# Sufficient Statistics for Dynamic Spatial Economics

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# Motivation

- How does the spatial distribution of economic activity respond to local shocks? (e.g. productivity, transport infrastructure, trade)
  - This response can be gradual because of migration frictions for mobile factors and the accumulation of immobile factors (capital structures)

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- We make four main contributions:

1. Incorporate forward-looking capital investments in a dynamic spatial model with migration and characterize existence / uniqueness
2. Linearize this model and derive a closed-form solution for the economy's entire transition path in terms of sufficient statistics
3. Use our linearization to analytically characterize transition path in terms of the eigenvalues and eigenvectors of a transition matrix
4. Apply our framework to examine the reallocation US economic activity from the "Rust Belt" to the "Sun Belt"

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  - Use an eigendecomposition of $\boldsymbol{P}$ to isolate the locations exposed to particular shocks and the shocks that impact particular locations
- Applications: US state data 1965-2015; state-industry data 1999-2015
  - Decline of the "Rust-Belt" and rise of the "Sun-Belt"
  - Slow convergence and heterogeneous impact of local shocks

# Related Literature

- Theoretical work on economic geography
  - Krugman (1991, 1992), Helpman (1998), Fujita et al. (1999), Baldwin (2001)
- Static quantitative spatial trade models
  - Armington (1969), Eaton & Kortum (2002), Redding & Sturm (2008), Allen & Arkolakis (2014), Ramondo et al. (2016), Redding (2016), Donaldson (2018), Caliendo et al. (2018), Fajgelbaum et al. (2019), Fajgelbaum & Gaubert (2020)
- Dynamic Models of Trade and Geography
  - Artuç et al. (2010), Desmet & Rossi-Hansberg (2014), Desmet et al. (2018), Caliendo et al. (2019), Caliendo & Parro (2020), Peters (2019), Peters & Walsh (2019), Walsh (2019), Allen & Donaldson (2020), Greaney (2020)
- Research on sufficient statistics for welfare in international trade
  - Arkolakis et al. (2012), Adão et al. (2017), Adão et al. (2019), Baqaee & Farhi (2019), Galle et al. (2019), Huo et al. (2019), Barthelme et al. (2019), Kleinman et al. (2020)
- Empirical evidence on the U.S. Rust Belt and local labor market shocks
  - Blanchard & Katz (1992), Holmes (1998), Kovak (2013), Feyrer et al. (2007), Topolova (2010), Kovak (2013), Autor et al. (2013, 2014, 2020), Dix-Carneiro & Kovak (2017), Charles et al. (2018), Fort et al. (2018), Amior & Manning (2018), Pavcnik & McCaig (2018), Pierce & Schott (2019), Alder et al. (2019)

# Outline

- Dynamic Spatial Model
- Extensions
- Data
- Empirical Results
- Conclusions

# Model Setup

- Multi-location, single-sector Armington model (extensions later)
- Economy consists of a set of locations $i \in \{1, \ldots, N\}$
- Locations differ in productivity, amenities, bilateral goods trade costs, and bilateral migration costs
- Two types of agents: workers and landlords
- Continuum of workers
  - Endowed with one unit of labor
  - Geographically mobile subject to migration costs
  - No savings-investment technology ("hand to mouth")
  - Make dynamic forward-looking migration decisions to maximize intertemporal utility
- Continuum of landlords in each location
  - Own the stock of local capital
  - Geographically immobile
  - Make dynamic forward-looking consumption-investment choices to maximize intertemporal utility

# Worker Migration (CDP)

- At the beginning of period $t$, mass of workers $\ell_{it}$ in location $i$:
  - Produce and consume
  - Observe extreme value idiosyncratic mobility shocks $\{\epsilon_{gt}\}$
  - Choose optimal location for period $t+1$ given mobility costs $\kappa_{git}$
- Expected value of living in location $i$ in period $t$ depends on wage ($w_{it}$), cost of living ($p_{it}$), amenities ($b_{it}$) and the expected value of optimal location choice

$$v_{it} = \ln\left(\frac{w_{it}}{p_{it}}\right) + \ln b_{it} + \rho \ln \sum_{g=1}^{N} \left(\exp\left(\beta v_{gt+1}\right)/\kappa_{git}\right)^{1/\rho}$$

- Location choice probabilities

$$D_{igt} = \frac{\left(\exp\left(\beta v_{gt+1}\right)/\kappa_{git}\right)^{1/\rho}}{\sum_{k=1}^{N}\left(\exp\left(\beta v_{kt+1}\right)/\kappa_{kit}\right)^{1/\rho}}$$

- Population flow condition

$$\ell_{gt+1} = \sum_{i=1}^{N} D_{igt}\ell_{it}$$

# Trade and Production

- Armington differentiation of goods by location of origin

$$p_{nt} = \left[ \sum_{i=1}^{N} p_{nit}^{-\theta} \right]^{-1/\theta}, \qquad \theta = \sigma - 1, \qquad \sigma > 1$$

- Competitive production and iceberg trade costs $\tau_{nit} \geq 1$
- Cost in location $n$ of sourcing a variety from location $i$ is

$$p_{nit} = \frac{\tau_{nit} w_{it}^{\lambda} r_{it}^{1-\lambda}}{z_{it}}, \qquad 0 < \lambda < 1$$

- Using profit maximization to substitute for equilibrium labor input, landlord income is linear in capital

$$\Pi_{it} = \lambda \left( p_{it} z_{it} \right)^{\frac{1}{\lambda}} \left( \frac{1-\lambda}{w_{it}} \right)^{\frac{1-\lambda}{\lambda}} k_{it}$$

# Landlord Investment

- Landlords have the same preferences as workers but have access to an investment technology for local capital (we also solve CRRA case)

$$v_{it}^{k} = \sum_{t=0}^{\infty} \beta^{t} \ln c_{it}^{k}$$

- Landlords in a location can produce one unit of capital in that location using one unit of the local consumption index
- Local capital is geographically immobile once installed and depreciates at constant rate $\delta$
- Intertemporal budget constraint

$$r_{it} k_{it} = p_{it} c_{it}^{k} + p_{it} \left( k_{it+1} - (1 - \delta) k_{it} \right)$$

- Logarithmic utility and linear income in capital together imply a constant saving rate (as in Moll 2014) ▸ more ▸ CRRA

$$k_{it+1} = \beta \left( r_{it} / p_{it} + (1 - \delta) \right) k_{it}$$

# Existence and Uniqueness

- Dynamic spatial model with many locations, rich geography of trade and migration costs, and two sources of dynamics

## Proposition

*There exists a unique steady-state equilibrium* $\{w_i^*, v_i^*, \ell_i^*, k_i^*\}$ *(up to a numeraire) given time-invariant location characteristics* $\{z_i, b_i, \tau_{ni}, \kappa_{ni}\}$ *that is independent of the economy's initial conditions* $\{\ell_{i0}, k_{i0}\}$. ▸ Proof

- When we introduce agglomeration forces
  - Derive condition on parameters for the existence of unique equilibrium
  - Show this condition satisfied for sufficiently small agglomeration forces

# Steady-state Sufficient Statistics

- Totally differentiating the general equilibrium conditions of the model and stacking them in matrix form

## Proposition

*The steady-state response of the endogenous variables to productivity and amenity shocks satisfies the linear system:*

$$\begin{bmatrix} \mathrm{d}\ln \boldsymbol{\ell}^* \\ \mathrm{d}\ln \boldsymbol{k}^* \\ \mathrm{d}\ln \boldsymbol{w}^* \\ \mathrm{d}\ln \boldsymbol{v}^* \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}^{z*} \\ \boldsymbol{K}^{z*} \\ \boldsymbol{W}^{z*} \\ \boldsymbol{V}^{z*} \end{bmatrix} \mathrm{d}\ln \boldsymbol{z} + \begin{bmatrix} \boldsymbol{L}^{b*} \\ \boldsymbol{K}^{b*} \\ \boldsymbol{W}^{b*} \\ \boldsymbol{V}^{b*} \end{bmatrix} \mathrm{d}\ln \boldsymbol{b}$$

*where the $N \times N$ matrices $\{\boldsymbol{L}^{z*}, \boldsymbol{K}^{z*}, \boldsymbol{W}^{z*}, \boldsymbol{V}^{z*}, \boldsymbol{L}^{b*}, \boldsymbol{K}^{b*}, \boldsymbol{W}^{b*}, \boldsymbol{V}^{b*}\}$ are functions of the four observed matrices of expenditure shares ($\boldsymbol{S}$), income shares ($\boldsymbol{T}$), outmigration shares ($\boldsymbol{D}$) and inmigration shares ($\boldsymbol{E}$) and the structural parameters of the model $\{\beta, \theta, \rho, \lambda, \delta\}$.* more

- Element $[\boldsymbol{L}^{z*}]_{in} = \mathrm{d}\ln \ell_i^* \,/\, \mathrm{d}\ln z_n$
  - Elasticity of steady-state population in location $i$ ($\ell_i^*$) with respect to an increase in productivity in location $n$ ($z_n$)

# Transition Dynamics

- Suppose that the economy at time $t = 0$ is on a convergence path towards an initial steady-state with constant fundamentals ($\boldsymbol{z}$, $\boldsymbol{b}$, $\boldsymbol{\kappa}$, $\boldsymbol{\tau}$)
- Characterize transition dynamics given shocks to fundamentals

1. At time $t = 0$, agents learn about one-time, permanent shocks to fundamentals ($\widetilde{\boldsymbol{f}} = \begin{bmatrix} \widetilde{\boldsymbol{z}} \\ \widetilde{\boldsymbol{b}} \end{bmatrix}$) from time $t = 1$ onwards that are revealed under perfect foresight
2. At time $t = 0$, agents learn about a convergent sequence of future shocks to fundamentals $\left\{\widetilde{\boldsymbol{f}}_s\right\}_{s\geq 1} = \left\{\begin{bmatrix} \widetilde{\boldsymbol{z}}_s \\ \widetilde{\boldsymbol{b}}_s \end{bmatrix}\right\}_{s\geq 1}$ from time $t = 1$ onwards that are revealed under perfect foresight
3. Consider an economy with an arbitrary initial value of the state variables at time $t = 0$ ($x_0$). Suppose that productivity and amenities evolve stochastically according to the AR(1) process and agents have rational expectations

- Transition path: 2nd-order difference equation in state variables ($\widetilde{\boldsymbol{\ell}}_t$, $\widetilde{\boldsymbol{k}}_t$) that solve with method of undetermined coefficients (Uhlig 1999)

# Transition Dynamics

## Proposition

*Suppose that the economy at time $t = 0$ is on a convergence path towards an initial steady-state with constant fundamentals $(\boldsymbol{z}, \boldsymbol{b}, \boldsymbol{\kappa}, \boldsymbol{\tau})$. At time $t = 0$, agents learn about one-time, permanent shocks to productivity and amenities ($\widetilde{\boldsymbol{f}} = \begin{bmatrix} \widetilde{\boldsymbol{z}} \\ \widetilde{\boldsymbol{b}} \end{bmatrix}$) from time $t = 1$ onwards. There exists a $2N \times 2N$ transition matrix $(\boldsymbol{P})$ and a $2N \times 2N$ impact matrix $(\boldsymbol{R})$ such that the second-order difference equation system has a closed-form solution of the form:*

$$\widetilde{\boldsymbol{x}}_{t+1} = \boldsymbol{P}\widetilde{\boldsymbol{x}}_t + \boldsymbol{R}\widetilde{\boldsymbol{f}} \quad \textit{for } t \geq 1.$$

*where* $\widetilde{\boldsymbol{x}}_t \equiv \begin{bmatrix} \widetilde{\boldsymbol{\ell}}_t \\ \widetilde{\boldsymbol{k}}_t \end{bmatrix}$ *and a tilde denotes a log deviation from the initial steady-state:* $\tilde{\boldsymbol{\ell}}_t \equiv \ln \boldsymbol{\ell}_t - \ln \boldsymbol{\ell}^*_{initial}$

▸ more ▸ convseq ▸ stochfund ▸ CDP

# Exact Additive Decomposition

- Use our linearization to obtain an exact additive decomposition of the dynamics of the spatial distribution of economic activity: more

$$\ln \boldsymbol{x}_t - \ln \boldsymbol{x}_{-1} = \underbrace{\sum_{s=0}^{t} \boldsymbol{P}^s \left(\ln \boldsymbol{x}_0 - \ln \boldsymbol{x}_{-1}\right)}_{\substack{\text{convergence given} \\ \text{initial fundamentals}}} + \underbrace{\sum_{s=0}^{t-1} \boldsymbol{P}^s \boldsymbol{R} \widetilde{\boldsymbol{f}}}_{\substack{\text{dynamics from} \\ \text{fundamental shocks}}} \qquad \text{for all } t \geq 1,$$

- With no shocks to productivity and amenities ($\widetilde{\boldsymbol{f}} = \boldsymbol{0}$), we have:

$$\ln \boldsymbol{x}^*_{\text{initial}} = \lim_{t \to \infty} \ln \boldsymbol{x}_t = \ln \boldsymbol{x}_{-1} + \left(\boldsymbol{I} - \boldsymbol{P}\right)^{-1} \left(\ln \boldsymbol{x}_0 - \ln \boldsymbol{x}_{-1}\right),$$

- Using only initial state variables (for $t = 0$ and $t = -1$) and trade and migration matrices, we can compute implied steady-states

# Spectral Analysis

- Use our linearization to characterize the economy's transition path in terms of lower-dimensional components
- Undertake an eigendecomposition of the transition matrix

$$\boldsymbol{P} \equiv \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{V},$$

- where $\boldsymbol{\Lambda}$ is a diagonal matrix of eigenvalues arranged in decreasing order by absolute values, and $\boldsymbol{V} = \boldsymbol{U}^{-1}$
- For each eigenvalue $\lambda_k$, the left-eigenvectors ($\boldsymbol{u_k}$) and right-eigenvectors ($\boldsymbol{v_k'}$) satisfy

$$\lambda_k \boldsymbol{u}_k = \boldsymbol{P}\boldsymbol{u}_k, \qquad \lambda_k \boldsymbol{v}_k' = \boldsymbol{v}_k'\boldsymbol{P}$$

- Define an eigen-shock as a shock to productivity and amenities ($\widetilde{\boldsymbol{f}}_k$) for which the initial impact of these shocks on the state variables ($\boldsymbol{R}\widetilde{\boldsymbol{f}}_k$) coincides with a real eigenvector of the transition matrix ($\boldsymbol{u}_k$)

$$\widetilde{\boldsymbol{f}}_k = \boldsymbol{R}^{-1}\boldsymbol{u}_k$$

# Spectral Analysis

## Proposition

*Consider an economy that is initially in steady-state at time $t = 0$ when agents learn about one-time, permanent shocks to productivity and amenities ($\widetilde{\boldsymbol{f}} = \begin{bmatrix} \widetilde{\boldsymbol{z}} \\ \widetilde{\boldsymbol{b}} \end{bmatrix}$) from time $t = 1$ onwards. The transition path of the state variables can be written as a linear combination the eigenvalues ($\lambda_k$) and eigenvectors ($\boldsymbol{u}_k$) of the transition matrix:*

$$\widetilde{\boldsymbol{x}}_t = \sum_{s=0}^{t-1} \boldsymbol{P}^s \boldsymbol{R} \widetilde{\boldsymbol{f}} = \sum_{k=1}^{2N} \frac{1-\lambda_k^t}{1-\lambda_k} \boldsymbol{u}_k \boldsymbol{v}_k' \boldsymbol{R} \widetilde{\boldsymbol{f}} = \sum_{k=1}^{2N} \frac{1-\lambda_k^t}{1-\lambda_k} \boldsymbol{u}_k a_k$$

*where the weights this linear combination ($a_k$) can be recovered from a linear projection of the observed shocks ($\widetilde{\boldsymbol{f}}$) on the eigenshocks ($\widetilde{\boldsymbol{f}}_k$).*

- Use this spectral analysis to distinguish shocks and exposure to shocks
- Empirical shocks expressed as linear combinations of eigen-shocks

▸ more

# Speed of Convergence

## Proposition

*Consider an economy that is initially in steady-state at* $t = 0$ *when agents learn about one-time, permanent shocks to productivity and amenities (*$\widetilde{\boldsymbol{f}} = \begin{bmatrix} \widetilde{\boldsymbol{z}} \\ \widetilde{\boldsymbol{b}} \end{bmatrix}$*) from* $t = 1$ *onwards. Suppose the initial impact of the shock to fundamentals on the state variables at* $t = 1$ *coincides with an eigenvector (*$\boldsymbol{R}\widetilde{\boldsymbol{f}} = \boldsymbol{u}_k$*) of the transition matrix (*$\boldsymbol{P}$*) (eigen-shock). The transition path of the state variables (*$\widetilde{\boldsymbol{x}}_t \equiv \begin{bmatrix} \widetilde{\boldsymbol{\ell}}_t \\ \widetilde{\boldsymbol{k}}_t \end{bmatrix}$*) reduces to:*

$$\widetilde{\boldsymbol{x}}_t = \frac{1-\lambda_k^t}{1-\lambda_k}\boldsymbol{u}_k,$$

*and the half-life is given by:*

$$t_i^{(1/2)}\left(\widetilde{\boldsymbol{f}}\right) = -\left\lceil \frac{\ln 2}{\ln \lambda_k} \right\rceil$$

*for all state variables* $i = 1, \cdots, 2N$*, where* $\lceil \cdot \rceil$ *is the ceiling function.*

▸ more

# Outline

- Dynamic Spatial Model
- Extensions
  - CRRA utility ▸ CRRA
  - Trade deficits ▸ more
  - Shocks to trade and migration costs ▸ more
  - Agglomeration and dispersion forces ▸ more ▸ existuniqagglom
  - Housing capital ▸ more
  - Multi-sector ▸ more
  - Multi-sector and input-output linkages ▸ more
- Data
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# Data

- Two empirical implementations
  - State-time data from 1965-2015 (decline Rust Belt and rise Sun Belt)
  - State-industry-time data from 1999-2015

- U.S. State GDP, population and capital stock
  - Bureau of Economic Analysis (BEA) 1965-2015

- Bilateral value of shipments between U.S. states
  - Commodity Flow Survey (CFS)
  - Commodity Transportation Survey (CTS)

- Bilateral migration flows between U.S. states
  - Population census and American Community Survey (ACS) 1960-2010
  - Five-year migration matrices

- Foreign imports and exports of U.S. states
  - Foreign exports by origin of movement (OM) state 1999-2015
  - Foreign imports by state of destination (SD) 1999-2015

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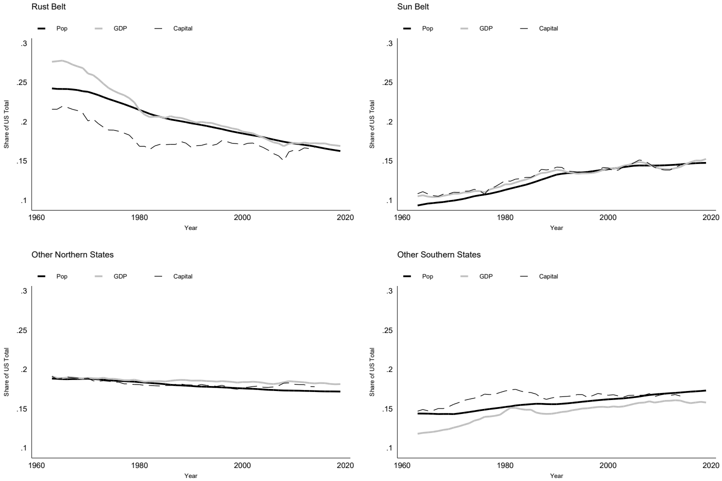
# Shares of U.S. Economic Activity

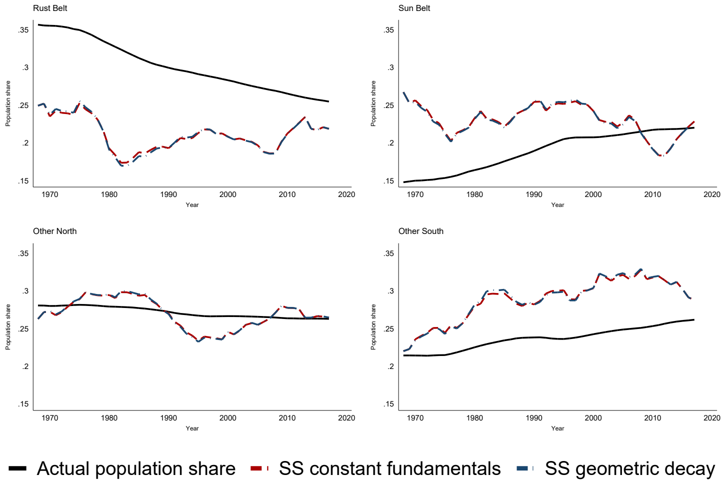



Rust Belt: Illinois, Indiana, Michigan, New York, Ohio, Pennsylvania, West Virginia and Wisconsin. Sun Belt: Arizona, California, Florida, New Mexico and Nevada. Other Southern all other former members of the Confederacy. Other Northern all other Union states during the Civil War

- Capital and GDP dynamics differ from population dynamics ▸ migrants

# Exact Additive Decomposition for Transition Path

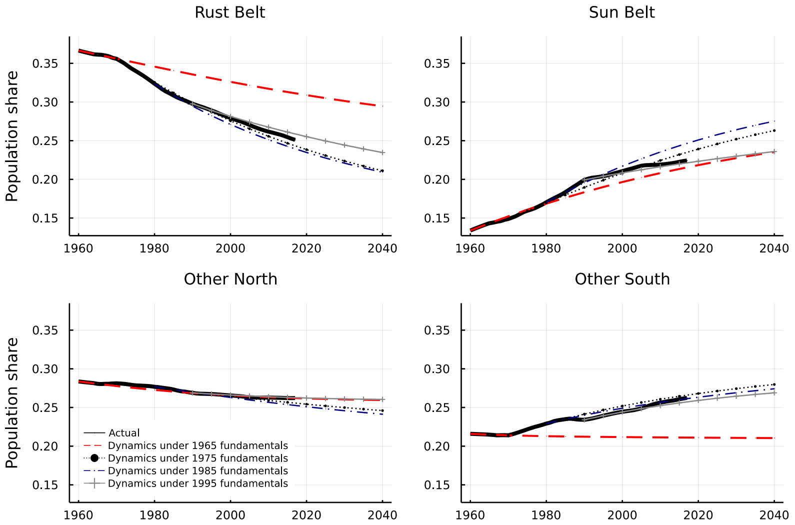
# Population Gap from Steady-State



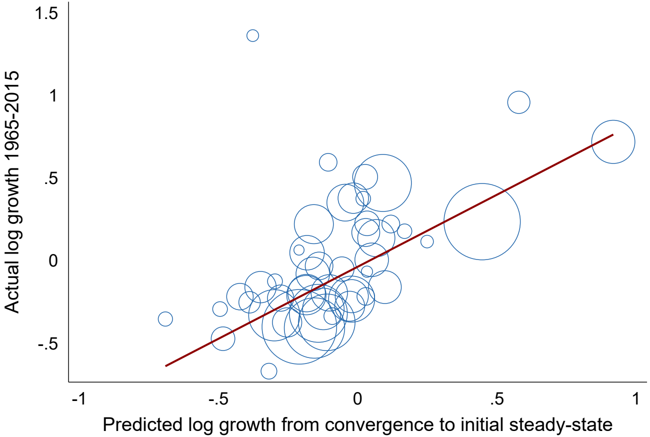

Rust Belt: Illinois, Indiana, Michigan, New York, Ohio, Pennsylvania, West Virginia and Wisconsin. Sun Belt: Arizona, California, Florida, New Mexico and Nevada. North and South definitions based on Federal and Confederacy states

▸ NJ

# Transition Dynamics and Shocks



# Predictive Power Initial Steady-State

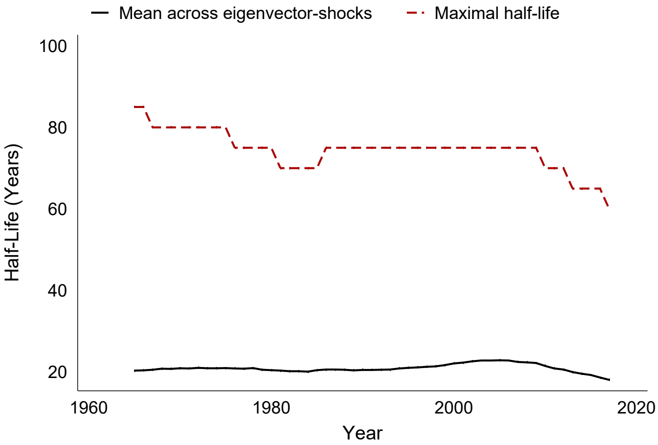



Note: Slope coefficient: 0.8709; standard error: 0.1081; R-squared: 0.5035.

- Robust to controlling for initial log population and capital stock and initial log population growth ▸ more

# Spectral Analysis

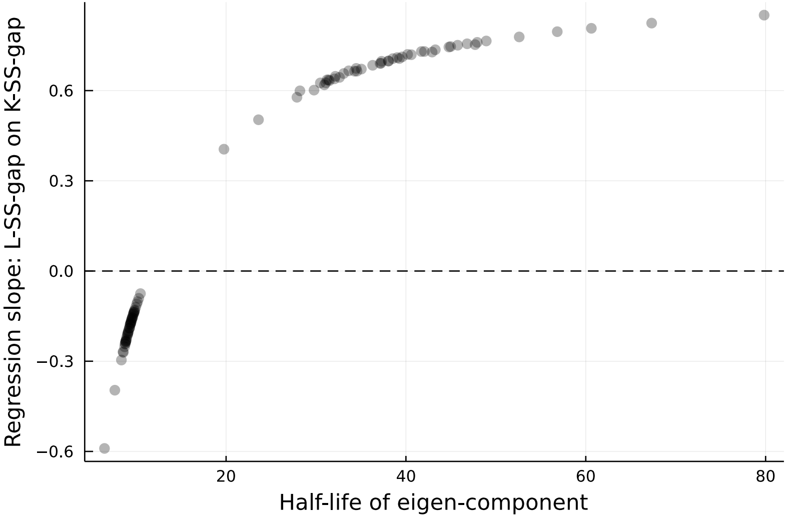
# Half-lifes



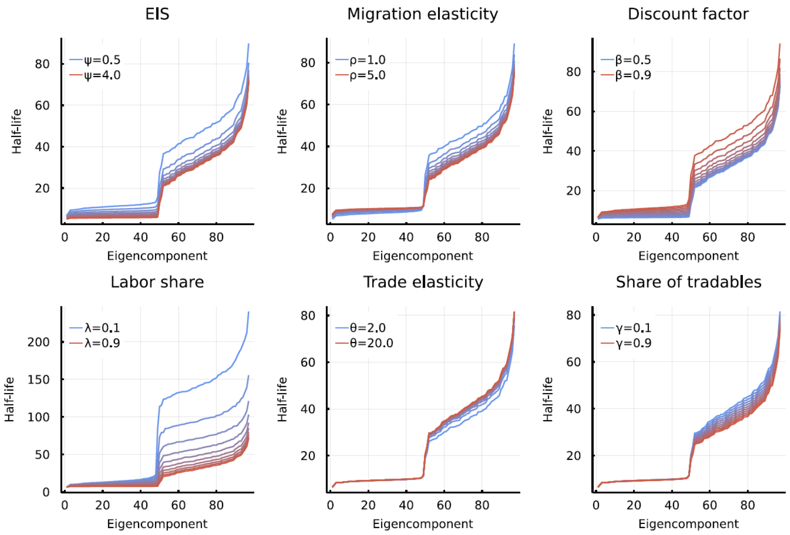

Note: Half-life Corresponds to the time in years for the state variables to converge half of the way towards steady-state for a shock to productivity and amenities for which its initial impact on the state variables ($\widetilde{\boldsymbol{Rf}}$) corresponds to an eigenvector ($\boldsymbol{u}_k$) of the transition matrix ($\boldsymbol{P}$); figure shows mean and maximum half-life across eigenvectors of the transition matrix in each year from 1965-2015.

nocap

# Heterogeneity in Half Lives

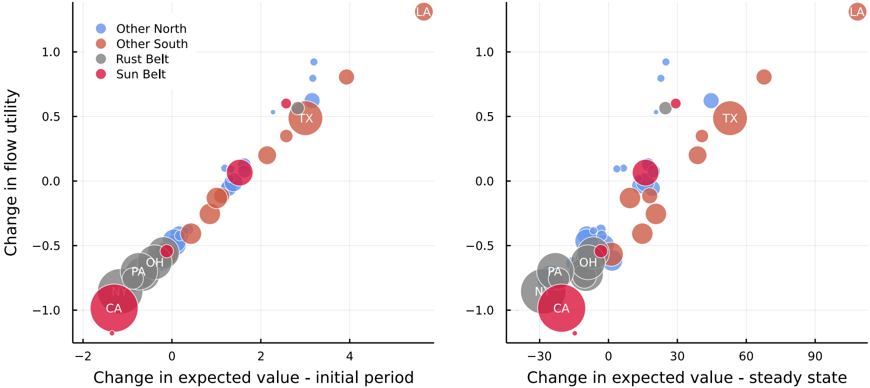


# Parameters and Speed of Convergence



# Distributional Effects

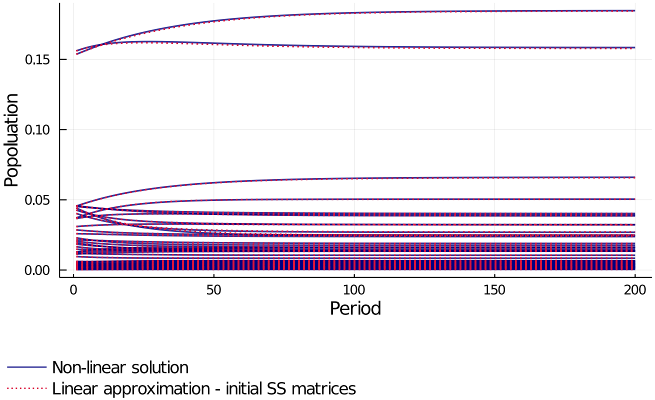
- Compare time path of welfare effects *by location*
- Start from the observed data in 1965
- Shock with vector of productivity shocks from 1965-2015



# Are we Missing Important Non-linearities?

# Approximation Quality (Transition)

- Invert non-linear model (prod., amenities, trade & migration costs)
- Start from steady-state implied by these 1990 fundamentals
- Shock by vector of productivity shocks 1990-2000
- Compare transition paths in our linearization and non-linear model

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# Conclusions

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- Empirical setting features both capital dynamics and migration
- Use our linearization to provide new evidence on slow convergence, labor v. capital dynamics, and heterogeneous impact of local shocks

# Thank You