Trade Wars, Currency Wars*

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Abstract

For most of the post WWII period, trade protectionism followed a declining trend, contained within international agreements. Even after the 2008-9 Great Recession, most countries forwent the temptation to restrict trade flows, and the key policy conflicts revolved around monetary policy spillovers, or 'currency wars'. Recently however, there has been a sharp shift towards unilateral, discretionary trade policy focused on short term macroeconomic objectives, and as a consequence, the phenomenon of 'trade wars' has become entangled with 'currency wars'. This paper explores the interaction of non-cooperative trade policy and monetary policy within a standard DSGE open economy macroeconomic model. We find that a non-cooperative trade policy can significantly worsen macroeconomic conditions. Moreover, the stance of monetary policy has major implications for the degree of protection in a non-cooperative equilibrium. In particular, cooperative determination of monetary policy (by eliminating 'currency wars') may significantly reduce welfare by increasing the size of trade restrictions. By contrast, when the exchange rate is pegged by one country, equilibrium rates of protection are generally lower, but in this case, there are multiple asymmetric equilibria in tariff rates which benefit one country relative to another.

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1 Introduction

Until quite recently, research on optimal trade policy was mostly divorced from that of optimal macroeconomic policy. Traditional studies on trade policy focused on the different degrees of bilateral or multilateral cooperation imposed by trade agreements within regions or more widely orchestrated by the WTO[1]. But in recent years, we have seen a progressive breakdown of existing trade agreements and a rise in protectionism among many countries. A particular aspect of this trend is the tendency to engage in protectionism to achieve short term macroeconomic objectives, such as targeting the trade balance. In this environment, the distinction between trade policy and macroeconomic policy has been blurred. This paper is intended to explore the consequences of the breakdown in cooperation between countries in trade policy, and to investigate how non-cooperative trade policy interacts with macro policy, and in particular monetary policy.

In the aftermath of the Global Financial Crisis, many writers described the spillovers of non-cooperative monetary policy across countries as an outcome of 'currency wars'.[2] Much of the discussion of currency wars emphasized the spillover of monetary policy from large countries to small countries, and the need for some form of monetary policy coordination. But with the increasing aggressive use of protectionist policies, it becomes important to focus on 'trade wars', and to understand the interaction between currency wars and trade wars.[3] In the case of the US for example, persistent threats of trade protection have been made after accusations of 'currency manipulation' by US trade partners. In light of these linkages, it may not be tenable to focus on trade policy determination independent of the stance of monetary policy.

Our paper explores the relationship between 'currency wars' and 'trade wars' within a standard two country New Keynesian DSGE model. Aside from the exploration of endogenous, non-cooperative trade policy, the model is quite standard. Households consume and supply labour, and monopolistic competitive firms maximize profits subject to costs of price adjustment. Prices are set in producer currencies. Policymakers in each country are assumed to have the choice of inflation rates through endogenous monetary policy but also trade protection in the form of tariffs.

[1]For a recent survey, see Bagwell and Staiger (2016).

[2]For instance, Mishra and Rajan (2018) argue that " Aggressive monetary policy actions by one country can lead to significant adverse cross-border spillovers on others, especially as countries contend with the zero lower bound. If countries do not internalize these spillovers, they may undertake policies that are collectively suboptimal. "

[3]For recent evaluation of the policy issues, see for instance Bénassy-Quéré, Bussière, and Wibaux (2018), Fajgelbaum et al. (2019), and Eichengreen (2019).

We first explore the consequences of currency wars on their own. That is, we compare a non-cooperative outcome of a game between discretionary policy makers who set monetary policy by choosing their domestic inflation rates with a similar game where inflation rates are set cooperatively. Here, there is a well-known tension between the desire to follow a contractionary monetary policy so as to improve the terms of trade, and an expansionary policy to eliminate domestic monopoly distortions. Moreover, as shown by Rogoff (1985) and others, cooperation in monetary policy can then be counterproductive, unless monopoly distortions are separately removed by subsidies to firms.

When we extend the model to allow for the joint determination of tariffs and inflation rates in a non-cooperative discretionary equilibrium, we find very different outcomes. In the non-cooperative game, each country imposes a very high tariff rate, and because tariffs focus on the manipulation on the terms of trade, equilibrium rates of inflation are substantially higher than in the 'currency war' equilibrium alone. Welfare is much lower when trade wars are combined with currency wars. Moreover, when countries cooperate in monetary policy, eliminating the possibility of currency wars, equilibrium tariffs rise even further, leading to higher equilibrium inflation rates, as each country attempts to further exploit its terms of trade advantage, given that under cooperation, monetary policy plays no role in affecting the terms of trade. In this case, monetary policy cooperation generates large welfare losses, much larger than in the case of currency wars alone. In addition, this negative impact of cooperation remains, even when monopoly pricing distortions are removed by subsidies to firms.

An important dimension of the debate on currency wars is the asymmetry in country size. According to conventional wisdom self-oriented policy-making by large countries imposes large negative spillovers on smaller countries. In our model, we find the opposite: a rise in country's size generally reduces its welfare - large countries don't gain an advantage in the currency war. This is because larger countries generally choose higher equilibrium inflation rates. But when we combine trade wars with currency wars, size wins out. With endogenous trade policy and monetary policy, a rise in a country's size increases its welfare, because it becomes more protectionist, increasing its tariff rate, while the smaller country reduces its tariff. Hence, in the currency and trade war, larger countries gain.

The main analysis of the paper assumes discretionary policy-making. Both trade and monetary policy is determined without committing to future policy decisions. But it may be realistic to consider that trade policy engenders some degree of commitment. In a later sub-section we present a simplified version of the trade war where trade policy-makers choose tariffs non-cooperatively, but can commit to tariff rates in advance of the currency war. In this case, equilibrium tariff rates are much lower than in the baseline case. The reason is that with commitment, trade authorities take account of the impact of higher tariffs on inflation choices of the monetary authorities, which leads them to temper their desire for terms of trade manipulation.

In an economy with sticky prices, the nominal exchange rate becomes a critical channel through which both monetary and trade policy operate. How do our results change if exchange rates are fixed by one country? Here again, we find a dramatically different outcome between the currency war and the currency and trade war equilibrium. In a fixed exchange rate regime where inflation is the only policy instrument, only one country has an independent choice of instrument, and in a symmetric equilibrium inflation rates are equalized across countries. In fact, the fixed exchange rate outcome is identical to the steady state of the cooperative monetary policy outcome.

When we allow for independent choice of tariff rates combined with a fixed exchange rate, in a symmetric equilibrium, the most striking result is that equilibrium tariffs are much lower than under flexible exchange rates. For our calibration tariff rates are only 14 percent, as opposed to 50 percent in the baseline case. The intuitive reason for this large difference between fixed and flexible exchange rates comes from the fact that under fixed exchange rates, terms of trade manipulation can be done only by generating differences in inflation rates, which in itself imposes additional costs.

But there is a caveat. While a symmetric equilibrium with fixed exchange rates has a much lower tariff rate, there are also multiple asymmetric equilibria. Terms of trade movements under fixed exchange rates are determined by inflation dynamics, so the terms of trade displays persistent dynamics. This dynamic feature, in conjunction with non-cooperative choice of tariffs, gives rise to a continuum of asymmetric tariff equilibria. As a result, either country can gain or lose relative to the other in the trade war. There are equilibria where either the Home country or the Foreign country is more protectionist, and different equilibria imply substantially different welfare outcomes. However, for reasonable parameter values, we find that Nash equilibrium tariff rates are lower and welfare for each country higher than in the baseline model under flexible exchange rates.

Finally, we extend the analysis to a situation where countries do not have control over monetary policy due to the zero lower bound constraint on nominal interest rates. In this case, our model is extended to allow for interest rate determination of monetary policy. An equilibrium constrained by the ZLB delivers lower rates of inflation, lower consumption and output, and lower welfare than the outcome under currency wars. When the ZLB constraint is present in the trade war equilibrium however, we again find high equilibrium rates of protection. Each country attempts to use tariffs to raise its terms of trade and welfare. While the trade war under the ZLB does raise equilibrium rates of inflation, the higher degree of protection reduces equilibrium consumption and output, leading to much lower welfare for all countries.

One key message of the paper is that trade wars imply very high rates of protection in standard DSGE macro models. This may seem unrealistic, given that in recent history, observed tariffs among advanced economies have been much lower. But it is important to note that these historical observations refer to a period where WTO rules and other bilateral agreements were in place. By contrast, our paper explores the consequences of a breakdown of cooperation in trade policy. In this case, the tariff rates may not be so unrealistic. For instance, in a calibrated CGE trade model, Ossa (2014) finds that average tariffs would be over 60 percent in a full-scale world 'tariff war'. Also, recent experience suggests that tariff rates may go much higher. As an example, average US tariff on China, as measured by Bown (2019), rose from 8 percent in early 2018 to 26 percent at the end of 2019.

The interaction between trade policy and the macroeconomy has long been a subject of interest to economists (e.g. Eichengreen (1981) and Krugman (1982)). But recent events have seen a revival of interest in this area and an attempt to formalize the relationship within the modern macroeconomic toolkits. A number of recent papers look at the effects of trade policy in dynamic open economy macro models. Barattieri, Cacciatore, and Ghironi (2018) investigate empirically the impact of exogenous changes in tariffs in an SVAR framework, and show that they act as negative supply shocks, depressing GDP and raising inflation with little effects on the trade balance. A similar mechanism applies to our paper, as shown in Section 4 below. Barattieri, Cacciatore, and Ghironi (2018) develop a small open economy model with firm entry and endogenous tradability that successfully rationalizes the empirical evidence. We adopt an alternative approach, considering tariffs as endogenous, exploring the consequences of alternative strategic settings for both monetary policy and tariff formation. Another paper by Erceg, Prestipino, and Raffo (2018) looks at the impact of trade policies in the form of import tariffs and export subsidies. They find that the effects critically depend on the response of the real exchange rate, and that in turn depends on the expectations about future policies and potential retaliation from trade partners. A recent paper by Furceri et al. (2019) examines the macroeconomic consequences of tariff shocks, and shows that these shocks are generally contractionary. Lindé and Pescatori (2019) study the conditions under which Lerner symmetry holds, and how this affects the macroeconomic costs of a trade war.

Two papers that are more closely ours are Jeanne (2020) and Bergin and Corsetti (2020). Jeanne (2020) is closest in spirit to our paper. He explores the interaction between 'currency wars' and 'trade wars' in a simple analytical two period model with a continuum of small open economies with downward nominal wage rigidity and in some cases a global liquidity trap, and explores the benefits of international cooperation. By contrast, our study is focused on a more conventional infinite horizon two country DSGE model, where countries are large, and focuses on a discretionary Ramsey approach to policy-making.[4] Bergin and Corsetti (2020) develop a rich multi-country DSGE model with global value chains and look at the optimal response of monetary policy to exogenous tariff shocks. In addition, they focus on cooperative determination of monetary policy and consider tariffs as exogenous. Another relevant paper is Caballero, Farhi, and Gourinchas (2015), who investigate the interaction between an environment of low interest rate, financial imbalances and currency wars. Our paper does consider a binding ZLB constraint as one of the possible cases but deals with more generic environments, and considers the joint endogenous formation of tariff and monetary policies while neglecting imbalances by assuming balanced-trade restrictions. We thus consider our paper as an important complement, with a focus on the interaction between trade and monetary policy. In particular, we find that international cooperation may be significantly welfare reducing in this environment.[5]

Focusing more closely on the endogenous determination of trade policies, we noted above that there is a large empirical literature investigating the link between trade restrictions and the eco-

[4]This type of approach echoes the approach of Chari, Nicolini, and Teles (2018) or Auray, Eyquem, and Gomme (2018), although these papers focus on flex-price environments.

[5]Corsetti and Pesenti (2001) show how national welfare in open economies may depend on a terms of trade externality, using a two-country model with monopolistic competition. There are many papers analyzing optimal monetary policy in different open economy frameworks, among them Benigno and Benigno (2003), Galì and Monacelli (2005), Faia and Monacelli (2008), De Paoli (2009), Bhattarai and Egorov (2016), Groll and Monacelli (2016), Fujiwara and Wang (2017), or more recently Egorov and Mukhin (2019).

nomic cycle, and separately, the effect of real exchange rate undervaluation on trade policy (*e.g.* Oatley (2010), Gunnar and Francois (2006), Bown and Crowley (2013), among others). In a theoretical model Eaton and Grossman (1985) study optimal tariffs when international asset markets are incomplete and show that they can be used to partly compensate the lack of consumption insurance. Bergin and Corsetti (2008) also consider tariffs as policy instruments in addition to monetary policy but their focus is not specifically on tariffs, rather on the implications of monetary policy on the building of comparative advantages. Campolmi, Fadinger, and Forlati (2014) offer a detailed analysis of optimal non-cooperative policies with a large set of instruments, including tariffs.[6] In a rich model with endogenous location of firms and an extensive margin of trade, they show that the terms-of-trade externality remains the dominating incentive to apply positive tariffs. Bagwell and Staiger (2003) propose a trade model featuring potential terms-of-trade manipulation by governments, and trade agreements as means to restrict this policy option. Our paper is complementary to theirs. Most importantly, we incorporate endogenous tariff formation within a standard open economy macro model, showing the importance of price stickiness, the exchange rate regime, the extent of cooperation or the zero lower bound on nominal interest rates for the equilibrium degree of trade protection.

The rest of the paper is organized as follows. Section 2 sets out the basic model. Section 3 defines how inflation and tariffs are determined by policymakers. Section 4 sketches out a simple version of the model to give an intuition for the impact of tariffs. Section 5 develops the main results of the paper under currency wars and trade wars, where we explore the impact of cooperation, country size, and partial commitment. Section 6 analyzes the model under fixed exchange rates. Section 7 shows the results for trade wars under the zero lower bound constraint on monetary policy. Some conclusions are offered in Section 8.

2 The Model

We describe a two country model, denoted Home and Foreign, where agents supply labour and consume goods from both countries. The world is populated with a unit mass of agents and Home

[6] More generally, our paper also relates to the literature on tax and structural reforms to manipulate the exchange, which includes Correia, Nicolini, and Teles (2008), Hevia and Nicolini (2013), Farhi, Gopinath, and Itskhoki (2014), Eggertsson, Ferrero, and Raffo (2014), Cacciatore et al. (2016), Auray, Eyquem, and Ma (2017) or Barbiero et al. (2019).

has share n of these, with Foreign share $1-n$. Firms set prices in domestic currency (PCP), and adjust prices constrained by Rotemberg-style price adjustment costs. Agents in the Home country have preferences over consumption and hours given by

$$U = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{\chi}{1+\psi} H_t^{1+\psi} \tag{1}$$

We assume *no* financial market trading across countries, which implies that trade is balanced.[7]

2.1 Households

Absent international asset trading, the Home country budget constraint is

$$P_{h,t}C_{h,t} + (1+\tau_t)S_t P^*_{f,t} C_{f,t} = W_t H_t + \Pi_t + TR_t \tag{2}$$

where $P_{h,t}$ ($P^*_{f,t}$) is the Home (Foreign) goods price in Home (Foreign) currency, S_t is the exchange rate, $C_{h,t}$ ($C_{f,t}$) is the consumption of the Home (Foreign) good, τ_t is an import tariff imposed by the Home government, W_t is the Home nominal wage, Π_t represents the profits of the Home firm and TR_t is a lump sum transfer from the Home government. The elasticity of substitution between Home and Foreign goods is λ.

It is assumed that

$$C_t = \left(\varepsilon^{\frac{1}{\lambda}} C_{h,t}^{1-\frac{1}{\lambda}} + (1-\varepsilon)^{\frac{1}{\lambda}} C_{f,t}^{1-\frac{1}{\lambda}} \right)^{\frac{1}{1-\frac{1}{\lambda}}} \tag{3}$$

where $\varepsilon \geq n$, representing the possibility of home bias in preferences.[8]

The true price index for the Home consumer given the above preferences and the price definitions then becomes

$$P_t = \left(\varepsilon P_{h,t}^{1-\lambda} + (1-\varepsilon)((1+\tau_t)S_t P^*_{f,t})^{1-\lambda} \right)^{\frac{1}{1-\lambda}} \tag{4}$$

[7] Extending the model to allow for non-zero current accounts is left for future research.

[8] Letting $0 \leq x \leq 1$ represent the degree of home bias in preferences, where $x = 0$ ($x = 1$) represents zero (full) home bias, we can define $\varepsilon = n + x(1-n)$.

Optimal consumption of Home and Foreign goods for the Home consumer is

$$C_{h,t} = \varepsilon \left(\frac{P_{h,t}}{P_t}\right)^{-\lambda} C_t \tag{5}$$

$$C_{f,t} = (1-\varepsilon)\left(\frac{(1+\tau_t)S_t P^*_{f,t}}{P_t}\right)^{-\lambda} C_t \tag{6}$$

Optimal labour supply is described by

$$W_t = \chi P_t C_t^{\sigma} H_t^{\psi} \tag{7}$$

The preferences, budget constraints, and optimal choices for the Foreign economy are analogous. The presence of home bias in Foreign preferences then implies that the price index for the Foreign economy is

$$P^*_t = \left(\varepsilon^* P_{f,t}^{1-\lambda} + (1-\varepsilon^*)\left((1+\tau^*_t)\frac{P_{h,t}}{S_t}\right)^{1-\lambda}\right)^{\frac{1}{1-\lambda}} \tag{8}$$

2.2 Firms

A measure n of firms in the Home economy produce differentiated goods. The aggregate Home good is a composite of these differentiated goods, where the elasticity of substitution between individual goods is denoted as $\epsilon > 1$. The production function for firm i in the Home country is

$$Y_{i,t} = A_t L_{i,t}^{1-\alpha} X_{i,t}^{\alpha} \tag{9}$$

where A_t is an aggregate productivity term. Here, $X_{i,t}$ represents the use of intermediate goods on the part of the Home firm i and $L_{i,t}$ the use of labor. We allow that intermediate good inputs are composed of Home and Foreign goods in a different composition than that of the consumption aggregator. Namely

$$X_{i,t} = \left(\varepsilon_x^{\frac{1}{\lambda}} X_{i,h,t}^{1-\frac{1}{\lambda}} + (1-\varepsilon_x)^{\frac{1}{\lambda}} X_{i,f,t}^{1-\frac{1}{\lambda}}\right)^{\frac{1}{1-\frac{1}{\lambda}}}$$

where $X_{i,j,t}$ is the Home firm i's use of inputs from country $j = h, f$. The profits of Home firm i are then represented as

$$\Pi_{i,t} = \left((1+s_{i,t})P_{i,h,t} - MC_t\right) Y_{i,h,t} \tag{10}$$

where $MC_t = \frac{(1-\alpha)^{\alpha-1}\alpha^{-\alpha}}{A_t} W_t^{1-\alpha} P_{x,t}^{\alpha}$ denotes the firm's marginal cost, where $P_{x,t}$ is the price index relevant for the firm's use of intermediate inputs, and $s_{i,t}$ represents a subsidy that may be given to the firm to offset the monopoly distortion in pricing.[9]

Cost minimization by the firm implies:

$$A_t(1-\alpha)L_{i,t}^{-\alpha}X_{i,t}^{\alpha}MC_t = W_t \tag{11}$$

$$A_t\alpha L_{i,t}^{1-\alpha}X_{i,t}^{\alpha-1}\varepsilon_x^{\frac{1}{\lambda}}\left(\frac{X_{i,h,t}}{X_{i,t}}\right)^{-\frac{1}{\lambda}}MC_t = P_{h,t} \tag{12}$$

$$A_t\alpha L_{i,t}^{1-\alpha}X_{i,t}^{\alpha-1}(1-\varepsilon_x)^{\frac{1}{\lambda}}\left(\frac{X_{i,f,t}}{X_{i,t}}\right)^{-\frac{1}{\lambda}}MC_t = (1+\tau_t)S_tP_{f,t}^{*} \tag{13}$$

The firm chooses its price to maximize its present value of expected profits, net of price adjustment costs

$$E_t\sum_{j=0}\Omega_t\left(\Pi_{i,t}-\xi_t\left(\frac{P_{i,h,t}}{P_{i,h,t-1}}\right)P_{h,t}Y_{h,t}\right) \tag{14}$$

where Ω_t is the firm's nominal stochastic discount factor, and $\xi_t(.)$ represents a price adjustment cost function for the firm. We assume that $\xi'(.) > 0$, and $\xi''(.) > 0$. Price adjustment costs are proportional to the nominal value of Home output, to be consistent with the nominal profit objective function of the firm.

The first order condition for profit maximization for the Home firm i can be described as

$$\begin{aligned}(1+s_t)Y_{i,h,t} = \epsilon(P_{i,h,t}(1+s_t) &- MC_t)\frac{Y_{i,h,t}}{P_{i,h,t}} + \xi'\left(\frac{P_{i,h,t}}{P_{i,h,t-1}}\right)\frac{1}{P_{i,h,t}}P_{h,t}Y_{h,t} \\ &+ E_t\Omega_{t+1}\xi'\left(\frac{P_{i,h,t+1}}{P_{i,h,t}}\right)\frac{P_{i,h,t+1}}{P_{i,h,t}^2}P_{h,t+1}Y_{h,t+1} = 0\end{aligned} \tag{15}$$

2.3 Economic Policy

There are three separate levers of policy in this model. Fiscal policy may be used to subsidize monopoly firms. Trade policy may be used to levy tariffs on imports, and monetary policy may be used to either target inflation rates or exchange rates. In the case where firms are subsidized, we follow the literature in assuming that a fiscal authority chooses a subsidy to offset the steady-state monopoly markup. But we also allow for the possibility that the monopoly markup remains as a

[9] In particular, $P_{x,t} = \left(\varepsilon_x P_{h,t}^{1-\lambda} + (1-\varepsilon_x)((1+\tau_t)S_tP_{f,t}^{*})^{1-\lambda}\right)^{\frac{1}{1-\lambda}}$.

pre-existing distortion in the economy. As we see, this may have an important implication for both optimal monetary policy and trade policy.

2.3.1 Monetary Policy

Monetary policy is represented in two ways. In the first case, we assume that each country chooses its inflation rate independently, and the nominal exchange rate is flexible. In this case, it is implicitly assumed that the monetary authority can implement a desired inflation rate through an interest rate policy, but we abstract from the details of this policy. An alternative assumption is that monetary policy is represented by an exchange-rate target on the part of the Foreign government, leaving the Home country to independently choose an inflation rate. In this case, only the Home policymaker has an independent monetary instrument.

Nevertheless, it is both possible and realistic to imagine that neither monetary authority has the ability to achieve a desired inflation target due to the zero-bound constraint on nominal interest rates. We therefore explore a situation where monetary policy is temporarily constrained, and inflation rates are determined endogenously, given expectations about future monetary policy as well as the current stance of trade policy. In this case, the only policy tool available during the zero-bound period is that of trade policy. [10]

2.3.2 Trade Policy

Trade policy is represented by tariffs chosen by each country. We assume that tariff rates are chosen to maximize domestic welfare. In this scenario, countries engage in a 'trade war', where equilibrium tariff rates are determined in a Nash equilibrium. But in an economy with sticky prices and optimally determined monetary policy, an important determinant of the outcome of trade wars is the relationship between the domestic monetary authority and the trade authority. In the Nash equilibrium of the game between countries (as described below), we assume that both inflation and tariffs are chosen simultaneously by a domestic policymaker to maximize domestic welfare. We can think of this as a case where monetary and trade policy are determined by a single decision maker, although because both authorities are 'benevolent' in maximizing Home welfare, and act

[10] Since we are assuming that all monetary policy-makers lack commitment, we do not explore the consequences of Forward Guidance in monetary policy announcements.

at the same time, the result would be the same if we thought of monetary and trade policy as determined separately by a monetary and fiscal authority. In the above case, we have assumed that the authorities in each country act separately, without cooperation. However, investigating the effects international cooperation is of critical interest. In this case, we focus on monetary policy alone, assuming that tariffs are determined independently (and non-cooperatively) by domestic policymakers. This assumption is natural, since cooperative tariffs would always be zero in a symmetric equilibrium of our model, were trade policy to be determined jointly by policymakers. As we show below, whether monetary policy is cooperative versus non-cooperative delivers very different outcomes when trade policy is determined in a non-cooperative way.

It is possible however that trade policy is determined at a different frequency than monetary policy. Even though countries may engage in trade wars, the trade authorities may be able to pre-commit in a way that is not possible for monetary authorities. While a full investigation of the duration of commitment is beyond the scope of our analysis, we explore a simple version of this idea by asking whether equilibrium tariff rates would differ if tariff setters could internalize the ex-post response of monetary authorities to the tariff rates chosen by each country. We analyze this case below, by assuming that the 'trade war' takes place 'before' the 'currency war', with trade authorities taking account of the endogenous responses of the monetary authorities.

In all cases, regardless of the assumptions about trade and monetary policy, we assume that policy is discretionary. This means that policymakers maximize *current* welfare, taking as given that future policymakers will behave in a similar fashion.

2.3.3 Government Budget constraint

While the assumptions about the stance of policy differs, the representation of the consolidated government budget constraint is the same in all situations. The government in each country balances its budget. Tariffs generate revenues, while subsidies represent a cost paid to domestic firms. The difference is rebated back to domestic households in the form of lump-sum transfers. Hence, for the Home country we have

$$TR_t = \tau_t S_t P^*_{f,t} C_{f,t} + \tau_t S_t P^*_{f,t} X_{f,t} - s_t P_{h,t} Y_{h,t} \tag{16}$$

where the last expression on the right hand side represents total subsidies paid to all domestic firms.

2.4 The Competitive Equilibrium

2.4.1 Baseline model

The full description of the competitive equilibrium for this economy is available in the Appendix. As noted above, we assume that when monetary policy is active, it can be represented by the PPI inflation rates, $\pi_{h,t} = \frac{P_{h,t}}{P_{h,t-1}}$ and $\pi^*_{f,t} = \frac{P^*_{f,t}}{P^*_{fh,t-1}}$. In addition, we can define the terms of trade as $\mathcal{S}_t = \frac{S_t P_{f,t}}{P_{h,t}}$. Using this notation, and also noting that $\frac{P_{h,t}}{P_t} = \frac{1}{\mathcal{P}_t}$, where $\mathcal{P}_t = \left(\varepsilon + (1-\varepsilon)((1+\tau_t)\mathcal{S}_t)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}$, and likewise $\mathcal{P}_{x,t} = \left(\varepsilon_x + (1-\varepsilon_x)((1+\tau_t)\mathcal{S}_t)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}$. Then, the Appendix shows that, conditional on monetary policies $\left\{\pi_{h,t}, \pi^*_{f,t}\right\}$ and tariff policies $\{\tau_t, \tau^*_t\}$, the equilibrium can be written in the form of 5 equations in the 5 variables $Y_{h,t}$, $Y^*_{f,t}$, C_t, C^*_t and $\mathcal{S}_t$. These are expressed as follows:

$$Y_{h,t} - \xi(\pi_{h,t})Y_{h,t} = \mathcal{S}_t \left(\frac{(1+\tau_t)\mathcal{S}_t}{\mathcal{P}_t}\right)^{-\lambda} D^f_{h,t} + \left(\frac{1}{\mathcal{P}_t}\right)^{-\lambda} D^h_{h,t} \quad (17)$$

$$(1+s_t) = \epsilon\left((1+s_t) - \mathcal{P}_{x,t}\alpha^{-1}\mathcal{X}_t\right) + \xi'(\pi_{h,t})\,\pi_{h,t} + E_t\omega_t\xi'(\pi_{h,t+1})\,\pi_{h,t+1} \quad (18)$$

$$(1+s^*_t) = \epsilon\left((1+s^*_t) - \mathcal{P}^*_{x,t}\mathcal{S}_t^{-1}\alpha^{-1}\mathcal{X}^*_t\right) + \xi'\left(\pi^*_{f,t}\right)\pi^*_{f,t} + E_t\omega^*_t\xi'\left(\pi^*_{f,t+1}\right)\pi^*_{f,t+1} \quad (19)$$

$$Y_{h,t}(1-\xi(\pi_{h,t})) = \left(\frac{1}{\mathcal{P}_t}\right)^{-\lambda} D^h_{h,t} + \frac{1-n}{n}\left(\frac{(1+\tau^*_t)}{\mathcal{P}^*_t}\right)^{-\lambda} D^h_{f,t} \quad (20)$$

$$Y_{f,t}(1-\xi\left(\pi^*_{f,t}\right)) = \left(\frac{\mathcal{S}_t}{\mathcal{P}^*_t}\right)^{-\lambda} D^f_{f,t} + \frac{n}{1-n}\left(\frac{(1+\tau_t)\,\mathcal{S}_t}{\mathcal{P}_t}\right)^{-\lambda} D^f_{h,t} \quad (21)$$

where

$$D^f_{h,t} = (1-\varepsilon)C_t + (1-\varepsilon_x)\left(\frac{\mathcal{P}_t}{\mathcal{P}_{x,t}}\right)^{-\lambda}\mathcal{X}_t \text{ and } D^h_{h,t} = \varepsilon C_t + \varepsilon_x\left(\frac{\mathcal{P}_t}{\mathcal{P}_{x,t}}\right)^{-\lambda}\mathcal{X}_t \quad (22)$$

$$D^h_{f,t} = (1-\varepsilon^*)C^*_t + (1-\varepsilon^*_x)\left(\frac{\mathcal{P}^*_t}{\mathcal{P}^*_{x,t}}\right)^{-\lambda}\mathcal{X}^*_t \text{ and } D^f_{f,t} = \varepsilon^* C^*_t + \varepsilon^*_x\left(\frac{\mathcal{P}^*_t}{\mathcal{P}^*_{x,t}}\right)^{-\lambda}\mathcal{X}^*_t \quad (23)$$

stand for aggregate Home/Foreign demands for Home/Foreign goods, and

$$\mathcal{X}_t = \alpha\left(\frac{\mathcal{P}_t C^\sigma_t Y^\psi_{h,t}}{\mathcal{P}_{x,t} A_t^{(1+\psi)/(1-\alpha)}}\right)^{\frac{1-\alpha}{1-\alpha+2\alpha}} Y_{h,t}\Phi_1 \text{ and } \mathcal{X}^*_t = \alpha\left(\frac{\mathcal{P}^*_t C^{*\sigma}_t Y^\psi_{f,t}}{\mathcal{P}^*_{x,t} A_t^{*(1+\psi)/(1-\alpha)}}\right)^{\frac{1-\alpha}{1-\alpha+2\alpha}} Y_{f,t}\Phi_1 \quad (24)$$

for the Home and Foreign aggregate use of intermediate goods, with $\Phi_1 \equiv \left(\frac{1}{(\alpha^{\alpha})^2}\right)^{\frac{1}{1-\alpha+2\alpha}}$. Equation (17) represents the concentrated Home budget constraint, taking account of the demand for Home intermediate goods on the part of Home firms. Then, (18) and (19) are the Home and Foreign inflation equations, after substituting for the marginal cost functions, while (20) and (21) are the goods market clearing conditions for the two countries.

2.4.2 Fixed Exchange Rates

In the baseline model, we assume that the policymaker in the Home and Foreign country can choose $\pi_{h,t}$ and $\pi^*_{f,t}$ respectively. But if the Foreign country targets the nominal exchange rate, we must have

$$\pi_{h,t} = \pi^*_{f,t} \frac{\mathcal{S}_{t-1}}{\mathcal{S}_t} \tag{25}$$

This adds a state variable to the model in the form of the lagged terms of trade. Since the nominal exchange rate is pegged, the terms of trade can adjust only via differences in inflation rates. In addition, because the Foreign country is pegging the nominal exchange rate, it cedes control of $\pi^*_{f,t}$, so the Home country takes (25) as a constraint in its choice of $\pi_{h,t}$.

2.4.3 The Zero Bound

In the above formulation, we implicitly assumed that the monetary authority could freely adjust nominal interest rates to target its desired rate of domestic PPI inflation. When there is no constraint on nominal interest rate adjustment, it is not necessary to incorporate household's Euler equation conditions. But in order to capture the zero bound constraint, we have to incorporate households inter-temporal choice. Because we are in financial autarky, net national saving is zero, so household's Euler equations must be consistent with zero current account balance. Nonetheless, when nominal interest rates are constrained at zero, households inter-temporal savings decisions have an impact on aggregate demand and economic activity. In the case of the Home economy, defining R_t as the gross nominal interest rate, the Euler equation is:

$$1 = \beta \exp(-\zeta_t) E_t \frac{C_t^{\sigma} P_t}{C_{t+1}^{\sigma} P_{t+1}} R_t \tag{26}$$

We assume that outside of the ZLB, the interest rate is determined by the Taylor rule:

$$R_t^{Taylor} = \frac{1}{\beta} \exp(\zeta_t) \left(\frac{\pi_{h,t}}{\bar{\pi}} \right)^{\sigma_\pi} \tag{27}$$

where $\bar{\pi}$ is a target rate of inflation, which is set to mimic the steady state of the Nash equilibrium in the policy game defined below, and ζ_t is a time preference shock. We also assume that $\sigma_\pi > 1$. We will assume an 'MIT' shock process for the ζ_t shock. Initially, $\zeta = 0$, but then $\zeta < 0$ occurs without anticipation, and continues with probability μ, while it reverts to zero with probability $1 - \mu$. We assume identical ζ_t shocks in each country. We focus on a $\zeta_t < 0$ that is large enough in absolute value that, without restriction, $R_t < 1$ would be required so satisfy (26) and (27). In this case, we need to impose the interest rate non-negativity constraint:

$$R_t = \max\left(R_t^{Taylor}, 1 \right) \tag{28}$$

3 Monetary Policy and Trade Policy

3.1 Optimal Monetary Policy: Currency Wars

We first analyze the determination of optimal inflation rates in a discretionary Nash equilibrium. In a discretionary Nash equilibrium each government chooses its inflation rate to maximize domestic welfare, subject to (17)-(21). Since there is no trade in financial assets, there are no endogenous state variables in the model, so a discretionary (time-consistent) Nash equilibrium can be described simply by each government's choice of current-valued variables, taking future inflation rates, consumption levels, output levels and terms of trade as given. Hence, a discretionary Nash equilibrium in monetary policy can be described as:

$$\text{Max}_{\{C_t, C_t^*, Y_{h,t}, Y_{f,t} \mathcal{S}_t, \pi_{h,t}\}} \quad \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{\chi}{1+\psi} \left(\frac{Y_{h,t}}{A_t \mathcal{X}_t^{\alpha}} \right)^{\frac{1+\psi}{1-\alpha}} \tag{29}$$

subject to (17)-(21).

$$\text{Max}_{\{C_t, C_t^*, Y_{h,t}, Y_{f,t} \mathcal{S}_t, \pi_{f,t}\}}, \quad \frac{C_t^{*1-\sigma} - 1}{1-\sigma} - \frac{\chi}{1+\psi} \left(\frac{Y_{f,t}}{A_t^* \mathcal{X}_t^{*\alpha}} \right)^{\frac{1+\psi}{1-\alpha}} \tag{30}$$

subject to (17)-(21).

We compare the Nash discretionary equilibrium to a cooperative discretionary equilibrium. In this case, a global social planner chooses inflation rates in both countries to maximize the sum of Home and Foreign utility, subject to the competitive equilibrium conditions (17)-(21).

$$\begin{aligned} \text{Max}_{\{C_t, C_t^*, Y_{h,t}, Y_{f,t} \mathcal{S}_t, \pi_{h,t}, \pi_{f,t}\}} \quad & n\left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{\chi}{1+\psi}\left(\frac{Y_{h,t}}{A_t \mathcal{X}_t^{\alpha}}\right)^{\frac{1+\psi}{1-\alpha}}\right) \\ + \quad & (1-n)\left(\frac{C_t^{*1-\sigma}-1}{1-\sigma} - \frac{\chi}{1+\psi}\left(\frac{Y_{f,t}}{A_t^* \mathcal{X}_t^{*\alpha}}\right)^{\frac{1+\psi}{1-\alpha}}\right) \end{aligned} \tag{31}$$

subject to (17)-(21).

3.2 Optimal monetary policy under a fixed exchange rate

Under a fixed exchange rate regime, we must explicitly account for the initial conditions faced by the policymakers in the form of the lagged terms of trade $\mathcal{S}_{t-1}$. Since the peg itself represents the monetary policy of the Foreign country, we describe a fixed exchange problem as the problem of the Home country. In this case, we the Home country will choose $\pi_{h,t}$ to maximize its value $V(\mathcal{S}_{t-1})$. The problem can be stated as

$$\text{Max}_{\{C_t, C_t^*, Y_{h,t}, Y_{f,t}, \mathcal{S}_t, \pi_{h,t}, \pi_{f,t}\}} \quad V(\mathcal{S}_{t-1}) = \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{\chi}{1+\psi}\left(\frac{Y_{h,t}}{A_t \mathcal{X}_t^{\alpha}}\right)^{\frac{1+\psi}{1-\alpha}} + E_t \beta V(\mathcal{S}_t) \tag{32}$$

subject to (17)-(21) and (25).

3.3 Currency Wars and Trade Wars

Our main interest is in the determination of trade policy and its interaction with the monetary policy stance. To explore this we allow the policymakers to choose both tariffs and inflation rates in a Nash discretionary equilibrium. The definition of a Nash discretionary equilibrium with endogenous tariffs and inflation is extended in a straightforward way as follows:

$$\text{Max}_{\{C_t, C_t^*, Y_{h,t}, Y_{f,t}, \mathcal{S}_t, \pi_{h,t}, \tau_t\}} \quad \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{\chi}{1+\psi}\left(\frac{Y_{h,t}}{A_t \mathcal{X}_t^{\alpha}}\right)^{\frac{1+\psi}{1-\alpha}} \tag{33}$$

subject to (17)-(21).

$$\text{Max}_{\{C_t, C_t^*, Y_{h,t}, Y_{f,t}, \mathcal{S}_t, \pi_{f,t}, \tau_t^*\}} \quad \frac{C_t^{*1-\sigma} - 1}{1-\sigma} - \frac{\chi}{1+\psi}\left(\frac{Y_{f,t}}{A_t^* \mathcal{X}_t^{*\alpha}}\right)^{\frac{1+\psi}{1-\alpha}} \tag{34}$$

subject to (17)-(21).

As noted above, we will also analyze a case where monetary policy is chosen cooperatively as described by (31), but trade policy is chosen separately by each national government.

3.3.1 Currency Wars and Trade Wars under a fixed exchange rate

In the case of a fixed exchange rate, the Home country chooses both an inflation rate and its tariff rate to maximize Home welfare, while the Foreign country's only policy lever is its tariff rate. In this case, the Foreign country's problem is an amended version of (34) where it chooses only the tariff rate and has no control over its inflation rate, which is indirectly determined as a consequence of the Home country's inflation policy as well as the tariff rates chosen by both countries.

4 A simple illustration of the model mechanism

Before going to the full solution and optimal policy problem, we develop a simplified version of the model to illustrate the impact of 'currency wars' and 'trade wars' in the form of inflation choices and tariffs in the model. For this section we assume that $\alpha = 0$, so there are no intermediate goods, and we take countries to be equally sized, so that $n = \frac{1}{2}$. We take a linear approximation of the model around a steady state of zero inflation and zero tariffs. We assume that all changes in tariffs and inflation are i.i.d., so shocks are purely transitory. We can then represent the model in the form of log deviations from the steady state. Equations (17), (20) and (21) can then be represented as:

$$c = y - (1-\epsilon)s \tag{35}$$

$$y = \epsilon c + (1-\epsilon)c^* + 2\lambda\epsilon(1-\epsilon)s + \lambda\epsilon(1-\epsilon)(t - t^*) \tag{36}$$

$$y^* = \epsilon c^* + (1-\epsilon)c - 2\lambda\epsilon(1-\epsilon)s - \lambda\epsilon(1-\epsilon)(t - t^*) \tag{37}$$

Lower case letters represent log deviations from the steady state symmetric mean, where s is the log deviation of the terms of trade, and t and t^* represent log deviations of tariffs.

Equations (35)-(37) can be solved for relative consumption and the terms of trade as a function of relative output and relative tariffs:

$$\Delta c = \frac{2\lambda\varepsilon(1-\varepsilon)\Delta t + (2\varepsilon\lambda - 1)\Delta y}{2\varepsilon(\lambda - 1) + 1} \tag{38}$$

$$s = \frac{-\lambda\varepsilon\Delta t + \Delta y}{2\varepsilon(\lambda - 1) + 1} \tag{39}$$

where $\Delta c = c - c^*$, $\Delta t = t - t^*$, and $\Delta y = y - y^*$.

Equation (38) gives relative Home versus Foreign consumption as an increasing function of Home versus Foreign output, and Home versus Foreign tariff rates (all in terms of log deviations from steady state). Equation (39) gives the terms of trade as a negative function of relative tariffs and a positive function of relative output.

The linearized inflation equations from (18) and (19) when inflation shocks are transitory can be represented as:

$$\pi_h = \kappa(y + c + (1-\varepsilon)(s + t)) \tag{40}$$

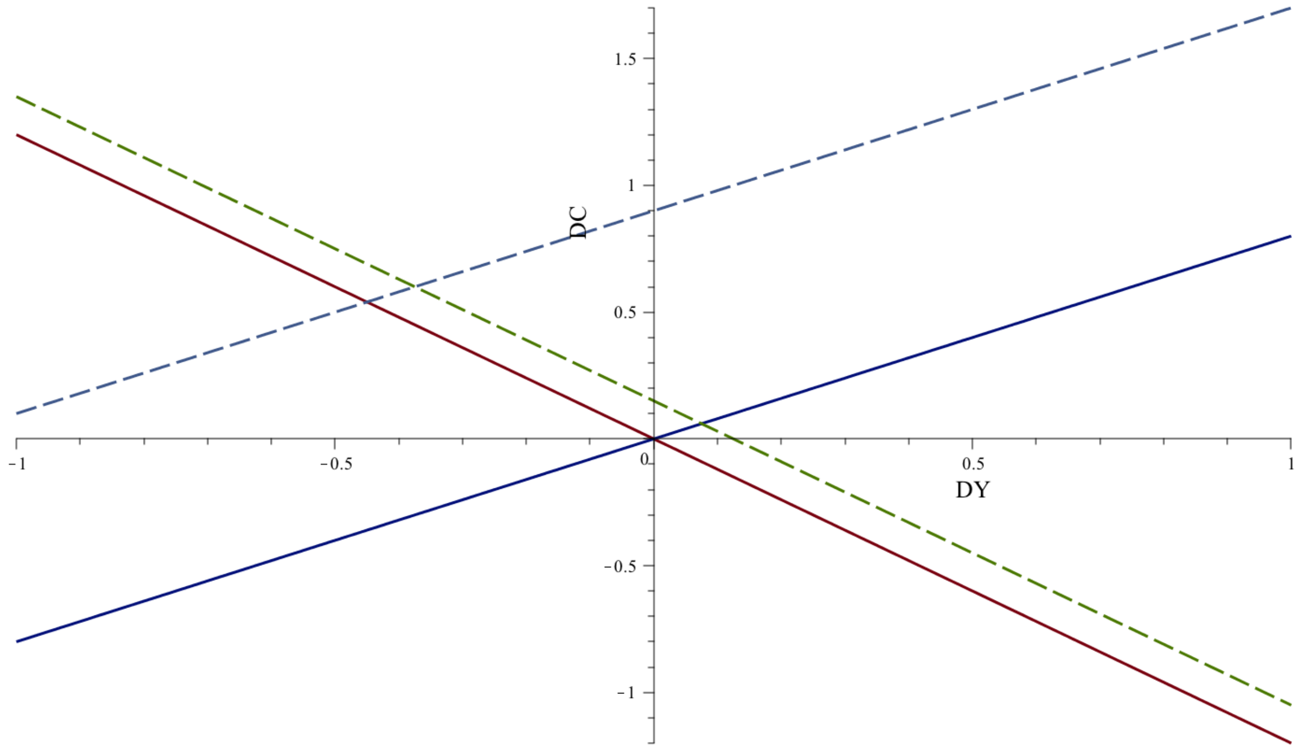
$$\pi_f^* = \kappa(y^* + c^* + (1-\varepsilon)(-s + t^*)) \tag{41}$$

Here κ represents a measure of price stickiness coming from the underlying price adjustment cost model. Substituting in the solutions (39) we get a relationship between relative consumption, relative output, the relative tariff rates, and relative inflation rates, as follows:

$$\Delta c = \frac{(2\varepsilon - 1)(1-\varepsilon)\Delta t - (2\varepsilon(\lambda - 2) + 3)\Delta y}{2\varepsilon(\lambda - 1) + 1} + \frac{\Delta\pi}{\kappa} \tag{42}$$

Equations (38) and (42) can be represented as the upward and downward sloping lines in Figure 1, respectively. Together they determine relative consumption and relative output as a function of relative inflation and relative tariff shocks. Since (38) is independent of inflation shocks, a rise in inflation, shifting up (42), will raise relative Home consumption, and relative Home output. By contrast, a rise in the Home tariff rate will shift up both curves, when $\epsilon > \frac{1}{2}$. The relative demand

Figure 1: A tariff shock in the simplified model



Relative Demand (blue) Relative Supply (red)

curve shifts up because tariffs redirect world demand towards the Home country, even accounting for the terms of trade appreciation. The relative supply curve shifts up because, given the impact of tariffs on the terms of trade, marginal costs for the Home relative to the Foreign country are reduced by a Home tariff rate increase when there is home bias in preferences. Figure 1 illustrates that for the calibration used in the full model solution below, a Home tariff will shift up (38) by more than (42). As a consequence, relative Home consumption rises, but relative Home output falls, for given $\Delta\pi$.

It is straightforward to use the same approximation of equations (35)-(37) and (40) and (41) to show that a rise in the Home tariff rate reduces overall world consumption and output as follows

$$c + c^* = y + y^* = -\frac{(1-\varepsilon)t}{2}$$

Since a rise in the Home tariff rate increases Home relative consumption but decreases Home relative output, and world consumption and output fall together, it follows that in this simple version of the model, a rise in the Home tariff will reduce Foreign consumption and reduce Home output. By contrast, the impact on Home consumption is positive, given by

$$c = \frac{1-\varepsilon}{2}\frac{\varepsilon t}{2\varepsilon(\lambda-1)+1} > 0.$$

Since Home welfare will therefore increase, this supports the conventional intuition that each country will have a welfare based incentive to raise tariffs, starting at a symmetric steady state with zero tariffs.

5 Optimal Monetary and Trade Policy

5.1 Calibration

We now derive the solution to the optimal policy games in the full non-linear model. We calibrate the model in a very straightforward manner. We let the trade elasticity $\lambda = 3$. This is on the higher end of the range estimated by Feenstra et al. (2018) , but is more appropriate for the evaluation of trade policy. Initially, we set $\sigma = 1$ to imply log utility, but also examine alternative values of σ. We set $\psi = 1$ to capture a Frisch elasticity of labor supply equal to unity, and normalize $\chi = 1$. We assume the individual firm's elasticity of demand $\epsilon = 6$, which implies a 20 percent price-cost markup. We initially assume equal sized countries so that $n = 0.5$. We then set $x = .3$, so that $\varepsilon = \varepsilon^* = .6$. In addition, following standard settings for Rotemberg pricing equations, we assume that $\xi(\pi) = \frac{\phi}{2}(\pi-1)^2$ and set $\phi = 70$. Finally, we assume that materials constitute 40 percent of gross output, so that $\alpha = 0.4$.

5.2 Currency Wars

Table 1 describes the steady state outcome of the Nash and Cooperative equilibrium where policymakers choose only inflation rates, without being constrained by the zero bound. In the Nash equilibrium each country faces a trade-off between choosing a positive rate of inflation in order to eliminate the monopoly pricing distortion on economic activity, and choosing disinflation to reduce output and appreciate the terms of trade vis-a-vis their trading partner, thus partly substituting for the absence of direct trade policy instruments. For the particular calibration in Table 1, the

Table 1: Currency wars

Variable	Non-cooperative	Cooperative	Non-coop/Subsidy	Coop/Subsidy
C	0.291	0.287	0.328	0.339
C^*	0.291	0.287	0.328	0.339
Y_h	0.437	0.460	0.554	0.555
Y_f	0.437	0.460	0.554	0.555
$\mathcal{S}$	1.00	1.00	1.00	1.00
π_h	1.014	1.036	0.973	1.00
π_f^*	1.014	1.036	0.973	1.00
U	-1.651	-1.699	-1.654	-1.604

Table showing equilibrium of Non-cooperative and Cooperative Monetary Policy, with and without offsetting subsidies for monopoly pricing

first motive dominates, and the Nash equilibrium inflation rate is 1.4 percent. By contrast, with cooperative monetary policy the terms of trade motive is eliminated, and each country chooses a much higher positive rate of inflation of 3.6 percent. The Table in fact indicates that monetary policy cooperation is welfare reducing. This is essentially the well-known Rogoff (1985) result that international cooperation may be counterproductive in face of discretionary monetary.

If subsidies are in place to offset the monopoly distortion, then Table 1 indicates that each country follows a sharply deflationary monetary policy in a Nash equilibrium, since the terms of trade motive then fully dominates the incentives for inflation in each country. Output and consumption are higher than in the absence of optimal subsidies, but the lower rate of inflation generates welfare costs which means that welfare is lower than in the Nash equilibrium without subsidies. Hence, eliminating the monopoly price distortion so much exacerbates the currency war that welfare *falls* for both countries. By contrast, if optimal subsidies are in place, and monetary policy is chosen cooperatively, inflation rates are zero, then the equilibrium is first-best, since all distortions are eliminated and inflation is zero.

To address the motivation discussed in the introduction, we conclude from these results that 'currency wars' may be either good or bad. If there is a pre-existing monopoly distortion, then, following the logic of Rogoff, we confirm that cooperation in monetary policy may be undesirable, whereas with optimal subsidies in place, cooperation supports the first best outcome.

Before we address the interaction of currency wars and trade wars, it is revealing to ask how the presence of exogenously determined tariffs would affect the outcome of the currency war in

Table 1. Tariffs are inherently distortionary and, when equally applied by both countries, they convey no terms-of-trade advantage to either country. But in the presence of a different set of distortions, second-best logic suggests that the welfare consequences of trade restrictions cannot be immediately inferred. Figure 2 illustrates this. The Figure shows the impacts of tariffs, on the horizontal axis, and inflation rates and welfare, on the vertical right and left axis respectively. In both the left and right panel, corresponding to the case where the monopoly pricing distortions are present or eliminated by an offsetting subsidy, we find that tariffs increase the Nash equilibrium inflation rate. Intuitively, the production distortion imposed by tariffs leads policymakers to choose a higher rate of inflation. When there is a monopoly pricing distortion, the left panel shows that even small tariffs reduce welfare, since the combination of an increased production distortion and higher inflation is unambiguously negative. But in the case where the monopoly pricing distortion is offset, the right panel of Figure 2 shows that a small mutual tariff (imposed by both countries) can actually increase welfare through its effect on endogenously reducing the deflation rate that is chosen in the currency war between the two countries. Thus, surprisingly, a small level of trade restrictions can be desirable in reducing the negative impacts of currency wars when monopoly distortions are eliminated.

5.3 Currency Wars and Trade Wars

We now compare the previous results to a situation where national governments choose both tariffs and inflation rates in a Nash discretionary equilibrium. Table 2 illustrates the allocations and welfare effects of the combined currency war and trade war. We start with the far right-hand panel in the Table, which shows the outcome where both countries choose inflation rates non-cooperatively, but the Home country chooses an optimal tariff unilaterally. Under the current calibration, the Home country chooses a tariff rate of 52 percent. This generate a 22 percent appreciation in its terms of trade and raises Home welfare at the expense of Foreign welfare, comparing the left panel of Table 1 with the right panel of Table 2. In addition, as suggested by the simplified model of the last section, the tariff raises relative Home consumption, but reduces relative Home output. In addition, the Home country unilateral tariff leads to an endogenous rise in PPI inflation rates in both countries, but more-so in the Home country.

The left-hand panel of Table 2 shows the results for a trade war, where both countries choose an

Figure 2: Effects of exogenous tariffs under currency wars

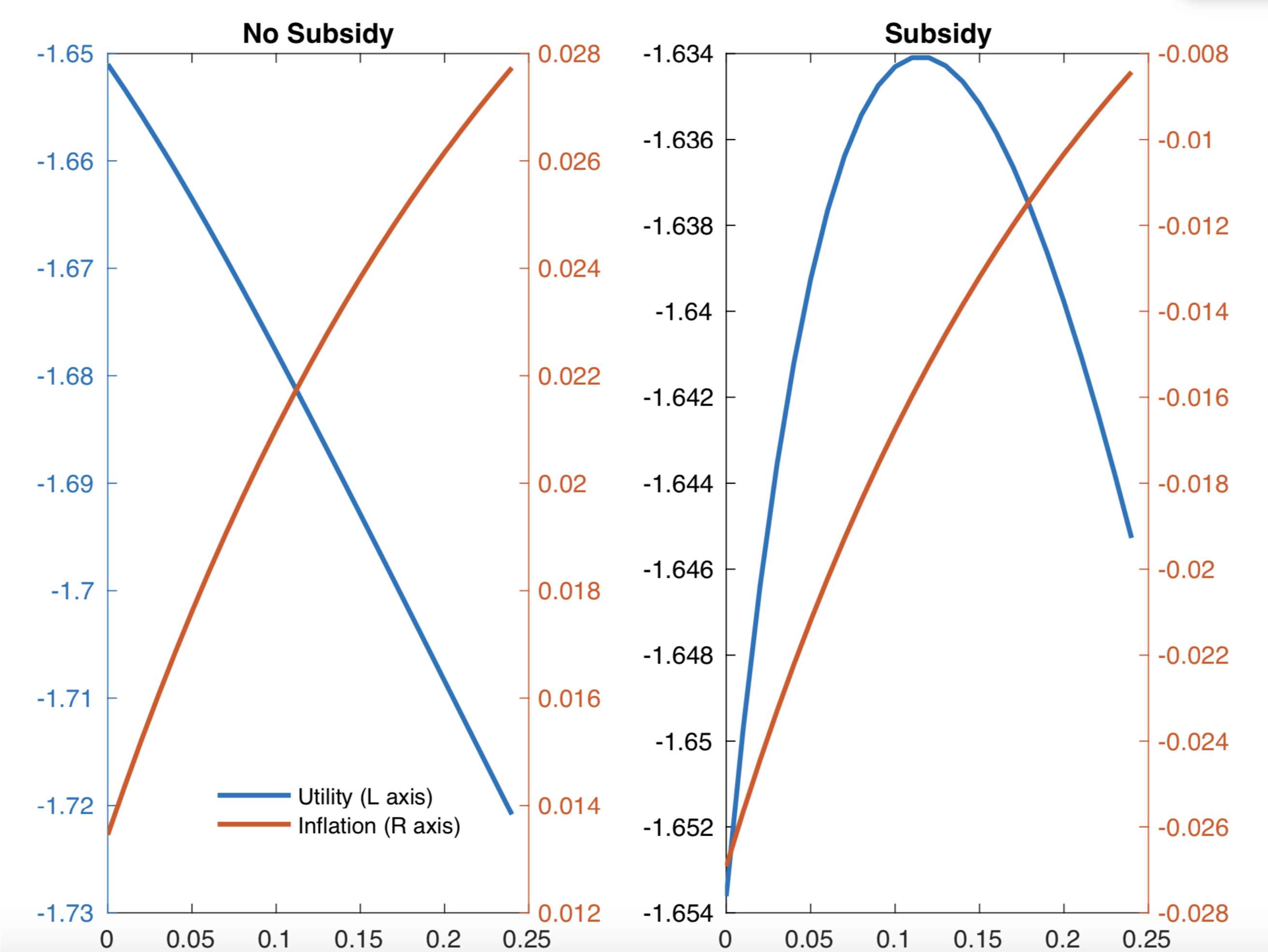


Table 2: Trade and Currency Wars

Variable	Non-coop M.	Coop M.	Subsidy, Non-coop M.	Subsidy, Coop M.	Unilateral
C	0.249	0.243	0.289	0.289	0.284
C^*	0.249	0.243	0.289	0.289	0.252
Y_h	0.399	0.416	0.478	0.487	0.402
Y_f	0.399	0.416	0.478	0.487	0.421
$\mathcal{S}$	1.00	1.00	1.00	1.00	0.777
π_h	1.034	1.049	1.00	1.016	1.033
π_f^*	1.034	1.049	1.00	1.016	1.022
τ	0.490	0.526	0.58	0.590	0.522
τ^*	0.490	0.526	0.58	0.590	0.00
U	-1.794	-1. 845	-1.713	-1.720	-1.6352
U^*	-1.794	-1.845	-1.713	-1.720	-1.808

Table showing equilibrium of Tariff war and Currency Wars, with and without offsetting subsidies for monopoly pricing optimal tariff rate, in addition to an optimal inflation rate, in a Nash equilibrium. The trade war leads to mutual tariff rates of 49 percent. In the symmetric Nash equilibrium, there is no change in the terms of trade, but the rise in domestic prices leads to a shift back in labor supply which reduces equilibrium employment and output. At the same time, the fall in consumption of imported goods distorts the composition of consumption and leads to a fall in aggregate consumption in both countries. Thus, the trade war has large negative effects on real activity.

Table 2 also shows, however, that the trade war causes a change in equilibrium inflation rates. Absent the trade war, Nash equilibrium inflation rates were 1.4 percent (see Table 1), which as described above, represented a balance between the desire to eliminate monopoly distortions and the desire to improve the national terms of trade. When countries engage in the trade war, optimal tariffs focus on the second objective – terms of trade manipulation – and monetary authorities redirect inflation rates towards the first objective. As a result, inflation rates are quite higher – at 3.4 percent – in the equilibrium with both trade and currency wars.

Table 2 further indicates that the trade war has major implications for welfare. Comparing the Nash discretionary equilibrium of the combined Trade and currency wars with that of the Nash equilibrium under the currency war alone (left panel of Table 2 compared with that of Table 1) leads to a fall in welfare of 14 percent in terms of consumption equivalents.

The second column of Table 2 documents the outcome where policymakers cooperate on monetary policy, but follow a trade war in the choice of tariffs. As we would anticipate, given the results of Table 1, monetary policy cooperation is, again, counter-productive. But this is now for two reasons. First, as before, the equilibrium inflation rates increase from 3 percent to 4.9 percent, as monetary policy focuses only on eliminating domestic distortions and ignores the impact on the terms of trade. Second, this adjustment in the focus of monetary policy leads to a redirection of tariffs: the trade war becomes more intense, as independent policymakers increase tariffs to more fully exploit a terms of trade advantage. Tariff rates increase to 53 percent – against 49 percent when monetary policy is non-cooperative – and aggregate consumption falls by 2.5 percent. As a result, the welfare costs of counter-productive cooperation in monetary policy are greater in the presence of endogenous tariffs.

When optimal subsidies are in place to eliminate the monopoly pricing distortion, inflation rates are no longer targeted at eliminating the distortion in equilibrium output. While Table 1 showed that, in this case, governments follow deflationary policies, Table 2 shows that inflation rates are zero in the trade war and currency war equilibrium. However, equilibrium tariffs rise further to 58 percent in this case. Intuitively, in the absence of the domestic distortion and with inflation rates set to zero, there is no equilibrium cost of price re-setting and each policymaker further attempts to exploit its terms of trade advantage, leading to a Nash equilibrium with lower output and aggregate consumption than in the analogous case with a currency war alone.

While Table 1 showed that the combination of an optimal subsidy and monetary policy cooperation delivered the first-best outcome with zero inflation and no distortions, the fourth column of Table 2 shows that monetary policy cooperation reduces welfare when tariffs are determined non-cooperatively, even in the presence of optimal subsidies. Indeed, comparing the 3rd and 4th column of Table 2, cooperation in monetary policy leads to a rise in equilibrium inflation rates, and a rise in equilibrium tariff rates. Not only do tariff rates rise again as in the no-subsidy case because tariffs alone can be directed towards improving the terms of trade, but inflation rates rise in both countries. The rise in inflation is intuitive: with joint determination of monetary policy, policymakers raise inflation to offset the distortion in output and consumption imposed by the trade war. Thus, monetary policy cooperation is counter-productive, even without the presence of monopoly pricing distortions.

These results illustrate that there is a significant interaction between trade policy and monetary policy in an environment with sticky prices. A natural question is how does trade policy in this environment differ from an economy without nominal rigidities; *i.e.* how do these results differ compared with a standard tariff war in a flexible price economy? Table 3 illustrates the outcome of a trade war in a flexible price economy. The second column shows that equilibrium tariffs are actually lower and welfare higher than in the economy with sticky prices and non-cooperative monetary policy (the first column of Table 2). Thus, trade wars are more intense in the equilibrium where there is a combined trade war and currency war.

These results indicate that trade wars imply very high rates of protection in standard DSGE macro models. We might question the relevance of this analysis, given that in recent history, observed tariffs among advanced economies have been much lower. For instance, the average degree of trade restriction (including both tariff and non-tariff barriers), reported by UNCTAD (2013) for advanced economies is approximately 10 percent. But it is important to note that these

Table 3: Trade War under flexible prices

Variable	Flexible Price	Flexible Price Trade War
C	0.290	0.254
C^*	0.290	0.254
Y_h	0.430	0.377
Y_f	0.430	0.377
$\mathcal{S}$	1.00	1.00
π	-	1.05
π^*	-	1.05
τ	0	0.448
τ^*	0	0.448
U	-1.646	-1.744
U^*	-1.646	-1.744

Table showing equilibrium of Flexible Price economy with and without Trade War

observations are taken from a period where WTO rules and other bilateral agreements governed the size of tariffs. The interpretation we follow here is to explore the consequences of a full scale breakdown of cooperation in trade policy. In this case, the tariff rates may not be so unrealistic. In fact, in a calibrated multi-country trade model, Ossa (2014) finds that average tariffs would be over 60 percent in a full-scale world 'tariff war'. In addition, we note that in the case of US China trade, average US tariff rates as measured by Bown (2019) rose from 8 percent in early 2018 to 26 percent at the end of 2019.

5.4 Country-size effects and alternative parameter values

All the previous derivations assumed equal sized countries. But discussion of currency wars and trade wars in the policy debate is most often focused on the role of large countries relative to small countries. Particularly in the discussion of monetary policy spillovers, it is often argued that smaller countries are more prone to the negative effects of uncoordinated policy spillovers from larger countries.

In the baseline model without endogenous policy choice, country size is irrelevant for real outcomes such as consumption, output, terms of trade or welfare. This is because as country size varies, so also does the range of goods that each country produces, so size has no implications for the terms of trade. Country size may however matter when countries engage in currency wars or trade wars. Table 4 illustrates the importance of large versus small countries in the case of currency

Table 4: Effects of country size on combined Trade and Currency Wars

Variable	Currency War	Currency War $n = .75$	Trade War	Trade War $n = .75$
C	0.291	0.289	0.249	0.272
C^*	0.291	0.291	0.249	0.218
Y_h	0.437	0.448	0.399	0.431
Y_f	0.437	0.430	0.399	0.362
$\mathcal{S}$	1.00	1.005	1.00	0.90
π	1.014	1.025	1.034	1.035
π^*	1.014	1.00	1.034	1.030
τ	0	0	0.490	0.564
τ^*	0	0	0.490	0.436
U	-1.651	-1.672	-1.794	-1.727
U^*	-1.651	-1.640	-1.794	-1.909

Table showing the effect of a change in country size on the Currency War and Trade War

wars, and currency and trade wars.

The first two panels on the left hand side illustrate the impact of an increase in the Home country from 50 percent to 75 percent of the world economy in the case of a currency war alone, with zero tariffs. Counter to received wisdom, the Home country actually suffers relative to the equal-size benchmark. The reason is, again, related to the trade-off between terms-of-trade manipulation and inflation. When the Home country is larger, it behaves more like a closed economy and focuses more on inflationary stimulus to offset the monopoly distortion. In a discretionary equilibrium, this leaves the Home country worse off. The Foreign country, by contrast, focuses more on terms-of-trade manipulation. In equilibrium, the Home ends up with a higher inflation rate and the Foreign country with zero inflation, at least for our calibration. So, in the currency war, country size is welfare reducing!

The two right-hand panels of Table 4 illustrate the impact of country size in the case of combined currency and trade wars. Relative to the equal-size Nash equilibrium, the Home country increases its tariff and the Foreign country reduces its tariff. Intuitively, since the larger country's consumption basket is more weighted towards its own goods, the cost of a tariff on domestic consumption is less, while conversely, that for the Foreign country is greater. The result is that the Home country is more protectionist, obtains a significant terms-of-trade advantage, and gains in welfare relative to the Foreign country. Hence, country size is an advantage in the combined currency and trade war environment, while it is a disadvantage in the currency war alone.

Table 8 in the Appendix illustrates the outcome under alternative parameter values for the Trade and currency war. For the degree of protection, not surprisingly the most important parameter is the trade elasticity. Our calibration uses $\lambda = 3$, which is on the high side of the trade elasticities used in the aggregate macro literature. But elasticities in the trade literature tend to be higher. For a value of $\lambda = 6$ we find that the symmetric Nash equilibrium of the current and trade war implies a tariff rate of 20 percent, substantially lower than that of Table 2. The consequent welfare impacts of the trade war are then less. But the main qualitative implications are the same as above. The trade war leads to a higher equilibrium rate of inflation in all countries, and monetary policy cooperation leads to a more severe trade war, as above.

5.5 Commitment in trade policy

So far we have assumed that both inflation and tariffs are chosen simultaneously by domestic policymakers to maximize own country welfare. A central assumption is that policy is discretionary, so that policy-makers cannot bind the hands of future policy-makers, rather take these future actions as given. But it could be argued that trade policy embodies more commitment than monetary policy. Trade policy is typically enacted by legislation, and this is not as easily changed as monetary policy decisions, which can be altered at the whim of an independent central bank.

In this subsection, we analyze a simplified game where trade policy is determined in a non-cooperative game between policy-makers, but assuming that the trade policy-makers can commit to their tariff choices. The general case where trade policy is made with commitment and monetary policy is discretionary in the two country setting involves a complicated dynamic interaction. We focus instead on a much simplified setting where trade authorities commit to a single tariff rate that remains constant. Moreover, we assume that in choosing tariffs, the trade authorities internalize the endogenous response of inflation rates to tariffs in the currency war game between monetary authorities.

Therefore, in the initial period trade authorities choose a tariff rate, taking the tariff rate of the other authority as given, but taking into account the equilibrium of the monetary policy game played by the monetary authorities, within each period. We focus on a steady state of this tariff game with commitment. Thus, given the initial tariff rate, monetary authorities choose their inflation rate in a currency war, without commitment. Given the constant tariff rates, which are equal in a symmetric equilibrium, inflation rates are constant over time, and also equal across countries in the symmetric equilibrium of the currency war.

Given that there is just a one-time choice of tariffs, and we focus on a steady state, the optimal tariff rates for this game can be chosen simply as a Nash equilibrium in τ and τ^* where each trade authority chooses to maximize one-period domestic utility, taking account of the competitive equilibrium, and internalizing the response of inflation in both countries to their tariff rate, but taking as given the tariff rate of the other country.

Somewhat more formally, define $V(\tau_t, \tau_t^*)$ and $V^*(\tau_t, \tau_t^*)$ as follows:

$$V(\tau_t, \tau_t^*) = \text{Max}_{\{C_t, C_t^*, Y_{h,t}, Y_{f,t} \mathcal{S}_t, \pi_{h,t}\}} \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{\chi}{1+\psi}\left(\frac{Y_{h,t}}{A_t \mathcal{X}_t^{\alpha}}\right)^{\frac{1+\psi}{1-\alpha}} \tag{43}$$

subject to (17)-(21).

$$V^*(\tau_t, \tau_t^*) = \text{Max}_{\{C_t, C_t^*, Y_{h,t}, Y_{f,t} \mathcal{S}_t, \pi_{f,t}\}}, \frac{C_t^{*1-\sigma} - 1}{1-\sigma} - \frac{\chi}{1+\psi}\left(\frac{Y_{f,t}}{A_t^* \mathcal{X}_t^{*\alpha}}\right)^{\frac{1+\psi}{1-\alpha}} \tag{44}$$

subject to (17)-(21).

Then a Nash equilibrium with commitment in tariff policy, τ_t^N, τ_t^{*N} is defined by the equilibrium to the conditions:

$$\text{Max}_{\tau_t} V(\tau_t, \tau_t^{*N}) \tag{45}$$

$$\text{Max}_{\tau_t^*} V^*(\tau_t^N, \tau_t^*) \tag{46}$$

Table 5 below illustrates the equilibrium of this game. The most striking feature of the Table is that tariff rates are significantly lower than those in the baseline case of the simultaneous move game. The Nash tariff rates for the calibrated model are 27 percent, compared with 49 percent in the baseline model. At the same time, the equilibrium inflation rates are lower, and consumption, output, and welfare for each country is higher.

What accounts for the difference between the commitment equilibrium and the baseline case? The key factor is that the trade authorities take account of the endogenous increase in inflation that will follow from a higher round of tariffs facing the monetary policy-makers in the second stage of the game. Because this inflation will be costly due to price adjustment costs, but have little benefit in terms of higher output, the trade authorities endogenously choose lower equilibrium tariff rates. Individually, monetary authorities choose a rate of inflation taking future inflation rates as given. In a steady state equilibrium, the future inflation rate is equal to the current inflation rate, so that from the firm's first order condition in the Home country, we have

$$(1+s) - \epsilon\left((1+s) - mc\right) - \xi'\left(\pi_h\right)\pi_h + \beta\xi'\left(\pi_h\right)\pi_h = 0$$

Since the trade authorities take account of the sequence of their tariff choices on π, they individually choose a lower degree of protection than in the tariff game without commitment, where both tariffs and inflation are taken as given.

This example highlights the implications of a loss of commitment in trade policy. Even in the absence of any international trade agreements, when tariffs are chosen without commitment, at the same frequency as monetary policy, there may be significant losses in welfare.

Table 5: Trade and Currency Wars with commitment in trade policy

Variable	Baseline.	Commitment
C	0.249	0.265
C^*	0.249	0.265
Y_h	0.399	0.408
Y_f	0.399	0.408
$\mathcal{S}$	1.000	1.000
π_h	1.034	1.029
π_f^*	1.034	1.029
τ	0.490	0.270
τ^*	0.490	0.270
U	-1.794	-1.730
U^*	-1.794	-1.730

Table showing equilibrium of Tariff war and Currency Wars, comparing baseline model with tariff commitment model

6 Fixed Exchange Rates

Since prices are sticky, the nominal exchange rate plays an important role in the link between tariffs and the terms of trade. What happens if the exchange rate is fixed? Table 6 illustrates the outcome of the trade war in the case of fixed exchange rates. When exchange rates are pegged by

Table 6: Trade war under fixed exchange rates

Variable	Fixed ER	Fixed ER,Trade War
C	0.287	0.271
C^*	0.287	0.271
Y_h	0.460	0.441
Y_f	0.460	0.441
$\mathcal{S}$	1.00	1.00
π	1.036	1.043
π^*	1.036	1.043
τ	-	0.144
τ^*	-	0.144
U	-1.699	-1.746
U^*	-1.699	-1.746

Table showing equilibrium of Trade War in the model with fixed exchange rates

the Foreign country, only the Home country has an independent monetary policy. Absent tariffs, the first column shows that the Home country will choose an inflation rate of 3.6 percent under our calibration, and the equilibrium is perfectly symmetric. Given unitary initial terms of trade, so that $\mathcal{S}_{t-1} = 1$, the Home country can only improve its terms of trade by a higher rate of inflation, relative to the Foreign country. This contrasts with the flexible exchange rate case, where, for a given Foreign rate of inflation, the terms of trade can be improved by a contractionary monetary policy and an exchange rate appreciation, giving rise to a downward bias in inflation rates in both countries. With a fixed exchange rate, the Home country instead focuses on removing the monopoly distortion for a given terms of trade. This leads to a symmetric equilibrium where both countries inflation rates are positive, and the terms of trade is unchanged. In fact, in comparing Table 6 with Table 1, we see that the fixed exchange rate case is identical to the equilibrium of the currency war with cooperation in monetary policy. This then implies that in welfare terms, the currency war equilibrium dominates the equilibrium with fixed exchange rates, absent the trade war.

The second column of Table 6 compares this to the case of a trade war under an exchange rate peg. This column also identifies a fully symmetric outcome, where the existing terms of trade facing each policymaker is unity. The Home country chooses its inflation rate and its tariff rate, and the Foreign country chooses only its tariff rate. In a symmetric equilibrium both inflation rates and tariff rates are equal. What is most striking about this outcome is the huge difference between non-cooperative tariff rates relative to the flexible exchange rate case. In the Nash equilibrium tariff rates in each country are only 14 percent, compared to 49 percent in the flexible price equilibrium. As a result, in a reversal of the above welfare ranking, when countries engage in trade wars, the symmetric equilibrium implies that an exchange rate peg dominates the equilibrium with flexible exchange rates.

What is the intuition for the substantial difference between fixed and flexible exchange rates with respect to equilibrium tariff rates? This can be best explained by focusing on equation (25), repeated here.

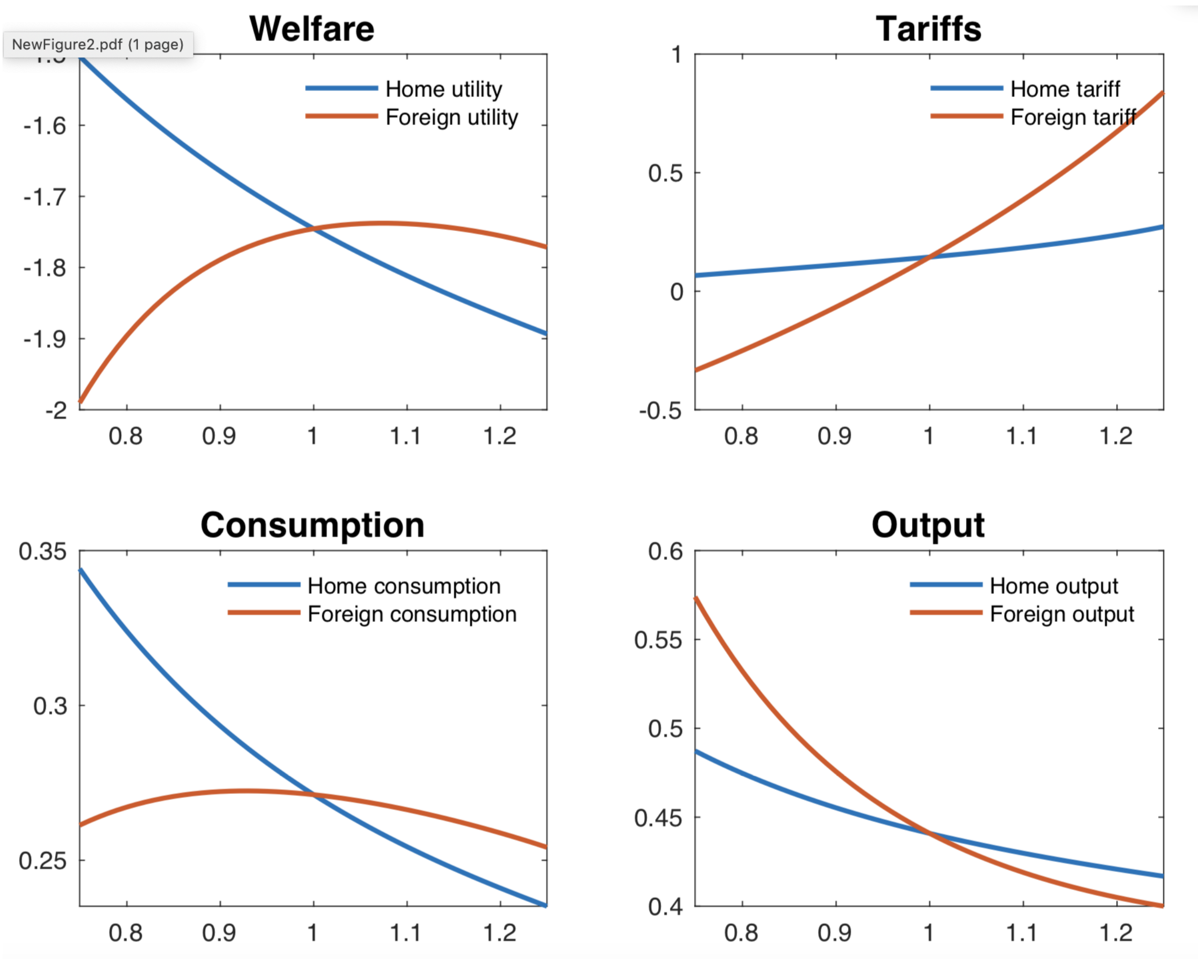
$$\pi_{h,t} = \pi^*_{f,t}\frac{\mathcal{S}_{t-1}}{\mathcal{S}_t}$$

Under the fixed exchange rate regime, the Home country is choosing both its tariff rate and its own inflation rate. If it chooses its tariff rate to appreciate the terms of trade, then this implies, given $\mathcal{S}_{t-1}$, that it must be increasing its inflation rate, relative to the Foreign country inflation rate. But the fact that the authority is simultaneously choosing $\pi_{h,t}$ subject to the costs of inflation adjustment, effectively reduces the benefits of an appreciated terms of trade. In a symmetric equilibrium where $\mathcal{S}_{t-1} = 1$ these factors exactly offset, so that it chooses an inflation rate identical to the Foreign rate, and a tariff rate identical to the Foreign tariff rate. For our calibration, the reduced benefit of tariff hikes under a peg leads to a very low equilibrium tariff rate.

However, there is a critical caveat in this case. In fact, there exists a continuum of equilibrium Nash equilibrium tariff rates in a symmetric equilibrium, conditioned on different values of $\mathcal{S}_{t-1}$. If we take an initial value $\mathcal{S}_{t-1} < 1$, then the Home country will choose a tariff rate higher than that of the Foreign country, so that in equilibrium $\mathcal{S}_t = \mathcal{S}_{t-1} < 1$, and equilibrium inflation rates are again equalized. Likewise for $\mathcal{S}_{t-1} > 1$, then the Home country chooses a lower tariff rate than the Foreign country, and again $\mathcal{S}_t = \mathcal{S}_{t-1} > 1$, with identical inflation rates. Thus, there is a continuum of Nash equilibrium tariff rates in which the Home country is more or less protectionist than the Foreign country, and each delivers a more or less appreciated terms of trade for the Home country.

Figure 3 illustrates the set of equilibria conditional on initial values of the terms of trade. In each panel, the equilibrium terms of trade is on the horizontal axis. The top right panel shows that a low initial terms of trade (to the advantage of the Home country) may be sustained by a low Home tariff, and a lower Foreign tariff. The Foreign tariff may even be negative, indicating an import subsidy (or an export subsidy, since this model satisfies Lerner equivalence). In this case, the top left panel shows that Home welfare exceeds Foreign welfare, and the bottom left panel shows that Home consumption exceeds Foreign consumption. By the same token, with a lower tariff rate, Foreign gross output exceeds Home output. But there are also equilibria in which the equilibrium terms of trade exceed unity, the Foreign tariff exceeds the Home tariff, Home welfare and consumption are lower than Foreign, and Home output exceeds Foreign output.

Figure 3: Trade wars under fixed exchange rates



The graphs report outcomes of a trade and currency war for given values of the initial value of terms of trade, *i.e.* $\mathcal{S}_{t-1}$.

In each of these equilibria, Home and Foreign inflation rates are equal, which implies that the initial terms of trade is maintained in steady state. Moreover, inflation is invariant to the equilibrium tariff rates chosen by the Home and Foreign country. Table 6 indicates that in the symmetric equilibrium, inflation rates in each country are 3.8 percent. For our calibration, this inflation remains the equilibrium outcome for all values of $\mathcal{S}_t$ and for all different equilibrium tariff

Table 7: Trade and Currency Wars under the zero lower bound

Variable	ZLB	ZLB,Trade War
C	0.249	0.225
C^*	0.249	0.225
Y_h	0.353	0.330
Y_f	0.353	0.330
$\mathcal{S}$	1.00	1.00
π	0.982	0.984
π^*	0.982	0.984
τ	-	0.378
τ^*	-	0.378
$U = U^*$	-1.703	-1.782

Table showing equilibrium of Trade War in the model where monetary policy is constrained by the ZLB.

rates reported in Figure 3.

7 Trade Wars under the Zero Lower Bound

Table 6 illustrates the impact of the zero lower bound on the trade war. We assume that the zero bound is forced in the manner as discussed in 2.4.3. In our numerical computation, the zero bound is generated by a fall in the subjective discount rate of the private sector so that $\beta = 1.25$, and we assume this persists with probability 0.5. As discussed above, in this case, the monetary authority has no control of current rates of inflation, and inflation is determined by aggregate demand, given forward looking consumers and the expectation that the economy will revert to the Nash equilibrium of the currency and trade war as described in Table 1. In the absence of trade policy, the ZLB outcome leads to an equilibrium with deflation rates of 2 percent in both countries, with consumption and output significantly below the Nash equilibrium of the currency war levels as shown in Table 1.

When countries engage in a trade war under the zero lower bound, the outcome is substantially worse. Each country levies tariffs in the Nash equilibrium, but this has little effect in increasing the inflation rate, and leads to lower levels of consumption, output, and welfare. Interestingly, the Nash equilibrium tariff rates are only 38 percent, notably lower than in the full currency war and trade war equilibrium under flexible exchange rates. They key explanation for this is similar to the example of commitment in trade policy discussed above. In the environment of the ZLB, trade policy-makers take account of their choice of tariffs on the endogenous rates of inflation in the two countries. This leads them to limit the size of their tariff choices relative to the case where inflation and tariff rates are chosen simultaneously in the full discretionary equilibrium.

8 Conclusions

This paper is primarily a theoretical exploration of the links between trade policy and monetary policy from the point of view of international strategic policy interaction. There is a large literature both on international macroeconomic policy coordination/non-coordination on the one hand and the determinants of trade policy and tariff setting in strategic environments on the other hand. In our labeling, we denote the first topic as pertaining to 'currency wars', and the second related to 'trade wars'. Our paper represents a first pass at combining 'currency wars' and 'trade wars' within a simple New Keynesian open economy framework. In the introduction, we argued that contemporary developments in global economic policy made the interaction of these two dimensions of policy-making of much greater relevance than in the past. The results of our analysis show that in many ways, currency wars and trade wars are very closely linked to one another, and differences in policy settings can lead to major differences in macroeconomic outcomes, the overall degree of trade protection, and welfare.

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A Appendix: The Competitive Equilibrium

When we combine the description of optimal behaviour for the Home economy with the analogous conditions for the Foreign economy, and impose market clearing conditions, we obtain a competitive equilibrium which can be described by the following equations:

$$\varepsilon \left(\frac{P_{h,t}}{P_t}\right)^{-\lambda} C_t + \varepsilon_x \left(\frac{P_{h,t}}{P_{x,t}}\right)^{-\lambda} X_t + (1-\varepsilon)\frac{S_t P^*_{f,t}}{P_{h,t}} \left(\frac{(1+\tau_t) S_t P^*_{f,t}}{P_t}\right)^{-\lambda} C_t + (1-\varepsilon_x)\frac{S_t P^*_{f,t}}{P_{h,t}} \left(\frac{(1+\tau_t) S_t P^*_{f,t}}{P_{x,t}}\right)^{-\lambda} X_t$$

$$= Y_{h,t} - \xi(\frac{P_{h,t}}{P_{h,t-1}}) Y_{h,t} \tag{A.47}$$

$$(1+s_t) Y_{i,h,t} - \epsilon (P_{i,h,t}(1+s_t) - MC_t) \frac{Y_{i,h,t}}{P_{i,h,t}}$$

$$- \xi'(\frac{P_{i,h,t}}{P_{i,h,t-1}}) \frac{P_{h,t} Y_{h,t}}{P_{i,h,t-1}} + E_t \Omega_{t+1} \xi'(\frac{P_{i,h,t+1}}{P_{i,h,t}}) \frac{P_{i,h,t+1}}{P^2_{i,h,t}} P_{h,t+1} Y_{h,t+1} = 0 \tag{A.48}$$

$$A_t (1-\alpha) L_t^{-\alpha} X_t^{\alpha} MC_t = W_t \tag{A.49}$$

$$A_t \alpha L_t^{\alpha} X_t^{\alpha-1} MC_t = P_{x,t} \tag{A.50}$$

$$W_t = \chi P_t C_t^{\sigma} H_t^{\psi} \tag{A.51}$$

$$(1+s^*_t) Y_{i,f,t} - \epsilon (P^*_{i,f,t}(1+s_t) - \frac{MC^*_t}{A^*_t}) \frac{Y_{i,f,t}}{P^*_{i,f,t}}$$

$$- \xi'(\frac{P^*_{i,f,t}}{P^*_{i,f,t-1}}) \frac{P^*_{f,t} Y_{f,t}}{P^*_{i,f,t-1}} + E_t \Omega^*_{t+1} \xi'(\frac{P^*_{i,f,t+1}}{P^*_{i,f,t}}) \frac{P^*_{i,f,t+1}}{P^{*2}_{i,h,t}} P^*_{f,t+1} Y_{f,t+1} = 0 \tag{A.52}$$

$$A^*_t (1-\alpha) L_t^{*-\alpha} X_t^{*\alpha} MC^*_t = W^*_t \tag{A.53}$$

$$A_t \alpha L_t^{*\alpha} X_t^{*\alpha-1} MC_t^* = P_{x,t}^* \tag{A.54}$$

$$W_t^* = \chi P_t^* C_t^{*\sigma} H_t^{*\psi} \tag{A.55}$$

$$Y_{h,t}(1-\xi(\frac{P_{h,t}}{P_{h,t-1}})) = \varepsilon \left(\frac{P_{h,t}}{P_t}\right)^{-\lambda} C_t + \varepsilon_x \left(\frac{P_{h,t}}{P_{x,t}}\right)^{-\lambda} X_t$$
$$+ \frac{(1-n)}{n}\left((1-\varepsilon^*)\left(\frac{(1+\tau_t^*)P_{h,t}}{S_t P_t^*}\right)^{-\lambda} C_t^* + (1-\varepsilon_x^*)\left(\frac{(1+\tau_t^*)P_{h,t}}{S_t P_{x,t}^*}\right)^{-\lambda} X_t^*\right) \tag{A.56}$$

$$Y_{f,t}(1-\xi(\frac{P_{f,t}^*}{P_{f,t-1}^*})) = \frac{n}{1-n}\left((1-\varepsilon)\left(\frac{(1+\tau_t)S_t P_{f,t}}{P_t}\right)^{-\lambda} C_t + (1-\varepsilon_x)\left(\frac{(1+\tau_t)S_t P_{f,t}}{P_{x,t}}\right)^{-\lambda} X_t\right)$$
$$+ \varepsilon^* \left(\frac{P_{f,t}}{P_t^*}\right)^{-\lambda} C_t^* + \varepsilon_x^* \left(\frac{P_{f,t}}{P_{x,t}^*}\right)^{-\lambda} X_t^* \tag{A.57}$$

Equation (A.47) is the Home country budget constraint after netting out the government budget constraint. Equation (A.48) and equations (A.49)-(A.51) are the profit maximizing and cost minimizing relationships for each Home firm i, and the Home labour supply equations. Equations (A.52) and equations (A.53)-(A.55) are the analogous conditions for the Foreign firm. Then equations (A.56) and (A.57) are the Home and Foreign goods market clearing conditions.

The system (A.47)-(A.57) can be simplified and rewritten into the 5 equations (17) - (21) of the text.

B Appendix: Alternative parameter values

Table 8 describes the results of the currency and trade war under alternative parameter values. For a larger trade elasticity, assuming $\lambda = 6$, equilibrium tariffs in the trade war are substantially lower. Tariffs are higher than the baseline when the monopoly markup is lower ($\epsilon = 11$, implying a 10 percent markup), and lower in the case of greater home bias in preferences and production. In addition, a smaller weight of intermediate goods, and a lower elasticity of intertemporal substitution

Table 8: Trade and Currency Wars under alternative parameter values

Variable	$\lambda = 6$	$\epsilon = 11$	$\varepsilon = 0.75$	$\alpha = .2$	$\sigma = 2$	$\psi = 0$
C	0.270	0.270	0.265	0.234	0.389	0.220
C^*	0.270	0.270	0.265	0.215	0.389	0.220
Y_h	0.431	0.425	0.425	0.359	0.620	0.366
Y_f	0.431	0.425	0.425	0.390	0.620	0.366
$\mathcal{S}$	1.00	1.00	1.00	1.00	1.00	1
π	1.035	1.017	1.002	1.025	1.031	1.043
π^*	1.035	1.017	1.002	1.025	1.031	1.043
τ	0.204	0.513	0.460	0.53	0.543	0.503
τ^*	0.204	0.513	0.460	0.53	0.543	0.503
U	-1.738	-1.729	-1.754	-1.257	-2.499	-2.184
U^*	-1.738	-1.729	-1.754	-1.257	-2.499	-2.184

Table showing equilibrium of Tariff War and Currency War, for varying parameter values.

also leads to higher Nash equilibrium tariff rates. A lower Frisch elasticity of substitution in labor supply has minimal effects on equilibrium tariff rates, but leads to a 1 percentage point rise in the equilibrium inflation rate.