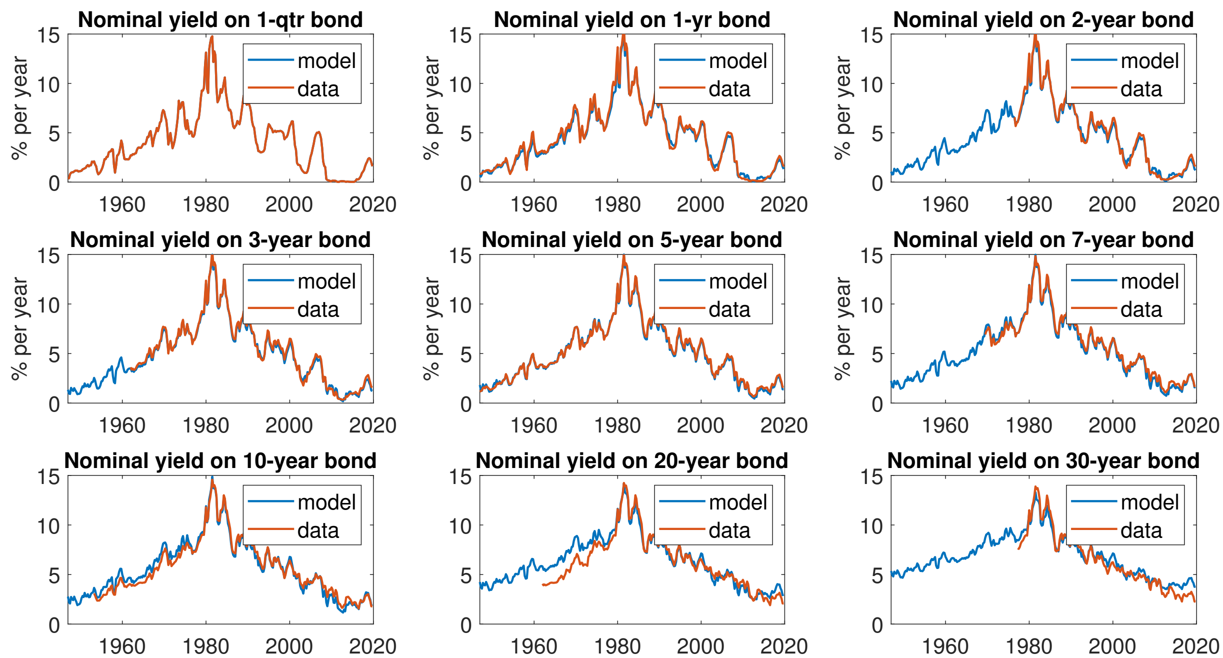
The scalar $A^{\$}(\tau)$ and the vector $B^{\$}_{\tau}$ follow ordinary difference equations (ODE) that depend on the properties of the state vector and on the market prices of risk. Real bond yield are also exponentially affine with coefficients that follow their own ODEs. We will price the cross-section of nominal and real bond yields (price levels), putting more weight on matching the time series of one- and twenty-quarter nominal bond yields since those yields are part of the state vector $\boldsymbol{z}_t$. We also fit the dynamics of 20-quarter nominal bond risk premia (price changes).

Figure D1 plots the nominal bond yields on bonds of maturities 1 quarter, 1-, 2-, 3-, 5-, 7-, 10-, 20-, and 30-years. These are all available bond yields in the data. The 20-, and 3-year bond yields are not available in parts of the sample, but the estimation minimizes the distance between observed and model-implied yields for every period where data is available. The model matches the time series of bond yields in the data closely. It matches nearly perfectly the 1-quarter and 5-year bond yield which are part of the state space.

Figure D2 shows that the model also does a good job matching real bond yields. These yields are available over a much shorter sample in the data, and we only plot the relevant subsample for the model-implied yields as well.

The top panels of Figure D3 show the model's implications for the average nominal (left panel) and real (right panel) yield curves at longer maturities. These long-term yields are well behaved. The bottom left panel shows that the model matches the dynamics of the nominal bond risk premium, defined as the expected excess return on five-year nominal bonds. The compensation for interest rate risk varies substantially over time, both in data and in the model. The bottom right panel shows a decomposition of the yield on a five-year nominal bond into the five-year real bond yield, annual expected inflation over the next five years, and the five-year inflation risk premium. The importance of these components fluctuates over time. This graph shows the secular rise and fall of real bond yields, with a peak in the early 1980s.

Figure D1: Dynamics of the Nominal Term Structure of Interest Rates




*Note:* The figure plots the observed and model-implied nominal bond yields. Data are from FRED: constant-maturity Treasury yields, daily averages within the quarter.

### E.2.2 Equity Factors and Housing Wealth Pricing

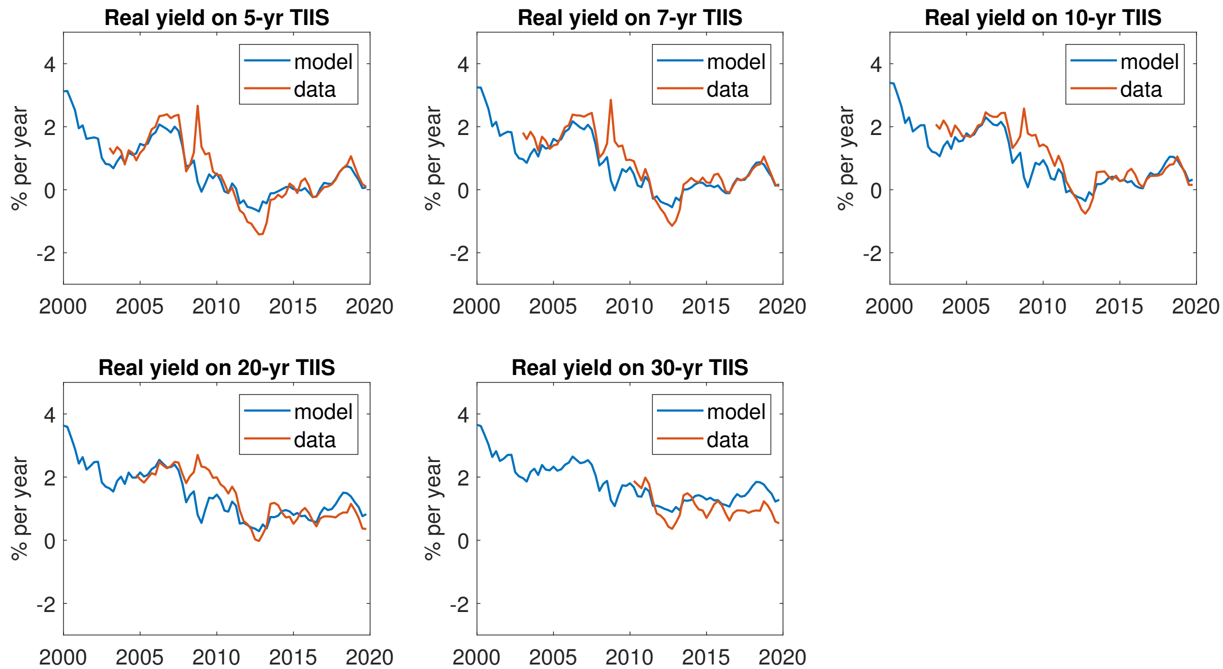
The VAR contains both the log price-dividend ratio and log dividend growth for five equity risk factors (the aggregate stock market, small stocks, growth stocks, value stocks, and infrastructure stocks), and residential real estate wealth. Together these two time-series imply a time-series for log returns through the definition of a log stock return. Hence, the VAR implies linear dynamics for the expected excess stock return, or equity risk premium, for each of these seven assets. We estimate market prices of risk to match the VAR-implied risk premium levels and dynamics.

The price of a stock equals the present-discounted value of its future cash-flows. By value-additivity, the price of the aggregate stock index, $P_t^m$, is the sum of the prices to each of its future cash-flows $D_t^m$. These future cash-flow claims are the so-called market dividend strips or zero-coupon equity (Wachter, 2005). Dividing by the current dividend $D_t^m$:

$$\frac{P_t^m}{D_t^m} = \sum_{\tau=1}^{\infty} P_{t,\tau}^d \tag{25}$$

$$\exp\left(\overline{pd} + e'_{pd^m} z_t\right) = \sum_{\tau=0}^{\infty} \exp\left(A_\tau^m + B_\tau^{m\prime} z_t\right), \tag{26}$$

Figure D2: Dynamics of the Real Term Structure of Interest Rates

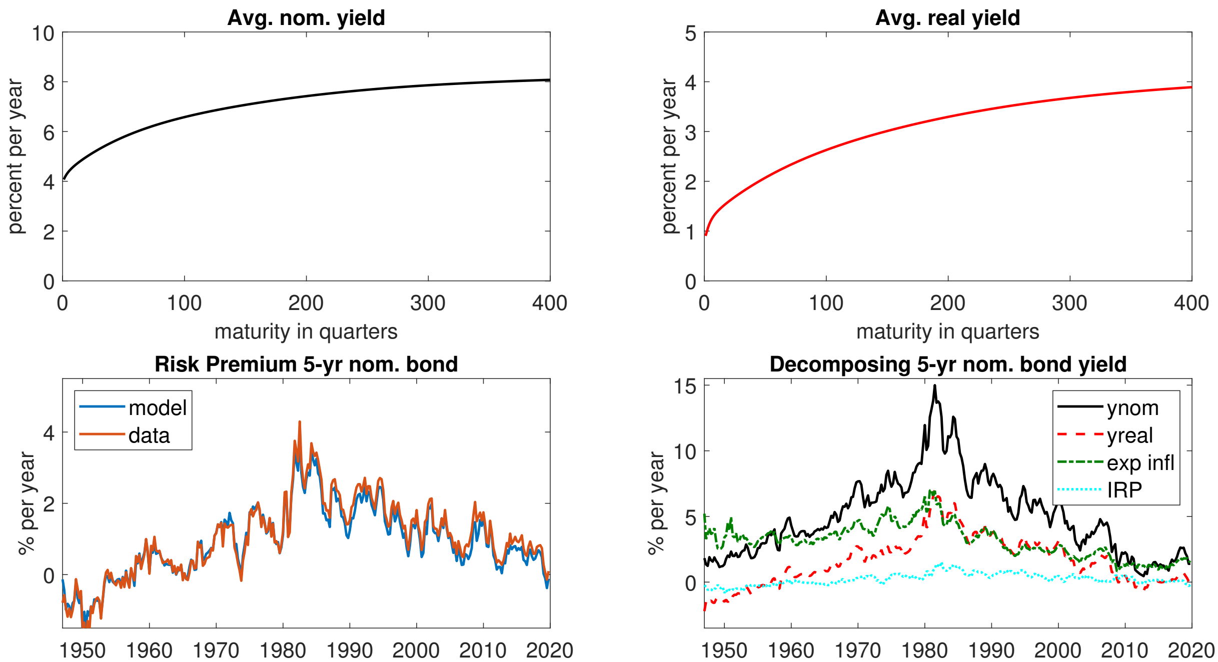



*Note:* The figure plots the observed and model-implied real bond yields. Data are from FRED: constant-maturity Treasury inflation-indexed bond yields, daily averages within the quarter.

where $P^d_{t,\tau}$ denotes the price of a $\tau$-period dividend strip divided by the current dividend. The log price-dividend ratio on each dividend strip, $p^d_{t,\tau} = \log\left(P^d_{t,\tau}\right)$, is affine in the state vector and the coefficients $\left(A^m_\tau, \boldsymbol{B}^m_\tau\right)$ follow an ODE. Since the log price-dividend ratio on the stock market is an element of the state vector, it is affine in the state vector by assumption. Equation (26) restates the present-value relationship from equation (25). It articulates a non-linear restriction on the coefficients $\{(A^m_\tau, B^m_\tau)\}_{\tau=1}^{\infty}$ at each date (for each state $z_t$), which we impose in the estimation. Analogous present value restrictions are imposed for each of the other four equity factors, and for housing wealth.

If dividend growth were unpredictable and its innovations carried a zero risk price, then dividend strips would be priced like real zero-coupon bonds. The strips' dividend-price ratios would equal yields on real bonds with the coupon adjusted for deterministic dividend growth. All variation in the price-dividend ratio would reflect variation in the real yield curve. In reality, the dynamics of real bond yields only account for a small fraction of the variation in the price-dividend ratio, implying large prices of risk associated with shocks to dividend growth that are orthogonal to shocks to bond yields. Hence, matching price-dividend ratios (price levels) and expected returns (price changes) allow us to pin down the market prices of risk associated with orthogonal dividend growth shocks (shocks to the state variables in rows 6, 8, 10, 12, 14, 16, and 18 of the VAR).

Figure D3: Long-term Yields and Bond Risk Premia



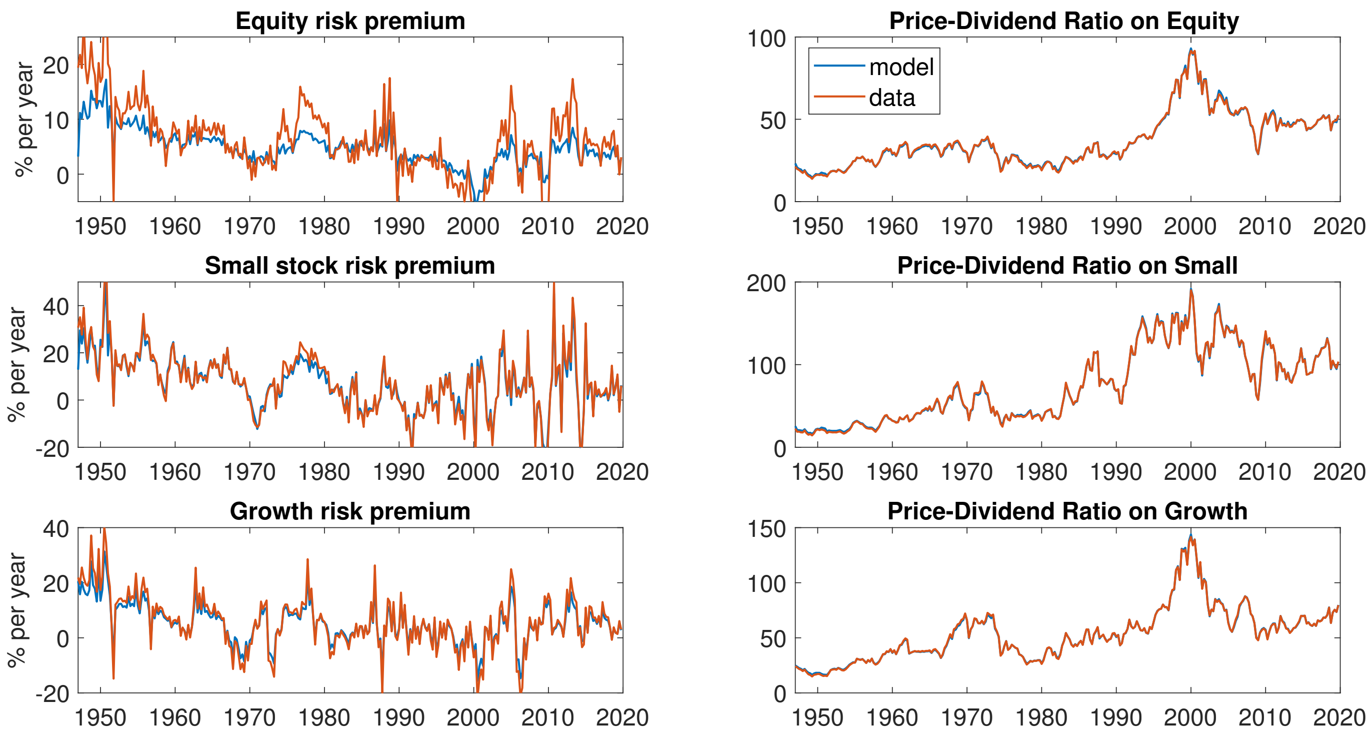

*Note:* The top panels plot the average bond yield on nominal (left panel) and real (right panel) bonds for maturities ranging from 1 quarter to 400 quarters. The bottom left panel plots the nominal bond risk premium in model and data. The bottom right panel decomposes the model's five-year nominal bond yield into the five-year real bond yield, the five-year inflation risk premium and the five-year real risk premium.

Figures D4 and D5 show the equity risk premium, the expected excess return, in the left panels and the price-dividend ratio in the right panels. The various rows cover the five equity indices and the housing wealth series we price. The dynamics of the risk premia in the data are dictated by the VAR. The model chooses the market prices of risk to fit these risk premium dynamics as closely as possible alongside with the price-dividend ratio levels. The price-dividend ratios in the model are formed from the price-dividend ratios on the strips of maturities ranging from 1 to 3600 quarters, as explained above. The figure shows an excellent fit for price-dividend levels and a good fit for risk premium dynamics. Some of the VAR-implied risk premia have outliers which the model does not fully capture. This is in part because the good deal bounds restrict the SDF from becoming too volatile and extreme. We note large level differences in valuation ratios across the various stock factors, as well as big differences in the dynamics of both risk premia and price levels, which the model is able to capture well.

### E.2.3 Pricing Claims to Aggregate Consumption and Labor Income

Shocks to the growth rate in consumption/GDP (labor income/GDP) ratio are priced only to the extent that they are correlated with other priced sources of risk. The innovation to the change

Figure D4: Equity Risk Premia and Price-Dividend Ratios (1/2)

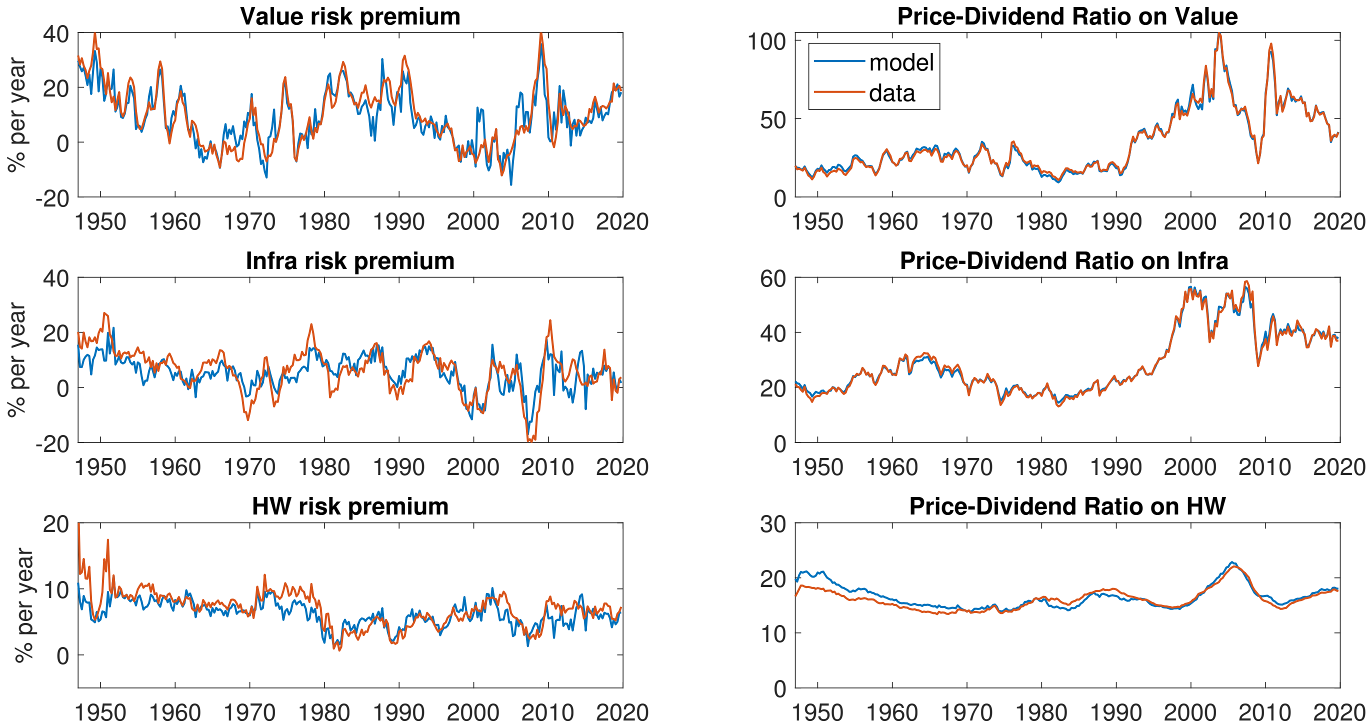



*Note:* The figure plots the observed and model-implied equity risk premium on the overall stock market, small stocks, and growth stocks in the left panels, as well as the corresponding price-dividend ratio in the right panels. The model is the blue line, the data are the red line.

in the consumption/GDP (labor income/GDP) ratio that is orthogonal to all prior shocks is not priced. Since consumption/GDP growth and labor income/GDP growth appear last in the VAR and the model includes many sources of priced aggregate risk, those innovations are as small as possible.

Figure D6 plots the annual price-dividend ratios on the claims to GDP, aggregate consumption, and aggregate labor income. It contrasts these valuation ratios to those for the aggregate stock market, and housing wealth. The valuation ratios of GDP, aggregate consumption, and aggregate labor income claims are all highly correlated. They are high at the start of the sample, low in the early 1980s, and high at the end of the sample. Since total wealth is a claim to aggregate consumption, this suggests that expected returns on total wealth were highest in the early 1980s and have been falling ever since.

Figure D5: Equity Risk Premia and Price-Dividend Ratios (2/2)




*Note:* The figure plots the observed and model-implied equity risk premium on value stocks, infrastructure stocks, and housing wealth in the left panels, as well as the corresponding price-dividend ratio in the right panels. The model is the blue line, the data are the red line.

### E.2.4 Cash-flow Duration

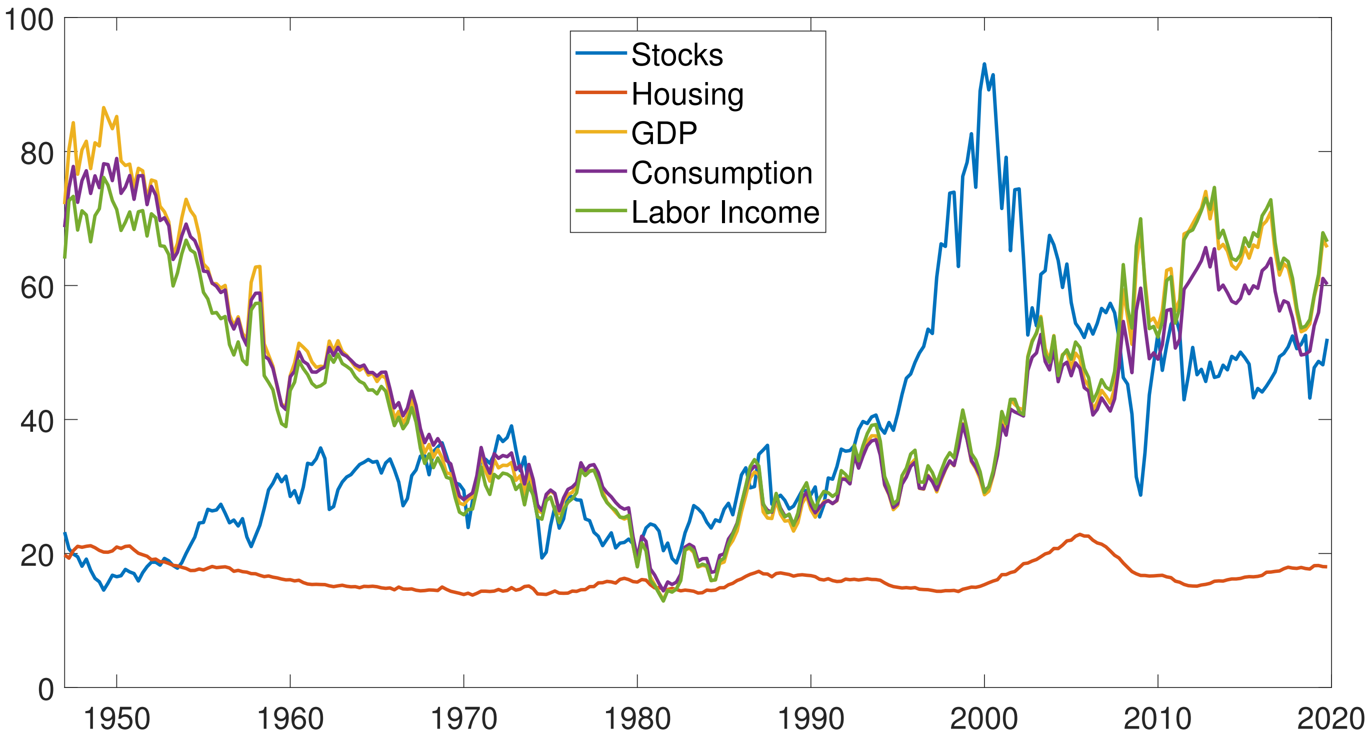
The (McCauley) duration is the weighted average time for an investor to receive cash flows. For the aggregate stock market, this measure is computed as follows:

$$D_t^{CF,m} = \sum_{\tau=1}^{\infty} w_{t,\tau}\tau, \qquad w_{t,h} = \frac{P_{t,\tau}^d}{\frac{P_t^m}{D_t^m}} = \frac{\exp\left(A_\tau^m + B_\tau^{m\prime} z_t\right)}{\exp\left(\overline{pd} + e_{pd^m}' z_t\right)}$$

where $P_{t,\tau}^d$ is the price-dividend ratio of a $\tau$-period dividend strip. Since durations are usually expressed in years while time runs in quarters in our model, we divide by 4. Duration is defined analogously for the other four equity indices, housing wealth, and for the GDP, consumption, and labor income claims. Note that for a nominal or real zero-coupon bond of maturity $\tau$, $D_t^{CF} = \tau$.

Figure D7 The figure plots the model-implied time series of cash-flow durations on the overall stock market, small stocks, growth stocks, value stocks, infrastructure stocks, housing wealth, the GDP claim, the aggregate consumption claim, and the aggregate labor income claim. Durations tend to be positively correlated with the price-dividend ratios: high at the start of the sample, lowest in the early 1980s, and high at the end of the sample. The duration of housing wealth is

Figure D6: Valuation Ratios




*Note:* The figure plots the annual price-dividend ratios on the aggregate stock market, housing wealth, and on claims to GDP, aggregate consumption, and aggregate labor income.

highest during the housing boom in 2003–2007 when the valuation ratio of housing peaks. It then falls sharply in the housing bust before rising again in the housing boom that starts in 2013.

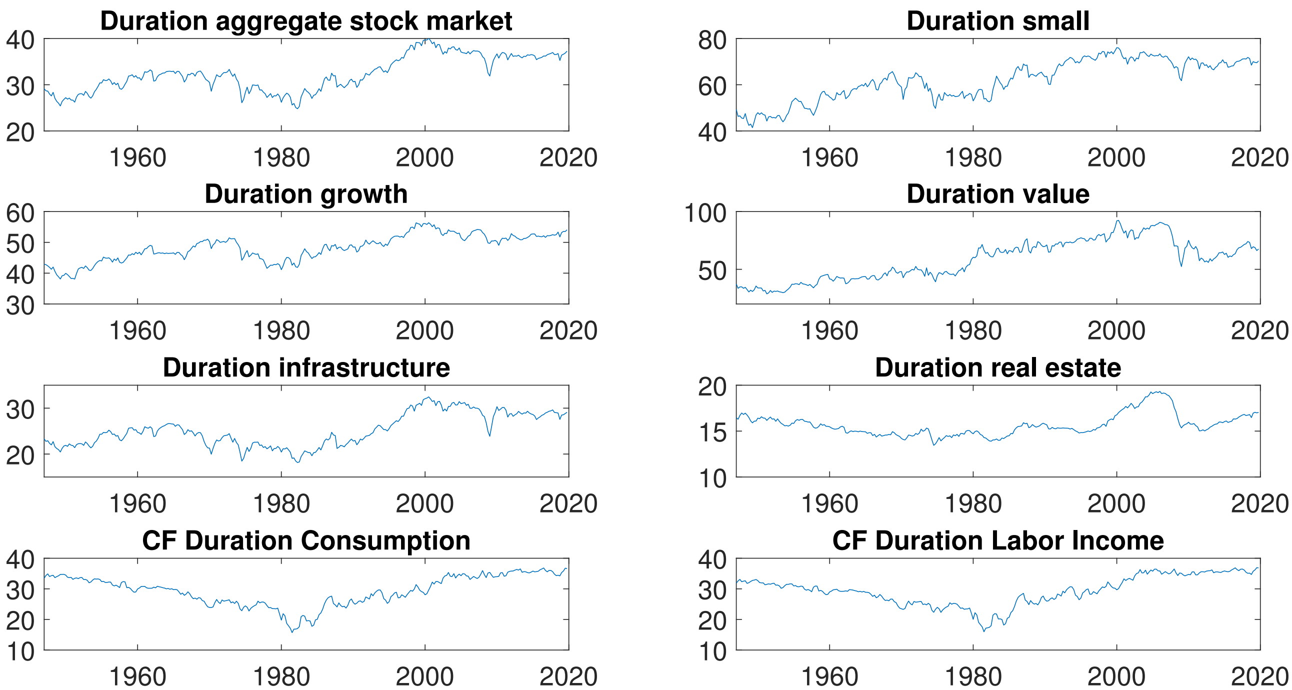
### E.2.5 Market Price of Risk Estimates

The market prices of risk are pinned down by the moments discussed in the main text. Here we report and discuss the point estimates. Note that the prices of risk are associated with the orthogonal VAR innovations $\varepsilon \sim \mathcal{N}(0, I)$. Therefore, their magnitudes can be interpreted as (quarterly) Sharpe ratios. The constant in the market price of risk estimate $\widehat{\Lambda_0}$ is:

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.11 | 0.00 | -0.36 | 0.06 | 0.00 | 0.43 | 0.00 | -0.01 | 0.00 | 0.12 | 0.00 | 0.25 | 0.00 | 0.26 | 0.00 | 2.76 | 0.00 | 0.00 | 0.00 |

The matrix that governs the time variation in the market price of risk is estimated to be $\widehat{\Lambda_1} =$:

Figure D7: Cash-Flow Duration




*Note:* The figure plots the model-implied time series of cash-flow durations on the overall stock market, small stocks, growth stocks, value stocks, infrastructure stocks, housing wealth, the GDP claim, the aggregate consumption claim, and the aggregate labor income claim. The duration is expressed in years.

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0. |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0. |
| 19.3 | 16.5 | -31.7 | -250.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0. |
| 37.5 | 15.1 | 0.0 | 3.7 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 2.2 | 19.5 | 0.9 | 0. |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0. |
| 23.3 | 4.0 | -29.7 | -160.0 | -0.9 | 12.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.4 | 3.6 | 0.1 | 1. |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0. |
| -5.0 | -1.6 | 1.5 | -21.1 | 0.6 | 2.0 | -0.5 | 8.8 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 2.9 | -4.0 | 0.2 | 0. |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0. |
| 1.1 | 34.3 | -23.7 | 14.0 | 0.9 | -11.2 | -0.1 | 1.8 | -1.1 | 16.8 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.8 | 1.2 | -0.1 | -0. |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0. |
| 0.0 | 34.7 | -11.1 | 34.1 | 0.8 | -15.6 | 0.1 | -1.1 | -0.3 | 0.5 | -2.0 | 0.6 | 0.0 | 0.0 | 0.0 | 0.0 | -1.5 | 1.3 | 17.6 | 0. |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0. |
| 10.1 | 46.0 | -40.9 | -20.8 | 7.6 | -10.9 | 1.0 | -2.4 | -5.9 | -2.3 | 0.5 | 1.6 | -4.4 | 0.1 | 0.0 | 0.0 | 3.3 | 3.7 | -5.2 | -2. |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0. |
| 8.6 | 27.2 | -113.3 | 66.6 | 5.7 | 1.3 | -2.2 | -5.7 | -0.4 | -3.0 | 0.8 | 0.2 | -3.3 | 0.0 | 1.3 | 0.4 | 0.1 | -0.2 | -15.0 | -6. |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0. |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0. |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0. |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0. |