Setbacks, Shutdowns, and Overruns*

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March 2021

Abstract

We study optimal project management in a setting where random setbacks arise naturally during development (e.g., software, construction, or manufacturing). The contractor can shirk, and the sponsor cannot observe the occurrence of setbacks and must rely on unverifiable reports. The optimal dynamic mechanism provides incentives via a cost-plus-award-fee contract featuring a soft deadline or time budget and a terminal payment that is linear in the time remaining on the schedule. Late-stage setbacks require randomization between project cancellation and extension. Because randomization may happen repeatedly, the project can run far beyond its original expected duration and budget and yet be canceled yielding no value. If commitment to randomization probabilities is not possible, the sponsor optimally commits *more* time and resources to the project, even though it is less valuable to her. Our analysis suggests that although overruns and cancellations are commonly viewed as failures of project governance, such outcomes are necessary features of optimal project management.

JEL Classifications: C72, D21, D86, M11
Key Words: Project Management, Dynamic Agency, Soft Deadline, Cost-Plus-Award-Fee Contract, Schedule Slippage, Optional Stopping Theorem.

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Hofstadter's Law: *It always takes longer than you expect, even when you take into account Hofstadter's Law.*

—Douglas Hofstadter, *Gödel, Escher, Bach: An Eternal Golden Braid* (1979)

1 Introduction

Broad and expanding swaths of the modern economy are dedicated to the planning and execution of projects, "temporary endeavor[s] undertaken to create a unique product, service or result . . . The development of software for an improved business process, the construction of a building or bridge, the relief effort after a natural disaster, the expansion of sales into a new geographic market all are projects."[1] Given the current and growing significance of this mode of production, it is important to understand its intrinsic characteristics and – in particular – how best to improve its efficacy. Indeed, the annals of project management are rife with jobs that ran notoriously over time and over budget, some of which were ultimately canceled by their sponsors resulting in little if any residual value. For example, what might be "the most highly publicized software failure in history" (Goldstein, 2005) is the FBI's contracting debacle with the Science Applications International Corporation, (SAIC) to develop a virtual-case-file-(VCF) system. Irigoyen (2017) summarizes the VCF project failure as going through "significant management and implementation problems and cost overruns, which culminated in the cancellation of the project in 2005, with little to show for the USD170 million investment." The FBI Director at the time, Robert S. Mueller, III, testified before a Congressional subcommittee that he was disheartened by "the setbacks which have plagued this program" but that he was confident that a future attempt to develop a VCF system would be successful.[2]

The FBI is hardly alone in its project management woes. For instance, "According to a 2017 report from the Project Management Institute, 14 percent of IT projects fail. However, that number only represents the total failures. Of the projects that didn't fail outright, 31 percent didn't meet their goals, 43 percent exceeded their initial budgets, and 49 percent were late" (Greene, 2019). The same pattern exists in large scale construction and industrial manufacturing, as illustrated by the high profile cases listed in Table 1.

[1]Excerpted from What is Project Management? (Project Management Institute, 2020)

[2]In his testimony, Mueller blamed the type of contract, saying "The contract [with SAIC] was based on hours worked – cost plus an award fee. We now know these types of contracts are difficult to manage."

Table 1

Project	Time Overrun	Cost Overrun	Source
Boston's Big Dig	9 years	190%	Haynes (2007)
Scottish Parliament Building	3 years	1000%	Ahiaga-Dagbui and Smith (2014)
Sydney Opera House	10 years	1400%	Wild (2015)
Boeing 787 DreamLiner	3 years	200%	Shenhar et al. (2016)
Berlin's Brandenberg Airport	10 years	300%	Brandt (2020)

According to Lineberger and Hussain (2016), "The combined cost overrun for Major Defense Acquisition programs in 2015 was $468 billion ... with an average schedule delay of 29.5 months." Importantly, "setbacks are a near-universal, and universally costly, experience ... large capital projects are typically 20 months late, and 80% over the original authorized budget" (Billante, 2017). More, setbacks can cause a project to be canceled leaving the sponsor with huge bills and often nothing else. A prime recent example is South Carolina's V.C. Summer nuclear power plant construction project, canceled in 2017 after a series of major setbacks and cost overruns, saddling taxpayers with a bill of $9 billion and "nothing to show for it" (Lacy, 2019).

In this paper we argue that project setbacks, overruns, and cancellations are not always the product of incompetence or inattention, but – at least to some degree – are unavoidable consequences of optimal project governance in the face of agency frictions. In particular, we introduce a model of project development in which setbacks arise naturally as part of the production process. Examples include discovering: adverse site conditions (construction), a design feature won't work as intended (manufacturing), or incompatibility of certain off-the-shelf subroutines (software engineering). Due to unforeseeable contingencies such as these, the amount of time and resources required to complete the project are necessarily uncertain. Hence, the timeline as well as the initial budget consist of estimates rather than known quantities. Indeed, the inherently random nature of project schedules and costs has been recognized by planners and engineers for decades (Malcolm, Roseboom, Clark, and Fazar, 1959). For instance, PERT (Program Evaluation and Review Technique) is a still frequently used method for project planning and management that was first developed in 1957 by the U.S. Navy's Special Projects Office to manage the scheduling and cost uncertainties of the nascent Polaris nuclear submarine program (Navy, 1958a,b).

In our model, as in practice, the sponsor (the principal) must hire a contractor (the agent) to run the project on her behalf. Both parties are risk-neutral, but the agent is protected by limited liability. Setbacks arrive randomly according to a Poisson process with known intensity. There is a flow cost of running the project, and the project is completed whenever a span of time $\bar{X}$ passes without the arrival of a setback. The first-best policy in this environment is straightforward. The project should be started and run until completed if and only if the value of the finished project to the sponsor exceeds the flow cost of operation times the expected duration.

The contractual setting we investigate is marked by both hidden actions and hidden states. The principal is unable to observe the progress of the project or the occurrence of setbacks herself, and must rely on unsubstantiated reports from the agent. However, delivery of the completed and working project is verifiable – the principal can use the software, fly the plane, or work in the building once it is complete. Because the principal cannot observe the status of the unfinished project, the agent may surreptitiously divert the flow of operating capital to garner private benefits instead of advancing the project.[3] The combination of hidden actions and hidden states gives the agent broad scope for committing moral hazard without fear of detection. Specifically, he may cover up the interruption of progress associated with resource diversion either by submitting false reports of setbacks or delaying the reports of real ones. Thus, the principal's problem is to write a contract, contingent only on the passage of time and potential project delivery, that induces the agent to work efficiently and report honestly.

The crucial incentive constraint is what we label the *No-Postponed-Setbacks* (NPS) condition. This constraint requires that whenever a setback occurs, the agent prefer to report it immediately rather than divert resources for any length of time and report it later. We show that (NPS) always binds under an optimal incentive scheme for the principal. This has several important implications. First, it implies that the agent also never prefers to cover up resource diversion with claims of false setbacks; that is, binding (NPS) is necessary and sufficient for incentive compatibility. Second, it allows us to fully characterize the optimal contract. Interestingly, Director Mueller's 2005 congressional testimony not withstanding, the optimal incentive mechanism in this context can be implemented with a cost-plus-award-fee contract of the kind the FBI signed with SAIC.[4]

Under the optimal contract, the principal gives the agent a soft deadline or time budget

[3]Interestingly, Project Management Institute identifies that "team member procrastination" accounts for 11% of all IT project failures.

[4]"A cost-plus-award-fee contract is a cost-reimbursement contract that provides for a fee consisting of (a)a base amount (which may be zero) fixed at inception of the contract and (b) an award amount, based upon a judgmental evaluation by the Government, sufficient to provide motivation for excellence in contract performance." (The U.S. General Service Administration FAR 16.401). We comment on this implementation more formally in the discussion following Proposition 1 below.

for delivering the completed project and commits to pay the flow cost of operation. The contract ultimately ends for one of two reasons, either because the agent successfully delivers the completed project or because it is canceled by the principal. If the agent delivers the project prior to cancellation, then he is paid a *linear* reward consisting of a fixed fee and an incentive award proportional to the time remaining before the soft deadline is exhausted. If the time remaining on the soft deadline is greater than $\bar{X}$ (the required uninterrupted development time), the principal takes no action. If, the time remaining is less than $\bar{X}$ and a setback occurs, then project completion before the clock runs out is impossible. At this point, the contract calls for a random termination procedure under which the project is either canceled with a terminal payment of zero to the agent or the deadline is extended to $\bar{X}$. All subsequent reports of setbacks are treated similarly.

Thus, while the optimal incentive contract induces the agent to work diligently and report honestly, it may, nevertheless, result in the type of unfortunate outcomes observed in our leading examples. In other words, schedule and cost overruns, and even cancellations that yield no useful output, are *features* of an optimal contract. The feature exist because randomization is necessary: if the contract had a deterministic deadline, then a late-stage setback would render project completion impossible and the agent would *shirk out the clock.* On the other hand, if there was some sequence of reports that enabled the project to run indefinitely, then the agent would make those reports and shirk forever. The only solution is random termination, which yields the possibility of both overruns and inefficient cancellations.

Our analysis utilizes novel methods that do not involve the usual dynamic programming and differential equation techniques to characterize the principal's value function. In particular, we identify two fundamental martingales and invoke the optional stopping theorem. The principal's expected payoff equals the probability of project completion times the first-best value of the project net of expected agency rents. The probability of project completion is increasing, concave, and approaches 1 as the length of the soft deadline, S, tends to infinity. On the other hand, agency rents increase linearly in S. Hence, there exists a unique optimal initial time budget S^* to assign to the agent at project inception. Except in the case when $S^* = \bar{X}$ is optimal, the soft deadline is strictly longer than the expected duration of the project. In other words, the principal builds in an allowance for some schedule *slippage* in the initial contract.

Interestingly, the principal's value function is a concave polynomial with kinks at $S = n\bar{X}$. These kinks imply that the optimal initial time budget S^* is an integer multiple of $\bar{X}$ for a non-negligible set of parameters – there are *focal contract lengths* that are multiples of the best case duration. We use these observations to identify the conditions under which $S^* = \bar{X}$ is optimal; this is a *short-leash* contract in which the soft deadline equals the expected duration of the project and every reported setback results in cancellation with positive probability. Although a short-leash contract has expected duration of $\bar{X}$, the support of the stopping time is unbounded due to the probability of project extension. Hence, even when the principal commits to keep the agent on a short leash, arbitrarily large cost and schedule overruns occur with positive probability. Importantly, every optimal contract possesses a short-leash phase that is triggered whenever a setback occurs sufficiently late in the schedule (i.e., when $S_t < \bar{X}$).

The optimal contract requires the principal to commit to randomized extension or cancellation with explicit probabilities that are a function of the time remaining when a setback occurs. However, absent commitment, the principal would strictly prefer to keep the agent working on the project by extending it rather than canceling it. So, we vary the baseline model by relaxing the assumption of full commitment and investigate a setting in which randomization is feasible but not verifiable. In this case, randomization by the principal is incentive compatible if and only if the agent, upon receiving an extension, himself randomizes between continuing to work and shirking out the clock.

While relaxing commitment is clearly harmful to the principal, we are able to use our methods for characterizing the value function to show that she optimally grants the agent a *longer* initial schedule in this setting. Intuitively, she does so in order to raise the likelihood that the project will be completed before the short-leash phase of the schedule is ever reached. Put differently, granting large S^* not only increases the chance the agent will successfully complete the project, it decreases the chance that lack of commitment will ever come into play. Although this logic is compelling, one might easily have expected the reverse finding. Absent commitment, the project is completed with lower probability under any given schedule and is, therefore, worth less to the principal. Given this, it is somewhat surprising that the principal optimally responds to her lack of commitment by devoting more – rather than less – time and money to the less valuable endeavor.

The remainder of the paper is organized as follows. Related literature is reviewed below. In Section 2 the model of setbacks is presented and the corresponding first-best policy is characterized. The agency environment is described in Section 3. In Section 4 the optimal contract is identified and the associated value function for the principal and its properties are derived. We investigate the variant of the model under which randomization is not verifiable in Section 5. Section 6 contains some concluding remarks. The proofs of all results can be found in the appendix.

Literature Review The literature on the optimal provision of incentives in dynamic environments is extensive and active. Pioneering articles responsible for moving it forward at various stages include Green (1987), Spear and Srivastava (1987), Phelan and Townsend (1991), Quadrini (2004), Clementi and Hopenhayn (2006), DeMarzo and Sannikov (2006), Biais, Mariotti, Plantin, and Rochet (2007) and Sannikov (2008).

Bergemann and Hege (1998) investigate venture capital financing in a discreet-time model where the arrival of revenues depend on whether the project is good or bad and whether the entrepreneur (agent) works or shirks. The dynamic agency costs may be high and lead to an inefficient early termination of the project. Biais et al. (2010) analyze a model in which large observable losses may arrive via a Poisson process, and an agent must exert hidden effort in order to minimize the likelihood of their arrival. Toxvaerd (2006) considers a setting in which a finite number of observable arrivals are needed in order to complete a project. In his setting, the agent is risk averse and the optimal contract trades off optimal risk-sharing for incentive provision, but does not involve deadlines or inefficient termination. In contrast with these papers, the Poisson shocks in our proposed model are privately observed by the agent and arise as an unavoidable consequence of the production process – that is they are *discoveries*. The potential for their occurrence essentially gives the agent cover to commit moral hazard; i.e., to make plausible excuses for why project completion has been delayed.

Some other recent papers share features with the environment we study. Lewis (2012) investigates a model of delegated search in which the agent is motivated by a hard deadline and a reward that decreases over time. Mason and Välimäki (2015) study a dynamic moral hazard model where the agent's effort in each period corresponds to the probability that the project succeeds in that period. Rahmani, Roels, and Karmarkar (2017) investigate multi-agent collaboration in a discrete time model with a finite horizon where progress is observable and contractible. Vasama (2017) analyze contracting in an environment where the agent can secretly divert output for private benefit. The optimal contract relies on an inefficient termination threat to give the agent incentives not to skim, which relaxes over time following good performance. Varas (2018) studies a setting in which an agent chooses at each instant between working to make a breakthrough that results in a high quality product and shirking that results in a low quality one. The principal in his model optimally defers compensation upon delivery in order to acquire evidence regarding product quality. In a similar vein, Hoffmann, Inderst, and Opp (2020) consider a setting in which an agent takes a one-time action and the principal subsequently observes an informative dynamic process that is used to determine his ultimate reward.

More closely related are four recent papers (one publication and three working papers) that explore the optimal deadline for a project in the context of dynamic agency. The published article is Green and Taylor (2016) who study a setting where a project must have two Poisson breakthroughs in order to be completed. The agent hired to run the project privately observes the occurrence of the first breakthrough, or what the authors call progress. As in our setting, the agent can surreptitiously divert the principal's flow of investment in the project for private benefit, which delays project completion. However, once the agent reports arrival of the first breakthrough in Green and Taylor (2016), there is no turning back, which limits his scope for further manipulation. We consider a richer environment where progress corresponds to a continuum of states and in which a potentially infinite number of setbacks may occur in rout to project completion. Thus our agent may repeatedly report the occurrence of false setbacks or repeatedly postpone reporting the occurrence of real ones, or any combination of such behavior.

In an insightful working paper, Madsen (2020) studies how an organization should optimally manage a project of uncertain scope when advised by an expert with private information about the project's state who prefers to prolong his employment. In this model, a project turns from "good" to "bad" stochastically over time. The agent is a "advisor", who possesses information regarding whether the project quality has changed. By contrast, in the setting we investigate, it is common knowledge that the state of the project is "good" from the outset, and never changes. Our agent is not an advisor hired to monitor whether project quality has declined – His expertise resides in the ability to operate the project itself.

Mayer (2019) presents a dynamic contracting model in which a project succeeds if it survives until the completion date. The completion date is the outcome of a random variable that is unknown to both the principal and agent. While the project is in operation, the agent exerts unobservable precautionary effort in order to reduce the arrival rate of a failure shock that will kill the project before it reaches completion. Project completion is contractually verifiable, but the arrival of the failure shock is privately observed by the agent. As in Madsen (2020), the principal must provide incentives for the agent to report that the project has gone bad and should be terminated. Our environment differs along a number of salient dimensions. Rather than the arrival of a single failure shock, our agent may observe numerous setbacks, none of which render project completion infeasible. Because the completion date in the absence of setbacks is common knowledge in our setting, the agent is never rewarded for reporting one. Indeed, if a setback occurs in our model, then the agent will eventually have to report it or he will otherwise be fired for not delivering the project when the principal expects it to be finished.

Yet a third recent paper featuring a single privately observed transition is Sinander and Curello (2020). Similar in spirit but opposite in application to Madsen (2020), in this model,, a technological breakthrough occurs exogenously at some random time witnessed only by the agent. The principal would like to adopt the innovation as soon as possible, but the agent prefers the *status quo* technology, other things equal. Hence, the agent must be incentivized, through non-monetary means, to disclose the arrival of the innovation. As in our setting, a deadline can play an important part in inducing timely revelation. The environment we investigate is, however, very different both in terms of application and technical details. The agent in our model is a contractor hired to complete a project who may privately observe numerous setbacks an who is tempted to use his informational advantage to cover up lack of progress due to his own shirking.

2 The Model and The First-Best

2.1 The Project

A risk-neutral principal (she) hires a risk-neutral agent (he) over an infinite horizon to work on a project. The principal has deep pockets, and the agent has no wealth and is protected by limited liability. The project requires accumulated progress $\bar{X}$ before it is completed; $\bar{X}$ is the project's *scope*. As the agent works on the project, *progress* X accumulates deterministically. However, *setbacks* occur, following a Poisson process with arrival rate λ, which is the setback *frequency*. A setback at t resets progress from X_t to 0. When progress reaches $\bar{X}$, the project is complete and results in a monetary payoff of R to the principal. While the project is in operation, the principal must pay a flow cost of c to keep it moving forward.

Two points are worth highlighting. First, for simplicity we assume that an incomplete project has no value to the principal. Second, setbacks result naturally as a result of unforeseeable contingencies, and, in particular, setbacks are not due to the negligence or indolence of the agent. Examples include discovering: adverse site conditions (construction), a design feature won't work as intended (manufacturing), or incompatibility of certain off-the-shelf subroutines (software engineering). Such problems can be unearthed only through working on the project.

The potential for moral hazard in this setting stems from the ability of the agent to surreptitiously divert the resource flow c to his own private benefit and cover the resulting cessation in progress by misinforming the principal about the occurrence of setbacks. Formally, the project's true progress follows

$$dX_t = a_t(dt - X_t dN_t)$$

where $a_t \in \{0,1\}$ denotes the agent's private action. $a_t = 0$ represents shirking, which corresponds to diversion of the resource stream c, while $a_t = 1$ represents working, which corresponds to using the funds to develop the project. Shirking yields the agent a private flow benefit of b:

Assumption 1 $b < c$*, so diverting funds (shirking) is socially inefficient.*

Whenever the agent shirks, progress on the project remains constant; i.e., setbacks are discovered only if the agent is working. Both the principal and agent are perfectly patient and possess outside options of zero.[5]

2.2 The First-Best

We begin by characterizing the first-best policy and expected payoff for the principal. If the agent's actions are publicly observable, then the principal can induce his compliance without incurring additional cost. Clearly, if it is worth starting the project in the first place, then it is worth running it until it is eventually completed. Suppose that the project is operated until completed and let F^{FB} be the value to the principal at inception. Then we have

$$F^{\mathrm{FB}} = \int_0^{\bar{X}} \lambda e^{-\lambda X}(-cX + F^{\mathrm{FB}})dX + e^{-\lambda \bar{X}}(R - c\bar{X}). \tag{1}$$

The integral in this expression corresponds to the possibility that a setback occurs before the project is finished, resetting progress X to 0, at which point the project must re-start. The time between setbacks is exponentially distributed with intensity λ, with the principal paying c for the duration of development. The project is completed without incurring a

[5] Our results hold if the principal and agent share subjective discount rate, $r > 0$. The optimal contract is a time budget (soft deadline) with random extensions and termination, just as with $r = 0$. In fact, our economy with $r = 0$, including the principal's payoffs and policies, is attained as the limit of economies as $r \to 0$. These results are available upon request.

setback with probability $e^{-\lambda \bar{X}}$, in which case the principal's net payoff is $R - c\bar{X}$. Performing the integration and solving yields

$$F^{\mathrm{FB}} = R - \frac{c}{\lambda}\left(e^{\lambda \bar{X}} - 1\right), \tag{2}$$

This expression is easily interpreted. Because the project is operated until it is complete, the principal eventually obtains R for sure. Her expected cost when initiating the project is the flow cost c times the project's total expected duration $\frac{1}{\lambda}\left(e^{\lambda \bar{X}} - 1\right)$. It is straightforward to verify that expected project duration is increasing in $\bar{X}$ and λ and that

$$\lim_{\lambda \to 0} \frac{e^{\lambda \bar{X}} - 1}{\lambda} = \bar{X}.$$

Thus, a project with larger scope or higher frequency of setbacks has a longer expected duration, while a project for which setbacks never occur has a deterministic duration of $\bar{X}$.

It follows immediately that the first-best policy is to start the project and run it until completed if and only if the right side of (2) is positive. However, in the second-best, incentivizing the agent involves paying him rents, so a somewhat stronger assumption on the gross value of the project to the principal is required:

Assumption 2

$$R > \frac{c+b}{\lambda}\left(e^{\lambda \bar{X}} - 1\right) \tag{3}$$

As we shall see, this condition is both necessary and sufficient for the principal to be willing to hire the agent to run the project. Interestingly, although Assumption 2 implies that the principal is willing to incur the flow cost $c + b$ until the project is eventually completed, this is not, in fact, the outcome implemented by an optimally designed contract.

3 Unobservable Progress and Incentive Compatibility

3.1 Contracts and Reports

The agent's expertise and work on the project give him an ability to observe the project's status that the principal lacks. Instead, the principal must rely on status reports by the agent. We assume:

Assumption 3 *The principal cannot observe the agent's choice of action* $a_t \in \{0, 1\}$*, the state of the project* X_t*, or the occurrence of setbacks. The principal can observe project completion only upon delivery, which is contractually verifiable.*

This assumption provides the agent with a great deal of latitude to commit malfeasance without detection. For instance, he could shirk for some time and then falsely claim a setback to cover up the lack of progress; or, following a real setback, the agent could shirk for a time before reporting it.

However, the fact that the principal knows how long it takes to complete the project in the absence of a setback and that a completed project is verifiable does place some discipline on the agents actions and reports. We assume

Assumption 4 *If the agent is verifiably detected misallocating resources or lying about the project's progress, then he is terminated at that point without severence.*

In other words, if the agent deviates from the principal's recommended actions, (shirks or lies), then the outcome generated must be consistent with *some* feasible path under the recommended actions. For example, the agent cannot shirk for an $\bar{X}$-length of time without reporting a false setback or he will be fired for not delivering the completed project.

The agent makes a report of the project's current state, $\hat{X}_t$. Given the project's true evolution (1), reporting the path of $\hat{X}$ implicitly reports actions ($\hat{a}$) and setbacks ($\hat{N}$), with

$$d\hat{X}_t = \hat{a}_t(dt - \hat{X}_t d\hat{N}_t) \tag{4}$$

In fact, as long as the agent implicitly reports taking the recommended action, he needs only report the occurrence of setbacks with the understanding that "no news is good news" regarding project progress.

The principal possesses two instruments for providing incentives for the agent to faithfully run the project and honestly report progress and setbacks. She can cancel the project prior to completion (i.e., *fire* the agent), or she can provide the agent with a reward when the project is complete. We also allow the principal to provide the agent with rewards based on reported project status; however, we will show that because both parties are risk-neutral and are equally patient, it is without loss of generality to backload all monetary payments into a single reward granted upon successful completion.

Definition 1 (Contract) *Denote the probability space as* $(\Omega, \mathcal{F}, P)$*, and the filtration as* $\{\mathcal{F}_t\}_{t\geq 0}$ *generated by the history of reports* $\{\hat{X}_t\}_{t\geq 0}$*. Contingent on the filtration, a contract* *specifies a stopping time τ when the contract is terminated, a terminal reward K_τ to the agent, and cumulative intermediate rewards $\{C\}_{t\geq 0}$. All quantities are assumed to be integrable and measurable under the usual conditions.*

Contracts are characterized using the agent's continuation utility as the state variable. Given a contract, the agent chooses actions $\{a_t\}_{t\geq 0}$ and reports $\{\hat{X}\}_{t\geq 0}$. His continuation utility is the expected value of the reward from project completion plus private benefits from any shirking:

$$W_t^{a,X} = E_{a,X}\left[\int_t^\tau b(1-a_s)ds + \int_t^\tau dC_s + K_\tau \middle| \mathcal{F}_t\right] , \tag{5}$$

The principal's objective function F_t is the expected value of the benefit from a completed project net of the expected operating cost and the expected reward to the agent:

$$F_t^{a,X} = E_{a,X}\left[-\int_t^\tau cds + R_\tau - \int_t^\tau dC_s - K_\tau \middle| \mathcal{F}_t\right] , \tag{6}$$

where $R_\tau = R$ if the project is completed and 0 if it is not.

Before we go on to characterize general incentive compatibility, we can simplify the contracting space:

Lemma 1 (High Action and Prizes) *The principal will always choose to implement the high action ($a_t = 1$). The principal will reward the agent consumption only upon successful completion of the project ($K_\tau > 0$ iff success; $dC_t = 0$).*

The first result holds because it is always more efficient to award the agent intermediate consumption than to implement inefficient diversion. The second result holds because both the principal and agent are equally patient and so payments can always be delayed.

A contract is incentive compatible if the agent chooses the high action and accurately reports the status of the project:

Definition 2 (Incentive Compatibility) *A contract is incentive compatible if the agent maximizes his objective (5) by choosing $a_t = 1$ and $\hat{X}_t = X_t$ for all $t \geq 0$.*

Then, in an incentive compatible contract, the agent's continuation utility is the expected value of the terminal prize. The principal's utility is the expected payoff of the project minus the running cost and expected prize.

A contract is optimal if it maximizes the principal's objective function within the class of feasible, incentive compatible contracts:

Definition 3 (Optimal Contract) *A contract is optimal if it maximizes the principal's objective function (6) over the set of contracts that 1) are incentive compatible, 2) grant the agent his initial level of utility W_0, and 3) honor $W_t \geq 0$.*

3.2 Incentive Compatibility

In this subsection, we introduce a necessary incentive constraint, the *No-Postponed-Setbacks* (NPS) constraint. This constraint provides the necessary incentives for the agent to report any setbacks immediately, rather than delaying the report and shirking in the meantime. Later, we will show that this constraint is also sufficient to prevent any other deviation.

We now summarize the evolution of the agent's continuation utility, W:

Lemma 2 (Incentive Compatibility) *Given any contract and any sequence of the agent's choices, there exists a predictable, finite, non-negative process J_t $(0 \leq t \leq \tau)$ such that W_t evolves according to*

$$dW_t = J_t \left(\lambda dt - dN_t\right) \tag{7}$$

Between setbacks, J is deterministic. A necessary condition for incentive compatibility is that between setbacks, we have

$$-J_t \geq b\delta + \int_0^\delta \lambda J_{t+s} ds - J_{t+\delta}, \quad \forall \delta \in (0, \bar{X} - X). \tag{NPS}$$

The contract is terminated if $W_t = 0$.

The evolution of the agent's continuation utility under an incentive compatible contract is a martingale. Hence, it drifts up deterministically as the agent accumulates steady progress toward project completion but jumps down by J whenever there is a setback.

To understand the (NPS) incentive constraint, suppose the project is in state $X_t \in (0, \bar{X})$ when a setback occurs. Consider two possible futures:

- [Work] The agent reports the setback immediately, and then works as desired.
- [Shirk] The agent delays reporting the setback and shirks for time $\delta \leq \bar{X} - X_t$. Then, he reports a setback and works as desired.

A critical feature of this shirk path is that after the postponed setback is finally reported, the agent has dissipated his persistent private information about the status of the project. The agent and the principal both believe that X_t is 0 and have the same information about the project and contract going forward. Thus, the agent's continuation utility and the principal's beliefs about it coincide.

Now, we compare the two paths, with working first. Since working is optimal, the agent's continuation utility is a martingale, and we have[6]

$$E\left[W_\tau\right] - W_{t-} = -J_t \ .$$

The only difference between the agents expected utility when the project ends and his utility at $t-$ is the jump down from reporting the setback.

Next, we consider shirking. In this case, the change in continuation utility is

$$E\left[W_\tau\right] - W_{t-} = \int_0^\delta \lambda J_{t+s} ds - J_{t+\delta} \ , \tag{8}$$

with an additional private benefit due to shirking of $b\delta$. The first term accounts for the upward drift in the principal's beliefs about the agent's continuation utility as he (falsely) reports progress while shirking; and the second term captures the jump down in the principal's beliefs about the agent's continuation utility when he finally stops shirking and reports a setback. Adding the private benefit $b\delta$ we obtain (NPS), which simply says that the value from the working path is at least as high as the value from the shirk path.

The (NPS) constraint requires that the agent's loss of utility between setbacks is at least equal to the time he could have spent shirking between them. Thus, there is a round trip penalty imposed on the agent between any two truthfully reported setbacks. We call this a "round trip" because the agent goes from $X = 0$ through some path and back to $X = 0$. To see the penalty, imagine that the agent starts at time t with $X_t = 0$ and works until $t + \delta$ and $X_{t+\delta} = \delta$ when he receives a setback. With truthful reporting, (NPS) implies that the agent's continuation utility is

$$W_{t+\delta} = W_t + \int_0^\delta \lambda J_s ds - J_\delta \leq W_t - J_0 - b\delta = W_t - b\delta \tag{9}$$

where we have used in the final step that a setback at $X = 0$ has no effect and no penalty. If the (NPS) constraint binds, then the agent's utility-drop between setbacks grows linearly

[6] We adopt the standard convention of indexing the value of a process immediately prior to a jump with $t-$.

with time: after each setback, the agent is re-started with a continuation utility that is lower by the amount of time he could have spent shirking.

3.3 Termination and Randomization

We now consider how the agent is terminated when the project is still incomplete.

First, termination is required. Imagine not; then, there is some path of X that would result in the project being funded without end. However, the agent could simply mimic that path with his reports while shirking, and thus obtain infinite utility. The agent would prefer this to any incentive compatible path.

Put differently, the NPS round trip penalty implies that between any two setbacks, the agent loses continuation value at least proportional to the elapsed time. However, termination must occur if $W_t = 0$ because, given limited liability, termination is the only way for the principal to deliver $W_t = 0$. Because the agent's initial utility W_0 is finite, and he must eventually report either a setback or succeed, then the agent must also eventually run out of time.

Second, termination is random and not deterministic. We reason based on the NPS round trip utility penalty (9) and the fact that the agent has limited liability. Imagine that a setback occurs at t resulting in $W_t \in (0, b\bar{X})$. In this case, if the agent continues to work but then suffers another setback after making progress $\delta > \frac{W_t}{b}$, the drop in his continuation utility required by the (NPS) constraint would result in $W_{t+\delta} < 0$, which is not feasible. The agent would prefer to shirk rather than to report the second setback. What can the principal do about this? One option is simply to terminate the contract at t and give the agent a severance payment of W_t. However, allowing the agent to shirk or giving the agent a severance payment is never optimal (Lemma 1).

Instead, there is a better alternative: the principal can use randomization to either fire the agent without severance (generating $W_t = 0$) or increase W_t enough to restore incentive compatibility. Randomization preserves the agent's expected continuation utility – and thus the principal's expected payout to the agent – but it allows for a positive probability that the project will be completed.

To preserve consistency and incentive compatibility, the agent must be randomly assigned a utility equal to 0 or greater than $b\bar{X}$. We assume now (and verify in Proposition 3) that the principal's value function is concave so that she wishes to use the least disperse randomization procedure possible.

Lemma 3 *If $W_t < b\bar{X}$ following a reported setback, then incentive compatibility and concavity of the principal's value function require that the agent is assigned utility of $b\bar{X}$ with probability $p = \frac{W_t}{b\bar{X}}$ or utility 0 with probability $1 - p$.*

4 Optimal Contract: A Time Budget

4.1 The Principal's Problem

As we demonstrate below, the concept of a *time budget* plays a crucial role in the implementation of an optimal contract. A time budget is a stochastic deadline that may either count down deterministically or jump up or down randomly such that the expected value of the jump is zero. Formally we have the following:

Definition 4 (Time Budget) *A time budget S_t is a non-negative process S_t satisfying $E[dS_t] = -dt$ ($dS_t = -dt$, absent a setback). The principal initially grants the agent S_0 and then, if the project does not succeed, cancels the project iff $S_t = 0$.*

A time budget creates a random stopping time τ when the contract is terminated (on the event of success or cancellation). Define

$$\tau(S) = \mathrm{E}_t\left[\tau | S_t = S, X_t = 0\right] \tag{10}$$

to be the expected time to contract termination, given that there is no intermediate progress ($X_t = 0$). Notice that $\tau(S)$ is weakly less than S because the project can be completed early.

Our first result is that the NPS constraint is binding and sufficient, and thus the optimal contract can be implemented as a time budget. In other words, the agent's loss of utility between setbacks grows linearly with time, and this is enough to generate full effort and prevent any mis-reporting by the agent. Intuitively, the best the agent can do by lying to the principal is to gain time to divert resources from the project (shirk), and reducing the agent's expected prize by the amount he could have diverted is enough to deter such malfeasance. In turn, this means that the optimal contract can be implemented as a time budget:

Proposition 1 *The optimal contract has the following properties:*

i. The (NPS) constraint binds, and the agent's utility penalty for reporting a setback J_t is a function of X_t only:

$$J_t = J(X_t) = \frac{b}{\lambda}\left(e^{\lambda X_t} - 1\right). \tag{11}$$

ii. The contract can be implemented with a time budget which is set such that $bS_0 = W_0$ and the agent is terminated if $S_t = 0$. If $S_{t-} < \bar{X}$ and a setback is reported, then S_t is set to either 0 with probability $1-p$ or $\bar{X}$ with probability p where

$$p = \frac{S_{t-}}{\bar{X}}. \tag{12}$$

iii. The agent's continuation utility under the optimal contract is

$$W_t = bS_t + \frac{b}{\lambda}\left(e^{\lambda X_t} - 1\right). \tag{13}$$

If the agent completes the project at time τ, he receives a reward of

$$K_\tau = bS_\tau + \frac{b}{\lambda}\left(e^{\lambda \bar{X}} - 1\right) \tag{14}$$

This result says that it is optimal for the principal to assign the agent an expected amount of time S_0 to complete the project. The agent works on the project and makes continuous progress reports including the occurrence of any setbacks. If $S_t < \bar{X}$ remains on the clock and a setback is reported, then there is not enough time remaining to complete the project. At this point, the randomization procedure is invoked in which the project is either canceled with probability $1 - \frac{S_{t-}}{\bar{X}}$ or the schedule is extended to $S_t = \bar{X}$ with the complementary probability. Any subsequent setbacks are treated analogously, until the project is ultimately either canceled or completed.

The payment to the agent for delivering the completed project at time τ consists of a fixed component $\frac{b}{\lambda}\left(e^{\lambda \bar{X}} - 1\right)$ plus a *bonus* that is proportional to the remaining time on the clock bS_τ. The bonus term is the inverse of the NPS constraint: because the agent's utility declines between setbacks, he must receive an incentive payment if the project succeeds before he reports another setback. The fixed portion of the reward represents the value to the agent of a fully mature project. It is calibrated so that he is just indifferent between delivering the finished project and falsely reporting a last minute setback and then shirking (i.e., diverting resources) for a spell of time $\bar{X}$ before making delivery. In particular, if the agent reports a setback in state X, then his continuation utility drops by $J(X) = \frac{b}{\lambda}\left(e^{\lambda X} - 1\right)$ because he loses all accumulated progress.

As noted in the introduction, the implementation of the optimal incentive mechanism characterized in Proposition 1 is a cost-plus-award-fee contract. In particular,the principal commits: (i) to cover the operating cost of the project $c\tau$, (ii) to pay a fixed fee $\frac{b}{\lambda}\left(e^{\lambda \bar{X}} - 1\right)$ upon project completion, and (iii) to pay an incentive award bS_τ for timely completion.

It is also worth noting that optimal incentives can be implemented with less stringent reporting requirements than the ones assumed. Rather than requiring continuous progress reports, at project inception the principal can announce a *soft deadline* $T = S_0 - \bar{X}$ and then commit to fund the project until this date *no-questions-asked*. If the agent delivers the completed project at $\tau \leq T$ then he receives K_τ as given in the proposition. Once the soft deadline has passed, the principal requires setbacks to be reported, and she follows the random termination procedure specified in Proposition 1 from that point on.

4.2 Initial Value of the Project

Given the form of the optimal contract, we can now write the principal's value function $F(S, X)$. We are most interested in her valuation of a given time budget when starting from scratch: $F(S, 0)$. This is summarized in the following proposition: [7]

Proposition 2 *The principal's initial valuation of a given time budget S is*

$$F(S,0) = \left(\frac{\lambda R}{e^{\lambda \bar{X}} - 1} - c\right)\tau(S) - bS \tag{15}$$

where $\tau(S)$ is defined in (10). $F(S, 0)$ is concave and hump-shaped in S.

More, the probability that the project is completed as a function of S, given $X_t = 0$, is

$$P(S) = \frac{\lambda \tau(S)}{e^{\lambda \bar{X}} - 1} \ . \tag{16}$$

This result has an intuitive interpretation. Using the first-best value (2), we can write

$$F(S,0) = P(S)F^{\text{FB}} - bS \ , \tag{17}$$

[7] For $X > 0$ we have $F(S,X) = \int_0^{\bar{X}-X} \lambda e^{-\lambda t}\left(F(S-t,0) - ct\right)dt + e^{-\lambda(\bar{X}-X)}(R - c(\bar{X} - X))$.

Written this way, we see that asymmetric information harms the principal for two related reasons. First, $P(S)$ is the probability that the agent eventually delivers a completed project starting with an initial time budget of S – the probability that the project is not inefficiently canceled. The second way in which the principal is harmed is that she has to pay an expected agency rent of $W_t = bS_t$ to induce the agent to work and report honestly.

Intuitively, the larger the time budget S, the more likely it is that the agent will complete the project; i.e., $P(S)$ is increasing and $\lim_{S\to\infty} P(S) = 1$. But, of course the principal will not commit to pay the agent an unboundedly large rent to obtain a payoff bounded by F^{FB}. In particular, she faces a tradeoff when setting the initial time budget S_0 between higher probability of project completion, $P(S_0)$, and paying higher agency rents, bS_0. This tradeoff manifests in the hump shape of the value function $F(S, 0)$. At low levels of S both the principal and agent prefer a larger time budget. However, as S grows, diminishing marginal returns to the probability of project completion $P(S)$ are eventually dominated by the linear agency cost bS, and $F(S, 0)$ peaks at some critical value S^* beyond which it decreases.

Interestingly, we can derive the formula for the principal's value function given in (15) without using the usual techniques involving ODEs, PDEs, and dynamic programming. Instead, we accomplish this by use of two martingales. This allows for a more fundamental understanding of the contracts and the associated characteristics. The goal is to evaluate the principal's expected welfare (6), which is the payoff to a completed project minus the running cost:

$$
\begin{aligned}
F(S,0) =& P(S)E\left[R - K \,|\, S_t = S,\, X_\tau = \bar{X}\right] - c\tau(S) && (18)\\
=& P(S)E\left[R - \frac{b}{\lambda}\left(e^{\lambda\bar{X}} - 1\right) - bS_\tau \,\middle|\, S_t = S,\, X_\tau = \bar{X}\right] - c\tau(S) && (19)
\end{aligned}
$$

To compute this, we need to pin down $P(S)$ and $E\left[S_\tau | S_t = S,\, X_\tau = \bar{X}\right]$, the probability of success and the expected time budget remaining after success.

First, since $dX_t = dt - X_t dN_t$, we have that $e^{\lambda X_t} - \lambda t$ is a martingale:

$$
d[e^{\lambda X_t} - \lambda t] = (1 - e^{\lambda X_t})(dN_t - \lambda dt) \tag{20}
$$

Then, using the optional stopping theorem, the definition of $\tau(S)$ (10), and $X_0 = 0$, we have

$$\begin{aligned} 1 =& E\left[e^{\lambda X_\tau} - \lambda\tau\right] \\ =& P(S)E\left[e^{\lambda X_\tau}|S_t = S,\, X_\tau = \bar{X}\right] + (1 - P(S))E\left[e^{\lambda X_\tau}|S_t = S,\, X_\tau = 0\right] - \lambda\tau(S) \\ =& P(S)e^{\lambda\bar{X}} + (1 - P(S)) - \lambda\tau(S) \end{aligned} \tag{21}$$

where we have used the fact that X_τ ends on $\bar{X}$ after success and 0 after failure. Solving for $P(S)$ yields the probability of success (16).

Second, since S_t declines deterministically on average (Proposition 1), we have that $S_t + t$ is a martingale. Again using the optional stopping theorem, we have

$$S_0 = E\left[S_\tau + \tau\right] \tag{22}$$

$$= P(S)E\left[S_\tau|S_t = S,\, X_\tau = \bar{X}\right] + (1 - P(S))E\left[S_\tau|S_t = S,\, X_\tau = 0\right] + \tau(S) \tag{23}$$

$$= P(S)E\left[S_\tau|S,\, X_\tau = \bar{X}\right] + \tau(S) \tag{24}$$

where we have used the fact $E\left[S_\tau|S_t = S,\, X_\tau = 0\right] = 0$: if the project ends in failure and the agent is fired, the time budget has been exhausted. Solving for $E\left[S_\tau|S_t = S,\, X_\tau = \bar{X}\right]$ and substituting that into the principal's welfare (19) yields the statement of the proposition (15).

4.3 Value Function Characterization

In the randomization region, where $S_t \le \bar{X}$, we can calculate $\tau(S)$ and $F(S,0)$ explicitly, yielding the following result:

Corollary 1 *For $S \le \bar{X}$,*

$$\tau(S) = S \tag{25}$$

$$F(S,0) = S\left(\frac{\lambda R}{e^{\lambda\bar{X}} - 1} - c - b\right). \tag{26}$$

Thus, in the randomization region the principal's expected payoff following any setback is proportional to the time remaining on the clock. Also, observe that – as indicated earlier – $F(S,0) \ge 0$ if and only if Assumption 2 holds. In other words, the principal must be willing to incur the flow cost $c + b$ as if the project were being run until completed, even though the expected duration of the contract in this region is only $\tau(S) = S \le \bar{X}$.

It is also possible to solve explicitly for $\tau(S)$ if $S > \bar{X}$ by applying an iterative procedure. Using (15), we can then obtain the principal's value function in closed form as a piecewise polynomial in S. The important properties of $\tau(S)$ are summarized as follows:

Proposition 3 (Value Function Properties) *$\tau(S)$ is continuous in S. For all $n \geq 1$, we have*

(i) *For $S \in ((n-1)\bar{X}, n\bar{X}]$, $\tau(S)$ is a concave, increasing polynomial of order n.*

(ii) $\lim_{S\to\infty} \frac{\partial}{\partial S}\tau(S) = 0$ *and* $\lim_{S\to\infty} \tau(S) = \frac{1}{\lambda}\left(e^{\lambda \bar{X}} - 1\right)$.

(iii) $\lim_{S\uparrow n\bar{X}} \frac{\partial}{\partial S}\tau(S) > \lim_{S\downarrow n\bar{X}} \frac{\partial}{\partial S}\tau(S) > 0$.

The first observation is a straightforward implication of the iterative procedure detailed in the appendix: each round adds a higher order term but the function is always concave and increasing. Combining this observation with Corollary 1 we see that

$$\tau(S) \begin{cases} = S, & \text{if } S \leq \bar{X} \\ < S, & \text{if } S > \bar{X}. \end{cases}$$

In other words, the expected duration of the project at inception, $\tau(S_0)$, is weakly less than the initial time budget, S_0, allotted to the agent. $\tau(S_0) < S_0$ means that the principal builds some slack or *slippage* time into the contract: she initially gives the agent more expected time to complete the project than its actual expected duration. This implies that whenever the project is extended such that $\tau > S_0$, a schedule and cost overrun must have happened. That is, the project runs longer than it's initial expected duration of $\tau(S_0)$ and costs more than the initial estimate of $c\tau(S_0) + bS_0$.

The second observation in Proposition 3 is a restatement of the fact that the marginal benefit from increasing S becomes arbitrarily small as $P(S)$ goes to 1. This along with the fact that the marginal cost remains constant at b implies that there exists a unique schedule S^* at which $F(S, 0)$ is maximized.

The third observation identifies kinks in the value function at positive integer multiples of $\bar{X}$. One implication is that it is optimal for the principal to assign an initial time budget equal $n\bar{X}$ for a non-negligible set of parameters. We illustrate this possibility for an important case in the following subsection.

4.4 A Short-Leash Project

Here we use the first kink in the principal's value function identified in part (iii) of Proposition 3 to derive conditions under which it is optimal for her to set an initial time budget of $S_0 = \bar{X}$. Applying the iterative procedure in the appendix gives

$$\lim_{S\uparrow\bar{X}} \frac{\partial}{\partial S}\tau(S) = 1 > 1 - e^{-\lambda\bar{X}} = \lim_{S\downarrow\bar{X}} \frac{\partial}{\partial S}\tau(S). \tag{27}$$

Using these observations to evaluate the derivative of the value function in (15) directly yields the following result:

Corollary 2 *The optimal contract involves a minimal time budget of $S_0 = \bar{X}$ and a fixed prize of $\frac{b}{\lambda}\left(e^{\lambda\bar{X}} - 1\right)$ iff*

$$\frac{c+b}{\lambda}\left(e^{\lambda\bar{X}} - 1\right) < R < \frac{c + \frac{b}{1-e^{-\lambda\bar{X}}}}{\lambda}\left(e^{\lambda\bar{X}} - 1\right) \tag{28}$$

The first inequality in (28) is a restatement of Assumption 2 (that the project is feasible), and it ensures that $F(S,0)$ is increasing for $S < \bar{X}$. The second inequality then ensures that $F(S,0)$ is decreasing for $S > \bar{X}$. Hence, when (28) holds, the kink in the value function at $S = \bar{X}$ corresponds to the peak and it is optimal for the principal to set an initial time budget of $S_0 = \bar{X}$. In other words, she should keep the agent on a short leash, granting him in expectation only the minimal amount of time necessary to complete the project, requiring him to report every setback, and canceling the project with positive probability each time one is reported. The key parameter in (28) is b, the agent's per-period benefit from shirking. If b is too large, then the left inequality fails and moral hazard precludes the project from ever getting off the ground. On the other hand, if b is too small, then the right inequality fails. In this case, moral hazard is less concerning, and the principal prefers to give the agent more than the minimal expected time to complete the project.

At inception of a short-leash project, the expected duration is $\tau(S = \bar{X}) = \bar{X}$, and the expected cost to the principal is $(c+b)\bar{X}$. However, if the agent reports a setback with S_t left on the schedule, then he is granted an extension of $\bar{X} - S_{t-}$ (i.e., the clock is reset) with probability $\frac{S_{t-}}{\bar{X}}$. Hence, the support of the stopping time τ is unbounded, implying that the project may run arbitrarily long, incur arbitrarily large costs, and yet may still be canceled.

In fact, it is possible to determine explicitly the probability of an overrun, $\Pr\{\tau > \bar{X}\}$, under a short-leash contract.[8] Define $\mu \equiv \lambda \bar{X}$ to be the expected number of setbacks experienced while the project is in operation. Figure 1 plots the probabilities of three exhaustive events:

1. $P_{\mathrm{EC}}(\mu)$ is the probability that the project is canceled early, before its initial expected duration of $\bar{X}$, (left panel, blue solid curve).[9] It is zero when $\mu = 0$ because no setbacks are possible in this case. It is increasing because as the expected number of setbacks rises, it becomes ever more likely that the project will not survive the requisite randomizations before time $\bar{X}$ has elapsed. Indeed, $\lim_{\mu\to\infty} P_{\mathrm{EC}}(\mu) = 1$ because a steady stream of setbacks must result in early project cancellation for any $\bar{X} > 0$.

2. $P_{\mathrm{OT}}(\mu) = e^{-\mu}$ is the probability that the project is completed on time (left panel, red dotted curve). This function is obviously one when $\mu = 0$ because the project is always completed on time when setbacks are not possible (the agent has no plausible excuse for late delivery). It is decreasing because as the expected number of setbacks rises, the probability that none occur falls. Indeed, $\lim_{\mu\to\infty} P_{\mathrm{OT}}(\mu) = 0$ because when setbacks are a virtual certainty, the project cannot be completed on time.

3. $P_{\mathrm{OR}}(\mu) = 1 - P_{\mathrm{EC}}(\mu) - P_{\mathrm{OT}}(\mu)$ is the probability of an over run (right panel). It is low for small values of μ because the project will most likely be completed on time. It rises until achieving a maximum of approximately 0.39 when $\mu = 3.34$ and then decreases as the probability of early cancellation becomes ever more likely.

4.5 The Value of Randomization

We can now address the value to the principal of the reporting process: why not simply assign a deadline of S_0 and stick with it? The answer is that the value of reporting derives directly from application of the randomization procedure once the contract enters the short-leash phase, $S_t \le \bar{X}$. Assume instead that the principal does offer a fixed deadline of S_0 and the prize (14) upon completion, and then does not solicit or review reports. The agent will

[8] Note that we define an overrun to be the event that the project ends, either from completion or cancelation, after the initial expected duration of the project has elapsed. An alternative common usage of the term refers to only those projects that are completed late.

[9] This can be obtained analytically from $P_{\mathrm{EC}}(\mu) = p(x = 1; \mu)$, where $p(x; \mu)$ is the solution to the second-order ODE $p''(x; \mu) + \mu p'(x; \mu) + \mu p(x; \mu) = \mu$ with boundary conditions $p(x = 0; \mu) = p'(x = 0; \mu) = 0$.

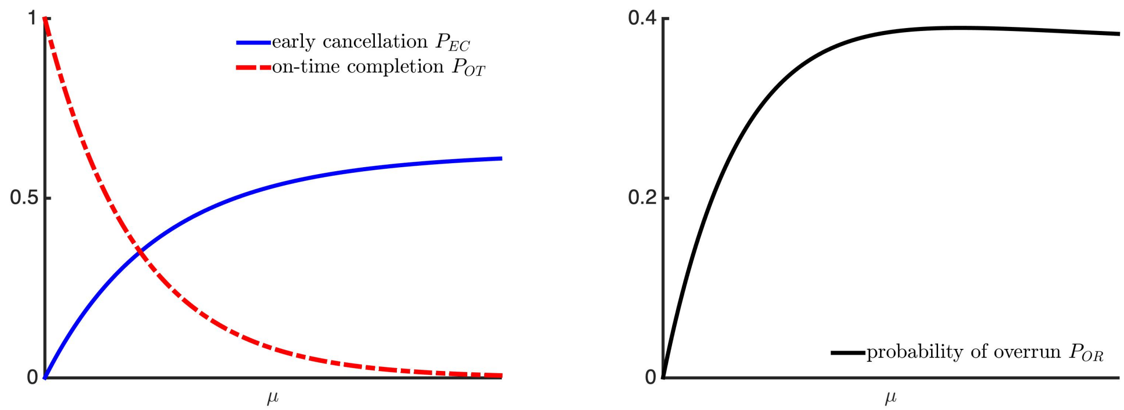

Figure 1: **Probability of an Overrun Under a Short-leash Contract**
The left panel of this figure plots the probability of early cancellation (P_{EC}, blue solid line) and the probability of on-time completion (P_{OT}, red dash-dot line). The right panel of this figure plots the probability of an overrun (P_{OR}) $\mu \equiv \lambda \bar{X}$ is the expected number of setbacks experienced while the project is in operation.

use high effort until experiencing a setback at $S_t < \bar{X}$ and will then shirk for the remainder of the contract, only reporting the setback at the last possible second.

Under the optimal contract, when $S_t \geq \bar{X}$, the time budget behaves just like a deadline, counting down naturally (i.e. $dS_t = -dt$), and the principal effectively ignores all reports. In fact, the (NPS) constraint binds, implying that the agent is always indifferent between working or shirking, and hence, willing to work for $S_t \geq \bar{X}$. However, if a setback occurs at $S_t < \bar{X}$ and the deadline is fixed, then it is impossible for the agent to complete the project and receive payment in the remaining time. Because he can only realize any positive utility through the final reward at the time of project completion if he chooses to work, he prefers to shirk out the clock and report a last-second setback to obtain bS_t at this point.

In other words, a fixed deadline and prize can be used to induce full effort until the short-leash phase of the contract. At that point, there isn't enough time left to complete the project if a setback occurs, and the agent postpones reporting the setback until the end of the contract and shirks. The reports are useful precisely because they enable the principal to create a soft deadline instead of a hard one: the principal stochastically extends the agent's time or terminates him, in place of inefficient shirking.

5 The Role of Commitment

A potential shortcoming of the optimal contract characterized in Proposition 1 is that it involves the principal committing to randomly cancel the project if a setback is reported when $S_t < \bar{X}$. In particular, if the agent will continue to work after receiving an extension, then the principal strictly prefers extending the project to canceling it, and – lacking commitment to randomize – she will grant an extension with probability 1. On the other hand, if the agent believes that the project will always be extended, then he will shirk and report false setbacks *ad infinitum*, leading the principal to prefer project cancellation. This is the familiar logic underpinning a mixed-strategy equilibrium. To relax the required level of commitment, we modify the baseline model in this section by assuming that randomization by either party is possible but not contractually verifiable. All other aspects of the environment remain intact.

Consider again the situation in which a setback results in $W_t \in (0, b\bar{X})$. As we noted in subsection 3.3, one way to give the agent his promised continuation utility is simply to cancel the project and make a severance payment. This option has value $-W_t$ to the principal. We showed that the principal could do better by randomizing between project cancellation without severance ($W_t = 0$) and the minimally feasible project extension ($W_t = b\bar{X}$). Indeed, because the agent continues to work if granted an extension, this randomization has strictly positive net value to the principal. However, as per the argument in the previous paragraph, this scenario can only be implemented when randomization is verifiable.

Nevertheless, it is possible to implement an outcome that delivers expected value of zero to the principal, which still dominates payment of severance. To see how, suppose that rather than working with probability 1 when granted an extension, the agent randomizes at that moment between continuing to work and shirking out the clock, two options over which he is indifferent. If the agent randomizes such that the principal's expected value from extending the project is 0, then she will be indifferent between canceling the project and extending it, and will be willing to randomize herself. In terms of payoffs, the only difference between this setting and the baseline model is that the occurrence of a setback inside the randomization region drops the principal's value function down to zero. The agent's continuation utility W_t is still a martingale and (NPS) still binds.

Proposition 4 (Non-verifiable Randomization) *If randomization is not verifiable, then the optimal contract for the principal can still be implemented with a time budget and the same prize structure K_τ. If $S_{t-} < \bar{X}$ and a setback is reported, then the principal extends the schedule to $\bar{X}$ with probability $\frac{S_{t-}}{\bar{X}}$ and cancels the project with probability $1 - \frac{S_{t-}}{\bar{X}}$. Upon* *receiving an extension, the agent randomizes between shirking out the clock with probability* $q = \frac{\hat{F}(\bar{X},0)}{\hat{F}(\bar{X},0)+c\bar{X}}$*, and continuing to work with probability* $1-q$*, where*

$$\hat{F}(S=\bar{X},0) \equiv Re^{-\lambda\bar{X}} - \frac{c+b}{\lambda}\left(1-e^{-\lambda\bar{X}}\right). \tag{29}$$

Note that this version of the contract need not specify specific probabilities of cancellation or extension – it need only specify that the principal has the right to cancel or extend the project *at will*. This is somewhat more realistic than the contract characterized in Proposition 1 where the exact probabilities of cancellation and extension were necessarily an explicit part of the agreement. Of course, the inability to commit to explicit probabilities harms the principal. In particular, $\hat{F}(S=\bar{X},0)$ is the value she derives from a short-leash contract when she cannot commit and

$$\hat{F}(S=\bar{X},0) = F(S=\bar{X},0)\left(\frac{1-e^{-\lambda\bar{X}}}{\lambda\bar{X}}\right). \tag{30}$$

The fraction on the right side of this equation is evidently less than 1 for $\lambda\bar{X} > 0$. This makes sense: a setback during the course of a short-leash contract under full commitment still leaves the principal with a positive expected payoff as shown in Corollary 1, whereas a setback in the randomization region absent commitment results in an expected payoff of zero for the principal.

Note that while we specified that the agent randomizes between continuing to work and shirking out the clock after receiving an extension, there are other strategies the agent could pursue that are expected payoff equivalent for both parties. For example, while shirking the agent could mimic nature by randomly reporting setbacks at rate λ, continuing to shirk after each and every extension. Relative to shirking out the clock, this would increase the variance of τ, though not its expected value.

Intriguingly, the lack of commitment to randomize leads the principal to grant the agent *more* initial time:

Proposition 5 (Optimal Initial Time Budget) *Define* S^* *to be the principal's optimal initial time budget when the randomization is verifiable, and* $\hat{S}^*$ *to be the principal's optimal initial time budget when the randomization is not verifiable. Assume that the agent's initial outside option value is sufficiently low such that his participation constraint is met in both economies. Then,* $S^* \leq \hat{S}^*$*.*

Because the inability to commit to explicit randomization harms the principal (i.e. $\hat{F}(S,0) < F(S,0)$), a reasonable conjecture is that she would prefer to grant the agent less time. After all, for any given value of S, lack of commitment implies lower probability of project completion and reduces the value of the project to the principal. It is, therefore, somewhat surprising that she responds by devoting more time and money to the less valuable project. (Note that the agent benefits from the principal's lack of commitment because his expected payoff is proportional to schedule length.)

The intuition is actually straightforward. Lack of commitment power only harms the principal if a setback occurs in the short-leash randomization region, $S_t < \bar{X}$. By granting the agent a longer initial time budget, the principal raises the probability that the project will be completed before the schedule enters this problematic phase. That is, she reduces the likelihood that her inability to commit will even come into play. In a sense, the principal doubles down on the part of the contract to which she can commit (the length of the schedule) in order to reduce the impact of the part to which she cannot (explicit probabilities of project cancelation and extension). Put differently, granting large $\hat{S}_0$ not only increases the chance the agent will successfully complete the project, it decreases the chance that lack of commitment will become a problem.

Finally, we point out that fully renegotiation-proof contracts are not reasonable in our setting. To be renegotiation-proof, the principal must not gain from a one-time grant of utility to the agent, meaning that the principal's value function must be decreasing in the agent's utility. However, the agent has his lowest utility (zero) when he is terminated and the project fails. Thus, without additional assumptions or explicit commitment mechanisms, a fully renegotiation proof contract requires that the principal attains her highest utility when the project fails.

6 Conclusion

At a very general level, projects are usually viewed as possessing three defining features, scope, schedule, and budget – the so-called "iron triangle." (Wyngaard et al., 2012) The scope of a project is the quality of the deliverable, be it a software application, a power plant, or a doctoral thesis; The schedule is the time allotted to production of the deliverable; and the budget is the monetary or other physical resources committed to it. However, because projects are by definition – at least somewhat – unique, their implementation typically involves considerable uncertainty. In this paper we held scope fixed, and presented a model of project implementation focussing on what appears to be the most common sources of project uncertainty, schedule setbacks and the concomitant cost overruns.

Whether a project is under taken in-house (e.g., the Boeing Dreamliner) or outsourced (e.g., South Carolina's V.C. Summer nuclear plant or the FBI's virtual-case-file system), its progress will almost surely be hampered to some degree by agency frictions. To study this, We embedded a natural model of production with random setbacks into a dynamic agency environment and solved for the optimal contract from the principal's perspective. This analysis yielded a number of novel insights and conclusions. Among the most robust are: 1) an optimal contract can always be implemented with a time budget and a linear terminal payment corresponding to a cost-plus-award-fee contract; 2) penalties for reports of setbacks or delays are generally more severe the later they occur in project development; and 3) mishaps that are reported near the end of the allotted schedule either result in project cancelation or minimally feasible project extension.

There are a number of open avenues for future research. First, it would be edifying (albeit technically demanding) to investigate a setting where partial setbacks were possible. Such an environment would endow the agent with yet another dimension of private information, further exacerbating the principal's problem of designing an optimal dynamic incentive scheme. Also, expanding our current treatment to incorporate common strategies for dealing with the time pressure created by unanticipated setbacks seems promising. After all, the completion of projects is frequently especially time-sensitive. For example, Lewis and Bajari (2011) investigate the procurement of highway construction projects where completion delays can have large social costs. In this vein, exploring the possibility of speeding up production through *fast-tracking* (running several phases in parallel) or *crashing* (deploying more resources) to make up for unanticipated setbacks is a potentially important consideration (Monnappa, 2020). Finally, there is the question of scope itself. Throughout we supposed that the project was either incomplete (worth zero to the sponsor) or complete (worth a fixed amount). In reality, the ultimate quality of many projects varies along a continuum. Indeed, *scope creep* on the part of sponsors (demanding a higher quality deliverable than originally specified) is often cited as a contributing factor to project failure.[10] We leave these considerations and others for future work, having judged this particular project to be deliverable as complete.

[10] Ely and Szydlowski (2020) considers a dynamic moral hazard problem in which the principal uses scope creep to entice the agent to exert effort on a project that he would not have agreed to work on if he had known the full scope of the project at the outset.

Appendix – Proofs and Derivations

A Proof of Lemmas 1, 2, and 3

A.1 Proof of Lemma 1

First, the principal always induces the high action ($a_t = 1$). Imagine there is an interval of time in which the principal induces shirking. The project does not advance, nor is there a setback. The principal can award the agent intermediate consumption during the shirking interval without changing the agent's continuation utility. This makes the agent indifferent and the principal better off, because $c > b$ implies that assigning the agent any positive amount of utility by allowing shirking is more costly for the principal than directly paying the agent.

Second, any contract with intermediate payment can be weakly improved by one without. Because the principal and the agent share the same discount rate (0), the principal can simply delay any intermediate payments until the end, leaving both participants indifferent.

Third, any contract with severance pay upon termination can be (weakly) improved by one that pays only on the event of success. Notice that the principal can re-start any existing contract and both participants will have positive value going forward. Any contract that ends with a severance payment can be replaced with one that randomizes between re-starting the contract and termination with zero payment. The probability of re-starting the project can be set to make the agent indifferent to the randomization. The principal is better off because she receives zero (termination) or a positive value (re-start) instead of making a severance payment.

A.2 Proof of Lemma 2

First, by Lemma 1, $dC_t = 0$ for all $t < \tau$. Therefore (5) becomes

$$W_t^{a,X} = E_{a,X}\left[\int_0^\tau b(1-a_s)ds + K_\tau \,|\mathcal{F}_t\right],$$

where $\mathcal{F}_t$ is the filtration generated by the agent's report $\{\hat{X}_t\}_{t\in[0,\tau]}$. Note that W_t is an $\mathcal{F}_t$-martingale. Thus, by the martingale representation theorem for jump processes, there exists a $\mathcal{F}_t$-predictable, integrable process J such that

$$dW_t^{a,X} = J_t(\lambda dt - d\hat{N}_t) \tag{31}$$

If the contract is incentive compatible, the agent will exert high effort and report the setback N_t truthfully. In this case, the analysis following the statement of Lemma 2 applies, and the

(NPS) condition

$$-J_t \geq b\delta + \int_0^\delta \lambda J_{t+s} ds - J_{t+\delta}, \quad \forall \delta \in (0, \bar{X} - X). \tag{32}$$

must hold as a necessary condition.

Next, J_t must be weakly positive. If not, then the agent would gain from falsely reporting a setback.□

A.3 Proof of Lemma 3

Suppose $W_t < b\bar{X}$ following a reported setback. Then randomization between $W_{t'} = 0$ and $W_{T'} \geq b\bar{X}$ is necessary. To see why, suppose that immediately after randomization (including degenerate randomization) or at the beginning of a phase of Poisson termination, the agent's continuation utility is $W_{t'} \in (0, b\bar{X})$. Imagine the agent continues to work for $\delta \in (W_{t'}/b, \bar{X})$ and then suffers a setback which he truthfully reports. By the round-trip property of (NPS) we have

$$W_{t'+\delta} = W_{t'} + \int_0^\delta \lambda J_s ds - J_\delta \tag{33}$$

$$\leq W_{t'} - J_0 - b\delta = W_{t'} - b\delta < 0, \tag{34}$$

which violates limited liability. Finally, concavity of the principal's value function (established in Proposition 3) implies that randomization should involve minimal dispersion. Hence, to deliver W_t to the agent, he should receive $W_{t'} = 0$ with probability $(1 - p)$ and $W_{t'} = b\bar{X}$ with probability $p = \frac{W_t}{b\bar{X}}$. □

B Proof of Proposition 1

The proof of Proposition 1 consists of four parts. Part 1 demonstrates that the (NPS) constraint can be re-written in a more tractable way. Part 2 shows that (NPS) binds with equality in an optimal contract. Part 3 shows that if (NPS) holds with equality, then the optimal contract can be implemented with a simple time budget. Part 4 shows that a simple time budget is sufficient to make truthful reporting optimal for the agent. We will also take as given that the principal's value function is concave with respect to W (or, S), which is verified in Proposition 2 and the analysis in Section 4.2.

We begin with the following lemma which is helpful several times:

Lemma 4 (Lower bound for the marginal value of agent's utility) *Let $F(W, X)$ be the principal's value function (6), then $F_W(W, X) \geq -1$.*

Proof: Imagine not: $F_W(W, X) < -1$. Then the principal would gain by giving the agent intermediate consumption. But this cannot be the case (Lemma 1).□

B.1 Re-writing (NPS)

An important fact is that J can only vary with time (or, the progress of X) between t and $t+\delta$. Because no setbacks are being reported, the passage of time is the only thing the principal can observe. Thus, we can write J as a function of current project progress and all of history prior to the previous setback, $J(X,\cdot)$. We will suppress the $(\cdot)$ notation for convenience. Then, (NPS) becomes

$$J(X_t) \leq -b\delta - \int_0^{\delta} \lambda J(X_t+s)ds + J(X_t+\delta),\ \forall X_t \in [0,\bar{X}) \text{ and } \delta > 0 \tag{35}$$

First, we reformulate (NPS) so that J can be written as the sum of its minimum, binding value and a term capturing the excess. If (NPS) binds everywhere, (35) holds with equality and we can take the derivative with respect to δ to obtain

$$0 = -b - \lambda J(X+\delta) + J'(X+\delta) \tag{36}$$

This has the (general) solution $J(X) = C_1 e^{\lambda X} - \frac{b}{\lambda}$, where C_1 is a constant. Because $J(X) \geq 0$, the minimum value of C_1 is $\frac{\beta}{\lambda}$. Thus, the minimum, binding value of $J(X)$ is

$$J^{min}(X) = \frac{b}{\lambda}e^{\lambda X} - \frac{b}{\lambda} \tag{37}$$

Next, we define the functions $f(X)$ and $g(X)$ such that

$$f(X) = \frac{1}{b}\left[J(X) - J^{min}(X)\right] \tag{38}$$

$$g(X) = f(X) - \int_0^X \lambda f(u)du \tag{39}$$

So, $f \geq 0$ captures the difference between J and its minimum, and g is a convenient summarizing function. Notice that $f(0) = g(0)$, so $f(X)$ can be fully recovered if we know $g(X)$.

Second, we show that (NPS) is satisfied if and only if $g(X) \geq 0$ and f and g are weakly increasing. Substituting the definition of $f(X)$ into (35) yields

$$f(X) \leq -\int_0^{\delta} \lambda f(X+u)du + f(X+\delta),\ \forall X_t \in [0,\bar{X}) \text{ and } \delta > 0 \tag{40}$$

Then, from the definition of g, we have

$$g(X+\delta)-g(X)=f(X+\delta)-\int_0^{X+\delta}\lambda f(u)du-f(X)+\int_0^{X}\lambda f(u)du \tag{41}$$

$$=f(X+\delta)-\int_X^{X+\delta}\lambda f(u)du-f(X) \tag{42}$$

$$\geq 0 \tag{43}$$

where (40) shows that (42) is non-negative, so $g(X)$ is weakly increasing. More, $f(0)=g(0)$ and g weakly increasing imply $g(X)\geq 0$. Then, using the definition of g and $f\geq 0$, we see that we must also have f weakly increasing. Using the definitions of f and g, we see that f and g positive and weakly increasing are also sufficient to show (35). Thus, we can use f and g instead of (40) to characterize (NPS).

We can now write the change in the agent's continuation utility as X starts at $X_t=0$ and progresses until a setback is experienced at $X=X_{t+s}$. Using $X_{t+u}=u$ and the f and g notation, we have

$$\begin{aligned}\int_0^{s}\lambda J(X_{t+u})du-J(X_{t+s}) &= \int_0^{s} be^{\lambda u}du-b\bar{X}-\frac{b}{\lambda}e^{\lambda s}+\frac{b}{\lambda}-bg(X_{t+s})\\ &=\frac{b}{\lambda}\left(e^{\lambda s}-1\right)-bs-\frac{b}{\lambda}e^{\lambda s}+\frac{b}{\lambda}-bg(X_{t+s})\\ &=-bs-bg(X_{t+s}) \end{aligned}\tag{44}$$

Similarly, we can write down the prize the agent receives, given that the agent starts at time t with W_t and $X_t=0$ and progresses to project completion:

$$W_t+\int_0^{\bar{X}}\lambda J(x)dx=W_t+\int_0^{\bar{X}} be^{\lambda x}dx-b\bar{X}+b\int_0^{\bar{X}}\lambda f(x)dx \tag{45}$$

B.2 The Optimality of Binding the (NPS)

We show that the optimal contract has $f=g=0$. To proceed, using (44 and 45), and that the probability that any particular try at the project is successful is $e^{-\lambda\bar{X}}$, we have that $F(W,X=0)$ can be written as

$$F(W,X=0)=e^{-\lambda\bar{X}}\left[R-\frac{b}{\lambda}\left(e^{\lambda\bar{X}}-1\right)+(b-c)\bar{X}-W-b\int_0^{\bar{X}}\lambda f(u)du\right] \tag{46}$$

$$+\int_0^{\bar{X}}\lambda e^{-\lambda u}F(W-bu-bg(u),X=0)du \tag{47}$$

with constraints $g'(u)\geq 0$ and $g(u)\geq 0$. We will use the Hamiltonian maximization method with $\zeta(u)=g'(u)$ as the control variable, $f(u)$ and $g(u)$ as the state variables, and $f'(u)=$

$\lambda f(u) + \zeta(u)$ and $g'(u) = \zeta(u)$ as the laws of motion. The constraints are $\zeta(u) \geq 0$ and $g(u) \geq 0$. The objective function, ignoring constant terms, is

$$\max \int_0^{\bar{X}} \left[\lambda e^{-\lambda u} F\left(W - bu - bg(u), X = 0\right) - b\lambda e^{-\lambda \bar{X}} f(u)\right] du \tag{48}$$

Then, the Hamiltonian is

$$\begin{aligned} \mathcal{H} =& e^{-\lambda u} \lambda F\left(W - bu - bg, X = 0\right) - b\lambda e^{-\lambda \bar{X}} f \\ &+ \gamma_1(\lambda f + \zeta) + \gamma_2 \zeta + \eta_1(\zeta - 0) + \eta_2(g - 0) \end{aligned} \tag{49}$$

The optimality conditions are

$$0 = \frac{\partial \mathcal{H}}{\partial \zeta} = \gamma_1 + \gamma_2 + \eta_1 \tag{50}$$

$$-\gamma_1' = \frac{\partial \mathcal{H}}{\partial f} = -b\lambda e^{-\lambda \bar{X}} + \gamma_1 \lambda \tag{51}$$

$$-\gamma_2' = \frac{\partial \mathcal{H}}{\partial g} = -e^{-\lambda X} b\lambda F_W\left(W - bu - bg, X = 0\right) + \eta_2 \tag{52}$$

We can solve for $\gamma_1(u)$ directly:

$$\gamma_1(u) = k_1 e^{-\lambda u} + b e^{-\lambda \bar{X}} \tag{53}$$

for some constant k_1.

We now work through the various cases with respect to η_1 and η_2:

- Imagine that neither constraint binds for some X, so $\eta_1 = \eta_2 = 0$. Then, $\gamma_2 = -\gamma_1 = -k_1 e^{-\lambda u} + b e^{-\lambda \bar{X}}$ and so $-\gamma_2' = -\lambda k_1 e^{-\lambda u}$. Plugging that back in (52), we obtain $bF_W\left(W - bu - bg, X = 0\right) = k_1$ which implies $u + g$ is a constant, which violates g weakly increasing.
- Imagine that $g' \geq 0$ binds but $g \geq 0$ does not, so $\eta_1 > 0$ and $\eta_2 = 0$. Then, $-\gamma_2' = \gamma_1' + \eta_1' = -\lambda k_1 e^{-\lambda u} + \eta_1'$. This is a valid differential equation solution.
- Imagine that $g' \geq 0$ does not bind but $g \geq 0$ does, so $\eta_2 > 0$ and $\eta_1 = 0$. This is the solution in which $g(u) = 0$ and is valid.

Thus, we need only to consider the case in which $g(u) = g(0)$ is constant and optimize over that constant.

To proceed, we differentiate the definition of g and solve for f to obtain

$$f(u) = e^{\lambda u} \left[g(0) + \int_0^u e^{-\lambda v} g'(v) dv\right] \tag{54}$$

If g is constant, we have $f(u) = e^{\lambda u} g(0)$. Using the definition of $g(u)$ to show that $\int_0^u \lambda f(v)dv = g(\bar{X}) - f(\bar{X})$, the objective function for the principal can be written as

$$\max \int_0^{\bar{X}} \left[\lambda e^{-\lambda u} F\left(W - bu - bg(0), X = 0\right) du\right] + b(g(\bar{X}) - f(\bar{X})) \tag{55}$$

$$= \max \int_0^{\bar{X}} \left[\lambda e^{-\lambda u} F\left(W - bu - bg(0), X = 0\right) du\right] + bg(0)(1 - e^{\lambda \bar{X}}) \tag{56}$$

Given the concavity of F in W, the first-order condition for $g(0)$ is

$$- \int_0^{\bar{X}} \left[\lambda e^{-\lambda u} b F_W\left(W - bu - bg(0), X = 0\right) du\right] - b(e^{\lambda \bar{X}} - 1) \tag{57}$$

By Lemma 4, $F_W(\cdot, X = 0) > -1$, then this expression is negative and the optimal value of $g(0)$ is 0. Since g is constant, we also have $f = g = 0$, so

$$J(X) = J^{min} = \frac{b}{\lambda}\left(e^{\lambda X} - 1\right) \tag{58}$$

B.3 Implementation

The optimal contract can be implemented with a time budget with the following properties:

1. $S_0 = W_0/b$
2. $dS_t = -dt - M_t dN_t$. $M_t = 0$ if $S_t \geq \bar{X}$. If $S_t < \bar{X}$, M_t is a binary random variable that takes value $\bar{X} - S_t$ with probability $p = S_t/\bar{X}$ and $-S_t$ with probability $1 - p$.
3. The contract ends at $t = \tau$ if $S_\tau = 0$.

Part I and II of the proof shows that if a setback is reported at any time t, the agent's continuation utility is $W_t = W_0 - bt$. Therefore, $W_t = b(S_0 - t) = bS_t$, where the first equality comes from Properties 1 above and the second equality utilizes Property 2, respectively. Randomization occurs when a setback is reported and $W_t < b\bar{X}$ where W_t jumps to $b\bar{X}$ with probability $p = W_t/b\bar{X}$ or 0 with probability $1-p$. Because $W_t = bS_t$ following any setback, randomization occurs if $S_t < \bar{X}$, and S_t jumps to $\bar{X}$ with probability $p = bS_{t-}/b\bar{X} = S_{t-}/\bar{X}$ or 0 with probability $1 - p$ (Property 2 and 3). Finally, combining (44) and (45) and using the fact that $f = g = 0$, the prize for project completion is

$$K_\tau = W_0 - b\tau + \frac{b}{\lambda}\left(e^{\lambda \bar{X}} - 1\right) \tag{59}$$

$$= bS_\tau + \frac{b}{\lambda}\left(e^{\lambda \bar{X}} - 1\right) \tag{60}$$

B.4 Truthful Reporting

Next we show that the linear time-budget is sufficient to induce the agent to report truthfully and take the high action. The agent's objective is to solve (5), subject to the evolution of state variables:

$$dX_t = a_t(dt - X_t dN_t) \tag{61}$$

$$dS_t = -dt - M_t d\hat{N}_t \tag{62}$$

where

$$M_t = \begin{cases} 0 & \text{if } S_{t-} \geq \bar{X} \\ \bar{X} - S_{t-} & \text{if } S_{t-} < \bar{X} \text{ , with probability } \frac{S_{t-}}{\bar{X}} \\ -S_{t-} & \text{if } S_{t-} < \bar{X} \text{ , with probability } 1 - \frac{S_{t-}}{\bar{X}} \end{cases} \tag{63}$$

Let $U(X,S)$ denote the agent's value function. Then $U(X,S)$ satisfies

$$\begin{aligned} 0 = \max \Big\{ & \mathrm{E}U(X, S-M) - U(X,S), \\ & U_X a + (1-a)b - U_S \\ & + \lambda \max\left\{\mathrm{E}U(0, S-M) - U(X,S), U(0,S) - U(X,S)\right\} \Big\} \end{aligned} \tag{64}$$

plus the following boundary conditions:

$$U(\bar{X}, S) = bS + \frac{b}{\lambda}\left(e^{\lambda \bar{X}} - 1\right) \tag{65}$$

$$U(X, 0) = 0 \tag{66}$$

The first line of (64) represents the change of utility from reporting a false setback or not. The second line represents the flow utility from working or shirking, and the third line represents the change of utility from postponing the report of a true setback or not. The two boundary conditions come from project completion and termination, respectively, both of which are verifiable events for the principal.

The idea behind (64) is that the agent is making a branched choice. The first max is over whether to announce a false setback or not to do that and instead to see what happens over dt. If there is no false setback, the agent chooses to work or shirk, and whether to postpone the report of a setback when one occurs.

Outside the randomization region, $M = 0$ which implies $U(X, S-M) - U(X,S) = 0$. That is, the agent does not benefit from falsely reporting a setback. Furthermore, when

$M = 0$, (64) implies

$$0 \geq \max_a \left[U_X a + (1-a)b - U_S + \lambda \max\left(U(0,S) - U(X,S)\right)\right] \tag{67}$$

where the equality is achieved when $a = 1$ and

$$U(X,S) = bS + \frac{b}{\lambda}\left(e^{\lambda X} - 1\right) \tag{68}$$

Moreover, (68) satisfies the boundary conditions (65) and (66). Therefore, the agent achieves the highest utility from exerting the high effort and reporting actual setbacks immediately outside the randomization region.

Inside the randomization region, M_t equals $\bar{X} - S_{t-}$ with probability $S_{t-}/\bar{X}$ and equals $-S_{t-}$ with probability $1 - S_{t-}/\bar{X}$. Then equation (64) becomes

$$\begin{aligned} 0 = \max \Bigg\{ & \left(1 - \frac{S}{\bar{X}}\right) U(X,0) + \frac{S}{\bar{X}} U(X,\bar{X}) - U(X,S), \\ & U_X a + (1-a)b - U_S \\ & \quad + \lambda \max\left\{\left(1 - \frac{S}{\bar{X}}\right) U(0,0) + \frac{S}{\bar{X}} U(0,\bar{X}) - U(X,S), U(0,S) - U(X,S)\right\}\Bigg\} \end{aligned} \tag{69, 70}$$

With boundary conditions $U(X,0) = 0$ and $U(X,\bar{X}) = b\bar{X} + \frac{b}{\lambda}\left(e^{\lambda X} - 1\right)$ (from (68) at $S = \bar{X}$). Replacing $U(X,0)$ and $U(X,\bar{X})$ in (70) with these boundary conditions and substituting in $U(X,S) = bS + \frac{b}{\lambda}\left(e^{\lambda X} - 1\right)$ yields

$$0 = \max\left\{\left(\frac{S}{\bar{X}} - 1\right)\frac{b}{\lambda}\left(e^{\lambda X} - 1\right), b\left(e^{\lambda X} - 1\right)(a-1)\right\} \tag{71}$$

where we have used the fact that $S/\bar{X} < 1$ inside the randomization region. Equation (71) implies that the agent prefers working if $X > 0$ and is indifferent if $X = 0$. The agent strictly prefers not announcing a false setback, and is indifferent between postponing the report of a setback or not when one actually occurs. $\square$

C Proof of Proposition 3

Recall the definition of $\tau(S)$ from (10). From the randomization for $S \in [0, \bar{X}]$, we have $\tau(S) = \frac{S}{\bar{X}}\tau(\bar{X})$ for $S \in [0, \bar{X}]$.

We will use an iterative procedure to find $\tau(S) = \tau_n(S)$ on interval $S \in (\bar{X} + n\bar{X}, \bar{X} +$

$(n+1)\bar{X}]$ for $n \geq 0$. We start by observing that for any $S \geq \bar{X}$,

$$\tau(S) = \int_0^{\bar{X}} \lambda e^{-\lambda t}(t + \tau(S-t))dt + e^{-\lambda \bar{X}}\bar{X} \tag{72}$$

Further define

$$\phi_n(\nu) = \int_{\nu - n\bar{X}}^{\bar{X}} \lambda e^{-\lambda t}\tau_{n-1}(\nu - t)dt \tag{73}$$

$$\xi = \int_0^{\bar{X}} \lambda e^{-\lambda t} t dt + e^{-\lambda \bar{X}}\bar{X} = \frac{1}{\lambda}\left(1 - e^{-\lambda \bar{X}}\right) \tag{74}$$

To continue,

$$\begin{aligned}
\tau(S) &= \int_0^{S-n\bar{X}} \lambda e^{-\lambda t}(t + \tau(S-t))dt + \int_{S-n\bar{X}}^{\bar{X}} \lambda e^{-\lambda t}(t + \tau(S-t))dt + e^{-\lambda \bar{X}}\bar{X} \\
&= \int_0^{S-n\bar{X}} \lambda e^{-\lambda t}\tau(S-t)dt + \int_0^{S-n\bar{X}} \lambda e^{-\lambda t} t dt \\
&\quad + \int_{S-n\bar{X}}^{\bar{X}} \lambda e^{-\lambda t}(t + \tau(S-t))dt + e^{-\lambda \bar{X}}\bar{X} \\
&= \int_0^{S-n\bar{X}} \lambda e^{-\lambda t}\tau(S-t)dt + \int_0^{\bar{X}} \lambda e^{-\lambda t} t dt + \int_{S-n\bar{X}}^{\bar{X}} \lambda e^{-\lambda t}\tau(S-t)dt + e^{-\lambda \bar{X}}\bar{X} \\
&= \int_0^{S-n\bar{X}} \lambda e^{-\lambda t}\tau(S-t)dt + \phi_n(S) + \xi
\end{aligned} \tag{75}$$

Differentiating with respect to S and then integrating by parts yields

$$\tau'(S) = \lambda\phi_n(S) + \phi_n'(S) + \lambda\xi \tag{76}$$

So,

$$\tau(S) = \tau(n\bar{X}) + \int_{n\bar{X}}^{S} \tau'(\nu)d\nu = \tau(n\bar{X}) + \int_{n\bar{X}}^{S} (\lambda\phi_n(\nu) + \phi_n'(\nu) + \lambda\xi)d\nu \tag{77}$$

where $\nu - t \leq n\bar{X}$ and $\tau(\nu), \nu \leq \bar{X}$ is known from iteration in the previous round.

C.1 Part (i)

First, $\tau(S)$ is increasing. For $S \leq \bar{X}$, we have $\tau(S) = \frac{\tau_1(\bar{X})}{\bar{X}} S$, and hence $\tau'(S) > 0$. For $S > \bar{X}$,

$$\tau(S) = \int_0^{\bar{X}} \lambda e^{-\lambda t}(t + \tau(S-t))dt + e^{-\lambda \bar{X}} \bar{X} \tag{78}$$

and taking the derivative yields

$$\tau'(S) = \int_0^{\bar{X}} \lambda e^{-\lambda t} \tau'(S-t)dt \tag{79}$$

Hence $\tau'(S) > 0$ because $\tau'(S-t) < 0$ for $\forall 0 < t < S$.

Second, $\tau(S)$ is concave for $S \in (n\bar{X}, (n+1)\bar{X}]$. When $n = 1$, we have $\tau_1(S) = \frac{\tau_1(\bar{X})}{\bar{X}} S$, which is weakly concave in S. Then, suppose concavity holds for $n-1$, we show that it holds for n as well. Since

$$\tau_n'(S) = \lambda \phi_n(S) + \phi_n'(S) + \lambda \xi \tag{80}$$

differentiating yields

$$\tau_n''(S) = \lambda \phi_n'(S) + \phi_n''(S) \tag{81}$$

Differentiating ϕ yields

$$\phi_n'(S) = - \lambda e^{-\lambda(S-n\bar{X})} \tau_{n-1}(n\bar{X}) + \int_{S-n\bar{X}}^{\bar{X}} \lambda e^{-\lambda t} \tau_{n-1}'(S-t)dt \tag{82}$$

$$\phi_n''(S) = \lambda^2 e^{-\lambda(S-n\bar{X})} \tau_{n-1}(n\bar{X}) - \lambda e^{-\lambda(S-n\bar{X})} \tau_{n-1}'(n\bar{X}) \tag{83}$$

$$- \lambda e^{-\lambda(S-n\bar{X})} \tau_{n-1}'(n\bar{X}) + \int_{S-n\bar{X}}^{\bar{X}} \lambda e^{-\lambda t} \tau_{n-1}''(S-t)dt \tag{84}$$

and

$$\begin{aligned}
\tau_n''(S) &= \lambda \phi_n'(S) + \phi_n''(S) \\
&= \lambda \int_{S-n\bar{X}}^{\bar{X}} \lambda e^{-\lambda t} \tau_{n-1}'(S-t)dt - 2\lambda e^{-\lambda(S-n\bar{X})} \tau_{n-1}'(n\bar{X}) + \lambda e^{-\lambda t} \tau_{n-1}''(S-t)dt \\
&\leq \lambda \int_{S-n\bar{X}}^{\bar{X}} \lambda e^{-\lambda t} \tau_{n-1}'\left(S - (S-n\bar{X})\right) dt - 2\lambda e^{-\lambda(S-n\bar{X})} \tau_{n-1}'(n\bar{X}) + 0 \\
&= -\lambda e^{-\lambda \bar{X}} \tau_{n-1}'(n\bar{X}) - \lambda e^{-\lambda(S-n\bar{X})} \tau_{n-1}'(n\bar{X}) < 0
\end{aligned} \tag{85}$$

The last step is from the fact that τ_{n-1} is a increasing function. So, by induction we have

$\tau_n(S)$ is concave on interval $S \in (n\bar{X}, (n+1)\bar{X}]$ for every n.

Third, we show that $\tau_n(S)$ is a polynomial of order n. We have

$$\begin{aligned}
\lambda\phi_n(S) + \phi_n'(S) = {} & \lambda \int_{S-n\bar{X}}^{\bar{X}} \lambda e^{-\lambda t}\tau_{n-1}(S-t)dt - \lambda e^{-\lambda(S-n\bar{X})}\tau_{n-1}(n\bar{X}) \\
& + \int_{S-n\bar{X}}^{\bar{X}} \lambda e^{-\lambda t}\tau_{n-1}'(S-t)dt \\
= {} & \lambda \int_{S-n\bar{X}}^{\bar{X}} \lambda e^{-\lambda t}\tau_{n-1}(S-t)dt - \lambda e^{-\lambda(S-n\bar{X})}\tau_{n-1}(n\bar{X}) \\
& \quad + \lambda e^{-\lambda(S-n\bar{X})}\tau_{n-1}(n\bar{X}) - \lambda e^{-\lambda\bar{X}}\tau_{n-1}(S-\bar{X}) \\
& \quad - \lambda \int_{S-n\bar{X}}^{\bar{X}} \lambda e^{-\lambda t}\tau_{n-1}(S-t)dt \\
= {} & -\lambda e^{-\lambda\bar{X}}\tau_{n-1}(S-\bar{X})
\end{aligned} \tag{86, 87}$$

and therefore (77) implies

$$\tau_n(S) = \tau(n\bar{X}) + \int_{n\bar{X}}^{S} \left(\lambda\xi - \lambda e^{-\lambda\bar{X}}\tau_{n-1}(S-\bar{X})\right) d\nu \tag{88}$$

since τ_1 is linear and hence polynomial of order 1, by (88) τ_2 is quadratic and τ_n is polynomial of order n.

C.2 Parts (ii) and (iii)

Because

$$\tau(S) = \frac{1}{\lambda}\left[1 - P(S)e^{\lambda\bar{X}} - (1-P(S))\right] < \infty \tag{89}$$

$\tau(S)$ is a bounded function. Since every monotone and bounded function in $\mathbb{R}$ converges, let the limit be

$$\tau(\infty) = \lim_{S\to\infty}\tau(S) \tag{90}$$

Since

$$\tau(S) = \int_0^{\bar{X}} \lambda e^{-\lambda t}(t + \tau(S-t))dt + e^{-\lambda\bar{X}}\bar{X} \tag{91}$$

taking $S \to \infty$ yields

$$\tau(\infty) = \int_0^{\bar{X}} \lambda e^{-\lambda t}(t + \tau(\infty))dt + e^{-\lambda \bar{X}} \bar{X} = \frac{1}{\lambda}(1 - e^{-\lambda \bar{X}}) + \tau(\infty)(1 - e^{-\lambda \bar{X}}) \tag{92}$$

hence we have

$$\tau(\infty) = \frac{e^{\lambda \bar{X}} - 1}{\lambda} \tag{93}$$

Moreover, by the convergence of $\tau(S)$, we have

$$\lim_{S\to\infty} \tau'(S) = \lim_{S\to\infty} \lim_{\Delta S \to 0} \frac{\tau(S + \Delta S) - \tau(S)}{\Delta S} = 0 \tag{94}$$

Then, at $S = n\bar{X}$, the left slope is

$$\begin{aligned}
\tau'_{n-1}(n\bar{X}) &= \lambda \phi_{n-1}(n\bar{X}) + \phi'_{n-1}(n\bar{X}) + \lambda\xi \\
&= \lambda \int_{\bar{X}}^{\bar{X}} \lambda e^{-\lambda t} \tau_{n-1}(n\bar{X} - t)dt - \lambda e^{-\lambda \bar{X}} \tau_{n-1}(n\bar{X}) \\
&\quad + \int_{\bar{X}}^{\bar{X}} \lambda e^{-\lambda t} \tau'_{n-1}(n\bar{X} - t)dt + \lambda\xi \\
&= -\lambda e^{-\lambda \bar{X}} \tau_{n-1}(n\bar{X}) + \lambda\xi
\end{aligned} \tag{95}$$

The right slope is

$$\begin{aligned}
\tau'_n(n\bar{X}) &= \lambda \phi_n(n\bar{X}) + \phi'_n(n\bar{X}) + \lambda\xi \\
&= \lambda \int_0^{\bar{X}} \lambda e^{-\lambda t} \tau_{n-1}(n\bar{X} - t)dt - \lambda \tau_{n-1}(n\bar{X}) + \lambda\xi
\end{aligned} \tag{96}$$

Hence

$$\begin{aligned}
\tau'_{n-1}(n\bar{X}) - \tau'_n(n\bar{X}) &= -\lambda e^{-\lambda \bar{X}} \tau_{n-1}(n\bar{X}) - \lambda \int_0^{\bar{X}} \lambda e^{-\lambda t} \tau_{n-1}(n\bar{X} - t)dt + \lambda \tau_{n-1}(n\bar{X}) \\
&> -\lambda e^{-\lambda \bar{X}} \tau_{n-1}(n\bar{X}) - \lambda \int_0^{\bar{X}} \lambda e^{-\lambda t} \tau_{n-1}(n\bar{X})dt + \lambda \tau_{n-1}(n\bar{X}) \\
&= -\lambda e^{-\lambda \bar{X}} \tau_{n-1}(n\bar{X}) - \lambda \tau_{n-1}(n\bar{X}) + \lambda \tau_{n-1}(n\bar{X}) - \lambda \tau_{n-1}(n\bar{X}) \\
&= 0
\end{aligned}$$

where the inequality comes from the fact that τ_{n-1} is a (strictly) increasing function. $\square$

D Proof of Proposition 4 and Proposition 5

D.1 Value Function Without Commitment to Randomization

The same steps used to prove Proposition 1 apply in this setting, except that upon receiving an extension the agent must randomize between working and shirking such that the principal's expected payoff from granting the extension is the same as from canceling the project, namely 0.

First observe that the agent's expected utility from working immediately following an extension is $b\bar{X}$, which is the same as his utility from shirking out the clock – so he is willing to randomize at this point. Now, suppose the agent receives an extension and works. Let $s \in [0, \bar{X}]$ be the amount of time since the extension was granted. Any setback during the extension will reset the principal's expected utility to 0. Therefore her expected utility when the agent works as a function of s is

$$\left(\mathcal{R} - c(\bar{X} - s)\right) e^{-\lambda(\bar{X}-s)} - \int_0^{\bar{X}-s} ct\lambda e^{-\lambda t} dt \tag{97}$$

$$= \mathcal{R} e^{-\lambda(\bar{X}-s)} - \frac{c}{\lambda}\left(1 - e^{-\lambda(\bar{X}-s)}\right) \equiv \hat{F}(\bar{X} - s, s), \tag{98}$$

where $\mathcal{R} \equiv R - \frac{b}{\lambda}\left(e^{\lambda\bar{X}} - 1\right)$ is the principal's surplus at the completion of the project.

Let q be the probability the agent shirks out the clock following an extension. Then randomization between cancelation and granting an extension is incentive compatible for the principal iff

$$(1 - q)\hat{F}(\bar{X}, 0) - qc\bar{X} = 0, . \tag{99}$$

Solving for q yields the formula given in the claim.

There is one further item that must be checked: the principal must be willing to continue the contract if the agent does not report a setback. This must be checked because the principal does not know if the agent is working or shirking out the clock and shirking does not generate setbacks. The probability that the principal believes the agent is shirking conditional on having reported no setbacks by s is

$$q(s) = \frac{\hat{F}(\bar{X}, 0)}{\hat{F}(\bar{X}, 0) + c\bar{X}e^{-\lambda s}}.$$

Then, the principal is willing to let the clock run during the extension iff for all $s \in [0, \bar{X}]$,

$$
\begin{aligned}
& (1 - q(s))\hat{F}(\bar{X} - s, s) - q(s)c(\bar{X} - s) \geq 0 \\
\iff & \bar{X}e^{-\lambda s}\hat{F}(\bar{X} - s, s) \geq (\bar{X} - s)\hat{F}(\bar{X}, 0) \\
\iff & \bar{X}e^{-\lambda\bar{X}}\left(\mathcal{R}e^{-\lambda(\bar{X}-s)} - \frac{c}{\lambda}\left(1 - e^{-\lambda(\bar{X}-s)}\right)\right) \geq (\bar{X} - s)\left(\mathcal{R}e^{-\lambda\bar{X}} - \frac{c}{\lambda}\left(1 - e^{-\lambda\bar{X}}\right)\right) \\
\iff & \frac{c}{\lambda}\left(\bar{X} - s + se^{-\lambda\bar{X}} - \bar{X}e^{-\lambda s}\right) \geq -s\mathcal{R}e^{-\lambda\bar{X}}.
\end{aligned}
$$

The right side is non-positive, so the claim will hold if the left side is non-negative for $s \in [0, \bar{X}]$:

$$
\bar{X} - s + se^{-\lambda\bar{X}} - \bar{X}e^{-\lambda s} \geq 0.
$$

At $s \in \{0, \bar{X}\}$ the inequality above evidently binds. For $s \in (0, \bar{X})$, differentiating the left side w.r.t. s yields $-1 + e^{-\lambda\bar{X}} + \lambda\bar{X}e^{-\lambda s}$. This is positive at $s = 0$, negative at $s = \bar{X}$, and 0 at a single point between 0 and $\bar{X}$.□

D.2 Optimal Initial Time Budget

Define $\hat{P}(S_0)$ to be the probability that the project succeeds given an initial time budget S_0 and initial progress $X_0 = 0$. Then, the principal's value function can be written

$$
\hat{F}(S_0, X = 0) = \hat{P}(S_0)E\left[R - bS_{\hat{\tau}^{proj}} - \frac{b}{\lambda}(e^{\lambda\bar{X}} - 1)\middle| S_0, X_{\hat{\tau}^{proj}} = \bar{X}\right] - cE[\hat{\tau}^{proj}|S_0] \tag{100}
$$

The first term is the expected reward minus payment, the second term is expected running cost.

Define $\hat{\tau}^{work}$ as the random time the agent stops working (either by completion of project, or by the agent shirking) and $\hat{\tau}^{proj}$ as the random time the project stops (either by completion of project, or by termination of project). Both $\hat{\tau}^{work}$ and $\hat{\tau}^{proj}$ are $\mathcal{F}_t^N-$ stopping times, and $\hat{\tau}^{work} \leq \hat{\tau}^{proj}$.

Next, we continue with the martingale analysis, following the main text. For $t \leq \hat{\tau}^{work}$, we have that $dX_t = dt - X_t dN_t$ implies $d(e^{\lambda X_t} - \lambda t) = (1 - \lambda e^{\lambda X_t})(dN_t - \lambda dt)$, which is a martingale. Otherwise, for $\hat{\tau}^{work} < t \leq \hat{\tau}^{proj}$, we have $dX_t = 0$, which implies that $e^{\lambda X_t} - \lambda t$ is not a martingale. Applying the optional stopping theorem regarding $e^{\lambda X_t} - \lambda t$, we have

$$
\begin{aligned}
1 &= \mathrm{E}[e^{\lambda X_t} - \lambda t | S_0, t = \hat{\tau}^{work}] \\
&= \hat{P}(S_0)\mathrm{E}[e^{\lambda X_{\hat{\tau}^{work}}}|S_0, X_{\hat{\tau}^{work}} = \bar{X}] + (1 - \hat{P}(S_0))\mathrm{E}[e^{\lambda X_{\hat{\tau}^{work}}}|S_0, X_{\hat{\tau}^{work}} = 0] - \lambda\mathrm{E}[\hat{\tau}^{work}|S_0] \\
&= \hat{P}(S_0)e^{\lambda\bar{X}} + (1 - \hat{P}(S_0)) - \lambda E[\hat{\tau}^{work}|S_0] \qquad (101)
\end{aligned}
$$

Notice that we have used the fact that success or failure of the project is fully determined at $t = \hat{\tau}^{work}$ – one does not have to wait until $t = \hat{\tau}^{proj}$. Solving, we have

$$\hat{P}(S_0) = \frac{\lambda}{e^{\lambda\bar{X}} - 1}\mathrm{E}[\hat{\tau}^{work}|S_0] \tag{102}$$

Next, apply the optional stopping theorem regarding $S_t + t$

$$\begin{aligned} S_0 &= \mathrm{E}[S_t + t|S_0, t = \hat{\tau}^{proj}] \\ &= \hat{P}(S_0)\mathrm{E}[S_{\hat{\tau}^{proj}}|S_0, X_{\hat{\tau}^{proj}} = \bar{X}] + (1 - \hat{P}(S_0))\mathrm{E}[S_{\hat{\tau}^{proj}}|S_0, X_{\hat{\tau}^{proj}} = 0] + \mathrm{E}[\hat{\tau}^{proj}|S_0] \\ &= \hat{P}(S_0)\mathrm{E}[S_{\hat{\tau}^{proj}}|S_0, X_{\hat{\tau}^{proj}} = \bar{X}] + \mathrm{E}[\hat{\tau}^{proj}|S_0] \end{aligned} \tag{103}$$

The third line follows from that $\mathrm{E}[S_{\hat{\tau}^{proj}}|S_0, X_{\hat{\tau}^{proj}} = 0] = 0$. If the project has ended and it is incomplete, there must not be any time remaining.

Examining the terms in the principal's value function one at a time, we have

$$\hat{P}(S_0)R = \frac{R\lambda}{e^{\lambda\bar{X}} - 1}\mathrm{E}[\hat{\tau}^{work}|S_0] \tag{104}$$

$$-\hat{P}(S_0)E[bS_{\hat{\tau}^{proj}}|S, X_{\hat{\tau}^{proj}} = \bar{X}] = b\mathrm{E}[\hat{\tau}^{proj}|S_0] - bS_0 \tag{105}$$

$$-\hat{P}(S_0)\frac{b}{\lambda}(e^{\lambda\bar{X}} - 1) = -b\mathrm{E}[\hat{\tau}^{work}|S_0] \tag{106}$$

$$-cE[\hat{\tau}^{proj}|S_0] = -cE[\hat{\tau}^{proj}|S_0] \tag{107}$$

Adding these up and re-arranging, we obtain

$$\begin{aligned} \hat{F}(S_0, X = 0) &= R\frac{\lambda\mathrm{E}[\hat{\tau}^{work}|S_0]}{e^{\lambda\bar{X}} - 1} - cE[\hat{\tau}^{proj}|S_0] - bS_0 + b\left(\mathrm{E}[\hat{\tau}^{proj}|S_0] - \mathrm{E}[\hat{\tau}^{work}|S_0]\right) \\ \hat{F}(S_0, X = 0) &= \left(\frac{R\lambda}{e^{\lambda\bar{X}} - 1} - c\right)\mathrm{E}[\hat{\tau}^{proj}|S_0] - bS_0 + \left(\frac{R\lambda}{e^{\lambda\bar{X}} - 1} + b\right)\left(\mathrm{E}[\hat{\tau}^{work}|S_0] - \mathrm{E}[\hat{\tau}^{proj}|S_0]\right) \end{aligned}$$

Next, we observe that upon entering the short-leash region, we have $E[\hat{\tau}^{proj}|S = \bar{X}, X = 0] = E[\tau|S = \bar{X}, X = 0] = \bar{X}$. This means that commitment does not change the average project duration. In addition, outside the short-leash region, the agent is not shirking, and the evolution of X and S are the same with and without commitment. Thus, we have $E[\hat{\tau}^{proj}|S_0] = E[\tau|S_0]$, and we can write the principal's value function without commitment as

$$\hat{F}(S_0, X = 0) = F(S_0, X = 0) - \left(\frac{R\lambda}{e^{\lambda\bar{X}} - 1} + b\right)\left(\mathrm{E}[\hat{\tau}^{proj}|S_0] - \mathrm{E}[\hat{\tau}^{work}|S_0]\right) \tag{108}$$

As a reminder, F is the principal's value function with commitment. We want to show that $\left(\mathrm{E}[\hat{\tau}^{proj}|S_0] - \mathrm{E}[\hat{\tau}^{work}|S_0]\right)$ is decreasing in S_0.

Next, we will use the fact that outside of the short-lease region, the agent never shirks. Thus, if the project succeeds before entering the short-lease region, we have $\hat{\tau}^{work} = \hat{\tau}^{proj}$.

Then, we can write the difference between $E[\hat{\tau}^{proj}|S_0]$ and $E[\hat{\tau}^{work}|S_0]$ entirely as a function of the probability of entering the short-lease region and the expected time spent there:

$$E[\hat{\tau}^{proj}|S_0] - E[\hat{\tau}^{work}|S_0] = (1 - \Pr(X_{\hat{\tau}^{proj}} = \bar{X}, S_{\hat{\tau}^{proj}} > \bar{X})) \\ \times \left(\mathrm{E}[\hat{\tau}^{proj}|S_{\hat{\tau}^{proj}} \leq \bar{X}] - \mathrm{E}[\hat{\tau}^{work}|S_{\hat{\tau}^{work}} \leq \bar{X}]\right) \tag{109}$$

where $(1-\Pr(X_{\hat{\tau}^{proj}} = \bar{X}, S_{\hat{\tau}^{proj}} > \bar{X})) = (1-\Pr(X_{\hat{\tau}^{work}} = \bar{X}, S_{\hat{\tau}^{work}} > \bar{X}))$ is the probability of entering the short lease region. Then we notice: $\mathrm{E}[\hat{\tau}^{proj}|S_{\hat{\tau}^{proj}} \leq \bar{X}] - \mathrm{E}[\hat{\tau}^{work}|S_{\hat{\tau}^{work}} \leq \bar{X}]$, the difference in the expected time taken by the end of work and the end of the project in the short leash region, does not depend on the starting value of S_0.

Since the probability that the agent does not succeed before the short-leash region is decreasing in S, we must have $E[\hat{\tau}^{proj}|S_0] - \mathrm{E}[\hat{\tau}^{work}|S_0]$ is decreasing in S_0. Thus, from (108), the principal's value function without commitment is equal to the value function with commitment minus a term that is decreasing in S_0; without-commitment and with-commitment have increasing differences in S_0. Both value functions are bounded from above and are $-\infty$ for $S_0 \to \infty$. By the standard logic of increasing differences, we must have that the optimal S_0 without commitment is weakly larger than the optimal S_0 with commitment. $\square$

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