The Real Costs of International Financial Dis-Integration

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PRELIMINARY AND INCOMPLETE, NOT FOR QUOTATION

Abstract

The real costs of deviations or "wedges" from efficient international financial markets depend upon exchange rates, consumption, and returns identified in the data. We show how to infer the costs of inefficient financial markets using the identifying assumptions that are implicit in asset pricing and macroeconomic models. We measure and decompose these costs in their individual components, including exchange rates. This decomposition demonstrates that the ability of goods markets to respond to financial markets through exchange rate adjustment has significant implications for welfare. For this reason, we explore how different views of exchange rate determination impact these costs. Our analysis illustrates that standard assumptions about the exchange rate behavior implicit in price aggregators used in the macro and finance literature lead to very different implications about the costs of inefficient international markets.

The financial dislocations implied by events such as the on-going COVID crisis has led to important questions about the economic toil on individual households. These questions prompt a sharper focus on economic models to ask what they imply about the degree to which these households are insured against such dislocations. However, the real impacts in the international financial markets are particularly difficult to assess because individuals living in different parts of the world typically face different prices. Since prices across countries are known to deviate for extended time periods, these relative prices can also generate implications for the costs of financial market inefficiencies.

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In this paper, we show that the real costs of financial market inefficiencies can be measured by combining standard aggregate data with the intertemporal decisions embedded in the economic model under investigation. Specifically, this identification arises from the intertemporal decision of the marginal investor, a key building block in standard studies. For example, in representative agent models, the discount factor of this marginal investor is presumed to directly price asset returns. In heterogeneous agent models, this role may alternate among multiple investors or else may be determined by one investor while other investors are inframarginal. Given the intertemporal decision of the marginal investor, this paper demonstrates that the cost of inefficient financial markets can be summarized by the ratio of the value of lifetime resources to the current household consumption; that is, a wealth price-to-consumption ratio. We demonstrate below that in many cases this price ratio can be directly measured with consumption and real exchange rate data alone.

The cost calculation follows from a straightforward implication of this intertemporal decision. That implication begins by recognizing that the intertemporal decision embodied in the Euler equation is constrained by a lifetime budget constraint. Therefore the value of an asset paying the equivalent of this constraint, based upon consumption and prices relevant for this investor, provides a welfare-based measure of wealth.[1] To connect this observation with a cost measure, our analysis considers two different valuations of these lifetime resources. First, in order to directly value lifetime consumption, we treat consumption data as the equilibrium outcome of a savings and portfolio decision made by the investor facing prices and risks. We then use this data identification to construct a return on wealth and then use the Euler equation of the marginal investor to value the lifetime consumption implied by this return. Given that this measure is implied directly from existing consumption data, we call the economy that generates this valuation the "Data Economy."

We then ask whether this "Data" valuation of lifetime resources reflects efficient use of available financial markets by comparison to a counterfactual valuation implied when risks are optimally shared. To make this comparison, we consider the lifetime valuation of the same data if households are optimally diversifying risks internationally. This diversifiication can be replicated by the decisions of a social planner who faces potential constraints in the goods market. We call this valuation of lifetime resources the "Risk Sharing Economy." Comparing these two valuations between the

[1] The insight that the value of wealth can be determined from the intertemporal decision of investors has been used in a large literature on consumption-based asset pricing. See for example, Campbell (1993) and Lettau and Ludvigson (2001).

"Data" economy and "Risk-sharing" economies then provides a certainty equivalent consumption cost due to the lack of international integration; that is, the international "dis-integration."

Given this theoretical framework, we illustrate how this approach can be connected to the data using two standard identifying assumptions from the literature to discipline the "Data Economy" Euler equations. The first approach assumes that investors in each country are the marginal investors who price their own asset returns. This assumption is the benchmark in a large consumption-based asset pricing literature.[2] The second approach assumes that the domestic investor Euler equation prices foreign assets, an assumption that provides the cornerstone for the equally voluminous foreign exchange risk premium literature. These two approaches are equivalent when financial markets are complete, but not when markets are incomplete. Therefore, we use data to highlight a range of results depending upon the underlying assumption of the marginal investor that prices asset returns. Given these valuations of life-time consumption implied by the data, we determine the optimal "Risk-Sharing Economy" version in which households hold an optimal set of Arrow-Debreu securities that replicate a planner's consumption allocation subject to various goods markets frictions. Using the two measures, we calculate a "total wedge" that measures the deviation between the value of lifetime consumption in the data and its counterpart under efficient Arrow-Debreu markets. We decompose this deviation into components that can be disciplined by asset return and exchange rate moments in the data. Specifically, the total wedge decomposes into parts due to wedges in the current intertemporal marginal utilities, in the value of future lifetime consumption, and in the exchange rate.

Importantly, we show that the wedges depend critically on the nature of the goods market frictions that impact exchange rate behavior. For example, when exchange rate variations arise solely from frictions that restrict goods market clearing to the extent that prices are sticky and effectively exogenous, the social planner faces this same constraint. In this case, financial markets cannot alter goods prices and there is no exchange rate wedge. By contrast, if some goods markets are perfectly functioning but others are not, as under the standard tradeables versus nontradeables view of exchange rate determination, the exchange rate wedge can provide an important component of dis-integration costs. Intuitively, if some goods markets do not equilibrate, then financial

[2] In the Related Literature subsection at the end of this introduction, we provide a longer description of this and other related literatures.

markets can shift resources to investors facing country-specific relative prices in non-tradeables. In the extreme, if some relative goods prices appear sufficiently sticky that they appear exogenous to financial markets, then this variability will always reduce the benefits of financial market integration. All told, the results below demonstrate that the costs of inefficient financial markets depend critically on the adjustment of prices in the goods market.

To determine the efficient degree of integration across countries, we use the well-known optimality condition that the intertemporal marginal rate of substitution is equalized across investors once converted into a common price; that is, under complete markets, price-adjusted stochastic discount rates are equalized. To connect this relationship to the impact of exchange rates in the costs of dis-integration, our analysis highlights a new relationship between countries under complete markets. That is, under complete markets, the state value of wealth returns across countries are equalized once adjusted by the differences in weights in the consumption sharing rule. Importantly, this weight adjustment is uniquely determined by exchange rates for many standard models. As such, we introduce a new benchmark for comparing data to the counterfactual complete markets world.

The format of the paper is as follows. Section 1 sets up the theoretical framework for the real costs of financial market inefficiencies by valuing the wealth-to-consumption in the economy. This section also describes the connection to recent studies of wedges due to exchange rates. Section 2 provides a simple two-country symmetric example to show how this wedge and its decomposition can be measured with consumption and price data using standard assumptions in macroeconomics and asset pricing.[3] Section 3 shows how the relationships generalize to a multi-country framework using an example with Penn-World Tables data for four countries. This section also describes how the framework can be used in other more general applications. Concluding remarks are in Section 4. All proofs and derivations are provided in the Appendices.

Related Literature:

This paper is related to a number of important literatures in macroeconomics and finance. Given the size of these related literatures, we mention only representative papers within each literature.

First, this paper relates to the growing international asset pricing literature based upon complete

[3] For example, this section considers both a standard Constant-Relative-Risk-Aversion case with i.i.d. consumption as often used in macroeconomics as well as a recursive Epstein-Zin case with long-run risk that is popular in asset pricing.

markets that identifies the exchange rate with the ratio of stochastic discount factors. Following the observation of this relationship by Backus, Foresi, and Telmer (2001), a number of studies have used this identity including, among others, Colacito and Croce (2011) and Colacito, Croce, Gavazzoni, and Ready (2018) with long run risk, Farhi and Gabaix (2016) with disaster risk, and Lustig, Stathopoulos, and Verdelhan (2018) for the term structure of exchange rate returns. In contrast with these papers, the analysis below begins with the presumption that markets might be incomplete and uses data to ask how important the deviation may be. We also highlight a novel connection between wealth returns across countries under complete markets.

Second, our paper is related to the literature on consumption risk sharing and exchange rates, an issue popularized by the puzzle illustrated in Brandt, Cochrane, and Santa Clara (2006). Focusing upon the complete markets identity between the exchange rate and the ratio of stochastic discount rates noted earlier, a number of papers have used the data counterparts to infer information about the incomplete markets wedge. For example, Lustig and Verdelhan (2019) use wedges in the Euler equation of the risk-free rate to consider implications for exchange rates while Backshi, Cerrato, and Crosby (2019) evaluate exchange rates and a portfolio of international returns. Sandulescu, Trojani, and Vedolin (2019) demonstrate in a model-free framework that the ratio of stochastic discount factors do not correspond to the equilibrium exchange rate in complete financial markets when financial markets are incomplete. Burnside and Graveline (2018) show that evaluating the costs from imperfect financial market risk-sharing inherent in exchange rate and consumption data requires an economic framework that depends upon goods markets, but do not specify that framework. Instead, our approach specifies a framework that can be used to connect data to economic theories. By contrast to all of these studies, our analysis provides a framework to directly value these implicit wedges. It also allows a decomposition of these wedges into their respective components.

Third, our paper is related to the wide literature on consumption insurance. These studies include household level analysis as in Mace (1991) and Cochrane (1991), and the welfare costs of business cycles (Lucas (1987), Obstfeld (1994a), Alvarez and Jermann (2005)). It is also related to the implications of that consumption insurance across countries noted by Obstfeld (1994b), Tesar (1995), Lewis (1996), Kalemli-Ozcan, Sorensen, and Yosha (2003), and Lewis and Liu (2015). However, these papers do not directly focus upon the implications of goods market prices in affecting the welfare costs from imperfect risk-sharing. Cole and Obstfeld (1991) provides an important exception that we reconsider below.

Finally, this paper is connected to the vast literature that examines the general connection between exchange rates and consumption aggregators used in international macroeconomics and finance. In highlighting the role of exchange rates, these papers range from Backus, Kehoe and Kydland (1992), Coeurdacier and Rey (2012), and Berka, Devereux, and Engel (2018) in macroeconomics to Pavlova and Rigobon (2008), Verdelhan (2010), and Ready, Roussanov, and Ward (2018) in financial economics. This literature focuses upon understanding or explaining regularities in the data. By contrast, we take these models as given and provide a framework to ask what they say about the costs of real financial dis-integration.

1 Measuring the Costs of Inefficient Market Wedges

We begin by describing the general framework for valuing the inefficient markets wedge components and how to measure their economic costs using standard Euler equation solutions. In a model-free environment, we show how these costs can be decomposed into three components: current marginal utility, future lifetime wealth, and real exchange rates. In this section, we take as given that there exist data measures of key economic variables such as stochastic discount rates and the return on wealth. Section 2 describes how these variables can be determined using the identifying assumptions about the marginal investor and the nature of goods market frictions.

1.1 Incomplete markets and the Euler Equation

In order to measure the costs of imperfect risk-sharing using asset market wedges, we begin with the standard first-order condition of intertemporal consumption optimization given in the Euler equation:

$$E_t \left\{ M_{t+1} R_{a,t+1} \right\} = 1 \tag{1}$$

where $R_{a,t+1}$ is the gross return on any asset, indexed by a, and where M_{t+1} is the intertemporal marginal rate of substitution in consumption, typically called the stochastic discount factor. For example, in models with CRRA preferences, $M_{t+1} \equiv (C_{t+1}/C_t)^{-\gamma}$ where C_t is the consumption at time t in units of the domestic price level and γ is the relative risk aversion parameter in utility.

1.1.1 Defining Wedges

Backus, Foresi, and Telmer (2001), hereafter BFT, noted a convenient connection between this standard Euler equation based upon complete markets and its counterpart used in empirical analysis. This connection arises when a domestic investor who consumes in local goods units evaluates an asset with return payouts in the units of a foreign good. For simplicity and without loss of generality, we will treat this foreign price level as the numeraire below. Then, defining the return of any asset available to domestic investors in units of the numeraire as $\widetilde{R}_{a,t+1}$, this return in units of domestic goods is: $\widetilde{R}_{a,t+1}\left(S_{t+1}/S_t\right)$ where S_t is the relative price of foreign goods in units of domestic goods. That is, S_t is the real exchange rate. Then the Euler equation for that foreign asset return to domestic investors and to foreign investors, respectively, can be written using equation (1) as:

$$\begin{aligned} E_t\left\{M_{t+1}\widetilde{R}_{a,t+1}\left(S_{t+1}/S_t\right)\right\} &= 1 \\ E_t\left\{\widetilde{M}_{t+1}\widetilde{R}_{a,t+1}\right\} &= 1 \end{aligned} \tag{2}$$

Given these valuations, BFT combine two relationships. The first is that the stochastic discount factor under complete markets is the same across countries once translated into the local goods prices.[4] Denoting with asterisk $*$ the complete markets counterparts to the variables above, the relationship between stochastic discount factors and prices can be written:

$$M^*_{t+1} = \widetilde{M}^*_{t+1}/\left(S^*_{t+1}/S^*_t\right) \tag{3}$$

Substituting this relationship into the above Euler equations clearly demonstrates that under complete markets the two equations are equivalent.

The second relationship pointed out by BFT is that these complete markets identities can be rewritten as deviations from their counterparts in the data.[5] To see this connection, we denote with a superscript "D" the counterparts to the variables above that would be measured in the data

[4] The equivalence between between stochastic discount factors once converted into common pricing units has been established in a long literature. See for example, Ingersol (1987).

[5] For now, we take as given there there exist counterparts in the data, but describe their identification in detail below.

and define the proportional "wedges" ζ through the following identities:

$$\begin{aligned} M^D_{t+1} &\equiv \zeta_{M,t+1} M^*_{t+1};\ \widetilde{M}^D_{t+1} \equiv \zeta_{\widetilde{M},t+1} \widetilde{M}^*_{t+1} \\ \left(\frac{S^D_{t+1}}{S^D_t}\right) &\equiv \zeta_{S,t+1} \left(\frac{S^*_{t+1}}{S^*_t}\right) \\ R^D_{a,t+1} &\equiv \zeta_{R_a,t+1} R^*_{a,t+1};\ \widetilde{R}^D_{a,t+1} \equiv \widetilde{\zeta}_{R_a,t+1} \widetilde{R}^*_{a,t+1} \end{aligned} \tag{4}$$

The data-implied variables on the left-hand side are the natural objects of interest because they are key components in pricing relationships in equation (2). Moreover, their data counterparts are either directly observable or else their properties can be inferred using asset return data . Similarly, the defined wedges provide measures of the ratio of the data-implied variables and the complete markets alternative. Thus, $\zeta_{M,t}$, $\widetilde{\zeta}_{M,t+1}$ are, respectively, the wedges between the stochastic discount factors identified in the data M^D_{t+1}, $\widetilde{M}^D_{t+1}$ and under complete markets M^*_{t+1}, $\widetilde{M}^*_{t+1}$ measured in each country's price units. By contrast, $\zeta_{S,t+1}$ is the wedge between the relative prices across countries in the data and under complete markets. Finally, $\zeta_{R_a,t+1}$ is the wedge between returns in the data and the returns on assets with equivalent pay-outs, but priced under complete markets. Clearly, these definitions impose no restrictions since, if markets are indeed complete in the data, then these wedges will all be equal to one; i.e., $\zeta_{M,t+1} = \widetilde{\zeta}_{M,t+1} = \zeta_{S,t+1} = \zeta_{R_a,t+1} = 1$. Nevertheless, allowing for these wedges provide an opportunity to decompose the impact of each component in the data, as we show below. For expositional simplicity, we generally refer to these wedges according to the variable that potentially deviates from optimality; i.e., the M-wedge is ζ_M, the S-wedge is ζ_S, etc.

Substituting equations (4) into the relationship between complete markets discount factors in equation (3) implies:[6]

$$\left(\frac{S^D_{t+1}}{S^D_t}\right) = \frac{\widetilde{M}^D_{t+1}}{M^D_{t+1}} \left(\frac{\zeta_{M,t+1}\zeta_{S,t+1}}{\widetilde{\zeta}_{M,t+1}}\right) = \frac{\widetilde{M}^D_{t+1}}{M^D_{t+1}} \exp(\eta_{t+1}) \tag{5}$$

[6] As noted earlier, the connection between stochastic discount factors in the data and their relationship to complete markets have been studied in several papers using financial returns including Lustig and Verdelhan (2019), Bakshi, Cerrato, and Crosby (2019) and Sandulscu, Trojani, and Vedolin (2019). The next section describes the connection between our data analysis and those papers.

For expositional convenience, we rewrite this "wedge" relationship in logarithmic terms as:

$$g_{S,t+1}^{D} = \widetilde{m}_{t+1}^{D} - m_{t+1}^{D} + \eta_{t+1}$$

where $g_{S,t+1}^{D} \equiv \ln(S_{t+1}^{D}/S_{t}^{D})$, lower case m, $\widetilde{m}$ refers to the logarithm of their counterparts M, $\widetilde{M}$, respectively, and $\eta_{t+1} \equiv \ln(\zeta_{M,t+1}\zeta_{S,t+1}/\widetilde{\zeta}_{M,t+1})$. Since the joint wedge η provides the deviation between the data and complete markets counterparts in the identity equation (3), BFT focused upon this variable as the so-called "perturbation" of interest. Nevertheless, below we show that each component in the joint wedge η can have distinct and sometimes offsetting implications for welfare.

1.1.2 Wedges and Life-Time Consumption

Above, we defined the wedges between the data components of the standard Euler equation and their counterparts under complete markets. At this level of generality, the wedges are purely definitional. For a given set of data counterparts, the wedges are unique since the complete markets components of the stochastic discount rates M_{t+1}^{*}, $\widetilde{M}_{t+1}^{*}$ and their relative prices measured in S_{t}^{*} are unique for a specified equilibrium. Thus, the wedge analysis requires an identifying assumption about the data measures of stochastic discount factors and exchange rates; that is, measures for M_{t}^{D}, $\widetilde{M}_{t}^{D}$, and S_{t}^{D}. Conveniently, the data counterpart of the exchange rate S_{t}^{D} is directly observable. However, it is well-known that if markets are incomplete, there will not be a unique stochastic discount rate in the economy once converted into common prices. For this section, we take as given a measure for these variables given the state at date t. In the next section describes identifications of the "Data Economy" variables implicit in standard studies as examples.

To see how to price the value of the wedges, consider the two basic versions of the economy to evaluate the deviations above. Recall that the first version we termed the "Data economy." This version derives from the standard presumption that asset returns are determined by an Euler equation given consumption data and prices to be made explicit below. The second version that we called the "Risk-sharing economy" is the allocation of resources based upon optimal diversification with complete markets Arrow-Debreu claims.[7] Using this simple dichotomy of "data" and "risk-

[7] For much of our analysis we focus upon planner reallocations of consumption although Section 3 describes the implications when output is reallocated.

sharing" economies, we can determine whether there are any wedges and if so, their economic importance as viewed through the lens of standard modeling approaches.

The economic costs of insufficient financial market diversification depends upon how their wedge deviations in the data impact the value of lifetime consumption. Based upon the intertemporal budget constraint, this value of lifetime consumption is equal in equilibrium to wealth.[8] Thus, defining C_t^D as consumption measured in the data and C_t^* as the possibly counterfactual consumption under complete markets, the value of lifetime consumption as measured by data, W_t^D, and under complete markets, W_t^*, can be written, alternatively, as:

$$W_t^D \equiv C_t^D + \Gamma_t^D; \; W_t^* \equiv C_t^* + \Gamma_t^* \tag{6}$$

where Γ_t^D and Γ_t^* are the sum of future consumption in the two scenarios discounted by the stochastic discount factors described above. Thus, Γ_t measures wealth excluding current consumption and therefore we call it the value of future wealth below. In particular, $\Gamma_t^D \equiv E_t \sum_{\tau=1}^{\infty} Q_{t+\tau}^D C_{t+\tau}^D$, and $\Gamma_t^* \equiv E_t \sum_{\tau=1}^{\infty} Q_{t+\tau}^* C_{t+\tau}^*$ where the discount factors are the intertemporal stochastic discount factors between $t+1$ and future periods $t+\tau$.[9]

The value of the future consumption sequence may be calculated using the Euler equations above by treating the realization of consumption as the return on an asset given by $R_{c,t+1} \equiv (C_{t+1} + \Gamma_{t+1})/\Gamma_t$. We call this return the "wealth return" since it is the return on a claim to future wealth Γ_t.[10] Using this definition, the value of wealth can be priced in the "data economy" and the "risk-sharing economy" using Euler equation (1) for the wealth return. In particular,

$$E_t \left\{ M_{t+1}^D R_{c,t+1}^D \right\} = 1 \tag{7}$$

$$E_t \left\{ M_{t+1}^* R_{c,t+1}^* \right\} = 1 \tag{8}$$

where

$$R_{c,t+1}^D \equiv (C_{t+1}^D + \Gamma_{t+1}^D)/\Gamma_t^D; \; R_{c,t+1}^* \equiv (C_{t+1}^* + \Gamma_{t+1}^*)/\Gamma_t^* \tag{9}$$

[8] This constraint and its implications for lifetime wealth can be found in texts such as Obstfeld and Rogoff (1996) for international macroeconomics and Ingersoll (1987) and Cochrane (2005) for financial economics.

[9] That is, $Q_{t+\tau}^D \equiv \Pi_{j=1}^{\tau} M_{t+j}^D$ and $Q_{t+\tau}^* \equiv \Pi_{j=1}^{\tau} M_{t+j}^*$.

[10] This variable is often called the return on the "consumption asset" since it values the stream of lifetime consumption. See for example Bansal and Yaron (2006).

These Euler equations and their returns provide the basis for valuing all of the costs of the wedges. For example, given data processes for consumption C^D_{t+1} and the stochastic discount factor M^D_{t+1}, the price of a claim on future wealth can be determined by substituting the definition for wealth returns $R^D_{c,t+1}$ in equation (9) into the Euler equation (7) and solving for Γ^D_{t+1}. With similar steps, these equations determine the price of future wealth under complete markets, Γ^*_{t+1}.

Since the value of these prices differ according to their wedges defined in equations (4), we can directly value the deviations between complete and incomplete markets in the data. As an example of the data identifications we describe in the next section, consider the standard assumption that the domestic investor prices domestic assets. For example, in the standard Euler equation (1), the returns R_a refer to domestic assets. As in much of the consumption-based asset pricing literature, therefore, the domestic risk-free rate would be priced as:

$$E_t \left\{ M^D_{t+1} \right\} = 1/R_{rf,t} \tag{10}$$

Using this identifying assumption, one could use data to evaluate domestic asset returns to infer data implications about M^D_{t+1} and consider how these measures deviate from complete markets. Given this value and a corresponding complete markets value M^*_{t+1} determines the wedges $\zeta^j_{M,t+1}$. With the same identification for the other wedges, ζ, we can connect the state-by-state the risk-sharing return on wealth for the domestic country to its counterpart in the data economy as:

$$M^*_{t+1} R^*_{c,t+1} \equiv M^D_{t+1} R^D_{c,t+1} \left(\zeta_{R_C,t+1} \zeta_{M,t+1} \right)^{-1} \tag{11}$$

Note that these wedge components are measured in domestic price units and thus do not provide information about the exchange rate wedge.

For this purpose, we can alternatively identify the marginal investor for domestic assets as the foreign investor. In this case, equation (2) could be used for example to price the foreign risk-free rate as:

$$E_t \left\{ \widetilde{M}^D_{t+1} \left(S^D_{t+1}/S^D_t \right)^{-1} \right\} = 1/R_{rf,t} \tag{12}$$

This is the identification often used in the foreign exchange literature. Therefore, under the assumption that the foreign investor is marginal, the wedge differentials can be written relative to a data measure of $M_{t+1}^{D}\left(S_{t+1}^{D}/S_{t}^{D}\right)$ and $\widetilde{M}_{t+1}^{D}/\left(S_{t+1}^{D}/S_{t}^{D}\right)$. With this identification, consider the wedge differences in domestic relative to foreign exchange rates using the complete markets exchange rate identity in equation (3). That is, this same asset return can be restated as a return on the foreign consumption asset in the data economy using the definition of the wedge in exchange rates, $\zeta_{S,t+1}$, and the wedge in the foreign consumption asset return, $\widetilde{\zeta}_{R_C,t+1}^{j}$, as[11]:

$$M_{t+1}^{*}R_{c,t+1}^{*} \equiv \left[\widetilde{M}_{t+1}^{D}/\left(S_{t+1}^{D}/S_{t}^{D}\right)\right] R_{c,t+1}^{D}\left(\widetilde{\zeta}_{M,t+1}\zeta_{R_C,t+1}/\zeta_{S,t+1}\right)^{-1} \tag{13}$$

As we show below, this relationship allows us to decompose the impact of each of the various individual wedges.

1.1.3 Certainty Equivalent Consumption Wedges

An approach often taken to consider the costs of incomplete markets or wedges is to evaluate the reduction in risk in portfolios. The costs due to inadequate diversification provide one such measure commonly found in empirical measures of international portfolios.[12] While this approach provides a useful metric of imperfect diversification, it does not address how much an economic agent values that loss. Put simply, a highly risk averse agent will care a great deal about under diversification while a relatively risk neutral one will care little. For this reason, it is useful to evaluate any costs of wedges in the context of an economic agent's preferences, given by a utility function.

To compare these consumption processes in utility units requires the value function of consumption given the wealth of each country. For this purpose, consider a general utility function $U(C_t)$ for an agent with a set of assets and resources that determine an intertemporal budget constraint. The optimization of preferences given this constraint then implies a sequence of lifetime consumption $\{C_t, C_{t+1,}...C_{t+\tau}, ...\}$ and a value function, $V(W_t)$, where as before W_t is the value of the sequence of lifetime consumption. Assuming that preferences are homogeneous, this value function can be written as: $V(W_t) = C_t V(W_t/C_t)$. Using the definitions above, that wealth in the data economy is W_t^D, and that wealth in the optimal risk-sharing economy is W_t^*, we can calculate the costs of

[11] Clearly, the relationship for the foreign country can similarly be rewritten as: $\widetilde{M}_{t+1}^{*}\widetilde{R}_{c,t+1}^{*} = \left[M_{t+1}^{D}\left(S_{t+1}^{D}/S_{t}^{D}\right)\right]\widetilde{R}_{c,t+1}^{D}\left(\zeta_{M,t+1}\widetilde{\zeta}_{R_C,t+1}\zeta_{S,t+1}\right)^{-1}$

[12] See, for example, the risk measures in Brandt, Cochrane, and Santa Clara (2006), Bekaert et al (2011), Christopherson et al (2012), and Pukthuanthong and Roll (2009).

incomplete risk-sharing to investors given their utility functions.

To understand these costs for general homogeneous preferences, consider an agent living under complete markets with current consumption C_t^* and wealth W_t^*. Now suppose the agent with these same preferences instead lives in the data economy world with consumption C_t^D and wealth W_t^D. What is the value of their loss in permanent consumption units? Clearly, the values to the agent of the two economies are equivalent when the consumption by the agent in the risk-sharing economy is reduced by Δ in the expression: $(1-\Delta)C_t^* V(W_t^*/C_t^*) = C_t^D V(W_t^D/C_t^D)$. Thus, given solutions for consumption and wealth, we can solve for this cost according to Δ in the following equation:

$$(1-\Delta) = \frac{V(W_t^D/C_t^D)}{V(W_t^*/C_t^*)}\frac{C_0^D}{C_0^*} \tag{14}$$

In other words, Δ measures the loss in permanent consumption to a household with access to perfect risk-sharing who instead is required to live in the data economy.

1.2 Quantifying Costs from Inefficient Market Wedges

Given preferences, the consumption processes can be used to construct the costs from incomplete markets described above. We describe both these preferences and the implied value of wedges here. In Section 2, we return to the question of how to discipline these preferences to match asset return moments.

1.2.1 Preferences

While the discussion to this point has been general, we now assume a class of preferences that can nest a number of standard asset pricing models to be described below. For this purpose, we assume a continuum of identical households each living in the domestic country and in the foreign country, respectively. All of the domestic households have preferences over the aggregate consumption baskets, C, measured in its local prices. Similarly the foreign household consumes aggregate consumption baskets denoted as $\widetilde{C}$ that are measured in foreign prices. In much of our quantitative analysis we assume that these households have recursive preferences following Epstein and Zin (1989) and Weil (1989), although we also consider constant-relative risk aversion (CRRA) as a special case. Further, to ensure that our risk-sharing results do not arise from differences in country-specific views toward risk, we also assume all countries have the same preference parameters over an aggregate consumption basket of goods.[13] Then, the utility for the domestic country at time t over the general consumption basket can be written:

$$U(C_t, U_{t+1}) = \left\{ C_t^{\frac{1-\gamma}{\theta}} + \beta E_t \left[\left(U_{t+1} \right)^{1-\gamma} \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{15}$$

where U_{t+1} is the utility function at $t+1$; $0 < \beta < 1$ is the time discount rate; $\gamma \geq 0$ is the risk-aversion parameter; $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$ for $\psi \geq 0$, the intertemporal elasticity of substitution; and where $\mathrm{E}_t(\cdot)$ is the expectation operator conditional on the information set at time t.[14] This utility function specializes to standard time-additive CRRA preferences when $\gamma = \frac{1}{\psi}$ as in:

$$U(C_t, U_{t+1}) = E_t \sum_{\tau=0}^{\infty} \beta^{\tau} (C_{t+\tau})^{1-\gamma} \tag{16}$$

In order to allow for country-specific preferences in individual goods within the consumption aggregate, C_t, we consider below various differences in individual consumption baskets reflecting, for example, home bias towards a domestic good or nontradeables versus tradeables. As a general characterization of these aggregates, then, there are N commodities so that:

$$C_t \equiv C(C_{1t}, C_{2t}, ..., C_{Nt}) \tag{17}$$

where C_{it} is the consumption of good i by the domestic country at time t. Moreover, households may face country-specific prices of their aggregate consumption good due either to goods market frictions or to differences in their relative consumption aggregators. We define the price index of the aggregate consumption C_t for the domestic country in units of the foreign numeraire as P_t. Thus, the real exchange rate can be written as the relative price of consumption in the domestic country relative to the foreign numeraire consumption good with price at time t given as $\widetilde{P}_t$. We then define this real exchange rate as: $S_t \equiv (P_t/\widetilde{P}_t)$.

[13] To consider differences in commodity price effects, however, we will allow for potential differences in preferences within these consumption baskets as various special cases to be described further below.

[14] In equation (15), we follow the form in Epstein and Zin (1989), equation (5.3). While we use this equation for parsimonious price and the value functions, our gains and return calculations are also consistent with the Epstein and Zin (1991) form.

1.2.2 The Cost of Wedges using Preferences

To compare these consumption processes in utility units requires the value function solution for each country. Defining $Z_t = \Gamma_t / C_t$, Epstein and Zin (1991) show that the value function of the preferences in equation (15) can be written as:[15]

$$V(C_t, W_t) = (W_t/C_t)^{\frac{1}{1-\left(\frac{1}{\psi}\right)}} C_t = (1 + Z_t)^{\frac{1}{1-\left(\frac{1}{\psi}\right)}} C_t \tag{18}$$

where W_t is the present value of all future expected consumption.[16]

Therefore, rewriting the solution for the cost of wedges in equation (14) using equation (18) provides a general form for the costs from imperfect risk sharing as Δ in:

$$(1 - \Delta) = \left\{ \frac{W_0^D / C_0^D}{W_0^* / C_0^*} \right\}^{\left(\frac{1}{1-\left(\frac{1}{\psi}\right)}\right)} \left(\frac{C_0^D}{C_0^*} \right) = \left\{ \frac{1 + Z^D}{1 + Z^*} \right\}^{\left(\frac{1}{1-\left(\frac{1}{\psi}\right)}\right)} \left(\frac{C_0^D}{C_0^*} \right) \tag{19}$$

Below, we use conventional views about goods markets and asset markets to calculate the price-to-dividend ratios Z using standard Euler equation methods. As these variables are key drivers of the cost measures, we simply call them "price ratios" throughout the rest of the paper.

1.2.3 Consumption and Wealth in the Data Economy

As noted above, solving for the value function in the data economy requires the solution for wealth in the data; that is: $W_t^D \equiv C_t^D + \Gamma_t^D$. Since C_t^D can be observed in the data, we only require a value for the price of the future consumption process, Γ_t^D Fortunately, that solution can be derived from the Euler equation using the return on future wealth. In particular, Epstein and Zin (1989) show that for the recursive preferences in equation (15), the stochastic discount factor M_t is given by:

$$M_{t+1} = \beta^\theta (C_{t+1}/C_t)^{\left(-\frac{\theta}{\psi}\right)} (R_{c,t+1})^{\theta - 1} \tag{20}$$

[15] This connection with the value function has been used to connect asset returns to wealth and/or utility in a number of studies including Campbell (1993) and Bansal and Yaron (2001).

[16] The second equality follows since $W_t \equiv C_t + \Gamma_t$, so that $W_t/C_t = 1 + Z_t$ where $Z_t = \Gamma_t/C_t$, the price-to-dividend ratio for the future wealth asset.

where $R_{c,t+1} \equiv (C_{t+1} + \Gamma_{t+1})/\Gamma_t = \left(C_{t+1}/C_t\right)(1 + Z_{t+1})/Z_t$ is the return on future wealth. Therefore, substituting these expressions for the data economy into equation (1) yields:

$$E_t \left\{ \beta^\theta (C_{t+1}^D/C_t^D)^{\left(-\frac{\theta}{\psi}\right)} (R_{c,t+1}^D)^\theta \right\} = 1. \tag{21}$$

Substituting the return on the consumption process in the data economy, i.e., $R_{c,t+1} \equiv \left(C_{t+1}^D/C_t^D\right)(1+ Z_{t+1}^D)/Z_t^D$, into this Euler equation and solving for the future price ratio Z_t^D then determines wealth as a function of data moments.

For example, suppose that preferences are $CRRA$ so that $\gamma = 1/\psi$ and that consumption growth is i.i.d. so that the price ratio Z is constant. Then, the Euler equation in (21) simplifies to:

$$\beta E_t \left\{ \left(C_{t+1}^D/C_t^D\right)^{1-\gamma} \right\} = Z^D/(1 + Z^D)$$

Given moments of the distribution for the consumption process, this equation can be solved for Z^D to determine the value of wealth.

1.2.4 Consumption and Wealth in the Risk-sharing Economy

In order to measure the importance of this value function from the data economy, we next consider the counterfactual consumption and prices that would be implied in the complete markets "risk sharing" economy. For this purpose, we first determine the optimal consumption process for each country and then follow the same steps to measure wealth as in the data economy.[17] All of the details are given in Appendix A.

Under complete markets with common preferences across agents, the consumption of each agent follows a sharing rule. This rule is the outcome of a planner's problem facing any relevant goods market constraints.[18] To see that rule in the current context, first define the world aggregate consumption and world wealth both measured in the foreign numeraire country price units as: $\widetilde{C}_t^w \equiv S_t^* C_t^D + \widetilde{C}_t^D$ and $\widetilde{W}_t^{w*} \equiv S_t^* W_t^* + \widetilde{W}_t^*$ where $S_t^* \equiv (P_t^*/\widetilde{P}_t^*)$ is the real exchange rate between the domestic and foreign country when markets are complete. Then optimal consumption for the

[17] The more general multi-country form of this solution is stated below in Proposition 1.

[18] This rule is based upon sharing consumption although we can in principle based our analysis on sharing income as described in Section 3.

domestic country and the foreign country is given by the sharing rule:

$$C_t^* = \omega_t \widetilde{C}_t^w ;\ \widetilde{C}_t^* = (1 - \omega_t)\widetilde{C}_t^w, \forall t. \tag{22}$$

Thus, equation (22) gives the standard consumption risk-sharing rule that individual consumption is a share of aggregate consumption, $\widetilde{C}_t^w$.[19]

Since the price of consumption potentially differs across countries, the aggregate consumption $\widetilde{C}_t^w$ is specified in units of the numeraire foreign good. Thus, the time-varying shares ω_t capture the effects of real exchange rate changes due to country-specific price effects. In the next section, we detail how to recover these effects from the data depending upon the researcher's view of exchange rate behavior.

Calculating the loss due to wedges in certainty equivalent units in equation (19) requires a value in the risk-sharing economy of Z^*, the ratio of the future price of wealth-to-consumption, as well as C^*, the optimal allocation of the consumption level. This calculation is straightforward following the same approach as in the data economy. In particular, we first solve for the price ratio Z^* by substituting the risk-sharing rule in equation (22) into the Euler equation (20) to obtain:

$$E_t \left\{ \beta^\theta \left(\frac{\omega_{t+1} \widetilde{C}_{t+1}^w}{\omega_t \widetilde{C}_t^w} \right)^{\left(-\frac{\theta}{\psi}\right)} \left(R_{c,t+1}^*\right)^\theta \right\} = 1. \tag{23}$$

where $R_{c,t+1}^* \equiv \left(C_{t+1}^*/C_t^*\right)(1+Z_{t+1}^*)/Z_t^* = (\widetilde{C}_{t+1}^w/\widetilde{C}_t^w)\left(\omega_{t+1}/\omega_t\right)(1+Z_{t+1}^*)/Z_t^*$. Given solutions for the optimal home country consumption growth rate of consumption C_{t+1}^*/C_t^*, the Euler equation uniquely pins down the value of the price ratio of future wealth Z_t^*. In the next section we describe examples of these equilibrium consumption growth rates.

To consider a simple example of consumption and preferences, suppose that preferences are $CRRA$ and that consumption growth in the two countries are i.i.d. Then, the Euler equation for the home country based upon optimal consumption growth C_{t+1}^*/C_t^* in (23) simplifies to:

$$\beta E_t \left[\left(C_{t+1}^*/C_t^*\right)^{1-\gamma} \right] = Z^* / \left(1 + Z^*\right).$$

[19] In general, this share ω is time-varying when agents differ due to preferences, prices, or information. For example, Dumas, Lewis and Osambelo (2017) show how this sharing rule varies due to differences in beliefs about observed information.

Given the implied values for Z_t^* implies that the certainty equivalent loss for the home country is:

$$1 - \Delta = \left\{ \frac{1 + Z_0^D}{1 + Z_0^*} \right\}^{\left(\frac{1}{1 - \left(\frac{1}{\psi} \right)} \right)} \left(\frac{C_0^D}{C_0^*} \right) \tag{24}$$

where $C_0^* = \omega_0 \widetilde{C}_0^w$. The loss for the foreign country has the same form but measured in numeraire price units.

The initial value of shares in aggregate consumption, ω_0, depend upon the initial relative wealth of each country's consumption process in the "Data" economy. In order to focus on the impact of exchange rates over time, we assume that all countries have the same initial wealth for much of the rest of the paper.[20]

1.2.5 Calculating the Contributions of Individual Wedges

Given the connection between the costs from incomplete market wedges Δ and the price ratio of claims on future wealth Z_0^D in equation (24), we can now evaluate the contribution of each of the individual wedges. These individual effects upon the costs may be calculated as a straightforward extension of the Euler equation analysis above.

1.2.6 State Prices with Individual Wedges

For this purpose, it is useful to reconsider the relationship between the complete markets state price and its relationship with the data economy state price adjusted by the wedges defined in equation (4).

First, recall that the state-by-state complete markets valuation of the future wealth return can be written in terms of its counterpart in the data economy adjusted by wedges according to equation (11) rewritten here for convenience:

$$M_{t+1}^* R_{c,t+1}^* = M_{t+1}^D R_{c,t+1}^D \left(\zeta_{M,t+1} \zeta_{R,t+1} \right)^{-1}.$$

[20] As shown in the appendix, unequal initial shares result from the constraint to the planner's problem that agents in each country cannot consume more than the value of their resources. The value of these resources under complete markets may be more valuable to some countries than others, depending upon the consumption process in the data. See, for example, Lewis and Liu (2015). In this paper, we instead focus on the dynamics of exchange rate movements rather than the level of wealth.

Clearly, since the total cost Δ determines the value of the state price in the data economy of $M^D_{t+1}R^D_{c,t+1}$ relative to the risk-sharing economy counterpart of $M^*_{t+1}R^*_{c,t+1}$, this cost measures the total welfare value of the deviation given by $\zeta_{M,t+1}\zeta_{R,t+1}$.

This same logic can therefore be used to evaluate each component separately. To illustrate this approach, consider as an example the effect of the M-wedge; that is, the wedge on stochastic discount factors from $\zeta_{M,t+1}$ to a risk sharing economy investor. Applying this wedge to equation (11) implies that the state price becomes: $M^*_{t+1}R^*_{c,t+1}\zeta_{M,t+1} = M^D_{t+1}R^*_{c,t+1}$. Put differently, the effect of the M-wedge, $\zeta_{M,t+1}$, on the valuation of the return to future wealth to a complete markets investor can be measured by the data economy investor's valuation of that same future wealth. Thus, the value of this marginal wedge can be measured in certainty equivalent units by calculating the price-to-dividend ratio Z_M that solves the Euler equation:

$$E_t\left\{M^D_{t+1}R^*_{c,t+1}\right\} = 1. \tag{25}$$

For example, using the $CRRA$ preferences as an example, the price ratio Z_M can be calculated as: $\beta E_t\left\{\left(C^D_{t+1}/C^D_t\right)^{-\gamma}\left(C^*_{t+1}/C^*_t\right)\right\} = Z_M/\left(1+Z_M\right)$. Similarly, the cost arising from the R-wedge can be determined by calculating the price ratio Z_{R_c} that solves the Euler equation for an investor distorted only by ζ_R and can be valued with the price ratio Z_R determined by the Euler equation:

$$E_t\left\{M^*_{t+1}R^D_{c,t+1}\right\} = 1. \tag{26}$$

Note as above that these wedges measure the marginal impact of the stochastic discount factor and the consumption asset in local consumption units.

Alternatively, this state-by-state valuation can be rewritten as its data counterpart in the foreign country using equation (13). This condition relates the domestic economy's risk-sharing state price value of the return on wealth with the valuation of the foreign investor of the domestic wealth return converted at the exchange rate rewritten here as:

$$\begin{aligned} M^*_{t+1}R^*_{c,t+1} &= \left[\widetilde{M}^D_{t+1}/\left(S^D_{t+1}/S^D_t\right)\right]R^D_{c,t+1}\left(\zeta_{\widetilde{M},t+1}\zeta_{R_C,t+1}/\zeta_{S,t+1}\right)^{-1} \\ &= \widetilde{M}^D_{t+1}R^D_{c,t+1}\left(S^D_{t+1}/S^D_t\right)^{-1}\left(\zeta_{s,t+1}\right)\left(\zeta_{\widetilde{M}R,t+1}\right)^{-1} \end{aligned} \tag{27}$$

where $\zeta_{\widetilde{M}R,t+1} \equiv \widetilde{M}^D_{t+1} R^D_{c,t+1} \left(\widetilde{M}^*_{t+1} R^*_{c,t+1}\right)^{-1}$; that is, the wedge between state price of the consumption asset in the numeraire economy. Without wedges, the valuation of domestic wealth by the foreign investor in the data would follow the Euler equation:

$$E_t \left\{ \widetilde{M}^D_{t+1} \widetilde{R}^D_{c,t+1} \left(S^D_{t+1}/S^D_t\right)^{-1} \right\} = 1 \tag{28}$$

where $\widetilde{R}^D_{c,t+1} \equiv \left(\widetilde{C}^D_{t+1}/\widetilde{C}^D_t\right)(1+\overline{Z}^D_{t+1})/\overline{Z}^D_t$. Again, the marginal impact of each component on the total cost can be determine by the price-to-dividend ratio that solves the Euler equation for an investor facing each wedge. That is, the contribution of the S-wedge can be calculated by applying the exchange rate wedge to the state price:

$$M^*_{t+1} R^*_{c,t+1} \zeta^{-1}_{S,t+1} = \widetilde{M}^*_{t+1} R^*_{c,t+1} \left(S^D_{t+1}/S^D_t\right)^{-1} \tag{29}$$

and then valuing the price ratio Z_S using the Euler equation:

$$E_t \left\{ \widetilde{M}^*_{t+1} R^*_{c,t+1} \left(S^D_{t+1}/S^D_t\right)^{-1} \right\} = 1. \tag{30}$$

Thus, Z_S is the price ratio to a hypothetical complete markets foreign investor who values the domestic economy's wealth under complete markets but must translate that wealth return at the data exchange rate rather than the complete market counterpart.

By contrast, the complement of this wedge is the impact on the state price value of future wealth in the foreign country:

$$M^*_{t+1} R^*_{c,t+1} \zeta_{\widetilde{M}R,t+1} = \widetilde{M}^D_{t+1} R^D_{c,t+1} \left(S^*_{t+1}/S^*_t\right)^{-1} \tag{31}$$

valued with the price ratio $Z_{\widetilde{M}R}$ in the Euler equation:

$$E_t \left\{ \widetilde{M}^D_{t+1} R^D_{c,t+1} \left(S^*_{t+1}/S^*_t\right)^{-1} \right\} = 1 \tag{32}$$

Thus, $Z_{\widetilde{M}R}$ is the value to a foreign investor who invests in the domestic economy's wealth measued in the data but converts this return at the exchange rate that would be implied under complete markets.

We can use these valuations to decompose the overall costs from dis-integration using the price ratios: Z^* and Z^D for domestic wealth to the domestic investor under complete markets and the date, respectively; $\overline{Z}^D$ for domestic wealth to the foreign investor in the data; Z_M for the domestic SDF wedge; Z_R for the domestic wealth return wedge, and Z_S for the exchange rate wedge.

Table 1 summarizes the impact on complete markets arising from each of these wedges. As defined there and used throughout the rest of the paper, we follow the notational convention that Z_ζ signifies the price ratio for the value of a return under complete markets distorted by wedge ζ.

1.2.7 Decomposing Costs with Price Ratios

The price ratios used to value the wedges can in turn be used to measure the relative contribution of each wedge in the total cost measure Δ in equation (24). For simplicity, we assume for now that the domestic and foreign countries are symmetric so that in this case they have equal shares in consumption and therefore $C_0^D = C_0^*$. Thus, the costs are only impacted by the relative valuations of the two economies, given by Z^D and Z^*.

To understand this decomposition, suppose first that we calculate the impact on certainty equivalent consumption to, alternatively, a complete markets investor and a data economy investor who faces the M-wedge. That is,

$$1 - \Delta_{M*} = \left\{ \frac{1 + Z_M}{1 + Z^*} \right\}^{\left(\frac{1}{1 - \left(\frac{1}{\psi} \right)} \right)} ; \; 1 - \Delta_{MD} = \left\{ \frac{1 + Z^D}{1 + Z_M} \right\}^{\left(\frac{1}{1 - \left(\frac{1}{\psi} \right)} \right)} . \tag{33}$$

Thus, Δ_{M*} measures the certainty equivalent consumption loss to a complete markets investor who faces the M-wedge. This cost will generally be positive because the wedge generates a departure of optimality. By contrast, Δ_{MD} is the same loss for an investor who only faces the M-wedge but now must face all other wedges in the data, in this case the R_c-wedge. Since Z^D and Z_M in this cost both reflect wedges, they compare second-best outcomes and as such there is no presumption for whether they are positive or negative. Nevertheless, they each illustrate in the same local price consumption units the impact upon the total wedge. Clearly, these two marginal costs can be combined to provide the overall cost Δ since: $1 - \Delta = (1 - \Delta_{MD}) / (1 - \Delta_{M*})$.

We make use of this basic intuition to provide a full decomposition that includes all relevant wedges. Consider first the wedges generated by the difference between the domestic risk-sharing and data economies, all in local prices, as given in equation (24). For this purpose, note that this cost can be written as a sequence of three wedges: (a) the M-Wedge to risk-sharing economy as given by Δ_{M*}, (b) the R_c-Wedge to the data economy as given by $\Delta_{R,D}$; and (c) the difference in welfare arising from the M-Wedge relative to the R-wedge. This latter value measures the importance of the distortion due to the current stochastic discount factor M and the return on future wealth R_c. In this way, the total wedge can be rewritten according to this decomposition as: $1-\Delta = (1-\Delta_{M*})(1-\Delta_{M,R})(1-\Delta_{RD})$ and similarly for the other wedges. Thus, since $\ln(1-\Delta) \approx -\Delta$, the impact of a particular wedge ζ^{+} relative to other wedges $\zeta^{\times}$ in the total cost can be decomposed as : $\Delta = \Delta_{\zeta^{+},*} + \Delta_{\zeta^{+},\zeta^{\times}} + \Delta_{\zeta^{\times},D}$. For example, to evaluate the impact of the R_c-Wedge and the S-Wedge the decompositions are, respectively:

$$\begin{aligned} \Delta &= \Delta_{M,*} + \Delta_{M,Rc} + \Delta_{R_c,D} \qquad (34) \\ \Delta &= \Delta_{R,*} + \Delta_{R,M} + \Delta_{M,D} \end{aligned}$$

In order to value the impact of the exchange rate wedge, we make use of the foreign investor valuation of domestic wealth in equation (28). Note that in this case, the data valuation of wealth including all of the wedges is for a hypothetical foreign investor, not the domestic investor. Therefore, to connect this measure to the data valuation of the domestic investor, we require the relative cost:

$$1-\Delta_{\eta,D} = \left\{\frac{1+Z^{D}}{1+\overline{Z}^{D}}\right\}^{\left(\frac{1}{1-\left(\frac{1}{\psi}\right)}\right)} \qquad (35)$$

As indicated by the notation, this relative cost is the implied difference in valuation due to the η wedge in BFT. To see why, note that Z^{D} values $M^{D}_{t+1}R^{D}_{c,t+1}$ while $\overline{Z}^{D}$ values $\widetilde{M}^{D}_{t+1}R^{D}_{c,t+1}\left(S^{D}_{t+1}/S^{D}_{t}\right)^{-1}$. Then, using equations (11) and (13), these state prices can be written as:

$\left(M^{D}_{t+1}R^{D}_{c,t+1}\right)/\left(\widetilde{M}^{D}_{t+1}R^{D}_{c,t+1}\left(S^{D}_{t+1}/S^{D}_{t}\right)^{-1}\right) = \zeta_{M,t+1}\zeta_{s,t+1}\zeta^{-1}_{\widetilde{M},t+1} = \exp(\eta_{t+1})$ [21] Therefore, be-

[21] In particular, $M^{D}_{t+1}R^{D}_{c,t+1} = M^{*}_{t+1}R^{*}_{c,t+1}\left(\zeta_{M,t+1}\zeta_{R,t+1}\right)$ and $\widetilde{M}^{D}_{t+1}R^{D}_{c,t+1}\left(S^{D}_{t+1}/S^{D}_{t}\right)^{-1} = M^{*}_{t+1}R^{*}_{c,t+1}\left(\zeta_{\widetilde{M},t+1}\zeta_{R,t+1}\right)\zeta^{-1}_{s,t+1}$. Thus, the data valuation of wealth to the domestic investor relative to the data valuation of domestic wealth to the foreign investor is: $\left(M^{D}_{t+1}R^{D}_{c,t+1}\right)/\left(\widetilde{M}^{D}_{t+1}R^{D}_{c,t+1}\left(S^{D}_{t+1}/S^{D}_{t}\right)^{-1}\right) =$

low, we call the cost due to the difference in these data valuations the η wedge.

These observations therefore inform an alternative decomposition using domestic versus foreign prices according to:

$$\begin{aligned}\Delta &= \Delta_{S,*} + \Delta_{S,\widetilde{M}R} + \Delta_{\widetilde{M}R,\eta} + \Delta_{\eta,D} \\ \Delta &= \Delta_{MR,*} + \Delta_{MR,S} + \Delta_{S,\eta} + \Delta_{\eta,D}\end{aligned} \tag{36}$$

The next section illustrates the impact of this decomposition with a simple example using basic aggregate data.

2 A Two-Country Symmetric Example

So far, we have demonstrated how standard Euler equation relationships can be used to derive the costs of inefficient international financial markets generally. One approach to using these relationships would be to evaluate asset return data directly to attempt to uncover implications for the various wedges. In this paper, we follow the approach in the macro and consumption-based asset pricing literature of using asset return data moments to discipline parameters used to value consumption and prices. In this section in particular, we consider the impact of relative prices across countries to evaluate the costs from exchange rate variability. We show that standard assumptions about the exchange rate behavior implicit in price aggregators used in the macro and finance literature lead to very different implications about the costs of inefficient international markets.To illustrate these effects, we consider a simple two country symmetric example and show how the costs can be disciplined by consumption and real exchange rate data. To develop this idea, we first examine these data assuming their innovations are i.i.d. However, since these assumptions do not generate relationships that will match asset returns, we later consider the impact of more persistent consumption growth processes that provide a better fit to financial market data.

$\zeta_{M,t+1}\zeta_{s,t+1}\zeta^{-1}_{\widetilde{M},t+1} = \exp(\eta_{t+1})$.

2.1 The Basic Structure using Data

In order to illustrate how costs can be calculated using only standard data on consumption and their relative prices, we begin with a very simple stylized symmetric two-country model with i.i.d. consumption growth. The growth rates in the data for consumption in each country are defined as $g_{c,t+1} \equiv \ln(C^D_{t+1}/C^D_t)$, and $\widetilde{g}_{c,t+1} \equiv \ln(\widetilde{C}^D_{t+1}/\widetilde{C}^D_t)$ and that of their relative price as $g^D_{s,t+1} \equiv \ln(S^D_{t+1}/S^D_t)$. By symmetry, the domestic and foreign country each have the same i.i.d. consumption processes in local price units. In other words, the distribution of the growth rate of the data economy consumption is:

$$g^D_{c,t+1} = \mu + \sigma_c \upsilon_{c,t+1} \tag{37}$$

where $\upsilon_{c,t+1} \backsim N(0,1)$, with counterparts for the foreign country denoted as $\widetilde{g}^D_{c,t+1}$. This approach is used in many macroeconomic models that detrend the economy around a steady state.[22] Similarly, assume that the data real exchange rate, S^D_{t+1}, is lognormally distributed with a mean growth rate of zero.[23] Thus, the data exchange rate process is given by:

$$g^D_{s,t+1} = \sigma_s \upsilon_{s,t+1} \tag{38}$$

where $\upsilon_{s,t+1} \backsim N(0,1)$ and where the consumption and exchange rate growth rates are jointly distributed.[24] Again, these assumptions are made for simplicity and a persistent process in the real exchange rate could be introduced as we describe below. To provide numbers for the example, we assume the volatility of consumption, σ_c, is 1.52% per year and the correlation of consumption across the two countries in local prices is $Corr(\upsilon^1_c, \upsilon^2_c) = 0.574$.

To highlight the role of diversification without exchange rate movements, we begin by assuming the relative prices do not vary so that $\sigma_s = 0$. To further simplify our example, suppose that preferences are given by CRRA so that $\gamma = 1/\psi$. Then the stochastic discount factor in the data economy is simply $M^D_{t+1} = \beta \left(C^D_{t+1}/C^D_t\right)^{-\gamma}$ and in the risk-sharing economy is $M^*_{t+1} = \beta \left(C^w_{t+1}/C^w_t\right)^{-\gamma}$ for

[22] See for example Lucas (1978), Hansen and Singleton (1983), Cochrane and Hansen (1992), and much of the international real business cycle literature such as Backus, Kehoe, and Kydland (1991).

[23] In the endogenous versions of the model, we consider homothetic preferences so that relative prices determine constant shares of individual goods within the consumption basket. In the exogenous exchange rate version, we assume the i.i.d. to be in line with the other versions of the model as well as other exchange rate literature.

[24] In Section 2 below, we descirbe how this exchange rate process may be an endogenous result of differences in preferences due to home bias, non-tradeables, or a combination.

$\widetilde{C}_t^w \equiv C_t^D + \widetilde{C}_t^D$. Thus all variables can be constructed from the data using the distribution of consumption in the data from equation (37) and preferences parameters β and γ, the costs can be directly calculated.

Figure 1 first illustrates the decomposition for $CRRA$ preferences for the M-wedge and the R-wedge, respectively. The left-hand range considers the standard macro preferences with $\gamma = (1/\psi) = 2$. Since the preferences constrain the intertemporal elasticity of consumption to be the reciprocal of risk aversion, this elasticity declines from $\psi = 0.5$ to 0.1 as γ increases from 2 to 10. As a result, the impact from higher risk aversion on costs is muted as the "Total Delta" Δ increases slightly before leveling off near 0.12%.[25] Moreover the costs due to the M-wedge valuing the future risk-sharing consumption wealth from the data economy stochastic discount factor declines with risk aversion and conversely increases with I.E.S. Similarly, the R-wedge which values the future data economy consumption from the perspective of a risk-sharing stochastic discount factor declines with risk aversion.

Figures 2 show these same decompositions when the IES and risk aversion are allowed to differ with general recursive preferences. In particular, Figure 2a and 2b show the effects on wedges from varying risk aversion when IES is 0.5 and 1.5, respectively. In both cases, the total costs now increase proportionally with the risk aversion γ. By contrast, the M-wedge cost, $\Delta_{M,*}$, that measures the data economy valuation of the risk-sharing economy wealth does not depend upon risk aversion, but rather the intertemporal elasticity of subsitution ψ. When $\psi = 0.5$ as in Figure 2a, the benefit of the future risk-sharing consumption cannot be easily substituted into the present and therefore the contribution of this wedge is a cost. However, when $\psi = 1.5$ as in Figure 2b, the less risky future consumption path is more valuable and the "cost" is negative; that is, the wedge contributes positively to welfare. The figures also graph these same patterns for the R-wedge cost $\Delta_{R,*}$, valuing the data economy wealth using the risk-sharing discount rate. Again, the impact of the IES parameter ψ is critical. As Figure 2a illustrates for the low elasticity case, the risk-sharing economy value of future consumption in the riskier data economy increases with risk-aversion and contributes significantly to the overall costs. When intertemporal substitution is instead high as in Figure 2b, the contribution of the R-wedge cost is to reduce the costs.

[25] Obstfeld (1994) noted the difficulty of using CRRA with welfare analysis since higher risk aversion necessarily dampens intertemporal substitution.

2.2 The Role of Different Price Aggregates

We now extend this simple two country symmetric example to include volatility in exchange rates, thereby impacting the consumption aggregator given in equation (17) and hence, the relative price of this goods basket across countries. The literature treats the equilibrium in these individual markets differently according to the view about the determinants of exchange rate behavior.

To motivate how these approaches impact the costs of inefficient financial markets, we begin with two polar views regarding exchange rate variability. The appendix describes the specific assumptions behind each case in detail. On one extreme, exchange rates do not depend upon the degree of financial market integration. According to this view, even if financial markets are complete, goods markets prices do not equilibrate across countries. If so, a social planner who attempts to allocate consumption efficiently would have to face the constraint that prices are given by their values in the data; that is, $S_t = (P_t^D / \widetilde{P}_t^D)$ or in other words, by the constraint, $S^* = S^D$. Since exchange rates move independently of financial market completeness, we call this view the "exogenous exchange rates" case.

On the other extreme, goods markets function frictionlessly so that prices move to clear individual commodity markets. According to this view, the real exchange rate changes because households have a preference for home goods produced in their own country.[26] This goods market view we call the "home bias exchange rates" case following the literature. For example, suppose the two goods are indexed country 1 and 2 and that the consumption aggregator is Cobb-Douglas. Then, if each country i prefers its own good i, then the aggregator can be written as:

$$C_t^1 = \left(C_{1,t}^1\right)^a \left(C_{2,t}^2\right)^{1-a}; C_t^2 = \left(C_{1,t}^1\right)^{1-a} \left(C_{2,t}^2\right)^a \tag{39}$$

where $a > 1/2$. Then, defining the relative price of commodity i relative to the numeraire country price index and arbitrarily designating country 2 as the numeraire, the real exchange rate is: $S_t = (\frac{P_t}{\widetilde{P}_t}) = \frac{(P_{1,t})^a \left(P_{2,t}\right)^{1-a}}{(P_{1,t})^{1-a}\left(P_{2,t}\right)^a} = \left(\frac{P_{1,t}}{P_{2,t}}\right)^{2a-1}$. Note that since each commodity market allocates goods across countries efficiently, the prices in goods markets are also in equilibrium. We leave the wedge analysis of the home bias case for the next version of this paper.

[26] Studies that have used this approach to explain exchange rate behavior include Colacito and Croce (2011), Coeurdacier and Rey (2012), and Stathopoulos (2017), among many others.

These two extreme views imply that the same exchange rate in the data has different welfare implications. When exchange rates are exogenous, the variability of prices generate a dead-weight loss to financial markets and hence exchange rate variation generally increases the costs of financial market integration. By contrast when exchange rates vary due to consumption home bias, then this variation may improve efficiency, a case made with common preferences by Cole and Obstfeld (1991).

Although these two extreme perspectives on goods markets provide useful benchmarks, other views of exchange rates would suggest that goods markets function somewhere in between. That is, some markets equilibrate rather well, while others face more significant frictions. An example of this view can be found in the standard nontradeable goods model that treats exchange rates as the relative price of traded goods to nontraded goods. Continuing the two country Cobb Douglas example above and defining $C_{T,t}$ and $C_{N,t}$ as the consumption of tradeables, T, and nontradeables, N, respectively, the consumption aggregator in this case would be:

$$C_t^j \equiv C(C_{T,t}^j, C_{N,t}^j) = \left(C_{T,t}^j\right)^{\alpha} \left(C_{N,t}^j\right)^{1-\alpha} \tag{40}$$

for $j = 1, 2$. Only the traded goods market is assumed to clear internationally while the nontradeables market does not. Thus, defining $P_{T,t}$ and $P_{N,t}$ as the price of tradeables and nontradeables, respectively, the real exchange rate with country 2 as numeraire becomes $S_t \equiv \frac{(P_{T,t})^{\alpha}(P_{N,t}^1)^{1-\alpha}}{(P_{T,t})^{\alpha}(P_{N,t}^2)^{1-\alpha}} = \left(\rho_{N,t}^2/\rho_{N,t}^1\right)^{1-\alpha}$ where $\rho_{N,t}^j \equiv \left(P_{N,t}^j/P_{T,t}\right)$, the relative price of non-tradeables to tradeables and where we have made use of the fact that $P_{T,t}^1 = P_{T,t}^2$. Since one of the goods markets does not clear but the other does, financial market efficiency will change the equilibrium prices by reallocating aggregate consumption across countries. Therefore, as we illustrate below, the exchange rate in the data given by: $S_t^D = (P_t^D/\widetilde{P}_t^D) \neq S_t^* = (P_t^*/\widetilde{P}_t^*)$. In this case, the exchange rate wedge can play an important role in altering the costs of financial market incompleteness.

2.3 A Simple Example with Different Price Aggregates

As this discussion suggests, through the lens of each version of the standard aggregators and their price adjustments, the same process for exchange rate data will imply very different welfare costs. To see this relationship, we now combine the simple example above with basic data moments.

Since we require a data set with consumption series along with comparable relative prices across countries and time, we study several OECD countries in the Penn World Tables 9 described in detail by Feenstra et al (2015). Table 2, Panel A provides key information for two representative countries: the United States (US) and Canada (Can). We extend our analysis to the United Kingdom (UK) and Australia (Aus) in Section 3.

Panel A shows basic information about consumption growth for these two countries measured in per capita units. The first two columns show the mean and standard deviation of the growth rates in real local consumption units corresponding to $g^D_{c,t}$ in the analysis above. Both countries have had historically similar consumption growth rates and for this reason we set their growth rates to be equal in the following analysis. The following two columns provide the standard deviations for the same per capita consumption deflated by two world numeraire price indices described by Feenstra et al (2015). The first measure, P^e, provides a price deflator for the expenditure basket of each country alone. The second measure, P^o, is the price deflator for output including the trade balance.[27] The last three columns show the consumption correlation matrix measured in local price units across countries. As typically found in the literature, these correlations are low at 0.57, suggesting imperfect diversification. Given these data moments, we take the average across countries and assume that the volatility of the relative prices in equation (38) is 1.12% and its correlation with local consumption $Corr(\upsilon_c, \upsilon_s) = -0.18$.

Connecting these data moments to the real costs of dis-integration in equation (24) requires a measure of the consumption shares ω_t in order to value the price ratios in the Euler equation (23). To see this connection, we first use the complete markets first order condition for the planner given exchange rates under complete markets given by equation (3) as $\widetilde{M}^*_{t+1}/M^*_{t+1} = S^*_{t+1}/S^*_t$. Using the solution for the stochastic discount factor for Epstein-Zin preferences in equation (20) and the definition of the return on wealth, $R_{c,t+1} = (C_{t+1}/C_t)(1 + Z_{t+1})/Z_t$ then $M_{t+1} = \beta^{\theta}(C_{t+1}/C_t)^{\left(-\frac{\theta}{\psi}\right)}(R_{c,t+1})^{\theta-1}$. Thus, combining this complete markets condition with the

[27] These price deflators correspond to the GDP^e and GDP^o series in Feenstra et al (2015), respectively. The appendix details the use of these data below.

sharing rule, $C_t^* = \omega_t C_t^w$ the relationship between stochastic discount factors can be restated as:

$$\begin{aligned}\frac{\widetilde{M}_{t+1}^*}{M_{t+1}^*} &= \left(\frac{\frac{\widetilde{C}_{t+1}^*}{\widetilde{C}_t^*}}{\frac{C_{t+1}^*}{C_t^*}}\right)^{-\gamma}\left(\frac{(1+\widetilde{Z}_{t+1}^*)/\widetilde{Z}_t^*}{(1+Z_{t+1}^*)/Z_t^*}\right)^{(\theta-1)} \qquad (41)\\ &= \left(\frac{\frac{\widetilde{\omega}_{t+1}}{\widetilde{\omega}_t}}{\frac{\omega_{t+1}}{\omega_t}}\right)^{-\gamma}\left(\frac{(1+\widetilde{Z}_{t+1}^*)/\widetilde{Z}_t^*}{(1+Z_{t+1}^*)/Z_t^*}\right)^{(\theta-1)} = \frac{S_{t+1}^*}{S_t^*}.\end{aligned}$$

When the consumption processes are i.i.d, then the price ratios of future to current consumption are constant so that $Z_{t+1} = Z$. Moreover, under symmetry, $Z^* = \widetilde{Z}^*$. Writing equation (41) in log growth terms then implies:

$$\begin{aligned}g_{c,t+1}^* &= \widetilde{g}_{c,t+1}^* + \frac{1}{\gamma} g_{s,t+1}^* \qquad (42)\\ &= \frac{1}{2} g_{c,t+1}^{wD} - \frac{1}{2}(1-\frac{1}{\gamma}) g_{s,t+1}^*\end{aligned}$$

where in the second line we have used the resource constraint that $1 = \omega_t + \widetilde{\omega}_t$. Note that in the absence of real exchange rate movements (i.e., when $g_{s,t+1} = 0$), the optimal consumption growth for the home country could be inferred directly from the data using the growth rate of world consumption in numeraire $g_{c,t+1}^{wD} \equiv \ln(C_{t+1}^w / C_t^w)$. However, relative price movements change the desired allocation of consumption. Moreover, exchange rate movements will have different implications for welfare depending upon the nature of goods market frictions. Therefore, in order to connect the relationship to the data and its implications for welfare, we consider as examples the three views of exchange rates noted above and their implications for the exchange rate growth solution $g_{s,t+1}^*$. The appendix provides solutions for each of these cases.

For the same data moments above, Panels B and C of Table 2 illustrate the costs of financial dis-integration given three views of the exchange rate. For the purposes of our illustration, we focus only on the case where $\gamma = 10$ and $\psi = 1.5$, a common parameter set in the asset pricing literature.Panel B reports the costs given by equation (19) under the column heading "Total Cost Δ" as well as the decomposition measured in local prices given by equation (34) under the column headings "Components." For example, "Components: M to R_c" provides the measures of the cost relative to complete markets of first introducing the M-Wedge, measured by $\Delta_{M,*}$ and the deviation in welfare from consuming wealth at the R_c-Wedge relative to the data, measured by $\Delta_{R,D}$. Since these two costs add to the total cost Δ once the deviation between the two wedges is included, we ommit this redundant measure, $\Delta_{M,R}$, for expositional clarity. The counterpart decomposition for the costs due to first introducing the R-Wedge is given under "Components: R_c to M." Panel C provides the same breakdown using foreign to domestic prices to uncover the effects of the exchange rate through the decomposition in equation (36).[28]

Consider first the case when goods prices are unaffected by financial market completeness as given by the "Exogenous" exchange rate case. In this case, the exchange rate in the risk-sharing equilibrium is trivially equal to the exchange rate observed in the data; that is, $(S^*_{t+1}/S^*_t) = \left(S^D_{t+1}/S^D_t\right)$ or equivalently, $g^*_{s,t+1} = g^D_{s,t+1}$. In this case, optimal consumption growth in equation (42) as measurable in the data becomes:

$$g^*_{c,t+1} = \frac{1}{2} g^{wD}_{c,t+1} - \frac{1}{2}(1 - \frac{1}{\gamma}) g^D_{s,t+1} \tag{43}$$

Pricing the value of this risk-sharing consumption growth rate in the Euler equation (23), implies the optimal price Z^* and the corresponding costs of the wedges.

The first row of Panels B and C report the results for the Exogenous exchange rate case. Panel B shows that the certainty equivalent costs of imperfect risk-sharing is 2.16% of permanent consumption. However, the decomposition of these costs provide insights into the relative importance of the wedges.

Panel B shows that the loss from insufficient risk-sharing arises from the value of future wealth rather the SDF; i.e., through the R_c-Wedge rather than the M-Wedge. In particular, the loss from the M-Wedge relative to complete markets $\Delta_M, *$ is actually a gain of -0.38% but this gain is more than compensated by the cost due to the R_c-Wedge relative to the Data economy, $\Delta_{R,D}$ of 13.60%. The columns under "Components: R_c to M" show that similar patterns considering the marginal impact of deviations for the R_c-Wedge relative to complete market first. This relationship is reversed when the IES parameter is less than one (not shown), a pattern that mimics the findings in Figures 1 and 2.

Decomposing the impact due to exchange rates in Panel C shows that since exchange rates

[28] In this case, we subsume the residual deviation between arising from the S-Wedge and the $\widetilde{MR}$-Wedge; that is, $\Delta_{S,\widetilde{MR}}$

are exogenous to financial markets by assumption, there is no S-Wedge. More surprisingly, the wedges in the valuation by the foreign country of the domestic wealth, given by the $\widetilde{MR}$-Wedge is very high and, on its own, reflects a cost relative to complete markets of about 22% of certainty equivalent consumption. The reason is that this marginal wedge considers the cost evaluated from the perspective of the foreign investor and domestic wealth in the data, without allowing the exchange rate to impact the diversified portfolio of world wealth in the future, a point we come back to in the next subsection. This effect is largely offset, however, by the impact once these wedges are converted to domestic price units through the S-$Wedge$. Overall, then, for standard parameter values, the costs when exchange rates are exogenous arise from significant deviations between the return on wealth and discount rates that are not offset by goods price movements.

Finally, the table demonstrates the case when relative prices vary due to the relative price of tradeables and nontradeables as in equation (40). Again, the planner's efficient consumption rule across countries can be written in units of the consumption and exchange rate growth in the data. In this case, the planner is restricted from redistributing nontradeables across countries since only claims on tradeables goods are shared. Moreover, the optimal exchange rate in the risk sharing equilibrium is no longer the same as the exchange rate in the data economy. In this case, the appendix shows that the optimal risk-sharing exchange rate is:

$$g^*_{s,t+1} = (1-\alpha)\varkappa(g^D_{c,t+1} - \widetilde{g}^D_{c,t+1}) - \alpha\varkappa g^D_{s,t+1} \tag{44}$$

where $\varkappa \equiv \left((1-\alpha)\frac{1}{\gamma} - \alpha\right)^{-1}$. Clearly, in this case, there is an S-Wedge because a reallocation of tradeables consumption impacts the relative valuation of tradeables to nontradeables goods within each country. Specifically, the risk-sharing exchange rate in equation (44) has two terms that relate to the data. The difference between domestic and foreign consumption $g^D_{c,t+1} - \widetilde{g}^D_{c,t+1}$ affects the equilibrium according to the share of consumption allocated to non-tradeables, $(1-\alpha)$. The quantities of this expenditure is offset by the relative price of non-tradeables observed in the data, $g^D_{s,t+1}$ according to expenditures on tradeables α. The effects are amplified by $(1-\alpha)\frac{1}{\gamma} - \alpha$ because the planner cannot reallocate nontradeables with share $(1-\alpha)$. Indeed, when preferences for nontradeables are sufficiently high, the planner may not be able to reallocate tradeables in a welfare-improving way if over-all consumption is positively correlated across countries.[29] For this reason, we assume $\alpha > 1/2$ in our example.

Table 2 shows that the effects of these exchange rate adjustments when the share of tradeables, α, is 0.7 and 0.9. Since households can only trade claims on the tradeable consumption, the higher is α, the more consumption can be potentially diversified. As a result, the total costs from dis-integration increase with α, from 1.78% to 2.13% in Panel B. The decomposition shows important differences compared to the Exogenous case. In particular, Panel B shows that the M-wedge generates a loss relative to complete markets with $\Delta_M, *$ as an unambiguous loss while the value of future wealth increases by between 6% to 10%.

Panel C demonstrates the impact from the equilibrium exchange rates. In this case, the S-Wedge is positive and contributes between 4.01% and 1.28% of certainty equivalent costs, for $a = 0.7$ and 0.9, respectively. However, as with the exogenous exchange rates case, these contributions are swamped by the impacts of the wedges in the foreign valuation of domestic wealth, through the $\widetilde{MR}$ wedge. Indeed, these effects contribute positively to welfare although again, converting these valuations back into domestic price units through the "Combined η-Wedge" reduced the costs by 22% to yield the overall total costs of 1.78% and 2.13%, as reported in Panel B.

All told, the results in Table 2 demonstrate that the costs of inefficient financial markets depend critically on the adjustment of prices in the goods market. Furthermore, looking "under the hood" at the decompositions shows a great deal of interactions between current and future tradeoffs in wealth through the M and R wedges and between valuations across countries through the S, $\widetilde{MR}$, and η wedges.

One feature of this closer inspection highlights the large deviations in current period valuations across countries evident in the S-Wedge calculations in Panel C. However, as we explain next, part of these large effects are generated by focusing exclusively upon wedges in the current period exchange rate alone. In the next subsection, we introduce a different version that addresses this issue.

[29] Intuitively, if $\alpha < 1/2$, a negative shock to home consumption will fall disproportionately on nontradeables. Allocating more tradeables at a time when domestic residents want more nontradeables will only push the price of nontradeables higher and reduce welfare.

2.4 A Total S-Wedge

To understand the relationship between the current wedge and the exchange rate and the impact on valuations of future wealth, recall the basic complete markets relationship in equation (13) and restated here for convenience:

$$
\begin{aligned}
M_{t+1}^{*}R_{c,t+1}^{*} &\equiv \widetilde{M}_{t+1}^{*}R_{c,t+1}^{*}\left(S_{t+1}^{*}/S_{t}^{*}\right)^{-1} \\
&= \widetilde{M}_{t+1}^{*}R_{c,t+1}^{*}\left(S_{t+1}^{D}/S_{t}^{D}\right)^{-1}\zeta_{S,t+1}
\end{aligned}
$$

In this way, applying the exchange rate wedge $\zeta_{S,t+1}$ to the state price allows a valuation on the impact of the exchange rate on the current period's valuation of wealth. However, this ignores the impact of wedges in exchange rates on the value of future wealth. In the appendix we show that under complete markets, the return on wealth across countries are related according to:

$$
\frac{R_{c,t+1}^{*}}{\widetilde{R}_{c,t+1}^{*}} = \left(\frac{S_{t+1}^{*}}{S_{t}^{*}}\right)^{\frac{1}{\gamma}}\left[\frac{(1+Z_{t+1}^{*})/Z_{t}^{*}}{(1+\widetilde{Z}_{t+1}^{*})/\widetilde{Z}_{t}^{*}}\right]^{\frac{\theta}{\gamma\psi}}
$$

In our example, countries are symmetric of that $Z_{t}^{*} = \widetilde{Z}_{t}^{*}$. Furthermore, consumption growth innovations are i.i.d. so that these price ratios are constant. In this case, the return on wealth in the two countries are related according to:

$$
R_{c,t+1}^{*} = \widetilde{R}_{c,t+1}^{*}\left(S_{t+1}^{*}/S_{t}^{*}\right)^{\frac{1}{\gamma}} \tag{45}
$$

Thus, we can write the total relationship between the return on wealth across countries as:

$$
\begin{aligned}
M_{t+1}^{*}R_{c,t+1}^{*} &\equiv \widetilde{M}_{t+1}^{*}\widetilde{R}_{c,t+1}^{*}\left(S_{t+1}^{*}/S_{t}^{*}\right)^{-(1-\frac{1}{\gamma})} \\
&= \widetilde{M}_{t+1}^{*}\widetilde{R}_{c,t+1}^{*}\left(S_{t+1}^{D}/S_{t}^{D}\right)^{-(1-\frac{1}{\gamma})}\left(\zeta_{S,t+1}\right)^{-(1-\frac{1}{\gamma})}
\end{aligned} \tag{46}
$$

Following the same steps as above, we can then look at the effects of the wedge in the *Total* S-Wedge on the dis-integration costs.

Panel D of Table 2 shows the effect when the *Total* S-wedge values the impact on future consumption wealth. Under the Exogenous Exchange Rate view, a wedge in exchange rates unambiguously reduces welfare. Moreover the total η wedge, here redefined as η^T is more muted. Under the Nontradeables View, the exchange rate wedge contributes to welfare through $\Delta_{S,*}$, unlike the result in Panel B. However, the wedge on foreign valuation of wealth through $\Delta_{\widetilde{MR},*}$ contributes to the costs. Overall, domestic agents value the future impact of exchange rates on wealth, thereby mitigating the costs on exchange rate wedges in the short run.

2.5 A Simple Example with Persistent Consumption Risk

The example in Table 2 highlights the role of goods markets in determining the benefits of financial market integration. However, we have assumed so far that consumption growth has i.i.d. innovations. It is well-known that i.i.d. consumption growth implies counter-factual asset pricing implications under this assumption. Therefore, we now extend our analysis to richer consumption processes with persistent risk that can match asset returns. To that end, we first examine asset returns in the data and then their counterparts in the model based upon various parameter assumptions. In doing so, we must make an assumption about the identity of the marginal investor who prices a given asset. As noted in Section 1, if market are incomplete then the Euler equation will not hold for agents who are inframarginal in a given asset market. For example, if domestic investors are marginal in the domestic risk-free rate market, then the Risk-free rate is given by equation (10) while if foreign investors are marginal in this market, then the Risk-free rate is given by equation (12). Clearly, these investors may be marginal in different states and time periods.[30] Nevertheless, some papers use the identification that one investor is marginal over some periods in order to provide insights on the implicit wedges in asset return data directly.[31] In contrast to those papers that use asset return data directly, we use data on consumption and prices and use asset return data indirectly to discipline the model implications. For this purpose, we first consider the assumption that domestic investors are the marginal investors in their own assets, the standard assumption in much of the consumption-based asset pricing literature. In the next version, we intend to consider alternative assumptions.

Table 3 shows the relationship between the analysis above and asset returns in the data under

[30] See for example Telmer (1993).

[31] Lustig and Verdelhan (2019) assume that the foreign investor is always the marginal investor for the domestic risk-free rate and examines implications for η relative to that benchmark. Bakshi, Cerrato, and Crosby (2017) consider investments across many assets and thereby analyze a bound on the range of these wedges across assets and states. Sandulescu, Trojani, and Vedolin (2020) consider the minimum wedge across a range of investable assets.

the identifying assumption that the domestic investor prices domestic assets. As in the model, the returns are measured as real returns in domestic price units. The asset return solutions are described in the appendix. Panel A.I of Table 3 provides the basic asset pricing moments for the US and Canada. Mean equity returns in the data are in the range of 7% to 8.5% and display considerably greater variation than the risk-free rate. Panel A.II of Table 3 illustrates the implications of the framework above assuming i.i.d. consumption processes and standard preference parameters from macroeconomics. In particular, risk aversion is assumed to be $\gamma = 2$ and the intertemporal elasticity of substitution $\psi = 0.5$. As the table illustrates, standard asset pricing puzzles hold in this framework. In all cases, the equity returns are too low, the risk free rate is too high and all asset returns are insufficiently volatile. In fact, the model implies a constant risk free rate that is counterfactual to the data.

As the literature on consumption asset-pricing has shown, persistent consumption risk can help address these puzzles. This persistence may arise due to fears of a disaster (e.g., Wachter (2013), Barro (2007)), habit persistence (Campbell and Cochrane (2009)), and long run risk (Bansal and Yaron (2004)), to name a few. Considering all of these models is clearly outside the scope of this paper. Therefore, we consider a model with parameters and consumption processes that fit most easily with the i.i.d structure above. For this purpose, we use the long run risk structure and assume that instead of the i.i.d. process, consumption in the data is given by:

$$
\begin{aligned}
g^{D}_{c,t+1} &= \mu + x_t + \sigma_c \upsilon_{c,t+1} \\
x_{t+1} &= \rho x_t + \varphi_e \sigma e_{t+1}
\end{aligned}
\tag{47}
$$

and $\upsilon_{c,t+1}, e_{t+1} \backsim N(0,1)$ and are mutually independent. In this case, the price ratios Z_t are time varying and depend upon the persistent component, x_t.

To focus upon the effect of the persistent risk in consumption, we continue to assume that the exchange rate follows the random walk process in equation (38). This assumption could be modified by including a persistent component that would induce mean reversion over time thereby allowing for a long run real exchange rate.[32] Instead, here we adopt an incremental approach to this line

[32] Mean reversion of the long run real exchange rate has been highlighted in a long literature in macro and finance ranging from Abuaf and Jorion (1990), Froot and Rogoff (1995), to more recently, Lustig, Stathapoulous, and Verdelhan (2019) and Chernov and Creal (forthcoming.)

of inquiry by first adding persistence to consumption growth alone and then adding persistence in exchange rates as a future step. Thus, for now, we focus on the i.i.d. version of the exchange rate process. Nevertheless, it carries important implications for the relative importance of the implied S-Wedge below.

For the analysis with these processes, the value of lifetime wealth based upon the data on consumption and exchange rates is calculated as before, but now includes the persistent consumption term. We use the preference parameters from Bansal and Yaron (2004) of $\gamma = 10$ and $\psi = 1.5$, as in some of our analysis above. Equity is treated as an asset that pays dividends in the form of a levered consumption process following Abel (1999). That leverage parameter, ϕ, is therefore also an important parameter used to match equity return moments. We discipline the model by fitting this leverage parameter as well as the consumption and exchange rate moment parameters in order to match asset returns. These data moment parameters are the growth rate of consumption μ, the standard deviation of transitory consumption, σ^j, the autocorrelation in persistent consumption, ρ, the relative volatility of persistent consumption, φ^e, and the standard deviation of the real exchange rate, σ_s. To acheive this fit, the growth rate of consumption and the standard deviation of the exchange rate and calibrated to their data counterparts. We then identify the remaining parameters by implementing Simulated Method of Moments to provide the best match between the asset return moments in the model and in the data. The foreign exchange risk premium is calculated directly from the implied foreign exchange return.The appendix provides details on this implementation in the Empirical Methods section.

Panel A.III of Table 3 demonstrates the asset returns in the model using the parameters matched using SMM. First, the risk free rate is indeed much lower than what is implied by the iid consumption model, and while it is generally still too high, it comes closer to the data. Similarly, the risk-free rate volatility is no longer constant. Finally, the implied equity premia range from 5% to almost 7%, which reasonably matches the equity return data for Canada, but falls short for the United States. The foreign exchange risk premium volatility is not a targeted moment as noted earlier, and therefore the foreign exchange risk premium implied by the model is somewhat lower than the data..

Panels B of Table 3 shows the costs from dis-integration now calculated using these parameters generated from Simulated Method of Moments analysis, as well as using the data correlations discussed earlier. One critical assumption that has significant impact on the cost of imperfect risk-sharing is the correlation of the long run risk component in consumption, a moment that is not directly observable in the data. However, Lewis and Liu (2015) show that this moment can be identified using the model together with the data correlations of equity return and consumption. Using a typical estimate for most countries from that paper, we assume here a correlation of 0.9 in the long run component of consumption risk.

Panel B shows that for all three versions of the exchange rate, the total costs are somewhat larger due to the presence of persistent risk and are above 3% of permanent consumption. However, this modest increase in costs belies important differences between the wedge from the M-Wedge and the R-Wedge. For example, starting with the final two columns for the Exogenous case, introducing a wedge on future wealth as measure by $\Delta_{R,*}$ implies significant costs on the order of 142% of permanent consumption while a wedge on the current M implies a gain measured by $\Delta_{M,*}$ of 21%. The differences in the directions of contributions are a reflection of the high IES as noted earlier. However, the magnitudes of these effects are significantly higher due to the persistent risk in all exchange rate versions.

Panel C shows that the decomposition using the *Total* S-Wedge. As the panel highlights, the large adjustment in the wedge components arises from the impact of exchange rates relative to the value of foreign wealth, that is, the $\widetilde{MR}$-Wedge. When exchange rates are exogenous to financial market adjustment, the cost from exchange rate adjustment, $\Delta_{S,*}$, is zero and therefore all of the wedge arises from the difference between stochastic discount factors and the wedges in foreign wealth returns, $\Delta_{\eta,D}$ and $\Delta_{\widetilde{MR},*}$. Moreover, when exchange rates can adjust as in the case on non-tradeables, the contribution of the S-Wedge is very small with cost contributions of $\Delta_{S,*}$ of 0.31% and 0.02% for α of 0.7 and 0.9, respectively. However, the "cost" from the $\widetilde{MR}$-Wedge implies a large gain that is only offset by the difference between the two wedges. These large effects from stochastic discount factors and returns relative to the exchange rate are a reflection of the i.i.d. assumption of exchange rates, noted above.

Overall, fitting the wedges to asset return data shows significant interactions between the contributions from exchange rates, wealth returns, and stochastic discount factors. Although we have only presented examples here, they suggest important takeaways. First, different views about the source of exchange rate variation imply differences in overall real costs of international financial dis-integration. Perhaps more significantly, however, the underlying assumptions about the exchange rate process within an asset pricing framework generates more significant differences. If the exchange rate is presumed to follow a random walk, the impact of wedges in the real wealth to investors are only transitory. When investors face persistent consumption risk in local prices but only transitory risk in the exchange rate, implied wedges in the local price stochastic discount factors are more important than the exchange rate wedges.

3 Multiple Countries and Other Generalizations

We have so far considered a simple two-country example to illustrate how the real costs of international financial dis-integration may be decomposed and valued using standard data and asset return pricing identifications. This section describes how this framework may be generalized in a number of ways. In particular, we show that the approach extends naturally to multiple countries and illustrate with data for four countries. Further, while our analysis above focused upon the diversification of consumption risk, this section describes the extension to production economies.

3.1 The Costs of Wedges with Multiple Countries

It is straightforward to generalize the approach in Section 2 to a set of countries $j = 1, ..., J$. For this purpose, we now denote with a superscript, the variables specific to a given country j, where $j = 1, ...J$ where as above we arbitrarily choose the last country J to be the numeraire. Thus, C_t^j and $S_t^j = (P_t^j / \widetilde{P}_t^J)$ are respectively the aggregate consumption level and real exchange rate for country j below. This multi-country version requires extending the resource constraints to J countries so that world consumption units measured in the numeraire country is now:

$$\sum_{j=1}^{J} S_t^{jD} C_t^{j,D} = \widetilde{C}_t^w \tag{48}$$

Then the optimal consumption policy for each country is a sharing rule summarized by the following proposition. The formal statement of the proposition and its proof are given in the Appendix.

Proposition 1: *The complete risk-sharing consumption level for country j at time t that maximizes equation (15) given (i) the wealth constraints in equation (6) for each country $j =$*

$1,...,J$; *(ii) the world resources contraint in equation (48) for each period* $t = 0,...,\infty$, (iii) *and a participation contraint at an initial period, arbitrarity set at time* 0, *is given by*:

$$C_t^{j*} = \omega_t^j \widetilde{C}_t^w, \forall t, \tag{49}$$

where $\omega_0^j = W_o^{j,*}/\widetilde{W}_0^{w,*}$ and $\left(\omega_{t+1}^j/\omega_t^j\right) = \left(S_{t+1}^{j*}/S_t^{j*}\right)^{(1-(1/\gamma))}\left(1+Z_{t+1}^{j*}/Z_t^{j*}\right)^{(\theta-1)/\gamma} D_t$ where $D_t = \Sigma_{i=1}^J(\omega_{t+1}^i/\omega_t^i)$.$\circ$

To illustrate the implications of this result, we return to the case when countries have i.i.d. consumption processes. In this case, Z_t^{j*} are constant so that: $\left(\omega_{t+1}^j/\omega_t^j\right) = \left(S_{t+1}^{j*}/S_t^{j*}\right)^{(1-(1/\gamma))}$. Thus, evaluating the impact on consumption requires determining the solution for the risk-sharing exchange rate S_t^{j*}. For this purpose, we return to the two versions of goods markets described above: Exogenous exchange rates and Nontradeables. The analysis in the rest of this subsection illustrates the extension to multiple countries for these two cases. The quantitative results below assume for now that the consumption rules are constant and therefore should be viewed as suggestive only. We leave the time-varying case for the next version of the paper.

3.1.1 The Costs with Exogenous Exchange Rates

We first extend the data series from the United States (US) and Canada (Can) to include the United Kingdom (UK) and Australia (Aus). The Appendix shows basic information about consumption growth for this extended group of countries countries measured in per capita units. In order to focus upon the impact of multiple countries, we show the implications for the total cost of dis-integration Δ.

To illustrate the impact of exchange rate variability, we first assume counterfactually that the exchange rate volatility is zero; that is, $g_{s,t}^{j,D} = 0$. Table 4 Panel A reports the results from these calculations using the correlations for real consumption growth in Table 2. These levels are shown for a base case set of typical macro parameters of $\gamma = 2$ and $\psi = 0.5$, for a "High IES" case of $\psi = 1.5$, for a "High RA" case of $\gamma = 10$, and finally for a "High Both" case that is typical in asset pricing of $\gamma = 10$ and $\psi = 1.5$. In all cases, the benefits of risk-sharing imply positive certainty equivalent consumption wedge Δ^j. Moreover, these costs increase with risk aversion and the intertemporal elasticity of substitution, IES.

By contrast, Panel B shows the impact of exchange rate volatility when the variance of $g_{s,t}^{j,D}$ is given by the expenditure price data. In all cases, the benefits are lower and are even negative for Canada, meaning that if there were no adjustment, it would not participate in additional international risk-sharing.

Table 4 Panel C includes the effects of differences in wealth shares, ω^j, calculated using the value of each country's consumption stream in the risk-sharing economy equilibrium. In the interest of brevity, we report only the two cases of "macro" and "high both" parameters. These equilibrium wealth shares are close to one so that the basic relationships continue to hold. That is, international risk-sharing is unambigously beneficial in the absence of exchange rate volatility. But with exogenous exchange rate volatility, the standard gains from trade must overcome the adverse price variability to make further international risk-sharing beneficial.

3.1.2 Non-tradeable and Tradeable Goods

An alternative view of real exchange rate changes derives from the literature on non-tradeables.[33] The multi-country extension implies a rewriting of consumption for country j as: $C_t^j \equiv C(C_{T,t}^j, C_{N,t}^j) = \left(C_{T,t}^j\right)^{\alpha}\left(C_{N,t}^j\right)^{1-\alpha}$ where as before $C_{T,t}^j$ and $C_{N,t}^j$ are consumption by country j of tradeables and nontradeables, respectively.

Table 5 illustrates the costs of imperfect integration using the same data as in Table 4. In this case, the analysis depends upon the share of tradeables and therefore we report the results for a range of α using the set of preference parameters used before. In particular, we consider a "low" share of tradeables, $\alpha = 0.1$, and a "high" share $\alpha = 0.9$ as well as an intermediate case with $\alpha = 0.7$, closer to numbers used in the literature.

Several patterns are apparent. First, for the low share case of $\alpha = 1$, the costs are almost zero. As described above, the low level of costs arise because household have a strong preference for nontradeables which in the limit implies no benefits to risk-sharing and therefore no implicit costs from imperfect risk-sharing. As the share increases, however, the costs are monotonically moving away from zero. Second, the benefits to risk-sharing for Canada are negative as before due to the negative correlation of its exchange rate with the rest of the world. However, in this case, the mechanism arises because during periods when its nontradeables price is high due to scarcity,

[33] For examples, see Cole and Obstfeld (1991), Engel (1999), and Asea and Mendoza (1994), among many others.

other countries are relatively abundant in nontradeables, thereby increasing their relative price of tradeables worldwide. This exchange rate effect is sufficiently strong to overcome the standard gains to risk-sharing for Canada.

3.2 The Costs of Wedges using Production

The analysis in this paper has focused upon using consumption data because it is the driver in many asset pricing models. Moreover, it relates to a large literature on consumption risk-sharing. Nevertheless, the approach can easily be generalized to consider the implications of inefficient allocations in production. One approach would be to suppose that production is linear in technology. For example, consider a model in which output in each firm is produced with linear technology:

$$y_t(z) = Z_t z l_t(z) \tag{50}$$

where $l_t(z)$ is the amount of labor employed by the firm and where Z_t is a stochastic process generating aggregate productivity. In this case, if domestic consumption depends upon claims to this output across countries, the total world consumption, C_t^w would be replaced to total world output, Y_t^w in the data. In this way, the same analysis of the real costs of dis-integration can be calculated.

4 Concluding Remarks

The question of how much frictions in financial markets affect the welfare of economic agents is an important one. In this paper, we consider this question by connecting standard approaches to explain exchange rates through international goods market and financial frictions in a conventional macro-asset pricing framework. Our analysis provides several important contributions. First, and foremost, it develops a framework for calculating the real costs of insufficient integration of markets across countries. We have showed how the identifying assumptions for this analysis is typically a natural implication of the model under consideration by the researcher. Second, the paper shows that the costs from imperfect financial market risk-sharing depend critically upon the structure of the goods market and the reasons why real exchange rates vary. If the exchange rates vary due to goods market trading frictions that are exogenous to the financial market, then these variations in prices create a constraint upon the ability of financial market integration to improve risk-sharing.

By contrast, in the more realistic case that goods markets are neither perfectly efficient nor are price variations exogenous, we show that financial market integration would endogenously alter the goods market prices. As a standard example of such a view, we consider the case when exchange rates vary due to the price of nontradeables versus tradeables. In this case, the exchange rates in the data differ from the counterfactual exchange rates in a financially integrated world because asset trade reallocates total consumption across countries with idiosyncratic nontradeables risk.

We illustrate how this approach may be used to decompose the overall costs of international dis-integration into components due to exchange rates, return on wealth, and the stochastic discount factor. The paper showed that the impact of exchange rates on financial market wedges may be uncovered from standard consumption and exchange rate data, using the underlying assumptions of the model. As an illustration of how these component costs may be disciplined with asset return data, we fit a simple long-run risk model to the data. This example highlighted the role of persistence in the exchange rate process. That is, when the exchange rate follows an i.i.d process and consumption growth is persistent, then the component of dis-integration costs due to exchange rates is dwarfed by the wedges in the asset return components.

Overall, our paper provides an important step toward connecting the behavior of asset returns, exchange rates, and the costs of international dis-integration.

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Table 1 Wedge Cost Valuation Summary

A. Risk-sharing State Price Benchmark						
Wedge		General Pricing Relationships		Example:	CE Cost relative to:	
		Data State Price	Price Ratio[1]	CRRA pricing[2]	Risk-Share	Data
0		$M_t^* R_{c,t}^*$	Z^*	$\left(\frac{C_t^*}{C_{t-1}^*}\right)^{1-\gamma}$	0	Δ^{-1}
B. Variables for Cost Decomposition in Local Prices						
Data counterpart		$M_t^* R_{c,t}^* = M_t^D R_{c,t}^D \left(\zeta_{M,t}\zeta_{R,t}\right)^{-1}.$				
Wedge	Notation	General Pricing Relationships		Example:	CE Cost relative to:	
		Data State Price	Price Ratio[1]	CRRA pricing[2]	Risk-Share	Data
$Total$	$\zeta_{M,t}\zeta_{R,t}$	$M_t^D R_{c,t}^D$	Z^D	$\left(\frac{C_t^D}{C_{t-1}^D}\right)^{1-\gamma}$	Δ	0
M	$\zeta_{M,t}$	$M_t^D R_{c,t}^*$	Z_M	$\left(\frac{C_t^D}{C_{t-1}^D}\right)^{-\gamma} \frac{C_t^*}{C_{t-1}^*}$	$\Delta_{M,*}$	$\Delta_{M,D}$
R	$\zeta_{R,t}$	$M_t^* R_{c,t}^D$	Z_{R_c}	$\left(\frac{C_t^*}{C_{t-1}^*}\right)^{-\gamma} \frac{C_t^D}{C_{t-1}^D}$	$\Delta_{R,*}$	$\Delta_{R,D}$
C. Variables for Cost Decomposition relative to Foreign Prices						
Data counterpart		$M_t^* R_{c,t}^* = \widetilde{M}_t^D R_{c,t}^D \left(\frac{S_t^D}{S_{t-1}^D}\right)^{-1} \left(\zeta_{s,t}\right) \left(\zeta_{\widetilde{M}R,t}\right)^{-1}$				
Wedge	Notation	General Pricing Relationships		Example:	CE Cost relative to:	
		Data State Price[3]	Price Ratio	CRRA pricing	Risk-Share	Data
$\widetilde{Total}$	$\zeta_{s,t}\zeta_{\widetilde{M}R,t}^{-1}$	$\widetilde{M}_t^D R_{c,t}^D (\frac{S_t^D}{S_{t-1}^D})^{-1}$	$\overline{Z}^D$	$\left(\frac{\widetilde{C}_t^D}{\widetilde{C}_{t-1}^D}\right)^{1-\gamma} (\frac{S_t^D}{S_{t-1}^D})^{-1}$	$\Delta_{\eta,*}$	$\Delta_{\eta,D}$
S	$\zeta_{s,t}$	$\widetilde{M}_t^* R_{c,t}^* (\frac{S_t^D}{S_{t-1}^D})^{-1}$	Z_S	$\left(\frac{\widetilde{C}_t^*}{\widetilde{C}_{t-1}^*}\right)^{1-\gamma} (\frac{S_t^D}{S_{t-1}^D})^{-1}$	$\Delta_{S,*}$	$\Delta_{\widetilde{M}R,D}$
$\widetilde{M}R$	$\zeta_{\widetilde{M}R,t}$	$\widetilde{M}_t^D R_{c,t}^D (\frac{S_t^*}{S_{t-1}^*})^{-1}$	$Z_{\widetilde{M}R}$	$\left(\frac{\widetilde{C}_t^D}{\widetilde{C}_{t-1}^D}\right)^{1-\gamma} (\frac{S_t^*}{S_{t-1}^*})^{-1}$	$\Delta_{\widetilde{M}R,*}$	$\Delta_{S,D}$

Notes: [1]Recursive Value function determined by $V(W) = (1+Z)^{1/(1-(1/\psi))}$; [2]CRRA pricing determined by Z in each row that equates the $E_{t-1}(\text{cell}) = Z/\beta(1+Z)$; [3]For iid, $\kappa = (1-\gamma^{-1})$.

Table 2 Wedge Cost-Symmetric Two-Country Example

A: Summary Data Statistics

Country	Mean	St Dev (Local P)	St Dev (World P^e)	St Dev (World P^o)	Consumption Correlation		
						US	Can
US	1.91%	1.56%	1.96%	1.84%	US	1.00	0.57
Can	1.89%	1.52%	2.16%	1.97%	Can	0.57	1.00

B. Total Cost and Decomposition in Local Prices

Exchange Rates View	Tradeables Share α	Total Cost Δ		Components: M to R_c		Components: R_c to M	
				M-Wedge $\Delta_{M,*}$	R_c -Wedge $\Delta_{R,D}$	M -Wedge $\Delta_{M,D}$	R_c -Wedge $\Delta_{R,*}$
Exogenous	NA	2.16		-0.38	13.60	2.54	-11.44
Non-tradeables	0.7	1.78		2.90	7.87	-1.12	-6.09
Non-tradeables	0.9	2.13		0.84	12.13	1.28	-10.00

C. Decomposition with Foreign to Domestic Prices: S-Wedge

Exchange Rates View	Tradeables Share α		Combined η-Wedge $\Delta_{\eta,D}$	Components: S to $\widetilde{MR}$		Components: $\widetilde{MR}$ to S	
				S-Wedge $\Delta_{S,*}$	$\widetilde{MR}$ -Wedge $\Delta_{S,\widetilde{MR}}$	S-Wedge $\Delta_{S,D}$	$\widetilde{MR}$ -Wedge $\Delta_{\widetilde{MR},*}$
Exogenous	NA		22.09	0.0	0.0	-19.93	-19.93
Non-tradeables	0.7		22.09	4.01	-43.52	-24.31	23.21
Non-tradeables	0.9		22.09	1.28	-52.25	-21.24	32.29

D. Decomposition with Foreign to Domestic Prices: Total S-Wedge

Exchange Rates View	Tradeables Share α		Combined η^T-Wedge $\Delta_{\eta^T,D}$	Components: S to $\widetilde{MR}$		Components: $\widetilde{MR}$ to S	
				S-Wedge $\Delta_{S,*}$	$\widetilde{MR}$-Wedge $\Delta_{S,\widetilde{MR}}$	S-Wedge $\Delta_{S,D}$	$\widetilde{MR}$-Wedge $\Delta_{\widetilde{MR},*}$
Exogenous	NA		0.20	0.0	0.0	1.96	1.96
Non-tradeables	0.7		0.20	-5.16	-0.37	6.75	1.96
Non-tradeables	0.9		0.20	-3.01	-0.09	4.93	2.02

Panel B, C, and D: σ_c= 1.52 , $Corr(\upsilon_c,\widetilde{\upsilon}_c) = 0.574$, σ_s= 1.12, , $Corr(\nu_c, \nu_s) = -0.18$; for $\psi = 1.5$, $\gamma = 10$

Table 3: Asset Prices and Costs of Imperfect Risk Sharing

A. Asset Return Moments

	Equity Mean	Equity vol	Risk-free Mean	Risk-free vol	Equity Premium	FX Prem Vol
I. Data Moments						
US	8.32%	18.62%	2.88%	1.48%	5.44%	2.77%
Can	7.65%	21.22%	0.74%	2.06%	6.91%	
II. IID Model Moments						
US	5.24%	5.82%	5.41%	0.00%	-0.17%	1.12 %
Can	5.10%	8.15%	5.33%	0.00%	-0.24%	
III. LRR Model Moments						
US	7.13%	17.99%	2.17%	0.47%	4.96%	1.39%
Can	8.47%	21.12%	1.55%	0.67%	6.92%	

B. Total Cost and Decomposition in Local Prices

Exchange Rates View	Tradeables Share α	Total Cost Δ		Components: M to R$_c$ M-Wedge $\Delta_{M,*}$	Components: M to R$_c$ R_c-Wedge $\Delta_{R,D}$	Components: R$_c$ to M M-Wedge $\Delta_{M,D}$	Components: R$_c$ to M R_c-Wedge $\Delta_{R,*}$
Exogenous	NA	3.58		-21.3	-148.2	25.0	141.9
Non-tradeables	0.7	3.18		-131.5	-356.2	134.8	359.4
Non-tradeables	0.9	3.76		-122.3	-350.5	126.1	354.3

C. Decomposition with Foreign to Domestic Prices: Total S-Wedge

Exchange Rates View	Tradeables Share α		Combined η^T-Wedge $\Delta_{\eta,D}$	Components: S to $\widetilde{MR}$ S-Wedge $\Delta_{S,*}$	Components: S to $\widetilde{MR}$ $\widetilde{MR}$-Wedge $\Delta_{\widetilde{MR},D}$	Components: $\widetilde{MR}$ to S S-Wedge $\Delta_{S,D}$	Components: $\widetilde{MR}$ to S $\widetilde{MR}$-Wedge $\Delta_{\widetilde{MR},*}$
Exogenous	NA		0.19	0.0	0.0	3.39	3.39
Non-tradeables	0.7		0.19	0.31	61.16	2.72	-58.13
Non-tradeables	0.9		0.19	0.02	60.48	3.62	-56.83

Table 4: Costs of Wedges with Exogenous Rates

$$(1+\Delta^j) = \left\{ \frac{W_0^{j,D}/C_0^{j,D}}{W_0^{j,*}/C_0^{j,*}} \right\}^{(1-(1/\psi))^{-1}} \left(\frac{C_0^{j,D}}{C_0^{j,*}} \right)$$

A. Δ^j Without Wealth Shares $(C_0^{j,*} = C_0^{j,D})$					
1. Without Exchange Rate Volatility		ConstantWeight Estimates			
Version	Parameters	US	Can	UK	Aus
Macro	$\gamma = 2, \psi = 0.5$	0.15%	0.13%	0.23%	0.26%
High IES	$\gamma = 2, \psi = 1.5$	0.44%	0.39%	0.67%	0.75%
High RA	$\gamma = 10, \psi = 0.5$	1.39%	1.23%	2.13%	2.38%
High Both	$\gamma = 10, \psi = 1.5$	3.80%	3.37%	5.75%	6.38%
2. With Exchange Rate Volatility Δ^j					
Macro	$\gamma = 2, \psi = 0.5$	0.07%	-0.01%	0.03%	0.09%
High IES	$\gamma = 2, \psi = 1.5$	0.19%	-0.03%	0.08%	0.25%
High RA	$\gamma = 10, \psi = 0.5$	0.61%	-0.10%	0.26%	0.82%
High Both	$\gamma = 10, \psi = 1.5$	1.66%	-0.26%	0.69%	2.20%
B. Δ^j With Wealth Shares $(C_0^{j,*}/C_0^{j,D}) \equiv$ "$Weights$"					
1. Without Exchange Rate Volatility		ConstantWeight Estimates			
Version	Parameters	US	Can	UK	Aus
Macro	$\gamma = 2, \psi = 0.5$	0.10%	0.14%	0.22%	0.33%
	Weights	1.001	1.000	1.000	0.999
High Both	$\gamma = 10, \psi = 1.5$	3.53%	4.05%	5.17%	6.54%
	Weights	1.003	0.993	1.006	0.998
2. With Exchange Rate Volatility					
Macro	$\gamma = 2, \psi = 0.5$	0.00%	-0.05%	-0.01%	0.23%
	Weights	1.001	1.000	1.000	0.999
High Both	$\gamma = 10, \psi = 1.5$	2.52%	1.28%	-1.03%	1.54%
		0.991	0.985	1.017	1.007

Table 5: Costs of Imperfect Risk Sharing with NonTradeables Goods

$$(1+\Delta^j) = \left\{ \frac{W_0^{j,D}/C_0^{j,D}}{W_0^{j,*}/C_0^{j,*}} \right\}^{(1-(1/\psi))^{-1}} \left(\frac{C_0^{j,D}}{C_0^{j,*}} \right)$$

Constant Weights Estimates		Certainty Equivalent Costs Δ^j			
Macro Parameters $\gamma = 2, \psi = 0.5$					
Version	Tradeables Share	US	Can	UK	Aus
Low	$\alpha = 0.1$	0.01%	0.00%	0.00%	0.01%
Medium	$\alpha = 0.7$	0.04%	-0.01%	0.02%	0.06%
High	$\alpha = 0.9$	0.06%	-0.01%	0.02%	0.08%
High IES Parameters $\gamma = 2, \psi = 1.5$	Tradeables Share				
Low	$\alpha = 0.1$	0.02%	-0.01%	0.00%	0.02%
Medium	$\alpha = 0.7$	0.13%	-0.03%	0.05%	0.18%
High	$\alpha = 0.9$	0.17%	-0.03%	0.07%	0.23%
High RA Parameters $\gamma = 10, \psi = 0.5$	Tradeables Share				
Low	$\alpha = 0.1$	0.05%	-0.02%	0.02%	0.08%
Medium	$\alpha = 0.7$	0.41%	-0.09%	0.16%	0.57%
High	$\alpha = 0.9$	0.54%	-0.09%	0.22%	0.74%
High Both Parameters $\gamma = 10, \psi = 1.5$	Tradeables Share				
Low	$\alpha = 0.1$	0.15%	-0.05%	0.04%	0.21%
Medium	$\alpha = 0.7$	1.13%	-0.23%	0.42%	1.52%
High	$\alpha = 0.9$	1.48%	-0.26%	0.59%	1.97%

Note:All results equally weighted for parsimony i.e., $C_0^{j,*} = C_0^{j,D}$

A International Risk Sharing with Exchange Rates

<Reader Note: Appendix under revision for typos and other mistakes.>

In this appendix we show how international risk sharing can be calculated under various assumptions about the sources of exchange rate variations.

A.1 General Solution to Risk Sharing Economy

We begin with a standard utilitarian social planner's problem, allocating consumption across multiple agents in each state and time. In order to simplify notation, we denote the realization of a variable in a given state at a point in time with a time t subscript, subsuming the state within the time identification.

Assumption 1: Agents' Utility There exist J representative agents in each of J countries with utility functions given by

$$U(C_t^j, U_{t+1}^j) = \left\{ C_t^{j\,\frac{1-\gamma}{\theta}} + \beta E_t \left[\left(U_{t+1}^j \right)^{1-\gamma} \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}, \; j = 1, ..., J. \tag{51}$$

where C_τ^j is the consumption of the representative agent in country j at time τ and $U_{\tau+1}^j \equiv U(C_{\tau+1}^j, U_{\tau+2}^j)$ is his next period utility. Furthermore, $C_\tau^j \equiv C^j(\underline{C}_\tau^j)$ is a homogeneous function of individual consumption goods $\underline{C}_\tau^j \equiv \left\{ C_{1,\tau}^j, ..., C_{K,\tau}^j \right\}$ where $C_{k,\tau}^j$ is consumption of good k by country j at time τ. Note that the choice of aggregator includes special cases such as countries (a) all share the same aggregator function, $C_\tau^j \equiv C^j(\underline{C}_\tau^j)$; (b) consume a single good, $K = 1$; (c) each consume a bundle of tradeables and non-tradeables, $K = 2$; and (d) each country has a "home bias" preference toward its own goods, that is, $K = J$ with greater weighting for country i on home good C_τ^i. (These special cases will be described in the following subsections.)

Assumption 2: Feasible Consumption Each agent $j = \{1, .., J\}$ is endowed at time 0 with a feasible "benchmark" consumption process, $\mathbf{C}^{jB} \equiv \left\{ C_0^{jB}, C_1^{jB}, ..., C_t^{jB}, ..., C_\infty^{jB} \right\}$ with individual consumption goods in their aggregators.

Assumption 3: Bounded and Rational Utility There exist initial consumption and next period utility allocations, $\left(C_0^j, U_1^j \right)$, such that:

$$U(C_0^j, U_1^j) \in R, < \infty, \forall j \tag{52}$$

Assumption 4: Asset Market Trade In the risk-sharing equilibrium, contingent claims are traded on a set of the consumption goods outputs. For expositional convenience and without loss in generality, assume that the consumption goods with contingent asset claims are ordered as the first L units $\widehat{\underline{C}}_{\tau}^{j} \equiv \left\{C_{1,\tau}^{j}, ..., C_{L,\tau}^{j}\right\}$ where $L \leq K$. Accordingly, the consumption good outputs that do not have contingent asset trade are: $\overline{\underline{C}}_{\tau}^{j} \equiv \left\{C_{L+1,\tau}^{j}, ..., C_{K,\tau}^{j}\right\}$ Note that this assumption admits a range of possibilities such as asset trade in all assets, $L = K$, or asset trade in only a subset such as in the case of non-tradeable goods, $L < K$. We consider both cases below.

Definition 1: Exchange Rates An exchange rate is the relative price of the numeraire country consumption aggregator in units of another country consumption aggregator. Thus, assuming that the numeraire country is $j = 1$, the exchange rate is S_t^j where:

$$S_t^j \equiv P_t^j / P_t^1 = \frac{P^j(P_{1,\tau}^j, ..., P_{K,\tau}^j)}{P^1(P_{1,\tau}^1, ..., P_{K,\tau}^1)} \tag{53}$$

given $P_{k,\tau}^j$ is the price of good k in country j at time τ and $P^j(P_{1,\tau}^j, ..., P_{K,\tau}^j)$ is the price index implied by the consumption aggregator $C_\tau^j \equiv C^j(\underline{C}_\tau^j)$.

Definition 2: Numeraire Consumption Units The consumption of country j in units of the numeraire country is: $\widetilde{C}_\tau^j = S_t^j C_\tau^j$.

Definition 3: State prices The state price the equalize the marginal utilities of all agents in units of the numeraire consumption is Q_τ. The state price in units of country j consumption is Q_τ^j.

General Framework: Given that Assumptions 1, 2, 3 and 4 hold and Definitions 1,2, and 3, then the solution to the planner's problem*:*

$$\underset{\substack{\left\{C_{i,t}^j\right\} \\ \forall t,i,j}}{Max} \sum_{j=1}^{J} a^j \sum_{\tau=0}^{\infty} U(C_\tau^j, U_{\tau+1}^j) \tag{54}$$

$$\text{s.t.} \sum_{j=1}^{J} \widetilde{C}_{i,t}^j = \sum_{j=1}^{J} \widetilde{C}_{i,t}^{jB}, \forall t, i = 1, ..., K \tag{55}$$

$$E_0 \sum_{\tau=0}^{\infty} Q_\tau \widetilde{C}_\tau^j = E_0 \sum_{\tau=0}^{\infty} Q_\tau \widetilde{C}_\tau^{jB}, \forall j \tag{56}$$

is given by:

$$C_\tau^{j*} \equiv C^j(\widehat{\underline{C}}_\tau^{j*}, \overline{\underline{C}}_\tau^{jB}), \forall t,$$

where $\widehat{\underline{C}}_\tau^{j*} = \varpi^j C_t^w$ *for* $C_t^w \equiv \sum_{j=1}^{J} \sum_{i=1}^{K} P_{i,t}^j \widehat{C}_{i,t}^{jB}$, *the aggregate world consumption of the goods traded* *in asset markets. And where:*

$$\varpi^j = \frac{\widehat{W}_0^{jB}}{\widehat{W}_0^w} \tag{57}$$

for $\widehat{W}_0^{jB} = E_0 \sum_{\tau=0}^{\infty} Q_\tau S_{i,t}^{j*} \widehat{C}_{i,\tau}^{jB}$ *and* $W_0^w = E_0 \sum_{\tau=0}^{\infty} Q_\tau \widehat{C}_\tau^w$*, the present value of country j's benchmark consumption and the world consumption that is traded at the world asset markets, respectively, evaluated at the optimal world stochastic discount factor,* Q_τ*.*

Below we show how this framework can be used in specific examples.

B General Framework with Exogenous Exchange Rates

This section considers the solution with exogenous exchange rates.

B.1 Planner Problem

Proposition 1: *Let* Q_τ *be the state-price measured in units of the numeraire country* 1 *at time* τ*, and* P_t^j *be the price of the consumption good for country* j *at time* t *in units of country* j*. Then the solution to the utilitarian planner's problem:*

$$\underset{\substack{\{C_t^j\} \\ \forall t}}{Max} \sum_{j=1}^{J} U(C_\tau^j, U_{\tau+1}^j) \tag{58}$$

for

$$U(C_t^j, U_{t+1}^j) = \left\{ C_t^{j\,\frac{1-\gamma}{\theta}} + \beta E_t \left[\left(U_{t+1}^j \right)^{1-\gamma} \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}, \; j = 1, ..., J. \tag{59}$$

$$\text{s.t.} \tag{60}$$

$$E_0 \sum_{\tau=0}^{\infty} Q_\tau P_t^j C_\tau^j = E_0 \sum_{\tau=0}^{\infty} Q_\tau P_t^1 C_\tau^{j,D} \equiv W_o^j, \forall j \tag{61}$$

$$\sum_{j=1}^{J} P_t^j C_t^j = \sum_{j=1}^{J} P_t^1 C_t^{j,D} \equiv P_t^1 \widetilde{Y}_t^W, \forall t \tag{62}$$

is given by:

$$C_t^{j*} = \omega^j \frac{\widetilde{Y}_t^w}{S_t^j}, \forall t, \tag{63}$$

where $S_t^j = \left(P_t^j / P_t^1\right)$.

$$\varpi^j = \frac{W_o^j}{\sum_{i=1}^{J} W_o^i} \equiv \frac{E_0 \sum_{\tau=0}^{\infty} Q_\tau S_\tau^j Y_\tau^j}{E_0 \sum_{\tau=0}^{\infty} Q_\tau \sum_{i=1}^{J} S_t^i Y_t^i}. \tag{64}$$

Proof: To be a solution, the consumption rules defined in equations (63) and (64) must maximize the utility function in equations (59) given the wealth constraints for each country in equation (61), the resource constraints for goods for each period in equation (62)).Thus the consumption rule must maximize intertemporal utility across countries given the common state price density Q_t measured in units of the numeraire consumption aggregate. Note that the maximum consumption for each country j given its respective wealth constraint in equation (61) is given by the value function in equation (67) repeated here for convenience:

$$V(C_t^j, W_t^j) = C_t^j \left(\frac{W_t^j}{C_t^j}\right)^{\frac{1}{1-(1/\psi)}} = C_t^j \left[\left(\frac{\Gamma_t^j}{C_t^j}\right) + 1\right]^{\frac{1}{1-(1/\psi)}}$$

Therefore, a common complete markets state price density will determine wealth in all countries if it can be shown to price the return on a claim to consumption in each country. Using the definition of the return on that claim in equilbrium as: $R_{C,t+1}^{j*} = (C_{t+1}^{j*} + \Gamma_{t+1}^{j*})/\Gamma_t^{j*}$, then defining the stochastic discount factor $M_{t+1} \equiv (Q_{t+1}/Q_t) = \beta^\theta (C_{t+1}^j / C_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{C,t+1}^j)^{(\theta-1)}$ the Euler equation is priced in equilibrium by:

$$M_{t+1}^{j*} = \beta^\theta (C_{t+1}^{j*} / C_t^{j*})^{\left(-\frac{\theta}{\psi}\right)} (R_{C,t+1}^{j*})^{(\theta-1)} \tag{65}$$

Then adjusting for the difference in weights ω^i the consumption rule relationship can be rewritten for country j relative to arbitrarily chosen numeriare country 1:

$$C_t^{j*} = \frac{\omega^j C_t^{1*}}{\omega^1 S_t^{j*}} \tag{66}$$

thereby verifying the consumption rule equation (63).

While this rule establishes the equivalence of same price growth rates, the level of consumptions differ according to the initial wealth levels implied by ω^j. To determine these levels, note that wealth given the consumption rule can be rewritten:

$$E_0 \sum_{\tau=0}^{\infty} Q_\tau C_\tau^{j*} S_t^j = E_0 \sum_{\tau=0}^{\infty} Q_\tau \omega_t^j Y_\tau^W \equiv W_o^j, \forall j, \forall j$$

Then, given the consumption rule, the constraints in equations (61) and (62) can be combined as in:

$$E_0 \sum_{\tau=0}^{\infty} Q_\tau C_\tau^{j*} = \omega^j W_o^w = W_o^j, \forall j$$

where $\omega^j \equiv \left(\frac{W_o^j}{W_o^w}\right)$, thereby verifying equation (64). QED

B.2 Value Function and Consumption Asset Pricing:

To calculate the gains from risk sharing, we require solutions to the value function under the benchmark economy and the full risk-sharing economy as in:

$$V(C_t^j, W_t^j) = C_t^j \left(\frac{W_t^j}{C_t^j}\right)^{\frac{1}{1-(1/\psi)}} = C_t^j \left[\left(\frac{\Gamma_t^j}{C_t^j}\right) + 1\right]^{\frac{1}{1-(1/\psi)}} \tag{67}$$

for $W_t^j \equiv C_t^j + \Gamma_t^j$ where Γ_t^j is the time t expected value of lifetime consumption for investor j. Moreover, all returns and therefore asset prices are determined by the Epstein and Zin (1989) Euler equation:

$$E_t \left\{\beta^\theta (C_{t+1}^j / C_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{C,t+1}^j)^{(\theta-1)} R_{\ell,t+1}\right\} = 1 \tag{68}$$

where $R_{C,t+1}^j = (C_{t+1}^j + \Gamma_{t+1}^j)/\Gamma_t^j$ is the return on the asset that pays out the consumption process and $R_{\ell,t+1}$ is the return on any asset. Then clearly the Euler equation for the "consumption asset," that is, the asset the pays out the consumption process, can written by substituting $R_{C,t+1}^j = R_{\ell,t+1}$ to obtain:

$$E_t \left\{\beta^\theta (C_{t+1}^j / C_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{C,t+1}^j)^{\theta}\right\} = 1 \tag{69}$$

Using this general framework, we determine the returns under the benchmark economy by substituting $C_t^{j,D} = C_t^j$ and under the risk sharing economy by substituting $C_t^{j*} = C_t^j$ as detailed below.

B.3 Benchmark Economy Value Function

This subsection describes the solution of the value function in the benchmark case.

B.3.1 Consumption Asset Price in Local Prices

In the economy with incomplete markets, households in each country hold a portfolio of assets that generates their consumption. Following the framework above, the problem solved by individuals as observed in the data may be written as:

$$\underset{\{C_t^j\}_{\forall t}}{Max}\ U(C_\tau^j, U_{\tau+1}^j) = \left\{ C_t^{j\,\frac{1-\gamma}{\theta}} + \beta E_t \left[\left(U_{t+1}^j \right)^{1-\gamma} \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{70}$$

for

s.t.

$$E_t \sum_{\tau=0}^{\infty} Q_{t+\tau}^j P_{t+\tau}^j C_{t+\tau}^j = E_0 \sum_{\tau=0}^{\infty} Q_{t+\tau}^j P_{t+\tau}^1 C_{t+\tau}^{j,D} \equiv W_t^j, \forall j \tag{71}$$

$$\sum_{j=1}^{J} P_t^j C_t^j = \sum_{j=1}^{J} P_t^1 C_t^{j,D} \equiv P_t^1 \widetilde{Y}_t^W, \forall t \tag{72}$$

Epstein and Zin (1991) show that the solution to this problem given consumption C_t^j is:

$$V(C_t^j, W_t^j) = C_t^j \left(\frac{W_t^j}{C_t^j} \right)^{\frac{1}{1-(1/\psi)}}$$

where W_t^j is defined by equation (71). Moreover, by the goods market constraint in equation (72), consumption in each period is:

$$C_t^j = \frac{P_0^1 C_t^{j,D}}{P_t^j} = \frac{C_t^{j,D}}{S_t^j} \tag{73}$$

for

$$S_t^j = \frac{\left(P_t^j\right)}{\left(P_0^1\right)}, \forall j \tag{74}$$

Thus, the wealth for the benchmark economy can be determined by solving for the Euler equation (69) using the benchmark consumption in equation (73) as $C_t^{j,D} = S_t^j\ C_t^j$. For this purpose, we follow the literature in defining consumption growth as $g_{c,t+1}^j \equiv \lg(C_{t+1}^{j,D}/C_t^{j,D})$ where:

$$\begin{aligned} g_{c,t+1}^j &= \mu^j + x_t^j + \sigma^j \eta_{t+1}^j \\ x_{t+1}^j &= \rho^j x_t^j + \varphi_e^j \sigma^j e_{t+1}^j \end{aligned} \tag{75}$$

and $\eta_{t+1}^j, e_{t+1}^j \backsim N(0,1)$ with η_{t+1}^j, e_{t+1}^j are mutually independent. For expositional simplicity, we adopt the more parsimonious notation that $\sigma_e^j \equiv \varphi_e^j \sigma^j$ where possible. Furthermore, we assume that the relative price of consumption baskets over time is i.i.d. Thus, defining the relative price growth as $g^j_{s,t+1} \equiv \lg(S^j_{t+1}/S^j_t)$, then:

$$g^j_{s,t+1} \equiv \ln(S^j_{t+1}/S^j_t) = \sigma^{sj}\eta^{sj}_{t+1} \tag{76}$$

To value the asset return series, we use equation (**??**) and substitute the process for:

$$\ln(C^j_{t+1}/C^j_t) = \ln(\frac{C^{j,D}_{t+1}}{S^j_{t+1}} \div \frac{C^{j,D}_t}{S^j_t}) = g^j_{c,t+1} - g^j_{s,t+1}$$

into the Euler equation (69) and then solve for the implicit returns that generate consumption as in: $R^j_{C,t+1} = (C^j_{t+1} + \Gamma^j_{t+1})/\Gamma^j_t$, described earlier.

We follow Bansal and Yaron (2004) in assuming these returns can be approximated using the Campbell and Shiller (1988) approximation. This approximation then implies that equity returns are a function of the log of the benchmark consumption asset price-to-consumption ratio, $z^j_t \equiv \ln(\Gamma^j_t/C^j_t)$, and of consumption growth $g^j_{c,t+1}$ and relative price growth, $g^j_{s,t+1}$ specified by:

$$R^j_{C,t+1} = k^j_0 + k^j_1 z^j_{t+1} - z^j_t + g^j_{c,t+1} - g^j_{s,t+1} \tag{77}$$

where k^j_0 and k^j_1 represent approximating constants. Note that:

$$\exp(z^j_t) = (\Gamma^j_t/Y^j_t) \tag{78}$$

providing a measure of the consumption price-consumption ratio required to measure the value function in equation (67). We use this relationship to solve for the value function below.

B.3.2 Benchmark Economy Welfare

Since the value function and the return process depends upon the price-to-payout ratio, it is necessary to solve for this ratio given in equation (78). Following Bansal and Yaron (2004), we conjecture that the log price-to-consumption ratio is linear in the persistent risk, x_t. Thus,

$$z^j_t = A^j_0 + A^j_1 x^j_t. \tag{79}$$

Then, we can rewrite the Euler equation (69) by using the fact that the variables are joint log normally distributed, the Euler equation can be rewritten:

$$\exp\left[E_t\left\{\theta\ln(\beta)-\frac{\theta}{\psi}g^{jB}_{c,t+1}+\theta r^j_{C,t+1}\right\}+\frac{1}{2}Var_t\left\{\theta\ln(\beta)-\frac{\theta}{\psi}g^{jB}_{c,t+1}+\theta r^j_{C,t+1}\right\}\right]=1$$

or

$$E_t\left\{\theta\ln(\beta)-\frac{\theta}{\psi}g^{jB}_{c,t+1}+\theta r^j_{C,t+1}\right\}+\frac{1}{2}Var_t\left\{\theta\ln(\beta)-\frac{\theta}{\psi}g^{jB}_{c,t+1}+\theta r^j_{C,t+1}\right\}=0 \tag{80}$$

Then substituting the log price-to consumption ratio (**??**), the consumption growth (75), and the relative price growth (38) into the return process in equation (77) and the result into the Euler equation (80), we can solve for A^j_0 and A^j_1. Doing so implies:

$$A^j_1=\frac{1-\frac{1}{\psi}}{1-k^j_1\rho} \tag{81}$$

$$A^j_0=\frac{\ln\beta+k^j_0+(1-\frac{1}{\psi})\mu^j+\frac{1}{2}\theta\left\{(1-\frac{1}{\psi})^2[(\sigma^j)^2+(\sigma^{sj})^2-2\sigma^j_{\eta,s}]+(k^j_1A^j_1\sigma^j\varphi^j_e)^2\right\}}{1-k^j_1} \tag{82}$$

where $\sigma^j_{\eta,s}\equiv Cov(\sigma^{sj}\eta^{sj}_{t+1},\sigma^j\eta^j_{t+1})$, $k^j_0=log(1+exp(\bar{z}^j))-k^j_1\bar{z}_j$ and $k^j_1=exp(\bar{z}^j)/\left(1+exp(\bar{z}^j)\right)$. Note that the approximating constants k^j_0, k^j_1 depend upon the solution to the long run value of z^j_t so our solution solves for the fixed point between the z^j_t equation (79) and the constant A^j_0 in equation (82).

Defining $Z^j_t\equiv\exp(z^j_t)$, the value function can be found by substituting the solution for the price-to-consumption ratio into the wealth equation giving:

$$V(C^{jB}_t,W^{jB}_t)=Y^j_t\left(1+\frac{\Gamma^j_t}{Y^j_t}\right)^{\frac{1}{1-(1/\psi)}}=C^{jB}_t\left(1+Z^{jB}_t\right)^{\frac{1}{1-(1/\psi)}}$$

B.4 Risk Sharing Economy Value Function with Exogenous Exchange Rates

In this subsection, we describe the solution for the value function in the risk-sharing economy when exchange rates are exogenous.

B.4.1 Consumption in Numeraire Country Prices

When markets are complete, countries are allocated shares of aggregate world consumption in a numeraire index. In growth rates, this process is given by: $\ln(S^j_{t+1}Y^j_{t+1}/S^j_tY^j_t) = \ln(S^j_{t+1}/S^j_t) + \ln(Y^j_{t+1}/Y^j_t)$. To characterize this process, we use the process from the benchmark economy that Y^j_t follows a long run risk process as in equation (75). Therefore, we specify these series as following a long-run risk process as $g^j_{c,t+1} \equiv \ln(Y^j_{t+1}/Y^j_t)$ where:

$$\begin{aligned} g^j_{c,t+1} &= \mu^j + x^j_t + \sigma^j\eta^j_{t+1} \\ x^j_{t+1} &= \rho^j x^j_t + \varphi^j_e\sigma^j e^j_{t+1} \end{aligned} \tag{83}$$

and $\eta^j_{t+1}, e^j_{t+1} \backsim N(0,1)$ and are mutually independent, as before.

Furthermore, as in much of the empirical literature, exchange rate growth follows an i.i.d. random walk process.As solved earlier, in the full risk sharing economy, the consumption for country j is a share of world consumption converted back into the consumption price index of country j:

$$C^{j*}_t = \varpi^j\widetilde{Y}^w_t/S^j_t \tag{84}$$

where $\varpi^j = \left(\widetilde{Y}^j_0 + \Gamma^{j*}_0\right) / \left(\widetilde{Y}^w_0 + \Gamma^{w*}_0\right)$ and where $\widetilde{Y}^w_t \equiv \Sigma^J_{j=1}\widetilde{Y}^j_t$.

B.4.2 Risk Sharing Economy Welfare

To determine the consumption-to-price ratio under the risk-sharing equilibrium, we require the price to consumption ratio in equation (84). Note that this requires a process for:

$$\ln(C^{j*}_{t+1}/C^{j*}_t) = \ln(\varpi^j Y^w_{t+1}/S^j_{t+1}) - \ln(\varpi^j Y^w_t/S^j_t) = \ln(Y^w_{t+1}/Y^w_t) - \ln(S^j_{t+1}/S^j_t) \tag{85}$$

Assuming the approximation that:

$$\ln(Y^w_{t+1}/Y^w_t) = \ln(\Sigma^J_{j=1}\widetilde{Y}^j_{t+1}/\Sigma^J_{j=1}\widetilde{Y}^j_t) \approx \Sigma^J_{j=1}\widetilde{g}^j_{c,t+1}, \tag{86}$$

we can then write the consumption process for each country as:

$$\ln(C^{j*}_{t+1}/C^{j*}_t) = \Sigma^J_{j=1}\widetilde{g}^j_{c,t+1} - \ln(S^j_{t+1}/S^j_t) = g^*_{c,t+1} - g^j_{S,t+1} \tag{87}$$

We can then price the consumption asset for each country using the Euler equation and the

Campbell-Shiller approximation. For example, the common growth rate across countries is the weighted sum of the country growth rates:

$$
\begin{aligned}
g^*_{c,t+1} &= \mu^* + x^*_t + \sigma^* \eta^*_{t+1} \\
x^*_{t+1} &= \rho^* x^*_t + \varphi^*_e \sigma^* e^*_{t+1}
\end{aligned}
\tag{88}
$$

where $\mu^* \equiv \frac{1}{J}\Sigma^J_{j=1}\ \mu^j$, $x^*_t \equiv \frac{1}{J}\Sigma^J_{j=1}\ x^j_t$, $\eta^*_t \equiv \frac{1}{J}\Sigma^J_{j=1}\ \eta^j_t$ and $e^*_t \equiv \frac{1}{J}\Sigma^J_{j=1} e^j_t$ so that $\sigma^{*2} = \left(\frac{1}{J}\right)^2 \iota'\ \Sigma\ \iota$ and $\varphi^{*2}_e = \left(\frac{1}{J}\right)^2 \iota'\ \Sigma_e\ \iota$ for Σ and Σ_e, the variance-covariance matrix of transitory and persistent shocks, respectively, and ι, a J-dimensional unit vector. Below, we use the notation $\sigma^*_e \equiv \varphi^*_e \sigma^*$. Similarly, the exchange rate process is given as in equation (38).

Then we require the price-to-consumption raio in the risk-sharing economy for each country. Defining this ratio as:

$$
\exp(z^{jw}_t) = (\Gamma^{jw}_t / C^{j*}_t) \tag{89}
$$

where $C^{j*}_t$is given by equation (84) for all t, j and where

$$
\Gamma^{jw}_t = \varpi^j E_t \sum_{\tau=1}^{\infty} Q_{t+\tau} \left(\widetilde{Y}^w_{t+\tau} / S^j_{t+\tau} \right) \tag{90}
$$

Note: Since ϖ^j appears in both the denominator and numerator of $\exp(z^{jw}_t)$, we omit it in the solution of the price-to-consumption ratio below.

To solve for the logarithm of this ratio, conjecture that it has the form:

$$
z^{iw}_t = A^{iw}_0 + \sum_{j=1}^{J} A^{iw}_j x^j_t - A^{iS} x^S_t \tag{91}
$$

For expositional clarity but with some abuse of notation, rewrite this expression as:

$$
z^{iw}_t = A^{iw}_0 + \underline{A}^{iw}_1 \overline{x}_t - A^{iS} x^S_t \tag{92}
$$

so that $\underline{A}^{iw}_1 \overline{x}_t \equiv \sum_{j=1}^J A^{iw}_j x^j_t$. Since this variable is a weighted sum of world long run risk components exactly as in LL1, we use the notation as given in equation (88).

To solve for this ratio, use the Campbell-Shiller decomposition as in equation (**??**) but now using the risk-sharing consumption solution:

$$
R^{iw}_{C,t+1} = k^{iw}_0 + k^{iw}_1 z^{iw}_{t+1} - z^{iw}_t + g^{iw}_{c,t+1} \tag{93}
$$

where

$$g^{iw}_{c,t+1} = g^{*}_{c,t+1} - g^{j}_{S,t+1} \tag{94}$$
$$k^{iw}_0 = \ln(1+\exp(\overline{z}^{iw})) - k^{iw}_1 \overline{z}^{iw} \tag{95}$$
$$k^{iw}_1 = \exp(\overline{z}^{iw})/(1+\exp(\overline{z}^{iw})) \tag{96}$$

To solve for the coefficients in equation (92), use the fact that with joint lognormality, the Euler equation (69) can be rewritten:

$$E_t(\theta \ln\beta - \frac{\theta}{\psi} g^{iw}_{c,t+1} + \theta r^{iw}_{c,t+1}) + \frac{1}{2} Var_t(\theta\ln\beta - \frac{\theta}{\psi} g^{iw}_{c,t+1} + \theta r^{iw}_{c,t+1}) = 0 \tag{97}$$

Then substitute into the return process in equation(93), the price-to-consumption ratio in equation (92), the processes for the world process $g^{*}_{c,t+1}$ in equation (88), and the change in the exchange rate $g^{j}_{S,t+1}$ in equation (**??**). Substituting the expectations and variances of these returns and the growth rates of consumptions into the Euler equation in equation (97) implies:

$$A^{iw}_1 = A^{is}_1 = \frac{1-\frac{1}{\psi}}{1-k^{iw}_1\rho} \tag{98}$$

$$A^{iw}_0 = \left\{ \begin{array}{l} \ln\beta + k^{iw}_0 + (1-\frac{1}{\psi})\mu * + \frac{1}{2}\theta[(1-\frac{1}{\psi})^2((\sigma^*)^2 + (\sigma^{is})^2 - 2\sigma_{*,is}) \\ +(k^{iw}_1 A^{iw}_1 \sigma^* \varphi^*_e)^2 + (k^{is}_1 A^{is}_1 \sigma^{is}\varphi^{is}_e)^2 - 2\sigma_{*,is}\sigma^*\varphi^*_e k^{iw}_1 A^{iw}_1 k^{is}_1 A^{is}_1] \end{array} \right\} /1 - k^{w}_1 \tag{99}$$

where $\sigma_{*,is}$ is the covariance between the aggregate world consumption growth innovation, η^*_{t+1}, and the exchange rates for country i innovation, η^{sj}_{t+1}.The solution to this fixed point problem determines $Z^{iw}_t \equiv \exp(z^{iw}_t)$.

B.4.3 Country Weights

The country weights as given in equation (**??**) are given by:

$$\varpi^j = \frac{W^j_o}{\sum\limits_{i=1}^{J} W^i_o} \equiv \frac{S^j_0 Y^j_0 + \Gamma^{j*}_0}{\sum\limits_{i=1}^{J} S^i_0 Y^i_0 + \Gamma^{w*}_0} = \frac{\widetilde{Y}^j_0 + \Gamma^{j*}_0}{\widetilde{Y}^w_0 + \Gamma^{w*}_0}. \tag{100}$$

where

$$\Gamma_0^{j*} = E_0 \sum_{\tau=1}^{\infty} Q_\tau S_\tau^j Y_\tau^j \tag{101}$$

$$\Gamma_0^{w*} = E_0 \sum_{\tau=0}^{\infty} Q_\tau \sum_{i=1}^{J} S_t^i Y_t^i \tag{102}$$

where we use the prior definition that $\widetilde{Y}_\tau^j \equiv S_\tau^j Y_\tau^j$ and $\widetilde{Y}_\tau^w \equiv \Sigma_{j=1}^J S_\tau^j Y_\tau^j$. Note that equation (101) gives the prices of a claim on country j endowment consumption measured in the numeraire country prices evaluated at the price density in numeraire country price units, Q_τ. Therefore, it can be calculated using the Euler equation in equation (68) using the numeraire country pricing kernel:

$$E_t \left\{ \beta^\theta (C_{t+1}^{1*}/C_t^{1*})^{\left(-\frac{\theta}{\psi}\right)} (R_{C,t+1}^{1*})^{(\theta-1)} R_{Y,t+1}^j \right\} = E_t \left\{ \beta^\theta (\widetilde{Y}_{t+1}^w/\widetilde{Y}_t^w)^{\left(-\frac{\theta}{\psi}\right)} (R_{C,t+1}^{1*})^{(\theta-1)} R_{Y,t+1}^j \right\} = 1 \tag{103}$$

where $R_{C,t+1}^{1*} = (C_{t+1}^{1*} + \Gamma_{t+1}^{1*})/\Gamma_t^{1*}$ is the return on the consumption asset for country 1 previously solved using the Euler equation (69) and $R_{Y,t+1}^j = (Y_{t+1}^j + \Gamma_{t+1}^j)/\Gamma_t^j$ is the return on a claim to the endowment process from country j. The price-to-dividend ratio can be solved using the same steps as above with the Campbell Shiller return approximation in equation (93) and the Euler equation for the return on the claim to endowment j in equation (103). Similarly, equation (102) is the price of the aggregate world consumption evaluated at the same price density. Thus, the aggregate world wealth, $\sum_{i=1}^{J} W_o^i$ can be solved by summing over the countries. The ratio of these two wealth levels provide the country weights, ϖ^j.

C General Framework with Non-Tradeables

In this section, we describe the solution for the value function in the risk-sharing economy under non-tradeables.

C.1 Planner Problem

Proposition 2: *Let a^j be the planner weights on the utility of the representative agent of country j, Q_τ^j be the state-price for country j at time τ, and $P_{T,\tau}^j$ and $P_{N,\tau}^j$ be the prices of tradeables and non-tradeables, respectively, for country j at time t in units of the numeraire country 1 consumption* *aggregate. Then the solution to the planner's problem:*

$$\underset{\substack{\{C^j_{T,t}\}\\ \forall t}}{Max}\sum_{j=1}^{J}a^j\sum_{\tau=0}^{\infty}U(C^j_\tau,U^j_{\tau+1}) \tag{104}$$

for

$$U(C^j_t,U^j_{t+1})=\left\{C^{j\,\frac{1-\gamma}{\theta}}_t+\beta E_t\left[\left(U^j_{t+1}\right)^{1-\gamma}\right]^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}},\ j=1,...,J. \tag{105}$$

$$C^j_\tau\equiv C(C^j_{T,\tau},C^j_{N,\tau})=\left(C^j_{T,\tau}\right)^{\alpha}\left(C^j_{N,\tau}\right)^{1-\alpha} \tag{106}$$

s.t.

$$\begin{aligned}
E_0\sum_{\tau=0}^{\infty}Q_\tau(P_{T,\tau}C^j_{T,\tau}+P^j_{N,\tau}C^j_{N,\tau}) &= E_0\sum_{\tau=0}^{\infty}Q_\tau(P_{T,\tau}C^{j,D}_{T,\tau}+P^j_{N,\tau}C^{j,D}_{N,\tau})\equiv W^j_o,\forall j && (107)\\
\sum_{j=1}^{J}P_{T,\tau}C^j_{T,\tau} &= \sum_{j=1}^{J}P_{T,\tau}C^{j,D}_{T,t}\equiv P_{T,\tau}Y^W_{T,t},\forall t && (108)\\
C^j_{N,t} &= Y^j_{N,t},\forall j,t && (109)
\end{aligned}$$

is given by:

$$C^{j*}_t=\left(\omega^jY^w_{T,t}\right)^{\alpha}\left(Y^j_{N,\tau}\right)^{1-\alpha}=\frac{\omega^j}{\omega^1}\frac{\left(\omega^1Y^w_{T,t}\right)^{\alpha}\left(Y^1_{N,\tau}\right)^{1-\alpha}}{S^{j,*}_t},\forall t, \tag{110}$$

where

$$S^{j,*}_t=\frac{\left(P^{j,*}_{T,\tau}\right)^{\alpha}\left(P^{j,*}_{N,\tau}\right)^{1-\alpha}}{\left(P^{1,*}_{T,\tau}\right)^{\alpha}\left(P^{1,*}_{N,\tau}\right)^{1-\alpha}}=\frac{\left(\omega^j/Y^j_{N,\tau}\right)^{1-\alpha}}{\left(\omega^1/Y^1_{N,\tau}\right)^{1-\alpha}} \tag{111}$$

$$\varpi^j=\frac{W^j_o}{W^w_o}\equiv\frac{E_0\sum\limits_{\tau=0}^{\infty}Q_\tau C^{j*}_\tau}{E_0\sum\limits_{\tau=0}^{\infty}Q_\tau C^{W*}_t} \tag{112}$$

where $W^w_{T,o}\equiv\sum\limits_{j=1}^{J}W^j_o$ *and* $C^{W*}_t\equiv\sum\limits_{j=1}^{J}C^{j*}_t$ *the present value of world wealth and aggregate consumption in units of the numeraire country, all evaluated at the complete markets stochastic discount factor,* Q_τ*.*

Proof: To be a solution, the consumption rules defined in equations (110), (111), and (112) must maximize the utility function in equations (105) and (106) given the wealth constraints for each country in equation (107), the resource constraints for tradeables for each period in equation

(108) and given the allocation of non-tradeables in equation (109).

We begin by demonstrating that the exchange rate solution is given by equation (111). Since non-tradeables are given as an endowment, and since tradeables consumption is given by the data, for notational convenience, we set $C_{N,t}^{j,D} = Y_{N,t}^{j}$ and define $Y_{T,t}^{j} = C_{T,t}^{j,D}$ below. Note that in general the price index of the consumption aggregator in equation (106) is given by: $P_t^j = \left(P_{T,t}^j\right)^{\alpha}\left(P_{N,t}^j\right)^{1-\alpha}$ and therefore relative to the numeraire country

$$S_t^j \equiv \frac{\left(P_{T,\tau}^j\right)^{\alpha}\left(P_{N,\tau}^j\right)^{1-\alpha}}{\left(P_{T,\tau}^1\right)^{\alpha}\left(P_{N,\tau}^1\right)^{1-\alpha}} = \frac{\left(P_{T,\tau}^j\right)\left(\rho_{N,\tau}^j\right)^{1-\alpha}}{\left(P_{T,\tau}^1\right)\left(\rho_{N,\tau}^1\right)^{1-\alpha}} \tag{113}$$

where $\rho_{N,\tau}^j \equiv \left(P_{N,\tau}^j/P_{T,\tau}^j\right)$, the relative price of non-tradeables to tradeables. Each country maximizes utility and lifetime constraint in equation (107) and the sequence of non-tradeables outputs (109). They also face the budget constraint:

$$E_0\sum_{\tau=0}^{\infty}Q_\tau^j(C_{T,\tau}^j + \rho_{N,\tau}^j C_{N,\tau}^j) = E_0\sum_{\tau=0}^{\infty}Q_\tau^j(Y_{T,\tau}^j + \rho_{N,\tau}^j Y_{N,\tau}^j) \equiv W_o^j, \forall j$$

In this case, the first-order conditions of tradeables and non-tradeables imply:

$$\begin{aligned}\frac{\partial U_t^j}{\partial C_t^j}\frac{\partial C_t^j}{\partial C_{T,t}^j} &= \lambda_t^j P_{T,t}^j \\ \frac{\partial U_t^j}{\partial C_t^j}\frac{\partial C_t^j}{\partial C_{N,t}^j} &= \lambda_t^j P_{N,t}^j\end{aligned}$$

where λ_t^j is the Lagrangian on the budget contraint at time t for country j, or alternatively:

$$\frac{\left(\frac{\partial C_t^j}{\partial C_{N,t}^j}\right)}{\left(\frac{\partial C_t^j}{\partial C_{T,t}^j}\right)} = \frac{P_{N,t}^j}{P_{T,t}^j} \equiv \rho_{N,t}^j$$

In the case of Cobb-Douglas, this relationship reduces to:

$$\frac{(1-\alpha)}{\alpha}\frac{C_{T,t}^j}{C_{N,t}^j} = \rho_{N,t}^j \tag{114}$$

Now using the consumption rule in equation (110) and solving for the intratemporal marginal rate of substitution between non-tradeables and tradeables implies that in equilibrium the relative price is:

$$\rho^{j,*}_{N,\tau} = \frac{1-\alpha}{\alpha}\left(\omega^j Y^W_{T,\tau}/Y^j_{N,\tau}\right) \tag{115}$$

Substituting this solution into the exchange rate equation (113) and using the fact that the tradeables goods market clears so that $P^j_{T,\tau} = P^1_{T,\tau}$ implies:

$$S^{j*}_t = \frac{\left(P^{j*}_{T,\tau}\right)\left(\rho^{j*}_{N,\tau}\right)^{1-\alpha}}{\left(P^{1*}_{T,\tau}\right)\left(\rho^{1*}_{N,\tau}\right)^{1-\alpha}} = \frac{\left(\rho^{j}_{N,\tau}\right)^{1-\alpha}}{\left(\rho^{1}_{N,\tau}\right)^{1-\alpha}} = \frac{\left(\omega^j/Y^j_{N,\tau}\right)^{1-\alpha}}{\left(\omega^1/Y^1_{N,\tau}\right)^{1-\alpha}},$$

thereby verifying the exchange rate solution in equation (111).

Next, we demonstrate that the rule maximizes intertemporal utility across countries given the common state price density Q_t measured in units of the numeraire consumption aggregate. Note that the maximum consumption for each country j given its respective wealth constraint in equation (107) is given by the value function in equation (67) repeated here for convenience:

$$V(C^j_t, W^j_t) = C^j_t\left(\frac{W^j_t}{C^j_t}\right)^{\frac{1}{1-(1/\psi)}} = C^j_t\left[\left(\frac{\Gamma^j_t}{C^j_t}\right)+1\right]^{\frac{1}{1-(1/\psi)}}$$

Therefore, a common complete markets state price density will determine wealth in all countries if it can be shown to price the return on a claim to consumption in each country. Using the definition of the return on that claim in equilbrium as: $R^{j*}_{C,t+1} = (C^{j*}_{t+1} + \Gamma^{j*}_{t+1})/\Gamma^{j*}_t$, then defining the stochastic discount factor $M_{t+1} \equiv (Q_{t+1}/Q_t) = \beta^\theta (C^j_{t+1}/C^j_t)^{\left(-\frac{\theta}{\psi}\right)}(R^j_{C,t+1})^{(\theta-1)}$ the Euler equation is priced in equilibrium by:

$$M^{j*}_{t+1} = \beta^\theta (C^{j*}_{t+1}/C^{j*}_t)^{\left(-\frac{\theta}{\psi}\right)}(R^{j*}_{C,t+1})^{(\theta-1)} \tag{116}$$

The optimal consumption rule for each country in units of the numeraire converted into country j prices is:

$$C^{j*}_t = \left(\omega^j Y^w_{T,t}\right)^\alpha \left(Y^j_{N,\tau}\right)^{1-\alpha} \tag{117}$$

Similarly, consumption in the numeraire country converted into country j prices is:

$$\begin{aligned} C^{1*}_t/S^{j*}_t &= \left(\omega^1 Y^w_{T,t} P^1_{T,\tau}/P^j_{T,\tau}\right)^\alpha \left(Y^1_{N,\tau} P^1_{N,\tau}/P^j_{N,\tau}\right)^{1-\alpha} \\ &= \left(\omega^1 Y^w_{T,t}\right)^\alpha \left(Y^1_{N,\tau}\rho^{1,*}_{N,\tau}/\rho^{j,*}_{N,\tau}\right)^{1-\alpha} \\ &= \left(\omega^1 Y^w_{T,t}\right)^\alpha \left(Y^j_{N,\tau}\right)^{1-\alpha} \end{aligned} \tag{118}$$

Then adjusting for the difference in weights ω^i the relationship can be rewritten:

$$C_t^{j*} = \frac{\omega^j C_t^{1*}}{\omega^1 S_t^{j*}} \tag{119}$$

thereby verifying the consumption rule equation (111).

Given the equivalence between equations (118) and (117) once converted into same prices and adjusted by weights implies that a single pricing kernel jointly prices all country wealths. In particular, considering the two components of the stochastic discount factor in equation (116), they are equalized after adjusting for prices. In particular, the component that depends upon consumption growth is:

$$\beta^\theta (C_{t+1}^{j*}/C_t^{j*})^{\left(-\frac{\theta}{\psi}\right)} = \beta^\theta \frac{\left(Y_{T,t+1}^w\right)^\alpha \left(Y_{N,t+1}^j\right)^{1-\alpha}}{\left(Y_{T,t}^w\right)^\alpha \left(Y_{N,t}^j\right)^{1-\alpha}} = \beta^\theta \frac{\left(Y_{T,t+1}^w\right)^\alpha \left(Y_{N,t+1}^1\right)^{1-\alpha}}{\left(Y_{T,t}^w\right)^\alpha \left(Y_{N,t}^1\right)^{1-\alpha}} / \left(S_{t+1}^{j*}/S_t^{j*}\right), \forall j \tag{120}$$

Moreover, since the price of the consumption asset is the same for all countries in units of the numeraire, $R_{C,t+1}^{j*} = R_{C,t+1}^{i*}$, $\forall j$, i.

While equation (120) establishes the equivalence of same price growth rates, the level of consumptions differ according to the initial wealth levels implied by ω^j. To determine these levels, note that wealth given the consumption rule can be rewritten:

$$E_0 \sum_{\tau=0}^{\infty} Q_\tau C_\tau^{j*} = \sum_{\tau=0}^{\infty} Q_\tau \left(Y_{T,t+1}^w\right)^\alpha \left(Y_{N,t+1}^j\right)^{1-\alpha} \equiv W_o^j, \forall j$$

Then, given the consumption rule, the constraints in equations (107), (108) and (109) can be combined as in:

$$E_0 \sum_{\tau=0}^{\infty} Q_\tau C_\tau^{j*} = \omega^j W_o^w = W_o^j, \forall j$$

where $\omega^j \equiv \left(\frac{W_o^j}{W_o^w}\right)$, thereby verifying equation (112). QED

C.1.1 Identifying Risk-Sharing Gains

As we showed for the exogenous exchange rate case, the gains can be calculated as Δ^i, in the ratio of value functions:

$$1 + \Delta^i = \left(\frac{C_0^{i*}}{C_0^i}\right) \left\{\frac{W_0^{i*}/C_0^{i*}}{W_0^{iB}/C_0^{iB}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} = \left(\frac{C_0^{i*}}{C_0^i}\right) \left\{\frac{1 + Z_t^{j*}}{1 + Z_t^{jB}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}$$

where $W_t^j \equiv C_t^j + \Gamma_t^j$ for Γ_t^j defined as the time t expected value of lifetime consumption for investor j and superscript B refers to the benchmark incomplete markets economy while $*$ refers to the equilibrium risk-sharing economy. Then, we need to identify $Z_t^j \equiv \Gamma_t^j / C_t^j$ as before as well as the risk-sharing weight, ω^j. For this purpose, consider the exchange rates and consumption in the data using the Cobb Douglas assumption and denote them with superscript B. In the data, consumption using the notation above is identified as:

$$C_t^{j,D} \equiv \left(Y_{T,t}^j\right)^{\alpha} \left(Y_{N,t}^j\right)^{1-\alpha}. \tag{121}$$

However, the risk-sharing equilibrium requires measures to calculate the aggregate world tradeable output as given in equation (108):

$$\sum_{j=1}^{J} Y_{T,t}^{j,D} \equiv Y_{T,t}^{W}.$$

Therefore, we work backwards from the implied relationships in the data to infer the consumption of tradeables separate from non-tradeables given the aggregator in equation (121). For this purpose, solve equation (121) for $Y_{T,\tau}^j$ using the relationship between relative prices and consumption in equation (114) as in:

$$Y_{T,t}^j = C_t^{jD} \left(\frac{Y_{N,t}^j}{Y_{T,t}^j}\right)^{(1-\alpha)} = C_t^{jD} \left(\rho_{N,t}^{jD} \frac{\alpha}{(1-\alpha)}\right)^{(1-\alpha)} \tag{122}$$

Then, substituting the fact that the exchange rate in the data is

$$S_t^{jB} = \frac{\left(P_{T,t}^{jB}\right)\left(\rho_{N,t}^{jB}\right)^{1-\alpha}}{\left(P_{T,t}^{1B}\right)\left(\rho_{N,t}^{1*}\right)^{1-\alpha}} = \frac{\left(\rho_{N,t}^{jB}\right)^{1-\alpha}}{\left(\rho_{N,t}^{1B}\right)^{1-\alpha}} \tag{123}$$

into the solution for the tradeables good in equation (122), implies that the component of consumption due to tradeables is:

$$Y_{T,t}^j = \left(\frac{\alpha}{(1-\alpha)}\right)^{1-\alpha} \left(\rho_{N,t}^{1B}\right)^{1-\alpha} C_t^{jB} S_t^{jB}. \tag{124}$$

Thus, aggregate world tradeables can be inferred from the data as:

$$\widetilde{Y}_{T,t}^{W} = \left(\frac{\alpha}{(1-\alpha)}\right)^{1-\alpha} \left(\rho_{N,t}^{1B}\right)^{1-\alpha} \sum_{i=1}^{J} C_t^{iB} S_t^{iB} \tag{125}$$

To calculate the level of non-tradeables in the data substitute the solution for tradeables in equation (124) into the consumption aggregator in equation (121) and solve for $Y^j_{N,\tau}$ yielding:

$$Y^j_{N,t} = \left(\frac{\alpha}{(1-\alpha)}\right)^{-\alpha} \left(\rho^{1B}_{N,t}\right)^{-\alpha} C^{jB}_t \left(S^{jB}_t\right)^{-\frac{\alpha}{1-\alpha}} \tag{126}$$

This equation then identifies the consumption of non-tradeables in the consumption data using only observed exchange rates and aggregate consumption.

C.1.2 Benchmark Consumption Growth with Nontradeables

To relate benchmark consumption growth to the data, we take the growth rate of the consumption aggregator in the data given by equation (121) allowing for consumption of tradeables versus nontradeables. In this case, the observed consumption growth in local country j prices is:

$$\ln(C^{jB}_{t+1}/C^{jB}_t) = g^j_{c,t+1} = \alpha \ln(Y^j_{T,t+1}/Y^j_{T,t}) + (1-\alpha)\ln(Y^j_{N,t+1}/Y^j_{N,t}) \tag{127}$$

Note that to be consistent with our exogenous exchange rate analysis, we require under long run risk the same process in numeraire prices as before as in equation (75), repeated here:

$$\begin{aligned} g^{jB}_{c,t+1} &= \mu^j + x^j_t + \sigma^j \eta^j_{t+1} \\ x^{jB}_{t+1} &= \rho^j x^j_t + \varphi^j_e \sigma^j e^j_{t+1} \end{aligned}$$

and $\eta^j_{t+1}, e^j_{t+1} \backsim N(0,1)$ and are mutually independent, as before.

C.2 Consumption in Numeraire Country Prices

In the complete risk-sharing equilibrium, countries are allocated shares of aggregate tradeables good in the numeraire country goods index. Thus, taking the growth rate of the consumption rule in equation (84) implies:

$$\ln(C^{j*}_{t+1}/C^{j*}_t) = \alpha \ln\left(\frac{\widetilde{Y}^w_{T,t+1}}{\widetilde{Y}^w_{T,t}}\right) + (1-\alpha)\ln\left(\frac{Y^j_{N,t+1}}{Y^j_{N,t}}\right), \forall j \tag{128}$$

Substituting the solution for tradeables in equations (125) for $\ln\left(Y^w_{T,t+1}/Y^w_{T,t}\right)$ implies:

$$\begin{aligned}
\ln\left(\frac{\widetilde{Y}^w_{T,t+1}}{\widetilde{Y}^w_{T,t}}\right) &= \ln\left(\frac{\left(\rho^{1D}_{N,t+1}\right)^{1-\alpha}\sum\limits_{i=1}^{J} C^{iD}_{t+1}S^{iD}_{t+1}}{\left(\rho^{1D}_{N,t}\right)^{1-\alpha}\sum\limits_{i=1}^{J} C^{iD}_{t}S^{iD}_{t}}\right) \\
&\simeq (1-\alpha)\ln\left(\frac{\rho^{1D}_{N,t+1}}{\rho^{1D}_{N,t}}\right) + \sum_{i=1}^{J} g^{iD}_{c,t+1} + \sum_{i=1}^{J} g^{iD}_{s,t+1}
\end{aligned} \tag{129}$$

Similarly, substituting the solution for nontradeables in equations (126) into the consumption growth rate in equation (128) implies

$$\begin{aligned}
\ln\left(\frac{Y^j_{N,t+1}}{Y^j_{N,t}}\right) &= \ln\left(\frac{\left(\rho^{1D}_{N,t+1}\right)^{-\alpha} C^{jD}_{t+1}\left(S^{jD}_{t+1}\right)^{-\frac{\alpha}{1-\alpha}}}{\left(\rho^{1D}_{N,t}\right)^{-\alpha} C^{jD}_{t}\left(S^{jD}_{t}\right)^{-\frac{\alpha}{1-\alpha}}}\right) \\
&= -\alpha\ln\left(\frac{\rho^{1D}_{N,t+1}}{\rho^{1D}_{N,t}}\right) + g^{jD}_{c,t+1} - \frac{\alpha}{1-\alpha} g^{jD}_{s,t+1}, \forall j
\end{aligned} \tag{130}$$

Substituting equations (129) and (130) into the total consumption growth in equation (128) yields the solution:

$$\ln\left(\frac{C^{j*}_{t+1}}{C^{j*}_{t}}\right) = \alpha\sum_{i=1}^{J} g^{iD}_{c,t+1} + (1-\alpha)g^{jD}_{c,t+1} + \alpha\sum_{i=1}^{J} g^{iD}_{s,t+1} - \alpha g^{jD}_{s,t+1} \tag{131}$$

Risk Sharing Economy Welfare To determine the consumption-to-price ratio under the risk-sharing equilibrium, we can follow the same steps as in the exogenous exchange rate case. We substitute the consumption growth rates determined in the data above into the Campbell-Shiller return process. In particular, we solve for A^{iw}_0 and A^{iw}_j in the following formulation

$$z^{iw}_{t+1} = A^{iw}_0 + \sum_{\substack{j=1 \\ j\neq i}}^{J} A^{jw}_1 x^{jB}_{t+1}\alpha + A^{iw}_2 x^{iB}_{t+1} \tag{132}$$

Following this form and the expectations and variances of these returns and the growth rates of consumptions into the Euler equation in equation (97) implies:

$$A^{jw}_1 = A^{iw}_2 = \frac{1-\frac{1}{\psi}}{1-k^{iw}_1\rho}\forall j \tag{133}$$

$$A_0^{iw} = \left\{ \begin{array}{l} \ln\beta + k_0^{iw} + (1-\frac{1}{\psi})\mu * + \frac{1}{2}\theta[(1-\frac{1}{\psi})^2((\sigma^*)^2 + (\sigma^{is})^2 - 2\sigma_{*,is}) \\ +(k_1^{iw}A_1^{iw}\sigma^*\varphi_e^*)^2 + (k_1^{is}A_1^{is}\sigma^{is}\varphi_e^{is})^2 - 2\sigma_{*,is}\sigma^*\varphi_e^* k_1^{iw}A_1^{iw}k_1^{is}A_1^{is}] \end{array} \right\} /1 - k_1^w \tag{134}$$

where now $\mu^* \equiv \alpha\frac{1}{J}\Sigma_{j=1}^J\ \mu^j + (1-\alpha)\mu^i$, $x_t^* \equiv \alpha\frac{1}{J}\Sigma_{j=1}^J\ x_t^j + (1-\alpha)x_t^i$, $\eta_t^* \equiv \alpha\frac{1}{J}\Sigma_{j=1}^J\ \eta_t^j + (1-\alpha)\eta_t^i$ and $e_t^* \equiv \alpha\frac{1}{J}\Sigma_{j=1}^J e_t^j + (1-\alpha)e_t^i$. This solution is thus the same as in the exogenous consumption case except that the growth rate of consumption for each country is given by equation (131). As before, the solution to this fixed point problem determines $Z_t^{iw} \equiv \exp(z_t^{iw})$.

C.3 Country Weights

The country weights as given in equation (112) are given by:

$$\varpi^j = \frac{W_o^j}{W_o^w} \equiv \frac{E_0\sum\limits_{\tau=0}^{\infty} Q_\tau C_\tau^{j*}}{E_0\sum\limits_{\tau=0}^{\infty} Q_\tau C_t^{W*}} = \frac{\widetilde{Y}_0^j + \Gamma_0^{j*}}{\widetilde{Y}_0^w + \Gamma_0^{w*}}. \tag{135}$$

where using the relationship in the data from equation (121) that $C_\tau^{jB} \equiv \left(Y_{T,\tau}^j\right)^\alpha \left(Y_{N,\tau}^j\right)^{1-\alpha}$ we have that the lifetime budget constraint holds then:

$$\Gamma_0^{j*} = E_0\sum_{\tau=1}^{\infty} Q_\tau S_\tau^j C_\tau^{jD} \tag{136}$$

$$\Gamma_0^{w*} = E_0\sum_{\tau=1}^{\infty} Q_\tau \sum_{i=1}^{J} S_\tau^i C_\tau^{jD} \tag{137}$$

as before. Note that equation (136) gives the prices of a claim on country j endowment consumption measured in the numeraire country prices evaluated at the price density in numeraire country price units, Q_τ. Therefore, it can be calculated using the Euler equation in equation (68) using the numeraire country pricing kernel:

$$E_t\left\{\beta^\theta (C_{t+1}^{1*}/C_t^{1*})^{\left(-\frac{\theta}{\psi}\right)} (R_{C,t+1}^{1*})^{(\theta-1)} R_{Y,t+1}^j\right\} = E_t\left\{\beta^\theta (\widetilde{Y}_{t+1}^w/\widetilde{Y}_t^w)^{\left(-\frac{\theta}{\psi}\right)} (R_{C,t+1}^{1*})^{(\theta-1)} R_{Y,t+1}^j\right\} = 1 \tag{138}$$

where $R_{C,t+1}^{1*} = (C_{t+1}^{1*} + \Gamma_{t+1}^{1*})/\Gamma_t^{1*}$ is the return on the consumption asset for country 1 previously solved using the Euler equation (69) and $R_{Y,t+1}^j = (Y_{t+1}^j + \Gamma_{t+1}^j)/\Gamma_t^j$ is the return on a claim to the endowment process from country j. The price-to-dividend ratio can be solved using the same steps as above with the Campbell Shiller return approximation in equation (93) and the Euler equation for the return on the claim to endowment j in equation (138). Similarly, equation (137) is the price of the aggregate world consumption evaluated at the same price density. Thus, the aggregate world wealth, $\sum_{i=1}^{J} W_o^i$ can be solved by summing over the countries. The ratio of these two wealth levels provide the country weights, ϖ^j. Note that although the reasons are different, the form of this solution is identical to the exogenous exchange rate case because they both price the value of the consumption level in the data as measured by the exchange rates in the data.

D Empirical Methods

In this appendix, we describe the data and empirical approach.

D.1 Measurement and Data

Each country's consumption is measured both in units of its own local prices as well as a numeraire world price. Thus, each country's consumption is measured in units of a numeraire: $\widetilde{C}_{t+1}^{j,D} = S_{t+1}^j C_{t+1}^j$ where $S_{t+1}^j = P_t^j / \widetilde{P}_t$ where P_t^j is the price index in country j and $\widetilde{P}_t$ is the price index in the numeraire country. In Penn World Tables, this consumption series is given as consumption at the relative prices of the same goods in different countries converted at the exchange rate. To make the series comparable over time, we use data through 2010 and benchmark the series to the price level in the numeraire in 1950. Feenstra, Inklaar, and Timmer (2015) describe issues related to the comparability of prices over countries and time.

Our analysis also requires data for asset returns, and dividends. For dividend and equity return data, we use quarterly data from the Total Market Indices in Datastream-Thomson Financial from 1970 to 2009. For the risk-free rates we update the series in Campbell (2003) using the IMF's International Financial Statistics. To be consistent with the annual consumption data, we first aggregate the quarterly data to annual. We then deflate using the consistent price deflators from Penn World Tables to form real annual equity returns, risk-free rates, and dividend growth rates.

D.2 Solutions and Simulated Method of Moments

We solve for the consumption process parameters in our model by fitting target annual moments from a reduced Simulated Method of Moments (SMM) analysis. The model is estimated at the monthly level and therefore the simulated data from the model must be time-aggregated to match the annual data moments.

Our approach requires a set of preference parameters. For this purpose, we use parameter estimates that have been found to fit asset returns best in the US. We therefore take the parameters from Bansal and Yaron (2004) of IES = 1.5, $\gamma = 10$, and monthly $\beta = .998$ or annualized $\beta = .985$. As is required from our model, these parameters are the same across all countries.

To generate the parameter values, we first calibrate the monthly growth rates μ and μ_d to the annual means of consumption growth and dividend growth, depending on the model. To simplify, we calculate the mean annual growth rates from the data and divide by 12. In trial runs of the SMM procedure described below, we find that this calibration assumption makes little difference in the estimates of the remaining parameters but greatly decreases the computation time.

We then use the reduced SMM to fit the remaining parameters for each country: $\left[\sigma^j, \varphi_e^j, \rho^j, \phi^j, \varphi_d^j\right]$ for the "dividend asset" case. Implementing the SMM procedure involves the following steps. For every set of parameter values, we first solve the model using the analytical solutions for returns in the benchmark economy. We choose a set of targeted moments to best represent both consumption and asset pricing data[34]. We then compute a weighted difference between a targeted set of model-generated moments and the data moments using a weighting matrix. To treat all targets equally, we report the estimates using the identity matrix.[35] The set of parameter values that minimizes this difference is the SMM estimate.

To time-aggregate, we compute the growth between the levels at t and $t + 12$, given the realizations of 12 monthly growth rates.[36] To match our annual consumption, dividend growth and asset return moments, we then time-aggregate the model-generated data from monthly to annual frequency. Parameter estimates and simulated model moments are the averages of 500 simulations, each with 840 time-aggregated monthly observations.

For the asset return model, we choose the following set of target data moments for each country: the standard deviation of log consumption growth, the first order auto-correlation of log consumption growth, the mean equity premium, the mean risk-free rate, the standard deviation of the market return, and the standard deviation of the risk-free rate. Using these six moments per country, we estimate the three parameters capturing the transitory risk, σ^j, persistent risk, φ_e^j, and degree of persistence, ρ^j. As a practical matter, we find that fitted values of ρ^j are quite similar across countries so we equate them in the analysis reported in the paper. To fit these parameters to dividends, we augment the number of targets in the six moment "consumption asset" set to include the standard deviation of dividend growth, and the first order auto-correlation of log dividend growth. Using these eight moments per country, we fit the same three consumption

[34] See Gallant and Tauchen (1999) for a discussion on efficient method of moments and problems with moment selection.

[35] We also implemented the reduced SMM procedure using a diagonal matrix with typical components equal to the sample variance. This procedure gave qualitatively similar results.

[36] By comparison, we multiply monthly rates times 12 when we annualize as opposed to time aggregate.

parameters along with two new parameters, the relative volatility of monthly dividend growth φ_d^j, and the consumption leverage parameter, ϕ^j. In fitting these two parameters, we impose bounds from the literature. Specifically, Abel (1999) suggests the leverage parameter can be guided by the ratio of the standard deviation of dividend growth over that of two variables: income growth and consumption growth. These measures calibrated in Abel (1999) using U.S. data suggested a range of $\phi \in \{2.5, 3.6\}$. Similarly, we impose a range on $\varphi_d^j \in \{1, 6\}$ allowing for significantly higher volatility in dividends than consumption. For example, Bansal and Yaron (2004) consider $\varphi_d^j = 4.5$. Once again, the ρ^j values obtained are similar across countries so that we set them equal in the reported results. For comparison, we also consider a version of the model restricting $\phi^j = 3$ for all countries. For the reported results, we choose $\phi^j = 3$.

D.3 Equity Returns and Consumption

In the observed economy, consumption growth, defined by $g_{c,t+1}^j \equiv \lg(\widetilde{C}_{t+1}^{jB}/\widetilde{C}_t^{jB})$, is given by:

$$\begin{aligned} g_{c,t+1}^j &= \mu^j + x_t^j + \sigma^j \eta_{t+1}^j \\ x_{t+1}^j &= \rho^j x_t^j + \varphi_e^j \sigma^j e_{t+1}^j \end{aligned} \tag{139}$$

where $\eta_{t+1}^j, e_{t+1}^j \backsim N(0,1)$ and where η_{t+1}^j, e_{t+1}^j are mutually independent. For expositional simplicity, we adopt the more parsimonious notation that $\sigma_e^j \equiv \varphi_e^j \sigma^j$ where possible. To value the asset return series, we therefore substitute the the process for $\exp(g_{c,t+1}^j)$ into the Euler equation (69).

We follow Bansal and Yaron (2004) in assuming these returns can be approximated using the Campbell and Shiller (1988) approximation.[37] In the case where equity is assumed to pay out the consumption process, the equity return is also the return on the benchmark consumption asset so that $R_{P,t+1}^j \equiv (C_{t+1}^{jB} + P_{t+1}^{jB})/C_t^{jB}$ where P_t^{jB} is the price of the asset paying out the country j benchmark consumption process forever. The Campbell-Shiller approximation then implies that equity returns are a function of the log of the benchmark consumption asset price-to-consumption ratio, $z_t^j \equiv \ln(P_t^{jB}/C_t^{jB})$, and of consumption growth, $g_{c,t+1}^j$, specified by:

$$R_{P,t+1}^j = k_0^j + k_1^j z_{t+1}^j - z_t^j + g_{c,t+1}^j \tag{140}$$

where k_0^j and k_1^j represent approximating constants. Appendix D provides more details about

[37] However, these approximations can lead to misleading conclusions. As pointed out by Hansen (2012), the true returns from recursive preferences depend upon a nontrivial factorization.

the solutions to these constants. Note that $\exp(z_t^j) = (P_t^{jB}/C_t^{jB})$, providing a measure of the consumption price-consumption ratio required to measure the value function in equation (67). We will use this relationship below.

While the consumption asset returns in equation (69) characterize the price-consumption ratio required to identify the value function, we also consider two other types of asset returns in our analysis: (a) the "dividend asset" returns when equity is assumed to pay out the dividend process in the data, and (b) the risk free rate.

In the case when equity is assumed to be the dividend asset, equity returns have a similar Campbell-Shiller representation to equation (140) but now are a function of the equity price-to-dividend ratio, $z_{m,t}^j \equiv \ln(P_t^j/D_t^j)$, and dividend growth, $g_{d,t+1}^j \equiv \lg(D_{t+1}^j/D_t^j)$. Using the subscript m to signify the dividend asset, these equity returns follow:

$$R_{m,t+1}^j = k_0^j + k_1^j z_{m,t+1}^j - z_{m,t}^j + g_{d,t+1}^j \tag{141}$$

where $k_{m,0}^j$ and $k_{m,1}^j$ are again approximating constants defined more fully in Appendix D[38]. We use these relationships below to determine the welfare in the benchmark and the risk sharing economy.

For the risk-free rate, we define $R_{f,t}^j$ as the return on an asset purchased at time t that pays one unit of consumption for sure at $t+1$. To value this return, we again apply the Euler equation (68). Since the risk-free rate is known at time t, the Euler equation can be written as:

$$E_t \left\{ \beta^\theta (C_{t+1}^{jB}/C_t^{jB})^{\left(-\frac{\theta}{\psi}\right)} (R_{P,t+1}^j)^{(\theta-1)} \right\} R_{f,t}^j = 1 \tag{142}$$

Given the solution for the return on the consumption asset, $R_{P,t+1}^j$, the conditional expectation in the equation (142) can be solved directly using the consumption process equation (75). This substitution implies the risk-free rate has the form:

$$R_{f,t}^j = -\theta \ln \beta + \frac{\theta}{\psi} E_t(g_{c,t+1}^j) + (1-\theta) E_t(R_{P,t+1}^j) - 0.5 * Var_t(-\frac{\theta}{\psi} g_{c,t+1}^j + (\theta - 1) R_{P,t+1}^j) \tag{143}$$

Given the solution for $R_{P,t+1}^j$ and definition of $g_{c,t+1}^j$, we solve for each term in the above risk-free equation.

[38] The constants are $k_1^j = \frac{exp(\bar{z}^j)}{1+exp(\bar{z}^j)}$ and $k_0^j = log(1 + exp(\bar{z}^j)) - k_1^j \bar{z}_j$, where $\bar{z}^j$ is the steady state log price to consumption ratio.

Figure 1 Cost Decomposition for M-Wedge and CRRA

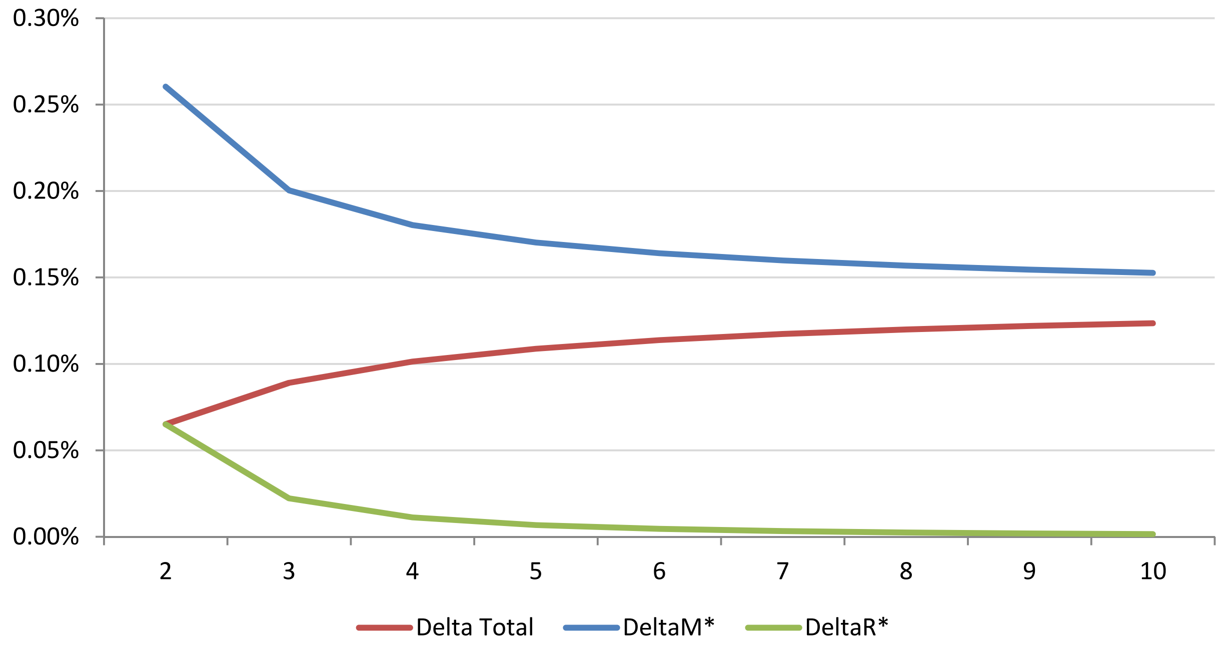

Figure 2a Cost Decomposition for IES = 0.5

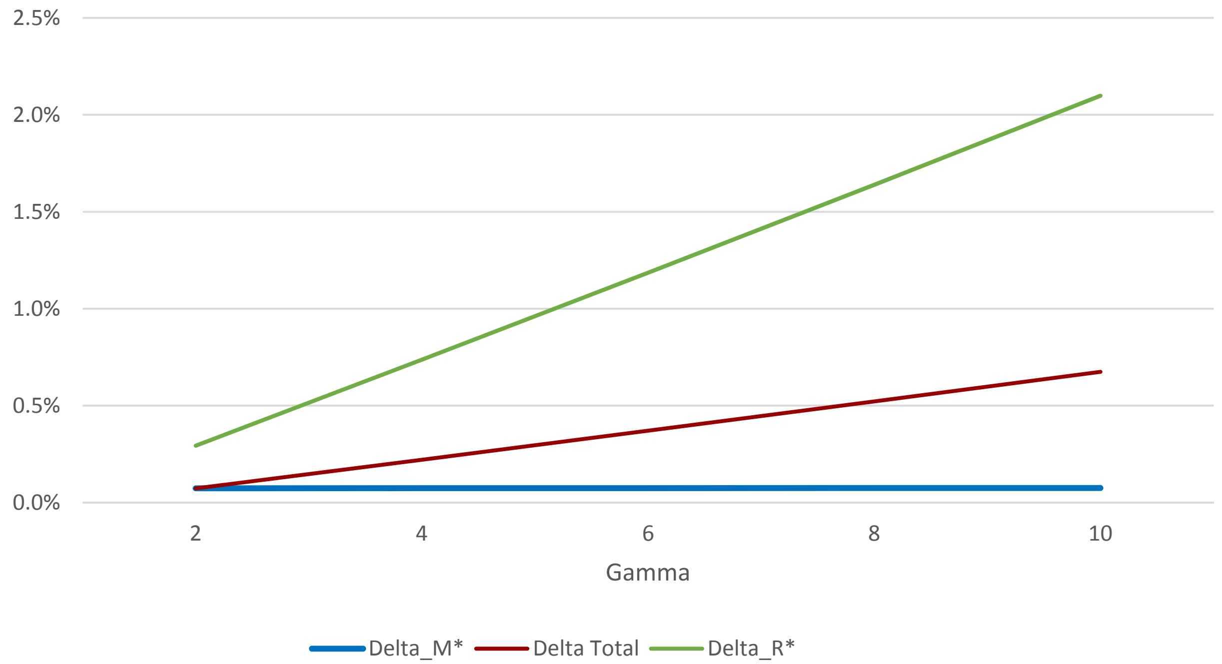

Figure 2b Cost Decomposition for IES = 1.5