Foreign Shocks as Granular Fluctuations[1]

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[1]The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of New York.

International business cycle shock propagation

- Textbook: cross-country propagation through relative prices, representative firm
  - e.g. BKK (1992), Kose and Yi (2006), Johnson (2014), ...
- Data: importing and exporting i) relatively rare; ii) strongly concentrated among largest firms
  - e.g. Freund and Pierola (2015), di Giovanni et al. (2017, 2018), ...
- "Micro of Macro": role of large firms, idiosyncratic shocks in aggregate fluctuations
  - Gabaix (2011), di Giovanni et al. (2014), Carvalho an Grassi (2015), ...

This paper

- A firm-level view of international shock propagation
- Foreign shocks (even purely aggregate) affect firms differentially depending on the extent and nature of their international linkages
- Census of French firms, 1993–2007
  - Value added/sales, bilateral imports and exports
  - Appended with WIOD: 40 countries, 32 sectors
- Quantitative model with heterogeneous firms, multiple countries, multiple sectors
  - Implemented directly on firm-level data
  - Simulate hypothetical and actual foreign shocks

Main results

1. Data: Larger French firms are significantly more sensitive to foreign GDP growth

2. Quantitative model, micro: Foreign shocks are mostly granular fluctuations

$$d \ln Y^F = \mathcal{E}^F + \Gamma^F$$

   Γ^F is $40 - 85\%$ of the total effect of a foreign shock

3. Quantitative model, macro: Firm heterogeneity in importing dampens the GDP impact of foreign shocks
   - Shock reallocates market shares towards larger firms...
   - ... That have a lower influence on domestic GDP, conditional on their size

   $\Rightarrow$ Heterogeneity reduces the propagation by 15%

Literature

- Micro origins of macro fluctuations
  - **Input-output networks:** Carvalho (2010), Acemoglu et al. (2012), Foerster et al. (2011) Acemoglu et al. (2016) Atalay (2017) Grassi (2017), Baqaee (2018) Baqaee and Farhi (2019a,b) Bigio and La'O (2019), Foerster et al. (2019)
  - **Large firms:** Gabaix (2011), di Giovanni et al. (2014), Carvalho and Grassi (2018), Gaubert and Itskhoki (2020)
  - **Firms + networks**: Barrot and Sauvagnat (2016), Carvalho et al. (2016), Huneeus (2018), Lim (2018), Taschereau-Dumouchel (2019), Dhyne et al. (2020).
- International
  - **Input linkages:** Hummels et al. (2001), Yi (2003), Johnson and Noguera (2012, 2017), Burstein et al. (2008), Bems et al. (2010), Johnson (2014), Eaton et al. (2016a,b)
  - **Network analytics:** Baqaee and Farhi (2019c), Huo et al. (2020)
  - **Firms:** di Giovanni and Levchenko (2012), Kleinert et al. (2015), Cravino and Levchenko (2017), Blaum et al. (2016), Blaum (2018), Boehm et al. (2019), di Giovanni et al. (2018); Ghironi and Melitz (2005), Alessandria and Choi (2007)

Data sources

- France (firm level):
  - Fiscal administration: firm tax forms from INSEE-Ficus: value added, sales, intermediate usage, industry ▸ Statistics by sector
  - Customs: partner-country exports and imports (Trade in *goods*)
- World (sector level):
  - WIOD: global input-output matrix, 40 countries, 32 sectors
- Firm-level coefficients normalized to match WIOD at the sector level

Larger firms more sensitive to foreign GDP

	(1)	(2)	(3)	(4)
	Dep. Var.: Log change in firm VA			
Firm's size×World GDP growth	0.139[a]	0.160[a]	0.078[a]	0.077[a]
	(0.020)	(0.020)	(0.021)	(0.022)
Firm's size	-0.015[a]	-0.015[a]	-0.015[a]	-0.015[a]
	(0.001)	(0.001)	(0.001)	(0.001)
World GDP growth	-0.962[a]			
	(0.121)			
Firm's size×French GDP growth				0.003[b]
				(0.017)
Observations	1,345,729	1,345,729	1,345,729	1,345,729
# years	11	11	11	11
# firms	122,339	122,339	122,339	122,339
Adjusted R^2	0.004	0.012	0.019	0.019
Fixed Effects	–	Year	Sector×Year	Sector×Year

- A doubling of firm size increases the elasticity of firm growth to world GDP by about 0.08
- Model's explanation: Large firms' exposure to foreign shocks through trade More

Main ingredients

- Heterogeneous-firm, multi-country, multi-sector model of trade
  - Countries m, n, k, sectors i, j, firms f, g
- Rest of the world: no firm-level data $\Rightarrow$ no heterogeneity within a sector
- France: heterogeneity in i) productivity, ii) input linkages, iii) export patterns, and iv) labor shares
- General structure: Armington + Melitz + IO linkages

Preferences of the representative household

- Increasing in consumption and leisure (Greenwood et al, 1988)
- Cobb-Douglas across sectors
- CES across countries within a sector (elasticity σ)
- CES across firms within a sector$\times$country (elasticity ρ)

$\Rightarrow$ Demand addressed to firm f by market n:

$$X_{f,mn,j} = \pi_{f,mn,j} X_{mn,j}$$

Firms

- Monopolistically competitive
- Productivity: a_f
- Taste shocks: $\{\xi_{f,mn,j}\}_n$
- Firm-specific input bundle cost:

$$b_{f,m,j} = \left[\alpha_{f,m,j} w_m^{1-\phi} + (1-\alpha_{f,m,j}) \left(P^M_{f,m,j}\right)^{1-\phi}\right]^{\frac{1}{1-\phi}}$$

$$P^M_{f,m,j} = \left[\sum_i \sum_k \gamma_{f,km,ij} P^{1-\eta}_{km,i}\right]^{\frac{1}{1-\eta}}$$

Firms

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- Firm-specific input bundle cost:

$$b_{f,m,j} = \left[\alpha_{f,m,j} w_m^{1-\phi} + (1-\alpha_{f,m,j})\left(P_{f,m,j}^M\right)^{1-\phi}\right]^{\frac{1}{1-\phi}}$$

$$P_{f,m,j}^M = \left[\sum_i \sum_k \gamma_{f,km,ij} P_{km,i}^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

- Heterogeneity:

$$\pi_{f,mn,j} = \frac{\xi_{f,mn,j} a_f^{\rho-1} b_{f,m,j}^{1-\rho}}{\sum_{g\in\Omega_{mn,j}} \xi_{g,mn,j} a_g^{\rho-1} b_{g,m,j}^{1-\rho}}$$

The role of heterogeneity

- Aggregate and firm-level value added:

$$Y_m = \sum_f Y_{f,m} \qquad \rightarrow \qquad d \ln Y_m^F = \sum_f \omega_{f,m,-1} d \ln Y_{f,m}^F$$

$$d \ln Y_m^F = \underbrace{\frac{1}{N} \sum_f d \ln Y_{f,m}^F}_{\mathcal{E}^F} + \underbrace{\sum_f \omega_{f,m,-1} d \ln Y_{f,m}^F - \frac{1}{N} \sum_f d \ln Y_{f,m}^F}_{\Gamma^F \propto \mathrm{Cov}\left(\omega_{f,m,-1}, d \ln Y_{f,m}^F\right)}$$

The role of heterogeneity

- Aggregate and firm-level value added:

$$Y_m = \sum_f Y_{f,m} \qquad \rightarrow \qquad d\ln Y_m^F = \sum_f \omega_{f,m,-1} d\ln Y_{f,m}^F$$

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- Firm value added growth:

$$d\ln Y_{f,m}^F \quad \approx \quad (1-\rho)\underbrace{\left[\pi^l_{f,m,j,-1} d\ln w_m + \sum_i \sum_k (1-\pi^l_{f,m,j,-1})\pi^x_{f,km,ij,-1} d\ln P_{km,i}\right]}_{\Delta \text{unit cost}}$$

$$+\sum_n s_{f,mn,j,-1}\, \underbrace{d\ln\left[\xi_{f,mn,j}\left(\frac{\tau_{mn,j}}{P_{mn,j}}\right)^{1-\rho} X_{mn,j}\right]}_{\Delta \text{demand}}$$

Calibration

- Solve the model in GE, transform to growth rates, use sales shares data directly (Dekle, Eaton, and Kortum, 2008) ▸ DEK
- Use GDP deflator to express results in real terms ▸ GDP deflator

Param.	Value	Source	Related to
ρ	3	Broda and Weinstein (2006)	subst. elasticity btw. firms
σ	1.5	Feenstra et al. (2018)	Armington elasticity
η	1	standard	subst. elasticity btw. inputs
ϕ	1	standard	subst. elasticity btw. inputs and labor
$\overline{\psi}$	3	Chetty et al. (2012)	Frisch elasticity
$\pi^{l}_{f,n,i}$, $\pi^{x}_{f,mn,ji}$	}	Our calculations based on French data and WIOD	labor and intermediate shares
ϑ_j			final consumption shares
$\pi^{c}_{mn,j}$			final trade shares
$\pi_{f,nk,j}$			intermediate use trade shares

World shocks

	$d \ln Y^F$	$\mathcal{E}^F$	Γ^F	$d \ln Y^F$	$\mathcal{E}^F$	Γ^F
Shock:	**Productivity**			**Demand**		
Baseline	2.66	0.39	2.27	0.35	0.20	0.15
Share:		*0.148*	*0.852*		*0.572*	*0.428*

Sector-Level Decomposition

	$d \ln Y^F$	$\mathcal{E}^F_J$	Γ^F_J	$d \ln Y^F$	$\mathcal{E}^F_J$	Γ^F_J
Baseline	2.66	2.05	0.60	0.35	0.60	-0.25
Share:		*0.773*	*0.227*		*1.699*	*-0.699*

$d \ln Y_{f,m}$ distribution
$d \ln Y_{f,m}$ and firm size
$d \ln Y_{f,m}$ and imported input share
$d \ln Y_{f,m}$ and export intensity
Actual TFP shocks

Larger firms and foreign shocks: data vs model

	(1)	(2)	(3)
	Dep. Var.: $d \ln Y_{m,j,t+1}(f)$		
	Data	Model	
		World Prod. Shock	World Pref. Shock
$\ln Y_{m,j,t}(f) \times d \ln Y_{W,t}$	0.077^a	0.020^a	0.333^a
	(0.022)	(0.0001)	(0.001)
$\ln Y_{m,j,t}(f)$	-0.015^a		
	(0.001)		
Observations	1,345,729	385,928	385,928
# years	11	1	1
# firms	122,339	385,928	385,928
Adjusted R^2	0.019	0.444	0.432
Fixed Effects	Sector$\times$Year	Sector	Sector

Macro: dampening effect of heterogeneity

	$d \ln Y^F$	$\mathcal{E}^F$	Γ^F	$d \ln Y^F$	$\mathcal{E}^F$	Γ^F
Shock:	**Productivity**			**Demand**		
Baseline	2.66	0.39	2.27	0.35	0.20	0.15
Share:		*0.148*	*0.852*		*0.572*	*0.428*
Homogeneous firms	3.13	3.07	0.06	0.37	0.38	-0.01
Share:		*0.982*	*0.018*		*1.025*	*-0.025*

Intuition for the dampening effect

- Source of the dampening is heterogeneity in firms' production functions
  - Under homogeneity in input and labor shares across firms within a sector, $d \ln Y^F$ is invariant to the distribution of sales shares $\pi_{f,nk,j}$

- Firms importing inputs
  1. Are more sensitive to foreign shocks
  2. Have a lower influence on the domestic GDP, conditional on their size

$\Rightarrow$ In the fully heterogeneous model, foreign shocks reallocate market shares towards large, low-influence firms. This pushes the elasticity down

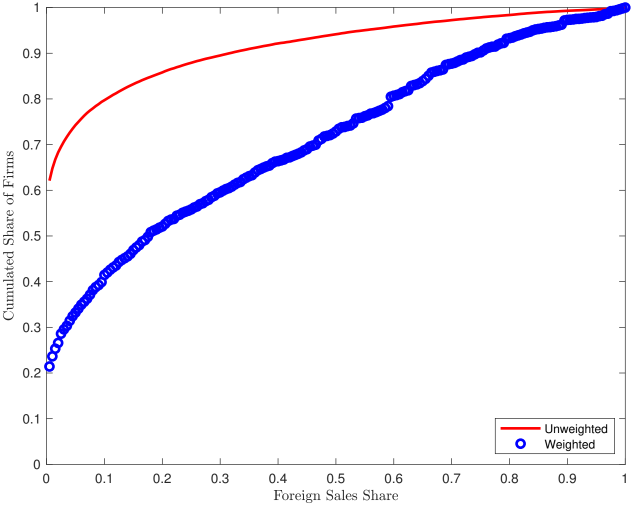
▸ Example in a 2×2×2 model ▸ Details

Conclusion

- A firm-level view of international shock propagation
- Reduced-form: large firms are more sensitive to foreign GDP growth
- Micro: Propagation of foreign shocks largely granular
- Macro: Heterogeneity dampens the GDP response to aggregate foreign shocks

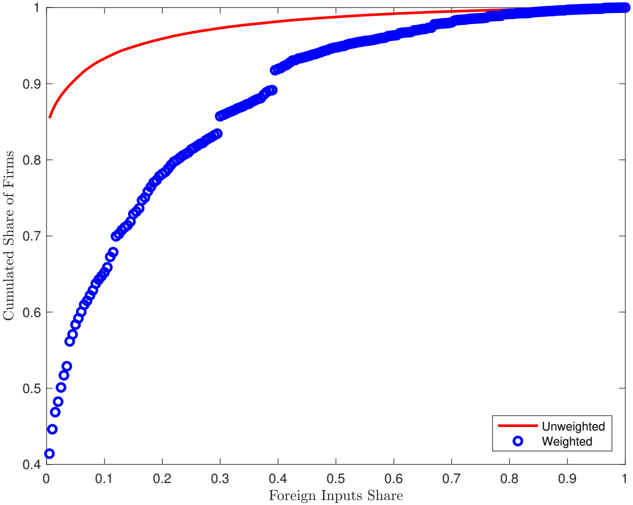
Thank you!

Fact 2a: Larger firms more likely to export



Sources: French customs and balance-sheet data, for 2005. Restricted to T sectors. Foreign sales share is the share of exports in total sales

Fact 2b: Larger firms more likely to import



Sources: French customs and balance-sheet data, for 2005. Foreign inputs share is the share of foreign inputs in firms' total input expenditure

Price level and input shares

$$P_{mn,j} = \left[\sum_{f \in \Omega_{mn,j}} \xi_{f,mn,j} \left(\frac{\rho}{\rho - 1} \frac{\tau_{mn,j} b_{f,m,j}}{a_f} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

$$\pi^l_{f,m,j} = \frac{\alpha_{f,m,j} w_m^{1-\phi}}{\alpha_{f,m,j} w_m^{1-\phi} + (1 - \alpha_{f,m,j}) \left(P^M_{f,m,j} \right)^{1-\phi}}$$

$$\pi^M_{f,km,ij} = \frac{\gamma_{f,km,ij} P^{1-\eta}_{km,i}}{\sum_i \sum_n \gamma_{f,nm,ij} P^{1-\eta}_{nm,i}}$$

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GDP accounting in the model

Real GDP:

$$Y_m = \sum_{j=1}^{J} \left(P_{m,j,-1} Q_{m,j} - P^M_{m,j,-1} M_{m,j} \right),$$

GDP change:

$$\widehat{Y}_m = \sum_{j=1}^{J} \omega^D_{m,j,-1} \left(\widehat{Q}_{m,j} - \pi^M_{m,j,-1} \widehat{M}_{m,j} \right)$$

Where:

$$\widehat{Q}_{m,j} = \frac{1}{\widehat{P}_{m,j}} \times \frac{P_{m,j} Q_{m,j}}{P_{m,j,-1} Q_{m,j,-1}}$$

$$\widehat{M}_{m,j} = \frac{1}{\widehat{P}^M_{m,j}} \times \frac{P^M_{m,j} M_{m,j}}{P^M_{m,j,-1} M_{m,j,-1}}$$

GDP deflator:

$$\widehat{P}^{GDP}_m = \frac{\widehat{Y}^{NOM}_m}{\widehat{Y}_m}.$$

DEK (2008) formulation

$$\widehat{X}_{mn,j}X_{mn,j,-1} = \pi^c_{mn,j}\pi^c_{n,j}\left[\widehat{w}_n\left(\frac{\widehat{w}_n}{\widehat{P}_n}\right)^{\frac{1}{\bar{\psi}-1}} s^L_{n,-1} + \widehat{\Pi}_n s^{\Pi}_{n,-1} + \widehat{D}_n s^D_{n,-1}\right] P_{n,-1}C_{n,-1}$$
$$+ \sum_i \frac{\rho-1}{\rho}\sum_{f\in i}(1-\pi^l_{f,n,i})\pi^M_{f,mn,ji}\sum_k \pi_{f,nk,i}\widehat{X}_{nk,i}X_{nk,i,-1}$$

$$\pi^c_{mn,j} = \frac{\widehat{P}^{1-\sigma}_{mn,j}\pi^c_{mn,j,-1}}{\sum_k \widehat{P}^{1-\sigma}_{kn,j}\pi^c_{kn,j,-1}}$$

$$\pi_{f,nk,j} = \frac{\widehat{\xi}_{f,nk,j}\left(\widehat{b}_{f,n,j}\widehat{a}_f^{-1}\right)^{1-\rho}\pi_{f,nk,j,-1}}{\sum_{g\in\Omega_{nk,j}}\widehat{\xi}_{g,nk,j}\left(\widehat{b}_{g,n,j}\widehat{a}_g^{-1}\right)^{1-\rho}\pi_{g,nk,j,-1}}$$

$$\widehat{b}_{f,m,j} = \left[\pi^l_{f,m,j,-1}\widehat{w}_m^{1-\phi} + (1-\pi^l_{f,m,j,-1})\left(\widehat{P}^M_{f,m,j}\right)^{1-\phi}\right]^{\frac{1}{1-\phi}}$$

$$\widehat{P}^M_{f,m,j} = \left[\sum_i\sum_k \pi^M_{f,km,ij,-1}\widehat{P}^{1-\eta}_{km,i}\right]^{\frac{1}{1-\eta}}$$

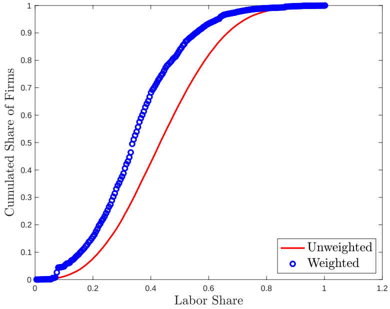
$$\pi^l_{f,m,j} = \frac{\pi^l_{f,m,j,-1}\widehat{w}_m^{1-\phi}}{\pi^l_{f,m,j,-1}\widehat{w}_m^{1-\phi} + (1-\pi^l_{f,m,j,-1})\left(\widehat{P}^M_{f,m,j}\right)^{1-\phi}}; \quad \pi^M_{f,km,ij} = \frac{\pi^M_{f,km,ij,-1}\widehat{P}^{1-\eta}_{km,i}}{\sum_i\sum_n \pi^M_{f,nm,ij,-1}\widehat{P}^{1-\eta}_{nm,i}}$$

$$\sum_j\sum_{f\in j}\sum_k \frac{\rho-1}{\rho}\pi^l_{f,n,j,-1}\pi_{f,nk,j,-1}X_{nk,j,-1}\left[\widehat{\pi}^l_{f,n,j}\widehat{\pi}_{f,nk,j}\widehat{X}_{nk,j} - \widehat{w}_n^{\frac{\bar{\psi}}{\bar{\psi}-1}}\widehat{P}_n^{\frac{1}{1-\bar{\psi}}}\right] = 0$$

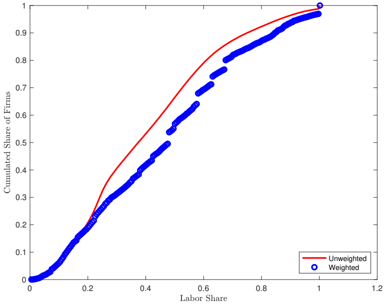
Summary statistics by sector (2005 data)

WIOD sector	# firms	Share VA	Traded/ non-traded
Agriculture, Hunting, Forestry, Fishing	7,718	.0067	T
Mining, Quarrying	1,022	.0041	T
Food, Beverages, Tobacco	10,883	.0354	T
Textile Products	1,684	.0039	T
Leather, Footwear	2,501	.0058	T
Wood Products	3,045	.0044	T
Pulp, Paper, Publishing	7,721	.0202	T
Coke, Refined Petroleum, Nuclear Fuel	50	.0056	T
Chemical Products	2,051	.0358	T
Rubber and Plastics	2,992	.0155	T
Other Non-Metallic Minerals	2,607	.0127	T
Basic and Fabricated Metals	14,561	.0373	T
Machinery n.e.c.	6,442	.0243	T
Electrical, Optical Equipment	6,599	.0288	T
Transport Equipment	1,804	.0315	T
Manufacturing n.e.c.	4,946	.0086	T
Electricity, Gas, Water Supply	321	.0364	NT
Construction	54,428	.0664	NT
Wholesale and Retail Motor Vehicles and Fuel	25,975	.0218	NT
Wholesale Trade	49,166	.0867	NT
Retail Trade	76,069	.0739	NT
Hotels and restaurants	29,135	.0259	NT
Inland Transport	9,244	.0401	NT
Water Transport	171	.0017	NT
Air Transport	66	.0085	NT
Other Transport Activities	2,068	.0256	NT
Post and Telecommunications	276	.0488	NT
Real Estate	7,726	.0425	NT
Business Activities	31,605	.1849	NT
Education	1,569	.0037	NT
Health and Social Work	6,200	.0200	NT
Other Personal Services	15,283	.0324	NT

Distribution of labor shares across French firms



Tradable Sectors



Non-tradable Sectors

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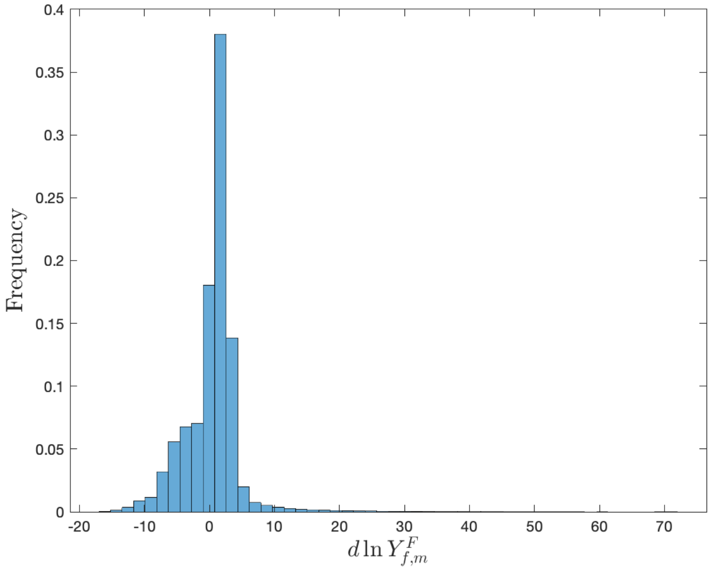
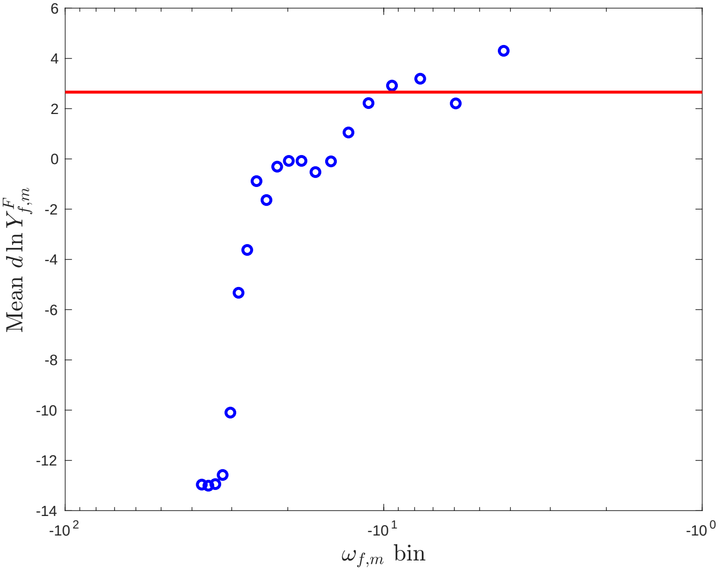
Distribution of ϵ^f

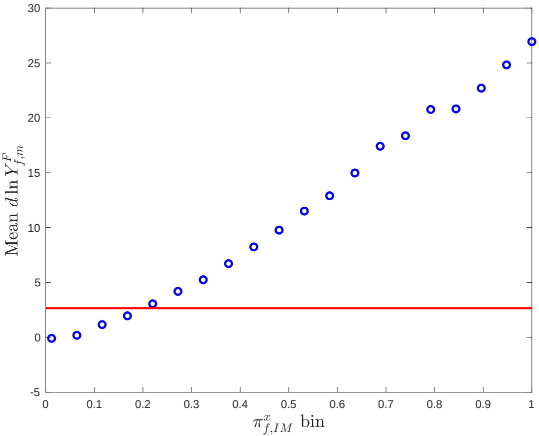


ϵ^f and firm size



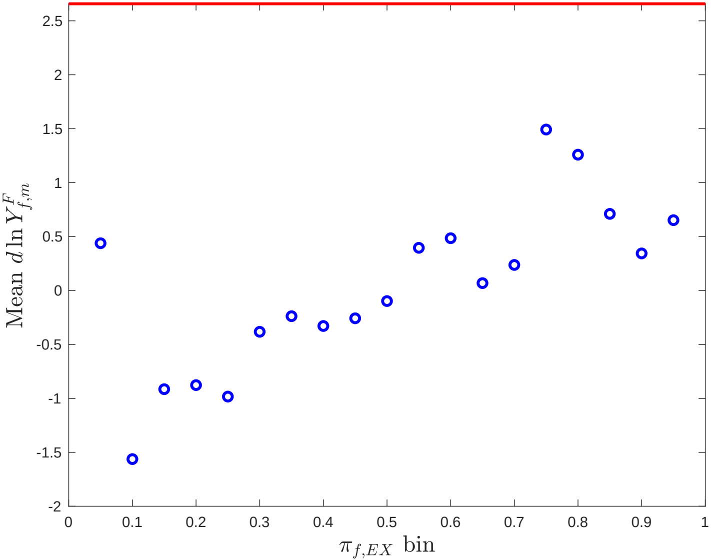
Note: red horizontal line indicates baseline value of ϵ^Y; x-axis is log-scale Back

ϵ^f and imported input share



Notes: red horizontal line indicates baseline value of ϵ^Y Back

ϵ^f and export intensity



Note: red horizontal line indicates baseline value of ϵ^Y Back

Feeding actual foreign shocks

- Firm value added following a foreign shock:

$$d \ln Y^F_{f,m} = \sum_n \epsilon_{f,n} d \ln a_n$$

- GDP change:

$$d \ln Y^F_m = \sum_f \omega_{f,m,-1} d \ln Y^F_{f,m}$$

Period	Data		Foreign TFP			Foreign GDP		
	$d \ln Y_m$	Γ	$d \ln Y^F_m$	Γ^F	$\mathcal{E}^F$	$d \ln Y^F_m$	Γ^F	$\mathcal{E}^F$
1975–2014	1.54		0.28	0.26	0.05	0.11	0.10	0.01
1991–2007	1.11	0.96	0.27	0.25	0.06	0.07	0.06	0.02

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Intuition for the dampening effect

- By definition, GDP change:

$$d \ln Y^F = \sum_f \lambda_f d \ln \widetilde{a}_f$$

where λ_f is firm f's "influence:"

$$\lambda_f \equiv \frac{d \ln Y_m}{d \ln a_f}$$

and $d \ln \widetilde{a}_f$ is a synthetic TFP shock that leads to the same change in value added as the foreign shock

$$d \ln \widetilde{a}_f = \frac{1}{(1-\rho)} d \ln Y_{f,m,j}$$

- In comparison with the homogeneous model, influence of high $d \ln \widetilde{a}_f$ firms is reduced due to the production involving less domestic value added

◂ Back

Aggregate impact

- GDP change:

$$d \ln Y_m = \sum_f \lambda_f d \ln a_f$$

where λ_f is firm f's "influence:"

$$\lambda_f \equiv \frac{d \ln Y_m}{d \ln a_f}$$

- Following a foreign productivity shock:

$$d \ln Y_{f,m,j} = (1-\rho) d \ln \widetilde{a}_f$$

where $\widetilde{a}_f$ is a "synthetic" shock to firms that leads to the same change in value added as the foreign shock

- Ex.: reduction in input prices:

$$d \ln Y_{f,m,j} = (1-\rho) \sum_i \sum_k (1-\pi^I_{f,m,j,-1}) \pi^x_{f,km,ij,-1} d \ln P_{km,i} = (1-\rho) d \ln \widetilde{a}_f$$

Back

$2 \times 2 \times 2$ example

- Experiment: start with 2 symmetric firms in each sector, and then give more and more imported inputs to one of the firms

Input	Tradeable		Non-tradeable	
share	Firm 1	Firm 2	Firm 1	Firm 2
France	0.76	0.76	0.92	0.92
ROW	0.24	0.24	0.08	0.08

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Input share	Tradeable		Non-tradeable	
	Firm 1	Firm 2	Firm 1	Firm 2
France	0.76	0.76	0.92	0.92
ROW	0.24	0.24	0.08	0.08

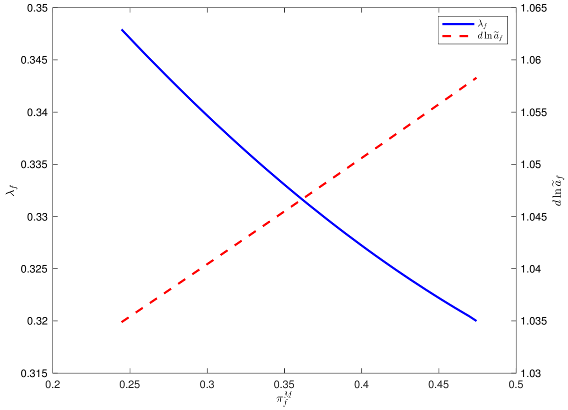
$$\Downarrow$$

France	0.53	0.99	0.86	0.99
ROW	0.47	0.01	0.14	0.01

Back

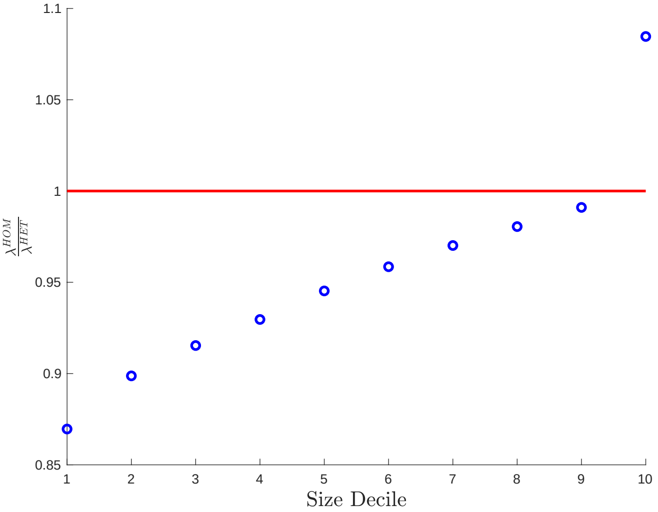
$2 \times 2 \times 2$ example

- Experiment: start with 2 symmetric firms in each sector, and then give more and more imported inputs to one of the firms
- Isolate a negative covariance between a firm's influence λ_f and its elasticity to the shock $d \ln \tilde{a}_f$



Back

Actual model: heterogeneity in firm size



◂ Back

Households

- M countries and J sectors
- $\bar{L}_n$ households in country n (supply of primary factors)
- GHH preferences (Greenwood et al, 1988):

$$U\left(c_n, l_n\right) = \nu\left(c_n - \frac{\psi_0}{\bar{\psi}} l_n^{\bar{\psi}}\right)$$

$$c_n = \prod_j c_{n,j}^{\vartheta_j}$$

$$c_{n,j} = \left[\sum_m \mu_{mn,j}^{\frac{1}{\sigma}} c_{mn,j}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

Sectors and firms

- CES aggregate of firms from m selling to n in sector j:

$$Q_{mn,j} = \left[\sum_{f \in \Omega_{mn,j}} \xi_{f,mn,j}^{\frac{1}{\rho}} Q_{f,mn,j}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$$

- Demand faced by firm f, expressed in expenditures:

$$X_{f,mn,j} = \underbrace{\xi_{f,mn,j} \frac{p_{f,mn,j}^{1-\rho}}{P_{mn,j}^{1-\rho}}}_{\pi_{f,mn,j}} X_{mn,j}$$

Equilibrium

- Goods market clearing:

$$X_{mn,j} = \pi^{c}_{mn,j} \vartheta_j P_n C_n + \sum_i \sum_{f \in i} \frac{\rho - 1}{\rho} (1 - \pi^{I}_{f,n,i}) \pi^{M}_{f,mn,ji} \sum_k X_{f,nk,i}$$

▸ Price level and input shares

- Factor market clearing:

$$\left(\frac{1}{\psi_0} \frac{w_n}{P_n} \right)^{\frac{1}{\bar{\psi}-1}} \bar{L}_n = \sum_j L_{n,j}$$

$$= \frac{\rho - 1}{\rho} \frac{1}{w_n} \sum_j \sum_{f \in j} \pi^{I}_{f,n,i} \sum_k X_{f,nk,i}$$