Finite Resources and the World Economy

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Abstract

We build and evaluate a global macroeconomic model incorporating natural-resource scarcity. The model features low short-run substitutability between the natural resource and other inputs, while in the longer run endogenous directed technical change—allowing the economy to save on scarce resources—generates much higher substitutability, with rather stable cost shares. A nontrivial feature of the framework is secularly increasing resource use: initially, when the resource is abundant, much less is used of it, and as physical and human capital are accumulated, its use increases. The model is also able to generate highly volatile prices at higher frequencies.

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1 Introduction

The principal aim of this paper is to develop quantitative theory for use in global macroeconomic analysis, both aimed at understanding business-cycle and longer-run movements in aggregates. Our key contribution is the explicit incorporation of key natural resources in finite supply. In particular, we propose a way of matching some of the key facts for natural resources: (i) a secular increase in use (but, in some cases, a peak at some point that is arguably due to the finiteness); (ii) very significant movements in prices in the shorter run; and (iii) relatively stable movements in cost shares in the long run while shares vary significantly at higher frequencies. Though our points, we believe, apply to most natural resources in finite supply, our quantitative application is to fossil energy, in particular oil.

The core model proposed in the paper builds on our recent work: Hassler, Krusell, and Olovsson (2021). This paper constructs a model of directed, energy-saving technical change aimed at understanding the long-run energy share. In the model, the substitution elasticity between a Cobb-Douglas composite and fossil fuel is near zero (i.e., Leontief) in the short run. However, because technology is endogenous in the medium and longer run, the mix of capital/labor vs. energy-augmenting technical change varies in response to need: as a fossil energy becomes more scarce, energy-augmenting technical change rises in response. Hence, unlike in the short run, the long-run cost share is rather stable in response to shocks.

In the present paper, we consider a world-economy version of the model and, more importantly, consider transitional dynamics. This allows us to make the point that the model naturally produces secularly increasing resource use, while of course eventually finiteness binds and the use of the resource declines toward zero. As we argue in the paper, the prediction of secular increase in resource use is valuable given that standard growth models imply decreasing use over time. This model feature, which is needed to match the data, is intimately connected with the endogeneity of technical change in the model and with the resource being "abundant" initially in the appropriate sense: the state variables of the model—the capital stock, the resource stock, and the two input-augmenting levels—need to satisfy a certain inequality whereby capital and the capital/labor-augmenting technology are low relative to the resource stock and the energy-augmenting technology level.

We do not explicitly consider shocks in this paper, but because there is very high, near-Leontief complementarity in the short run, changes in demand and supply can change resource prices significantly. In fact, in a separate paper, Bornstein et al. (2021) looks at short-run fluctuations in a world equilibrium model of the oil market based precisely on the production function used here and document that the model indeed can match the high price volatility.

We begin the paper, after a very brief literature review in Section 2, in Section 3 by describing some facts; the current version of the paper is preliminary and the selection of facts is somewhat arbitrary. In particular, we display oil/fossil-fuel facts as well as facts for some metals, but the time ranges as well as the coverages vary greatly (facts on metal quantities, for example, are missing). Section 4 then goes through some simple models of limited resources to make the point that there is a robust result in standard setting: the use of the resource will be largest in the beginning and then declining over time toward zero, quite unlike what we see in the data. Section 5 then displays the core framework in this paper from the production side and uses the model to back out the trends in input-augmenting technologies using the case of fossil fuel and U.S. data. These document negative correlation and how energy-saving is faster when the fossil price rises: there appears to be endogenous directed technical change. Section 6 then states the full world model and solves for a transition path. We also consider an extension featuring a new energy resource (such as "green energy") that is initially expensive but that is not limited in supply. This material is contained in Section 7. Section 8 concludes.

2 Literature

Our paper is related to the literature that is concerned with the well-known "cake-eating" problem that analyses how a finite resource should be depleted over time. This problem was first studied by Hotelling (1931) and later by Gale (1967). Many studies have evaluated its implications against data, typically using oil as an example. Heal and Barrow (1980) look at metal prices. Livernois (2009) offers a review of the empirical performance of Hotelling's theory.

The question of how natural resources are and should be depleted then received renewed interest after the first oil-price shock in 1973. Indeed, the Review of Economic Studies featured a special issue on the economics of exhaustible resources already in 1974, and it contained important contributions from Dasgupta and Heal, Solow and from Stiglitz among others. These papers are all concerned with the question of how economic growth is affected by the presence of an input that is depleted over time. Later contributions that also are concerned with the growth implications of natural resource scarcity but allows for endogenous technical change include Barbier (1999), Scholz and Ziemes (1999), Smulders and Nooij (2003), Grimaud and Rouge (2003), and Groth (2007). Most of these papers consider a Cobb-Douglas production function. A paper that is related to ours is Tahvonen and Salo (2001) that sets up a theoretical model with endogenous technical change and consider the energy transition where the use of non-renewables starts from zero, reaches a maximum and then approaches zero.

3 Some facts

Figure 1 shows U.S. data on the uses of different energy sources over a long time span. We see clear upward trends, while there are also peaks in use for oil and coal.

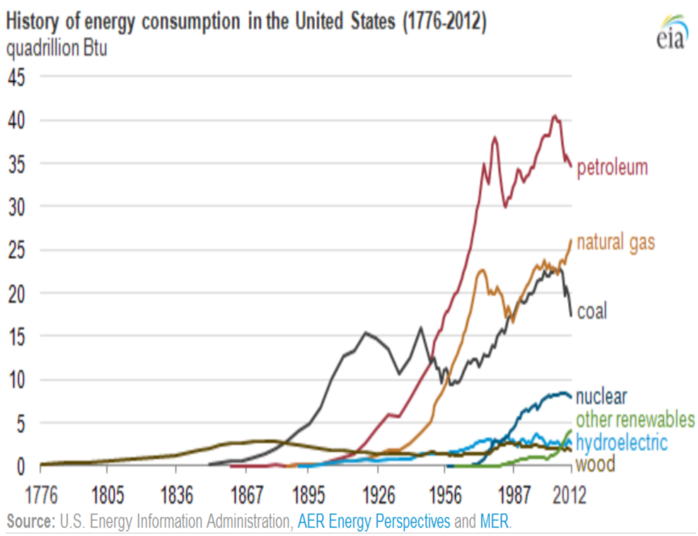

Figure 1: Graph borrowed from Almenhali et al. (2017)

Figure 2 focuses on oil consumption per capita: it peaked in the 1970s and it has been decreasing since then. This is true both in the United States and in the world as a whole. The picture is also very much the same if we instead look at U.S. consumption of a fossil-fuel composite instead: it also grew until the early 1970s when it peaked and it has since then been falling.

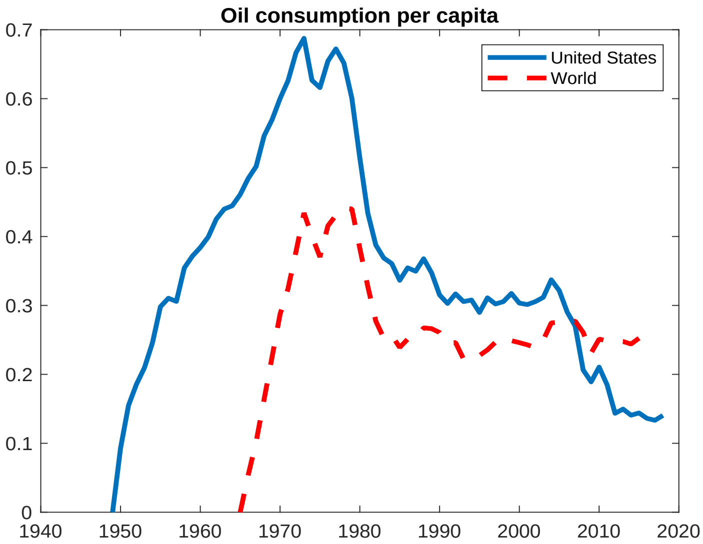

Figure 2

Moving to prices, Figure 3 plots the evolution of oil prices for a very long period in time. What stands out the most clearly from this graph is the enormous volatility, a feature that we will also see present for other natural resources. Second, over this long a horizon, the prices appear to have no trend.

However, if one—as is often the case in quantitative macroeconomic studies—limits attention to the postwar period, a different picture emerges. Figure 4 displays growth rates during sub-periods for a fossil-fuel composite.

The figure reveals a positive price growth for all sub-periods considered at somewhere between one and two percent per year. For conventional oil, whose marginal cost of production is only a small fraction of its market price, standard Hotelling reasoning (as illustrated in the theory below) implies that for this oil to be produced at any two consecutive dates, its price growth would have to equal the real interest rate. The real interest rate path during the postwar period reveals fluctuations too (in particular during the inflation and disinflation episodes around 1980), but the mean is around two percent. Thus, the postwar oil/fossil-fuel price data is at least not in wild contradiction with the Hotelling rule. However, an argument can be made that the safe real

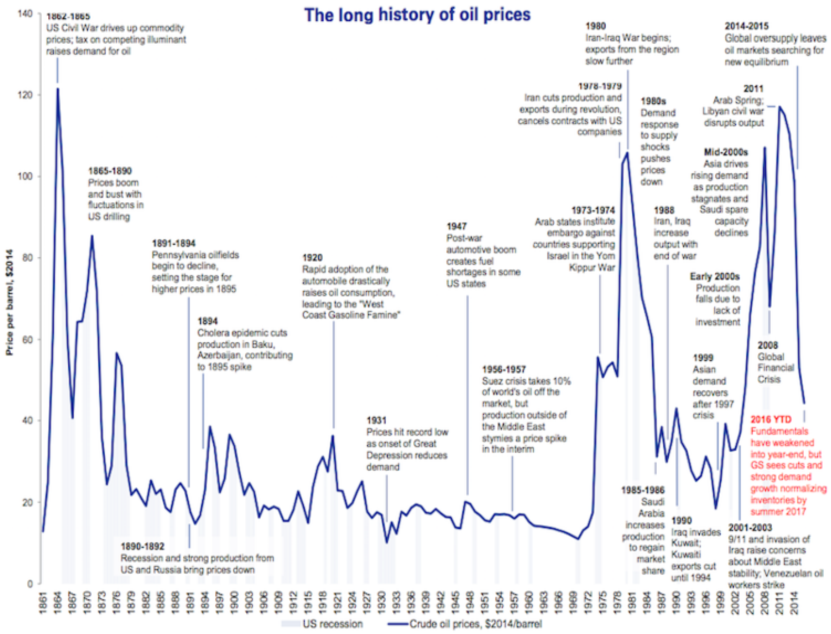

Figure 3: Graph borrowed from Business Insider (2016)

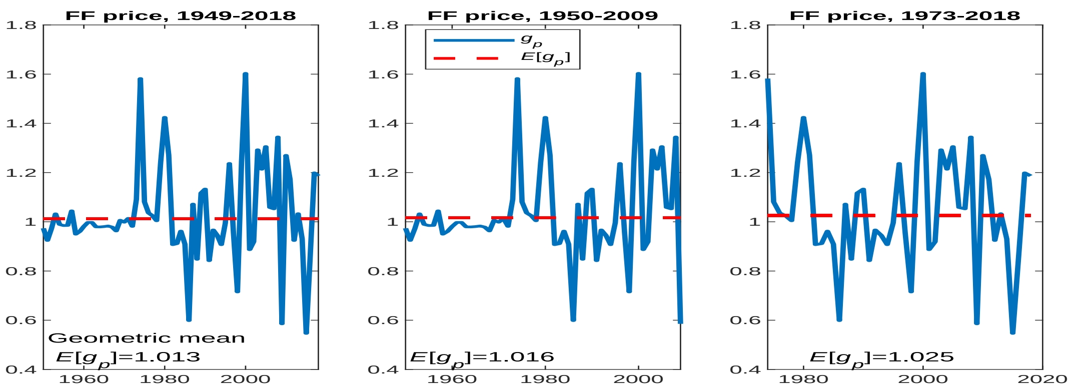

Figure 4

interest rate is not the right measure to relate to the growth rate of the oil price. The Hotelling result can be interpreted as an arbitrage condition. The return on oil, its price growth, should be equal to the return on alternative investments. In a model with risk, risk premia thus need to be factored in. Thus, the stock market, with a markedly higher expected return, may be a better comparison. In any case, the focus on this paper purpose of this paper is on the demand side of the market for natural resources in finite supply.

Figures 6–7 show the prices of coal, lead, zinc, and copper.

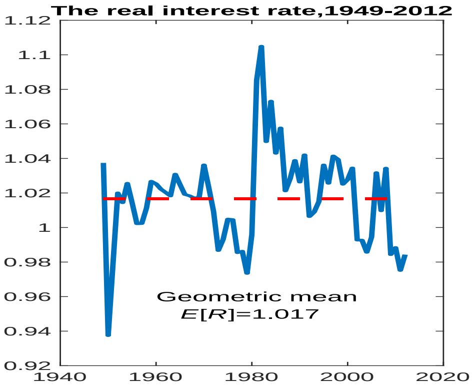

Figure 5

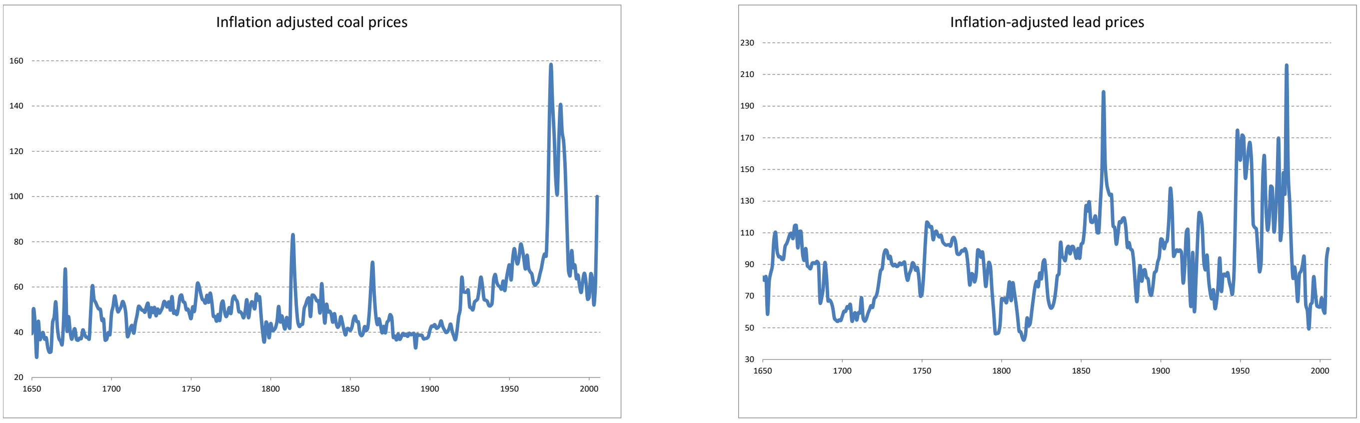

Figure 6

Coal's price resembles that of oil, but the metals are somewhat different. They share the stationarity of a longer time horizon but, unlike for fossil fuels, there is no visible upward trend in the postwar period. This is arguably because the marginal-cost behavior for metals is different than it is for oil. Hotelling implies

$$\frac{p_{t+1}}{p_t} = 1 + r_t + \frac{1}{p_t}\left(mc_{t+1} - (1 + r_t)mc_t\right)$$

so that if the marginal cost is rising faster than the rate of interest, the price has to rise faster to compensate. Thus, if metals (i) have a marginal cost that is high relative to the price (so that the "rent" is small) and (ii) have

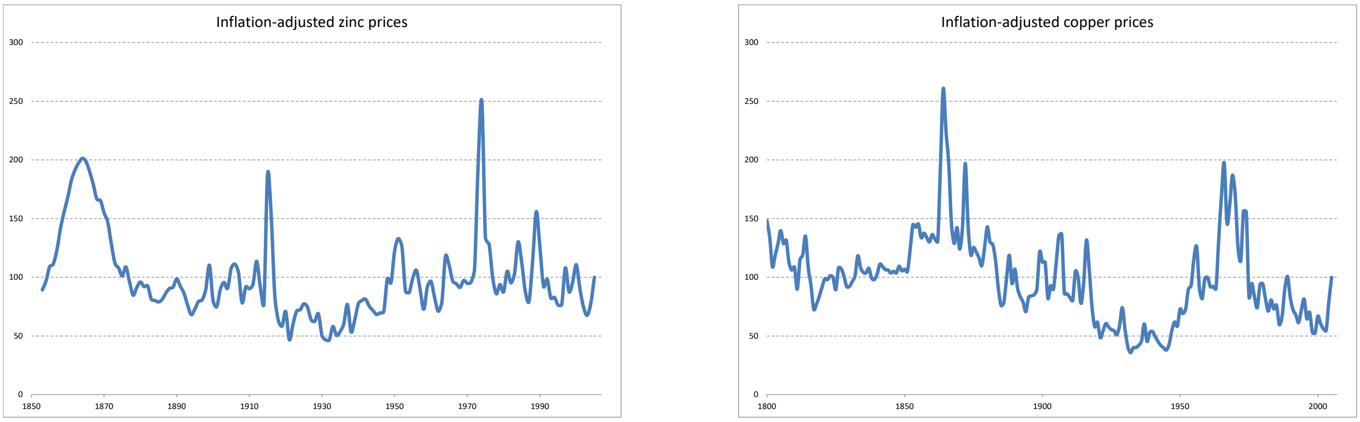

Figure 7

technological change in production exceeding the real interest rate, then their price growth will be lower. We have not direct measurements of technological change in metal production but plausibly the rent is lower there than for oil and there has been significant technological change in extraction. However, these statements represent a “hopeful” rather than “convinced” perspective on the power of Hotelling’s rule for explaining average price growth.

Finally, we emphasize that all natural resources seem to display massive price volatility. Figure 8 tabulates price volatilities for a range of objects, including natural resources. We see that these volatilities are of the same magnitude as that of the U.S. stock market. The most volatile of those listed are coffee, cocoa, and sugar, all produced commodities.

To summarize: fossil fuels like oil, and arguably natural resources more generally, possess a number of noteworthy features. Their quantities go up secularly over time (at least since the Industrial Revolution). Whether we see an eventual “peak” or not may vary, to the extent one is close to exhaustion or production costs have risen significantly. Second, they display very high price volatility. Third, many natural resources display no trend in their prices, though for oil/fossil fuel a trend increase has been observed during the postwar period.

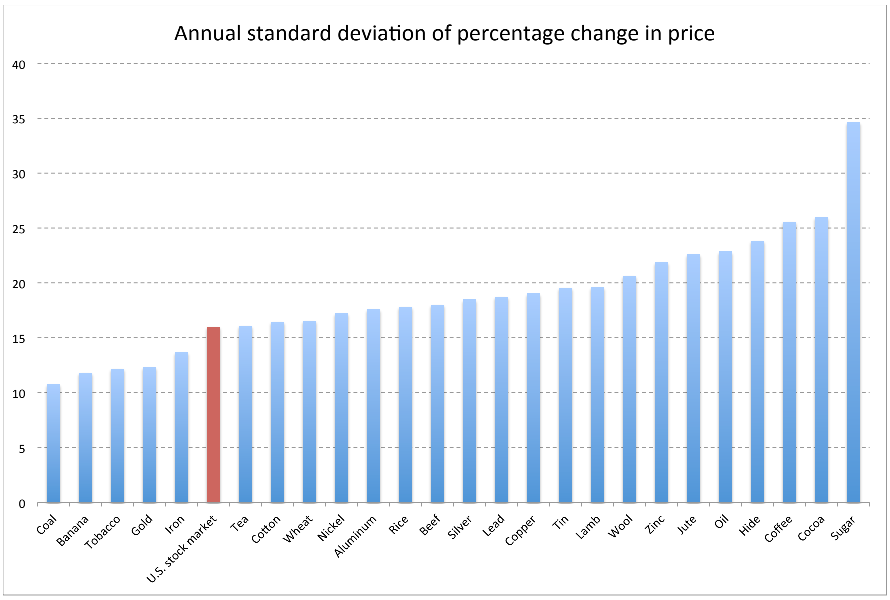

Figure 8

4 The path of natural-resource use

Throughout the analysis we will consider an infinitely lived representative household with the following utility function

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}, \tag{1}$$

where c_t is the one and only consumption good and β is the discount factor. The population structure is not key; the dynasty assumption is very often used, however, as it offers a way of incorporating how bequests are intentional. We focus on planning problems throughout, because they deliver—in the absence of frictions—the same allocations as in market equilibria with perfect competition. In the case below where we study endogenous directed technical change, we also restrict attention to planning problems, even though market equilibria may involve inefficiencies in those cases. These inefficiencies, however, are expected to be minor, or even nil, based on arguments we put forth in Hassler, Krusell, and Olovsson (2021).

We will present a sequence of simple models, from extremely simple to slightly less simple, as a means of illustrating the key point here: that it is difficult to derive a secular increase in the use of the natural resource, which will eventually be exhausted (and therefore the increase must, at some point, turn into a decrease). The models are well known and there is no substantive contribution here; the material should be viewed as a background for our main model to be presented below.

4.1 Cake eating, I

As a starting point, consider now a simple cake eating problem where consumption has to come from a natural resource in finite supply. The stock of the resource at time t is denoted by R_t, and the resource constraint is given by

$$\sum_{t=0}^{\infty} c_t = R_0. \tag{2}$$

We can think of this as R being oil with a production function of final output that is linear in oil, and where oil is extracted at zero costs.

The social planning problem involves maximizing (1) subject to (2). The solution provides the optimal depletion path for R and it is given by

$$c_t = (1 - \beta) R_t,$$

where $R_{t+1} = R_t - c_t$. Alternatively, we can write the solution as

$$c_t = (1 - \beta) \beta^t R_0,$$

which reveals that consumption is exponentially falling in R_0.

Hence, in summary, the simple cake eating problem cannot account for periods with increasing consumption if the natural resource, and the intuitive explanation for this result is simply discounting, which is a core part of macroeconomic modeling and as such hard to dispense with. It is, among other things, hard to otherwise explain how on average real interest rates are positive.

4.2 Cake eating, II: the resource as a production input

We now expand the model to include production and assume that the final good is produced with capital, k, labor, l, and the resource, e, a notation we use because our main application is to think about oil, or fossil fuel or more generally energy. For now, we will refer to e as oil and energy interchangeably.[1]

Oil exists in finite supply and, as in the previous section, we assume that it is extracted at zero cost; this is a decent approximation for conventional oil (think Saudi oil), where the price far exceeds the marginal cost. Mostly, however, the zero cost assumption here is made because it more easily illustrates the core points. The resource constraint for oil is thus given by

$$\sum_{t=0}^{\infty} e_{o,t} - R_0 \tag{3}$$

Labor is inelastically supplied by the representative consumer and, for simplicity, there is no population growth.

4.2.1 Logarithmic utility and full depreciation of capital

To be able to derive closed-form solutions, we now set $\sigma = 1$, which implies logarithmic utility, and assume full depreciation of capital, i.e., $\delta = 1$. Both these assumptions are commonly used in medium- to long-run macroeconomic analysis so one is still able to view the resulting analysis as having some quantitative relevance. We also start with the case without any technological progress and assume that the initial level of total factor productivity (TFP) is constant at a value A.

The production function for final goods is Cobb-Douglas in all three inputs, i.e.,

$$y_t \equiv F(A, k_t, l, e_t) = A k_t^{\alpha} l^{1-\alpha-\nu} e_t^{\nu}. \tag{4}$$

Because we assume that the production of fossil energy requires no inputs, the production function, which is gross in nature, also represents GDP. The analysis is greatly simplified by not having to consider the allocation of capital, labor, and energy across sectors.

The aggregate resource constraint is given by

$$c_t + k_{t+1} = y_t. \tag{5}$$

The social planning problem can now be stated. It is given by

$$\max_{\{k_{t+1}, e_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log(y_t - k_{t+1}) - \lambda \left[\sum_{t=0}^{\infty} e_{o,t} - R_0 \right]. \tag{6}$$

[1] In the United States, oil use and usage of the fossil fuel composite are highly correlated.

The first-order conditions imply the following relationships:

$$\frac{c_{t+1}}{c_t} = \beta\alpha\frac{y_{t+1}}{k_{t+1}} \tag{7}$$

and

$$\frac{y_{t+1}/e_{t+1}}{y_t/e_t} = \alpha\frac{y_{t+1}}{k_{t+1}}. \tag{8}$$

Equation (7) is the standard Euler equation and (8) is the Hotelling equation that demands the return to the two assets capital and oil to be the same.[2] Specifically, the left-hand side features the ratio of the marginal products of energy in periods t and $t+1$, whereas the right-hand side is the return to capital. Because the marginal product of energy is the price of energy in a decentralized version of this economy, the Hotelling equation delivers the well-known result that the price of the finite resource should grow at the rate of interest.

It is straightforward to show that the solution to (6) features a constant savings rate, $s = \alpha\beta$, and a growth rate of oil use that is given by

$$\frac{e_{t+1}}{e_t} = \beta, \quad \forall t. \tag{9}$$

As in the simple cake eating problem in Section 4, oil use is thus *always* exponentially falling at rate β. Hence, the model is again unable to account for *any* time period with increasing oil use.

On a balanced growth path, the growth rate of capital, consumption and output—which we denote by g—is constant and can be shown to be given by

$$g = g^{\alpha}\beta^{\nu} = \beta^{\frac{\nu}{1-\alpha}} < 1.$$

In this setting, the economy is thus shrinking over time. However, from the Euler equation it follows that the balanced growth real interest rate is given by $\beta^{\frac{\nu}{1-\alpha}-1} > 1$, which again implies that the oil price increases over time.

In the Appendix, we modify the Cobb-Douglas production function to also allow for exogenous technological progress. The main difference relative to the previous case is that the growth rate of the economy now can be positive if technology growth is high enough. Also with exogenous growth, the savings rate is constant and given by $s = \alpha\beta$ and the depletion rate is still given by (9).

[2] See Hotelling (1931).

In conclusion, a straightforward extension of the consumption cake model to one where the cake is an input into production in an otherwise neoclassical model delivers the same core prediction—the resource is depleted at a constant rate from date zero—in the case where the aggregate production function is Cobb-Douglas. The assumptions we made in the analysis on the preference side, and that for capital depreciation, are special, but we do not consider them problematic, as argued briefly in the next subsection. However, as we will argue later, the Cobb-Douglas assumption, when relaxed, will give very different predictions.

4.2.2 CRRA utility and less than full depreciation of capital

It is straightforward to relax the assumptions of logarithmic utility and full depreciation of capital to show that the model with Cobb-Douglas production function still is not able to account for periods with increasing usage of energy. The intuition here is that the key difference between this case and the former case is that the initial capital stock will be a more nontrivial state variable. In particular, it will affect the saving rate. However, this effect is well known to be rather minor. Thus, the capital accumulation path will not differ markedly compared to the previous case. Under the assumption of Cobb-Douglas production, equation (8) still holds and since capital accumulation is not very different, neither is the path of oil use.

4.3 CES technology

There are two reasons to go outside the Cobb-Douglas case. One is that it has an implication that is clearly counterfactual: it predicts energy's share of income to be constant and equal to ν in all periods, whereas energy's share of income in the data closely follows the energy price in the short to medium run.[3] This indicates a lower elasticity of substitution between energy and the remaining inputs than one.

The other, and for our purposes more direct, reason is that a CES technology that does have low substitution elasticity between energy and the other inputs will offer a way of accounting for secularly increasing resource use. So suppose we look at the most extreme case: Leontief production. In particular, assume that

$$y_t = F(k_t, l, e_t) = \min\left\{Ak_t\alpha l^{1-\alpha}, A_e e_t\right\}, \tag{10}$$

where we focus, for simplicity, on the case without technology growth. There

[3] See Hassler, Krusell, and Olovsson (2021).

are two input-augmenting levels in this formulation—A and A_e—but only their relative levels play a role.

This economy gives a much more important role for initial conditions than did our previous cases. In the very first period, e_0 will be chosen to satisfy

$$e_0 \leq \frac{Ak_0\alpha l^{1-\alpha}}{A_e},$$

for any value above that would clearly be wasteful: this economy will clearly have a positive marginal value of oil at all points in time. Then, if k_0 is sufficiently low the oil path must be increasing, at least between periods 0 and 1: capital will be built up initially and only eventually fall, as e must eventually fall toward zero.

More generally, if A is low relative to A_e and tk_0 is low in relation to R_0, the path for capital as well as for energy use will be rising over time, peak, and then fall.

Of course, in this economy, the energy share display strong trends as well. Provided $\beta < 1$, the energy share will in the long run approach unity (see Hassler, Krusell, and Olovsson, forthcoming). If the initial condition is the one we entertained—k_0 is low so that energy is abundant and capital scarce—the capital share will start out positive implying an energy share strictly below unity with an increasing trend. In the data, however, once one removes the high-frequency fluctuations, the share is surprisingly stable. Thus, this production function also does not satisfy the stylized facts at hand. This is why we must allow some more generality still.

4.4 Other models

Before going to our main model, let us briefly mention that secularly increasing resource can potentially be accounted for with models that have fundamentally different features than those entertained here. For example, if at each point in time market participants misjudge the total stock of available natural resources, then an increasing path may follow: at each point in time will the economy then plan to go through a decreasing path, as prescribed above in the cake-eating models, but the decreasing path is then continuously revised upward, de facto generating an upward path. Since revisions upward in total stocks have been noted, this kind of explanation is perhaps plausible at first thought. However, our preliminary assessment is that expectations would have to be so far from rational—the mistakes would have to continuously occur in the same direction over a long period of time—that this explanation is not plausible, at least not as the sole explanation for increased resource use. An arguably more reasonable deviation from rationality is analyzed in Spiro (2014), where it is assumed that resource owners have a finite planning horizon and disregard the possibility of selling after the end of the horizon. Then, if the resource is not fully extracted within the planning horizon, it is priced at marginal extraction cost."”

A different explanation could be found in capacity constraints: the idea would be that the extraction of a resource at each point in time is severely constrained by convex costs. The convex costs would be short-run costs, such as those involved in oil extraction: it is hard to significantly change the outtake of oil from any given well once the machinery for pumping has been put in place. Thus, the marginal costs of oil may be low in an ex-ante sense (because exploring/drilling for oil and then building the pumps is mostly time-consuming but does not involve high costs) but high ex post. This kind of approach has been taken recently to explain fluctuations of prices, quantities, and investments in the world oil market; see Bornstein et al. (2021). However, it would then still be necessary to explain why more oil exploration and drilling has not taken place historically. The benchmark model below gives an answer based on weak oil demand: oil has not been the scarce resource historically, but capital (along with capital-augmenting technology) has been.

5 CES and time-varying input saving

As a first step toward our full model, we now modify the simple CES technology with one that has time-varying input-augmenting technology levels. Thus, the final good is produced with the following production function

$$y_t \equiv F\left(A_t k_t^{\alpha}, A_{e,t} e_t\right) = \left[\left(A_t k_t^{\alpha} l_t^{1-\alpha}\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(A_{e,t} e_t\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{11}$$

where A and A_e are productivity components that respectively affects capital/labor and energy. In this setting, A is similar to the standard TFP measure, whereas A_e specifically captures energy-saving technical change.

Let us first comment on how this production function can be used to interpret the data. Hassler, Krusell, and Olovsson (2021) uses this function, and the assumption of perfect competition in input markets, to solve explicitly for the two technology trends A_t and A_{et} in terms of observable output, inputs, and their prices (and production-function parameters). Formally, these technology trends are given by

$$A_t = \frac{y_t}{k_t^{\alpha} l_t^{1-\alpha}} \left(\frac{l_t^{share}}{1-\alpha}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{12}$$

and

$$A_{et} = \frac{y_t}{e_t} \left(e_t^{share}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{13}$$

where $l_t^{share} \equiv w_t l_t / y_t$ and $e_t^{share} = p_t e_t / y_t$.

Figure 9 plots y_t and e_t together with the two technology trends A and A_e, based on postwar U.S. data.[4] A key input is the substitution elasticity parameter, which is estimated to be near zero, i.e., almost Leontief, in consistency with the fact that the fossil price and the fossil share move in lockstep.

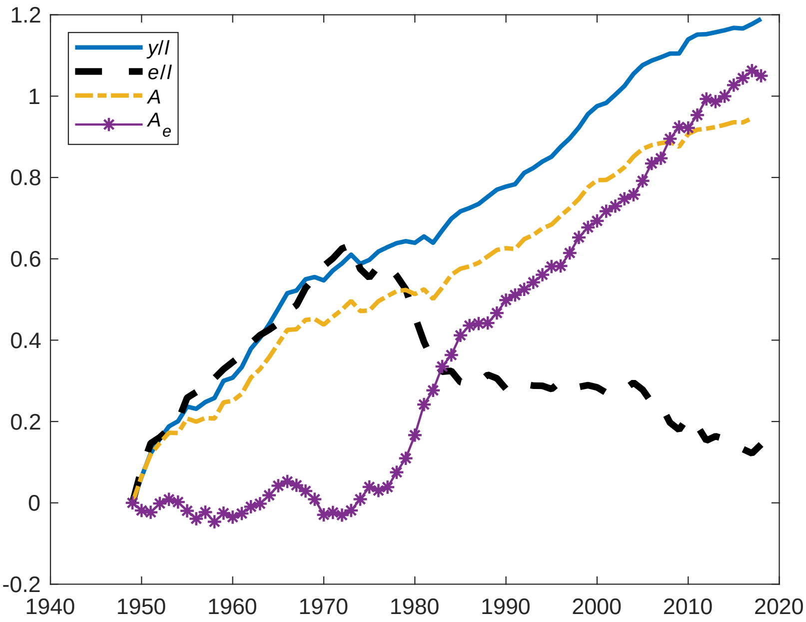

Figure 9

In the figure, e/l is (the normalized logarithm of) a fossil-fuel composite consisting of oil, coal, and natural gas per unit of labor input instead of just oil per person, but the picture for energy use is the same as in Figure 2: energy use per person increased up to the early 1970s and it has been decreasing since. The variable y/l is (the normalized logarithm of) labor productivity. Note also that e/l closely follows y/l between 1949 and 1973 and that, over this period, energy-saving technical change was roughly zero, i.e., A_e remains almost constant during the period. Since 1973, however, A_e is rising at a high rate, while e/l has been falling; energy use in *efficiency units*, $A_e e/l$, have however not decreased.[5] Instead, the reduction in physical

[4]See the Appendix for the data sources.

[5]In fact, fossil-fuel use in efficiency units closely follows output.

energy use has thus been offset by faster energy-saving technical change.

There are two important takeaways. One is that, in the figure, the input-augmenting series display dramatic changes over time, and they thus need to be incorporated into our analysis in order to have a chance to match the data. Second, though the fossil price is not plotted in the same graph, it is clear that when the price started rising fast in the mid-1970s, energy-saving took off, from previously having been dormant. That is, the input-saving technology series really appear like endogenous responses to scarcity, as measured by the fossil price. This motivates our main model that is now presented.

6 Directed technical change

To account for changes in A and A_e, we now endogenize the direction of research by considering a fixed stock of researchers, n, that can be allocated to improve the productivity of capital/labor and/or energy services. Specifically, we assume that R&D can affect the growth rates of A and A_e. Formally, we have

$$A_{t+1}/A_t \equiv g_{A,t} = f(n_t) \tag{14}$$
$$A_{e,t+1}/A_{et} \equiv g_{A_e,t} = f_e(1-n_t). \tag{15}$$

Following Hassler, Krusell, and Olovsson (2021), the R&D functions are assumed to have the following functional forms

$$f(n_t) = 1 + Bn_t^{\phi}, \tag{16}$$
$$f_e(1-n_t) = 1 + B_e(1-n_t)^{\phi} \tag{17}$$

The specification defined by (14)-(17) implies a trade-off in that an increase in n implies a higher growth rate for A at the cost of a lower growth rate for A_e.

The law of motion for the stock of oil is still given by (3). With less than full depreciation of capital, the aggregate resource constraint is given by

$$c_t + k_{t+1} = y_t + (1-\delta)k_t. \tag{18}$$

The equilibrium properties in the above model are laid out in detail in Hassler, Krusell, and Olovsson (2021). Specifically, Theorems 2 and 3 in that paper states that the economy will converge to an exact balanced growth path with an interior choice for technology, where following features must hold:

1. The two arguments of the aggregate production function, $A_t k_t^{\alpha}$ and $A_{et} e_t$, both grow at the rate of output g.

2. Energy use falls at a constant rate: $e_{t+1}/e_t = \beta g^{1-\sigma}$.

3. Technology effort n and the consumption growth factor g are determined by $f_e(1-n)\beta = f(n)^{\frac{\sigma}{1-\alpha}} = g^{\sigma}$.

4. Energy's share of income is exclusively determined by how costly it is to enhance energy efficiency in terms of lost capital/labor efficiency.

As stated by property 2, energy use is falling on a BGP. With $\sigma = 1$ (i.e., with logarithmic utility) the rate of depletion is exclusively determined by the discount factor β. An important difference relative to the Cobb-Douglas specification, however, is that the depletion rate now potentially can differ substantially from β during periods when the economy is not on the BGP. We now turn to quantify the importance of this mechanism.

6.1 Transformation

To simulate the model and evaluate the predictions, we transform it to make it stationary. To simplify the notation, we use the general notation $G(A_{t+1}/A_t, A_{e,t+1}/A_{et}) = 0$ to capture the technology constraint imposed by (14)-(15). The transformation follows the transformation carried out in Hassler, Krusell, and Olovsson (2021), but an important difference is that the fuel price here is endogenous. Now, define $x_t \equiv k_t^{\alpha} A_t$, $\hat{c}_t = c_t/x_t$, $\hat{k}_t = k_t/x_t$, $a_{et} = A_{e,t} R_t / x_t$, and $\hat{e}_t = e_t/R_t$. Moreover, let $g_{x,t+1} = x_{t+1}/x_t$. In addition, because $A_t = x_t k_t^{-\alpha} = x_t \left(\hat{k}_t x_t\right)^{-\alpha} = x_t^{1-\alpha} \hat{k}_t^{-\alpha}$ and $A_{e,t} = a_{et} x_t / R_t$ and $R_{t+1}/R_t = 1 - \hat{e}_t$, we have $A_{t+1}/A_t = g_{x,t+1}^{1-\alpha} \left(\hat{k}_{t+1}/\hat{k}_t\right)^{-\alpha}$, and $A_{e,t+1}/A_{e,t} = g_{x,t+1}\left(a_{et+1}/a_{et}\right)/\left(1-\hat{e}_t\right)$.

Note here that x and a_{et} are state variables in a true sense: all their components are predetermined. Then, using $e_t = \hat{e}_t R_t$, we have

$$\max_{\left\{\hat{c}_t, \hat{k}_{t+1}, g_{x,t+1}, a_{et+1}, \hat{e}_t\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{(\hat{c}_t x_t)^{1-\sigma}}{1-\sigma}$$

subject to

$$\hat{c}_t + \hat{k}_{t+1} g_{x,t+1} = F(1, a_{et}\hat{e}_t) + (1-\delta)\hat{k},$$

and

$$G\left(g_{x,t+1}^{1-\alpha}\left(\frac{\hat{k}_{t+1}}{\hat{k}_t}\right)^{-\alpha}, \frac{g_{x,t+1}\frac{a_{et+1}}{a_{et}}}{1-\hat{e}_t}\right) = 0.$$

Noting that

$$F\left(1, a_{et}\widehat{e}_t\right) = \left[1 + \left(a_{et}\widehat{e}_t\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{19}$$

the problem can now be written as

$$\max_{\left\{\hat{k}_{t+1}, x_{t+1}, a_{et+1}, \hat{e}_t\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{\left(\left(F(1, a_{et}\hat{e}_t) + (1-\delta)\hat{k}_t - \hat{k}_{t+1}\frac{x_{t+1}}{x_t}\right) x_t\right)^{1-\sigma}}{1-\sigma} \tag{20}$$

subject to

$$\begin{gathered} G\left(g_{x,t+1}^{1-\alpha}\left(\frac{\hat{k}_{t+1}}{\hat{k}_t}\right)^{-\alpha}, \frac{g_{x,t+1}\frac{a_{et+1}}{a_{et}}}{1-\hat{e}_t}\right) \equiv \\ \frac{g_{x,t+1}\frac{a_{et+1}}{a_{et}}}{1-\hat{e}_t} - 1 - B_e\left(1 - \left(\frac{g_{x,t+1}^{1-\alpha}\left(\hat{k}_{t+1}/\hat{k}_t\right)^{-\alpha} - 1}{B}\right)^{\frac{1}{\phi}}\right)^{\phi} = 0. \end{gathered} \tag{21}$$

The first order conditions to the above problem form a dynamic system with the state vector $(\hat{k}_t, a_{et})$ and the control vector $(g_{x,t+1}, \hat{e}_t)$. For reasons of tractability, the first order conditions and the characterization of the steady state are placed in the Appendix [6]

Two variables that we are particularly interested in are the growth rate of fossil fuel use,$g_{e,t}$, and the oil price, $g_{p,t}$. With the oil price given by the marginal product of oil, $p_t = A_t^e F_2\left(1, a_{et}\hat{e}_t\right)$, these variables are, with the definitions stated above, given by

$$g_{e,t} \equiv \frac{e_{t+1}}{e_t} = \frac{\hat{e}_{t+1}R_{t+1}}{\hat{e}_t R_t} = \frac{\hat{e}_{t+1}}{\hat{e}_t}(1-\hat{e}_t). \tag{22}$$

$$g_{p,t} \equiv \frac{A_{t+1}^e F_2\left(1, a_{et+1}\hat{e}_{t+1}\right)}{A_t^e F_2\left(1, a_{et}\hat{e}_t\right)} = \frac{g_{x,t+1}\left(a_{et+1}/a_{et}\right)}{1-\hat{e}_t}\frac{F_2\left(1, a_{et+1}\hat{e}_{t+1}\right)}{F_2\left(1_t, a_{et}\hat{e}_t\right)}, \tag{23}$$

where F_2 denotes the derivative of F w.r.t. the second argument.

6.2 Calibration

Given that the model here is similar to the model in Hassler, Krusell, and Olovsson (2021), we use the estimated values reported in that paper. Specifically, the calibration is in laid out in detail in Table 1.

[6] Formally, the system includes equations (21)–(34) and the transformed resource constraint $\hat{c}_t = F(1, a_{et}\hat{e}_t) + (1-\delta)\hat{k}_t - \hat{k}_{t+1}\frac{x_{t+1}}{x_t}$.

Table 1: Calibration

Parameter	α	β	ε	σ	B	B_e	ϕ
Value	0.2632	0.985	0.02	1	0.016	0.19	0.92

Arguably the most important aspect of the calibration for our results is a low value for ε.

To compute the transition path, we use our estimated model and set initial conditions so that a_e (the state variable indicating the transformed level of energy efficiency) initially is 3.5 times larger than its steady-state value whereas $\hat{k}_t$ is at its steady state value. This is aimed at representing the idea that in the beginning of the simulation period (say the beginning of the 20th century), oil was not a scarce factor of production—in relation to capital—taking into account the technology factors A and $A_{e.}$ The exact values are selected mostly as an example to illustrate the quantitative magnitudes involved but it is central that the initial value of a_e is larger than its steady state value.

6.3 Results

The results are plotted in Figure 10 below.

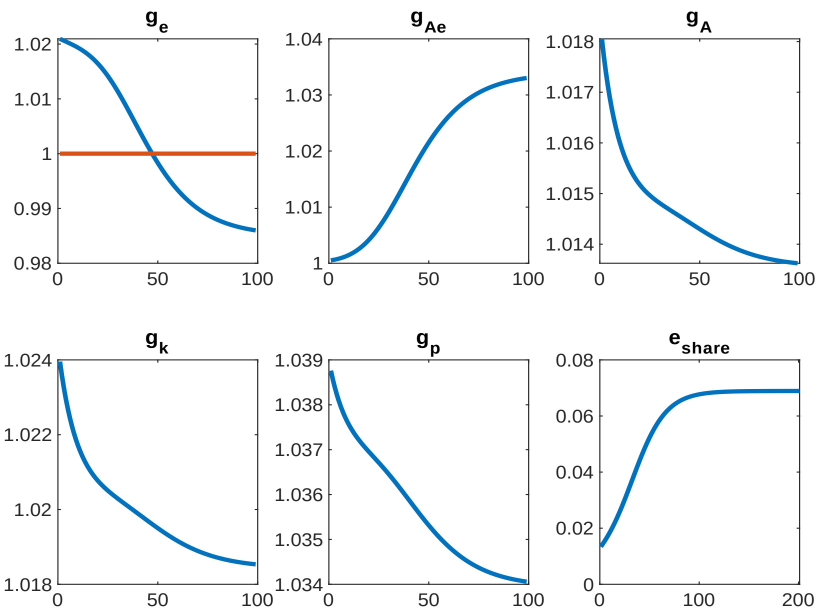

Figure 10

The upper left graph in the figure shows the main result, which is that oil use increases for 46 years before it starts to fall (its gross growth rate g_e is larger than one). In the beginning of the transition and as in the data presented in Figure 9, the increase in energy use is accompanied by a low growth rate of energy-saving technical change (A_e). In contrast, the capital/labor-augmenting technology is initially growing relatively fast. Also capital is accumulated at a relatively fast pace.

The intuition for the results is as follows. With a close to zero short-run elasticity of substitution between energy use and the capital/labor composite, the two inputs must be used in almost fixed proportions. Because the energy-saving technology initially is high relative to the existing amount of oil in the ground, the efficient solution is to allocate most of the R&D resource to improving the productivity of capital and labor, with an accompanying accumulation of physical capital.[7] As the capital input is growing and technology-augmented, more energy in efficiency units is needed and because energy-saving technology is relatively stagnant, this energy has to come from the input of more physical oil. Hence, we obtain a path with increasing oil use for a number of periods before oil starts to fall.

The growth rate of the fuel price is initially high and it then falls somewhat during the transition. The reason is that the since capital is scarce in the beginning, the marginal product of capital is high implying a high interest rate and thus via the Hotelling logic a high growth rate of the fuel price. As the economy approaches its balanced growth path, the growth rate of the fuel price converges to its steady state value from above.

The growth rate of the fuel price is higher than in the data but the difference is arguably not major. As shown in Figure **??**, the average growth rate for the oil price is just below three percent between 1973 and 2018, whereas it is somewhat above three percent in the model.

7 Green energy finally takes over

The model just proposed does, we think, offer a decent account of past data. However, going into the future it arguably lacks an important element: other sources of energy, besides those based on natural resources in finite supply. In this section, we therefore extend the model to this case. We will thus entertain the same gross aggregate production function as before but model its energy input as follows:

$$e_t = G\left(e_{1,t}, e_{2,t}\right) = \left[\left(A_1 e_{1,t}\right)^{\frac{\rho-1}{\rho}} + \left(A_2 e_{2,t}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \tag{24}$$

[7] Human capital would be accumulated too in this phase if the setup allowed it.

where e_1 represents oil (or fossil energy more generally) and e_2 other, renewable energy sources. We assume $\rho > 1$ so that none of the two resources are strictly needed.

We think of oil as before, but we assume that the renewable source is produced at a cost. Specifically, let its production be given by

$$e_{2,t} = Bc_{e,t}^{\mu} L^{1-\mu}. \tag{25}$$

Here, c_e represent consumption goods devoted to the production of green energy and L represents a fixed factor, like land. We assume that μ is strictly between zero and one so that the production of green energy at any point in time has convex costs (where costs are measured in output units). This equation can be used to solve for c_e as a function of e_2 to yield a resource constraint for the economy that spells

$$c_t + k_{t+1} = y_t - Me_{2,t}^{\chi}, \tag{26}$$

where $\chi = 1/\mu > 1$ and M is a constant. Thus, $e_{2,t}$ is solved for in a static manner by maximizing $y - Me_2^{\chi}$, which is a well-defined problem. Of course, y has time-varying components so the path for e_2 will be chosen to vary and be interdependent with oil use.

7.1 Transformation

We now transform the model to make it stationary. The key here is to realize that the long-run path will be characterized by e_1 asymptotically reaching zero, i.e., the relevant long-run growth path is one with green energy only. Such a path can be characterized as follows, along the lines of the reasoning for the model above. Output will need to grow at the same rate as $Ak\alpha$ and as $A_e e_2$, and from the resource contraint it follows that $e_2\chi$ will grow at the rate of output. This gives a condition relating the growth rates of A and A_e, which combined with the menu for choosing directed technology growth will give a unique value for the growth rates of these variables, and hence for all the other variables in the model.

The transformation is based on these insights. Define $x_t \equiv k_t^{\alpha} A_t$, $\hat{c}_t = c_t/x_t$, $\hat{k}_t = k_t/x_t$, $a_{et} = A_{e,t}/x_t^{1-1/\chi}$, $\hat{e}_{1t} = e_{1t}/x_t^{1/\chi}$, $\hat{e}_{2t} = e_{2t}/x_t^{1/\chi}$, $\hat{e}_t = e_t/x_t^{1/\chi}$, and $\widetilde{R}_t = R_t/x_t^{1/\chi}$. In addition, because $A_t = x_t k_t^{-\alpha} = x_t \left(\hat{k}_t x_t\right)^{-\alpha} = x_t^{1-\alpha} \hat{k}_t^{-\alpha}$ and $A_{e,t} = a_{et} x_t^{1-1/\chi}$, we have $A_{t+1}/A_t = (x_{t+1}/x_t)^{1-\alpha} \left(\hat{k}_{t+1}/\hat{k}_t\right)^{-\alpha}$, and $A_{e,t+1}/A_{e,t} = \frac{a_{et+1}}{a_{et}} (x_{t+1}/x_t)^{\frac{\chi-1}{\chi}}$. The planning problem can now be written as

$$\max_{\{\hat{k}_{t+1},x_{t+1},a_{e,t+1},\hat{e}_{1t},\hat{e}_{2t},\tilde{R}_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{\left(\left(F(1,a_{et}\hat{e}_t)+(1-\delta)\hat{k}_t-\hat{k}_{t+1}\frac{x_{t+1}}{x_t}-M\hat{e}_{2t}^{\chi}\right)x_t\right)^{1-\sigma}}{1-\sigma} \tag{27}$$

subject to (21) and

$$\widetilde{R}_{t+1}\left(\frac{x_{t+1}}{x_t}\right)^{\frac{1}{\chi}}-\widetilde{R}_t+\hat{e}_{1t}=0. \tag{28}$$

7.2 Calibration and results

The calibration is here the same as in Table 1, except that we now set $\varepsilon = 0.05$.[8] We further set $\mu = 0.5$ (which implies $\chi = 2$), and $M = 10$. This implies a relatively high value of M so that the share of energy that is coming from green energy initially is low. The results are presented in Figure 11.

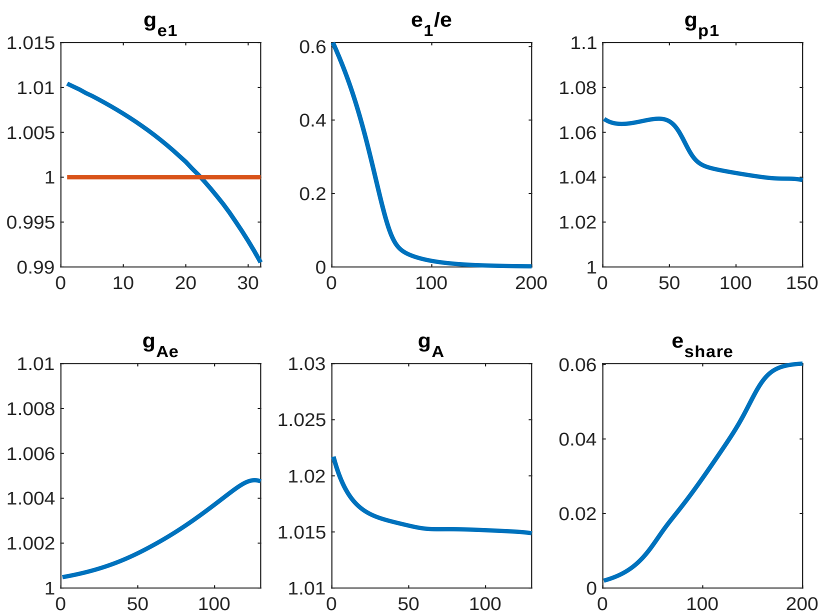

Figure 11

As can be seen in the top left graph, it is still a feature of the equilibrium to display a number of periods with increasing oil use, although this number

[8]The difference in results for $\varepsilon = 0.02$ and $\varepsilon = 0.05$ are small, but finding the numerical equilibrium is harder in the former case due to the higher curvature.

is lower than without the renewable energy source. The middle top graph shows that the share of energy that is coming from oil decreases relatively fast throughout the transition. As in Section 6.3, the oil price increases fast in the beginning of the transition. The transition paths for the technology components A and A_e and energy's share of income are also similar to those in the model with just oil as the energy source.

8 Conclusion

We propose a quantitative-theory framework for modeling a world economy with limited natural resources. The challenge is to make sure our framework is consistent with the stylized resource facts, such as secularly increasing use (and an eventual peak), highly volatile prices, with cost shares that are highly correlated with prices while not drifting toward zero or infinity. The framework we propose has, as a key component, very low substitutability between the resource and other inputs in the short run but, through directed input-saving technical change, much higher substitutability in the longer run. What then generates secularly increasing resource use is the initial scarcity of physical capital: energy was abundant and the increasing path of energy over time simply reflects energy's keeping up with capital growth. We complement this setting with one that is more "forward-looking", namely a richer energy input formulation allowing green energy to play a more and more important role over time.

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A Appendix

A.1 Data

Data on oil use per capita for the World is taken from Our World in Data (https://ourworldindata.org). Data on oil use is for the U.S. taken from the U.S. Energy Information Administration (EIA) at https://www.eia.gov/totalenergy/data/annual; Table 1.3 "Primary Energy Consumption by Source". Data on GDP is taken from the FRED Database. As stated in Section 4.2, fossil energy used directly as a final good should be subtracted from GDP. For the United States, fossil fuel used directly as a final good largely consists of auto fuel, which amounts to about 10% of the total fossil-fuel consumption. Because of a lack of a consistent time series on auto fuel, we set this number to be zero. Energy used for heating homes is considered an intermediary good in the production of housing services, and is therefore an imputed part of GDP. We do, however, subtract net export of fossil fuel.

The data on employment, l, is taken from the U.S. Bureau of Labor Statistics (https://www.bls.gov/webapps/legacy/cpsatab10.htm, Table A-1: Employment status of the civilian population by sex and age).

For the oil price, we used data from EIA: Oil first purchase price (https://www.eia.gov/dnav/pet/pet_pri_dfp1_k_a.htm). The series is converted to a real price series with a GDP deflator from the FRED Database.

The source for the real one-year interest rate is Robert Shiller Online Data, (http://www.econ.yale.edu/~shiller/data.htm; Long term stock, bond, interest rate and consumption data).

The fossil fuel composite, e, the fossil-fuel composite price, A and A_e are all taken from Hassler, Krusell, and Olovsson (2021) and the methods for computing these variables are described in detail there.

A.2 A Cobb-Douglas production function without growth

The first-order conditions w.r.t. e_t and e_{t+1} are, respectively, given by

$$\beta^t c_t^{-1} \nu A k_t^{\alpha} l^{1-\alpha-\nu} e_t^{\nu-1} - \lambda = 0, \tag{29}$$

and

$$\beta^{t+1} c_{t+1}^{-1} \nu A k_{t+1}^{\alpha} l^{1-\alpha-\nu} e_{t+1}^{\nu-1} - \lambda = 0. \tag{30}$$

Combining (29) and (30) delivers:

$$\frac{c_{t+1}}{c_t} = \beta \frac{k_{t+1}^{\alpha} e_{t+1}^{\nu-1}}{k_t^{\alpha} e_t^{\nu-1}}.$$

Combining the above equation with (7) gives the Hotelling equation.

To derive (9), guess that $k_{t+1} = \beta\alpha A k_t^{\alpha} e_t^{\nu}$,and substitute this expression into (8).

A.3 A Cobb-Douglas production function with growth

$$y_t \equiv F(A, k_t, l, e_t) = A\gamma^t k_t^{\alpha} l^{1-\alpha-\nu} e_t^{\nu}. \tag{31}$$

The social planning problem and the equilibrium conditions are respectively is still given by (4), (7), and (8). The savings rate is still constant at $s = \alpha\beta$, and the depletion rate is given by (9). With $k_{t+1} = \alpha\beta A\gamma^t k_t^{\alpha} l^{1-\alpha-\nu} e_t^{\nu}$, gross capital growth is constant and given by

$$g = \gamma g^{\alpha}\beta^{\nu} = (\gamma\beta^{\nu})^{\frac{1}{1-\alpha}} .$$

For large enough γ, $g > 1$.

A.4 A Cobb-Douglas production function with growth and less than full depreciation

The production function is given by (31) and the social planing problem is given by

$$\max_{\{k_{t+1},e_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{(y_t + (1-\delta)\, k_t - k_{t+1})^{1-\sigma} - 1}{1-\sigma} - \lambda \left[\sum_{t=0}^{\infty} e_{o,t} - R_0 \right].$$

The Euler equation and the Hotelling equation are now respectively given by

$$\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \left[\alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \delta\right]$$

and

$$\frac{\beta\gamma k_{t+1}^{\alpha} e_{t+1}^{\nu-1}}{k_t^{\alpha} e_t^{\nu-1}} = \alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \delta.$$

$$c_t + k_{t+1} = A\gamma^t k_t^{\alpha} l^{1-\alpha-\nu} e_t^{\nu} + (1-\delta)\, k_t$$

A.5 The CES production function

The first-order conditions to the problem defined by (20) can, after some manipulation, be written as

$$\begin{gathered} -g_{x_{t+1}} + \frac{F_{2t} a_{et} (1-\hat{e}_t)^2}{G_{2t} \frac{a_{et+1}}{a_{et}}} G_{1t} \left(g_{x_{t+1}}\right)^{-\alpha} \hat{k}_t^{\alpha} (\alpha \hat{k}_{t+1}^{-\alpha-1}) \\ +\beta \left(g_{x_{t+1}}\right)^{1-\sigma} \left(\frac{\hat{c}_{t+1}}{\hat{c}_t}\right)^{-\sigma} (1-\delta) \\ -\beta \left(g_{x_{t+1}}\right)^{1-\sigma} \left(\frac{\hat{c}_{t+1}}{\hat{c}_t}\right)^{-\sigma} \frac{F_{2,t+1} a_{et+1} (1-\hat{e}_{t+1})^2}{G_{2,t+1} \frac{a_{et+2}}{a_{et+1}}} G_{1,t+1} \left(g_{x_{t+2}}\right)^{-\alpha} \hat{k}_{t+2}^{-\alpha} \alpha \hat{k}_{t+1}^{\alpha-1} = 0, \end{gathered} \tag{32}$$

$$
\begin{aligned}
-\hat{k}_{t+1} - \frac{F_{2t} a_{et}(1-\hat{e}_t)^2}{G_{2t} g_{x_{t+1}} \frac{a_{et+1}}{a_{et}}} \left(G_{1t}(1-\alpha) \left(\frac{1}{g_{x_{t+1}}}\right)^{\alpha} \left(\frac{\hat{k}_{t+1}}{\hat{k}_t}\right)^{-\alpha} + G_{2t} \frac{\frac{a_{et+1}}{a_{et}}}{1-\hat{e}_t} \right) \\
+\beta \left(\frac{\hat{c}_{t+1}}{\hat{c}_t} g_{x_{t+1}}\right)^{-\sigma} \left(\hat{c}_{t+1} + \hat{k}_{t+2} g_{x_{t+2}}\right) \\
-\beta \left(g_{x_{t+1}} \frac{\hat{c}_{t+1}}{\hat{c}_t}\right)^{-\sigma} \frac{F_{2,t+1} a_{et+1}(1-\hat{e}_{t+1})^2}{G_{2,t+1} g_{x_{t+2}} \frac{a_{et+2}}{a_{et+1}}} \left(-G_{1,t+1}(1-\alpha) \left(g_{x_{t+2}}\right)^{1-\alpha} \left(\frac{\hat{k}_{t+2}}{\hat{k}_{t+1}}\right)^{-\alpha} - G_{2,t+1} \frac{g_{x_{t+2}} \frac{a_{et+2}}{a_{et+1}}}{1-\hat{e}_{t+1}} \right) \\
= 0.
\end{aligned}
\tag{33}
$$

and

$$
\beta \left(g_{x_{t+1}}\right)^{1-\sigma} \left(\frac{\hat{c}_{t+1}}{\hat{c}_t}\right)^{-\sigma} F_{2,t+1} = \frac{F_{2t}(1-\hat{e}_t)}{\frac{a_{et+1}}{a_{et}}}. \tag{34}
$$

A.6 Steady state

To solve for a steady state, let us evaluate the expressions at constant values. First, equation (21) becomes

$$
G\left({g_x}^{1-\alpha}, \frac{g_x}{1-\hat{e}}\right) = \bar{G}. \tag{35}
$$

Second, equation (32) can be written as

$$
\hat{k} = (1-\beta g_x^{1-\sigma})\alpha \frac{F_2 a_e \left(1-\hat{e}\right)^2 G_1}{G_2} g_x^{-(1+\alpha)} + \beta(1-\delta) g_x^{-\sigma} \hat{k}. \tag{36}
$$

$$
\hat{k} = \frac{1-\beta g_x^{1-\sigma}}{1-\beta(1-\delta)g_x^{-\sigma}} \alpha \frac{F_2 a_e \left(1-\hat{e}\right)^2 G_1}{G_2} g_x^{-(1+\alpha)}
$$

Third, equation (33) simplifies to

$$
\hat{k} g_x - \beta g_x^{1-\sigma} \left(F(1, a_e \hat{e}) + (1-\delta)\hat{k}\right) + \left(1-\beta g_x^{1-\sigma}\right) \frac{F_2 a_e \left(1-\hat{e}\right)^2}{G_2} \left(G_1(1-\alpha) g_x^{-\alpha} + G_2 \frac{1}{1-\hat{e}}\right) = 0. \tag{37}
$$

Finally, equation (34) delivers

$$
\beta g_x^{1-\sigma} = 1-\hat{e}. \tag{38}
$$

Note that for $\sigma = 1$ (38) implies $\hat{e} = 1-\beta$.

The steady state is obtained by solving equations (28)–(38) for the steady-state vector $(\hat{k}, a_e, g_x, \hat{e})$.

A.7 The model with green energy

As above, we let $\beta^t \lambda_{1t}$ be the multiplier of the constraints (21) and $\beta^t \lambda_{2t}$ be the multiplier on (28).

The first order condition w.r.t. $\hat{k}_{t+1}$ and a_{et} are the same as before and so are the aggregate resource constraint and the technology constraint.

The foc w.r.t. x_{t+1} is now given by

$$(\hat{c}_t x_t)^{-\sigma} (-\hat{k}_{t+1}) -$$

$$\lambda_{1t} \left(G_{1t}(1-\alpha) x_{t+1}^{-\alpha} x_t^{\alpha-1} \left(\frac{\hat{k}_{t+1}}{\hat{k}_t} \right)^{-\alpha} + G_{2t} \left(\frac{\chi - 1}{\chi} \right) \frac{a_{et+1}}{a_{et}} \left(\frac{x_{t+1}}{x_t} \right)^{-1/\chi} \frac{1}{x_t} \right) -$$

$$\lambda_{2t} \frac{1}{\chi} \widetilde{R}_{t+1} \left(\frac{x_{t+1}}{x_t} \right)^{\frac{1}{\chi}} \frac{1}{x_{t+1}} +$$

$$\beta \left(\hat{c}_{t+1} x_{t+1} \right)^{-\sigma} \left(\hat{c}_{t+1} + \hat{k}_{t+2} \frac{x_{t+2}}{x_{t+1}} \right) -$$

$$\beta \lambda_{1t+1} \left(-G_{1,t+1}(1-\alpha) x_{t+2}^{1-\alpha} x_{t+1}^{\alpha-2} \left(\frac{\hat{k}_{t+2}}{\hat{k}_{t+1}} \right)^{-\alpha} - G_{2,t+1} \left(\frac{\chi-1}{\chi} \right) \frac{a_{et+2}}{a_{et+1}} \left(\frac{x_{t+2}}{x_{t+1}} \right)^{-1/\chi} \frac{x_{t+2}}{x_{t+1}^2} \right) +$$

$$\beta \lambda_{2t+1} \widetilde{R}_{t+2} \frac{1}{\chi} \left(\frac{x_{t+2}}{x_{t+1}} \right)^{\frac{1}{\chi}} \frac{1}{x_{t+1}}.$$

The derivative with respect to $\hat{e}_{1t}$ reads

$$\lambda_{2t} = (\hat{c}_t x_t)^{-\sigma} F_{2t} a_{et} \hat{e}_t^{\frac{1}{\psi}} \hat{e}_{1t}^{-\frac{1}{\psi}} x_t. \tag{39}$$

The derivative with respect to $\hat{e}_{2t}$ is now

$$F_{2t} a_{et} \hat{e}_t^{\frac{1}{\psi}} \hat{e}_{2t}^{-\frac{1}{\psi}} - \chi M \hat{e}_{2t}^{\chi-1} = 0.$$

The derivative with respect to $\widetilde{R}_{t+1}$ reads

$$-\lambda_{2t} \left(\frac{x_{t+1}}{x_t} \right)^{\frac{1}{\chi}} + \beta \lambda_{2t+1} = 0. \tag{40}$$

Rearranging (39) and (40), we get

$$\widetilde{\lambda}_{2t} = \hat{c}_t^{-\sigma} F_{2t} a_{et} \hat{e}_t^{\frac{1}{\psi}} \hat{e}_{1t}^{-\frac{1}{\psi}}, \tag{41}$$

and

$$\beta\lambda_{2t+1} = \lambda_{2t}\left(\frac{x_{t+1}}{x_t}\right)^{\frac{1}{\chi}}, \tag{42}$$

where $\widetilde{\lambda}_{2t} \equiv \lambda_2 / x_t^{1-\sigma}$.

Rearranging, and using (41) and (42), and defining $g_{x,t+1} \equiv x_{t+1}/x_t$, $\widetilde{\lambda}_{1t} \equiv \lambda_{1t}/x_t^{1-\sigma}$, we can write these foc:s as follows.

$$\begin{gathered}
0 = -\hat{c}_t^{-\sigma}\hat{k}_{t+1} \\
-\widetilde{\lambda}_{1t}\left(G_{1t}(1-\alpha)g_{x,t+1}^{-\alpha}\left(\tfrac{\hat{k}_{t+1}}{\hat{k}_t}\right)^{-\alpha} + G_{2t}\left(\tfrac{\chi-1}{\chi}\right)\tfrac{a_{et+1}}{a_{et}}g_{x,t+1}^{-1/\chi}\right) + \\
\beta\left(\hat{c}_{t+1}g_{x,t+1}\right)^{-\sigma}\left(\hat{c}_{t+1} + \hat{k}_{t+2}g_{x,t+2}\right) - \\
\beta\widetilde{\lambda}_{1t+1}g_{x,t+1}^{-\sigma}\left(-G_{1,t+1}(1-\alpha)g_{x,t+2}^{1-\alpha}\left(\tfrac{\hat{k}_{t+2}}{\hat{k}_{t+1}}\right)^{-\alpha} - G_{2,t+1}\left(\tfrac{\chi-1}{\chi}\right)\tfrac{a_{et+2}}{a_{et+1}}g_{x,t+2}^{1-1/\chi}\right) - \\
(1/\chi)\,\hat{c}_t^{-\sigma}F_{2t}a_{et}\hat{e}_t^{\frac{1}{\psi}}\hat{e}_{1t}^{-\frac{1}{\psi}}g_{x,t+1}^{\frac{1-\chi}{\chi}}\hat{e}_{1t+1},
\end{gathered} \tag{43}$$

where we used that $\widetilde{R}_{t+1}g_{x,t+1}^{\frac{1}{\chi}} = \left(\widetilde{R}_t - \hat{e}_{1t}\right)$ and $\widetilde{R}_{t+2}g_{x,t+2}^{\frac{1}{\chi}} = \widetilde{R}_{t+1} - \hat{e}_{1t+1}$;

$$\hat{e}_{2t} = \left(\frac{F_{2t}a_{et}\hat{e}_t^{\frac{1}{\psi}}}{\chi M}\right)^{\frac{\psi}{1+(\chi-1)\psi}}; \tag{44}$$

$$\left(\frac{\hat{c}_{t+1}}{\hat{c}_t}\right)^{\sigma} = \beta\frac{F_{2t+1}}{F_{2t}}\frac{a_{et+1}}{a_{et}}\left(\frac{\hat{e}_{t+1}}{\hat{e}_t}\right)^{\frac{1}{\psi}}\left(\frac{\hat{e}_{1t+1}}{\hat{e}_{1t}}\right)^{-\frac{1}{\psi}}g_{x,t+1}^{\frac{\chi(1-\sigma)-1}{\chi}}; \tag{45}$$

$$\widetilde{R}_{t+1}g_{x,t+1}^{\frac{1}{\chi}} - \widetilde{R}_t + \hat{e}_{1t} = 0; \tag{46}$$

and

$$\hat{e}_t = \left[\hat{e}_{1t}^{\frac{\psi-1}{\psi}} + \hat{e}_{2t}^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}. \tag{47}$$