Sufficient Statistics for Dynamic Spatial Economics*

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Abstract

We model the dynamic response of the spatial economy to shocks to productivity, amenities, trade costs and migration costs in the presence of migration frictions and forward-looking capital investments. We derive closed-form solutions for the first-order general equilibrium response of the economy's entire transition path to these shocks. Our sufficient statistics depend on four observable trade and migration matrices, the initial values of the state variables in each location (population and the capital stock), and the structural parameters of the model. We show that these sufficient statistics are exact for small shocks and provide a close approximation to the full non-linear model solution for the empirical distribution of shocks. We provide an analytical characterization of the economy's transition path in terms of an impact matrix, which captures the initial impact of the shocks, and a transition matrix, which governs the subsequent evolution of the state variables. We show that the speed of convergence to the steady-state depends on the eigenvalues of this transition matrix. We implement our approach empirically using data on U.S. states from 1965-2015 and data on U.S. state-sectors from 1999-2015. We find slow average speeds of convergence, with U.S. states much closer to steady-state at the end of our sample period than at the beginning. We use an eigendecomposition of the transition matrix to characterize the heterogeneous impact of local shocks.

Keywords: dynamics, economic geography, trade, welfare

JEL Classification: F14, F15, F50

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1 Introduction

A key research question in economics is understanding the dynamic response of the spatial distribution of economic activity to shocks. We develop a model of spatial dynamics that allows for gradual adjustment because of both migration costs for mobile factors (labor) and forward-looking investments in immobile factors (capital structures). We derive closed-form solutions for the elasticity of economic activity to shocks to productivities, amenities, trade costs and migration costs, both in steady-state and along the entire transition path. These closed-form solutions depend on four observable matrices for expenditure shares, income shares, outmigration shares and inmigration shares, the initial values of the state variables in each location (population and the capital stock), and the structural parameters of the model. We show that these sufficient statistics are exact for small shocks and provide a close approximation to the full non-linear model solution for the empirical distribution of shocks. Our closed-form solutions yield an analytical characterization of the economy's transition path in terms of an impact matrix, which captures the initial impact of the shocks, and a transition matrix, which governs the subsequent evolution of the state variables. We show that the speed of convergence to the steady-state depends on the eigenvalues of this transition matrix. We use our eigendecomposition of the transition matrix to characterize the heterogeneous impact of local shocks, both across locations and different types of shocks. We show that our approach admits a large number of extensions and generalizations, including agglomeration forces, multiple sectors, and input-output linkages.

One of the key challenges in dynamic spatial models is incorporating forward-looking dynamic decisions in realistic environments with many locations, because in general the investment decision in each location depends on economic activity in all locations in all future time periods. With high-dimensional state spaces of many locations, this can introduce a curse of dimensionality, which can make computing the equilibrium allocations challenging, or can make it difficult to distinguish between alternative possible future trajectories for the economy. We develop a framework that incorporates this forward-looking behavior for both migration decisions for the mobile factor and investment decisions for the immobile factor. Even with these two sources of dynamics, we provide an analytical characterization of the existence and uniqueness of the steady-state equilibrium of the full non-linear model, and of the comparative statics of the spatial distribution of economic activity in each future time period in response to shocks.

To illustrate our approach, we begin with a baseline single-sector Armington model of trade, with dynamic discrete choice migration decisions, and investment determined as the solution to an intertemporal consumption-investment problem. The economy consists of many locations that differ in productivity, amenities, bilateral trade costs and bilateral migration costs. There are two types of agents: workers and landlords. Workers are geographically mobile but do not have access to an investment technology (and hence live "hand to mouth"). They make forward-looking migration decisions, taking into account migration costs and the expected continuation value from optimal future location decisions, as in Caliendo et al. (2019). Landlords are geographically immobile but have access to an investment technology for accumulating local capital. They make forward-looking consumption-investment decisions to maximize intertemporal logarithmic utility, as in the macroeconomics literature following Moll (2014).

We show that the first-order general equilibrium effect of shocks to productivities, amenities, trade costs or migration costs can be evaluated using only four observed trade and migration matrices, the initial value of the state variables (population and the capital stock), and the structural parameters of the model. These four matrices include: (i) an expenditure share matrix ($\boldsymbol{S}$) that reflects the expenditure share of each importer on each exporter; (ii) an income share matrix ($\boldsymbol{T}$) that captures the share of each exporter's value-added derived from each importer; (iii) an outmigration share matrix ($\boldsymbol{D}$) that reflects the share of people in a given origin that migrate to each destination; (iv) an inmigration share matrix ($\boldsymbol{E}$) that corresponds to the share of people in a given destination that migrate from each origin. Although for expositional clarity we focus for most of our analysis on shocks to productivity and amenities, we show that our results naturally extend to shocks to trade and migration costs.

We linearize the general equilibrium conditions of the model to derive a closed-form solution for the transition path of economic activity in each location with respect to a shock in any location. We consider an economy that is somewhere along the transition path towards an unobserved steady-state at a given point in time. Using the observed state variables (population and the capital stock) in each location, and the trade and migration share matrices, we begin by solving for the economy's transition path to the implied steady-state, in the absence of any further shocks to fundamentals. Starting with the same initial conditions, we next show how to solve for the economy's transition path in response to any convergent sequence of future shocks under perfect foresight. In both cases, the economy's transition path corresponds to the solution to a second-order difference equation, which can be solved using the method of undetermined coefficients. We obtain a closed-form solution for this transition path in terms of an impact matrix ($\boldsymbol{R}$), which captures the impact of shocks in the initial period in which they occur, and a transition matrix ($\boldsymbol{P}$), which governs the evolution of the state variables from one period to the next in response to these shocks. We show that the speed of convergence to steady-state, as measured by the half-life, is determined by the eigenvalues of this transition matrix. We use an eigendecomposition of this transition matrix to isolate the locations exposed to particular shocks and the shocks that impact particular locations.

Our closed-form solutions correspond to the elasticity of the endogenous variables in each location with respect to a shock in any location at each future point in time. This closed-form solution involves a single matrix inversion and diagonalization, which yields our bilateral measures of each location's exposure to productivity and amenity shocks along the transition path. The main advantage of this approach is that we can exploit the transition matrix $\boldsymbol{P}$ to provide an analytical characterization of the economy's transition path, including the speed of convergence and heterogeneous impact of shocks. A secondary advantage is that we can use our closed-form solutions to evaluate (to first-order) any number of counterfactuals for different shocks in different locations. In contrast, using conventional methods, one must solve each counterfactual separately using a computationally costly shooting algorithm.

Although for simplicity we begin with our baseline single-sector Armington model, we show that our approach admits a large number of extensions and generalizations. We incorporate agglomeration forces in both production (productivity spillovers) and residential decisions (amenity spillovers). In the presence of these agglomeration forces, we show that the conditions for the existence of a unique steady-state require that these agglomeration forces are sufficiently weak relative to the model's dispersion forces, which include idiosyncratic worker preferences. More generally, we show that our results hold for an entire class of constant elasticity trade models, including models of perfect competition and constant returns to scale, and models of monopolistic competition and increasing returns to scale. For simplicity, we assume perfect foresight in our baseline specification, but we show that our results generalize to allow for stochastic fundamentals and rational expectations. We also assume for simplicity that capital is only used in production, but we show that our results naturally extend to the case in which capital is also used residentially (housing). Finally, we demonstrate that our approach also extends to incorporate multiple sectors (as in Costinot et al. 2012) and both multiple sectors and input-output linkages (as in Caliendo and Parro 2015 and Caliendo et al. 2019).

In our main empirical application, we use data on U.S. states from 1965-2015 to examine the decline of the "Rust Belt" in the North-East and Mid-West and the rise of the "Sun Belt" in the South and West. We show that this setting features convergence dynamics in capital and both net and gross migration, highlighting the relevance of a framework such as ours that features both forward-looking investments and dynamic migration decisions. We use the observed trade and migration shares to compute our impact and transition matrices, which determine the elasticity of economic activity in each location with respect to any combination of shocks in any combination of locations (up to first-order). To recover empirically-realistic shocks, we invert the full non-linear solution of the model to solve for the unobserved productivities, amenities, trade costs and migration costs implied by the observed data on population, income, trade and migration. We show that these unobservables can be recovered under our baseline assumption of perfect foresight, without imposing additional assumptions on where the economy lies on the transition path to steady-state or the expected path of future fundamentals. Implicitly, the observed migration flows control for the expected future path of fundamentals within the structure of the model, related to recent insights in the literature estimating conditional choice probabilities. Although our linearization is only exact for small shocks, we show that it provides a close approximation to the full non-linear model solution for these empirically-realistic shocks, both for comparative statics between steady-states and for the entire transition path towards steady-state. In contrast to the full non-linear solution, our linearization yields analytical solutions for the full transition path, which use to evaluate the determinants of the economy's dynamic response to shocks.

Using our closed-form solution for the transition matrix, we compute the unobserved steady-state distribution of economic activity across U.S. states implied by the observed data for each year from 1965-2015. We compute this implied steady-state both under the assumption of no further changes in fundamentals and projected changes in fundamentals based on an assumed autoregressive process for productivity and amenities. Already in 1965, we find that U.S. states in the Rust Belt were substantially above their steady-state populations, while those in the Sun Belt were substantially below their steady-state populations. By 2015, we find that U.S. states are typically much closer to their steady-state populations than they were at its beginning, suggesting that convergence towards steady-state contributes to the modest observed decline in geographical mobility during our sample period. We also show that the initial distance of population from steady-state in 1965 has substantially explanatory power for subsequent population growth from 1965-2015, even after controlling for initial values of population, the capital stock and population growth. Therefore, a substantial component of the observed reallocation of economic activity across U.S. states can be explained by convergence to steady-state. Of the remainder, we find that a fall in relative productivity plays an important role for the 8 percentage point decline in the Rust Belt's population share over our sample period, while a rise in relative amenities makes a substantial contribution to the 5 percentage point increase in the Sun Belt's population share.

We next use our eigendecomposition of the transition matrix to evaluate the speed of convergence to steady-state. For productivity and amenity shocks that correspond to an eigenvector of the transition matrix, we show that the half-life of convergence to steady-state can be recovered from the associated eigenvalue. Furthermore, any empirical combination of shocks to productivities and amenities can be expressed as a weighted average of these eigenvectors of the transition matrix. Using these results for our baseline single-sector model, we find slow convergence to steady-state, with an average half-life across the eigenvectors of around 20 years, which is consistent with recent empirical findings of persistent impacts of local labor market shocks. Nevertheless, we observe considerable heterogeneity in this speed of convergence across the eigenvectors, depending on the pattern and magnitude of the shocks. We show that this slow convergence to steady-state is primarily driven by labor dynamics (migration) rather than by capital dynamics (investment), although capital accumulation plays an important role in amplifying the impact of local shocks. We find that average half-lifes are relatively constant over time, if anything falling towards the end of sample period, which contrasts with the modest observed decline in geographical mobility over time. This pattern of results again reinforces the idea that an observed decline in geographical mobility need not necessarily imply a rise in barriers to this mobility, since it can also reflect convergence towards steady-state, or changes in the pattern and magnitude of the shocks to fundamentals across locations.

In a final empirical exercise, we implement our multi-sector extension using region-sector data on U.S. states and foreign countries from 1999-2015. In this multi-sector extension, we continue to find relatively slow convergence towards steady-state, but the average half-life is notably lower than our baseline single-sector specification. This pattern of results reflects the property of the data that movements of people between sectors within U.S. states occur much more frequently than movements of people between U.S. states. This finding is consistent with the view that the extent to which local labor market shocks are persistent is likely to be heterogeneous across both locations and different shocks. The speed with which the economy adjusts to these shocks depends crucially on their incidence, and in particular the extent to which they affect one industry relative to another industry within the same location, versus the extent to which that affect all industries in one location relative to all industries in another location.

Our research is related to several strands of existing work. First, our paper contributes to a long line of research on economic geography, including Krugman (1991b), Krugman and Venables (1995) and Helpman (1998), as synthesized in Fujita et al. (1999), and reviewed in Duranton and Puga (2004) and Redding (2020). Early theoretical research on economic geography considered static models or assumed myopic migration decisions, as in Krugman (1992). Exceptions include a small number of theoretical trade and geography papers that have considered forward-looking decisions under perfect foresight, including Krugman (1991a), Matsuyama (1991) and Baldwin (2001). In contrast, most recent research on quantitative spatial models has often considered static specifications, including Redding and Sturm (2008), Allen and Arkolakis (2014), Ahlfeldt et al. (2015), Allen et al. (2017), Ramondo et al. (2016), Redding (2016), Caliendo et al. (2018) and Monte et al. (2018), as surveyed in Redding and Rossi-Hansberg (2017).

A key reason that quantitative spatial models have frequently focused on static specifications is that locations in these models are connected through rich bilateral networks of trade and migration flows. Once dynamic decisions are introduced, the optimal decision in each location depends through these networks on the entire spatial distribution of economic activity across all locations in all future periods of time, as emphasized in Desmet and Rossi-Hansberg (2010). One approach to this challenge has been to consider specifications in which dynamic decisions reduce to static problems. In the innovation models of Desmet and Rossi-Hansberg (2014), Desmet et al. (2018) and Peters (2019), the incentive to invest in innovation each period depends on the comparison of static profits and innovation costs. In the overlapping generations model of Allen and Donaldson (2020), adults make migration decisions to maximize their own adult utility, and do not consider the utility of the next generation of youths. Another approach is to capture forward-looking migration decisions using dynamic discrete choice models, including Artuç et al. (2010) and Caliendo et al. (2019, 2020).[1] Relative to all of these studies, we develop a dynamic spatial model that incorporates both dynamic migration decisions for the mobile factor and forward-looking investments for the immobile factor. Under our assumption of logarithmic intertemporal preferences, we show that the optimal investment policy function takes the tractable form of a constant saving rate out of income net of depreciation. This constant saving rate, the gravity equation for migration flows and our linearization are the three key features of our approach that enable us to derive our analytical characterization of the model's transition dynamics.

Second, our work is related to the literature on sufficient statistics in static international trade models, including Arkolakis et al. (2012), Caliendo et al. (2017), Baqaee and Farhi (2019), Galle et al. (2018), Huo et al. (2019), Bartelme et al. (2019), Adão et al. (2019), Bonadio et al. (2020), and Kim and Vogel (2020). Using a class of constant elasticity international trade models, Kleinman et al. (2020) show that the first-order comparative statics can be stacked as a matrix inversion problem, which yields closed-form solutions of the elasticity of the endogenous variables in each country with respect to shocks in any other country. Although these existing studies have developed sufficient statistics for *static* spatial models, our key contribution is to develop these sufficient statistics for *dynamic* spatial models, incorporating both dynamic migration decisions for the mobile factor and forward-looking investment decisions for the immobile factor.

Third, our research is related to an empirical literature on local labor markets, including Autor et al. (2013), Kovak (2013), Kline and Moretti (2014), Dix-Carneiro and Kovak (2015), and Diamond (2016), Hornbeck and Moretti (2018) and Eriksson et al. (2019), as reviewed in Moretti (2011) and Autor et al. (2016). One strand of this literature has examined the reallocation of U.S. economic activity from the Rust Belt to the Sun Belt, such as Blanchard and Katz (1992), Feyrer et al. (2007), Rappaport (2007), Glaeser and Ponzetto (2010), Hartley (2013), Yoon (2017) and Alder et al. (2019). Another strand of this literature has emphasized the persistent impact of negative local labor market shocks, including in particular Dix-Carneiro and Kovak (2017), Amior and Manning (2018) and Autor et al. (2020). We contribute to this research by using our closed-form solutions

[1] See Glaeser and Gyourko (2005) and Greaney (2020) for models in which population dynamics are shaped by durable housing. See Walsh (2019) for a model in which innovation takes the form of the creation of new varieties.

for the economy's transition path to quantify the speed of convergence to steady-state and evaluate the roles of migration costs for mobile factors (labor) and the gradual accumulation of immobile factors (capital structures) in generating persistent impacts of local shocks. We use our eigendecomposition of the transition matrix to characterize the heterogeneity in the impact of these local shocks.

The remainder of the paper is structured as follows. In Section 2, we introduce our baseline quantitative spatial model with dynamics from forward-looking migration and investment decisions. In Section 3, we derive our main sufficient statistics results for the impact of productivity and amenity shocks on the entire transition path of the spatial distribution of economic activity. In Section 4, we show that our analysis admits a number of extensions and generalizations, including shocks to trade and migration costs, agglomeration forces, stochastic fundamentals and rational expectations, multiple sectors, and input-output linkages, among others. In Section 5, we implement our baseline specification for U.S. states from 1965-2015 and our multi-sector extension for U.S. states and foreign countries from 1999-2015. Section 6 summarizes our conclusions.

2 Dynamic Spatial Model

In this section, we introduce our baseline dynamic quantitative spatial model. We combine a specification of trade between locations with a constant trade elasticity, a formulation of migration decisions with a constant migration elasticity, and optimal consumption-investment decisions with logarithmic intertemporal utility. We derive our main sufficient statistics results for the comparative statics of the spatial distribution of economic activity, both in steady-state and along the entire transition path.

For simplicity, we model trade between locations as in Armington (1969), in which goods are differentiated by origin. In Section C of the online appendix, we establish a number of isomorphisms, in which we show that our results hold throughout the class of models with a constant trade elasticity considered in Arkolakis et al. (2012). To streamline the exposition, we focus in this section on shocks to productivities and amenities. In Section 4 below, we show that our approach naturally incorporates shocks to trade and migration costs, and admits a large number of extensions and generalizations, including agglomeration economies, multiple sectors, and input-output linkages.

We consider an economy with many locations indexed by $i \in \{1, \ldots, N\}$. Time is discrete and is indexed by t. There are two types of infinitely-lived agents: workers and landlords. Workers are endowed with one unit of labor that is supplied inelastically and are geographically mobile subject to migration costs. Workers do not have access to an investment technology, and hence live "hand to mouth," as in Kaplan and Violante (2014). Landlords are geographically immobile and own the capital structures in their location. They make forward-looking decisions over consumption and investment in this local stock of capital structures. We assume that capital is geographically immobile once installed, but depreciates gradually at a constant rate δ. In our baseline specification, we assume that workers and landlords have perfect foresight over all location characteristics, but we present an extension to stochastic fundamentals and rational expectations in Section 4 below.

In Subsections 2.1-2.5, we introduce our specifications of worker migration and landlord investment decisions. In Subsection 2.6, we provide a characterization of the general equilibrium of the model. The derivations for all expressions and results in this section are reported in Section B of the online appendix.

2.1 Worker Migration Decisions

At the beginning of period t, the economy inherits a mass of workers in each location i (ℓ_{it}), with its total labor endowment given by $\bar{\ell} = \sum_{i=1}^{N} \ell_{it}$. Workers produce and consume in their current location during period t, before observing mobility shocks $\{\epsilon_{gt}\}$ for all possible locations $g \in \{1, \ldots, N\}$, and deciding where to move for the next period $t+1$, given bilateral migration costs $\{\kappa_{git}\}$. Therefore, the value function for a worker in location i in period t ($\mathbb{V}_{it}^{w}$) is equal to the current flow of utility in that location plus the expected continuation value next period from the optimal choice of location:

$$\mathbb{V}_{it}^{w} = \ln u_{it}^{w} + \max_{\{g\}_1^N} \left\{ \beta \mathbb{E}\left[\mathbb{V}_{gt+1}^{w}\right] - \kappa_{git} + \rho \epsilon_{gt} \right\}, \tag{1}$$

where we use the superscript w to denote workers; we assume logarithmic utility ($\ln u_{it}^{w}$); β is the discount rate; $\mathbb{E}[\cdot]$ denotes an expectation taken over the distribution for idiosyncratic mobility shocks; ρ controls the dispersion of these idiosyncratic mobility shocks; and we assume $\kappa_{iit} = 1$ and $\kappa_{nit} > 1$ for $n \neq i$.

We make the conventional assumption that the idiosyncratic mobility shocks are drawn from an extreme value distribution: $F(\epsilon) = e^{-e^{(-\epsilon-\bar{\gamma})}}$,where $\bar{\gamma}$ is the Euler-Mascheroni constant. Under this assumption, the expected value for a worker of living in location i at time t (v_{it}^{w}) is equal to:

$$v_{it}^{w} = \ln u_{it}^{w} + \rho \ln \sum_{g=1}^{N} \left(\exp\left(\beta v_{gt+1}^{w}\right) / \kappa_{git} \right)^{1/\rho}. \tag{2}$$

The corresponding probability that a worker migrates from location i to location g is given by:

$$D_{igt} = \frac{\left(\exp\left(\beta v_{gt+1}^{w}\right) / \kappa_{git} \right)^{1/\rho}}{\sum_{m=1}^{N} \left(\exp\left(\beta v_{mt+1}^{w}\right) / \kappa_{mit} \right)^{1/\rho}}. \tag{3}$$

In this gravity equation, bilateral migration flows between each pair of locations depend not only on bilateral frictions (κ_{git}) in the numerator ("bilateral resistance") but also on frictions with all possible locations (κ_{mit}) in the denominator ("multilateral resistance"). We refer to D_{igt} as the *outmigration* probability, because it captures share of workers in origin i at time t that departed to destination g at time $t+1$. The presence of idiosyncratic mobility shocks implies that the steady-state equilibrium of the model exhibits ongoing gross migration flows between locations. Along the transition path towards the steady-state equilibrium, there are also net migration flows between locations, as population gradually reallocates across locations.

2.2 Worker Consumption

Worker preferences are modeled as in the standard Armington model of trade with constant elasticity of substitution (CES) preferences. As workers do not have access to an investment technology, they choose their consumption of varieties to maximize their utility each period. The indirect utility function each period depends on the worker's wage (w_{nt}), the cost of living (p_{nt}) and amenities (b_{nt}):

$$\ln u_{nt}^{w} = \ln b_{nt} + \ln w_{nt} - \ln p_{nt}. \tag{4}$$

The cost of living (p_{nt}) in location n depends on the price of the variety sourced from each location i (p_{nit}):

$$p_{nt} = \left[\sum_{i=1}^{N} p_{nit}^{-\theta} \right]^{-1/\theta}, \tag{5}$$

where $\sigma > 1$ is the constant elasticity of substitution and $\theta = \sigma - 1 > 0$ is the constant trade elasticity. Amenities (b_{nt}) capture characteristics of a location that make it a more attractive place to live regardless of goods consumption (e.g. scenic views). In our baseline specification in this section, we assume that these amenities are exogenous locational fundamentals. In an extension in Section 4 below, we allow amenities to have both an endogenous component that reflects agglomeration / dispersion forces and an exogenous component that captures locational fundamentals.

Using the properties of these CES preferences, the share of expenditure in importer n on the goods supplied by exporter i takes the standard form:

$$S_{nit} = \frac{(p_{nit})^{-\theta}}{\sum_{m=1}^{N} (p_{nmt})^{-\theta}}. \tag{6}$$

2.3 Production

Firms in each location use labor (ℓ_{it}) and capital (k_{it}) to produce output (y_{it}) of the variety supplied by that location. Production is assumed to occur under conditions of perfect competition and subject to the following constant returns to scale technology:

$$y_{it} = z_{it} \left(\frac{\ell_{it}}{\lambda} \right)^{\lambda} \left(\frac{k_{it}}{1-\lambda} \right)^{1-\lambda}, \qquad 0 < \lambda < 1, \tag{7}$$

where z_{it} denotes productivity in location i at time t. In our baseline specification in this section, we also assume that productivity is an exogenous locational fundamental. In an extension in Section 4 below, we allow productivity to have both an endogenous component that reflects agglomeration / dispersion forces and an exogenous component that captures locational fundamentals.

We assume that trade between locations is subject to iceberg variable trade costs, such that $\tau_{nit} \geq 1$ units of a good must be shipped from location i in order for one unit to arrive in location n, where $\tau_{nit} > 1$ for $n \neq i$ and $\tau_{iit} = 1$. From profit maximization, the cost to a consumer in location n of sourcing the good produced by location i depends solely on these iceberg trade costs and constant marginal costs:

$$p_{nit} = \tau_{nit} p_{iit} = \frac{\tau_{nit} w_{it}^{\lambda} r_{it}^{1-\lambda}}{z_{it}}, \tag{8}$$

where p_{iit} is the "free on board" price of the good supplied by location i before trade costs.

2.4 Capital Accumulation

Landlords in each location choose their consumption and investment to maximize their intertemporal utility subject to their budget constraint. Landlords' intertemporal utility equals the present discounted value of their flow utility, which is assumed to take the same logarithmic form as for workers:

$$v_{it}^{k} = \sum_{t=0}^{\infty} \beta^{t} \ln c_{it}^{k}, \tag{9}$$

where we use the superscript k to denote landlords; c_{it}^k is the consumption index dual to the price index (5); and β is the discount rate. Since landlords are geographically immobile, we omit the term in amenities from their flow utility, because this does not affect the equilibrium in any way, and hence is without loss of generality.

We assume that the investment technology in each location uses the varieties from all locations with the same functional form as consumption. We assume that landlords can only invest in their own location and that one unit of capital can be produced using one unit of the consumption index in that location.[2] We interpret capital as buildings and structures, which are geographically immobile once installed. Capital is assumed to depreciate at the constant rate δ and we allow for the possibility of negative investment. The intertemporal budget constraint for landlords in each location requires that total income from the existing stock of capital ($r_{it}k_{it}$) equals the total value of their consumption ($p_{it}c_{it}^k$) plus the total value of net investment ($p_{it}\left(k_{it+1}-(1-\delta)\,k_{it}\right)$):

$$r_{it}k_{it} = p_{it}\left(c_{it}^k + k_{it+1} - (1-\delta)\,k_{it}\right). \tag{10}$$

From the first-order condition to landlords' intertemporal optimization problem, our assumption of logarithmic intertemporal utility implies that landlords' optimal policy function features a constant saving rate, as in Moll (2014). Therefore, although landlords are making optimal consumption-saving decisions, the evolution of capital between periods is similar to that in the conventional Solow-Swan model:

$$k_{it+1} = \beta\left[r_{it}/p_{it} + (1-\delta)\right]k_{it}, \tag{11}$$

where the saving rate is here endogenously determined by the discount rate (β).

We show below that there exists a steady-state equilibrium level of the capital-labor ratio in each location, towards which the economy gradually converges in the absence of further shocks. This constant equilibrium saving rate is a key feature of the model that permits a tractable characterization of its transition dynamics, despite the high-dimensional state space of many locations. Along the transition path towards steady-state, the real rental rate in terms of the consumption good used for investment can differ across locations, but these differences are limited by the tradeability of the consumption varieties used for investment. In steady-state, the real rental rate in terms of the consumption good is equalized across all locations.[3]

2.5 Market Clearing

Goods market clearing implies that income in each location, which equals the sum of the income of workers and landlords, is equal to expenditure on the goods produced by that location:

$$\left(w_{it}\ell_{it} + r_{it}k_{it}\right) = \sum_{n=1}^{N} S_{nit}\left(w_{nt}\ell_{nt} + r_{nt}k_{nt}\right). \tag{12}$$

Capital market clearing implies that the rental rate for capital is determined by the requirement that landlords' income from the ownership of capital equals payments for its use. Using profit maximization and zero

[2]This specification can be extended to allow landlords to invest in other locations at the cost of additional complication. Although we make the standard assumption that consumption and investment use goods in the same proportions, similar results hold in an alternative specification in which investment uses only the good produced by each location.

[3]In particular, in steady-state, we have: $r_i^*/p_i^* = \left(1-\beta\,(1-\delta)\right)/\beta$.

profits, this capital market clearing condition can be expressed as follows:

$$r_{it}k_{it} = \frac{1-\lambda}{\lambda} w_{it}\ell_{it}. \tag{13}$$

2.6 General Equilibrium

Given the state variables $\{\ell_{i0}, k_{i0}\}$, the general equilibrium of the economy is the path of allocations and prices such that firms in each location choose inputs to maximize profits, workers make consumption and migration decisions to maximize utility, landlords make consumption and investment decisions to maximize utility, and prices clear all markets. For expositional clarity, we collect the equilibrium conditions and express them in terms of a sequence of four endogenous variables $\{\ell_{it}, k_{it}, w_{it}, v_{it}\}_{t=0}^{\infty}$. All other endogenous variables of the model can be recovered as a function of these variables.

Capital Accumulation: Using capital market clearing (13), the price index (5) and the equilibrium pricing rule (8), the capital accumulation equation (11) becomes:

$$k_{it+1} = \beta \frac{1-\lambda}{\lambda} \frac{w_{it}}{p_{it}} \ell_{it} + \beta (1-\delta) k_{it}. \tag{14}$$

$$p_{nt} = \left[\sum_{i=1}^{N} \left(w_{it} \left(\frac{1-\lambda}{\lambda} \right)^{1-\lambda} (\ell_{it}/k_{it})^{1-\lambda} \tau_{ni}/z_i \right)^{-\theta} \right]^{-1/\theta}. \tag{15}$$

Goods Market Clearing: Using the equilibrium pricing rule (8), the expenditure share (6) and the capital market clearing condition (13) in the goods market clearing condition (12), the requirement that income equals expenditure on the goods produced by a location can be written solely in terms of labor income:

$$w_{it}\ell_{it} = \sum_{n=1}^{N} S_{nit} w_{nt} \ell_{nt}, \tag{16}$$

$$S_{nit} = \frac{\left(w_{it} (\ell_{it}/k_{it})^{1-\lambda} \tau_{ni}/z_i \right)^{-\theta}}{\sum_{m=1}^{N} \left(w_{mt} (\ell_{mt}/k_{mt})^{1-\lambda} \tau_{nm}/z_m \right)^{-\theta}}, \qquad T_{int} \equiv \frac{S_{nit} w_{nt} \ell_{nt}}{w_{it}\ell_{it}}, \tag{17}$$

where we have used the property that capital income is a constant multiple of labor income; S_{nit} is the expenditure share of importer n on exporter i at time t; we have defined T_{int} as the corresponding income share of exporter i from importer n at time t; and note that the order of subscripts switches between the expenditure share (S_{nit}) and the income share (T_{int}), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Population Flow: Using the outmigration probabilities (3), the population flow condition for the evolution of the population distribution over time is given by:

$$\ell_{gt+1} = \sum_{i=1}^{N} D_{igt} \ell_{it}, \tag{18}$$

$$D_{igt} = \frac{\left(\exp\left(\beta v_{gt+1}^{w}\right)/\kappa_{git}\right)^{1/\rho}}{\sum_{m=1}^{N}\left(\exp\left(\beta v_{mt+1}^{w}\right)/\kappa_{mit}\right)^{1/\rho}}, \qquad E_{git} \equiv \frac{\ell_{it} D_{igt}}{\ell_{gt+1}}, \tag{19}$$

where D_{igt} is the outmigration probability from location i to location g between time t and $t{+}1$; we have defined E_{git} as the corresponding inmigration probability to location g from location i between time t and $t+1$; again note that the order of subscripts switches between the outmigration probability (D_{igt}) and the inmigration probability (E_{git}), because the first and second subscripts will correspond below to rows and columns of a matrix, respectively.

Worker Value Function: Using the worker indirect utility function (4) in the value function (2), the expected value from living in location n at time t can be written as:

$$v_{nt}^{w} = \ln b_{nt} + \ln\left(\frac{w_{nt}}{p_{nt}}\right) + \rho \ln \sum_{g=1}^{N} \left(\exp\left(\beta v_{gt+1}^{w}\right)/\kappa_{gnt}\right)^{1/\rho}. \tag{20}$$

Properties of General Equilibrium: Given the state variables in each location i in a given time period t $\{\ell_{it},\, k_{it}\}$, the general equilibrium for consumption, production and trade is determined as in a standard static international trade model. Between periods, the evolution of the stock of capital $\{k_{it}\}$ is determined by the constant equilibrium saving rate, and the dynamics of the population distribution $\{\ell_{it}\}$ are determined by the gravity equation for migration. Given the state variables in an initial time period, we define a sequential equilibrium as the sequence of four endogenous variables $\{\ell_{it}, k_{it}, w_{it}, v_{it}\}_{t=0}^{\infty}$ that solves the general equilibrium conditions of the model.

Definition 1. Sequential Equilibrium. Given the state variables $\{\ell_{i0},\, k_{i0}\}$ in each location in an initial period $t = 0$, a *sequential equilibrium* of the economy is a set of wages, expected values, mass of workers and stock of capital in each location in all subsequent time periods $\{w_{it}, v_{it}, \ell_{it}, k_{it}\}_{t=0}^{\infty}$ that solves the value function (20), the population flow condition (18), the goods market clearing condition (16), and the capital market clearing and accumulation condition (14).

In a steady-state equilibrium of the economy, the four endogenous variables in each location $\{\ell_i^*, k_i^*, w_i^*, v_i^*\}$ are constant across time periods, where we use an asterisk to denote the steady-state value of variables.

Definition 2. Steady State. A *steady-state* of the economy is an equilibrium in which all location-specific variables (wages, expected values, mass of workers and stock of capital in each location) are time invariant: $\{w_i^*, v_i^*, \ell_i^*, k_i^*\}$.

Our model features rich spatial interactions between locations in both goods and factor markets and forward-looking migration and investment decisions. Nevertheless, the absence of agglomeration forces and diminishing marginal returns to capital accumulation ensure the existence of a unique steady-state spatial distribution of economic activity (up to a choice of numeraire), given time-invariant values of the locational fundamentals of productivity (z_i), amenities (b_i), goods trade costs (τ_{ni}) and migration frictions (κ_{ni}).

Proposition 1. *There exists a unique steady-state spatial distribution of economic activity* $\{w_i^*, v_i^*, \ell_i^*, k_i^*\}$ *(up to a choice of numeraire) given time-invariant locational fundamentals* $\{z_i, b_i, \tau_{ni}, \kappa_{ni}\}$ *that is independent of the economy's initial conditions* $\{\ell_{i0}, k_{i0}\}$*.*

Proof. See Section B.7 of the online appendix. □

Trade and Migration Share Matrices: We now introduce the trade and migration share matrices that we use in the next section to characterize the model's comparative statics. Let $\boldsymbol{S}$ be the $N \times N$ matrix with the ni-th element equal to importer n's expenditure on exporter i. Let $\boldsymbol{T}$ be the $N \times N$ matrix with the in-th element equal to the fraction of income that exporter i derives from selling to importer n. We refer to $\boldsymbol{S}$ as the *expenditure share* matrix and to $\boldsymbol{T}$ as the *income share* matrix. Intuitively, S_{ni} captures the importance of i as a supplier to location n, and T_{in} captures the importance of n as a buyer for country i. Note the order of subscripts: in matrix $\boldsymbol{S}$, rows are buyers and columns are suppliers, whereas in matrix $\boldsymbol{T}$, rows are suppliers and columns are buyers. Both matrices have rows that sum to one across columns.

Similarly, let $\boldsymbol{D}$ be the $N \times N$ matrix with the ni-th element equal to the share of outmigrants from origin n to destination i. Let $\boldsymbol{E}$ be the $N \times N$ matrix with the in-th element equal to the share of inmigrants to destination i from origin n. We refer to $\boldsymbol{D}$ as the *outmigration* matrix and to $\boldsymbol{E}$ as the *inmigration* matrix. Intuitively, D_{ni} captures the importance of i as a destination for origin n, and E_{in} captures the importance of n as an origin for destination i. Note again the order of subscripts: in matrix $\boldsymbol{D}$, rows are origins and columns are destinations, whereas in matrix $\boldsymbol{E}$, rows are destinations and columns are origins. Both matrices again have rows that sum to one across columns.

These $\boldsymbol{S}$, $\boldsymbol{T}$, $\boldsymbol{D}$ and $\boldsymbol{E}$ matrices are equilibrium objects that can be obtained directly from observed trade and migration data. We derive our comparative statics results using these observed matrices. Using $\boldsymbol{S}^k$ to represent the matrix $\boldsymbol{S}$ raised to the k-th power, we impose the following technical assumptions on these matrices, which are satisfied in our observed trade and migration data for U.S. states.

Assumption 1. ***(i)*** *For any* i, n, *there exists* k *such that* $\left[\boldsymbol{S}^k\right]_{in} > 0$ *and* $\left[\boldsymbol{D}^k\right]_{in} > 0$. ***(ii)*** *For all* i, $\boldsymbol{S}_{ii} > 0$ *and* $\boldsymbol{D}_{ii} > 0$. ***(iii)*** *The matrices* $\boldsymbol{S}$ *and* $\boldsymbol{D}$ *are of rank* $N-1$*.*

The first part of this assumption states that all locations are connected with each other directly or indirectly, through flows of goods and migrants. That is, in the language of graph theory, the trade and migration networks are *strongly connected.* The second part of the assumption ensures that each location consumes a positive amount of domestic goods and has a positive amount of own migrants. The third part of the assumption ensures that $N-1$ rows and columns of the trade and migration share matrices are linearly independent, with the final row and column determined by the requirement that the trade and migration shares sum to one.

In our comparative static exercises, we solve for changes in wages and populations in each location. As the expenditure shares ($\boldsymbol{S}$) and income shares ($\boldsymbol{T}$) are homogeneous of degree zero in factor prices, we require a choice of units or numeraire in order to solve for changes in wages. We choose the total income of all locations as our numeraire: $\sum_{i=1}^N w_{it}\ell_{it} = \sum_{i=1}^N q_{it} = \bar{q}_t = 1$, which implies $\sum_{t=1}^N q_i^* \, \mathrm{d}\ln q_i^* = \sum_{t=1}^N q_i^* \frac{\mathrm{d}q_i^*}{q_i^*} = \sum_{t=1}^N \mathrm{d}q_i^* = 0$. Similarly, the outmigration shares ($\boldsymbol{D}$) and inmigration shares ($\boldsymbol{E}$) are homogeneous of degree zero in the total population of all locations, which requires a choice of units to solve for population levels. We solve for population shares, imposing the requirement that the population shares sum to one: $\sum_{i=1}^{N} \ell_{it} = \overline{\ell} = 1$, which implies $\sum_{i=1}^{N} \ell_i^* \, \mathrm{d}\ln \ell_i^* = \sum_{i=1}^{N} \ell_i^* \frac{\mathrm{d}\ell_i^*}{\ell_i^*} = \sum_{i=1}^{N} \mathrm{d}\ell_i^* = 0$.

3 Sufficient Statistics

We now derive our main sufficient statistics results for the response of the spatial distribution of economic activity to shocks to the economic environment. We use our baseline assumption that agents have perfect foresight for all location characteristics, except for these unanticipated (MIT) shocks. We show in Section 4 below that our analysis generalizes to allow for stochastic fundamentals and rational expectations. We derive our sufficient statistics for steady-state changes in Subsection 3.1 and for the entire transition path in Subsection 3.2. We report the derivations of all results in Subsection B.8 of the online appendix. Throughout the following, we use bold math font to denote a vector (lowercase letters) or matrix (uppercase letters). For expositional clarity, we focus in this section on shocks to productivity ($\mathrm{d}\ln \boldsymbol{z}$) and amenities ($\mathrm{d}\ln \boldsymbol{b}$), but we show in Section 4 below that our approach naturally accommodates shocks to trade and migration costs.

3.1 Changes in Steady-States

We start by characterizing the change in the spatial distribution of economic activity between steady-states in response to small shocks to productivity ($\mathrm{d}\ln \boldsymbol{z}$) and amenity shocks ($\mathrm{d}\ln \boldsymbol{b}$), holding constant the economy's aggregate labor endowment ($\mathrm{d}\ln \overline{\ell} = 0$), trade costs ($\mathrm{d}\ln \boldsymbol{\tau} = 0$) and commuting costs ($\mathrm{d}\ln \boldsymbol{\kappa} = 0$). Totally differentiating the general equilibrium conditions of the model, we derive a system of four equations that fully characterizes the steady-state changes in population ($\mathrm{d}\ln \boldsymbol{\ell}^*$), the capital stock ($\mathrm{d}\ln \boldsymbol{k}^*$), wages ($\mathrm{d}\ln \boldsymbol{w}^*$), and worker value functions ($\mathrm{d}\ln \boldsymbol{v}^*$) in terms of the initial steady-state matrices of expenditure shares ($\boldsymbol{S}$), income shares ($\boldsymbol{T}$), outmigration shares ($\boldsymbol{D}$) and inmigration shares ($\boldsymbol{E}$).

Capital Accumulation. First, from the capital accumulation condition (14), we see that the steady-state level of the capital-labor ratio k_i^*/ℓ_i^* in each location is proportional to the real wage w_i^*/p_i^* in that location. Therefore, the log change in the steady-state capital-labor ratio must satisfy:

$$\mathrm{d}\ln \boldsymbol{k}^* = \underbrace{\mathrm{d}\ln \boldsymbol{\ell}^*}_{\text{change in population}} + \underbrace{\mathrm{d}\ln \boldsymbol{w}^*}_{\text{change in wages}} - \underbrace{\mathrm{d}\ln \boldsymbol{p}^*}_{\text{change in the price index}}. \tag{21}$$

Intuitively, producer optimization implies that capital income in each location is always proportional to wage income. Therefore, an increase in population (first term on the right-hand side) or the wage rate (second term on the right-hand side) in each location raises the income of landlords in that location, and hence their savings through capital accumulation. The rate at which a landlord in location n turns these savings into the capital stock depends on the location's price index, the change of which is captured by the third term on the right-hand side, and is equal to:

$$\mathrm{d}\ln \boldsymbol{p}^* = \boldsymbol{S} \underbrace{\left[\mathrm{d}\ln \boldsymbol{w}^* - (1-\lambda)\left(\mathrm{d}\ln \boldsymbol{k}^* - \mathrm{d}\ln \boldsymbol{\ell}^*\right) - \mathrm{d}\ln \boldsymbol{z}\right]}_{\text{change in the production cost in each region}}. \tag{22}$$

That is, the change in the price index of a given location n ($\mathrm{d}\ln p_n^*$) is equal to a weighted average of the change in the production cost in each location i (in square brackets), where the weights are the share of expenditure in location n on the goods produced by location i (S_{ni}). In turn, the steady-state production cost in each location inside the parentheses is increasing in the wage (captured in vector form in equation (22) by $\mathrm{d}\ln \boldsymbol{w}^*$); decreasing in its capital-labor ratio (captured by $(1-\lambda)(\mathrm{d}\ln \boldsymbol{k}^* - \mathrm{d}\ln \boldsymbol{\ell}^*)$), because a higher capital-labor ratio implies a lower rental rate; and decreasing in its productivity (captured by $\mathrm{d}\ln \boldsymbol{z}$).

Goods Market Clearing. Second, from the goods market clearing condition (12), the log change in labor income in each location is equal to the log change in the wage ($\mathrm{d}\ln \boldsymbol{w}^*$) plus the log change in population ($\mathrm{d}\ln \boldsymbol{\ell}^*$):

$$\mathrm{d}\ln \boldsymbol{w}^* + \mathrm{d}\ln \boldsymbol{\ell}^* = \underbrace{\boldsymbol{T}\left(\mathrm{d}\ln \boldsymbol{w}^* + \mathrm{d}\ln \boldsymbol{\ell}^*\right)}_{\text{market size}} + \underbrace{\theta\left(\boldsymbol{TS}-\boldsymbol{I}\right)\left[\mathrm{d}\ln \boldsymbol{w}^* - (1-\lambda)\left(\mathrm{d}\ln \boldsymbol{k}^* - \mathrm{d}\ln \boldsymbol{\ell}^*\right) - \mathrm{d}\ln \boldsymbol{z}\right]}_{\text{cross-substitution}}. \tag{23}$$

The first term on the right-hand side captures a *market-size effect*. If the productivity and amenity shocks raise income in another location n (through either a higher per capita income (w_n^*) or a higher population (ℓ_n^*)), this raises income in location i through increased expenditure on its goods. The magnitude of this effect depends on the share of income that location i derives from location n (as captured by the matrix $\boldsymbol{T}$). Only labor income appears on both sides of the equation, because capital income is a constant multiple of labor income, and this constant multiple cancels from both sides of the goods market clearing condition (as shown in equation (16)).

The second term on the right captures a *cross-substitution* effect. If these productivity and amenity shocks reduce production costs in another location n (which are captured again by the terms inside the square brackets, as in equation (21)), consumers in all markets m substitute towards location n and away from other locations $i \neq n$, thereby reducing income in location i and raising it in location n.

Population Flow. Third, from the population flow condition (18), the log change in a location's own population depends on the log change in populations in surrounding locations and the change in migration shares:

$$\mathrm{d}\ln \boldsymbol{\ell}^* = \underbrace{\boldsymbol{E}\,\mathrm{d}\ln \boldsymbol{\ell}^*}_{\text{population access}} + \underbrace{\frac{\beta}{\rho}\left(\boldsymbol{I}-\boldsymbol{ED}\right)\mathrm{d}\boldsymbol{v}^*}_{\text{expected value}}. \tag{24}$$

The first term on the right-hand side captures a *population access effect* for given initial migration shares. If the population in surrounding locations i increases because of productivity and amenity shocks, this in turn leads to a rise in population in location g for any given inmigration shares (E_{gi}) in the initial steady state. The second term on the right-hand side captures an *expected value effect* for given initial migration shares. If the expected value of living in location g (v_g) increases as a result of productivity and amenity shocks, this makes it more attractive for workers to inmigrate and less attractive for workers to outmigrate, which raises the population of that location by an amount that depends on the inmigration shares ($\boldsymbol{E}$) and outmigration shares ($\boldsymbol{D}$).

Worker Value Function. Finally, from the worker value function (2), the change in the expected value of living in each location depends on the change in the flow utility and the change in the continuation value for that location:

$$\mathrm{d}\boldsymbol{v}^{*} = \underbrace{\mathrm{d}\ln\boldsymbol{b} + \mathrm{d}\ln\boldsymbol{w}^{*} - \mathrm{d}\ln\boldsymbol{p}^{*}}_{\text{flow utility}} + \underbrace{\beta\boldsymbol{D}\,\mathrm{d}\boldsymbol{v}^{*}}_{\text{continuation value}}. \tag{25}$$

The first term on the right-hand side captures the change in *flow utility*, which depends on the change in amenities ($\mathrm{d}\ln\boldsymbol{b}$), the change in the wage ($\mathrm{d}\ln\boldsymbol{w}^{*}$), and the change in the cost of living ($\mathrm{d}\ln\boldsymbol{p}^{*}$). This change in the cost of living, as captured by (22), is again the expenditure-share weighted average of the changes in production costs, which in turn depend on the changes in wages, capital-labor ratios, and productivities. The second term on the right-hand side captures the change in *continuation value*, which depends on the outmigration shares ($\boldsymbol{D}$) and the change in the expected value of living in each location ($\mathrm{d}\boldsymbol{v}^{*}$).

Sufficient Statistics. Together these four key equilibrium conditions (21), (23), (24) and (25) completely determine the response of the steady-state spatial distribution of economic activity to the shocks to productivities ($\mathrm{d}\ln\boldsymbol{z}$) and amenities ($\mathrm{d}\ln\boldsymbol{b}$).

Proposition 2. *The steady-state elasticities of economic activity* $\{w_i^*, v_i^*, \ell_i^*, k_i^*\}$ *with respect to small productivity* ($\mathrm{d}\ln\boldsymbol{z}$) *and amenity shocks* ($\mathrm{d}\ln\boldsymbol{b}$) *are uniquely determined by the matrices* $\{\boldsymbol{L}^{z*}, \boldsymbol{K}^{z*}, \boldsymbol{W}^{z*}, \boldsymbol{V}^{z*}, \boldsymbol{L}^{b*}, \boldsymbol{K}^{b*}, \boldsymbol{W}^{b*}, \boldsymbol{V}^{b*}\}$*, which depend solely on the structural parameters* $\{\theta, \beta, \rho, \lambda\}$ *and the observed matrices of expenditure shares* ($\boldsymbol{S}$)*, income shares* ($\boldsymbol{T}$)*, outmigration shares* ($\boldsymbol{D}$) *and inmigration shares* ($\boldsymbol{E}$)*:*

$$\begin{bmatrix} \mathrm{d}\ln\boldsymbol{\ell}^{*} \\ \mathrm{d}\ln\boldsymbol{k}^{*} \\ \mathrm{d}\ln\boldsymbol{w}^{*} \\ \mathrm{d}\ln\boldsymbol{v}^{*} \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}^{z*} \\ \boldsymbol{K}^{z*} \\ \boldsymbol{W}^{z*} \\ \boldsymbol{V}^{z*} \end{bmatrix} \mathrm{d}\ln\boldsymbol{z} + \begin{bmatrix} \boldsymbol{L}^{b*} \\ \boldsymbol{K}^{b*} \\ \boldsymbol{W}^{b*} \\ \boldsymbol{V}^{b*} \end{bmatrix} \mathrm{d}\ln\boldsymbol{b}. \tag{26}$$

Proof. See Section B.8.3 of the online appendix. □

The elements of the matrices $\{\boldsymbol{L}^{z*}, \boldsymbol{K}^{z*}, \boldsymbol{W}^{z*}, \boldsymbol{V}^{z*}, \boldsymbol{L}^{b*}, \boldsymbol{K}^{b*}, \boldsymbol{W}^{b*}, \boldsymbol{V}^{b*}\}$ correspond to the steady-state elasticities of the endogenous variables in one location with respect to a small change in productivity or amenities in another location, incorporating all first-order general equilibrium effects. For instance, the ni-th entry of $\boldsymbol{L}^{z}$ is the steady-state elasticity of population in location n with respect to a small change in productivity in location i. The elements of these matrices capture bilateral exposure to shocks in other locations and can be computed from our observed matrices of expenditure shares ($\boldsymbol{S}$), income shares ($\boldsymbol{T}$), outmigration shares ($\boldsymbol{D}$) and inmigration shares ($\boldsymbol{E}$). Therefore, these observed matrices of trade and migration shares are sufficient statistics for the first-order general equilibrium effect of productivity and amenity shocks on the steady-state distribution of economic activity.

Naturally, locations with higher productivity (z_i) and higher amenities (b_i) have higher steady-state populations (ℓ_i^*) and capital stocks (k_i^*), other things equal. More subtly, the elasticities of steady-state population (ℓ_i^*) and capital stocks (k_i^*) with respect to productivity and amenities in both the own location (z_i and b_i) and other locations (z_n and b_n for $n \neq i$) depend on the entire networks of spatial linkages between locations in both goods markets (as determined by trade costs τ_{ni} and reflected in observed bilateral trade flows) and also labor markets (as determined by migration frictions κ_{ni} and captured in observed bilateral migration flows). Since the steady-state capital-labor ratio (k_i^*/ℓ_i^*) in each location is proportional to the real wage (w_i^*/p_i^*), differences in real wages across locations reflect not only primitive differences in productivity (z_i) but also endogenous investments in capital accumulation (k_i^*). These endogenous investments depend not only on own productivity and amenities, but also on the geography of access to other locations in goods and labor markets.

3.2 Transition Dynamics

This section derives first-order sufficient statistics for the entire transition path of the spatial distribution of economic activity in response to small shocks to fundamentals (productivity and amenities). We derive these results starting from arbitrary initial values of the state variables, assuming that the economy is somewhere on a convergence path towards the steady-state implied by the initial values for its fundamentals. We first solve in closed-form for the transition path of the economy following a one-time, unexpected and permanent shock to fundamentals. We then derive analogous closed-form solutions for an unexpected convergent sequence of shocks to fundamentals, assuming that once agents learn about this convergent sequence of shocks, the future time path of fundamentals is revealed under perfect foresight. We show that the economy's transition path depends on a transition matrix ($\boldsymbol{P}$), which governs the evolution of the state variables from one period to the next, and an impact matrix ($\boldsymbol{R}$), which captures the initial impact of the shocks to fundamentals. We use these results to quantify the contributions of convergence to steady-state and shocks to fundamentals to the observed changes in economic activity during our sample period and to undertake counterfactuals. We show that the economy's speed of convergence to steady-state depends on the eigenvalues of the transition matrix ($\boldsymbol{P}$), which in turn depends on our observed trade and migration share matrices $\{\boldsymbol{S}, \boldsymbol{T}, \boldsymbol{D}, \boldsymbol{E}\}$. We use the transition ($\boldsymbol{P}$) and impact ($\boldsymbol{R}$) matrices to undertake a spectral analysis that decomposes the shocks to fundamentals into lower-dimensional components. We are thus able to isolate locations that are sensitive to particular types of shocks and types of shocks that disproportionally affect particular locations.

3.2.1 Dynamic Sufficient Statistics: the Transition ($\boldsymbol{P}$) and Impact ($\boldsymbol{R}$) Matrices

Suppose that we observe the state variables $\{\boldsymbol{\ell_t}, \boldsymbol{k_t}\}$ and the trade and migration share matrices $\{\boldsymbol{S}, \boldsymbol{T}, \boldsymbol{D}, \boldsymbol{E}\}$ of the economy at time $t = 0$. The economy need not be in steady-state at $t = 0$, but we assume that it is on a convergence path towards a steady-state with constant fundamentals $\{\boldsymbol{z}, \boldsymbol{b}, \boldsymbol{\kappa}, \boldsymbol{\tau}\}$. We begin by considering the case in which agents learn at time $t = 0$ about a one-time, unexpected, and permanent change in fundamentals (productivity and amenities) from time $t = 1$ onwards. We derive sufficient statistics for the transition path of the entire spatial distribution of economic activity in response to this shock. We refer to the steady-state implied by the initial fundamentals (productivity, amenities, trade and migration costs) at $t = 0$ as the *initial steady-state*, and we refer to the steady-state following the shock to productivity and amenities as the *new steady-state*. Throughout the following, we use a tilde above a variable to denote a log deviation from the initial steady-state ($\widetilde{\chi}_{it+1} = \ln \chi_{it+1} - \ln \chi_i^*$) for all variables except for the worker value function, for which with a slight abuse of notation we use the tilde to denote a deviation in levels ($\widetilde{v}_{it} \equiv v_{it} - v_t^*$). Under our assumption of a one-time change in fundamentals, we can write the shocks to productivity and amenities as $\left(\widetilde{\boldsymbol{z}}_t, \widetilde{\boldsymbol{b}}_t\right) = \left(\widetilde{\boldsymbol{z}}, \widetilde{\boldsymbol{b}}\right)$ for $t \geq 1$.

Totally differentiating our four equilibrium conditions around the implied initial steady-state, holding constant the aggregate labor endowment, trade costs and migration costs, the transition dynamics of the economy for time $t \geq 1$ onwards are characterized, to first order, by the following system of equations:

$$\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} = \frac{\beta}{\rho} \left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D}\right) \widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}} + \boldsymbol{E}\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} \tag{27}$$

$$\widetilde{\boldsymbol{k}}_{t+1} = \left[\begin{array}{c} \left[\beta\left(1-\delta\right)\boldsymbol{I} + \left(1-\beta\left(1-\delta\right)\right)\left(1-\lambda\right)\boldsymbol{S}\right]\left(\widetilde{\boldsymbol{k}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}\right) \\ + \left(1-\beta\left(1-\delta\right)\right)\left(\boldsymbol{I}-\boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \left(1-\beta\left(1-\delta\right)\right)\boldsymbol{S}\widetilde{\boldsymbol{z}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} \end{array} \right], \tag{28}$$

$$\left[\boldsymbol{I} - \boldsymbol{T} + \theta\left(\boldsymbol{I} - \boldsymbol{T}\boldsymbol{S}\right)\right]\widetilde{\boldsymbol{w}}_t = \left[-\left(\boldsymbol{I}-\boldsymbol{T}\right)\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \theta\left(\boldsymbol{I}-\boldsymbol{T}\boldsymbol{S}\right)\left(\widetilde{\boldsymbol{z}} + \left(1-\lambda\right)\left(\widetilde{\boldsymbol{k}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}\right)\right)\right]. \tag{29}$$

$$\widetilde{\boldsymbol{v}}_{\boldsymbol{t}} = \left(\boldsymbol{I}-\boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \boldsymbol{S}\widetilde{\boldsymbol{z}} + \left(1-\lambda\right)\boldsymbol{S}\left(\widetilde{\boldsymbol{k}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}\right) + \widetilde{\boldsymbol{b}} + \beta\boldsymbol{D}\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}}, \tag{30}$$

as shown in Subsection B.8.4 of the online appendix.

In this system of equations, the state variables are the population ($\widetilde{\boldsymbol{\ell}}_t$) and capital stock ($\widetilde{\boldsymbol{k}}_t$) in each location. The labor equation (27) is forward-looking, because the future population in each location depends on the current population and the expected future value of living in each location. The capital equation (28) is backward-looking, such that the constant saving rate pins down the future capital stock as a function of the current period state variables and the shocks to productivities and amenities. The wage equation (29) depends only on current-period state variables and the shocks to productivities and amenities, through the static trade model. The value function (30) is forward-looking, because the expected value of living in each location depends on current-period state variables and the shocks to productivities and amenities, as well as the expected future value of living in each location.

Substituting the wage equation (29) and the value function (30) into the labor dynamics equation (27) and the capital accumulation condition (28), we can reduce the model's transition dynamics to a linear system of two second-order difference equations. We start by using the wage equation (29) to express the value function (30) in terms of the state variables and the productivity and amenity shocks:

$$\widetilde{\boldsymbol{v}}_t = \boldsymbol{\mathcal{L}}\left(\widetilde{\boldsymbol{\ell}}_t, \widetilde{\boldsymbol{k}}_t, \widetilde{\boldsymbol{z}}, \widetilde{\boldsymbol{b}}\right) + \beta\boldsymbol{D}\widetilde{\boldsymbol{v}}_{t+1}, \tag{31}$$

where $\boldsymbol{\mathcal{L}}_t \equiv \boldsymbol{\mathcal{L}}\left(\widetilde{\boldsymbol{\ell}}_t, \widetilde{\boldsymbol{k}}_t, \widetilde{\boldsymbol{z}}, \widetilde{\boldsymbol{b}}\right)$ is a multilinear map that corresponds to a linear combination of the changes in the state variables and the shocks to productivities and amenities $\{\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}, \widetilde{\boldsymbol{k}}_t, \widetilde{\boldsymbol{z}}, \widetilde{\boldsymbol{b}}\}$ relative to the initial steady-state, such that $\boldsymbol{\mathcal{L}}_t = \boldsymbol{\mathcal{L}}^{\boldsymbol{\ell}}\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \boldsymbol{\mathcal{L}}^{\boldsymbol{k}}\widetilde{\boldsymbol{k}}_t + \boldsymbol{\mathcal{L}}^{\boldsymbol{z}}\widetilde{\boldsymbol{z}} + \boldsymbol{\mathcal{L}}^{\boldsymbol{b}}\widetilde{\boldsymbol{b}}$, where $\boldsymbol{\mathcal{L}}^{\boldsymbol{\ell}}$, $\boldsymbol{\mathcal{L}}^{\boldsymbol{k}}$, $\boldsymbol{\mathcal{L}}^{\boldsymbol{z}}$, $\boldsymbol{\mathcal{L}}^{\boldsymbol{b}}$ are $N \times N$ matrices that can be constructed from the structural parameters $\{\theta, \beta, \rho, \lambda, \delta\}$ and the trade and migration share matrices $\{\boldsymbol{S}, \boldsymbol{T}, \boldsymbol{D}, \boldsymbol{E}\}$, as shown in Section B.8.4 of the online appendix.

We next iterate the value function (31) forward in time to express the current expected value of living in each location in terms of all future expected values of living in each location as follows:

$$\widetilde{\boldsymbol{v}}_t = \sum_{s=0}^{\infty} \left(\beta\boldsymbol{D}\right)^s \boldsymbol{\mathcal{L}}_{t+s}.$$

Substituting this expression into the labor equation (27), we can express the future population in each location in terms of its current population and all future expected values of living in that location:

$$\begin{aligned}\widetilde{\boldsymbol{\ell}}_{t+1} - \boldsymbol{E}\widetilde{\boldsymbol{\ell}}_t &= \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{ED}\right)\sum_{s=0}^{\infty}\left(\beta\boldsymbol{D}\right)^s \boldsymbol{\mathcal{L}}_{t+1+s}, \\ &= \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{ED}\right)\left(\boldsymbol{\mathcal{L}}_{t+1} + \beta\boldsymbol{D}\sum_{s=0}^{\infty}\left(\beta\boldsymbol{D}\right)^s \boldsymbol{\mathcal{L}}_{t+2+s}\right).\end{aligned}$$

Taking differences between this equation for periods t and $t+1$, we obtain a second-order difference equation that governs population dynamics, which captures the role of both past state variables and future expectations in influencing migration decisions:

$$\widetilde{\boldsymbol{\ell}}_{t+2} = \left(\boldsymbol{I} + \boldsymbol{E}\right)\widetilde{\boldsymbol{\ell}}_{t+1} - \boldsymbol{E}\widetilde{\boldsymbol{\ell}}_t - \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{ED}\right)\boldsymbol{\mathcal{L}}_{t+1}. \tag{32}$$

We obtain another difference equation by substituting the wage equation (29) into the capital accumulation equation (28), to express next period's capital stock in terms of the current period's state variables and the shocks to productivities and amenities. Stacking these migration and capital accumulation equations in matrix form, we obtain the following system of second-order difference equations in the state variables:

$$\boldsymbol{\Psi}\widetilde{\boldsymbol{x}}_{\boldsymbol{t+2}} = \boldsymbol{\Gamma}\widetilde{\boldsymbol{x}}_{\boldsymbol{t+1}} + \boldsymbol{\Theta}\widetilde{\boldsymbol{x}}_{\boldsymbol{t}} + \boldsymbol{\Pi}\widetilde{\boldsymbol{f}}, \tag{33}$$

where $\widetilde{\boldsymbol{x}}_{\boldsymbol{t}} = \begin{bmatrix}\widetilde{\boldsymbol{\ell}}_t \\ \widetilde{\boldsymbol{k}}_t\end{bmatrix}$ is a $2N \times 1$ vector of the state variables; $\widetilde{\boldsymbol{f}} = \begin{bmatrix}\widetilde{\boldsymbol{z}} \\ \widetilde{\boldsymbol{b}}\end{bmatrix}$ is a $2N \times 1$ vector of the shocks to fundamentals; and $\boldsymbol{\Psi}$, $\boldsymbol{\Gamma}$, $\boldsymbol{\Theta}$, and $\boldsymbol{\Pi}$ are $2N \times 2N$ matrices that depend only on the structural parameters $\{\theta$, β, ρ, λ, $\delta\}$ and the observed trade and migration matrices $\{\boldsymbol{S}, \boldsymbol{T}, \boldsymbol{D}, \boldsymbol{E}\}$.

This system of second-order difference equations (33) can be solved using standard techniques from the time-series macroeconomics literature. Using the method of undetermined coefficients following Uhlig (1999), we obtain a closed-form solution for the transition path of the spatial distribution of economic activity in terms of a transition matrix ($\boldsymbol{P}$), which governs the evolution of state variables from one period to the next, and an impact matrix ($\boldsymbol{R}$), which captures the initial impact of the shocks to fundamentals.[4]

Proposition 3. *Suppose that the economy at time $t = 0$ is on a convergence path towards an initial steady-state with constant fundamentals ($\boldsymbol{z}$, $\boldsymbol{b}$, $\boldsymbol{\kappa}$, $\boldsymbol{\tau}$). At time $t = 0$, agents learn about one-time, permanent shocks to productivity and amenities ($\widetilde{\boldsymbol{f}} = \begin{bmatrix}\widetilde{\boldsymbol{z}} \\ \widetilde{\boldsymbol{b}}\end{bmatrix}$) from time $t = 1$ onwards. There exists a $2N \times 2N$ transition matrix ($\boldsymbol{P}$) and a $2N \times 2N$ impact matrix ($\boldsymbol{R}$) such that the second-order difference equation system in (33) has a closed-form solution of the form:*

$$\widetilde{\boldsymbol{x}}_{t+1} = \boldsymbol{P}\widetilde{\boldsymbol{x}}_t + \boldsymbol{R}\widetilde{\boldsymbol{f}} \quad \textit{for } t \geq 0. \tag{34}$$

Proof. See Section B.8.4 of the online appendix. □

[4]Relative to the time-series macroeconomics literature, our dynamic spatial model features a much larger state space of many locations or location-sectors over time. Nevertheless, the use of standard linear algebra techniques allows our approach to accommodate large state spaces, while remaining computationally efficient and easy to implement.

From Proposition 3, we have a closed-form solution for the evolution of the state variables in each location in all future time periods. Therefore, we can compute the time path of the entire spatial distribution of economic activity in response to the shock to productivity and amenities, using only the initial observed values of the state variables $\{\boldsymbol{\ell}_t, \boldsymbol{k}_t\}$, the transition matrix ($\boldsymbol{P}$) and the impact matrix ($\boldsymbol{R}$). The key to obtaining this closed-form is our linearization of the model's transition dynamics. Although this linearization again captures first-order general equilibrium effects, and hence is only exact for small changes, we show below that it provides a close approximation not only for the change in steady-state but also for the entire transition path for the full non-linear model solution, even for changes in productivity and amenities of the magnitude implied by the observed data. In Section 3.2.3 below, we derive analogous closed-form solutions for any convergent sequence of shocks to productivity and amenities under our baseline assumption of perfect foresight. In Section 4.3, we further generalize these results to allow for stochastic fundamentals under rational expectations.

We now use this analytical characterization of the transition path in Proposition 3 to decompose the evolution of the spatial distribution of economic activity across locations into the contributions of convergence towards the initial steady-state and shocks to fundamentals:

$$\ln \boldsymbol{x}_t - \ln \boldsymbol{x}_{-1} = \underbrace{\sum_{s=0}^{t} \boldsymbol{P}^s \left(\ln \boldsymbol{x}_0 - \ln \boldsymbol{x}_{-1}\right)}_{\substack{\text{convergence given} \\ \text{initial fundamentals}}} + \underbrace{\sum_{s=0}^{t-1} \boldsymbol{P}^s \boldsymbol{R} \widetilde{\boldsymbol{f}}}_{\substack{\text{dynamics from} \\ \text{fundamental shocks}}} \qquad \text{for all } t \geq 1, \tag{35}$$

where the first term on the right-hand side captures convergence to the steady-state implied by the initial fundamentals at time $t = 0$. In the absence of any shocks to productivity and amenities ($\widetilde{\boldsymbol{f}} = \boldsymbol{0}$), the second term on the right-hand side is zero, and the state variables converge to:

$$\ln \boldsymbol{x}^*_{\text{initial}} = \lim_{t \to \infty} \ln \boldsymbol{x}_t = \ln \boldsymbol{x}_{-1} + \left(\boldsymbol{I} - \boldsymbol{P}\right)^{-1} \left(\ln \boldsymbol{x}_0 - \ln \boldsymbol{x}_{-1}\right), \tag{36}$$

where $\left(\boldsymbol{I} - \boldsymbol{P}\right)^{-1} = \sum_{s=0}^{\infty} \boldsymbol{P}^s$ is well-defined under the condition that the spectral radius of $\boldsymbol{P}$ is smaller than one, a property that we verify empirically.

Using this implication of Proposition 3 in equation (36), we can compute the steady-state spatial distribution of economic activity implied by the initial fundamentals, in the absence of any further changes to those fundamentals. All we require to solve for this implied steady-state is the observed values of the state variables for two initial periods ($t = -1, 0$), the trade and migration share matrices for one initial period $\{\boldsymbol{S}, \boldsymbol{T}, \boldsymbol{D}, \boldsymbol{E}\}$, and the structural parameters $\{\theta, \beta, \rho, \lambda, \delta\}$. Importantly, we can compute the transition path towards this implied steady-state without having to solve for the unobserved fundamentals themselves $\{\boldsymbol{z}_t, \boldsymbol{b}_t, \boldsymbol{\tau}_t, \boldsymbol{\kappa}_t\}$. We use this exercise to assess the contribution of convergence given initial fundamentals to the observed changes in the spatial distribution of economic activity. In our extension to stochastic fundamentals and rational expectations, we relax the assumption of constant fundamentals to solve for convergence to steady-state, given agents' initial forecasts for the evolution of fundamentals under rational expectations.

The second term on the right-hand-side of (35) captures the dynamics introduced by the shocks to productivity and amenities from time $t = 1$ onwards. The matrix $\boldsymbol{R}$ captures the initial impact of these shocks $\widetilde{\boldsymbol{f}}$ on the state variables, and the $\boldsymbol{P}$ matrix governs how this impact propagates over time. If the economy is initially in a steady-state at time 0, then the first term is zero, and the path of the state variables follows:

$$\widetilde{\boldsymbol{x}}_t = \sum_{s=0}^{t-1} \boldsymbol{P}^s \boldsymbol{R} \widetilde{\boldsymbol{f}} = \left(\boldsymbol{I} - \boldsymbol{P}^t\right) \left(\boldsymbol{I} - \boldsymbol{P}\right)^{-1} \boldsymbol{R} \widetilde{\boldsymbol{f}} \quad \text{for all } t \geq 1. \tag{37}$$

In this case, the initial response is $\widetilde{\boldsymbol{x}}_1 = \boldsymbol{R}\widetilde{\boldsymbol{f}}$, and, taking the limit as $t \to \infty$, we obtain the steady-state response:

$$\lim_{t \to \infty} \widetilde{\boldsymbol{x}}_t = \ln \boldsymbol{x}^*_{\text{new}} - \ln \boldsymbol{x}^*_{\text{initial}} = \left(\boldsymbol{I} - \boldsymbol{P}\right)^{-1} \boldsymbol{R} \widetilde{\boldsymbol{f}}, \tag{38}$$

where $\left(\boldsymbol{I} - \boldsymbol{P}\right)^{-1} \boldsymbol{R} \widetilde{\boldsymbol{f}}$ coincides with our explicit solution for the change in steady-states in Section 3.1 above.

Proposition 3 and our subsequent generalizations to allow for any convergent sequence of shocks under perfect foresight (Section 3.2.3) and rational expectations (Section 4.3) play an important role in our empirical analysis. First, given the observed spatial distribution of economic activity—population and the capital stock—and trade and migration share matrices, we can use these results to recover the initial unobserved steady-state. Second, given empirical measures of shocks to productivity and amenities during the sample period, we can use these results to assess the relative contributions of transition dynamics and these shocks to fundamentals to the observed evolution of the spatial distribution of economic activity. Third, we can undertake counterfactuals for policy changes and other out of sample shocks, and examine the relative importance of transition dynamics versus changes in steady-state in shaping the counterfactual impact of these shocks. In the next section, we use our closed-form solutions to further analyze the determinants of the economy's response to shocks.

3.2.2 Dynamics and the Spectral Properties of the Transition Matrix $\boldsymbol{P}$

We now provide a further analytical characterization of the model's transition path in terms of the spectral properties (i.e., the eigenvalues and eigenvectors) of the transition matrix ($\boldsymbol{P}$). We use this analytical characterization to examine the determinants of the speed of convergence to steady-state and the heterogeneous impact of shocks, both across locations and across different types of shocks. Since we have already shown in Proposition 3 that we can decompose the dynamic path of the economy into one component capturing shocks to fundamentals and another component capturing convergence to the initial steady-state, we focus for expositional simplicity in this and the remaining subsections on an economy that is initially in steady-state.

We begin by undertaking an eigendecomposition of the transition matrix, $\boldsymbol{P} \equiv \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{V}$, where $\boldsymbol{\Lambda}$ is a diagonal matrix of eigenvalues arranged in decreasing order by absolute values, and $\boldsymbol{V} = \boldsymbol{U}^{-1}$. For each eigenvalue λ_k, the k-th column of $\boldsymbol{U}$ ($\boldsymbol{u_k}$) and the k-th row of $\boldsymbol{V}$ ($\boldsymbol{v'_k}$) are the corresponding right- and left-eigenvectors of $\boldsymbol{P}$, respectively, such that

$$\lambda_k \boldsymbol{u}_k = \boldsymbol{P}\boldsymbol{u}_k, \qquad \lambda_k \boldsymbol{v}'_k = \boldsymbol{v}'_k \boldsymbol{P}.$$

That is, $\boldsymbol{u}_k$ ($\boldsymbol{v}'_k$) is the vector that, when left-multiplied (right-multiplied) by $\boldsymbol{P}$, is proportional to itself but scaled by the corresponding eigenvalue λ_k. We construct the right-eigenvectors such that the 2-norm of $\boldsymbol{u}_k$ is equal to 1 for all k, where note that $\boldsymbol{v}'_i \boldsymbol{u}_k = 1$ for $i = k$ and $\boldsymbol{v}'_i \boldsymbol{u}_k = 0$ otherwise. We refer to $\boldsymbol{u}_k$ simply as eigenvectors. Both $\{\boldsymbol{u}_k\}$ and $\{\boldsymbol{v}'_k\}$ are bases that span the $2N$-dimensional vector space.

Using this eigendecomposition, we are able to characterize the transition path of the entire spatial distribution of economic activity simply in terms of the eigenvectors and eigenvalues of the transition matrix.

Proposition 4. *Consider an economy that is initially in steady-state at time $t = 0$ when agents learn about one-time, permanent shocks to productivity and amenities ($\widetilde{\boldsymbol{f}} = \begin{bmatrix} \widetilde{\boldsymbol{z}} \\ \widetilde{\boldsymbol{b}} \end{bmatrix}$) from time $t = 1$ onwards. The transition path of the state variables can be written in terms of the eigenvalues (λ_k) and left and right eigenvectors ($\boldsymbol{u}_k$, $\boldsymbol{v}_k'$) of the transition matrix as follows:*

$$\widetilde{\boldsymbol{x}}_t = \sum_{s=0}^{t-1} \boldsymbol{P}^s \boldsymbol{R} \widetilde{\boldsymbol{f}} = \sum_{k=1}^{2N} \frac{1-\lambda_k^t}{1-\lambda_k} \boldsymbol{u}_k \boldsymbol{v}_k' \boldsymbol{R} \widetilde{\boldsymbol{f}}. \tag{39}$$

Proof. The proposition follows from the eigendecomposition of the transition matrix: $\boldsymbol{P} \equiv \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{V}$, as shown in Section B.8.4 of the online appendix. □

In this Proposition, we use the eigendecomposition of the transition matrix ($\boldsymbol{P}$) to convert the cumulative impact of fundamental shocks $\widetilde{\boldsymbol{f}}$ up until time t from a summation across time into a summation across eigencomponents. An implication is that, if the initial impact of the shock to productivity and amenities on the state variables ($\boldsymbol{R}\widetilde{\boldsymbol{f}}$) coincides with a real eigenvector $\boldsymbol{u}_k$, then over time the changes in the state variables decay exponentially:

$$\widetilde{\boldsymbol{x}}_t = \sum_{j=1}^{2N} \frac{1-\lambda_j^t}{1-\lambda_j} \boldsymbol{u}_j \boldsymbol{v}_j' \boldsymbol{u}_k = \frac{1-\lambda_k^t}{1-\lambda_k} \boldsymbol{u}_k \qquad \Longrightarrow \qquad \ln \boldsymbol{x}_{t+1} - \ln \boldsymbol{x}_t = \lambda_k^t \boldsymbol{u}_k.$$

That is, when the initial impact of the shock on the state variables is the eigenvector $\boldsymbol{u}_k$, the subsequent changes in these state variables across periods are always proportional to $\boldsymbol{u}_k$, and decay exponentially at a rate determined by the corresponding eigenvalue λ_k, as the economy converges to the new steady-state. These eigenvalues fully summarize the impact of migration frictions for the mobile factor and gradual accumulation for the immobile factor on the model's transition dynamics.[5]

Although there is no necessary reason why an individual shock to productivity and amenities should correspond to an eigenvector $\boldsymbol{u}_k$, we show below that the initial impact of *any* productivity or amenity shock on the state variables can be expressed as a linear combination of these eigenvectors. Another advantage of this eigendecomposition is that we obtain an analytical characterization of the speed of convergence to steady-state, even with the high-dimensional state space and multiple sources of dynamics in the model. We measure the speed of convergence to steady-state using the conventional measure of the half-life. In particular, we define the half-life of a shock $\widetilde{\boldsymbol{f}}$ for the i-th state variable as the time it takes for that state variable to converge half of the way towards steady-state:

$$t_i^{(1/2)}\left(\widetilde{\boldsymbol{f}}\right) \equiv \arg \max_{t \in \mathbb{Z}_{>0}} \frac{|\widetilde{x}_{it} - \widetilde{x}_{i\infty}|}{\max_s |\widetilde{x}_{is} - \widetilde{x}_{i\infty}|} \geq \frac{1}{2},$$

where $\widetilde{x}_{i\infty} = x_{i,\text{new}}^* - x_{i,\text{initial}}^*$. For a shock to productivity and amenities ($\widetilde{\boldsymbol{f}}$) whose initial impact on the state variables ($\boldsymbol{R}\widetilde{\boldsymbol{f}}$) coincides with an eigenvector $\boldsymbol{u}_k$ of the transition matrix $\boldsymbol{P}$, its half-life can be expressed solely in terms of the corresponding eigenvalue λ_k, as summarized in the following proposition.

[5] In general, these eigenvectors and eigenvalues can be complex-valued. If the initial impact is the real part of a complex eigenvector $\boldsymbol{u}_k$ ($\boldsymbol{R}\widetilde{\boldsymbol{f}} = \text{Re}\,(\boldsymbol{u}_k)$), then $\ln \boldsymbol{x}_{t+1} - \ln \boldsymbol{x}_t = \text{Re}\left(\lambda_k^t \boldsymbol{u}_k\right) \neq \text{Re}\,(\lambda_k) \cdot \text{Re}\left(\lambda_k^{t-1} \boldsymbol{u}_k\right)$. That is, the impact no longer decays at a constant rate λ_k. Instead, the complex eigenvalues introduce oscillatory motion as the dynamical system converges to the new steady-state. For expositional purposes, we focus on real-valued eigenvalues and eigenvectors. In our empirical application, the imaginary components of $\boldsymbol{P}$'s eigenvalues are small, implying that oscillatory effects are small relative to the effects that decay exponentially.

Proposition 5. *Consider an economy that is initially in steady-state at time $t = 0$ when agents learn about one-time, permanent shocks to productivity and amenities ($\widetilde{\boldsymbol{f}} = \begin{bmatrix} \widetilde{\boldsymbol{z}} \\ \widetilde{\boldsymbol{b}} \end{bmatrix}$) from time $t = 1$ onwards. Suppose the initial impact of the shocks to productivity and amenities on the state variables at time $t = 1$ coincides with an eigenvector ($\boldsymbol{R}\widetilde{\boldsymbol{f}} = \boldsymbol{u}_k$) of the transition matrix ($\boldsymbol{P}$). The transition path of the state variables ($\boldsymbol{x_t}$) reduces to:*

$$\widetilde{\boldsymbol{x}}_t = \frac{1-\lambda_k^t}{1-\lambda_k}\boldsymbol{u}_k,$$

and the half-life is given by:

$$t_i^{(1/2)}\left(\widetilde{\boldsymbol{f}}\right) = -\left\lceil \frac{\ln 2}{\ln \lambda_k} \right\rceil$$

for all state variables $i = 1, \cdots, 2N$, where $\lceil \cdot \rceil$ is the ceiling function.

Proof. The proposition follows from the eigendecomposition of the transition matrix ($\boldsymbol{P} \equiv \boldsymbol{U\Lambda V}$), for the case in which the initial impact of the shocks to productivity and amenities on the state variables at time $t = 1$ coincides with an eigenvector ($\boldsymbol{R}\widetilde{\boldsymbol{f}} = \boldsymbol{u}_k$) of the transition matrix ($\boldsymbol{P}$), as shown in Section B.8.4 of the online appendix. □

Therefore, for productivity and amenity shocks whose initial impact corresponds to an eigenvector of the transition matrix ($\boldsymbol{P}$), the associated eigenvalue is a sufficient statistic for the speed of convergence to steady-state. As the transition matrix ($\boldsymbol{P}$) can be recovered from the observed trade and migration share matrices $\{\boldsymbol{S}, \boldsymbol{T}, \boldsymbol{D}, \boldsymbol{E}\}$ and the structural parameters of the model $\{\theta, \beta, \rho, \lambda, \delta\}$, we can compute this measure of the speed of convergence from the observed data. We refer to fundamental shocks of this form as *eigen-shocks*. If the impact matrix ($\boldsymbol{R}$) is invertible, which we verify empirically later, these eigen-shocks can be written as $\widetilde{\boldsymbol{f}}_k \equiv \boldsymbol{R}^{-1}\boldsymbol{u}_k$ and form a basis of $\mathbb{R}^{2N}$. This property in turn implies that any arbitrary fundamental shock vector can be decomposed into a linear combination of the eigen-shocks. Therefore, we can use these eigen-shocks to characterize the impact of observed and counterfactual shocks. In particular, we show in the following proposition that the left-eigenvectors can be used to retrieve the coordinates of any vector of shocks to productivity and amenities on the eigen-shock basis.

Proposition 6. *If the impact matrix ($\boldsymbol{R}$) is invertible, the impact of any arbitrary vector of shocks to productivity and amenities ($\widetilde{\boldsymbol{f}}$) can be written as a linear combination of the eigen-shocks:*

$$\boldsymbol{R}\widetilde{\boldsymbol{f}} = \sum_{k=1}^{2N} a_k \boldsymbol{R}\widetilde{\boldsymbol{f}}_k,$$

where weights $\boldsymbol{a} \equiv (a_1, \cdots, a_{2N})'$ satisfy $\boldsymbol{a} = \boldsymbol{VR}\widetilde{\boldsymbol{f}}$.

Proof. Note $\boldsymbol{v}_k'\boldsymbol{R}\widetilde{\boldsymbol{f}} = \boldsymbol{v}_k' \sum_{i=1}^{2N} a_i \boldsymbol{R}\widetilde{\boldsymbol{f}}_i = \sum_{i=1}^{2N} a_i \boldsymbol{v}_k'\boldsymbol{u}_i = a_k$. □

Using this proposition, we now show that we can decompose the time path of each state variable along each eigencomponent. This result enables us to undertake a spectral analysis to isolate, for different time horizons, the locations that are affected by particular shocks (as captured by the eigenvectors) and the types of shocks that affect particular locations (as captured by the loadings onto the eigencomponents).

Proposition 7. *Consider an economy that is initially in steady-state at time $t = 0$ when agents learn about one-time, permanent shocks to productivity and amenities ($\widetilde{\boldsymbol{f}} = \begin{bmatrix} \widetilde{\boldsymbol{z}} \\ \widetilde{\boldsymbol{b}} \end{bmatrix}$) from time $t = 1$ onwards. The law of motion for the i-th state variables ($\widetilde{\boldsymbol{x}}_{it}$) is:*

$$\begin{aligned} \boldsymbol{e}_i' \widetilde{\boldsymbol{x}}_t &= \sum_{k=1}^{2N} \frac{1-\lambda_k^t}{1-\lambda_k} \boldsymbol{e}_i' \boldsymbol{u}_k \boldsymbol{v}_k' \boldsymbol{R} \widetilde{\boldsymbol{f}} \\ &= \sum_{k=1}^{2N} \frac{1-\lambda_k^t}{1-\lambda_k} u_{ik} a_k, \end{aligned}$$

where u_{ik} is the ik-th entry of the matrix $\boldsymbol{U}$ of eigenvectors, a_k is the k-th coordinate of the fundamental shocks $\widetilde{\boldsymbol{f}}$ on the eigen-shock basis, and $\boldsymbol{e}_i$ is the i-th standard basis vector.

Proof. Note $\boldsymbol{v}_k' \boldsymbol{R} \widetilde{\boldsymbol{f}} = a_k$ from Proposition 6 and that $\boldsymbol{e}_i' \boldsymbol{u}_k a_k = u_{ik} a_k$. □

Proposition 7 decomposes the dynamic exposure of the state variable in each location ($\widetilde{x}_{it}$) to productivity and amenity shocks ($\widetilde{\boldsymbol{f}}$) along individual eigencomponents. As such, this decomposition has both similarities and differences with empirical shift-share Bartik decompositions. A key similarity is that there a direct measure of exposure for each location to a given shock and any given shock affects different locations unevenly. A key difference is that our decomposition is derived from the closed-form solution for the economy's transition path. Additionally, our decomposition is dynamic, and hence evolves over time as the economy gradually adjusts to the shock. The exposure of the state variable in location i to the k-th eigen-shock is u_{ik}, and the exposure to the fundamental shock $\widetilde{\boldsymbol{f}}$ along the k-th component is the product between u_{ik} and $\widetilde{\boldsymbol{f}}$'s coordinate on the eigencomponent, a_k.

In our empirical analysis below, we show that for the state variables in any location i, the distribution of u_{ik}'s vary substantially across eigencomponents k, with some entries much larger than others. A key implication of this empirical finding is that only a few eigen-shocks matter quantitatively for the dynamics of the state variables in any given location. Additionally, we find that the dynamic effects of these eigen-shock vectors decay over time at differential rates, as determined by the corresponding eigenvalues λ_k. Therefore, the decomposition in Proposition 7 provides a lower-dimensional representation of which shocks matter for which state variables — the population and capital stock in different locations — and over what time horizons.

3.2.3 Dynamic Response to a Convergent Sequence of Fundamental Shocks

In this section, we now generalize our analysis of the model's transition dynamics to any convergent sequence of future shocks to productivities and amenities, under our baseline assumption of perfect foresight. In particular, we consider an economy that is initially in steady-state at time $t = 0$, when agents learn about a convergent sequence of future shocks to fundamentals $\left\{ \widetilde{\boldsymbol{f}}_s \right\}_{s \geq 1}$, where $\widetilde{\boldsymbol{f}}_s$ is a vector of log differences in productivity and amenities for each location between times s and 0.

Proposition 8. *Consider an economy that is initially in steady-state at time $t = 0$ when agents learn about a convergent sequence of future shocks to productivity and amenities $\left\{ \widetilde{\boldsymbol{f}}_{\boldsymbol{s}} \right\}_{s \geq 1} = \left\{ \begin{bmatrix} \widetilde{\boldsymbol{z}}_{\boldsymbol{s}} \\ \widetilde{\boldsymbol{b}}_{\boldsymbol{s}} \end{bmatrix} \right\}_{s \geq 1}$ from time $t = 1$*

onwards. There exists a $2N \times 2N$ transition matrix ($\boldsymbol{P}$) and a $2N \times 2N$ impact matrix ($\boldsymbol{R}$) such that the dynamic path of state variables relative to the initial steady-state follows:

$$\widetilde{\boldsymbol{x}}_t = \sum_{s=t+1}^{\infty} \left(\boldsymbol{\Psi}^{-1}\boldsymbol{\Gamma} - \boldsymbol{P}\right)^{-(s-t)} \boldsymbol{R} \left(\widetilde{\boldsymbol{f}}_s - \widetilde{\boldsymbol{f}}_{s-1}\right) + \boldsymbol{R}\widetilde{\boldsymbol{f}}_t + \boldsymbol{P}\widetilde{\boldsymbol{x}}_{t-1} \qquad \text{for all } t \geq 1, \tag{40}$$

with initial condition $\widetilde{\boldsymbol{x}}_0 = \boldsymbol{0}$ and where $\boldsymbol{\Psi}, \boldsymbol{\Gamma}$ are matrices from the second-order difference equation (33) and are derived in Section B.8.4 of the online appendix.

Proof. See Section B.8.4 of the online appendix. □

Therefore, even though we consider a general convergent sequence of shocks to productivity and amenities in a setting with many locations connected by a rich geography, and with multiple sources of dynamics from migration and capital accumulation, we obtain a closed-form for the transition path of the spatial distribution of economic activity. We use our linearization to take log deviations between the state variables at each future point in time $s \geq 1$ and the initial steady-state at time $t = 0$, taking into account the intervening changes in fundamentals in between those times. The transition matrix $\boldsymbol{P}$ and impact matrix $\boldsymbol{R}$ are exactly the same as in the previous subsection, and can be recovered from our observed trade and migration share matrices {$\boldsymbol{S}$, $\boldsymbol{T}$, $\boldsymbol{D}$, $\boldsymbol{E}$} and the structural parameters of the model {θ, β, ρ, λ, δ}.

Proposition 8 encompasses a number of special cases. First, the permanent shock to productivity and amenities in period 1 considered in the previous subsection is naturally a special case of the convergent sequence of shocks to fundamentals considered here: if $\widetilde{\boldsymbol{f}}_s = \widetilde{\boldsymbol{f}}$ for all $s \geq 1$, equations (B.98) and (34) coincide. Second, we encompass a permanent shock to productivity and amenities in any period $\hat{t} \geq 1$, with $\widetilde{\boldsymbol{f}}_s = \widetilde{\boldsymbol{f}}$ for $s \geq \hat{t}$ and zero before $\hat{t}$. In this case, the dynamic evolution of the state variables in equation (B.98) simplifies to

$$\widetilde{\boldsymbol{x}}_t = \begin{cases} \boldsymbol{R}\widetilde{\boldsymbol{f}}_t + \boldsymbol{P}\widetilde{\boldsymbol{x}}_{t-1} & t \geq \hat{t}, \\ \left(\boldsymbol{\Psi}^{-1}\boldsymbol{\Gamma} - \boldsymbol{P}\right)^{-(\hat{t}-t)} \boldsymbol{R}\widetilde{\boldsymbol{f}} + \boldsymbol{P}\widetilde{\boldsymbol{x}}_{t-1} & t < \hat{t}. \end{cases}$$

Third, and finally, another special case is when the convergent sequence of future shocks to productivity and amenities is characterized by a constant rate of decay, with $\widetilde{\boldsymbol{f}}_{s+1} - \widetilde{\boldsymbol{f}}_s = \eta\left(\widetilde{\boldsymbol{f}}_s - \widetilde{\boldsymbol{f}}_{s-1}\right)$ for all periods $s \geq 1$ and an initial fundamental shock in period one of $\widetilde{\boldsymbol{f}}_1 = (1-\eta)\,\widetilde{\boldsymbol{f}}$. In this case, the long-run change in productivity and amenities in the new steady-state is $\widetilde{\boldsymbol{f}}$, and the dynamic evolution of the state variables in equation (B.98) reduces to

$$\widetilde{\boldsymbol{x}}_t = \left(1 - \eta^{t-1}\right) \boldsymbol{R}\widetilde{\boldsymbol{f}} + (1-\eta)\,\eta^{t-1} \left(\boldsymbol{I} - \eta\left(\boldsymbol{\Psi}^{-1}\boldsymbol{\Gamma} - \boldsymbol{P}\right)\right)^{-1} \boldsymbol{R}\widetilde{\boldsymbol{f}} + \boldsymbol{P}\widetilde{\boldsymbol{x}}_{t-1} \quad \text{for all } t \geq 1. \tag{41}$$

More generally, Proposition 8 allows for an arbitrary convergent sequence of future shocks to productivity and amenities, without restricting the dynamic evolution of these shocks.

3.3 Distributional Consequences of Shocks

The presence of gradual adjustment in the model from migration frictions and capital accumulation has two important implications for the welfare effects of shocks to productivity and amenities. First, these welfare effects depend not only on the change in steady-state, but also on the transition dynamics. Second, there is a distribution of these welfare effects, both across landlords because they are geographically immobile, and across workers because of migration frictions, which imply that a worker's initial location matters for the welfare impact of these shocks.

As our approach provides sufficient statistics for the economy's transition path in response to shocks to fundamentals, it also provides sufficient statistics for the welfare effects of these shocks. In the remainder of this subsection, we illustrate these sufficient statistics for welfare, using changes in migration flows to reveal information about continuation values. In particular, we suppose that the economy starts from steady-state at time $t = 0$, at which point agents become aware of a permanent change in fundamentals $(\widetilde{\boldsymbol{f}})$ at time $t = 1$. Since fundamentals change from time $t = 1$ onwards, the change in workers' welfare at time $t = 0$ is completely determined by the change in the continuation value from their optimal location choice:

$$\widetilde{\boldsymbol{v}}_0 = \beta \boldsymbol{D} \widetilde{\boldsymbol{v}}_1, \tag{42}$$

where this change in continuation value $(\beta \boldsymbol{D} \widetilde{\boldsymbol{v}}_1)$ depends on workers' initial location at time $t = 0$, because of migration frictions, as captured by the outmigration matrix $(\boldsymbol{D})$.

We now show that the expression for population dynamics in equation (27) can be used to infer relative changes in continuation values in response to shocks to fundamentals from these population movements:

$$\widetilde{\boldsymbol{\ell}}_1 = \boldsymbol{E} \widetilde{\boldsymbol{\ell}}_0 + \frac{\beta}{\rho} \left(\boldsymbol{I} - \boldsymbol{E} \boldsymbol{D} \right) \left(\widetilde{\boldsymbol{v}}_1 + \varsigma \right),$$

where the first term $(\boldsymbol{E} \widetilde{\boldsymbol{\ell}}_0)$ is equal to zero, because of our assumption that the economy starts from an initial steady state at time $t = 0$ $(\widetilde{\boldsymbol{\ell}}_0 = \ln \boldsymbol{\ell}_{\boldsymbol{0}} - \ln \boldsymbol{\ell}^{\boldsymbol{*}} = 0)$; the presence of the constant ς reflects the fact that migration decisions depend on relative expected values across locations, and hence are invariant to a common change in expected values across all locations.

To compute the impact on the overall level of welfare, we set this constant equal to the average change in expected values across all locations weighted by population shares $(\boldsymbol{\ell}^{*\prime} \cdot \widetilde{\boldsymbol{v}}_1)$, where we stack the $\boldsymbol{\ell}^{*\prime}$ vector N times into an $N \times N$ matrix $\boldsymbol{L} \equiv [\boldsymbol{\ell}^{*\prime}, \ldots, \boldsymbol{\ell}^{*\prime}]$, such that $\varsigma = -\boldsymbol{L} \widetilde{\boldsymbol{v}}_1$. This convenient choice has two simplifying properties: (i) $\boldsymbol{L}^2 = \boldsymbol{L}$; (ii) $\boldsymbol{L} \boldsymbol{D} = \boldsymbol{L}$, because $\boldsymbol{\ell}^{*\prime}$ is the Perron-eigenvector of $\boldsymbol{D}$.[6] Using these properties, we can re-write the above population dynamics equation as follows:[7]

$$\left(\boldsymbol{I} - \boldsymbol{L} \right) \widetilde{\boldsymbol{v}}_1 = \frac{\rho}{\beta} \left(\boldsymbol{I} - \boldsymbol{E} \boldsymbol{D} + \boldsymbol{L} \right)^{-1} \widetilde{\boldsymbol{\ell}}_1.$$

Combining this result with equation (42), we obtain the following key implication that population movements at time $t = 1$ in response to these shocks to fundamentals are sufficient statistics for their impact on relative expected values for workers in different locations at time $t = 0$:[8]

$$\left(\boldsymbol{I} - \boldsymbol{L} \right) \widetilde{\boldsymbol{v}}_0 = \rho \boldsymbol{D} \left(\boldsymbol{I} - \boldsymbol{E} \boldsymbol{D} + \boldsymbol{L} \right)^{-1} \widetilde{\boldsymbol{\ell}}_1,$$

[6] Since $\boldsymbol{\ell}^{*\prime}$ is the Perron-eigenvector of $\boldsymbol{D}$ and $\boldsymbol{E}$, we have $\boldsymbol{L}\boldsymbol{D} = \boldsymbol{D}\boldsymbol{L} = \boldsymbol{L}\boldsymbol{E} = \boldsymbol{E}\boldsymbol{L} = \boldsymbol{L}$. Since population share sum to one, $\boldsymbol{L} \times \widetilde{\boldsymbol{l}}_1 = \boldsymbol{0}$.

[7] In particular, we use $\left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D} \right) \left(\widetilde{\boldsymbol{v}}_1 - \boldsymbol{L} \widetilde{\boldsymbol{v}}_1 \right) = \left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D} + \boldsymbol{L} \right) \left(\widetilde{\boldsymbol{v}}_1 - \boldsymbol{L} \widetilde{\boldsymbol{v}}_1 \right)$, because $\boldsymbol{L}^2 = \boldsymbol{L}$.

[8] We pre-multiply both sides of equation (42) by $\left(\boldsymbol{I} - \boldsymbol{L} \right)$ and use $\left(\boldsymbol{I} - \boldsymbol{L} \right) \boldsymbol{D} \widetilde{\boldsymbol{v}}_1 = \boldsymbol{D} \left(\boldsymbol{I} - \boldsymbol{L} \right) \widetilde{\boldsymbol{v}}_1$.

where $\boldsymbol{L}\widetilde{\boldsymbol{v}}_0$ is again a constant vector that represents the average change in expected values across all locations weighted by initial population shares, and the right-hand side captures relative changes in expected values across locations, as revealed by the first-period population movements.

Finally, we can connect these first-period population movements ($\widetilde{\boldsymbol{\ell}}_1$) to the productivity ($\widetilde{\boldsymbol{z}}$) and amenity ($\widetilde{\boldsymbol{b}}$) shocks using our closed-form solution for the economy's transition path (37), which yields our sufficient statistic for workers' welfare exposure to these shocks.

Proposition 9. *Consider an economy that is initially in steady-state at time $t = 0$ when agents learn about one-time, permanent shocks to productivity and amenities ($\widetilde{\boldsymbol{f}} = \begin{bmatrix} \widetilde{\boldsymbol{z}} \\ \widetilde{\boldsymbol{b}} \end{bmatrix}$) from time $t = 1$ onwards.*

1. *The relative welfare impact for agents initially in each location at time 0 is*

$$\widetilde{\boldsymbol{v}}_0 - \boldsymbol{L}\widetilde{\boldsymbol{v}}_0 = \rho \boldsymbol{D} \left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D} + \boldsymbol{L}\right)^{-1} \boldsymbol{R}^{\ell} \widetilde{\boldsymbol{f}},$$

where $\boldsymbol{R}^{\ell}$ is the matrix representing the first N rows of $\boldsymbol{R}$.

2. *The average welfare impact on all agents, weighted by initial population shares, is*

$$\boldsymbol{L}\widetilde{\boldsymbol{v}}_0 = \frac{1}{1-\beta}\boldsymbol{L}\left(\underbrace{\begin{bmatrix} \boldsymbol{\mathcal{L}}^{\boldsymbol{z}} & \boldsymbol{\mathcal{L}}^{\boldsymbol{b}} \end{bmatrix} \widetilde{\boldsymbol{f}}}_{\substack{\text{direct effects from} \\ \text{changes in fundamentals}}} + \underbrace{\begin{bmatrix} \boldsymbol{\mathcal{L}}^{\boldsymbol{\ell}} & \boldsymbol{\mathcal{L}}^{\boldsymbol{k}} \end{bmatrix} \left(\boldsymbol{I} - (1-\beta)\left(\boldsymbol{I} - \beta\boldsymbol{P}\right)^{-1}\right)\left(\boldsymbol{I} - \boldsymbol{P}\right)^{-1}\boldsymbol{R}\widetilde{\boldsymbol{f}}}_{\substack{\text{indirect effects from} \\ \text{changes in state variables}}} \right),$$

where $\boldsymbol{\mathcal{L}}^{\boldsymbol{z}}, \boldsymbol{\mathcal{L}}^{\boldsymbol{b}}, \boldsymbol{\mathcal{L}}^{\boldsymbol{\ell}}, \boldsymbol{\mathcal{L}}^{\boldsymbol{k}}$ are matrices representing the multilinear map defined in equation (31).

Proof. See Section B.8.4 of the online appendix. □

In the first part of the proposition, workers' initial location matters for their exposure to these productivity and amenity shocks, because of migration frictions and gradual capital accumulation. In the second part of the Proposition, we derive the sufficient statistic for the average change in welfare across all locations, which incorporates the expectation of all future migration decisions and all future capital accumulation, discounted to the present. This average impact can be decomposed into two components: one that captures the impact of changes in fundamentals (holding constant the state variables of population and the capital stock) and the other that captures the impact of the adjustment in the state variables (through migration and capital accumulation).

4 Extensions

We now consider a number of extensions of our sufficient statistics for spatial dynamics. In Subsection 4.1, we show that our approach naturally accommodates shocks to trade and migration costs. In Subsection 4.2, we allow productivity and amenities to have an endogenous component that reflects agglomeration forces, as well as an exogenous component of fundamentals. In Subsection 4.3, we show that our framework also can be extended to allow for stochastic fundamentals and rational expectations instead of perfect foresight. In Subsection 4.4, we discuss extensions with multiple final goods sectors (Costinot et al. 2012) and input-output linkages (Caliendo and Parro 2015 and Caliendo et al. 2019). In Subsection 4.5, we discuss a number of additional extensions that can be incorporated into our approach, including trade deficits and residential capital (housing).

4.1 Shocks to Trade and Migration Costs

In this subsection, we show that our analysis naturally accommodates shocks to trade and migration costs, where the derivation for all results in this section is reported in Section D.1 of the online appendix. Whereas the productivity and amenity shocks considered in the previous section are common across all partner locations, trade and migration cost shocks are bilateral, which implies that our comparative static results now have a representation as a three tensor. To reduce these three tensors down to a matrix (two tensor) representation, we aggregate bilateral trade and migration shocks across partner locations, using the appropriate weights implied by the model. In particular, we define two measures of outgoing and incoming trade costs, which are trade-share weighted averages of the bilateral trade costs across all export destinations and import sources, respectively. We define *outgoing* trade costs for location i as $\ln \tau_{it}^{\text{out}} \equiv \sum_{n=1}^{N} T_{int} \ln \tau_{nit}$, where the weights are the income share (T_{int}) that location i derives from selling to each export destination n. We define *incoming* trade costs for location n as $\ln \tau_{nt}^{\text{in}} \equiv \sum_{i=1}^{N} S_{nit} \ln \tau_{nit}$, where the weights are the expenditure share (S_{nit}) that location n devotes to each import source i. Similarly, we define *outgoing* migration costs for location i as $\ln \kappa_{it}^{\text{out}} \equiv \sum_{n=1}^{N} D_{int} \ln \kappa_{nit}$, where the weights are the outmigration shares from location i to each destination n. We define *incoming* migration costs for location n as $\ln \kappa_{nt}^{\text{in}} \equiv \sum_{i=1}^{N} E_{nit} \ln \kappa_{nit}$, where the weights are the inmigration shares (E_{nit}) to location n from each origin i. Using these definitions, the system of equations for the model's transition dynamics can be written as:

$$\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t+1}} = \boldsymbol{E}\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \frac{\beta}{\rho}\left(\boldsymbol{I} - \boldsymbol{E}\boldsymbol{D}\right)\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}} - \frac{1}{\rho}\left(\widetilde{\boldsymbol{\kappa}}^{\text{in}} - \boldsymbol{E}\widetilde{\boldsymbol{\kappa}}^{\text{out}}\right), \tag{43}$$

$$\widetilde{\boldsymbol{k}}_{\boldsymbol{t+1}} = \left[\begin{array}{c} \left[\beta\left(1-\delta\right)\boldsymbol{I} + \left(1-\beta\left(1-\delta\right)\right)\left(1-\lambda\right)\boldsymbol{S}\right]\left(\widetilde{\boldsymbol{k}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}\right) - \left(1-\beta\left(1-\delta\right)\right)\widetilde{\boldsymbol{\tau}}^{\text{in}} \\ + \left(1-\beta\left(1-\delta\right)\right)\left(\boldsymbol{I}-\boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \left(1-\beta\left(1-\delta\right)\right)\boldsymbol{S}\widetilde{\boldsymbol{z}} + \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} \end{array}\right], \tag{44}$$

$$\left[\boldsymbol{I} - \boldsymbol{T} + \theta\left(\boldsymbol{I} - \boldsymbol{T}\boldsymbol{S}\right)\right]\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} = \left[\; -\left(\boldsymbol{I}-\boldsymbol{T}\right)\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} + \theta\left(\boldsymbol{I}-\boldsymbol{T}\boldsymbol{S}\right)\left(\widetilde{\boldsymbol{z}} + \left(1-\lambda\right)\left(\widetilde{\boldsymbol{k}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}\right)\right) + \theta\left[\boldsymbol{T}\widetilde{\boldsymbol{\tau}}^{\text{in}} - \widetilde{\boldsymbol{\tau}}^{\text{out}}\right]\;\right], \tag{45}$$

$$\widetilde{\boldsymbol{v}}_{\boldsymbol{t}} = \left[\; \left(\boldsymbol{I}-\boldsymbol{S}\right)\widetilde{\boldsymbol{w}}_{\boldsymbol{t}} + \boldsymbol{S}\widetilde{\boldsymbol{z}} + \left(1-\lambda\right)\boldsymbol{S}\left(\widetilde{\boldsymbol{k}}_{\boldsymbol{t}} - \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}\right) - \widetilde{\boldsymbol{\tau}}^{\text{in}} + \widetilde{\boldsymbol{b}} - \widetilde{\boldsymbol{\kappa}}^{\text{out}} + \beta\boldsymbol{D}\widetilde{\boldsymbol{v}}_{\boldsymbol{t+1}}\;\right], \tag{46}$$

where recall that we use a tilde above a variable to denote a log deviation from the initial steady-state. Note that equations (27)-(30) in our baseline specification above correspond to the special case of equations (43)-(46) in which $\widetilde{\boldsymbol{\tau}}^{in} = \widetilde{\boldsymbol{\tau}}^{out} = \widetilde{\boldsymbol{\kappa}}^{in} = \widetilde{\boldsymbol{\kappa}}^{out} = \boldsymbol{0}$.

4.2 Agglomeration Forces

In this subsection, we generalize our baseline specification from Section 2 to introduce agglomeration forces, where the derivation for all results in this subsection is reported in Section D.2 of the online appendix. We allow productivity and amenities to have both exogenous and endogenous components. The exogenous component captures locational fundamentals, such as climate and access to natural water. The endogenous component captures agglomeration forces and depends on the surrounding concentration of economic activity. Following the standard approach in the economic geography literature, we model these agglomeration forces as constant elasticity functions of a location's own population: $z_{it} = \bar{z}_{it}\ell_{it}^{\eta^z}$ and $b_{it} = \bar{b}_{it}\ell_{it}^{\eta^b}$, where $\eta^z > 0$ and $\eta^b > 0$ parameterize the strength of agglomeration forces for productivity and amenities respectively.[9]

In this extension, the general equilibrium conditions of the model remain the same as in Section 2.6 above, substituting for the terms in productivity and amenities (z_{it} and b_{it}) using the terms in location fundamentals and agglomeration forces ($z_{it} = \bar{z}_{it}\ell_{it}^{\eta^z}$ and $b_{it} = \bar{b}_{it}\ell_{it}^{\eta^b}$). The introduction of these agglomeration forces magnifies the impact of exogenous differences in fundamentals on the spatial distribution of economic activity. Furthermore, depending on the strength of these agglomeration forces, there can either be a unique steady-state equilibrium or multiple steady-state equilibria for the spatial distribution of economic activity. Again we obtain an analytical characterization of the conditions for the existence and uniqueness of the general equilibrium of the model, as summarized in the following proposition.

Proposition 10. ***(A)*** *There exists a unique steady-state spatial distribution of economic activity* $\{w_i^*, v_i^*, \ell_i^*, k_i^*\}$ *(up to a choice of numeraire) given the exogenous fundamentals* $\{\bar{z}_i, \bar{b}_i, \tau_{ni}, \kappa_{ni}\}$ *if the largest absolute value of the following vector of eigenvalues is less than one.*

$$\begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{pmatrix} = \begin{pmatrix} \frac{1-\frac{\beta\eta^b}{\rho}}{\frac{\beta}{\rho}\left(1-\eta^b-\eta^z\theta-\eta^b\lambda\theta\right)+(1+\lambda\theta)} \\ \frac{(1+\lambda\theta)}{\frac{\beta}{\rho}\left(1-\eta^b-\eta^z\theta-\eta^b\lambda\theta\right)+(1+\lambda\theta)} \\ \beta \\ (1-\lambda) \end{pmatrix}.$$

(B) *For sufficiently small agglomeration forces* ($\eta^b + \eta^z\theta + \eta^b\lambda\theta < 1$)*, the largest absolute value of this vector of eigenvalues is necessarily less than one.*

Proof. See Section D.2 of the web appendix. ☐

In general, from part (A) of Proposition 10, whether there is an unique steady-state in the model depends on the strength of agglomeration forces (η^z, η^b), trade elasticity (θ), capital intensity (λ), the discount rate (β) and the dispersion of idiosyncratic preferences (ρ). From part (B) of Proposition 10, a sufficient condition for the existence of a unique steady-state is that the agglomeration forces (η^z, η^b) are sufficiently small. As these parameters converge towards zero, we obtain our baseline specification without agglomeration forces, in which there is necessarily a unique steady-state equilibrium, as shown in Section 2.6 above. Additionally, an increase in the dispersion of idiosyncratic preferences (a larger value for ρ) necessarily increases the range of values for the other parameters for which a unique steady-state equilibrium exists, as shown in the proof of the proposition.

After making the appropriate adjustments for the additional agglomeration parameters in the model's general equilibrium conditions in Section 2.6 above, we obtain analogous steady-state elasticities of the endogenous variables with respect to shocks to productivity and amenities, as in Proposition 2 in our baseline specification

[9]Although for simplicity we assume that agglomeration and dispersion forces only depend on a location's own population, our framework can be further generalized to incorporate spillovers across locations, as in Ahlfeldt et al. (2015) and Allen et al. (2020). While we focus on agglomeration forces ($\eta^z > 0$ and $\eta^b > 0$), it is straightforward to also allow for additional dispersion forces ($\eta^z < 0$ and $\eta^b < 0$).

above. Similarly, the transition dynamics of the model can be characterized in terms of an analogous transition matrix ($\boldsymbol{P}$) and impact matrix ($\boldsymbol{R}$), as in Proposition 3 in our baseline specification above.

4.3 Stochastic Location Fundamentals and Rational Expectations

In our baseline specification, we assume that agents have perfect foresight for all location characteristics, except for one-time unanticipated (MIT) shocks to the sequence of fundamentals. In this subsection, we generalize our analysis to allow for stochastic location fundamentals and rational expectations. For simplicity, we focus on stochastic productivity and amenities, although we can also incorporate stochastic trade and migration costs following a similar approach to that used in Subsection 4.1 above. We allow these shocks to productivity and amenities to affect the steady-state values of the endogenous variables of the model, by assuming that changes in productivity and amenities evolve stochastically over time according to the following AR(1) structure:

$$\begin{aligned} \ln z_{it+1} - \ln z_{it} &= \rho^{z}\left(\ln z_{it} - \ln z_{it-1}\right) + \varpi_{it}^{z}, \qquad && \left|\rho^{z}\right| < 1, \\ \ln b_{it-1} - \ln b_{it} &= \rho^{b}\left(\ln b_{it} - \ln b_{it-1}\right) + \varpi_{it}^{b}, \qquad && \left|\rho^{b}\right| < 1, \end{aligned} \tag{47}$$

where ϖ_{it}^{z} and ϖ_{it}^{b} are mean zero and independently and identically distributed innovations.

In our baseline specification with perfect foresight, when workers choose where to locate at the end of period t, they know the future path of all fundamentals (except for an unanticipated MIT shock), as well as the realization of the idiosyncratic mobility shock. In contrast, in our generalization here, when workers choose where to locate at the end of period t, we assume that they observe the productivity and amenity shocks for period $t+1$ ($\{z_{it+1}\}$, $\{b_{it+1}\}$), as well as the realization of the idiosyncratic mobility shock, and form rational expectations about the future path of fundamentals. Using these assumptions, our characterization of the general equilibrium of the model remains exactly the same as in our baseline specification in Section 2 above, except that the expected continuation value ($\mathbb{E}\left[\mathbb{V}_{gt+1}^{w}\right]$) now depends on expectations about future location characteristics rather than known values of these location characteristics under perfect foresight. As the linearization methods that we use to derive our closed-form solution for the model's transition dynamics in Section 3.2.1 above are taken from the Dynamic Stochastic General Equilibrium (DSGE) literature in macroeconomics, they apply with stochastic location fundamentals, and we can adapt our earlier results to compute the entire transition path of the *expected* value of the models' endogenous variables.

In particular, upon observing fundamental shocks at time t, agents in the economy no longer expect future fundamental shocks to be zero. Instead, given the AR(1) structure for changes in fundamentals in equation (47) and rational expectations, they expect future shocks to fundamentals to decay to zero over time:

$$\begin{aligned} \mathbb{E}_{t}\left[\widetilde{z}_{it+s} - \widetilde{z}_{it+s-1}\right] &= \left(\rho^{z}\right)^{s}\left(\widetilde{z}_{it} - \widetilde{z}_{it-1}\right), \\ \mathbb{E}_{t}\left[\widetilde{b}_{it+s} - \widetilde{b}_{it+s-1}\right] &= \left(\rho^{b}\right)^{s}\left(\widetilde{b}_{it} - \widetilde{b}_{it-1}\right), \end{aligned} \tag{48}$$

where $\mathbb{E}_{t}\left[\cdot\right]$ is the expectation conditional on the realizations of shocks up to time t; and we continue to use a tilde above a variable to denote a log deviation relative to the initial steady-state.

Using this property, we can extend Proposition 8 for the economy's transition path under perfect foresight to obtain the economy's expected transition path under stochastic fundamentals and rational expectations. In particular, consider an economy that is initially in steady-state at time $t = 0$, and suppose that agents observe realizations $\widetilde{\boldsymbol{f}}_1$ for shocks to productivity and amenities at time $t = 1$. Under our assumed shock process (47), we obtain the following closed-form solution for the expected transition path of the economy's state variables:

$$\mathbb{E}_1\left[\widetilde{\boldsymbol{x}}_t\right] = \sum_{s=t+1}^{\infty}\left(\boldsymbol{\Psi}^{-1}\boldsymbol{\Gamma} - \boldsymbol{P}\right)^{-(s-t)}\boldsymbol{R}\left(\mathbb{E}_1\left[\widetilde{\boldsymbol{f}}_s - \widetilde{\boldsymbol{f}}_{s-1}\right]\right) + \boldsymbol{R}\mathbb{E}_1\left[\widetilde{\boldsymbol{f}}_t\right] + \boldsymbol{P}\mathbb{E}_1\left[\widetilde{\boldsymbol{x}}_{t-1}\right] \qquad \text{for all } t \geq 1, \quad (49)$$

with initial condition $\widetilde{\boldsymbol{x}}_0 = \boldsymbol{0}$ and where $\boldsymbol{\Psi}, \boldsymbol{\Gamma}$ are matrices from Proposition 8 and are derived in Section B.8.4 of the online appendix.

Using our assumed shock process (47), we can also generalize Proposition 2 to obtain an analogous closed-form solution for changes in the expected steady-state values of variables in response to fundamental shocks, recognizing that the expectations at time t of productivities and amenities at a future time $(t + s)$ converge (as $s \to \infty$) to limits that depend on the current levels and changes in fundamentals:

$$\mathbb{E}_t\left[\ln z_i^*\right] \equiv \lim_{s\to\infty}\mathbb{E}_t\left[\ln z_{it+s}\right] = \ln z_{it} + \frac{\rho^z}{1-\rho^z}\left(\ln z_{it} - \ln z_{it-1}\right), \quad (50)$$

$$\mathbb{E}_t\left[\ln b_i^*\right] \equiv \lim_{s\to\infty}\mathbb{E}_t\left[\ln b_{it+s}\right] = \ln b_{it} + \frac{\rho^b}{1-\rho^b}\left(\ln b_{it} - \ln b_{it-1}\right).$$

Therefore, conditional on known initial realizations for shocks to productivities and amenities, it is straightforward to implement our closed-form solutions, and characterize the economy's expected future transition path with stochastic fundamentals and rational expectations. In our empirical analysis below, we invert the full non-linear model to recover productivities and amenities in each location. Undertaking this model inversion with stochastic fundamentals and rational expectations is substantially more complicated, because of the difference between realized and expected values of fundamentals, without much additional insight. Therefore, for the recovery of shocks to fundamentals from the full non-linear model, we focus on the perfect foresight case, as in the existing quantitative spatial literature.

4.4 Multi-sectors and Input-Output Linkages

Our sufficient statistics extend naturally to environments with multiple final goods sectors and with input-output linkages. We briefly discuss these extensions here and formally derive them in the online appendix.

As in our baseline specification in the previous section, we assume that capital is geographically immobile across locations once installed. For the multi-sector model, we consider two different assumptions about the mobility of installed capital across sectors within locations. In Section D.3 of the online appendix, we assume that installed capital is specific to a location, but mobile across sectors within locations. In Section D.4 of the online appendix, we assume installed capital is specific to both a location and a sector. This second specification corresponds to a dynamic spatial version of the traditional specific-factors model from the international trade literature, in which there are migration frictions for the mobile factor across locations and sectors, and there is endogenous accumulation of the factor specific to each location and sector over time. In both cases, we follow Costinot et al. (2012) and assume Cobb-Douglas preferences across sectors and CES preferences across the good supplied by each location within sectors. In Section D.5 of the online appendix, we further generalize these specifications to allow for input-ouput linkages, where the production technology in each sector now uses the two primary factors of labor and capital together with intermediate inputs according to a Cobb-Douglas functional form, as in Caliendo and Parro (2015).

In all of these specifications, consumption, production, trade and migration are modelled in a similar way as in our baseline, single-sector specification. The set of equilibrium conditions on value function (20), population flow (18), and goods market clearing (16) extends naturally from describing each location to describing each location-industry, and the capital market clearing and accumulation condition (14) extends depending on the capital mobility specification. We obtain analogous steady-state elasticities of the endogenous variables with respect to productivity and amenities shocks to those in Proposition 2, and a similar closed-form linear solution for the model's transition dynamics to that in Proposition 3. Again these sufficient statistics for the impact of shocks to fundamentals on the spatial distribution of economic activity depend only on the observed trade and migration share matrices $\{\boldsymbol{S}, \boldsymbol{T}, \boldsymbol{D}, \boldsymbol{E}\}$, the initial values of the state variables $\{\boldsymbol{\ell^j}, \boldsymbol{k^j}\}$, and the model's structural parameters $\{\theta, \beta, \rho, \lambda^j, \delta\}$.

In these multi-sector extensions, the trade and migration share matrices are defined from location-sector to location-sector. In addition, a key difference in the input-output specification is that the expenditure share ($\boldsymbol{S}$) and income share ($\boldsymbol{T}$) matrices must be adjusted to take into account the network structure of production: the gross value of trade from exporter i to importer n in industry k includes not only the direct value-added created in this exporter and industry but also the indirect value added created in previous stages of production, and that the effect of a foreign productivity shock in one country on any other country now differs depending on the extent to which it reduces input prices (and hence production costs) or reduces output prices.

4.5 Other Extensions

Our approach also accommodates a number of other extensions and generalizations. In Section D.6 of the online appendix, we incorporate trade deficits following the conventional approach of the quantitative international trade literature in treating these deficits as exogenous. In Section D.7 of the online appendix, we allow capital to be used residentially (for housing) as well as commercially (in production). In each case, we derive analogous steady-state elasticities with respect to shocks to fundamentals to those in Proposition 2 for our baseline specification, and a similar closed-form linear solution for the model's transition dynamics to that in Proposition 3 for our baseline specification.

5 Quantitative Analysis

In this section, we report our main empirical results for the dynamics of the spatial distribution of economic activity across U.S. states from 1965-2015. We choose U.S. states as our spatial units, because of the availability of data on bilateral shipments of goods, bilateral migration flows and capital stocks over this long historical time period, and because of the substantial changes in the observed distribution of economic activity across states over time. For the same reasons, we focus for most of our empirical analysis on a version of our baseline single-sector model, augmented to take account of the empirically-relevant distinction between traded and non-traded goods.[10] To examine the extent to which sectoral specialization influences the exposure of states to shocks, we also implement our multi-sector extension with location-sector-specific capital from Section 4.4 above for the shorter time period from 1999-2015 for which the sector-level data are available.

In Subsection 5.1, we introduce our data sources and definitions. In Subsection 5.2, we provide reduced-form evidence on the substantial reorientation of economic activity that occurred across U.S. states over our sample period, including the decline of the Rust Belt and rise of the Sun Belt. In Subsection 5.3, we invert the full non-linear model to recover the shocks to productivity, amenities, trade costs and migration frictions implied by the observed state variables ($\boldsymbol{\ell}_t$, $\boldsymbol{k}_t$) and the trade and migration share matrices ($\boldsymbol{S}$, $\boldsymbol{T}$, $\boldsymbol{D}$, $\boldsymbol{E}$). Using this empirical distribution of shocks, we show that our linearization provides a close approximation to the full non-linear solution of the model, even for productivity and amenity shocks of the magnitude implied by the observed data. In contrast to the full non-linear model solution, we have an analytical characterization of our linearization, which we use to explore the dynamic response of the economy to shocks.

In Subsection 5.4, we provide evidence on the relative contributions of convergence to steady-state versus shocks to fundamentals in explaining the observed evolution of economic activity across U.S. states. We solve for the unobserved steady-state distribution of economic activity across U.S. states in each year. We evaluate the speed of convergence towards this unobserved steady-state, and examine the roles of capital and labor dynamics in determining this speed of convergence. In Subsection 5.5, we undertake a spectral analysis to provide evidence on the heterogeneous impact of shocks, both across locations and across different types of shocks. In Subsection 5.6, we report the results of our multi-sector extension, and provide evidence on the role of sectors in shaping the speed of convergence to steady-state and the heterogeneous impact of shocks.

5.1 Data

Our main source of data for our baseline quantitative analysis from 1965-2015 is the national economic accounts of the Bureau of Economic Analysis (BEA), which report population, gross domestic product (GDP) and the capital stock for each U.S. state.[11] We focus on the 48 contiguous U.S. states plus the District of Columbia, excluding Alaska and Hawaii, became they only became U.S. states in 1959 close to the beginning of our sample period, and could be affected by idiosyncratic factors as a result of their geographical separation. We deflate GDP and the capital stock to express them in constant (2012) prices. We construct bilateral five-year migration flows between U.S. states from the U.S. population census from 1960-2000 and from the American Community Survey (ACS) after 2000. We interpolate between census decades to obtain five-year migration flows for each year of our sample period. We construct the value of bilateral shipments between U.S. states from the Commodity Flow Survey (CFS) from 1993-2017 and the Commodity Transportation Survey (CTS) for 1977. We again interpolate between reporting years and extrapolate the data backwards in time before 1977 using relative changes in the income of origin and destination states, as discussed in further detail in Section H of the online appendix. For our baseline quantitative analysis with a single traded and non-traded sector, we abstract from direct shipments to and from foreign countries, because of the relatively low level of U.S. trade openness,

[10]Therefore, our single-sector model in our empirical implementation features a single traded sector and a single non-traded sectors, as developed in detail in Section E of the online appendix.

[11]For further details on the data sources and definitions, see the data appendix in Section H of the online appendix.

particularly towards the beginning of our sample period.

For our multi-sector extension from 1999-2015, we construct data for the 48 contiguous U.S. states, 22 foreign countries and 19 economic sectors, yielding a total of 1,520 region-sector combinations, where a region is either a U.S. state or a foreign country. We allow for trade across all region-sectors, and for migration across all U.S. states and sectors. We obtain sector-level data on value added, employment and the capital stock for each U.S. state from the national economic accounts of the Bureau of Economic Analysis (BEA). We construct migration flows between U.S. states in each sector by combining data from the U.S. population census, American Community Survey (ACS), and Current Population Survey (CPS), as discussed in further detail in Section H of the online appendix. We use the value of bilateral shipments between U.S. states in each sector from the Commodity Flow Survey (CFS), interpolating between the years for which the data are reported. We measure foreign trade for each U.S. state and sector using the data on foreign exports by origin of movement (OM) and foreign imports by state of destination (SD) from the U.S. Census Bureau.[12] For each foreign country and sector, we obtain data on value added, employment and the capital stock from the World Input-Output Tables (WIOT).

5.2 Reduced-Form Evidence on the Rust and Sun-Belt

One the most striking features of economic activity in the United States over our sample period is its reorientation away from the "Rust Belt" in the mid-west and north-east towards the "Sun Belt" in the south and west. Although we implement our quantitative analysis for U.S. states, we begin by reporting some aggregate results for four groupings of states to illustrate this large-scale reorientation. Following Alder et al. (2019), we define the Rust Belt as the states of Illinois, Indiana, Michigan, New York, Ohio, Pennsylvania, West Virginia and Wisconsin, and the Sun Belt as the states of Arizona, California, Florida, New Mexico and Nevada. We group the remaining states into two categories to capture longstanding differences between the North and South: Other Southern States, which includes all former members of the Confederacy, except those in the Sun Belt; and Other Northern States, which comprises all the Union states from the U.S. Civil War, except those in the Rust Belt or Sun Belt.[13]

In Figure 1, we display the shares of these four groups of states in the U.S. population over time. As shown in the top-left panel, the Rust Belt exhibits by far the largest decline in population share, which falls by around 8 percentage points from 24.2-16.5 percent from 1965-2015. As shown in the top-right panel, the Sun Belt displays the largest increase in population share, which rises by around 5 percentage points from 9.3-14.7 percent over the same period. In contrast, the trends in population shares for the other two groups of states are much flatter. The population share of Other Northern States falls by 1.6 percentage points from 18.8-17.2 percent, while that for Other Southern States rises by 2.8 percentage points from 14.4-17.2 percent.

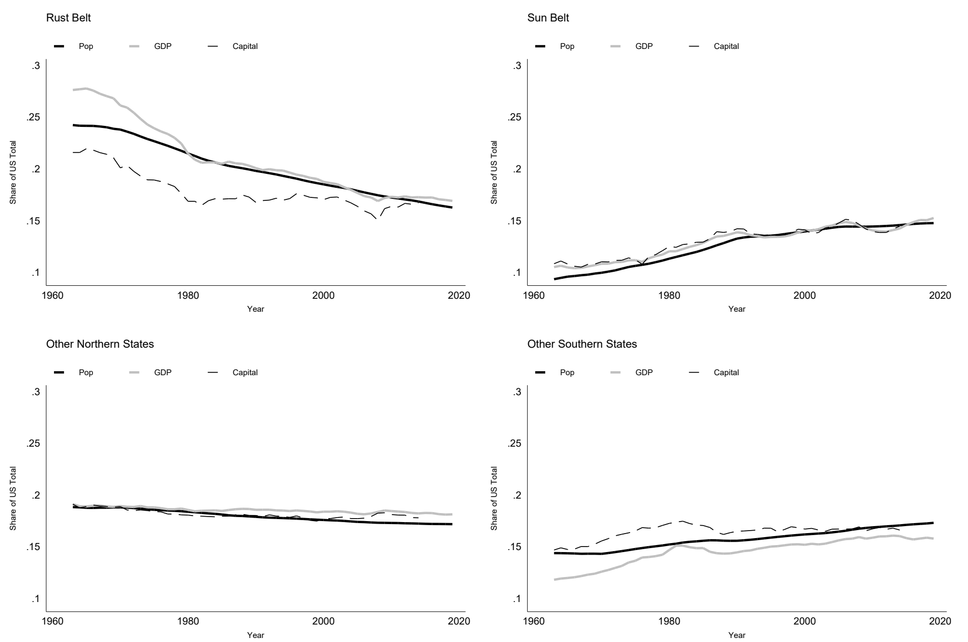
Figure 1 also shows the corresponding shares of these four groups of states in real GDP and the real cap-

[12]The Census Bureau constructs these data from U.S. customs transactions, aiming to measure the origin of the movement of each export shipment and the destination of each import shipment. Therefore, these data differ from measures of exports and imports constructed from port of exit/entry, and from the data on the exports of manufacturing enterprises (EME) from the Annual Survey of Manufactures (ASM). See https://www.census.gov/foreign-trade/aip/elom.html and Cassey (2009).

[13]Therefore, "Other Southern" includes Alabama, Arkansas, Georgia, Louisiana, Mississippi, North Carolina, South Carolina, Tennessee, Texas, and Virginia. "Other Northern" includes Colorado, Connecticut, Delaware, District of Columbia, Idaho, Iowa, Kansas, Kentucky, Maine, Maryland, Massachusetts, Minnesota, Missouri, Montana, Nebraska, New Hampshire, New Jersey, North Dakota, Oklahoma, Oregon, Rhode Island, South Dakota, Utah, Vermont, Washington and Wyoming.

ital stock in the United States over time.[14] We find that GDP and capital stock shares show some differences from population shares, highlighting the potential role for capital accumulation and productivity growth in understanding the observed reorientation of economic activity. In the Rust Belt, we find that capital and GDP shares fall more rapidly than population in the 1960s and 1970s. In contrast, in the Sun Belt, GDP and capital shares lie above population shares from the early 1960s to the 1990s, before population shares ultimately converge towards them. In Other Northern States, population shares fall marginally below GDP and capital shares from the mid-1980s onwards. Finally, in Other Southern States, GDP and capital shares rise substantially more sharply than population shares in the 1960s and 1970s, consistent with a role for income convergence.

Figure 1: Shares of Population, Gross Domestic Product (GDP) and the Capital Stock in the United States over Time



Rust Belt: Illinois, Indiana, Michigan, New York, Ohio, Pennsylvania, West Virginia and Wisconsin. Sun Belt: Arizona, California, Florida, New Mexico and Nevada. Other Southern all other former members of the Confederacy. Other Northern all other Union states during the Civil War

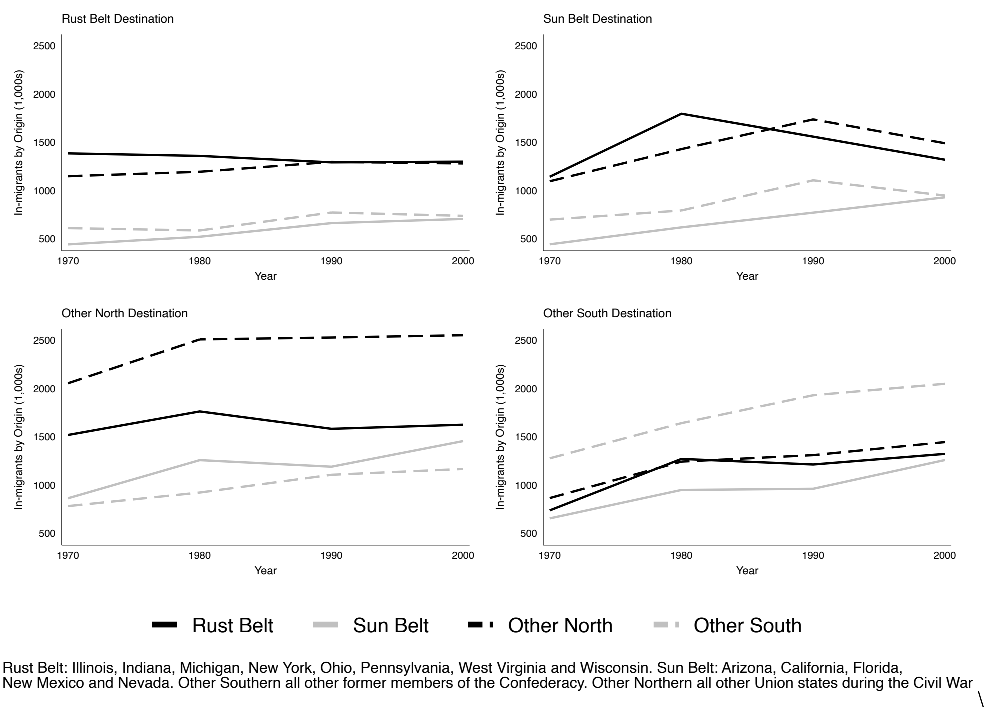
Notes: Shares of total population, real gross domestic product (GDP) and the real capital stock in the United States data from the national economic accounts of the Bureau of Economic Analysis (BEA). Real GDP and capital stock in 2012 prices.

In principle, the observed changes in population shares in Figure 1 can be explained by either internal migration, differences in fertility or international migration. In Figure 2, we provide evidence on internal migration for the four groups of states, where internal migration includes movements of people within the United States, and excludes international migration. We focus on internal in-migration, measured as inflows of people (in thousands) into each destination region, separated out by origin region. Three features are noteworthy. First, geographical proximity matters for migration flows, such that other Rust Belt states are one of the leading sources of in-migrants in the Rust Belt (top-left panel), consistent with our model's gravity equation predictions. Second, all groups of states receive non-negligible in-migration flows, such that gross migration flows are larger than net migration flows, in line with the idiosyncratic mobility shocks in our model. Third, despite the

[14] We find similar patterns whether we use real or nominal shares of GDP and the capital stock.

role for geography, the Rust Belt and Other Northern states are the two largest sources of in-migrants for the Sun Belt, consistent with internal migration contributing to the observed reorientation of population shares. Finally, although not shown in these figures, we find a modest decline in rates of internal migration between states in the later years of our sample, which is in line the findings of a number of studies, including Kaplan and Schulhofer-Wohl (2017) and Molloy et al. (2011). Consistent with the comparison of several different sources of administrative data in Hyatt et al. (2018), we find that this decline in rates of internal migration between states is smaller in the population census data than in Current Population Survey (CPS) data.

Figure 2: In-migrants for each Destination Region by Origin Region over Time



Notes: Internal in-migration to each destination region by source region from the population census (for 1960-2000) and the American Community Survey (ACS) (for 2000); internal migration includes all movements of people within the United States and excludes international migration; in-migration to each destination region excludes internal movements within U.S. states.

5.3 Quality of the Approximation

Although our linearization is only exact for small changes, we now show that it provides a close approximation to the full nonlinear solution of the model for changes in productivity and amenities of the magnitude implied by the observed data. First, we invert the non-linear model to recover the unobserved changes in productivity, amenities, trade costs and migration frictions (z_{it}, b_{it}, τ_{nit}, κ_{git}) implied by the observed changes in the state variables (ℓ_{it}, k_{it}) and the trade and migration shares (S_{nit}, T_{int}, D_{igt}, E_{git}). Second, we undertake counterfactuals for changes in steady-states in response to the empirical productivity and amenity shocks, comparing our linearization to the full non-linear model solution. Third, we undertake counterfactuals for transition dynamics in response to these empirical productivity and amenity shocks, again comparing our linearization to the full non-linear model solution. Finally, we compare the computational performance of the two approaches.

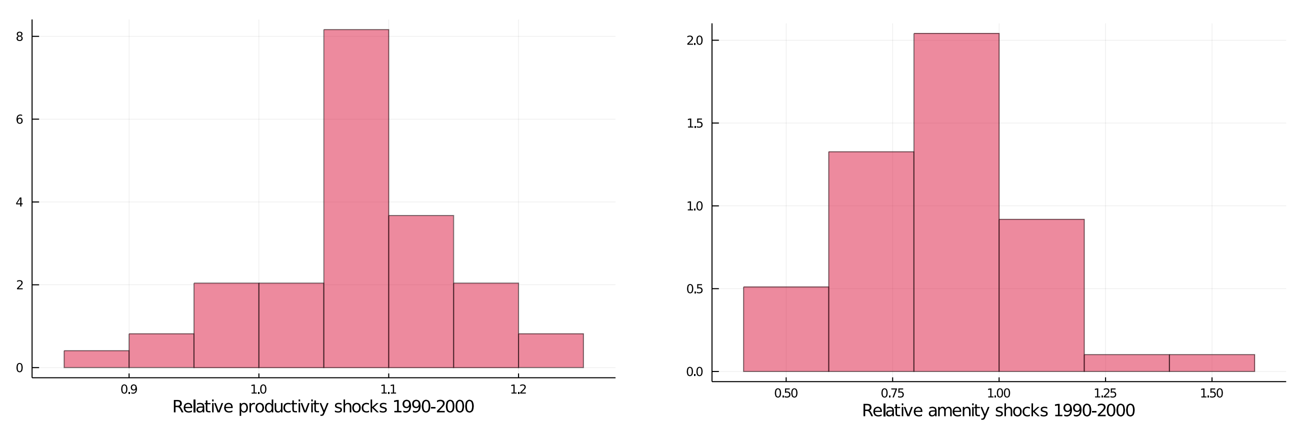
Empirical Distribution of Fundamental Shocks. We start by inverting the non-linear model to recover the unobserved changes in fundamentals (z_{it}, b_{it}, τ_{nit}, κ_{git}) implied by the observed changes in the data. Using our assumption of perfect foresight, we recover these changes in fundamentals, without making assumptions about where the economy lies on the transition path to steady-state or about the particular path of fundamentals, as shown in Section F of the online appendix. We choose periods in the model to correspond to five-year frequencies in the data to match our five-year migration matrices. As our model inversion requires data for two subsequent time periods, we can recover these unobserved changes in fundamentals from 1965-2015, using our data from 1960 to 2015.

We choose central values for the model's structural parameters from the existing empirical literature. We assume a trade elasticity of $\theta = 5$, as in Costinot and Rodríguez-Clare (2014). We set the 5-year discount rate equal to the conventional value of $\beta = 0.95^5$. We assume a value for the migration elasticity of $\rho = 3\beta$, which is in line with the values in Bryan and Morten (2019), Caliendo et al. (2019) and Fajgelbaum et al. (2019). We set the share of labor in value added to $\lambda = 0.65$, as a central value in the macro literature. We assume a five percent annual depreciation rate, such that the 5-year depreciation rate is $\delta = 1 - 0.95^5$, which is again a conventional value in the macro and productivity literatures.

In the left and right panels of Figure 3, we show the empirical distributions of relative changes in productivity ($\widehat{z}_i = z_{i2000}/z_{i1990}$) and amenities ($\widehat{b}_i = b_{i2000}/b_{i1990}$) across U.S. states for 1990-2000 from our model inversion. We find that relative changes in productivity and amenities are clustered around their geometric mean of one, although individual states can experience substantial changes in relative productivity and amenities over a period of a decade. In Section G.1 of the online appendix, we provide further evidence that we find an intuitive pattern of changes in productivity, amenities, bilateral trade costs, and bilateral migration frictions.

Figure 3: Relative Productivity and Amenity Shocks from 1990-2000 from our Model Inversion

(a) Relative Productivity Shocks ($\widehat{z}_i = z_{i2000}/z_{i1990}$) (b) Relative Amenity Shocks ($\widehat{b}_i = b_{i2000}/b_{i1990}$)

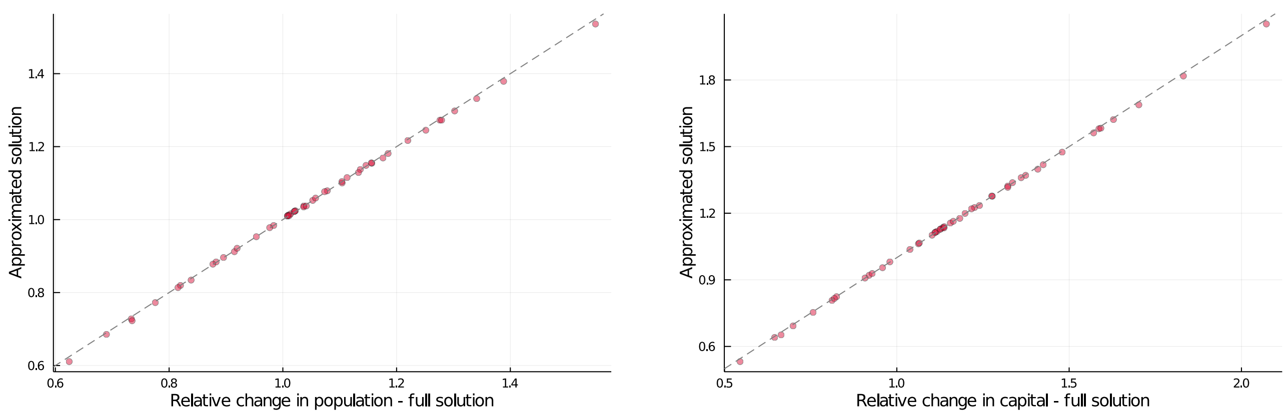


Note: Histograms of the distributions of relative changes in productivity ($\widehat{z}_i = z_{i2000}/z_{i1990}$) and amenities ($\widehat{b}_i = b_{i2000}/b_{i1990}$) from 1990-2000 from our model inversion, as discussed in Section F of the online appendix. Relative changes in productivity ($\widehat{z}_i = z_{i2000}/z_{i1990}$) and amenities ($\widehat{b}_i = b_{i2000}/b_{i1990}$) normalized to have a geometric mean of one.

Steady-state Approximation. To examine the quality of our approximation for steady-state changes, we first use Proposition 3 and equation (36) to solve for the steady-state values of the state variables $\{\ell_i^*, k_i^*\}$ implied by the 1990 values of the fundamentals recovered from our model inversion $\{z_i, b_i, \tau_{ni}, \kappa_{ni}\}$. Starting from this implied steady-state, we next undertake counterfactuals using the empirical distribution of relative changes in productivity from 1990-2000. We compare the predicted changes in steady-state from our linearization in Proposition 2 to those from solving the full non-linear model using the dynamic exact-hat algebra approach. Despite the substantial changes in relative productivity implied by the observed data, we find that our linearization provides a close approximation to the full non-linear model solution, as shown for population in Figure 4a and capital in Figure 4b. Regressing the two sets of counterfactual predictions for population changes on one another, we find a regression slope of 1.003 and a coefficient of correlation of 0.999. We find a similar pattern of results for the response of both state variables to amenity shocks.[15] Taken together, these results suggest that our linearization is not only exact for small changes, but provides a close approximation to the full non-linear model solution for the empirical distribution of changes in location fundamentals.

Figure 4: Steady-State Predictions of Our Approximation Versus the Full Non-Linear Solution for Counterfactual Changes in Productivity

(a) Counterfactual Changes in Population (b) Counterfactual Changes in Capital



Note: We first solve for the steady-state values of the state variables $\{\ell_i^*, k_i^*\}$ implied by the 1990 values of the fundamentals recovered from our model inversion $\{z_i, b_i, \tau_{ni}, \kappa_{ni}\}$. We next undertake counterfactuals using the empirical distribution of relative changes in productivity over the period 1990-2000 from our model inversion, as discussed in Section F of the online appendix. We compare the predicted changes in the steady-state state variables from our linearization in Proposition 2 to those from the full non-linear model solution using the dynamic exact-hat algebra approach.

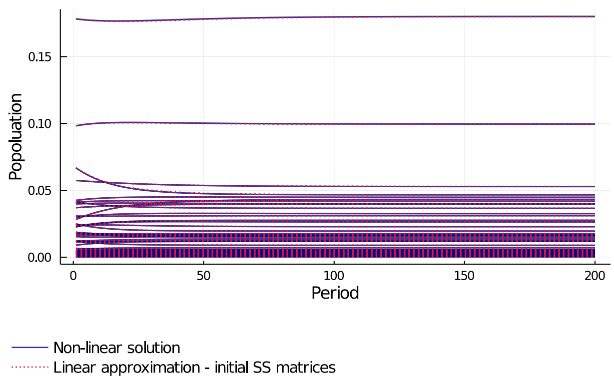
Transition Path Approximation. To examine the quality of our approximation for the transition path, we follow a similar approach. We first use Proposition 3 and equation (36) to solve for the steady-state values of the state variables $\{\ell_i^*, k_i^*\}$ implied by the 1990 values of the fundamentals recovered from our model inversion $\{z_i, b_i, \tau_{ni}, \kappa_{ni}\}$. Starting from this implied steady-state, we next undertake counterfactuals using the empirical distribution of relative changes in productivity from 1990-2000. We compare the predicted transition path from our linearization in Proposition 3 to that from the full non-linear model solution using the dynamic exact-hat algebra approach. Remarkably, we find that our linearization provides a close approximation to the full non-linear model solution along the entire transition path. This approximation is somewhat better for population shares (Figure 5a) than for population relative to the initial steady-state (Figure 5b), but remains close in both

[15]For amenity shocks, regressing the two sets of counterfactual changes in population on one another, we find a regression slope of 1.067 and a coefficient of correlation of 0.995.

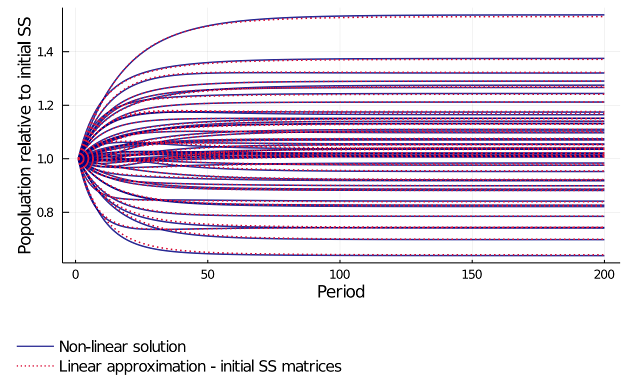
cases. Even over time periods of more than one hundred years (recall that we choose a five-year frequency in the model to match our observed migration matrices), we find that our linearization continues to track the full non-linear model solution at each point along the transition path. We find a similar pattern of results for the capital stock and for the response of both state variables to amenity shocks. Therefore, our linearization is not only exact for small changes, but provides a close approximation to the dynamic response of the spatial economy for the empirical distribution of changes in location fundamentals.

Figure 5: Transition Path Predictions of Our Approximation Versus the Full Non-Linear Solution for Counterfactual Changes in Productivity

(a) Transition Path for Population Shares



(b) Transition Path for Population Relative to Initial Steady-State Levels



Note: We first solve for the steady-state values of the state variables $\{\ell_i^*, k_i^*\}$ implied by the 1990 values of the fundamentals recovered from our model inversion $\{z_i, b_i, \tau_{ni}, \kappa_{ni}\}$. We next undertake counterfactuals using the empirical distribution of relative changes in productivity over the period 1990-2000 from our model inversion, as discussed in Section F of the online appendix. We compare the predicted transition path in the state variables from our linearization in Proposition 3 to those from the full non-linear model solution using the dynamic exact-hat algebra approach. Each period in the figure corresponds to a five-year period as in our observed five-year migration matrices.

Computational Speed. In comparing our (first-order) linearization to the conventional dynamic exact-hat algebra, another relevant dimension is computational efficiency. Our linearization involves a single matrix inversion and diagonalization, which yields a closed-form solution for the elasticity of the endogenous variables in each location in each time period with respect to a shock in any location. The main advantage of this approach is that we can exploit the transition matrix $\boldsymbol{P}$ to provide an analytical characterization of the economy's transition path, including the speed of convergence and heterogeneous impact of shocks. A secondary advantage is that we can use our closed-form solutions to evaluate (to first-order) any number of counterfactuals for different shocks in different locations. In contrast, using conventional methods, one must solve each counterfactual separately using a computationally costly shooting algorithm. This shooting algorithm involves first guessing the entire transition path for population and the capital stock towards the new steady-state, and then iterates over subsequent updates of this entire transition path.

In Table 1, we compare computation speed for the two approaches on a contemporary laptop for different numbers of states (N), based on randomly sampling state-sectors from our multi-sector extension. In our application, we are interested in bilateral exposure to productivity and amenity shocks. With N locations,

Table 1: Relative Computation Speed in Seconds for Our Linearization and the Full Non-Linear Model Solution

Number of states	Linearization	Non-linear Solution		
		1 Counterfactual	N Counterfactuals	2^N Counterfactuals
40	0.01	10	390	1.07177E+13
80	0.04	83	6606	9.98321E+25
160	0.15	242	38646	3.53005E+50
320	0.61	517	165319	1.1035E+99
640	4.29	2375	1520256	1.0838E+196

Note: Relative computation speed in seconds on a contemporary laptop for different numbers of states, based on randomly sampling location-sectors from our multi-sector extension, and using the empirical distribution of productivity shocks. In our linearization, we obtain a closed-form solution for the steady-state and the transition path from a single matrix inversion and diagonalization, which enables the bilateral impact of all combinations of productivity and amenity shocks in each period of time to be computed. In contrast, solving the full non-linear model solutions involves N counterfactuals to evaluate a productivity shock in each location separately, and 2^N counterfactuals to evaluate all possible combinations of productivity shocks across locations.

computing bilateral exposure involves N counterfactuals shocking each location separately, or 2^N counterfactuals if one considers shocks to each possible combination of locations. In our baseline single sector model, we have $N = 49$ locations in each year, while in our multi-sector extension, we have $N \approx 1,500$ location-sectors in each year. Across all numbers of state-sectors and counterfactuals, we find that our linearization is notably faster than the full non-linear solution, with the difference in computation time increasing with the state space. For a single counterfactual and only 40 states, this difference in computation speed is not too burdensome, since solving the full non-linear model solution still takes only 10 seconds, compared to 0.01 seconds for our linearization. With N counterfactuals and 640 states, the full non-linear model solution takes approximately 1,520,256 seconds or 25,338 hours, compared to 4.29 seconds for our linearization. In some empirical settings, only a few counterfactuals may be required. In contrast, in applications such as ours that involve a large number of counterfactuals, the gains in computational efficiency can be of practical relevance.

5.4 Convergence to Steady-state and Fundamental Shocks

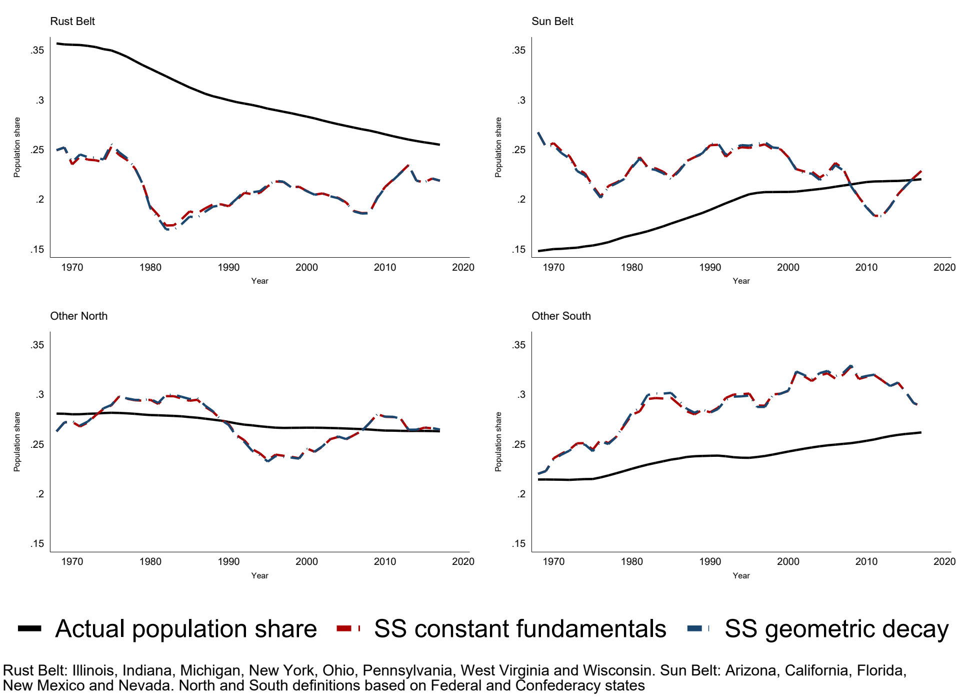
We now use our closed-form solution for the economy's transition path from Section 3.2 above to provide evidence on the role of shocks to fundamentals versus convergence to steady-state in explaining the observed evolution of economic activity across U.S. states over our sample period.

First, we compute the implied steady-state levels of economic activity for each U.S. state and year, given observed state variables and trade and migration share matrices, using Proposition 3. We compute these implied steady-states for each year t under two different assumptions about the future path of fundamentals: (i) no further changes in fundamentals for years $s > t$, as in equation (36); (ii) future changes in productivity and amenities for years $s > t$ follow geometric decay given the initial changes in these fundamentals between periods t and $t-1$, as in equation (41). This second specification corresponds to rational expectations assuming that fundamentals evolve according to the AR(1) process in equation (47), except that we recover fundamentals and estimate the autoregressive coefficients ρ^z and ρ^b, using our model inversion from Section F of the online appendix, under our assumption of perfect foresight.

In Figure 6, we summarize the state-level results by displaying actual and steady-state population shares for each of our four groupings of states. The black line shows the actual data for each year; the dashed red line shows the implied steady-state assuming no further changes in fundamentals; the dashed blue line shows the implied steady-state assuming that future changes in fundamentals follow geometric decay. In practice, we find that these two specifications are close to one another, because we estimate autoregressive coefficients for changes in fundamentals (ρ^z and ρ^b) in equation (47) that are close to zero.

From the top-left panel, we find that actual population in the Rust Belt states was already substantially above its steady-state value in 1965, and only begins to approach its steady-state value towards the end of our sample period. In the top-right panel, actual population in the Sun Belt states was substantially below its steady-state value in 1965, but converges towards its steady-state value by the late 2000s. In the bottom-right panel, actual population in Other Southern states rises substantially above its steady-state value in the middle of our sample period, before the two sets of population shares converge towards one another by the end of our sample period. Finally, in the bottom-left panel, actual and steady-state population shares in Other Northern states lie relatively close to one another throughout our sample period. Across these four panels, we find that actual population shares can remain persistently either above or below their steady-state values for decades, implying slow convergence to steady-state. Actual and steady-state population shares are closer together at the end of our sample period than at its beginning, suggesting that one potential reason for the modest observed decline in population mobility over time could be that the economy is now closer to steady-state.

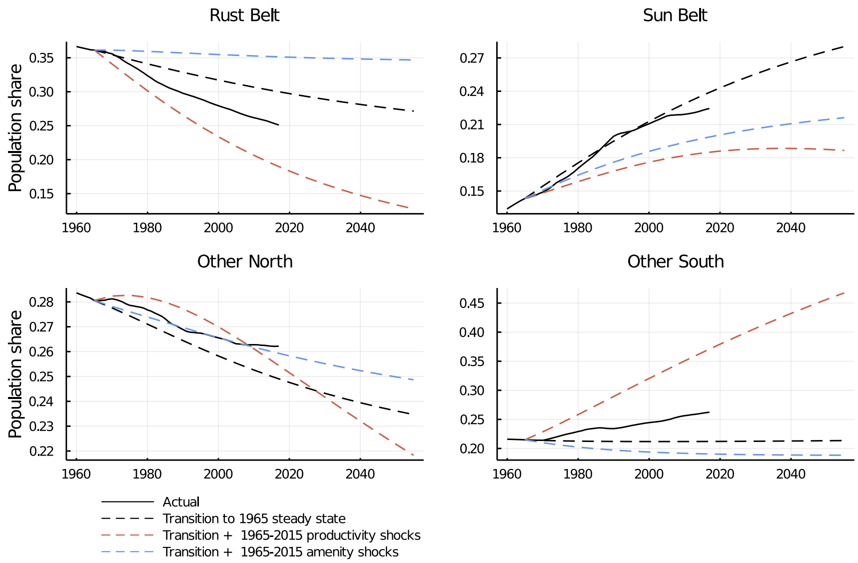
Figure 6: Actual and Implied Steady-State Population Shares by Region



Note: Black line shows actual population shares; two dashed lines show implied steady-state population share based on the observed state variables $\{\boldsymbol{l}_t, \boldsymbol{k}_t\}$ and trade and migration share matrices $\{\boldsymbol{S}_t, \boldsymbol{T}_t, \boldsymbol{D}_t, \boldsymbol{E}_t\}$ in each year using Proposition 3 and equation (35); red dashed line assumes no further changes in fundamentals, as in equation (36); blue dashed line assumes that future changes in fundamentals follow geometric decay, according to equation (41), and using estimated autoregressive coefficients of $\rho^z = 0.02$ and $\rho^b = 0.02$ from estimating the AR(1) process in equation (47).

Second, we compute the counterfactual transition path of economic activity implied by convergence towards the initial steady-state at the beginning of our sample period in 1965, again using Proposition 3. In

Figure 7: Actual Population Shares and Counterfactual Population Shares Implied by Convergence Towards the Initial Steady-State in 1965 and Shocks to Fundamentals

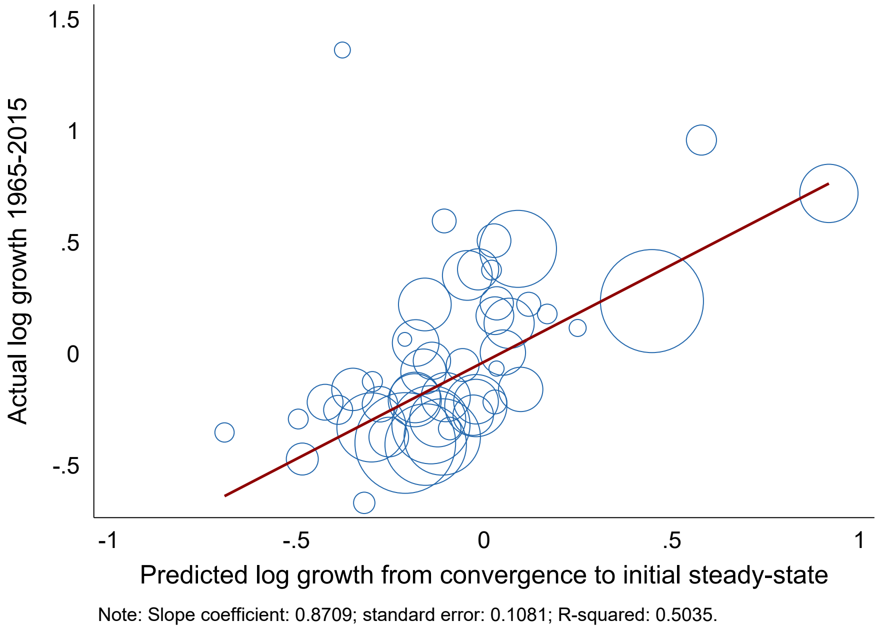


Note: Solid black line shows actual population shares; dashed black line shows counterfactual population shares based on convergence to the initial steady-state in 1965 from equation (36), assuming no further changes in fundamentals; red dashed line shows counterfactual population shares based on convergence to the initial steady-state in 1965 and the empirical distribution of productivity shocks from equation (35), assuming no further changes in other fundamentals; blue dashed line shows counterfactual population shares based on convergence to the initial steady-state in 1965 and the empirical distribution of amenity shocks from equation (35), assuming no further changes in other fundamentals.

the light of the similarity of our two specifications above, we focus on the specification assuming no further changes in fundamentals. In Figure 7, we compare this counterfactual transition path from convergence towards the initial steady-state (dashed black line) to the actual evolution of population shares (solid black line). We also include the counterfactual transition paths once we also incorporate productivity shocks (dashed red line) or amenity shocks (dashed blue line), using Proposition 3. We extend each of these counterfactual transition paths beyond the end of our sample period to again highlight the model's implied slow convergence towards steady-state. We find that the transition path implied by convergence towards the initial steady-state has substantial predictive power for the trajectory of actual population shares, particularly for the Rust Belt, Sun Belt and Other Northern States. Incorporating productivity shocks (dashed red line) helps to capture the observed decline in the population share of the Rust Belt.

To provide further evidence on this predictive power of convergence to steady-state, Figure 8 graphs actual population growth from 1965-2015 against predicted population growth based on convergence towards the initial steady-state at the beginning of our sample period in 1965, assuming no further changes in fundamentals. The sizes of the circles for each state are proportional to their initial population size. We find a strong regression relationship between the two variables, with a regression slope (standard error) of 0.8709 (0.1081) and a R-squared of 0.5035. In Section G.2 of the online appendix, we show that controlling for initial log population in 1965, log capital stock in 1965, and log population growth from 1965-66 has relatively little impact on either the estimated coefficient or the regression R-squared. Therefore, the strength of the relationship in Figure 8 does not simply reflect mean reversion, because we continue to find substantial independent information in

Figure 8: Actual Growth in Population Shares for each U.S. State from 1965-2015 Versus Predicted Growth Based on Convergence to the Implied Initial Steady-State in 1965



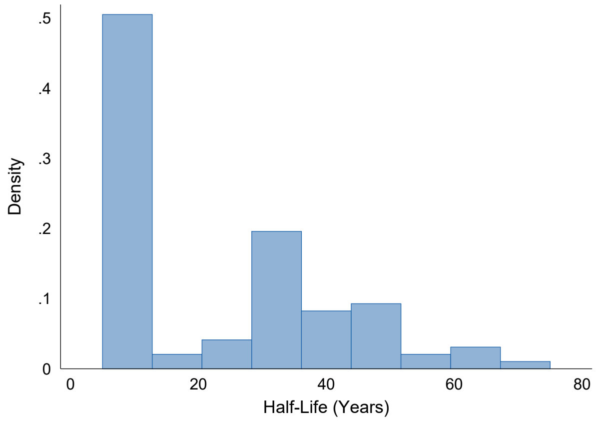
Note: Vertical axis is actual log population growth from 1965-2015; horizontal axis is predicted log population growth based on convergence to the implied initial steady-state at the beginning of our sample period in 1965 using equation (36), assuming no further changes in fundamentals; size of circles for each U.S. state is proportion to initial population size.

predicted population growth based on convergence towards the initial steady-state, even after controlling for these measures of initial levels and rates of growth of economic activity.

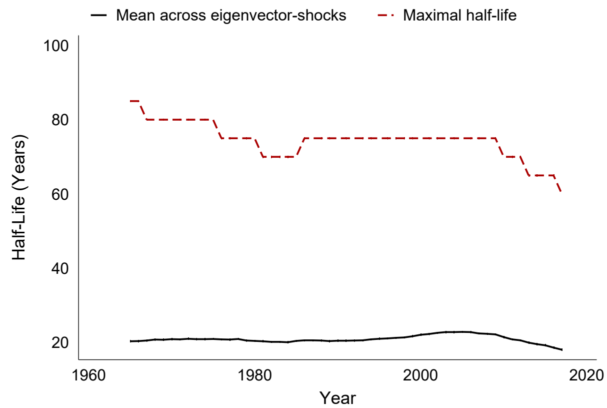
Third, we evaluate the speed of convergence towards steady-state by using Proposition 5 to compute half-lifes. In particular, we compute the number of years for the state variables to converge half of the way towards steady-state for a shock to productivity or amenities, such that the initial impact on the state variables $(\boldsymbol{R}\widetilde{\boldsymbol{f}})$ corresponds to an eigenvector $(\boldsymbol{u_k})$ of the transition matrix $(\boldsymbol{P})$. We have as many half-lifes as eigenvectors of the transition matrix $(\boldsymbol{P})$, each of which corresponds to a different pattern of shocks to productivity and amenities across states. Furthermore, the transition matrix $(\boldsymbol{P})$ changes over time with underlying changes in the trade and migration share matrices $(\boldsymbol{S}, \boldsymbol{T}, \boldsymbol{D}, \boldsymbol{E})$, and hence we can compute these distributions of half-lifes across eigenvectors of the transition matrix for each year of our sample period.

Figure 9: Half-lifes for Convergence Towards Steady-State

(a) Histogram of Half-lifes for Shocks to Productivity and Amenities that Correspond to Eigenvectors of the Transition Matrix ($\boldsymbol{P}$) in 2000



(b) Mean and Maximum Half-lifes for Shocks to Productivity and Amenities that Correspond to Eigenvectors of the Transition Matrix ($\boldsymbol{P}$) in Each Year



Note: Half-life corresponds to the time in years for the state variables to converge half of the way towards steady-state for a shock to productivity or amenities, such that its initial impact on the state variables ($\boldsymbol{R}\widehat{\boldsymbol{f}}$) corresponds to an eigenvector ($\boldsymbol{u_k}$) of the transition matrix ($\boldsymbol{P}$); left panel shows the distribution of half-lifes across eigenvectors of the transition matrix in 2000, where each eigenvector corresponds to a different pattern of productivity and amenity shocks; right panel shows the mean and maximum half-life across eigenvectors of the transition matrix in each year from 1965-2015.

In Figure 9a, we display the distribution of half-lifes across eigenvectors of the transition matrix ($\boldsymbol{P}$) in the year 2000. Unsurprisingly, we find substantial heterogeneity in half-lifes, depending on the pattern of shocks to productivity and amenities across states, as captured by each eigenvector. In general, we find that convergence to steady-state is slow, with an average half-life of around 20 years, and a maximum half-life of about 80 years. Nevertheless, for some vectors of shocks to productivity and amenities, which may involve only small changes in productivity and amenities, we find that the half-life can be as short as 5 years.

In Figure 9b, we display the evolution over time of the mean and maximum of this distribution of half-lifes across eigenvectors of the transition matrix. Although we observe a modest decline in geographical mobility in the data on state-to-state migration, we find that the speed of convergence towards steady-state is relatively constant over time, with the mean and maximum half-life declining somewhat towards the end of our sample period. This juxtaposition of a decline in geographical mobility and faster convergence towards steady-state again highlights the idea that a decline in geographical mobility does not necessarily imply a rise in migration frictions and hence slower convergence towards steady-state. This decline in geographic mobility instead can be explained by the economy being closer to steady-state at the end of our sample period than at its beginning and/or by a change to the pattern of shocks to productivity and amenities across locations.

5.5 Spectral Analysis and Distributional Consequences

Building on our characterization of the economy's transition path in the previous sections, we now undertake a spectral analysis of the transition matrix ($\boldsymbol{P}$) to provide further evidence on the respective contributions of migration and capital accumulation to gradual adjustment and on the extent to which different locations are exposed to similar productivity and amenity shocks. We also examine the implications of this gradual adjustment for the distributional consequences of shocks to fundamentals across workers depending on their initial location.

We begin with our spectral analysis of the transition matrix ($\boldsymbol{P}$). Using our eigendecomposition, we can express the transition matrix in terms of the matrix of right eigenvectors ($\boldsymbol{U}$), the matrix of left eigenvectors ($\boldsymbol{V} = \boldsymbol{U}^{-1}$) and the diagonal matrix of eigenvalues ($\boldsymbol{\Lambda}$):

$$\boldsymbol{P} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{V},$$

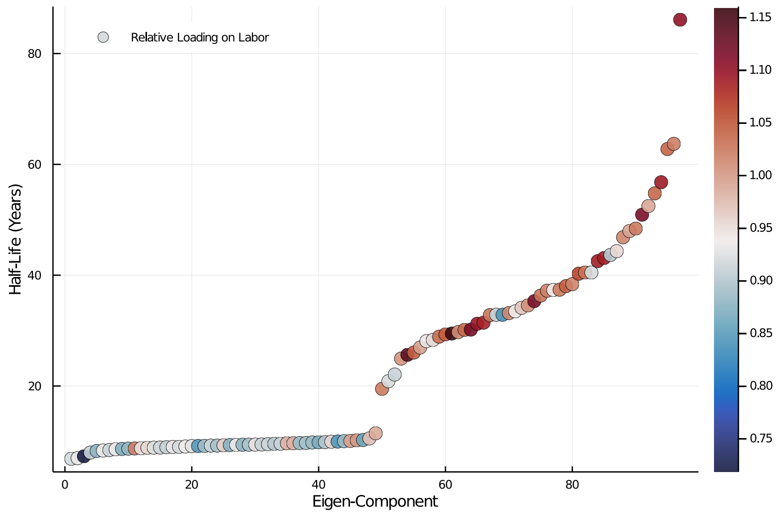
where the first $2N$ rows and columns of the transition matrix correspond to the labor state variables in each location and the remaining $2N$ rows and columns correspond to the capital state variables in each location.

First, we use this eigendecomposition to evaluate the extent to which the slow speed of convergence towards steady-state is driven by labor dynamics (migration) versus capital dynamics (investment). We use the property that we can compute a loading ($\boldsymbol{\alpha}$) on the eigenvectors ($\boldsymbol{U}$) for any linear combination of the state variables ($\boldsymbol{w}$): $\boldsymbol{\alpha'} = \boldsymbol{w'}\boldsymbol{U}$. In particular, we measure labor's loading on the eigenvectors ($\boldsymbol{w'}_{\text{labor}}$) using a vector of weights with population shares in all labor elements and zeros in all capital elements (i.e. $\boldsymbol{w'}_{\text{labor}} = [\boldsymbol{l}, \boldsymbol{0}_{1\times N}]$). Similarly, we measure capital's loading on the eigenvectors ($\boldsymbol{w'}_{\text{capital}}$) using a vector of weights with zeros in all labor elements and capital shares in all capital elements ($\boldsymbol{w'}_{\text{capital}} = [\boldsymbol{0}_{1\times N}, \boldsymbol{k}]$). Combining these two measures, we compute the relative loading of labor and capital on the eigenvectors using the relative magnitude of their individual loadings:

$$\text{Relative labor loading for } k = \frac{||\boldsymbol{\alpha'_{k,\text{labor}}}||}{||\boldsymbol{\alpha'_{k,\text{capital}}}||}. \tag{51}$$

We connect these relative loadings on labor and capital to the speed of convergence by using the property that each eigenvector of the transition matrix has a corresponding half-life of convergence to steady-state that is determined by its associated eigenvalue, as shown in Proposition 5. In Figure 10, we display the distribution of half-lifes across the eigenvectors of the 2000 transition matrix, sorted in terms of increasing half-life, and shaded in terms of their relative loading on labor. We use red shading to denote greater loading on labor and blue shading to represent greater loading on capital. As apparent from the figure, we find that eigenvectors that load relatively more on labor typically have slower convergence to steady-state than those that load relatively more on capital. Therefore, our findings of slow rates of convergence to steady-state are primarily driven by labor dynamics (migration) rather than by capital dynamics (investment). Eigenvectors that load relatively more capital (as shown towards the bottom left of the figure) tend to have relatively similar half-lifes, which is driven by the common savings rate out of income net of depreciation, as determined by the discount rate β.

Figure 10: Relative Loadings of the Eigenvectors of the Transition Matrix on Labor



Notes: Eigencomponents from the 2001 transition matrix ($\boldsymbol{P}$), sorted by increasing half-life, and colored based on their relative loadings on labor, as defined in equation (51) in the main text; blue shading denotes greater relative loading on capital; and red shading denotes greater relative loading on labor.

Second, we use this eigendecomposition to examine the extent to which different locations are exposed to similar productivity and amenity shocks. We compute the loading of an individual location i on the eigenvectors ($\boldsymbol{w_i'}$) using a vector of weights equal to that location's population share for its state variables and zero otherwise: $\boldsymbol{\alpha_i'} = \boldsymbol{w_i'U}$. Using these loadings for two different locations i and n, we compute the similarity of their exposure to productivity and amenity shocks as the correlation between these loadings:

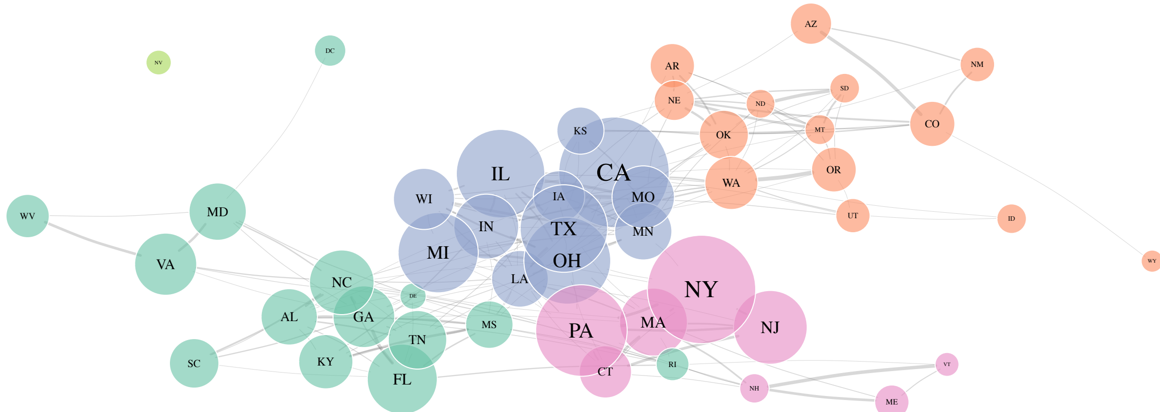
$$\text{Similarity}_{i,n} \equiv \frac{\boldsymbol{\alpha_i'\alpha_n}}{||\boldsymbol{\alpha_i}|| \times ||\boldsymbol{\alpha_n}||}, \tag{52}$$

where the stronger this correlation, the greater the extent to which these two locations are exposed to more similar productivity and amenity shocks.

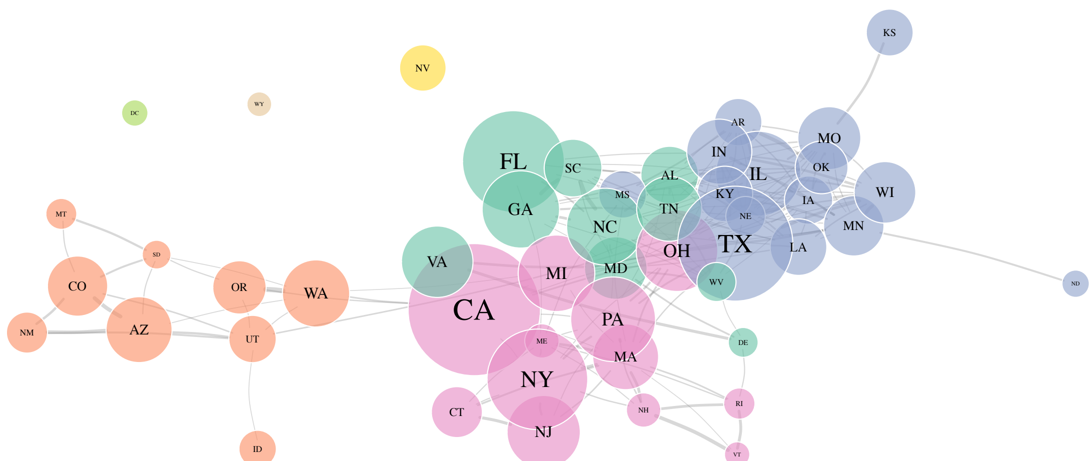
In Figure 11, we display the similarity of exposure to productivity and amenity shocks for U.S. states in 1965 (Panel A) and 2015 (Panel B) using a network graph. The nodes correspond to U.S. states, with the size of these nodes reflecting the population shares of the states. The thickness of each edge captures the degree of similarity in exposure to productivity and amenity shocks, where we focus on the 200 edges with the highest degrees of similarity for reasons of legibility. States are grouped to maximize modularity (the fraction of edges within the groups minus the expected fraction if the edges were distributed at random).

Figure 11: Network of State Exposure to Productivity and Amenity Shocks

Panel A: Year 1965



Panel B: Year 2015



Note: Network of bilateral similarity of state exposure to productivity and amenity shocks in 1965 (Panel A) and 2015 (Panel B), as defined in the main text; states grouped to maximize modularity (the fraction of edges within the groups minus the expected fraction if the edges were distributed at random); the two-letter codes correspond to the postal codes for each U.S. state (e.g. CA represents California); colors indicate distinct groupings of states.

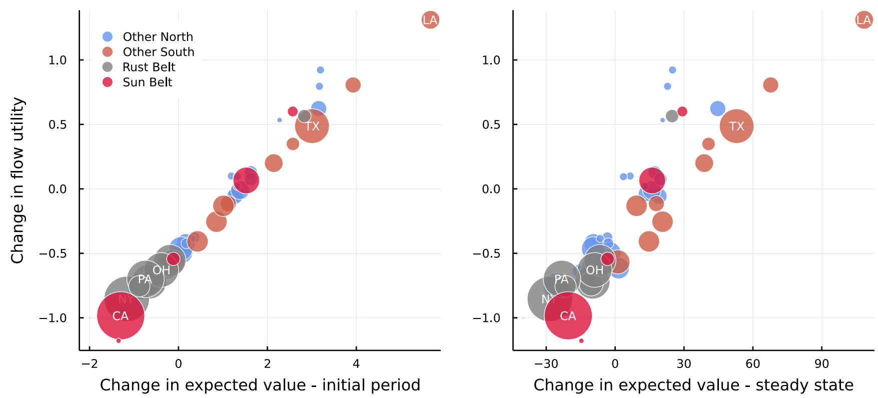
As shown in the figures, we find a powerful role for state size and geography in shaping the similarity of exposure to productivity and amenity shocks in both years, with the network graph approximately corresponding to a map of the United States, especially in 1965. Nevada and Wyoming are the most disconnected to the network of bilateral exposure in both years, consistent with their remote location and sparse population. Overall, we find that the network is more tightly clustered in 2015 than in 1965, consistent with an increase in the integration of the U.S. economy over time. We find that California and Texas become more central to the network over time, in line with the substantial increase in their relative economic size over this time period. In contrast, we find that the other Western and Mountain states remain relatively closely connected to one another, but the most disconnected from the remaining U.S. states.

Finally, we turn to the distributional consequences of these shocks in the presence of gradual adjustment from costly migration and capital accumulation. Starting at the observed values of the state variables (population and the capital stock) at the beginning of our sample period in 1965, we undertake a counterfactual for a one-time permanent productivity shock equal to the accumulated empirical distribution of productivity shocks from 1965-2015. In this counterfactual, we solve for the transition path of the spatial distribution of economic activity towards the new steady-state in response to this one-time shock. Using this solution for the transition path and the results from Proposition 9, we evaluate the uneven impact of these productivity shocks on the welfare of workers initially located in each state.

In Figure 12, we contrast the effects of these shocks on worker flow utility ($w_i b_i / p_i$) versus expected value (v_i^w) for the workers initially located in each state. In both panels, the vertical axis shows the change in the flow utility ($w_i b_i / p_i$) in the initial period. In the left-panel, the horizontal axis shows the change in the expected value (v_i^w) in the initial period. In contrast, in the right-panel, the horizontal axis shows the change in the expected value (v_i^w) in the new steady-state following these productivity shocks. Both the change in expected value and the change in flow utility are normalized to have a mean of zero across states.

In both panels, we find substantial heterogeneity in the welfare effects of the productivity shocks, depending on the state in which workers are initially located. This heterogeneity is driven by the migration frictions, which imply that it is costly and takes time for workers in states that experience relative reductions in expected values to reallocate towards those that experience relative increases in expected values. In general, the changes in relative expected values are much larger than the changes in relative flow utilities in the initial period, because these expected values correspond to the net present value of the stream of expected future flow utilities. Comparing the two panels, the change in the initial period flow utility is much more strongly correlated with the change in the initial period expected value (left panel) than the change in the expected value in the new steady-state (right panel). Again this intuitive pattern reflects the reallocation of some workers from states that initially experience relative reductions in expected values towards other states that initially experience relative increases in expected values.

Figure 12: Counterfactual Impact of Productivity and Amenity Shocks on Flow Utility and Expected Value in the Initial Period and the New Steady-State



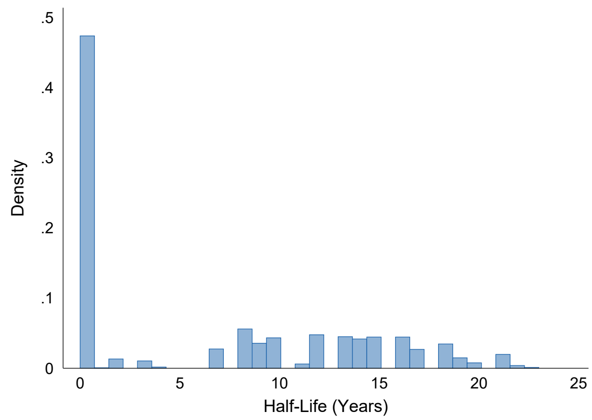
Note: Starting at the observed values of the state variables at the beginning of our sample in 1965, we undertake a counterfactual for a one-time permanent productivity shock in each state, equal to the accumulated empirical productivity shocks from 1965-2015; the left panel shows that change in flow utility and expected value in the initial period; the right panel shows the change in flow utility in the initial period and the expected value in the new steady-state; both the change in flow utility and expected value are normalized to have a mean of zero across U.S. states; the two-letter codes correspond to the postal codes for each U.S. state (e.g. CA represents California); the colors indicate the states in each of our four groups (e.g. gray corresponds to Rust Belt).

5.6 Multi-sector Quantitative Analysis

In a final empirical exercise, we implement our multi-sector extension from Section 4.4 above, using our region-sector data from 1999-2015, as discussed in Section 5.1 above. We compute our closed-form solutions for the comparative statics of economic activity in each region-sector with respect to productivity and amenity shocks in any region-sector, both in steady-state and along the transition path. In the interests of brevity, we focus largely on the speed of convergence towards steady-state, and on the spectral analysis of the transition matrix ($\boldsymbol{P}$) that determines this speed of convergence towards steady-state.

We begin by using Proposition 5 to compute half-lifes of convergence towards steady-state for shocks to productivity or amenities for which the initial impact on the state variables ($\boldsymbol{R}\widetilde{\boldsymbol{f}}$) corresponds to an eigenvector ($\boldsymbol{u_k}$) of the transition matrix ($\boldsymbol{P}$). In Figure 13, we display the distribution of these half-lifes across eigenvectors of the transition matrix in the year 2000. As apparent from the figure, we find substantially more rapid convergence to steady-state in our multi-sector extension, with an average half-life of 6.7 years and a maximum half-life of 23 years (compared to around 20 and 75 years in our baseline single-sector model). This finding is driven by the property of the region-sector migration matrices that flows of people between sectors within the same U.S. state are much larger than those between different U.S. states. A key implication of these findings is that there is likely to heterogeneity in the persistence of the impact of local shocks, depending on whether they induce reallocation across industries within the same location or reallocation across different locations.

Figure 13: Half-lifes for Convergence Toward Steady-State in the Multi-Sector Model in 2000



Note: Half-life corresponds to the time in years for the state variables to converge half of the way towards steady-state for a shock to productivity or amenities, such that its initial impact on the state variables ($\boldsymbol{R}\widehat{\boldsymbol{f}}$) corresponds to an eigenvector ($\boldsymbol{u_k}$) of the transition matrix ($\boldsymbol{P}$); figure shows the distribution of half-lifes across eigenvectors of the transition matrix in 2000 for our multi-sector extension, where each eigenvector corresponds to a different pattern of productivity and amenity shocks.

We next use our eigendecomposition to examine the extent to which different sectors are exposed to similar productivity shocks. We compute the loading of an individual sector j on the eigenvectors ($\boldsymbol{w^{j\prime}}$) using a vector of weights equal to the region-sector employment share for its state variables and zero otherwise: $\boldsymbol{\alpha^{j\prime}} = \boldsymbol{w^{j\prime}}\boldsymbol{U}$. Using these loadings for two different sectors j and k, we compute the similarity of their exposure to productivity shocks as the correlation between these loadings:

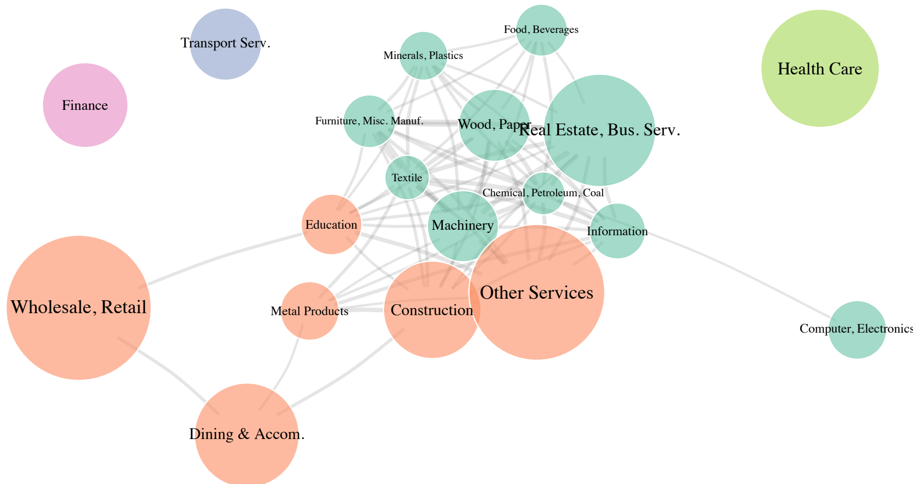
$$\text{Similarity}^{j,k} \equiv \frac{\boldsymbol{\alpha^{j\prime}}\boldsymbol{\alpha^{k}}}{||\boldsymbol{\alpha^{j}}|| \times ||\boldsymbol{\alpha^{k}}||}, \tag{53}$$

where the stronger this correlation, the greater the extent to which these two industries are exposed to more similar productivity shocks.

In Figure 14, we display the similarity of sector exposure to productivity shocks in 2000 using a network graph. The nodes now correspond to sector, with the size of these nodes now reflecting the employment shares of the sectors. The thickness of each edge captures the degree of similarity in exposure to productivity shocks, where we again focus on the 200 edges with the highest degrees of similarity for reasons of legibility. As for U.S. states above, we group sectors to maximize modularity.

As evident from the figure, we find two main groupings of sectors. The first of these groupings corresponds to largely service sectors (shown in orange), which all disproportionately depend on local final demand within each region. The second of these groupings corresponds to largely manufacturing sectors (shown in green), which have relatively close connections to one another through input-output linkages. Machinery and Metal products are the two manufacturing sectors that have the strongest links with both groupings, which is consistent with these sectors lieing relatively upstream in input-output networks. Finally, the Finance and Transport Services sectors form separate groups by themselves, with relatively weak connections to the other sectors.

Figure 14: Network of Sector Exposure to Productivity and Amenity Shocks 2000



Note: Network of bilateral similarity of sector exposure to productivity and amenity shocks in 2000, as defined in the main text; sectors grouped to maximize modularity (the fraction of edges within the groups minus the expected fraction if the edges were distributed at random); colors indicate distinct groupings of sectors.

6 Conclusions

A classic question in economics is the response of the spatial distribution of economic activity to local shocks. In general, this response can be gradual, because of migration frictions and endogenous investments in the accumulation of durable factors. A key challenge in modeling these dynamics, is that agents' forward-looking decisions depend on the entire spatial distribution of economic activity across all locations in all future periods of time. Our first main contribution is to develop a tractable dynamic general equilibrium framework that incorporates both migration costs for mobile factors (labor) and endogenous investments in durable immobile factors (capital structures). Despite the many locations connected by a rich geography of trade and migration frictions, and the multiple sources of dynamics in the model, we provide an analytical characterization of the conditions for the existence and uniqueness of the steady-state equilibrium.

Our second main contribution is to derive closed-form solutions for the first-order general equilibrium effect of shocks to fundamentals (productivities, amenities, trade costs and migration costs) on the full transition path of the spatial distribution of economic activity. These sufficient statistics depend on four observable matrices for expenditure shares, income shares, outmigration shares and inmigration shares, the initial values of the state variables (population and the capital stock) in each location, and the structural parameters of the model. We show that our sufficient statistics are exact for small changes and provide a close approximation to the full non-linear model solution for empirically-reasonable shocks to fundamentals. In contrast to conventional methods, we obtain an analytical characterization for our linearization, which we use to explore the determinants of the dynamic response of the economy to shocks. Although for expositional simplicity we focus in our baseline specification on a single sector, we show that our approach admits a large number of extensions and generalizations, including agglomeration forces, multiple sectors, and input-output linkages.

We show that the economy's transition path is fully characterized by a second-order difference equation in the state variables (population and the capital stock). This second-order difference equation has a closed-form solution in terms of an impact matrix, which captures the initial impact of the shocks, and a transition matrix, which governs the subsequent evolution of the state variables in response to these shocks. We show that the speed of convergence to steady-state, as measured by the half-life, is determined by the eigenvalues of this transition matrix. We use an eigendecomposition of this transition matrix to isolate the locations exposed to particular shocks and the shocks that impact particular locations.

In our main empirical application, we use data on U.S. states from 1965-2015 to examine the decline of the "Rust Belt" in the North-East and Mid-West and the rise of the "Sun Belt" in the South and West. We show that this setting features convergence dynamics in capital and both net and gross migration, highlighting the relevance of a framework such as ours that incorporates both forward-looking investments and dynamic migration decisions. Already at the beginning of our sample period in 1965, we find that Rust Belt and Sun Belt states were substantially below and above their steady-state populations, respectively. By the end of sample period, all states are much closer to their steady-state population than they were at its beginning. We show that the initial distance of a state's population from its steady-state has substantial predictive power for subsequent population growth from 1965-2015, even after controlling for the initial levels of population and the capital stock and initial population growth. We find slow convergence to steady-state, with an average half-life in our baseline specification of around 20 years, which is consistent with recent empirical findings of persistent impacts of local labor demand shocks. We show that this slow rate of convergence towards steady-state is primarily driven by labor dynamics (migration) rather than by capital dynamics (investment), although capital accumulation plays an important role in amplifying the impact of local shocks.

In a final empirical exercise, we implement our multi-sector extension using region-sector data on U.S. states and foreign countries from 1999-2015. We find lower average half-lifes in our multi-sector extension, which reflects the property of the data that there is greater mobility of labor across sectors within states than across states. Nevertheless, we find substantial variation in these half-lifes, highlighting the heterogeneity in the impact of local labor demand shocks, depending on their spatial and sectoral incidence. We find that these average half-lifes are relatively constant over time, which contrasts with the modest observed decline in geographical mobility. This pattern of results highlights that this decline in geographical mobility does not necessarily imply a rise in spatial frictions, since it is also influenced by convergence towards steady-state, and changes in the pattern and magnitude of shocks.

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