Market Competition and Political Influence: An Integrated Approach*

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Abstract

The operation of markets and of politics are in practice deeply intertwined. Political decisions set the rules of the game for market competition and, conversely, market competitors participate in and influence political decisions. We develop an integrated model to capture the circularity between the two domains. We show that a positive feedback loop emerges such that market power begets political power in a positive feedback loop, but that this feedback loop is bounded. With too much market power, the balance between politics and markets itself becomes lopsided and this drives a wedge between the interests of a policymaker and the dominant firm. Although such a wedge would seem pro-competitive, we show how it can exacerbate the static and dynamic inefficiency of market outcomes. More generally, our model demonstrates that intuitions about market competition can be upended when competition is intermediated by a strategic policymaker.

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1 Introduction

The operation of markets and of politics are in practice deeply intertwined. Political decisions set the rules of the game for market competition and, conversely, market competitors participate in and influence political decisions. Since at least the time of Stigler (1971), the connection between the two domains has been formalized in economics, and the flourishing literature that emerged has deepened our understanding of how special interests can distort political outcomes and how political decisions shape market outcomes.

What has been less explored is the circularity of this connection. If political decisions affect market structure, and that market structure, in turn, determines the power of firms to participate in and influence political decisions, a circularity develops in which market and political outcomes are codetermined. The endogeneity of both market and political outcomes leads to sharp questions about the origins, persistence, and welfare effects of market power.

These questions have come to the forefront of debate in recent years in both academic writing and the public forum. Recent evidence establishes that market power has increased in the US in the past few decades (De Loecker, Eeckhout & Unger 2020). An open question is why. Was the increase due to efficiency gains that were rewarded with market leadership—what Covarrubias, Gutierrez & Philippon (2020) refer to as "good concentration"—or is it "bad concentration" derived from anti-competitive practices and, in particular, the wielding of political power to handicap market rivals?

In this paper we develop a model to explore and analyze the circularity between markets and politics. Competition in the market is Cournot between two firms repeated without end. The essential element of the model is that firms can obtain market power from two distinct sources. Market power can come from a competitive advantage that firms invest in, be it through R&D and technological superiority, from higher managerial competence, or some combination thereof. This *capability* based market power builds a competitive advantage that makes the market as a whole more efficient.

The second source of market power is political protection. We endow a self-interested policymaker with the ability to intervene in the market to advantage one firm over its competitors. For concreteness, we model this power via a minimum standard, a regulatory tool common in practice. The policymaker can impose a standard to separate the firms, choosing a level that only the leader can meet and excluding the trailing firm from the market. The protected firm benefits from the removal of competition and passes along a share of the surplus that is gained as payment to the policymaker.[1] This *political* based market power enables a competitive ad-

[1]This tool can only separate firms that have a technological difference. The tool(s) available to the policymaker are fundamental to the outcome of market and political interaction. We return to this point later in the paper.

vantage by disabling competition, which, in contrast to capability-based market power, comes at the expense of efficiency.

We study this model dynamically. We show that a positive feedback loop emerges between the two sources of market power—that market power begets political power that begets market power in a reinforcing cycle. In this way an initial capability advantage can be parlayed over time into a larger advantage and a dominant market position.

We show, however, that this feedback loop is bounded and conditional on market power itself. We identify a threshold in capability-based advantage beyond which the feedback loop turns negative. Beyond this threshold, therefore, greater capability-based market power leads to the removal of protection and less politically-based market power. This removal restores a degree of competition and bounds the ability of firms to dominate the market through political protection.

The core insight driving this result is that the interests of the market leading firm and the policymaker are aligned but not perfectly aligned. Within each period their interests are aligned on political protection—monopoly power maximizes the surplus available for them to share. Across periods, however, the degree of market power changes, and so too does the balance of power in their relationship. If the market leader gains a large capability advantage on its competitor, the value of political protection declines, and as this declines, the ability of the policymaker to extract rents from the market leader declines. Capability-based and politically-based market power are substitutes, in effect, such that the more the market leader has of one, the less it needs of the other.

This generates dynamic incentives for the policymaker that are very different from her static incentives. Dynamically, the policymaker seeks to "manage competition." She wants to protect the leading firm so that she can extract rents, but she doesn't want the leader to get so far ahead technologically that political protection becomes obsolete. It is her desire to remain relevant that causes her to stop protecting the leader and encourage competition, hoping that this allows the trailing firm to catch up and make her protection valuable once more.

At first blush, managed competition appears promising as it bounds political intervention in the market and restores a semblance of competition. We show, however, that this is not the case. In an otherwise standard model of duopolistic competition, we show that managed competition can lead to the worst of both worlds. We characterize the unique renegotiation-proof subgame perfect equilibrium and show that play eventually stabilizes at a configuration in which technology stagnates and the policymaker protects the leading firm. The steady state is inefficient both because the leading firm is a protected monopolist and because investment stops at a low level. In fact, the capability level at the steady state is never greater than, and typically lower, than if the policymaker always protected the leading firm. Investment with political interventions is lower, therefore, than if monopoly were even guaranteed.

This result shares a deep connection with Arrow's (1962) well-known "replacement effect" from markets. Arrow observed that investment in technology will be higher with competition than in monopoly. The reason is that a monopolist obtains only an efficiency gain from investment whereas a duopolist has the additional benefit of capturing greater market share.[2]

The connection of Arrow to politics is that, by intervening in the market, the policymaker affects the degree of competition and, thus, the firms' incentive to invest. Our result shows that political intervention turns Arrow's logic on its head, creating what we refer to as a *reverse Arrow effect*. Precisely because the policymaker wants to manage competition—to remove protection should the leading firm's advantage exceed a threshold—the leading firm is incentivized to stop investing early. At the threshold, investment will not decrease competition, as Arrow suggests, rather it will increase as the policymaker removes protection, allowing the follower firm to enter the market. With Arrow's logic reversed, the leading firm stops investing at the precipice of the threshold, and as the policymaker continues to protect, the market stabilizes at a steady state with no competition and low investment.

A general lesson from this analysis is that the impact of political intervention on markets is a function of the structure of market competition itself. The insight from managed competition is that a self-interested policymaker seeks market competition not for its own sake, but so that the threat of even more competition increases the value of protection to the leading firm. This implies that a standard market intuition—that competitive pressure translates into more efficient markets—need not hold when that competition is intermediated by a strategic policymaker.

To explore this idea, we consider a market in which competitive pressure is reduced. Specifically, we suppose that a firm will give up and leave the market permanently if it has been excluded by political protection for some period of time. This change nominally reduces competitive pressure on the leading firm as exit by the trailing firm removes competition altogether. However, to understand the impact of this change on a market intermediated by a policymaker, we must understand how it changes the incentives of the policymaker.

We show that this reduction in competition pressure weakens the leverage of the policymaker and improves market outcomes. In fact, we show that in the steady state investment is higher than in monopoly and even duopoly. The reason for this reversal and efficiency gain again comes back to Arrow. The reverse Arrow effect still emerges in this setting, although now only temporarily, and the problem of underinvestment that it causes is eventually, albeit slowly, overcome. On top of this, we show that a separate, distinct variant of the Arrow effect emerges—what we refer to as the *politically enhanced Arrow effect*—in which political protection serves as a reward to investment rather than punishment. In this way political in-

[2] A duopolist "escapes competition" in the terminology of Aghion et al. (2005). We discuss this idea in more detail in Section 2.

tervention enhances investment and is able to correct, in part, the standard market failure in which firms underinvest.

These results illuminate a novel economic mechanism when markets and politics intersect. This mechanism goes beyond the truism that politics affect markets. Rather, it lays out a specific channel through which the structure of market competition links to the degree of political influence. We show how the power of this mechanism rests on the substitutability of the two sources of market power, that the value of political power varies inversely with the technological state of the market. Tracing through the logic of this mechanism, we then see how heightened competitive pressure can generate political inefficiency, and to such a degree that it overshadows the standard market-based efficiency of increased competition, leaving society worse off. This result poses a challenge to the standard benchmark of a competitive market. If more competition only provides fertile ground for a self-interested policymaker to extract rents, there is little reason to expect that overall efficiency will increase. As Lerner (1972, p.259) observed, "An economic transaction is a solved political problem." When politics is itself a live variable—a yet unsolved problem—the market transaction must be viewed through a broader lens.[3]

A more practical and immediate lesson from our model is for the current debates around market power and politics. One line of argument is that concentration must be "good" if a dominant firm has a capability advantage over competitors. What our results show is that the path to dominance matters. If the leading firm benefited from political protection along the path to dominance, the resulting market concentration need not be "good." Indeed, in a market with softer competitive pressure, we show that the steady state of the industry involves a monopolist with a high level of capability and no political protection. Yet that position was not earned through market competition. Rather, the outcome was preordained from the moment the firm gained an initial capability advantage. In equilibrium, the firm parlays that initial advantage into lasting dominance through political protection in a reinforcing cycle. Our model informs this debate by providing a structure to understand the empirical and practical connection between market concentration and political power over time.

1.1 Connections to the Literature

Competition within the market and the dynamics of market structure have been extensively analyzed in the economics literature. While government intervention to affect market structure has been a core element of economic models, for instance in analyzing the effects of antitrust policies (e.g., Segal & Whinston 2007, Asker & Bar-Isaac 2020), most of these analyses

[3]Lerner (1972, p.259) goes further and argues that "Economics has gained the title of queen of the social sciences by choosing *solved* political problems as its domain."

assume a benevolent social planner or simply exogenous government interventions. Our contribution is to introduce politically motivated strategic market interventions into the standard model of firm competition. Once political economy considerations are incorporated, standard intuitions about the evolution of market structures are altered. Similarly, firms and industries have been at the core of political economy models, as actors who lobby for favored policies. Yet their interests and capabilities have been generally taken as given without accounting for how they coevolve dynamically with policy (e.g., Grossman & Helpman 1994). Our paper is a small step toward bringing these literatures closer together and exploring their interdependence.[4]

Our model is closest in spirit to Coate & Morris (1999). They explicitly connect lobbying and political influence to private sector investment, showing how political choices influence private sector decisions that, in turn, influence politics. In their model there is a single firm that decides which of two sectors to operate in.[5] We differ in emphasizing competition between firms and the dynamics of competition within a single market, showing the importance to a policymaker of deciding when and not just whether to extract rents.

More broadly, our model relates to the literature on the role of commitment in political economy. Our result that investment stops at a level lower than monopoly is, at heart, a commitment problem. A higher capability level for the leader would increase the surplus available, but because the firm and the policymaker can't commit to a sharing of the surplus going forward, the policymaker attempts to manage competition and the reverse Arrow effect take hold. Acemoglu (2003) views the lack of commitment in politics through the lens of the Coase theorem and argues that a political version of the theorem does not hold. Closer to our work, Acemoglu & Robinson (2000), and more thoroughly, Acemoglu (2006) and Acemoglu, Golosov & Tsyvinski (2008), build models in which economic outcomes impact politics and show how the inability to commit to who holds political power—and, therefore, the inability to commit to the sharing of future surplus—distorts economic outcomes away from efficiency.[6]

Relative to this literature, our contribution is to enrich the private sector and study competition *within* the market. This allows us to see how private sector actors compete for political influence, and how a strategic policymaker can trade their interests off against each other to maximize her own gain. By building a market structure, our model provides a microfoundation for how and why political and market outcomes are codetermined. Indeed, in contrast to

[4]A more distant connection is to the small literature that combines industrial organization with organizational economics (Barron & Powell (2018) provide an overview). In particular, Powell (2019) focuses on commitment and how the interplay of current and future rents affects market performance.

[5]Besley & Coate (1998) present a related idea in a repeated elections model in which private investment and public tax decisions are interrelated.

[6]Shleifer & Vishny (1994) develop a related model of state owned firms in which the government directly participates in the market.

the literature, formal political power in our model is not contested—it is always held by the same policymaker. What changes instead is the *value* of policy making power itself. As the conditions of market competition change, as the leading firm gains a capability-based advantage, the value of political power declines, and we show how the stakeholders wage a contest to manage that balance.

A benefit to modeling the market microstructure is that we can better identify where inefficiencies come from and how they can be corrected. We pinpoint the mechanism through which higher competitive pressure in the market may augment political influence in that market. By identifying this specific mechanism of codetermination of market and political outcomes, we can consider policies to address market failures and improve overall efficiency.

The feedback loop between politics and markets has recently come into focus in the empirical literature, as most clearly and forcefully articulated in Zingales (2017) (see Philippon (2019) and Wu (2018) for related book-length treatments). Zingales provides many historical examples of inefficient outcomes caused by market participants' ability to capture political power. He coins the phrase a "Medici vicious cycle" to describe a positive feedback loop and identifies six broad factors that drive this feedback.[7] We develop a formal model of market and political competition that complements Zingales and we identify a novel channel through which the feedback loop operates. By including the policymaker as a strategic self-interested player, we show that the feedback loop is bounded, that it varies in the structure of market competition, and that it can potentially be harnessed to improve rather than harm market efficiency. At a more abstract level, the insights from our model reinforce and put structure to Zingales's (2017) argument that a 'goldilocks' balance is required between the power embedded in politics and in markets for the system to have any hope of efficient and fair progress.

2 The Model

The environment consists of two firms, indexed $i \in \{1, 2\}$, and a policymaker, P. In each period $t = 1, 2, \ldots$ the firms compete in the market and lobby the policymaker for protection.

The Market: Competition is Cournot in each period. Firm i in period t chooses quantity q_t^i, where $q_t^i = 0$ if the firm does not compete in the market. Total market quantity is then $Q_t = q_t^1 + q_t^2$. Market demand is constant across periods and the inverse demand function is given by:

$$p = a - b \cdot Q.$$

[7] Zingales's (2017) six factors are: the main source of political power, the conditions of the media market, the independence of the prosecutorial and judiciary power, the campaign finance laws, and the dominant ideology.

Each firm has a technology level τ that allows it to produce at constant marginal cost $\mu(\tau)$. We denote technology levels by l and f for the leader and follower firms, respectively, where $l \geq f$. The state of the market in period t is then $(l_t, f_t) \in \mathbb{R}_+ \times \mathbb{R}_+$. Within-period Cournot profits are $\pi^L(l, f)$ and $\pi^F(l, f)$, respectively. A monopolist's profit at technology level l is denoted $\hat{\pi}^M(l)$.

The firms can improve their technology level through investment. Investment incurs a fixed cost $c(\tau) > 0$ that is increasing and convex with $c'(\tau) \to \infty$ as τ gets large. Technological advancement is deterministic and one-step per investment.[8] The step sizes in technology are small in the sense that $\hat{\pi}^M(l) > \pi^L(l+1, f)$ for all f; that is, the leader prefers to have the follower firm excluded from the market than advance a technology level and have to compete.

We generally consider the situation in which both firms begin at technology level 0, although the analysis holds should the market begin at any state of technology. Indeed, one can view a different starting state as resulting from a disruptive innovation, after which the model describes incremental competition thereafter.

Political Influence: The policymaker can intervene in the market and impose a minimum technology standard. The standard can be adjusted from period to period. It is outcome relevant only if it separates the firms.[9] When a standard is imposed, the trailing firm is excluded from the market, earning zero profit, and the leader obtains monopoly power.[10]

The protected firm pays rents to the policymaker, which we assume to be a fixed share of the value of protection. The value is the difference between monopoly and duopoly profits, which for the policymaker's share $\rho \in [0, 1]$ and technology levels l and f, gives rents of:

$$\pi^P(l, f) = \rho \cdot \left[\hat{\pi}^M(l) - \pi^L(l, f)\right]. \tag{1}$$

The protected firm's profit is then monopoly profit less rents:

$$\pi^M(l, f) = \hat{\pi}^M(l) - \pi^P(l, f), \tag{2}$$

which, by construction, exceeds the duopoly profit. Note that the policymaker and leading

[8]Step-by-step advancement is standard in the literature; see Aghion et al. (2005). In the appendix we prove that our results are robust to stochastic advance in capability.

[9]It could, in principle, exclude both firms. As that delivers zero rents to the policymaker and is a dominated strategy, we set this possibility aside.

[10]Formally, investment in our model is a cost reduction and so we model the regulatory intervention as a technology or capability standard. Modeling investment as a quality improvement on the final goods would permit an analogous application to quality floor regulations. As Tirole (1997, p. 389) points out, "a product innovation can generally be regarded as a process innovation—imagine that the new product existed prior to the innovation, and that the innovation simply reduced its production cost."

firm cannot commit to a rent-sharing agreement beyond the present period. We return to this important assumption in the discussion.

Timing & Equilibrium: The timing of the play within each period is as follows. For $l_t > f_t$:

1. *Investment*. The leading firm invests ($i_t = 1$) or not ($i_t = 0$) and the state is $(l_t + i_t,\ f_t)$.

2. *Protection*. The policymaker imposes a technology standard ($a_t = 1$) or not ($a_t = 0$).

3. *Market competition*. The firms compete (if $a_t = 0$) or L is a monopolist (if $a_t = 1$).

4. *Transition*. The state in period $t + 1$ will begin at $(l_t + i_t,\ f_t + 1 - a_t)$.

The transition in stage 4 implies that the trailing firm moves up one technological step whenever it competes in the market. This captures the idea that catch-up growth is easier than frontier-expanding innovation. The follower firm can more easily imitate the leading firm than the leader can come up with new ideas. A version of this assumption appears in many other models of competition and innovation, such as in the influential work by Aghion et al. (2005) and Bessen & Maskin (2009). We assume this catch-up growth is dependent on market participation reflecting the fact that much innovation, and even imitation, comes from market interactions and experience, as documented in the literature on learning-by-doing. Our results not require this distinction to be so sharp, nor that catch-up growth is guaranteed. We require only that catch-up growth is more likely when a firm is in the market than outside. This is important as it implies political protection impacts the market in two ways: It removes competition *and* it restrains technological catch-up by the trailing firm. Both aspects will play a role in our analysis.

When the firms are equal technologically and $l_t = f_t$, nature selects in step 1 one of the firms to invest, and play proceeds identically otherwise. This is a simple tie-breaking rule that creates the opportunity for capability gaps to open up between the firms.[11]

Competition and the Incentive to Invest: The incentives of firms to invest depend on market structure and political intervention. In a purely market setting, Arrow (1962) argues that the incentive to invest is lower in monopoly than with competition. This has come to be known as the Arrow replacement effect (Tirole 1997) and led to an enormous amount of research on the impact of competition on investment and innovation. In our model, as in Arrow (1962), only a single firm has the opportunity to invest and, by so doing, it lessens the degree of competition with the trailing firm, thereby "escaping competition" (Aghion et al.

[11] Alternatively, if success is stochastic we could allow both firms the opportunity to invest as this would permit a capability gap to open up.

2005).[12] Empirical evidence strongly points to competition increasing the incentive to invest and innovate in this context (Shapiro 2012, Holmes & Schmitz 2010).

Arrow's effect is intuitive although it does not follow directly from Cournot competition, and so we impose the following condition on relative profits.

Assumption 1: $\frac{\partial}{\partial l}\hat{\pi}^M(l) \leq \frac{\partial}{\partial l}\pi^L(l, f)$.

Arrow's effect is simply that, for a technology level l, the marginal gain from stepping up a level is higher for the duopolist than the monopolist. The duopolist improves efficiency *and* gains market share from its competitor. This implies that the gap between profits in monopoly and duopoly is narrowing. That as the leader's technology level grows, competition restrains its profits to a lesser degree.[13]

This property is important as it the gap in profit between monopoly and duopoly that determines the rents paid to the policymaker. The policymaker receives a share of the value of protection, which is exactly this difference in profit. That this gap declines in the leader's technology level implies, therefore, that the policymaker's rents also decline in leading firm's capability level.

Assumption 1 is for a duopolist and a pure monopolist. The case of a protected monopolist—who shares rents with the policymaker—lies between these cases. The fixed proportion rent sharing rule we assume implies, immediately from Assumption 1, that the incentive to invest of a protected monopolist satisfies: $\frac{\partial}{\partial l}\hat{\pi}^M(l) \leq \frac{\partial}{\partial l}\pi^M(l, f) \leq \frac{\partial}{\partial l}\pi^L(l, f) \leq 0$ for each f, and increases from the lower to the upper bound as the policymaker's share of the surplus of protection, ρ, varies from zero to one.

Final Details: The policymaker and the firms discount utility at rates, δ and β, respectively. Throughout our analysis the policymaker is far-sighted with $\delta \in (0, 1)$. For simplicity, we present the model when the firms are short-sighted ($\beta = 0$). In the appendix we establish the robustness of the results for any $\beta \in (0, 1)$. Note that the firms receive the benefit of investment within a period, so even when myopic, investment can have a positive return.

We identify a renegotiation proof equilibrium and fully characterize it. We prove in the appendix that the equilibrium is unique within this class and that it has the structure of a Markov Perfect Equilibrium. We refer to it as the equilibrium throughout the paper.

[12] Shapiro (2012) provides a thorough overview of these ideas. He identifies the critical property of market *contestability*—that the prospect of gaining or protecting profitable sales spurs innovation. This property holds for investment in our model.

[13] To the extent that competition is relaxed completely for a large enough capability gap (if, for example, the monopoly price for the leading firm is below the trailing firm's cost of production), then the Arrow effect *must* hold for at least large parts of the technology range.

3 Market Incentives

To illuminate the market incentives in the model we begin by shutting down the policymaker as a strategic actor. We consider two benchmarks. One in which the policymaker does not exist or, equivalently, never intervenes in the market, and a second in which the policymaker always intervenes to protect the leading firm.

The Policymaker Never Intervenes: Without political intervention, both firms compete in each period and the market is a duopoly. A firm invests if the improvement in capability increases profit enough to justify the cost. For firms with a single period horizon, investment is profitable if:

$$\pi^{L}(l+1, f) - c(l) \geq \pi^{L}(l, f). \tag{3}$$

The decision to invest depends on the capability level of the leader as well as the follower. This generates a threshold capability level for the follower, denoted by $\mathrm{IC}_D(l)$, at which equality holds in (3) and the leader is indifferent between investing and not. We then have the following result.

Lemma 1 *The leader invests if and only if* $f < IC_D(l)$*, where* $\frac{d}{dl} IC_D(l) < 0$*.*

The leading firm's willingness to invest is decreasing in its own capability level and also the capability of the trailing firm. The more capable is the leader itself, the cost of further advancement is higher and the increase in profit it produces is decreasing. The impact of the trailing firm's capability level is solely through the market effect. As the trailing firm catches up to the leader, competition is more intense and the leader is able to capture less of the value of its investment and, thus, is less willing to invest. The $\mathrm{IC}_D(l)$ threshold is depicted in the left panel of Figure 1, where each point in the positive quadrant corresponds to a state (l, f).

The figure also depicts the dynamic path of the market when starting at the origin. One firm invests in the first period, becoming the market leader. In every subsequent period that firm invests, advancing one level, and the following firm also advances while remaining one step behind. This continues until the state reaches the $\mathrm{IC}_D(l)$ threshold, at which point the leader no longer finds it worthwhile to invest and stops. The follower catches up one final step and the market stabilizes at equal technology levels, as marked by the dot.

The Policymaker Always Intervenes: In this case the leading firm benefits from political protection in every period and operates as a monopolist. Investing at capability level l is profitable if:

$$\pi^{M}(l+1, f) - c(l) \geq \pi^{M}(l, f). \tag{4}$$

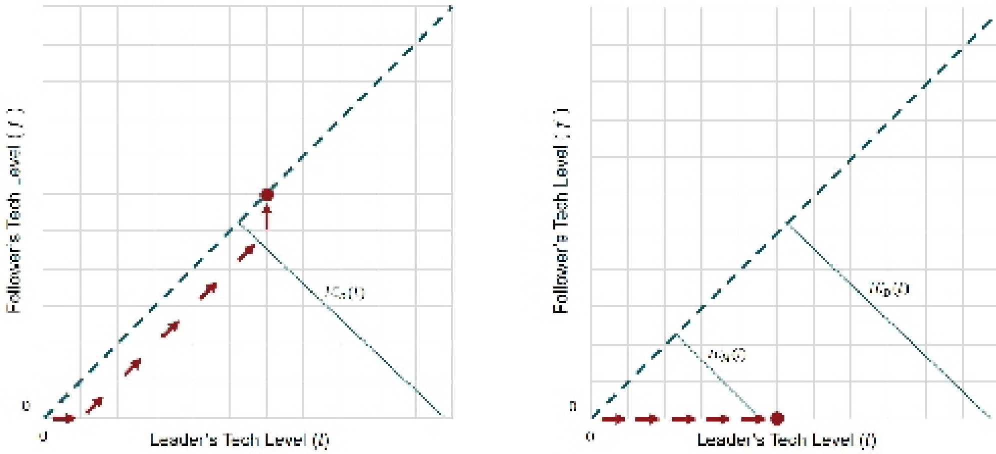

Figure 1: Market Incentives when the Policymaker (i) Never Intervenes, (ii) Always Intervenes

Although the leader is a monopolist whether it invests or not, the profitability of investment depends on the follower's capability level. This is because we are considering a protected monopolist. The leader pays rents to the policymaker proportional to the value of protection, and this depends on profitability should the leader have to compete. As in the duopoly case, this leads to a threshold in investment, denoted by $\mathrm{IC}_M(l)$, at which equality holds in (4) and the leader is indifferent between investing and not.

Lemma 2 *The leader invests if and only if* $f < IC_M(l)$, *where* $\frac{d}{dl} IC_M(l) < 0$ *and* $IC_M(l) < IC_D(l)$.

This is depicted in the right-side panel of Figure 1. The threshold is downward sloping as it is for duopoly. The leader is more willing to invest the further behind is the following firm as it then pays smaller rents to the policymaker and captures more of the efficiency gains of investment. The leader's willingness to invest is lower than in duopoly, as implied by Arrow's effect, and the protected monopolist stops investing earlier than does the duopolist.

The dynamic path of the market moves only horizontally (as the follower is never in the market and never catches up). Starting at the origin, the market moves along the horizontal axis and stabilizes at the first capability level beyond the IC_M threshold, as marked by the dot.[14]

[14] This steady state is inefficient relative to that for duopoly as the market is uncompetitive. It also involves a lower level of investment than duopoly when ρ, the share of surplus that goes to the policymaker, is sufficiently small. We consider welfare comparisons in detail in Section 4.2.

4 Market & Political Equilibrium

To market competition we now add the strategic policymaker. The policymaker will choose to protect only when it is in her interests. Protection delivers rents today, but it also excludes the trailing firm from the market and this gives the leading firm an opportunity to advance its capability advantage, which lowers the policymaker's rents in future periods. The policymaker's optimal strategy depends, therefore, on the investment decisions of the firms which, in turn, depend on the policymaker's decision to protect or not. The market and political equilibrium is the balance between these different incentives.

Our main result is that in equilibrium this balance leads to the worst of both worlds. The policymaker's effort to extract rents from the leading firm causes that firm to stop investing when it is at a low capability level, often at a level strictly lower than in duopoly and even monopoly. Moreover, the policymaker protects the leader in every period. The equilibrium path, therefore, is inefficient both within period and across periods. When the policymaker protects and the leader does not invest, the market stabilizes and remains in a steady state thereafter.

Proposition 1 *Suppose there exists* $\hat{l} \geq 0$ *such that* $\pi^P\left(\hat{l}, 0\right) = \delta \cdot \pi^P\left(\hat{l}, 1\right)$.[15] *Then, beginning at the origin, the steady state is* $(l^*, 0)$, *where* $l^* \leq \min\left\{IC_M^{-1}(0), \hat{l}\right\}$.

The dynamic equilibrium path starting at the origin is depicted in Figure 2. The steady state is given by the green triangle. The proposition establishes that this steady state is at or lower than the investment level under protected monopoly, given by the red dot. As depicted, the steady state may be strictly lower than the monopoly level and the under investment caused by political intervention can be severe.

The equilibrium path represents a positive feedback loop between markets and politics. One firm gains an initial capability advantage and uses that advantage to obtain political protection that is parlays into a larger capability advantage. The entire market outcome, including the steady state, is preordained once the identity of the firm with the initial advantage is realized.[16]

The positive feedback loop does raise the question of why investment stops at such a low capability level. If a monopolist invests at this capability level, why wouldn't a protected monopolist invest?

[15] See Appendix for the result where $\pi^P\left(\hat{l}, 0\right) < \delta \cdot \pi^P\left(\hat{l}, 1\right)$ for all $\hat{l} \geq 0$.

[16] The path dependence of the equilibrium is particularly stark here and there is no overtaking in equilibrium. This strict determinancy can be relaxed with the addition of appropriate noise without upsetting the core intuition of the result.

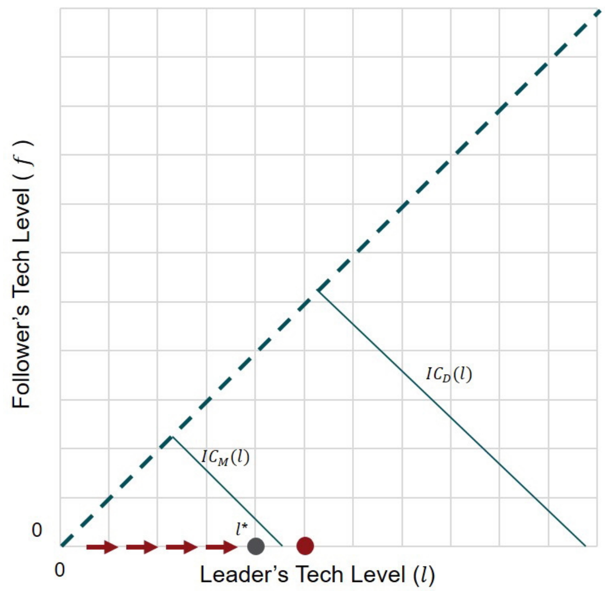

Figure 2: Market and Political Equilibrium

The reason why investment stops is because the feedback loop turns negative. A crucial feature of the equilibrium is that at state $(l^* + 1, 0)$ the equilibrium calls for the policymaker to not protect. Therefore, if the leader obtains one more step of capability-based market power, the feedback loop will reverse and the firm's political-based market power will be removed.

It is at this state that the policymaker tries to "manage competition." At this state she decides that forgoing rents today is worth the benefit of allowing the trailing firm to stay in touch with the leader. By not protecting, the policymaker ensures a higher degree of potential competition tomorrow that will, therefore, allow her to extract higher rents. The policymaker manages competition not for competition's sake but to ensure her own relevance.

Managed competition undermines investment as it generates a reverse Arrow effect. Because the policymaker will remove protection at state $(l^* + 1, 0)$, the leading firm anticipates at state $(l^*, 0)$ that investment will cause it to lose protection and switch from a protected monopolist to a duopolist. This flips Arrow's logic on its head. Contra Arrow, investment does not reduce competition—it doesn't allow the firm to "escape competition"—and, in fact, it actually increases competition. This pro-competition effect suppresses investment and induces market stagnation at a low level of firm capability.

To this point we have explained why the steady state exists given the equilibrium behavior at higher states, but we haven't yet explained why that equilibrium behavior is what it is. If the leader instead invested at state $(l^*, 0)$, won't the policymaker be tempted to take the rents on offer? Why is her threat to not protect credible? Why does the simple condition in Proposition 1 reflect a one-period trade-off for the policymaker when she is far-sighted?

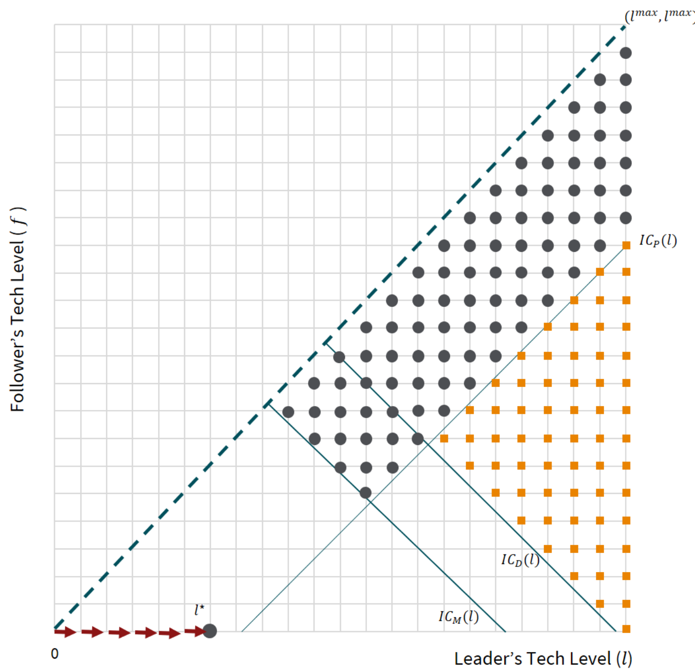

Figure 3: Equilibrium Behavior for High Capability States

The answers to these questions depend on the full structure of the equilibrium. The policymaker is credible in her desire to manage competition because of what she anticipates in the future. To see what that is, we need to solve the model via backward induction. This produces the full characterization of equilibrium, allowing for an understanding of dynamic paths starting at any state. The details are provided in the appendix. We focus here on the intuition that generates the path in Proposition 1.

It is helpful to begin with the policymaker's incentives. The condition on π^P in Proposition 1 is the limit case of a general condition, given by the following:

$$\pi^P(l, f) = \delta \pi^P(l, f+1). \tag{5}$$

This defines, for each l, the level of f at which the policymaker is indifferent between the rents available today from protection and the higher rents available tomorrow should she not protect and the follower catches up one step. Denote this critical value by $\mathrm{IC}_P(l)$, reflecting that this is the policymaker's incentive constraint. It is straightforward to show that the threshold $\mathrm{IC}_P(l)$ is strictly increasing in l and that above the threshold the policymaker prefers to take rents today whereas below the threshold she is willing to be patient and prefers the higher discounted rents tomorrow. This threshold, along with the investment thresholds for the firms, are depicted in Figure 3.

A one period trade-off for the policymaker matters because of equilibrium steady states.

At a high enough capability level, the leading firm will not invest regardless of the state of competition and political protection.[17] Denote this state by $l^{\max}$. It follows that state $(l^{\max}, l^{\max})$ is stable. Neither firm invests, the policymaker cannot protect, and there is no catch-up in capabilities. From here, it follows that state $(l^{\max}, l^{\max}-1)$ is also stable. If the policymaker doesn't protect the follower catches up, the state transitions to $(l^{\max}, l^{\max})$, and the policymaker never obtains rents again. It is therefore optimal to protect today to obtain rents $\pi^P(l^{\max}, l^{\max}-1)$, and as the leader doesn't invest at capability level $l^{\max}$, state $(l^{\max}, l^{\max}-1)$ is stable.

The one-period trade-off represented by the threshold $\mathrm{IC}_P(l)$ matters as we backward induct from this point. Consider state $(l^{\max}, l^{\max}-2)$. If the policymaker does not protect she receives no rents today, the state transitions to $(l^{\max}, l^{\max}-1)$, which is stable, and she receives rents $\pi^R(l^{\max}, l^{\max}-1)$ tomorrow and every period thereafter. If she protects, she receives rents $\pi^P(l^{\max}, l^{\max}-2)$ this period, and as this state is then stable, the same rents thereafter. Her choice reduces down to the simple one-period comparison of $\pi^P(l^{\max}, l^{\max}-2)$ and $\delta\pi^P(l^{\max}, l^{\max}-1)$.[18]

This trade-off recurs as we backward induct for lower values of f and the leader at $l^{\max}$. This leads to a unique transition point in equilibrium behavior, as depicted in Figure 3. Above the value of $\mathrm{IC}_P(l^{\max})$, the policymaker protects, takes rents today, and each state is stable. For lower levels of f, the policymaker is more patient, does not protect, forgoes rents, and allows the follower to catch up one step. These states are not stable, therefore, and the state transitions until it reaches the first state above the $\mathrm{IC}_P(l)$ threshold, at which it stabilizes.

Backward inducting from here is relatively straightforward for the upper regions of the state space. Equilibrium behavior is shown in Figure 3. For states above the $\mathrm{IC}_P(l)$ threshold and above the monopoly threshold $\mathrm{IC}_M(l)$, the firm does not invest, the policymaker protects in equilibrium, and the state is stable, as represented by green circles. Below the $\mathrm{IC}_P(l)$ threshold and above the duopoly threshold $\mathrm{IC}_D(l)$, the firm doesn't invest and the policymaker does not protect in equilibrium. These states are not stable, represented by blue circles.

Throughout this region the leader does not invest in steady states because it is protected if it does not and, because the state is above the $\mathrm{IC}_M(l)$ threshold, investment as a monopolist is not profitable.[19] The leader likewise does not invest when it isn't protected because doing so will only transition (to the right) to another unprotected state and, above the $\mathrm{IC}_D(l)$ threshold, investment as a duopolist is not profitable. This behavior by the firms supports the equilibrium

[17] That such a state exists follows from our assumptions on the cost of investment in capability.

[18] Protection delivers the stream of rents $\sum \delta^t \pi^P(l^{\max}, l^{\max}-2) = \frac{1}{1-\delta}\pi^P(l^{\max}, l^{\max}-2)$, whereas not protecting delivers rents $\sum \delta^{t+1}\pi^P(l^{\max}, l^{\max}-1) = \frac{\delta}{1-\delta}\pi^P(l^{\max}, l^{\max}-1)$.

[19] This holds even though there are reachable states that we have yet to characterize equilibrium behavior for; i.e., states just below $\mathrm{IC}_D(l)$ and $\mathrm{IC}_P(l)$. As the state is above $\mathrm{IC}_M(l)$ and the policymaker protects, it follows that the leader does not invest regardless of the policymaker's behavior should the leader not invest.

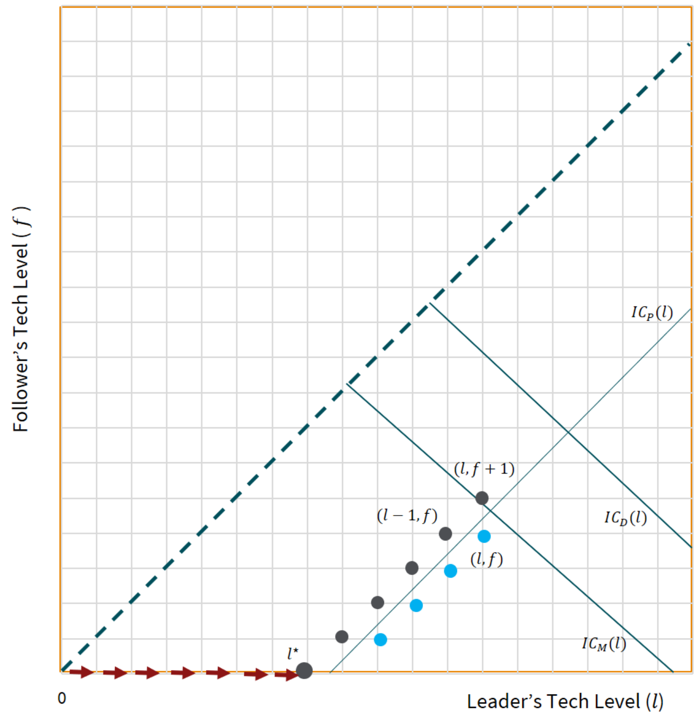

Figure 4: Market and Political Equilibrium – Backward Induction

behavior by the policymaker. Throughout this region she faces the same trade-off as she faces when the leader's technology level is $l^{\max}$. Either protect and take rents today and thereafter, or forgo rents today to receive a higher stream of rents beginning tomorrow. Consequently, the single-period trade-off represented by $\text{IC}_P(l)$ demarcates behavior throughout the region.

We are now in a position to explain the genesis of the reverse Arrow effect. Consider the steady state $(l, f+1)$ immediately above the $\text{IC}_M(l)$ and $\text{IC}_P(l)$ thresholds, as marked in Figure 4. Now consider state (l, f) that is below both thresholds. The policymaker will not protect at this state because she prefers to defer rents today, allow the state to transition to $(l, f+1)$, and receive $\pi^P(l, f+1)$ in every period thereafter.

This is important not for what the leading firm will do at state (l, f), rather it is important for what the firm can expect if it invests when at state $(l-1, f)$. If the firm invests at $(l-1, f)$ it knows it will not be protected. The state is below the monopoly, and thus the duopoly, threshold, and investment without protection remains profitable. This choice, however, must be compared to what happens should the leader not invest at $(l-1, f)$.

If it does not invest, the leader anticipates that the policymaker will protect. Not protecting will transition to the steady state $(l-1, f+1)$ and a higher stream of rents beginning tomorrow, whereas protecting will earn rents today and cause the present state, $(l-1, f)$, to be stable. It is optimal for the policymaker to protect as the state is above the $\text{IC}_P(l)$ threshold and her trade-off is the same as discussed above for higher states.[20]

This simple calculus creates the reverse Arrow effect. The leading firm would invest at state $(l-1,\ f)$ if it were sure of being a monopoly or duopoly regardless of its investment. But its investment causes the level of competition to change by changing whether the policymaker imposes protection or not. If the leader invests, protection is removed and competition is high, whereas if it doesn't invest, protection is applied and the firm has monopoly power. As the value of monopoly outweighs the benefit of a higher capability level, the firm doesn't invest. This is the reverse of Arrow's classic argument and reveals the distortion to market outcomes from political intervention.

The behavior at state $(l-1,\ f)$ is interesting but by itself of marginal impact. Its true importance is in how, by backward induction, it affects behavior at preceding states. The logic of the reverse Arrow effect recurs. Because state $(l-1,\ f)$ is stable, the policymaker will not protect at state $(l-1,\ f-1)$ and, in turn, this induces the leader to not invest and for the policymaker to protect at state $(l-2,\ f-1)$.

We establish in the proof that this unraveling continues all the way to the horizontal axis where the trailing firm has a capability level of zero. The unraveling argument is more subtle when the slopes of thresholds $\mathrm{IC}_M(l)$ and $\mathrm{IC}_P(l)$ are not as neatly arranged as they are in Figure 4, but nevertheless complete unraveling always occurs.

The steady state in Proposition 1 depends on where the $\mathrm{IC}_P(l)$ and $\mathrm{IC}_M(l)$ thresholds intersect the horizontal axis and involves the same or a lower level of investment than the minimum of these two values. In the case in which $\mathrm{IC}_P(l)$ intersects at a lower level, as is depicted in Figure 4, the level of investment can be substantially below that for a protected monopolist.[21]

4.1 Relaxing Competitive Pressure

The insight of "managed competition" is that the policymaker seeks market competition purely for the threat value. She allows competition only because it enables the follower firm to catch up and increase the threat of further competition. In this sense, competitive pressure translates not into more efficient markets, but into leverage for the policymaker to extract rents. This induces the reverse Arrow effect that undermines market efficiency.

The lesson from this is that standard intuitions about market competition need not hold

[20]This argument does not depend on the particular arrangement of states relative to the thresholds depicted in the figure. The policyumaker protects today if the leader invests at state $(l-1,\ f+1)$ as forgoing rents today does not cause the follower to close the capability gap with the lader and increase rents tomorrow; we make this argument precise in the appendix.

[21]Investment at the steady state may be strictly lower than both thresholds depending on the nature of unraveling in equilibrium.

when markets and politics are intertwined. Counterintuitively, therefore, it may be that outcomes are improved if the degree of competition is relaxed. We explore this possibility in this section.

One dimension of competitive pressure is the willingness of the trailing firm to enter into the market and compete if political protection is removed. It is a striking feature of the equilibrium in Proposition 1 that the trailing firm never competes in the market yet nevertheless stands ever ready to do so. Although this is a reasonable description of some markets (e.g., foreign competitors and trade barriers), it is less appropriate in other markets, and one might think that the trailing firm will, at some point, give up and abandon the market altogether.

To formalize this idea, we amend the model as follows. We suppose that a firm that has been *excluded* from the market for κ consecutive periods will permanently *exit* the market. That is to say, if a firm has not been allowed to compete for κ periods it gives up and pursues opportunities elsewhere.

This assumption captures competitive pressure in a way that is simple and realistic and that resonates with the interdependence of politics and markets that is the focus of our paper.[22] A firm exits the market if political intervention is excessive, but if it does so, the value of political protection disappears, costing the policymaker her leverage over the remaining firm. Although this is a simple variant, it complicates the analysis considerably. The state space is now the technology levels of the firms plus the number of periods of consecutive protection. It is possible for the firms to remain at technology levels for multiple periods before advancing. We say a state is steady only if the capability levels have been stable for κ periods and are permanently stable. For this environment we characterize the steady states of market competition but do not provide a full description of the equilibrium path.

A market with potential exit changes the incentives of the policymaker. The policymaker must now remove protection at least once every κ periods else she loses her leverage. The wisdom of doing this is clear when the difference in capability levels of the firms is more than a single step. In this case, after protecting for $(\kappa - 1)$ periods, the policymaker faces a simple trade-off: Protect and receive rents for a final period or forgo rents today, allow competition, and renew a fresh stream of rents for κ periods. Indeed, if the leader doesn't invest while protection is off, tomorrow's rents are certain to be higher. For κ and δ not too small, the benefit of waiting is clear.[23] This implies, therefore, that the market cannot stabilize unless the firms' capability levels are close.

Lemma 3 *For κ and δ sufficiently large, the firm capability pair (l, f) is not a steady state if $f < l - 1$.*

[22]We consider several other measures of competitive pressure in the discussion section.

[23]Similar forces are at work for κ small or δ small, although the analysis is more complicated and identifying steady states would require the full characterization of the equilibrium path.

The need for the policymaker to refresh competition also changes the incentives of the firms. Because in some periods the leading firm knows that protection will be removed *regardless* of whether it invests, the reverse Arrow effect is relaxed in those periods. In those periods, the leader invests knowing it will compete and, therefore, the relevant threshold is that of duopoly, $IC_D(l)$. This is not to say the reverse Arrow effect does not bind in other periods when protection is a choice for the policymaker, only that in some periods it is relaxed, and that is enough to ensure that eventually market investment advances to the duopoly level.

Lemma 4 *For κ and δ sufficiently large, the firm capability pair $(l,\ f)$ is not a steady state if $f\ < IC_D(l)$.*

Combining the two lemmas provides a broader picture of equilibrium behavior. Lemma 3 shows that a steady state must be either on or adjacent to the 45 degree line where firm capabilities are equal, and Lemma 4 shows that a steady state cannot exist below the duopoly threshold.

The logic of Lemma 3 falters within one step of the 45 degree line as if the leader doesn't invest when protection is removed, the state transitions to the 45 degree line and the technology standard has no bite. This matters because the policymaker cannot extract any rents without being able to protect one firm. The optimal behavior of the policymaker in this case depends, then, on the investment strategy of the leading firm.

A reasonable conjecture is that investment stops as soon as the duopoly threshold is passed. Were this true, the policymaker would, upon first reaching a state $(l, l-1)$ beyond the 45 degree line, take the k^{th} period of rents and let the follower exit the market, all to avoid reaching the 45 degree line and stagnation.

We show, however, that this does not occur as the conjecture about investment is not true. In this environment, the firms are willing to invest beyond the duopoly threshold, although only on the 45 degree line when their capability levels are equal. The reason for this comes, again, from Arrow. However, the logic of Arrow is not reversed in this case, rather it is enhanced. The firms, if given the chance, are willing to invest on the 45 degree line because investment is the only way they can obtain protection.

To see this, note again that the policymaker cannot protect when capabilities are equal as the firms cannot be separated by a technology standard. Should one firm invest and attain an advantage, however, then protection is possible and the policymaker can extract rents. This means that a firm will benefit from protection if—and only if—it invests. This enhances the standard Arrow effect as not only is competition reduced by investment, it is entirely eliminated and the investing firm becomes a monopolist. We refer to this as the *politically enhanced Arrow effect*.

In this case, investment is profitable for a firm on the 45 degree line if:

$$\pi^{M}(l+1, l) - c(l) \geq \pi^{L}(l, l). \tag{6}$$

This is similar to the conditions for duopoly and monopoly in Equations (3) and (4), respectively. The difference here is that the firm receives the profit of a protected monopolist when it invests but the duopoly profit otherwise. Denote by IC_{EA} for 'enhanced Arrow' the threshold level at which (6) is satisfied by equality.

The enhanced Arrow effect applies only on the 45 degree line and, thus, the IC_{EA} threshold is defined only in that case, as depicted in Figure 5. With firms willing to invest on the 45 degree line, it implies that the logic of Lemma 3 holds one step away from the 45 degree line as long as condition (6) holds. This delivers the following result.

Proposition 2 *With relaxed competitive pressure, for κ and δ sufficiently large, every steady state is given by $(l^{**}, l^{**}-1)$ for some $l^{**} > IC_{EA} - 1$, and the follower firm exits the market.*

This result can be seen in Figure 5. It depicts a potential dynamic path for the equilibrium in which investment passes the duopoly threshold with the leading firm holding a large capability advantage. Beyond the duopoly threshold the equilibrium behavior becomes clear. The leader no longer finds it profitable to invest and the state transitions vertically until reaching the 45 degree line. Progress to this point is staggered, with stretches of protection and temporary stability interspersed with periods of competition as the policymaker renews her leverage. As this path intersects the 45 degree line below the threshold IC_{EA}, the policymaker is happy to let the trailing firm fully catch up. She knows, through the enhanced Arrow effect, that the firms will invest on the 45 degree line when given the opportunity. This creates a ratchet effect as the state moves off the 45 degree line and back to it repeatedly, with investment increasing along the path. This sequence finally ends once the IC_{EA} threshold is crossed. The steady state, $(l^{**}, l^{**}-1)$, is off the diagonal and the trailing firm permanently exits the market. At this state the policymaker protects the leader for a full κ periods and accepts the exit of the following firm as she knows that, should she remove protection and let the follower catch up, neither firm will invest any more, the technology standard will not have any bite, and she wouldn't be able to extract any more rents.

The steady state is striking for what it implies about competition and protection. In contrast to Proposition 1, the leading firm is not protected in the steady state. Moreover, it faces no competition—as the trailing firm exits the market—and it has attained a high level of capability. To an observer, this outcome would suggest a firm that has earned market dominance from having a high capability. However, the full equilibrium path belies this interpretation. Political intervention is a mainstay along the equilibrium path and the final outcome is prede-

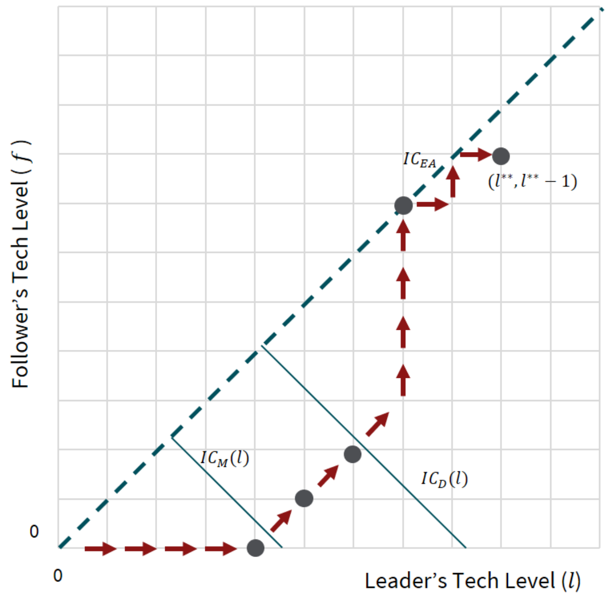

Figure 5: Relaxed Competitive Pressure: Equilibrium Path with Enhanced Arrow Effect

termined once the initial advantage is obtained even when, as in our model, it is determined by luck.[24] This is not to deny the efficiency of the final steady state, but it does imply that fairness had little to do with it.

4.2 Welfare

In the steady states of both Propositions 1 and 2 there is no competition and the leading firm operates as a monopolist; in Proposition 1 because the leader is protected politically, and in Proposition 2 because the trailing firm has exited the market. Therefore, a welfare comparison between these states depends only on the level of investment by the leading firm.

We establish here that the steady state in Proposition 2 strictly dominates that in Proposition 1 when ρ, the share of surplus going to the policymaker, is sufficiently small. In this case, weaker competitive pressure leads to strictly greater market efficiency.

This comparison is not immediate more generally—even though the steady state in Proposition 2 is above the duopoly threshold and the steady state in Proposition 1 is below the monopoly threshold—as the thresholds themselves are downward sloping. However, for ρ

[24]With myopic firms this conclusion depends on the tie-breaking rule when the state returns to the 45 degree line. If tie-breaking is random then the crucial stroke of luck for the firms is the final random selection of a firm to invest rather than the first. Our preferred tie-breaking rule is that the firm that was leading previously is given the opportunity to invest. This ensures the importance of the stroke of luck at the origin of the market. We prefer this rule as it reflects the market outcome when firms are forward-looking. If the tie-breaking rule at the 45 degree line were random, a leading firm that is forward-looking would invest preemptively to always maintain its lead and avoid being subject to that randomization.

sufficiently small the welfare ranking is clear. In this case the thresholds are sufficiently separated that the right-most point of the monopoly threshold involves a smaller capability level than the left-most point of the duopoly threshold.

The logic for this result follows from the original Arrow effect. Assumption 1 requires the monopolist to have a lower incentive to invest than a duopolist, regardless of the follower firm's capability level. This assumption is for unregulated markets, however, whereas the $\text{IC}_M(l)$ constraint of Lemma 2 is for a regulated monopolist who shares rents with the policymaker. As discussed in Section 2, the incentive to invest in this case is higher than for the unconstrained monopolist. The following strengthens Assumption 1 to cover this case.[25]

Assumption 1': $\frac{\partial}{\partial l}\left((1-\rho)\,\hat{\pi}^M(l)+\rho\pi^L(l,\,f)\right)\leq\frac{\partial}{\partial l}\pi^L\left(l,\,f'\right)$ for all $l,\ f,\ f'$.

This assumption converges to Assumption 1 as ρ converges to zero. This implies that the difference in incentive between a protected and an unconstrained monopolist disappears for ρ small. Conversely, therefore, if Arrow's replacement effect holds, then for values of ρ sufficiently small the steady state in Proposition 1 is strictly dominated by that in Proposition 2. We state this formally as follows. Recall that the leader's steady state capability level is l^* in Proposition 1 and l^{**} in Proposition 2.

Proposition 3 *Suppose the premises of Propositions 1 and 2 hold. When Assumption 1' additionally holds, $l^{**}>l^*$.*

The range of ρ values for which $l^{**}>l^*$ holds is broad. Assumption 1' is a sufficient condition for this to be true but it is not necessary. Assumption 1' implies the stronger result that the leading firm's capability level in duopoly strictly dominates that in monopoly; that is, it guarantees that the leader's investment is strictly higher in Lemma 1 than in Lemma 2. This isn't necessary for $l^{**}>l^*$ as l^* may be strictly less than the monopoly threshold and l^{**} is strictly above the duopoly threshold. Indeed, as ρ decreases, the threshold IC_{EA} (that provides the lower bound for l^{**}) increases, whereas the monopoly threshold decreases, expanding the potential gap between investment levels in the steady states.

Proposition 3 implies that relaxing competitive pressure can improve market efficiency when that market is intermediated by a strategic policymaker. This ordering reflects a balance of distortions. When competitive pressure is reduced there is a direct negative effect on welfare through the standard economic forces (if the competitor disappears the market switches from competitive to monopoly). In addition there is an indirect political effect, which is that the power of the policymaker is weakened. Both forces affect market efficiency, with the

[25] Recall that the profit for the regulated monopolist can be rearranged as $\pi^M(l,\,f)=(1-\rho)\,\hat{\pi}^M(l)+\rho\pi^L(l,\,f)$.

economic distortion decreasing efficiency whereas the political distortion increases it. Proposition 3 establishes that the latter effect dominates when the policymaker's ability to extract rents is not too great.

Proposition 3 compares the steady states but not the path to reach these points. A welfare analysis that includes the path only strengthens the ranking in Proposition 3. Along the path the market is competitive for many periods when competition is relaxed in Proposition 2, whereas it is never competitive on the path in Proposition 1.[26] The same is true for the comparison of duopoly and an always-protected monopoly (Lemma 1 vs. Lemma 2). Comparison of duopoly in Lemma 1 with the relaxed competition case in Proposition 2 is less clear as duopoly delivers constant competition but a lower capability level for the leading firm.[27]

5 Discussion

Model Robustness. Our model provides a simple framework to illustrate a mechanism through which market and political outcomes are linked. This mechanism requires only some basic ingredients: a policymaker who extracts rents by intervening in markets and where those rents depend on the state of market competition. These ingredients are sufficient to incentivize the policymaker to want to manage competition. Our model captures this mechanism and we explored how the mechanism manifests in two different market contexts. The exact nature of the equilibrium depends on the details of the model, and changing those details will, naturally, lead to different equilibria. The underlying mechanism that we identify remains relevant more broadly and, to this end, we discuss here (and consider formally in the appendix) several variations on our basic model. We begin with variations of the market environment, before turning to the political environment.

Far-sighted firms. The results of our model are qualitatively unchanged if we relax the assumption that firms are myopic (see the appendix for details). One impact of far-sighted firms is standard: the firms have a stronger incentive to invest as they internalize the long-run benefit, and the investment thresholds for both duopoly and monopoly increase. A second impact is more subtle. A far-sighted firm anticipates the policymaker's desire to manage competition. The firm cannot avoid the policymaker managing competition, and this continues to distort the firm's incentive to invest. Nevertheless, investment does not stop immediately when the policymaker first manages competition as the firm may invest through some of these periods. When the firm is forward looking it has preferences over which state proves stable,

[26] This comparison is unambiguous under our assumption that catch-up growth is costless. A small cost would not upset this conclusion, although such a cost may alter the equilibrium itself; see the discussion in the following section.

[27] We do not pursue this comparison in detail here as comparison of these cases is not our focus.

and it strategically invests to move the market to that point before. Ultimately, the policymaker's desire to manage competition puts a stop to investment, and does so at, or below, the investment level of a protected monopolist.

Catch-up costs. Anticipating the policymaker's desire to manage competition can also be relevant to the follower firm. We have assumed that the follower catches up to the leading firm automatically when it participates in the market. This may hold in some industries, where imitation is easier and more prevalent, but is less descriptive of other industries. A cost of investment makes it more difficult for the follower firm to challenge the leader and represents a decrease in competitive pressure. We show in the appendix that this change does not overcome the inefficiency of Proposition 1. Although the steady state investment level may improve, it remains bounded by the level for an always-protected monopolist (Lemma 2).

The logic for this result again flows through managed competition and the reverse Arrow effect. In the equilibrium of Proposition 1, the follower is poised to enter and close the technological gap with the leading firm. However, a forward-looking follower anticipates that it will never be allowed to fully catch up. That once it gets close technologically, the policymaker will again install protection and extract rents. Being reluctant to play the foil perennially, this dampens the follower's incentive to invest and compete in the market, to a greater extent than if even if the market were a standard duopoly. In turn, this undermines the ability of the policymaker to manage competition. She forgoes rents should she remove protection, but with the follower not investing, she does not gain greater rents tomorrow. The leader can then see that the policymaker's threat to remove protection no longer binds in the same way.

This result does not depart from the equilibria of Proposition 1 as radically as that in Proposition 2. This reinforces the conclusion that while political intervention into a market may upend standard intuitions, it does not reverse them. It is not that lower competitive pressure necessarily leads to more efficient market outcomes. Rather, that the effect is contingent on how it affects the incentives of the policymaker and that thinking through her incentives is a necessary step to understanding market competition.

Partially self-interested policymaker. We have assumed for simplicity that the policymaker cares only about the rents she can extract from protection. In precluding market competition, political protection harms consumer surplus, and a socially (or electorally) minded policymaker would weigh that effect against the rents she receives. Our results are robust to such motivations for the policymaker. In the appendix we show that all of our results hold if the weight the policymaker places on consumer welfare is small (that is, if the policymaker is sufficiently selfish). The conclusion for general policymaker preferences is more difficult to pin down. The mechanism that we identify in our main model carries through to this general case, although as the cost of protection for the policymaker then varies in the firms' capability levels, her willingness to protect will vary more the greater weight she places on consumer welfare.[28]

Policymaker tools. Our model is built on one particular tool that is available to the policymaker—a minimum standard. This allows us to see how the value of that tool—and the balance of power between the policymaker and firms—ebbs and flows depending on the degree of market competition. In practice, other tools are also available to policymakers, and the predictions of the model will depend on the nature of these tools, and not just the degree of market competition.[29]

The policymaker's toolkit can be used to serve the industry's interests or those of the policymaker. Stigler (1971) provides the example of an ascendant oil industry using regulatory power to not only capture government subsidy but also protect industry from new entrants.[30] A more common example is the creation of licensing standards that protect incumbents by making entry costly.

The ideal tool for a policymaker is one that maintains a heightened threat of entry. One such regulatory structure would allow the follower firm some but not total access to the market, thereby allowing the policymaker to extracts rents from protection while keeping the follower firm incentivized to invest and not exit the market altogether. Such a market may seem competitive—the follower would appear as a competitive fringe—viewed through the lens of our model, limited market access like this can be seen as serving only the interests of the policymaker. We discuss below the example of the home construction industry that exhibits such a form of restrained competition.

Political bargaining. The balance of power between the firms and the policymaker depends also on the bargaining protocol. In Stigler's (1971) original view, it was the industry cartel that held all of the bargaining power, making demands of policymakers and extracting all of the benefit of political protection. McChesney (1987) shows that if instead the policymakers were proactive and could make demands of the firms, then they would extract all of the surplus. Reality lies somewhere between these extremes. We have sought to thread the gap by fixing the relative bargaining power with the parameter ρ, which we model in reduced form. The size of ρ reflects many factors, including the willingness of the policymaker to accept a bribe or the firm to pay a bribe, of the cost to the policymaker of protection, as well as the degree of political competition. We set this to be a constant for simplicity. Fixing bargaining power also enables us to focus on how outcomes vary in the degree of market competition, independent of other institutional features. Modeling the process by which institutional design influences the bargaining power of a policymaker and, therefore, how that feeds through to

[28] In this case it is possible that this change would feed through to a change in ρ. See the discussion of ρ below.

[29] To see this, one need only see the focused attention Washington received from the tech industry once the prospect was raised of directly regulating or breaking up the largest firms.

[30] Government mandated barriers to entry also offer the advantage of avoiding antitrust scrutiny.

market outcomes, offers many interesting possibilities.

The Model and Practice. We have developed and analyzed a stylized model that steps back from specific contexts in order to capture a mechanism that is broadly applicable. One takeaway from this exercise is that the details do matter, that while the underlying mechanism may appear in many settings, the outcome it produces depends on the details of market and political competition. The key variable is the extent of the competitive threat that the policymaker can hold over a firm or industry. When this threat is high, the power of political intervention is high, and the policymaker's impact on markets is its most extensive.

International competition and trade protection offers perhaps the cleanest example of a market in which the threat of competition is unwavering. The historical example of the East India Company illustrates this structure. Throughout the long history of the company, competitors were eager to begin trade but blocked by the grant of monopoly to the company (Erikson 2016, Zingales 2017). A striking feature of this history is that the grant of monopoly was repeatedly threatened but consistently renewed, and each renewal was made only once those with political power had extracted some surplus. This cycle repeated up until the time that the Company's interests clashed directly with those of the Crown. The history of the East India Company resonates with the equilibrium in Proposition 1 and the policymaker's need to retain relevance in the bargaining relationship with the market leader.

Other examples of markets in which competitive pressure is high are those with neighboring industries that employ similar technology. A firm with a profitable home market remains a threat to neighboring markets even when excluded for lengthy periods of time. A similar threat emerges for industries that are geographically separated. A striking example of this is the home construction industry (Schmitz 2020). In this case the distinction is between industry segments rather than individual firms, with the relevant actors the trade association for each segment. The stick-built housing sector (homes constructed on-site), through their trade association, the National Association of Home Builders (NAHB), has successfully won political protection for over a century from competition from the factory-built (modular) housing industry, with regulations imposing minimum standards on housing quality that effectively block factory-built housing other than in specially designated areas (e.g., trailer parks). Interestingly, and in accordance with our model, the ability of the factory-built industry to operate at the margins of the industry and in some isolated locations, ensured that it remained a competitive threat to the stick-built industry throughout this time.

The connection between market dominance and political protection is also evident at the aggregate level. Faccio & McConnell (2020) provide evidence from 75 countries showing that when an incumbent dominates a market for an extended period of time it is most frequently due to political protection. This resonates with our equilibrium prediction of industry lock-in, that an early advantage is a strong predictor of long-term dominance. Evidence from the trade literature shows, consistent with these findings, that the frequency and intensity of lobbying varies with the degree of product market competition (Bombardini & Trebbi 2012, Kim 2017).

Our model reinforces the idea that political intervention into markets has real impacts on firms beyond the degree of competition in markets. Evidence of real distortions in resource allocation within firms is more difficult to identify, although it has begun to accumulate. Huneeus & Kim (2020) show that politically connected firms in the U.S. attract capital and grow larger. From a different angle, Aghion, Bergeaud & Reenen (2021) begin with a specific labor regulation in France and show how it distorts the development of firms at both the intensive and extensive margin of innovation.[31]

Policy Implications. In practice, the connection between the market's competitive structure and political influence is a tricky identification problem. Is concentration good because of investment and technological breakthrough? Or is concentration bad because of lobbying and other anti-competitive behavior? In allowing for both sources of market power, our model provides a theoretical structure to inform that decision. A core insight of our model is that standard market analyses may not hold up when political intervention is allowed for. Pointing to smaller players in a market to support a merger is less compelling once it is understood that the competitive fringe may serve the interests of a policymaker rather than consumers.

That political power and monopoly power are intertwined has been long acknowledged in practice, and particularly so in discussions around one type of policy, that of antitrust. Franklin Roosevelt's head of the Antitrust Division in the Department of Justice, Thurman Arnold, wrote in 1943 that:[32]

> "Monopolies enter into politics using money and economic coercion to maintain themselves in power, making alliances with other powerful groups against the interests of consumers and independent producers."

This connection has fallen out of focus over the years, and only recently has it that it has returned to the forefront of economists' and policymakers' minds. Wu (2018) and Philippon (2019) argue that lobbying and lax antitrust enforcement are a causal factor in the increased market concentration of recent decades (see also Gutierrez & Philippon (2019)).

The challenge for antitrust policy is that it is not the only relevant policy intervention. In practice, other policymakers have the ability to intervene in markets, less dramatically perhaps but no less significantly. Our analysis emphasizes that when antitrust policy is set, even if it

[31] At the macro level, Parente & Prescott (1999) show how concentration of market power restrains economic development.

[32] Arnold (1943); quoted in Schmitz (2020).

is evaluated through the lens of welfare maximization, it must account for the intervention of other, potentially less socially minded, policymakers into the market.

Another implication of our model for antitrust policy is that a static analysis is insufficient to understand the degree to which market concentration is good or bad. As noted in the previous section, the steady state in Proposition 2 describes a leading firm that has attained a high capability and faces no market competition. It does not benefit from political protection at the steady state, but only because it doesn't need it. It got to this position, however, due to protection along the equilibrium path. For instance, therefore, even though the political patronage that a dominant company receives today does not seem to matter so much to its market performance and its lobbying expenditures are relatively small, it does not follow that political power was not instrumental to its rise, helping critically when the technology gap was smaller and the competitive threat greater.

An intriguing corollary to this observation is that it may provide a rationale for the long-standing puzzle as to why there is so little money in politics (Ansolabehere, de Figueiredo & Snyder 2003). Although large amounts are spent, the absolute levels are small given what is at stake. Proposition 2 describes a case in which, at the steady state, there is no money spent on politics, not because the firms wouldn't spend it and political intervention couldn't be important, but because the market has reached a state of asymmetry in capability-based market power that political intervention has simply lost its relevance. The equilibrium is stark, to be sure, although in aggregate it resonates with practice. This suggests that insight into lobbying, as well as market competition, may come from more closely matching political spending to the state of market competition and the threats policymakers hold over firms.[33]

6 Conclusion

The focus of the political economy literature is on the choice of a policy, in which political power varies in the design of institutions and the identity of those who make the decisions. We have shown in this paper that the value of political power—the power of politics itself—varies when the market environment varies. For a fixed set of political tools, the command of policymakers over the economy and society can itself vary. This, in turn, changes the power of business in society. Eighty years ago, Thurman Arnold (1943) argued that the power of business had grown to such an extent that it subsumed part of the functions of the state:

> "In short, they will become a sort of independent state within a state, making treaties and alliances, expanding their power by waging industrial war, dealing

[33] Suggestive evidence in this regard is that the industries with the highest political expenditures anecodtally are those with high capital expenditures, where competition is fierce and political advantage most valuable. Examples include the airline and package delivery (FedEx/UPS) industries.

> on equal terms with the executive and legislative branches of the government and defying governmental authority if necessary with the self-righteousness of an independent sovereign."

This point has been picked up recently by Zingales (2017) from the perspective of the firm and Wu (2018) in critiquing the application of antitrust policy.[34] Our paper builds a formal structure that allows more detailed analysis of the problem and potential remedies.

The core insight of our model is that when markets and politics coevolve, the interests of firms and the policymaker are aligned but not perfectly aligned. This has ramifications for the outcomes in both domains. We build a model to capture these incentives and characterize the outcomes they produce. The model distills this mechanism to its essence, and many practical details are left out or included in a reduced form. The essential elements are that the policymaker cares about rents and the firms care about market power. The market and political outcomes then reflect how these forces balance out. Adding richness to the model will affect this balance and add nuance to the predictions, to be sure, but not fundamentally change the logic for how markets and politics interact.

There are many natural ways to extend our model beyond those discussed in the previous section. In practice there is more than a single policymaker who compete for rents, and the hierarchy of authority—from legislator down to regulator—creates an agency problem, opening up the question of where and not just how much firms lobby and transfer rents. The number of firms, the nature of competition, and the motivations of the policymaker are all promising directions for follow on work.

A particularly intriguing possibility is to explore a balance between political and economic goals. In our model political power is the means to the ends of market power. Some important political goals stand aside from economic outcomes, and market power may be the means toward those ends. Exploring the interdependence of politics and markets more deeply is of considerable importance.

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[34]Zingales (2017, p.117) argues that economics has erred in focusing on the technological rather than the power dimension of firms. Wu (2018), channeling Thurman Arnold and Louis Brandeis, argues for a return to an antitrust policy in which political concerns are dominant.

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Appendix

A Proofs from Section 3

A.1 Proof of Lemma 1

The duopoly profit is $\pi^L(l, f) = \left[(a - 2\mu(l) + \mu(f))^2\right]/9b$. Notice that

$$\operatorname{sign}\left(\pi^L_{lf}(l, f)\right) = -\operatorname{sign}\left[\mu'(l) \cdot \mu'(f)\right] \leq 0.^{35} \tag{7}$$

We define $IC_D(l)$ such that, for each l, at $f = IC_D(l)$,

$$\pi^L(l+1, f) - c(l) = \pi^L(l, f). \tag{8}$$

Then, conditions (7) and (8) imply that the leader invests if and only if $f < IC_D(l)$. In addition, $c'(l) > 0$ implies $\frac{d}{dl} IC_D(l) < 0$.

A.2 Proof of Lemma 2

The monopoly profit is $\hat{\pi}^M(l) = (a - \mu(l))^2/4b$. The policymaker's rent is $\pi^P(l, f) = \rho\left(\hat{\pi}^M(l) - \pi^L(l, f)\right)$, and the leader's payoff is

$$\pi^M(l, f) = \hat{\pi}^M(l) - \pi^P(l, f).$$

Notice that, by (7),

$$\operatorname{sign}\left(\pi^M_{lf}(l, f)\right) = \operatorname{sign}\left(\frac{d^2}{dldf}\left(\hat{\pi}^M(l) - \pi^P(l, f)\right)\right) = \operatorname{sign}\left(\pi^L_{lf}(l, f)\right) < 0. \tag{9}$$

We define $IC_M(l)$ such that, for each l, at $f = IC_M(l)$,

$$\pi^M(l+1, f) - c(l) = \pi^M(l, f). \tag{10}$$

Then, conditions (9) and (10) imply that the leader invests if and only if $f < IC_M(l)$. Moreover, since $c'(l) > 0$, we have $\frac{d}{dl} IC_M(l) < 0$.

[35] Each subscript denotes that the derivative is taken with respect to that variable.

B Proof of Proposition 1

B.1 Preliminaries

We prove Proposition 1 in a more general setup, where the policymaker's utility is a convex combination of the rent and social welfare (the relative weight on social welfare is not zero), with weight α on social welfare. Now the policymaker's utility depends on the leader's technology level l, the follower's technology level f, and the policymaker's choice $a \in \{0, 1\}$ such that

$$\pi^P(l, f, a) = \begin{cases} \alpha \cdot SW^D(l, f) & \text{if } a = 0 \text{ (no protection)} \\ \alpha \cdot SW^M(l) + (1-\alpha) \cdot \pi^P(l, f) & \text{if } a = 1 \text{ (protection),} \end{cases}$$

where $SW^D(l, f)$ denotes social welfare under duopoly given (l, f), and $SW^M(l)$ denotes social welfare under monopoly given l. The payoffs for the firms stay the same as in the main text. The product market is Cournot, which leads to

$$\begin{aligned} SW^M(l) &= \frac{3}{8b}(a - \mu(l))^2; \\ SW^D(l) &= \frac{(a - 2\mu(l) + \mu(f))^2}{9b} + \frac{(a - 2\mu(f) + \mu(l))^2}{9b} + \frac{1}{18b}(2a - \mu(l) - \mu(f))^2. \end{aligned}$$

Derivatives

We assume $\mu''(t) = 0$ for all t. We list the derivatives of the profit functions that will be referenced in the proofs that follow:[36]

$$\begin{aligned} \text{sign}\left(\pi^P_{l,f}(l, f)\right) &= \text{sign}\left(\frac{d^2}{dldf}\rho\left(\hat{\pi}^M(l) - \pi^L(l, f)\right)\right) \\ &= -\text{sign}\left(\pi^L_{lf}(l, f)\right) \\ &= -\text{sign}\left(\frac{d^2}{dldf}(a - 2\mu(l) + \mu(f))^2\right) \\ &= \text{sign}\mu'(l)\mu'(f) > 0. \end{aligned} \tag{11}$$

$$\begin{aligned} \text{sign}\left(\pi^M_{lf}(l, f)\right) &= \text{sign}\left(\frac{d^2}{dldf}\left(\hat{\pi}^M(l) - \pi^P(l, f)\right)\right) \\ &= -\text{sign}\left(\pi^P_{lf}(l, f)\right) = \text{sign}\left(\pi^L_{lf}(l, f)\right) < 0. \end{aligned} \tag{12}$$

[36] Each subscript denotes that the derivative is taken with respect to that variable.

$$\pi_l^P(l, f, 0) = -\alpha \cdot \frac{1}{9b} \cdot (4a - 4\mu(l) + 7(\mu(f) - \mu(l))) \cdot \mu'(l) > 0. \tag{13}$$

$$\pi_f^P(l, f, 0) = -\alpha \cdot \left(2\frac{a - 2\mu(l) + \mu(f)}{9b} + 4\frac{a - 2\mu(f) + \mu(l)}{9b}\right. \tag{14}$$
$$\left. + \frac{2}{18b}(2a - \mu(l) - \mu(f))\right)\mu'(f) > 0.$$

$$\pi_f^P(l, f, 1) = -(1 - \alpha) \cdot \rho \cdot 2 \cdot \frac{(a - 2\mu(l) + \mu(f))}{9b} \cdot \mu'(f) > 0. \tag{15}$$

$$\pi_{lf}^P(l, f, 0) = \alpha \cdot \frac{7}{9b} \cdot \mu'(f) \cdot \mu'(f) > 0.$$

$$\pi_{lf}^P(l, f, 1) = (1 - \alpha) \cdot \rho \cdot \frac{4}{9b} \cdot \mu'(l) \cdot \mu'(f) > 0. \tag{16}$$

$$\pi_{ff}^P(l, f, 1) = -(1 - \alpha) \cdot \rho \cdot \frac{2}{9b} \cdot \left(\mu'(f)\right)^2 < 0. \tag{17}$$

Policymaker and Investment Thresholds

For the policymaker, for each l, we define $IC_P(l)$ such that, at $f = IC_P(l)$,

$$\frac{\pi^P(l, f, 1)}{1 - \delta} = \pi^P(l, f, 0) + \delta \cdot \frac{\pi^P(l, f + 1, 1)}{1 - \delta}. \tag{18}$$

That is, at $f = IC_P(l)$, the policymaker is indifferent between protecting the leader or not-protecting the leader in the current period (given that she will protect the leader from next period on and the leader's technology level is not changing).

Next, given an Markov perfect equilibrium, for each l, we define $IC_P^*(l)$ as the set of the follower's technology levels f such that

$$\pi^P(l, f, 0) + \delta \cdot V^P(l, f + 1) \leq \frac{\pi^P(l, f, 1)}{1 - \delta},$$

where V^P is the equilibrium value for the policy maker.

Finally, for each l, we define $IC_P^{**}(l)$ such that, at $f = IC_P(l)$,

$$\pi^P(l, f, 1) + \frac{\delta}{1 - \delta} \cdot \pi^P(l + 1, f, 1) = \pi^P(l, f, 0) + \frac{\delta}{1 - \delta} \cdot \pi^P(l, f + 1, 1).$$

That is, at $f = IC_P^{**}(l)$, the policymaker is indifferent between protecting the leader or not-protecting the leader currently (given that she will protect the leader from next period on), even if (i) the current protection makes the leader invest and (ii) the current non-protection makes the leader not invest.

Let $\bar{L}$ be the smallest l such that there does not exist f such that

$$\pi^{M}(l+1, f) - c(l) \geq \pi^{L}(l, f). \tag{19}$$

The myopic leader never invests if $l \geq \bar{L}$. Hence, we will focus on $(l, f) \in [0, \bar{L}]^2$ and omit this condition throughout the appendix.

Conditions on parameters

We analyze the model under the following assumptions:

A1 For each (l, f), a stronger leader reduces the policymaker's utility:

$$\pi^{P}(l, f, 1) > \pi^{P}(l+1, f, 1). \tag{20}$$

A2 For each (l, f), the benefit of a stronger follower is higher when the policymaker protects the leader:

$$\pi_{f}^{P}(l, f, 0) - \pi_{f}^{P}(l, f, 1) < 0, \tag{21}$$

and the cost of a stronger leader with protection is higher than the benefit of a stronger leader without protection (under proper discounting):

$$(1-\delta)\,\pi_{l}^{P}(l, f, 0) + \delta\pi_{l}^{P}(l, f+1, 1) < 0. \tag{22}$$

A3 For each Markov perfect equilibrium and for each (l, f) with $f \geq IC_{P}(l)$, the policymaker prefers the leader not investing:

$$\frac{\pi^{P}(l, f, 1)}{1-\delta} > V^{P}(l+1, f). \tag{23}$$

A4 For each (l, f), if investment leads to competition while non-investment leads to monopoly, the leader does not invest:

$$\pi^{L}(l+1, f) - c(l) - \pi^{M}(l, f) < 0. \tag{24}$$

A5 For each f, given the smallest l with $f \geq IC_{P}(l)$,

$$f - 1 \leq IC_{P}^{**}(l). \tag{25}$$

First, we will show that conditions (20)–(23) hold for sufficiently small $\alpha > 0$.

Lemma B.1 *Suppose Assumption 1 (in the main text) holds. Then, for sufficiently small* $\alpha > 0$*, for each* (l, f)*, the conditions (20)–(23) hold.*

Proof. All proofs for the auxiliary lemmas are in Section B.4. ■

Second, we will show conditions (24) and (25) hold if we consider a proper limit of the step-size getting smaller. Suppose we consider the model where the step size is Δ and discount factor is $\delta = e^{-r\Delta}$. We assume that the cost of investment is proportional to Δ: to increase l to $l + \Delta$, the cost is $c(l)\Delta$. Given the model where the step size is Δ, we can create another model with step size 1 with the same strategic incentive: simply re-define the marginal cost at technology level at t in the new model as the marginal cost at technology level Δt in the model with step size Δ and re-define cost of investment from technology level l to $l+1$ in the new model as the cost of investment from the technology level Δl to $\Delta l + \Delta$ in the original model. Given this equivalence between the models, we will show that our assumptions hold in the model with a small step size Δ.

In particular, in the model with step size Δ, the threshold $IC_{P,\Delta}(l)$ is the solution for

$$\frac{\pi^P(l, x, 1)}{1 - e^{-r\Delta}} = \pi^P(l, x, 0) + e^{-r\Delta}\frac{\pi^P(l, x + \Delta, 1)}{1 - e^{-r\Delta}}.$$

As $\Delta \to 0$, we have $IC_{P,\Delta}(l) \to IC_{P,0}(l)$, where $IC_{P,\Delta}(l)$ is the solution for

$$r\left(\pi^P(l, x, 1) - \pi^P(l, x, 0)\right) = \pi^P_f(l, x, 1). \tag{26}$$

Similarly, we have $IC^{**}_{P,\Delta}(l) \to IC^{**}_{P,0}(l)$, where $IC^{**}_{P,0}$ is the solution for

$$r\left(\pi^P(l, x, 1) - \pi^P(l, x, 0)\right) = -\pi^P_l(l, x, 1) + \pi^P_f(l, x, 1). \tag{27}$$

For the leader, $\bar{L}_\Delta$ is the smallest l such that, for each f,

$$c(l)\Delta > \pi^M(l + \Delta, f) - \pi^L(l, f).$$

We have $\bar{L}_\Delta \to \bar{L}$, where $\bar{L}$ is the solution for

$$c(l) = \max_{f \le l}\left(\pi^M(l, f) + \pi^M_l(l, f) - \pi^L(l, f)\right).$$

We restrict attention to $(l, f) \in \left[0, \bar{L} + \varepsilon\right] \times \left[0, \bar{L} + \varepsilon\right]$ since, given Δ, we restrict attention to $(l, f) \in \left[0, \bar{L}_\Delta\right] \times \left[0, \bar{L}_\Delta\right]$.

In addition, $IC_{M,\Delta}(l)$ is the solution for

$$\pi^M(l+\Delta,\ f) - c(l)\,\Delta = \pi^M(l,\ f)\,.$$

We have $IC_{M,\Delta}(l) \to IC_{M,0}(l)$, where $IC_{M,0}(l)$ is the solution for $\pi_l^M(l,x) = c(l)$. Similarly, $IC_{D,\Delta}(l) \to IC_{D,0}(l)$, where $IC_{D,0}(l)$ is the solution for $\pi_l^L(l,x) = c(l)$.

The proper analogue for (24) and (25) is

$$\pi^L(l+\Delta,\ f) - c(l)\,\Delta - \pi^M(l,\ f) < 0 \tag{28}$$

and

$$IC^{**}_{P,\Delta}(l) \le IC_{P,\Delta}(l) - 2\Delta. \tag{29}$$

Lemma B.2 *For sufficiently small Δ, in the model with step size Δ, we have (24) and (25) for each $(l,\ f) \in \left[0, \bar{L}+\varepsilon\right] \times \left[0, \bar{L}+\varepsilon\right]$.*

Results about the Thresholds

We derive the two results about $IC_P(l)$. First, $IC_P(l)$ works as a proper threshold even if $\alpha > 0$.

Lemma B.3 *The following inequality is satisfied if and only if $f \ge IC_P(l)$:*

$$\frac{\pi^P(l,\ f,1)}{1-\delta} \ge \pi^P(l,\ f,0) + \delta\frac{\pi^P(l,\ f+1,1)}{1-\delta}.$$

Moreover, $\frac{d}{dl}IC_P(l) \ge 0$.

We next prove that $f = IC_P(l)$ crosses the 45-degree line at most once from above (when we represent l along the x-axis).

Lemma B.4 *There exists at most one l such that $IC_P(l) = l$. Moreover, at such l, $\frac{d}{dl}IC_P(l) < 1$.*

In addition, condition (23) implies

$$IC^*_P(l) \subseteq \{f : f \ge IC_P(l)\}. \tag{30}$$

We then prove the following result about $IC^{**}_P(l)$.[37]

[37] Recall that, at $f = IC_P(l)$, the policymaker is indifferent between protecting the leader or not-protecting the leader currently (given that she will protect the leader from next period on), keeping the leader's technology

Lemma B.5 *We have*

$$\pi^P(l,\ f,1)+\frac{\delta}{1-\delta}\pi^P(l+1,\ f,1)\geq\pi^P(l,\ f,0)+\frac{\delta}{1-\delta}\pi^P(l,\ f+1,1)$$

if and only if $f \geq IC_P^{**}(l)$. *Moreover, we have* $IC_P^{**}(l) > IC_P(l)$.

Finally, we prove that, if the leader always invests, then the policymaker protects the leader if $f \geq IC_P(l)$.

Lemma B.6 *If the leader always invests until* $l \leq \bar{L}$, *then protection is offered if* $f \geq IC_P(l)$.[38]

Cutoffs

We define the cutoff $(l_1,\ f_1)$ as the intersection of the 45-degree line with the curve

$$\{(l,\ f) : f = IC_P(l)\}.$$

If the intersection does not exist, then define $(l_1,\ f_1) = \emptyset$. We next define $(l_2, 0)$ as the intersection of the l-axis and $\{(l,\ f) : f = IC_P(l)\}$. If the intersection does not exist, then define $l_2 = \emptyset$.

Given Lemmas B.3 and B.4, we have exactly one of the following two conditions satisfied: $(l_1,\ f_1) = \emptyset$ or $l_2 = \emptyset$. If $(l_1,\ f_1) = \emptyset$, define $l_3 = l_2$. If $l_2 = \emptyset$, define

$$l_3 = \max_{l \geq l_1} l$$

level fixed. By contrast, at $f = IC_P^{**}(l)$, the policymaker is indifferent between protecting the leader or not-protecting the leader currently (given that she will protect the leader from next period on), even if (i) the current protection makes the leader invest and (ii) the current non-protection makes the leader not invest. This "even if" part makes protection less attractive, and hence the threshold $IC_P^{**}(l)$ requires that the follower is stronger compared to the threshold $IC_P(l)$.

[38] Here, l represents the leader's technology level upon the policymaker's protection decision. Since the leader is always investing, this is one step higher than the leader's technology level at the beginning of the period.

subject to: there exists $\left(\hat{l}, \hat{l}-1\right)$ with $\hat{l} < l_1$ and $\tilde{l}$ with $\hat{l}+1 \leq \tilde{l} \leq l_2$such that

$$\underbrace{\sum_{t=1}^{\tilde{l}-\hat{l}-1} \delta^{t-1} \pi^P \left(\hat{l}+t, \hat{l}-1+t, 0\right) + \delta^{\tilde{l}-\hat{l}-1} \frac{\pi^P \left(\tilde{l}, \tilde{l}-1, 1\right)}{1-\delta}}_{\text{the payoff when the state transitions to } \left(\hat{l}, \hat{l}-1\right) \to \left(\hat{l}+1, \hat{l}\right) \to \cdots \to \left(\tilde{l}, \tilde{l}-1\right)}$$

$$\leq \underbrace{\sum_{t=1}^{l-\hat{l}-1} \delta^{t-1} \pi^P \left(\hat{l}+t, \hat{l}-1, 1\right) + \delta^{l-\hat{l}-1} \frac{\pi^P \left(l, \hat{l}-1, 1\right)}{1-\delta}}_{\text{the payoff when the state transitions to } \left(\hat{l}, \hat{l}-1\right) \to \left(\hat{l}+1, \hat{l}-1\right) \to \cdots \to \left(l, \hat{l}-1\right)}.$$

For $l = l_1$, $\hat{l} = l_1 - 1$ and $\tilde{l} = l_1$ satisfy the constraint. Hence, l_3 is non-empty. In words, suppose the policymaker is at $\left(\hat{l}, \hat{l}-1\right)$, where $\hat{l}-1 \leq IC_P\left(\hat{l}\right)$. She knows that, if she keeps not protecting the leader, then the state transitions to $\left(\tilde{l}, \tilde{l}-1\right)$, before $\tilde{l}$ hits l_2 (that is, before the state exceeds the intersection between the 45-degree line and IC_P curve). She also knows that, if she keeps protecting the leader, then the state transitions to $\left(l, \hat{l}-1\right)$. The threshold l_3 is defined as a highest l with which the policy maker prefers the latter path.

To determine the l_3 threshold, notice that $\hat{l}-1 \leq IC_P\left(\hat{l}\right)$, so letting the follower grow is profitable. In addition, the leader keeps investing.[39] These two forces push l_3 down. At the same time, if the leader keeps growing, then it is profitable to extract rents before the leader gets stronger. This force pushes l_3 up.

Let l_4 be the solution for $IC_P(l) = IC_D(l)$. Again, if the solution does not exist in $\left[0, \bar{L}\right]$, define $l_4 = \emptyset$. There is at most one solution since $\frac{d}{dl} IC_P(l) \geq 0$ and $\frac{d}{dl} IC_D(l) \leq 0$ by Lemmas B.3 and 1.

B.2 Equilibrium Concept and Uniqueness

We consider the subgame perfect equilibrium that satisfies the following form of *renegotiation proofness*: after each history h (this can be either at the timing of the leader's investment decision or at the timing of the policymaker's protection decision), if there are two equilibria that are Pareto ranked for the policymaker and the leader, then we pick the Pareto efficient one. We show that the outcome of SPE satisfying renegotiation proofness is unique:

Lemma B.7 *The set of subgame perfect equilibrium payoffs that satisfy renegotiation proofness is unique after each* (l, f). *Moreover, in this renegotiation-proof subgame perfect equi-*

[39]The threshold $IC_P\left(\hat{l}\right)$ is calculated assuming that the leader's technology level stays at $\hat{l}$. As the leader's technology level increases, as $\frac{d}{dl} IC_P(l) \geq 0$, it is even more important to let the follower grow.

librium, the strategy is Markov and at each state (l, f), given the continuation play, if (non-investment, protection) is incentive compatible, then (non-investment, protection) is the equilibrium outcome.

Given this result, in what follows, we refer to "equilibrium" as the unique renegotiation proof SPE. Let eqm $(l, f) \in \{I, NI\} \times \{P, NP\}$ be the equilibrium outcome.

We state our main result as follows:

Proposition B.1 *There exists $l^* \leq l_3$ such that the leader's technology level at the steady state is l^* in equilibrium. In addition, the policymaker is protecting the leader in the steady state. Moreover, when $l_3 = l_2$, the steady state satisfies $(l^*, 0)$ with $l^* \leq l_2$.*

Note that, when $\alpha = 0$, this is the restatement of our main result in the main text. Note also that since the state space is effectively finite, the steady state exists. Moreover, the leader is protected at the steady state, as otherwise the follower's state would move. Thus, we will focus on proving that (i) $l^* \leq l_3$ in the steady state and (ii) $l_3 = l_2$, the steady state satisfies $(l^*, 0)$ with $l^* \leq l_2$.

For the rest of the proof, we will prove this proposition. For simplicity, we assume that there is no $(l, f) \in \mathbb{N}^2$ such that $f = IC_P(l), IC_M(l)$, or $f = IC_D(l)$. Without this assumption, all the proofs go through with more tedious tie-breaking analysis based on renegotiation proofness.

B.3 Equilibrium Dynamics

Classify the state (l, f) into the following three regions.

1. Region 1: $f \leq IC_P(l)$ and $f \geq IC_D(l)$. In this region, the leader does not invest and the policymaker does not protect.

2. Region 2: $f \geq IC_P(l)$ and $f \geq IC_M(l)$. In this region, the leader does not invest and the policymaker protects.

3. Region 3: $f \leq IC_P(l)$ and $f \leq IC_D(l)$ or $f \geq IC_P(l)$ and $f \geq IC_M(l)$. In this region, a complication arises.

We first prove that, in Region 1, eqm $(l, f) = (NI, NP)$.

Lemma B.8 *For each (l, f) with $f \leq IC_P(l)$ and $f \geq IC_D(l)$, then* eqm $(l, f) = (NI, NP)$.

Next, we will prove that, in Region 2, eqm $(l, f) = (NI, P)$.

Lemma B.9 *For each (l, f) with $f \geq IC_P(l)$ and $f \geq IC_M(l)$, then* eqm $(l, f) = (NI, P)$.

Region 3 For Region 3, we first show that, at state $(l,\ f)$, if $f \geq IC_P(l)$ and the policymaker does not protect the leader after his non-investment at state $(l+1,\ f)$, then the equilibrium outcome at $(l,\ f)$ is $(NI,\ P)$.

Lemma B.10 *For each* $(l,\ f)$ *satisfying* $f \geq IC_P\ (l)$*, if either* $\text{eqm}(l+1,\ f) \in \{(I,\ P),\ (I,\ NP)\}$ *and the policymaker does not protect after the leader's non-investment (deviation) or (ii)* $\text{eqm}(l+1,\ f) = (NI,\ NP)$*, then* $\text{eqm}(l,\ f) = (NI,\ P)$.

Next, we show that, once the policymaker protects the leader after his investment at $(l,\ f)$, then the protection will be on for the rest of the game.

Lemma B.11 *Suppose the policymaker protects the leader after his investment at* $(l,\ f)$*. Then, at* $(l+1,\ f)$*, we have either* $\text{eqm}\ (l+1,\ f) = (NI,\ P)$ *or* $\text{eqm}\ (l+1,\ f) = (I,\ P)$.

Further, at $(l,\ f)$ with $f \leq IC_P^{**}\ (l)$, if the leader does not invest once f increases to $f+1$, then at state $(l,\ f)$, the leader does not invest or he does but the policymaker does not protect the leader if he deviates to non-investment.

Lemma B.12 *For each* $(l,\ f)$*, if* $\text{eqm}\ (l,\ f+1) \in \{(NI,\ P),\ (NI,\ NP)\}$ *and* $f \leq IC_P^{**}\ (l)$*, then either* $\text{eqm}\ (l,\ f) \in \{(NI,\ P),\ (NI,\ NP)\}$ *or the policymaker does not protect the leader after non-investment at* $(l,\ f)$.

We show that, if the policymaker does not protect the leader after non-investment at $(l+1,\ f)$, then the policymaker does not protect the leader after investment at $(l,\ f)$. Note that the statement holds even if non-investment is an off-path action at $(l+1,\ f)$, or even if investment is an off-path action at $(l,\ f)$.

Lemma B.13 *For each* $(l,\ f)$*, if the policymaker does not protect the leader after non-investment at* $(l+1,\ f)$*, then the policymaker does not protect the leader after investment at* $(l,\ f)$*. Moreover, either the equilibrium outcome is non-investment at* $(l,\ f)$ *or the policymaker does not protect the leader after non-investment at* $(l,\ f)$.

The results in these lemmas lead to the following unravelling results (Proposition B.1 and hence Proposition 1):

Lemma B.14 *We have* $\text{eqm}\ (l_1, l_1 - 1) = (NI,\ P)$ *if* $l_1 \neq \emptyset$ *and* $\text{eqm}\ (l_2, l_2 - 1) = (NI,\ P)$ *if* $l_1 = \emptyset$.

When $l_3 = l_2$: $IC_P\ (l)$ does not intersect 45-degree line.

Lemma B.15 *When* $l_3 = l_2$*, the steady state satisfies* $(l^*, 0)$ *with* $l^* \leq l_2$.

When $l_3 \neq l_2$: $IC_P\ (l)$ intersects 45-degree line.

Lemma B.16 *When* $l_1 \neq \emptyset$*, the steady state* $(l^*,\ f^*)$ *satisfies* $l^* \leq l_3$.

B.4 Proofs of Auxiliary Lemmas

B.4.1 Proof of Lemma B.1

Since we focus on $(l,\ f) \in \left[0, \bar{L}\right]^2$ and all values are continuous in α, it suffices to show that given $\alpha = 0$, for each $(l,\ f)$, the conditions (20)–(23) assumed in A1–A3 hold. First, with $\alpha = 0$, Assumption 1 (in the main text) implies

$$\begin{aligned} 0 &> \left(\pi^M(l+1,\ f) - \pi^M(l,\ f)\right) - \left(\pi^L(l+1,\ f) - \pi^L(l,\ f)\right) \\ &= \hat{\pi}^M(l+1) - \hat{\pi}^M(l) - \rho\left(\hat{\pi}^M(l+1) - \pi^L(l+1,\ f)\right) + \rho\left(\hat{\pi}^M(l) - \pi^L(l,\ f)\right) \\ &\quad - \left(\pi^L(l+1,\ f) - \pi^L(l,\ f)\right) \\ &= \frac{1-\rho}{\rho}\left(\pi^P(l+1,\ f, 1) - \pi^P(l,\ f, 1)\right). \end{aligned}$$

Hence, (20) and (22) hold.

Second, with $\alpha = 0$, we have $\pi_f^P(l,\ f, 0) - \pi_f^P(l,\ f, 1) = -\pi_f^P(l,\ f, 1) < 0$ by (15). Hence, (21) holds.

Third, for (23), we will show that, for each $f \geq IC_P(l)$, we have $\frac{\pi^P(l,\ f,1)}{1-\delta} > V^P(l+1,\ f)$. Note that there exists $\{l_t,\ f_t, a_t\}_{t=0}^{\infty}$ such that $l_0 = l+1$ and

$$V^P(l+1,\ f) = \sum_{t=0}^{\infty} \delta^t \pi^P(l_t,\ f_t, a_t).$$

Define t_τ as the period in which the policymaker take $a_t = 1$ τ^{th} time (with the convention that $t_0 = -1$ and $f_{t_0} = f$). Then,

$$\begin{aligned} V^P(l+1,\ f) &= \sum_{\tau=1}^{\infty} \delta^{t_\tau} \pi^P\left(l_{t_\tau},\ f_{t_\tau}, 1\right) \text{ since } \alpha = 0 \\ &< \sum_{\tau=1}^{\infty} \delta^{t_\tau} \pi^P\left(l,\ f_{t_\tau}, 1\right) \text{ by (20).} \end{aligned}$$

Thus, defining $\tilde{V}^P(l,\ f) = \sum_{\tau=1}^{\infty} \delta^{t_\tau} \pi^P\left(l,\ f_{t_\tau}, 1\right)$, it suffices to show that $\frac{\pi^P(l,\ f,1)}{1-\delta} \geq \tilde{V}^P(l,\ f)$.

Since (i) f_t increases if and only if $a_t = 0$ and (ii) $t_\tau - t_{\tau-1} - 1$ is the number of periods with $a_t = 0$ between $t_{\tau-1}$ and t_τ, we can write

$$\tilde{V}^P(l,\ f) = \sum_{\tau=1}^{\infty} \delta^{t_\tau} \pi^P\left(l,\ f_{t_{\tau-1}} + t_\tau - t_{\tau-1} - 1, 1\right). \tag{31}$$

Since $f \geq IC_P(l)$, we have

$$\begin{aligned}\delta^{t_\tau}\pi^P\left(l,\ f_{t_{\tau-1}}+t_\tau-t_{\tau-1}-1,1\right) &= \delta^{t_{\tau-1}}\delta^{t_\tau-t_{\tau-1}}\pi^P\left(l,\ f_{t_{\tau-1}}+t_\tau-t_{\tau-1}-1,1\right)\\ &\leq \delta^{t_{\tau-1}+1}\pi^P\left(l,\ f_{t_{\tau-1}},1\right)\end{aligned}$$

and hence

$$\begin{aligned}\tilde{V}^P(l,\ f) &= \pi^P(l,\ f,1)+\delta\sum_{\tau=2}^{\infty}\delta^{t_{\tau-1}}\pi^P\left(l,\ f_{t_{\tau-1}},1\right)\\ &= \pi^P(l,\ f,1)+\delta\sum_{\tau=1}^{\infty}\delta^{t_\tau}\pi^P\left(l,\ f_{t_\tau},1\right)\\ &= \pi^P(l,\ f,1)+\delta\tilde{V}^P(l,\ f).\end{aligned}$$

Thus, we have

$$\frac{\pi^P(l,\ f,1)}{1-\delta}\geq\tilde{V}^P(l,\ f),$$

as desired.

B.4.2 Proof of Lemma B.2

First, (28) is equivalent to

$$\frac{\pi^L(l+\Delta,\ f)-\pi^L(l,\ f)}{\Delta}-c(l)-\frac{\pi^M(l,\ f)-\pi^L(l,\ f)}{\Delta}<0.$$

Note that the first term converges to $\pi^L_l(l,\ f)$ while the last term diverges to ∞ as $\Delta\to 0$. Since $\left[0,\bar{L}+\varepsilon\right]$ is compact, we have (28) for sufficiently small Δ. Since π^P is continuously differentiable and $\left[0,\bar{L}+\varepsilon\right]$ is compact, by (26) and (27), there exist Π and $\bar{\Delta}$ such that, for $\Delta\leq\bar{\Delta}$, we have

$$-\Pi\Delta\leq r\left(\pi^P\left(l,IC_{P,\Delta}(l),1\right)-\pi^P\left(l,IC_{P,\Delta}(l),0\right)\right)-\pi^P_f\left(l,IC_{P,\Delta}(l),1\right)\leq\Pi\Delta$$

and

$$-\Pi\Delta\leq\left(\begin{array}{c} r\left(\pi^P\left(l,IC^{**}_{P,\Delta}(l)+2\Delta,1\right)-\pi^P\left(l,IC^{**}_{P,\Delta}(l)+2\Delta,0\right)\right)\\ +\pi^P_l\left(l,IC^{**}_{P,\Delta}(l)+2\Delta,1\right)-\pi^P_f\left(l,IC^{**}_{P,\Delta}(l)+2\Delta,1\right)\end{array}\right)\leq\Pi\Delta.$$

Since $\pi_l^P(l, f, 1) < 0$ for all $(l, f) \in [0, \bar{L} + \varepsilon] \times [0, \bar{L} + \varepsilon]$, there exists $\eta > 0$ such that $\pi_l^P(l, f, 1) \leq -\eta$ for all $(l, f) \in [0, \bar{L} + \varepsilon] \times [0, \bar{L} + \varepsilon]$. Thus,

$$\begin{aligned} & r\left(\pi^P\left(l, IC^{**}_{P,\Delta}(l) + 2\Delta, 1\right) - \pi^P\left(l, IC^{**}_{P,\Delta}(l) + 2\Delta, 0\right)\right) - \pi_f^P\left(l, IC^{**}_{P,\Delta}(l) + 2\Delta, 1\right) \\ \leq \; & -\eta + \Pi\Delta. \end{aligned}$$

By taking Δ sufficiently small such that $-\eta + \Pi\Delta \leq -\Pi\Delta$, we have $IC^{**}_{P,\Delta}(l) \leq IC_{P,\Delta}(l) - 2\Delta$. Thus, (25) holds.

B.4.3 Proof of Lemma B.3

Note that

$$\begin{aligned} & \frac{d}{df}\left(\frac{\pi^P(l, f, 1)}{1-\delta} - \pi^P(l, f, 0) - \delta\frac{\pi^P(l, f+1, 1)}{1-\delta}\right) \\ = \; & \frac{d}{df}\left(-\left(\pi^P(l, f, 0) - \pi^P(l, f, 1)\right) - \delta\frac{\pi^P(l, f+1, 1) - \pi^P(l, f, 1)}{1-\delta}\right). \end{aligned}$$

Since $\frac{d}{df}\left(\pi^P(l, f, 0) - \pi^P(l, f, 1)\right) \leq 0$ by (21) and $\frac{d}{df}\left(\pi^P(l, f+1, 1) - \pi^P(l, f, 1)\right) \leq 0$ by (17), we have

$$\frac{d}{df}\left(\frac{\pi^P(l, f, 1)}{1-\delta} - \pi^P(l, f, 0) - \delta\frac{\pi^P(l, f+1, 1)}{1-\delta}\right) \geq 0.$$

Together with the definition of $IC_P(l)$, we have $\frac{\pi^P(l, f, 1)}{1-\delta} \geq \pi^P(l, f, 0) + \delta\frac{\pi^P(l, f+1, 1)}{1-\delta}$ if and only if $f \geq IC_P(l)$.Moreover, by the implicit function theorem, the slope of $IC_P(l)$ satisfies

$$\frac{d}{dl}IC_P(l) = \frac{(1-\delta)\left(\pi_l^P(l, f, 0) - \pi_l^P(l, f, 1)\right) + \delta\left(\pi_l^P(l, f+1, 1) - \pi_l^P(l, f, 1)\right)}{-(1-\delta)\left(\pi_f^P(l, f, 0) - \pi_f^P(l, f, 1)\right) - \delta\left(\pi_f^P(l, f+1, 1) - \pi_f^P(l, f, 1)\right)}. \tag{32}$$

Since $\pi_l^P(l, f, 0) - \pi_l^P(l, f, 1) \geq 0$ by (13) and (20), $\pi_{lf}^P(l, f, 1) \geq 0$ by (16),

$$\frac{d}{df}\left(\pi^P(l, f, 0) - \pi^P(l, f, 1)\right) \leq 0$$

by (21), and $\pi_{ff}^P(l, f, 1) \leq 0$ by (17), (32) is non-negative.

B.4.4 Proof of Lemma B.4

It suffices to prove that, if $IC_P(l) = l$, then, at such l, $\frac{d}{dl} IC_P(l) < 1$. The numerator of the fraction in (32) equals

$$(1-\delta)\cdot\left(\pi_l^P(l, f, 0) - \pi_l^P(l, f, 1)\right) + \delta \cdot \pi_{lf}^P(l, f, 1) \geq 0$$

and the denominator equals

$$-(1-\delta)\cdot\left(\pi_f^P(l, f, 0) - \pi_f^P(l, f, 1)\right) - \delta \cdot \pi_{ff}^P(l, f, 1) \geq 0.$$

Since we assume $\mu(t)$ is linear, these equalities exactly hold. When $f = IC_P(l) = l$, we can write $\mu(l) = \mu(f) = c$ and $\mu'(l) = \mu'(f) = \gamma$. Then, the numerator minus the denominator equals

$$\begin{aligned}
& (1-\delta)\cdot\left(\pi_l^P(l, f, 0) - \pi_l^P(l, f, 1)\right) + \delta \cdot \pi_{lf}^P(l, f, 1) \\
& + (1-\delta)\cdot\left(\pi_f^P(l, f, 0) - \pi_f^P(l, f, 1)\right) + \delta \cdot \pi_{ff}^P(l, f, 1) \\
= \;& (1-\delta)\cdot\left[-\frac{\alpha}{9b}(4a-4c) + \alpha\frac{3}{4b}(a-c) + (1-\alpha)\rho\frac{1}{18b}(a-c)\right]\gamma \\
& + (1-\delta)\cdot\left[-\alpha\left(2\frac{a-c}{9b} + 4\frac{a-c}{9b} + \frac{2}{18b}(2a-2c)\right)(1-\alpha)\rho 2\frac{a-c}{9b}\right]\gamma \\
& + \delta\cdot\alpha\cdot\frac{7}{9b}\cdot\gamma^2 - \delta\cdot(1-\alpha)\cdot\rho\cdot\frac{2}{9b}\gamma^2. \qquad (33)
\end{aligned}$$

We would like to show that this is less than zero. Given $f = IC_P(l) = l$, we have

$$(1-\delta)\cdot\left(\pi^P(l, l, 1) - \pi^P(l, l, 0)\right) = \delta\cdot\pi_f^P(l, l, 1).$$

Solving this equality for l and substituting it to (33), we obtain that (33) equals

$$-\frac{2(1-\alpha)\rho\gamma^2\delta}{9b} < 0,$$

as desired.

B.4.5 Proof of Lemma B.5

Note that

$$\begin{aligned}
& \frac{d}{df}\left(\pi^P(l,f,1)+\frac{\delta}{1-\delta}\pi^P(l+1,f,1)-\pi^P(l,f,0)-\frac{\delta}{1-\delta}\pi^P(l,f+1,1)\right) \\
= \; & \frac{d}{df}\Big(\pi^P(l,f,1)-\pi^P(l,f,0)+\frac{\delta}{1-\delta}\pi^P(l+1,f,1) \qquad (34)\\
& -\frac{\delta}{1-\delta}\pi^P(l,f,1)+\frac{\delta}{1-\delta}\pi^P(l,f,1)-\frac{\delta}{1-\delta}\pi^P(l,f+1,1)\Big).
\end{aligned}$$

Recall that

$$\pi_f^P(l,f,1)-\pi_f^P(l,f,0)\geq 0, \pi_{lf}^P(l,f,1)\geq 0, \pi_{ff}^P(l,f,1)\leq 0$$

by (21), (16), and (17). Hence, the sign of (34) is positive. Thus, we have

$$\pi^P(l,f,1)+\frac{\delta}{1-\delta}\pi^P(l+1,f,1)\geq \pi^P(l,f,0)+\frac{\delta}{1-\delta}\pi^P(l,f+1,1)$$

if and only if $f\geq IC_P^{**}(l)$. Moreover, at $f=IC_P(l)$, we have

$$\pi^P(l,f,1)+\frac{\delta}{1-\delta}\pi^P(l,f,1)=\pi^P(l,f,0)+\frac{\delta}{1-\delta}\pi^P(l,f+1,1).$$

Since $\pi_l^P(l,f,1)\leq 0$, we have

$$\pi^P(l,f,1)+\frac{\delta}{1-\delta}\pi^P(l+1,f,1)<\pi^P(l,f,0)+\frac{\delta}{1-\delta}\pi^P(l,f+1,1),$$

which implies $IC_P^{**}(l)>IC_P(l)$.

B.4.6 Proof of Lemma B.6

For each l, let $\bar{l}=\min\{\bar{L},l\}$. Suppose the statement of the lemma does not hold: there exists a period t in which protection is not on in period t but $f_t\geq IC_P(l_t+1)$. Let $\tau+1$ be the next period in which protection is on. Such τ must exists since once l hits $\bar{L}$, it is optimal to protect the leader if $f=\bar{L}-1$ (recall that the protection becomes infeasible once $l=f=\bar{L}$ is realized).

The state (l,f,a) transits from $(l_t+1,f_t,0)$, $\left(\overline{l_t+2},f_t+1,0\right)$, ..., $\left(\overline{l_t+\tau-t+1},f_t+\tau-t,0\right)$,

$\left(\overline{l_t+\tau-t+2},\ f_t+\tau-t,1\right)$. The policymaker's payoff is

$$\sum_{t'=0}^{\tau-t-1}\delta^{t'}\pi^P\left(\overline{l_t+t'+1},\ f_t+t',0\right)+\delta^{\tau-t}\pi^P\left(\overline{l_t+\tau-t+2},\ f_t+\tau-t,1\right)$$
$$+\delta^{\tau-t+1}V^P\left(\overline{l_t+\tau-t+2},\ f_t+\tau-t\right). \tag{35}$$

Consider the following deviation that the protection is on in period t and then protection is not on until period $\tau+1$. The state transits from $(l_t+1,\ f_t,1)$, $\left(\overline{l_t+2},\ f_t,0\right)$, ..., $\left(\overline{l_t+\tau-t+1},\ f_t+\tau-t-1,0\right)$, $\left(\overline{l_t+\tau-t+2},\ f_t+\tau-t,0\right)$. Then, the policymaker's payoff is

$$\pi\left(l_t+1,\ f_t,1\right)+\sum_{t'=1}^{\tau-t}\delta^{t'}\pi\left(\overline{l_t+t'+1},\ f_t+t'-1,0\right)$$
$$+\delta^{\tau-t+1}V^P\left(\overline{l_t+\tau-t+2},\ f_t+\tau-t\right). \tag{36}$$

Since $f_t\geq IC_P\left(l_t+1\right)$, we have

$$(1-\delta)\,\pi^P\left(l_t+1,\ f_t,0\right)+\delta\pi^P\left(l_t+1,\ f_t+1,1\right)\leq\pi^P\left(l_t+1,\ f_t,1\right).$$

Again, since $f_t+1\geq IC_P\left(l_t+1\right)$, we have

$$\begin{aligned}&(1-\delta)\,\pi^P\left(l_t+1,\ f_t,0\right)+(1-\delta)\,\delta\pi^P\left(l_t+1,\ f_t+1,0\right)+\delta^2\pi^P\left(l_t+1,\ f_t+2,1\right)\\ \leq\ &\pi^P\left(l_t+1,\ f_t,1\right).\end{aligned}$$

By (22), this implies

$$\begin{aligned}&(1-\delta)\,\pi^P\left(l_t+1,\ f_t,0\right)+(1-\delta)\,\delta\pi^P\left(\overline{l_t+2},\ f_t+2,0\right)+\delta^2\pi^P\left(\overline{l_t+2},\ f_t+2,1\right)\\ \leq\ &\pi^P\left(l_t+1,\ f_t,1\right).\end{aligned}$$

Recursively, we have

$$\begin{aligned}&(1-\delta)\sum_{t'=0}^{\tau-t-1}\delta^{t'}\pi^P\left(\overline{l_t+t'+1},\ f_t+t',0\right)+\delta^{\tau-t}\pi^P\left(\overline{l_t+\tau-t+1},\ f_t+\tau-t,1\right)\\ \leq\ &\pi^P\left(l_t+1,\ f_t,1\right).\end{aligned}$$

Thus, (36) minus (35) is no less than

$$
\begin{aligned}
&(1-\delta)\sum_{t'=0}^{\tau-t-1}\delta^{t'}\pi^{P}\left(\overline{l_t+t'+1},\, f_t+t',0\right)+\delta^{\tau-t}\pi^{P}\left(\overline{l_t+\tau-t+1},\, f_t+\tau-t,1\right)\\
&+\sum_{t'=1}^{\tau-t}\delta^{t'}\pi\left(\overline{l_t+t'+1},\, f_t+t'-1,0\right)\\
&-\sum_{t'=0}^{\tau-t-1}\delta^{t'}\pi^{P}\left(\overline{l_t+t'+1},\, f_t+t',0\right)+\delta^{\tau-t}\pi^{P}\left(\overline{l_t+\tau-t+2},\, f_t+\tau-t,1\right)\\
=\;&\delta^{\tau-t}\left[\pi^{P}\left(\overline{l_t+\tau-t+1},\, f_t+\tau-t,1\right)-\pi^{P}\left(\overline{l_t+\tau-t+2},\, f_t+\tau-t,1\right)\right]\\
&+\sum_{t'=0}^{\tau-t-1}\delta^{t'+1}\pi\left(\overline{l_t+t'+2},\, f_t+t',0\right)-\sum_{t'=1}^{\tau-t-1}\delta^{t'+1}\pi^{P}\left(\overline{l_t+t'+1},\, f_t+t',0\right)\\
=\;&\delta^{\tau-t}\left[\pi^{P}\left(\overline{l_t+\tau-t+1},\, f_t+\tau-t,1\right)-\pi^{P}\left(\overline{l_t+\tau-t+2},\, f_t+\tau-t,1\right)\right]\\
&+\sum_{t'=0}^{\tau-t-1}\delta^{t'+1}\left[\pi^{P}\left(\overline{l_t+t'+2},\, f_t+t',0\right)-\pi^{P}\left(\overline{l_t+t'+1},\, f_t+t',0\right)\right].
\end{aligned}
$$

By (13) and (20), the deviation is profitable, which is a contradiction.

B.4.7 Proof of Lemma B.7

For sufficiently large l such that $\pi^{M}(l+1,\, f)-\pi^{L}(l,\, f)<c(l)$ for each f, the leader never invests. Hence, the policymaker is the single decision maker and the result holds. Fix $(l,\, f)$. Suppose the result holds for each $\left(l',\, f'\right)$ with $\left(l',\, f'\right)\geq(l,\, f)$ and $\left(l',\, f'\right)\neq(l,\, f)$. Let $\mathcal{V}^{P}(l,\, f)$ be the set of SPE payoffs at $(l,\, f)$ for the policymaker.

1. After the leader invests, $V^{P}(l+1,\, f+1)$ and $V^{P}(l+1,\, f)$ are determined by the inductive hypothesis. Hence, the continuation payoff for the policymaker is unique. Since the leader prefers protection, in the renegotiation proof SPE, we break the tie for protection.

2. After the leader does not invest, the policymaker's payoff without protection is determined by $\delta V^{P}(l,\, f+1)$. With protection, it will be $\pi^{P}(l,\, f,1)+\delta v$ for some $v\in\mathcal{V}^{P}(l,\, f)$. If $\min_{v\in\mathcal{V}^{P}(l,f)}\pi^{P}(l,\, f,1)+\delta v\geq\pi^{P}(l,\, f,0)+\delta V^{P}(l,\, f+1)$ or $\max_{v\in\mathcal{V}^{P}(l,f)}\pi^{P}(l,\, f,1)+\delta v\leq\pi^{P}(l,\, f,0)+\delta V^{P}(l,\, f+1)$, then the equilibrium payoff for the policymaker is unique (again, in case of the policymaker's indifference, we break the tie for protection). Hence, we assume that $\min_{v\in\mathcal{V}^{P}(l,f)}\pi^{P}(l,\, f,1)+\delta v<\pi^{P}(l,\, f,0)+\delta V^{P}(l,\, f+1)<\max_{v\in\mathcal{V}^{P}(l,f)}\pi^{P}(l,\, f,1)+\delta v$. For v satisfying $\pi^{P}(l,\, f,1)+\delta v>\pi^{P}(l,\, f,0)+\delta V^{P}(l,\, f+1)$, the policymaker protects the leader.

Hence, $v^* \in \arg\max_{v \in \mathcal{V}^P(l,f)} \pi^P(l, f, 1) + \delta v$ should be attained by protection. Given this property of the policymaker's value and incentives, it suffices to show that the equilibrium path that achieves $\max_{v \in \mathcal{V}^P(l,f)} \pi^P(l, f, 1) + \delta v$ also maximizes the leader's payoff. This holds since the leader prefers the protection. Therefore, after the leader's investment decision, the renegotiation proof SPE is unique.

B.4.8 Proof of Lemma B.8

The statement clearly holds for $l = \bar{L}$. Suppose the statement holds for $l + 1$. We now prove that the statement holds for l.

We first show that, at (l, f), after the leader invests, that is, at interim state $(l + 1, f)$,[40] the policymaker does not protect the leader. With protection, the policymaker obtains

$$\pi^P(l + 1, f, 1) + \delta^2 V^P(l + 1, f + 1)$$

since the leader will not invest and the policymaker will not protect him in the next period (this follows from $IC_P(l) \leq IC_P(l + 1)$ and the inductive hypothesis). By contrast, without protection, she obtains the payoff at least

$$\pi^P(l + 1, f, 0) + \delta\pi^P(l + 1, f + 1, 1) + \delta^2 V^P(l + 1, f + 1)$$

(by protecting the leader in the next period). Since $f \leq IC_P(l + 1)$, it is better not to protect at interim state $(l + 1, f)$.

In addition, since the policymaker does not protect the leader after investment, by definition of $IC_D(l)$, the leader does not invest (regardless of the policymaker's choice after non-investment). Given this specification of the leader's strategy, we will show that the policymaker does not protect the leader after his non-investment, that is, at interim state (l, f), for each $f \in [IC_D(l), IC_P(l)]$.

Suppose otherwise: the protection is one of the optimal action at state (l, f) after the leader's non-investment. Then, the policymaker's equilibrium value is $\frac{1}{1-\delta}\pi^P(l, f, 1)$. By contrast, given the specification of the continuation play, she obtains the payoff $\pi^P(l, f, 0) + \delta V^P(l, f + 1, 1)$ without protection. It remains for us to show that

$$\frac{1}{1-\delta}\pi^P(l, f, 1) \leq \pi^P(l, f, 0) + \delta V^P(l, f + 1, 1) \tag{37}$$

for each $f \in [IC_D(l), IC_P(l)]$.

We prove (37) inductively for each $f \in [IC_D(l), IC_P(l)]$. For $f = IC_P(l)$, since

[40] We call the technology profile after the leader's investment "the interim state."

$f+1 \geq IC_P(l)$ implies that $f+1 \in IC_P^*(l+1)$ by (30), we have (37). Thus, the statement of the lemma holds for $f = IC_P(l)$.

Suppose that (37) and the statement of the lemma hold for $f+1$. Since $f \geq IC_D(l)$ implies $f+1 \geq IC_D(l)$, the leader will not invest at state $(l, f+1)$. Since the policymaker can obtain at least $\frac{1}{1-\delta}\pi^P(l, f+1, 1)$ by always protecting the leader at state $(l, f+1)$, we have $\delta V^P(l, f+1, 1) \geq \frac{\delta}{1-\delta}\pi^P(l, f+1, 1)$. Thus, for $f \leq IC_P(l)$, we have (37), as desired.

B.4.9 Proof of Lemma B.9

As seen in the proof of Lemma B.7, it suffices to show that $\mathrm{eqm}(l, f) = (NI, P)$ is an equilibrium outcome. Given this conjecture of $\mathrm{eqm}(l, f)$ and the specification of the continuation payoff, with protection, the policymaker obtains $\frac{1}{1-\delta}\pi^P(l, f, 1)$ while without protection, she obtains $\pi^P(l, f, 0) + \frac{\delta}{1-\delta}\pi^P(l, f+1, 1)$. By (18), it is better to protect. Therefore, the policymaker protects the leader if he does not invest. Therefore, he can at least guarantee $\pi^M(l, f)$ without investment. Since $f \geq IC_M(l)$, even if investment will lead to protection, he will not invest. Hence, $\mathrm{eqm}(l, f) = (NI, P)$ is an equilibrium outcome.

B.4.10 Proof of Lemma B.10

Since the leader will lose protection after investment, the leader does not have an incentive to deviate. In addition, since $f \geq IC_P(l)$ implies $f \in IC_P^*(l)$, the policymaker does not have an incentive to deviate.

B.4.11 Proof of Lemma B.11

Since the policymaker protects the leader after his investment at (l, f), we have

$$\pi^P(l+1, f, 1) + \delta V^P(l+1, f) \geq \pi^P(l+1, f, 0) + \delta V^P(l+1, f+1),$$

which implies that the policymaker protects the leader after his non-investment at $(l+1, f)$. Hence, either $\mathrm{eqm}(l+1, f) = (NI, P)$ or the leader invests. In the latter case, since the leader invests when he is protected after non-investment, (24) implies that the policymaker has to protect him after investment. Hence we have $\mathrm{eqm}(l+1, f) = (I, P)$.

B.4.12 Proof of Lemma B.12

Suppose $\mathrm{eqm}(l, f) = (I, P)$. Suppose the leader deviates and does not invest. Then, protection gives the policymaker the payoff no more than $\pi^P(l, f, 1) + \frac{\delta}{1-\delta}\pi^P(l+1, f, 1)$ (by recursively applying Lemma B.11, once eqm $(l, f) = (I, P)$, the state transits to (l, f), $(l+1, f)$, ...) while non-protection gives the policymaker the payoff no less than $\pi^P(l, f, 0) + \frac{\delta}{1-\delta}\pi^P(l, f+1, 1)$. By definition of $IC_P^{**}(l)$, if $f \leq IC_P^{**}(l)$, then the latter is optimal.

Suppose eqm $(l, f) = (I, NP)$. Since the leader invests even though he loses protection after investment, the policymaker must take non-protection after non-investment at (l, f).

B.4.13 Proof of Lemma B.13

If the policymaker does not protect the leader after non-investment at $(l+1, f)$, then

$$\pi^P(l+1, f, 0) + \delta V^P(l+1, f+1) \geq \pi^P(l+1, f, 1) + \delta V^P(l+1, f).$$

This condition implies that the policymaker does not protect the leader after investment at (l, f). Since the policymaker does not protect the leader after investment at (l, f), by (24), the leader invests on equilibrium path only if the policymaker does not protect the leader after non-investment at (l, f).

B.4.14 Proof of Lemma B.14

Define $\tilde{l} = l_1$ if $l_1 \neq \emptyset$ and $\tilde{l} = l_2$ if $l_1 = \emptyset$. If $\tilde{l} \leq l_4$, then the result follows since $\left(\tilde{l}, \tilde{l}-1\right)$ is in Region 2. If $l_4 > \tilde{l}$, then take (l, f) sufficiently close to $(l_4, IC_P(l_4))$ such that

$$\max\{IC_P(l), IC_M(l)\} \leq f \text{ and } f-1 \leq IC_P^{**}(l). \tag{38}$$

The existence of such (l, f) follows from (25).

We will show that, for each $0 \leq n \leq f - \left(\tilde{l}-1\right)$, there exists $k \geq 0$ such that eqm $(l-k, f-n) = (NI, P)$, $f-n-1 \leq IC_P^{**}(l-k)$, and $f-n \geq IC_P(l-k)$. Note that this statement is sufficient for eqm $\left(\tilde{l}, \tilde{l}-1\right) = (NI, P)$ by taking $n = f - \left(\tilde{l}-1\right)$.

For $n = 0$, by (38) and Lemma B.9, with $k = 0$, we have eqm $(l-k, f-n) = (NI, P)$, $f-n-1 \leq IC_P^{**}(l-k)$, and $f-n \geq IC_P(l-k)$.

We next prove that, if the statement holds for $n-1$, then it holds for n. Given this inductive hypothesis, there exists $\hat{l}$ such that eqm $\left(\hat{l}, f-n+1\right) = (NI, P)$ and $f-n \leq IC_P^{**}\left(\hat{l}\right)$.

$$\begin{array}{c}\left(\hat{l}, f-n+1\right)\\ \circlearrowleft\\ \left(\hat{l}, f-n\right)\end{array}$$

Since eqm $\left(\hat{l}, f-n+1\right) = (NI, P)$ and $f-n \leq IC_P^{**}\left(\hat{l}\right)$, Lemma B.12 implies that, at $\left(\hat{l},\ f-n\right)$, we have either (i) eqm $\left(\hat{l},\ f-n\right) \in \{(I, P), (I, NP)\}$ and the policymaker does not protect after the leader's non-investment (deviation), (ii) eqm $\left(\hat{l},\ f-n\right) = (NI, NP)$, or (iii) eqm $\left(\hat{l},\ f-n\right) = (NI, P)$. Moreover, (iii) implies $f-n \geq IC_P\left(\hat{l}\right)$ since eqm $\left(\hat{l},\ f-n+1\right) = (NI, P)$.

Suppose (i) or (ii) is the case. Then, by Lemma B.13, at $\left(\hat{l}-2,\ f-n\right)$, we have either (i) eqm $\left(\hat{l}-2,\ f-n\right) \in \{(I, P), (I, NP)\}$ and the policymaker does not protect after the leader's non-investment (deviation), (ii) eqm $\left(\hat{l}-2,\ f-n\right) = (NI, NP)$, or (iii) eqm $\left(\hat{l}-2,\ f-n\right) = (NI, P)$. Recursively, if (i) or (ii) is the case for $\left(\hat{l}-k,\ f-n\right)$, then at $\left(\hat{l}-k-1,\ f-n\right)$, we have either (i) eqm $\left(\hat{l}-k-1,\ f-n\right) \in \{(I, P), (I, NP)\}$ and the policymaker does not protect after the leader's non-investment (deviation), (ii) the equilibrium eqm $\left(\hat{l}-k-1,\ f-n\right) = (NI, NP)$, or (iii) eqm $\left(\hat{l}-k-1,\ f-n\right) = (NI, P)$. Moreover, by Lemma B.10, if $f-n \geq IC_P\left(\hat{l}-k-1\right)$, then (i) or (ii) at $(l-k,\ f-1)$ implies eqm$(l-k-1,\ f-1) = (NI, P)$. In total, there exists $k \geq 0$ such that eqm $\left(\hat{l}-k,\ f-n\right) = (NI, P)$ and either [$k = 0$ and $f-n \geq IC_P\left(\hat{l}\right)$] or k is the smallest integer with $f-n \geq IC_P\left(\hat{l}-k\right)$. If $k = 0$, since $f-n \leq IC_P^{**}\left(\hat{l}\right)$, we have $f-n-1 \leq IC_P^{**}\left(\hat{l}-k\right)$. Otherwise, since k is the smallest integer with $f-n \geq IC_P\left(\hat{l}-k\right)$, we have $f-n-1 \leq f-n \leq IC_P^{**}\left(\hat{l}-k\right)$. Therefore, for n, there exists $k \geq 0$ such that eqm $\left(\hat{l}-k,\ f-n\right) = (NI, P)$ and $IC_P\left(\hat{l}-k\right) \leq f-n$ and $f-n-1 \leq IC_P^{**}\left(\hat{l}-k\right)$, as desired.

B.4.15 Proof of Lemma B.15

Suppose $l^* > l_3$. We first claim that there exists $(\hat{l}, \hat{l}-1)$ with $\hat{l} \leq l_3$ such that, in equilibrium, the state transitions from $(2, 1)$ to $(3, 2)$, from $(3, 2)$ to $(4, 3)$, ..., to $(\hat{l}, \hat{l}-1)$ and we have eqm$(\hat{l}, \hat{l}-1) = (I, P)$. To see why such $(\hat{l}, \hat{l}-1)$ exists, suppose otherwise. Then, as long as $l \leq l_3$, we have eqm$(l,\ f) \neq (I, P)$ and eqm$(l,\ f) \neq (NI, P)$ on equilibrium path. That is, eqm$(l,\ f) \in \{\{NI, NP\}, \{I, NP\}\}$. If eqm$(l,\ f) \in \{NI, NP\}$ for some $l \leq l_3$ on the equilibrium path, then since the leader does not invest even though the current state leads to NP, it means that $f \geq IC_D(l)$. Thus, the future state will be either in Region 1 or Region 2, which implies $l^* \leq l \leq l_3$. The remaining case is that eqm$(l,\ f) \in \{I, NP\}$ for each $l \leq l_3$ on the equilibrium path. In this case, the state would transit from $(2, 1)$ to $(3, 2)$, from $(3, 2)$ to $(4, 3)$, ..., to (l_3, l_3-1). If so, then we would have $l^* = l_3$, as desired. This finishes the proof of the existence of $(\hat{l}, \hat{l}-1)$.

After the state reaches such $(\hat{l}, \hat{l}-1)$, by Lemma B.11, the state transitions from $(\hat{l}, \hat{l}-1)$

to $(\hat{l}+1, \hat{l}-1)$, from $(\hat{l}+2, \hat{l}-1)$ to $(\hat{l}+3, \hat{l}-1)$, until it stops at $(l^*, \hat{l}-1)$. Hence, the policymaker's payoff is

$$V^P = \sum_{l=2}^{\hat{l}} \delta^{l-2} \pi(l, l-1, 0) + \sum_{l=\hat{l}+1}^{l^*-1} \delta^{l-2} \pi(l, \hat{l}-1, 1) + \delta^{l^*-2} \frac{\pi(l^*, \hat{l}-1)}{1-\delta}.$$

Since $l^* > l_3$, this payoff means that, after $(\hat{l}, \hat{l}-1)$, keeping the equilibrium strategy of the policymaker, her payoff is the same after we replace the leader's strategy with $\hat{\sigma}^L$ such that he invests regardless of the policymaker's strategy until l hits l_3 (and then follow the equilibrium strategy). By allowing the policymaker to best respond to $\hat{\sigma}^L$, the policymaker's payoff increases. In that case, the policymaker would prefer protecting the leader until l hits l_3 by the same argument as Lemma B.6 since $f \geq IC_P(l)$ trivially holds for each (l, f) with $l \leq l_3 = l_2$. However, since eqm $(l_3, 1) = (NI, P)$, by protecting the leader until the leader's technology level hits l_3, the policymaker could guarantee this payoff. This is a contradiction.

B.4.16 Proof of Lemma B.16

Suppose otherwise. By the same proof as Lemma B.15, there exists $\hat{l} \leq l_3$ such that the state transitions from $(2, 1)$ to $(3, 2)$, from $(3, 2)$ to $(4, 3)$, ..., to $(\hat{l}, \hat{l}-1)$, then $(\hat{l}, \hat{l}-1)$ to $(\hat{l}+1, \hat{l}-1)$, from $(\hat{l}+2, \hat{l}-1)$ to $(\hat{l}+3, \hat{l}-1)$, until it stops at $(l^*, \hat{l}-1)$. Hence, the policymaker's payoff is

$$V^P = \sum_{l=2}^{\hat{l}} \delta^{l-2} \pi^P(l, l-1, 0) + \sum_{l=\hat{l}}^{l^*-1} \delta^{l-2} \pi^P(l, \hat{l}-1, 1) + \delta^{l^*} \frac{\pi^P(l^*, \hat{l}-1, 1)}{1-\delta}.$$

By contrast, suppose the policymaker at $(\hat{l}, \hat{l}-1)$ deviates to the following strategy: She does not protect the leader until $\left(\tilde{l}, \tilde{l}-1\right)$ hits the first state with eqm $\left(\tilde{l}, \tilde{l}-1\right) \in \{(NI, P), (NI, NP)\}$. Once $\left(\tilde{l}, \tilde{l}-1\right)$ hits such a state, then protect the leader forever. Her payoff from this deviation is

$$\sum_{l=2}^{\hat{l}} \delta^{l-2} \pi^P(l, l-1, 0) + \sum_{l=\hat{l}}^{\tilde{l}-2} \delta^{l-1} \pi^P(l, l-1, 0) + \delta^{\tilde{l}-2} \frac{\pi^P(\tilde{l}, \tilde{l}-1, 1)}{1-\delta}.$$

Since the deviation should not be profitable, the definition of l_3 implies $l^* \leq l_3$.

C Proofs from Section 4.1

This section presents the argument to prove Lemmas 3 and 4, and Proposition 2.

Thresholds and Assumptions

We define $IC_{EA}(l)$ as the solution for $\pi^M(l+1,x) - \pi^L(l,x) - c(l) = 0$. By the same proof as Lemmas 1 and 2, we can show that

$$\pi^M(l+1,x) - \pi^L(l,x) - c(l) \geq 0$$

if and only if $f \leq IC_{EA}(l)$. Given the definition of $\bar{L}$ in (19), we have $IC_{EA}(l) \neq \emptyset$ if and only if $l \leq \bar{L} - 1$. Relatedly, let IC^*_{EA} be the leader's technology level such that $\pi^M(l+1,l) - \pi^L(l,l) - c(l) = 0$ at $l = IC^*_{EA}$.

For a simple analysis, we assume

$$IC_P(l) \leq l - 2 \tag{39}$$

for all l. In addition, to avoid a tedious tie-breaking, we assume that there is no $(l, f) \in \mathbb{N}^2$ such that $f = IC_D(l)$.

We also make the following assumptions. Define

$$\pi^P(l,l,1) = \alpha SW^M(l) + (1-\alpha) + \rho\left(\hat{\pi}^M(l) - \pi^L(l,l)\right).$$

That is, even though protection is not feasible if $l = f$, we calculate what would be the policymaker's payoff if it were feasible.[41] In the statement of the assumptions below, we allow $l = f$.

A1 For each (l, f), the stronger leader reduces the policymaker's utility given protection:

$$\pi^P(l, f, 1) > \pi^P(l+1, f, 1). \tag{40}$$

A2 α is sufficiently small, π_l is sufficiently small, and δ is sufficiently large: For each

[41]In the baseline model, the state with $l = f$ is never reached. By contrast, in this model of relaxed competition, it is reached on the equilibrium path. Thus, we need additional assumptions involving $\pi^P(l,l,1)$.

(l, f), we have

$$\sum_{t=1}^{\kappa} \delta^t \pi^P (l+t, f+1, 1) + \min_{(l_\tau, f_\tau) \text{ with } \bar{L} \geq l_\tau \geq l \text{ and } \bar{L} \geq f_\tau \geq f+1} \sum_{\tau=\kappa+1}^{\infty} \delta^\tau \pi^P (l_\tau, f_\tau, 0)$$

$$\geq \pi^P (l, f, 1) + \max_{(l_t, f_t) \text{ with } \bar{L} \geq l_t \geq l \text{ and } \bar{L} \geq f_t \geq f} \sum_{t=1}^{\infty} \delta^t \pi^P (l_t, f_t, 0). \tag{41}$$

A3 For each (l, f), protection with a slightly weaker follower brings a higher payoff than non-protection with a slightly stronger follower:

$$\pi^P (l, f-1, 1) > \pi^P (l, f, 0) \tag{42}$$

By the same reasoning, the leader never invests if $l \geq \bar{L}$. Thus, we focus on (l, f) with $0 \leq f \leq l \leq \bar{L}$. We use renegotiation proof subgame perfect equilibrium as our equilibrium concept.

We show that Assumptions (40)–(42) hold with small α.

Lemma C.1 *Suppose Assumption 1 (in the main text) holds. Then, for sufficiently small $\alpha > 0$, sufficiently small π_l, and sufficiently large δ, for each (l, f), conditions (40)–(42) hold.*

The proof to this lemma and all auxiliary lemmas is provided in Section C.1.

Equilibrium steady state characterization

In equilibrium, we show that the steady state technology level is no less than $IC^*_{EA} - 1$. Given the above lemma, this then implies Proposition 2 for $\alpha = 0$. To prove this result, we first provide a counterpart of Lemma B.7.

Lemma C.2 *The unique renegotiation-proof subgame perfect equilibrium is Markov perfect. Moreover, we use renegotiation proofness only to break a tie in favor for the other agent.*

Given this lemma, we write the policymaker's value function as $V^P (l, f, k)$ for each (l, f) and k. Here, $k \in \{0, ..., \kappa\}$ indicates how many consecutive periods the leader has been protected. $k = \kappa$ means that the follower has disappeared.

We next pin down the state transition for (l, f) with $f \geq IC_P (l)$ and $f > IC_D (l)$.

Lemma C.3 *For each feasible state (l, f, k) with $f \geq IC_P (l)$ and $f > IC_D (l)$, (i) if two firms have different technology level ($l > f$), the leader does not invest, (ii) in head-to-head competition ($l = f$), the firm with an investment opportunity invests if $l \leq IC^*_{EA}$, (iii)* *if $k = \kappa - 1$ and either $l - 1 > f$ or $l \leq IC^*_{EA}$, the policymaker does not protect the leader, (iv) if $k < \kappa - 1$, then the policymaker protects the leader, and (v) $V^P(l, f, k)$ is decreasing in k and $V^P(l, l, k) \leq \frac{\pi^P(l, l-1, 1)}{1-\delta}$ for each k and $V^P(l, f, k) \leq \frac{\pi^P(l, f, 1)}{1-\delta}$ for each $l - 1 \geq f \geq IC_P(l)$ and k.*

Given Lemma C.3, to show that, in the long run, the leader's technology level is no less than $IC^*_{EA} - 1$, it suffices to show that the equilibrium path reaches a state (l, f, k) with $f \geq IC_P(l)$, $f > IC_D(l)$, and $k \leq \kappa - 1$.

We use the following two auxiliary lemmas, which will be useful for the final step of the proof.

Lemma C.4 *For each state $(l, f, \kappa - 1)$ and the leader's investment decision ι, if the leader invests in $(l + \iota, f + 1, 0)$ or $l + \iota > f + 1$, the policymaker does not protect the leader.*

Lemma C.5 *For each state $(l, f, \kappa - 1)$ and the leader's investment decision ι, if $f + 1 < IC_D(l + \iota)$, then the policymaker does not protect the leader.*

Finally, the following lemma concludes the proof that in the absorbing state, the leader's technology level is no less than $IC^*_{EA} - 1$:

Lemma C.6 *The equilibrium path reaches a state (l, f, k) either with $l \geq IC^*_{EA} - 1$ or with $f \geq IC_P(l)$, $f > IC_D(l)$, and $k \leq \kappa - 1$.*

C.1 Proofs of Auxiliary Lemmas

C.1.1 Proof of Lemma C.1

Since we focus on $(l, f) \in [0, \bar{L}]^2$ and all values are continuous in α, it suffices to show that given $\alpha = 0$, for each (l, f), the assumptions (40)–(42) hold hold. First, with $\alpha = 0$, Assumption 1 (in the main text) implies (40), as in the proof of Lemma B.1. Second, with $\alpha = 0$, (41) is equivalent to

$$\sum_{t=1}^{\kappa} \delta^t \pi^P(l + t, f, 1) \geq \pi^P(l, f, 1).$$

For sufficiently small π_l and large δ, the left hand side exceeds the right hand side, as desired. Third, (42) clearly holds with $\alpha = 0$ since $\pi^P(l, f, 0) = 0$.

C.1.2 Proof of Lemma C.2

For $l = \bar{L}$, the leader never invests. Hence, the policymaker is the only decision maker and the result holds. We now proceed inductively. Fix $l \leq \bar{L} - 1$ arbitrarily. Suppose the result holds for $l + 1, ..., \bar{L}$. In particular, $V^P(l + 1, f, k)$ is well-defined value function for each f and k.

Given the current state (l, f, k), suppose the leader invests. If $k = \kappa$, then the policymaker has no choice. If $k \leq \kappa - 1$, the policymaker protects the leader if and only if

$$\pi^P(l + 1, f, 1) + \delta V^P(l + 1, f, k + 1) \geq \pi^P(l + 1, f, 0) + \delta V^P(l + 1, f + 1, 0).$$

Note that all the values are determined. Hence, by letting the policymaker break her tie in favor for the leader, the result holds for l.

Given the current state (l, f, k), suppose the leader does not invest. For a fixed l, for each (f, k), we want to show that the result holds for (l, f, k). We proceed inductively with respect to k. If $k = \kappa$, then the policymaker has no choice. Hence, the result holds.

For each $k \leq \kappa - 1$, suppose that the result holds for $k + 1, ..., \kappa$. In particular, $V^P(l, f, k + 1)$ is well-defined value function for each f. The policymaker protects the leader if and only if

$$\pi^P(l, f, 1) + \delta V^P(l, f, k + 1) \geq \pi^P(l, f, 0) + \delta V^P(l, f + 1, 0).$$

Note that all the values are determined. Hence, by letting the policymaker break her tie in favor for the leader, the result holds for l.

Given that the policymaker's strategy is determined, the leader's optimal strategy is determined since he is myopic. We break his tie in favor for the policymaker.

C.1.3 Proof of Lemma C.3

As seen in Lemma C.2, the only source of multiplicity in the subgame perfect equilibrium is the tie breaking and renegotiation proofness breaks the tie in favor for the other player. The policymaker breaks the tie in favor for the leader (namely, she protects him unless $k = \kappa - 1$). Thus, it suffices to prove the optimality of each player's action and the strictness of the leader's incentive, assuming that the other player acts as prescribed in the statement of the Lemma. For $l = \bar{L}$, the statement obviously holds. For each $l \leq \bar{L} - 1$, suppose all the statements (i)–(iv) hold for $l + 1$. We now prove that statements (i)–(iv) hold for l. This will conclude the proof by mathematical induction.

Proof of Statement (i). If $k < \kappa - 1$, then since the leader expects that the policymaker always protects him regardless of his investment decision, given $f > IC_D(l) \geq IC_M(l)$, the leader does not invest. If $k = \kappa - 1$, then since the leader expects that the policymaker does not protect him regardless of his investment decision, given $f \geq IC_D(l)$, the leader does not invest.

Proof of Statement (ii). Given the current state (l, l, k) with $k \leq \kappa - 1$, feasibility implies $k = 0$ since the current head-to-head competition implies that the policymaker did not protect the leader in the previous period (either f increased from the previous period or the previous state was (l, l) and the regulation was not feasible). Hence, without investment, the firm obtains the payoff of $\pi^L(l, l)$, while with investment, he obtains the payoff of $\pi^M(l+1, l) - c(l)$. Hence, it is optimal to invest if and only if $l \leq IC^*_{EA}$.

Proof of Statements (iii). With protection, the policymaker obtains the payoff no more than

$$\pi^P(l,\ f, 1) + \max_{(l_t,\, f_t)\text{ with } \bar{L} \geq l_t \geq l \text{ and } \bar{L} \geq f_t \geq f} \sum_{t=1}^{\infty} \delta^t \pi^P(l_t,\ f_t, 0)\,.$$

We next calculate the lower bound of the payoff without protection. If $l - 1 > f$, then the next period state will be $(l,\ f+1)$ since the leader will not invest. If $l - 1 \leq f$ but $l \leq IC^*_{EA}$, then the next period starts with (l, l) and then the firm with an investment opportunity will invest. Thus, the next period interim state will be $(l+1,\ f+1)$. Thus, the policymaker can obtain the payoff at least

$$\sum_{t=1}^{\kappa} \delta^t \pi^P(l+t,\ f+1, 1) + \min_{(l_\tau,\, f_\tau)\text{ with } \bar{L} \geq l_\tau \geq l \text{ and } \bar{L} \geq f_\tau \geq f+1} \sum_{\tau=\kappa+1}^{\infty} \delta^\tau \pi^P(l_\tau,\ f_\tau, 0)\,.$$

By (41), the policymaker does not protect the leader.

Proof of Statements (iv) and (v). Suppose $l = f > IC_D(l)$. Again, feasibility implies $k = 0$. When the leader invests, with protection, the current payoff for the policymaker is $\pi^P(l+1, l, 1)$. Then, since $\frac{d}{dl} IC_D \leq 0$ by Lemma 1, we have $l > IC_D(l) \geq IC_D(l+1)$ and hence the leader will not invest at state $(l+1, l)$. Thus, by taking non-protection at state $(l+1, l)$, her total payoff is at least $\pi^P(l+1, l, 1) + \delta\pi^P(l+1, l, 0) + \delta^2 V^P(l+1, l+1, 0)$. Without protection, the payoff is $\pi^P(l+1, l, 0) + \delta V^P(l+1, l+1, 0)$.

The former is no less than the latter since

$$\begin{aligned}
& \pi^P(l+1,l,1)+\delta\pi^P(l+1,l,0)+\delta^2 V^P(l+1,l+1,0) \\
& -\pi^P(l+1,l,0)-\delta V^P(l+1,l+1,0) \\
= \; & \pi^P(l+1,l,1)+\delta\pi^P(l+1,l,0)-\pi^P(l+1,l,0) \\
& -\delta(1-\delta)V^P(l+1,l+1,0) \\
\geq \; & \pi^P(l+1,l,1)+\delta\pi^P(l+1,l,0)-\pi^P(l+1,l,0) \\
& -\delta\pi^P(l+1,l,1) \\
& \text{by the inductive hypothesis, } (1-\delta)V^P(l+1,l+1,0) \\
& \text{is no more than } \pi^P(l+1,l,0) \\
= \; & (1-\delta)\left(\pi^P(l+1,l,1)-\pi^P(l+1,l,0)\right) \\
\geq \; & 0 \text{ by (42).}
\end{aligned}$$

For $l=f$, only feasible k is $k=0$ or $k=\kappa$. We have $V^P(l,l,0)\geq V^P(l,l,\kappa)$. To see why, for $V^P(l,l,\kappa)$, given Assumption 1 of the main text, the remaining firm will not invest given $l\geq IC_D(l)$. Thus, the policymaker's payoff is $\frac{1}{1-\delta}$ times the social welfare under monopoly. She can obtain this payoff at state $(l,l,0)$ by not protecting the leader forever.

Moreover,

$$\begin{aligned}
V^P(l,l,0) &= \pi^P(l,l,0)+\delta V^P(l+1,l,1) \\
&\leq \pi^P(l,l,0)+\delta\frac{\pi^P(l+1,l,1)}{1-\delta} \text{ by inductive hypothesis.}
\end{aligned}$$

Since $l-1\geq IC_P(l)$ implies

$$\frac{\pi^P(l,l-1,1)}{1-\delta}\geq\pi^P(l,l,0)+\delta\frac{\pi^P(l,l,1)}{1-\delta},$$

given $\pi_l^P\leq 0$, we have

$$\frac{\pi^P(l,l-1,1)}{1-\delta}\geq\pi^P(l,l,0)+\delta\frac{\pi^P(l+1,l,1)}{1-\delta}.$$

In total, we have $V^P(l,l,k)\leq\frac{\pi^P(l,l-1,1)}{1-\delta}$ for each feasible k, as desired.

For $f=l-1\geq IC_D(l)$, suppose statements (i)–(v) hold for each $f'\geq f+1$. Since the leader will not invest at state $(l,l-1)$, with protection, by not protecting the leader in the next period, the policymaker at least obtains the payoff of

$$\begin{cases} \pi^P(l, f, 1) + \delta \pi^P(l, f, 0) + \delta^2 V^P(l, f+1, 0) & \text{if } l \leq IC^*_{EA}, \\ \pi^P(l, l-1, 1) + \delta \pi^P(l, l-1, 0) + \delta^2 \frac{\pi^P(l,l,0)}{1-\delta} & \text{otherwise.} \end{cases} \tag{43}$$

Without protection, the policymaker's payoff is

$$\begin{cases} \pi^P(l, f, 0) + \delta V^P(l, f+1, 0) & \text{if } l \leq IC^*_{EA}, \\ \pi^P(l, l-1, 0) + \delta \frac{\pi^P(l,l,0)}{1-\delta} & \text{otherwise.} \end{cases} \tag{44}$$

Suppose $l \leq IC^*_{EA}$. We first prove (iv). By inductive hypothesis, we have

$$\delta^2 V^P(l, f+1, 0) - \delta V^P(l, f+1, 0) \geq -\delta \pi^P(l, f+1, 1).$$

Since $f \geq IC_P(l)$, we have

$$-\delta \pi^P(l, f+1, 1) \geq -\pi^P(l, f) + (1-\delta)\pi^P(l, f+1, 0).$$

Hence, (43) minus (44) is no less than 0. Therefore, protection is optimal.

We next prove (v). As before, it is clear that $V^P(l, f, k) \geq V^P(l, f, \kappa)$ for each $k \leq \kappa - 1$. Moreover, by the equilibrium strategy,

$$V^P(l, f, k) = \sum_{t=0}^{\kappa-1-k} \delta^t \pi^P(l, f, 1) + \delta^{\kappa-k}\pi^P(l, f, 0) + \delta^{\kappa-k+1} V^P(l, f+1, 0).$$

Thus, for each $k \leq \kappa - 2$,

$$\begin{aligned} V^P(l, f, k) - V^P(l, f, k+1) \;=\;& \delta^{\kappa-1-k}\pi^P(l, f, 1) + \delta^{\kappa-k}\pi^P(l, f, 0) \\ & + \delta^{\kappa-k+1} V^P(l, f+1, 0) \\ & - \delta^{\kappa-k-1}\pi^P(l, f, 0) - \delta^{\kappa-k} V^P(l, f+1, 0). \end{aligned}$$

By inductive hypothesis, we have

$$\delta^{\kappa-k+1} V^P(l, f+1, 0) - \delta^{\kappa-k} V^P(l, f+1, 0) \leq -\delta^{\kappa-k}\pi^P(l, f+1, 1).$$

Thus,

$$\begin{aligned} V^P(l, f, k) - V^P(l, f, k+1) \;\geq\;& \delta^{\kappa-1-k}\pi^P(l, f, 1) + \delta^{\kappa-k}\pi^P(l, f, 0) \\ & - \delta^{\kappa-k-1}\pi^P(l, f, 0) - \delta^{\kappa-k}\pi^P(l, f+1, 1). \end{aligned}$$

Since $f \geq IC_P(l)$, we have $\pi^P(l, f, 1) \geq (1-\delta)\pi^P(l, f, 0) + \delta\pi^P(l, f+1, 1)$. Therefore,

$$V^P(l, f, k) - V^P(l, f, k+1) \geq 0$$

and the value $V^P(l, f, k)$ is decreasing in k. Moreover, for each k,

$$\begin{aligned}
V^P(l, f, k) &\leq V^P(l, f, 0) \\
&= \sum_{t=0}^{\kappa-1} \delta^t \pi^P(l, f, 1) + \delta^\kappa \pi^P(l, f, 0) + \delta^{\kappa+1} V^P(l, f+1, 0) \\
&\leq \sum_{t=0}^{\kappa-1} \delta^t \pi^P(l, f, 1) + \delta^\kappa \pi^P(l, f, 0) + \delta^{\kappa+1} \frac{\pi^P(l, f+1, 1)}{1-\delta} \\
&\qquad \text{by inductive hypothesis} \\
&\leq \sum_{t=0}^{\kappa-1} \delta^t \pi^P(l, f, 1) + \delta^\kappa \frac{\pi^P(l, f, 1)}{1-\delta} \text{ since } f \geq IC_P(l) \\
&\leq \frac{\pi^P(l, f, 1)}{1-\delta},
\end{aligned}$$

as desired.

Suppose next $l > IC^*_{EA}$. Again, we first prove (iv). If $l > IC^*_{EA}$, (43) minus (44) equals

$$\begin{aligned}
& \pi^P(l, l-1, 1) + \delta\pi^P(l, l-1, 0) + \delta^2 \frac{\pi^P(l, l, 0)}{1-\delta} \\
& - \left(\pi^P(l, l-1, 0) + \delta \frac{\pi^P(l, l, 0)}{1-\delta}\right) \\
\geq\ & \pi^P(l, l-1, 1) + \delta\pi^P(l, l-1, 0) - \delta\pi^P(l, l, 0) - \pi^P(l, l-1, 0) \\
\geq\ & \pi^P(l, l-1, 1) + \delta\pi^P(l, l-1, 0) - \delta\pi^P(l, l-1, 1) - \pi^P(l, l-1, 0) \\
& \text{by (42)} \\
=\ & (1-\delta)\left(\pi^P(l, l-1, 1) - \pi^P(l, l-1, 0)\right) \\
\geq\ & 0.
\end{aligned}$$

Hence, protection is optimal.

We next prove (v). Given the equilibrium strategy, for each k, we have

$$V^P(l, l-1, k) = \sum_{t=0}^{\kappa-1-k} \delta^t \pi^P(l, l-1, 1) + \delta^{\kappa-k} \frac{\pi^P(l, l, 0)}{1-\delta}.$$

This is decreasing in k and bounded by $\frac{\pi^P(l, l-1, 1)}{1-\delta}$, as desired.

The proof for f with $IC_D(l) \leq f \leq l-2$ is the same as the proof for $f = l-1$ and $l \leq IC^*_{EA}$.

C.1.4 Proof of Lemma C.4

If $l + \iota = f$, then the protection is not feasible by definition. If $l + \iota > f$, then after non-protection, the leader invests if $l + \iota = f + 1$. Thus, the policymaker can obtain at least

$$\underbrace{\sum_{t=1}^{\kappa} \delta^t \pi^P (l+t,\ f, 1)}_{\substack{\text{A lower bound of the protection payoff can be attained} \\ \text{by assuming that the leader always invests given (40)}}} \qquad + \min_{(l_\tau, f_\tau) \text{ with } \bar{L} \geq l_\tau \geq l \text{ and } \bar{L} \geq f_\tau \geq f} \sum_{\tau=\kappa+1}^{\infty} \delta^\tau \pi^P (l_\tau,\ f_\tau, 0).$$

By contrast, with protection, she obtains at most

$$\pi^P (l,\ f, 1) + \max_{(l_t, f_t) \text{ with } \bar{L} \geq l_t \geq l \text{ and } \bar{L} \geq f_t \geq f} \sum_{t=1}^{\infty} \delta^t \pi^P (l_t,\ f_t, 0).$$

Thus, non-protection is optimal by (41).

C.1.5 Proof of Lemma C.5

Given Lemma C.4, it suffices to show that the leader invests in $(l+\iota,\ f+1, 0)$ if $l+\iota = f+1$. This follows since, at state $(l+\iota,\ f+1)$, (i) the policymaker cannot protect the leader if the leader does not invest from $l+\iota$ by feasibility and (ii) $f+1 < IC_D(l+\iota)$.

C.1.6 Proof of Lemma C.6

Consider the steady state $(l,\ f, k)$ (the existence is obvious). If $l > f$ and $k < \kappa - 1$, then the state $(l,\ f, k)$ is not a steady state since either protection is on and k increases or protection is off and f increases. Thus, we have either $l = f$ or $k = \kappa$ in the steady state. Suppose we have $l < IC^*_{EA} - 1$. We will prove either (i) we have $f \geq IC_P(l)$ and $f > IC_D(l)$, or (ii) it leads to a contradiction.

Suppose $l = f < IC^*_{EA}$ but $k \neq \kappa$. Then since the protection is not feasible at state $(l,\ f)$ with $l = f$, we have $k = 0$. Since the leader is not investing, we have $f > IC_D(l)$. Moreover, (39) implies that, at $(l,\ f)$ with $l = f$, we have $f \geq IC_P(l)$, as desired.

Suppose next that $k = \kappa$. We will prove that this would lead to a contradiction. Let $\left(\hat{l},\ \hat{f}, \kappa - 1\right)$ be the last state before the equilibrium transitions to $k = \kappa$. Note that $\hat{l} < IC^*_{EA}$ since we have assumed $l < IC^*_{EA} - 1$. Let ι be the investment decision in that state. By

Lemma C.4, we have $\hat{l} + \imath = \hat{f} + 1$ and the leader does not invest in $\left(\hat{l} + \imath,\ \hat{f} + 1, 0\right)$. (39) and $\hat{l} + \imath = \hat{f} + 1$ imply that $\hat{f} \geq IC_P\left(\hat{l}\right)$ and $\hat{f} + 1 \geq IC_P\left(\hat{l} + \imath + 1\right)$.

Since $\hat{l} + \imath = \hat{f} + 1$, protection is not feasible in state $\left(\hat{l} + \imath,\ \hat{f} + 1, 0\right)$ if the leader does not invest in $\left(\hat{l} + \imath,\ \hat{f} + 1, 0\right)$. Thus, the fact that the leader does not invest in $\left(\hat{l} + \imath,\ \hat{f} + 1, 0\right)$ implies that $\hat{f} + 1 > IC_D\left(\hat{l} + \imath\right)$. By contrast, since $\hat{l} < IC^*_{EA} - 1$, we have $\hat{l} + \imath < IC^*_{EA}$. Hence, the fact that the leader does not invest in $\left(\hat{l} + \imath,\ \hat{f} + 1, 0\right)$ implies that he will not be protected after investment. Since we have $\hat{f} + 1 \geq IC_P\left(\hat{l} + \imath + 1\right)$, Lemma C.3 implies that $\hat{f} + 1 < IC_D\left(\hat{l} + \imath + 1\right) \leq IC_D\left(\hat{l} + \imath\right)$. This is a contradiction.

D Proof of Proposition 3

By Propositions 1 and 2, l^* is no more than the solution for

$$IC_M(l) = 0 \Leftrightarrow \frac{\partial}{\partial l}\left((1-\rho)\,\hat{\pi}^M(l) + \rho\pi^L(l, 0)\right) = c_l(l),$$

while l^{**} is no less than the solution for

$$IC_D(l) = l \Leftrightarrow \frac{\partial}{\partial l}\pi^L(l, l) = c_l(l).$$

Thus, Assumption 1' implies $l^{**} > l^*$.

E Extension: Adding Costly Catch-Up

We assume that, in each period t, the follower's technology level increases from f_t to $f_t + 1$ at the beginning of period $t+1$ if and only if protection is off in period $t+1$ (with a positive probability). We call this variant of the model "the model of costly catch-up." The rest of the model is the same as the baseline model. In particular, we focus on renegotiation proof subgame perfect equilibrium, and assume that, for each $(l,\ f)$ with $f \geq IC_P(l)$, for any MPE, we have

$$\frac{\pi^P(l,\ f, 1)}{1-\delta} > V^P(l+1,\ f). \tag{45}$$

Note that (45) implies $\frac{\pi^P(l,f,1)}{1-\delta} > \frac{\pi^P(l+1,f,0)}{1-\delta}$ since the policymaker can obtain at least $\frac{\pi^P(l+1,f,0)}{1-\delta}$ by not protecting the leader forever (recall that $\pi^P(l,\ f, 0)$ is increasing in both l

and f). Thus,

$$\pi^P(l, f, 1) > \pi^P(l, f, 0). \tag{46}$$

We assume that, for each l such that $l - 1 > IC_M(l)$ (that is, if the leader is not willing to invest if he is protected regardless of the investment decision and the follower is at $l - 1$, then), we have $l - 1 > IC_P(l)$ (that is, the policymaker prefers to protect the leader if the follower is sufficiently strong and the leader's technology level stays at l).

Recursively, we will show that the follower's technology level never increases and protection is always on.

Lemma E.1 *In the model of costly catch-up, the protection is on for all (l, f) with $l > f$ and hence f never increases.*

E.1 Proof of Lemma E.1

For $l \geq \bar{L}$, since the leader never invests, even if the follower catches up, the policymaker protects the leader if $f \geq IC_P(l)$. Hence, eqm $(l, f) = (I, P)$ for $f \geq IC_P(l)$. Given this continuation play, at $f \geq IC_P(l) - 1$, the follower does not catch up. Hence, it is optimal for the policymaker to protect the leader given discounting at $f \geq IC_P(l) - 1$. Recursively, the protection is always on and the follower does not catch up.

For each l, suppose the statement is correct for $l + 1$. Then, for $(l, l - 1)$, suppose (i) the leader invests with probability one. Then, the policymaker protects the leader since the protection will be on at $(l + 1, l)$ by the inductive hypothesis and hence the follower does not catch up from $(l + 1, l - 1)$ to $(l + 1, l)$.

Suppose (ii) with a positive probability, the leader does not invest. After investment, by the proof above, protection is always on. Thus, we focus on proving that, after the leader's non-investment, protection is on.

With investment, by the inductive hypothesis, the leader would obtain the payoff of

$$\pi^M(l + 1, l - 1) - c(l).$$

Thus, we have to have

$$\pi^M(l + 1, l - 1) - c(l) \leq \max\left\{\pi^M(l, l - 1), \pi^L(l, l - 1)\right\} = \pi^M(l, l - 1),$$

which implies that $l - 1 > IC_M(l)$.

Given $l - 1 > IC_M(l)$, the leader does not have an incentive to deviate from eqm $(l, l - 1) = (NI, P)$. Thus, it remains to show that the policymaker's payoff after non-protection is less than $\frac{\pi^P(l,l-1,1)}{1-\delta}$. Without protection, her payoff is

$$\pi^P(l,l-1,0)+\delta\frac{\pi^P(l,l,0)}{1-\delta} \text{ if the firm with an opportunity does not invest at } (l,l)$$
$$\pi^P(l,l-1,0)+\delta\frac{\pi^P(l+1,l,1)}{1-\delta} \text{ otherwise, given the inductive hypothesis.}$$

Both of them are bounded by $\pi^P(l,l-1,0)+\delta\frac{\pi^P(l,l,1)}{1-\delta}$ (for the first case, this follows from (46) and for the second case, this follows from $\pi_l^P < 0$). As $l-1 > IC_M(l)$ implies $l-1 > IC_P(l)$, we have $\frac{\pi^P(l,l-1,1)}{1-\delta} > \pi^P(l,l-1,0)+\delta\frac{\pi^P(l,l,1)}{1-\delta}$, as desired.

Given protection at $(l,l-1)$, together with the inductive hypothesis, the follower does not catch up at $(l,l-2)$ (regardless of the leader's investment decision). Hence, protection is offered. Recursively, protection is always offered at (l,f).