Effective Demand Failures and the Limits of Monetary Stabilization Policy*

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Abstract

The COVID-19 pandemic presents a challenge for stabilization policy that is different from those resulting from either "supply" or "demand" shocks that similarly affect all sectors of the economy, owing to the degree to which the necessity of temporarily suspending some (but not all) economic activities disrupts the circular flow of payments, resulting in a failure of what Keynes (1936) calls "effective demand." In such a situation, economic activity in many sectors of the economy can be much lower than would maximize welfare (even taking into account the public health constraint), and interest-rate policy cannot eliminate the distortions — not because of a limit on the extent to which interest rates can be reduced, but because monetary stimulus fails to stimulate demand of the right sorts. Fiscal transfers are instead well-suited to addressing the fundamental problem, and can under certain circumstances achieve a first-best allocation of resources without any need for a monetary policy response.

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The COVID-19 pandemic has presented substantial challenges both to policymakers and to macroeconomists, and these go beyond the simple fact that the disturbance to economic life has been unprecedented in both its severity and its suddenness. The nature of the disturbance has also been different from those typically considered in discussions of business cycles and stabilization policy, and this has raised important questions about how to think about an appropriate policy response.

Among the more notable features of the economic crisis resulting from the pandemic has been the degree to which its effects have been concentrated in particular sectors of the economy, with some activities having to shut down completely for the sake of public health, while others continue almost as normal. A consequence of this asymmetry is a significant disruption of the "circular flow" of payments between sectors of the economy. In a stationary equilibrium of the kind to which an economy tends in the absence of shocks, each economic unit's payment outflows are balanced by its inflows, over any interval of time; this makes it possible for the necessary outflows to be financed at all times, without requiring the household or firm to maintain any large liquid asset balances.

Economic disturbances, regardless of whether these are "supply shocks" or "demand shocks," do not change this picture, as long as they affect all sectors of the economy in the same way: whether activity of all types is temporarily higher or lower, as long as the co-movement of the different sectors is sufficiently close, it continues to be the case that inflows and outflows should balance, so that financing constraints do not bind, even when many individual units maintain low liquid asset balances. Under such circumstances, the market mechanism should do a good job of ensuring an efficient allocation of resources. It is only necessary for policy to ensure that intertemporal relative prices (i.e., real interest rates) incentivize economic units to allocate expenditure over time in a way that is in line with variations in the efficient level of aggregate activity; in an economy where the prices of goods and services are fixed in advance in monetary units, this requires the central bank to manage the short-term nominal interest rate in an appropriate way. But it is often supposed that a reasonably efficient allocation of resources can be assured as long as interest-rate policy is adjusted in response to aggregate disturbances in a suitable way.

A disturbance like the COVID-19 creates difficulties of a different kind. The efficient level of some activities is now different, once public health concerns are taken into account. But in addition, the cessation of payments for the activities that need to be shut down interrupts the flow of payments that would ordinarily be used to finance other activities, even though these latter activities are still socially desirable (if one compares the utility that consumers can get from them to the disutility required to supply them). As a result, many activities may take place at a lower than efficient level, owing to insufficiency of what Keynes (1936) calls "effective demand" — the ability of people to signal in the marketplace the usefulness of goods to them, through their ability to pay for them. While it may well be efficient for restaurants or theaters to suspend the supply of their services for a period (because their usual customers cannot safely consume these services while the disease is rampant),[1] the loss of their normal source of revenue may leave them unable to pay their rent; the loss

[1] We do not here attempt an analysis of the cost-benefit calculations involved in such a determination. In the discussion below of alternative possible policies, it is taken as a constraint that certain activities must be suspended on public health grounds; both the activities and the length of time for which they must be suspended are taken as given.

of rental income may then require the real-estate management companies to dismiss their maintenance staff and fail to pay their property taxes; the furloughed maintenance staff may be unable to buy food or pay their own rent, the municipal government that does not receive a normal level of tax revenue may have to lay off city employees, and so on.[2] The later steps in this chain of effects are all suspensions of economic transactions that are in no way required by the need to stop supplying in-restaurant meals and theater performances.

An effective demand failure of this kind can result in a reduction in economic activity that is much greater than would occur in an efficient allocation of resources, even taking into account the public health constraint. Yet the problem is not simply that aggregate demand is too low, at existing (predetermined) prices, relative to the economy's aggregate productive capacity; in such a case one would expect the problem to be cured by a monetary policy that sufficiently reduces the real rate of interest. But as Leijonhufvud (1973) stresses, in a situation of sufficiently generalized effective demand failure, arising because financing constraints have temporarily become binding for a large number of economic units, the usual mechanisms of price adjustment in a market economy do not suffice to achieve an efficient allocation. The market-determined real rate of interest in a flexible-price economy will not achieve this; and neither, in the more realistic case of an economy with nominal rigidities, will a central bank that adjusts its policy rate to bring about the real rate of interest that would be associated with a flexible-price equilibrium, be able to do so.

Here we present a simple (and highly stylized) model to illustrate the nature of the problem presented by a disturbance like the COVID-19 pandemic. In our model, the fact that economic activity is much lower than in an optimal allocation of resources, in the absence of any policy response, does not necessarily imply that interest rates need to be reduced. While the model is one in which (owing to nominal rigidities) a reduction of the central bank's policy rate increases economic activity, the particular ways in which it increases activity need not correspond at all closely with the particular activities that it would most enhance welfare to increase. Instead, fiscal transfers directly respond to the fundamental problem preventing the effective functioning of the market mechanism, and can bring about a much more efficient equilibrium allocation of resources, even when they are not carefully targeted. And when fiscal transfers of a sufficient size are made in response to the pandemic shock, there is no longer any need for interest-rate cuts, which instead will lead to excessive current demand.

We are not the first to note that a crucial feature of the COVID-19 pandemic has been the degree to which its effects are sectorally concentrated; in particular, this is emphasized by both Guerrieri *et al.* (2020) and Baqaee and Farhi (2020). Indeed, the framework used here to consider alternative possible responses to a pandemic owes much of its structure to the pioneering work of Guerrieri *et al.* The emphases here are somewhat different, however, than in either of those earlier studies. We abstract altogether from either preference-based or technological complementarities between sectors, of the kind emphasized in the papers just cited, in order to focus more clearly on the consequences of the network structure of payments even in the absence of those other reasons for spillovers between activity in different sectors of the economy to exist. Because a key issue examined here is the effects of different

[2]See, for example, Goodman and Magder (2020) and Gopal (2020) on the problems created by effective demand failures of this kind in New York City during the current crisis.

possible network structures of payments, we consider a model in which there can be more than two sectors (and hence more than one sector still active in the case of a pandemic), unlike the baseline model of Guerrieri *et al.* And unlike either of these papers, we do not assume that all consumers choose to consume the same basket of goods; as we show below, non-uniformity in the way expenditure is allocated across goods by economic units that also have different sources of income can play an important role in amplifying the magnitude of the effective demand shortfall resulting from a pandemic.

The macroeconomics of a shock like COVID-19 is the subject of a rapidly expanding literature, already too large to easily summarize. Many interesting contributions focus on different issues than those of concern here. For example, Bigio *et al.* (2020) do not consider what can be achieved with conventional interest-rate policy, instead comparing the effects of lump-sum transfers with those of central-bank credit policies, in a model with less tight borrowing constraints than those assumed here. Caballero and Simsek (2020) consider the possible amplification of the effects of the shock through the effects of income reductions on endogenous financial constraints, from which we abstract here. Céspedes *et al.* (2020) primarily emphasize the longer-run costs of firms having to shed workers during the crisis; here we abstract from such effects, and show that transfers can be beneficial even when they are not taken into account. Auerbach *et al.* (2020) similarly emphasize the increased effects of transfer policies when there is endogenous exit of firms, and focus on channels through which transfers matter even in the absence of financing restrictions. None of these papers give much attention to the effects of conventional interest-rate policy in the case of a pandemic shock.

The paper proceeds as follows. Section 1 explains the structure of the model, and derives the first-best allocation of resources, both for the case of shocks that affect all sectors identically, and the case of a "pandemic shock" that requires one sector to be shut down entirely for one period only. This section also shows that if there are only aggregate shocks, interest-rate policy suffices to achieve the first-best allocation as a decentralized equilibrium outcome, while lump-sum transfers are not only unnecessary, but also ineffective as a tool of aggregate demand management. Section 2 analyzes the effects of a pandemic shock in the absence of any monetary or fiscal policy response, showing how a collapse of effective demand occurs, in the absence of ex ante insurance against such a shock. Section 3 considers what can be achieved by an adjustment of interest-rate policy in response to the pandemic shock, while section 4 instead considers what can be achieved using lump-sum fiscal transfers. Section 5 considers the conditions under which policies that increase aggregate demand also increase welfare, and compares the effectss of interest-rate cuts with those of fiscal transfers in this regard; the respective importance of these two tools is reversed, relative to the conclusions of section 1. Section 6 concludes.

1 An N-sector Model

Let us consider an N-sector "yeoman farmer" model, in which the economy is made up of producer-consumers that each supply goods or services for sale (subject to a disutility of supplying them), and also purchase and consume the goods or services supplied by other such units. Each such unit belongs to one of N sectors (where $N \geq 2$) and specializes in the supply of the good produced by that sector, but consumes the goods produced by multiple sectors. We assume that there is a continuum of unit length of infinitesimal units in each of the sectors. We further order the sectors on a circle, and use modulo-N arithmetic when adding or subtracting numbers from sectoral indices (thus "sector $N + 1$" is the same as sector 1, "sector -2" is the same as sector $N - 2$, and so on).

1.1 Preferences and the network structure of payments

A producer-consumer in sector j seeks to maximize a discounted sum of utilities

$$\sum_{t=0}^{\infty} \beta^t U^j(t) \tag{1.1}$$

where $0 < \beta < 1$ is a common discount factor for all sectors, and the utility flow each period is given by

$$U^j(t) \;=\; \sum_{k \in K} \alpha_k u(c^j_{j+k}(t)/\alpha_k;\, \xi_t) \;-\; v(y_j(t);\, \xi_t), \tag{1.2}$$

where $c^j_k(t)$ is the quantity consumed in period t of the goods produced by sector k, and $y_j(t)$ is the unit's production of its own sector's good. The non-negative coefficients $\{\alpha_k\}$ allow a given sector to have asymmetric demands for the goods produced by the other sectors; K is the subset of indices k for which $\alpha_k > 0$ (so that j wishes to consume goods produced in sector $j + k$). The vector ξ_t represents aggregate disturbances that may shift either the utility from consumption or the disutility of supplying goods (or both);[3] note that these shocks are assumed to affect all goods and all consumers in the same way, as in standard one-sector New Keynesian models.

For any possible vector of aggregate shocks ξ, the utility functions are assumed to satisfy the following standard conditions: $u(0) = 0$, and $u'(c) > 0$, $u''(c) < 0$ for all $c > 0$; $\lim_{c\to 0} u'(c) = \infty$, and $\lim_{c\to\infty} u'(c) = 0$; and finally, $v(0) = 0$, and $v'(y) > 0$, $v''(y) \geq 0$ for all $y > 0$. The Inada conditions imply that the socially optimal supply of each good will be positive but finite (except in the case of a sector affected by a pandemic shock, as discussed below); at the same time, the assumptions that $u(0) = v(0) = 0$ imply that utility will still be well-defined when there is zero supply in some sector. Moreover, the additively separable form (1.2) implies that closing down one sector (preventing either production or consumption of that good) has no effect on either the utility from consumption or disutility of supplying any of the other goods. Thus we abstract entirely from complementarities between sectors owing either to preferences or production technologies, of the kind stressed by Guerrieri *et al.* (2020), in order to focus more clearly on the linkages between sectors resulting from the circular flow of payments.

The coefficients $\{\alpha_k\}$ are important for our analysis, as they determine the network structure of the flow of payments in the economy. We assume that $\alpha_k \geq 0$ for each k, and that $\sum_{k=0}^{N-1} \alpha_k = 1$. Then if all goods have the same price in some period t (and no markets

[3] These may include aggregate productivity shocks, represented here as a shift in the disutility of effort required to produce a given quantity of output.

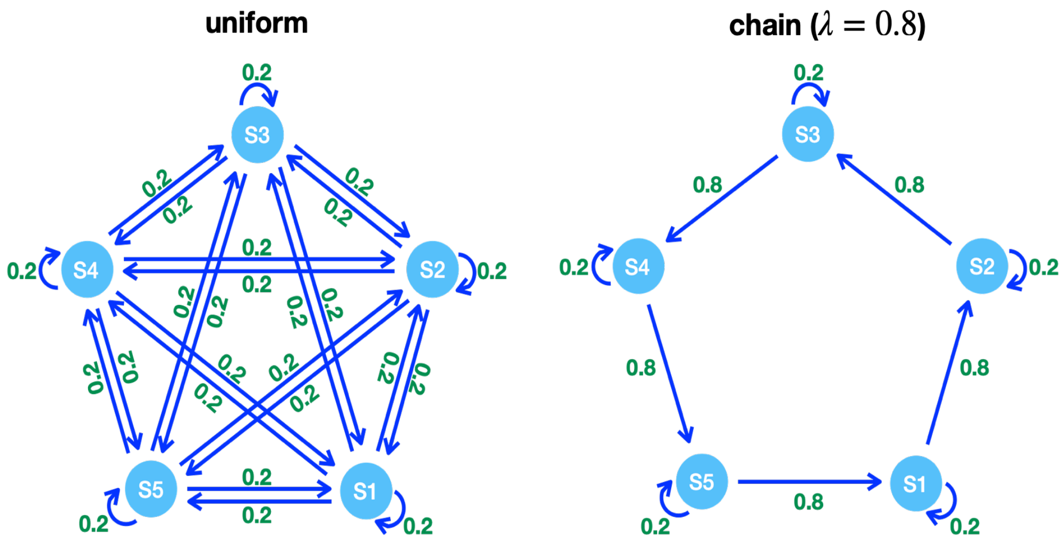

Figure 1: Two possible network structures when $N = 5$. The number on the arrow from sector j to sector k indicates the value of the coefficient α_{k-j}.

have been closed down by a pandemic), the optimal intra-temporal allocation of expenditure by any sector j will be given by

$$c_k^j(t) \;=\; \alpha_{k-j} \cdot c^j(t) \tag{1.3}$$

for each good k, where $c^j(t) \equiv \sum_{k=1}^N c_k^j(t)$ is total real expenditure by the sector in period t.

Note that the coefficients $\{\alpha_k\}$ are assumed to be the same for all sectors j; this means that when there is no pandemic shock, the model has a rotational symmetry: it is invariant under any relabeling of the sectors in which each sector j is relabeled $j+r \pmod N$, for some integer r. We also assume that $\alpha_0, \alpha_1 > 0$ (given an appropriate ordering of the sectors), to ensure that the network structure is indecomposable.[4]

Figure 1 illustrates two of the possible network structures allowed by our notation, for the case $N = 5$. Note that in either case, the numbers on the arrows leaving any sector sum to 1; these indicate the share of that sector's spending allocated to each of the sectors to which arrows lead, in the case that all goods prices are the same. Because of the rotational symmetry, the numbers on the arrows leading to any sector also sum to 1. If in addition to prices being the same, each sector spends the same amount, then all sectors' revenues will be the same, and each sector's inflows and outflows will be balanced. This illustrates the "circular flow" of payments in an equilibrium in which only aggregate shocks occur (discussed further below).

The left panel shows the case of a uniform network, in which $\alpha_k = 1/N$ for all k. In this case, each sector has the same preferences over consumption bundles as any other sector, and these preferences treat all goods symmetrically; if the prices of all goods are the same, each individual unit will purchase the same quantity from each sector. The right panel instead

[4] We exclude, for example, cases in which N is even and even-numbered sectors purchase only from other even-numbered sectors, while odd-numbered sectors purchase only from odd-numbered sectors.

shows the case of a "chain" network, in which $\alpha_0 = 1 - \lambda$ and $\alpha_1 = \lambda$, for some $0 < \lambda < 1$, while all other α_k are zero. In this case, each sector purchases only from its own sector and the sector immediately following it on the circle. In the numerical example shown in the figure, $\lambda = 0.8$, so that in both examples the fraction of own-sector purchases (in the case that prices of all goods are equal) is the same (i.e., 20 percent). But in the left panel, out-of-sector purchases are uniformly distributed over all of the other sectors, while in the right panel they are concentrated on one other sector. We show below that the network structure has important consequences for both the effects of a pandemic shock and the effects of fiscal transfers in response to the shock.

We suppose that the entire sequence $\{\xi_t\}$ for $t \geq 0$ is revealed at time $t = 0$; for simplicity, we suppose there is no further uncertainty about these disturbances to reveal after that. In addition to these aggregate shocks, we consider the possibility of a "pandemic shock," also revealed at time $t = 0$ if it occurs, as a result of which some sector p may be required for reasons of public health to suspend all supply of its good in period zero, though activity in the sector is able to resume as normal in period 1 and thereafter. The effects of the pandemic are thus assumed to last for only one period, and this is known at the time that the shock occurs; thus also in the case of this kind of shock, all uncertainty is resolved in period zero.

We assume that ex ante (before period zero), there is an equal probability $\pi < 1/N$ that any of the sectors will be required to shut down as a result of such a shock. (We further assume that these probabilities are independent of the realization of the aggregate shocks.) While somewhat artificial, this assumption implies that despite the very asymmetric ex post effects of the pandemic shock, the model continues to be rotationally symmetric ex ante. This is convenient both because it simplifies the solution for equilibrium outcomes, and because it provides us with an unambiguous ex ante welfare ranking of the outcomes associated with different stabilization policies, despite the differing situations of producer-consumers in the different sectors ex post.

1.2 The first-best optimal allocation of resources

As a benchmark for discussion of what stabilization policy can achieve, it is useful to define the first-best allocation that would be chosen by a social planner, given only the constraints of preferences and technology (including public health concerns). We can separately consider optimal policy for each possible realization of the sequence $\{\xi_t\}$. We further consider only ex-ante symmetric allocations in which, if no pandemic shock occurs, $c^j_{j+k}(t)$ is the same for all j (but may depend on k and t); and if a pandemic shock occurs, $c^{p+j}_{p+k}(t)$ depends only on j, k and t (independent of p). Units in all sectors agree about the ex ante ranking of all such allocations. For each sequence $\{\xi_t\}$, the allocation should be chosen to maximize

$$\sum_{t=0}^{\infty} \beta^t \left[\sum_{j=1}^{N} U^j(t) \right], \tag{1.4}$$

where $U^j(t)$ is defined in (1.2), under the constraints (a) that $\sum_j c^j_k(t) = y_k(t)$ for each sector k at each date t, and (b) that if a pandemic shock occurs, $c^j_p(0) = 0$ for all j.

The welfare objective (1.4) can be written as a sum of separate terms for each good g at each date t. We thus obtain a separate problem for each good and date, of choosing $y_g(t)$

and the $\{c_g^j(t)\}$ for $j = 1, \ldots, N$ to maximize

$$\sum_{k \in K} \alpha_k u(c_g^{g-k}(t)/\alpha_k;\, \xi_t) \;-\; v(y_g(t);\, \xi_t),$$

subject to the constraints that $\sum_j c_g^j(t) = y_g(t)$, and that all quantities must equal zero if g is a sector that is shut down by a pandemic shock.

If no pandemic shock occurs, and there are only aggregate shocks, the solution to this problem sets $y_k(t) = y_t^*$ for each sector k, where y_t^* (the "natural rate of output") is implicitly defined by the first-order condition

$$u'(y_t^*;\, \xi_t) \;=\; v'(y_t^*;\, \xi_t). \tag{1.5}$$

Note that this condition is the same as in a single-sector model. (And our preference assumptions guarantee a unique interior solution for y_t^*, for any vector of aggregate shocks ξ_t.) This supply of each good k is then allocated to consumers according to the shares

$$c_k^j(t) \;=\; \alpha_{k-j} \cdot y_t^*. \tag{1.6}$$

If instead a pandemic shock occurs, the allocation problem is trivial for good p at date zero: no one can produce or consume it. But for any good $k \neq p$ in period zero, the optimal allocation continues to be given by $y_k(0) = y_0^*$ and (1.6); and in all periods $t \geq 1$, the optimal allocation continues to be the same as in the case of no pandemic shock. Thus a first-best optimal allocation of resources requires that the occurrence of a pandemic shock should have no effects on the production or consumption of any goods, except for the necessary effect of preventing all consumption of good p in period zero. It remains to be considered under what conditions this ideal outcome is achievable in a decentralized economy.

1.3 The decentralized economy

Because all uncertainty is resolved at time $t = 0$, the allocation of resources from period zero onward (conditional on the shocks revealed at that time) can be modeled as a perfect foresight equilibrium of a deterministic model. Each period, there are spot markets for the goods produced by sectors that have not been closed down by a pandemic shock, with $p_k(t)$ the money price of good k in period t. There is also trading in a one-period nominal bond, that pays a nominal interest rate $i(t)$ between periods t and $t+1$. (Because there is only one possible future path for the economy conditional on the state in period zero, allowance for more than one financial asset in any period $t \geq 0$ would be redundant.) The price $p_k(t)$ of each sectoral good is assumed to be predetermined one period in advance, at a level that is expected at that earlier time to clear the market for good k in period t; this temporary stickiness of prices allows monetary policy to affect real activity in period zero.

Let $a^j(t)$ be the nominal asset position of units in sector j at the beginning of period t (after any taxes or transfers), and $b^j(t)$ the nominal asset position at the end of the period (after period t payments for goods are settled). Then in any period $t \geq 0$, a unit in sector j chooses expenditures $\{c_{j+k}^j\}$ (for $k \in K$) and end-of-period assets $b^j(t)$ subject to the flow budget constraint

$$\sum_{k \in K} p_{j+k}(t) c_{j+k}^j(t) \;+\; b^j(t) \;=\; a^j(t) \;+\; p_j(t) y_j(t) \tag{1.7}$$

and the borrowing constraint

$$b^j(t) \;\geq\; 0, \tag{1.8}$$

where $y_j(t)$ is the quantity sold by the unit of its product.

In period zero, $a^j(0) \geq 0$ is given as an initial condition for each sector; this quantity reflects not only wealth brought into the period (before shocks are realized), but also any transfers from the government in response to the shocks realized at time zero, and the payoffs from any private insurance contracts conditional on those shocks.[5] In any subsequent period, $a^j(t+1)$ is given by

$$a^j(t+1) \;=\; (1+i(t))b^j(t) \;-\; \tau(t+1), \tag{1.9}$$

where $\tau(t+1)$ is a lump-sum nominal tax obligation, assumed to be the same for all sectors. (We consider the possibility of sector-specific taxes or transfers only in period zero, in response to a pandemic shock.) A unit in sector j takes as given the value of $a^j(0)$, and the sequences $\{\xi_t, p_k(t), y^j(t), i(t), \tau(t+1)\}$ for all $t \geq 0$, and chooses sequences $\{c^j_k(t), b^j(t)\}$ consistent with constraints (1.7)–(1.9) for all $t \geq 0$ (together with the constraint that $c^j_p(0) = 0$ if sector p is temporarily shut down by a pandemic shock), so as to maximize (1.1).

In equilibrium, the sales by units in each sector are given by

$$y_k(t) \;=\; \sum_{j=1}^{N} c^j_k(t). \tag{1.10}$$

The assumption that prices are set in advance at a level expected to clear markets means that for each j, the sequence $\{y_j(t)\}$ for $t \geq 1$ must be the sequence that a unit in sector j would choose, if it were also to choose that sequence at time $t = 0$, taking as given the values of $a^j(0)$ and $y_j(0)$ and the sequences $\{\xi_t, p_k(t), y^j(t), i(t), \tau(t+1)\}$ for all $t \geq 0$. The value of $y_j(0)$, however, need not be the one that units in sector j would choose, given the shocks realized at $t = 0$, because the price $p_k(0)$ is determined prior to the realization of those shocks. Because of the ex ante symmetry of our model, the predetermined prices $p_k(0)$ will all be set equal to some common price $\bar{p} > 0$. The exact determinants of $\bar{p}$ are not relevant to our results below; it is only important that the price is the same for all sectors, and that it cannot be changed by any policy response to shocks realized at time $t = 0$.

Conditions (1.7), (1.9) and (1.10) imply that the total supply of liquid assets $a(t) \equiv \sum_{j=1}^{N} a^j(t)$ must evolve according to a law of motion

$$a(t+1) \;=\; (1+i(t))a(t) \;-\; \tau(t+1) \tag{1.11}$$

for all $t \geq 0$, which can be regarded as a flow budget constraint of the government. We shall consider only equilibria in which $a(t) > 0$ for all $t \geq 0$, so that there exist public-sector liabilities, the interest rate on which is controlled by the central bank.[6]

The path of interest rates $\{i(t)\}$ for $t \geq 0$ is determined by monetary policy; the initial asset positions $\{a^j(0)\}$ for each sector and the path of tax obligations $\{\tau(t+1)\}$ for $t \geq 0$

[5] In most of the discussion, the quantities $\{a^j(0)\}$ are taken as exogenously given, but in section 2.1 we consider an extension of the model in which these quantities are endogenously determined.

[6] See Woodford (2003, chap. 2) for discussion of the conduct of monetary policy by setting the nominal interest yield on an outside nominal asset of this kind.

are determined by fiscal policy. As in an analysis of Ramsey tax policy, we are interested in the set of allocations (specifications of the sequences $\{c_k^j(t), y_k(t)\}$ consistent with (1.10) for all $t \geq 0$) that can be supported as an equilibrium (that is, that are such that the allocation is individually optimal for units in each sector j) by some initial asset positions $\{a^j(0)\}$, sequences $\{p_k(t)\}$ for $t \geq 1$, and sequences $\{i(t), \tau(t+1)\}$ for all $t \geq 0$, taking as given the initial prices $p_k(0) = \bar{p}$ and the shocks realized at time $t = 0$.

1.4 Optimal policy if only aggregate shocks occur

Let us consider first the case in which pandemic shocks do not occur, but different sequences $\{\xi_t\}$ for the aggregate disturbances may be revealed in period $t = 0$. Can the first-best allocation of resources (characterized above) be supported as an equilibrium, using only the policy instruments listed in the previous section? It is easily seen that this is possible, regardless of the predetermined price level $\bar{p}$ and the particular aggregate shock sequence $\{\xi_t\}$.

The first-best allocation, in which $c_k^j(t) = \alpha_{k-j} y_t^*$ and $y_k(t) = y_t^*$ for all j, k, and all $t \geq 0$, will represent an optimal plan for each sector j as long as the following conditions are all simultaneously satisfied: (a) the initial asset positions of all sectors are the same ($a^j(0) = a(0)/N$ for some aggregate initial asset supply $a(0) > 0$); (b) the price $p_k(t) = P(t)$ is the same for each sector in each period $t \geq 1$; (c) the real rate of interest each period satisfies

$$(1 + i(t))\frac{P(t)}{P(t+1)} \;=\; 1 + r^*(t) \;\equiv\; \frac{1}{\beta}\frac{u'(y^*(t);\, \xi(t))}{u'(y^*(t+1);\, \xi(t+1))} \tag{1.12}$$

for each $t \geq 0$; and (d) the path of tax collections $\{\tau(t+1)\}$ implies an evolution for the total supply of liquid assets such that $a(t+1) > 0$ each period, and the transversality condition

$$\lim_{t\to\infty} \beta^t u'(y^*(t);\, \xi(t))\frac{a(t)}{P(t)} \;=\; 0 \tag{1.13}$$

is satisfied. Note that (1.7)) implies that under these conditions, the plan chosen by each sector will satisfy $b^j(t) = a^j(t) = a(t)/N > 0$ each period, so that the borrowing constraint (1.8) never binds.

It is easily seen that for any sequence $\{\xi_t\}$ of aggregate disturbances, a value of $a(0)$ and sequences $\{P(t+1), i(t), \tau(t+1)\}$ for all $t \geq 0$ can be chosen that satisfy the above conditions, and hence that support the first-best allocation as a perfect foresight equilibrium. One way that we might imagine the optimal policy being implemented is as follows. First, the central bank sets the interest rate in accordance with a Taylor rule of the form

$$\log(1 + i(t)) \;=\; \log(1 + r_t^*) \;+\; \pi^*(t+1) \;+\; \phi \cdot [\log(P(t)/P(t-1)) - \pi^*(t)] \tag{1.14}$$

with $\phi > 1$, where $\{\pi^*(t)\}$ is a sequence of target inflation rates for $t \geq 0$, and r_t^* is the "natural rate of interest" defined in (1.12).[7] The target inflation rate $\pi^*(0)$ is chosen to equal the predetermined inflation rate $\log(\bar{p}/P(-1))$, and subsequent targets are chosen so that

$$\log(1 + r_t^*) \;+\; \pi^*(t+1) \;\geq\; 0 \tag{1.15}$$

[7] The rule can be applied even when the prices of different goods are not all equal, by defining the price index $P(t)$ as $(1/N)\sum_k p_k(t)$, an equally-weighted average of the sectoral prices.

for all $t \geq 0$.[8] Second, the fiscal authority chooses period-zero transfers and a uniform tax obligation $\tau(t+1)$ each period after that so as to make (1.11) consistent with a target path $\{a(t)\}$ for the nominal public debt. This target path requires that $a(t) > 0$ for each $t \geq 0$, and satisfies (1.13) if the price level $P(t)$ grows at the target inflation rate.

This is essentially identical to the way in which optimal stabilization policy can be conducted in response to aggregate shocks in a one-sector model.[9] Note that the monetary policy reaction function (1.14) must adjust in response to aggregate shocks (specifically, in response to shifts in the natural rate of interest), but that in general no change in either the initial supply of nominal assets $a(0)$ or the subsequent target path $\{a(t)\}$ is needed in order to accommodate a different shock sequence $\{\xi_t\}$. Moreover, not only is a fiscal policy response not necessary in order to achieve the first-best outcome, but fiscal transfers have *no effect* in this case.

For example, suppose that the target asset supply $a(t)$ is increased by some multiplicative factor $\mu > 1$ for all $t \geq 0$, with the increase in $a(0)$ achieved through a lump-sum transfer to all sectors in period zero, while the central bank's reaction function (1.14) is unchanged; then the set of perfect foresight equilibrium paths for the variables $\{c_k^j(t), y_k(t), i(t), P(t+1)\}$ is exactly the same under the new policy regime as under the previous one. Thus in the case of only aggregate shocks (and fiscal policies that affect all sectors uniformly), fiscal transfers are irrelevant as a tool of stabilization policy, even if monetary policy is sub-optimal; and monetary policy alone suffices to allow the first-best allocation of resources to be supported as an equilibrium outcome. As we shall see, our conclusions about the relative usefulness of the two types of policy are quite different in the case of a pandemic shock.

2 The Effects of a Pandemic Shock

We next consider the case in which a pandemic shock occurs, and some sector p must be shut down for public health reasons in period zero, though neither the disutility of supplying or utility from consuming goods other than p are affected, and the fundamentals in sector p are also assumed to be unaffected in periods $t \geq 1$. To simplify the discussion, we suppose that $\xi_t = \bar{\xi}$ for all $t \geq 0$, and let $\bar{y}$ be the constant value of the efficient level of production y_t^* in this case.[10]

We have already derived the first-best optimal resource allocation in the case of a pandemic shock. Can this again be achieved in equilibrium, under policies of the kind discussed in the previous section? Prices in period zero are predetermined, and so must be equal for all goods. This means that units in any sector j will still wish to allocate their purchases among sectors $k \neq p$ that are not shut down in proportion to the coefficients α_{k-j}; but because purchases from sector p are no longer possible, the first-best pattern of purchases

[8] This condition ensures that in the perfect foresight equilibrium solution corresponding to the first-best resource allocation, $i(t) \geq 0$ each period.

[9] See, for example, Woodford (2003, chaps. 2, 4).

[10] This is simply to make the notation less cumbersome; the results below are easily extended to the case in which the pandemic shock is accompanied by a change in the path of the aggregate disturbances. In that more general case, optimal policy continues to require that interest-rate policy track changes in the natural rate of interest, as in section 1.4; but apart from this the conclusions for policy remain closely analogous to those derived here.

will no longer correspond to a balanced circular flow of payments. The flow of payments required to sustain the first-best allocation as a decentralized equilibrium is consistent with an arbitrarily small value of $a(0)$ if no pandemic shock occurs, because each sector receives as much income as it spends. This balanced circular flow is disrupted by the pandemic shock: the efficient allocation requires sector p to spend more than its income in period zero (which is now zero), while other sectors spend less than their income. This is not possible if the initial level of liquid assets held by sector p is not large enough to sustain the efficient level of expenditure over the course of the pandemic.

2.1 Equilibrium with ex ante insurance

One case in which the monetary/fiscal regime discussed in section 1.4 would continue to be adequate is if there were an efficient ex ante market for "pandemic insurance." Suppose that before the state in period zero is revealed, units are able to trade state-contingent claims paying off in period zero, in a competitive market. Units in any sector j can then choose a state-contingent initial asset position, $a^j(0|s)$, where s is the state realized in period zero, subject to a budget constraint

$$\sum_s q(s) a^j(0|s) \;\leq\; \tilde{a}^j, \tag{2.1}$$

where $q(s)$ is the price in the ex ante market of a claim that pays off if and only if state s occurs, and $\tilde{a}^j$ is financial wealth of sector j at the time of the ex ante market. The market clears if the prices $\{q(s)\}$ induce demands such that $\sum_{j=1}^N a^j(0|s) = \sum_{j=1}^N \tilde{a}^j$ for each state s. Once the state s is realized, the conditions for a perfect foresight equilibrium from $t = 0$ onward are the same as those stated above, with the value of $a^j(0)$ for each sector given by the quantity $a^j(0|s)$ contracted in the ex ante market.

Because of the model's ex ante rotational symmetry, we assume that $\tilde{a}^j = a(0)/N$ for all sectors. And for simplicity, let us suppose that it is already known with certainty that ξ_t will equal $\bar{\xi}$ for all $t \geq 0$, but that it is not yet known whether a pandemic shock will occur, or which sector will be the impacted sector if one does. (We let $\bar{y}$ denote the natural rate of output when $\xi_t = \bar{\xi}$.) The possible states are then $s = \emptyset$, the state in which no pandemic occurs, and states $s = 1, \ldots, N$ in which a pandemic occurs that impacts sector s.

The first-best optimal resource allocation, in each of the possible states s, can then be supported as an equilibrium by the same prices and interest rates as in section 1.4, if the initial asset positions arranged through ex ante contracting are equal to $a^j(0|s) = (a(0)/N) + \bar{a}^j(s)$, where $\bar{a}^j(\emptyset) = 0$ for all j if no pandemic shock occurs, and

$$\bar{a}^p(p) \;\equiv\; (1 - \alpha_0)\bar{p}\bar{y}, \qquad \bar{a}^j(p) \;\equiv\; -\,\alpha_{p-j}\bar{p}\bar{y} \quad \text{for all } j \neq p \tag{2.2}$$

if a pandemic shock occurs that impacts sector p. That is, if a pandemic shock occurs, net insurance payments must make up for the interruption to the normal circular flow of payments, replacing the income that sector p would ordinarily receive from out-of-sector sales and collecting payments from the other sectors of the funds that they would otherwise spend on the product of sector p. If insurance payments in these precise amounts occur, the monetary and fiscal policies described in section 1.4 again suffice to bring about the first-best optimal allocation of resources.

And these are exactly the insurance contracts that should be arranged, in an equilibrium of the ex ante market, if people correctly understand the consequences of pandemic shocks of each type and assign correct ex ante probabilities to their occurrence. Let $V^j(a; s)$ be the discounted utility (1.1) that a unit in sector j can expect if state s occurs, the prices and interest rates in periods $t \geq 0$ are the ones that support the first-best allocation, the demand for its output is $y_j(t) = \bar{y}$ in each period in which the sector is not shut down, and the initial assets of this individual (not necessarily everyone in the sector) are given by $a^j(0|s) = a$. With these expectations, sector j chooses state-contingent initial assets $\{a^j(0|s)\}$ for the different possible states s so as to maximize $\sum_s \pi(s) V^j(a^j(0|s); s)$ subject to (2.1), where $\pi(s)$ is the ex ante probability of state s.

One can show that[11]

$$V^j(a; s) \;=\; \psi^j(s) u\left(\bar{y} \,+\, \frac{a - \bar{a}^j(s) - (a(0)/N)}{\bar{p}\psi^j(s)}; \,\bar{\xi}\right) \,-\, \tilde{\psi}^j(s) v(\bar{y}; \,\bar{\xi}), \tag{2.3}$$

for all a satisfying the bound

$$a \;\geq\; \frac{a(0)}{N} \,+\, \bar{a}^j(s) \,-\, (\beta^{-1} - 1)\psi^j(s)\frac{a(t)}{N} \tag{2.4}$$

for all $t \geq 0$, where

$$\psi^j(p) \;\equiv\; \frac{1}{1-\beta} \,-\, \alpha_{p-j} \quad \text{in a pandemic state,} \qquad \psi^j(\emptyset) \;\equiv\; \frac{1}{1-\beta},$$

$$\tilde{\psi}^j(j) \;=\; \frac{\beta}{1-\beta}, \qquad \tilde{\psi}^j(s) \;=\; \frac{1}{1-\beta} \quad \text{for any } s \neq j.$$

Here the bound (2.4) guarantees that the unit's borrowing limit (1.8) will not bind in any period.[12]

Then if the state prices are given by $q(s) = \pi(s)$ for each state, the ex ante problem of units in any sector j has a unique interior optimum, given by $a^j(0|s) = (a(0)/N) + \bar{a}^j(s)$ for each state. Since these asset demands clear the ex ante market for each state, these are market-clearing state prices, and the first-best allocation of resources is shown to be an equilibrium.

2.2 The possibility of a collapse of effective demand

In practice, however, little insurance of this kind existed in the case of the COVID-19 pandemic, and there are good reasons to doubt that such markets will come into existence or function efficiently, simply because the possibility of such a pandemic is now more evident. Let us now consider equilibrium in the case of a pandemic shock, under the assumption that

[11] See Appendix A.1 for details of the demonstration.

[12] The value function can also be defined for lower values of a, but this is not necessary, as we find that the optimal choice satisfies the constraint (2.4) for all states s. In order to verify that the asserted solution is indeed an optimum, it suffices to observe that for all feasible values of a, the value function is necessarily no larger than the expression given in (2.3), since binding borrowing constraints can only lower the unit's maximum achievable utility.

there is no ex ante insurance market, so that $a^j(0) = a(0)/N$ for all j. We consider first the case in which neither monetary nor fiscal policy responds to the pandemic shock: these continue to be specified in the way discussed in section 1.4. This means that since the path of $\{r_t^*\}$ does not change, there is no change in the monetary policy reaction function, either immediately or later; and there is no change in the target path of the public debt.

As discussed further in section 4, the effect of the shock depends on the existing level of liquid assets. We first discuss the case in which the collapse of effective demand is most dramatic, which is when liquid assets are low. In this section, we simplify the analysis by considering the limiting case in which $a(0) \to 0$. Note that even in this limiting case, no inefficiency would result as long as only aggregate shocks occur; thus we can imagine an economy choosing to operate with a very low level of liquid assets, if the ex ante probability assigned to the occurrence of a pandemic shock has been quite small. Without loss of generality, we assume that the sector impacted by the shock is sector 1.

As noted above, the fact that $p_k(0) = \bar{p}$ for all sectors that are still able to operate means that units in sector j continue to distribute expenditure in period zero across these sectors in proportion to the value of the coefficient α_{k-j} for each sector. However, because it is no longer possible to purchase from sector 1, the coefficients $\{\alpha_{k-j}\}$ may no longer sum to 1, when summed over the sectors k that are still open. Thus instead of (1.3) we now have

$$c_k^j(0) \;=\; A_{kj} \cdot c^j(0), \tag{2.5}$$

in period zero, where the coefficients A_{kj} (elements of an $N \times N$ matrix $\boldsymbol{A}$) are given by

$$A_{1j} \;=\; 0 \quad \text{for all } j, \qquad A_{kj} \;=\; \frac{\alpha_{k-j}}{1-\alpha_{1-j}} \quad \text{for any } k \neq 1, \text{ and all } j.$$

It follows that total demand for the product of sector k will be given by

$$y_k(0) \;=\; \sum_{j=1}^{n} c_k^j(0) \;=\; \sum_{j=1}^{N} \;=\; A_{kj} \cdot c^j(0).$$

In vector notation, we can write

$$\boldsymbol{y}(0) \;=\; \boldsymbol{Ac}(0), \tag{2.6}$$

where $\boldsymbol{y}(0)$ is the N-vector indicating the output of each of the N sectors and $\mathbf{c}(0)$ is the n-vector indicating the total real spending of each of the sectors.

Clearing of the asset market in period zero requires that in equilibrium, $\sum_{j=1}^{N} b_j(0) = a(0)$. It follows that if $a(0) \to 0$, the only way in which constraint (1.8) can be satisfied for all j is if $b^j(0) \to 0$ for each sector. Thus in equilibrium, each sector must spend all of its income, so that $c^j(0) = y^j(0)$ for each j. It then follows from (2.6) that $\boldsymbol{c}(0) \;=\; \boldsymbol{Ac}(0)$. Thus $\mathbf{c}(0)$ must be a right eigenvector of $\boldsymbol{A}$, with an associated eigenvalue of 1.

Such an eigenvector must exist. $\boldsymbol{A}$ is a non-negative matrix such that $\boldsymbol{e}'\boldsymbol{A} \;=\; \boldsymbol{e}'$, where $\boldsymbol{e}$ is an N-vector of ones; thus it is a stochastic matrix, and necessarily has an eigenvalue equal to 1. Using the properties of stochastic matrices discussed in Gantmacher (1959, sec. XIII.6), we can further establish[13] that 1 is the maximal eigenvalue of $\boldsymbol{A}$ (all of its $N-1$

[13]See Appendix B.1 for details of the application of these results.

other eigenvalues have modulus less than 1), and that the right eigenvector $\boldsymbol{\pi}$ associated with this maximal eigenvalue is non-negative in all elements.[14] If we normalize the eigenvector so that $\boldsymbol{e}'\boldsymbol{\pi} = 1$, then $\boldsymbol{\pi}$ corresponds to the stationary probability distribution of an N-state Markov chain for which $\boldsymbol{A}$ defines the transition probabilities.

Since this is the unique right eigenvector with an associated eigenvalue of 1, equilibrium requires that $\boldsymbol{c}(0) = \theta\boldsymbol{\pi}$ for some scalar coefficient $\theta \geq 0$. In order to determine the value of θ, we observe that intertemporal optimization requires that the Euler condition

$$u'\left(\frac{c^j(0)}{1-\alpha_{1-j}}; \bar{\xi}\right) \;\geq\; u'(\bar{y}; \bar{\xi}) \tag{2.7}$$

must hold for each sector j, and must hold with equality for any sector with $b^j(0) > 0$. (The left-hand side is the marginal utility of purchasing an additional unit of any of the goods $k \neq 1$ from which units in sector j derive utility in period zero, when expenditures are given by (2.5); the right-hand side is the marginal utility of purchasing an additional unit of any good in period 1, when expenditures are given by (1.6). Note that since negligible assets are carried into period 1 by any sector, the equilibrium from period 1 onward is the symmetric equilibrium characterized in section 1.4. Both the interest rate $i(0)$ and the price level $P(1)$ determined by monetary policy from $t = 1$ onward continue to be those of an equilibrium in which no pandemic shock occurs; hence the real interest rate between periods zero and 1 is given by $(1 + i(0))(\bar{p}/P(1)) = \beta^{-1}$, and the two marginal utilities must be equal if units in sector j choose $b^j(0) > 0$. The Euler condition can hold as an inequality if $b^j(0) = 0$, because of the borrowing constraint (1.8).)

Because of the strict concavity of $u(c)$, condition (2.7) can equivalently be written as

$$c^j(0) \;\leq\; c^{*j} \;\equiv\; (1-\alpha_{1-j})\bar{y}. \tag{2.8}$$

The value of θ must be small enough for each element of $\boldsymbol{c}(0)$ to be consistent with this upper bound; but at the same time it must be large enough for the inequality to hold with equality for at least one sector. Thus we must have

$$\frac{1}{\theta} \;=\; \max_j \frac{\pi_j}{1-\alpha_{1-j}} \cdot \frac{1}{\bar{y}} \;>\; 0. \tag{2.9}$$

In this solution, there will necessarily be at least one sector (the sector or sectors j for which the maximum value is achieved in the problem on the right-hand side of (2.9)) for which the borrowing constraint does not bind, and as a consequence period zero expenditure $c^j(0)$ is at the level c^{*j} required for the first-best allocation. (The allocation of any such sector's expenditure across the different goods will also be consistent with the first-best allocation.) But at the same time, there will necessarily be at least one sector (sector 1) in which expenditure is constrained to a level much less than the first-best optimal level.

The severity of the collapse of effective demand depends critically on the network structure of payments. The two cases shown in Figure 1 provide contrasting examples. Figure 2 shows the equilibrium consumption vector $\boldsymbol{c}(0)$ for each of these numerical examples, and compares it to the first-best consumption allocation. In each panel of the figure, the five

[14] Of course, the left eigenvector associated with the maximal eigenvalue is $\boldsymbol{e}'$.

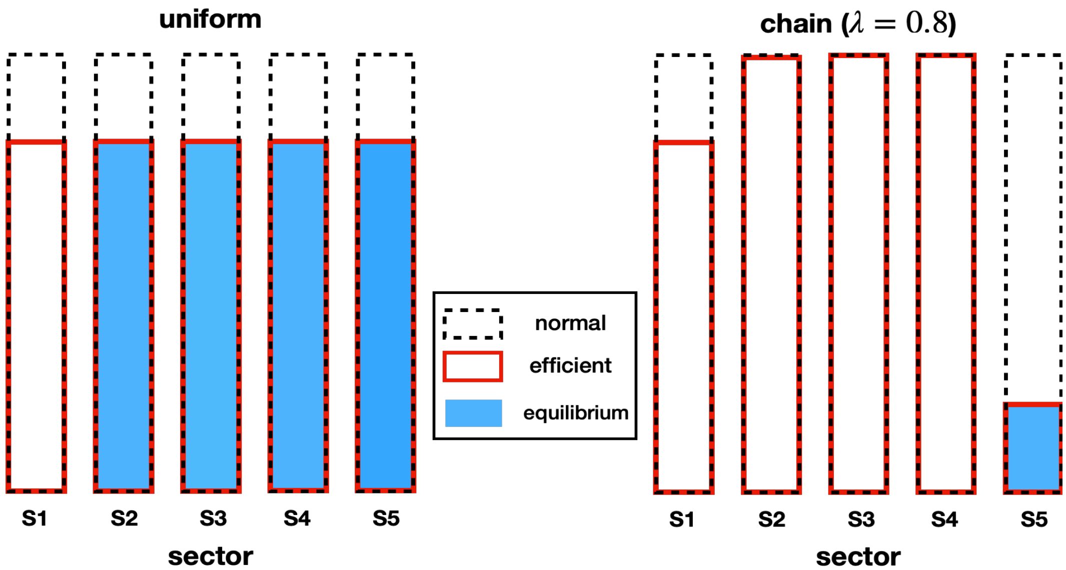

Figure 2: Equilibrium and first-best optimal sectoral expenditure levels in the case of a pandemic shock that requires sector 1 to be shut down, in the case of the two network structures shown in Figure 1.

columns represent total expenditure by units in each of the five sectors. The height of the dashed black border indicates the "normal" level of expenditure (equal to $\bar{y}$ for each sector) — the equilibrium level of expenditure if no pandemic shock occurs, which is also the optimal allocation in that case. (This is necessarily no higher than the normal level and smaller for at least some sectors; thus it is optimal for expenditure and production to decline if a pandemic shock occurs.) The height of the solid red outline for each sector indicates the level c^{*j} that would be optimal given the occurrence of the pandemic shock. The height of the filled blue bar instead indicates the equilibrium level of expenditure $c^j(0)$. This is necessarily no higher than the optimal level for any sector, owing to the Euler constraint (2.8).

In the case of a uniform network structure, $A_{kj} = 1/(N-1)$ for all j and any $k \neq 1$; the Frobenius-Perron maximal right eigenvector is then easily seen to be

$$\boldsymbol{\pi} \;=\; (0 \quad 1/(N-1) \quad \ldots \quad 1/(N-1))'.$$

(Because every sector spends the same amount in each sector $k \neq 1$, but nothing in sector 1, the eigenvector must have this property as well.) Furthermore, $1 - \alpha_{1-j} \;=\; (N-1)/N$ for all j. Hence the maximal value in the problem on the right-hand side of (2.9) is achieved by all sectors $j \neq 1$, and the equilibrium expenditure vector is given by

$$\boldsymbol{c}(0) \;=\; (0 \quad (N-1)/N \;\ldots\; (N-1)/N)' \cdot \bar{y},$$

as shown in the left panel of Figure 2 for the case $N = 5$. In this example, expenditure collapses completely in sector 1 (which no longer receives any income), but it is reduced in sectors $j \neq 1$ only to the extent that it is efficient for these sectors to reduce their spending (given that they no longer can or should buy sector-1 goods).

The collapse of effective demand is much more severe (and the inefficiency much greater) in the case of a "chain" network. In this case, one can show that the Frobenius-Perron maximal right eigenvector is given by

$$\boldsymbol{\pi} \; = \; (0 \; \ldots \; 0 \quad 1)'.$$

(Sector 1 cannot spend at all, because it receives no income. Given that sector 1 cannot spend, sector 2 receives no income other than its own within-sector spending. But because sector 2 does not spend all of its income within-sector, an eigenvalue with eigenvector 1 must involve zero spending by this sector as well. Continuing iteratively in this way, one can show that every sector but sector N must have zero expenditure.

The argument no longer goes through in the case of sector N, because — given that they can no longer buy sector 1 goods — units in sector N spend all of their income within-sector. Hence all elements of $\boldsymbol{\pi}$ but the final one must equal zero.) In the problem on the right-hand side of (2.9), sector N achieves the maximum. Then given that $c^{*N} = (1-\alpha_1)\bar{y} = (1-\lambda)\bar{y}$, the equilibrium expenditure vector is given by

$$\boldsymbol{c}(0) \; = \; (0 \; \ldots \; 0 \quad 1-\lambda)' \cdot \bar{y},$$

as shown in the right panel of Figure 2 for the case $N = 5, \lambda = 0.8$.

These two cases illustrate the two extremes with regard to the degree of collapse of aggregate expenditure and output in the limiting case in which $a(0) \to 0$. For a general network structure with a fraction $\alpha_0 = 1/N$ of within-sector spending by all sectors, we can show that aggregate spending $c^{agg}(0) \equiv \sum_{j=1}^{N} c^j(0)$ must fall within the bounds

$$(1/N)\bar{y} \;\; \leq \;\; c^{agg}(0) \;\; \leq \;\; (N-2+(1/N))\bar{y} \;\; < \;\; (N-1)\bar{y} \; = \; y^* \; \equiv \; \sum_{j=1}^{N} c^{*j}.$$

Here the lower bound is established by the fact that at least one sector must spend its optimal level c^{*j}, and since that sector cannot be sector 1, its first-best level of spending must at least equal $\alpha_0\bar{y}$, its efficient level of within-sector spending. The upper bound is established by the fact that spending by sector 1 must be zero, and that spending in every other sector must be bounded above by (2.8). Both of these bounds are achievable, since (as just shown) the chain network achieves the lower bound while the uniform network achieves the upper bound. Note that even in the most benign case (the uniform network of payments), aggregate spending and output are inefficiently low, because spending by sector 1 is inefficiently low.[15]

Thus effective demand failure can result in a substantially greater reduction of economic activity than is efficient. In the numerical example shown in the figures, it is efficient for output to decline by 20 percent in response to the pandemic shock; but in equilibrium, if liquid asset balances are low and there is no policy response, the output decline must be somewhere between a 36 percent reduction (the left panel of Figure 2) and a 96 percent

[15] It is also clear that the empirically relevant network structure is not completely uniform, so that sector 1 will almost certainly not be the only borrowing-constrained sector in the limits as $a(0) \to 0$. See, for example, Danieli and Olmstead-Rumsey (2020), who document for the US economy that the sectoral composition of the reduction of demand in the COVID-19 crisis was non-uniform in ways that go beyond the simple fact that people in contact-intensive occupations were no longer able to work.

reduction (the right panel of Figure 2). But these contractions occur under the assumption of no policy response. To what extent can macroeconomic stabilization policy achieve a different outcome?

3 What Can a Monetary Policy Response Achieve?

Given the conclusions of section 2.2, it might seem natural to suppose that the most important kind of policy response should be an adjustment of the central bank's interest-rate target in response to the real disturbance. And our model is one in which equilibrium activity in period zero can be increased or decreased by interest-rate policy; thus to the extent that one thinks about the policy problem in terms of an aggregate output gap, it should be possible to eliminate the gap entirely by a sufficiently large cut in interest rates, assuming that the zero lower bound does not preclude this. Thus it is often supposed that if counter-cyclical fiscal policy is also needed, this is only because (especially in a low-inflation environment) the lower bound may not allow interest rates to be reduced to the degree needed in the case of a severe disturbance. And in fact our model is one in which the zero lower bound never constrains how much it should be possible to reduce the real interest rate (which is what matters for aggregate demand), if the central bank is willing to commit itself to more inflationary policy in the future.

Nonetheless, monetary policy remains a decidedly second-best policy instrument for dealing with the inefficiencies created by an effective demand failure resulting from a pandemic shock. The problem is not so much that monetary policy cannot increase economic activity in these circumstances — the elasticity of aggregate output with respect to changes in the real interest rate is determined by the intertemporal elasticity of substitution, in the same way as in the case of the response to aggregate disturbances discussed in section 1.4. It is rather that the composition of the added expenditure that can be stimulated by interest-rate cuts will necessarily be inefficient, and (depending on the network structure of payments) may be severely so.

Let us consider how the analysis of the previous section is changed if we suppose that the central bank cuts $i(0)$ in response to the pandemic shock. As above, the expenditure vector $\boldsymbol{c}(0)$ must be a right eigenvector of the matrix $\boldsymbol{A}$ with eigenvalue 1, and hence we must have $\boldsymbol{c}(0) = \theta\boldsymbol{\pi}$ for some $\theta \geq 0$. The difference is that the Euler condition (2.7) now becomes

$$u'\left(\frac{c^j(0)}{1-\alpha_{1-j}};\,\bar{\xi}\right) \;\geq\; \frac{1+i(0)}{1+\bar{\imath}}u'(\bar{y};\,\bar{\xi}), \tag{3.1}$$

where $\bar{\imath} > 0$ is the value of $i(0)$ assumed in (2.7), the nominal interest rate implied by a policy rule of the form (1.14) that would be optimal if there were only aggregate shocks. This in turn implies an upper bound for expenditure by each sector,

$$c^j(0) \;\leq\; \hat{c}^j(i(0)) \;\equiv\; (1-\alpha_{1-j})\hat{y}(i(0)), \tag{3.2}$$

generalizing (2.8), where $\hat{y}(i(0))$ is the quantity implicitly defined by

$$u'(\hat{y}(i(0));\,\bar{\xi}) \;=\; \frac{1+i(0)}{1+\bar{\imath}}u'(\bar{y};\,\bar{\xi}). \tag{3.3}$$

Because $u(c)$ is strictly concave, $\hat{y}(i(0))$ is a monotonically decreasing function.

Once again, θ must be such that (3.2) holds for all sectors, and holds with equality for at least one sector. Hence the solution for θ is given by

$$\frac{1}{\theta} \;=\; \max_j \frac{\pi_j}{1-\alpha_{1-j}} \cdot \frac{1}{\hat{y}(i(0))} \;>\; 0, \tag{3.4}$$

generalizing (2.9). Thus if $i(0)$ is reduced, each of the $\{c^j(0)\}$ is scaled up by the same factor, the factor by which $\hat{y}(i(0))$ is greater than $\bar{y}$.

It follows that our model implies that $c^{agg}(0)$, and correspondingly aggregate output $y^{agg}(0) \equiv \sum_{j=1}^{N} y_j(0)$, increases in proportion to $\hat{y}(i(0))$ if the interest-rate target is changed. This is the same interest-rate elasticity of aggregate output as exists in the case that only aggregate disturbances exist (so that equilibrium output and expenditure are the same in all sectors, and borrowing constraints do not bind for any sector): in that case, $c^j(0) = y_j(0) = \hat{y}(i(0))$ for every sector, so that also in that case $y^{agg}(0)$ grows in proportion to $\hat{y}(i(0))$. Thus the fact that borrowing constraints bind for many sectors need not imply any lower interest-elasticity of output in the case of an effective demand failure. And the Inada conditions assumed for the function $u(c)$ imply that $\hat{y}$ can be driven arbitrarily close to zero by raising the real interest rate enough, and made arbitrarily large by lowering the real interest rate enough;[16] thus the model implies that a very great degree of control over aggregate output (in the short run) is possible using monetary policy, even during the crisis created by a pandemic shock.

Nonetheless, monetary policy is not well-suited to correct the distortions created by the pandemic shock. While lowering interest rates should increase spending and hence output, the sectoral composition of the spending and output that are stimulated need not correspond to the kinds are most needed in order to increase welfare. In the limiting case in which $a(0) \to 0$, we have seen that the expenditure vector $\boldsymbol{c}(0)$ continues to be a multiple of the eigenvector $\boldsymbol{\pi}$, regardless of the value of $i(0)$. This means that spending is increased in each sector only in proportion to the extent to which that sector is already spending when $i(0) = \bar{\imath}$; thus spending is increased most in those sectors where it was already highest (which tend to be sectors for which the marginal utility of additional spending is lower[17]).

In fact, an interest-rate cut need not increase welfare at all. Consider the (admittedly extreme) case of a chain network (the kind shown in the right panel of Figure 1) in which the disutility of supplying output is linear: $v(y;\bar{\xi}) = \nu \cdot y$ for some $\nu > 0$. Then the natural rate of output satisfies $u'(\bar{y};\bar{\xi}) = \nu$. Suppose that a pandemic shock occurs when liquid asset balances are negligible ($a(0) \to 0$). Then regardless of the value of $i(0)$, spending is non-zero only by units in sector N, which purchase only within-sector goods, in the amount $c_N^N(0) = y_N(0) = (1-\lambda)\hat{y}(i(0))$. The welfare measure (1.4) is equal to

$$W_0 \;\equiv\; \sum_{j=1}^{N} U^j(0) \;=\; (1-\lambda)u(\hat{y}(i(0));\,\bar{\xi}) \;-\; \nu \cdot [(1-\lambda)\hat{y}(i(0))] \tag{3.5}$$

[16] Because of the zero lower bound on the nominal interest rate, of course, very low real interest rates can be achieved only by creating an expectation of high inflation.

[17] Note however that the ranking of sectors according to the marginal utility of additional spending need be precisely the inverse of their ranking with regard to the level of $c^j(0)$; instead, the marginal utility of additional real expenditure is given by $u'(c^j(0)/(1-\alpha_{1-j}))$, while it is not increased at all in sectors (like sector 1) that cannot spend at all in the absence of a policy response.

plus the terms for the utility flows received in periods $t \geq 1$ (which are unaffected by monetary policy).

It is clear from (3.5) that W_0 is a concave function of $\hat{y}(i(0))$; moreover, it is locally increasing or locally decreasing according to whether $u'(\hat{y}(i(0)); \bar{\xi})$ is greater or less than ν. Thus W_0 reaches its maximum when $\hat{y}(i(0)) = \bar{y}$, which is to say, when $i(0) = \bar{\imath}$. In this case, the welfare-maximizing monetary policy response is *not to change the interest rate at all* in response to the pandemic shock. Any interest-rate cut would actually be a Pareto-inferior policy even from an ex-post perspective: not only would it lower the ex ante welfare measure W_0, but ex post it would not improve welfare in any sector, while it would lower welfare for some (the units in sector N).

As it happens, the same is true if we assume a uniform network (the kind shown in the left panel of Figure 1), but again assume a linear disutility of supply. In this case, $c^j_k(0) = \hat{y}(i(0))/N$ for all $j, k \neq 1$, but there is no spending by units in sector 1, so that $y_k(0) = ((N-1)/N)\hat{y}(i(0))$ for all $k \neq 1$. The contribution to welfare from period-zero utility flows is then

$$W_0 \;\equiv\; \sum_{j=1}^{N} U^j(0) \;=\; \frac{(N-1)^2}{N} u(\hat{y}(i(0)); \bar{\xi}) \;-\; (N-1)\cdot\nu\cdot\left[\frac{N-1}{N}\hat{y}(i(0))\right],$$

which again is a concave function of $\hat{y}(i(0))$ that is maximized when $\hat{y}(i(0)) = \bar{y}$.

Nonetheless, the result that it does not help anyone to adjust interest-rate policy at all requires quite special assumptions. Even for network structures of these two kinds, if $v(y)$ is strictly convex, the fact that $u'(c^j(0)) = v'(\bar{y})$ in the sectors that are not borrowing-constrained when $i(0) = \bar{\imath}$ no longer means that the disutility of further supply by these sector will exceed the increased utility from additional spending; because $y^j(0) < \bar{y}$ when $i(0) = \bar{\imath}$, one will have

$$u'(c^j(0); \bar{\xi}) \;>\; v'(y_j(0); \bar{\xi}) \tag{3.6}$$

even for the unconstrained sectors.

Moreover, these network structures are special in implying that the non-borrowing-constrained sectors purchase nothing from borrowing-constrained sectors. In general, even if the utility of supply is linear, π_j will typically be positive for some sectors that are borrowing-constrained (that is, that do not achieve the maximum value in the problem on the right-hand side of (2.9)), and in these borrowing-constrained sectors, $u'(c^j(0)) > \nu = v'(y_j(0))$. Thus in most cases, reducing $i(0)$ below the level $\bar{\imath}$ will increase $c^j(0) = y_j(0)$ in some sectors j such that (3.6) holds when $i(0) = \bar{\imath}$. It follows that some degree of reduction in $i(0)$ will raise welfare in those sectors. And since (2.7) requires that

$$u'(c^j(0); \bar{\xi}) \;\geq\; v'(y_j(0); \bar{\xi})$$

for *all* sectors when $i(0) = \bar{\imath}$, it follows that a sufficiently small reduction in $i(0)$ will in most cases increase W_0.

Even when this is true, however, the optimal interest-rate reduction can be much less than the size of reduction in the real interest rate that would be required to eliminate the "output gap" — that is, one large enough to make $y^{agg}(0)$ equal to aggregate output in the first-best allocation, y^*. Suppose that in addition to the assumptions made about the utility functions above, we also assume that $cu'(c)$ is a strictly concave function of c, and that $yv'(y)$ is a weakly concave function of y.[18] Then regardless of the network structure, we can show[19] that

$$\frac{\partial W_0}{\partial \theta}(\theta = \bar{\theta}) \; < \; 0. \tag{3.7}$$

Here $W_0(\theta)$ is the contribution to welfare from period-zero utility flows, computed for the expenditure allocation in which $c_k^j(0) = \theta \cdot A_{kj}\pi_j$ for all j, k; note that the alternative allocations that can be achieved using monetary policy (in the case in which $a(0) \to 0$) are all of this form, with the value of θ corresponding to a particular monetary policy given by (3.4). The derivative is evaluated at the value $\theta = \bar{\theta} \equiv (N-1)\bar{y}$, which is the value required to achieve $y^{agg}(0) = y^*$.

Because the expenditure allocation is linear in θ, $W_0(\theta)$ is a concave function of θ. The fact that the derivative is negative when $\theta = \bar{\theta}$ means that W_0 is maximized by a value $\theta < \bar{\theta}$, corresponding to a real interest rate higher than the one that would fully eliminate the output gap. The second-best optimal monetary policy involves lower aggregate activity than the first-best allocation, but further interest-rate reduction would increase the welfare losses from excess expenditure by the less-constrained sectors by more than it can reduce the welfare losses due to insufficient expenditure by the more-constrained sectors.

This conclusion is obtained under the assumption that liquid assets are initially low, and that there are no fiscal transfers in response to the pandemic shock. If on the other hand borrowing constraints do not bind, either because the level of initial liquid assets is large enough or because fiscal transfers are large enough for them not to bind, one would conclude that there is no benefit from cutting interest rates — because sufficient liquid asset balances and/or fiscal transfers eliminate the problem of effective demand failure without requiring any change in monetary policy. We turn now to this case.

4 Fiscal Transfers and Effective Demand

Let us next consider what can be achieved using lump-sum taxes and transfers that are adjusted in response to the occurrence of the pandemic shock. We begin by supposing, as in section 2, that there is no monetary policy response to the pandemic shock, meaning that $i(0) = \bar{\imath}$, and that the central-bank reaction function (1.14) for periods $t \geq 1$ continues to be the one appropriate to an environment in which no pandemic shocks occur.

4.1 Fiscal policy as "retrospective insurance"

If there is sufficient flexibility in the adjustment of lump-sum taxes and transfers, fiscal policy can achieve the first-best allocation of resources; indeed, it can do this, in principle, without any increase in the public debt. One obvious way is to use fiscal policy to provide all sectors with the same budgets as they would have in the equilibrium with ex ante insurance contracts described in section 2.1. This simply requires that units in sector 1 each receive

[18] Examples of utility functions that would satisfy these assumptions, along with all of those listed earlier, would be $u(c) = c^a$ for some $0 < a < 1$, and $v(y) = \nu \cdot y^b$, for some $\nu > 0, b \geq 1$.

[19] See Appendix C.2 for details of the proof.

a lump-sum transfer in the amount $(1-\alpha_0)\bar{p}\bar{y}$, while units in each sector $j \neq 1$ are taxed an amount $\alpha_{1-j}\bar{p}\bar{y}$. In words, units in the sector that is required to shut down for public health reasons receive transfers sufficient to replace the income that they would otherwise have received from out-of-sector spending on the goods that they produce. These are paid for by taxes on the sectors that are not required to shut down, in the amount of the money that they would otherwise have spent on sector 1 goods.

Because the aggregate value of the taxes equal the amount of income that must be replaced for sector 1, no government deficit is required, and there is no need to change the anticipated path of tax obligations $\{\tau(t+1)\}$ in later periods. As in the case where the transfers occur as a result of ex ante insurance against the possibility of a pandemic shock, the first-best allocation can then be supported as an equilibrium, with the same prices and interest rates as in the case where no pandemic shock occurs. Hence no change in monetary policy would be needed to achieve the first-best outcome in this case, and indeed, any change in $i(0)$ (not offset by a corresponding change in the inflation target for period 1, so as to leave the real interest rate in period zero the same) would lower welfare.

Under this approach, the first-best outcome is achieved by a fiscal policy that can be described as "retrospective insurance" of the kind called for by Milne (2020);[20] fiscal policy implements the state-contingent transfers that would have been privately agreed upon in (counter-factual) ideal ex ante contracting. It should be noted that the optimal policy does not require full replacement of the level of income that units in sector 1 would have had in the absence of the shock. This is not because of moral hazard or adverse selection problems created by more complete insurance; we abstract from all such issues here. Rather, it is because an ex-ante-optimal insurance contract would not replace the income that units in sector 1 do not need if they cannot purchase sector 1 goods as they normally would (just as it would ask units in other sectors to give up the income that they no longer need to spend on sector 1 goods). An alternative statement of the relevant principle would say that units that have been required to close down for public health reasons should receive transfers sufficient to allow them to cover those expenses (rent, etc.) that it remains efficient for them to be able to pay for.

There is an advantage, however, to thinking of the relevant principle as replacing income that has been lost owing to the fact that some goods and services can no longer safely be supplied or consumed, rather than as paying for expenditures that are judged to remain appropriate despite the pandemic. This is that the optimal "retrospective insurance" policy replaces (part of) the income lost by units in sector 1, but does not make transfers to any other sectors — despite the fact that, in the absence of any policy response, incomes of many others may collapse as well (as illustrated in the right panel of Figure 2). Our model shows how an interruption of the circular flow of payments can result in other sectors suffering a loss of income and hence being constrained in their ability to spend at the efficient level, not just the sector that is precluded from selling its product for public health reasons; but these other sectors' incomes should be restored once the income of the directly impacted sector is replaced.

It is also worth noting that the case for retrospective insurance made here is independent of the central argument of Milne (2020) or Saez and Zucman (2020), that it is important to

[20] See also Saez and Zucman (2020) for a similar proposal.

prevent business failures, because of the costs involved in re-establishing such businesses once they have failed. Our intention is not to deny that this should also be an important concern; the COVID-19 pandemic seems likely to result in a massive wave of business failures, in the absence of further fiscal measures than those announced at this time, and these are indeed likely to have significant costs. However, this is not the *only* reason why a retrospective insurance policy would be efficient (at least if we abstract from the cost of administration of such a policy). Even if there were no cost at all of restarting economic activities in period 1 that did not take place in period zero, as assumed in our simple model, the retrospective insurance transfers increase welfare by increasing the efficiency of the allocation of resources in period zero — both by allowing greater utilization of available productive capacity, and by allowing a more equitable distribution of the goods that are produced. Nor would introducing a concern for the costs of business failures increase the degree of insurance that would be optimal in our model; even without any such costs, it is optimal to maintain the same level of production of all goods (and the same pattern of distribution of those goods) in period zero as will be desired in later periods, with the single exception of sector 1 goods — and it would not be efficient for these goods to be produced and consumed even if costs of business exit were introduced.

4.2 A multidimensional "Keynesian cross"

While retrospective insurance provides a conceptually straightforward solution to the problem of the disruption of the circular flow of payments in a pandemic, it might not be politically acceptable in practice. In particular, it might be difficult to get those who should be taxed under the prescription above to agree to pay such taxes simply because (according to an economic theory) they "would have" consented to pay such insurance premiums in an ideal ex ante contracting situation. This leads us to consider what can be achieved if only non-negative lump-sum transfers to each sector are possible in response to the pandemic shock. In general, we shall suppose that these can be sector-specific (and as we shall see, the most efficient transfer policy will be sectorally targeted). However, it might also be considered simplest on both administrative and political grounds to simply make uniform transfers to everyone in the economy, regardless of how they have been impacted by the pandemic. Thus it is also of interest to consider what can be achieved using fiscal transfers when these are constrained to be uniform in period zero, just as the tax obligations $\{\tau(t+1)\}$ in subsequent periods are assumed always to be the same for units in all sectors.

Lump-sum transfers in period zero adjust the initial liquid assets $\{a^j(0)\}$ for units in the different sectors; thus we can consider the effects of different possible transfer policies in response to the pandemic shock simply by characterizing equilibrium in the case of different initial asset balances. The specification of these initial balances can be summarized by $\boldsymbol{a}(0)$, the N-vector with jth element $a^j(0)$. For each specification of $\boldsymbol{a}(0)$, we must choose a specification of subsequent lump-sum tax obligations $\{\tau(t+1)\}$, or alternatively a path for the outstanding public debt $\{a(t+1)\}$, that is consistent with the transversality condition (1.13). In addition, we suppose that tax obligations in periods $t \geq 1$ are chosen in accordance with a fiscal rule that ensures that

$$\frac{a(t)}{P(t)} \;\geq\; \beta(1+i(0))\frac{a(0)}{P(1)} \tag{4.1}$$

in each period $t \geq 1$. (Thus there is a sense in which any increase in the public debt as a result of transfers in response to the pandemic shock is assumed to be permanent, though it is not allowed to grow so rapidly as to violate the transversality condition.)

Apart from these stipulations, the exact way in which tax obligations are determined in periods $t \geq 1$ do not matter for our conclusions. Condition (4.1) suffices to ensure that the borrowing constraint (1.8) cannot bind for any units in periods $t \geq 1$.[21] Given this, and a fiscal rule that ensures satisfaction of the transversality condition, the feasible expenditure plans by units in any sector j are defined by an intertemporal budget constraint, that depends (only) on the paths of relative prices and real interest rates in periods $t \geq 1$, and the real pre-tax wealth $\tilde{a}^j(1) \equiv (1+i(0))b^j(0)/P(1)$ carried into period 1 by units in this sector. We can thus parameterize the complete range of different fiscal transfer policies under consideration by different choices of the vector $\boldsymbol{a}(0)$.

Regardless of a unit's initial assets $a^j(0)$, the fact that the prices of all goods are equal in period zero implies that purchases of each good $k \neq 1$ will be proportional to α_{k-j}. It follows that (2.5) and (2.6) continue to hold, as above. The unit's flow budget constraint (1.7) in period zero further requires that

$$c^j(0) \;=\; y_j(0) + \frac{a^j(0) - b^j(0)}{\bar{p}} \;=\; \sum_k A_{jk} c^k(0) + \frac{a^j(0) - b^j(0)}{\bar{p}}. \tag{4.2}$$

In addition, the unit's expenditure in period zero must be consistent with (1.8), and a generalized version of the Euler condition (2.7).

Let Λ^j be the marginal utility for a unit in sector j of additional real pre-tax wealth carried into period 1; that is, the amount by which the unit's continuation utility (anticipated utility flows in periods $t \geq 1$, discounted back to period 1) is increased by a unit increase in $\tilde{a}^j(1)$, or alternatively, the amount that it is increased by an additional $P(1)$ units of money (that is, the amount needed to buy $1/N$ units of each of the goods) carried into period 1. As noted above, the equilibrium from period $t = 1$ onward depends only on the vector $\tilde{\boldsymbol{a}}(1)$ of real pre-tax asset balances carried into period 1. The marginal utility Λ^j then depends on the vector $\tilde{\boldsymbol{a}}(1)$ (which determines equilibrium prices and interest rates for all $t \geq 1$), but also on the individual unit's wealth $\tilde{a}$, which may differ from the average wealth of units in its sector (specified by the jth element of the vector $\tilde{\boldsymbol{a}}(1)$). Thus we can let $\Lambda^j(\tilde{a}; \tilde{\boldsymbol{a}}(1))$ denote this marginal utility.

The Euler condition (2.7) can then be stated more generally as

$$u'\left(\frac{c^j(0)}{1-\alpha_{1-j}}; \bar{\xi}\right) \;\geq\; \Lambda^j(\tilde{a}^j(1); \tilde{\boldsymbol{a}}(1)). \tag{4.3}$$

This reduces to (2.7) in the case considered in section 3, because

$$\Lambda^j(\tilde{a}^j(1); \tilde{\boldsymbol{a}}(1)) \;\to\; u'(\bar{y}; \bar{\xi}) \tag{4.4}$$

in the limiting case in which $\tilde{\boldsymbol{a}}(1) \to \mathbf{0}$. The inequality (4.3) must hold along with (1.8), and at least one of these inequalities must hold with equality.

[21]See Appendix A.2 for details of the argument.

We can simplify the analysis of equilibrium when $a(0) > 0$ by considering the limiting case in which $\beta \to 1$ (meaning that the equilibrium real rate of interest is negligible relative to the length of time for which a pandemic lasts). In this limit as well, (4.4) holds, regardless of the values of the arguments $\tilde{a}^j(1)$ and $\tilde{\boldsymbol{a}}(1)$. (When the real interest rate in periods $t \geq 1$ is negligible, the degree to which greater wealth $\tilde{a}^j(1)$ leads a unit to increase its spending in any single period is negligible, as a result of which the relative-price changes required for equilibrium when the vector $\tilde{\boldsymbol{a}}(1)$ changes are also negligible.[22]) As a consequence, the Euler condition (4.3) again takes the simpler form (2.7), which again is equivalent to (2.8).

Thus in the limiting case in which $\beta \to 1$, the unit's optimal choices for $c^j(0)$ and $b^j(0)$ are jointly determined by equation (4.2), inequalities (1.8) and (2.8), and the requirement that at least one of the inequalities hold with equality. This implies a consumption function

$$c^j(0) \;=\; \min\left\{\frac{a^j(0)}{\bar{p}} \;+\; \sum_k A_{jk}c^k(0),\; c^{*j}\right\} \tag{4.5}$$

for each sector j. The collection of these conditions, one for each j, can be written in vector form as

$$\boldsymbol{c}(0) \;=\; \mathbf{min}\left\{\frac{1}{\bar{p}}\boldsymbol{a}(0) \;+\; \boldsymbol{A}\boldsymbol{c}(0),\; \boldsymbol{c}^*\right\}, \tag{4.6}$$

where **min** is the operator that maps two N-vectors to an N-vector, each element of which is the minimum of the corresponding elements of its arguments,[23] and $\boldsymbol{c}^*$ is the vector of optimal expenditure levels $\{c^{*j}\}$. For any vector $\boldsymbol{a}(0)$ of liquid asset balances, equilibrium requires that $\boldsymbol{c}(0)$ be a fixed point of (4.6). This is a multidimensional generalization of the "Keynesian cross" diagram commonly used to explain the derivation of the fiscal multipliers in Keynes (1936, chap. 3).

For any vector $\boldsymbol{a}(0) >> \mathbf{0}$, the right-hand side of (4.6) defines a positive concave mapping of the kind for which the results summarized in Cavalcante *et al.* (2016) allow one to establish that there is a unique fixed point.[24] Thus equation (4.6) has a unique solution $\boldsymbol{c}(0) = \bar{\boldsymbol{c}}(\boldsymbol{a}(0))$. There is furthermore a uniquely defined extension of this vector-valued function $\bar{\boldsymbol{c}}(\boldsymbol{a}(0))$ to the entire domain of vectors such that $\boldsymbol{a}(0) \geq \mathbf{0}$. In the case of zero initial liquid assets, for example, this means defining $\bar{\boldsymbol{c}}(\mathbf{0})$ to be the vector of expenditure levels in the limit as $\boldsymbol{a}(0) \to \mathbf{0}$, characterized in section 2.2.

Associated with this solution for the vector of sectoral expenditure levels $\boldsymbol{c}(0)$ are corresponding solutions for sectoral output levels and end-of-period financial positions. The vector $\boldsymbol{y}(0)$ of sectoral output levels is given by (2.6). If we let $\boldsymbol{b}(0)$ be the N-vector the jth element of which is $b^j(0)$, then the vector of sectoral end-of-period financial positions is given by

$$\boldsymbol{b}(0) \;=\; \boldsymbol{a}(0) \;+\; \bar{p}\cdot(\boldsymbol{y}(0) \;-\; \boldsymbol{c}^*).$$

[22]See Appendix A.3 for a more formal demonstration of the result.

[23]That is, it is the meet of the two vectors, if $\mathbb{R}^n$ is treated as a lattice with the partial order $\leq$.

[24]See Appendix B.2 for details of the proof of this claim and the further characterization of the solution in the next section.

4.3 The matrix of fiscal transfer multipliers

The dependence of the solution for $\boldsymbol{c}(0)$ on the vector of initial asset balances $\boldsymbol{a}(0)$ allows us to compute the Keynesian multipliers associated with fiscal transfers of different types. Note that rather than there being a single "transfer multiplier," in our model there is an $N \times N$ matrix $\boldsymbol{M}$ of multipliers, with an element M_{jk} to indicate the effect of an additional dollar of transfers to sector k on total expenditure by sector j for each pair $j, k = 1, 2, \ldots, N$. The elements of the matrix are partial derivatives,

$$M_{jk} \equiv \bar{p} \cdot \frac{\partial c^j(0)}{\partial a^k(0)},$$

understood to be right derivatives (which are everywhere well-defined, even though the right derivatives can differ from the corresponding left derivatives at values of $\boldsymbol{a}(0)$ where the borrowing constraint just ceases to bind for some sector).[25]

The matrix of multipliers can easily be determined from the solution to the "Keynesian cross" equation system (4.6). For any vector $\boldsymbol{a}(0)$, there is a set C of sectors that are borrowing-constrained, in the sense that $c^j(0) < c^{*j}$ (note that this may be the empty set), and a complementary set (containing at least one sector) that is unconstrained. For all vectors $\boldsymbol{a}(0)$ that are close enough to $\boldsymbol{0}$, we have shown that the set of unconstrained sectors is U_0, the set of sectors j for which the maximum value is achieved in the problem on the right-hand side of (2.9); hence at such points the set of constrained sectors will be C_0, the complement of U_0. We can further show that increasing any element of $\boldsymbol{a}(0)$ can only reduce the set of sectors that are borrowing-constrained; hence for any $\boldsymbol{a}(0) \geq \boldsymbol{0}$, the set of constrained sectors C is necessarily an element of $\mathcal{C}$, the set of all subsets of C_0.

Now suppose that we know the set $C \in \mathcal{C}$ that specifies the set of constrained sectors in the case of initial assets $\boldsymbol{a}(0)$. (Because our definition specifies that $c^j(0)$ is strictly less than c^{*j} for each of the constrained sectors, these will continue to be the constrained sectors in the case of any small enough increase in the vector of initial assets.) Then we know, for each sector j, which of the two terms on the right-hand side of (4.5) $c^j(0)$ is equal to; this allows us to replace the nonlinear equation system (4.6) by a system of linear equations, that must hold locally at $\boldsymbol{a}(0)$ and for any $\boldsymbol{a} \geq \boldsymbol{a}(0)$ close enough to it.

If we order the sectors so that all of the sectors in C (if any) come first, then the matrices $\boldsymbol{A}$ and $\boldsymbol{M}$ can be partitioned as

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_{CC} & \boldsymbol{A}_{CU} \\ \boldsymbol{A}_{UC} & \boldsymbol{A}_{UU} \end{bmatrix}, \qquad \boldsymbol{M} = \begin{bmatrix} \boldsymbol{M}_{CC} & \boldsymbol{M}_{CU} \\ \boldsymbol{M}_{UC} & \boldsymbol{M}_{UU} \end{bmatrix}, \tag{4.7}$$

where submatrix $\boldsymbol{A}_{CC}$ measures the share of spending by each of the constrained sectors on the products of other constrained sectors, and so on.[26] In terms of this notation, the local

[25] We could alternatively (and perhaps more traditionally) define a matrix $\boldsymbol{M}^{\boldsymbol{Y}}$ of multipliers indicating the effects of transfers to each of the sectors on the output of each of the sectors. But the matrix $\boldsymbol{M}^{\boldsymbol{Y}}$ of output multipliers can easily be computed from the matrix $\boldsymbol{M}$ of expenditure multipliers, using the relation $\boldsymbol{M}^{\boldsymbol{Y}} = \boldsymbol{A}\boldsymbol{M}$ which follows from (2.6); the reverse is not true, because the matrix $\boldsymbol{A}$ is non-invertible. Hence we focus on the matrix of expenditure multipliers $\boldsymbol{M}$.

[26] Note that the ordering of sectors required for this notation, as well as the partitioning of the rows and columns, depends on the set C. Thus $\boldsymbol{A}_{CC}$ has a different meaning for different vectors $\boldsymbol{a}(0)$, even though the matrix $\boldsymbol{A}$ can be given a representation that is independent of $\boldsymbol{a}(0)$.

version of (4.6) can be written as

$$\begin{bmatrix} \hat{\boldsymbol{c}} \\ \check{\boldsymbol{c}} \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{a}} \\ \check{\boldsymbol{a}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{A}_{CC} & \boldsymbol{A}_{CU} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{c}} \\ \check{\boldsymbol{c}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{c}}^* \\ \check{\boldsymbol{c}}^* \end{bmatrix}.$$

Here $\hat{\boldsymbol{c}}$ is the vector of expenditures $c^j(0)$ for the sectors $j \in C$, $\check{\boldsymbol{c}}$ is the vector of expenditures for sectors $j \notin C$; the vectors $\hat{\boldsymbol{a}}$ and $\check{\boldsymbol{a}}$ similarly collect the values of $a^j(0)/\bar{p}$ for the two groups of sectors; and the vectors $\hat{\boldsymbol{c}}^*$ and $\check{\boldsymbol{c}}^*$ collect the values of c^{*j} for the two groups of sectors.

For any $C \in \mathcal{C}$, we can show that $\boldsymbol{A}_{CC}$ has all of its eigenvalues inside the unit circle. It then follows that the matrix $\boldsymbol{I} - \boldsymbol{A}_{CC}$ must be invertible, and the linear local system has a unique solution of the form

$$\boldsymbol{c}^{loc}(\boldsymbol{a}(0);\, C) \;=\; \boldsymbol{M} \begin{bmatrix} \hat{\boldsymbol{a}} \\ \check{\boldsymbol{a}} \end{bmatrix} + \boldsymbol{N} \begin{bmatrix} \hat{\boldsymbol{c}}^* \\ \check{\boldsymbol{c}}^* \end{bmatrix}, \tag{4.8}$$

where

$$\boldsymbol{M} \;=\; \begin{bmatrix} (\boldsymbol{I} - \boldsymbol{A}_{CC})^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \qquad \boldsymbol{N} \;=\; \begin{bmatrix} \boldsymbol{0} & (\boldsymbol{I} - \boldsymbol{A}_{CC})^{-1}\boldsymbol{A}_{CU} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix}.$$

Here we have written the set C as an argument of the function, because there is a separate function of this kind for each possible choice of $C \in \mathcal{C}$.

Note that the matrix $\boldsymbol{M}$ is just the matrix of transfer multipliers for which we wish to solve. (Under our definition of C, the set C must remain the same in the case of small increases in any of the elements of $\boldsymbol{a}(0)$. Hence the solution (4.8) continues to apply, and the matrix $\boldsymbol{M}$ in this solution is the matrix of right derivatives.) Note that the multipliers are all zero, except those in the block $\boldsymbol{M}_{CC}$, indicating the effects of transfers to borrowing-constrained sectors on the spending by other borrowing-constrained sectors. This submatrix can equivalently be written in the form

$$\boldsymbol{M}_{CC} \;=\; \boldsymbol{I} + \boldsymbol{A}_{CC} + (\boldsymbol{A}_{CC})^2 + (\boldsymbol{A}_{CC})^3 + \ldots \tag{4.9}$$

Since $\boldsymbol{A}_{CC} \geq \boldsymbol{0}$, it follows that $\boldsymbol{M}_{CC} \geq \boldsymbol{0}$ as well.

The solution (4.9) expresses the total "multiplier effect" of fiscal transfers as the sum of a "first-round" effect (a unit effect on spending by the sector receiving the transfer), a "second-round" effect (additional spending resulting from the increases in income due to the "first-round" effects), and so on. Note, however, that in our model even the "first-round" effects exist only in the case of transfers to borrowing-constrained units; "second-round" effects exist only to the extent that "first-round" spending increases involve purchases from borrowing-constrained units, and so on.[27] This implies that effects beyond the "first-round" effect can exist only if $N > 2$;[28] hence Guerrieri *et al.* (2020) find no such effects in their baseline model.

[27]The conclusion that the multiplier is zero in the case of transfers to sectors that are not borrowing-constrained depends on the simplification of considering the limiting case in which $\beta \to 1$. When $\beta < 1$, there is instead a small effect on the current spending of unconstrained units of receiving greater-than-average transfers, so that their intertemporal budget increases despite the anticipation of higher future taxes. Nonetheless, the effects on current spending by such units remain small, under realistic assumptions about the discount factor, because of their desire to smooth expenditure over a long horizon.

[28]If $N = 2$, the set of constrained sectors C must consist only of sector 1, as there must be at least one unconstrained sector, regardless of the size of $a(0)$. And $A_{11} = 0$, since no one can purchase sector-1 goods during the pandemic. Hence $A_{CC} = 0$, and all higher-order terms vanish.

We see that we can solve for the complete matrix of (local) multipliers if we know the set of constrained sectors C for any vector of initial assets $\boldsymbol{a}(0)$. This can be determined in the following way. The global solution function $\bar{\boldsymbol{c}}(\boldsymbol{a}(0))$ is just the lower envelope of the finite collection of candidate local solutions specified in (4.8):

$$\bar{\boldsymbol{c}}(\boldsymbol{a}(0)) \;=\; \min_{C \in \mathcal{C}} \boldsymbol{c}^{loc}(\boldsymbol{a}(0); C). \tag{4.10}$$

The set C of borrowing-constrained sectors for any vector $\boldsymbol{a}(0)$ is then given by the solution to the minimization problem on the right-hand side of (4.10). When this problem has a unique solution (the generic case), this is the set C that defines the matrix of multipliers $\boldsymbol{M}$. If there are multiple choices of C that equally minimize the objective, one is at a boundary between two different piecewise-linear regions of the solution function; in this case the values of the derivatives depend on the direction in which one changes $\boldsymbol{a}(0)$, and this also determines the appropriate choice of C from among the set of minimizers. (If one wants to compute right derivatives, as assumed above, then one must select the set C that continues to minimize the objective when elements of $\boldsymbol{a}(0)$ are slightly increased. This corresponds to the definition of C given above.)

Equation (4.10) implies that each of the component functions $\bar{c}^j(\boldsymbol{a}(0))$ of the vector-valued solution function is a continuous, piecewise linear function; weakly increasing in each of the elements of the vector $\boldsymbol{a}(0)$; a concave function of $\boldsymbol{a}(0)$; non-negative and bounded above by c^{*j}. (Some examples of the effects of increasing $\boldsymbol{a}(0)$ in a particular direction are plotted in Figure 3 below.) As one further increases the size of the period-zero transfers, progressively fewer sectors come to be borrowing-constrained; once a given sector j ceases to be borrowing-constrained, it continues to be unconstrained in the case of additional transfers of any size and composition. As additional sectors cease to be borrowing-constrained, the set C shrinks, and the number of non-zero elements in the sums (4.9) shrinks; hence all elements of the matrix of multipliers are non-increasing as the size of transfers is increased. If transfers are large enough (in a sense made precise below), all sectors are unconstrained ($C = \emptyset$), and $\boldsymbol{c}(0) = \boldsymbol{c}^*$. Beyond this point, all multipliers fall to zero.

4.4 Examples

Our model implies that the multiplier effects of fiscal transfers can be quite different, depending both on the sectors receiving the transfer and which sectors' expenditure we are concerned with. One reason why it matters which sectors receive the transfers is that the marginal propensity to consume out of additional income is different for units in different sectors, owing to differences in the degree to which sectors are borrowing-constrained, as stressed by Oh and Reis (2012). But this is not the only difference: in our model, the MPC is 1 for all constrained sectors and zero for all unconstrained sectors, but it also generally matters how transfers are allocated among the different borrowing-constrained sectors, as the size of "higher-round effects" are generally different for different sectors. Moreover, it is not only the effect of transfers on aggregate spending that matters for welfare.

As an example, consider again the chain network with fraction λ of out-of-sector purchases, and suppose that we compute the multipliers for small additional transfers, starting from liquid asset balances that satisfy the inequality

$$\sum_{j \neq N} a^j(0) \ < \ \lambda \bar{p} \bar{y}. \qquad (4.11)$$

When this inequality is satisfied, sector N is the only unconstrained sector (as we showed above was true in the limit as $a(0) \to 0$), so that the set C consists of sectors $\{1, 2, \ldots, N-1\}$. The matrix of transfer multipliers is in this case equal to

$$\boldsymbol{M} \ = \ \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 & 0 \\ \lambda^{-1} & \lambda^{-1} & 0 & \ldots & 0 & 0 \\ \lambda^{-1} & \lambda^{-1} & \lambda^{-1} & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \lambda^{-1} & \lambda^{-1} & \lambda^{-1} & \ldots & \lambda^{-1} & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 \end{bmatrix}.$$

The aggregate expenditure multiplier for transfers to sector k can be obtained by summing column k of the matrix $\boldsymbol{M}$; this is largest (equal to $1 + (N-2)/\lambda$) for transfers to sector 1, and smallest (zero) for transfers to sector N. The multiplier effect of uniformly distributed transfers on sector j spending can be obtained by averaging the elements of row j of the matrix; these are largest (equal to $(N-1)/(\lambda N)$) for sector $N-1$, and smallest (again zero) for sector N.

This latter type of differentiation of alternative multipliers is relevant for calculation of the welfare effects of transfer policies, since the marginal utility of additional spending varies across sectors. For example, when (4.11) holds, the marginal utility of additional real expenditure by any sector $j \neq N$ is given by

$$\mu^j \ \equiv \ u'(c^j(0)/(1-\alpha_{1-j}) \ = \ u'(\lambda^{-1} \sum_{k=1}^{j} a^k(0)),$$

while for sector N it is

$$\mu^N \ \equiv \ u'(c^N(0)/(1-\alpha_1)) \ = \ u'(\bar{y}).$$

It follows that in the case of any $\boldsymbol{a}(0) >> \boldsymbol{0}$ satisfying (4.11),

$$\mu^1 \ > \ \mu^2 \ \ldots \ \mu^{N-1} \ > \ \mu^N.$$

The welfare effect of transfers to each of the different sectors is then given not by $\boldsymbol{e}'\boldsymbol{M}$ (the vector of column sums), but by $\boldsymbol{\mu}'\boldsymbol{M}$, where $\boldsymbol{\mu}$ is the N-vector with jth element equal to μ^j.

The above example illustrates that it is possible, in principle, for fiscal transfer multipliers to be sizeable. (In the case of the numerical example with $N = 5$ and $\lambda = 0.8$ shown in the right panel of Figure 1, the aggregate expenditure multiplier for a uniformly distributed lump-sum transfer when asset balances satisfy (4.11) is equal to 2.45, while the aggregate expenditure multiplier for transfers targeted to units in sector 1 only would equal 4.75.) But their size depends not only on how they are targeted, but on the network structure of payments. In the case of the uniform network structure shown in the left panel of Figure 1, the aggregate expenditure multiplier for a uniformly distributed lump-sum transfer is only 0.20, even when initial asset balances are extremely small.

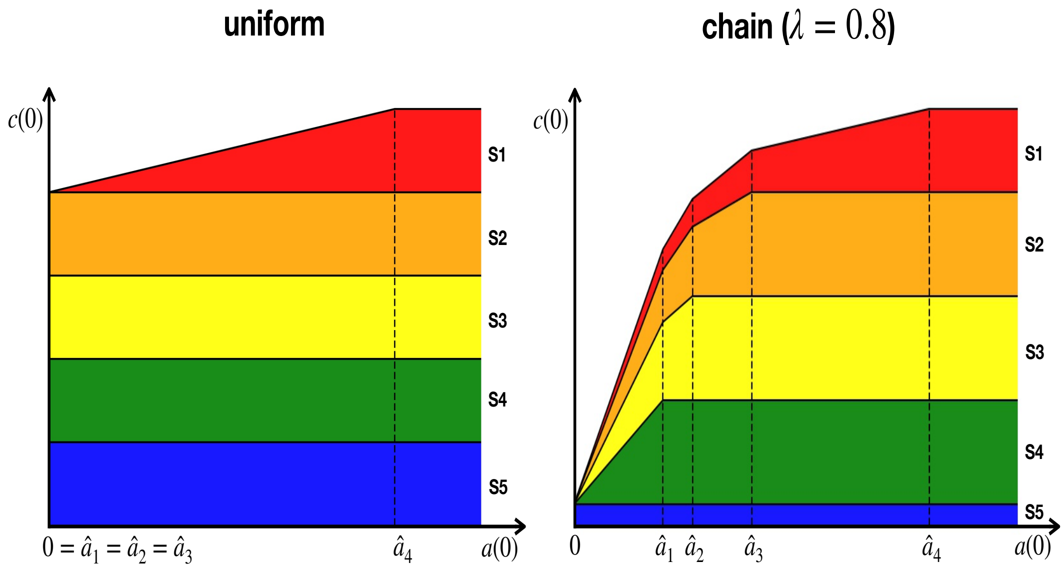

Figure 3: Equilibrium expenditure as a function of total liquid assets $a(0)$ after any transfers in response to the shock, in the case of the two network structures shown in Figure 1. Here liquid assets are assumed to be equally divided among the 5 sectors.

For a given network structure, the fiscal transfer multipliers depend on the existing level and distribution of initial asset balances, as these determine the sector for which the borrowing constraint binds. Figure 3 indicates how the level and composition of aggregate expenditure $c^{agg}(0)$ change as total initial liquid assets $a(0)$ (including fiscal transfers in response to the pandemic shock) are increased, under the assumption that initial liquid assets are equally distributed across sectors ($a^j(0) = a(0)/N$ for each j). The two panels present the results for the two different network structures in shown in the corresponding panels of Figure 1. In each panel, the upper envelope plots $c^{agg}(0)$ as a function of $a(0)$, and the differently shaded regions decompose aggregate expenditure into the contributions from expenditure by each of the sectors. Thus the figure shows the effects on both aggregate and sectoral expenditure levels of uniformly distributed lump-sum transfers, assuming that one starts from some uniform distribution of liquid asset balances. The left-most point in each panel shows the composition of expenditure in the limit as $a(0) \to 0$, already reported in Figure 2. The transfer multipliers can be seen from the rate at which the vertical height of each of the regions increases as one moves to the right in the figure (corresponding to increasing $a(0)$).

In the case of the chain network (the right panel of the figure), in the limiting case of a negligible quantity of liquid assets, aggregate expenditure is only 4 percent of its normal level, and only 5 percent of the efficient level (shown by the height of the upper envelope in the rightmost part of the figure); essentially all expenditure is by sector 5, as shown in Figure 2. The multiplier effects of a uniformly distributed transfer are in this case substantial (except that there is no increase in spending by sector 5: in the limit as $\beta \to 1$, as assumed here, all of the transfer to units in sector 5 is saved). This continues to be the case for all values of $a(0)$ less than the critical value $\hat{a}_1$. Once $a(0) = \hat{a}_1$, however, the borrowing constraint ceases to bind for sector 4. Additional transfers beyond that point no longer increase spending by either sector 4 or sector 5, and the aggregate multiplier falls from 2.45 to 1.45. Once $a(0) = \hat{a}_2$, the borrowing constraint ceases to bind for sector 3 as well, and the aggregate multiplier falls to 0.70; for $a(0) \geq \hat{a}_3$, the borrowing constraint ceases to bind for sector 2, and the aggregate multiplier falls to only 0.20. Finally, for values of $a(0) \geq \hat{a}_4$, no borrowing constraints bind, and additional transfers beyond that point have no further effect on spending by any sector (the aggregate multiplier and all sectoral multipliers are zero). Note that in this case, the first-best allocation of resources is achieved, so that no further demand stimulus is needed.

In the case of the uniform network (the left panel of Figure 3), the logic is similar, but the values $\hat{a}_1 = \hat{a}_2 = \hat{a}_3$ have already all been reached at $a(0) = 0$, since even when liquid assets are negligible, it is already the case that borrowing constraints bind only in sector 1, as shown in the left panel of Figure 2. For any values $0 < a(0) < \hat{a}_4$, additional uniformly distributed transfers increase spending only in sector 1, and since sector 1 receives only 1/5 of the transfers, the aggregate multiplier is only 0.20 (as is also true for levels of liquid assets between $\hat{a}_3$ and $\hat{a}_4$ in the right panel). Once $a(0) \geq \hat{a}_4$, borrowing constraints no longer bind in any sector, the first-best allocation of resources (as indicated in the left panel of Figure 2) is achieved, and additional transfers have no effect.

5 Stabilization and Welfare

We have seen that fiscal transfers can have a significant effect on the equilibrium allocation of resources, with at least some multipliers being quite large. But we should care, not only about whether aggregate demand can be increased, but about whether the policy increases welfare; we have seen in section 3 that these two questions are not equivalent, even when aggregate output is below the efficient level. We turn to the effects on welfare of policies that involve fiscal transfers, again limiting our attention to the calculations for the limiting case in which $\beta \to 1$.

of fiscal transfers on welfare. In the limiting case in which $\beta \to 1$, we can neglect the effects of alternative policies on the allocation of resources in periods $t \geq 1$, and focus on the effects of policies on the value of W_0, the period-zero contribution to (1.4).

5.1 Transfer policy and welfare

We have noted in section 3 that use of monetary policy to increase aggregate demand in response to a pandemic shock may decrease welfare, even though y^{agg} remains less than y^*. Instead, we can show that when $\beta \to 1$, fiscal transfers cannot reduce our ex-ante welfare measure (1.4). And as long as at least part of the transfers to go to sectors that are borrowing-constrained (so that the transfers have an effect), lump-sum transfers in period zero necessarily increase welfare, regardless of how they are targeted. In this section, we suppose that monetary policy does not respond to the shock, so that $i(0) = \bar{\imath}$, as assumed in section 4.

The ex-ante welfare measure (1.4) can be written as

$$\sum_{j=1}^{n} U^j(0) \;+\; \frac{\beta}{1-\beta}\sum_{j=1}^{N}[\bar{U}^j - U^*],$$

where $\bar{U}^j$ is the stationary level of the utility flow $U^j(t)$ for all $t \geq 1$ in the stationary equilibrium for $t = 1$ onward associated with a given policy,[29] and U^* is the value of that stationary utility flow (the same for all j) in the first-best allocation of resources. Subtracting a constant from the welfare measure does not affect the welfare ranking of alternative allocations of resources, but has the advantage of providing a measure with a well-behaved limit as $\beta \to 1$. In particular, we can show that[30]

$$\lim_{\beta\to 1} \frac{\beta}{1-\beta}\sum_{j=1}^{N}[\bar{U}^j - U^*] \;=\; 0 \tag{5.1}$$

for all of the policies that we consider. (As noted above, in this limiting case, the marginal utility of additional wealth transfer to period $t = 1$ is a constant, the same for all j, and the same regardless of the assets that each of the sectors carry over to that period. Thus the average continuation utility for the different sectors depends only on the average assets carried into the period, in excess of the present value of tax collections in periods $t \geq 1$; but the latter quantity must exactly cancel the former one, when one averages across sectors.) It follows that in the limit as $\beta \to 1$, the welfare ranking of alternative policies depends only on the value of $W_0 \;=\; \sum_{j=1}^{N} U^j(0)$, the period-zero contribution to (1.4).

We next show that lump-sum transfers cannot reduce the value of W_0. Note that we can express this quantity as a sum of terms

$$\begin{aligned} W_0 \;&=\; \sum_{j=1}^{n} U^j(0) \\ &=\; \sum_{j=1}^{n}[\sum_{k\in K} \alpha_k u(c^j_{j+k}(0)/\alpha_k;\, \bar{\xi}) \;-\; v(y_j(0);\, \bar{\xi})] \\ &=\; \sum_{j=1}^{n}\sum_{k\in K}[\alpha_k u(c^j_{j+k}(0)/\alpha_k;\, \bar{\xi}) - u'(\bar{y};\, \bar{\xi})c^j_{j+k}(0)] \;+\; \sum_{j=1}^{N}[v'(\bar{y};\, \bar{\xi})y_j(0) - v(y_j(0);\, \bar{\xi})] \end{aligned}$$

Each of the terms of the form

$$\alpha_k u(c^j_{j+k}(0)/\alpha_k;\, \bar{\xi}) - u'(\bar{y};\, \bar{\xi})c^j_{j+k}(0) \tag{5.2}$$

is necessarily increased if $c^j_{j+k}(0)$ is increased, because of the strict concavity of $u(c;\, \bar{\xi})$ in c, and the fact that if $c^j_{j+k}(0)$ can be increased, it must be to a level no greater than $\alpha_k\bar{y}$, since (2.5) and (2.8) must both hold regardless of the level of fiscal transfers. Since $c^j_{j+k}(0)$ cannot

[29] See Appendix A.2 for the calculation of this quantity.

[30] See Appendix A.4 for the demonstration.

be decreased by fiscal transfers, however they may be targeted, it follows that terms of this kind necessarily are not decreased. Similarly, each of the terms of the form

$$v'(\bar{y};\,\bar{\xi})y_j(0) - v(y_j(0);\,\bar{\xi})$$

is necessarily at least weakly increasing in $y^j(0)$, because of the convexity of $v(y;\,\bar{\xi})$ in y, and the fact that if $y_j(0)$ can be increased, it must be to a level no greater than $\bar{y}$, since (as just argued) the post-transfer values of each of the quantities $c_j^h(0)$ must be no greater than $\alpha_{j-h}\bar{y}$, so that we must have

$$y_j(0) \;=\; \sum_{h=1}^{N} c_j^h(0) \;\leq\; \sum_{h=1}^{N} \alpha_{j-h}\bar{y} \;=\; \bar{y}$$

post-transfer. And since none of the $c_j^h(0)$ can be decreased by fiscal transfers, $y_j(0)$ cannot be decreased, and it follows that terms of this kind necessarily are not decreased either.

Thus we have expressed W_0 as a sum of terms, none of which can be decreased as a result of any non-negative fiscal transfers in period zero. Furthermore, if $a^j(0)$ is increased for some sector j that is borrowing-constrained prior to the transfers, in the sense that $c^j(0) < c^{*j}$ (which is necessary if the transfers are to have any effect on the allocation of resources in period zero at all), then the transfers increase $c^j(0)$ and hence each of the $c_{j+k}^j(0)$ such that $k \neq 1$ and $k \in K$. In particular, $c_j^j(0)$ will necessarily increase. Thus at least one of the quantities $c_{j+k}^j(0)$ will increase, and so at least one of the terms of the form (5.2) will increase. Since no other terms can decrease, the sum W_0 must increase. Thus as stated above, lump-sum transfers in period zero must increase welfare, regardless of how they are targeted and how large they may be, as long as at least some of the transfers go to a borrowing-constrained sector.

It follows that in the case of a transfer policy that gives some positive fraction of the transfers to each sector, welfare will be continually increasing as the size of the transfers is increased, until one reaches a situation in which no sector is borrowing-constrained any longer. In the case of equal initial liquid balances in all sectors and uniform transfers, so that $a^j(0) = a(0)/N$ for each j, this occurs when the size of the transfers satisfies

$$a(0) \;\geq\; (1-\alpha_0)N\bar{p}\bar{y}. \tag{5.3}$$

(When $N = 5$, this is the quantity called $\hat{a}_4$ in Figure 3.) Beyond this point, additional transfers have no further effect; but because $c^j(0) = c^{*j}$ for all j in this case, the first-best allocation of resources (and hence the maximum feasible value of W_0) has been achieved.

While the uniform transfer policy is a simple one, it is not necessary in order for the first-best allocation of resources to be achieved. In fact, one can show that a necessary and sufficient condition for the first-best allocation of resources to be achieved is that[31]

$$a^1(0) \;\geq\; (1-\alpha_0)\bar{p}\bar{y}. \tag{5.4}$$

(Note that under the assumption that $a^j(0) = a(0)/N$ for each j, this implies condition (5.3).) What is crucial is that sector 1 (the one impacted by the pandemic shock) receive

[31] See Appendix B.3 for the demonstration.

transfers to make up the income that, in the absence of the lockdown, units in sector 1 would receive from sales to buyers outside their sector; or alternatively, that units in sector 1 receive transfers of the size that they would receive under the "retrospective insurance" scheme discussed in section 4.1. It is not important that units in any other sectors receive transfers (though it also does not matter if they do, since any unnecessary transfers are simply saved). Indeed, as shown in section 4.1, it would even be possible to assign negative transfers to (that is, to tax) those sectors $j \neq 1$ that would purchase sector-1 services in the absence of the lockdown.

Thus unlike what can be achieved with monetary policy, sufficiently large transfers in period zero can bring about the first-best allocation of resources. Moreover, the transfers need not be carefully targeted to have this effect. Lump-sum transfers are thus more effective than monetary policy as a means of preventing the distortions that would otherwise be created by a pandemic shock.

Transfer policy has two advantages over monetary policy. First, as long as the sector directly impacted by the pandemic receives at least some of the transfers, fiscal transfers increase spending by that sector, which monetary policy cannot. And second, even if some transfers also go to sectors that are already able to spend at the efficient level, this will not increase current spending by those sectors beyond that level (leading to inefficient over-consumption and/or inflationary pressures);[32] looser monetary policy instead necessarily increases spending to an undesirable extent in all of the non-borrowing-constrained sectors.

Of course, our conclusion (based on a highly simplified analysis) that the first-best allocation is necessarily achievable through lump-sum transfers, no matter how indiscriminately they may be targeted, as long as they are large enough, neglects two considerations. First, it is only true that the sectoral distribution of asset balances at the end of period zero does not matter for the ex ante welfare objective in the limiting case $\beta \to 1$ (assumed above to simplify the calculations). Under the more realistic assumption that $\beta < 1$, achieving the first-best allocation of resources requires not only that period zero liquid assets be large enough for borrowing constraints not to bind in any sector, but also that end-of-period financial assets $b^j(0)$ be equal for all sectors.

While the first of these conditions is satisfied by any transfer policy that results in a vector of initial assets satisfying (5.4), the second condition is only satisfied under much more special circumstances: the vector $\boldsymbol{a}(0)$ must belong to the one-parameter family

$$a^j(0) \;=\; \frac{a(0)}{N} \;+\; \bar{a}^j(s=1) \tag{5.5}$$

for arbitrary $a(0) > 0$, where $\bar{a}^j(s)$ is defined in (2.2). This requires a particular sectoral composition of transfers. (For example, in the case of the chain network illustrated in Figure

[32]Coibion *et al.* find that only a minority of the households receiving transfers under the CARES Act used them entirely to increase current spending, and that a substantial fraction saved the entire amount; they argue that this indicates that the transfers were relatively ineffective, and suggest that because of this such policies should not be the government's only response to such a crisis. Discussions of this kind seem to assume that the goal of policy is simply to increase spending and output, regardless of who consumes more of what; the analysis here shows that the fact that many transfers will be saved (when they are indiscriminately targeted) is actually what makes this kind of policy more efficient than other ways of stimulating aggregate demand.

1, it requires that sector 1 receive the largest transfers and sector 5 the smallest.) Note however that the welfare-optimal composition of transfers is not generally the one with the largest aggregate expenditure multiplier. (In the chain network example, that would mean giving all transfers to sector 1.)

Second, our welfare calculations have assumed that the public debt issued in period zero to finance lump-sum transfers in response to the pandemic shock can be serviced using funds raised by lump-sum taxes $\{\tau(t+1)\}$ in later periods. In practice, the only available sources of government revenue in later periods will be distorting taxes of one type or another, and an increase in the deadweight losses associated with such taxes must be counted among the costs of lump-sum transfers in response to the pandemic shock. While we do not propose to undertake here an analysis of optimal policy taking such deadweight losses explicitly into account, it is clear that among the transfer policies consistent with conditions (5.5), a lower value of $a(0)$ will result in higher welfare (since deadweight losses in periods $t \geq 1$ are lower, while borrowing constraints continue not to bind for any sector in period zero).

Thus targeted transfer policies have a second advantage over indiscriminate transfers: not only do they help to avoid the distortions resulting from an unequal sectoral distribution of assets at the end of period zero, but they reduce the extent to which the public debt must be increased in order to bring about an equilibrium in which borrowing constraints do not bind, despite the asymmetric effects of the pandemic shock. If transfers must be distributed uniformly across all sectors, then (5.4) can be satisfied only if $a(0) \geq (1-\alpha_0)N\bar{p}\bar{y}$; if instead initial liquid assets are $\tilde{a}(0)/N$ for each sector, but all transfers in response to the pandemic shock are given to sector 1, then (5.4) will be satisfied as long as

$$a(0) \;\geq\; \frac{N-1}{N}\tilde{a}(0) \,+\, (1-\alpha_0)\bar{p}\bar{y}.$$

This only requires an increase in the public debt that is $1/N$ the size of the increase required in the case of the uniformly distributed transfers.

If it is possible to levy taxes on sectors other than sector 1 to pay for the transfers to units in sector 1, this will be even better: one can use lump-sum taxes and transfers to prevent borrowing constraints from binding for any sector, without any increase in the public debt (and hence any increase in deadweight losses in future periods). In particular, the "retrospective insurance" policy discussed in section 2.1 results in a vector of initial liquid asset positions that satisfies both conditions (5.4) and (5.5), without requiring any increase in the public debt.

5.2 Monetary easing combined with fiscal transfers

If one grants that it is likely not costless to provide transfers of the magnitude necessary to satisfy condition (5.4) and achieve the first-best allocation of resources (again considering for simplicity the case in which $\beta \to 1$), is there then a case for monetary easing along with some more modest level of fiscal transfers? In section 3, we have considered the effect of an interest-rate cut in the case of a negligible level of liquid assets. We now consider interest-rate policy in the case that initial liquid assets are larger (possibly as a result of fiscal transfers in response to the pandemic shock), but still not large enough to ensure that (5.4) is satisfied.

As discussed in section 3, a change in $i(0)$ results in condition (2.8) being replaced by (3.2). If we let $f_j \equiv (1-\alpha_{1-j})$ be the factor for each sector such that $\hat{c}^j(i(0)) = f_j \cdot \hat{y}(i(0))$, and let $\boldsymbol{f}$ be the N-vector the jth element of which is f_j, then the equilibrium conditions (4.6) can be written more generally as

$$\boldsymbol{c}(0) \;=\; \mathbf{min}\left\{\frac{1}{\bar{p}}\boldsymbol{a}(0) \;+\; \boldsymbol{A}\boldsymbol{c}(0),\;\; \hat{y}(i(0))\cdot\boldsymbol{f}\right\}, \tag{5.6}$$

which reduces to (4.6) when $i(0) = \bar{\imath}$. The same argument as before can be used to show for any vector $\boldsymbol{a}(0) >> \mathbf{0}$ and any value $\hat{y}(i(0)) > 0$, (5.6) has a unique solution $\boldsymbol{c}(\boldsymbol{a}(0);\, \hat{y}(i(0)))$. Moreover, the homogeneity of the mapping defined by the right-hand side of (5.6) implies that the solution function $\boldsymbol{c}(\boldsymbol{a};\, \hat{y})$ is homogeneous degree one in its arguments. This means that we can write

$$\boldsymbol{c}(\boldsymbol{a};\, \hat{y}) \;=\; \frac{\hat{y}}{\bar{y}}\cdot\bar{\boldsymbol{c}}\left(\frac{\bar{y}}{\hat{y}}\boldsymbol{a}\right),$$

where $\bar{\boldsymbol{c}}(\boldsymbol{a}(0))$ is the solution to (4.6).

We can differentiate this solution function to determine the effects of an interest-rate cut on spending by the various sectors. The effects of the interest-rate cut depend entirely on the extent to which $\hat{y}(i(0))$ is increased. If we again order the sectors so that the borrowing-constrained sectors are ordered first, and partition the vector $\boldsymbol{f}$ into parts $\hat{\boldsymbol{f}}$ and $\check{\boldsymbol{f}}$ corresponding to the constrained and unconstrained sectors respectively, then the effects of a small increase in $\hat{y}(i(0))$ on the equilibrium expenditure vector are given by[33]

$$\frac{\partial\boldsymbol{c}(0)}{\partial\hat{y}} \;=\; \begin{bmatrix} \boldsymbol{M}_{CC}\boldsymbol{A}_{CU} \\ \boldsymbol{I} \end{bmatrix}\cdot\check{\boldsymbol{f}}. \tag{5.7}$$

The terms in (5.7) have a simple interpretation. The effects on $c^j(0)$ for sectors $j \in U$ come from the from the increase in the quantity $\hat{c}^j(i(0)) = f_j \cdot \hat{y}(i(0))$, the level of spending required to satisfy the Euler condition with equality. The interest-rate cut affects spending by sectors $j \in C$ only to the extent that there are demand spill-overs from the increased spending by the unconstrained sectors (indicated by the elements of $\boldsymbol{A}_{CU}$); in the case that such spill-overs exist, the effect on expenditure by the borrowing-constrained sectors is then amplified by the Keynesian multipliers indicated by the elements of $\boldsymbol{M}_{CC}$.[34]

As in the more special case considered in section 3, interest-rate policy affects economic activity, but this does not necessarily mean that an interest-rate cut must increase welfare

[33]Note once again that in the case of a sector for which the two terms on the right-hand side of (4.5) are exactly equal, the sector must be assigned to C or its complement depending on the direction of the derivative that one wishes to compute. If one wishes to calculate the effect of a small *increase* in $i(0)$, the partition of sectors is defined as above (since this means an increase in the elements of $(\bar{y}/\hat{y})\boldsymbol{a}(0)$); but if one wishes to calculate the effect of a further small *decrease* in $i(0)$, this means a decrease in the elements of $(\bar{y}/\hat{y})\boldsymbol{a}(0)$. In the latter case, one must define C as the set of sectors j for which $b^j(0) = 0$, whether or not $c^j(0)$ is smaller than c^{*j}, so that the same sectors belong to C in the case of a small increase in the elements of $\boldsymbol{c}^*$.

[34]Note that result (5.7) is consistent with the analysis in section 3 of the limiting case in which $a(0) \to 0$; in that limit, the right-hand side of (5.7) is equal to the quantity $\theta/\hat{y}$ defined in (3.4) times the eigenvector $\boldsymbol{\pi}$, or equivalently, to $1/\hat{y}$ times $\boldsymbol{c}(0)$. See Appendix C.1 for a demonstration of this equivalence.

when output is inefficiently low. Equation (5.7) implies that an interest-rate cut must increase spending in each of the unconstrained sectors (of which there may be more when liquid asset balances are larger); but because these sectors are not borrowing-constrained, increased spending by them need not increase welfare. Let us again consider the case in which $v(y) = \nu \cdot y$, and suppose that we evaluate derivatives at an initial situation in which $i(0) = \bar{\imath}$. Then

$$u'(c^j(0); \bar{\xi}) \;=\; u'(\bar{y}; \bar{\xi}) \;=\; \sum_{k=1}^{N} A_{kj} v'(y_k(0); \bar{\xi})$$

for each of the sectors $j \notin C$. Then the first-order effect of an increase in spending in any of these sectors is zero.

If in addition $A_{CU} = 0$ (as will necessarily be true in the case of either of the two networks shown in Figure 1, regardless of the assumed vector of liquid asset balances), then (5.7) implies that there will be no increase in spending in any sector $j \in C$. Hence we conclude that the right derivative of W_0 with respect to $\hat{y}$ (and hence the left derivative of W_0 with respect to $i(0)$) is zero when evaluated at $i(0) = \bar{\imath}$. But since W_0 is a strictly concave function of $\hat{y}$,[35] it follows that W_0 is necessarily reduced by any reduction in $i(0)$ below $\bar{\imath}$. In cases of this kind, an interest-rate cut necessarily reduces welfare, just as was concluded in section 3.

More generally, a modest reduction in the interest rate can increase welfare, but larger reductions need not be as effective. An important difference between fiscal transfers and monetary policy as ways of increasing demand is that while additional transfers progressively reduce the number of sectors that are borrowing-constrained (as illustrated in the right panel of Figure 3), additional interest-rate cuts progressively *increase* the number of constrained sectors. The set of constrained sectors C depends monotonically on the vector $(\bar{y}/\hat{y})\boldsymbol{a}(0)$; thus an increase in $\hat{y}(i(0))$ has the same effect on C as a proportional decrease in the vector $\boldsymbol{a}(0)$ (movement from right to left in Figure 3). As discussed above, large enough fiscal transfers (with some positive fraction going to sector 1) necessarily result in $C = \emptyset$; instead, a large enough reduction in the real interest rate in period zero necessarily results in $C = C_0$ (the maximal possible constrained set).

One can show quite generally that no amount of interest-rate reduction can achieve the first-best allocation. Moreover, interest-rate policy cannot reduce the size of the increase in the public debt that is required to eliminate the distortions created by the pandemic shock using lump-sum transfers. Regardless of the choice of $i(0)$, spending in sector 1 will be given by

$$c^1(0) \;=\; \min\left\{\frac{a^1(0)}{\bar{p}},\;\; \hat{y}(i(0)) \cdot f^1\right\}.$$

Thus sector 1 spends the amount required by the first-best allocation if and only if the two inequalities (5.4) and $i(0) \geq \bar{\imath}$ are both satisfied, and at least one holds with equality.

It follows (if only non-negative transfers are possible) that the minimum increase in the public debt consistent with achieving the first-best allocation must be the size of the transfer to sector 1 that is required to get $a^1(0)$ to the level consistent with (5.4).[36] On the other hand,

[35] See the introduction to Appendix C for further discussion.

[36] If instead one supposes that targeted lump-sum taxes can be levied in period zero, then the size of

once (5.4) is satisfied, no further transfers are needed in order for the first-best allocation to be achieved when $i(0) = \bar{\imath}$, as already noted in section 4.4. One can further show that when (5.4) is satisfied, the first-best allocation is achieved *only* if $i(0) = \bar{\imath}$: any higher interest rate will imply inefficiently low spending in all sectors, owing to (3.2), while any lower interest rate will imply inefficiently high spending in the unconstrained sectors.

Thus if we consider the problem of choosing transfer policy and monetary policy jointly, so as to achieve the maximum possible value of W_0, while keeping the public debt $a(0)$ as small as possible consistent with achievement of that maximum value for the period-zero contribution to welfare, we obtain a simple solution: make transfers to sector 1 only to the extent necessary to satisfy (5.4), and maintain $i(0) = \bar{\imath}$, just as under the monetary policy that would be appropriate in the absence of a pandemic shock. The fact that in this statement of the problem we also consider alternative monetary policy responses does not change the nature of the optimal policy.

6 Conclusion

Our results imply that common views about the relative importance of interest-rate policy and fiscal transfers as tools of macroeconomic stabilization require revision in the case of an economic crisis of the kind resulting from the COVID-19 pandemic. In the decades prior to the financial crisis of 2008 (the period of the so-called "Great Moderation"), it had become orthodox to suppose that adjustment of the central bank's interest-rate target alone in response to varying economic conditions should suffice to maintain a reasonable degree of stability of both prices and real activity; cyclical adjustment of the size of taxes and transfers was assigned no important role, though it was understood to be important for macroeconomic stability for there to be a general commitment to maintaining a public debt that did not exceed some maximum "sustainable" level. This view of matters is consistent with the implications of the model presented above, in the case that economic disturbances (whether on the supply side or the demand side) affect all sectors of the economy equally, as assumed in section 1.4.

Following the financial crisis, much more attention has been given to the possibility that the use of interest-rate policy for stabilization purposes can be constrained by the zero lower bound (or at any rate, an effective lower bound in the vicinity of zero) for short-term nominal interest rates; and in this case, both policymakers and academic macroeconomists have been more willing to consider an important role for "fiscal stimulus" policies. Yet even in this case, the research literature has primarily focused on the potential usefulness of counter-cyclical government purchases, rather than transfer policies (with the notable exception of Oh and Reis, 2012). Moreover, the need for counter-cyclical fiscal policy as a stabilization tool has remained controversial. To the extent that it has been considered only necessary in the case of an inability to lower real interest rates enough via the conventional method of immediate reduction of the central bank's target for its policy rate, the availability of other means by which monetary policy can lower real rates (especially the longer-term real rates

public debt required for achievement of the first-best allocation must at least equal the lower bound for $a^1(0)$ specified in (5.4). In either case, the lower bound on the public debt implied by (5.4) is independent of monetary policy.

that are important determinants of current spending decisions), such as forward guidance or central-bank asset purchases, provides a reason to doubt that counter-cyclical fiscal policy is ever really needed.

Current events provide an important reason to reconsider this view. The analysis above, in the context of an admittedly stylized model, implies that conventional interest-rate policy is poorly suited to respond to a pandemic shock. This is not because our model denies that interest-rate cuts should be as effective in stimulating aggregate demand as in the case of other types of shocks; at least in the limiting case analyzed in section 3, the elasticity of aggregate expenditure with respect to changes in the real interest rate is of exactly the same size following a pandemic shock as it is when calculating the appropriate monetary response to an aggregate disturbance of the kind considered in section 1.4. Nor is the problem that the size of real interest-rate reduction required to keep aggregate output at the level associated with an optimal allocation of resources comes into conflict with the interest-rate lower bound. In our model, the real interest rate required to achieve any desired level of aggregate output in period zero can always be implemented using monetary policy, by raising the rate of inflation expected between period zero and period 1 to the extent necessary.[37] The problem, instead, is that while interest-rate policy can increase aggregate spending, it does not generate a pattern of expenditure with the ideal sectoral composition. In particular, interest-rate cuts do nothing to replace the income lost by units in sector 1 as a result of being required to shut down for reasons of public health. Yet these are the economic units for which binding financing constraints create the greatest welfare losses.

Lump-sum fiscal transfers are instead an effective tool of macroeconomic stabilization following a pandemic shock, in a model of this kind. Not only are they able to increase aggregate demand for goods and services during the pandemic (unlike what would be true if they were to be used in response to a negative economy-wide demand or supply shock), but they can increase expenditure of precisely the kinds that are needed to bring about a more efficient allocation of resources (for example, spending by units in sector 1).

Nor does this desirable effect on the composition of demand require that transfers be targeted with any great precision. The reason is that in our model, transfers from the government are effectively a way of relaxing people's borrowing constraints. We assume that people understand that their future tax obligations will be higher as a result of the increase in the public debt; but giving them access to more funds immediately in exchange for expected future payment obligations amounts to allowing them to borrow from the government, at the rate at which the government is able to market its obligations.[38] Depending on the way in which the transfers are targeted, transfer policies may also redistribute from unconstrained sectors to financially-constrained ones; but even in the absence of any sectoral redistribution,

[37]There are a variety of reasons to doubt that the constraint imposed by the effective lower bound can be sidestepped so easily in practice, but we have conducted our analysis under very simple assumptions, both about the nature of nominal rigidities and the degree of central-bank control over expectations about the economy's evolution from period 1 onward, precisely in order to make it clear that the limits to monetary stabilization policy illustrated here are in no way dependent on an interest-rate lower bound becoming a binding constraint.

[38]This conclusion depends, of course, on the assumption that borrowing limits are not affected by the change in anticipated future tax obligations. If instead we were to assume the kind of endogenous borrowing limits analyzed by Bigio *et al.* (2020), transfers would not have this effect, as those authors discuss.

they effectively relax borrowing constraints.

Such a relaxation of borrowing constraints matters only for the sectors for which borrowing constraints would otherwise bind; thus current spending is stimulated only in those sectors where a further increase is needed to achieve an efficient allocation of resources — precisely the opposite of the heterogeneity in sectoral effects resulting from monetary stimulus. Note also that the reason why transfer policies have this felicitous compositional effect in the case of an effective demand failure is precisely the reason why they are relatively ineffective as a means of managing aggregate demand in response to aggregate demand and supply disturbances of the kind considered in section 1.4.

Thus our analysis implies that a simple doctrine according to which either monetary policy (as argued by monetarists, and the orthodoxy of the "Great Moderation" period) or cyclical variation in the government budget (as argued by Keynesians in the decades immediately following the Great Depression) should be the sole instrument of stabilization policy under all circumstances is inadequate. A more sophisticated view would recognize that each tool has its place — monetary policy for correcting imbalances in the intertemporal allocation of aggregate expenditure (of the kind that can arise from the aggregate shocks considered in section 1.4, owing to nominal rigidities), and fiscal policy for correcting imbalances in the circular flow of payments between sectors (when these are too large to be smoothed out using the shock absorbers provided by liquid asset balances, as can occur in the case of pandemic shock).

In the analysis presented here, we have for pedagogical purposes considered only two starkly opposed cases: either disturbances have precisely identical effects on all sectors (as assumed in section 1.4), or they are "pandemic shocks" of a very special sort. Actual economic disturbances are of course much more varied than this. Even in more typical business cycles, all economic disturbances are to some degree asymmetric in their effects on different regions and sectors of the economy; and even in the case of a pandemic, it is not possible to cleanly partition sectors into ones that must be closed down completely on the one hand, and ones whose activities are not affected at all on the other. A more complete theory would allow the cost of production and convenience of consumption of each sector's goods to be affected to differing extents by each possible type of disturbance, as in the rich model of Baqaee and Farhi (2020).

In this case, the relevant distinction would not be between purely symmetric aggregate disturbances and shocks that require complete shutdowns of part of the economy, but rather, between states of the world in which the imbalances in the circular flow of payments are modest enough to be accommodated by economic units' "self-insurance" using their liquid asset balances, and ones in which the severity of the asymmetry in the effects of disturbances are such that such self-insurance is inadequate, and binding financing constraints will result in a serious failure of effective demand, in the absence of fiscal transfers that can provide an additional degree of social insurance. Leijonhufvud (1973) refers to the range of economic situations of the former sort as "the corridor," and argues that it is only when large enough shocks take an economy outside this corridor that Keynesian policies are needed to restore stability. The kind of model sketched here is intended to provide further insight into the reasons why a different kind of policy is needed under less commonplace circumstances. Our account is in many ways consistent with the basic vision of Leijonhufvud's, but it implies that "the corridor" should be defined not as a state in which shocks to the economy are sufficiently small, but rather as one in which they are sufficiently symmetric in their effects on the income and spending of different parts of the economy.

It should also be evident that in a model of this kind, the width of "the corridor" is not determined purely by preferences and technology (the network structure of the payment flows implied by desired expenditures, elasticities of substitution, and so on). It also depends on the quantity of liquid assets held by the various economic units that can serve to buffer transitory variations in the flow of payments; and while the equilibrium distribution of those liquid assets depends on the choices made by the various individual economic units in the economy, in our model the aggregate supply of them depends on the size of the public debt.

Thus maintaining a larger real public debt has the virtue of reducing the extent to which financing constraints distort the equilibrium allocation of resources, as argued in Woodford (1990). Of course, a higher real public debt also has costs, when the revenues required to service it can only be raised using distorting taxes; thus a balance must be struck between these costs and the benefits resulting from a higher average level of liquid balances. But it would be a mistake to consider the optimal level of public debt without taking into account the advantages for macroeconomic stability of making it possible for people to maintain a higher level of liquid asset balances.

References

[1] Auerbach, Alan J., Yuriy Gorodnichenko, and Daniel Murphy, "Fiscal Policy and COVID19 Restrictions in a Demand-Determined Economy," NBER Working Paper no. 27366, June 2020.

[2] Baqaee, David Rezza and Emmanuel Farhi, "Supply and Demand in Disaggregated Keynesian Economies with an Application to the Covid-19 Crisis," NBER Working Paper no. 27152, revised June 2020.

[3] Bigio, Saki, Mengbo Zhang, and Eduardo Zilberman, "Transfers vs. Credit Policy: Macroeconomic Policy Trade-offs During Covid-19," NBER Working Paper no. 27118, May 2020.

[4] Caballero, Ricardo J., and Alp Simsek, "A Model of Asset Price Spirals and Aggregate Demand Amplification of a 'Covid-19' Shock," NBER working paper no. 27044, revised May 2020.

[5] Cavalcante, Renato L.G., Yuxiang Shen, and Sławomir Stańczak, "Elementary Properties of Positive Concave Mappings with Applications to Network Planning and Optimization," *IEEE Transactions on Signal Processing* 64: 1774-1783 (2016).

[6] Céspedes, Luis Felipe, Roberto Chang, and Andrés Velasco, "The Macroeconomics of a Pandemic: A Minimalist Model," NBER Working Paper no. 27228, May 2020.

[7] Coibion, Olivier, Yuriy Gorodnichenko, and Michael Weber, "How Did U.S. Consumers Spend Their Stimulus Payments?" NBER Working Paper no. 27693, August 2020.

[8] Danieli, Ana, and Jane Olmstead-Rumsey, "Sector-Specific Shocks and the Expenditure Elasticity Channel During the COVID-19 Crisis," working paper, Northwestern University, May 2020.

[9] Gantmacher, F.R., *The Theory of Matrices,* volume II, Providence: AMS Chelsea Publishing, 1959.

[10] Goodman, Laurie, and Dan Magder, "Avoiding a COVID-19 Disaster for Renters and the Housing Market," Urban Institute policy brief, April 2020.

[11] Gopal, Prashant, "NYC Rental Market Pushed to Breaking Point by Tenant Debts," *Bloomberg Businessweek,* posted July 8, 2020.

[12] Guerrieri, Veronica, Guido Lorenzoni, Ludwig Straub, and Iván Werning, "Macroeconomic Implications of COVID-19: Can Negative Supply Shocks Cause Demand Shortages?" NBER Working Paper no. 26918, April 2020.

[13] Keynes, John Maynard, *The General Theory of Employment, Interest and Money,* London: MacMillan, 1936.

[14] Leijonhufvud, Axel, "Effective Demand Failures," *Swedish Journal of Economics* 75: 27-48 (1973).

[15] Milne, Alistair, "A Critical Covid 19 Economic Policy Tool: Retrospective Insurance," working paper, Loughborough University, UK, posted on *SSRN*, March 2020.

[16] Oh, Hyunseung, and Ricardo Reis, "Targeted Transfers and the Fiscal Response to the Great Recession," *Journal of Monetary Economics* 59: S50-S64 (2012).

[17] Saez, Emmanuel, and Gabriel Zucman, "Keeping Business Alive: The Government Will Pay," posted online at http://gabriel-zucman.eu/files/coronavirus2.pdf, March 2020.

[18] Woodford, Michael, "Public Debt as Private Liquidity," *American Economic Review* 80(2): 382-388 (1990).

[19] Woodford, Michael, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton: Princeton University Press, 2003.

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APPENDIX

"Effective Demand Failures and the Limits of Monetary Stabilization Policy"

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Here we present additional details of several of the arguments in the main text.

A Value Functions for Stationary Equilibria

Here we consider equilibrium in the case of a pandemic shock in period 0, under the assumption (as in section 2 and after in the main text) that $\xi_t = \bar{\xi}$ at all times with certainty. All uncertainty is then resolved in period 0; it follows that the economy's evolution from period 1 onward must be perfectly predictable in period 0. Because prices are set one period in advance at a level that is expected to clear the market for the good in question, all prices in periods $t \geq 1$ will be set so as to clear markets (just as if they were determined in competitive spot markets). In period 0 instead, the prices of all goods are equal to $\bar{p}$, if the goods are offered for sale, regardless of the sector in which a pandemic shock occurs.

A.1 The case of an ex-ante insurance market

We first consider, as in section 2.1, the case in which state-contingent initial asset positions $\{a^j(0|s)\}$ are determined in an ex-ante insurance market. We assume that monetary policy determines a nominal interest rate $i(0)$ and a price level $P(1)$ such that

$$(1+i(0))\frac{\bar{p}}{P(1)} \;=\; \frac{1}{\beta}, \tag{A.1}$$

as must be true in a decentralized equilibrium that supports the first-best allocation of resources (characterized in section (1.2)). (We further assume that the price-level target $P(1)$ is chosen so that (A.1) can be satisfied by a nominal interest rate $i(0) \geq 0$.)

We wish to show that in this case the first-best allocation of resources represents a decentralized equilibrium, regardless of policy with regard to lump-sum taxes and transfers, and regardless of any other aspects of monetary policy from $t = 1$ onward. In such an equilibrium, we must have equal prices for all goods in every period $t \geq 1$; thus the price of any good in period t must be equal to the price index $P(t)$. (This is necessary for units in each sector j to choose to allocate their expenditure in period t in proportion to the coefficients $\{\alpha_{k-j}\}$, as required by (1.6).) Assuming that in equilibrium borrowing constraints never bind (as we will show to be true), interest rates must satisfy

$$(1+i(t))\frac{P(t)}{P(t+1)} \;=\; \frac{1}{\beta}$$

for each $t \geq 1$, in addition to (A.1).

With these prices and interest rates, units in sector j will indeed choose $c_k^j(t) = \alpha_{k-j}\bar{y}$ for each k in each period $t \geq 1$, as required by the first-best allocation, and would wish to supply a quantity $y_j(t) = \bar{y}$ for each $t \geq 1$, as required for markets to clear at these prices, as long as they enter period 1 with assets $a^j(1) = a(1)/N$. That is, the outstanding public debt must be held in equal quantities by each of the N sectors. This in turn will be true if and only if $b^j(0) = a(0)/N$ for each sector at the end of period zero; and if it is true, units in each sector will choose to hold assets $b^j(t) = a(t)/N$ at the end of each period $t \geq 1$, so that borrowing constraints never bind for any sector. It follows that the assumed prices and interest rates will indeed clear markets in all periods $t \geq 1$.

It thus remains only to show that all units will choose the first-best consumption allocation in period zero, given the predetermined prices $\bar{p}$ for all goods produced by sectors that do not have to shut down, and that they will end period zero with assets $b^j(0) = a(0)/N$, as required in the previous paragraph. If $s = \emptyset$ (no pandemic shock occurs), this requires that $c_k^j(0) = \alpha_{k-j}\bar{y}$ for all k; this will be optimal (given the predetermined prices and the interest rate (A.1)) if and only if units in each sector begin period zero with assets

$$a^j(0|\emptyset) \;=\; a(0)/N.$$

If instead $s = p$ (a pandemic shock to sector p), the first-best allocation requires that $c_k^j(0) = \alpha_{k-j}\bar{y}$ for all $k \neq p$, but $c_p^j(0) = 0$ for each j. This will be optimal if and only if units begin period zero with assets

$$a^j(0|p) \;=\; (a(0)/N) \;-\; \alpha_{p-j}\bar{p}\bar{y}$$

for all $j \neq p$, and

$$a^p(0|p) \;=\; (a(0)/N) \;+\; (1-\alpha_0)\bar{p}\bar{y}.$$

Note that these are just the initial asset positions specified by (2.2) in the main text.

It then remains only to show that ex-ante insurance contracting will lead to the initial asset positions just specified. To consider the insurance chosen by a individual unit, we must compute the value function $V^j(a;s)$ for an individual unit in sector j that arranges to have an asset position $a^j(0|s) = a$ when state s is realized. We compute this under the assumption that market conditions once state s is realized are those specified in the description of equilibrium above; we do this in order to verify that the situation described above is indeed an equilibrium when there is ex-ante contracting.

Suppose that prices and interest rates in periods $t \geq 0$ are as specified above, but that an individual unit has an asset position a, not necessarily equal to the value of $a^j(0|s)$ for sector j as a whole, specified above. Then the individual unit will sell the quantity $y_j(t)$ each period specified above, because output is demand-determined. Assuming a value of a large enough to ensure that borrowing constraints never bind, the prices and interest rates imply that it will choose an expenditure pattern of the form $c_k^j(t) = \alpha_{k-j}\hat{y}$ for all goods k that are sold in period t, where the value of $\hat{y}$ is the same for all t; the value of $\hat{y}$ is the largest value consistent with the unit's intertemporal budget constraint (that depends on a).

This results in a value for the discounted objective (1.1) equal to

$$V^j(a;s) \;=\; \psi^j(s)u(\hat{y};\bar{\xi}) \;-\; \tilde{\psi}^j(s)v(\bar{y};\bar{\xi}), \tag{A.2}$$

where

$$\psi^j(s) \equiv \sum_{k\neq s} \alpha_{k-j} + \sum_{t=1}^{\infty} \beta^t \sum_{k=1}^{N} \alpha_{k-j}$$
$$= \sum_{k\neq s} \alpha_{k-j} + \frac{\beta}{1-\beta},$$

$$\tilde{\psi}^j(s) \equiv I(j \neq s) + \sum_{t=1}^{\infty} \beta^t$$
$$= I(j \neq s) + \frac{\beta}{1-\beta}.$$

Here $I(j \neq s)$ is an indicator function, taking the value 1 if $j \neq s$ and 0 otherwise (i.e., if $j = s$); and we interpret "$k \neq s$" to mean all sectors k in the case that $s = \emptyset$, and all sectors $k \neq p$ in the case that a pandemic shock occurs that impacts sector p. (That is, "$k \neq s$" is true if and only if sector k is allowed to sell its product in period zero.) Note that the positive weights $\psi^j(s), \tilde{\psi}^j(s)$ are the ones specified in (2.3) in the main text.

The value of $\hat{y}$ that just exhausts the unit's intertemporal budget constraint will be the solution to the equation

$$\psi^j(s)\bar{p}\hat{y} = a + \tilde{\psi}^j(s)\bar{p}\bar{y} - a(0)/N.$$

Here the left-hand side is the present value of the unit's spending in periods $t \geq 0$; the second term on the right-hand side is the present value of its sales in periods $t \geq 0$; and the final term represents the present value of tax obligations in periods $t \geq 1$, the same for all sectors. Hence we obtain

$$\hat{y} = \frac{a + \tilde{\psi}^j(s)\bar{p}\bar{y} - a(0)/N}{\tilde{\psi}^j(s)\bar{p}}$$
$$= \bar{y} + \frac{a + (\tilde{\psi}^j(s) - \psi^j(s))\bar{p}\bar{y} - a(0)/N}{\tilde{\psi}^j(s)\bar{p}}$$
$$= \bar{y} + \frac{a - \bar{a}^j(s) - a(0)/N}{\tilde{\psi}^j(s)\bar{p}},$$

where $\bar{a}^j(s)$ is defined as in (2.2). Substitution of this value for $\hat{y}$ into (A.2) then yields expression (2.3) in the main text.

In this calculation, we have assumed that borrowing constraints never bind for the unit. The spending plan that we have calculated implies that for this individual unit,

$$b^j(0) = a + I(j \neq s)\,\bar{p}\bar{y} - \sum_{k\neq s} \alpha_{k-j}\,\bar{p}\hat{y}$$
$$= a + (\tilde{\psi}^j(s) - (\beta/(1-\beta)))\bar{p}\bar{y} - (\tilde{\psi}^j(s) - (\beta/(1-\beta)))\bar{p}\hat{y}$$
$$= \frac{\beta}{1-\beta}\bar{p}(\hat{y} - \bar{y}) + a(0)/N.$$

Similarly, we can show for any period $t \geq 0$ that

$$b^j(t) \;=\; \frac{\beta}{1-\beta}\bar{p}(\hat{y} - \bar{y}) \;+\; a(0)/N.$$

The expression on the right-hand side is non-negative (so that the borrowing constraint is satisfied in period t) if and only if a satisfies the lower bound (2.4) stated in the main text. Thus if (2.4) is satisfied for all $t \geq 0$, the borrowing constraint never binds, the unit's optimal plan is the one described above, and expression (2.3) for the value function is correct.

Since by assumption $a(t) > 0$ for all t, the value $a = a^j(0|s) \equiv (a(0)/N) + \bar{a}^j(s)$ is necessarily large enough for (2.4) to be satisfied (with a strict inequality) for all $t \geq 0$. Hence $a = a^j(0|s)$ is in the interior of the set of values for which expression (2.3) is the correct formula for the value function. We accordingly find that the marginal value of a deviation from this level of initial assets is given by

$$\frac{\partial V^j}{\partial a}(a^j(0|s);\, s) \;=\; \frac{u'(\bar{y};\, \bar{\xi})}{\bar{p}},$$

for each state s.

The pattern of state-dependent initial assets required to support the first-best allocation as a decentralized equilibrium (calculated above) therefore satisfies the first-order condition

$$\pi(s)\frac{\partial V^j}{\partial a}(a^j(0|s);\, s) \;=\; \lambda q(s) \tag{A.3}$$

for each state s, where $\lambda > 0$ is a Lagrange multiplier, and also exhausts the budget constraint (2.1), when the state prices are given by $q(s) = \pi(s)$ for each state. These are necessary, though not sufficient, conditions for this choice of state-dependent initial assets to represent an optimum.

Finally, we observe that for any value of a that is feasible for a unit in sector j in state s, the value function $V^j(a; s)$ must have a value no greater than the value of the expression (2.3), the discounted utility calculated on the assumption that borrowing limits never bind. (This is because a binding borrowing limit can only reduce the maximum achievable value for the unit's discounted utility.) Furthermore, the expression (2.3) is a strictly increasing, strictly concave function of a, over the range of values of a for which the expression is defined. (We can define it to have a value of $-\infty$ for values of a too low for the function to be defined; the function will remain a concave function under this extension.) Hence the unit's objective in the ex ante insurance market, $V \equiv \sum_s \pi(s)V^j(a^j(0)|s);\, s)$, is bounded above by a strictly increasing, strictly concave function $\bar{V}$ of the vector of state-contingent initial asset positions, the function obtained by using expression (2.3) for V^j.

Because of the strict concavity of $\bar{V}$, the initial asset vector that satisfies the first-order condition (A.3) must be the one that maximizes $\bar{V}$. Since this asset vector is in the range where $V = \bar{V}$, and $\bar{V}$ is an upper bound for V, this initial asset vector must also be the one that maximizes the objective V subject to the budget constraint (2.1). Hence the prices and allocations discussed above represent an equilibrium of the model with an ex ante insurance market.

A.2 Equilibrium from $t = 1$ onward in the more general case

Next we consider the more general case in which the initial asset positions $\{a^j(0)\}$ are not necessarily the ones that would be chosen in an ex ante insurance market. We now consider equilibrium from period zero onward in the case of a single state s, given the initial asset positions in this state and the monetary and fiscal policies followed given that this state has occurred. We assume that the state is one in which a pandemic shock occurs, and without loss of generality we suppose that $p = 1$, as in section 2.2 and subsequently in the main text.

As in the previous section, the fact that all uncertainty is resolved in period zero means that the equilibrium allocation of resources in periods $t \geq 1$ must be the same as in an economy with flexible prices in those periods. Thus the equilibrium from period $t = 1$ onward must involve the same relative prices and real allocation of resources as the competitive equilibrium of a flexible-price model, in which we take as given the real pre-tax asset balances $\tilde{a}^j(1)$ with which units in each sector enter period 1. In this continuation economy, the government levies lump-sum taxes $\tau(t)$ each period, the same for each sector, with a time path of taxes that satisfies the transversality condition (whatever equilibrium interest rates may be). This means that the present value of tax obligations in periods $t \geq 1$, if market interest rates are used to discount the tax obligations back to an equivalent payment in period $t = 1$, must equal $\tau^{PV} \equiv \sum_{j=1}^{N} \tilde{a}^j(1)/N$ for each sector.

Given the stationary structure of this flexible-price economy, it is natural to look for an equilibrium from $t = 1$ onward in which quantities and relative prices are all stationary. That is, we expect the equilibrium allocation of resources to be of the form $c_k^j(t) = c_k^j$ and $y_j(t) = y_j$ for all $t \geq 1$, and for relative prices and real interest rates to be of the form

$$\frac{p_k(t)}{P(t)} \;=\; q_k, \qquad (1 + i(t))\frac{P(t)}{P(t+1)} \;=\; \frac{1}{\beta} \tag{A.4}$$

for all $t \geq 1$, where the stationary relative prices satisfy $q_k > 0$ for all j and $(1/N)\sum_{k=1}^{N} q_k = 1$ (by definition of the price index).

Given these relative prices, a stationary plan $(\{c_k^j\}, y_j)$ for units in sector j is consistent with their intertemporal budget constraint if it satisfies

$$\frac{1}{1-\beta}\sum_{k=1}^{N} q_k c_k^j \;+\; \tau^{PV} \;\leq\; \frac{1}{1-\beta} q_j y_j \;+\; \tilde{a}^j(1),$$

or alternatively,

$$\sum_{k=1}^{N} q_k c_k^j \;\leq\; q_j y_j \;+\; (1-\beta)(\tilde{a}^j(1) - \tau^{PV}). \tag{A.5}$$

It also implies a real asset position at the end of any period $t \geq 1$ that satisfies the equation

$$\beta^{t-1}\frac{b^j(t)}{P(t)} \;=\; \tilde{a}^j(1) \;+\; \frac{1-\beta^t}{1-\beta}\left(q_j y_j - \sum_{k=1}^{N} q_k c_k^j\right) \;-\; \left(\tau^{PV} - \beta^{t-1}\frac{a(t)}{NP(t)}\right).$$

Here the first term on the right-hand side is the pre-tax real asset position at the beginning of period 1; the next term is the present value (discounted back to period 1) of the amount by which sales exceed spending in periods 1 through t; and the final term is the present value of tax obligations per sector i periods 1 through t if real public debt is $a(t)/P(t)$ at the end of period t.

In the case of any plan for sector j consistent with (A.5), this result implies that

$$\beta^{t-1}\frac{b^j(t)}{P(t)} \;\geq\; \beta^t(\tilde{a}^j(1) - \tau^{PV}) \,+\, \beta^{t-1}\frac{a(t)}{NP(t)}.$$

Since the borrowing limit in period zero implies that $\tilde{a}^j(1) \geq 0$, it further implies that

$$\frac{b^j(t)}{P(t)} \;\geq\; \frac{a(t)}{NP(t)} \,-\, \beta\tau^{PV}.$$

If in addition $a(t)$ satisfies the lower bound (4.1), we necessarily have $b^j(t) \geq 0$. Thus a stationary plan consistent with (A.5) necessarily satisfies the borrowing limit (1.8) in all periods $t \geq 1$. It follows that the intertemporal budget constraint (A.5) alone defines the set of feasible stationary plans.

A stationary plan $(\{c^j_k\},\, y_j)$ for sector j is therefore optimal if and only if it maximizes the stationary utility level

$$\bar{U}^j \;\equiv\; \sum_{k\in K}\alpha_k u(c^j_{j+k}/\alpha_k;\,\bar{\xi}) \,-\, v(y_j(t);\,\bar{\xi}) \tag{A.6}$$

among all plans consistent with (A.5). A collection of stationary plans for each of the sectors and vector of relative prices then constitute a stationary equilibrium of the economy from $t = 1$ onward if (i) given the relative prices, the stationary plan for each sector j maximizes $\bar{U}^j$ from among all plans consistent with (A.5); and (ii) for each sector k,

$$\sum_{j=1}^{N} c^j_k \;=\; y_k. \tag{A.7}$$

The constraint (A.5) can be written more compactly as

$$\sum_{k=1}^{N} q_k c^j_k \;\leq\; q_j y_j \,+\, v^j,$$

where

$$v^j \;\equiv\; (1-\beta)(\tilde{a}^j(1) - \tau^{PV}) \;=\; (1-\beta)\left(\tilde{a}^j(1) - \frac{1}{N}\sum_{k=1}^{N}\tilde{a}^k(1)\right).$$

Then if we let $\boldsymbol{v}$ denote the N-vector with jth element v^j, we see that the collection of stationary allocations and relative prices that constitute a stationary equilibrium depend only on the vector $\boldsymbol{v}$, which is in turn completely determined by the vector $\tilde{\boldsymbol{a}}(1)$.

A.3 The Euler condition for period $t = 0$

Now let $\{c_k^j, q_k\}$ be the stationary equilibrium associated with a particular vector $\tilde{\boldsymbol{a}}(1)$, and consider the level of continuation utility (the utility flow in periods $t \geq 1$, discounted back to period 1) that can be achieved by a unit in sector j that enters period 1 with pre-tax real wealth $\tilde{a}$, not necessarily equal to the value $\tilde{a}^j(1)$ that represents the pre-tax real wealth of other units in its sector. Given the relative prices and real interest rates (A.4) for all $t \geq 1$, the unit's optimal expenditure plan for periods $t \geq 1$ will be a stationary plan $\{\tilde{c}_k^j\}$, the one that maximizes (A.6) among all those consistent with the intertemporal budget constraint

$$\sum_{k=1}^{N} q_k \tilde{c}_k^j \;\leq\; q_j y_j \;+\; (1-\beta)\left(\tilde{a} \;-\; \frac{1}{N}\sum_{k=1}^{N} \tilde{a}^k(1)\right).$$

This constraint corresponds to (A.5), where we take into account that both y_j and τ^{PV} are determined by the stationary equilibrium associated with the aggregate vector $\tilde{\boldsymbol{a}}(1)$, and are unaffected by the individual unit's choice of $\tilde{a}$.

The solution to this optimization problem defines a maximum achievable value of $\bar{U}^j$. If we multiply this quantity by $1/(1-\beta)$, we obtain the value of the discounted utility flow in periods $t \geq 1$ in the stationary equilibrium (discounted back to period 1), which we can denote $V^j(\tilde{a}; \tilde{\boldsymbol{a}}(1))$. This is an individual unit's value function for its continuation problem in period 1. The unit's optimization problem in period zero can then be written as a choice of an expenditure plan $\{c_k^j(0)\}$ for all $k \neq s$ and end-of-period asset balance $b^j(0)$ consistent with (1.7)–(1.8), so as to maximize the objective

$$U^j(0) \;+\; \beta V^j(\tilde{a}; \tilde{\boldsymbol{a}}(1)),$$

where in the second term we substitute the value

$$\tilde{a} \;=\; (1+i(0))\frac{b^j(0)}{P(1)}. \tag{A.8}$$

Here the price $P(1)$, the interest rate $i(0)$, and the aggregate vector $\tilde{\boldsymbol{a}}(1)$ are taken as given by the individual unit.

Among the first-order conditions for a solution to this optimization problem is the Euler condition

$$u'\left(\frac{c^j(0)}{\sum_{k\neq s}\alpha_{k-j}}\right) \;\geq\; \beta(1+i(0))\frac{\bar{p}}{P(1)}\,\Lambda^j(\tilde{a}^j(1);\, \tilde{\boldsymbol{a}}(1)), \tag{A.9}$$

where

$$\Lambda^j(\tilde{a};\, \tilde{\boldsymbol{a}}(1)) \;\equiv\; \frac{\partial V^j(\tilde{a};\, \tilde{\boldsymbol{a}}(1))}{\partial \tilde{a}}.$$

An optimal plan must satisfy both inequalities (1.8) and (A.9), and at least one of these must hold with equality for each sector j. Note that in the case of a monetary policy rule that would support the first-best allocation in the case that there were no pandemic shock (as assumed in section 4.2), (A.1) must hold, and condition (A.9) reduces to condition (4.3) given in the main text.

Given our definition of the value function V^j, the envelope theorem implies that

$$\Lambda^j(\tilde{a}^j(1);\, \tilde{\boldsymbol{a}}(1)) \;=\; u'(c^j;\, \bar{\xi}), \tag{A.10}$$

where c^j is the stationary level of per-period expenditure by units in sector j, in the stationary equilibrium associated with aggregate vector $\tilde{\boldsymbol{a}}(1)$. Note also that we have shown in the previous subsection that the stationary equilibrium allocation of resources from period $t = 1$ onwards depends only on the value of the vector $\boldsymbol{v}$; hence the marginal utility (A.10) can be expressed as a function $\Lambda^j(\boldsymbol{v})$. It is also worth noting that the way in which the stationary allocation — and hence the function $\Lambda^j(\boldsymbol{v})$ — depends on the elements of the vector $\boldsymbol{v}$ nowhere involves the parameter β. Thus if we vary the value of β, while keeping fixed the single-period utility function (1.2) and the stationary value $\bar{\xi}$ of the preference shifters, the functions $\{\Lambda^j(\boldsymbol{v})\}$ remain the same. (This is our reason for introducing the notation $\boldsymbol{v}$, rather than simply referring to the vector $\tilde{\boldsymbol{a}}(1)$.)

In the main text, we make repeated use of an asymptotic approximation to condition (A.9), for the case in which all elements of the vector $\boldsymbol{v}$ are small. It is easily seen that when $\boldsymbol{v} = \mathbf{0}$, the stationary equilibrium for periods $t \geq 1$ is one in which $q_k = 1$ for all k, $y_k = \bar{y}$ for all k, and $c_k^j = \alpha_{k-j}\bar{y}$ for all j and all k. The envelope condition (A.10) then implies that

$$\Lambda^j(\mathbf{0}) \;=\; u'(\bar{y};\, \bar{\xi}) \tag{A.11}$$

for all sectors j.

We can then use the implicit function theorem to show that for all values of $\boldsymbol{v}$ in a neighborhood of $\mathbf{0}$, there exists a continuously differentiable function $\Lambda^j(\boldsymbol{v})$ consistent with (A.11), such that for each $\boldsymbol{v}$ in this neighborhood there is a stationary equilibrium consistent with the vector $\boldsymbol{v}$ in which the marginal utility is given by the function $\Lambda^j(\boldsymbol{v})$. Hence if we consider a sequence of vectors $\boldsymbol{v}$ with the property that $\boldsymbol{v} \to \mathbf{0}$, we must have $\Lambda^j(\boldsymbol{v}) \to u'(\bar{y};\, \bar{\xi})$ for all j.

Thus for any value of $\boldsymbol{v}$ close enough to $\mathbf{0}$, the Euler condition (A.9) can be well approximated by the condition

$$u'\left(\frac{c^j(0)}{\sum_{k\neq s}\alpha_{k-j}}\right) \;\geq\; \beta(1+i(0))\frac{\bar{p}}{P(1)}\; u'(\bar{y};\, \bar{\xi}).$$

If we introduce the notation

$$1+\bar{\imath} \;\equiv\; \frac{1}{\beta}\frac{P(1)}{\bar{p}}$$

for the interest rate implied by (A.1), the Euler condition can alternatively be written in the form

$$u'\left(\frac{c^j(0)}{\sum_{k\neq s}\alpha_{k-j}}\right) \;\geq\; \frac{1+i(0)}{1+\bar{\imath}}u'(\bar{y};\, \bar{\xi}). \tag{A.12}$$

The approximation (A.12) is used in two ways in the main text. First, in sections 2.2 and 3, we consider a case in which the value of $a(0)$ is arbitrarily close to zero (though we assume that $a(0) > 0$). In this case, satisfaction of the borrowing constraint (1.8) by all sectors requires that $0 \leq b^j(0) \leq a(0)$ for each sector, and hence that each element of $\boldsymbol{v}$ must satisfy the bounds

$$-\frac{1}{N}(1-\beta)\frac{(1+i(0))}{P(1)}a(0) \quad \leq \quad v^j \quad \leq \quad \frac{N-1}{N}(1-\beta)\frac{(1+i(0))}{P(1)}a(0). \tag{A.13}$$

As $a(0) \to 0$, for fixed values of the other model parameters, both the upper and lower bounds converge to 0. Hence we can assure that in equilibrium, all elements of $\boldsymbol{v}$ must be as close as desired to zero, by fixing a sufficiently small value for $a(0)$. Hence in the limit as $a(0) \to 0$, the Euler condition (A.9) takes the limiting form (A.12). This explains the form (2.7) given in section 2.2, and the generalization (3.1) given in section 3.

Even if we do not assume that $a(0)$ is small (relative, say, to the value of $\bar{p}\bar{y}$), the asymptotic approximation remains relevant if we consider the case of a low rate of time preference (relative to the length of a "period", i.e., the length of time for which the pandemic shock is assumed to last in our model). Suppose that we fix the within-period utility function (1.2) and the values of the preference shifters $\bar{\xi}$, so that among other things, the value of $\bar{y}$ is fixed; we fix the initial price level $\bar{p}$ and the initial asset positions $\{a^j(0)\}$; and we fix the value of the ratio $(1+i(0))/(1+\bar{\imath})$; but we let $\beta \to 1$. Then any given expenditure choices $\{c_k^j(0)\}$ in period zero will be associated with particular values $\{\tilde{a}^j(1)\}$, that are independent of the value assumed for β; but the implied vector $\boldsymbol{v}$ can be made arbitrarily close to $\mathbf{0}$, by making β close enough to 1.

Thus the Euler condition approaches condition (A.12) as $\beta \to 1$. This is the case considered in section 4.2 of the main text. It is assumed there (until section 4.5) that monetary policy is such that $i(0) = \bar{\imath}$, so that (A.12) reduces again to (2.7), as in section 2.2. The form of the Euler condition assumed in section 4.5 is instead the more general form (A.12).

A.4 Average welfare in the stationary equilibrium for periods $t \geq 1$

In the stationary equilibrium characterized above, the contribution of utility flows in periods $t \geq 1$ to the ex-ante welfare objective (1.4) will equal

$$\frac{\beta}{1-\beta}\sum_{j=1}^{N}\bar{U}^j \;=\; \frac{\beta}{1-\beta}\sum_{j=1}^{N}V^j(\boldsymbol{v}).$$

Here we have suppressed the argument $\tilde{a}$ of the value function V^j, as we no longer need to consider the effect on an individual unit's discounted utility of a deviation from the average asset balance of units in its sector, only the common level of utility obtained in equilibrium by units in sector j. And we have written V^j as a function of $\boldsymbol{v}$ rather than the vector $\tilde{\boldsymbol{a}}(0)$, since as shown above, the stationary equilibrium allocation for $t \geq 1$ depends only on the vector $\boldsymbol{v}$.

If we subtract from this the value of these utility flows in the first-best allocation of resources (a change that, as explained in the text, does not affect the welfare ranking of different policies), we obtain instead the deviation measure

$$\Delta(\boldsymbol{v}) \;\equiv\; \frac{\beta}{1-\beta}\sum_{j=1}^{N}[\bar{U}^j - U^*] \;=\; \frac{\beta}{1-\beta}\sum_{j=1}^{N}[V^j(\boldsymbol{v}) - V^j(\mathbf{0})],$$

where for each j, $V^j(\mathbf{0}) = U^* \equiv u(\bar{y};\, \bar{\xi}) - v(\bar{y};\, \bar{\xi})$. We are interested in the value of this deviation measure in the limit as $\beta \to 1$, holding fixed the vector of real asset balances $\tilde{\boldsymbol{a}}(1)$ carried into period $t = 1$.

Since $\boldsymbol{v} \to \mathbf{0}$ as $\beta \to 1$, we see that $\Delta(\boldsymbol{v})$ is a quotient of two quantities that each converge to zero as $\beta \to 1$. The limiting value of the ratio can then be computed using L'Hôpital's rule, yielding

$$\Delta(\boldsymbol{v}) \;\to\; \sum_{j=1}^{N}\sum_{k=1}^{N} \frac{\partial V^j}{\partial v_k} \left[\tilde{a}^k - (1/N)\sum_{h=1}^{N} \tilde{a}^h\right], \tag{A.14}$$

where the partial derivatives are evaluated at $\boldsymbol{v} = \mathbf{0}$.

It follows from (A.6) that

$$\sum_{j=1}^{N} V^j(\boldsymbol{v}) \;=\; \sum_{j=1}^{N} \left[\sum_{k\in K} \alpha_k u(c^j_{j+k}/\alpha_k;\, \bar{\xi}) \;-\; v(y_j(t);\, \bar{\xi})\right],$$

and hence that

$$\begin{aligned}
\sum_{j=1}^{N} \frac{\partial V^j}{\partial v_\ell} \;&=\; \sum_{j=1}^{N}\sum_{k\in K} u'(\bar{y};\, \bar{\xi}) \frac{\partial c^j_{j+k}}{\partial v_\ell} \;-\; \sum_{j=1}^{N} v'(\bar{y};\, \bar{\xi}) \frac{\partial y_j}{\partial v_\ell} \\
&=\; u'(\bar{y};\, \bar{\xi}) \cdot \sum_{k=1}^{N} \left[\sum_{j=1}^{N} \frac{\partial c^j_k}{\partial v_\ell} \;-\; \frac{\partial y_k}{\partial v_\ell}\right] \\
&=\; 0.
\end{aligned}$$

Here the second line uses the fact that $v'(\bar{y};\, \bar{\xi}) = u'(\bar{y};\, \bar{\xi})$, and rearranges terms. The final line uses the fact that for any $\boldsymbol{v}$, the solutions for the $\{c^j_k(\boldsymbol{v})\}$ and $y_k(\boldsymbol{v})$ must satisfy (A.7); differentiation of this equation with respect to v_ℓ yields

$$\sum_{j=1}^{N} \frac{\partial c^j_k}{\partial v_\ell} \;-\; \frac{\partial y_k}{\partial v_\ell} \;=\; 0$$

for each k.

Thus (A.14) implies that

$$\lim_{\beta\to 1} \Delta(\boldsymbol{v}) \;=\; \sum_{k=1}^{N} \left[\sum_{j=1}^{N} \frac{\partial V^j}{\partial v_k}\right] \cdot \left[\tilde{a}^k - (1/N)\sum_{h=1}^{N} \tilde{a}^h\right] \;=\; 0,$$

for any vector $\tilde{\boldsymbol{a}}(1)$. This is the result (5.1) reported in the main text, which implies that comparison of the welfare associated with different policies only requires us to compare the implied values for W_0, the period-zero contribution to welfare.

B Existence and Uniqueness of Equilibrium at $t = 0$

Here we explain further how a unique equilibrium at $t = 0$ can be shown to exist for relatively general network structures. We restrict attention to cases in which the vector $\boldsymbol{v}$ is necessarily small (either because the total supply of liquid assets in period zero is small, or because the rate of time preference is low), and take it as given that the equilibrium from $t = 1$ onward is a stationary equilibrium of the kind defined in appendix section A.2.

More specifically, the equilibrium for $t \geq 1$ is the stationary equilibrium that is a small perturbation of the one for the case $\boldsymbol{v} = \mathbf{0}$ discussed in section A.3, in which the allocation of resources for all $t \geq 1$ coincides with the first-best allocation characterized in section 1.2. This perturbation solution can be shown to be uniquely defined for all small enough vectors $\boldsymbol{v}$ using the implicit function theorem, as discussed in section A.3. Given this, we now consider equilibrium determination in period $t = 0$, including the determination of the vector $\boldsymbol{v}$ that then determines the equilibrium for $t \geq 1$.

B.1 The limiting case in which $a(0) \to 0$

We first consider equilibrium determination in the case that $a(0)$ is very small, as assumed in sections 2.2 and 3 of the main text. In this case, the equilibrium for periods $t \geq 1$ is necessarily determined by a vector $\boldsymbol{v}$ near $\mathbf{0}$, as discussed in section A.3. Hence the Euler condition in period zero is of the form (A.12). On the assumption that a pandemic shock occurs that impacts sector 1, this reduces to

$$u'\left(\frac{c^j(0)}{1-\alpha_{1-j}}\right) \;\geq\; \frac{1+i(0)}{1+\bar{\imath}}u'(\bar{y};\bar{\xi}), \tag{B.1}$$

corresponding to (3.1) in the main text.

Given that $\beta < 1$, the bounds (A.13) can alternatively be written

$$-\frac{1}{N}a(0) \;\leq\; b^j(0) \;\leq\; \frac{N-1}{N}a(0).$$

Thus we observe that in the limit as $a(0) \to 0$, the equilibrium value of each of the $b^j(0)$ must approach zero. We thus calculate the equilibrium in period zero for the limiting case in which we must have $b^j(0) = 0$ for all j. It then follows, as discussed in section 2.2 of the main text, that the vector $\boldsymbol{c}(0)$ of sectoral expenditure levels must satisfy $\boldsymbol{c}(0) = \boldsymbol{A}\boldsymbol{c}(0)$.

We now show that under our assumptions, the matrix $\boldsymbol{A}$ must have a unique right eigenvector $\boldsymbol{\pi}$ with an associated eigenvalue of 1. We first note that the definition of the matrix $\boldsymbol{A}$ (following equation (2.5) in the main text) implies that $A_{kj} \geq 0$ for all k, j, and that $\sum_{k=1}^{N} A_{kj} = 1$ for every j, or in vector notation, that

$$\boldsymbol{e}'\boldsymbol{A} \;=\; \boldsymbol{e}'.$$

This indicates that 1 must be an eigenvalue of the matrix $\boldsymbol{A}$, with $\boldsymbol{e}'$ the associated left eigenvector. Any eigenvalue must also have at least one associated right eigenvector; thus it remains only to establish that the right eigenvector $\boldsymbol{\pi}$ is unique (up to normalization).

We observe from the properties noted in the previous paragraph that $\boldsymbol{A}$ is a non-negative matrix (Gantmacher, 1959, chap. XIII, Definition 1) that is furthermore a stochastic matrix (Definition 4).[39] Any non-negative matrix necessarily has a maximal (Frobenius-Perron)

[39] More precisely, the transpose $\boldsymbol{A}'$ is a stochastic matrix as defined in Gantmacher. Below we translate the properties of stochastic matrices established in Gantmacher into statements about the matrix $\boldsymbol{A}$.

eigenvalue $\bar{\lambda}$ with the properties that (i) $\bar{\lambda}$ is a non-negative real number, and (ii) $|\lambda| \leq \bar{\lambda}$ for all eigenvalues λ of the matrix (where $|\lambda|$ denotes the modulus of an eigenvalue that may be complex); moreover, (iii) the left and right eigenvectors associated with the maximal eigenvalue are real-valued and non-negative in all elements (Gantmacher, Theorem 3). In the more specific case of a stochastic matrix, the maximal eigenvalue is 1 (Gantmacher, p. 83). The associated left eigenvector is $\boldsymbol{e}'$, which is obviously non-negative in all elements; but there must be a (non-zero) right eigenvector $\boldsymbol{\pi}$ that is also non-negative in all of its elements. Because $\boldsymbol{\pi} \geq \mathbf{0}$, we can normalize the right eigenvector to satisfy $\boldsymbol{e}'\boldsymbol{\pi} = 1$.

To go further it is useful to write the matrix $\boldsymbol{A}$ in the normal form defined in Gantmacher (sec. XIII.4).[40] This involves partitioning the N sectors (the rows and columns of the matrix) into disjoint subsystems $\{S_1, \ldots, S_s\}$, each of which is irreducible, in the sense that any two sectors $j \neq z$ within the same subsystem can be linked by a sequence of sectors $(j, k, l, \ldots, y, z)$ all within the same subsystem, with the property that j buys goods produced in k, k buys goods produced in l, ..., and y buys goods produced in z. We further define a subsystem as "isolated" if each of the sectors $j \in S_i$ spend only on products of sectors in subset S_i. Then Gantmacher shows that one can order the subsystems so that the first $g \geq 1$ of them are the (only) isolated subsystems; and the subsystems S_i for $g+1 \leq i \leq s$ each have the property that each of the sectors $j \in S_i$ spends only on products produced in subsystems S_k with $k \leq i$. Thus if one re-orders the sectors in accordance with this ordering of the subsystems, the matrix $\boldsymbol{A}$ has a normal form representation that is upper block-triangular, with all off-diagonal blocks being blocks of zeroes in the first g block columns.

In our case, we must have $g = 1, s \geq 2$. The only isolated subsystem must be the one that contains sector N, since our assumption that $\alpha_1 > 0$ implies that 1 buys from 2 which buys from ... which buys from $N - 1$ which buys from N, so that any isolated subsystem must contain sector N. In addition, subsystem S_s must consist solely of sector 1, since no other sector buys anything from sector 1 (as a result of the pandemic); thus there must be at least two subsystems.

Gantmacher (Theorem 12) shows that a stochastic matrix $\boldsymbol{A}$ has a unique right eigenvector $\boldsymbol{\pi}$ with an associated eigenvalue of 1 if and only if $g = 1$ in the normal form representation (i.e., there is a unique isolated subsystem). In this case the Frobenius-Perron eigenvector $\boldsymbol{\pi}$ corresponds to the uniquely defined stationary long-run probabilities of occupying the N different states, if $\boldsymbol{A}$ is interpreted as the matrix of transition probabilities defining a homogeneous N-state Markov chain. The elements of this eigenvector satisfy $\pi_j > 0$ for all $j \in S_1$, and $\pi_j = 0$ for all other j.

This unique solution for $\boldsymbol{\pi}$ allows us then to solve uniquely for the vector $\boldsymbol{c}(0) = \theta\boldsymbol{\pi}$, where the value of $\theta > 0$ is given by (3.4), in order to satisfy the Euler condition (B.1). (In the case of a monetary policy that implies $i(0) = \bar{\imath}$, this reduces to the solution (2.9) given in section 2.2.) Because $\boldsymbol{c}(0)$ is a multiple of $\boldsymbol{\pi}$, it has the property that $c^j(0) > 0$ for all j in S_1, while $c^j(0) = 0$ for all other sectors. For example, in the case of the uniform network structure shown in the left panel of Figure 1, S_1 consists of sectors $\{2, 3, 4, 5\}$, while S_2 consists of $\{1\}$. In the case of the chain structure shown in the right panel of Figure 1, instead (and regardless of the value of the parameter λ), the irreducible subsystems are $S_1 = \{5\}$, $S_2 = \{4\}$, $S_3 = \{3\}$, $S_4 = \{2\}$, and $S_5 = \{1\}$, among which only S_1 is an isolated

[40] More precisely, we put $\boldsymbol{A}'$ in the form shown in Gantmacher (p. 75).

subsystem. This explains why, in Figure 2, we have $c^j(0) = 0$ only for $j = 1$ in the left panel, while instead $c^j(0) = 0$ for all $j \leq 4$ in the right panel.

B.2 The limiting case in which $\beta \to 1$

We next consider equilibrium determination in the case that the rate of time preference is very small, as assumed in section 4 of the main text. This limiting case also implies that $\boldsymbol{v} \to \boldsymbol{0}$, as discussed in section A.2 above, so that the Euler condition again takes the form (B.1). However, we can no longer (as in the previous section) assume that the equilibrium in period zero will involve $b^j(0) = 0$ for all j. We assume in this section that $a(0) > 0$, so that necessarily $b^j(0) > 0$ for at least one sector.

When $a(0)$ is non-negligible, the equilibrium in period zero depends not just on the aggregate level of liquid assets, but on how they are initially distributed across sectors (perhaps as a result of a sectorally-targeted transfer policy). We let $\boldsymbol{a}(0)$ be the vector of initial (post-transfer) asset balances, and assume that $\boldsymbol{a}(0) >> \boldsymbol{0}$. Then as explained in the main text, $\boldsymbol{c}(0)$ represents an equilibrium in period zero if and only if it satisfies the system of equations (5.6), where $\hat{y}(i(0))$ is the function defined by (3.3). (Note also that this definition implies that $\hat{y}(i(0)) > 0$ for any $i(0)$.) We further observe that the mapping defined by the right-hand side of (5.6) is homogeneous of degree one. This implies that if for any vector $\boldsymbol{c}(0)$ we define $\bar{\boldsymbol{c}} \equiv (\bar{y}/\hat{y})\boldsymbol{c}(0)$, the vector $\boldsymbol{c}(0)$ is a solution to (5.6) if and only if the vector $\bar{\boldsymbol{c}}$ is a solution to (4.6). Hence it suffices that we consider the set of solutions to (4.6) for any vector $\boldsymbol{a}(0) >> \boldsymbol{0}$.

B.2.1 Existence of a unique solution to the "Keynesian cross"

This problem can be shown to have a unique solution using properties of positive concave mappings that are reviewed in Cavalcante *et al.* (2016). For any vector $\boldsymbol{c}(0)$, let $\boldsymbol{F}(\boldsymbol{c}(0))$ be the vector defined by the right-hand side of (4.6); thus $\boldsymbol{F}(\cdot)$ maps N-vectors into N-vectors. If $\boldsymbol{a}(0) >> \boldsymbol{0}$, we can further show that $\boldsymbol{F}(\cdot)$ is a positive mapping, in the sense that for any $\boldsymbol{c}(0) \geq \boldsymbol{0}$, we have $\boldsymbol{F}(\boldsymbol{c}(0)) >> \boldsymbol{0}$. Let $F_j(\cdot)$ be the jth element of $\boldsymbol{F}(\cdot)$, that is, the implied value for $c^j(0)$. Then for each j, we need to show that for any $\boldsymbol{c}(0) \geq \boldsymbol{0}$, $F_j(\boldsymbol{c}(0)) > 0$. Since $\boldsymbol{A} \geq \boldsymbol{0}$, $\boldsymbol{c}(0) \geq \boldsymbol{0}$ implies that $\boldsymbol{A}\boldsymbol{c}(0) \geq \boldsymbol{0}$. Then under the hypothesis that $\boldsymbol{a}(0) >> \boldsymbol{0}$, we must have

$$\frac{1}{\bar{p}}\boldsymbol{a}(0) + \boldsymbol{A}\boldsymbol{c}(0) >> \boldsymbol{0}.$$

Thus the jth element of this vector must be positive, for any j. Since $c^{*j} > 0$ as well, the minimum of the two quantities must be positive. Thus $F_j(\boldsymbol{c}(0))$ is necessarily positive, as required.

We can further show that each of the functions $F_j(\cdot)$ is concave. This requires that for any vectors $\boldsymbol{c}_1$, $\boldsymbol{c}_2$, and any scalar $0 \leq \alpha \leq 1$,

$$F_j(\alpha\boldsymbol{c}_1 + (1-\alpha)\boldsymbol{c}_2) \geq \alpha F_j(\boldsymbol{c}_1) + (1-\alpha)F_j(\boldsymbol{c}_2). \tag{B.2}$$

Given the definition of $F_k(\cdot)$ in (4.6) as the minimum of two functions, this holds if and only if *both* of the inequalities

$$\frac{a^j(0)}{\bar{p}} + \sum_k A_{jk}[\alpha c_1^k + (1-\alpha)c_2^k] \;\geq\; \alpha F_j(\boldsymbol{c}_1) + (1-\alpha)F_j(\boldsymbol{c}_2), \tag{B.3}$$

$$c^{*j} \;\geq\; \alpha F_j(\boldsymbol{c}_1) + (1-\alpha)F_j(\boldsymbol{c}_2) \tag{B.4}$$

are necessarily satisfied. But inequality (B.3) follows from the fact that

$$F_j(\boldsymbol{c}_i) \;\leq\; \frac{a^j(0)}{\bar{p}} + \sum_k A_{jk} c_i^k$$

for each of the cases $i = 1, 2$; and inequality (B.4) follows from the fact that

$$F_j(\boldsymbol{c}_i) \;\leq\; c^{*j}$$

for each of the cases $i = 1, 2$. Hence (B.2) is satisfied, and $F_j(\cdot)$ is a concave function for each j. This in turn means that $\boldsymbol{F}(\cdot)$ is a concave mapping.

Thus $\boldsymbol{F}(\cdot)$ is a positive concave mapping. Moreover, there exists a finite upper bound $\bar{\boldsymbol{c}}$ with the property that $\boldsymbol{F}(\boldsymbol{c}(0)) \leq \bar{\boldsymbol{c}}$ for all $\boldsymbol{c}(0) \leq \bar{\boldsymbol{c}}$; this is the bound $\bar{\boldsymbol{c}} = \boldsymbol{c}^*$, where the elements of the vector $\boldsymbol{c}^*$ are defined in (2.8). It then follows from Cavalcante *et al.* (2016, Proposition 1 and Facts 4.1 and 4.2) that the mapping $\boldsymbol{F}(\cdot)$ has a unique fixed point. This means that the system of equations (4.6) has a unique solution $\boldsymbol{c}(0)$.

B.2.2 Properties of the solution: monotonicity

For any vector $\boldsymbol{a}(0) >> \boldsymbol{0}$, let this unique fixed point be denoted $\bar{\boldsymbol{c}}(\boldsymbol{a}(0))$. (Note that the mapping $\boldsymbol{F}(\cdot)$ depends on the vector $\boldsymbol{a}(0)$.) One can easily establish several features of the functional dependence of the fixed point on $\boldsymbol{a}(0)$. First, because $\boldsymbol{0} << \boldsymbol{F}(\boldsymbol{c}) \leq \boldsymbol{c}^*$ for all $\boldsymbol{c}$, it is clear that the fixed point must satisfy $0 << \bar{\boldsymbol{c}}(\boldsymbol{a}(0)) \leq \boldsymbol{c}^*$ for all $\boldsymbol{a}(0) \geq \boldsymbol{0}$.

We can also show that each of the component functions $\bar{\boldsymbol{c}}_j(\boldsymbol{a})$ must be at least weakly increasing in each of the elements of $\boldsymbol{a}$. Consider any two vectors $\boldsymbol{a}_1, \boldsymbol{a}_2$ such that $\boldsymbol{a}_2 \geq \boldsymbol{a}_1 >> \boldsymbol{0}$. Then we can show that we must have $\bar{\boldsymbol{c}}(\boldsymbol{a}_2) \geq \bar{\boldsymbol{c}}(\boldsymbol{a}_1)$ for each j. Let $\boldsymbol{F}_i(\cdot)$ be the mapping defined by the right-hand side of (4.6) when $\boldsymbol{a}(0) = \boldsymbol{a}_i$, for $i = 1, 2$, and further define the mapping

$$\tilde{\boldsymbol{F}}(\boldsymbol{\delta}) \;\equiv\; \boldsymbol{F}_2(\bar{\boldsymbol{c}}(\boldsymbol{a}_1) + \boldsymbol{\delta}) \;-\; \bar{\boldsymbol{c}}(\boldsymbol{a}_1),$$

defined for an arbitrary vector $\boldsymbol{\delta} \geq \boldsymbol{0}$. Then $\boldsymbol{c}$ will be a fixed point of $\boldsymbol{F}_2$ if and only if $\boldsymbol{\delta} = \boldsymbol{c} - \bar{\boldsymbol{c}}(\boldsymbol{a}_1)$ is a fixed point of $\tilde{\boldsymbol{F}}$.

It is evident that $\tilde{\boldsymbol{F}}(\cdot)$ is a continuous mapping, with the upper bound $\tilde{\boldsymbol{F}}(\boldsymbol{\delta}) \leq \boldsymbol{c}^* - \bar{\boldsymbol{c}}(\boldsymbol{a}_1)$ for all $\boldsymbol{\delta}$. Moreover, for any $\boldsymbol{\delta} \geq \boldsymbol{0}$, we must have

$$\begin{aligned}\tilde{\boldsymbol{F}}(\boldsymbol{\delta}) \;&\geq\; \boldsymbol{F}_2(\bar{\boldsymbol{c}}(\boldsymbol{a}_1)) \;-\; \bar{\boldsymbol{c}}(\boldsymbol{a}_1) \\ &\geq\; \boldsymbol{F}_1(\bar{\boldsymbol{c}}(\boldsymbol{a}_1)) \;-\; \bar{\boldsymbol{c}}(\boldsymbol{a}_1) \;=\; \boldsymbol{0}.\end{aligned}$$

Thus $\tilde{\boldsymbol{F}}(\cdot)$ maps the set of vectors satisfying the bounds

$$\boldsymbol{0} \;\leq\; \boldsymbol{\delta} \;\leq\; \boldsymbol{c}^* \;-\; \bar{\boldsymbol{c}}(\boldsymbol{a}_1)$$

into itself. These bounds define a compact, convex subset of $\mathbb{R}^N$. Hence by Brouwer's fixed-point theorem, there must be a vector $\boldsymbol{\delta}^*$ in this set that is a fixed point of the mapping $\tilde{\boldsymbol{F}}(\cdot)$. It follows that $\boldsymbol{c} = \bar{\boldsymbol{c}}(\boldsymbol{a}_1) + \boldsymbol{\delta}^*$ is a fixed point of $\boldsymbol{F}_2(\cdot)$, and since (as shown above) the latter mapping must have a unique fixed point, it follows that we must have

$$\bar{\boldsymbol{c}}(\boldsymbol{a}_2) \;=\; \bar{\boldsymbol{c}}(\boldsymbol{a}_1) \,+\, \boldsymbol{\delta}^* \;\geq\; \bar{\boldsymbol{c}}(\boldsymbol{a}_1).$$

Hence each of the functions $\bar{c}_j(\boldsymbol{a})$ must be weakly increasing in each of the elements of $\boldsymbol{a}$, as asserted.

This means that along any continuous expansion path $\boldsymbol{a}(s)$ for the vector $\boldsymbol{a}(0)$ of initial assets, where the real variable s indexes distance along the expansion path [not time], if each element of $\boldsymbol{a}(s)$ is at least weakly increasing in s, then each element of $\bar{\boldsymbol{c}}(\boldsymbol{a}(s))$ will be weakly increasing in s as well. Among other things, this means that along any such expansion path (representing situations that are reached through progressively larger lump-sum transfers in period zero), if the borrowing constraint no longer constrains some sector j for the level of transfers parameterized by s, then sector j will not be borrowing-constrained for any vector of transfers corresponding to a point $s' > s$ along the expansion path.

Thus for each sector, there will be a single point along the expansion path at which that sector shifts from being borrowing-constrained (for all levels of transfers below that point) to being unconstrained (for all levels of transfers beyond that point). This is illustrated for two different network structures in Figure 3 (where the labeled points $\{\hat{a}_i\}$ on the horizontal axis are levels of initial liquid assets at which another sector ceases to be borrowing-constrained). If we let C be the subset of the sectors that are borrowing-constrained in the case of a particular vector of initial asset positions, then as one proceeds along any monotonic expansion path, the set C remains the same except at a finite number of points, and at any point where C changes, increasing s can only result in the subtraction of elements from C. Along any path that eventually makes $a^1(0)$ large enough, the set C is reduced to the empty set $\emptyset$, as also illustrated in Figure 3.

The set of possible vectors $\boldsymbol{a}(0)$ can thus be partitioned into regions corresponding to different subsets C of borrowing-constrained sectors. We have already shown (in section 2.2 of the main text) that for all $\boldsymbol{a}(0)$ close enough to $\mathbf{0}$, the set of unconstrained sectors will be U_0, the set of sectors j for which the maximum value is achieved in the problem on the right-hand side of (2.9); hence at such points the set of constrained sectors will be C_0, the complement of U_0. Because the set of constrained sectors can only shrink as a result of additional initial transfers, it follows that for all $\boldsymbol{a}(0) >> \mathbf{0}$, C must be an element of $\mathcal{C}$, the set of all subsets of C_0 (including the empty set $\emptyset$ as well as C_0 itself).

B.2.3 A closed-form solution

If we know what the set C is for a given vector $\boldsymbol{a}(0)$, it is straightforward to compute the equilibrium expenditure vector $\bar{\boldsymbol{c}}(\boldsymbol{a}(0))$ at that point.[41] The solution vector $\boldsymbol{c}(0)$ must satisfy

$$c^j(0) \;=\; \frac{a^j(0)}{\bar{p}} + \sum_k A_{jk} c^k(0)$$

for all $j \in C$, and

$$c^j(0) \;=\; c^{*j}$$

for all $j \notin C$. This is a system of linear equations to solve for $\boldsymbol{c}(0)$.

The first of these sets of equations can be written in vector form as

$$\hat{\boldsymbol{c}} \;=\; \hat{\boldsymbol{a}} + \boldsymbol{A}_{CC}\hat{\boldsymbol{c}} + \boldsymbol{A}_{CU}\check{\boldsymbol{c}}^*,$$

where $\hat{\boldsymbol{c}}$ is the vector of elements of the solution $\boldsymbol{c}(0)$ corresponding to sectors $j \in C$,; $\hat{\boldsymbol{a}}$ is the vector collecting the values of $a^j(0)/\bar{p}$ for the sectors $j \in C$; $\check{\boldsymbol{c}}^*$ is the vector of elements of $\boldsymbol{c}^*$ corresponding to sectors $k \notin C$; and the matrix $\boldsymbol{A}$ has been partitioned as in (4.7) in the main text. This system of linear equations has a unique solution (and hence the complete system has a unique solution) if and only if the matrix $\boldsymbol{I} - \boldsymbol{A}_{CC}$ is non-singular.

This is necessarily the case for any $C \in \mathcal{C}$. Note that in order for $\boldsymbol{I} - \boldsymbol{A}_{CC}$ to be singular, there would have to exist a vector $\boldsymbol{u} \neq \boldsymbol{0}$ such that $\boldsymbol{A}_{CC}\boldsymbol{u} = \boldsymbol{u}$. This would require that the set of sectors C be an isolated subsystem (that is, sectors $j \in C$ spend only on the products of sectors $k \in C$). But we have shown in section A.2 that under our assumptions, the only isolated subsystem can be S_1, the one containing sector N. We have further shown that the eigenvector $\boldsymbol{\pi}$ has non-zero elements only for sectors $j \in S_1$. Hence the elements of U_0 (of which there must be at least one) belong to S_1; it follows that not all of S_1 can belong to C_0, and thus that not all of S_1 can belong to any $C \in \mathcal{C}$. We can therefore conclude that C cannot be an isolated subsystem, from which it follows that $\boldsymbol{I} - \boldsymbol{A}_{CC}$ must be non-singular.

We can show something stronger, which is that all eigenvalues of $\boldsymbol{A}_{CC}$ must be inside the unit circle (i.e., have modulus less than 1). We note that $\boldsymbol{A}_{CC}$ is a non-negative matrix, though no longer a stochastic matrix (because the set C cannot be an isolated subsystem, as just discussed). It follows from Gantmacher (1959, chap. XIII, Theorem 3) that $\boldsymbol{A}_{CC}$ has a non-negative real eigenvalue r, such that $|\lambda| \leq r$ for all of the other eigenvalues of the matrix. This maximal eigenvalue is bounded above by

$$r \;\leq\; \max_{j \in C} \sum_{k \in C} A_{kj} \;\leq\; 1.$$

[41]Note that it may be ambiguous whether to include a particular sector j in the set C or not, as in equilibrium the sector's income may be just enough to allow it to spend the optimal quantity c^{*j}, with end-of-period assets $b^j(0) = 0$. In this case, it does not matter whether we consider the set C to include the sector j or not; the solution obtained for $\bar{\boldsymbol{c}}(\boldsymbol{a}(0))$ will be the same in either case. In such a case, the value of $\boldsymbol{a}(0)$ lies on the boundary between two regions corresponding to different sets of constrained sectors; but since the function $\bar{\boldsymbol{c}}(\boldsymbol{a}(0))$ is continuous at such boundaries, it does not matter to which region the boundary case is assigned. Note that if instead we wish to compute the effect of a *change* in $\boldsymbol{a}(0)$, it will matter how we define the set of constrained sectors C; but in that case, the right answer will depend on the direction in which $\boldsymbol{a}(0)$ is to be changed.

Here the first inequality follows from Gantmacher (p. 68), and the second from the definition of the matrix $\boldsymbol{A}$.

However, we have just shown that 1 cannot be an eigenvector of $\boldsymbol{A}_{CC}$. Thus the maximal eigenvalue must satisfy $r < 1$, from which it follows that $|\lambda| < 1$ for every eigenvalue of $\boldsymbol{A}_{CC}$. This allows us to write

$$(\boldsymbol{I} - \boldsymbol{A}_{CC})^{-1} \;=\; \boldsymbol{I} \,+\, \boldsymbol{A}_{CC} \,+\, (\boldsymbol{A}_{CC})^2 \,+\, (\boldsymbol{A}_{CC})^3 \,+\, \ldots$$

where the infinite sum must converge because the eigenvalues of $\boldsymbol{A}_{CC}$ have modulus less than 1. Since each of the terms on the right-hand side is a non-negative matrix, it follows that

$$(\boldsymbol{I} - \boldsymbol{A}_{CC})^{-1} \;\geq\; \boldsymbol{0}. \tag{B.5}$$

The system of linear local equations therefore has a unique solution

$$\boldsymbol{c}^{loc}(\boldsymbol{a}(0); C) \;=\; \left[\begin{array}{c} (\boldsymbol{I} - \boldsymbol{A}_{CC})^{-1}(\hat{\boldsymbol{a}} + \boldsymbol{A}_{CU}\check{\boldsymbol{c}}^*) \\ \check{\boldsymbol{c}}^* \end{array} \right],$$

where the solution is partitioned as in (4.7); this is the solution (4.8 given in the text. Here we have written the set C as an argument of the function, because there is a separate function of this kind for each possible choice of $C \in \mathcal{C}$. We further see that for any C, this is an affine function of $\boldsymbol{a}(0)$, which is weakly increasing in each element of $\boldsymbol{a}(0)$ because of (B.5).

We see then that if we can determine which set of sectors C is the borrowing-constrained set in the case of any given vector of initial asset balances, we can determine the value of $\bar{\boldsymbol{c}}(\boldsymbol{a}(0))$ at that point. We next show how to do this. Fixing the vector $\boldsymbol{a}(0)$, let $\bar{C}$ be the set of constrained sectors in the solution to the "Keynesian cross" system (4.6), and let C instead be any other element of $\mathcal{C}$. Then let

$$\boldsymbol{\delta} \;\equiv\; \boldsymbol{c}^{loc}(\boldsymbol{a}(0); \bar{C}) \,-\, \boldsymbol{c}^{loc}(\boldsymbol{a}(0); C)$$

measure the difference between the linear solution under the assumption that sectors $\bar{C}$ are constrained and the linear solution under the assumption instead that C is the set of constrained sectors. (Also, in what follows, let us write $\boldsymbol{c}^{loc}(\boldsymbol{a}(0); C)$ simply as $\boldsymbol{c}$, and $\boldsymbol{c}^{loc}(\boldsymbol{a}(0); \bar{C})$ as $\bar{\boldsymbol{c}}$.)

In the case of any sector $j \in C$, we must have

$$c^j \;=\; \hat{a}^j \,+\, \sum_k A_{jk} c^k,$$

$$\bar{c}^j \;\leq \hat{a}^j \,+\, \sum_k A_{jk} \bar{c}^k,$$

where the second condition holds for all j given that $\bar{\boldsymbol{c}}$ is a solution to (4.6). Subtracting the first equation from the second yields the implication

$$\delta^j \;\leq\; \sum_k A_{jk} \delta^k$$

for all $j \in C$. Instead, in the case of any sector $j \notin C$, we must have

$$c^j = c^{*j},$$

$$\bar{c}^j \leq c^{*j},$$

where again the second condition holds for all j given that $\bar{\boldsymbol{c}}$ is a solution to (4.6). Subtracting the first equation from the second yields the implication

$$\delta^j \leq 0$$

for all $j \notin C$.

Then if we let $\hat{\boldsymbol{\delta}}$ be the vector of elements of $\boldsymbol{\delta}$ corresponding to sectors $j \in C$, and $\check{\boldsymbol{\delta}}$ the vector of elements corresponding to sectors $j \notin C$, we must have

$$\hat{\boldsymbol{\delta}} \leq \boldsymbol{A}_{CC}\hat{\boldsymbol{\delta}} + \boldsymbol{A}_{CU}\check{\boldsymbol{\delta}}, \qquad \check{\boldsymbol{\delta}} \leq \boldsymbol{0}.$$

If we let $\boldsymbol{u} \equiv (\boldsymbol{I} - \boldsymbol{A}_{CC})\hat{\boldsymbol{\delta}}$, then the first inequality implies that $\boldsymbol{u} \leq \boldsymbol{0}$, and hence that

$$\hat{\boldsymbol{\delta}} = (\boldsymbol{I} - \boldsymbol{A}_{CC})^{-1}\boldsymbol{u} \leq \boldsymbol{0},$$

using (B.5). This together with the second inequality implies that $\boldsymbol{\delta} \leq 0$, and hence that

$$\boldsymbol{c}^{loc}(\boldsymbol{a}(0); \bar{C}) \leq \boldsymbol{c}^{loc}(\boldsymbol{a}(0); C). \tag{B.6}$$

The fact that (B.6) must hold for any $C \in \mathcal{C}$ then implies that $\bar{C}$ must be the selection of borrowing-constrained sectors that implies that

$$\bar{\boldsymbol{c}}(\boldsymbol{a}(0)) = \min_{C \in \mathcal{C}} \boldsymbol{c}^{loc}(\boldsymbol{a}(0); C). \tag{B.7}$$

That is, for any $\boldsymbol{a}(0) >> \boldsymbol{0}$, $\bar{C}$ must be one of the elements of $\mathcal{C}$ that solve the minimization problem on the right-hand side of (B.7). Since (B.7) must hold for arbitrary $\boldsymbol{a}(0)$, this gives us a closed-form solution for the function $\bar{\boldsymbol{c}}(\boldsymbol{a}(0))$ for all $\boldsymbol{a}(0) >> \boldsymbol{0}$.

If for values of $\boldsymbol{a}(0)$ on the boundary of the positive orthant we select as the relevant solution to (4.6) the vector $\boldsymbol{c}(0)$ that can be reached as the limit of a sequence of solutions $\boldsymbol{c}_n \rightarrow \boldsymbol{c}(0)$ corresponding to a non-increasing sequence of vectors $\boldsymbol{a}_n \rightarrow \boldsymbol{a}(0)$ with $\boldsymbol{a}_n >> \boldsymbol{0}$ for each n, then also for these values of $\boldsymbol{a}(0)$ the solution for $\boldsymbol{c}(0)$ will be the one given by (B.7). (This follows immediately from the fact that the functions defined in (B.7) are all continuous functions of $\boldsymbol{a}(0)$.) Hence (B.7) is the desired solution for all $\boldsymbol{a}(0) \geq \boldsymbol{0}$, as stated in equation (4.10) of the main text.

B.3 The transfers required to support the first-best allocation

Here we show that, as asserted in the main text, condition (5.4) is both necessary and sufficient for the first-best allocation of resources to be achieved as the equilibrium outcome, under the assumption that $i(0) = \bar{\imath}$. It is easily seen that the first-best allocation of resources (characterized in section 1.2) is achieved if and only if $c^j(0) = c^{*j}$ for each sector j. Thus we wish to show that (5.4) is necessary and sufficient for the vector $\boldsymbol{c}^*$ to be the solution to the system of equations (4.6).

We first show that the condition is necessary. The vector $\boldsymbol{c}^*$ satisfies equations (4.6) if and only if

$$\frac{a^j(0)}{\bar{p}} + \sum_k A_{jk} c^{*k} \;\geq\; c^{*j} \tag{B.8}$$

for each sector j. For $j = 1$, we have $A_{jk} = 0$ for all k, so that condition (B.8) reduces to

$$a^1(0) \;\geq\; \bar{p} c^{*1}.$$

This is just the bound expressed in (5.4); thus (5.4) is necessary in order for (B.8) to be satisfied.

Next we show that this is also a sufficient condition for (B.8) to hold for all j. We have just shown that (5.4) is equivalent to (and thus implies) the case $j = 1$ of the equation system (B.8). But we can also show that (B.8) necessarily holds for any $j \neq 1$. For any $j \neq 1$,

$$\sum_k A_{jk} c^{*k} \;=\; \sum_k \alpha_{j-k} \bar{y} \;=\; \bar{y} \;\geq c^{*j}.$$

Hence (B.8) holds for any $a^j(0) \geq 0$. Thus if (5.4) is satisfied, (B.8) holds for all j, and the vector $\boldsymbol{c}^*$ satisfies the system of equations (4.6). Thus any non-negative transfers consistent with (5.4) imply that the solution to (4.6) is one in which the borrowing constraint binds for no sector, and the allocation of resources corresponds to the first-best optimum.

C Effects of Monetary Easing

We again focus for simplicity on the case in which $\beta \to 1$. As explained in the main text, for any choice of $i(0)$, the equilibrium pattern of expenditure in period zero is given by $\boldsymbol{c}(\boldsymbol{a}(0);\, \hat{y}(i(0)))$, where

$$\boldsymbol{c}(\boldsymbol{a};\, \hat{y}) \;=\; \frac{\hat{y}}{\bar{y}} \bar{\boldsymbol{c}} \left(\frac{\bar{y}}{\hat{y}} \boldsymbol{a} \right). \tag{C.1}$$

Equilibrium expenditure on the products of each sector are then given by

$$c_k^j(0) \;=\; A_{kj} c^j(0)$$

for each j, k. Finally, the period zero contribution to utility is given by

$$W_0 \;\equiv\; \sum_{j=1}^N U^j(0) \;=\; \sum_{j=1}^N \sum_{k \in K} \alpha_k u(c_{j+k}^j(0)/\alpha_k;\, \bar{\xi}) \;-\; \sum_{k=2}^N v\left(\sum_{j=1}^N c_k^j(0);\, \bar{\xi} \right). \tag{C.2}$$

Substituting the solution for the $\{c_k^j(0)\}$ into (C.2), we can obtain W_0 as a function of $\boldsymbol{a}(0)$ and $\hat{y}(i(0))$. We further note that (B.7) together with (C.1) implies that $\boldsymbol{c}(\boldsymbol{a};\, \hat{y})$ is the minimum of a finite collection of functions that are each linear functions of $\hat{y}$ (for a given vector $\boldsymbol{a}$; hence $\boldsymbol{c}(\boldsymbol{a};\, \hat{y})$ must be a (weakly) concave function of $\hat{y}$. It then follows from the strict concavity of $u(c;\, \bar{\xi})$ in c and the (at least weak) convexity of $v(y;\, \bar{\xi})$ in y that $W_0(\boldsymbol{a};\, \hat{y})$

must be a strictly concave function of $\hat{y}$, for any vector $\boldsymbol{a}$ (as stated in the main text in section 5.2).

From this it follows that we can find the value of $\hat{y}$ that maximizes $W_0(\boldsymbol{a}(0); \hat{y})$ for any specification of $\boldsymbol{a}(0)$ simply by evaluating the derivative $\partial W_0/\partial \hat{y}$ for different possible values of $\hat{y}$. We now consider some of the properties of this partial derivative.

C.1 Derivatives of sectoral expenditure with respect to $\hat{y}$

Differentiation of the function (C.1) yields

$$\begin{aligned} \frac{\partial \boldsymbol{c}}{\partial \hat{y}} &= \frac{1}{\bar{y}}\bar{\boldsymbol{c}} - \frac{1}{\bar{p}\hat{y}}\boldsymbol{M}\boldsymbol{a}(0) \\ &= \frac{1}{\bar{y}}\boldsymbol{N}\boldsymbol{c}^* \\ &= \frac{1}{\bar{y}}\begin{bmatrix} \boldsymbol{M}_{CC}\boldsymbol{A}_{CU} \\ \boldsymbol{I} \end{bmatrix}\check{\boldsymbol{c}}^* = \begin{bmatrix} \boldsymbol{M}_{CC}\boldsymbol{A}_{CU} \\ \boldsymbol{I} \end{bmatrix}\check{\boldsymbol{f}}, \end{aligned}$$

which is equation (5.7) in the main text. The first line of the derivation uses the notation $\boldsymbol{M}$ for the matrix of partial derivatives (defined in the main text); the second line substitutes the local solution (4.8) for the function $\bar{\boldsymbol{c}}(\boldsymbol{a})$; and the final line substitutes the definition of $\boldsymbol{N}$ given in the main text (following equation (4.8)). (Note that at a point where the function $\bar{\boldsymbol{c}}$ is not continuously differentiable, this formula still correctly computes the one-sided derivatives, with an appropriate choice of the partition of the sectors defined by C.)

In the limiting case in which $a(0) \to 0$, this derivation is also correct, regardless of whether β is near 1 (since all we actually need is an assumption that $\boldsymbol{v} \to \boldsymbol{0}$). In this case, $C = C_0$, as explained in section 2.2. Now let the Frobenius-Perron maximal eigenvector $\boldsymbol{\pi}$ (discussed in section B.1 above) be partitioned into two vectors $\hat{\boldsymbol{\pi}}$ and $\check{\boldsymbol{\pi}}$ that collect the elements π_j corresponding to borrowing-constrained and unconstrained sectors respectively. The definition of the set C_0 implies that the ratio π_j/f_j takes the same value $\zeta > 0$ for all of the sectors $j \notin C_0$ (the maximum value of this ratio over all sectors). Thus $\check{\boldsymbol{\pi}}$ is a multiple of the vector $\check{\boldsymbol{f}}$, which we can write as

$$\check{\boldsymbol{\pi}} = \zeta\check{\mathbf{f}}, \tag{C.3}$$

using the definition (3.4) of θ.

Moreover, using this same partition of the sectors, the eigenvector condition $\boldsymbol{A\pi} = \boldsymbol{\pi}$ can be written

$$\begin{bmatrix} \boldsymbol{A}_{CC} & \boldsymbol{A}_{CU} \\ \boldsymbol{A}_{UC} & \boldsymbol{A}_{UU} \end{bmatrix}\begin{bmatrix} \hat{\boldsymbol{\pi}} \\ \check{\boldsymbol{\pi}} \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{\pi}} \\ \check{\boldsymbol{\pi}} \end{bmatrix}.$$

This allows us to solve for the complete eigenvector as a function of the elements of $\check{\boldsymbol{\pi}}$,

$$\begin{bmatrix} \hat{\boldsymbol{\pi}} \\ \check{\boldsymbol{\pi}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{M}_{CC}\boldsymbol{A}_{CU} \\ \boldsymbol{I} \end{bmatrix}\check{\boldsymbol{\pi}} = \zeta\begin{bmatrix} \boldsymbol{M}_{CC}\boldsymbol{A}_{CU} \\ \boldsymbol{I} \end{bmatrix}\check{\boldsymbol{f}},$$

where the second expression substitutes (C.3).

Using this result, we see that when $a(0)$ is close enough to 0, equation (5.7) can alternatively be written as

$$\frac{\partial \boldsymbol{c}}{\partial \hat{y}} = \frac{1}{\zeta}\boldsymbol{\pi},$$

as implied by the solution $\boldsymbol{c}(0) = \theta\boldsymbol{\pi}$ given in section 3 of the main text, where θ is the function of $\hat{y}$ given in (3.4). Hence conclusion (5.7) is a generalization of our results about the effects of interest-rate policy on sectoral expenditure obtained in section 3 for the case in which $a(0) \to 0$.

C.2 Suboptimality of achieving a zero "output gap" with monetary policy

Now consider the case in which $a(0) \to 0$, and suppose in addition that $cu'(c)$ is a strictly concave function, while $yv'(y)$ is at least weakly convex (as proposed in the discussion of (3.7) in the main text). We wish to sign the derivative of W_0 with respect to $\hat{y}$, which is equivalent to signing the derivative with respect to θ (the formulation used in (3.7)). Specifically, we wish to show that if $\hat{y}$ is increased to the point where $y^{agg}(0) = y^*$ (as is possible by decreasing the real interest rate enough), we will have

$$\frac{\partial W_0}{\partial \hat{y}} < 0. \tag{C.4}$$

Here we are interested in the *left* derivative with respect to $\hat{y}$, because we want (C.4) to be a condition that implies that lowering $\hat{y}$ (by raising the real interest rate, relative to the policy required to achieve a zero output gap) will increase welfare.

If we differentiate (C.2) with respect to $\hat{y}$, we obtain

$$\frac{\partial W_0}{\partial \hat{y}} = \sum_{j=1}^{N}\sum_{k\in K} u'\left(\frac{c^j_{j+k}(0)}{1-\alpha_{1-j}};\bar{\xi}\right)\frac{\partial c^j_{j+k}(0)}{\partial \hat{y}} - \sum_{k=2}^{N} v'\left(\sum_{j=1}^{N} c^j_k(0);\bar{\xi}\right)\sum_{j=1}^{N}\frac{\partial c^j_k(0)}{\partial \hat{y}}. \tag{C.5}$$

Since in the case that $a(0) \to 0$, each of the $c^j_k(0)$ grows in proportion to $\hat{y}(i(0)$, we can write the partial derivative as

$$\frac{\partial c^j_k(0)}{\partial \hat{y}} = \frac{c^j_k(0)}{\hat{y}}.$$

Then noting also that

$$\frac{c^j_{j+k}(0)}{\alpha_k} = \frac{c^j(0)}{1-\alpha_{1-j}}$$

for each of the $k \in K$, and that $\sum_{j=1}^{N} c^j_k(0) = y_k(0)$ for each k, we can rewrite (C.5) as

$$\hat{y}\frac{\partial W_0}{\partial \hat{y}} = \sum_{j=1}^{N}(1-\alpha_{1-j})f\left(\frac{c^j(0)}{1-\alpha_{1-j}}\right) - \sum_{k=2}^{N} g(y_k(0)), \tag{C.6}$$

where we define

$$f(c) \equiv cu'(c;\bar{\xi}), \qquad g(y) \equiv yv'(y;\bar{\xi}).$$

Under the assumption that $f(c)$ is a strictly concave function, Jensen's inequality implies that

$$\begin{aligned}\sum_{j=1}^{N}(1-\alpha_{1-j})f\left(\frac{c^j(0)}{1-\alpha_{1-j}}\right) &< \sum_{j=1}^{N}(1-\alpha_{1-j})f\left((N-1)^{-1}\sum_{h=1}^{N}(1-\alpha_{1-h})\frac{c^h(0)}{1-\alpha_{1-h}}\right)\\ &= (N-1)f\left(\frac{\sum_{h=1}^{N}c^h(0)}{N-1}\right)\\ &= (N-1)f\left(\frac{y^{agg}(0)}{N-1}\right) \;=\; (N-1)f(\bar{y}),\end{aligned}$$

where the last equality uses the fact that $\hat{y}(i(0))$ has been chosen so as to ensure that $y^{agg}(0) = y^* = (N-1)\bar{y}$. Here the inequality must be strict except in the case that all of the arguments $c^j(0)/(1-\alpha_{1-j})$ of the function $f(\cdot)$ are equal in value. This can occur only if

$$c^j(0) \;=\; (1-\alpha_{1-j})\frac{\sum_{h=1}^{N}c^h(0)}{N-1} \;=\; (1-\alpha_{1-j})\bar{y} \;=\; c^{*j}$$

for all j — that is, only if the policy achieves the first-best allocation. But we have shown in section 5.2 of the main text that no amount of interest-rate reduction can achieve this; hence the inequality must be strict, as written above.

Similarly, under the assumption that $g(y)$ is at least weakly convex, Jensen's inequality implies that

$$\sum_{k=2}^{N} g(y_k(0)) \;\geq\; (N-1)g\left(\frac{\sum_{k=2}^{N}y_k(0)}{N-1}\right) \;=\; (N-1)g(\bar{y}).$$

Using these two results to bound the terms on the right-hand side of (C.6), we obtain

$$\begin{aligned}\hat{y}\frac{\partial W_0}{\partial \hat{y}} &< (N-1)f(\bar{y}) - (N-1)g(\bar{y})\\ &= (N-1)\bar{y}[u'(\bar{y}) - v'(\bar{y})] \;=\; 0.\end{aligned}$$

This establishes (C.4), and hence assertion (3.7) in the main text.