Tasks, Occupations, and Wage Inequality in an Open Economy*

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Abstract

This paper documents and theoretically explains a nexus between globalization and wage inequality within plants through internal labor market organization. We document that the dominant component of overall and residual wage inequality is within plant-occupations and, combining within-occupation task information from labor force surveys with linked plant–worker data for Germany, establish three interrelated facts: (1) larger plants and exporters organize production into more occupations, (2) workers at larger plants and exporters perform fewer tasks within occupations, and (3) overall and residual wages are more dispersed at larger plants. To explain these facts, we build a model in which the plant endogenously bundles tasks into occupations and workers match to occupations. By splitting the task range into more occupations, the plant assigns workers to a narrower task range per occupation, reducing worker mismatch while typically raising the within-plant dispersion of wages. Embedding this rationale into a Melitz model, where fixed span-of-control costs increase with occupation counts, we show that inherently more productive plants exhibit higher worker efficiency and wider wage dispersion and that economy-wide wage inequality is higher in the open economy for an empirically confirmed parametrization. Reduced-form tests confirm main predictions of the model, and simulations based on structural estimation suggest that trade induces a stricter division of labor at globalized plants with an associated change in wage inequality.

Keywords: Tasks; specialization; international trade; firm-internal labor allocation; heterogeneity

JEL Classification: F12, F16, J3, L23

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> "It is the great multiplication of the productions of all the different arts, in consequence of the division of labour, which occasions, in a well-governed society, that universal opulence which extends itself to the lowest ranks of the people."
>
> — Adam Smith (1776): The Wealth Of Nations, Book I, Chapter I

1 Introduction

Recent theories of international trade at the firm level have opened new insights into a nexus between globalization and wage inequality within sectors and occupations. Much of the emphasis to date has been on the wage dispersion between firms, given the wage premia that exporters pay to otherwise similar workers within sectors and occupations (Helpman, Itskhoki and Redding 2010, Egger and Kreickemeier 2009, Davis and Harrigan 2011, Amiti and Davis 2012). The empirical importance of the between-firm or between-plant dispersion of wages for changes in overall wage inequality has been documented for labor markets in general (Card, Heining and Kline 2013, Lopes de Melo 2013, Song et al. 2015) and for labor market outcomes in open economies in particular (Egger, Egger and Kreickemeier 2013, Coşar, Guner and Tybout 2016, Helpman et al. 2017, Eaton, Kortum and Kramarz 2015). In the cross section of workers, however, the commanding component of wage variation is within firms: studies such as Abowd et al. (2001) and Menezes-Filho, Muendler and Ramey (2008), for instance, control for worker and employer characteristics, as well as firm effects, in Mincer regressions and show a dominance of the residual wage component; Lemieux (2006) documents the response of the residual wage component to economic change. In this paper we relate back to the basic principle of the division of labor, within plants and within occupations across workers as well as across countries in the global economy. We explore how the large within-plant part of wage inequality responds to trade—through internal labor-market reorganization—and show that the size of a plant's global product market translates into its internal division of labor, so that global specialization affects inequality across the "ranks of the people."

In his foundational analysis of the division of labor, Adam Smith (1776, Book I, Chapter I) described tasks that a single worker could perform cumulatively or that the employer could alternatively assign to several workers:

> "[M]aking a pin is . . . divided into about eighteen distinct operations. . . . [T]en persons . . . could make among them upwards of forty-eight thousand pins in a day. But if they had all wrought separately and independently . . . they certainly could not each of them have made twenty, perhaps not one pin in a day."

To elicit information on cumulative tasks in Adam Smith's operational sense, and on the organization of the workplace in today's economy, we use the German Qualifications and Career Surveys (BIBB-BAuA surveys) to build time-consistent measures of workplace operations and multitasking. Importantly for an understanding of the evolving division of labor at employers, the BIBB-BAuA surveys also allow us to quantify in a time consistent manner how many tasks workers perform *within* their occupations. We combine the task information by occupation, industry, location and plant size with German linked plant–worker data (LIAB).

Three striking facts emerge. First, larger plants adopt more occupations. Second, workers at larger plants perform a narrower range of tasks within the same occupation. In other words, larger size in the product market is associated with a stricter internal division of labor. Third, both overall and residual wages are more dispersed within occupations at larger plants.[1] Our hypothesis is that workers differ in their ability to carry out the tasks of an occupation so that match quality determines labor efficiency within occupations. Ability mismatches generate wage inequality—in accordance with the empirical observation that the dominant share of residual wage inequality is within plants, within layers of hierarchy, and within occupations.

To explain these facts, we propose a model of endogenous occupation choice and task assignments by the employer. Employers can organize the full range of tasks required for production into fewer or into more occupations. A smaller count of occupations at a plant implies that workers have to carry out a wider range of tasks per occupation. Conversely, in plants with a larger count of occupations, each occupation comes with a narrower range of tasks to be performed. We postulate that workers have a core ability that makes them most efficient at one particular task in the full task range and monotonically less efficient at tasks that are more distant from their core ability. Workers assortatively match to task ranges that include their core ability. As a consequence, when a plant's task ranges are narrower, then the degree of mismatch between a worker and the tasks is smaller because all of an occupation's tasks are closer to a worker's core ability. Workers are therefore more efficient at plants with more occupations and a finer division of labor. Plants incur a span-of-control fixed cost of operation that increases with the count of occupations. In a Melitz (2003) model with heterogeneous producers, more productive plants can recover a larger span-of-control fixed cost with operating profits, so that in our framework more productive plants adopt of a high occupation count. In particular, productive plants that select into exporting choose more occupations with narrower task ranges compared to non-exporters. In our parameterized model, and empirically, plants with a stricter internal division of labor exhibit higher wage inequality within occupations because the plant–worker match quality affects surplus sensitively, and surplus is partly shared with workers through the wages.

A main prediction of the model is that a plant's count of occupations and the width of its average task range per occupation are inversely related and respond to the plant's product-market size. We document that this inverse relationship between occupation counts and the task range per occupation holds empirically, using an instrumental-variable approach in the spirit of Autor, Dorn and Hanson (2013) and Dauth, Findeisen and

[1]We use residual log daily wages from standard Mincer regressions, conditioning out demographic, education and tenure information as well as time, industry and region effects.

Suedekum (2014) to relate a plant's revenues to exogenous foreign-market shocks from China and Eastern Europe. Similar regressions document that the within plant-occupation wage dispersion is higher at more globalized plants.

To study inequality, we carefully specify the stochastic foundations of our model so we can separate the inherent dispersion of outcomes on the one hand from policy-induced change on the other hand. We structurally estimate our model with a maximum likelihood approach under the assumption that plant characteristics are jointly log-normally distributed (on log normality also see Helpman et al. 2017, Fernandes et al. 2018). We confront two challenges beyond previous implementations of structural heterogeneous-firm estimation, which is largely based on the Chaney (2008) version of the Melitz (2003) model. First, we show that joint log-normality imposes important restrictions on the possible frequency of endogenous switching (selection into export status) for given patterns in the conditional higher moments (variances of outcomes at exporters and non-exporters). For example, a higher variance of log revenues among exporters than among non-exporters implies that more than half of the plants must be exporters, unless there is censoring (selection into activity). We therefore derive an endogenous switching model with censoring for estimation, including the entry margin into activity as in the original Melitz model. As a second challenge, censoring in the Melitz model does not conform to a conventional Tobit approach such as in Carson and Sun (2007) because censoring occurs with respect to an unobserved plant characteristic (productivity) not with respect to an observed outcome. We show that our estimation model is point identified and derive an according two-step estimator that is widely implementable for versions of the Melitz (2003) model. We simulate the structurally estimated model to quantify the importance of trade opening for the intra-plant division of labor in Germany. We find that the simulated model predicts substantive changes that reflect Germany's economic globalization and its association with heightened wage inequality within plant-occupations.

Opening up to trade leads to a selection of more productive plants into exporting, raising welfare directly and indirectly through the stricter division of labor at larger plants. But trade opening results in an asymmetric response of occupation-level wage inequality. The model predicts that the dispersion of wages within occupations increases at exporting plants, if wage inequality was already high at these producers under autarky, while within-occupation wage inequality declines at non-exporters. Given the asymmetry in plant-level implications, access to foreign markets exerts counteracting effects on economy-wide wage inequality. However, we can show theoretically under plausible parameterizations and in simulations of the structurally estimated model that economy-wide wage inequality is higher in the more open economy.

Workplace tasks are an important, employer-driven characteristic of the labor market, and have been documented to relate closely to recent labor market changes including wage polarization (Autor, Katz and Kearney 2006, Goos, Manning and Salomons 2009) and the offshorability of jobs (Leamer and Storper 2001, Levy and Murnane 2004, Blinder 2006). The assignment of tasks in an open economy, and the implications for welfare and wage inequality, have been studied from a theoretical perspective in industry-level models, including the Heckscher-Ohlin (Grossman and Rossi-Hansberg 2008, 2010) and the Ricardian framework (Rodríguez-Clare 2010, Acemoglu and Autor 2011). Our model complements the industry-level perspective with a plant-level view. Beyond considerations of offshorability, our treatment of tasks emphasizes the quality of the worker-task match as a key determinant of plant performance and in this regard relates closely to studies of internal labor markets by Barron, Black and Loewenstein (1989), who show that the quality of worker-task matches reduces on-the-job training costs, as well as Meyer (1994) and Burgess et al. (2010), who document that the quality of worker-task matches raises team efficiency and the effectiveness of incentives.

Human resource management practices have been found to be an important determinant of the variation in plant and firm productivity within and across countries (Bloom and Van Reenen 2011). Yet, aspects of the internal labor market and residual wage inequality are difficult to observe directly. Recent studies of the firm's internal labor market have turned to the importance of observable hierarchies (Caliendo and Rossi-Hansberg 2012, Caliendo, Monte and Rossi-Hansberg 2015) and their response to firm-level trade. Our model complements the hierarchical approach to a firm's internal organization with a perspective on the horizontal differentiation of worker abilities and their tasks within hierarchical layers. In fact, we find that most employer-level residual wage inequality in the German data is also within hierarchies (and within occupation categories), suggesting that an important horizontal wage differentiation component acts within hierarchies. The internal organization of plants and firms also involves the motivation of workers to exert effort. Related studies analyze the response of employers' incentives for workers, and observable incentive pay in particular, when global competition changes (Guadalupe 2007, Cunat and Guadalupe 2009). Our paper complements the view on incentives for worker effort with a perspective on management responses to product-market opportunities, as employers adjust the observable count of occupations they offer and coordinate the observable range of tasks they assign within jobs.

An alternative approach to modelling worker-level wage dispersion within and between employers considers the employer-worker matching process (see e.g. Legros and Newman 2002, Eeckhout and Kircher 2011).[2] The potential efficiency gains from improved assortative matching have received according attention in the trade literature (Costinot and Vogel 2010, Sampson 2014). Several studies highlight trade-induced changes in match quality as a key aspect of trade in terms of welfare, employment and wage inequality (Amiti and Pissarides 2005,

[2]The literature estimating search models of the labor market more generally includes Burdett and Mortensen (1998), Cahuc, Postel-Vinay and Robin (2006), Postel-Vinay and Robin (2002), and Postel-Vinay and Turon (2010).

Davidson, Matusz and Shevchenko 2008, Davidson et al. 2014). More recent studies have started to complement the analysis of cross-industry and cross-firm matches with an analysis of within-firm matches.[3] Larch and Lechthaler (2011) study the assignment of workers across plants within multinational firms, and Bombardini, Orefice and Tito (2019) investigate the permissible ability ranges of workers at firms when worker-firm matches are formed.[4] Our model highlights that an additional source of efficiency gains for employers is to improve match quality by narrowly assigning tasks to workers with the best fit to those tasks (a core ability within the occupation's task range).[5] Our worker-reported task frequencies within occupations characterize empirically the assignment of workplace operations to workers at different plants.

In our model relatively more productive plants choose to augment their elemental productivity with a stricter division of labor that raises labor efficiency, thus concentrating the firm size distribution beyond the inherent productivity dispersion. While the principal selection of more productive firms into exporting remains a basic force in the model (as documented empirically by Clerides, Lach and Tybout 1998, e.g., and others), the feedback of exporting into worker efficiency through internal specialization at exporters is akin to a learning-by-exporting effect (for direct evidence on learning-by-exporting see, e.g., Crespi, Criscuolo and Haskel 2008). The labor market feedback effect in our model is similar to the outcome of screening in Helpman, Itskhoki and Redding (2010), the effect of investment into innovations in Aw, Roberts and Xu (2011), and the effect on team production in Chaney and Ossa (2013). In the Helpman, Itskhoki and Redding (2010) model, screening for higher ability workers raises the returns to exporting and vice versa; in the Aw, Roberts and Xu (2011) model, R&D investments raise the returns to exporting and vice versa; in the Chaney and Ossa (2013) model a larger number of more specialized teams increases the returns to exporting and vice versa; in our model, improving the worker-task match quality raises the returns to exporting and vice versa. The Chaney and Ossa (2013) model incorporates important features of the task model by Becker and Murphy (1992) and allows for the organization of production to depend on firm size. Our model shares with Chaney and Ossa (2013) the mechanism by which market size translates into a firm's incentives to reduce worker-task mismatches, but employers in their task model, as in Becker and Murphy (1992), are identical to each other and workers within a team are homogeneous. We introduce employer and

[3]In a review of the literature on the structure of wages within and across firms, Lazear and Shaw (2009) conclude that the wage structure appears to be more dependent on firm- or within-plant sorting of workers to occupations than on sorting of workers to firms or plants.

[4]Bombardini, Orefice and Tito (2019) isolate a permanent worker-specific and time invariant wage component (average lifetime earnings or a worker-fixed wage effect) and find that the long-term wage component is less dispersed at larger French manufacturing firms. In light of our model, the use of a permanent worker-specific and time invariant wage component to proxy worker efficiency is akin to working with a negative parameter of sensitivity of worker performance to task mismatch, which gives rise to the same result. The lack of full longitudinal worker data in the IAB-LIAB random sample of plants does not allow us to replicate the permanent wage component measures from Bombardini, Orefice and Tito (2019).

[5]An interpretation related to the core ability of workers, most suitable for specific tasks, is that human capital is occupation specific. Kambourov and Manovskii (2009) and Sullivan (2010) provide according empirical evidence.

worker heterogeneity to establish a link between labor efficiency across employers and wage inequality within their occupations.

The remainder of this paper proceeds as follows. In Section 2, we present our data and collect descriptive evidence for three main facts that motivate our model. In Section 3, we build a model of production with task assignment to occupations. We derive the equilibrium for a closed economy in Section 4 and for two symmetric open economies in Section 5. In Section 6 we use reduced-form estimation to test key relationships of the model. In Section 7 we present a structural estimation model, obtain parameter estimates, and use them to simulate the consequences of Germany's further opening to global trade between 1999 and 2012 for efficiency and inequality. Section 8 concludes.

2 Data and Descriptives

The two main sources for our novel micro-level data on employer-level task assignments are (i) the German Qualifications and Career Surveys (BIBB-BAuA surveys), and (ii) the Linked Plant–Worker Data provided by IAB (LIAB). In this section, we elicit three empirical facts from these two datasets to motivate a theory that can explain the division of labor at employers and the resulting wage dispersion within occupations. Additionally we use sector-level bilateral merchandise trade data from the United Nations Commodity Trade Statistics Database (Comtrade) and service trade from the trade in services database (TSD) from the World Bank to construct instrumental variables related to globalization shocks that are exogenous to the employers. We consolidate varying sector definitions and construct 39 longitudinally consistent industries for all data sources. Our industry definition is based on an aggregation of NACE 1.1 for the European Communities, which is equivalent to the German *Klassifikation der Wirtschaftszweige* WZ 2003 at the 2-digit level (see Becker and Muendler 2015).

2.1 Linked plant–worker data

To link workers to their employers, we use data at the German Federal Employment Office's Institute for Employment Research (IAB): the matched plant–worker data LIAB. The LIAB data combine detailed administrative records on workers from the German social security system with the IAB plant panel data.[6] On the employer side, LIAB provides detailed plant information from surveys on an annual basis since 1993. Information on plants in East Germany is only available since 1996. We therefore restrict the sample period to the years 1996-2014 to cover the German economy as a whole. At the plant level we use information on revenues, export status,

[6] See `http://fdz.iab.de/en/Integrated_Establishment_and_Individual_Data/LIAB.aspx`.

Table 1: Decompositions of the log Residual Wage Inequality in Linked Plant–Worker Data

		Subsamples			
Contribution of component (%)		high w	age ≥45	skilled	manager
within industry[a]	88	97	88	88	88
within occupation	84	87	82	86	92
within plant	71	88	68	79	77
within plant-layer	65	71	60	72	73
within plant-occupation[b]	54	60	46	61	63

[a]Mincer regression excludes industry effects ($R^2 = 0.42$).

[b]The within plant-occupation decomposition is identical to the within plant-layer-occupation decomposition because occupations at the 3-digit KldB-88 level are nested strictly within layers of hierarchy.

Source: LIAB 1996-2014.

Notes: The economy-wide variance is decomposed into a within-group and a between-group component for worker groups g using

$$(1/L)\sum_{i=1}^{L}\left(\ln w_i - \overline{\ln w}\right)^2 = (1/L)\sum_{g\in\mathbb{G}}\sum_{i=1}^{L_g}\left(\ln w_i - \overline{\ln w_g}\right)^2 + \sum_{g\in\mathbb{G}}(L_g/L)\left(\overline{\ln w_g} - \overline{\ln w}\right)^2.$$

The reported numbers show the former within components for varying worker groups. Residual log daily wage from standard Mincer regression, conditioning out demographic, education and tenure information as well as time, industry and region effects ($R^2 = 53\%$). Subsample (1): workers with above-median daily wage; (2): workers 45 years old and older; (3): high-skilled workers (Abitur or equivalent); (4): supervisors and managers. 357 occupations at the 3-digit KldB-88 level. The variance of the log daily wage w_i is linearly decomposed into a within and a between part. The reported percentages are the contribution of the within component to the total. Layers of hierarchy based on a mapping of the Caliendo, Monte and Rossi-Hansberg (2015) hierarchies to KldB-88 using ISCO-88.

export revenues and employment as well as region and industry categories. At the individual worker level, LIAB offers a comprehensive set of characteristics. We use demographic, tenure and education indicators, occupation characteristics, and data on daily wages.[7] Larger plants are over-represented in the plant panel. We therefore use the weighting factors provided by IAB and make our plant-level data representative for the German economy as a whole.

LIAB allows us to quantify sources of wage variation in the German labor market. To assess the dispersion in daily wages, we first remove observed demographic, education and tenure information together with time, industry and region effects from log daily wages in a Mincer regression, and obtain residual log daily wages. We remove observed worker characteristics because they are well explained by existing labor-market theories and have been addressed with classic trade theory and its extension to offshoring (see, e.g., Katz and Murphy 1992, Feenstra and Hanson 1999). Observed worker characteristics explain about 53 percent of the log wage variation (42 percent if we omit industry effects). Similar to other studies for both industrialized and developing countries (see e.g. Abowd et al. 2001, Menezes-Filho, Muendler and Ramey 2008), this finding implies that almost half of the wage dispersion remains unexplained at this level of analysis.

[7]Wage information in the social security records is right-censored, so we replace censored wages by imputed wages, following the procedure proposed by Baumgarten (2013). Hourly wages cannot be constructed. We therefore use daily wages as the most precise measure of earnings.

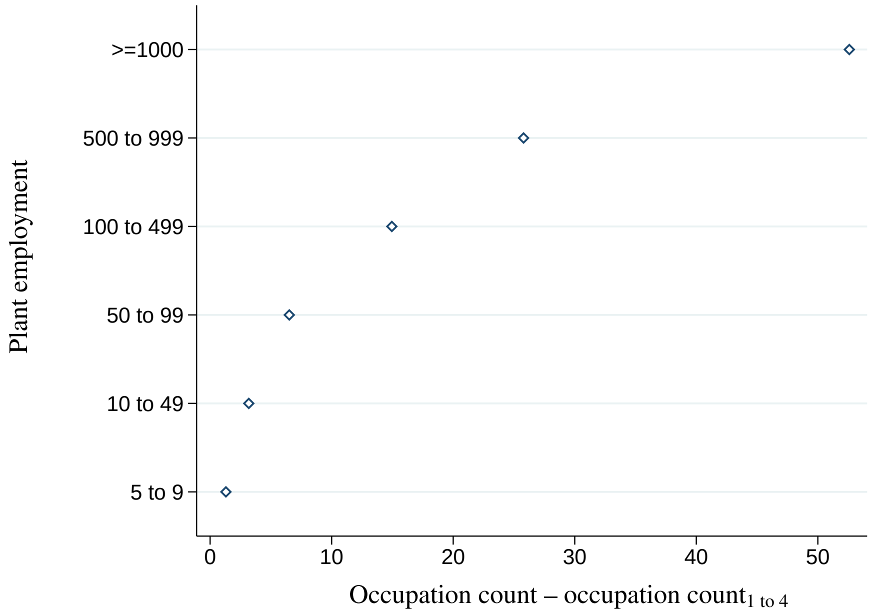
Table 1 follows up with further decompositions of the variance of the log daily wage residual.[8] Variation between industries and between occupations explains 12 and 16 percent of the variation in residual daily wages in 1996-2014 (first column). The remaining within-industry (88 percent) and within-occupation variation (84 percent) suggests that classic trade theory, which predicts cross industry-occupation differences in wages, is not a strong candidate to explain the part of wage variation that is unrelated to worker characteristics. Variation between plants is more successful and explains 29 percent of residual wage variation, but still leaves 71 percent of residual wage variability unexplained. Recent trade theories such as Helpman, Itskhoki and Redding (2010), Egger and Kreickemeier (2009), Davis and Harrigan (2011) and Amiti and Davis (2012) address ways in which globalization can affect the between-employer variation in wages, and Egger, Egger and Kreickemeier (2013), Coşar, Guner and Tybout (2016), Helpman et al. (2017) and Eaton, Kortum and Kramarz (2015) provide according empirical evidence. Looking at the variation between plants and between their managerial hierarchies brings the unexplained part of residual wage dispersion down by another 6 percentage points in 1996-2014 (to 65 percent). Recent models of the firm's internal labor market have elaborated the importance of hierarchies. Caliendo and Rossi-Hansberg (2012) and Caliendo, Monte and Rossi-Hansberg (2015) provide evidence on earnings responses across hierarchies to firm-level trade. Considering the residual daily wage variation between plants and between their occupations (357 occupations at the 3-digit KldB-88 level) pushes the unexplained part further down by another 11 percentage points. Occupations are perfectly nested within hierarchies (using the occupation-to-hierarchy mapping from Caliendo, Monte and Rossi-Hansberg 2015). We are not aware of theory or empirical work on within-employer reallocations across occupations in response to globalization shocks.

Table 1 also documents that for subsamples of more qualified workers, such as highly paid or skilled workers, the within-variation inside plants and their occupations is typically even more dominant. For example, workers with an above-median daily wage, supervisors and managers, and high-skilled workers with a college-qualifying secondary-education diploma (Abitur or equivalent) exhibit higher residual wage variability within plants and their occupations than the overall worker population. In contrast, more experienced workers at an age of 45 years or older face a lower residual wage dispersion than the overall population, perhaps because collective bargaining contracts become more binding at longer tenure. However, even for the subgroup with the lowest within-plant-occupation wage variation, the within components still accounts for almost half of residual log wage variation (46 percent compared to 54 percent in the first column).

In summary, for the worker population and within subsamples of relatively qualified workers about half or more of the residual daily wage variation remains unexplained even at the plant-occupation level. In other words,

[8] When using the exponentiated log daily wage residual, we find variance decompositions to be closely similar.

Figure 1: Count of Occupations by Plant Employment



Source: LIAB 1996-2014.
Notes: Prediction of occupation count n by plant employment category, controlling for sector, region, occupation and worker characteristics. Results are differences to smallest plant-size category (1 to 4 workers). Thick, medium, and thin lines represent the 99, 95, and 90 percent confidence intervals.

the dominant part of residual wage variation, not accounted for with worker characteristics, occurs within plant-occupations. This is the wage variation that we take on in this paper. The finding also suggests that the wage gaps between senior management and the median worker at an employer are less relevant for overall wage inequality than is the wage dispersion within (mainly horizontally differentiated) occupations.

In addition to the detailed decomposition of residual wage variation, the LIAB data allow us to establish

Fact 1. *The count of occupations at a plant increases with plant employment.*

We project the observed count of occupations n in worker i's plant on sector, region, occupation and worker characteristics. We then plot, in Figure 1, the so normalized count of occupations n (on the horizontal axis) against the plant's employment by size category (on the vertical axis). We normalize the occupation count (on the horizontal axis) by subtracting the count at the smallest plants (with 1 to 4 workers). We choose the depicted size categories (on the vertical axis) because they are the ones reported in our other data source. The figure shows that the occupations count n increases monotonically with plant size. Around the average occupation count per plant-size category, the figure draws thick, medium, and thin lines that represent the 99, 95, and 90 percent confidence intervals, but those lines are largely invisible given only minor dispersions of the normalized occupation counts within size categories. Of course, the monotonic increase of the occupation count in plant size is not necessarily evidence in favor of a finer division of labor in larger plants. Trivially, the simple fact that bigger plant have more workers to a assign to a larger count of occupations is also consistent with Fact 1.[9] To establish relevant facts about the varying division of labor across plants of different sizes we therefore need to look within occupations and into the tasks per occupation. German labor force survey data allow us just that look.

2.2 Labor force survey data

A meaningful analysis of the within-plant-occupation component requires measurable properties of occupations. We take information on the organization of the workplace from three German Qualifications and Career Surveys conducted over the years 1999 through 2012 by Germany's Federal Institute for Vocational Education and Training BIBB (most recently in collaboration with the think tank BAuA). Each wave is based on a frame that selects a random sample of around one-tenth of a percent of the German labor force with more than 20 hours of work during the survey week. The BIBB-BAuA data report detailed information on workplace properties, worker characteristics, the industry, occupation and earnings, as well as rudimentary information on the employer, such as the size of a worker's plant in seven size categories (as on the vertical axis of Figure 1). Most importantly, we observe workers' responses to survey questions that regard the tasks they perform in their occupation. Following the time consistent definitions in Becker and Muendler (2015), who used German Qualifications and Career Surveys conducted over the years 1979 through 2006, we append the 2012 survey data and make use of the questions that elicit *what* operations (tasks) a worker carries out on the job. A worker may report these operations as performed or not. We can discern 15 such workplace operations, surveyed in a time consistent manner throughout the three BIBB-BAuA waves: 1. Manufacture, Produce Goods; 2. Repair, Maintain; 3. Entertain, Accommodate, Prepare Foods; 4. Transport, Store, Dispatch; 5. Measure, Inspect, Control Quality; 6. Gather Information, Develop, Research, Construct; 7. Purchase, Procure, Sell; 8. Program a Computer; 9. Apply Legal Knowledge; 10. Consult and Inform; 11. Train, Teach, Instruct, Educate; 12. Nurse, Look After, Cure; 13. Advertise, Promote, Conduct Marketing and PR; 14. Organize, Plan, Prepare Others' Work; 15. Control Machinery and Technical Processes. These workplace operations are cumulative and exhibit a pronounced change towards more multitasking over time until the early 2000s (Becker and Muendler 2015), with a relatively stable level of multitasking from then on.[10]

[9] This caveat only applies to small and medium-sized plants, but not to the larger firm sizes where the number of workers exceeds the total number of occupations (357).

[10] Research into wage polarization (e.g. Autor, Katz and Kearney 2006, Goos, Manning and Salomons 2009) and offshorability (e.g. Leamer and Storper 2001, Levy and Murnane 2004, Blinder 2006) frequently considers a different dimension of "tasks," including the

Table 2: Multitasking in Simultaneous Workplace Operations

Number of Workplace Operations (Tasks)		Subsamples			
		high w	age $\geq$45	manager	skilled
up to 3	0.158	0.117	0.151	0.079	0.077
4 to 7	0.447	0.452	0.445	0.433	0.463
8 or more	0.396	0.432	0.405	0.488	0.460
Total Number of Tasks	6.667	7.010	6.750	7.442	7.286

Source: BIBB-BAuA 1999, 2006 and 2012 (inverse sampling weights).
Notes: Shares of performed workplace operations per worker, out of 15 possible workplace operations. Subsample (1): workers with above-median daily wage; (2): workers 45 years old and older; (3): supervisors and managers; (4): high-skilled workers (Abitur or equivalent). For a list of the 15 workplace operations see Appendix Table A1.

Table 2 shows that German workers perform on average 6.7 workplace operations (tasks), out of 15 possible such tasks. We report frequencies by individual task in Appendix Table A1. The most frequent number of tasks is 4 to 7, performed by almost half of the workers in the sample. Interestingly, the number of tasks performed is relatively stable across subsamples of workers. For example, workers with an above-median daily wage, more experienced workers at an age of 45 years or older, and high-skilled workers with a college-qualifying secondary-education diploma (Abitur or equivalent) all share the characteristic with the overall population that their most frequent number of tasks performed is between 4 and 7 tasks. The comparably more noticeable differences in multitasking occur for supervisors and managers, and partly for high-skilled workers (with Abitur or equivalent), who perform few tasks (up to 3 tasks) more rarely than the overall population. On average, supervisors and managers perform 7.4 tasks, 0.7 tasks more than the overall population of workers, and high-skilled workers 7.3 tasks. Multitasking occurs, with some variation, for all income groups, age groups, layers of hierarchy, and skill groups.

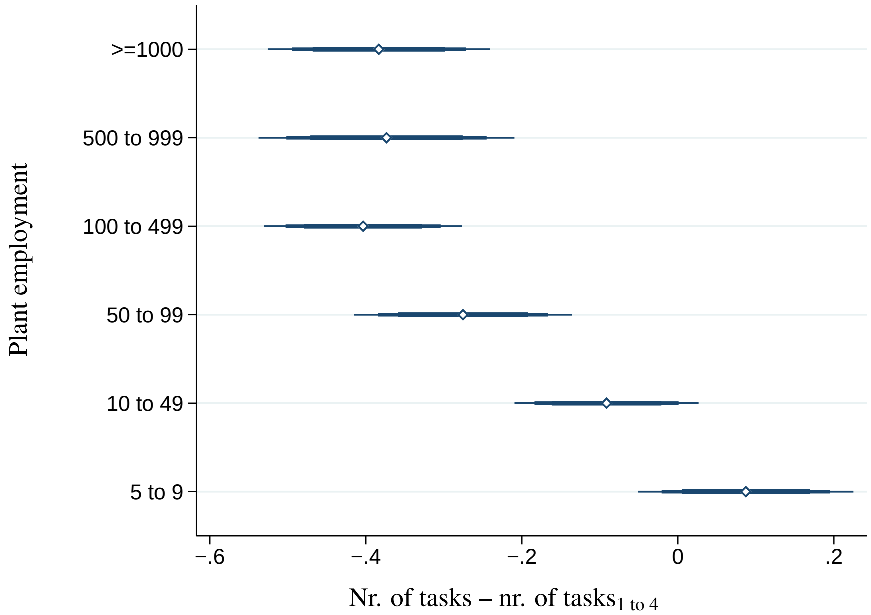
The BIBB-BAuA data allow us to revisit evidence on the division of labor and establish

Fact 2. *The number of tasks within an occupation at a plant decreases with plant employment.*

We compute the number of tasks that workers in their respective occupations report in the BIBB-BAuA data. We then project the reported number of tasks per occupations b on the same sector, region, occupation and worker characteristics as before. Figure 2 plots the so normalized number of tasks b per occupation (on the horizontal

routineness of work steps and codifiability of job descriptions, which are also reported in the BIBB-BAuA surveys. Becker and Muendler (2015) call those tasks, which are related to *how* workers conduct their work, performance requirements and document that those tasks exhibit little time variation even though they are not mutually exclusive tasks. In contrast to the (*what*) operations here, which increase from an average of 5.25 workplace operations (tasks) performed per worker in 1999 to 7.24 operations in 2006, German workers do not report clearly more simultaneous performance requirements over time (Becker and Muendler 2015, Tables 1 and 2). In the spirit of Adam Smith's division of labor, and for the purposes of our model of a plants' internal labor markets, we are most interested in operations that are empirically found to be cumulated at the workplace into multitasking. We therefore restrict our attention to the 15 (*what*) operations in the BIBB-BAuA data.

Figure 2: Number of Tasks per Occupation by Plant Employment



Source: BIBB-BAuA 1999, 2006 and 2012.
Notes: Prediction of number of tasks b within plant-occupation by plant employment category, controlling for sector, region, occupation and worker characteristics. Results are differences to smallest plant-size category (1 to 4 workers). Thick, medium, and thin lines represent the 99, 95, and 90 percent confidence intervals.

axis) against the plant's employment by size category (on the vertical axis). We normalize the occupation count (on the horizontal axis) by subtracting the count at the smallest plants (with 1 to 4 workers). The depicted size categories (on the vertical axis) are the ones reported in the BIBB-BAuA data. The figure shows that the number of tasks b strictly decreases with plant size, relative to the smallest plants, up to a plant employment of 500 workers and then remains constant. In other words, larger plants choose a finer division of labor and assign narrower task ranges to their workers (who fill more occupations by Fact 1). In magnitude, the reduction in the number of tasks from small plants, with 1 to 4 workers, to large plants, with 100 or more workers, is 0.4 tasks per worker out of 15 possible tasks.

Similar to Adam Smith's tenet, workers engage in less multitasking at more pin-factory like large plants. The differences in the numbers of tasks are statistically significant up to a plant size of about 500 workers. Around the average number of tasks per occupation in a plant-size category, the figure shows thick, medium, and thin lines that represent the 99, 95, and 90 percent confidence intervals. For plants with employment between 5 to 9 and 100 to 599 workers, the confidence bands do not overlap. From the threshold of about 500 workers on, plants assign roughly similar task ranges to their workers.

2.3 Data combination

Task information is not available in the linked plant–worker data LIAB. To conduct an employer-level analysis, we therefore need to combine the BIBB-BAuA labor force survey information with the LIAB linked plant–worker records through imputation. A large set of worker characteristics and plant attributes overlaps between the BIBB-BAuA survey and the LIAB records. We use these common variables to conduct imputations in both possible directions: task information from the BIBB-BAuA survey into LIAB in one direction, and plant-level information from LIAB into BIBB-BAuA in the alternate reverse direction.

For our plant-level analysis, the imputation of BIBB-BAuA task information into LIAB is most important. To combine BIBB-BAuA task information with the LIAB plant-worker data and preserve within-occupation and time variation with possibly much precision, we opt for regression-based imputation. Note that the imputation is based on the empirical covariation between common worker variables in both data sets and the tasks that the workers report in BIBB-BAuA, and this covariation preserves the statistically relevant task-related information from BIBB-BAuA in the LIAB data. We first run a linear (OLS) model on the BIBB-BAuA data, regressing the number of tasks (the sum over the 15 activity task indicators) on a set of worker, occupation and plant attributes that are jointly observed in the BIBB-BAuA and in the LIAB data.[11] With the estimated coefficients at hand we perform an out-of-sample linear prediction in the LIAB data using all common variables. Under this procedure we obtain, for 76% of the LIAB observations, an individual-specific number of tasks. Finally, by computing the mean over all individuals within a plant, we end up with a measure of the (mean) number of tasks b that workers perform per occupation within a plant.

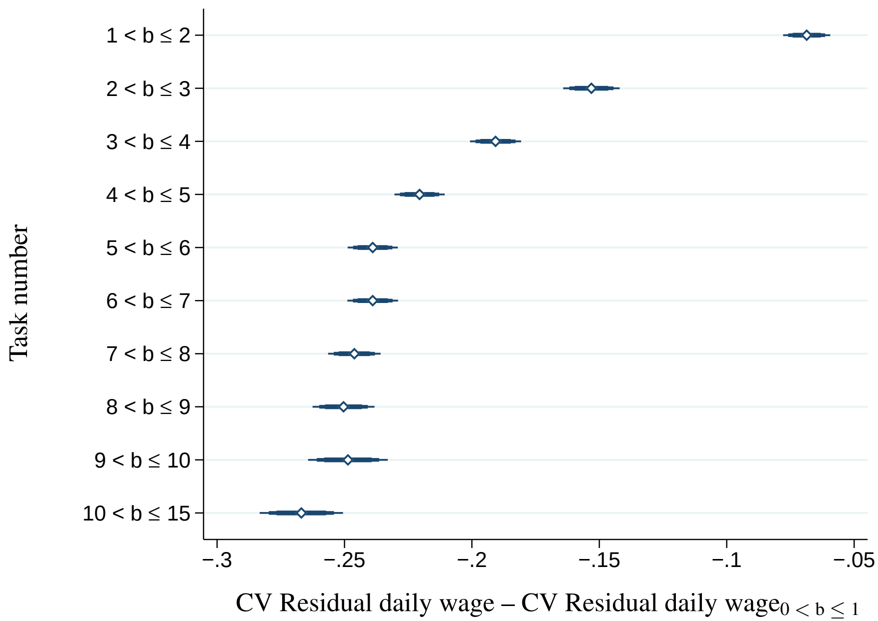
As reported in Appendix Table A4, the average number of tasks per occupation at the plant level varies between 0.32 and 8.87, with a mean of 3.96 and a standard deviation of 0.01.[12] The LIAB data also allow us to compute the coefficient of variation CV of the daily wages within plant-occupations. This coefficient of variation disregards the dispersion of wages across occupations, across plants, and across sectors. It instead isolates the within plant-occupation component in the wage variance, which was shown to be the dominant component of the residual wage in Table 1. The coefficient of variation is 0.3 at the mean (and the median) but is more than 4 at the plants with the most unequal wage distribution in the combined sample.

In addition to mapping information on the number of tasks from BIBB-BAuA to LIAB, we can also estimate

[11] The independent variables used in the regression are log daily wage, job experience, squared job experience together with indicators for (i) gender, (ii) 7 schooling and vocational training indicators, (iii) 16 regions, (iv) 34 sectors, (v) 7 plant-size categories, and (vi) 335 occupations. In the baseline regression we pool over the years 1992, 1999, 2006 and 2012. In an alternative specification we estimate the number of tasks separately for these four years and compute year-specific predictions from a moving average.

[12] In the BIBB-BAuA data the average number of tasks for 7 different plant-size categories varies between 4.77 and 5.32, with a mean of 4.92 and standard deviation 0.2. The differences in the task number intervals are mainly due to differences in the wage levels and perhaps the fact the BIBB-BAuA only covers workers with more than 20 hours of work per week.

Figure 3: Residual Wage Inequality per Plant-Occupation by Number of Tasks



Sources: LIAB 1996-2014, with imputed task information per plant-occupation from BIBB-BAuA 1999, 2006 and 2012.
Notes: Prediction of coefficient of variation of daily wage residual (exponentiated Mincer residual) CV within plant-occupation by task number, controlling for sector, region, occupation and worker characteristics. Results are differences to smallest task-number category (0 to 1 tasks). Thick, medium, and thin lines represent the 99, 95, and 90 percent confidence intervals.

the probability of performing a specific task in BIBB-BAuA and make an out-of-sample prediction regarding the probability that an individual worker perform this specific task in the LIAB data. For this purpose, we run 15 probit regressions (one for each task) with the same set of explanatory variables as in the regression for the number of tasks outlined above. With these out-of-sample predictions at hand, we can then construct a measure for the overall number of distinct tasks performed at a plant in LIAB. Due to the chosen estimation approach, the total number of distinct tasks must be smaller than 15 and it is larger than zero if our mapping was successful for at least one worker at the plant.[13] We then divide the average number of tasks (b) by the full number of distinct tasks observed (denoted with $\tilde{z}$ in the model below), to obtain a normalized measure of the number of tasks—a real number on the unit interval: $b/\tilde{z} \in (0, 1]$. As shown in Appendix Table A4, the normalized number of tasks $b/\tilde{z}$ varies between 0.03 and 0.70 with a mean of 0.36 and a tight standard deviation.

The imputation of BIBB-BAuA task information into the LIAB linked plant–worker data allows us to establish

Fact 3. *The coefficient of variation* CV *of daily wages within a plant-occupation decreases with the number of*

[13]The total number of distinct tasks varies between a minimum of 3.58 and a maximum of 15, with a mean of 11.2 and standard deviation 0.02.

tasks within a plant-occupation.

We project the coefficient of variation *CV* of the (exponentiated) residual daily wages within a plant-occupation on sector, region, occupation and worker characteristics, as before. Figure 3 plots the so normalized *CV* of daily wages within a plant-occupation (on the horizontal axis after subtracting the coefficient of daily wage variation in the range of less than one imputed task) against numbers of tasks (on the vertical axis), for plants with at least two workers. There is a clear inverse relationship with an S-like shape: wage variability drops strongly as the number of tasks per plant-occupation increases from one task to about six tasks, then it drops less pronounced, and drops again more sensitively in the upper ranges of more than nine tasks. Workers within the same occupation are subject to more wage inequality within their occupation at the same employer if they are assigned narrower task ranges. We can also relate the normalized coefficient of variation *CV* of daily wages within a plant-occupation to plant size, similar to Figures 1 and 2. As Appendix Figure A2 shows, using LIAB 1996-2014 only, wage variability within plant-occupations increases strongly with plant employment. Workers within the same occupation are subject to more wage inequality within their occupation at larger employers.

One consistent hypothesis is that workers who perform only a few tasks have a strong impact on the surplus that they generate at the employer. If workers who specialize in a narrow task range make mistakes, those mistakes weigh down surplus heavily, and their wages are lower. Conversely, workers who specialize in a narrow task range and perform strongly generate large surplus and receive high wage compensation. Under this hypothesis, surplus and wage payments will be particularly sensitive to worker mismatches in occupations with narrow task ranges. In other words, plants that behave more like Adam Smith's pin factory also exhibit more wage dispersion within plant-occupations. Our theory is devised to relate the more pronounced within plant-occupation wage dispersion back to the plant's internal division of labor, that is the plant's internal labor market organization.

2.4 Trade data

To link the plant-internal division of labor, and wage inequality, back to the plant's globalization status and predicted export sales, we need trade data. Our information on Germany's sector-level imports and exports with China and Eastern Europe comes from the United Nations Commodity Trade Statistics Database (Comtrade) and the trade in services database (TSD) at the World Bank. To construct instruments for German exports and imports we follow Autor, Dorn and Hanson (2013) and Dauth, Findeisen and Suedekum (2014) and use shipments between Australia, Canada, Japan, Norway, New Zealand, Sweden, Singapore, and the United Kingdom on the one hand and China and Eastern Europe on the other hand as the instrument group. Trade flows are converted into Euros (using annual exchange rates from Bundesbank). We map the SITC Rev. 2 sector information to a common sector definition across all waves of the German data. To create a concordance from SITC Rev. 2 to the 39 longitudinally consistent industries, we rely on existing mappings from SITC Rev. 2 to ISIC Rev. 3.1 and from ISIC Rev. 3.1 to WZ 2003.

3 A Model of Production with Task Assignment

3.1 Consumers

We consider an economy with a population of L individuals, who are risk neutral. As consumers, the individuals have homothetic preferences over a continuum of differentiated goods labelled $\omega \in \Omega$. The representative consumer maximizes utility

$$U = \left[\int_{\omega \in \Omega} c(\omega)^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}\omega \right]^{\frac{\sigma}{\sigma-1}}$$

subject to the economy-wide budget constraint $\int_{\omega \in \Omega} p(\omega) c(\omega) \, \mathrm{d}\omega = Y$, where $p(\omega)$ is the price of variety ω, Y is aggregate income, and $\sigma > 1$ is the elasticity of substitution between varieties. The resulting economy-wide demand for variety ω of the consumption good is:

$$c(\omega) = \left(\frac{p(\omega)}{P} \right)^{-\sigma} \frac{Y}{P}, \tag{1}$$

where $P \equiv \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} \, \mathrm{d}\omega \right]^{1/(1-\sigma)}$ is the CES price index. A producer of variety ω faces total demand $c(\omega)$ for its product. We introduce heterogeneity in individual consumers' budget sets given differentiated individual wages below.

3.2 Production

A plant ω is fully characterized with a tupel of three stochastic characteristics in our baseline model. We assume that the plant receives an *elemental productivity* draw $\tilde{\varphi}(\omega)$ from a lottery, as in Melitz (2003). To participate in the lottery, plants hire f_e workers at the going wage rate w. After the lottery, the cost is sunk. Depending on its elemental productivity draw and other characteristics in its tupel, a plant decides on whether to start production. Production requires the additional fixed employment of $f(\omega)$ workers for overhead services in operation; $f(\omega)$ is a function of the plant's choice of the number of occupations and reflects span-of-control costs.

The plant also receives from the lottery a draw of its required *full task range* $\tilde{z}(\omega)$, which the plant needs to cover in order to produce its output. All conceivable tasks are lined up around a circle, but plants need to cover different segments $\tilde{z}(\omega)$ of the circle. We consider the plant-specific full task range $\tilde{z}(\omega)$ to be the product of two parts

$$\tilde{z}(\omega) = \tilde{\zeta}(\omega) \cdot z(\omega),$$

where $\tilde{\zeta}(\omega) > 0$ is a plant's unobserved task range variability and $z(\omega) < 1$ is the observed total number of tasks at the plant—a fraction of the 15 possibly observable tasks. From imputation of the BIBB-BAuA task data into the LIAB plant–worker data, we can measure a plant's total number of tasks $z(\omega)$. However, it is unobserved what range of tasks a plant outsources to domestic suppliers and what range it offshores to foreign suppliers, in-house or at arm's length. A plant may need to cover only a fraction of our 15 benchmark tasks, then $\tilde{\zeta}(\omega)_t < 1$. Or the plant may need to cover a task range that exceeds our 15 observed benchmark tasks, then $\tilde{\zeta}(\omega)_t > 1$. We treat $\tilde{\zeta}(\omega)$ as a stochastic characteristic of the plant and allow $\tilde{\zeta}(\omega)$ to covary with the other characteristics in the plant's tupel in structural estimation, including with its elemental productivity $\tilde{\varphi}(\omega)$.

In addition, the plant receives a draw of its sensitivity of worker performance to task mismatch $\tilde{\eta}(\omega)$, which regulates how strongly a worker's surplus responds to the worker's average mismatch to her or his tasks at the plant. By allowing the sensitivity of performance to vary across plants, we can accommodate heterogeneity in the link between a plant's average number of tasks per occupation and its wage variability within occupations. For short, we will refer to the sensitivity of worker performance to task mismatch as *sensitivity of performance*. The sensitivity of performance $\tilde{\eta}(\omega)$ is not known but, under the structural relationships that we will derive, it can be recovered from observed variables.

Instead of carrying around the plant identifier ω, we will soon describe a plant's decisions given this tupel of three characteristics $(\tilde{\varphi}, \tilde{z}, \tilde{\eta})$. For empirics and estimation in Section 6, we will also allow plants to differ in one additional characteristic: plants will draw a stochastic fixed cost of exporting $\tilde{f}_x(\omega)$ as in Helpman et al. (2017) to break the deterministic link between elemental productivity and export-market participation, which exhibits variation in the data. In that extension, a plant's decisions will depend on its tupel of four stochastic characteristics $(\tilde{\varphi}, \tilde{z}, \tilde{\eta}, \tilde{f}_x)$. (To derive intuitive general-equilibrium relationships in closed form for the closed and open economy and to simplify exposition, however, in Sections 4 and 5 we will restrict the plant's tupel of characteristics to just the two stochastic components $(\tilde{\varphi}, \tilde{z})$ and comment on generalizations under current elaboration.) For now, consider the plant to be a tupel of three characteristics in the baseline model $(\tilde{\varphi}, \tilde{z}, \tilde{\eta})$.

Each variety of the consumption good is produced by a unique plant ω. When it comes to market structure, we assume that plants are monopolistic competitors. Labor is the only input. As workers, individuals are endowed with one unit of labor, which they supply inelastically to plants. Individual individuals differ in their core ability as workers.

Production requires that workers perform tasks in their respective occupations. A plant ω decides about three types of employment outcomes. First, the plant chooses the total number of occupations $n(\omega) + 1$ that it wants to offer (a plant's count of occupations in the data). We consider the possible count of occupations $[n(\omega) + 1] = 1, 2, \ldots$ to be countable and require a plant to offer at least one occupation—when $n(\omega) = 0$. Second, the plant assigns an occupation-invariant measure of tasks $b(\omega)$ that need to be performed within each occupation at the plant. For tractability, we make the total measure of tasks $b(\omega)$ a real number. By the technology we propose, the first choice of the total count of occupations $n(\omega) + 1$ will inversely determine the measure of tasks $b(\omega)$ as an outcome at the plant level. And third, the plant chooses a measure of workers $\ell(\omega)$ to hire into the occupations that it offers.

A plant ω with elemental productivity $\tilde{\varphi}(\omega)$ produces quantity $q(\omega)$ of its variety by combining the individual outputs $q_j(\omega)$ of its occupations $j = 1, \ldots, n(\omega) + 1$ into a Cobb-Douglas production function:

$$q(\omega) = \tilde{\varphi}(\omega)\, \tilde{z}(\omega)\, [n(\omega) + 1]\, \exp\left[\frac{1}{n(\omega) + 1} \sum_{j=1}^{n(\omega)+1} \ln q_j(\omega)\right], \tag{2}$$

where $q_j(\omega)$ is the output of occupation j and $n(\omega) + 1$ is the count of distinct occupations at the plant.[14] The plant-specific draw $\tilde{z}(\omega) < 1$ is the plant ω's full task range required to produce its outputs $q_j(\omega)$. The way in which the term $n(\omega) + 1$ enters the production function implies that, in the case of symmetric occupations, plants can raise their output by creating additional occupations, with an elasticity of one. Therefore, worker efficiency does not change in our model just because a plant adds new occupations. Only if workers get to specialize on a smaller range of tasks when new occupations are added, then worker efficiency increases with the addition of these occupations (see below).[15] To simplify notation for now, we suppress the variety label ω and consider a single plant.

3.3 Task assignments to occupations and labor efficiency

Workers have innate abilities to carry out tasks more or less efficiently. Complementing the knowledge view of the firm, there are no organizational hierarchies in our model. Instead, the worker abilities are *horizontally* differentiated and uniformly distributed over a circle with circumference 1. This ability circle simultaneously represents the technology space and characterizes the set of distinct possible tasks, which are also uniformly distributed with measure one. The location of a worker on the circle indicates the task that corresponds to his or

[14]Output in eq. (2) corresponds to to a Cobb-Douglas production function of the form $q = \tilde{\varphi}\tilde{z} \prod_j (q_j/\alpha_j)^{\alpha_j}$, with $\sum_j \alpha_j = 1$ and $\alpha_j = 1/(n + 1)$ under symmetry.

[15]A generalization to a non-unitary elasticity does not result in substantively different model predictions but would make the model more complicated and would require an additional parameter to discipline with data.

her core ability.

Plants cannot use the full circle of tasks for production. Instead they are restricted to select tasks from a subinterval with maximal length $\tilde{z}$. We do not observe a plant that is predicted to conduct all 15 tasks in our data (see Appendix Table A4). The length of the maximally feasible task range $\tilde{z}$ is plant-specific and exogenously given to the plant. However, plants have a choice to bundle adjacent tasks into occupations, which are then executed by the workers hired for these occupations. Plants must choose a common number (measure) of tasks $b < \tilde{z}$ for all occupations that they offer. In other words, we impose that plants choose symmetric divisions of the segment of the task circle on which they operate. We choose this restriction to reduce the number of parameters to estimate, so our focus in this paper rests on the average number tasks performed per occupation within a plant. We leave potential plant-and-occupation-specific task range choices for future work and start our analysis with only plant-specific task range choices. The measure of tasks b that a plant adopts is thus the same across all of the plant's occupations.

Suppose for a moment that tasks on the plant's segment of the unit circle are mutually exclusively assigned to n separate occupations with no overlap. The plant's chosen count of occupations n and its chosen measure of tasks b would then be linked to each other according to

$$b = \frac{\tilde{z}}{n+1}.$$

However, in practice and in our data, the same operations (*what* tasks) are typically performed across multiple occupations. We therefore introduce an exogenous degree of overlap ν—a common parameter beyond a plant's control. With a common degree of overlap, a plant's measure of tasks per occupation becomes

$$b = \frac{\tilde{z}}{\nu n+1} \qquad \text{for} \quad \nu \in (0,1]. \tag{3}$$

If $\nu = 1$, eq. (3) collapses to the simple case above with no overlap. If $\nu < 1$, the mapping of tasks to occupations is not unique and the task intervals overlap. In the limiting case of $\nu = 0$, each occupation uses the whole range of tasks of the plant, irrespective of n.

Workers must allocate the same amount of time to all tasks specified by the occupation, and worker efficiency falls in the distance of their core ability to a given task. The idea that workers spend equal time on the performance of all tasks is a common feature of task models, going back to at least Becker and Murphy (1992). We can therefore interpret the average distance of a worker at location i to the various tasks in interval $[0, b]$ within each occupation as a measure of mismatch. Provided that labor is not systematically misallocated so that workers have their core ability within their occupation's task range (see below), one convenient measure of mismatch $m(i,b)$ of a worker i with the b tasks in her occupation is

$$m(i,b) = \frac{1}{b}\left\{\int_0^i (i-t)\,\mathrm{d}t + \int_i^b (t-i)\,\mathrm{d}t\right\} = \frac{b^2-2i(b-i)}{2b}, \tag{4}$$

where t is the running index of the task location. The mismatch depends linearly on the worker's position in the interval and is lowest if the worker is located in the middle of the task interval. Mismatch is highest at the boundaries of the task interval. The results of our analysis are valid for more flexible functional forms.[16]

There needs to be an inverse link between mismatch $m(i,b)$ and a worker i's efficiency $\lambda(i,b)$, which we define as

$$\lambda(i,b) \equiv \frac{\tilde{\eta}}{\tilde{z}} + \frac{1}{m(i,b)} = \frac{\tilde{\eta}}{\tilde{z}} + \frac{2b}{b^2-2i(b-i)}. \tag{5}$$

For worker efficiency to be well defined, we impose that the sensitivity of performance satisfies $\tilde{\eta} > -2$, so that all workers from interval $[0,b]$ have positive efficiency for all possible outcomes $b \leq \tilde{z}$. The sensitivity of performance $\tilde{\eta}$ will play an important role below when it comes to the intra-plant dispersion of wages and how that wage dispersion varies between plants with different productivities. Note that we do not restrict $\tilde{\eta}$ to be positive.

The plant can choose to hire a measure $\ell_j(i,b)$ of workers with core ability i into occupation j given a task range b per occupation. Average worker efficiency in occupation j is then

$$\lambda_j(b) = \frac{1}{\ell_j(b)}\int_0^b \lambda(i,b)\ell_j(i,b)\,\mathrm{d}i, \qquad \text{where} \qquad \ell_j(b) \equiv \int_0^b \ell_j(i,b)\,\mathrm{d}i \tag{6}$$

denotes the total amount of labor hired for occupation j at a plant with task range b per occupation. Occupation-level output is then

$$q_j = \lambda_j(b)\ell_j(b). \tag{7}$$

Note that, if $\ell_j(i,b)$ is the same for all workers i in occupation j then $\ell_j(b) = \int_0^b \ell_j(i,b)\,\mathrm{d}i$ is the same across

[16] An important standardization in (4) is by the width of the task range $1/b$ to generate efficiency gains from specialization. We could alternatively specify

$$\hat{m}(i,b) \equiv \left(\frac{1}{b}\right)^{\beta}\left\{\int_0^i (i-t)\,\mathrm{d}t + \int_i^b (t-i)\,\mathrm{d}t\right\} = \frac{b^2-2i(b-i)}{2b^{\beta}} \qquad \text{and} \qquad \hat{\lambda}(i,b) \equiv \left(\frac{\tilde{\eta}}{\tilde{z}}\right)^{2-\beta} + \frac{1}{\hat{m}(i,b)},$$

with $\beta = 1$ our special case. Our results generalize to $\beta \in (0,2)$. For instance, the coefficient of variation of a plant's wages, shown in eq. (10) below, would become

$$\hat{CV}_w(b) = \sqrt{4-\pi(\pi-2)}\,\frac{[\tilde{z}/b]^{2-\beta}}{\tilde{\eta}^{2-\beta}+\pi[\tilde{z}/b]^{\beta}}.$$

occupations j because b is occupation invariant and $\lambda_j(b)$ is the same across all occupations j. Then q_j is the same at all occupations j of a plant by (7). (We will show in Subsection 4.1 below that it is optimal for the plant to make employment $\ell_j(i,b)$ constant across worker types i in occupation j.)

3.4 Hiring, production, and wage setting

Labor is employed in three different roles: for the sunk cost to make the productivity draw f_e, for the fixed input into overhead services $f(\omega)$ to manage and coordinate the occupations, and for the variable input into production. In the first two roles, workers have an efficiency of one, whereas in the third role their efficiency is given by $\lambda(i,b)$ and thus match specific. To hire workers for production, plants post occupations in a competitive labor market at the going wage w. The occupation posting provides a binary signal that informs workers about whether their core ability is within the occupations's task interval, or not, but not on their specific location within this interval. One way to think about this is that the location of the occupation's task interval on the unit circle is not part of the occupation description but that workers can receive a costless test report that reveals with certainty whether or not their ability is within the occupation's task range. The occupation posting does specify what the wage schedule will be for the worker upon accepting the occupation offer. Given their risk neutrality, workers will accept any wage schedule that pays an expected wage rate w.

In Appendix C, we show how Stole and Zwiebel (1996) wage bargaining in the presence of equilibrium unemployment can be embedded into our production model. For our baseline framework and its equilibrium relationships, we want to set aside unemployment and introduce an equivalent wage schedule to the one that would arise under Stole-Zwiebel wage bargaining by allowing for workers' endogenous effort choice. Production workers can choose an effort level e from interval $[0,1]$ and thus choose the time productively used in their occupation, so the output of worker i in occupation j is given by

$$q_j(i) = e(i)\lambda(i,b) \qquad \text{in every occupation } j = 1,\ldots,n(\omega)+1.$$

Suppose for a moment that full effort were enforceable through monitoring. Then a plant could not do better than offering a constant wage w to all workers that equals the going wage in the economy.

Now suppose the utility of workers is reduced by a constant factor $\varepsilon > 0$ per unit of effort. Then plants will link wage payments to the ex post output if the effort is unobservable for outsiders and hence not contractible. The lacking contractibility of effort rules out a uniform wage for all production workers. In fact, plants cannot do better than setting

$$w(i,b) = w\frac{\lambda(i,b)}{\lambda(b)} \tag{8}$$

for a constant going wage w, prompting workers to provide full effort $e = 1$ if ε is sufficiently small. To see that the going wage is occupation independent, note that all occupations inside a plant are symmetric in that they require the same task range b, so $w(i,b)/\lambda(i,b) = w_j/\lambda_j(b)$ for all occupations j, hence $w_j/\lambda_j(b) = w/\lambda(b)$ because workers of type i and of type $i+b$ have an equivalent degree of mismatch in their respective occupations. Following this reasoning, plants pay a constant wage per efficiency unit of $w/\lambda(b)$ to all of their production workers (and w to workers providing fixed inputs), implying that the efficiency differences of workers in the occupation translate one-to-one into wage differences between workers.[17]

Plants pay the same wage per efficiency unit of labor. Plants are therefore indifferent between all applicants. Furthermore, workers are ex ante indifferent between all occupations that correspond to their qualification, that is all occupations for which their core ability lies within the covered task interval. Plants therefore end up hiring workers whose abilities are uniformly distributed over the task intervals covered by their occupations, and $\ell_j(i,b)$ is the same for all workers i in occupation j. As a result, average worker efficiency is the same for all occupations in the plant and given by

$$\lambda(b) = \frac{1}{b}\int_0^b \lambda(i,b)\,\mathrm{d}i = \frac{1}{b}\left[\frac{\tilde{\eta}}{\tilde{z}}i + 2\arctan\left(\frac{2i-b}{b}\right)\right]_0^b = \frac{\tilde{\eta}}{\tilde{z}} + \frac{\pi}{b} \quad \text{or, equivalently, by} \tag{9a}$$

$$\lambda(\omega) = \frac{1}{\tilde{z}}\left[\tilde{\eta} + \pi(\nu n(\omega)+1)\right], \tag{9b}$$

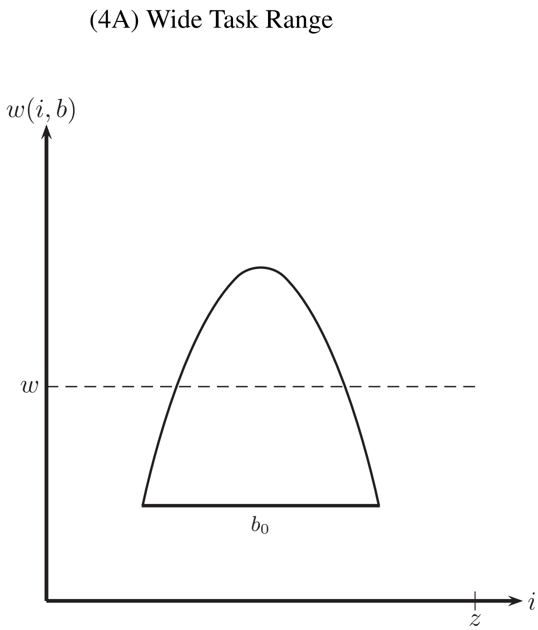
where the final equality follows when substituting $\tilde{z}/b(\omega) = \nu n(\omega) + 1$ from eq. (3). It is a consequence of constant $\lambda_j(b) = \lambda(b)$, as noted before, that q_j is the same at all occupations j of a plant by eqs. (6) and (7).

A crucial implication of eq. (9a) is Adam Smith's tenet that more specialized plants, with a narrower range of tasks and therefore a higher count of occupations in their internal labor market, exhibit higher worker efficiency: $\lambda'(b) < 0$.

It follows from these insights that the wage dispersion is the same in all occupations of a plant and linked to the plant ω's chosen task range per occupation. We measure the wage dispersion with the coefficient of variation

[17] Under the wage schedule (8), there are workers who earn less than the going wage rate w. These workers would benefit from quitting and searching for a new occupation elsewhere because, in expectation, a new occupation that covers their core ability would offer a payment w. By design, endogenous quits are not a problem in a static setting. However, if worker efficiency is revealed ex post, one can extend the model to a variant with involuntary unemployment under search frictions to ensure that quitting remains unattractive for workers even after their efficiency has been revealed. In Appendix C, we provide such an extension and consider Stole-Zwiebel bargaining instead of efficiency wages as a modelling strategy for worker-specific wages. There we show that the main results on wage dispersion within occupations remain unaffected by this modification, and that the extended model leads to a setting with involuntary unemployment in equilibrium.

Figure 4: Within-occupation Wage Schedule and Plant Choice of the Task Range



Notes: A plant receives a stochastic draw of its full task range $\tilde{z}$, which is required to produce its outputs. The index i denotes the core ability of a worker (the worker type) and designates the ideal task to which the worker is matched without any mismatch. The plant chooses its task range b. A worker with a core ability in the interior of a given task range b has a lower degree of mismatch the narrower the task range (mismatch is the average distance of a worker i from all the tasks that fall within the range b). It depends on the plant's sensitivity of performance $\tilde{\eta}$ how responsive its surplus is to the mismatch of workers to tasks: $w(i,b) = w \cdot \lambda(i,b)/\lambda(b)$ by eq. (8) and $\lambda(i,b)/\lambda(b)$ is a function of $\tilde{\eta}$ and given by the ratio of eqs. (5) and (9a).

(the standard deviation relative to the mean) of the wage at a plant ω:

$$CV_w(\omega) = \frac{\sqrt{\mathbb{V}(w(i,\omega)|\omega)}}{\mathbb{E}[w(i,\omega)]} = \frac{1}{w}\sqrt{\frac{w^2}{b}\int_0^b \left(\frac{\lambda(i,b)}{\lambda(b)}\right)^2 \mathrm{d}i - w^2} = \sqrt{4-\pi(\pi-2)}\,\frac{\nu n(\omega)+1}{\tilde{\eta}+\pi[\nu n(\omega)+1]}. \quad (10)$$

Graph A of Figure 4 illustrates the wage dispersion within a plant-occupation that spans a task range b_0, where a worker i's wage $w(i,b) = w \cdot \lambda(i,b)/\lambda(b)$ under eqs. (5) and (6). Now suppose the plant optimally adopts a narrower task range $b_1 < b_0$ as depicted in Graph B of Figure 4. The wage schedule will still vary around the unchanged economy-wide wage w, but it depends on the plant's sensitivity of worker performance to task mismatch $\tilde{\eta}$ whether the worker efficiency dispersion, and hence the wage dispersion around the economy-wide mean, stays constant, rises, or falls at the plant. For a positive sensitivity parameter $\tilde{\eta} > 0$, a narrower task range $b_1 < b_0$ magnifies the worker efficiency dispersion in that it induces more variation in any worker i's wage (hence our term *sensitivity of performance* for $\tilde{\eta}$). Larger plants with narrower task ranges will exhibit a wider wage dispersion within plant-occupation for $\tilde{\eta} > 0$. We consider it an empirical matter how task ranges should relate to wage outcomes across workers within a plant-occupation and therefore introduce the parameter $\tilde{\eta}$ for estimation. In practice, workers with badly matched abilities near the boundary of a narrow task range might exhibit a more than proportionally diminished efficiency, if their mistakes on the job can result in heavier losses to the employer than in wider task ranges. A priori, it is equally conceivable that badly matched workers in narrow task ranges suffer only a less than proportional reduction in efficiency, compared to their efficiency in wide task ranges, if their mistakes matter little to the employer, because narrower task ranges may have a lesser impact on overall production.

We will return to plant-level optimality conditions in our outline for structural estimation in Section 7 below, where we also recover the sensitivity of worker performance to mismatch $\tilde{\eta}(\omega)$ from the model's structural relationships.

4 Division of Labor in the Closed Economy

To derive equilibrium relationship in closed from, we now simplify our model and impose that the sensitivity of worker performance to task mismatch is constant across plants: $\tilde{\eta}(\omega) = \eta$. We maintain the mild condition that $\eta > -2$ from above, but do not require η to have a specific sign so that the adoption of narrower task ranges by larger plants may result in reduced or heightened within-occupation wage variability. In this section, a plant ω is a tupel of two characteristics $(\tilde{\varphi}, \tilde{z})$. We discuss extensions in Section 5.3.

To derive equilibrium in an intuitive form, we assume that the distribution of elemental productivity $\tilde{\varphi}$ is Pareto $G(\tilde{\varphi}) = 1 - \tilde{\varphi}^{-\theta}$ with shape parameter θ as in Helpman, Melitz and Yeaple (2004) and Chaney (2008). Plants draw their $\tilde{\varphi}(\omega)$ from a common Pareto distribution, where $\theta > 1$ to ensure a finite mean of productivity. We discuss an alternative parametrization with log normally distributed productivity in Section 5.3.

4.1 Profit maximization in the closed economy

Plants decide about entry and production in three stages. On stage one, a plant ω decides on paying the sunk cost of f_e units of labor for entering the elemental productivity draw. On stage two, the plant decides on starting production conditional on its productivity draw. Prior to production on stage three, the plant must also determine on stage two the count of occupations $n(\omega)$ and pay a fixed cost of $f(\omega)$ units of labor to operate. We set the plant's fixed cost of operation to

$$f(\omega) = f_0 + \{\eta + \pi([\nu n(\omega) + 1]\}^{\gamma}$$

with a semi-elasticity of the fixed cost with respect to occupation counts $\gamma > 0$, so that the overhead costs are positively linked to the count of occupations $n(\omega)$ at the plant. It is costly to the plants to create additional occupations (and have a narrower task range per occupations). This span-of-control cost for a plant is more convex for larger γ. Figure 2 (Fact 2) above documents that the number of tasks within an occupation at a plant decreases with plant employment, and the number of tasks becomes largely insensitive to further size increases of the plant above a threshold of about 500 workers. Our model can capture the invariance of task ranges to plant size beyond a threshold with highly convex fixed costs. To keep the model parsimonious, we choose a constant semi-elasticity γ of the span-of-control cost with respect to occupation counts.

On stage three, plants hire production workers $\ell(\omega)$, manufacture output $q(\omega)$ and sell this output to consumers. We solve the three-stage decision problem by backward induction. On stage three, a plant sets $\ell_j(\omega)$ to maximize its profits

$$\psi(\omega) = p(\omega)q(\omega) - w \sum_{j=1}^{n(\omega)+1} \ell_j(b(\omega)) - w \{\eta + \pi[\nu n(\omega) + 1]\}^{\gamma} - w f_0, \tag{11}$$

subject to aggregate consumer demand for their variety (1), the market clearing condition $c(\omega) = q(\omega)$ for their variety, the plant's production function (2)

$$q(\omega) = \tilde{\varphi}(\omega)[n(\omega) + 1] \{\eta + \pi[\nu n(\omega) + 1]\} \exp\left[\frac{1}{n(\omega)+1} \sum_{j=1}^{n(\omega)+1} \ln \ell_j(b(\omega))\right], \tag{12}$$

and common non-negativity constraints. Profit maximization on stage three results in the first-order condition for revenues and employment

$$r(\omega)[(\sigma - 1)/\sigma] = [n(\omega) + 1] w \ell_j(b(\omega)),$$

with $r(\omega) \equiv p(\omega)q(\omega)$. This first-order condition establishes the intuitive result that plant ω chooses the same employment level for all occupations j:

$$\ell_j(b(\omega)) = \hat{\ell}(\omega).$$

Furthermore, the profit-maximizing price can be expressed as a constant markup over the plant's marginal cost $p(\omega) = [\sigma/(\sigma - 1)]mc(\omega)$, given CES demand, for the endogenous (occupation-count dependent) marginal cost

$$mc(\omega) \equiv \frac{w}{\tilde{\varphi}(\omega)\{\eta + \pi[\nu n(\omega) + 1]\}}. \tag{13}$$

We now turn to stage two. Plants rationally anticipate the profit value on stage three as a function of their entry decisions and choice of the count of occupations. Substituting eq. (13) into eq. (11) and using eq. (1) yields profits of plant ω as a function of the count of occupations chosen by the plant $n(\omega)$:

$$\psi(\omega) = \frac{Y}{P^{1-\sigma}} \frac{1}{\sigma} \left[\frac{\sigma}{\sigma - 1} \frac{w}{\tilde{\varphi}(\omega) \left\{ \eta + \pi[\nu n(\omega) + 1] \right\}} \right]^{1-\sigma} - w \left\{ \eta + \pi[\nu n(\omega) + 1] \right\}^{\gamma} - w f_0. \tag{14}$$

Plants face the trade-off that increasing the count of occupations lowers marginal costs with a positive effect on profits, but at the same time raises the overhead costs with a negative effect on profits. This trade-off is similar to the one in Eckel (2009) and Bustos (2011), where producers can pay a fixed cost to reduce variable production costs.

Treating $n(\omega)$ as a continuous variable for purposes of exposition, the first-order condition for the profit-maximization problem at stage two is given by

$$r(\omega) \frac{\sigma - 1}{\sigma} = \gamma w \left\{ \eta + \pi[\nu n(\omega) + 1] \right\}^{\gamma}. \tag{15}$$

We assume that $\gamma > \sigma - 1$, a necessary condition for an interior solution to be a maximum. In addition, we assume that parameters are such that every plant benefits from specifying more than one occupation, i.e. from setting $n(\omega) > 0$. Think of plants with at least one worker in a more organizational (senior) role and another worker in a more operational (junior) role. The plant that gains least from increasing $n(\omega)$ is the plant with the lowest $\tilde{\varphi}(\omega)$. This is the plant that makes zero profits from production $\hat{\psi}(\omega) = 0$, provided that not all plants find it attractive to start production (see below). In an interior maximum, this zero-profit condition can be expressed as

$$\frac{\sigma - 1}{\gamma - \sigma + 1} f_0 = \{ \eta + \pi[\nu n(\omega) + 1] \}^{\gamma}, \tag{16}$$

and hence we can safely conclude that the maximization problem has an interior solution if eq. (16) holds for a strictly positive $n(\omega)$, that is for $[(\sigma - 1)/(\gamma - \sigma + 1)] f_0 > (\eta + \pi)^{\gamma}$. This latter inequality characterizes the parameter domain to which we restrict ourselves because, in combination with $\gamma > \sigma - 1$, it is sufficient for a unique maximum at stage two, with $n(\omega) > 0$ for all plants.

Eqs. (1) and (13) together with market clearing condition $c(\omega) = q(\omega)$ establish a first relationship between relative revenues of two plants and the relative count of distinct occupations in these plants, while eq. (15)

establishes a second relationship between these variables. We have:

$$\frac{r(\omega_1)}{r(\omega_2)} = \left(\frac{\tilde{\varphi}(\omega_1)\{\eta + \pi[\nu n(\omega_1) + 1]\}}{\tilde{\varphi}(\omega_2)\{\eta + \pi[\nu n(\omega_2) + 1]\}}\right)^{\sigma-1}, \qquad \frac{r(\omega_1)}{r(\omega_2)} = \left(\frac{\eta + \pi[\nu n(\omega_1) + 1]}{\eta + \pi[\nu n(\omega_2) + 1]}\right)^{\gamma} \tag{17}$$

respectively. These expressions allow us to express relative revenues and the relative count of occupations of two plants as a function of these plants' relative differences in their elemental productivity parameter $\tilde{\varphi}$:

$$\frac{r(\omega_1)}{r(\omega_2)} = \left(\frac{\tilde{\varphi}(\omega_1)}{\tilde{\varphi}(\omega_2)}\right)^{\xi}, \qquad \frac{\eta + \pi[\nu n(\omega_1) + 1]}{\eta + \pi[\nu n(\omega_2) + 1]} = \left(\frac{\tilde{\varphi}(\omega_1)}{\tilde{\varphi}(\omega_2)}\right)^{\xi/\gamma}, \tag{18}$$

where

$$\xi \equiv \gamma(\sigma - 1)/(\gamma - \sigma + 1) > 0$$

denotes the elasticity of revenues with respect to productivity parameter $\tilde{\varphi}$. Using eqs. (1) and (12), we can also determine relative output and relative plant-level employment of production workers as a function of the productivity differential between these plants:

$$\frac{q(\omega_1)}{q(\omega_2)} = \left(\frac{\tilde{\varphi}(\omega_1)}{\tilde{\varphi}(\omega_2)}\right)^{\frac{\sigma}{\sigma-1}\xi}, \qquad \frac{\ell(\omega_1)}{\ell(\omega_2)} = \left(\frac{\tilde{\varphi}(\omega_1)}{\tilde{\varphi}(\omega_2)}\right)^{\xi}, \tag{19}$$

where $\ell(\omega) \equiv [n(\omega) + 1]\hat{\ell}(\omega)$ is the employment of production workers at plant ω. Our framework shares with other models of heterogeneous employers the empirically well-documented property that workers in larger firms are more productive (see Idson and Oi 1999). However, in contrast to other contributions, the productivity differences are further scaled up under the plants' profit maximizing choice of the count of occupations, which raises worker efficiency.

It is an important insight from eqs. (18) and (19) that plant outcomes are fully characterized by exogenous differences in $\tilde{\varphi}$. Hence, we can drop ω and index plants by their elemental productivity parameter from now on. Denoting productivity of the marginal plant by φ^*, revenues and the count of occupations at the marginal plant are given by

$$r(\varphi^*) = \frac{\sigma\xi f_0}{\sigma - 1}w \qquad \nu n(\varphi^*) + 1 = \frac{1}{\pi}\left[\left(\frac{\xi f_0}{\gamma}\right)^{\frac{1}{\gamma}} - \eta\right] \tag{20}$$

by eqs. (15) and (16). Furthermore, the coefficient of variation of wages within an occupation at the marginal producer can be expressed as

$$CV_w(\varphi^*) = \frac{\sqrt{4 - \pi(\pi - 2)}}{\pi}\frac{(\xi f_0/\gamma)^{1/\gamma} - \eta}{(\xi f_0/\gamma)^{1/\gamma}} \tag{21}$$

by eqs. (10), (16) and (20). The coefficient of variation of wages in a plant with $\tilde{\varphi} > \varphi^*$ is then given by

$$CV_w(\tilde{\varphi}) = CV_w(\varphi^*) \frac{(\tilde{\varphi}/\varphi^*)^{\xi/\gamma}(\xi f_0/\gamma)^{1/\gamma} - \eta}{(\tilde{\varphi}/\varphi^*)^{\xi/\gamma}[(\xi f_0/\gamma)^{1/\gamma} - \eta]} \tag{22}$$

by eqs. (15), (18) and (21). For $\eta = 0$, the coefficient of variation of wages is the same at all plants. Otherwise, for any two plants with $\tilde{\varphi}_1 > \tilde{\varphi}_2$ we have $CV_w(\tilde{\varphi}_1) > CV_w(\tilde{\varphi}_2)$ iff $\eta > 0$. More productive plants have higher within-plant-occupation wage inequality iff $\eta > 0$.

To solve for the plants' problem at stage one of participation in the productivity lottery, we note that free entry is consistent with profit maximization if and only if the expected profit from participating in the productivity draw just compensates a plant for the sunk costs of economic activity. Using the superscript a to denote equilibrium outcomes in autarky, the zero-profit condition is

$$\int_{\varphi^{*,a}}^{\infty} \psi(\tilde{\varphi})\, \mathrm{d}G(\tilde{\varphi}) = w f_e.$$

We show in Appendix B.1 that $\int_{\varphi^{*,a}}^{\infty} \psi(\tilde{\varphi})\, \mathrm{d}G(\tilde{\varphi}) = [1 - G(\varphi^{*,a})] w f_0 \xi/(\theta - \xi)$, which allows us to solve for the productivity of the marginal plant:

$$\varphi^{*,a} = \left(\frac{f_0}{f_e}\frac{\xi}{\theta - \xi}\right)^{\frac{1}{\theta}} \tag{23}$$

that participates in the productivity draw, where we assume $\theta > \xi$ to ensure a positive and finite value of aggregate revenues and profits, and we assume $f_0/f_e > \theta/\xi - 1$ to ensure $\varphi^{*,a} > 1$ and hence an outcome by which only relatively more productive plants start production at stage two.

4.2 The autarky equilibrium

To solve for general equilibrium, we choose labor as the numéraire and set $w = 1$. We keep using the superscript a to denote equilibrium outcomes in autarky. Since profit income is used to pay for participation in the productivity lottery, the mass of producers is determined by the condition that economy-wide labor income, L, equals total consumption expenditures, Y, and thus aggregate revenues $R^a = M^a r(\varphi^{*,a})\theta/(\theta - \xi)$. Using eq. (20), we obtain

$$M^a = \frac{L(\sigma - 1)}{\sigma \xi f_0}\frac{\theta - \xi}{\theta}. \tag{24}$$

Welfare of the representative agent is (proportional to the) real wage and thus given by the inverse of the CES price index: $W^a = 1/P^a$. The price index can be expressed as $P^a = [\theta M^a/(\theta - \xi)]^{1/(1-\sigma)} p(\varphi^{*,a})$ and it therefore follows from eqs. (13), (23), (24) and constant markup pricing that welfare is given by

$$W^a = \left(\frac{L}{\gamma}\right)^{\frac{1}{\sigma-1}} \left(\frac{\gamma}{f_0\xi}\right)^{\frac{1}{\xi}} \left(\frac{\sigma-1}{\sigma}\right)^{\frac{\sigma}{\sigma-1}} \left(\frac{f_0}{f_e}\frac{\xi}{\theta-\xi}\right)^{\frac{1}{\theta}}. \tag{25}$$

To complete the characterization of the closed economy, we compute economy-wide wage inequality as the employment-share weighted average of the variances of wages at the plant level (equivalent to the computation of the within-plant-occupation component on the final line of Table 1 but for the wage and not its log):

$$\mathbb{V}_w^a = \frac{\theta-\xi}{\theta\ell(\varphi^{*,a})}\int_{\varphi^{*,a}}^{\infty} CV_w(\tilde{\varphi})^2\,\ell(\tilde{\varphi})\,\mathrm{d}G(\tilde{\varphi}), \tag{26}$$

where the plant-level wage variance is equal to the squared coefficient of variation $\mathbb{V}(w(i,\tilde{\varphi})|\tilde{\varphi}) = CV_w(\tilde{\varphi})^2$ by eq. (10) for $w = 1$ and average employment of production workers per plant is $\ell(\varphi^{*,a})\theta/(\theta-\xi)$. Solving the integral yields

$$\mathbb{V}_w^a = CV_w(\varphi^{*,a})^2 \left\{1 + \frac{2\xi/\gamma}{\theta-\xi+2\xi/\gamma}\frac{\eta}{(f_0\xi/\gamma)^{1/\gamma}-\eta}\left[1 + \frac{(f_0\xi/\gamma)^{1/\gamma}}{(f_0\xi/\gamma)^{1/\gamma}-\eta}\frac{\xi/\gamma}{\theta-\xi+2\xi/\gamma}\right]\right\}, \tag{27}$$

using eq. (22). The result implies that $\mathbb{V}_w^a > CV_w(\varphi^{*,a})^2$ if and only if $\eta > 0$.

5 Division of Labor in the Open Economy

To derive global equilibrium relationship in closed from under free trade, we maintain the simplifying assumption that the sensitivity of performance is plant-invariant $\tilde{\eta}(\omega) = \eta > -2$ and that a plant ω is a tupel of two characteristics $(\tilde{\varphi}, \tilde{z})$, as in the preceding section. We discuss variations in Subsection 5.3 below. We consider the case of two symmetric countries for the numéraire $w = w^* = 1$, with consumption and production as in the preceding section.

5.1 Fundamentals

There are two types of trade costs: fixed costs $f_x > 0$ (in units of labor) for setting up a foreign distribution network; and variable iceberg transport costs $\tau > 1$ with the usual interpretation that τ units of the consumption good must be shipped in order for one unit to arrive in the foreign economy. Both of these costs are also present in the Melitz (2003) framework and—in combination with plant heterogeneity in elemental productivity $\tilde{\varphi}$—they generate self-selection of the more productive producers into exporting, provided that the trade costs are finite and sufficiently high. Beyond the Melitz (2003) model, the decision to start exporting in our model also influences a plant's optimal choice of $n(\tilde{\varphi})$, raising plant productivity beyond its elemental level, and thus exerts a feedback effect on profits attainable in the domestic market. Due to this feedback effect, we have to distinguish between variables referring to exporters (denoted by superscript e) and non-exporters (denoted by superscript d). Furthermore, we use the subscript T in this section to refer to variables associated with total (domestic and foreign) market activities.

Holding economy-wide variables constant, access to exporting does not affect the profit maximization problem of a non-exporter. For an exporter, in contrast, export revenues in the foreign market alter the choice of specialization in the internal labor market. Denote an exporter's revenues in the domestic market with $r^e(\tilde{\varphi})$. For symmetric countries, an exporter's revenues in the foreign market are then $\tau^{1-\sigma} r^e(\tilde{\varphi})$. An exporting plant's profit-maximizing choice of $n^e(\tilde{\varphi})$ is given by

$$\left(1+\tau^{1-\sigma}\right) r^e(\tilde{\varphi}) \frac{\sigma-1}{\sigma} = \gamma \left\{\eta + \pi[\nu n^e(\tilde{\varphi}) + 1]\right\}^{\gamma} \tag{28}$$

instead of eq. (15). Eq. (28) is the same for all exporters, so the ratios in eqs. (18) and (19) carry over from the close to the open economy for any two plants with the same export status. However, when hypothetically comparing two plants with the same elemental productivity parameter $\tilde{\varphi}$ but differing export status, we obtain

$$\frac{r^e(\tilde{\varphi})}{r^d(\tilde{\varphi})} = \left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\gamma}}, \qquad \frac{\eta + \pi[\nu n^e(\tilde{\varphi}) + 1]}{\eta + \pi[\nu n^d(\tilde{\varphi}) + 1]} = \left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\gamma(\sigma-1)}}, \tag{29}$$

and

$$\frac{q^e(\tilde{\varphi})}{q^d(\tilde{\varphi})} = \left(1+\tau^{1-\sigma}\right)^{\frac{\sigma\xi}{\gamma(\sigma-1)}}, \qquad \frac{\ell^e_T(\tilde{\varphi})}{\ell^d_T(\tilde{\varphi})} = \left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}}, \tag{30}$$

where $\ell^e_T(\tilde{\varphi})$ and $\ell^d_T(\tilde{\varphi})$ denote total labor input of plant $\tilde{\varphi}$ for export and non-export status. The plant adopts one degree of specialization on its internal labor market regardless of the destinations of its products, so $n^e(\tilde{\varphi})$ and $n^d(\tilde{\varphi})$ carry no T subscript. From the closed economy case we know that larger plants choose more occupations. Exporting generates additional revenues and therefore induces a plant to adopt more occupations: $n^e(\tilde{\varphi}) > n^d(\tilde{\varphi})$. The resulting finer division of labor makes exporters more efficient and lowers their unit production costs by eq. (13). The added efficiency raises an exporter's sales in both the domestic and the foreign market, establishing $r^e(\tilde{\varphi}) > r^d(\tilde{\varphi})$ and $q^e(\tilde{\varphi}) > q^d(\tilde{\varphi})$ in eqs. (29) and (30). In summary, there is a positive feedback of exporting into domestic revenues and this effect raises a plant's incentives to export beyond the benchmark Melitz (2003) model. It follows from eq. (30) that exporting raises employment. While exporters raise their productivity by adopting more occupations, the associated increase in efficiency units of labor does not fully accommodate the added labor demand from higher overall sales so that an exporting plant expands employment.

Despite the feedback from exporting into domestic sales our model preserves key properties of the Melitz (2003) model for selection into export status. We use eqs. (15), (18), (20), (28) and (29) and state a plant $\tilde{\varphi}$'s added profit from exporting $\Delta\psi_T(\tilde{\varphi}) \equiv \psi_T^e(\tilde{\varphi}) - \psi_T^d(\tilde{\varphi})$, including the feedback effect, as follows:

$$\Delta\psi_T(\tilde{\varphi}) = \left[\left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}} - 1\right]\left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\xi} f_0 - f_x. \tag{31}$$

The added profit increases in the elemental productivity parameter $\tilde{\varphi}$, so there is selection of innately productive plants into exporting as in the benchmark Melitz model as long as the two trade costs f_x and τ are finite and sufficiently high. We can then identify the elemental productivity of a plant that is indifferent between exporting and non-exporting from $\Delta\psi_T(\tilde{\varphi}) = 0$. We denote the *elemental cutoff productivity* of this indifferent plant by $\tilde{\varphi}_x^*$, implying that a plant $\tilde{\varphi}$ is an exporter if $\tilde{\varphi} \geq \tilde{\varphi}_x^*$ and a non-exporter otherwise. Solving $\Delta\psi_T(\tilde{\varphi}_x^*) = 0$ for the ratio of the two elemental productivity cutoffs—$\tilde{\varphi}_x^*$ for exporting and φ^* for entry—and noting that the share of exporters is given by $\chi \equiv [1 - G(\varphi_x^*)]/[1 - G(\varphi^*)]$ under Pareto distributed elemental productivity with shape parameter θ, we can compute

$$\chi = \left\{\frac{f_0}{f_x}\left[\left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}} - 1\right]\right\}^{\frac{\theta}{\xi}} < 1. \tag{32}$$

Fixed trade costs f_x and variable trade costs τ raise the elemental productivity cutoff $\tilde{\varphi}_x^*$, thereby lowering the share χ of exporters among active plants.

5.2 The open economy equilibrium

We turn to the open economy equilibrium. Profit maximization in the open economy is described by a four-stage decision problem that is similar to the closed economy, but additionally involves the decision to export or be a non-exporter that exclusively sells to domestic consumers (in stage 2). Access to the export market raises profits of the most productive plants, and thus the expected profit prior to entry into the productivity lottery, which is given by

$$\int_{\varphi^*}^{\infty} \psi_T(\tilde{\varphi})\,\mathrm{d}G(\tilde{\varphi}) = [1 - G(\varphi^*)]\,\frac{\xi f_0}{\theta - \xi}\left(1 + \frac{\chi f_x}{f_0}\right) \tag{33}$$

in the open economy (see Appendix B.3). Free entry into the productivity lottery implies $\int_{\varphi^*}^{\infty} \psi_T(\tilde{\varphi})\,\mathrm{d}G(\tilde{\varphi}) = f_e$ and thus

$$\varphi^*/\varphi^{*,a} = \left(1 + \chi f_x/f_0\right)^{1/\theta},$$

where we use the superscript a to denote an autarky variable. Access to exporting increases expected profits from production, and hence the probability to be active $1-G(\varphi^*)$ must decrease in order to restore zero-profit condition for entry, so the elemental productivity cutoff for entry must increase. This mechanism is well understood from Melitz (2003) and points to asymmetric effects of openness at the plant level. Whereas highly productive plants see their profits increase with foreign-market access, low-productivity plants experience a profit loss due to stronger competition (for scarce labor), with the least productive plants being forced to cease activity.

To shed further light on the asymmetry in plant-level responses to trade, we can study how producers adjust their assignment of workers to tasks in the open economy. We start with non-exporting plants. The fixed overhead costs of the marginal producer $f(\varphi^*)$ remain to be determined by eq. (16). The marginally active plant in the open economy must have higher elemental productivity than the marginally active plant in the closed economy, and the marginally active plant's fixed costs are therefore lower with trade than in autarky. In view of eq. (18) fixed costs are lower for all non-exporting plants, implying that these plants reduce their count of occupations in response to trade. The intuition is that non-exporting plants command smaller market shares in the open economy and incur lower span-of-control overhead costs, adopting a smaller number of occupations. To compare span-of-control overhead costs in the open and closed economy for non-exporting plants, we compute the proportion

$$\frac{\eta+\pi[\nu n^d(\tilde{\varphi})+1]}{\eta+\pi[\nu n^a(\tilde{\varphi})+1]} = \left(\frac{1}{1+\chi f_x/f_0}\right)^{\frac{\xi}{\theta\gamma}} \equiv \rho^d(\tilde{\varphi}) < 1 \tag{34}$$

given eq. (18) and using $\varphi^*/\varphi^{*,a} = (1+\chi f_x/f_0)^{1/\theta}$ from above. Similarly, for the comparison for exporting plants we compute the proportion

$$\frac{\eta+\pi[\nu n^e(\tilde{\varphi})+1]}{\eta+\pi[\nu n^a(\tilde{\varphi})+1]} = \left(\frac{\left(1+\tau^{1-\sigma}\right)^{\frac{\theta}{\sigma-1}}}{1+\chi f_x/f_0}\right)^{\frac{\xi}{\gamma\theta}} = \left(\frac{\left(1+\chi^{\xi/\theta} f_x/f_0\right)^{\frac{\theta}{\xi}}}{1+\chi f_x/f_0}\right)^{\frac{\xi}{\gamma\theta}} \equiv \rho^e(\tilde{\varphi}) > 1, \tag{35}$$

where the first equality follows from eqs. (29) and (34) and the second equality from eq. (32). By $\theta > \xi$ it follows that $n^e(\tilde{\varphi}) > n^a(\tilde{\varphi})$ and thus $\rho^e(\tilde{\varphi}) > 1$. An exporting plant in the open economy generates higher revenues and thus raises its occupation count in the internal labor market. The asymmetric response of plants in their internal division of labor is the consequence of an asymmetric exposure to exporting. If all plants were to export ($\chi = 1$), the marginal plant would be the same as in the closed economy, $\varphi^* = \varphi^{*,a}$, implying $n^e(\tilde{\varphi}) = n^a(\tilde{\varphi})$ for all active producers. It is therefore the asymmetric exposure to exporting rather than the market size increase *per se* that is responsible for plant-level adjustments on the internal labor market. The following proposition summarizes these insights.

Proposition 1. *In the open economy, compared to autarky, exporting plants raise the count of occupations and narrow the task range per occupation in the internal labor market, resulting in lower mismatch and higher worker efficiency, whereas non-exporters reduce the count of occupations and widen the task range per occupation in the internal labor market, resulting in higher mismatch and lower worker efficiency.*

In the open economy, exporters become more like to Adam Smith's pin factory, whereas non-exporters become less like that. The asymmetric response of plants to trade in their internal labor markets has consequences for wage differences in the plant-occupations. Following the derivations for the closed economy, we can express the coefficient of variation of wages at non-exporters $CV_w^d(\tilde{\varphi})$ as a function of the coefficient of variation of wages at the marginally active plant $CV_w^d(\varphi^*)$ by eq. (22). The functional relationship in the marginally active plant is the same as in the closed economy. However, in the open economy marginally active plant is a plant with higher elemental productivity. Given $\varphi^*/\varphi^{*,a} = (1 + \chi f_x/f_0)^{1/\theta}$, the effect of openness on wage inequality at the plant-level for non-exporters is therefore

$$CV_w^d(\tilde{\varphi}) = CV_w^a(\tilde{\varphi}) \frac{(\tilde{\varphi}/\varphi^{*,a})^{\xi/\theta}(\xi f_0/\gamma)^{1/\gamma} - \eta/\rho^d(\tilde{\varphi})}{(\tilde{\varphi}/\varphi^{*,a})^{\xi/\theta}(\xi f_0/\gamma)^{1/\gamma} - \eta} \tag{36}$$

by eq. (34). Given $r_d(\tilde{\varphi}) < 1$, it follows from eq. (36) that $CV_w^d(\tilde{\varphi}) < CV_w^a(\tilde{\varphi})$ iff $\eta > 0$. For exporters, the relationship is

$$CV_w^e(\tilde{\varphi}) = CV_w^a(\tilde{\varphi}) \frac{(\tilde{\varphi}/\varphi^{*,a})^{\xi/\theta}(\xi f_0/\gamma)^{1/\gamma} - \eta/\rho^e(\tilde{\varphi})}{(\tilde{\varphi}/\varphi^{*,a})^{\xi/\theta}(\xi f_0/\gamma)^{1/\gamma} - \eta} \tag{37}$$

by eqs. (22) and (34). Given $r_e(\tilde{\varphi}) > 1$, it follows that $CV_w^e(\tilde{\varphi}) > CV_w^a(\tilde{\varphi})$ iff $\eta > 0$. Recall from the closed-economy derivations (Subsection 4.1) that more productive plants have higher wage inequality within plant-occupations iff $\eta > 0$. The following proposition summarizes the effects of trade on plant-level wage inequality.

Proposition 2. *In the open economy, compared to autarky, exporting plants raise within-plant-occupation wage inequality iff more productive plants used to have higher wage inequality in autarky (iff $\eta > 0$), whereas non-exporting plants lower within-plant-occupation wage inequality iff more productive plants used to have higher wage inequality in autarky (iff $\eta > 0$).*

Given the asymmetry in the plant-level implications, access to trade exerts counteracting effects on the general equilibrium variables of interest: welfare W and economy-wide wage inequality $\mathbb{V}_w$. Similar to autarky, welfare in the open economy is given by the real wage and hence inversely related to the CES price index $P = [gM(1 +$

$\chi f_x/f_0)/(\theta-\xi)]^{1/(1-\sigma)}p^d(\varphi^*)$. The mass of producers in the open economy is given by $M = M^a/(1+\chi f_x/f_0)$ and thus smaller than in the closed economy. Noting further that $p(\varphi^*) = p(\varphi^{*,a})(1+\chi f_x/f_0)^{-1/\theta}$, we can relate welfare in the open economy to welfare in the closed economy, according to

$$W = W^a \left(1 + \frac{\chi f_x}{f_0}\right)^{\frac{1}{\theta}}. \tag{38}$$

For plant entry in our model is allocationally efficient (similar to the case in Dhingra and Morrow 2016), a movement from autarky to trade is akin to lifting a technology barrier, which must be welfare enhancing.

We show in Appendix B.4 that economy-wide wage inequality in the open economy is given by

$$\mathbb{V}_w = \mathbb{V}_w^a + \frac{\eta\, CV_w^d(\varphi^*)^2}{[(\xi f_0/\gamma)^{1/\gamma} - \eta]^2} \frac{\theta - \xi}{\theta - \xi + 2\xi/\gamma} \frac{\chi^{1-\frac{\xi}{\theta}}}{1 + \chi f_x/f_0} V(\chi), \tag{39}$$

with

$$\begin{aligned} V(\chi) \equiv 2\left(\frac{\xi f_0}{\gamma}\right)^{1/\gamma} \frac{\theta - \xi + 2\xi/\gamma}{\theta - \xi + \xi/\gamma} & \left\{\chi^{\frac{\xi}{\theta\gamma}} - 1 + \left(1 + \chi^{\frac{\xi}{\theta}} \frac{f_x}{f_0}\right)\left[1 - \left(1 + \chi^{\frac{\xi}{\theta}} \frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}} \chi^{\frac{\xi}{\theta\gamma}}\right]\right\} \\ & - \eta\left\{\chi^{\frac{2\xi}{\theta\gamma}} - 1 + \left(1 + \chi^{\frac{\xi}{\theta}} \frac{f_x}{f_0}\right)\left[1 - \left(1 + \chi^{\frac{\xi}{\theta}} \frac{f_x}{f_0}\right)^{-\frac{2}{\gamma}} \chi^{\frac{2\xi}{\theta\gamma}}\right]\right\} > 0. \end{aligned} \tag{40}$$

Economy-wide wage inequality is therefore higher (lower) in the open than the closed economy iff $\eta > 0$. Iff $\eta > 0$ wage inequality within high-productivity plants increases while wage inequality within low-productivity plants falls. As a consequence, there are counteracting effects on economy-wide wage inequality. However, the combined effect is unambiguous for two reasons. On the one hand, aggregate overhead expenditures associated with the division of tasks into occupations increase, which raises wage inequality iff $\eta > 0$. On the other hand, exporters expand production and employment, whereas non-exporters contract production and employment. Hence, plants with higher wage inequality carry more weight in the computation of $\mathbb{V}_w$, which contributes to an increase in economy-wide wage inequality. We summarize the effects of trade on welfare and economy-wide wage inequality in the following proposition.

Proposition 3. *In the open economy, compared to autarky, welfare increases with the selection of more productive plants into exporting. Economy-wide wage inequality widens iff wage inequality widens at exporting plants in the open economy compared to their wage inequality in autarky.*

5.3 Variations and extensions

To derive clear and intuitive equilibrium relationship in Sections 4 and 5, we consider plants to be tupels of only two random characteristics—elemental productivity $\tilde{\varphi}$ and a required task range $\tilde{z}$ for production. In general equilibrium, the stochastic required task range $\tilde{z}$ plays no relevant role, and for elemental productivity $\tilde{\varphi}$ we assumed a Pareto distribution. In varying variations and extensions of our model, we have accounted for additional sources of plant heterogeneity, with the main purpose of achieving a better fit between the theoretical model and its quantitative implementation in Section 7, where we consider a plant to be tupel of four stochastic characteristics: elemental productivity $\tilde{\varphi}$, a required full task range $\tilde{z}$, fixed cost for exporting $\tilde{f}_x$, and the sensitivity of worker performance to task mismatch $\tilde{\eta}$.

Considering firm-specific realizations of performance sensitivity to mismatch $\tilde{\eta}$ does not affect the firm-level effects of trade but it weakens the relationship between plant-level revenues and wage dispersion. The reason is that two plants with the same elemental productivity φ can now exhibit different degrees of wage dispersion within plant-occupations. As a consequence of the dependence of wage dispersion on $\tilde{\eta}$ beyond φ, the distribution of $\tilde{\eta}$ severs the clear nexus between trade and economy-wide wage dispersion under a constant η and the sign of the mean of $\tilde{\eta}$ is no longer a sufficient condition for the nexus.

Accounting for plant heterogeneity in the fixed costs of exporting makes export selection less sharp as it no longer depends on a single cutoff for elemental productivity. As a consequence, the intensive margin takes on a more important role as a channel for trade effects in general equilibrium (see Armenter and Koren 2015). In our model, heterogeneity in fixed export costs heightens the labor efficiency gain from the plant-internal division of labor. The main qualitative insights from our analysis remain unchanged with the exception that heterogeneity in fixed export costs also weakens the conclusiveness of trade effects on economy wide wage dispersion.

Specifying productivity to be log-normally distributed, as in our structural estimation and simulation in Section 7, makes the analysis of general equilibrium considerably more intricate than under a Pareto distribution. However, the modification leaves most insights from our model unchanged, again with the exception that the impact of trade on economy-wide wage dispersion becomes weaker under log-normally distributed productivity than in a comparable model with a Pareto distribution. Our theoretical exercises compare the open economy to autarky. We leave it to a structural and simulation of the German economy between 1999 and 2012 in Section 7 to assess the effects of gradual trade opening within our model.

6 Empirics

6.1 Empirical characterization of the sensitivity of performance

A crucial plant-level parameter in our model is the plant characteristic $\tilde{\eta}(\omega)_t$ that regulates a plant ω's sensitivity of worker performance to task mismatch at time t. Similar to elemental plant productivity, the characteristic $\tilde{\eta}(\omega)_t$ is not directly observed in data. However, the plant's profit-maximizing conditions yield optimal worker efficiency, which translates into wage dispersion within plant-occupations through $\tilde{\eta}(\omega)_t$ by eq. (10). Combining eq. (10) with the definition of a plant's number of tasks per occupation in eq. (3), and using $\tilde{z}(\omega) = \tilde{\zeta}(\omega) \cdot z(\omega)$, we can recover a proxy to the plant-level sensitivity of performance from the data without estimation:

$$\tilde{\eta}(\omega)_t / \tilde{\zeta}(\omega)_t = \left[\sqrt{4 - \pi(\pi - 2)} / CV(\omega)_t - \pi \right] \Big/ [b(\omega)_t / z(\omega)_t]. \tag{41}$$

A plant's coefficient of residual wage variation $CV(\omega)_t$ within its occupations and the plant's normalized number of tasks per occupation $b(\omega)_t/z(\omega)_t$ are observed in our combined data. We acknowledge that this proxy itself captures both a plant's wage sensitivity to performance and its susceptibility to outsourcing and offshoring. We therefore condition on plant size categories and industry effects as well as detailed occupational categories to isolate a worker's performance-related variation within industry-occupation boundaries, whereas outsourcing or offshoring arguably occur by industry and occupation, not the individual job holder.

The BIBB-BAuA survey includes the question whether a worker's small mistakes in his or her occupation cause the employer financial losses ("Financial losses by small mistake," see Becker and Muendler 2015). By construction in eq. (41), the $\tilde{\eta}(\omega)_t$ proxy captures a plant's wage variability $CV(\omega)_t$ and its degree of specialization $b(\omega)_t/z(\omega)_t$. In the model, the higher $\tilde{\eta}$, the more wage variability is induced when the plant shrinks the task range. Consider two pin-making plants. Each plant turns from a simple workshop with little division of labor into a highly specialized pin factory, but one plant has an innately lower $\tilde{\eta}$ than the other plant and will experience a lesser increase in wage dispersion as it specializes internally. Our model explains this link between wage variability and internal specialization with the tenet that an employer's surplus ("financial losses") is more responsive to (bad) match quality when the division of labor is more specialized. If this tenet is correct then we should observe workers at the plant with high innate $\tilde{\eta}$—a high sensitivity of surplus to specialization—report that bad match quality (causing "mistakes") results in large swings in surplus (even "small mistakes" cause "financial losses").

Answers to the question "Financial losses by small mistake" in the BIBB-BAuA survey come in four degrees: "never," "seldom," "occasionally," and "frequently or almost always." We run a worker-level regression of the

Table 3: Sensitivity of Performance and Financial Losses from Small Mistakes

Dependent variable: $\tilde{\eta}$ proxy	for Wages (1)	for Wages (2)	for Wages (3)	for Residual wages (4)	for Residual wages (5)	for Residual wages (6)
Financial losses from small mistakes						
seldom	.164	.143	.012	.166	.145	.021
	(.034)***	(.030)***	(.020)	(.032)***	(.029)***	(.014)
occasionally	.151	.120	.002	.173	.142	.022
	(.044)***	(.041)***	(.025)	(.040)***	(.036)***	(.018)
frequently	.376	.318	.117	.358	.302	.104
	(.043)***	(.044)***	(.034)***	(.039)***	(.038)***	(.025)***
Occupation area (KldB-88 1-dgt) FE		yes			yes	
Occupation class (KldB-88 4-dgt) FE			yes			yes
Adj. R^2	.574	.603	.799	.660	.687	.860
Observations	44,733	44,610	32,895	44,733	44,610	32,895

Source: BIBB-BAuA 1999, 2006 and 2012, and LIAB 1996-2014. Plants with more than 2 full-time workers.
Notes: Plant-level performance sensitivity to task mismatch: $\tilde{\eta}(\omega)/\tilde{\zeta}(\omega) = \left[\sqrt{4-\pi(\pi-2)}/CV_w(\omega) - \pi\right] z(\omega)/b(\omega)$. Worker-reported financial losses from small mistakes in four categories, omitted category: never or almost never. All regressions conditional on 34 industry effects and 7 plant size categories. There are 6 occupation areas (KldB-88 1-dgt) and 1,144 occupation classes (KldB-88 4-dgt) in the sample. Standard errors clustered at the industry level in parentheses. Significance levels: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

employing plant's $\tilde{\eta}(\omega)_t$ on the three worker-reported categories of loss frequencies after mistakes, relative to the omitted category "never." In the regression, we control for plant size categories and industries and find the results reported in Table 3.

The results suggest that a worker's plant exhibits a significantly higher surplus sensitivity to mismatches $\tilde{\eta}$ if the worker reports more frequent financial losses to the employer when he or she makes a small mistake on the job. Moreover, the ranking of effects is as we would expect under our tenet: when a worker reports that their mistake seldom or only occasionally causes losses to the plant, then the plant's measured surplus sensitivity to mismatches $\tilde{\eta}$ is strictly higher than for the omitted category (when workers report that there is never a financial loss from their mistakes). The categories "seldom" and "occasionally" are statistically indistinguishable from each other; only once we condition on 1,144 occupation classes (in Columns 3 and 6) do those two categories also become statistically indistinguishable from the "never a loss" category that is omitted. However, workers who report that their small mistakes "frequently cause" financial losses to the employer are employed at plants that have strictly higher surplus sensitivity to mismatches than in any of the other three categories, and this effect remains statistically significant even within 1,144 narrowly defined occupation classes. The implication is that, at plants whose workers report more frequent financial losses to the employer from their mistakes, our measure of surplus sensitivity to worker performance is higher, and worker mismatches tend to translate task specialization

Table 4: Predictors of the Number of Tasks

	Dependent variable: log Normalized number of tasks $\ln b/z$						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	OLS	OLS	OLS	IV	IV	IV	IV
log Revenues	-0.091***	-0.057***	-0.051***	-0.021	-0.021*	-0.259***	-0.257***
	(0.003)	(0.007)	(0.009)	(0.013)	(0.013)	(0.077)	(0.076)
log Count of occupations		-0.257***	-0.328***			4.363**	4.428**
		(0.037)	(0.075)			(1.975)	(2.010)
log Revenues		0.009***	0.013**			-0.226**	-0.230**
$\times$ log Count of occupations		(0.003)	(0.005)			(0.110)	(0.112)
Plant FE	no	no	yes	no	no	no	no
R^2	0.234	0.244	0.845				
Adj. R^2	0.234	0.243	0.793				
Hansen J (p-val.)					0.288		0.872
Observations	126,488	126,488	126,488	64,907	64,616	64,777	64,563

Sources: LIAB 1996-2014 and BIBB-BAuA 1999, 2006 and 2012, all sectors. Plants with more than 2 full-time workers, 1996-2014.
Notes: Specifications include time, region, and sector fixed effects. IV estimation is based on GMM. Standard errors clustered at the plant level in parentheses. Significance levels: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

into higher wage dispersion through a higher $\tilde{\eta}$.

6.2 Empirical tests of the model

The model of plant-level optimization presented in Sections 4 and 5 takes a plant as a tupel of two characteristics: its elemental productivity $\tilde{\varphi}(\omega)_t$ and its innate task range $\tilde{z}(\omega)_t$ required to produce output, holding the performance sensitivity to mismatch constant $\tilde{\eta}(\omega) = \eta$. The model gives rise to two testable hypotheses at the plant level:

Hypothesis 1. The number of tasks and revenues are inversely related.

Hypothesis 2. The within-plant wage dispersion is positively related to plant revenues iff the sensitivity of worker performance to task mismatch is positive, $\eta \geq 0$.

To test the hypotheses and see whether their patterns are robust, we run a series of regressions, in which we vary the set of explanatory variables and instrument those regressors, whose endogeneity is suggested by our theoretical model. We control in all specification for time, region and sector fixed effects. Standard errors are clustered at the plant level.

Test of hypothesis 1: Table 4 reports the results from estimating the plant-level link between the normalized number of tasks $b(\omega)_t/z(\omega)_t$ and revenues. The first three columns of the table present the outcome of OLS regressions, which support our theoretical hypothesis of a negative link between plant-level revenues and the number of tasks: larger plants are internally more specialized. The baseline specification in Column 1 suggests that a ten percent increase in plant-level revenues is associated with a one percent decline in the (normalized) number of tasks $b(\omega)_t/z(\omega)_t$. This effect gets smaller when we add the log count of distinct occupations in a plant and the interaction term of the log count of occupations and log revenues as further explanatory variables. A negative impact of the count of distinct occupations on the number of tasks is in line with our theoretical model. However, from our model one may expect that the count of occupations and revenues are perfectly correlated, which is not the case.[18] But this should not be interpreted as evidence against the formal structure of our model because the model does not predict a linear relationship between log revenues (or the log count of occupations) and the log normalized number of tasks $b(\omega)_t/z(\omega)_t$, and hence the fact that we are able to estimate significant effects of all three explanatory variables in Column 2 could simply reflect non-linearities in the relationship between these variables and the log normalized number of tasks. Overall, the marginal effect of an increase in log revenues on the log normalized number of tasks is still negative and amounts to -0.042, when evaluated at the mean of the log count of occupations, 1.648. The negative relationship between revenues and the count of occupations is robust to adding plant fixed effects (Column 3).

Our model implies that plant-level revenues, the count of occupations, and the number of tasks carried out by workers, are jointly endogenous to the plant's market conditions. The OLS estimates in Columns 1 through 3 therefore do not have a causal interpretation. We use an instrumental variable (IV) approach and estimate the relationship between revenues and the number of tasks, using GMM. The second-stage results of the respective regressions are reported in Columns 4 through 7 of Table 4, with the related first-stage results collected in Table 5.

Our choice of instruments is guided by insights from our model, which predicts that globalization as measured by additional industry exports and heightened import competition affects plant-level revenues, the count of distinct occupations, and thus the number of tasks carried out by workers. This suggests using exports and imports at the industry level as instruments. However, these industry aggregates themselves depend on the plants' common decisions. Therefore, we follow the reasoning of Autor, Dorn and Hanson (2013) and use other high-income countries' exports to and imports from China (CHN) as instruments for German exports and imports at the industry level. In the selection of other high-income countries, we follow Dauth, Findeisen and Suedekum (2014) and use Australia, Canada, Japan, Norway, New Zealand, Sweden, Singapore, and the United Kingdom as peer group. Since shipments to China affect exporters differently from non-exporters, we interact the log exports to China with a dummy capturing the export status of a plant in the preceding year. With three potentially en-

[18]The correlation coefficient of these two variables is 0.671.

Table 5: PREDICTORS OF THE NUMBER OF TASKS: FIRST STAGE

Dep. Variationable:	log Revenues (4.1)	log Revenues (5.1)	log Revenues (6.1)	log Cnt. occ. (6.2)	Interact. (6.3)	log Revenues (7.1)	log Cnt. occ. (7.2)	Interact. (7.3)
Export indic.$_{t-1}$	0.116***	0.116***	0.219***	2.428***	0.129***	0.212***	2.373***	0.126***
$\times$ log Exports$_{CHN}$	(0.007)	(0.007)	(0.020)	(0.243)	(0.015)	(0.018)	(0.239)	(0.015)
log Imports$_{CHN}$		-0.017				-0.193***	-1.201***	-0.069***
		(0.010)				(0.007)	(0.088)	(0.006)
Export indic.$_{t-1}$			-0.201***	-2.007***	-0.104***	-0.191***	-1.935***	-0.101***
$\times$ log Exports$_{EE}$			(0.021)	(0.259)	(0.016)	(0.019)	(0.253)	(0.016)
Miltll. Rev. dis.$_{t-1}$			0.001***	0.007***	0.0004***	0.001***	0.007***	0.0004***
$\times$log Imports$_{EE}$			(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
F-stat.	312.3	160.5	6290.4	1416.8	1074.1	4674.5	1044.3	800.5
Observations	64,907	64,616	64,563	64,563	64.563	64,563	64,563	64,563

Sources: LIAB 1996-2014 and BIBB-BAuA 1999, 2006 and 2012, all sectors. Plants with more than 2 full-time workers, 1996-2014.
Notes: Column numbers refer to column in Table 4: (6.3), for instance, refers to third endogenous regressor in Column 3 of Table 4. Specifications include time, region, and sector fixed effects. IV estimation is based on GMM. Standard errors clustered at the plant level in parentheses. Significance levels: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

dogenous regressors in Columns 2 and 3, it is not sufficient to specify just two instruments. We therefore add log exports to Eastern Europe (EE) interacted with a plant's export status in the preceding year and the log of imports from Eastern Europe interacted with a plant's millentile position in the revenue distribution from the preceding year as additional instruments. Accounting for exports to and imports from Eastern Europe as an additional set of instruments is motivated by the work of Dauth, Findeisen and Suedekum (2014) who show that trade exposure to China and Eastern Europe tend to have opposite consequences for the German economy.[19]

Column 4 reports the IV results when considering only log revenues as an explanatory variable (in addition to time, region, and industry fixed effects). In this case, we only need one instrument, for which we choose the interaction of log exports to China and the lagged exporter dummy. The first stage regression in Table 5 suggests that this interaction term has a positive and statistically significant effect on log revenues indeed, and the F-test statistic shows no evidence of weak instruments. However, for this specification we do not find a statistically significant impact of log revenues on the log normalized number of tasks on the second stage at conventional levels of significance. In an additional specification we add log imports from China as a second instrument. As findings in Column 2 of Table 5 suggest, this second instrument itself does not exert a significant impact on log revenues. Again, the F-statistic does not indicate the presence of weak instruments and Hansen's J overidentification test fails to reject validity of our instruments. More importantly, with the additional instrument, the estimated impact of log revenues on the log of normalized tasks on the second stage becomes statistically significant. In a further regression, we add the log count of distinct occupations and its interaction with log revenues as additional explanatory variables and instrument the now three endogenous regressors with the interaction of log exports to CHN with the lagged exporter dummy, the interaction of log exports to EE with the lagged exporter dummy, and log imports from EE with the lagged millentile position of a plant in the revenue distribution. Columns 3 through 5 of Table 5 indicate that the three instruments are statistically significant predictors in all three regressions, and the F-tests of excluded instruments pass conventional levels in all three first-stage regressions.[20] On the second stage we find a (now larger) negative and statistically significant effect of log revenues on the log normalized number of tasks, whereas the coefficients of the log count of distinct occupations and its interaction with log revenues change their signs when using an IV approach. In a final specification, we add the log of imports as additional instrument. This allows us to test for overidentification and the high p-value reported in the Table 4 indicates that we fail to reject validity of our instruments. Adding the additional instrument has only

[19]As in their study, we associate Eastern Europe with Bulgaria, the Czech Republic, Hungary, Poland, Romania, Slovakia, Slovenia, the former USSR and its successor Russian Federation, Belarus, Estonia, Latvia, Lithuania, Moldova, Ukraine, Azerbaijan, Georgia, Kazakhstan, Kyrgyzstan, Tajikistan, Turkmenistan, and Uzbekistan.

[20]With multiple endogenous regressors the F-tests on the first stage are not sufficient for rejecting the null that instruments are weak. Unfortunately, clustered standard errors do not offer a straightforward alternative to testing for weak instruments. The Kleibergen-Paap LM test rejects the null of underidentification.

minor effects on the parameter estimates on the second stage (Column 7).

Overall, our findings strongly support that both globalization-induced increases to revenues and globalization-induced division of labor in the internal labor market through additional occupations lead plants to adopt narrower task ranges. In other words, favorable global product-market conditions contribute to the internal division of labor at plants—generating production sites akin to Adam Smith's pin factory as plants expand.

Test of hypothesis 2: To test the novel link from plant performance to wage inequality within plant-occupations, we aim to test the sign of the correlation between wage dispersion inside a plant and its occupation count, conditional on $\tilde{\eta}(\omega)_t = \eta > 0$ in the model. We first establish that the performance sensitivity proxy $\tilde{\eta}(\omega)_t/\tilde{\zeta}(\omega)_t$ from (41) is overwhelmingly positive in our plant data. Note that $\tilde{\zeta}(\omega)_t$ is strictly positive by definition, so a positive sensitivity proxy $\tilde{\eta}(\omega)_t/\tilde{\zeta}(\omega)_t$ must imply a positive underlying sensitivity $\tilde{\eta}(\omega)_t > 0$. In our sample, 57.8 percent of plants have a strictly positive $\tilde{\eta}(\omega)_t 0$, and the sample mean $\tilde{\eta}(\omega)_t/\tilde{\zeta}(\omega)_t$ is positive. We proceed to evaluate the link between within-plant-occupation wage inequality and the internal division of labor at the sample mean.

We estimate the relationship between the within-plant-occupation standard deviation of the residual wage dispersion and the occupation count as a measure of the intra-plant division of labor, conditional on revenues, and test whether the relationship is positive. As a measure of wage dispersion we use the standard deviation of residual daily wages within plant-occupations. Results are reported in Table 6. (In Appendix Table A5 we repeat the exercises for the dispersion of the total daily wage.) We use similar empirical specifications and the same instruments as before. We do not report the first-stage results for the IV specifications since they are closely similar to those reported in Table 5—except for minor changes in the number of observations. The results in Table 6 indicate a clear positive relationship between revenues and residual wage dispersion within plant-occupations for OLS, as well as a clearly positive relationship between residual wage dispersion within plant-occupations and the occupation count. In the long specification of Column 2, a 10 percent increase in the occupation count predicts a more than ten percentage-point increase in the standard deviation of residual wages within plant-occupations. After controlling for plant fixed effects, this positive association is further strengthened; in Column 3, a 10 percent increase in the occupation count predicts a more than twelve percentage-point increase in the standard deviation of residual wages within plant-occupations.

In Columns 4 and 5 of Table 6 we report estimates when instrumenting log revenues as the single endogenous regressor. Test statistics are consistent with the hypothesis that the interaction of log exports to China with the plant's lagged exporter indicator and the log of imports from China provide strong instruments, and the positive effect of revenues on wage variation within plant-occupations remains robust to the change in estimation strategy.

Table 6: Predictors of Within-Plant Residual Daily Wage Dispersion

	Dependent variable: log StDev Residual daily wage						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	OLS	OLS	OLS	IV	IV	IV	IV
log Count of occupations		1.042***	1.232***			5.783***	6.006**
		(0.078)	(0.173)			(2.429)	(2.458)
log Revenues	0.185***	0.174***	0.104***	0.295***	0.293***	0.065	0.059
	(0.005)	(0.013)	(0.019)	(0.025)	(0.025)	(0.093)	(0.093)
log Revenues		-0.055***	-0.067***			-0.309***	-0.321**
$\times$ log Count of occupations		(0.006)	(0.012)			(0.135)	(0.137)
Plant FE	no	no	yes	no	no	no	no
Hansen J (p-val.)					0.165		0.685
R^2	0.293	0.345	0.836				
Adj. R^2	0.292	0.345	0.781				
Observations	126,483	126,483	126,483	64,905	64,614	64,775	64,561

Sources: LIAB 1996-2014, all sectors. Plants with more than 2 full-time workers.
Notes: Specifications include time, region, and sector fixed effects. IV estimation is based on GMM. Standard errors clustered at the plant level in parentheses. Significance levels: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

In Columns 6 and 7 we treat all three regressors—log revenues, the log count of occupations and the interaction term of these two variables—as endogenous variables and instrument them with the variables reported in Table 5. Log revenues lose statistical significance at conventional levels, whereas the association between the occupation count and residual wage dispersion within plant-occupations becomes stronger than under OLS.

We interpret the overall evidence as suggestive of a direct reorganization channel in the plant's internal labor market, by which product-market expansions in the wake of globalization trigger a more specialized division of labor, which in turn leads to more residual wage inequality within plant-occupations.

7 Structural Estimation and Simulation

We turn to a structural estimation model for plant-level outcomes grounded in our theory of trade and internal labor-market organization. Given our interest in inequality, we adopt a maximum likelihood (ML) approach because it disciplines the distributional foundations. Specifying a distribution of plant characteristics alongside the theory model is an important part of the overall framework when we consider not just predictions of aggregate variables (such as employment or the economy-wide wage bill) but the dispersion of outcomes. To specify the distribution of plant characteristics is to establish the stochastic fundamentals of the economy so that we can study inequality in addition to per-capita welfare.

We specify joint normality of the plant's four stochastic characteristics that may change over the course of the sample period:[21] the (log of scaled) elemental productivity $\xi \ln \tilde{\varphi}(\omega)_t$, the (log of) the plant's required full task range $\ln \tilde{z}(\omega)_t = \ln[\tilde{\zeta}(\omega)_t z(\omega)_t]$, the (log of) fixed cost for exporting $\ln \tilde{f}_x(\omega)_t$, and the sensitivity of worker performance to task mismatch $\tilde{\eta}(\omega)_t$, which relates the plant's number of tasks per occupation to its within-occupation wage inequality. The resulting log normality of revenues and other plant characteristics is plausible (see e.g. Helpman et al. 2017, Fernandes et al. 2018).

There are two challenges to structural estimation in our context, beyond previous implementations such as by Helpman et al. (2017). First, we cannot use the partial estimation model for observed outcomes only. We show below for such a conventional system with no censoring that, under joint normality, the variance of a variable such as log revenues among exporters must be less than the variance among non-exporters iff the share of exporters is less than one-half. In our German data as in data for other countries that we have explored, less than one-third of plants export in any given year but the log revenue variance among exporters exceeds that among non-exporters (log employment and the log wage bill exhibit a similar variance ranking between exporters and non-exporters). A participation equation for selection into existence, similar to the Melitz (2003) model, is necessary under joint normality to reconcile the variance ranking between exporters and non-exporters with an exporter share of less than one-half. Second, the entry threshold consistent with our model does not conform to conventional Tobit estimation such as in Carson and Sun (2007) because the censoring threshold is defined with respect to the unobserved productivity in a Melitz (2003) model, not with respect to an observed outcome. We establish that our model is nevertheless point identified and derive an according two-step ML estimator for endogenous switching and censoring. Once estimated, we simulate the model and quantify the importance of globalization for the intra-plant division of labor and economy-wide inequality.

7.1 Estimation model

We revisit the general model specification in Sections 3 and 4 when a plant is a tupel of four stochastic characteristics $(\xi \ln \tilde{\varphi}(\omega)_t, \ln \tilde{\zeta}(\omega)_t, \ln \tilde{f}_x(\omega)_t, \tilde{\eta}(\omega)_t)$. The plant's profit-maximizing conditions from the four-stage optimization problem in Section 4.1 imply a set of estimable equations. One characterization is a four-equation system that involves plant revenues $r(\omega)_t$, the plant's within-occupation coefficient of variation of the daily wage residual $CV_w(\omega)_t$ combined with its normalized number of tasks per occupation $b(\omega)_t/z(\omega)_t$, and an export

[21] We do not formally consider $\tilde{f}_x$ and $\tilde{\eta}$ to be plant specific in Subsection 4.1, so as to characterize general-equilibrium subsequently in closed form. However, the plant-level optimality conditions in Section 4.1 for stages 2 through 4 of the plant's decision problem also hold for the tupel of four stochastic characteristics under any distributional assumption.

indicator $\mathbf{1_x}(\omega)_t$ as the observed variables $\mathbf{x}(\omega)_t \equiv [r(\omega)_t, CV_w(\omega)_t, b(\omega)_t/z(\omega)_t, \mathbf{1_x}(\omega)_t]^T$:

$$\ln r(\omega)_t = \alpha_{0,t} + \alpha_{1,t}\mathbf{1_x}(\omega)_t + \xi_t \ln\tilde{\varphi}(\omega)_t, \tag{42a}$$

$$\ln CV_w(\omega)_t b(\omega)_t/z(\omega)_t = \beta_{0,t} - (1/\gamma_t)\ \ln r(\omega)_t + \ln\tilde{\zeta}(\omega)_t, \tag{42b}$$

$$\mathbf{1_x}(\omega)_t = 1 \ \Leftrightarrow\ \delta_{0,t} \geq \ln\tilde{f}_x(\omega)_t - \xi_t \ln\tilde{\varphi}(\omega)_t, \tag{42c}$$

$$\mathbf{x}(\omega)_t \text{ missing } \ \Leftrightarrow\ \xi_t \ln\tilde{\varphi}(\omega)_t < a_t. \tag{42d}$$

Eq. (42a) follows from eqs. (18) and (29) and the symmetry of the domestic and foreign economies for $\alpha_{0,t} \equiv \ln r^d(\varphi_t^*) - \xi_t \ln\varphi_t^*$ and $\alpha_{1,t} \equiv (1+\xi_t/\gamma_t)\ln(1+\tau_t^{1-\sigma_t})$. Eq. (42b) follows from eqs. (10) and (15) under $\tilde{z}(\omega) = \tilde{\zeta}(\omega)z(\omega)$ and for $\beta_{0,t} \equiv (1/2)\ln[4-\pi(\pi-2)] + (1/\gamma_t)\ln\gamma_t + (1/\gamma_t)\ln[(\sigma_t-1)/\sigma_t]$. Eq. (42c) follows from eq. (31) for $\delta_{0,t} \equiv \ln[(1+\tau_t^{1-\sigma_t})^{\xi_t/(\sigma_t-1)} - 1] + \ln f_0 - \xi_t \ln\varphi_t^*$. Eq. (42d) is a plant's presence condition in the sample for $a_t \equiv \xi_t \ln\varphi_t^*$.

The estimation parameter γ_t is the elasticity of the span-of-control fixed cost and it is time varying for consistency with time varying $\alpha_{0,t}$, $\alpha_{1,t}$ and $\beta_{0,t}$ in the estimation model that we implement. The parameters $\alpha_{0,t}$, $\alpha_{1,t}$, $\beta_{0,t}$ and $\delta_{0,t}$ are composites of model fundamentals including time-varying trade costs τ_t, the time-varying fixed cost of operation $f_{0,t}$, and the elasticity of substitution σ_t. Domestic and foreign market sizes do not enter under our assumption of symmetric domestic and foreign countries in Section 5 for the numéraire $w = w^* = 1$ but we could alternatively specify a small open economy and the model parameters would reflect the relative country sizes in addition to τ_t.

This equation system involves only three of the four stochastic terms that characterize a plant: $\ln\tilde{\varphi}(\omega)_t$, $\ln\tilde{\zeta}(\omega)_t$ and $\ln\tilde{f}_x(\omega)_t$. The plant's log performance sensitivity to task mismatch $\tilde{\eta}(\omega)_t$ is known conditional on $\ln\tilde{\zeta}(\omega)_t$: from observed residual wage variation $CV_w(\omega)_t$ and the normalized number of tasks per occupation $b(\omega)_t/z(\omega)_t$ at the plant we know $\tilde{\eta}(\omega)_t/\exp\{\ln\tilde{\zeta}(\omega)_t\}$ by eq. (41) and can infer $\tilde{\eta}(\omega)_t$ after scaling with $\exp\{\ln\tilde{\zeta}(\omega)_t\}$.

To simplify notation, we drop time subscripts and the plant identifier ω now. We show in Appendix D that the equation system (42) can be rewritten into a multi-variate endogenous switching model with censoring for the observed variables $y = \ln r(\omega)$, $w = \ln CV_w(\omega)b(\omega)/z(\omega)$ and $I = \mathbf{1_x}(\omega)$ as well as the jointly normally distributed disturbances $u = \xi\ln\tilde{\varphi}(\omega)$, $v = \ln\tilde{\zeta}(\omega)$, $e = u - \ln\tilde{f}_x(\omega)$, where u is truncated from below at a.

The estimation system can be recast as

$$y = \begin{cases} \mu_Y^e + u & \text{if } I = 1 \\ \mu_Y^d + u & \text{if } I = 0 \end{cases}, \tag{42a$'$}$$

$$w = \begin{cases} \mu_W^e - (1/\gamma)u + v & \text{if } I = 1 \\ \mu_W^d - (1/\gamma)u + v & \text{if } I = 0 \end{cases}, \tag{42b$'$}$$

$$I = \begin{cases} 1 & \text{if } \mu_X + e \geq 0 \\ 0 & \text{if } \mu_X + e < 0 \end{cases}, \tag{42c$'$}$$

$$y, w, I = \text{missing} \qquad \text{if } u < a, \tag{42d$'$}$$

where $\mu_Y^e \equiv \alpha_0 + \alpha_1$, $\mu_Y^d \equiv \alpha_0$, $\mu_W^e \equiv \beta_0 - (1/\gamma)\alpha_0 - (1/\gamma)\alpha_1$, $\mu_W^d \equiv \beta_0 - (1/\gamma)\alpha_0$, $\mu_X \equiv \delta_0$, and $\gamma = -(\mu_Y^e - \mu_Y^d)(\mu_W^e - \mu_W^d)$ is not an independent parameter.

The joint distribution of the unobserved plant characteristics (disturbances) can then be stated as

$$(u, v, e)^T \sim \mathcal{N}_{\mathcal{T}}(\mathbf{0}, \tilde{\mathbf{\Sigma}}) \qquad \text{with} \quad \tilde{\mathbf{\Sigma}} = \begin{pmatrix} \sigma_u^2 & \rho_{uv}\sigma_u\sigma_v & \rho_{ue}\sigma_u \\ \rho_{uv}\sigma_u\sigma_v & \sigma_v^2 & \rho_{ve}\sigma_v \\ \rho_{ue}\sigma_u & \rho_{ve}\sigma_v & 1 \end{pmatrix}$$

for accordingly defined standard deviations σ_u, σ_v and $\sigma_e = 1$ as well as correlation coefficients ρ_{uv}, ρ_{ue} and ρ_{ve} as shown in Appendix D.

An important property of this multivariate normal model of plants and their market entry is that conditional higher moments relate back to selection into domestic and foreign markets (endogenous switching and censoring). Concretely, censoring in a Melitz (2003) model is exclusively based on elemental productivity (u), so in our case the relationships between higher moments and selection (endogenous switching and censoring) applies also to just the bivariate normal model of plants with elemental productivity and a stochastic fixed cost of exporting. Relevant higher moments are the observed variances of log revenues among exporters $\mathbb{V}(y|I = 1)$ and among non-exporters $\mathbb{V}(y|I = 0)$ as well as moments of the distribution of underlying plant characteristics (σ_u, ρ_{ue}). The following proposition summarizes the relevant implication for our case.

Proposition 4. *If elemental productivity u is not truncated and the variance of log revenues among exporters $\mathbb{V}(y|I = 1)$ exceeds the variance of log revenues among non-exporters $\mathbb{V}(y|I = 0)$, then an outcome with a* *well defined joint normal distribution (with $\sigma_u > 0, \rho_{ue} \neq 0$) exists if and only if the share of exporters exceeds one-half.*

Proof. See Appendix E. □

In our data less than one-third of plants export but the variance of log revenues y among exporters exceeds the variance among non-exporters (whereas the variance of the second composite outcome variable w is smaller among exporters than among non-exporters). As a consequence, we need to allow elemental productivity to be truncated, or in other words we have to consider the full Melitz (2003) model with selection into activity (censoring), not merely the Chaney (2008) model of selection into exporting (endogenous switching).

Allowing for truncation by elemental productivity (censoring) requires that we use the observed minimum log revenues in ML estimation to recover the truncation point for elemental productivity. Given the parameter estimates for first and second moments related to log revenues and elemental productivity, we can infer from the observed minimum log revenues the internally consistent cutoff of elemental productivity. This ML procedure in turn requires that we express the likelihood functions in a conditional manner: once conditioning on the case that the observed minimum of log revenues in the data occurs at a non-exporter and once conditioning on the alternative case that the observed minimum occurs at an exporter. The alternative case is possible if a plant draws a low elemental productivity but also draws an extremely favorable fixed export cost so that the plant sells (little) domestically and exports (little), resulting in the minimum of total log revenues in the sample. We derive and state the conditional likelihood functions in Appendix F.1.

7.2 Implementation of estimation model

To implement ML estimation under censoring on just one unobserved plant characteristic u, we can segment the full equation system in eq. (42) into two subsystems. Following an approach proposed by Murphy and Topel (1985) and also outlined in Greene (2012, ch. 14.7), we can estimate in a first step the subsystem related to revenues and selection (endogenous switching and censoring)—eqs. (42a), (42c), (42d)—, and then insert the parameter estimates from the first step into the second step. In the second step, we estimate the subsystem related to internal specialization and selection (endogenous switching and censoring)—eqs. (42b), (42c), (42d). This split into two steps is possible because the parameter vector $\boldsymbol{\theta}_v = (\mu^e_W, \mu^d_W, \sigma_v, \rho_{ve}, \rho_{uv})$ only appears in step 2 (step 1 lacks the error term v). In contrast, the parameter vector $\boldsymbol{\theta}_u = (\mu^e_Y, \mu^d_Y, \mu_X, \sigma_u, \rho_{ue})$ appears not only in step 1 but also in step 2. The reasons are that γ is a composite of μ^e_Y, μ^d_Y and selection involves μ_X, σ_u and ρ_{ue} in step 2. While we cannot estimate the subsystems separately, we can estimate $\boldsymbol{\theta}_u$ on a first step and then use the parameter estimates from model 1 in model 2 and estimate parameter vector $\boldsymbol{\theta}_v$ in a second step. We derive and state the conditional likelihood functions in Appendix F.2.

We strive to isolate the distributional foundations of plant characteristics from equilibrium outcomes under a changing trade environment over time. We therefore restrict the higher moments related to the stochastic and unobserved plant characteristics to be constant over time. But we allow the equilibrium related parameters that are functions of trade policy variables such as trade costs and mean fixed export costs to vary over time.

Table 7 shows the results of ML estimation. After estimating parameters μ_Y^d, μ_Y^e, μ_X by year and σ_u and ρ_{ue} as constants across years in the first step (on the pooled sample of 20,161 plants over the three years 1999, 2006, 2012), we compute the composite parameters $\underline{a}$ for censoring into missing using eq. (F.8) and χ for endogenous switching into export status evaluating eq. (D.10) in the Appendix. In the second step, we estimate μ_W^d and μ_W^e for each year and σ_v, ρ_{ve} and ρ_{uv} as time invariant constants, conditional on the estimates from the first step. After estimation of both steps, we compute γ using $\gamma = -(\mu_Y^e - \mu_Y^d)(\mu_W^e - \mu_W^d)$. In estimation, we restrict standard deviations to be strictly positive and correlation coefficients to fall in the range -1 and 1, and obtain standard errors for all constrained and composite parameters using the Delta method.

Parameter estimates are statistically significantly different from zero at conventional confidence levels. The parameter estimates most relevant for log revenues and export selection are not in all cases statistically different from each other over time, whereas the parameters on step 2 most closely related to inequality outcomes are statistically different from each other. The estimated censoring cutoff is low and implies that only a tiny fraction of plants fails to enter; the threshold estimate reflects the fact that the smallest plant in our sample has only minor revenues. The presence of a finite lower threshold ensures nevertheless that our estimator is internally consistent with the variance ranking of outcomes between exporters and non-exporters. Beyond its statistical properties, we aim to evaluate the economic importance of our model in predicting observed export-market outcomes and changes to the internal division of labor at German plants over the period 1999–2012. To do so, we need to infer each plant's unobserved performance sensitivity to task mismatch $\tilde{\eta}$ from our structural model and estimation. As a theory-implied variable in the data we can infer the composite $\tilde{\eta}(\omega)_t/\tilde{\zeta}_t$ from the (estimation independent) relationship $\tilde{\eta}(\omega)_t/\tilde{\zeta}_t = \left[\sqrt{4-\pi(\pi-2)}/CV_w(\omega)_t - \pi\right] \cdot z(\omega)_t/b(\omega)_t$.

Post estimation, we can infer $\tilde{\zeta}(\omega)_t = \exp\{v(\omega)_t\}$ from eq. (42c′) and therefore

$$\tilde{\eta}(\omega)_t = \left[\sqrt{4-\pi(\pi-2)}/CV_w(\omega)_t - \pi\right] \cdot (z(\omega)_t/b(\omega)_t) \cdot \exp\{v(\omega)_t\}.$$

We obtain the standard deviation σ_η. From eqs. (42a′) and (42c′) we recover $u(\omega)_t$ and $v(\omega)_t$ and correlate both with the constructed $\tilde{\eta}(\omega)$ to obtain $\rho_{u\eta}$ and $\rho_{v\eta}$. We compute the standard errors of σ_η, $\rho_{u\eta}$ and $\rho_{v\eta}$ using

Table 7: Maximum Likelihood Parameter Estimates

	1999	2006	2012		Time invariant
		Maximum Likelihood Step 1			
μ_Y^d	12.969	12.916	12.939	σ_u	1.606
	(.055)	(.059)	(.043)		(.046)
μ_Y^e	16.376	16.504	16.353	ρ_{ue}	.233
	(.154)	(.123)	(.131)		(.028)
μ_X	-.981	-.950	-.937		
	(.048)	(.040)	(.032)		
$\underline{a}$	-8.155	-6.428	-8.319		
χ	.163	.171	.174		
		Maximum Likelihood Step 2			
μ_W^d	-2.679	-2.255	-2.256	σ_v	1.090
	(.013)	(.011)	(.011)		(.009)
μ_W^e	-3.912	-3.514	-3.477	ρ_{ve}	.790
	(.016)	(.015)	(.015)		(.003)
				ρ_{uv}	.628
					(.003)
		Post Estimation			
γ	2.763	2.849	2.795	σ_η	14.509
	(.140)	(.117)	(.120)		(.072)
				$\rho_{u\eta}$	-.080
					(.007)
				$\rho_{v\eta}$	-.452
					(.006)
				$\rho_{e\eta}$	0
log Pseudo-Likelihood Step 1			-4,980,867		
log Likelihood Step 2			-82,034.33		
Observations			20,161		

Source: LIAB and BIBB-BAuA, 1999, 2006 and 2012. Plants with more than 2 full-time workers, weighted by sampling frequencies. *Notes*: Parameters $\underline{a}$ from eq. (F.8) and χ from eq. (D.10) in the Appendix. Standard errors in parentheses (using the Delta method for constrained and composite parameters).

the formulae for jointly normally distributed variables. The disturbance $e(\omega)_t$ cannot be recovered directly post estimation, so we assume that $\eta(\omega)_t$ and $e(\omega)_t$ are uncorrelated. As a result, we obtain the quadrivariate joint normal distribution[22]

$$(u, v, e, \tilde{\eta})^T \sim \mathcal{N}_{\mathcal{T}}(\mathbf{0}, \tilde{\boldsymbol{\Sigma}}) \qquad \text{with} \quad \tilde{\boldsymbol{\Sigma}} = \begin{pmatrix} \sigma_u^2 & \rho_{uv}\sigma_u\sigma_v & \rho_{ue}\sigma_u & \rho_{u\eta}\sigma_u\sigma_\eta \\ \rho_{uv}\sigma_u\sigma_v & \sigma_v^2 & \rho_{ve}\sigma_v & \rho_{v\eta}\sigma_v\sigma_\eta \\ \rho_{ue}\sigma_u & \rho_{ve}\sigma_v & 1 & 0 \\ \rho_{u\eta}\sigma_u\sigma_\eta & \rho_{v\eta}\sigma_v\sigma_\eta & 0 & \sigma_\eta^2 \end{pmatrix}.$$

7.3 Simulation

Using the estimated quadrivariate joint normal distribution of $(u, v, e, \tilde{\eta})^T$, we simulate a fixed population of 20,000 plants by generating the following four stochastic characteristics of plants: adjusted elemental productivity $\xi \ln \tilde{\varphi}(\omega) = u$, the plant's task range variability $\ln \tilde{\zeta}(\omega) = v$, the plant's fixed cost of exporting $\ln \tilde{f}_x(\omega) = u - e$ and the plant's performance sensitivity to task mismatch $\ln \tilde{\eta}(\omega)$. These stochastic plant characteristics are held constant over time so that we can subject the same fundamental plant population to economic change in the time varying parameters. To measure economic change between 1999 and 2012, we transform the estimation parameters from the estimation model (42′) back into those of the baseline model (42) and obtain $\alpha_{0,t}$, $\alpha_{1,t}$, $\beta_{0,t}$, γ_t, $\delta_{0,t}$ and a_t for $t = 1999, 2012$. With those parameters at hand, we simulate the main economic outcomes: plant revenues $r(\omega)_t$, a plant's within-occupation coefficient of variation of the daily wage residual $CV_w(\omega)_t$ combined with its normalized number of tasks per occupation $b(\omega)_t/z(\omega)_t$, and an export indicator $\mathbf{1_x}(\omega)_t$.

The export indicator reflects the extensive margin of export entry. To capture the intensive margin of exports, we compute the share of exports in total revenues at exporters on average in the data and obtain the corresponding measure from our estimates using

$$\frac{r_T^e(\omega)_t - r^e(\omega)_t}{r_T^e(\omega)_t} = \frac{\tau_t^{1-\sigma}}{1 + \tau_t^{1-\sigma}} = \exp\{\alpha_{1,t}\}^{-\frac{\gamma_t - \sigma + 1}{\sigma}},$$

where the first equality follows from symmetry of the foreign and domestic economy and the second equality by eq. (42a) and $\xi_t/\gamma_t = (\sigma - 1)/(\gamma_t - \sigma + 1)$. The measure depends on the unobserved elasticity of substitution

[22] Under the restriction $\rho_{e\eta} =$ and the other parameter estimates, we find that $\tilde{\boldsymbol{\Sigma}}$ has no valid Cholesky decomposition. To perform simulations, we choose the minimally admissible $\rho_{e\eta} = .011$.

Table 8: Simulation Results

	Data		Simulation	
	1999	2012	1999	2012
Exporter share	.133	.152	.157	.169
Export share in revenues at exporters	.228	.240	.228[a]	.252
Changes in				
mean Normalized number of tasks per occupation				-17.1%
mean Coefficient of variation of wages within plant-occupations				43.3%

[a]Calibriation of the elasticity of substitution to $\sigma = 3.50$ matches the observed export share in revenues at exporters in 1999.

Source: LIAB and BIBB-BAuA, 1999 and 2012. Plants with more than 2 full-time workers, weighted by sampling frequencies.
Notes: Simulation for population of 20,000 plants with constant stochastic characteristics but exposed to time-varying parameter changes.

σ. We calibrate σ so that we match the export share in revenues at exporters in the data.[23] This choice serves a dual purpose. First, we can simulate the intensive margin of exports in 2012 for a constant σ and check whether the change matches that in the data. Second, we can reuse the calibrated σ to decompose the combined variable $CV_w(\omega) \cdot b(\omega)_t / z(\omega)_t$ into its parts.

To separate a plant's normalized number of tasks per occupation $b(\omega)_t / z(\omega)_t$ from the coefficient of variation of the plant's wages $CV_w(\omega)$ we use (3) in (10) and rearrange terms to find

$$\frac{b(\omega)_t}{z(\omega)_t} = \frac{\pi \exp\{\ln \tilde{\zeta}(\omega)\}}{\left[\frac{1}{\gamma_t}\frac{\sigma-1}{\sigma} \exp\{\ln r(\omega)_t\}\right]^{1/\gamma_t} + \tilde{\eta}(\omega)},$$

keeping the stochastic fundamentals $\ln \tilde{\zeta}(\omega)$ and $\tilde{\eta}(\omega)$ constant over time.

Table 8 shows the results of our simulations and contrasts them with available moments in the data. The share of exporting plants in the data (weighted by sampling frequencies) rose from 13 to 15 percent between 1999 and 2012, and our simulations find an increase from 16 to 17 percent. In the data, the share of exports in exporter revenues rose from 23 to 24 percent and to 25 percent in our simulation. Our simulations suggest that this increasing export-market participation was related to internal labor-market adjustments at the plants, which reduced the number of tasks per occupation by 17 percent from 1999 to 2012, choosing a stricter division of labor. Our simulation also suggests that wage inequality within plant-occupations increased markedly at the

[23]Post estimation, we could also attempt to recover an estimate of the elasticity of substitution σ_t from eq. (42b) using

$$\sigma_t = \frac{1}{1-X} \quad \text{with} \quad X = \gamma_t \left[\frac{\sqrt{4-\pi(\pi-2)}}{\exp\{\beta_{0,t}\}}\right]^{\gamma_t}.$$

The crucial constant term $\beta_{0,t}$ for that inference, however, is likely affected by additional changes in the economy beyond our model, so we do not choose that approach.

plants.[24] We conclude that the estimated and simulated model for Germany can account for substantive changes that reflect economic globalization and its association with the advancing division of labor at employers and heightened within-plant inequality.

8 Concluding Remarks

We document empirically that workers in larger plants perform fewer tasks and that a dominant part of residual wage inequality materializes within plant-occupations. Based on these observations, we build a model of the internal labor market, where the employer chooses the division of labor by assigning task ranges to occupations, workers of different ability match to occupations and the match quality determines the wage dispersion within plant-occupations. We embed this rationale into a heterogeneous-firm model of trade to relate global product-market conditions to the employer's optimal choice of the internal division of labor. A plant that commands a larger market share can achieve a labor efficiency gain by narrowing the range of tasks performed per occupation and simultaneously raising the count of occupations to which it assigns tasks. In equilibrium, inherently more productive plants and exporters adopt a stricter division of labor and thus boost their elemental productivity. We use German plant–worker data, combined with detailed German survey information on time-varying tasks performed by workers within their occupations, to document the impact of Germany's further globalization during the 2000s on the plant-internal division of labor and wage inequality within plant-occupations. Reduced form-evidence and results from simulating the structurally estimated model suggest that the internal division of labor and the associated wage inequality have played an important part in the rising wage inequality in Germany.

Our framework isolates the within-plant and within-occupation changes that globalization induces. While a dominant part of residual wage inequality materializes within plant-occupations, other forces are simultaneously at work. The employer-size wage premium contributes to wage dispersion (Helpman et al. 2017), and Card, Heining and Kline (2013) document the importance of that component for Germany's rise in wage inequality. Beyond residual wage inequality, Trottner (2019) considers the differential demand for high-skilled labor at large and globalized plants with non-homothetic production and the consequence for skill-related wage payments. While further globalization can reduce inequality under the employer-size wage premium channel (Helpman et al. 2017), the skill-demand effect under non-homothetic production (Trottner 2019) and our residual-wage effect under internal specialization predict higher inequality from globalization. A joint consideration of those alternative mechanisms remains for future research.

[24] Card, Heining and Kline (2013, Table 1b of the working paper version) report an increase of the standard deviation of the log real daily wage for German male workers by about 17 percent over 10 year (from .458 in 1999 to .535 in 2009). Our simulation would suggest an increase of the within-plant-occupation component for wages (not log wages) by about 40 percent over 13 years.

References

Abowd, John M., Francis Kramarz, David N. Margolis, and Kenneth R. Troske. 2001. "The Relative Importance of Employer and Employee Effects on Compensation: A Comparison of France and the United States." *Journal of the Japanese and International Economies*, 15(4): 419–436.

Acemoglu, Daron, and David Autor. 2011. "Skills, Tasks and Technologies: Implications for Employment and Earnings." In *Handbook of labor economics*. Vol. 4B, , ed. David Card and Orley Ashenfelter, Chapter 12, 1043–1171. Amsterdam, New York and Oxford:Elsevier Science, North-Holland.

Amiti, Mary, and Christopher A. Pissarides. 2005. "Trade and Industrial Location with Heterogeneous Labor." *Journal of International Economics*, 67(2): 392–412.

Amiti, Mary, and Donald R. Davis. 2012. "Trade, Firms, and Wages: Theory and Evidence." *Review of Economic Studies*, 79(1): 1–36.

Armenter, Roc, and Miklos Koren. 2015. "Economies of Scale and the Size of Exporters." *Journal of the European Economic Association*, 13(3): 482–511.

Aumann, Robert J., and Lloyd S. Shapley. 1974. *Values of non-atomic games.* Princeton, New Jersey:Princeton University Press.

Autor, David H., David Dorn, and Gordon H. Hanson. 2013. "The China Syndrome: Local Labor Market Effects of Import Competition in the United States." *American Economic Review*, 103(6): 2121–68.

Autor, David H., Lawrence F. Katz, and Melissa S. Kearney. 2006. "The Polarization of the U.S. Labor Market." *American Economic Review: Papers and Proceedings*, 96(2): 189–194.

Aw, Bee Yan, Mark J. Roberts, and Daniel Yi Xu. 2011. "R&D Investment, Exporting, and Productivity Dynamics." *American Economic Review*, 101(4): 1312–1344.

Barron, John M., Dan A. Black, and Mark A. Loewenstein. 1989. "Job Matching and On-the-Job Training." *Journal of Labor Economics*, 7(1): 1–19.

Baumgarten, Daniel. 2013. "Exporters and the Rise in Wage Inequality: Evidence from German Linked Employer-Employee Data." *Journal of International Economics*, 90(1): 201–217.

Becker, Gary S., and Kevin M. Murphy. 1992. "The Division of Labor, Coordination Costs, and Knowledge." *Quarterly Journal of Economics*, 107(4): 1137–1160.

Becker, Sascha O., and Marc-Andreas Muendler. 2015. "Trade and Tasks: An Exploration Over Three Decades in Germany." *Economic Policy*, 30(84): 589–641.

Blinder, Alan S. 2006. "Offshoring: The Next Industrial Revolution?" *Foreign Affairs*, 85(2): 113–28.

Bloom, Nicholas, and John Van Reenen. 2011. "Human Resource Management and Productivity." In *Handbook of Labor Economics*. Vol. 4B of *Handbooks in Economics*, , ed. Orley Ashenfelter and David Card, Chapter 19, 1697–1767. Amsterdam:Elsevier.

Bombardini, Matilde, Gianluca Orefice, and Maria D. Tito. 2019. "Does Exporting Improve Matching? Evidence from French Employer-Employee Data." *Journal of International Economics*, 117: 229–241.

Brugemann, Bjorn, Pieter Gautier, and Guido Menzio. 2015. "Intra Firm Bargaining and Shapley Values." *NBER Working Paper*, 21508.

Burdett, Kenneth, and Dale T. Mortensen. 1998. "Wage Differentials, Employer Size, and Unemployment." *International Economic Review*, 39(2): 257–73.

Burgess, Simon, Carol Propper, Marisa Ratto, Stephanie von Hinke, Kessler Scholder, and Emma Tominey. 2010. "Smarter Task Assignment or Greater Effort: The Impact of Incentives on Team Performance." *Economic Journal*, 120(547): 968–89.

Bustos, Paula. 2011. "Trade Liberalization, Exports, and Technology Upgrading: Evidence on the Impact of MERCOSUR on Argentinian Firms." *American Economic Review*, 101(1): 304–40.

Cahuc, Pierre, Fabien Postel-Vinay, and Jean Marc Robin. 2006. "Wage Bargaining with On-the-Job Search: Theory and Evidence." *Econometrica*, 74(2): 323–64.

Caliendo, Lorenzo, and Esteban Rossi-Hansberg. 2012. "The Impact of Trade on Organization and Productivity." *Quarterly Journal of Economics*, 127(3): 1393–1467.

Caliendo, Lorenzo, Ferdinando Monte, and Esteban Rossi-Hansberg. 2015. "The Anatomy of French Production Hierarchies." *Journal of Political Economy*, 123(4): 809–52.

Card, David, Jörg Heining, and Patrick Kline. 2013. "Workplace Heterogeneity and the Rise of West German Wage Inequality." *Quarterly Journal of Economics*, 128(3): 967–1015.

Carson, Richard T., and Yixiao Sun. 2007. "The Tobit Model with a Non-zero Threshold." *Econometrics Journal*, 10(3): 488–502.

Chaney, Thomas. 2008. "Distorted Gravity: The Intensive and Extensive Margins of International Trade." *American Economic Review*, 98(4): 1707–21.

Chaney, Thomas, and Ralph Ossa. 2013. "Market Size, Division of Labor, and Firm Productivity." *Journal of International Economics*, 90(1): 177–80.

Clerides, Sofronis K., Saul Lach, and James R. Tybout. 1998. "Is Learning by Exporting Important? Micro-dynamic Evidence from Colombia, Mexico, and Morocco." *Quarterly Journal of Economics*, 113(3): 903–47.

Coşar, A. Kerem, Nezih Guner, and James Tybout. 2016. "Firm Dynamics, Job Turnover, and Wage Distributions in an Open Economy." *American Economic Review*, 106(3): 625–63.

Costinot, Arnaud, and Jonathan Vogel. 2010. "Matching and Inequality in the World Economy." *Journal of Political Economy*, 118(4): 747–86.

Crespi, Gustavo, Chiara Criscuolo, and Jonathan Haskel. 2008. "Productivity, Exporting and the Learning-by-Exporting Hypothesis: Direct Evidence from U.K. Firms." *Canadian Journal of Economics*, 41(2): 619–638.

Cunat, Vicente, and Maria Guadalupe. 2009. "Globalization and the Provision of Incentives inside the Firm: The Effect of Foreign Competition." *Journal of Labor Economics*, 27(2): 179–212.

Dauth, Wolfgang, Sebastian Findeisen, and Jens Suedekum. 2014. "The Rise of the East and the Far East: German Labor Markets and Trade Integration." *Journal of the European Economic Association*, 12(6): 1643–75.

Davidson, Carl, Fredrik Heyman, Steven Matusz, Fredrik Sjöholm, and Susan Chun Zhu. 2014. "Globalization and Imperfect Labor Market Sorting." *Journal of International Economics*, 94(2): 177–94.

Davidson, Carl, Steven J. Matusz, and Andrei Shevchenko. 2008. "Globalization and Firm Level Adjustment with Imperfect Labor Markets." *Journal of International Economics*, 75(2): 295–309.

Davis, Donald R., and James Harrigan. 2011. "Good Jobs, Bad Jobs, and Trade Liberalization." *Journal of International Economics*, 84(1): 26–36.

Dhingra, Swati, and John Morrow. 2016. "Monopolistic Competition and Optimum Product Diversity Under Firm Heterogeneity." London School of Economics, unpublished manuscript.

Eaton, Jonathan, Samuel Kortum, and Francis Kramarz. 2015. “Firm-to-Firm Trade: Imports, Exports, and the Labor Market.” Brown University, unpublished manuscript.

Eckel, Carsten. 2009. “International Trade, Flexible Manufacturing, and Outsourcing.” *Canadian Journal of Economics*, 42(4): 1449–72.

Eeckhout, Jan, and Philipp Kircher. 2011. “Identifying Sorting–In Theory.” *Review of Economic Studies*, 78(3): 872–906.

Egger, Hartmut, and Udo Kreickemeier. 2009. “Firm Heterogeneity and the Labor Market Effects of Trade Liberalization.” *International Economic Review*, 50(1): 187–216.

Egger, Hartmut, Peter Egger, and Udo Kreickemeier. 2013. “Trade, Wages, and Profits.” *European Economic Review*, 64: 332–350.

Feenstra, Robert C., and Gordon H. Hanson. 1999. “The Impact of Outsourcing and High-Technology Capital on Wages: Estimates for the United States, 1979-1990.” *Quarterly Journal of Economics*, 114(3): 907–40.

Felbermayr, Gabriel, and Julien Prat. 2011. “Product Market Regulation, Firm Selection, and Unemployment.” *Journal of the European Economic Association*, 9(2): 278–317.

Fernandes, Ana M., Peter J. Klenow, Sergii Meleshchuk, Martha D. Pierola, and Andrés Rodríguez-Clare. 2018. “The Intensive Margin in Trade.” *NBER Working Paper*, 25195.

Goos, Maarten, Alan Manning, and Anna Salomons. 2009. “Job Polarization in Europe.” *American Economic Review*, 99(2): 58–63.

Greene, William H. 2012. *Econometric Analysis.* . 7th ed., Upper Saddle River, New Jersey:Prentice Hall.

Grossman, Gene M., and Esteban Rossi-Hansberg. 2008. “Trading Tasks: A Simple Theory of Offshoring.” *American Economic Review*, 98(5): 1978–97.

Grossman, Gene M., and Esteban Rossi-Hansberg. 2010. “External Economies and International Trade Redux.” *Quarterly Journal of Economics*, 125(2): 829–58.

Guadalupe, Maria. 2007. “Product Market Competition, Returns to Skill, and Wage Inequality.” *Journal of Labor Economics*, 25(3): 439–74.

Helpman, Elhanan, Marc J. Melitz, and Stephen R. Yeaple. 2004. "Export versus FDI with Heterogeneous Firms." *American Economic Review*, 94(1): 300–316.

Helpman, Elhanan, Oleg Itskhoki, and Stephen J. Redding. 2010. "Inequality and Unemployment in a Global Economy." *Econometrica*, 78(4): 1239–1283.

Helpman, Elhanan, Oleg Itskhoki, Marc-Andreas Muendler, and Stephen J. Redding. 2017. "Trade and Inequality: From Theory to Estimation." *Review of Economic Studies*, 84(1): 357–405.

Idson, Todd L., and Walter Y. Oi. 1999. "Workers Are More Productive in Large Firms." *American Economic Review: Papers and Proceedings*, 89(2): 104–08.

Kambourov, Gueorgui, and Iourii Manovskii. 2009. "Occupational Specificity of Human Capital." *International Economic Review*, 50(1): 63–115.

Katz, Lawrence F., and Kevin M. Murphy. 1992. "Changes in Relative Wages, 1963-1987: Supply and Demand Factors." *Quarterly Journal of Economics*, 107(1): 35–78.

Larch, Mario, and Wolfgang Lechthaler. 2011. "Multinational Firms and Labor Market Pooling." *Review of International Economics*, 19(4): 728–49.

Lazear, Edward P., and Kathryn L. Shaw. 2009. "Wage Structure, Raises, and Mobility: An Introduction to International Comparisons of the Structure of Wages Within and Across Firms." In *The Structure of Wages: An International Comparison*. , ed. Edward P. Lazear and Kathryn L. Shaw, Chapter 1, 1–57. Chicago and London:University of Chicago Press.

Leamer, Edward E., and Michael Storper. 2001. "The Economic Geography of the Internet Age." *Journal of International Business Studies*, 32(4): 641–65.

Legros, Patrick, and Andrew F. Newman. 2002. "Monotone Matching in Perfect and Imperfect Worlds." *Review of Economic Studies*, 69(4): 925–42.

Lemieux, Thomas. 2006. "Increasing Residual Wage Inequality: Composition Effects, Noisy Data, or Rising Demand for Skill?" *American Economic Review*, 96(3): 461–498.

Levy, Frank, and Richard J. Murnane. 2004. *The New Division of Labor.* Princeton:Princeton University Press.

Lopes de Melo, Rafael. 2013. "Firm Wage Differentials and Labor Market Sorting: Reconciling Theory and Evidence." University of Chicago, unpublished manuscript.

Maddala, G. S. 1986. “Disequilibrium, Self-selection, and Switching Models.” In *Handbook of Econometrics*. Vol. 3 of *Handbooks in Economics*, , ed. Zvi Griliches and Michael D. Intriligator, Chapter 28, 1633–88. Amsterdam, Oxford and Tokyo:North-Holland.

Melitz, Marc J. 2003. “The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity.” *Econometrica*, 71(6): 1695–1725.

Menezes-Filho, Naércio Aquino, Marc-Andreas Muendler, and Garey Ramey. 2008. “The Structure of Worker Compensation in Brazil, with a Comparison to France and the United States.” *Review of Economics and Statistics*, 90(2): 324–346.

Meyer, Margaret A. 1994. “The Dynamics of Learning with Team Production: Implications for Task Assignment.” *Quarterly Journal of Economics*, 109(4): 1157–84.

Murphy, Kevin M., and Robert H. Topel. 1985. “Estimation and Inference in Two-Step Econometric Models.” *Journal of Business and Economic Statistics*, 3(4): 370–79.

Postel-Vinay, Fabien, and Helene Turon. 2010. “On-the-Job Search, Productivity Shocks, and the Individual Earnings Process.” *International Economic Review*, 51(3): 599–629.

Postel-Vinay, Fabien, and Jean Marc Robin. 2002. “Equilibrium Wage Dispersion with Worker and Employer Heterogeneity.” *Econometrica*, 70(6): 2295–2350.

Rodríguez-Clare, Andrés. 2010. “Offshoring in a Ricardian World.” *American Economic Journal: Macroeconomics*, 2(2): 227–58.

Sampson, Thomas. 2014. “Selection into Trade and Wage Inequality.” *American Economic Journal: Microeconomics*, 6(3): 157–202.

Smith, Adam. 1776. *An inquiry into the nature and causes of the wealth of nations.* London:Printed for W. Strahan and T. Cadell. reprinted 1966 by A. M. Kelley, New York.

Song, Jae, David J. Price, Fatih Guvenen, Nicholas Bloom, and Till von Wachter. 2015. “Firming Up Inequality.” *NBER Working Paper*, 21199.

Stole, Lars A., and Jeffrey Zwiebel. 1996. “Intra-firm Bargaining under Non-binding Contracts.” *Review of Economic Studies*, 63(3): 375–410.

Sullivan, Paul. 2010. "Empirical Evidence on Occupation and Industry Specific Human Capital." *Labour Economics*, 17(3): 567–80.

Trottner, Fabian. 2019. "Who Gains From Scale? Trade and Wage Inequality Within and Between Firms." Princeton University, unpublished manuscript.

Appendix

A Empirical Appendix

A.1 Workplace operations

Using the BIBB-BAuA labor force survey data for the three waves 1999, 2006 and 2012, Table A1 shows the frequency of workplace operations (tasks) for the universe of workers and for subsamples of workers: high-earning workers with an above-median daily wage; experienced workers with an age of 45 years and older; supervisors and managers; and high-skilled workers with a college-qualifying secondary-education diploma (Abitur or equivalent). We inversely weight the frequency of worker observations by their sampling frequency to achieve representativeness. A comparison across the columns suggests that German workers engage in multitasking to a relatively similar extent across skill groups and layers of hierarchy, performing 6.7 tasks on average in any occupation and about 7.4 tasks in managerial occupations. Salient differences in task frequencies between skills and layers of hierarchy are observed for tasks such as "Train, Teach, Instruct, Educate" or "Apply Legal Knowledge" and "Gather Information, Develop, Research, Construct." Supervisors and managers perform those operations with a higher frequency of 12 additional percentage points or more compared to the worker population. Managers exhibit higher frequencies in a majority of tasks, with the notable exception of typically more manual-work intensive operations such as "Manufacture, Produce Goods" as well as "Repair, Maintain" and "Transport, Store, Dispatch."

A.2 Comparison to GSOEP data

To gauge the plausibility of our multi-tasking measures, we seek a comparison to an alternative German data source. In 2013, the German Socioeconomic Panel (GSOEP), a longitudinal survey similar to the U.S. Panel Study of Income Dynamics (PSID), included questions on a total of 23 workplace operations: 1. Making, processing or assembling things; 2. Building, installing, or fitting things/objects; 3. Operating, controlling, setting up or maintaining; 4. Repairing or maintaining things/objects, renovating or modernizing buildings; 5. Selling; 6. Buying, purchasing; 7. Advertising, marketing, public relations; 8. Consulting and informing; 9. Organizing, planning, coordinating, managing and preparation of work processes; 10. Collecting information, researching, documenting, analyzing; 11. Measuring, checking, testing, quality control; 12. Designing, developing, researching, constructing, shaping; 13. Educating, teaching, nurturing; 14. Entertaining, accommodating, preparing food; 15. Nursing, caring, coaching, healing; 16. Securing, protecting, monitoring, directing traffic; 17. Clean-

Table A1: Frequency of Workplace Operations

Workplace Operations (Tasks)		Subsamples			
		high w	age $\geq$45	skilled	manager
1. Manufacture, Produce Goods	0.185	0.182	0.171	0.138	0.139
2. Repair, Maintain	0.360	0.350	0.344	0.286	0.310
3. Entertain, Accommodate, Prepare Foods	0.281	0.247	0.256	0.231	0.297
4. Transport, Store, Dispatch	0.451	0.435	0.452	0.361	0.391
5. Measure, Inspect, Control Quality	0.602	0.624	0.599	0.631	0.638
6. Gather Information, Develop, Research, Construct	0.748	0.819	0.785	0.907	0.900
7. Purchase, Procure, Sell	0.462	0.476	0.462	0.475	0.509
8. Program a Computer	0.102	0.123	0.091	0.163	0.149
9. Apply Legal Knowledge	0.526	0.591	0.596	0.681	0.654
10. Consult and Inform	0.844	0.883	0.856	0.938	0.938
11. Train, Teach, Instruct, Educate	0.509	0.566	0.520	0.646	0.643
12. Nurse, Look After, Cure	0.265	0.263	0.268	0.285	0.342
13. Advertise, Promote, Conduct Marketing and PR	0.397	0.454	0.417	0.541	0.516
14. Organize, Plan, Prepare Others' Work	0.655	0.705	0.645	0.765	0.766
15. Control Machinery and Technical Processes	0.352	0.347	0.331	0.283	0.313
Total Number of Tasks	6.667	7.010	6.750	7.286	7.442

Source: BIBB-BAuA 1999, 2006 and 2012 (inverse sampling weights).
Note: Frequencies of performing a workplace operation (task) at the worker level. Subsample (1): workers with above-median daily wage; (2): workers 45 years old and older; (3): high-skilled workers (Abitur or equivalent); (4): supervisors and managers.

ing, clearing, recycling; 18. Working with computers; 19. Packing, transporting, storing, shipping, delivering; 20. Writing/reading texts/documents/e-mails, editing forms; 21. Calculating, booking; 22. Reporting, publishing, entertaining, presenting; 23. Sorting, stocking, ticketing. We do not attempt to map those tasks into the 15 tasks from the BIBB-BAuA data; instead we proceed as before and simply count up the tasks per worker under the assumption that they form an exhaustive set of observable tasks within the survey. We consider a task as being performed in the GSOEP data 2013 if the worker reports that he or she conducts the workplace operation more than half of the time or almost always.

We select three predictors that are observed in both the GSOEP survey in 2013 and the BIBB-BAuA data in 2012 in a similar way: years of schooling (which we transform in BIBB-BAuA to become similar to the GSOEP convention), gross monthly income in Euros (observed in intervals in the BIBB-BAuA data), and the reported weekly work hours. Table A2 shows the results. In both data sets, educational attainment, income and work hours are individually positively associated with multi-tasking and statistically significantly so at the one-percent confidence level. Workers whose assignments require more multi-tasking are more educated, earn more, and work longer hours. The regression coefficients are of similar magnitude in both dat; even the measures of goodness of

Table A2: Worker-level Predictors of the Number of Tasks

	Dependent variable: Number of tasks				
	(1)	(2)	(3)	(4)	(5)
			GSOEP (up to 23 tasks)		
Years of education	0.104***			0.0583*	-0.0622
	(0.0304)			(0.0345)	(0.0440)
Gross monthly income		0.000232***		0.0000505	0.0000276
		(0.0000477)		(0.0000640)	(0.0000756)
Weekly work hours			0.0413***	0.0348***	0.0310***
			(0.00605)	(0.00763)	(0.00835)
FE					yes
R^2	0.013	0.029	0.052	0.057	0.221
Observations	864	801	848	788	788
			BIBB-BAuA (up to 15 tasks)		
Years of education	0.162***			0.139***	0.0518***
	(0.00791)			(0.0103)	(0.0113)
Gross monthly income		0.000236***		0.0000998***	0.000128***
		(0.0000477)		(0.0000148)	(0.0000148)
Weekly work hours			0.0447***	0.0382***	0.0465***
			(0.00240)	(0.00287)	(0.00279)
FE					yes
R^2	0.021	0.024	0.020	0.046	0.232
Observations	20,012	13,936	17,104	13,928	13,928

Sources: GSOEP 2013 and BIBB-BAuA 2012.
Notes: Number of tasks from count of reported individual tasks out of 23 in GSOEP 2013, out of 15 in BIBB-BAuA 2012. Years of education in BIBB-BAuA data transformed into GSOEP definition; gross monthly income in BIBB-BAuA reported in intervals; weekly work hours are reported actual hours in BIBB-BAuA. Occupations at the two-digit ISCO level in GSOEP and at the two-digit KldB-88 occupation group level in BIBB-BAuA. The fixed-effects (FE) specification conditions on *Bundesland*, industry and respective two-digit occupation. Significance levels: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

fit are closely similar. However, when including all regressors simultaneously and especially when conditioning on region (*Bundesland*), sector (39 longitudinally consistent industries) and occupation group effects (two-digit ISCO level in GSOEP and two-digit KldB-88 occupation group level in BIBB-BAuA), then the small number of only about 800 observations in GSOEP does not allow for statistically significant predictions except for the work hours predictor. In the BIBB-BAuA data in contrast, with roughly 14,000 valid observations in 2012, all three predictors remain statistically significant at the one-percent confidence level even after controlling for region, sector and occupation group effects.

Our empirical analysis and model emphasize the relationship between plant size and multi-tasking. To assess the similarity between GSOEP and BIBB-BAuA with respect to plant size, we use the same worker-level predictors as in Table A2 above to check the association with employment. In the GSOEOP, size categories of

Table A3: Worker-level Predictors of Plant Size

Dependent variable: Plant size (midpoint of respective employment category)	(1)	(2)	(3)	(4)	(5)
			GSOEP (up to 23 tasks)		
Years of education	205.1***			25.62	-139.2
	(58.59)			(67.45)	(85.63)
Gross monthly income		0.591***		0.693***	0.545***
		(0.0918)		(0.125)	(0.147)
Weekly work hours			30.47**	-22.27	-8.038
			(11.96)	(14.93)	(16.26)
FE					yes
R^2	0.014	0.049	0.008	0.056	0.228
Observations	864	801	848	788	788
			BIBB-BAuA (up to 15 tasks)		
Years of education	59.35***			-4.478	37.69***
	(7.573)			(10.05)	(11.80)
Gross monthly income		0.354***		0.350***	0.245***
		(0.0124)		(0.0146)	(0.0155)
Weekly work hours			33.66***	2.632	0.748
			(0.00150)	(0.00174)	(2.974)
FE					yes
R^2	0.003	0.057	0.011	0.057	0.142
Observations	18,881	13,246	16,185	13,238	13,238

Sources: GSOEP 2013 and BIBB-BAuA 2012, using LIAB 2013 and 2012 to compute the respective employment category midpoints. *Notes*: Dependent variable are the LIAB employment averages per size category in 2012 (for BIBB-BAuA) and 2013 (for GSOEP). Years of education in BIBB-BAuA data transformed into GSOEP definition; gross monthly income in BIBB-BAuA reported in intervals; weekly work hours are reported actual hours in BIBB-BAuA. Occupations at the two-digit ISCO level in GSOEP and at the two-digit KldB-88 occupation group level in BIBB-BAuA. The fixed-effects (FE) specification conditions on *Bundesland*, industry and respective two-digit occupation. Significance levels: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

plants are 1-4, 5-9, 10-19, 20-99, 100-199, 200-1999, and more than 2000 workers. In the BIBB-BAuA data, the size categories are 1-4, 5-9, 10-49, 50-99, 100-499, 500-999, and more than 1000 workers. To make the categories comparable, we compute the average employment midpoints within each range from the representative sample of plants in LIAB 2012 (for BIBB-BAuA) and 2013 (for GSOEP), and use those midpoints as the dependent variable in our descriptive regressions. Table A3 reports the results. All three predictors are positive and statistically significantly associated with plant size (at the one-percent confidence level) in both data, when used as individual predictors. More educated workers, higher-paid workers and workers with longer work weeks are employed at larger plants. Coefficients on educational attainment and income remain robustly positive in the BIBB-BAuA data also within region (*Bundesland*), sector and two-digit occupation group in a joint prediction, but not work hours. In the GSOEP data, the small observation numbers preserve only the positive association between a worker's income and the size of the worker's employer, while the other two predictors are no longer separately statistically significant when conditioning on region, sector and occupation group.

In summary, the GSOEP data for 2013 exhibit closely similar covariation patterns between main characteristics of workers and their jobs (educational attainment, pay, work hours) on the one-hand side and multitasking or employer size on the other hand. Importantly, the worker and job characteristics are positively associated with both multitasking and employer size.

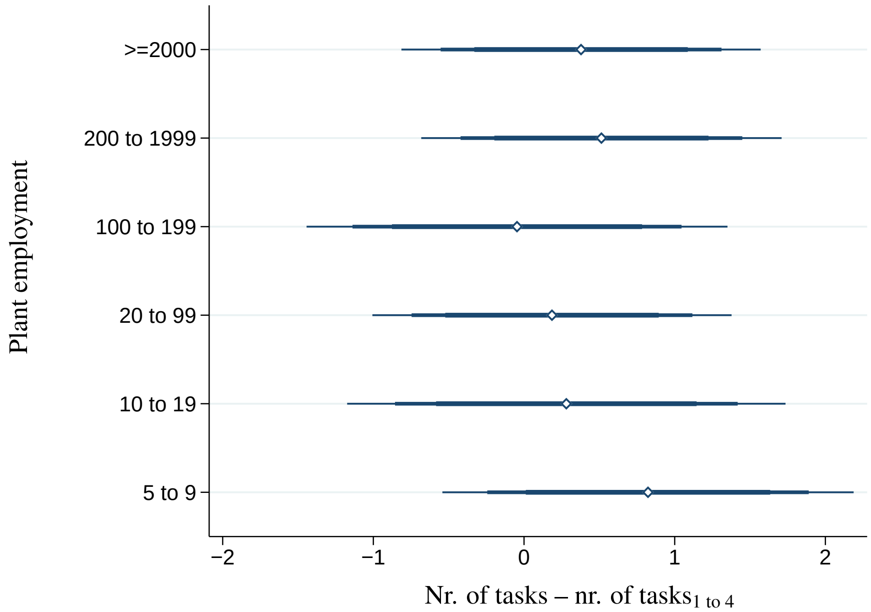
In the BIBB-BAuA data we find, however, that multitasking and employer size are negatively associated (Fact 2 and Figure 2) after conditioning on sector, region, occupation and worker characteristics. We construct a similar plot for the GSOEP 2013 data, now with the according GSOEP categories. Figure A1 depicts the graph. We broadly find an indication of a negative association between the number of tasks and plant employment up to a size of about 200 workers, but point estimates become inconclusive above for workforces above that size. Moreover, confidence bands are so wide that we cannot reject equal task numbers for any pair of size categories. Given the evidence from sample comparisons in Tables A2 and A3, a consistent interpretation is that the small sample size of only about 800 workers in GSOEP 2013 with reported tasks, compared to 13,000 in the BIBB-BAuA data, make confidence bands too wide for conclusive statistical inference. The wide confidence bands in the GSOEP sample nevertheless fail to reject the BIBB-BAuA evidence.

Other GSOEP-specific variables are statistically significantly associated with multitasking. For example, we find in the GSOEP 2013 data that an occupation's prestige (according to the KLAS scale magnitude) is stiatically significantly higher (at conventional confidence levels) for occupations in which workers report more multitasking and that Treiman's standard international occupation prestige score is also higher in occupations with more frequently reported multitasking. Similarly, the Erikson and Goldthorpe class category is lower in occupations, for which workers report more multitasking, again consistent with more prestigious jobs being multitasking jobs. Finally, workers who report more multitasking also report that they have more autonomy in their occupational activity. All these additional variables in the GSOEP 2013 data suggest that occupations with multitasking are more demanding, resulting in more worker autonomy and higher prestige.

A.3 Summary statistics

As described in Section 2.3, we combine the BIBB-BAuA labor force survey information with the LIAB linked plant–worker records. To include task information from BIBB-BAuA alongside the LIAB linked plant–worker data, we use the within occupation variance of log daily wage by plant, job experience, squared job experience, indicators for (i) gender, (ii) 7 schooling and vocational training indicators, (iii) 16 regions, (iv) 34 sectors, (v)

Figure A1: Number of Tasks per Occupation by Plant Employment in GSOEP 2013



Source: GSOEP 2013.
Notes: Prediction of number of tasks b within plant-occupation by plant employment category, controlling for sector, region, occupation and worker characteristics. Results are differences to smallest plant-size category (1 to 4 workers). Thick, medium, and thin lines represent the 99, 95, and 90 percent confidence intervals.

7 plant-size categories, and (vi) 335 occupations over the years 1992, 1999, 2006 and 2012. We predict using a probit estimation the probability that a worker reports performing a given task in the BIBB-BAuA sample and, using the same regressors, the probability that a worker in the LIAB linked plant–worker sample performs the task. Table A4 shows the raw data from LIAB as well as the imputed task information.

Table A4 also reports summary statistics on revenues and other relevant plant attributes from the combined LIAB and BIBB-BAuA data. Excluding plants for which we lack relevant information as well as plants with employment of two or fewer workers (for which we cannot compute meaningful measures of wage dispersion), our sample covers 116,931 plant-year observations, with 36,473 of these observations referring to exporters.

A.4 Residual wage inequality per plant-occupation by plant employment

We project the coefficient of variation CV of the (exponentiated) residual daily wages within a plant-occupation on sector, region, occupation and worker characteristics. Figure A2 plots the so normalized CV of daily wages within a plant-occupation in logs (on the horizontal axis after subtracting the coefficient of daily wage variation at plants with up to four workers) against numbers of tasks (on the vertical axis). We use the logarithm on the horizontal axis to treat idiosyncratic variability and to align the graph with structural estimation of eq. (42b).

Table A4: Descriptive Statistics for Combined Data

	Obs.	Mean	Median	StDev.	Min.	Max.
log Revenues	116,931	13.98	13.76	0.01	8.88	24.63
log Export revenues	36,473	17.48	17.33	0.03	10.92	29.01
Export indicator	116,933	0.17	0	.	0	1
Employment	116,933	18.48	6	0.12	3	44,419
log Daily wage	116,933	4.13	4.14	.	1.96	5.76
StDev Residual daily wage	116,933	23.07	19.82	0.19	.	1,167.85
CV Daily wage	116,933	0.32	0.31	.	.	4.02
Count 2-digit occupations n	116,933	3.5	2	0.01	1	63
Average number of tasks b	116,933	3.96	3.91	0.01	0.32	8.87
Normalized number of tasks $b/\tilde{z}$	116,933	0.36	0.36	.	0.03	0.70

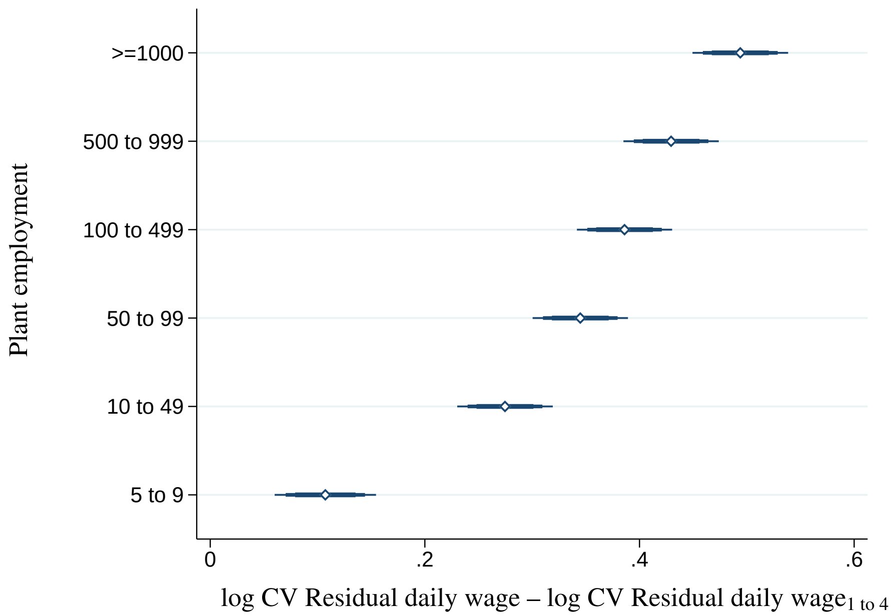
Sources: LIAB 1996-2014 and BIBB-BAuA 1992-2012. Sample restricted to plants with more than 2 full-time workers.
Notes: Descriptive statistics based on annual plant observations, using inverse probability weights to make plant sample representative of Germany economy, as suggested by the Research Data Centre at the IAB. *CV* is coefficient of variation of daily wage within a plant-occupation. *StDev Residual daily wage* measures the standard deviation of the (exponentiated) daily (log) wage residual from a Mincer regression (in logs), including demographic, education and tenure information as well as time, sector and region fixed effects and plant revenues.

There is a clearly positive relationship: wage variability within plant-occupations increases strongly with plant employment. Workers within the same occupation are subject to more wage inequality within their occupation at larger employers.

A.5 Predictors of within-plant daily wage dispersion

In Table A5, we repeat the empirical exercises from Table 6 in the text but now consider total rather than residual wages and specify as the dependent variable the coefficient of variation of wages as a measure of within-plant-occupation wage dispersion. OLS estimates in Columns 1 and 3 are closely comparable to those in Table 6 in the text. In Column 3, a 10 percent increase in the occupation count predicts a more than 14 percentage-point increase in the coefficient of variation of wages within plants. In Columns 4 and 5 of Table A5 we report estimates when instrumenting log revenues as the single endogenous regressor. Similar to Table 6 in the text, test statistics are consistent with the hypothesis that the interaction of log exports to China with the plant's lagged exporter dummy and the log of imports from China provide valid instruments, and the positive effect of revenues on wage variation within plant-occupations remains robust to the change in estimation strategy. In Columns 6 and 7 we treat all three regressors—log revenues, the log count of occupations and the interaction term of these two variables—as endogenous variables and instrument them with the variables reported in Table 5. In contrast with Table 6 in the text, all regressors now lose statistical significance.

Figure A2: Residual Wage Inequality per Plant-Occupation by Plant Employment



Source: LIAB 1996-2014.
Notes: Prediction of (log) coefficient of variation of daily wage residual (exponentiated Mincer residual) CV within plant-occupation by plant employment category, controlling for sector, region, occupation and worker characteristics. Results are differences to smallest plant-size category (1 to 4 workers). Thick, medium, and thin lines represent the 99, 95, and 90 percent confidence intervals.

B Mathematical Appendix

B.1 Derivation of expected profits in the closed economy

Using eqs. (14), (15), and (20) we can write profits in the closed economy as follows

$$\psi(\tilde{\varphi}) = \frac{r(\tilde{\varphi})}{\sigma}\frac{\gamma - \sigma + 1}{\gamma} - wf_0 = \frac{r(\tilde{\varphi}) - r(\varphi^*)}{r(\varphi^*)} wf_0.$$

Suppose elemental productivity $\tilde{\varphi}$ is Pareto distributed with shape parameter $\theta > \xi$. Substituting $r(\tilde{\varphi})/r(\varphi^*) = (\tilde{\varphi}/\varphi^*)^{\xi}$ from eq. (18), we can then compute

$$\int_{\varphi^*}^{\infty} \psi(\tilde{\varphi})\, \mathrm{d}G(\tilde{\varphi}) = wf_0(\varphi^*)^{\xi}\theta \int_{\varphi^*}^{\infty} \tilde{\varphi}^{\xi-\theta-1}\, \mathrm{d}\tilde{\varphi} - wf_0\theta \int_{\varphi^*}^{\infty} \tilde{\varphi}^{-\theta-1}\, \mathrm{d}\tilde{\varphi} = (\varphi^*)^{-\theta} wf_0 \frac{\xi}{\theta - \xi}.$$

Accounting for $1 - G(\varphi^*) = (\varphi^*)^{-\theta}$ then gives the respective expression (23) in the main text.

Table A5: Predictors of Within-Plant Daily Wage Dispersion

	Dependent variable: log CV Daily wage						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	OLS	OLS	OLS	IV	IV	IV	IV
log Count of occupations		-0.827***	1.425***			0.118	0.221
		(0.083)	(0.208)			(2.111)	(2.148)
log Revenues	0.086***	0.056***	0.067***	0.129***	0.127***	0.038	0.026
	(0.005)	(0.013)	(0.021)	(0.027)	(0.027)	(0.067)	(0.067)
log Revenues × log Count of occupations		-0.040***	-0.075**			0.003	-0.001
		(0.006)	(0.014)			(0.121)	(0.123)
Plant FE	no	no	yes	no	no	no	no
Hansen J (p-val.)					0.172		0.196
R^2	0.156	0.195	0.767				
Adj. R^2	0.156	0.195	0.688				
Observations	126,483	126,483	126,483	64,905	64,614	64,775	64,561

Sources: LIAB 1996-2014, all sectors. Plants with more than 2 full-time workers.
Notes: Specifications include time, region, and sector fixed effects. IV estimation is based on GMM. Standard errors clustered at the plant level in parentheses. Significance levels: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

B.2 Derivation of eqs. (26) and (27)

Suppose elemental productivity $\tilde{\varphi}$ is Pareto distributed with shape parameter $\theta > \xi$. In autarky equilibrium (denoted with superscript a), the average employment of production workers at a plant is

$$\int_{\varphi^{*,a}}^{\infty} \ell(\tilde{\varphi}) \frac{\mathrm{d}G(\tilde{\varphi})}{1 - G(\varphi^{*,a})} = \ell(\varphi^{*,a})(\varphi^{*,a})^{\theta-\xi}\theta \int_{\varphi^{*,a}}^{\infty} \tilde{\varphi}^{\xi-\theta-1}\,\mathrm{d}\tilde{\varphi} = \ell(\varphi^{*,a}) \frac{\theta}{\theta - \xi}$$

by eq. (19). The employment-share weighted average of the coefficient of variation of wages at the plant is then given by eq. (26). Using $CV_w(\tilde{\varphi})$ from eq. (22) and $\ell(\tilde{\varphi}) = (\tilde{\varphi}/\varphi^{*,a})^{\xi}\ell(\varphi^{*,a})$ from eq. (19) then establishes

$$\mathbb{V}_w^a = CV_w(\varphi^{*,a})^2 \frac{\theta - \xi}{(\beta - \eta)^2} \left\{ \beta^2 (\varphi^{*,a})^{\theta-\xi} \int_{\tilde{\varphi}}^{\infty} \tilde{\varphi}^{\xi-\theta-1}\,\mathrm{d}\tilde{\varphi} - 2\beta\eta(\varphi^{*,a})^{\theta+\frac{\xi}{\gamma}-\xi} \int_{\varphi^{*,a}}^{\infty} \tilde{\varphi}^{\xi-\frac{\xi}{\gamma}-\theta-1}\,\mathrm{d}\tilde{\varphi} \right.$$
$$\left. + \eta^2 (\tilde{\varphi}*)^{\theta+\frac{2\xi}{\gamma}-\xi} \int_{\varphi^{*,a}}^{\infty} \tilde{\varphi}^{\xi-\frac{2\xi}{\gamma}-\theta-1}\,\mathrm{d}\tilde{\varphi} \right\},$$

where $\beta \equiv (\xi f_0/\gamma)^{1/\gamma}$. Solving for the integral gives

$$\mathbb{V}_w^a = \frac{CV_w(\varphi^{*,a})^2}{[(\xi f_0/\gamma)^{1/\gamma} - \eta]^2} \left[\left(\frac{\xi f_0}{\gamma}\right)^{\frac{2}{\gamma}} - 2\eta \left(\frac{\xi f_0}{\gamma}\right)^{\frac{1}{\gamma}} \frac{\theta - \xi}{\theta - \xi + \xi/\gamma} + \eta^2 \frac{\theta - \xi}{\theta - \xi + 2\xi/\gamma} \right]$$

and eq. (27) in the text follows.

B.3 Derivation of expected profits in the open economy

Using eqs. (14), (15) and (20), we can write profits of non-exporters as $\psi_T^d(\tilde{\varphi}) = [r^d(\tilde{\varphi})/r^d(\varphi^*)]f_0 - f_0$, where $w = 1$ is the numéraire. Total profits of exporters are therefore given by

$$\psi_T^e(\tilde{\varphi}) = \left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}} \frac{r^d(\tilde{\varphi})}{\sigma}\frac{\gamma-\sigma+1}{\gamma} - f_0 - f_x = \left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}} \frac{r^d(\tilde{\varphi})}{r^d(\varphi^*)}f_0 - f_0 - f_x$$

by eqs. (14), (20), (28), and (29). Suppose elemental productivity $\tilde{\varphi}$ is Pareto distributed with shape parameter $\theta > \xi$. Plugging in for $r^d(\tilde{\varphi})/r^d(\varphi^*) = (\tilde{\varphi}/\varphi^*)^{\xi}$ from eq. (18) then allows us to compute

$$\begin{aligned}\int_{\varphi^*}^{\infty} \psi_T(\tilde{\varphi})\,\mathrm{d}G(\tilde{\varphi}) &= \int_{\varphi^*}^{\tilde{\varphi}_x^*} \psi_T^d(\tilde{\varphi})\,\mathrm{d}G(\tilde{\varphi}) + \int_{\tilde{\varphi}_x^*}^{\infty} \psi_T^e(\tilde{\varphi})\,\mathrm{d}G(\tilde{\varphi}) \\ &= (\varphi^*)^{-\xi} f_0\theta \int_{\varphi^*}^{\tilde{\varphi}_x^*} \tilde{\varphi}^{\xi-\theta-1}\,\mathrm{d}\tilde{\varphi} + \left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}} (\varphi^*)^{-\xi} f_0\theta \int_{\tilde{\varphi}_x^*}^{\infty} \tilde{\varphi}^{\xi-\theta-1}\,\mathrm{d}\tilde{\varphi} \\ &\quad - f_0\theta \int_{\varphi^*}^{\infty} \tilde{\varphi}^{-\theta-1}\,\mathrm{d}\tilde{\varphi} - f_x\theta \int_{\tilde{\varphi}_x^*}^{\infty} \tilde{\varphi}^{-\theta-1}\,\mathrm{d}\tilde{\varphi}\end{aligned}$$

Solving the integrals yields

$$\begin{aligned}\int_{\varphi^*}^{\infty} \psi_T(\tilde{\varphi})\,\mathrm{d}G(\tilde{\varphi}) &= f_0(\varphi^*)^{-\theta}\frac{\theta}{\theta-\xi}\left\{1 + \left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{\xi-\theta}\left[\left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}} - 1\right]\right\} \\ &\quad - (\varphi^*)^{-\theta} f_0\left[1 + \left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{-\theta}\frac{f_x}{f_0}\right],\end{aligned}$$

and eq. (33) in the text after using $1 - G(\varphi^*) = (\varphi^*)^{-\theta}$, $\chi = (\tilde{\varphi}_x^*/\varphi^*)^{-\theta}$ and eq. (32).

B.4 Derivation of eqs. (39) and (40)

Suppose elemental productivity $\tilde{\varphi}$ is Pareto distributed with shape parameter $\theta > \xi$. Average employment of production workers per plant in the open economy can then be computed as

$$\begin{aligned}\int_{\varphi^*}^{\infty} \ell_T(\tilde{\varphi})\frac{\mathrm{d}G(\tilde{\varphi})}{1-G(\varphi^*)} &= \int_{\varphi^*}^{\tilde{\varphi}_x^*} \ell_T^d(\tilde{\varphi})\frac{\mathrm{d}G(\tilde{\varphi})}{1-G(\varphi^*)} + \int_{\tilde{\varphi}_x^*}^{\infty} \ell_T^e(\tilde{\varphi})\frac{\mathrm{d}G(\tilde{\varphi})}{1-G(\varphi^*)} \\ &= \ell_T^d(\varphi^*)(\varphi^*)^{\theta-\xi}\theta\left[\int_{\varphi^*}^{\tilde{\varphi}_x^*} \tilde{\varphi}^{\xi-\theta-1}\,\mathrm{d}\tilde{\varphi} + (1+\tau^{1-\sigma})^{\frac{\xi}{\sigma-1}} \int_{\tilde{\varphi}_x^*}^{\infty} \tilde{\varphi}^{\xi-\theta-1}\,\mathrm{d}\tilde{\varphi}\right] \\ &= \ell_T^d(\varphi^*)\frac{\theta}{\theta-\xi}\left(1 + \frac{\chi f_x}{f_0}\right), \qquad \text{(B.1)}\end{aligned}$$

where the second equality follows from eq. (30) and the third equality from eq. (32). The economy-wide variance of wages can then be computed in analogy to the closed economy:

$$\mathbb{V}_w = \frac{\theta-\xi}{\theta\ell_T^d(\varphi^*)}\left(1+\frac{\chi f_x}{f_0}\right)^{-1}\left[\int_{\varphi^*}^{\tilde{\varphi}_x^*} CV_w^d(\tilde{\varphi})^2\ell_T^d(\tilde{\varphi})\frac{\mathrm{d}G(\tilde{\varphi})}{1-G(\varphi^*)}+\int_{\tilde{\varphi}_x^*}^{\infty} CV_w^e(\tilde{\varphi})^2\ell_T^e(\tilde{\varphi})\frac{\mathrm{d}G(\tilde{\varphi})}{1-G(\varphi^*)}\right].$$

We look at the integrals on the right-side separately. Following the derivation steps of the closed economy and defining $\beta \equiv (\xi f_0/\gamma)^{1/\gamma}$, we compute

$$\begin{aligned}\int_{\varphi^*}^{\tilde{\varphi}_x^*} CV_w^d(\tilde{\varphi})^2\ell_T^d(\tilde{\varphi})\frac{\mathrm{d}G(\tilde{\varphi})}{1-G(\varphi^*)} &= \ell_T^d(\varphi^*)\frac{CV_w^d(\varphi^*)^2}{(\beta-\eta)^2}\theta\left\{\beta^2(\varphi^*)^{\theta-\xi}\int_{\varphi^*}^{\tilde{\varphi}_x^*}\tilde{\varphi}^{\xi-\theta-1}\,\mathrm{d}\tilde{\varphi}\right.\\ &\qquad\left.-2\beta\eta(\varphi^*)^{\theta+\frac{\xi}{\gamma}-\xi}\int_{\varphi^*}^{\tilde{\varphi}_x^*}\tilde{\varphi}^{\xi-\theta-\frac{\xi}{\gamma}-1}\,\mathrm{d}\tilde{\varphi}+\eta^2(\varphi^*)^{\theta-\xi-\frac{2\xi}{\gamma}}\int_{\varphi^*}^{\tilde{\varphi}_x^*}\tilde{\varphi}^{\xi-\theta-\frac{2\xi}{\gamma}-1}\,\mathrm{d}\tilde{\varphi}\right\}\\ &= \ell_T^d(\varphi^*)\frac{CV_w^d(\varphi^*)^2}{(\beta-\eta)^2}\frac{\theta}{\theta-\xi}\left\{\left[\beta^2+2\beta\eta\frac{\theta-\xi}{\theta-\xi+\xi/\gamma}+\eta^2\frac{\theta-\xi}{\theta-\xi+2\xi/\gamma}\right]\left[1-\left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{\xi-\theta}\right]\right.\\ &\qquad+2\beta\eta\frac{\theta-\xi}{\theta-\xi+\xi/\gamma}\left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{\xi-\theta}\left[\left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{-\frac{\xi}{\gamma}}-1\right]\\ &\qquad\left.-\eta^2\frac{\theta-\xi}{\theta-\xi+2\xi/\gamma}\left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{\xi-\theta}\left[\left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{-\frac{2\xi}{\gamma}}-1\right]\right\}.\end{aligned}\tag{B.2}$$

For the second integral, we obtain

$$\begin{aligned}\int_{\tilde{\varphi}_x^*}^{\infty} CV_w^e(\tilde{\varphi})^2\ell_T^e(\tilde{\varphi})\frac{\mathrm{d}G(\tilde{\varphi})}{1-G(\varphi^*)} &= \ell_T^d(\varphi^*)\frac{CV_w^d(\varphi^*)^2}{(\beta-\eta)^2}\left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}}\theta\left\{\beta^2(\varphi^*)^{\theta-\xi}\int_{\tilde{\varphi}_x^*}^{\infty}\tilde{\varphi}^{\xi-\theta-1}\,\mathrm{d}\tilde{\varphi}\right.\\ &\qquad\left.-2\beta\eta(\varphi^*)^{\theta+\frac{\xi}{\gamma}-\xi}\int_{\tilde{\varphi}_x^*}^{\infty}\tilde{\varphi}^{\xi-\theta-\frac{\xi}{\gamma}-1}\,\mathrm{d}\tilde{\varphi}+\eta^2(\varphi^*)^{\theta-\xi-\frac{2\xi}{\gamma}}\int_{\tilde{\varphi}_x^*}^{\infty}\tilde{\varphi}^{\xi-\theta-\frac{2\xi}{\gamma}-1}\,\mathrm{d}\tilde{\varphi}\right\}\\ &= \ell_T^d(\varphi^*)\frac{CV_w^d(\varphi^*)^2}{(\beta-\eta)^2}\frac{\theta}{\theta-\xi}\left(1+\tau^{1-\sigma}\right)^{\frac{\xi}{\sigma-1}}\left\{\left[\beta^2+2\beta\eta\frac{\theta-\xi}{\theta-\xi+\xi/\gamma}+\eta^2\frac{\theta-\xi}{\theta-\xi+2\xi/\gamma}\right]\left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{\xi-\theta}\right.\\ &\qquad+2\beta\eta\frac{\theta-\xi}{\theta-\xi+\xi/\gamma}\left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{\xi-\theta}\left[\left(1+\tau^{1-\sigma}\right)^{-\frac{\xi}{\gamma(\sigma-1)}}\left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{-\frac{\xi}{\gamma}}-1\right]\\ &\qquad\left.-\eta^2\frac{\theta-\xi}{\theta-\xi+2\xi/\gamma}\left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{\xi-\theta}\left[\left(1+\tau^{1-\sigma}\right)^{-\frac{2\xi}{\gamma(\sigma-1)}}\left(\frac{\tilde{\varphi}_x^*}{\varphi^*}\right)^{-\frac{2\xi}{\gamma}}-1\right]\right\}.\end{aligned}\tag{B.3}$$

Substituting eqs. (B.2) and (B.3) into (B.1) and accounting for $\tilde{\varphi}_x^*/\varphi^* = \chi^{-1/\theta}$ and eqs. (27) and (32), we arrive at eq. (39) in the text with $V(\chi)$ given by eq. (40). From $\chi < 1$ it follows that $\chi^{\frac{\xi}{\theta\gamma}}-1 > \chi^{\frac{2\xi}{\theta\gamma}}-1$. Noting further that $\left(\frac{\xi f_0}{\gamma}\right)^{1/\gamma} > \eta$, it follows from eq. (40) that

$$2\left[1-\left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}}\chi^{\frac{\xi}{\theta\gamma}}\right] > \left[1-\left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)^{-\frac{2}{\gamma}}\chi^{\frac{2\xi}{\theta\gamma}}\right] \tag{B.4}$$

or, equivalently,

$$2 > \left[1+\left(1+\chi^{\frac{\xi}{\theta}}\frac{f_x}{f_0}\right)^{-\frac{1}{\gamma}}\chi^{\frac{\xi}{\theta\gamma}}\right] \tag{B.5}$$

is sufficient for $V(\chi) > 0$ because $[1+\chi^{\xi/\theta}f_x/f_0]^{-1/\gamma}\chi^{\xi/(\theta\gamma)} < 1$.

C Extension to Stole-Zwiebel Bargaining

A plant ω's revenues are

$$r(\omega) = A^{\frac{1}{\sigma}}\left\{\tilde{\varphi}(\omega)\tilde{z}(\omega)[n(\omega)+1]\exp\left[\frac{1}{n(\omega)+1}\sum_{j=1}^{n(\omega)+1}\ln\left(\int_0^{b(\omega)}\ell_j(i,b(\omega))\,\lambda(i,b(\omega))\,\mathrm{d}i\right)\right]\right\}^{1-\frac{1}{\sigma}} \tag{C.1}$$

where $n(\omega)+1$ is the plant's occupation count, $b(\omega)$ is its task range per occupation, $\tilde{z}(\omega)$ is its full task range required for production, $\ell_j(i,b(\omega))$ is employment of workers of type (core ability) i in the task interval of job j, $\lambda(i,b)$ is the labor efficiency of type-i workers in a task interval with range $b(\omega)$, $\tilde{\varphi}(\omega)$ is plant-specific elemental productivity, and A is a constant that captures demand shifters. We assume that hiring is subject to search frictions and wage setting is the result of individual bargaining of the employer with a continuum of workers as derived by Stole and Zwiebel (1996). We can distinguish $n(\omega)+1$ groups of workers by their occupation j and characterize the bargaining outcome at the employer with two equations of the following form:[25]

$$\begin{aligned}\psi(\omega) &= \frac{1}{\ell(\omega)}\int_0^{\ell(\omega)} r\left[k\,\mathbf{s}(\omega)\right]\,\mathrm{d}k, &\text{(C.2)}\\ \frac{\partial\psi(\omega)}{\partial\ell_j(i,b)} &= w_j(i,b), &\text{(C.3)}\end{aligned}$$

where $\psi(\omega)$ is the plant's operating profit (and equal to each worker's share in revenues under Stole and Zwiebel (1996) bargaining), k denotes a proportional increase in employment symmetrically over all the plant's occupations $n(\omega)+1$, $r[\cdot]$ are the plant's revenues as a function of its occupational employment-shares vector $\mathbf{s}(\omega)$, $\ell_j(\omega) \equiv \int_0^b \ell_j(k,b)\mathrm{d}k$ is employment in a task interval with range b, $\ell(\omega) \equiv \sum_{j=1}^{n(\omega)+1}\ell_j(\omega)$ is the plant's total

[25] Existence and uniqueness of this solution follow from Theorem 9 in Stole and Zwiebel (1996).

employment, $w_j(i,b)$ is type-i worker's wage in an occupation j with task range b, and each occupation j's employment share at the plant $s_j(\omega) \equiv \ell_j(\omega)/\ell(\omega)$ enters the occupational employment-share vector

$$\mathbf{s}(\omega) \equiv (s_1(\omega), ..., s_{n(\omega)+1}(\omega))^T.$$

The first eq. (C.2) links the result of the employer-worker bargaining outcome to the Aumann-Shapley value (Aumann and Shapley 1974).[26] Intuitively, eq. (C.2) assures that the employer's entire revenues are fully exhausted through bargaining. By eq. (C.3), the employer and every worker split the surplus equally so that revenues are divided by the mass of all workers and the employer but, since the employer is non-atomic, it does not affect the mass $\ell(\omega)$ and revenues are divided by $\ell(\omega)$. The plant's operating profit is therefore $\psi(\omega)$.

Employers allocate workers symmetrically over the task range of jobs, so $\ell_j(i, b(\omega)) = \ell_j(0, b(\omega)) = \ell_j(b(\omega), b(\omega))$ for all $i \in (0, b(\omega))$, we obtain

$$\psi(\omega) = \frac{\sigma}{2\sigma - 1} r(\omega), \tag{C.4}$$

where revenues $r(\omega)$ are restated in the notation from (C.1). Substitution into eq. (C.3) yields

$$w_j(i,b) = \frac{\sigma - 1}{2\sigma - 1} \frac{r(\omega)\lambda(i,b)}{\lambda_j(\omega)\ell_j(\omega)} \frac{1}{n(\omega)+1}, \tag{C.5}$$

where occupation-level labor efficiency is

$$\lambda_j(\omega) \equiv \frac{1}{\ell_j(\omega)} \int_0^b \ell_j(k,b)\lambda(k,b)\,\mathrm{d}k.$$

Combining eqs. (C.4) and (C.5) establishes

$$\frac{w_j(i,b)}{\lambda_j(i,b)} \lambda_j(\omega)\ell_j(\omega) = \frac{\sigma - 1}{\sigma} \frac{\psi(\omega)}{n(\omega)+1}. \tag{C.6}$$

[26] Brugemann, Gautier and Menzio (2015) point to a conceptual problem with Stole and Zwiebel bargaining because, unlike the argument in the original paper, the order in which workers bargain with the employer does matter for the payoff they receive. As a result, the outcome of the Stole and Zwiebel game differs from the equilibrium prescribed by Aumann-Shapley values. As a remedy, Brugemann, Gautier and Menzio (2015) propose to replace the Stole and Zwiebel game with by a Rolodex game, by which workers are randomly picked to bargain from a Rolodex shuffle, so as to anchor the bargaining outcome of Stole and Zwiebel (1996) in non-cooperative game theory. The outcome of the Rolodex game remains the same as the one posited in Stole and Zwiebel (1996), so we acknowledge the correction but refer to Stole and Zwiebel (1996) when discussing the solution concept.

Every worker in occupation j therefore receives the same wage per efficiency unit of labor:

$$w_j^e(\omega) \equiv w_j(i,b)/\lambda_j(i,b),$$

and this condition is sufficient to guarantee a symmetric allocation of workers over their task range, if worker types are uniformly distributed over the employer's full task range $\tilde{z}(\omega)$ and an employer gets a random draw of the workers.

With the bargaining solution at hand, we can turn to hiring. We assume that hiring takes place prior to the wage negotiation and involves the costs of advertising jobs for employers. Risk-neutral workers apply for those jobs that promise the highest expected return given the imperfect signal they receive regarding their suitability for executing the tasks required in an occupation, according to a posted vacancy. We assume that the signal the workers receive through a vacancy posting only informs them about whether their core ability i falls within the respective task range, but does not provide further details regarding their core ability's exact position within the task interval. Vacancy posting costs are given by sb, where s is a service fee equal to the return on labor used for providing services. Following Helpman, Itskhoki and Redding (2010), we propose that vacancy posting costs are positively related to labor market tightness, and decrease in the unemployment rate u. The ex ante probability of workers to be matched with an employer is $(1-u)$. Vacancy posting costs are specified to equal $sb = sB(1-u)^\varepsilon$, where $B > 1$ is a constant parameter and $\varepsilon > 0$ is the elasticity of vacancy posting costs with respect to the employment rate. The hiring problem of the employer can therefore be stated as follows:

$$\max_{\ell_j(\omega)} \psi(\omega) - \sum_{j=1}^{n(\omega)+1} sB(1-u)^\varepsilon \ell_j(\omega) - s\lambda(\omega)^\gamma - sf_0. \tag{C.7}$$

The first-order condition of this optimization problem is equivalent to

$$[n(\omega)+1]\ell_j(\omega) = \frac{\sigma-1}{\sigma}\frac{\tilde{\psi}(\omega)}{sB(1-u)^\varepsilon} = \ell(\omega), \tag{C.8}$$

so that employers hire the same number of workers for all of their (symmetric) jobs. Combining the results yields

$$r(\omega) = A\,[mc(\omega)]^{1-\sigma}, \qquad mc(\omega) \equiv \frac{w}{\tilde{\varphi}(\omega)\,\{\eta + \pi[\nu n(\omega)+1]\}}, \tag{C.9}$$

$$\lambda(\omega) = \frac{1}{b(\omega)}\int_0^{b(\omega)} \lambda(k, b(\omega))\mathrm{d}k = \frac{\eta}{\tilde{z}} + \frac{\pi}{b(\omega)} = \frac{1}{\tilde{z}}\left\{\eta + \pi[\nu n(\omega)+1]\right\}, \tag{C.10}$$

$$\psi(\omega) = \frac{r(\omega)}{2\sigma - 1} - s\left\{\eta + \pi[\nu n(\omega) + 1]\right\}^{\gamma} - sf_0, \quad \text{and} \quad \lambda(\omega)w^e(\omega) = sB(1-u)^{\varepsilon} = \frac{\sigma - 1}{2\sigma - 1}\frac{r(\omega)}{l(\omega)} \equiv w. \tag{C.11}$$

The optimal count of occupations is then determined by maximizing $\psi(\omega)$ with respect to $n(\omega)$, which yields

$$r(\omega)\frac{\sigma - 1}{\gamma(2\sigma - 1)} = s\left\{\eta + \pi[\nu n(\omega) + 1]\right\}^{\gamma}. \tag{C.12}$$

The zero-cutoff profit condition then establishes

$$r(\omega) = sf_0\frac{\gamma(2\sigma - 1)}{\gamma - \sigma + 1} \qquad \Longleftrightarrow \qquad \frac{f_0(\sigma - 1)}{\gamma - \sigma + 1} = \left\{\eta + \pi[\nu n(\omega) + 1]\right\}^{\gamma}. \tag{C.13}$$

The rest of the analysis follows as in the main text in Section 3.

However, the derivations of equilibrium in the closed (Section 4) and open economy (Section 5) differ because, under Stole-Zwiebel bargaining, there is unemployment in equilibrium. Risk-neutral workers must be indifferent between applying for jobs in the production sector (with an ex-ante expected wage w) or providing service inputs at a pay s (which is associated with self-employment so that production workers do not switch to the service sector ex post). The unemployment rate (of production workers) is then given by the requirement that $s = (1-u)w$, establishing $B(1-u)^{1+\varepsilon}$ from eq. (C.11). This equal-pay condition implies for the employment rate $1 - u = B^{-1/(1-\varepsilon)} < 1$, which is a constant in our model because labor is used for production as well as services provision.[27] Finally, we need to check that the wages paid to production workers are (weakly) higher than their expected income outside the job $(1-u)w$. The wage of the least productive worker at employer ω is given by

$$w(0, b(\omega)) = \frac{w\lambda(0, b(\omega))}{\lambda(\omega)} = w\frac{\eta + 2[\nu n(\omega) + 1]}{\eta + \pi[\nu n(\omega) + 1]} \equiv \underline{w}(n(\omega)). \tag{C.14}$$

Note that $\underline{w}'(n(\omega)) < 0$ and that $\lim_{n(\omega)\to\infty} \underline{w}((n(\omega)) = 2w/\pi$. It follows that $\underline{w}(n(\omega)) > (1-u)w$ is satisfied for all employers if $B < (\pi/2)^{1+\varepsilon}$. In this case, no workers who is matched to a production job will quit ex post. Therefore, we can maintain the parameter constraint $B \in (1, (\pi/2)^{1+\varepsilon})$ throughout our extended analysis.

[27] Alternatively, we could use final output as a services input. However, in that case, we would need to constrain the external economies of scale in order to ensure a stable interior solution (see Felbermayr and Prat 2011).

D Structural Estimation Model

D.1 From theory to structural estimation

Starting point is the equation system (42) in the main text, from which we drop time subscripts for clarity:

$$
\begin{aligned}
\ln r(\omega) &= \alpha_0 + \alpha_1 \mathbf{1}_x(\omega) + \xi \ln \tilde{\varphi}(\omega), \\
\ln CV(\omega) + \ln b(\omega)/x(\omega) &= \beta_0 - (1/\gamma)\, \ln r(\omega) + \ln \tilde{\zeta}(\omega), \\
\mathbf{1}_x(\omega) = 1 &\Leftrightarrow \delta_0 \geq \ln \tilde{f}_x(\omega) - \xi \ln \tilde{\varphi}(\omega), \\
\ln r(\omega) = . &\Leftrightarrow \xi \ln \tilde{\varphi}(\omega) < a.
\end{aligned}
$$

The estimation model in (42) captures an endogenous switching model, with two outcomes—log revenues $y(\omega) = \ln r(\omega)$ in (42a) and the composite coefficient of wage variation plus the normalized task number per occupation $w(\omega) = \ln CV(\omega) + \ln b(\omega)/x(\omega)$ in (42b)—and a selection equation (42c) for endogenous switching. Equation (42d) represents a further selection equation for presence in the sample and captures the important insight from the Melitz model that only sufficiently productive plants start production. The maintained assumption is that we do not observe plants with productivity below a truncation threshold in our data. This refers to a problem that can be addressed in the spirit of a Tobit model. However, our selection into sample presence differs from the conventional Tobit model because the censoring of our data is due to a threshold for the unobservable disturbance and not a threshold for an observable variable.

The structure of our model can be simplified to:

$$
y(\omega) = \begin{cases} \mu_Y^e + u(\omega) & \text{if } I(\omega) = 1 \\ \mu_Y^d + u(\omega) & \text{if } I(\omega) = 0 \end{cases}, \tag{D.2a}
$$

$$
w(\omega) = \begin{cases} \mu_W^e - (1/\gamma)u(\omega) + v(\omega) & \text{if } I(\omega) = 1 \\ \mu_W^d - (1/\gamma)u(\omega) + v(\omega) & \text{if } I(\omega) = 0 \end{cases}, \tag{D.2b}
$$

$$
I(\omega) = \begin{cases} 1 & \text{if } \mu_X + u(\omega) - x(\omega) \geq 0 \\ 0 & \text{if } \mu_X + u(\omega) - x(\omega) < 0 \end{cases}, \tag{D.2c}
$$

where $\mu_Y^e \equiv \alpha_0 + \alpha_1$, $\mu_Y^d \equiv \alpha_0$, $u(\omega) \equiv \xi \ln \tilde{\varphi}(\omega)$, $I(\omega) \equiv \mathbf{1}_x(\omega)$, $\mu_W^e \equiv \beta_0 - (1/\gamma)\alpha_0 - (1/\gamma)\alpha_1$,

$\mu_W^d \equiv \beta_0 - (1/\gamma)\alpha_0$, $v(\omega) \equiv \ln \tilde{\zeta}(\omega)$, $\mu_X \equiv \delta_0$, and $x(\omega) \equiv \ln \tilde{f}^X(\omega)$. We then compute

$$\gamma = -\frac{\mu_Y^e - \mu_Y^d}{\mu_W^e - \mu_W^d}, \tag{D.3}$$

which needs to be time variant as long as we allow one of the parameters $\mu_Y^e, \mu_Y^d, \mu_W^e . \mu_W^d$ to vary over time. Regarding the distribution of stochastic parameters, we impose the common assumption of normality:

$$(u, v, x)^T \sim \mathcal{N}_{\mathcal{T}}(\mathbf{0}, \mathbf{\Sigma}) \qquad \text{and} \quad \mathbf{\Sigma} = \begin{pmatrix} \sigma_u^2 & \rho_{uv}\sigma_u\sigma_v & \rho_{ux}\sigma_u\sigma_x \\ \rho_{uv}\sigma_u\sigma_v & \sigma_v^2 & \rho_{vx}\sigma_v\sigma_x \\ \rho_{ux}\sigma_u\sigma_x & \rho_{vx}\sigma_v\sigma_x & \sigma_x^2 \end{pmatrix}, \tag{D.4}$$

where, in contrast to other applications, u is truncated from below at a: $u \geq a$. The density function of $\mathcal{N}_{\mathcal{T}}(\mathbf{0}, \mathbf{\Sigma})$ is then given by

$$f_{u,v,x} \equiv \frac{1}{P(u \geq a)\sqrt{(2\pi)^3 \det(\mathbf{\Sigma})}} \exp\left[-\frac{1}{2}\mathbf{z}^T \mathbf{\Sigma}^{-1}\mathbf{z}\right],$$

where $\mathbf{z} = \left(\begin{smallmatrix} u \\ v \\ x \end{smallmatrix}\right)$,

$$\mathbf{\Sigma}^{-1} = \frac{1}{\det(\mathbf{\Sigma})} \begin{pmatrix} \sigma_v^2\sigma_x^2(1-\rho_{vx}^2) & -\sigma_u\sigma_v\sigma_x^2(\rho_{uv}-\rho_{ux}\rho_{vx}) & -\sigma_u\sigma_v^2\sigma_x(\rho_{ux}-\rho_{uv}\rho_{vx}) \\ -\sigma_u\sigma_v\sigma_x^2(\rho_{uv}-\rho_{ux}\rho_{vx}) & \sigma_u^2\sigma_x^2(1-\rho_{ux}^2) & -\sigma_u^2\sigma_v\sigma_x(\rho_{vx}-\rho_{uv}\rho_{ux}) \\ -\sigma_u\sigma_v^2\sigma_x(\rho_{ux}-\rho_{uv}\rho_{vx}) & -\sigma_u^2\sigma_v\sigma_x(\rho_{vx}-\rho_{uv}\rho_{ux}) & \sigma_u^2\sigma_v^2(1-\rho_{uv}^2) \end{pmatrix}$$

and

$$P(u \geq a) = \int_a^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty \frac{1}{\sqrt{(2\pi)^3 \det(\mathbf{\Sigma})}} \exp\left[-\frac{1}{2}\mathbf{z}^T \mathbf{\Sigma}^{-1}\mathbf{z}\right] \mathrm{d}v \mathrm{d}x \mathrm{d}u.$$

D.2 Transformation into simpler estimation problem

To simplify our estimation problem, we can define the auxiliary stochastic variable $e(\omega) \equiv u(\omega) - x(\omega)$ and reformulate equation system (D.2) as follows

$$y(\omega) = \begin{cases} \mu_Y^e + u(\omega) & \text{if } I(\omega) = 1 \\ \mu_Y^d + u(\omega) & \text{if } I(\omega) = 0 \end{cases}, \tag{D.2a$'$}$$

$$w(\omega) = \begin{cases} \mu_W^e - (1/\gamma)u(\omega) + v(\omega) & \text{if } I(\omega) = 1 \\ \mu_W^d - (1/\gamma)u(\omega) + v(\omega) & \text{if } I(\omega) = 0 \end{cases}, \tag{D.2b$'$}$$

$$I(\omega) = \begin{cases} 1 & \text{if } \mu_X + e(\omega) \geq 0 \\ 0 & \text{if } \mu_X + e(\omega) < 0 \end{cases}, \tag{D.2c$'$}$$

The joint distribution of the stochastic parameters can then be derived from equation (D.4) and is given by

$$(u, v, e)^T \sim \mathcal{N}_{\mathcal{T}}(\mathbf{0}, \tilde{\mathbf{\Sigma}}) \qquad \text{and} \quad \tilde{\mathbf{\Sigma}} = \begin{pmatrix} \sigma_u^2 & \rho_{uv}\sigma_u\sigma_v & \rho_{ue}\sigma_u\sigma_e \\ \rho_{uv}\sigma_u\sigma_v & \sigma_v^2 & \rho_{ve}\sigma_v\sigma_e \\ \rho_{ue}\sigma_u\sigma_e & \rho_{ve}\sigma_v\sigma_e & \sigma_e^2 \end{pmatrix} \tag{D.4$'$}$$

with $\sigma_e \equiv \sigma_u\sqrt{1 - 2\rho_{ux}\frac{\sigma_x}{\sigma_u} + \left(\frac{\sigma_x}{\sigma_u}\right)^2}$, $\rho_{ue} \equiv \frac{\sigma_u}{\sigma_e} - \frac{\rho_{ux}\sigma_x}{\sigma_e}$, $\rho_{ve} \equiv \rho_{uv}\frac{\sigma_v}{\sigma_e} - \frac{\rho_{vx}\sigma_x}{\sigma_e}$, and u being truncated from below at a: $u \geq a$. The density function of $\mathcal{N}_{\mathcal{T}}(\mathbf{0}, \tilde{\mathbf{\Sigma}})$ can be expressed as

$$f_{u,v,e} \equiv \frac{1}{P(u \geq a)\sqrt{(2\pi)^3 \det(\tilde{\mathbf{\Sigma}})}} \exp\left[-\frac{1}{2}\tilde{\mathbf{z}}^T\tilde{\mathbf{\Sigma}}^{-1}\tilde{\mathbf{z}}\right], \tag{D.5}$$

with $\tilde{\mathbf{z}} \equiv \left(\begin{smallmatrix} u \\ v \\ e \end{smallmatrix}\right)$.

For use in later derivations we note that the marginal distribution of $\mathcal{N}_{\mathcal{T}}(\mathbf{0}, \tilde{\mathbf{\Sigma}})$ for the parameter tupel (u, e) is truncated bivariate normal and given by $\mathcal{N}_{\mathcal{T}}(\mathbf{0}, \tilde{\mathbf{\Sigma}}_{ue})$ with

$$\tilde{\mathbf{\Sigma}}_{ue} = \begin{pmatrix} \sigma_u^2 & \rho_{ue}\sigma_u\sigma_e \\ \rho_{ue}\sigma_u\sigma_e & \sigma_e^2 \end{pmatrix}.$$

The density function of the truncated bivariate normal distribution is

$$f_{u,e} = \frac{1}{2\pi\sigma_u\sigma_e\sqrt{1-\rho_{ue}^2}P(u \geq a)} \exp\left\{-\frac{1}{2(1-\rho_{ue}^2)}\left[\left(\frac{u}{\sigma_u}\right)^2 - 2\frac{ue\rho_{ue}}{\sigma_u\sigma_e} + \left(\frac{e}{\sigma_e}\right)^2\right]\right\}. \tag{D.6}$$

To derive the marginal distribution, note that

$$\det(\tilde{\mathbf{\Sigma}}) = \sigma_u^2\sigma_v^2\sigma_e^2\left(1-\rho_{uv}^2-\rho_{ue}^2-\rho_{ve}^2+2\rho_{uv}\rho_{ue}\rho_{ve}\right)$$

and

$$\begin{aligned}\tilde{\mathbf{z}}^T\tilde{\mathbf{\Sigma}}^{-1}\tilde{\mathbf{z}} = \frac{1}{\det(\tilde{\mathbf{\Sigma}})}\Big\{&u^2\sigma_v^2\sigma_e^2(1-\rho_{ve}^2) - 2uv\sigma_u\sigma_v\sigma_e^2(\rho_{uv}-\rho_{ue}\rho_{ve}) - 2ue\sigma_u\sigma_v^2\sigma_e(\rho_{ue}-\rho_{uv}\rho_{ve}) \\ &- 2ve\sigma_u^2\sigma_v\sigma_e(\rho_{ve}-\rho_{uv}\rho_{ue}) + v^2\sigma_u^2\sigma_e^2(1-\rho_{ue}^2) + e^2\sigma_u^2\sigma_v^2(1-\rho_{uv}^2)\Big\},\end{aligned} \tag{D.7}$$

whereas $\det(\tilde{\mathbf{\Sigma}}_{ue}) = \sigma_u^2\sigma_e^2(1-\rho_{ue}^2)$ and

$$\tilde{\mathbf{z}}_{ue}^T\tilde{\mathbf{\Sigma}}_{ue}^{-1}\tilde{\mathbf{z}}_{ue} = \frac{1}{\det(\tilde{\mathbf{\Sigma}}_{ue})}\Big\{u^2\sigma_e^2 - 2ue\rho_{ue}\sigma_u\sigma_e + e^2\sigma_u^2\Big\}$$

for $\tilde{\mathbf{z}}_{ue} \equiv \left(\begin{smallmatrix}u\\e\end{smallmatrix}\right)$. Defining $\Delta \equiv \tilde{\mathbf{z}}^T\tilde{\mathbf{\Sigma}}^{-1}\tilde{\mathbf{z}} - \tilde{\mathbf{z}}_{ux}^T\tilde{\mathbf{\Sigma}}_{ux}^{-1}\tilde{\mathbf{z}}_{ux}$ for convenience, we compute

$$\Delta = \frac{1}{\det(\tilde{\mathbf{\Sigma}})(1-\rho_{ue}^2)}\Big\{u\sigma_v\sigma_e(\rho_{uv}-\rho_{ue}\rho_{ve}) + e\sigma_u\sigma_v(\rho_{ve}-\rho_{uv}\rho_{ue}) - v\sigma_u\sigma_e(1-\rho_{ue}^2)\Big\}^2$$

and thus $\Delta = [(v-\mu_v)/\tilde{\sigma}_v]^2$, with

$$\mu_v \equiv u\frac{\sigma_v}{\sigma_u}\frac{\rho_{uv}-\rho_{ue}\rho_{ve}}{1-\rho_{ue}^2} + e\frac{\sigma_v}{\sigma_x}\frac{\rho_{ve}-\rho_{uv}\rho_{ue}}{1-\rho_{ue}^2}, \quad \tilde{\sigma}_v \equiv \sigma_v\sqrt{\frac{1-\rho_{uv}^2-\rho_{ue}^2-\rho_{ve}^2+2\rho_{uv}\rho_{ue}\rho_{ve}}{1-\rho_{ue}^2}}.$$

Using eq. (D.5), we can then write

$$\begin{aligned}f_{u,e} &= \int_{-\infty}^{\infty} f_{u,v,e}\mathrm{d}v \\ &= \frac{1}{2\pi\sigma_u\sigma_e\sqrt{1-\rho_{ue}^2}P(u \geq a)} \exp\left\{-\frac{1}{2(1-\rho_{ue}^2)}\left[\left(\frac{u}{\sigma_u}\right)^2 - 2\frac{ue\rho_{ue}}{\sigma_u\sigma_e} + \left(\frac{e}{\sigma_e}\right)^2\right]\right\} \\ &\qquad\qquad \times \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\tilde{\sigma}_v^2}} \exp\left[-\frac{1}{2}\left(\frac{v-\mu_v}{\tilde{\sigma}_v}\right)^2\right]\mathrm{d}v,\end{aligned}$$

which simplifies to (D.6). With this result at hand, we can also compute

$$
\begin{aligned}
P(u \geq a) &= \int_a^\infty \int_{-\infty}^\infty \frac{1}{2\pi\sigma_u\sigma_e\sqrt{1-\rho_{ue}^2}} \exp\left\{-\frac{1}{2(1-\rho_{ue}^2)}\left[\left(\frac{u}{\sigma_u}\right)^2 - 2\frac{ue\rho_{ue}}{\sigma_u\sigma_e} + \left(\frac{e}{\sigma_e}\right)^2\right]\right\} \mathrm{d}e\mathrm{d}u \\
&= \int_a^\infty \frac{1}{\sqrt{2\pi\sigma_u^2}} \exp\left[-\frac{1}{2}\left(\frac{u}{\sigma_u}\right)^2\right] \underbrace{\int_{-\infty}^\infty \frac{1}{\sqrt{2\pi\sigma_x^2(1-\rho_{ue}^2)}} \exp\left[-\frac{1}{2}\left(\frac{e - \frac{\rho_{ue}\sigma_e}{\sigma_u}u}{\sigma_e\sqrt{1-\rho_{ue}^2}}\right)^2\right] \mathrm{d}e}_{=1} \,\mathrm{d}u \\
&= \int_a^\infty \frac{1}{\sigma_u}\phi\left(\frac{u}{\sigma_u}\right)\mathrm{d}u = 1 - \Phi\left(\frac{a}{\sigma_u}\right). \qquad \text{(D.8)}
\end{aligned}
$$

As a final result, we can determine the share of exporters among active producers. This share is the conditional probability $P(e \geq -\mu_X, u \geq a)/P(u \geq a) \equiv \chi$ and can be computed as

$$
\chi = \int_a^\infty \int_{-\mu_X}^\infty f_{u,e}\mathrm{d}e\mathrm{d}u. \qquad \text{(D.9)}
$$

To simplify the problem, we define the auxiliary variable $g \equiv \frac{1}{\sqrt{1-\rho_{ue}^2}}\left(\frac{e}{\sigma_e} - \frac{\rho_{ue}u}{\sigma_u}\right)$ and make use of $\mathrm{d}e = \sigma_e\sqrt{1-\rho_{ue}^2}\mathrm{d}x$ (for given u),

$$
\left(\frac{u}{\sigma_u}\right)^2 - 2\frac{ux\rho_{ux}}{\sigma_u\sigma_x} + \left(\frac{x}{\sigma_x}\right)^2 = (1-\rho_{ue}^2)\left[g^2 + \left(\frac{u}{\sigma_u}\right)^2\right]
$$

and

$$
\underline{e} \equiv -\mu_X \quad \Longrightarrow -\frac{\mu_X}{\sigma_e} - \frac{\rho_{ue}u}{\sigma_u} = \underline{g}\sqrt{1-\rho_{ue}^2} \quad \Longleftrightarrow \quad \underline{g} = -\frac{\mu_X + u\frac{\rho_{ue}\sigma_e}{\sigma_u}}{\sigma_u\sqrt{1-\rho_{ue}^2}} \equiv h(u).
$$

Then, making use of eqs. (D.6) and (D.9), we obtain

$$
\begin{aligned}
\chi &= \frac{1}{1-\Phi\left(\frac{a}{\sigma_u}\right)} \int_a^\infty \int_{h(u)}^\infty \frac{1}{2\pi\sigma_u} \exp\left\{-\frac{1}{2}\left[g^2 + \left(\frac{u}{\sigma_u}\right)^2\right]\right\} \mathrm{d}g\mathrm{d}u \\
&= \frac{1}{1-\Phi\left(\frac{a}{\sigma_u}\right)} \int_a^\infty \frac{1}{\sigma_u}\phi\left(\frac{u}{\sigma_u}\right)\Phi\left(\frac{u\frac{\rho_{ue}\sigma_e}{\sigma_u} + \mu_X}{\sigma_e\sqrt{1-\rho_{ue}^2}}\right)\mathrm{d}u. \qquad \text{(D.10)}
\end{aligned}
$$

In the limiting case of an untruncated u, we obtain $\lim_{a\to-\infty} P(u \geq a) = 1$ and

$$
\lim_{a\to-\infty} \chi = \int_{-\infty}^\infty \frac{1}{\sigma_u}\phi\left(\frac{u}{\sigma_u}\right)\Phi\left(\frac{u\frac{\rho_{ue}\sigma_e}{\sigma_u} + \mu_X}{\sigma_e\sqrt{1-\rho_{ue}^2}}\right)\mathrm{d}u \equiv I(\mu_X)
$$

Differentiation gives

$$I'(\mu_X) = -\frac{1}{\sqrt{2\pi\sigma_e^2}} \exp\left[-\frac{1}{2}\left(\frac{\mu_X}{\sigma_e}\right)^2\right] \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_u^2(1-\rho_{ue}^2)}} \exp\left[-\frac{1}{2}\left(\frac{u+\mu_X\frac{\rho_{ue}\sigma_u}{\sigma_e}}{\sigma_u\sqrt{1-\rho_{ue}^2}}\right)^2\right] \mathrm{d}u$$
$$= -\frac{1}{\sqrt{2\pi\sigma_e^2}} \exp\left[-\frac{1}{2}\left(\frac{\mu_X}{\sigma_e}\right)^2\right]$$

and allows us to compute

$$I(\mu_X) = -\int_{\mu_X}^{\infty} \frac{1}{\sqrt{2\pi\sigma_e^2}} \exp\left[-\frac{1}{2}\left(\frac{g}{\sigma_e}\right)^2\right] \mathrm{d}g = \int_{-\infty}^{\mu_X} \frac{1}{\sqrt{2\pi\sigma_e^2}} \exp\left[-\frac{1}{2}\left(\frac{g}{\sigma_e}\right)^2\right] \mathrm{d}g.$$

This establishes $\lim_{a\to-\infty} \chi = \Phi\left(\frac{\mu_X}{\sigma_e}\right)$.

E Proof of Proposition 4

The specification of normally distributed disturbances imposes a constraint on data moments, which can be overly restrictive unless one allows for censoring (such as through the truncation of u due to positive selection of high-productivity plants into activity). To illustrate this, we consider the conditional variance of log revenues y by export status I, which for exporters and non-exporters can be expressed as

$$\mathbb{V}[y|e \geq -\mu_X] = \mathbb{E}[u^2|e \geq -\mu_X, u \geq a] - \{\mathbb{E}[u|e \geq -\mu_X, u \geq a]\}^2,$$
$$\mathbb{V}[y|e < -\mu_X] = \mathbb{E}[u^2|e < -\mu_X, u \geq a] - \{\mathbb{E}[u|e < -\mu_X, u \geq a]\}^2,$$

respectively. These variances are complicated expressions under truncation but simplify to

$$\mathbb{V}[y|e \geq -\mu_X] = \sigma_u^2 \left\{1 - \rho_{ue}^2 \frac{\phi\left(\frac{\mu_X}{\sigma_e}\right)}{\Phi\left(\frac{\mu_X}{\sigma_e}\right)} \left[\frac{\phi\left(\frac{\mu_X}{\sigma_e}\right)}{\Phi\left(\frac{\mu_X}{\sigma_e}\right)} + \frac{\mu_X}{\sigma_e}\right]\right\} \quad \text{and} \tag{E.1a}$$

$$\mathbb{V}[y|e < -\mu_X] = \sigma_u^2 \left\{1 - \rho_{ue}^2 \frac{\phi\left(\frac{\mu_X}{\sigma_e}\right)}{1-\Phi\left(\frac{\mu_X}{\sigma_e}\right)} \left[\frac{\phi\left(\frac{\mu_X}{\sigma_e}\right)}{1-\Phi\left(\frac{\mu_X}{\sigma_e}\right)} - \frac{\mu_X}{\sigma_e}\right]\right\} \tag{E.1b}$$

if the distribution of u is not truncated (in the limiting case of $a \to -\infty$). Eqs. (E.1) can be used to solve for σ_u^2 and ρ_{ue}^2. Setting $x \equiv \frac{\mu_X}{\sigma_e}$, $\lambda_1 \equiv \frac{\phi(x)}{\Phi(x)}$, and $\lambda_2 \equiv \frac{\phi(x)}{1-\Phi(x)}$, we compute

$$\sigma_u^2 = \frac{\mathbb{V}[y|e \geq -\mu_X]\lambda_2(\lambda_2 - x) - \mathbb{V}[y|e < -\mu_X]\lambda_1(\lambda_1 + x)}{\lambda_2(\lambda_2 - x) - \lambda_1(\lambda_1 + x)}$$
$$\rho_{ue}^2 = \frac{\mathbb{V}[y|e \geq -\mu_X] - \mathbb{V}[y|e < -\mu_X]}{\mathbb{V}[y|e \geq -\mu_X]\lambda_2(\lambda_2 - x) - \mathbb{V}[y|e < -\mu_X]\lambda_1(\lambda_1 + x)}$$

In our data, for instance, we observe $\mathbb{V}[y|e \geq -\mu_X] - \mathbb{V}[y|e < -\mu_X] > 0$ in every sample year—implying that $\lambda_2(\lambda_2 - x) - \lambda_1(\lambda_1 + x) > 0$ is necessary and sufficient for $\sigma_u^2, \rho_{ue}^2 > 0$. This insight motivates the following result. Proposition 4 states: *If u is not truncated and $\mathbb{V}[y|e \geq -\mu_X] - \mathbb{V}[y|e < -\mu_X] > 0$, an outcome with $\sigma_u^2, \rho_{ue}^2 > 0$ exists iff $\chi = \Phi(\frac{\mu_X}{\sigma_e}) \geq 0.5$.*

We begin the proof of Proposition 4 with the observation that

$$\lambda_2(\lambda_2 - x) - \lambda_1(\lambda_1 + x) = \frac{\phi(x)}{\Phi(x)^2[1 - \Phi(x)]^2} \left\{-[1 - 2\Phi(x)]\phi(x) - x\Phi(x)[1 - \Phi(x)]\right\}.$$

Hence

$$\lambda_2(\lambda_2 - x) - \lambda_1(\lambda_1 + x) > 0 \Leftrightarrow A(x) \equiv -[1 - 2\Phi(x)]\phi(x) - x\Phi(x)[1 - \Phi(x)] > 0.$$

Moreover, we have $\lim_{x\to-\infty} A(x) = \lim_{x\to\infty} A(x) = A(0) = 0$ and

$$A'(x) = 2\phi(x)^2 - \Phi(x)[1 - \Phi(x)], \quad A''(x) = \phi(x)a(x),$$

with $a(x) \equiv -4x\phi(x) + 2\Phi(x) - 1$ and $a'(x) = 2\phi(x)(2x^2 - 1)$. We have $a'(x) < 0$ if $x \in (-\sqrt{1/2}, \sqrt{1/2})$, $a'(x) = 0$ if $x = -\sqrt{1/2}$ or $x = \sqrt{1/2}$, and $a'(x) > 0$ otherwise.

Note that $\lim_{x\to-\infty} a(x) = -1$, $\lim_{x\to\infty} a(x) = 1$ and $a(0) = 0$, so we can define a threshold $\underline{x} < -\sqrt{1/2}$, such that $a(x) < 0$ holds for all $x < \underline{x}$, whereas $a(x) > 0$ holds for all $x \in (\underline{x}, 0)$. Similarly, we can define a second threshold $\overline{x} > \sqrt{1/2}$, such that $a(x) < 0$ holds for all $x \in (0, \overline{x})$, whereas $a(x) > 0$ holds for all $x > \overline{x}$. Note that $\lim_{x\to-\infty} A'(x) = \lim_{x\to\infty} A'(x) = 0$ and $A'(0) = \pi^{-1} - 4^{-1} > 0$. It therefore follows from the properties discussed above that $A(x)$ has a unique minimum $x_{min} < 0$ and a unique maximum $x_{max} > 0$. Accordingly, $A(x) > 0$ iff $x > 0$. Since $\Phi(0) = 1/2$ and $\Phi'(x) > 0$, we can conclude that $\chi = \Phi\left(\frac{\mu_X}{\sigma_e}\right)$ is necessary and sufficient for $\sigma_u^2, \rho_{ue}^2 > 0$ if $\mathbb{V}[y|e \geq -\mu_X] > \mathbb{V}[y|e < -\mu_X]$.

This restrictive property of our model can be alleviated if we allow u to be truncated.

F Implementation of Estimation Model

F.1 Conditional likelihood functions

We want to estimate system (D.2′), using a maximum likelihood (ML) estimator. Substituting eq. (D.7) into eq. (D.5) and following the derivation steps from Appendix D.2, we can rewrite the density function in eq. (D.5) as follows:

$$f_{u,v,e} = \frac{1}{2\pi\sigma_u\sigma_v\sqrt{1-\rho_{uv}^2}P(u \geq a)} \exp\left\{-\frac{1}{2(1-\rho_{uv}^2)}\left[\left(\frac{u}{\sigma_u}\right)^2 - 2\frac{uv\rho_{uv}}{\sigma_u\sigma_v} + \left(\frac{v}{\sigma_v}\right)^2\right]\right\} \times \frac{1}{\sqrt{2\pi\tilde{\sigma}_e^2}} \exp\left[-\frac{1}{2}\left(\frac{e-\mu_e}{\tilde{\sigma}_e}\right)^2\right]$$

with

$$\mu_e \equiv u\frac{\sigma_e}{\sigma_u}\frac{\rho_{ue}-\rho_{uv}\rho_{ve}}{1-\rho_{uv}^2} + v\frac{\sigma_e}{\sigma_v}\frac{\rho_{ve}-\rho_{ue}\rho_{uv}}{1-\rho_{uv}^2}, \quad \tilde{\sigma}_e \equiv \sigma_e\sqrt{\frac{1-\rho_{ue}^2-\rho_{uv}^2-\rho_{ve}^2+2\rho_{ue}\rho_{uv}\rho_{ve}}{1-\rho_{uv}^2}}. \tag{F.1}$$

The density of u for exporters ($I_i = 1$) can then be computed according to

$$\begin{aligned} f_{u,v}^e = \int_{-\mu_X}^{\infty} f_{u,v,e}\,de &= \frac{1}{2\pi\sigma_u\sigma_v\sqrt{1-\rho_{uv}^2}P(u \geq a)} \exp\left\{-\frac{1}{2(1-\rho_{uv}^2)}\left[\left(\frac{u}{\sigma_u}\right)^2 - 2\frac{uv\rho_{uv}}{\sigma_u\sigma_v} + \left(\frac{v}{\sigma_v}\right)^2\right]\right\} \\ &\quad \times \int_{-\mu_X}^{\infty} \frac{1}{\sqrt{2\pi\tilde{\sigma}_e^2}} \exp\left[-\frac{1}{2}\left(\frac{e-\mu_e}{\tilde{\sigma}_e}\right)^2\right] de \\ &= \frac{1}{2\pi\sigma_u\sigma_v\sqrt{1-\rho_{uv}^2}P(u \geq a)} \exp\left\{-\frac{1}{2(1-\rho_{uv}^2)}\left[\left(\frac{u}{\sigma_u}\right)^2 - 2\frac{uv\rho_{uv}}{\sigma_u\sigma_v} + \left(\frac{v}{\sigma_v}\right)^2\right]\right\} \Phi\left(\frac{\mu_X+\mu_e}{\tilde{\sigma}_e}\right). \end{aligned} \tag{F.2}$$

Similarly for non-exporters ($I_i = 0$), we can compute

$$\begin{aligned} f_{u,v}^d = \int_{-\infty}^{-\mu_X} f_{u,v,e}\,de &= \frac{1}{2\pi\sigma_u\sigma_v\sqrt{1-\rho_{uv}^2}P(u \geq a)} \exp\left\{-\frac{1}{2(1-\rho_{uv}^2)}\left[\left(\frac{u}{\sigma_u}\right)^2 - 2\frac{uv\rho_{uv}}{\sigma_u\sigma_v} + \left(\frac{v}{\sigma_v}\right)^2\right]\right\} \\ &\quad \times \int_{-\infty}^{-\mu_X} \frac{1}{\sqrt{2\pi\tilde{\sigma}_e^2}} \exp\left[-\frac{1}{2}\left(\frac{e-\mu_e}{\tilde{\sigma}_e}\right)^2\right] de \\ &= \frac{1}{2\pi\sigma_u\sigma_v\sqrt{1-\rho_{uv}^2}P(u \geq a)} \exp\left\{-\frac{1}{2(1-\rho_{uv}^2)}\left[\left(\frac{u}{\sigma_u}\right)^2 - 2\frac{uv\rho_{uv}}{\sigma_u\sigma_v} + \left(\frac{v}{\sigma_v}\right)^2\right]\right\} \Phi\left(-\frac{\mu_X+\mu_e}{\tilde{\sigma}_e}\right). \end{aligned} \tag{F.3}$$

Lemma 1. *Denote the observed data with the vectors* $(\mathbf{y}, \mathbf{w}, \mathbf{I})$ *whose characteristic elements are* (y_i, w_i, I_i)*, denote the minimum observable* y_i *with* $y_{min} \equiv \min\{y_i\}$*, let* N *be the number of observations, and set* $\sigma_e = 1$*. We replace the truncation point* a *by* $\min\{y_i^d\} - \mu_Y^d$ *if* y_{min} *is observed for a non-exporter and by* $\min\{y_i^e\} - \mu_Y^e$ *otherwise. Then, the conditional likelihood function for system* (42) *is denoted*

$$\hat{\mathcal{L}}^j\big(\,\cdot\,\big|y,w,I\big) = \hat{\mathcal{L}}^j\big(\mu_Y^e, \mu_Y^d, \mu_W^e, \mu_W^d, \mu_X, \sigma_u, \sigma_v, \rho_{ue}, \rho_{uv}, \rho_{ve}\big|y,w,I\big), \qquad j \in \{e,d\}.$$

If $y_{min} = \min y_i^d$ *(minimum revenues at non-exporter), the conditional likelihood function is given by*

$$\begin{aligned}\hat{\mathcal{L}}^d\big(\,\cdot\,\big|y,w,I\big) = \prod_{i=1}^N &\left\{\frac{1}{\sqrt{2\pi}\sigma_u\sigma_v\sqrt{1-\rho_{uv}^2}}\frac{1}{1-\Phi\left(\frac{\min\{y_i^d\}-\mu_Y^d}{\sigma_u}\right)}\phi\left(x_i^e\right)\left[1-\Phi\left(-\frac{\mu_X+\mu_i^1}{\tilde{\sigma}_e}\right)\right]\right\}^{I_i} \\ &\times\left\{\frac{1}{\sqrt{2\pi}\sigma_u\sigma_v\sqrt{1-\rho_{uv}^2}}\frac{1}{1-\Phi\left(\frac{\min\{y_i^d\}-\mu_Y^d}{\sigma_u}\right)}\phi\left(x_i^d\right)\Phi\left(-\frac{\mu_X+\mu_i^2}{\tilde{\sigma}_e}\right)\right\}^{1-I_i},\end{aligned} \tag{F.4}$$

If $y_{min} = \min y_i^e$ *(minimum revenues at exporter), the conditional likelihood function is given by*

$$\begin{aligned}\hat{\mathcal{L}}^e\big(\,\cdot\,\big|y,w,I\big) = \prod_{i=1}^N &\left\{\frac{1}{\sqrt{2\pi}\sigma_u\sigma_v\sqrt{1-\rho_{uv}^2}}\frac{1}{1-\Phi\left(\frac{\min\{y_i^e\}-\mu_Y^e}{\sigma_u}\right)}\phi\left(x_i^e\right)\left[1-\Phi\left(-\frac{\mu_X+\mu_i^1}{\tilde{\sigma}_e}\right)\right]\right\}^{I_i} \\ &\times\left\{\frac{1}{\sqrt{2\pi}\sigma_u\sigma_v\sqrt{1-\rho_{uv}^2}}\frac{1}{1-\Phi\left(\frac{\min\{y_i^e\}-\mu_Y^e}{\sigma_u}\right)}\phi\left(x_i^d\right)\Phi\left(-\frac{\mu_X+\mu_i^2}{\tilde{\sigma}_e}\right)\right\}^{1-I_i}.\end{aligned} \tag{F.5}$$

We have used

$$\begin{aligned}x_i^j &\equiv \frac{1}{\sqrt{1-\rho_{uv}^2}}\sqrt{\left(\frac{y_i-\mu_Y^j}{\sigma_u}\right)^2 - 2\frac{(y_i-\mu_Y^j)[w_i-\mu_W^j+\frac{1}{\gamma}(y_i-\mu_Y^j)]\rho_{uv}}{\sigma_u\sigma_v} + \left(\frac{w_i-\mu_W^j+\frac{1}{\gamma}(y_i-\mu_Y^j)}{\sigma_v}\right)^2} \\ &= \frac{1}{\sqrt{1-\rho_{uv}^2}}\sqrt{\left(\frac{y_i-\mu_Y^j}{\sigma_u}\right)^2 - 2\rho_{uv}\left(\frac{y_i-\mu_Y^j}{\sigma_u}\right)\left[\left(\frac{w_i-\mu_W^j}{\sigma_v}\right)+\frac{\sigma_u}{\gamma\sigma_v}\left(\frac{y_i-\mu_Y^j}{\sigma_u}\right)\right] + \left[\left(\frac{w_i-\mu_W^j}{\sigma_v}\right)+\frac{\sigma_u}{\gamma\sigma_v}\left(\frac{y_i-\mu_Y^j}{\sigma_u}\right)\right]^2} \\ &= \frac{1}{\sqrt{1-\rho_{uv}^2}}\sqrt{\left[\left(\frac{y_i-\mu_Y^j}{\sigma_u}\right)\left(1-\frac{\sigma_u}{\gamma\sigma_v}\right)-\left(\frac{w_i-\mu_W^j}{\sigma_v}\right)\right]^2 + 2(1-\rho_{uv})\left(\frac{y_i-\mu_Y^j}{\sigma_u}\right)\left[\frac{\sigma_u}{\gamma\sigma_v}\left(\frac{y_i-\mu_Y^j}{\sigma_u}\right)+\left(\frac{w_i-\mu_W^j}{\sigma_v}\right)\right]},\end{aligned}$$

$$\mu_i^j \equiv \left(\frac{y_i-\mu_Y^j}{\sigma_u}\right)\frac{\rho_{ue}-\rho_{uv}\rho_{ve}}{1-\rho_{uv}^2} + \left[\left(\frac{w_i-\mu_W^j}{\sigma_v}\right)+\frac{\sigma_u}{\gamma\sigma_v}\left(\frac{y_i-\mu_Y^j}{\sigma_u}\right)\right]\frac{\rho_{ve}-\rho_{uv}\rho_{ue}}{1-\rho_{uv}^2}$$

for $j \in \{e, d\}$ and

$$\tilde{\sigma}_e \equiv \sqrt{\frac{1-\rho_{ue}^2-\rho_{uv}^2-\rho_{ve}^2+2\rho_{ue}\rho_{uv}\rho_{ve}}{1-\rho_{uv}^2}}.$$

Furthermore,

$$\gamma = -\frac{\mu_Y^e - \mu_Y^d}{\mu_W^e - \mu_W^d}, \tag{F.6}$$

is not an independently estimable parameter.

Proof. The conditional likelihood functions (F.4) and (F.5) follow from eqs. (F.2) and (F.3) after substituting $u_i = y_i - \mu_y^1$, $v_i = w_i - \mu_W^e + \frac{1}{\gamma}(y_i - \mu_Y^e)$ for exporters and $u_i = y_i - \mu_y^2$, $v_i = w_i - \mu_W^d + \frac{1}{\gamma}(y_i - \mu_Y^d)$ for non-exporters, setting $\sigma_e = 1$, and accounting for eq. (F.1). Eq. (F.6) follows from eq. (D.3). $\square$

For tractability we introduce three further auxiliary variables:

$$a_i^j \equiv \left(\frac{y_i - \mu_Y^j}{\sigma_u}\right), \qquad b_i^j \equiv \left(\frac{w_i - \mu_W^j}{\sigma_v}\right), \qquad c_i^j \equiv a_i^j \frac{\sigma_u}{\gamma\sigma_v} + b_i^j. \tag{F.7}$$

These auxiliary variables lead to the simplifications

$$x_i^j = \sqrt{\frac{\left(a_i^j - c_i^j\right)^2 + 2(1-\rho_{uv})a_i^j c_i^j}{1-\rho_{uv}^2}}$$

and

$$\mu_i^j = a_i^j \frac{\rho_{ue} - \rho_{uv}\rho_{ve}}{1-\rho_{uv}^2} + c_i^j \frac{\rho_{ve} - \rho_{uv}\rho_{ue}}{1-\rho_{uv}^2},$$

Introducing yet two more auxiliary variables,

$$d_i^j \equiv \frac{\mu_X + \mu_i^j}{\tilde{\sigma}_e} \qquad \text{and} \qquad \underline{a}^j \equiv \frac{\min\{y^j\} - \mu_Y^j}{\sigma_u}, \tag{F.8}$$

helps us rewrite the conditional likelihood functions as follows:

$$\hat{\mathcal{L}}^d\big(\,\cdot\,\big|y, w, I\big) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\sigma_u\sigma_v\sqrt{1-\rho_{uv}^2}} \left\{\frac{\phi\left(x_i^e\right)}{\Phi\left(-\underline{a}^d\right)}\Phi\left(d_i^e\right)\right\}^{I_i} \left\{\frac{\phi\left(x_i^d\right)}{\Phi\left(-\underline{a}^d\right)}\Phi\left(-d_i^d\right)\right\}^{1-I_i}, \tag{F.9}$$

$$\hat{\mathcal{L}}^e\left(\,\cdot\,\middle|y,w,I\right)=\prod_{i=1}^{N}\frac{1}{\sqrt{2\pi}\sigma_u\sigma_v\sqrt{1-\rho_{uv}^2}}\left\{\frac{\phi\left(x_i^e\right)}{\Phi\left(-\underline{a}^e\right)}\Phi\left(d_i^e\right)\right\}^{I_i}\left\{\frac{\phi\left(x_i^d\right)}{\Phi\left(-\underline{a}^e\right)}\Phi\left(-d_i^d\right)\right\}^{1-I_i}. \tag{F.10}$$

F.2 Estimation in two steps

To simplify the estimation problem we can make use of the specific structure of our model—with censoring based on only one unobserved characteristic u—and estimate the parameters of interest in two steps. For this purpose, we separate the full equation system (42) into the two sub-models, model 1:

$$y_i=\begin{cases}y_i^e=\mu_Y^e+u_i & \text{if } I_i=1\\ y_i^d=\mu_Y^d+u_i & \text{if } I_i=0\end{cases}, \tag{F.11a}$$

$$I_i=\begin{cases}1 & \text{if } I_i^*=\mu_X+e_i\geq 0\\ 0 & \text{if } I_i^*=\mu_X+e_i<0\end{cases}, \tag{F.11b}$$

$$y_i,I_i=\text{missing}\qquad \text{if } u_i<a, \tag{F.11c}$$

and model 2:

$$w_i=\begin{cases}w_i^1=\mu_W^e-(1/\gamma)u_i+v_i & \text{if } I_i=1\\ w_i^d=\mu_W^d-(1/\gamma)u_i+v_i & \text{if } I_i=0\end{cases}, \tag{F.12a}$$

$$I_i=\begin{cases}1 & \text{if } I_i^*=\mu_X+e_i\geq 0\\ 0 & \text{if } I_i^*=\mu_X+e_i<0\end{cases}, \tag{F.12b}$$

$$w_i,I_i=\text{missing}\qquad \text{if } u_i<a. \tag{F.12c}$$

The parameter vector $\boldsymbol{\theta}_u=(\mu_Y^e,\mu_Y^d,\mu_X,\sigma_u,\rho_{ue})$ appears in model 1 as well as model 2 because γ is a composite of μ_Y^e,μ_Y^d and because endogenous switching and censoring involve μ_X, σ_u and ρ_{ue} in model 2. In contrast, the parameter vector $\boldsymbol{\theta}_v=(\mu_W^e,\mu_W^d,\sigma_v,\rho_{ve},\rho_{uv})$ only appears in model 2 because model 1 lacks the error term v. As a consequence, we can estimate $\boldsymbol{\theta}_u$ from model 1 first and then use the parameter estimates from model 1 in model 2 and estimate parameter $\boldsymbol{\theta}_v$ in a second step.

The density function for realization (u,e) in model 1 is given by eq. (D.6). Using eq. (D.8) we obtain

$$f_{u,e}=\frac{1}{2\pi\sigma_u\sigma_e\sqrt{1-\rho_{uv}^2}P_0}\exp\left\{-\frac{1}{2(1-\rho_{ue}^2)}\left[\left(\frac{u}{\sigma_u}\right)^2-2\frac{ue\rho_{ue}}{\sigma_u\sigma_e}+\left(\frac{e}{\sigma_e}\right)^2\right]\right\}. \tag{F.13}$$

The export selection disturbance e is unobservable but it generates the endogenous switching between exporter and non-export status. We can integrate eq. (F.13) over e to determine the likelihood functions for exporters and non-exporters, relying on revenue observations from the data (see Maddala 1986). We compute for exporters ($I_i=1$)

$$\begin{aligned}f_{u,e}^1&=\int_{-\mu_X}^{\infty}f_{u,e}\mathrm{d}e=\frac{1}{\sqrt{2\pi}\sigma_u}\exp\left[-\frac{1}{2}\left(\frac{u}{\sigma_u}\right)^2\right]\int_{-\mu_X}^{\infty}\frac{1}{\sqrt{2\pi}\sigma_e\sqrt{1-\rho_{ue}^2}}\exp\left[-\frac{1}{2}\left(\frac{e-u\frac{\rho_{ue}\sigma_e}{\sigma_u}}{\sigma_e\sqrt{1-\rho_{ue}^2}}\right)^2\right]\mathrm{d}e\\&=\frac{1}{\sigma_u}\phi\left(\frac{u}{\sigma_u}\right)\int_{-\mu_X}^{\infty}\frac{1}{\sigma_e\sqrt{1-\rho_{ue}^2}}\phi\left(\frac{e-u\frac{\rho_{ue}\sigma_e}{\sigma_u}}{\sigma_e\sqrt{1-\rho_{ue}^2}}\right)\mathrm{d}e=\frac{1}{\sigma_u}\phi\left(\frac{u}{\sigma_u}\right)\Phi\left(\frac{\mu_X+u\frac{\rho_{ue}\sigma_e}{\sigma_u}}{\sigma_e\sqrt{1-\rho_{ue}^2}}\right).\end{aligned} \tag{F.14}$$

Similarly, for non-exporters ($I_i=0$) we compute

$$\begin{aligned}f_{u,e}^2&=\int_{-\infty}^{-\mu_X}f_{u,e}\mathrm{d}e=\frac{1}{\sqrt{2\pi}\sigma_u}\exp\left[-\frac{1}{2}\left(\frac{u}{\sigma_u}\right)^2\right]\int_{-\mu_X}^{\infty}\frac{1}{\sqrt{2\pi}\sigma_e\sqrt{1-\rho_{ue}^2}}\exp\left[-\frac{1}{2}\left(\frac{e-u\frac{\rho_{ue}\sigma_e}{\sigma_u}}{\sigma_e\sqrt{1-\rho_{ue}^2}}\right)^2\right]\mathrm{d}e\\&=\frac{1}{\sigma_u}\phi\left(\frac{u}{\sigma_u}\right)\int_{-\infty}^{-\mu_X}\frac{1}{\sigma_e\sqrt{1-\rho_{ue}^2}}\phi\left(\frac{e-u\frac{\rho_{ue}\sigma_e}{\sigma_u}}{\sigma_e\sqrt{1-\rho_{ue}^2}}\right)\mathrm{d}e=\frac{1}{\sigma_u}\phi\left(\frac{u}{\sigma_u}\right)\Phi\left(-\frac{\mu_X+u\frac{\rho_{ue}\sigma_e}{\sigma_u}}{\sigma_e\sqrt{1-\rho_{ue}^2}}\right).\end{aligned} \tag{F.15}$$

The conditional likelihood function for observed realizations u does not permit separate identification of σ_e from μ_X. We therefore set $\sigma_e=1$ or, put differently, we estimate the normalized export cutoff μ_X/σ_e.

Lemma 2. *Denote the observed data with the vectors* $(\mathbf{y},\mathbf{w},\mathbf{I})$ *whose characteristic elements are* (y_i,w_i,I_i)*, denote the minimum observable* y_i *with* $y_{min}\equiv\min\{y_i\}$*, let* N *be the number of observations, and set* $\sigma_e=1$*. We can replace the truncation point* a *with* $\min\{y_i^d\}-\mu_Y^d$ *if the* y_{min} *is observed for a non-exporter and with* $\min\{y_i^e\}-\mu_Y^e$ *if* y_{min} *is at an exporter. Then the conditional likelihood function for system* (F.11) *is*

$$\mathcal{L}^e\big(\,\cdot\,\big|\mathbf{y},\mathbf{I}\big)=\mathcal{L}^j\big(\mu_Y^e,\mu_Y^d,\mu_X,\sigma_u,\rho_{ue}\big|\mathbf{y},\mathbf{I}\big),\qquad j\in\{e,d\}.$$

If $y_{min} = \min\{y_i\}$ *occurs at a non-exporter, the conditional likelihood function is*

$$\mathcal{L}^d(\,\cdot\,|\mathbf{y},\mathbf{I}) = \prod_{i=1}^{N} \frac{1}{\sigma_u} \left\{ \frac{\phi\left(\bar{x}_i^e\right)}{\Phi\left(-\underline{a}^d\right)} \Phi\left(\bar{d}_i^e\right) \right\}^{I_i} \left\{ \frac{\phi\left(\bar{x}_i^d\right)}{\Phi\left(-\underline{a}^d\right)} \Phi\left(-\bar{d}_i^d\right) \right\}^{1-I_i} \tag{F.16}$$

If $y_{min} = \min\{y_i\}$ *occurs at a non-exporter, the conditional likelihood function is*

$$\mathcal{L}^e(\,\cdot\,|\mathbf{y},\mathbf{I}) = \prod_{i=1}^{N} \frac{1}{\sigma_u} \left\{ \frac{\phi\left(\bar{x}_i^e\right)}{\Phi\left(-\underline{a}^e\right)} \Phi\left(\bar{d}_i^e\right) \right\}^{I_i} \left\{ \frac{\phi\left(\bar{x}_i^d\right)}{\Phi\left(-\underline{a}^e\right)} \Phi\left(-\bar{d}_i^d\right) \right\}^{1-I_i} \tag{F.17}$$

We have used $\bar{\sigma}_e = \sqrt{1-\rho_{ue}^2}$*,*

$$\bar{x}_i^j \equiv a_i^j = \frac{y_i - \mu_y^j}{\sigma_u}, \quad \bar{d}_i^j \equiv \frac{\mu_X + \bar{\mu}_i^1}{\bar{\sigma}_e}, \quad \bar{\mu}_i^j \equiv \frac{\rho_{ue}(y_i - \mu_Y^e)}{\sigma_u},$$

and $\underline{a}^j = \left(\min\{y_i\} - \mu_Y^j\right)/\sigma_u$ *for* $j \in \{e, d\}$*.*

Proof. The conditional likelihood functions (F.16) and (F.17) follow from eqs. (F.14) and (F.15) after substituting $u_i = y_i^1 - \mu_Y^e$ for exporters and $u_i = y_i^2 - \mu_Y^d$ for non-exporters and setting $\sigma_e = 1$. □

Once $\hat{\theta}_u$ is determined, we can proceed with estimating parameter vector $\boldsymbol{\theta}_v$ from model 2 in eq. (F.12). The marginal likelihood for the observed realizations of (u, v, e) in model 2 is given by Lemma 1 with parameters $\mu_Y^e, \mu_Y^d, \mu_X, \sigma_u$, and ρ_{ue} replaced by their estimated counterparts.

Online Supplement to

Tasks, Occupations, and Wage Inequality in an Open Economy

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This Online Supplement presents additional empirical evidence.

S1 Comparison to GSOEP data

To gauge the plausibility of our multi-tasking measures, in Appendix A.2 we compare the BIBB-BAuA based measures of 15 tasks in 2012 to the German Socioeconomic Panel (GSOEP) that reports 23 tasks in 2013 in a one-time supplemental survey. To enhance comparability of the multi-tasking measures from two data source, BIBB-BAuA and GSOEP even further, in this supplemental section we present a multi-tasking count from 14 individually comparable workplace operations. For this purpose, we omit task 9. Apply Legal Knowledge from BIBB-BAuA, which has no counterpart in the GSOEP. Conversely, we omit from GSOEP the tasks 16. Securing, protecting, monitoring, directing traffic; 17. Cleaning, clearing, recycling; 20. Writing/reading texts/documents/e-mails, editing forms; 22. Reporting, publishing, entertaining, presenting; and 23. Sorting, stocking, ticketing. In addition, in GSOEP we map the tasks 1. Making, processing or assembling things; and 2. Building, installing, or fitting things/objects into a single BIBB-BAuA-consistent task (1. Manufacture, Produce Goods); we map 5. Selling; and 6. Buying, purchasing into a single task (7. Purchase, Procure, Sell); and we map 10. Collecting information, researching, documenting, analyzing; and 12. Designing, developing, researching, constructing, shaping into a single task (6. Gather Information, Develop, Research, Construct). With these adjustments, we derive from both BIBB-BAuA and GSOEP a multitasking measure based on 14 closely comparable tasks.

We select three predictors that are observed in both the GSOEP survey in 2013 and the BIBB-BAuA data in 2012 in a similar way: years of schooling (which we transform in BIBB-BAuA to become similar to the GSOEP convention), gross monthly income in Euros (observed in intervals in the BIBB-BAuA data), and the reported weekly work hours. Table S.1 shows the results. In both data sets, educational attainment, income and work hours

Table S.1: Worker-level Predictors of the Number of Tasks

	Dependent variable: Number of tasks				
	(1)	(2)	(3)	(4)	(5)
	GSOEP (up to 23 tasks)				
Years of education	0.104***			0.0583*	-0.0622
	(0.0304)			(0.0345)	(0.0440)
Gross monthly income		0.000232***		0.0000505	0.0000276
		(0.0000477)		(0.0000640)	(0.0000756)
Weekly work hours			0.0413***	0.0348***	0.0310***
			(0.00605)	(0.00763)	(0.00835)
FE					yes
R^2	0.013	0.029	0.052	0.057	0.221
Observations	864	801	848	788	788
	BIBB-BAuA (up to 15 tasks)				
Years of education	0.162***			0.139***	0.0518***
	(0.00791)			(0.0103)	(0.0113)
Gross monthly income		0.000236***		0.0000998***	0.000128***
		(0.0000477)		(0.0000148)	(0.0000148)
Weekly work hours			0.0447***	0.0382***	0.0465***
			(0.00240)	(0.00287)	(0.00279)
FE					yes
R^2	0.021	0.024	0.020	0.046	0.232
Observations	20,012	13,936	17,104	13,928	13,928

Sources: GSOEP 2013 and BIBB-BAuA 2012.
Notes: Number of tasks from count of reported individual tasks out of 23 in GSOEP 2013, out of 15 in BIBB-BAuA 2012. Years of education in BIBB-BAuA data transformed into GSOEP definition; gross monthly income in BIBB-BAuA reported in intervals; weekly work hours are reported actual hours in BIBB-BAuA. Occupations at the two-digit ISCO level in GSOEP and at the two-digit KldB-88 occupation group level in BIBB-BAuA. The fixed-effects (FE) specification conditions on *Bundesland*, industry and respective two-digit occupation. Significance levels: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

are individually positively associated with multi-tasking and statistically significantly so at the one-percent confidence level. Workers whose assignments require more multi-tasking are more educated, earn more, and work longer hours. The regression coefficients are of similar magnitude in both dat; even the measures of goodness of fit are closely similar. However, when including all regressors simultaneously and especially when conditioning on region (*Bundesland*), sector (39 longitudinally consistent industries) and occupation group effects (two-digit ISCO level in GSOEP and two-digit KldB-88 occupation group level in BIBB-BAuA), then the small number of only about 800 observations in GSOEP does not allow for statistically significant predictions except for the work hours predictor. In the BIBB-BAuA data in contrast, with roughly 14,000 valid observations in 2012, all three predictors remain statistically significant at the one-percent confidence level even after controlling for region, sector and occupation group effects.

Table S.2: Worker-level Predictors of Plant Size

Dependent variable: Plant size (midpoint of respective employment category)					
	(1)	(2)	(3)	(4)	(5)
			GSOEP (up to 23 tasks)		
Years of education	205.1***			25.62	-139.2
	(58.59)			(67.45)	(85.63)
Gross monthly income		0.591***		0.693***	0.545***
		(0.0918)		(0.125)	(0.147)
Weekly work hours			30.47**	-22.27	-8.038
			(11.96)	(14.93)	(16.26)
FE					yes
R^2	0.014	0.049	0.008	0.056	0.228
Observations	864	801	848	788	788
			BIBB-BAuA (up to 15 tasks)		
Years of education	59.35***			-4.478	37.69***
	(7.573)			(10.05)	(11.80)
Gross monthly income		0.354***		0.350***	0.245***
		(0.0124)		(0.0146)	(0.0155)
Weekly work hours			33.66***	2.632	0.748
			(0.00150)	(0.00174)	(2.974)
FE					yes
R^2	0.003	0.057	0.011	0.057	0.142
Observations	18,881	13,246	16,185	13,238	13,238

Sources: GSOEP 2013 and BIBB-BAuA 2012, using LIAB 2013 and 2012 to compute the respective employment category midpoints.
Notes: Dependent variable are the LIAB employment averages per size category in 2012 (for BIBB-BAuA) and 2013 (for GSOEP). Years of education in BIBB-BAuA data transformed into GSOEP definition; gross monthly income in BIBB-BAuA reported in intervals; weekly work hours are reported actual hours in BIBB-BAuA. Occupations at the two-digit ISCO level in GSOEP and at the two-digit KldB-88 occupation group level in BIBB-BAuA. The fixed-effects (FE) specification conditions on *Bundesland*, industry and respective two-digit occupation. Significance levels: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Our empirical analysis and model emphasize the relationship between plant size and multi-tasking. To assess the similarity between GSOEP and BIBB-BAuA with respect to plant size, we use the same worker-level predictors as in Table A2 above to check the association with employment. In the GSOEOP, size categories of plants are 1-4, 5-9, 10-19, 20-99, 100-199, 200-1999, and more than 2000 workers. In the BIBB-BAuA data, the size categories are 1-4, 5-9, 10-49, 50-99, 100-499, 500-999, and more than 1000 workers. To make the categories comparable, we compute the average employment midpoints within each range from the representative sample of plants in LIAB 2012 (for BIBB-BAuA) and 2013 (for GSOEP), and use those midpoints as the dependent variable in our descriptive regressions. Table A3 reports the results. All three predictors are positive and statistically significantly associated with plant size (at the one-percent confidence level) in both data, when used as individual predictors. More educated workers, higher-paid workers and workers with longer work weeks are employed at larger plants. Coefficients on educational attainment and income remain robustly positive in the BIBB-BAuA data also within region (*Bundesland*), sector and two-digit occupation group in a joint prediction, but not work hours. In the GSOEP data, the small observation numbers preserve only the positive association between a worker's income and the size of the worker's employer, while the other two predictors are no longer separately statistically significant when conditioning on region, sector and occupation group.

In summary, the GSOEP data for 2013 exhibit closely similar covariation patterns between main characteristics of workers and their jobs (educational attainment, pay, work hours) on the one-hand side and multitasking or employer size on the other hand. Importantly, the worker and job characteristics are positively associated with both multitasking and employer size.