Sovereign default and the decline in interest rates

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The puzzle?

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- Valuation ratios have not increased at the same pace.

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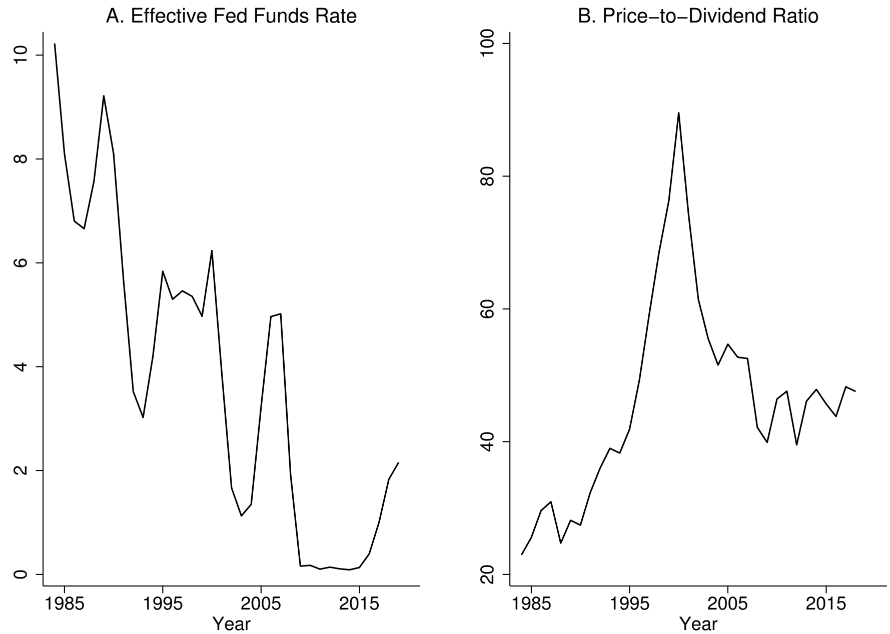

What are the current explanations?

- The secular stagnation literature (Hansen, 1939; Summers, 2015; Gordon, 2015) provides myriad potential reasons for the reduction in interest rates.
    1. Higher demand for savings coming from changing population demographics and growing inequality.
    2. A "savings glut" coming from foreign demand for safe assets (Bernanke, 2005; Caballero et al., 2008).
    3. A prolonged regime of low growth.
    4. Increased risk leading to a higher demand for precautionary savings.
- Farhi and Gourio (2018) combine these to quantitatively match several macro and asset pricing facts, including the price-dividend ratio.
- These explanations have trouble matching long-run trends and data outside the US.

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Our explanation?

- Our explanation: the risk premium on sovereign debt has been falling with little reduction in the true riskfree rate.
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What does the current approach say?

- Standard i.i.d. consumption-based asset pricing model, with a similar flavor of Farhi and Gourio (2018).
- Calibrate to match moments from 1984–2000 and 2001–2016, respectively.
- Results come from an increase in patience β and the probability of disaster p.

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The model

- Agents have Epstein and Zin (1989) utility and choose consumption and to maximize the present value of utility in a complete market in an endowment economy.
- Endowment follows

$$\Delta c_{t+1} = \mu + \eta_{t+1},$$

where η_{t+1} is an independent and identically distributed “disaster” term of the form

$$\eta_{t+1} = \begin{cases} 0 & \text{with probability} \quad 1-p \\ -Z & \text{with probability} \quad p \end{cases}$$

and $Z > 0$.

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Asset prices

- Using the Euler equation for the return on wealth, the price-consumption ratio, κ, is given by

$$\kappa = \frac{\beta e^{(1-\frac{1}{\psi})\mu}\left[1 + p(e^{-(1-\gamma)Z} - 1)\right]^{\frac{1}{\theta}}}{1 - \beta e^{(1-\frac{1}{\psi})\mu}\left[1 + p(e^{-(1-\gamma)Z} - 1)\right]^{\frac{1}{\theta}}}.$$

- The return on the riskfree asset is given by

$$r_f = -\log\beta + \frac{1}{\psi}\mu - \log(1 + p(e^{\gamma Z} - 1)) + \left(\frac{\theta - 1}{\theta}\right)\log(1 + p(e^{-(1-\gamma)Z} - 1)).$$

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Calibration

	Values	
	1984–2000	2001–2016
Panel A: Moments in the data		
Price-dividend ratio κ	42.34	50.11
Riskfree rate r_f	0.0279	-0.0035
Panel B: Baseline Model Parameters		
μ	0.0350	0.0282
β	0.967	0.979
p	0.0343	0.0663
Panel C: $\psi = 0.5$		
μ	0.0350	0.0282
β	0.997	0.983
p	0.0343	0.0667

Notes: Unless otherwise noted, risk aversion $\gamma = 12$, EIS $\psi = 2$, and disaster size (decline in log consumption in the event of a disaster) $Z = -\log(.85)$.

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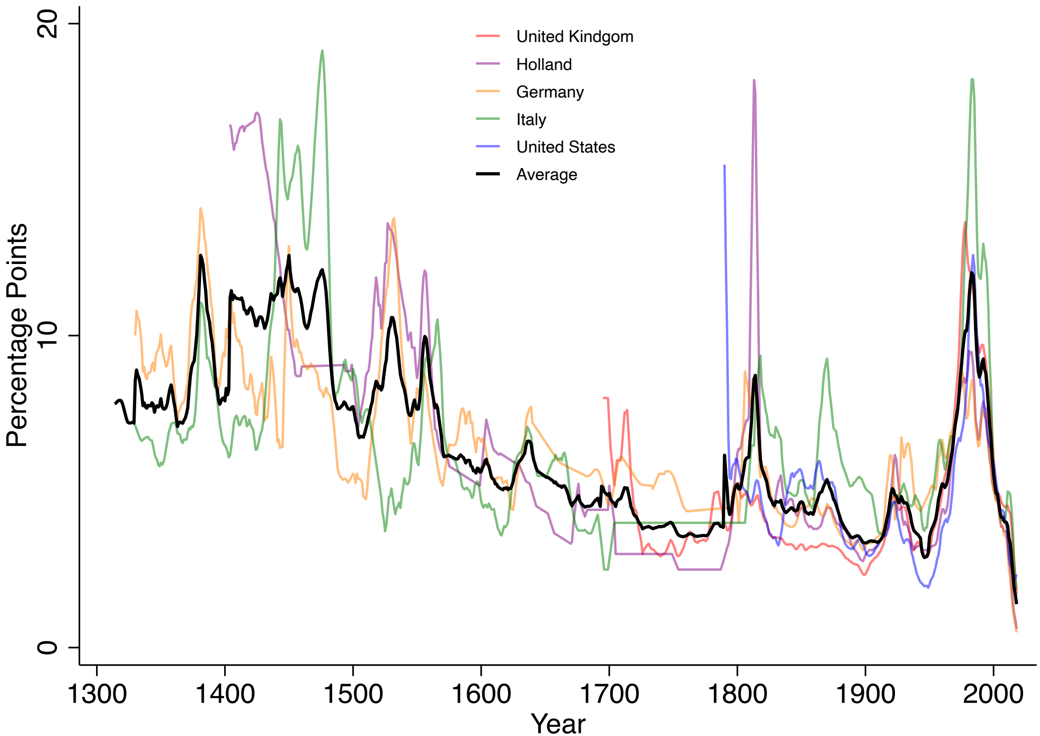
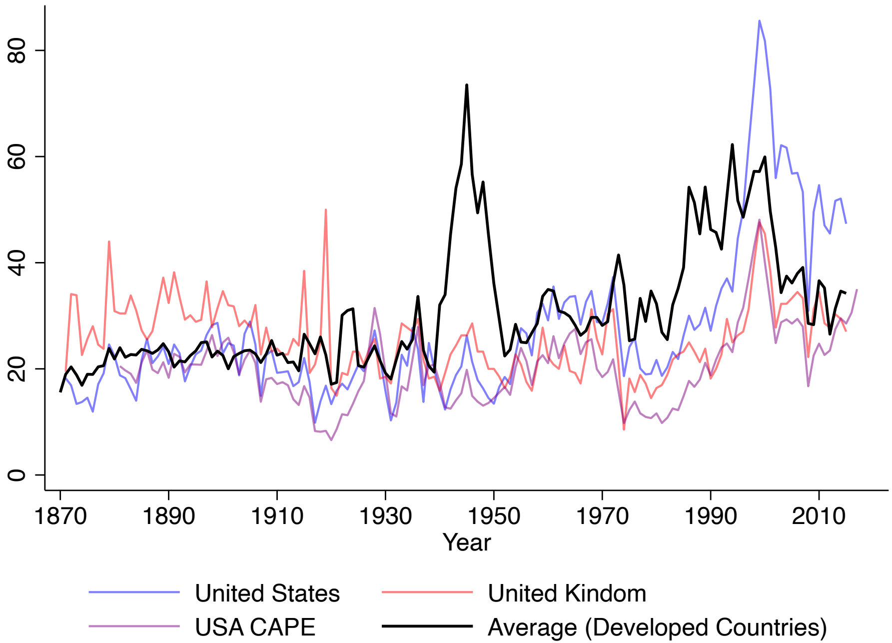
Long-run trends

- The reduction in interest rates has been ongoing for nearly 700 years!



Valuation ratios have been flat

- Price-earnings ratios in the US and price-dividend ratios in the UK (and the rest of the developed world) not risen much in the last 150 years.



Calibrating to different moments

	Values	
	1984–2000	2001–2016
Panel A: Moments in the data		
US CAPE ratio κ_{US}^{PE}	25.97	26.73
UK Price-dividend ratio κ_{UK}	27.78	30.86
US riskfree rate r_f^{US}	0.0279	-0.0035
UK riskfree rate r_f^{UK}	0.0503	0.0042
Panel B: US Moments, CAPE Ratio		
μ	0.0350	0.0282
β	0.957	0.968
p	0.0556	0.101
Panel C: UK Moments, PD Ratio		
μ	0.0278	0.0156
β	0.955	0.971
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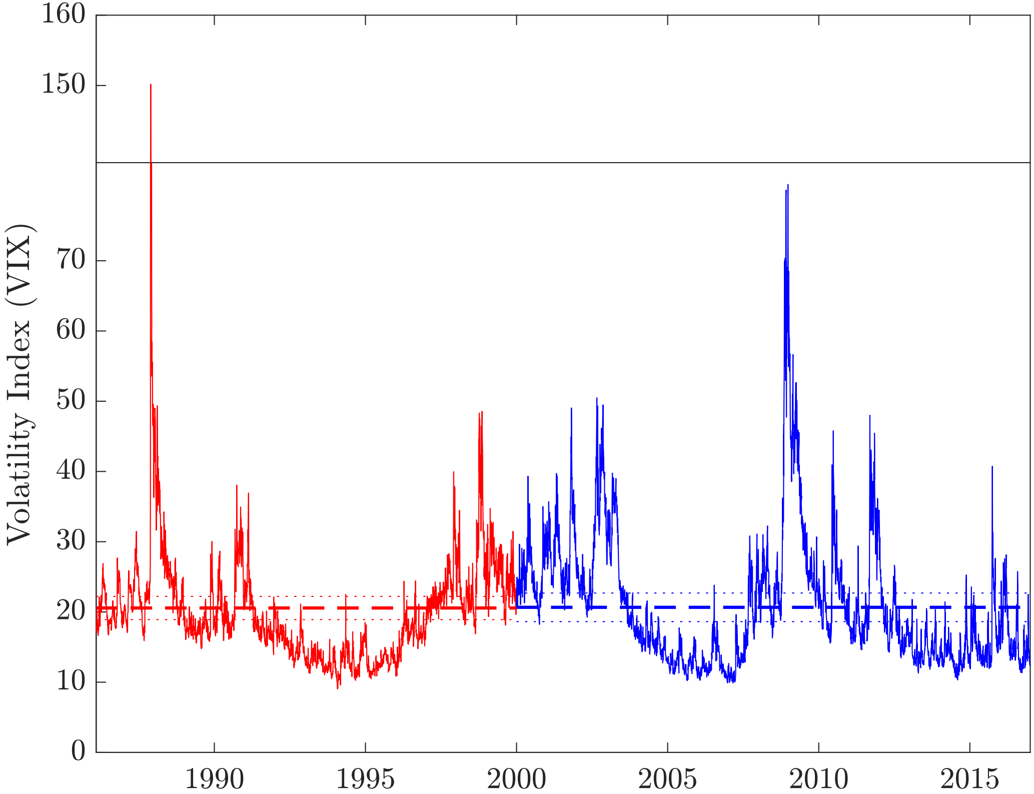
Has the disaster probability really increased?



Our approach

- Two main changes to the baseline set-up in our specification.

1. Government bonds are not riskfree: they include default and/or inflation risk.
2. Add production and allow for riskless storage (inventory) which bounds the real interest rate at zero. Agents can costlessly move consumption from one period to the next in lieu of investment.

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Default risk

- The government bond is equal to $e^{\eta^b_{t+1}}$ where

$$\eta^b_{t+1} = \begin{cases} 0 & \text{with probability } 1-p \\ -\zeta Z & \text{with probability } p \end{cases}$$

- This can be thought of as either outright (partial) default or as default through inflation.
- In the baseline endowment economy, the expected return on the government bond is given by

$$\log E_t\left[R_{b,t+1}\right] = r^f + \underbrace{\log\left(\frac{(1+p(e^{-\zeta Z}-1))(1+p(e^{\gamma Z}-1))}{1+p(e^{-(\zeta-\gamma)Z}-1)}\right)}_{\text{risk premium}}.$$

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Default calibration

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	1984–2000	2001–2016
Panel A: Moments in the data		
US CAPE ratio κ_{US}^{PE}	25.97	26.73
US bond rate r_b^{US}	0.0279	-0.0035
Panel B: Model with default		
β	0.963	0.964
ζ	0.305	-0.118

Notes: Unless otherwise noted, risk aversion $\gamma = 5$, EIS $\psi = 1$, disaster size (decline in log consumption in the event of a disaster) $Z = -\log(.70)$ and the probability of disaster $p = 0.04$. All panels are calibrated to the riskfree rate and growth used in Farhi and Gourio (2018) and the cyclically adjusted price-to-earnings ratio of Shiller (2000). In each panel, the 1984–2000 period has $\mu = 0.0350$ and the 2001–2016 period has $\mu = 0.0282$.

The model

- Households purchase capital and rent it to the firm. The firm produces the consumption good according to a linear, CRS production function

$$Y_t = AK_t$$

- Agents have log utility and face the budget constraint

$$C_t + I_t + \tilde{K}_{t+1} = I_{t-1} + (1 - \delta + A)K_t$$

where I_t is inventory and $\tilde{K}_{t+1} \equiv \frac{K_{t+1}}{e^{\eta_{t+1}}}$ is planned capital and η_{t+1} is a "capital quality shock."

- Motion of capital follows

$$\tilde{K}_{t+1} = (1 - \delta)K_t + X_t$$

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Solving as a portfolio choice problem

- Defining $W_t \equiv I_{t-1} + (1-\delta+A)\tilde{K}_t e^{\eta_t}$, this problem can be re-written as a portfolio choice problem where the budget constraint becomes

$$W_{t+1} = (W_t - C_t)(1 + (1-\theta_t)r_{f,t+1} + \theta_t r_{K,t+1})$$

where $\theta_t \equiv \frac{\tilde{K}_{t+1}}{W_t - C_t}$, $r_{K,t+1} \equiv (1-\delta+A)e^{\eta_{t+1}} - 1$, and $r_{f,t+1}$ is the net return on the riskfree bond.

- Because of log utility, the consumption-wealth ratio is constant and given by here $\frac{C_t}{W_t} = 1 - \beta$.

- This yields the Euler equations

$$E_t\left[\frac{1}{1+r_{f,t+1}+\theta(r_{K,t+1}-r_{f,t+1})}(1+r_{i,t+1})\right] = 1,$$

$$E_t\left[\frac{1}{1+r_{f,t+1}+\theta(r_{K,t+1}-r^f_{t+1})}(r_{i,t+1}-r_{f,t+1})\right] = 0.$$

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No inventory case

- Agent's choice depends on the unconstrained risk free rate

$$R^*_{f,t+1} = E_t\left[(1+r_{K,t+1})^{-1}\right]^{-1} = (1-\delta+A)(1+p(e^Z-1))^{-1}.$$

which is the return on the zero-net supply riskfree bond in the model without inventory.

- When $r^*_{f,t+1} > 0$, then the agent chooses not to hold inventory, as she obtains a higher return from investing in the riskfree asset.

- Entire portfolio in risky capital (i.e. $\theta = 1$) and the results are the same as in Barro (2009), but with log utility.

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ZLB with inventory

- When $r^*_{f,t+1} < 0$, the agent chooses to hold a positive position in inventory.
- In this case, $r_f = 0$ and the Euler equations become

$$E_t\left[\frac{1}{1+\theta r_{K,t+1}}(1+r_{i,t+1})\right] = 1$$

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- Taking $r_{i,t+1} = r_f = 0$ in the first Euler equation gives the solution for θ

$$\theta = -\frac{(1-p)r_{K,0} + p r_{K,Z}}{r_{K,0} r_{K,Z}}.$$

where $r_{K,0} = (1-\delta+A) - 1$ and $r_{K,Z} = (1-\delta+A)e^{-Z} - 1$.

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- Taking $r_{i,t+1} = r_f = 0$ in the first Euler equation gives the solution for θ

$$\theta = -\frac{(1-p)r_{K,0} + p r_{K,Z}}{r_{K,0} r_{K,Z}}.$$

where $r_{K,0} = (1-\delta+A) - 1$ and $r_{K,Z} = (1-\delta+A)e^{-Z} - 1$.

Inventory stabilizes losses from disasters

- In this set-up, the growth of consumption and wealth are identical and given by

$$\frac{C_{t+1}}{C_t} = \frac{W_{t+1}}{W_t} = \beta(1 + \theta r_{K,t+1}) = \beta(\theta e^{\eta_{t+1}}(1 - \delta + A) + 1 - \theta)$$

- Note that the mean and volatility of the consumption and wealth growth processes are decreasing in θ.
- The SDF is less volatile and covaries less with the risky asset.
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Dynamics in an i.i.d. framework

- The growth rates of capital and output are also the same

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- Note that these depend on whether the disaster has occurred!
- The investment-capital ratio $\frac{X}{K}$ will also depend on whether the disaster has occurred.

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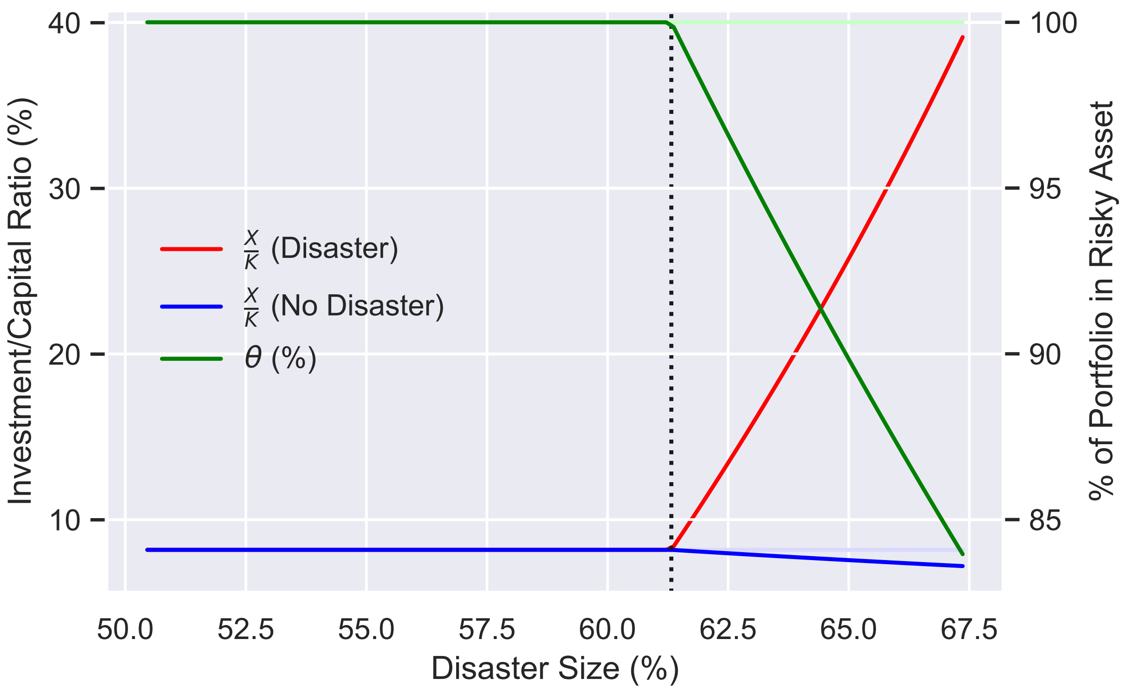
Comparative statics



Preferred calibration

	Values	
	1984–2000	2001–2016
Panel A: Moments in the data		
US CAPE ratio κ_{US}^{PE}	25.97	26.73
US bond rate r_b^{US}	0.0279	-0.0035
Growth rate μ	0.0368	0.0189
Panel B: With Inventory		
β	0.963	0.964
ζ	0.247	-0.036
δ	0.043	0.057
θ	1.000	0.898
$\frac{X}{K}$	0.080	0.076
Panel C: Without Inventory		
β	0.963	0.964
ζ	0.247	0.100
δ	0.043	0.061
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$\frac{X}{K}$	0.080	0.082

Notes: The model is solved under log-utility so risk aversion $\gamma = 1$ and EIS $\psi = 1$. The probability of a disaster is $p = .04$, the disaster size (decline in log consumption in the event of a disaster) $Z = -\log(.35)$ and the marginal product of capital $A = .12$.

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Conclusion

- Jointly explain patterns in interest rates and valuations ratios over the near- and long-term horizons.
- Decrease in risk premia on sovereign debt can explain the reduction in government interest rates over the short- and long-term.
- Allowing for consumption storage introduces a zero lower bound on real interest rates.
- In a model with production, this crowds out investment in productive capital.
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