Taming a Minsky Cycle*

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Abstract

Minsky famously argued that modern economies undergo recurring financial cycles. Episodes of complacency accompanied by rising asset prices and growing leverage come to an end in a "Minsky moment". At that point, risk is re-priced, asset prices crash, imprudent borrowers de-lever, aggregate demand falls, and a recession ensues. We take this narrative for granted and ask what policymakers should do about it. To answer this question, we build a stylized model with boom-bust financial cycles fueled by extrapolative expectations. We characterize optimal policy using Tinbergen's language of targets (macroeconomic and financial stability) and instruments (monetary and macroprudential policy). We derive lessons for the assignment of targets to instruments, the incompatibility of targets in the absence of enough instruments (without macroprudential policy), and how the remaining instruments (monetary policy) should then trade off the different targets.

1 Introduction

After the Great Recession, Minsky (1986)'s theory of financial cycles gained new prominence. His thesis is that modern economies recurrently go through recurring cycles. Episodes of complacency accompanied by rising asset prices and growing piles of debt come to an end in a "Minsky moment". At that point, risk is re-priced, asset prices crash, imprudent borrowers de-lever, aggregate demand falls, and a recession ensues. We take this narrative for granted and ask what policymakers should do about it.

To answer this question, we build a stylized three-period model of a Minsky cycle. There are two classes of agents: borrowers and savers. Borrowers borrow from savers to finance the

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purchase of a risky asset. They could be households borrowing to buy a house, or financial intermediaries borrowing to buy stocks. They have extrapolative expectations and believe that high past returns predict high future returns. Output is produced from labor and wages are sticky. Policymakers control monetary policy by setting the nominal interest rate, subject to the Zero Lower Bound (ZLB) constraint. They may also have access to macroprudential policy in the form of flexible taxes or quantity restrictions on the leverage of borrowers. The first period is the boom fueled in part by the extrapolative expectations of borrowers. The second period is the bust triggered in part by the realizations by borrowers that they were too optimistic. The ZLB does not bind during the boom but binds during the bust which produces a recession with output below potential.

Key to our analysis is a dual externality from leverage during the boom. First, there is an aggregate demand externality: borrowers do not take into account that by borrowing more during the boom, they amplify the asset crash, lower aggregate demand, and deepen the recession during the bust in general equilibrium. Second, there is a belief externality: because they are overly optimistic, borrowers do not internalize that when they borrow more, they actually reduce their welfare in partial equilibrium.

Our emphasis is on optimal policy during the boom. We use Tinbergen (1952)'s language of targets and instruments. We allow for either one or two instruments: monetary policy alone or together with macroprudential policy. We identify two targets: macroeconomic and financial stability. We make this language precise by deriving these notions endogenously from a welfare-based, micro-founded, general equilibrium approach. By macroeconomic stability, we mean no deviation of output from potential during the boom. By financial stability, we mean the correction of the dual externality from leverage during the boom.

We set the stage by analyzing the case where expectations are rational. When both monetary and macroprudential polices are available, macroprudential policy limits the leverage of borrowers and monetary policy ensures that output is at potential during the boom. Monetary policy therefore follows the principles of inflation targeting, and macroprudential policy acts as a Pigouvian curb on leverage designed to correct for the associated aggregate demand externality.[1] There is a natural assignment of targets to instruments: macroprudential policy targets financial instability and monetary policy targets macroeconomic stability. There is no tradeoff between macroeconomic stability and financial stability.

When macroprudential policy is not available, there is in general a tradeoff between macroeconomic and financial stability. However, it is not clear whether monetary policy should be

[1] We use the term inflation targeting a bit loosely. Our model features perfectly rigid wages and so there is no inflation. We take the inflation targeting prescription to be ensuring that output is at potential. The reason we feel that this abuse of terminology is warranted is that standard micro-foundations for nominal rigidities would yield the "divine coincidence" between ensuring that output is at potential and guaranteeing that there is no inflation. It is also well understood that the divine coincidence can break, but these considerations are not the focus of this paper.

more hawkish (tolerating a recession with output below potential) or more dovish (tolerating a boom with output above potential) than the inflation targeting prescription during the boom. Indeed, in partial equilibrium, holding output constant, higher interest rates create excess savings both by reducing how much borrowers want to borrow and by raising how much savers want to save. The resulting excess savings is resorbed in general equilibrium by a reduction in output, which acts both by increasing how much borrowers want to borrow and by reducing how much savers want to save. Partial equilibrium thinking leads to the unambiguous conventional wisdom conclusion that a hawkish monetary policy curbs credit flowing to borrowers. General equilibrium thinking exposes this line of reasoning as a fallacy. Indeed, in a realistic benchmark case, monetary policy has no effect on leverage whatsoever. Monetary policy should then exactly follow the inflation targeting prescription of exclusively targeting macroeconomic stability (ensuring that output is at potential) and ignoring financial stability. Away from this benchmark, monetary policy can either be more hawkish or more dovish depending on subtle features of the economy.

We then introduce extrapolative expectations during the boom. The policy prescriptions outlined under rational expectations generalize when both monetary and macroprudential policy are available, but not when macroprudential policy is unavailable. With both monetary and macroprudential policy, it is still optimal to use macroprudential policy to limit leverage and monetary policy to ensure that output is at potential. The only difference is that macroprudential must now correct for both the aggregate demand and the belief externality from leverage. Without macroprudential policy, a new mechanism operating through endogenous beliefs restores the conventional wisdom that hawkish monetary policy curbs credit flowing to borrowers: increases in interest rates depress asset prices, lower the optimism of borrowers, and reduce their desired leverage. In order to improve financial stability, monetary policy should sacrifice perfect macroeconomic stability by adopting a more hawkish stance than prescribed by inflation targeting and tolerate a recession with output below potential.

In the main text, we only consider the possibility that expectations are extrapolative during the boom. This captures complacency, irrational exuberance, or boom psychology. In the appendix, we analyze the case where expectations are instead extrapolative during the bust. This specification captures doubt, irrational pessimism, or bust psychology, and delivers some interesting new lessons. The key is that financial stability becomes multidimensional since it now not only depends on the leverage of borrowers but also on the beliefs of agents during the bust. The latter in turn depend on asset prices during the boom: lower asset prices during the boom imply higher realized returns and less pessimism during the bust. In a sense, there are now three targets for only one or two instruments.

It is no longer optimal to use monetary policy to exclusively target macroeconomic stability (ensuring that output is at potential) during the boom when macroprudential policy is available. Monetary policy is more hawkish than the inflation targeting prescription and tolerates a recession with output below potential. This reduces asset prices, which in turn mitigates pessimism, stimulates asset prices, aggregate demand and output during the bust. The same applies when macroprudential policy is not available.

These lessons offer an interesting contrast with the cases of rational expectations or extrapolative expectations during the boom. In these cases, when macroprudential policy is available, it is optimal to target financial stability with macroprudential policy and to target macroeconomic stability with monetary policy. It is only when macroprudential policy is not available that monetary policy must be willing to trade off macroeconomic and financial stability. Instead, with extrapolative expectations during the bust, this assignment of targets to instruments breaks down. Even when macroprudential policy is available, monetary policy must be conducted with an eye towards financial stability instead of focusing exclusively on macroeconomic stability. Financial stability becomes a multidimensional target influenced by leverage and beliefs/asset prices. Macroprudential policy cannot control both at the same time and monetary policy must shoulder a share of the burden. Limiting leverage falls on macroprudential policy, and managing beliefs/asset prices on monetary policy.

The paper is organized as follows. In Section 2, we set up the model. In Section 3, we characterize optimal policy under rational expectations. In Section 4, we derive optimal policy when expectations are extrapolative during the boom. In Section 5, we explain how the results change when expectations are extrapolative during the bust. Proofs and detailed derivations are in Appendix A. Extensions are in Appendix B.

Relation to the literature. This paper is closely related to our previous work. With rational expectations, the model is a particular case of the general framework that we developed in a previous paper (Farhi and Werning, 2016) to deal with jointly optimal monetary and macroprudential policies in the presence of aggregate demand externalities. In that paper, we did not analyze this particular appplication, and did not develop applications with segmented financial markets and financial intermediaries. We also did not conduct an analysis of optimal monetary policy without macroprudential policy.

The contribution of Section 3, which maintains rational expectations, is therefore: to flesh out how the theory developed in our previous paper can be applied to think about financial intermediaries levering up to purchase risky assets or savers financing risky house purchases with mortgages; and to characterize optimal monetary policy when macroprudential policy is not available. With extrapolative expectations, the model takes a further deviation from our previous work. The model then features both aggregate demand and belief externalities. The original contribution of Section 3 is therefore to work out optimal monetary policy and jointly optimal monetary and macroprudential policies in the presence of these two interacting externalities.

This paper is also related to several strands of literature. First, there is the extensive literature on optimal monetary policy and inflation targeting (see e.g. Woodford, 2011; Clarida et al., 1999; Galí, 2015). There is also the literature on monetary policy with heterogenous agents (Eggertsson and Krugman, 2012; Werning, 2015; Farhi and Werning, 2016; McKay et al., 2016; Guerrieri and Lorenzoni, 2017; Caballero and Farhi, 2017; Auclert, 2019; Kaplan et al., 2018; Kekre and Lenel, 2020). In particular Eggertsson and Krugman (2012) was also motivated by Minsky's ideas, but abstract away from the boom to focus on policy during the bust, while we instead focus on policy during the boom. Finally, there is the positive literature on monetary policy with non-rational expectations with a representative agent (Gabaix, 2016; Angeletos and Lian, 2018; Garcia-Schmidt and Woodford, 2019; Farhi and Werning, 2019).

Second, this paper is related to the normative literature on monetary and macroprudential policy in the presence of aggregate demand externalities and potentially non-rational expectations (Farhi and Werning, 2016; Korinek and Simsek, 2016; Caballero and Simsek, 2018, 2019). These papers characterize jointly optimal monetary and macroprudential policy allowing for exogenous subjective beliefs but not for endogenous beliefs such as extrapolative expectations. This distinction is essential since the policy channels that we emphasize exist only when beliefs are endogenous to policy. Still in this line with exogenous beliefs, Caballero and Simsek (2019) analyze optimal monetary policy when macroprudential policy is not available in a model with a binding borrowing constraint which depends on asset prices. In this setting, they show that monetary policy should be hawkish and tolerate a recession with output below potential during the boom. We obtain a similar result. Our mechanism, like theirs, works through a reduction in asset prices. But while in their case, the reduction in asset prices tightens the borrowing constraint, in ours, it reduces leverage by tempering optimism.

Third, this paper is related to the literature on macro-finance models with extrapolative or diagnostic expectations (Bordalo, Gennaioli and Shleifer, 2018; Bordalo, Gennaioli, Shleifer and Terry, 2019; Maxted, 2019). Within this literature, our paper is unique in taking a normative perspective, and in allowing for nominal rigidities, monetary policy, and macroprudential policy.

2 Model

In this section, we introduce our framework. We set up the model, introduce some definitions and conventions, and discuss our assumptions as well as different possible interpretations.

2.1 Setup

The model is deliberately stylized and contains only the minimal ingredients required to make our points. There are three periods $t \in \{0,1,2\}$. Risk is characterized by a binary aggregate state $\omega \in \{H, L\}$ with probability $\bar{\pi}_s$, with $\bar{\pi}_H + \bar{\pi}_L = 1$. The state ω governs the realization of the dividend $D_{2,\omega}$ of a risky asset (a Lucas tree) in inelastic unit supply, with $D_{2,H} > D_{2,L}$. All uncertainty is resolved at $t = 2$ and expectations under these probabilities are denote by $\mathbb{E}[\cdot]$.

There are two types of agents $i \in \{S, B\}$: savers (S) and borrowers (B). We let ϕ^S and ϕ^B denote, respectively, the share of savers and borrowers, with $\phi^S + \phi^B = 1$. Savers work, consume, and lend to borrowers. Borrowers consume and borrow from savers to finance the purchase of the risky asset.

In each period, there is a single output good which is used for consumption. It is produced from labor at dates 0 and 1, and from the dividend of the risky at date 2. The prices of these inputs are rigid and normalized to one.

Policy consists of monetary policy at dates 0, 1 and 2, possibly supplemented with macroprudential policy at date 0. Monetary policy selects the equilibrium with the maximal level of output, equal to the dividend of the risky asset, at date 2. Monetary policy may be constrained by the ZLB. We will assume that the ZLB does not bind at date 0, but that it binds at date 1. This will imply that there is a liquidity-trap recession at date 1. We will often refer to date 0 as the "boom" and to date 1 as the "bust".

Preferences. The preferences of a saver are given by

$$(1-\beta_0)[\log c_0^S - h(l_0^S)] + \beta_0(1-\beta_1)[\log c_1^S - h(l_1^S)] + \beta_0\beta_1\mathbb{E}[\log c_2^S],$$

where c_t^S and l_t^S are consumption and labor at date t.

The preferences of a borrower are given by

$$(1-\beta_0)\log c_0^B + \beta_0(1-\beta_1)\log c_1^B + \beta_0\beta_1\mathbb{E}[\log c_2^B],$$

where c_t^B is consumption at date t.

Technology. At dates 0 and 1, output Y_t is produced linearly one for one using labor l_t^S and used for consumption c_t^S and c_t^B according to the resource constraints

$$\phi^S c_t^S + \phi^B c_t^B \leq Y_t = \phi^S l_t^S + \phi^B l_t^B.$$

At date 2 in state $\omega \in \{H, L\}$, output $Y_{2,\omega}$ is produced linearly one for one using the exogenous dividend $D_{2,\omega}$ of the Lucas tree and used for consumption according to the resource constraints

$$\phi^S c_{2,\omega}^S + \phi^B c_{2,\omega}^B \leq Y_{2,\omega} = D_{2,\omega}.$$

Market structure. Output is produced from inputs by competitive firms. The prices of all inputs are rigid and normalized to one. At dates 0 and 1, savers accommodate labor demand at this unit price. At date 2, borrowers accommodate dividend demand up to their holdings.

Markets are incomplete because they are segmented. In every period, savers and borrowers can trade a risk-free bond. Borrowers can trade the Lucas tree but savers cannot, and each borrower is initially endowed with $1/\phi^B$ units of the Lucas tree at date 0.

The budget constraints of a saver are given by

$$c_0^S + \frac{b_1^S}{R_0} - b_0^S - l_0^S - t_0^S \leq 0 \quad \text{with} \quad l_0^S = \frac{Y_0}{\phi^S},$$

$$c_1^S + \frac{b_2^S}{R_1} - b_1^S - l_1^S \leq 0 \quad \text{with} \quad l_1^S = \frac{Y_1}{\phi^S},$$

$$c_{2,\omega}^S - b_2^S \leq 0,$$

where b_t^S and R_t are risk-free bond holdings and interest rates (nominal and real) at date t.

The budget constraints of a borrower are given by

$$c_0^B + \frac{b_1^B}{R_0}(1-\tau_0) - b_0^B + (x_1^B - \frac{1}{\phi^B})P_0 - t_0^B \leq 0,$$

$$c_1^B + \frac{b_2^B}{R_1} - b_1^B + (x_2^B - x_1^B)P_1 \leq 0,$$

$$c_{2,\omega}^B - b_2^B - x_2^B D_{2,\omega} \leq 0$$

where b_t^S and x_t^B are risk-free bond and risky Lucas tree holdings at date t, P_t is the price of the Lucas tree at date t, τ_0 is a macroprudential tax on borrower borrowing.

The budget constraints of the government are given by

$$\phi^S t_0^S + \phi^B t_0^B + \frac{b_1^B}{R_0}\tau_0 \leq 0.$$

Policy. Policy consists of monetary policy ($\{R_t\}$) and macroprudential policy ($\{\tau_0, t_0^i\}$). Note that the macroprudential tax raises revenues which must be rebated in some form, and we assume that there is no constraint on the way these revenues are rebated across savers and borrowers. This will ensure that the optimal macroprudential tax is not affected by distributive concerns.

Equilibrium. An equilibrium $\{c_t^i, l_t^i, x_t^i, b_t^i, R_t, P_t\}$ given initial debt $\{b_0^i\}$ and policy $\{R_t, \tau_0, t_0^i\}$ is defined by the requirements that: savers and borrowers maximize their utilities from consumption subject to their budget constraints taking output, interest rates, and asset prices as given; gross interest rates are above unity; and markets clear. Sometimes, we will constrain policy by requiring either that there be only monetary policy ($\tau_0 = t_0^S = t_0^B = 0$).

Note that we assume that all savers find themselves rationed at the rigid wage, working in proportion to the demand for labor. For simplicity, we assume this is the case whether or not workers would wish to increase or decrease their labor.

In what follows we will consider a number of different optimal policy problems. We will assume that for all these policy problems, at optimum, the the ZLB does not bind at date 0 ($R_0 > 1$) but binds at date 1 ($R_1 = 1$). This can be guaranteed by assuming that β_0 is low enough and that β_1 is high enough.

2.2 Terminology

Before proceeding, it is useful to define the notions of *boom* and *bust* and how they relate to the *labor wedge*. Because wages are sticky, savers are off their labor supply curves at dates 0 and 1, leading to potentially nonzero labor wedges[2]

$$\mu_t = 1 - c_t^S h'(l_0^S).$$

Labor wedges are nonlinear inverse equivalents of *output gaps* in log-linearized New Keynesian models. A positive labor wedge $\mu_t > 0$ at date t indicates that *actual output* is below *potential output*. Conversely, a negative labor wedge $\mu_t < 0$ at date t indicates that actual output is above potential. Assuming that the ZLB binds at date 1 implies that $\mu_1 > 0$, or in other words that actual output is below potential at date 1.

Throughout the paper, we refer to the doctrine of *inflation targeting*, even though our model features no inflation. We take it to mean that monetary policy at date t should ensure that actual output is equal to potential output ($\mu_t = 0$) as long as the ZLB does not bind at t. In particular, we will say that monetary policy during the boom at date 0 is conducted in accordance with the principles of inflation targeting if it delivers no output gap ($\mu_0 = 0$).

We feel that this abuse of terminology is warranted for the following reason. If we incorporated a standard New Keynesian micro-foundation for sticky but imperfectly rigid wages, with a standard time-invariant subsidy to undo the associated monopoly/monopsony problems, the model would deliver that ensuring that output is at potential is equivalent to guaranteeing that there is no inflation (the "divine coincidence"). It is also well understood that the divine coincidence can break, but these considerations are not the focus of this paper.

[2]Because of our prior assumption that monetary policy maximizes output at date 2, there is no such wedge for dividends.

2.3 Discussion

We wrap up this introductory section by discussing how to interpret the model and comment on the role of some simplifying assumptions that we have made in order to maximize the transparency of the model and the logic of the arguments.

Borrowers can be interpreted in different ways. They could be interpreted as households taking on mortgages to finance the purchase of a house. They could also be interpreted as financial intermediaries who finance the purchase of risky assets with risk-free debt and inside equity.

We have made a number of assumptions for the sole purpose of streamlining the analysis. For example, we have assumed that savers do not work at date 2, that borrowers do not work at all, and that the Lucas tree only pays a dividend at date 2. These assumptions are made only for simplicity. We show in Appendix B that they could be relaxed without substantively altering our results.

We have also assumed that wages were entirely rigid. This is obviously an extreme assumption, but one that allows us to introduce Keynesian elements into the model without having to take a particular stand on the form of Philipps curve. Nonetheless, our insights are robust to less extreme forms of price stickiness.

3 Optimal Policy with Rational Expectations

In this section, we characterize optimal policy with rational expectations. We first treat the case where there is only monetary policy. We then cover the case where there is both monetary and macroprudential policy. The analysis sets the stage for the results with extrapolative expectations in Section 4.

3.1 Debt as a State Variable

Equilibria admit a simple recursive representation which allow us to link date 0 and dates 1 and 2 via a single state variable: the risk-free bond holdings b_1^S of savers between dates 0 and 1, which are the mirror image of the risk-free debt issuance of borrowers ($b_1^B = -(\phi^S/\phi^B)b_1^S$). Policy $\{R_0, \tau_0, t_0^i\}$ at date 0 influences the equilibrium at dates 1 and 2 only through this state variable.

Indeed, the consumptions and labors of savers at dates 1 and 2 are given by

$$c_1^S = (1-\beta_1)\left(b_1^S + \frac{Y_1}{\phi^S}\right),$$

$$c^S_{2,\omega} = \beta_1 \left(b^S_1 + \frac{Y_1}{\phi^S} \right).$$

The first equation follows from the fact that with log utility, the consumption function is linear in wealth $b^S_1 + Y_1/\phi^S$ with marginal propensity to consume $1-\beta_1$. The second equation follows from the first given that the interest rate between dates 1 and 2 is equal to $R_1 = 1$. The labor of a savers date 1 is given by

$$l^S_1 = \frac{Y_1}{\phi^S}.$$

The consumptions of borrowers at dates 1 and 2 are given by

$$c^B_1 = \frac{1}{\frac{\beta_1}{1-\beta_1}\mathbb{E}\left[\frac{1}{\frac{D_2}{\phi^B} - \frac{\phi^S}{\phi^B}\beta_1\left(b^S_1 + \frac{Y_1}{\phi^S}\right)}\right]},$$

$$c^B_{2,\omega} = \frac{D_{2,\omega}}{\phi^B} - \frac{\phi^S}{\phi^B}\beta_1 \left(b^S_1 + \frac{Y_1}{\phi^S} \right).$$

The second equation is obtained by combining the equation for the consumption of savers at date 2 above and from the resource constraint at date 2. The first equation is obtained by combining the first equation with the Euler equation of borrowers for risk-free bonds between dates 1 and 2.

Combining the equations for consumptions of savers and borrowers at date 1 with the resource constraint at date 1 then yields a fixed-point equation for output at date 1

$$Y_1 = (1-\beta_1)\left(\phi^S b^S_1 + Y_1\right) + \frac{1}{\frac{\beta_1}{1-\beta_1}\mathbb{E}\left[\frac{1}{D_2 - \beta_1(\phi^S b^S_1 + Y_1)}\right]}.$$

We denote the solution of this fixed-point equation by $Y_1(b^S_1)$.

Plugging back into the equations above characterizes the entire equilibrium allocation at dates 1 and 2 as a function of b^S_1: $c^S_1(b^S_1)$, $c^S_{2,\omega}(b^S_1)$, $l^S_1(b^S_1)$, $c^B_1(b^S_1)$, and $c^B_{2,\omega}(b^S_1)$. We can also compute portfolio holdings as well as the price of the risky asset. For brevity, we only report the asset price at date 1, which is given by

$$P_1(b^S_1) = \frac{\beta_1}{1-\beta_1} Y_1(b^S_1).$$

This equation is obtained by combining the resource constraint at date 1, the equation for the consumption of savers at date 1, and another expression for the consumption of borrowers at date 1, namely $c^B_1 = (1-\beta_1)(b^B_1 + P_1/\phi^B)$ where $\phi^S b^S_1 + \phi^B b^B_1 = 0$. Finally we can compute the labor wedge

$$\mu_1(b_1^S) = 1 - c_1^S(b_1^S)h'\left(\frac{Y_1(b_1^S)}{\phi^S}\right).$$

It is easy to see that the functions $Y_1(b_1^S)$ and $P_1(b_1^S)$ are increasing in b_1^S with

$$\frac{dY_1}{db_1^S} = \frac{1-\beta_1}{\beta_1}\frac{dP_1}{db_1^S} = \frac{\phi^S(1-\beta_1)\left[1 - \frac{\mathbb{E}\left[\left(\frac{1}{c_2^B}\right)^2\right]}{\left(\mathbb{E}\left[\frac{1}{c_2^B}\right]\right)^2}\right]}{\beta_1 + (1-\beta_1)\frac{\mathbb{E}\left[\left(\frac{1}{c_2^B}\right)^2\right]}{\left(\mathbb{E}\left[\frac{1}{c_2^B}\right]\right)^2}} < 0,$$

where we have suppressed the explicit dependence on b_1^S to lighten the notation. Basically, an increase in the savings that savers bring into date 1 and a mirror increase in the debts that borrowers bring into date 1 increase the systematic pro-cyclicality of borrower consumption at date 2. This increases the risk premium and reduces the price of the risky asset at date 1. And this in turn reduces aggregate demand and output at date 1 via a wealth effect. The effect is stronger, the riskier is the consumption of the borrower at date 2, and hence the riskier is the dividend of the risky asset and the higher is the leverage of borrowers.

Remark. Borrowing and savings decision at date 0 *influences output at date* 1 *despite the fact the savers and borrowers have the same marginal propensity to spend at date* 1 *out of wealth at date* 1*. However, they have different marginal propensities to spend across states at date 2 out of wealth at date* 1*: borrowers have a higher marginal propensity to spend than savers at date* 2 *in state H and a lower marginal propensity to spend than savers at date* 2 *in state L. A reshuffling wealth at date* 1 *away from borrowers and towards savers therefore increases the demand for consumption at date* 2 *in state L and decreases the demand for consumption at date* 2 *in state H. And for given outputs at date* 2 *in both states, it increases the price of consumption at date* 2 *in state L and decreases the price of consumption at date* 2 *in stage H, i.e. it increases the risk premium.*

3.2 Aggregate Demand Externality

Building on the analysis of the previous section, we can characterize the general equilibrium welfare for savers and borrowers at dates 1 and 2 using the following value functions:

$$V^S(b_1^S) = (1-\beta_1)\left[\log\left((1-\beta_1)\left(b_1^S + \frac{Y_1(b_1^S)}{\phi^S}\right)\right) - h\left(\frac{Y_1(b_1^S)}{\phi^S}\right)\right] + \beta_1\log\left(\beta_1\left(b_1^S + \frac{Y_1(b_1^S)}{\phi^S}\right)\right),$$

$$V^B(b_1^S) = (1-\beta_1)\log\left((1-\beta_1)\left(-\frac{\phi^S}{\phi^B}b_1^S + \frac{P_1(b_1^S)}{\phi^B}\right)\right) + \beta_1\mathbb{E}\left[\log\left(\frac{D_{2,\omega}}{\phi^B} - \frac{\phi^S}{\phi^B}\beta_1\left(b_1^S + \frac{Y_1(b_1^S)}{\phi^S}\right)\right)\right].$$

It is important to realize that these value functions encode the welfare of agents in general equilibrium, not the usual private value functions from the agent's problems in partial equilibrium.

The derivatives of these value functions encode the social marginal values of marginal increases in saver risk-free bond holdings b_1^S between dates 0 and 1, taking into account general equilibrium effects, and not the private marginal values computed in partial equilibrium. They will turn out to play a crucial role. Suppressing the explicit dependence of b_1^S to lighten the notation, they can be expressed as a functions of the marginal utility of wealth of savers $(1-\beta_1)/c_1^S$ and borrowers $(1-\beta_1)/c_1^B$ at date 1, the labor wedge μ_1 at date 1, and of the derivative dY_1/db_1^S of output at date 1:

$$\phi^S \frac{dV^S}{db_1^S} = \phi^S \frac{1-\beta_1}{c_1^S}\left(1+\frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right),$$

$$\phi^B \frac{dV^B}{db_1^S} = -\phi^S \frac{1-\beta_1}{c_1^B}.$$

In turn, the social marginal rate of substitution between savers and borrowers at date 1 with respect to b_1^S is given by

$$-\frac{\lambda^S \phi^S \frac{dV^S}{db_1^S}}{\lambda^B \phi^B \frac{dV^B}{db_1^S}} = \frac{\lambda^S c_1^B}{\lambda^B c_1^S}\left(1+\frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right),$$

where λ^S and λ^B are welfare Pareto weights on savers and borrowers (see below). It differs from the corresponding private marginal rate of substitution $(\lambda^S c_1^B)/(\lambda^B c_1^S)$. The proportional wedge between the social and private marginal rates of substitution is given by $1+(\mu_1/\phi^S)(dY_1/db_1^S)$. The social marginal rate of substitution is below the private one if there is a recession at date 1 ($\mu_1 > 0$), which will always be the case since we have assumed that the ZLB binds at date 1. As we saw above, a reshuffling of wealth away from borrowers and towards savers increases aggregate demand at date 1. This is socially beneficial because there is under-employment at date 1.

By contrast, the private and social marginal rates of substitution between savers and borrowers at date 0 coincide because there is no aggregate demand externality since monetary policy can be freely adjusted. They are both given by

$$\frac{\lambda^S c_0^B}{\lambda^B c_0^S}.$$

All in all, the general equilibrium effects at date 1 from borrowing and savings decisions at date 0 are aggregate demand externalities that are not internalized by private agents in their borrowing and saving decisions at date 0, which are instead driven by private marginal utilities of wealth. As a result, private borrowing and savings decisions at date 0 are in general not socially optimal. Another intuition for the aggregate demand externality $(\mu_1/\phi^S)(dY_1/db_1^S)$ is that it is commensurate with the desirability of stimulating output at date 1 (captured by $\mu_1 > 0$) and of the effects of changes in debt on output at date 1 (captured by $dY_1/db_1^S < 0$). These observations have important implications for the conduct of monetary and macroprudential policy.

Remark. The aggregate demand externality and the resulting wedge between the private and social marginal rate of substitution between savers and borrowers at date 1 *is only present if the ZLB binds. In fact, even though the the formulas for the derivatives of the value functions and for social marginal rate of substitution above were derived under the assumption that the ZLB binds at date* 1*, they also hold when the ZLB does not bind and when monetary policy instead perfectly stabilizes the economy leading to* $\mu_1 = 0$*. This is consistent with the results in Farhi and Werning (2016): the aggregate demand externality because of the combination of a constraint on monetary policy (the ZLB at date* 1*) and heterogeneities across agents in marginal propensities to spend (across states at date* 2*).*

3.3 Monetary Policy Only

In this section, we analyze optimal monetary policy when macroprudential policy in not available ($\tau_0 = t_0^S = t_0^L = 0$). The only free policy instrument is the interest rate R_0 at date 0.

Equilibrium as a function of monetary policy. We start by characterizing the equilibrium as a function of R_0. To do so, we proceed in two steps. First, we use the recursive representation described above to compute c_0^S, c_0^B, Y_0, and P_0 as functions of the state variable b_1^S and of the interest rate R_0 at date 0. Recall that we have already characterized c_1^S, c_1^B, Y_1, P_1, $c_{2,\omega}^S$, and $c_{2,\omega}^B$ as functions of b_1^S in Section 3.2. Second, we derive a fixed-point equation describing the dependence of b_1^S on R_0. Combining these two steps allows us to complete our characterization.

Using the Euler equations for risk-free bonds between dates 0 and 1 and the consumption functions at date 1, we can express the consumptions c_0^S and c_0^B of savers and borrowers at date 0 as functions b_1^S and R_0:

$$c_0^S = \frac{1-\beta_0}{\beta_0 R_0}\frac{c_1^S(b_1^S)}{1-\beta_1} \quad \text{with} \quad c_1^S(b_1^S) = (1-\beta_1)\left(b_1^S + \frac{Y_1(b_1^S)}{\phi^S}\right),$$

$$c_0^B = \frac{1-\beta_0}{\beta_0 R_0}\frac{c_1^B(b_1^S)}{1-\beta_1} \quad \text{with} \quad c_1^B(b_1^S) = (1-\beta_1)\left(-\frac{\phi^S}{\phi^B}b_1^S + \frac{P_1(b_1^S)}{\phi^B}\right) \quad \text{and} \quad P_1(b_1^S) = \frac{\beta_1}{1-\beta_1}Y_1(b_1^S).$$

Combining these two equations with the resource constraint at date 0, we get the following expression for aggregate output Y_0 at date 0 as a function of b_1^S and R_0:

$$Y_0 = \frac{1-\beta_0}{\beta_0(1-\beta_1)}\frac{Y_1(b_1^S)}{R_0}.$$

Using the consumption function and the budget constraint for the savers at date 0, we get a different equation for aggregate output Y_0 at date 0 as a function of b_1^S and R_0:

$$c_0^S = \frac{Y_0}{\phi^S} + b_0^S - \frac{b_1^S}{R_0} \quad \text{with} \quad c_0^S = (1-\beta_0)\left(\frac{Y_0}{\phi^S} + \frac{1}{R_0}\frac{Y_1(b_1^S)}{\phi^S} + b_0^S\right).$$

Combining this equation with the previous one yields a fixed-point equation for b_1^S:

$$b_1^S = \beta_0 R_0 b_0^S + \frac{\beta_1(1-\beta_0)}{1-\beta_1}\frac{Y_1(b_1^S)}{\phi^S}.$$

To highlight its dependence on R_0, we denote the solution by $b_1^S(R_0)$. Plugging back this function into the previous equations defines $c_0^S(R_0)$, $c_0^B(R_0)$, and $Y_0(R_0)$ as functions of R_0. Similarly, the price P_0 of the risky asset can be expressed as a function of R_0

$$P_0(R_0) = \frac{\beta_1}{1-\beta_1}\frac{Y_1(b_1^S(R_0))}{R_0}.$$

Effects of monetary policy. Key to our subsequent analysis will be the comparative statics of the equilibrium with respect to interest rates R_0 at date 0. They can all be deduced from the effects of interest rates R_0 at date 0 on the debt due by borrowers to savers at date 1:

$$\frac{db_1^S}{dR_0} = \frac{\beta_0 b_0^S}{1 - \frac{\beta_1(1-\beta_0)}{1-\beta_1}\frac{1}{\phi^S}\frac{dY_1}{db_1^S}}.$$

Indeed, we can plug this back into the equations defining $c_0^S(R_0)$, $c_0^B(R_0)$, $Y_0(R_0)$, $P_0(R_0)$, $c_1^S(b_1^S(R_0))$, $c_1^B(b_1^S(R_0))$, $Y_1(b_1^S(R_0))$, $P_1(b_1^S(R_0))$, $c_2^S(b_1^S(R_0))$, $c_2^B(b_1^S(R_0))$, $Y_2(b_1^S(R_0))$ to compute their derivatives with respect to R_0.

We see that the effects of changes in interest rates R_0 on the debt b_1^S due by borrowers to savers at date 1 depend on the initial leverage of borrowers (b_0^S). We call this the initial leverage channel. To gain some intuition, we start with the case where borrowers have no initial debt ($b_0^S = 0$). In this case, it is easy to see that changes in the interest rate R_0 at date 0 have no impact on the debt b_1^S due by borrowers to savers ($db_1^S/dR_0 = 0$). At date 0, output Y_0, the consumptions of savers and borrowers c_0^S and c_0^B, labor l_0^S, and the price of the risky asset P_0, all move in inverse proportion to R_0. An increase in the interest rate R_0 reduces the savings of saver b_1^S/R_0 and the borrowing of borrowers $-(\phi^S/\phi^B)(b_1^S/R_0)$ at date 0, but does not affect the risk-free bond holdings of savers b_1^S and the debt due by borrowers $-(\phi^S/\phi^B)b_1^S$ at date 1.

This benchmark is useful to understand the cases where borrowers are initially debtors or creditors ($b_0^S \neq 0$). For example, when borrowers are initially debtors ($b_0^S > 0$), this initial leverage implies that the wealth of borrowers is more sensitive (in proportion) to changes in interest rate R_0 at date 0 than the wealth of savers. As a result, an increase in the interest rate R_0 at date 0 leads to a reduction in the debt $-(\phi^S/\phi^B)b_1^S$ due by borrowers to savers at date 1 ($db_1^S/dR_0 < 0$). All these arguments are reversed in the case where borrowers are initially creditors ($b_0^S < 0$), in which case increasing interest rates at date 0 increases the debt b_1^S due by borrowers to savers ($db_1^S/dR_0 > 0$).

As emphasize in Werning (2015), there is an intimate relationship between the effects of interest rates at date 0 on the debt due by borrowers to savers at date 1 (db_1^S/dR_0) and their effects on output and asset prices at dates 0 (dY_0/dR_0 and dP_0/dR_0) and 1 (dY_1/dR_0 and dP_1/dR_0). When borrowers have no initial debt ($b_0^S = 0$), changes in interest rates at date 0 have no effect on the debt due by savers to borrowers at date 1 ($db_1^S/dR_0 = 0$), no effect on output and asset prices at date 1 ($dY_1/dR_0 = 0$ and dP_1/dR_0), and one-for-one effects on output and asset prices at date 0 ($(R_0/Y_0)(dY_0/dR_0) = -1$ and $(R_0/P_0)(dP_0/dR_0) = -1$). The unitary elasticity of output and asset prices to interest rates at date 0 ($-(R_0/Y_0)(dY_0/dR_0) = 1$ and $-(R_0/P_0)(dP_0/dR_0) = 1$) is exactly the one that would obtain in a complete-markets economy where savers could also trade the risky assets. Similarly, when borrowers are initially debtors ($b_0^S > 0$), increases in interest rates at date 0 reduce the debt due by savers to borrowers at date 1 ($db_1^S/dR_0 < 0$), increase output and asset prices at date 1 ($dY_1/dR_0 > 0$ and $dP_1/dR_0 > 0$), and reduce output and asset prices at date 0 more than one for one ($(R_0/Y_0)(dY_0/dR_0) < -1$ and $(R_0/P_0)(dP_0/dR_0) < -1$). The elasticity of output to interest rates at date 0 is greater than the one that would obtain in a complete-markets economy ($-(R_0/Y_0)(dY_0/dR_0) > 1$). All these arguments are reversed when borrowers are initially creditors ($b_0^S > 0$).

Optimal monetary policy. The planning problem for optimal monetary policy is

$$\max_{R_0} \left\{ \lambda^S \phi^S \left[(1-\beta_0) \left(\log(c_0^S(R_0)) - h(\frac{Y_0(R_0)}{\phi^S}) \right) + \beta_0 V^S(b_1^S(R_0)) \right] \right.$$
$$\left. + \lambda^B \phi^B \left[(1-\beta_0) \log(c_0^B(R_0)) + \beta_0 V^B(b_1^S(R_0)) \right] \right\},$$

where λ^S and λ^L are welfare Pareto weights on savers and borrowers.

The first-order condition for optimality of the interest rate R_0 at date 0 can be expressed as

$$\mu_0 \frac{1}{\phi^S} \frac{dY_0}{dR_0} + \frac{1}{R_0} \frac{db_1^S}{dR_0} \frac{\mu_1}{\phi^S} \frac{dY_1}{db_1^S} + \left(1 - \frac{\lambda^B c_0^S}{\lambda^S c_0^B}\right) \frac{b_1^S}{R_0^2} = 0.$$

The three terms in this formula capture the three margins that are traded off by monetary policy at date 0. Contemplate a marginal increase in the interest rate at date 0. The first term captures that it reduces output Y_0 at date 0 which is detrimental if output is below potential at date 0 ($\mu_0 > 0$). The second term captures that it increases (decreases) output Y_1 at date 1 if it decreases (increases) the debt b_1^S due by borrowers to savers at date 1, which tends to be beneficial (detrimental) since output is below potential at date 1 ($\mu_1 > 0$). The third term captures that as long as borrowers are debtors at date 1 ($b_1^S > 0$), it redistributes from borrowers to savers in proportion to b_1^S / R_0^2 by Roy's identity, which is beneficial if borrowers are less valued than savers as captured by their relative private marginal utilities of wealth ($(\lambda^B c_0^S)/(\lambda^S c_0^B) < 1$), but detrimental if borrowers are more valued than savers ($(\lambda^B c_0^S)/(\lambda^S c_0^B) > 1$).

Distributive concerns are largely orthogonal to the efficiency issues that we wish to focus on. We neutralize them by choosing the welfare Pareto weights λ^S and λ^B so that at the optimum, they are proportional to the private marginal utilities of wealth of savers and borrowers at date 0 ($\lambda^S/\lambda^B = c_0^S/c_0^B$) or equivalently at date 1 ($\lambda^S/\lambda^B = c_1^S/c_1^B$).[3] Then the first-order condition for optimality of the interest rate R_0 at date 0 simplifies to

$$\mu_0 \frac{1}{\phi^S} \frac{dY_0}{dR_0} + \frac{1}{R_0} \frac{db_1^S}{dR_0} \frac{\mu_1}{\phi^S} \frac{dY_1}{db_1^S} = 0.$$

Another way to think about these two terms as two targets that are traded off by monetary policy at date 0 is as follows: the first term captures concerns for macroeconomic stabilization during the boom at date 0 (stabilizing μ_0 close to zero), and the second term captures concerns for financial stabilization (reducing b_1^S) to improve macroeconomic stabilization during the bust at date 1 (increasing Y_1) and thereby correct for the aggregate demand externality from leverage. We will refer to this language below.

Proposition 1. *Suppose that the only available policy is monetary policy and that expectations are rational. Suppose in addition that welfare Pareto weights are proportional to the private marginal utilities of wealth at date* 0 *($\lambda^S/\lambda^B = c_0^S/c_0^B$) at the optimum. Then under optimal monetary policy, the labor wedge μ_0 at date* 0 *satisfies the following equation*

$$\mu_0 = \left(\frac{\mu_1}{\phi^S} \frac{dY_1}{db_1^S} \right) \left(\frac{\phi^S \frac{db_1^S}{dR_0}}{-R_0 \frac{dY_0}{dR_0}} \right), \quad \textit{with} \quad \frac{db_1^S}{dR_0} = \frac{\beta_0 b_0^S}{1 - \frac{\beta_1(1-\beta_0)}{1-\beta_1} \frac{1}{\phi^S} \frac{dY_1}{db_1^S}}.$$

Recall that output is below potential during the bust at date 1 *($\mu_1 > 0$). Mild technical conditions guarantee that $dY_0/dR_0 < 0$, which we shall assume.*[4] *If borrowers have no initial debt ($b_0^S = 0$), then changes in interest rates at date* 0 *have no impact on the the debt due by borrowers to savers date* 1 *($db_1^S/dR_0 = 0$) and output is at potential during the boom at date* 0 *($\mu_0 = 0$). If borrowers are initially debtors ($b_0^S > 0$), then increases in interest rates at date* 0 *reduce the debt due by savers to borrowers at date* 1 *($db_1^S/dR_0 < 0$), and output is below potential during the boom at date* 0 *($\mu_0 < 0$). If instead*

[3] The corresponding point on the constrained Pareto frontier can be found by computing c_1^S/c_1^B at the optimum as a function of λ^S/λ^B and finding a point at which this curve crosses the 45 degree line. Since c_1^S/c_1^B has a finite limit when λ^S/λ^B tends to ∞, and a strictly positive limit as λ^S/λ^B tends to 0, such a point is guaranteed to exist.

[4] We have $dY_0/dR_0 < 0$ if $b_0^S \geq 0$ or if $b_0^S < 0$ and $|b_0^S|$ is small enough or there is not too much dividend risk.

borrowers are initially creditors ($b_0^S < 0$), then increases in interest rates at date 0 increase the debt due by borrowers to savers at date 1 ($db_1^S/dR_0 > 0$) and output is above potential during the boom at date 0 ($\mu_0 < 0$).

In general, the labor wedge at date 0 is proportional to $(\mu_1/\phi^S)(dY_1/db_1^S)$ and to db_1^S/dR_0. The former is the aggregate demand externality which captures the desirability of stimulating output Y_1 at date 1 (via $\mu_1 > 0$) and the effects of changes in debt on output at date 1 (via $dY_1/db_1^S > 0$). The latter, which is commensurate with the initial risk-free bond holdings b_0^S of savers at date 0, captures the effects of changes in interest rates R_0 at date 0 on the debt b_1^S due by borrowers to savers at date 1. All in all, increases in interest rates R_0 at date 0 stimulate output Y_1 at date 1 only to the extent that they reduce the debt b_1^S due by borrowers to savers at date 1.

When borrowers have no initial debt ($b_0^S = 0$), we have shown that changes in the interest rate R_0 at date 0 have no impact on the debt b_1^S due by borrowers to savers ($db_1^S/dR_0 = 0$). As a result, monetary policy at date 0 cannot stimulate output at date 1 and therefore has no prudential value and no effect on financial stability. Since we have neutralized distributive issues, it is not desirable to use monetary policy at date 0 for distributive purposes either. Monetary policy at date 0 is therefore only used for macroeconomic stabilization at date 0. In other words, optimal monetary policy during the boom at date 0 is implemented according to the the usual inflation targeting rule of stabilizing the output gap at date 0.

When borrowers are initially debtors ($b_0^S > 0$), we have shown that increases in the interest rate R_0 at date 0 reduce the debt b_1^S due by borrowers to savers ($db_1^S/dR_0 < 0$). Hence, increasing interest rates at date 0 (by tolerating $\mu_0 > 0$) now has prudential value by improving financial stability (reducing b_1^S) and improving macroeconomic stability at date 1 (increasing Y_1). Optimal monetary policy therefore deviates from perfect macroeconomic stabilization during the boom at date 0 by pushing output below potential in order to improve financial stability and stimulate the economy at date 1. In other words, optimal policy during the boom at date 0 is hawkish and leans against the wind.

All these arguments are reversed in the case where borrowers are initially creditors ($b_0^S < 0$). Then optimal monetary policy during the boom at date 0 is dovish and leans into the wind by pushing output above potential at date 0 in order to improve financial stability and stimulate the economy at date 1.

Remark. Proposition 1 assumes that the ZLB binds at date 1. If it does not, then monetary policy optimally stabilizes output at date 1 ($\mu_1 = 0$), and there is no longer any aggregate demand externality from private borrowing and saving decisions at date 0. The results in the proposition then still apply but with $\mu_1 = 0$. It is therefore optimal to also perfectly stabilize output at date 0 ($\mu_0 = 0$). Basically, since there is perfect macroeconomic stabilization at date 1, there is no financial stability concern at date 0, and hence monetary policy at date 0 focuses solely on macroeconomic stabilization in accordance with *the usual inflation targeting prescription.*

3.4 Monetary Policy and Macroprudential Policy

In this section, we analyze jointly optimal monetary and macroprudential policy. The free policy instruments are therefore the interest rate R_0 at date 0, the macroprudential tax τ_0 on the leverage of borrowers, and the rebates t_0^S and t_0^B.

Implementability conditions. The implementability conditions are as follows. First, we have the Euler equation of savers for the risk-free bond between dates 0 and 1, and the Euler equations of borrowers for the risk-free bond and for the risky asset between dates 0 and 1 :

$$\frac{1}{c_0^S} = \frac{\beta_0}{1-\beta_0} R_0 \frac{1-\beta_1}{c_1^S(b_1^S)},$$

$$\frac{1}{c_0^B} = \frac{\beta_0}{1-\beta_0} \frac{R_0}{1-\tau_0} \frac{1-\beta_1}{c_1^B(b_1^S)},$$

$$\frac{1}{c_0^B} = -\frac{\phi^S}{\phi^B} \frac{\beta_0}{1-\beta_0} \frac{P_1(b_1^S)}{P_0} \frac{1-\beta_1}{c_1^B(b_1^S)},$$

where

$$\frac{1-\beta_1}{c_1^S} = \frac{1}{1+\frac{\mu_1(b_1^S)}{\phi^S}\frac{dY_1(b_1^S)}{db_1^S}} \frac{dV^S(b_1^S)}{db_1^S} \quad \text{and} \quad \frac{1-\beta_1}{c_1^B(b_1^S)} = -\frac{\phi^S}{\phi^B}\frac{dV^S(b_1^S)}{db_1^S}.$$

Second, we have the budget constraint of savers and borrowers at date 0:

$$c_0^S + \frac{b_1^S}{R_0} = b_0^S + t_0^S,$$

$$c_0^B - \frac{\phi^S}{\phi^B}\frac{b_1^S}{R_0} = -\frac{\phi^S}{\phi^B} b_0^S + t_0^B.$$

Third, we have the resource constraint at date 0

$$\phi^S c_0^S + \phi^B c_0^B \leq Y_0.$$

The Euler equations of savers and borrowers between dates 1 and 2, their budget constraints at dates 1 and 2, and the resource constraints at dates 1 and 2, are all subsumed in the value functions $V^S(b_1^S)$ and $V^B(b_1^S)$. The budget constraints of the government are redundant by Walras' law.

It is easy to see that a necessary and sufficient condition for c_0^S, c_0^B, and b_1^S to be implementable as part of an equilibrium is that they satisfy the resource constraint at date 0. The necessary part is trivial, and so we only explain the sufficiency part. Take c_0^S, c_0^B, and b_1^S satisfying the resource constraint at date 0. We pick R_0 to satisfy the Euler equation of savers for the risk-free bond between dates 0 and 1. Given R_0, we pick τ_0 to satisfy the Euler equation of borrowers for the risk-free bond between dates 0 and 1. We pick P_0 to satisfy the Euler equation of borrowers for the risky asset between dates 1 and 2. We pick t_0^S to satisfy the budget constraint of savers. We pick t_0^S to satisfy the budget constraint of savers. Finally, we note that the resource constraint at date 0 is satisfied by assumption. This completes the argument.

Jointly optimal monetary and macroprudential policies. Building on our discussion of the implementability conditions, we set up the planning problem for jointly optimal monetary and macroprudential policy as a maximization over Y_0, c_0^S, c_1^S, and b_1^S subject to the resource constraint at date 0

$$\max_{\{Y_0,c_0^S,c_0^B,b_1^S\}} \left\{ \lambda^S\phi^S \left[(1-\beta_0)\left(\log(c_0^S) - h(\frac{Y_0}{\phi^S})\right) + \beta_0 V^S(b_1^S)\right] \right.$$
$$\left. + \lambda^B\phi^B\left[(1-\beta_0)\log(c_0^B) + \beta_0 V^B(b_1^S)\right]\right\},$$

subject to

$$\phi^S c_0^S + \phi^B c_0^B \le Y_0.$$

We use the other implementability conditions to deduce the values of R_0, P_0, τ_0, t_0^S, and t_0^B that implement the corresponding equilibrium.

The first-order conditions for optimality are

$$\frac{1}{c_0^S} - h'(\frac{Y_0}{\phi^S}) = 0,$$

$$\frac{\lambda^B}{c_0^B} - \frac{\lambda^S}{c_0^S} = 0,$$

$$\lambda^S\phi^S\frac{dV^S}{db_1^S} + \lambda^B\phi^B\frac{dV^B}{db_1^S} = 0,$$

where the values for dV^S/db_1^S and dV^B/db_1^S derived in Section 3.2 are

$$\phi^S\frac{dV^S}{db_1^S} = \phi^S\frac{(1-\beta_1)}{c_1^S}\left(1 + \frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right) \quad \text{and} \quad \phi^B\frac{dV^B}{db_1^S} = -\phi^S\frac{(1-\beta_1)}{c_1^B}.$$

The first equation immediately delivers that there is no labor wedge at date 0:

$$\mu_0 = 0.$$

Combining the second, third, and fourth equations yields that the (private or social) marginal rate of substitution $(\lambda^S c_0^B)/(\lambda^B c_0^S)$ between the consumptions of savers and borrowers at date 0 is equal to the social (not private) marginal rate of substitution between savers and borrowers at date 1:

$$\frac{\lambda^S c_0^B}{\lambda^B c_0^S} = -\frac{\lambda^S \phi^S \frac{dV^S}{db_1^S}}{\lambda^B \phi^B \frac{dV^B}{db_1^S}} = \frac{\lambda^S c_1^B}{\lambda^B c_1^S}\left(1 + \frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right),$$

which immediately implies that macroprudential taxes are required to offset the aggregate demand externality

$$1 - \tau_0 = \left(1 + \frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right).$$

Proposition 2. *Suppose that monetary policy and macroprudential policy are available and that expectations are rational. Then under jointly optimal monetary and macroprudential policy, the labor wedge μ_0 at date 0 is given by*

$$\mu_0 = 0,$$

and the macroprudential tax on the leverage of borrower satisfies

$$1 - \tau_0 = \left(1 + \frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right) \quad \text{or equivalently} \quad \tau_0 = -\frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S} > 0.$$

During the boom at date 0, output is always at potential at date 0 ($\mu_0 = 0$) and there is a positive macroprudential tax on the leverage of borrowers ($\tau_0 > 0$) which exactly offsets the aggregate demand externality $(\mu_1/\phi^S)(dY_1/db_1^S) < 0$.

When macroprudential policy is available, monetary policy focuses solely on macroeconomic stabilization during the boom at date 0 as prescribed by the usual inflation targeting rule. Monetary policy is used to maintain output at potential.

Financial stabilization falls entirely on macroprudential policy at date 0. Macroprudential policy leans against the wind by imposing a positive tax on the leverage of borrowers. The resulting macroprudential wedge $1 - \tau_0 < 1$ is exactly equal to the wedge $1 + (\mu_1/\phi^S)(dY_1/db_1^S) < 1$ between the social and private marginal rates of substitution between savers and borrowers at date 1 which reflects the aggregate demand externality. The macrorpudential tax $\tau_0 = -(\mu_1/\phi^S)(dY_1/db_1^S)$ is proportional to the severity of the recession at date 1 (measured by μ_1) and to the sensitivity of aggregate output at date 1 to a reduction in the leverage of borrowers (measured by $-dY_1/db_1^S$). The former indicates how desirable it is to stimulate output at date 1, and the latter how effective a reduction in the leverage of borrowers is at stimulating output at date 1.

Remark. Proposition 2 assumes that the ZLB binds at date If it does not, then monetary policy optimally stabilizes output at date 1 ($\mu_1 = 0$), and there is no longer any aggregate demand *externality from private borrowing and saving decisions at date* 0*. The results in the proposition then still apply but with* $\mu_1 = 0$*. There is no longer any need for macroprudential policy during the boom at date* 0 *(*$\tau_0 = 0$*).*

4 Optimal Policy with Extrapolative Expectations

We now allow for extrapolative expectations during the boom, between dates 0 and 1. We continue to assume rational expectations during the bust between dates 1 an 2. More specifically, we assume that borrowers hold extrapolative expectations during the boom, between dates 0 and 1. We continue to assume that borrowers are rational during the bust between dates 1 an 2, and that savers are rational throughout. That savers are rational throughout implies that borrowers and savers have different expectations during the boom between dates 0 and 1. We assume that borrowers and savers "agree to disagree" and do not try to learn from each other or from market signals. Since only borrowers can trade the risky asset, its price reflects their expectations. Savers may perceive that the asset is overpriced but cannot act on it.

We envision a policy planner who understands that borrowers have extrapolative expectations between dates 0 and 1 and who evaluates welfare at date 0 under the true probabilities. This implies that the planner is paternalistic. We believe that this makes sense given the problem that we are interested in. Indeed, it is consistent with the fact that in practice, policymakers frequently voice concerns of "irrational exhuberance" during booms. Furthermore, assuming that the policy planner is not paternalistic would have unappealing implications in our context where beliefs are endogenous: it would then make sense to conduct policy to influence beliefs for the sole purpose of making agents "feel better" by falsely expecting high welfare. Our assumption eliminates this undesirable policy incentive. It ensures that the policy planner may seek to influence beliefs, but only in order to influence the true allocation and the true welfare that agents ultimately experience.

4.1 Introducing Extrapolative Expectations

We take as given a past price P_{-1} of the risky asset, and assume that the expectations of borrowers regarding the future asset price P_1^e are extrapolative during the boom between dates 0 and 1 so that the return P_1^e/P_0 that borrowers expect on the risky asset between dates 0 and 1 is a weighted average of the true expected return P_1/P_0 and of the past return with respective weights $1-\rho$ and ρ:

$$\frac{P_1^e}{P_0} = (1-\rho)\frac{P_1}{P_0} + \rho\frac{P_0}{P_{-1}}.$$

At date 1, there is a "Minsky moment": borrowers realize that their expectations were wrong and start forming their expectations according to the true probabilities. Savers are rational throughout. The case where all agents hold rational expectations at all dates covered in Section 3 is obtained as the special case where $\rho = 0$.

Our model actually admits two different interpretations. Under the first interpretation, non-rational expectations are modeled as "wedges in the Euler equations" for the risky asset. Borrowers only hold extrapolative expectations regarding the price $P_1^e \neq P_1$ of the risky asset but not regarding the whole economy $Y_1^e = Y_1$. They do not realize that a high asset price at date 1 will increase aggregate demand and output, perhaps because of a limited ability to think through general equilibrium mechanisms. Extrapolative expectations are therefore confined to financial markets, and macroeconomic expectations remain rational. Since savers cannot trade the risky asset, it actually does not matter whether only borrowers or both borrowers and savers extrapolate. The price of the risky asset price reflects only reflects the extrapolative beliefs of borrowers. Savers may perceive that the asset is overpriced but cannot act on it.

Under the second interpretation, non rational expectations are modeled as "subjective probabilities" placed on the states of the world. Borrowers hold consistent extrapolative expectations regarding the price $P_1^e \neq P_1$ of the risky asset and regarding the whole economy $Y_1^e = ((1-\beta_1)/\beta_1)P_1^e \neq Y_1$. They realize that a high asset price at date 1 will increase aggregate demand and output. Macroeconomic expectations and financial expectations are both extrapolative. It then matters that only borrowers extrapolate and that savers remain rational. Savers do perceive that the asset is overpriced but cannot act on it.

Both paradigms are defensible. In all cases, compared to analysis that we conducted under rational expectations, the only relevant change is the extrapolative expectations of borrowers regarding asset prices because their optimism influences their desired borrowing during the boom at date 0. The optimism of savers which can be allowed under the first interpretation has no impact on their desired borrowing at date 0. The two key properties that are preserved by these different specifications are: the endogeneity of beliefs to policy during the boom at date 0; and the heterogeneity of their incidence on the borrowing decisions of savers and borrowers during the boom between dates 0 and 1.[5]

For simplicity only, we conduct the analysis under the first interpretation: we assume that only borrowers extrapolate but that savers remain rational; and we assume that borrowers only extrapolate the asset price but not the whole economy. It should be clear from this discussion that the results apply identically to the different variants that we have just outlined.

[5]If borrowers received labor income at date 1, there would be a difference between these two paradigms. Our qualitative results would hold under both.

4.2 Aggregate Demand and Belief Externalities

The allocation and the value functions at dates 1 and 2 given b_1^S are the same as under rational expectations described in Sections 3.1 and 3.2. This implies that b_1^S still summarizes all the links through which the allocation at date 0 influence the allocation at date 1.

As under rational expectations, the social marginal rate of substitution between savers and borrowers at date 1 with respect to b_1^S is given by

$$-\frac{\lambda^S \phi^S \frac{dV^S}{db_1^S}}{\lambda^B \phi^B \frac{dV^B}{db_1^S}} = \frac{\lambda^S c_1^B}{\lambda^B c_1^S}\left(1 + \frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right).$$

The difference is that now $(\lambda^S c_1^B)/(\lambda^B c_1^S)$ is no longer the private marginal rate of substitution. Indeed, because agents expect different consumptions, the private marginal rate of substitution is now $(\lambda^S c_1^{B,e})/(\lambda^B c_1^S)$. It is therefore more useful to write

$$-\frac{\lambda^S \phi^S \frac{dV^S}{db_1^S}}{\lambda^B \phi^B \frac{dV^B}{db_1^S}} = \frac{\lambda^S c_1^{B,e}}{\lambda^B c_1^S}\left(\frac{c_1^B}{c_1^{B,e}}\right)\left(1 + \frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right).$$

The private and social marginal rates of substitution between savers and borrowers at date 0 with respect to b_1^S coincide because there is no aggregate demand externality since monetary policy can be freely adjusted. They are both given by

$$\frac{\lambda^S c_0^B}{\lambda^B c_0^S}.$$

The wedge between the social and private marginal rates of substitution between savers and borrowers at date 1 now has has two distinct components: the aggregate demand externality captured by $1 + (\mu_1/\phi^S)(b_1^S)(dY_1/db_1^S)$ as under rational expectations, and a new belief externality captured by $(c_1^B/c_1^{B,e})$. The intuition for the aggregate demand externality is the same as under rational expectations spelled out in Section 3.2. The belief externality is new. It reflects the fact that borrowers make their borrowing and saving decisions at date 0 under the wrong expectations.

As explained above, the aggregate demand externality always pushes the social marginal rate of substitution between savers and borrowers below the private one at date 1. The belief externality reinforces this effect borrowers are optimistic regarding their consumption at date 1 $(c_1^{B,e}/c_1^B > 1)$. The interaction of aggregate demand and belief externalities will have important implications for the conduct of policy.

4.3 Monetary Policy Only

In this section, we analyze optimal monetary policy when macroprudential policy in not available ($\tau_0 = t_0^S = t_0^L = 0$). The only free policy instrument is the interest rate R_0 at date 0.

Equilibrium as a function of monetary policy. We start by characterizing the equilibrium as a function of R_0. To do so, we proceed in three steps. First, we compute P_0 and P_1^e as a function of b_1^S and R_0. Second, we compute c_0^S, c_0^B, Y_0, and P_0 as functions of b_1^S and R_0. Recall that we have already characterized c_1^S, c_1^B, Y_1, P_1, $c_{2,\omega}^S$, and $c_{2,\omega}^B$ as functions of b_1^S in Section 3.2. Third, we derive a system of fixed-point equations describing the dependence of b_1^S on R_0. Combining these two steps allows us to complete our characterization.

Combining the equation describing the extrapolation of the asset price above with the Euler equation of borrowers for the risk-free bond and for the risky asset yields the asset pricing equation:

$$P_0 = \frac{P_1^e}{R_0} \quad \text{where} \quad \frac{P_1^e}{P_0} = (1-\rho)\frac{P_1(b_1^S)}{P_0} + \rho\frac{P_0}{P_{-1}}.$$

This allows us to express P_0 and P_1^e as functions of b_1^S and R_0 :

$$P_0(b_1^S, R_0) = \frac{R_0 P_{-1} - \sqrt{(R_0 P_{-1})^2 - 4\rho(1-\rho)P_{-1}P_1(b_1^S)}}{2\rho},$$

$$P_1^e(b_1^S, R_0) = (1-\rho)P_1(b_1^S) + \rho\frac{P_0^2(b_1^S, R_0)}{P_{-1}},$$

where recall that $P_1(b_1^S) = (\beta_1/(1-\beta_1))Y_1(b_1^S)$. Below, we will need the derivatives of these functions with respect to b_1^S and R_0. It is easy to see that:

$$\frac{\partial P_0}{\partial R_0} = -\frac{P_{-1}}{2\rho}\left(\frac{1 - \sqrt{1 - 4\rho(1-\rho)\frac{P_1(b_S)}{R_0 P_{-1}}}}{\sqrt{1 - 4\rho(1-\rho)\frac{P_1(b_S)}{R_0 P_{-1}}}}\right) < 0,$$

$$\frac{\partial P_0}{\partial b_1^S} = (1-\rho)P_{-1}\left(\frac{1}{\sqrt{(R_0 P_{-1})^2 - 4\rho(1-\rho)P_{-1}P_1}}\right)\frac{dP_1}{db_1^S} < 0.$$

Using the Euler equations for risk-free bonds between dates 0 and 1 and the consumption functions at date 1, we can express the consumptions c_0^S and c_0^B of savers and borrowers at date

0 as functions b_1^S and R_0:

$$c_0^S = \frac{1-\beta_0}{\beta_0 R_0}\frac{c_1^S(b_1^S)}{1-\beta_1} \quad \text{with} \quad c_1^S(b_1^S) = (1-\beta_1)\left(b_1^S + \frac{Y_1(b_1^S)}{\phi^S}\right),$$

$$c_0^B = \frac{1-\beta_0}{\beta_0 R_0}\frac{c_1^{B,e}(b_1^S, R_0)}{1-\beta_1} \quad \text{with} \quad c_1^{B,e}(b_1^S, R_0) = (1-\beta_1)\left(-\frac{\phi^S}{\phi^B}b_1^S + \frac{P_1^e(b_1^S, R_0)}{\phi^B}\right).$$

The only but key difference with the case of rational expectations covered in Section 3.3 is that the consumption of borrowers at date 0 now also depends on the expected price P_1^e of the asset at date 1. Combining these two equations with the resource constraint at date 0 and the expression for $P_1^e(b_1^S)$ above, we get the following expression for aggregate output Y_0 at date 0 as a function of b_1^S and R_0:

$$Y_0 = \frac{1-\beta_0}{\beta_0 R_0}\left(\frac{1-\beta_1+\beta_1(1-\rho)}{1-\beta_1}Y_1(b_1^S) + \rho\frac{P_0^2(b_1^S, R_0)}{P_{-1}}\right).$$

Using the consumption function and the budget constraint for the savers at date 0, we get a different equation for aggregate output Y_0 at date 0 as a function of b_1^S and R_0:

$$c_0^S = \frac{Y_0}{\phi^S} + b_0^S - \frac{b_1^S}{R_0} \quad \text{with} \quad c_0^S = (1-\beta_0)\left(\frac{Y_0}{\phi^S} + \frac{1}{R_0}\frac{Y_1(b_1^S)}{\phi^S} + b_0^S\right).$$

Combining this equation with the previous one yields fixed-point equation for b_1^S:

$$b_1^S = \beta_0 R_0 b_0^S + (1-\beta_0)\left(\frac{\beta_1}{1-\beta_1}\frac{Y_1(b_1^S)}{\phi^S} + \rho\frac{1}{\phi^S}\left(\frac{P_0^2(b_1^S, R_0)}{P_{-1}} - P_1(b_S)\right)\right).$$

To highlight its dependence on R_0, we denote the solution by $b_1^S(R_0)$. Plugging back this function into the previous equations defines $c_0^S(R_0)$, $c_0^B(R_0)$, $Y_0(R_0)$, and $P_0(R_0)$.

Effects of monetary policy. Key to our subsequent analysis will be the comparative statics of the equilibrium with respect to interest rates R_0 at date 0. They can all be deduced from the effects of interest rates R_0 at date 0 on the debt due by borrowers to savers at date 1:

$$\frac{db_1^S}{dR_0} = \frac{\beta_0 b_0^S + (1-\beta_0)\frac{\rho}{\phi^S}\frac{2P_0}{P_{-1}}\frac{\partial P_0}{\partial R_0}}{1-(1-\beta_0)\left(\frac{1-\rho}{\phi^S}\frac{\beta_1}{1-\beta_1}\frac{dY_1}{db_1^S} + \frac{\rho}{\phi^S}\frac{2P_0}{P_{-1}}\frac{\partial P_0}{\partial b_1^S}\right)},$$

where $\partial P_1/\partial R_0 < 0$ and $\partial P_1/\partial b_1^S < 0$ are given above and $dY_1/db_1^S < 0$ given in Section **??**.

Indeed, we can plug this back into the equations defining $c_0^S(R_0)$, $c_0^B(R_0)$, $Y_0(R_0)$, $P_0(R_0)$, $c_1^S(b_1^S(R_0))$, $c_1^B(b_1^S(R_0))$, $Y_1(b_1^S(R_0))$, $P_1(b_1^S(R_0))$, $c_2^S(b_1^S(R_0))$, $c_2^B(b_1^S(R_0))$, $Y_2(b_1^S(R_0))$ to compute their derivatives with respect to R_0. to compute their derivatives with respect to R_0.

Changes in the interest rate R_0 at date 0 influence the debt b_1^S due by borrowers to savers at date 1 through two different channels corresponding to the two terms in the numerator on the right-hand side of the equation for db_1^S/dR_0.

The initial leverage channel (the first term in the numerator), is commensurate with the initial risk-free bond holdings of savers (b_0^S). It operates even borrowers have rational expectations ($\rho = 0$). It works through the different sensitivities of the wealths of savers and borrowers to changes in the interest rate R_0 at date 0. As discussed in Section 3.3, an increase in the interest rate R_0 at date 0 reduces the debt due by borrowers to savers at date 1 ($db_1^S/dR_0 < 0$) through this channel if and only if borrowers are initial debtors ($b_0^S > 0$).

By contrast, the beliefs channel (the second term in the numerator) is only present when borrowers have extrapolative expectations ($\rho > 0$). It operates independently of the first channel (for any b_0^S). When borrowers have extrapolative expectations, an increase in the interest rate R_0 at date 0 lowers the price P_0 of the risky at date 0. This reduces the realized return P_0/P_{-1} of the risky asset between dates -1 and 0. Because borrowers extrapolate returns, this in turn makes borrowers more pessimistic about the price P_1^e of the risky asset date 1. By contrast, savers do not become more pessimistic about their income Y_1/ϕ^S at date 1. As a result, borrowers borrow less and savers lends less at date 0. In other words, an increase in the interest rate R_0 at date 0 always reduces the debt due by borrowers to savers at date 1 ($db_1^S/dR_0 < 0$) through this channel as long as borrowers have extrapolative expectations ($\rho > 0$).

The same intimate relationship between the effects of interest rates at date 0 on the debt due by borrowers to savers at date 1 (db_1^S/dR_0) and their effects on output and asset prices at dates 0 (dY_0/dR_0 and dP_0/dR_0) and 1 (dY_1/dR_0 and dP_1/dR_0) exists as under rational expectations. However, the effects of interest rates at date 0 on the debt due by borrowers to savers at date 1 (db_1^S/dR_0) are no longer only determined by the initial leverage channel but also by the beliefs channel.

When changes in interest rates at date 0 have no effect on the debt due by savers to borrowers at date 1 ($db_1^S/dR_0 = 0$), they have no effect on output and asset prices at date 1 ($dY_1/dR_0 = 0$ and dP_1/dR_0), and one-for-one effects on output and asset prices at date 0 ($(R_0/Y_0)(dY_0/dR_0) = -1$ and $(R_0/P_0)(dP_0/dR_0) = -1$). The unitary elasticity of output and asset prices to interest rates at date 0 ($-(R_0/Y_0)(dY_0/dR_0) = 1$ and $-(R_0/P_0)(dP_0/dR_0) = 1$) is exactly the one that would obtain in a complete-markets economy where savers could also trade the risky assets. When increases in interest rates at date 0 reduce the debt due by savers to borrowers at date 1 ($db_1^S/dR_0 < 0$), they increase output and asset prices at date 1 ($dY_1/dR_0 > 0$ and $dP_1/dR_0 > 0$), and reduce output and asset prices at date 0 more than one for one ($(R_0/Y_0)(dY_0/dR_0) < -1$ and $(R_0/P_0)(dP_0/dR_0) < -1$). The elasticity of output to interest rates at date 0 is greater than the one that would obtain in a complete-markets economy ($-(R_0/Y_0)(dY_0/dR_0) > 1$). All these arguments are reversed when borrowers are initially creditors ($b_0^S > 0$).

Optimal monetary policy. The planning problem for optimal monetary policy is

$$\max_{R_0} \left\{ \lambda^S \phi^S \left[(1-\beta_0) \left(\log(c_0^S(R_0)) - h(\frac{Y_0(R_0)}{\phi^S}) \right) + \beta_0 V^S(b_1^S(R_0)) \right] \right.$$
$$\left. + \lambda^B \phi^B \left[(1-\beta_0) \left(\log(c_0^B(R_0)) \right) + \beta_0 V^B(b_1^S(R_0)) \right] \right\},$$

where λ^S and λ^L are welfare Pareto weights on savers and borrowers.

The first-order condition for optimality of the interest rate R_0 at date 0 can be expressed as

$$\mu_0 \frac{1}{\phi^S} \frac{dY_0}{dR_0} + \frac{1}{R_0} \frac{db_1^S}{dR_0} \left[\left(1 + \frac{\mu_1}{\phi^S} \frac{dY_1}{db_1^S} \right) - 1 + \frac{\lambda^B c_0^S}{\lambda^S c_0^B} \left(1 - \frac{c_1^{B,e}}{c_1^B} \right) \right] + \left(1 - \frac{\lambda^B c_0^S}{\lambda^S c_0^B} \right) \frac{b_1^S}{R_0^2} = 0.$$

Compared to the formula under rational expectations derived in Section 3.3, there is one main difference. The second term, which captures the efficiency (non-distributive) effects mediated by changes in the debt b_1^S due by borrowers to savers at date 1 now not only depends on the aggregate demand externality $1 + \mu_1 Y_1'(b_1^S)/\phi^S$ but also on the belief externality $c_1^B / c_1^{B,e}$. Indeed, as explained in Farhi and Gabaix (2020), Roy's identity does not hold for borrowers because they mis-optimize their experienced (not perceived) welfare. For example, suppose that borrowers are optimistic ($c_1^{B,e} > c_1^B$) and that an increase in the interest rate at date 0 reduces their borrowing ($db_1^S/dR_0 < 0$). Then it improves their experienced welfare because they borrowed too much at date 0 to begin with.

Once again, since distributive concerns are largely orthogonal to the efficiency issues that we wish to focus on, we neutralize them by choosing the welfare Pareto weights λ^S and λ^B so that at the optimum, they are proportional to the private marginal utilities of wealth of savers and borrowers at date 0 ($\lambda^S/\lambda^B = c_0^S/c_0^B$) or equivalently to the expected (not actual) private marginal utilities of wealth of savers and borrowers at date 1 ($\lambda^S/\lambda^B = c_1^S/c_1^B$).[6]

Then the first-order condition for optimality of the interest rate R_0 at date 0 simplifies to

$$\mu_0 \frac{1}{\phi^S} \frac{dY_0}{dR_0} + (1-\beta_1) \frac{1}{R_0} \frac{db_1^S}{dR_0} \left[\left(\frac{c_1^B}{c_1^{B,e}} \right) \left(1 + \frac{\mu_1}{\phi^S} \frac{dY_1}{db_1^S} \right) - 1 \right] \frac{c_1^{B,e}}{c_1^B} = 0.$$

We can think about these two terms as two targets that are traded off by monetary policy at date 0 is as follows: the first term captures concerns for macroeconomic stabilization during the

[6]The corresponding point on the constrained Pareto frontier can be found by computing c_1^S/c_1^B at the optimum as a function of λ^S/λ^B and finding a point at which this curve crosses the 45 degree line. Since c_1^S/c_1^B has a finite limit when λ^S/λ^B tends to ∞, and a strictly positive limit as λ^S/λ^B tends to 0, such a point is guaranteed to exist.

boom at date 0 (stabilizing μ_0 close to zero), and the second term captures concerns for financial stabilization (reducing b_1^S) to improve macroeconomic stabilization during the bust at date 1 (increasing Y_1) and reducing distortions from the heterogenous effects of expectation errors across agents. The difference with the case of rational expectations is that financial stabilization (reducing b_1^S) affects welfare not just because of the aggregate demand externality but also because of the belief externality. We will refer to this language below.

Proposition 3. *Suppose that the only available policy is monetary policy and that expectations are rational. Suppose in addition that welfare Pareto weights are proportional to the private marginal utilities of wealth at date* 0 *($\lambda^S/\lambda^B = c_0^S/c_0^B$) at the optimum. Then under optimal monetary policy, the labor wedge μ_0 at date* 0 *satisfies the following equation*

$$\mu_0 = \left(\left(\frac{c_1^B}{c_1^{B,e}}\right)\left(1 + \frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right) - 1\right)\left(\frac{(1-\beta_1)\phi^S \frac{c_1^{B,e}}{c_1^B}\frac{db_1^S}{dR_0}}{-R_0\frac{dY_0}{dR_0}}\right),$$

with

$$\frac{db_1^S}{dR_0} = \frac{\beta_0 b_0^S + (1-\beta_0)\frac{\rho}{\phi^S}\frac{2P_0}{P_{-1}}\frac{\partial P_0}{\partial R_0}}{1-(1-\beta_0)\left(\frac{1-\rho}{\phi^S}\frac{\beta_1}{1-\beta_1}\frac{dY_1}{db_1^S} + \frac{\rho}{\phi^S}\frac{2P_0}{P_{-1}}\frac{\partial P_0}{\partial b_1^S}\right)},$$

where $\partial P_1/\partial R_0 < 0$ and $\partial P_1/\partial b_1^S < 0$ are given above and $dP_1/db_1^S < 0$, and where recall that output is below potential during the bust at date 1 *($\mu_1 > 0$). Mild technical conditions guarantee that $dY_0/dR_0 < 0$, which we shall assume. If borrowers have no initial debt ($b_0^S = 0$) or are initial debtors ($b_0^S > 0$), then increases in interest rates at date* 0 *necessarily reduce the debt due by borrowers to savers date* 1 *($db_1^S/dR_0 = 0$). If in addition borrowers are optimistic at the optimum ($c_1^{B,e} > c_1^B$), then output is below potential during the boom at date* 0 *($\mu_0 = 0$).*

In general, the labor wedge at date 0 is proportional to $(c_1^B/c_1^{B,e})(1+(\mu_1/\phi^S)(dY_1/db_1^S)) - 1$ and to db_1^S/dR_0. The former is the wedge between the social and private marginal rates of substitution at date 1, and is given by the product of the aggregate demand externality and of the belief externality. The latter captures the effects of changes in interest rates R_0 at date 0 on the debt b_1^S due by borrowers to savers at date 1. All in all, increases in interest rates R_0 at date 0 stimulate output Y_1 at date 1 only to the extent that they reduce the debt b_1^S due by borrowers to savers at date 1.

In turn, db_1^S/dR_0 is determined by the initial leverage channel (through b_0^S) and by the beliefs channels (through ρ) discussed above. The initial leverage channel is present even when expectations are rational ($\rho = 0$) and was discussed extensively in Section 3.3. The beliefs channel is new. To gain intuition, we focus on the case where $b_0^S = 0$. As we already explained above, an increase in the interest rate R_0 at date 0 reduces the price P_0 of the risky asset at date 0, reduces the return on the risky asset between date -1 and 0, makes borrowers more pessimistic about the price P_1^e of the risky asset at date 1, and as a result reduces the debt b_1^S due by borrowers to savers at date 1. This in turn stimulates output Y_1 at date 1, thereby partly offsetting the aggregate demand externality. If in addition borrowers are optimistic at the optimum ($c_1^{B,e} > c_1^B$), this has the additional benefit of partly offsetting the belief externality. Hence, increasing interest rates at date 0 (by tolerating $\mu_0 > 0$) now has prudential value by improving financial stability (reducing b_1^S) and improving both macroeconomic stability at date 1 (increasing Y_1) and offsetting the over-borrowing of borrowers arising from their erroneous beliefs. Optimal monetary policy therefore deviates from perfect macroeconomic stabilization during the boom at date 0 by pushing output below potential in order to improve financial stability and stimulate the economy at date 1. In other words, optimal policy during the boom at date 0 is hawkish leans against the wind.

Remark. Proposition 1 assumes that the ZLB binds at date 1*. If it does not, then monetary policy optimally does not stabilize output at date* 1 *($\mu_1 \neq 0$) in general as long as the expectations of borrowers are incorrect ($c_1^{B,e} \neq c_1^B$) even though there is no longer any aggregate demand externality from private borrowing and saving decisions at date* 0*. The results in the proposition then still apply but with $\mu_1 = 0$.*

4.4 Monetary Policy and Macroprudential Policy

In this section, we analyze jointly optimal monetary and macroprudential policy. The free policy instruments are therefore the interest rate R_0 at date 0, the macroprudential tax τ_0 on the leverage of borrowers, and the rebates t_0^S and t_0^L.

The planning problem for jointly optimal monetary and macroprudential policy is

$$\max_{\{Y_0, c_0^S, c_0^B, b_1^S\}} \left\{ \lambda^S \phi^S \left[(1-\beta_0) \left(\log(c_0^S) - h(\frac{Y_0}{\phi^S}) \right) + \beta_0 V^S(b_1^S) \right] \right.$$
$$\left. + \lambda^B \phi^B \left[(1-\beta_0) \log(c_0^B) + \beta_0 V^B(b_1^S) \right] \right\},$$

subject to

$$\phi^S c_0^S + \phi^B c_0^B \leq Y_0.$$

It is exactly the same as under rational expectations. The reason is two-fold. First, because the objective function is the same because the planner evaluates welfare under the true probabilities, the planning problem continues to feature the true value functions $V^S(b_1^S)$ and $V^B(b_1^S)$. Second, the constraint set is the same and contains only the resource constraint at date 0. This is because the same arguments that we used to justify that this was the only binding implementability condition under rational expectations continue to apply under extrapolative expectations. Of course, the values of R_0, $R_0(1-\tau_0)$, and P_0 that support any particular choice of Y_0, c_0^S, c_0^B, and b_1^S, depend on the presence of extrapolative expectations since they must satisfy the Euler equations of the model between dates 0 and 1:

$$\frac{1}{c_0^S} = \frac{\beta_0}{1-\beta_0} R_0 \frac{1-\beta_1}{c_1^S(b_1^S)},$$

$$\frac{1}{c_0^B} = \frac{\beta_0}{1-\beta_0} \frac{R_0}{1-\tau_0} \frac{1-\beta_1}{c_1^{B,e}(b_1^S, R_0)},$$

$$\frac{1}{c_0^B} = -\frac{\phi^S}{\phi^B} \frac{\beta_0}{1-\beta_0} \frac{P_1^e(b_1^S, R_0)}{P_0} \frac{1-\beta_1}{c_1^{B,e}(b_1^S, R_0)},$$

where

$$\frac{1-\beta_1}{c_1^S(b_1^S)} = \frac{1}{1+\frac{\mu_1(b_1^S)}{\phi^S}\frac{dY_1(b_1^S)}{db_1^S}} \frac{dV^S}{db_1^S}(b_1^S) \quad \text{and} \quad \frac{1-\beta_1}{c_1^{B,e}(b_1^S, R_0)} = -\frac{\phi^S}{\phi^B} \frac{c_1^B}{c_1^{B,e}} \frac{dV^S(b_1^S)}{db_1^S}.$$

Proposition 4. *Suppose that monetary policy and macroprudential policy are available and that expectations are extrapolative. Then under jointly optimal monetary and macroprudential policy, the labor wedge μ_0 at date 0 is given by*

$$\mu_0 = 0,$$

and the macroprudential tax on the leverage of borrower satisfies

$$1 - \tau_0 = \left(\frac{c_1^B}{c_1^{B,e}}\right)\left(1 + \frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right).$$

During the boom at date 0, output is always at potential at date 0 ($\mu_0 = 0$) and there is a macroprudential tax on the leverage of borrowers ($\tau_0 \neq 0$) which exactly offsets the aggregate demand and belief externalities $(1 + \mu_1(dY_1/db_1^S)/\phi^S)(c_1^B/c_1^{B,e})$. The macroprudential tax is guaranteed to exceed the offset for the aggregate demand externality ($\tau_0 > -\mu_1(dY_1/db_1^S)/\phi^S$) if at the optimum, borrowers have an optimism bias ($c_1^{B,e} > c_1^B$).

As in the rational expectation case, when macroprudential policy is available, monetary policy focuses solely on macroeconomic stabilization during the boom at date 0 as prescribed by the usual inflation targeting rule. Monetary policy is used to maintain output at potential.

Financial stabilization and belief correction fall entirely on macroprudential policy at date 0. Macroprudential policy imposes a tax on the leverage of borrowers. The macroprudential wedge $1 - \tau_0$ is equal to the wedge $(1 + \mu_1(dY_1/db_1^S)/\phi^S)(c_1^B/c_1^{B,e})$ between the social and private marginal rates of substitution between savers and borrowers at date 1 which no longer only reflects the aggregate demand externality but also the belief externality. The macroprudential tax τ_0 is greater than the positive value $\mu_1(dY_1/db_1^S)/\phi^S$ that would be warranted by the aggregate demand externality if, at the optimum, borrowers have an optimism bias ($c_1^{B,e} > c_1^B$).

Macroprudential policy then leans against the wind ($\tau_0 > 0$) for two complementary reasons: to stimulate output at date 1 (to correct the aggregate demand externality) and to correct the expectation mistakes that lead borrowers to borrow too much from savers at any given interest rate (the belief externality).

Remark. Proposition 4 assumes that the ZLB binds at date 1. *If it does not, then macroprudential taxes are not zero in general ($\tau_1 \neq 0$) in general as long as the expectations of borrowers are incorrect ($c_1^{B,e} \neq c_1^B$) even though there is no longer any aggregate demand externality from private borrowing and saving decisions at date* 0. *The results in the proposition then still apply but with $\mu_1 = 0$.*

5 Extensions and Robustness

In the baseline model, we have assumed for simplicity that only savers work, and that dividends only accrue at date 2. In Appendices B.1 and B.2, we extend the model to allow for the possibility that both savers and borrowers work and that dividends accrue in all periods. All the main results go through with no or little modification. This shows that the lessons that we drew for the conduct of monetary and macroprudential policies do not depend on these simplifying assumptions.

In the baseline model, we only consider the possibility that expectations are extrapolative during the boom between dates 0 and 1. This captures complacency, irrational exuberance, or boom psychology. In Appendix B.3, we analyze the case where expectations are instead extrapolative during the bust between dates 1 and 2. This specification captures doubt, irrational pessimism, or bust psychology, and delivers some interesting new lessons. The key is that the leverage b_1^S of borrowers is no longer the only state variable at date 1. The price P_0 of the risky asset at date 0 acts as an additional state variable at date 1 because it influences beliefs. Policy at date 0 no longer only acts by influencing the leverage b_1^S of borrowers, but also by influencing their beliefs. In a sense, there are now three targets for only one or two instruments.

We first consider optimal monetary policy in the absence of macroprudential policy. We show that there is a new mechanism justifying leaning against the wind with monetary policy at date 0. Raising interest rates at date 0 reduces the price P_0 of the risky asset at date 0, increases the return P_1/P_0 of the risky asset between dates 0 and 1, makes borrowers more optimistic at date 1, increases the price P_1 of the risky asset at date 1, raises aggregate demand and output Y_1 at date 1, which in turn improves welfare because there is a recession at date 1 to begin with. Cooling asset prices at date 0 mitigates the asset price crash at date 1 and the ensuing recession. Optimal monetary policy at date 0 therefore deviates from inflation targeting and tolerates a recession at date 0.

We then consider jointly optimal monetary and macroprudential policy. We show that optimal monetary policy leans against the wind at date 0 even when macroprudential policy is available. Basically, monetary policy leans against the wind and tolerates a recession at date 0 in order to reduce the price P_0 of the risky asset at date 0 and improve beliefs at date 1, exactly as when macroprudential policy is not available. Macroprudential policy is hawkish and leans against the wind by reigning in the leverage b_1^S of borrowers. We show that the macroprudential tax τ_0 is lower than it would be under rational expectations (given wedges). This is because apart from directly raising the cost of borrowing for borrowers at date 0, the macroprudential tax lowers the asset price, makes borrowers more pessimistic, and hence also indirectly reduces their desire to borrow at a given cost.

These lessons offer an interesting contrast with the cases of rational expectations or extrapolative expectations during the boom. In these cases, when macroprudential policy is available, it is optimal to deal with financial stability with macroprudential policy and to let monetary policy deal with macroeconomic stability. It is only when macroprudential policy is not available that monetary policy must be willing to trade off macroeconomic and financial stability.

Instead, with extrapolative expectations during the bust, this assignment of targets to instruments breaks down. Even when macroprudential policy is available, monetary policy must be conducted with an eye towards financial stability instead of focusing exclusively on macroeconomic stability. In this setting, financial stability becomes a multidimensional target influenced by leverage and beliefs/asset prices. Macroprudential policy cannot control both at the same time and monetary policy must shoulder a share of the burden. Limiting leverage falls on macroprudential policy, and managing beliefs/asset prices on monetary policy.

6 Conclusion

We identify several avenues for future research. First, it would be interesting to allow other frictions such as borrowing constraints. If borrowers were up again their constraint during the bust, they would have higher marginal propensities to consume, and reducing their leverage during the boom would become even more important. Second, it would be valuable to embed our model into a full-fledged dynamic stochastic general equilibrium model. This would be interesting on two grounds: to refine our policy lessons when booms and bust unfold over time; to quantify of the forces that we have identified. Third, it would be valuable to consider other deviations from rational expectations such as inattention.

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A Proofs and Derivations

A.1 Proofs and Derivations for Section 3

Derivatives of the value functions. We have

$$
\begin{aligned}
\frac{dV^H}{db_1^S} &= (1-\beta_1)\left[\frac{(1-\beta_1)\left(1+\frac{1}{\phi^S}\frac{dY_1}{db_1^S}\right)}{c_1^S} - h'\frac{1}{\phi^S}\frac{dY_1}{db_1^S}\right] + \beta_1\frac{(1-\beta_1)\left(1+\frac{1}{\phi^S}\frac{dY_1}{db_1^S}\right)}{c_1^S}, \\
&= (1-\beta_1)\left[\frac{\left(1+\frac{1}{\phi^S}\frac{dY_1}{db_1^S}\right)}{c_1^S} - (1-\beta_1)h'\frac{1}{\phi^S}\frac{dY_1}{db_1^S}\right], \\
&= \frac{(1-\beta_1)}{c_1^S}\left[1+\left(1-c_1^S h'\right)\frac{1}{\phi^S}\frac{dY_1}{db_1^S}\right], \\
&= \frac{(1-\beta_1)}{c_1^S}(1+\mu_1\frac{1}{\phi^S}\frac{dY_1}{db_1^S}).
\end{aligned}
$$

We also have

$$
\begin{aligned}
\frac{dV^B}{db_1^S} &= (1-\beta_1)\frac{(1-\beta_1)\left(-\frac{\phi^S}{\phi^B}+\frac{\beta_1}{1-\beta_1}\frac{1}{\phi^S}\frac{dY_1}{db_1^S}\right)}{c_1^B} + \beta_1\mathbb{E}\left[\frac{-\frac{\phi^S}{\phi^B}\beta_1\left(1+\frac{1}{\phi^S}\frac{dY_1}{db_1^S}\right)}{c_2^B}\right], \\
&= (1-\beta_1)\frac{(1-\beta_1)\left(-\frac{\phi^S}{\phi^B}+\frac{\beta_1}{1-\beta_1}\frac{1}{\phi^B}\frac{dY_1}{db_1^S}\right)}{c_1^B} - \frac{\phi^S}{\phi^B}\beta_1\left(1+\frac{1}{\phi^S}\frac{dY_1}{db_1^S}\right)\beta_1\mathbb{E}\left[\frac{1}{c_2^B}\right], \\
&= (1-\beta_1)\frac{(1-\beta_1)\left(-\frac{\phi^S}{\phi^B}+\frac{\beta_1}{1-\beta_1}\frac{1}{\phi^B}\frac{dY_1}{db_1^S}\right)}{c_1^B} - \frac{\phi^S}{\phi^B}\beta_1\left(1+\frac{1}{\phi^S}\frac{dY_1}{db_1^S}\right)(1-\beta_1)\frac{1}{c_1^B}, \\
&= \frac{(1-\beta_1)}{c_1^B}\left[(1-\beta_1)\left(-\frac{\phi^S}{\phi^B}+\frac{\beta_1}{1-\beta_1}\frac{1}{\phi^B}\frac{dY_1}{db_1^S}\right) - \frac{\phi^S}{\phi^B}\beta_1\left(1+\frac{1}{\phi^S}\frac{dY_1}{db_1^S}\right)\right], \\
&= -\frac{\phi^S}{\phi^B}\frac{(1-\beta_1)}{c_1^B}.
\end{aligned}
$$

Planning problem for optimal monetary policy only. The planning problem for optimal monetary policy is

$$\max_{R_0} \left\{ \lambda^S \phi^S \left[(1-\beta_0) \left(\log(c_0^S(R_0)) - h(\frac{Y_0(R_0)}{\phi^S}) \right) + \beta_0 V^S(b_1^S(R_0)) \right] \right.$$
$$\left. + \lambda^B \phi^B \left[(1-\beta_0) \log(c_0^B(R_0)) + \beta_0 V^B(b_1^S(R_0)) \right] \right\},$$

where λ^S and λ^L are welfare Pareto weights on savers and borrowers.

$$\lambda^S \phi^S \left[(1-\beta_0) \left(\frac{1}{c_0^S} \frac{dc_0^S}{dR_0} - h' \frac{1}{\phi^S} \frac{dY_0}{dR_0} \right) + \beta_0 \frac{dV^S}{db_1^S} \frac{db_1^S}{dR_0} \right] + \lambda^B \phi^B \left[(1-\beta_0) \frac{1}{c_0^B} \frac{dc_0^B}{dR_0} + \beta_0 \frac{dV^B}{db_1^S} \frac{db_1^S}{dR_0} \right] = 0,$$

$$\lambda^S \phi^S (1-\beta_0) \left(\frac{1}{c_0^S} \frac{dc_0^S}{dR_0} - h' \frac{1}{\phi^S} \frac{dY_0}{dR_0} \right) + \lambda^B \phi^B (1-\beta_0) \frac{1}{c_0^B} \frac{dc_0^B}{dR_0}$$
$$+ \frac{db_1^S}{dR_0} \left[\lambda^S \phi^S \beta_0 \frac{(1-\beta_1)}{c_1^S} \left(1 + \frac{\mu_1}{\phi^S} \frac{dY_1}{db_1^S} \right) - \lambda^B \phi^S \beta_0 \frac{(1-\beta_1)}{c_1^B} \right] = 0,$$

$$\lambda^S (1-\beta_0) \left(\frac{\phi^S \frac{dc_0^S}{dR_0} + \phi^B \frac{dc_0^B}{dR_0}}{c_0^S} - h' \frac{dY_0}{dR_0} \right) + \phi^B (1-\beta_0) \left(\frac{\lambda^B}{c_0^B} - \frac{\lambda^S}{c_0^S} \right) \frac{dc_0^B}{dR_0}$$
$$+ \frac{db_1^S}{dR_0} \left[\lambda^S \phi^S \beta_0 \frac{(1-\beta_1)}{c_1^S} \left(1 + \frac{\mu_1}{\phi^S} \frac{dY_1}{db_1^S} \right) - \lambda^B \phi^S \beta_0 \frac{(1-\beta_1)}{c_1^B} \right] = 0,$$

$$\lambda^S (1-\beta_0) \left(1 - c_0^S h' \right) \frac{1}{c_0^S} \frac{dY_0}{dR_0} + \phi^B (1-\beta_0) \left(\frac{\lambda^B}{c_0^B} - \frac{\lambda^S}{c_0^S} \right) \frac{dc_0^B}{dR_0}$$
$$+ \frac{db_1^S}{dR_0} \left[\lambda^S \phi^S \beta_0 \frac{(1-\beta_1)}{c_1^S} \left(1 + \frac{\mu_1}{\phi^S} \frac{dY_1}{db_1^S} \right) - \lambda^B \phi^S \beta_0 \frac{(1-\beta_1)}{c_1^B} \right] = 0,$$

$$\lambda^S (1-\beta_0) \mu_0 \frac{1}{c_0^S} \frac{dY_0}{dR_0} + \phi^B (1-\beta_0) \left(\frac{\lambda^B}{c_0^B} - \frac{\lambda^S}{c_0^S} \right) \frac{dc_0^B}{dR_0}$$
$$+ \frac{db_1^S}{dR_0} \left[\frac{\lambda^S c_1^B}{\lambda^B c_1^S} \left(1 + \frac{\mu_1}{\phi^S} \frac{dY_1}{db_1^S} \right) - 1 \right] \lambda^B \phi^S \beta_0 \frac{(1-\beta_1)}{c_1^B} = 0,$$

$$\mu_0 \frac{1}{\phi^S}\frac{dY_0}{dR_0} + \frac{\phi^B}{\phi^S}\left(\frac{\lambda^B c_0^S}{\lambda^S c_0^B} - 1\right)\frac{dc_0^B}{dR_0} + \frac{1}{R_0}\frac{db_1^S}{dR_0}\frac{\lambda^B c_0^S}{\lambda^S c_0^B}\left[\frac{\lambda^S c_0^B}{\lambda^B c_0^S}\left(1 + \frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right) - 1\right] = 0,$$

$$\mu_0 \frac{1}{\phi^S}\frac{dY_0}{dR_0} + \frac{\phi^B}{\phi^S}\left(\frac{\lambda^B c_0^S}{\lambda^S c_0^B} - 1\right)\left(\frac{dc_0^B}{dR_0} - \frac{1}{R_0}\frac{\phi^S}{\phi^B}\frac{db_1^S}{dR_0}\right) + \frac{1}{R_0}\frac{db_1^S}{dR_0}\frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S} = 0.$$

We use

$$c_0^B + \frac{1}{R_0} b_1^B = b_0^B,$$

which can be re-expressed as

$$c_0^B - \frac{1}{R_0}\frac{\phi^S}{\phi^B} b_1^S = -\frac{\phi^S}{\phi^B} b_0^S.$$

Differentiating, we get

$$\frac{dc_0^B}{dR_0} - \frac{1}{R_0}\frac{\phi^S}{\phi^B}\frac{db_1^S}{dR_0} = -\frac{c_0^B}{R_0} - \frac{\phi^S}{\phi^B}\frac{b_0^S}{R_0} = -\frac{\phi^S}{\phi^B}\frac{b_1^S}{R_0^2}.$$

Plugging back in the first-order condition for optimality, we get

$$\mu_0 \frac{1}{\phi^S}\frac{dY_0}{dR_0} + \frac{1}{R_0}\frac{db_1^S}{dR_0}\frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S} + \left(1 - \frac{\lambda^B c_0^S}{\lambda^S c_0^B}\right)\frac{b_1^S}{R_0^2} = 0.$$

Assume that $\lambda^S/\lambda^B = c_0^S/c_0^B$. Then this simplifies to

$$\mu_0 \frac{1}{\phi^S}\frac{dY_0}{dR_0} + \frac{1}{R_0}\frac{db_1^S}{dR_0}\frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S} = 0,$$

$$\mu_0 = \left(\frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right)\left(\frac{\phi^S \frac{db_1^S}{dR_0}}{-R_0 \frac{dY_0}{dR_0}}\right).$$

A.2 Proofs and Derivations for Section 4

Derivatives of $Y_1^e(b_1^S, P_0)$. Since $Y_1^e(b_1^S, P_0)$ solves the following fixed point

$$Y_1 = (1 - \beta_1)\left(\phi^S b_1^S + Y_1\right) + \frac{1}{\frac{\beta_1}{1-\beta_1}\left(\frac{\pi_G(b_1^S, P_0)}{D_{2G} - \beta_1(\phi^S b_1^S + Y_1)} + \frac{\pi_B(b_1^S, P_0)}{D_{2B} - \beta_1(\phi^S b_1^S + Y_1)}\right)}.$$

This implies that

$$\frac{\beta_1^2}{1-\beta_1}\frac{\partial Y_1^e}{\partial P_0} = -\frac{\beta_1 \frac{\partial Y_1^e}{\partial P_0}\left(\frac{\pi_G}{\left(D_{2G}-\beta_1(\phi^S b_1^S+Y_1^e)\right)^2}+\frac{\pi_B}{\left(D_{2B}-\beta_1(\phi^S b_1^S+Y_1^e)\right)^2}\right)}{\left(\frac{\pi_G}{D_{2G}-\beta_1(\phi^S b_1^S+Y_1^e)}+\frac{\pi_B}{D_{2B}-\beta_1(\phi^S b_1^S+Y_1^e)}\right)^2} - \frac{\frac{\partial \pi_G}{\partial P_0}\left(\frac{1}{D_{2G}-\beta_1(\phi^S b_1^S+Y_1^e)}-\frac{1}{D_{2B}-\beta_1(\phi^S b_1^S+Y_1^e)}\right)}{\left(\frac{\pi_G}{D_{2G}-\beta_1(\phi^S b_1^S+Y_1^e)}+\frac{\pi_B}{D_{2B}-\beta_1(\phi^S b_1^S+Y_1^e)}\right)^2},$$

$$\frac{\partial Y_1^e}{\partial P_0} = \frac{\frac{\partial \pi_G}{\partial P_0}\left(\frac{1}{c_{2B}^e}-\frac{1}{c_{2G}^e}\right)}{\beta_1\left(\frac{\beta_1}{1-\beta_1}+\frac{\mathbb{E}^e\left[\left(\frac{1}{c_2^e}\right)^2\right]}{\left(\mathbb{E}^e\left[\frac{1}{c_2^e}\right]\right)^2}\right)\left(\mathbb{E}^e\left[\frac{1}{c_2^e}\right]\right)^2}.$$

We also use

$$\pi_G(b_1^S,P_0)D_{2G} + \left(1-\pi_G(b_1^S,P_0)\right)D_{2B} = (1-\rho)P_1(b_1^S) + \rho\frac{P_0^2}{P_{-1}}.$$

This implies that

$$\frac{\partial \pi_G}{\partial P_0} = \rho\frac{\frac{2P_0}{P_{-1}}}{D_{2G}-D_{2B}}.$$

We therefore get

$$\frac{\partial Y_1^e}{\partial P_0} = \frac{\rho\frac{\frac{2P_0}{P_{-1}}}{D_{2G}-D_{2B}}\left(\frac{1}{c_{2B}^e}-\frac{1}{c_{2G}^e}\right)}{\beta_1\left(\frac{\beta_1}{1-\beta_1}+\frac{\mathbb{E}^e\left[\left(\frac{1}{c_2^e}\right)^2\right]}{\left(\mathbb{E}^e\left[\frac{1}{c_2^e}\right]\right)^2}\right)\left(\mathbb{E}^e\left[\frac{1}{c_2^e}\right]\right)^2}.$$

We also get

$$\frac{\beta_1^2}{1-\beta_1}\frac{\partial Y_1^e}{\partial b_1^S} = -\frac{\beta_1\left(\phi^S+\frac{\partial Y_1^e}{\partial b_1^S}\right)\left(\frac{\pi_G}{\left(D_{2G}-\beta_1(\phi^S b_1^S+Y_1^e)\right)^2}+\frac{\pi_B}{\left(D_{2B}-\beta_1(\phi^S b_1^S+Y_1^e)\right)^2}\right)}{\left(\frac{\pi_G}{D_{2G}-\beta_1(\phi^S b_1^S+Y_1^e)}+\frac{\pi_B}{D_{2B}-\beta_1(\phi^S b_1^S+Y_1^e)}\right)^2} - \frac{\frac{\partial \pi_G}{\partial b_1^S}\left(\frac{1}{D_{2G}-\beta_1(\phi^S b_1^S+Y_1^e)}-\frac{1}{D_{2B}-\beta_1(\phi^S b_1^S+Y_1^e)}\right)}{\left(\frac{\pi_G}{D_{2G}-\beta_1(\phi^S b_1^S+Y_1^e)}+\frac{\pi_B}{D_{2B}-\beta_1(\phi^S b_1^S+Y_1^e)}\right)^2},$$

$$\frac{\partial Y_1^e}{\partial b_1^S} = \frac{-\beta_1 \phi^S \mathbb{E}^e\left[\left(\frac{1}{c_2^e}\right)^2\right] - \frac{\partial \pi_G}{\partial b_1^S}\left(\frac{1}{c_{2,G}^B} - \frac{1}{c_{2,B}^B}\right)}{\beta_1\left(\frac{\beta_1}{1-\beta_1}\left(\mathbb{E}^e\left[\frac{1}{c_2^e}\right]\right)^2 + \mathbb{E}^e\left[\left(\frac{1}{c_2^e}\right)^2\right]\right)}.$$

We also have

$$\frac{\partial \pi_G}{\partial b_1^S} = (1-\rho)\frac{\frac{dP_1}{db_1^S}}{D_{2G} - D_{2B}} < 0.$$

Therefore, we get

$$\frac{\partial Y_1^e}{\partial b_1^S} = \frac{-\phi^S \mathbb{E}^e\left[\left(\frac{1}{c_2^e}\right)^2\right] - (1-\rho)\left(\frac{1}{c_{2,B}^B} - \frac{1}{c_{2,G}^B}\right)\left(-\frac{dP_1}{db_1^S}\right)\left(\frac{1}{D_{2G}-D_{2B}}\right)\left(\frac{1}{\beta_1}\right)}{\beta_1\left(\frac{\beta_1}{1-\beta_1}\left(\mathbb{E}^e\left[\frac{1}{c_2^e}\right]\right)^2 + \mathbb{E}^e\left[\left(\frac{1}{c_2^e}\right)^2\right]\right)} < 0.$$

Planning problem for optimal monetary policy only. The planning problem for optimal monetary policy is

$$\max_{R_0}\left\{\lambda^S\phi^S\left[(1-\beta_0)\left(\log(c_0^S(R_0)) - h\left(\frac{Y_0(R_0)}{\phi^S}\right)\right) + \beta_0 V^S(b_1^S(R_0))\right]\right.$$
$$\left. + \lambda^B\phi^B\left[(1-\beta_0)\log(c_0^B(R_0)) + \beta_0 V^B(b_1^S(R_0))\right]\right\},$$

where λ^S and λ^L are welfare Pareto weights on savers and borrowers.

$$\lambda^S\phi^S\left[(1-\beta_0)\left(\frac{1}{c_0^S}\frac{dc_0^S}{dR_0} - h'\frac{1}{\phi^S}\frac{dY_0}{dR_0}\right) + \beta_0\frac{dV^S}{db_1^S}\frac{db_1^S}{dR_0}\right] + \lambda^B\phi^B\left[(1-\beta_0)\frac{1}{c_0^B}\frac{dc_0^B}{dR_0} + \beta_0\frac{dV^B}{db_1^S}\frac{db_1^S}{dR_0}\right] = 0,$$

$$\lambda^S\phi^S(1-\beta_0)\left(\frac{1}{c_0^S}\frac{dc_0^S}{dR_0} - h'\frac{1}{\phi^S}\frac{dY_0}{dR_0}\right) + \lambda^B\phi^B(1-\beta_0)\frac{1}{c_0^B}\frac{dc_0^B}{dR_0}$$
$$+ \frac{db_1^S}{dR_0}\left[\lambda^S\phi^S\beta_0\frac{(1-\beta_1)}{c_1^S}\left(1 + \frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right) - \lambda^B\phi^S\beta_0\frac{(1-\beta_1)}{c_1^B}\right] = 0,$$

$$\lambda^S(1-\beta_0)\left(\frac{\phi^S\frac{dc_0^S}{dR_0} + \phi^B\frac{dc_0^B}{dR_0}}{c_0^S} - h'\frac{dY_0}{dR_0}\right) + \phi^B(1-\beta_0)\left(\frac{\lambda^B}{c_0^B} - \frac{\lambda^S}{c_0^S}\right)\frac{dc_0^B}{dR_0}$$
$$+ \frac{db_1^S}{dR_0}\left[\lambda^S\phi^S\beta_0\frac{(1-\beta_1)}{c_1^S}\left(1 + \frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right) - \lambda^B\phi^S\beta_0\frac{(1-\beta_1)}{c_1^B}\right] = 0,$$

$$\lambda^S(1-\beta_0)\left(1-c_0^S h'\right)\frac{1}{c_0^S}\frac{dY_0}{dR_0}+\phi^B(1-\beta_0)\left(\frac{\lambda^B}{c_0^B}-\frac{\lambda^S}{c_0^S}\right)\frac{dc_0^B}{dR_0}$$
$$+\frac{db_1^S}{dR_0}\left[\lambda^S\phi^S\beta_0\frac{(1-\beta_1)}{c_1^S}\left(1+\frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right)-\lambda^B\phi^S\beta_0\frac{(1-\beta_1)}{c_1^B}\right]=0,$$

$$\lambda^S(1-\beta_0)\mu_0\frac{1}{c_0^S}\frac{dY_0}{dR_0}+\phi^B(1-\beta_0)\left(\frac{\lambda^B}{c_0^B}-\frac{\lambda^S}{c_0^S}\right)\frac{dc_0^B}{dR_0}$$
$$+\frac{db_1^S}{dR_0}\left[\frac{\lambda^S c_1^B}{\lambda^B c_1^S}\left(1+\frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right)-1\right]\lambda^B\phi^S\beta_0\frac{(1-\beta_1)}{c_1^B}=0,$$

$$\lambda^S(1-\beta_0)\mu_0\frac{1}{c_0^S}\frac{dY_0}{dR_0}+\phi^B(1-\beta_0)\left(\frac{\lambda^B}{c_0^B}-\frac{\lambda^S}{c_0^S}\right)\frac{dc_0^B}{dR_0}$$
$$+\frac{db_1^S}{dR_0}\left[\frac{\lambda^S c_1^B}{\lambda^B c_1^S}\left(1+\frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right)-1\right]\lambda^B\phi^S\beta_0\frac{(1-\beta_1)}{c_1^{B,e}}\frac{c_1^{B,e}}{c_1^B}=0,$$

$$\lambda^S(1-\beta_0)\mu_0\frac{1}{c_0^S}\frac{dY_0}{dR_0}+\phi^B(1-\beta_0)\left(\frac{\lambda^B}{c_0^B}-\frac{\lambda^S}{c_0^S}\right)\frac{dc_0^B}{dR_0}$$
$$+\frac{1}{R_0}\frac{db_1^S}{dR_0}\left[\frac{\lambda^S c_1^B}{\lambda^B c_1^S}\left(1+\frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right)-1\right]\lambda^B\phi^S(1-\beta_0)\frac{1}{c_0^B}\frac{c_1^{B,e}}{c_1^B}=0,$$

$$\mu_0\frac{\lambda^S}{c_0^S}\frac{1}{\phi^S}\frac{dY_0}{dR_0}+\frac{\phi^B}{\phi^S}\left(\frac{\lambda^B}{c_0^B}-\frac{\lambda^S}{c_0^S}\right)\frac{dc_0^B}{dR_0}$$
$$+\frac{1}{R_0}\frac{db_1^S}{dR_0}\frac{\lambda^B}{c_0^B}\left[\frac{\lambda^S c_1^B}{\lambda^B c_1^S}\left(1+\frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right)-1\right]\frac{c_1^{B,e}}{c_1^B}=0,$$

$$\mu_0\frac{1}{\phi^S}\frac{dY_0}{dR_0}+\frac{\phi^B}{\phi^S}\left(\frac{\lambda^B c_0^S}{\lambda^S c_0^B}-1\right)\frac{dc_0^B}{dR_0}$$
$$+\frac{1}{R_0}\frac{db_1^S}{dR_0}\frac{\lambda^B c_0^S}{\lambda^S c_0^B}\left[\frac{\lambda^S c_0^B}{\lambda^B c_0^S}\frac{c_1^B}{c_1^{B,e}}+\frac{\lambda^S c_0^B}{\lambda^B c_0^S}\frac{c_1^B}{c_1^{B,e}}\frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}-\frac{\lambda^S c_0^B}{\lambda^B c_0^S}\frac{c_1^B}{c_1^{B,e}}+\frac{\lambda^S c_0^B}{\lambda^B c_0^S}\frac{c_1^B}{c_1^{B,e}}-1\right]\frac{c_1^{B,e}}{c_1^B}=0,$$

$$\mu_0 \frac{1}{\phi^S}\frac{dY_0}{dR_0} + \frac{1}{R_0}\frac{db_1^S}{dR_0}\left[\left(1+\frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right)-1\right] + \frac{\lambda^B c_0^S}{\lambda^S c_0^B}\frac{1}{R_0}\frac{db_1^S}{dR_0}\left(1-\frac{c_1^{B,e}}{c_1^B}\right)$$
$$+\frac{\phi^B}{\phi^S}\left(\frac{\lambda^B c_0^S}{\lambda^S c_0^B}-1\right)\left(\frac{dc_0^B}{dR_0}-\frac{\phi^S}{\phi^B}\frac{1}{R_0}\frac{db_1^S}{dR_0}\right)=0.$$

We use
$$\frac{dc_0^B}{dR_0}-\frac{1}{R_0}\frac{\phi^S}{\phi^B}\frac{db_1^S}{dR_0} = -\frac{c_0^B}{R_0}-\frac{\phi^S}{\phi^B}\frac{b_0^S}{R_0} = -\frac{\phi^S}{\phi^B}\frac{b_1^S}{R_0^2}.$$

We get

$$\mu_0 \frac{1}{\phi^S}\frac{dY_0}{dR_0} + \frac{1}{R_0}\frac{db_1^S}{dR_0}\left[\left(1+\frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right)-1\right] + \frac{\lambda^B c_0^S}{\lambda^S c_0^B}\frac{1}{R_0}\frac{db_1^S}{dR_0}\left(1-\frac{c_1^{B,e}}{c_1^B}\right)$$
$$+\left(1-\frac{\lambda^B c_0^S}{\lambda^S c_0^B}\right)\frac{b_1^S}{R_0^2}=0.$$

Assume that $\lambda^S/\lambda^B = c_0^S/c_0^B$, which implies $\lambda^S/\lambda^B = c_1^S/c_1^{B,e}$. Then this simplifies to

$$\mu_0 \frac{1}{\phi^S}\frac{dY_0}{dR_0} + \frac{1}{R_0}\frac{db_1^S}{dR_0}\left[\frac{c_1^B}{c_1^{B,e}}\left(1+\frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right)-1\right]\frac{c_1^{B,e}}{c_1^B}=0.$$

$$\mu_0 = \left(\frac{c_1^B}{c_1^{B,e}}\left(1+\frac{\mu_1}{\phi^S}\frac{dY_1}{db_1^S}\right)-1\right)\left(\frac{(1-\beta_1)\phi^S \frac{c_1^{B,e}}{c_1^B}\frac{db_1^S}{dR_0}}{-R_0\frac{dY_0}{dR_0}}\right).$$

B Extensions

In this section, we develop two extensions. In the first extension in Appendices B.1 and B.2, we extend the model to allow for the possibility that both savers and borrowers work. In the second extension in Appendix B.3, we analyze the case of extrapolative expectations between 1 and 2.

Because we assume sticky wages, we need to specify the rationing rule for labor. We assume that the fractions of total labor respectively supplied by savers and borrowers are x^S and x^B. This means that each saver supplies x^S/ϕ^S units of labor, and each borrower supplies x^B/ϕ^B units of labor. We allow flexible dis-utilities of work h^B and h^S for borrowers and savers. This flexibility also allows us to capture the case output is divided into labor and dividend income in exogenous proportions $1-\delta_t$ and δ_t every period (by re-normalizing h^B and h^S).

In Appendix B.1, we analyze the case of rational expectations, echoing the analysis of Section 3. Then, in Appendix B.2, we analyze the case of extrapolative expectations between 0 and

1, echoing the analysis of Section 4. In both cases, the model can be analyzed along the same lines as the baseline model of Sections 3 and 4. We therefore only highlight the main differences. Finally, in Appendix B.3, we analyze the case of extrapolative expectations between 1 and 2.

B.1 Rational Expectations

B.1.1 Debt as a State Variable

The equilibrium at dates 1 and 2 can be expressed as a function of debt b_1^S, which acts as the only state variable:

$$c_1^S = (1-\beta_1)\left(b_1^S + \frac{x^S}{\phi^S} Y_1\right),$$

$$c_{2,\omega}^S = \beta_1 \left(b_1^S + \frac{x^S}{\phi^S} Y_1\right),$$

$$l_1^S = \frac{x^S}{\phi^S} Y_1,$$

$$c_1^B = \frac{1}{\frac{\beta_1}{1-\beta_1}\mathbb{E}\left[\frac{1}{\frac{D_2}{\phi_B} - \frac{\phi^S}{\phi_B}\beta_1\left(b_1^S + \frac{x^S}{\phi^S}Y_1\right)}\right]},$$

$$c_{2,\omega}^B = \frac{D_{2,\omega}}{\phi_B} - \frac{\phi^S}{\phi_B}\beta_1\left(b_1^S + \frac{x^S}{\phi^S} Y_1\right).$$

Output at date 1 is given by the fixed-point equation:

$$Y_1 = (1-\beta_1)\left(\phi^S b_1^S + x^S Y_1\right) + \frac{1}{\frac{\beta_1}{1-\beta_1}\mathbb{E}\left[\frac{1}{D_2 - \beta_1(\phi^S b_1^S + x^S Y_1)}\right]}.$$

We denote the solution by $Y_1(b_1^S)$. It can be plugged back into all the equations above to characterize the equilibrium as a function of b_1^S. For example, the asset price at date 1 is given by

$$P_1(b_1^S) = \frac{\beta_1}{1-\beta_1} Y_1(b_1^S).$$

It is easy to see that the functions $Y_1(b_1^S)$ and $P_1(b_1^S)$ are increasing in b_1^S with

$$\frac{dY_1}{db_1^S}\left[1-(1-\beta_1)x^S\right] = (1-\beta_1)\phi^S - \frac{1-\beta_1}{\beta_1}\frac{\left(\beta_1\phi^S+\beta_1 x^S\frac{dY_1}{db_1^S}\right)\mathbb{E}\left[\left(\frac{1}{D_2-\beta_1(\phi^S b_1^S+x^S Y_1)}\right)^2\right]}{\left(\mathbb{E}\left[\frac{1}{D_2-\beta_1(\phi^S b_1^S+x^S Y_1)}\right]\right)^2},$$

$$\frac{dY_1}{db_1^S} = \frac{1-\beta_1}{\beta_1}\frac{dP_1}{db_1^S} = \frac{\phi^S(1-\beta_1)\left[1-\frac{\mathbb{E}\left[\left(\frac{1}{c_2^B}\right)^2\right]}{\left(\mathbb{E}\left[\frac{1}{c_2^B}\right]\right)^2}\right]}{1-(1-\beta_1)x^S\left[1-\frac{\mathbb{E}\left[\left(\frac{1}{c_2^B}\right)^2\right]}{\left(\mathbb{E}\left[\frac{1}{c_2^B}\right]\right)^2}\right]} < 0.$$

B.1.2 Aggregate Demand Externality

The value functions for the welfare of savers and borrowers in general equilibrium at dates 1 and 2 are given by

$$V^S(b_1^S) = (1-\beta_1)\left[\log\left((1-\beta_1)\left(b_1^S+\frac{x^S}{\phi^S}Y_1(b_1^S)\right)\right) - h^S\left(\frac{x^S}{\phi^S}Y_1(b_1^S)\right)\right] + \beta_1\log\left(\beta_1\left(b_1^S+\frac{x^S}{\phi^S}Y_1(b_1^S)\right)\right)$$

$$V^B(b_1^S) = (1-\beta_1)\left[\log\left((1-\beta_1)\left(-\frac{\phi^S}{\phi^B}b_1^S+\frac{P_1(b_1^S)}{\phi^B}+\frac{x^B}{\phi^B}Y_1(b_1^S)\right)\right) - h^B\left(\frac{x^B}{\phi^B}Y_1(b_1^S)\right)\right]$$
$$+\beta_1\mathbb{E}\left[\log\left(\frac{D_{2,\omega}}{\phi_B}-\frac{\phi^S}{\phi^B}\beta_1\left(b_1^S+\frac{x^S}{\phi^S}Y_1(b_1^S)\right)\right)\right].$$

Their derivatives are given by

$$\phi^S\frac{dV^S}{db_1^S} = \phi^S\frac{(1-\beta_1)}{c_1^S}\left(1+\mu_1^S\frac{x^S}{\phi^S}\frac{dY_1}{db_1^S}\right),$$

$$\phi^B\frac{dV^B}{db_1^S} = -\phi^S\frac{(1-\beta_1)}{c_1^B}\left(1-\mu_1^B\frac{x^B}{\phi^S}\frac{dY_1}{db_1^S}\right),$$

where

$$\mu_t^S = 1-c_t^S h^{S\prime}\left(\frac{x^S}{\phi^S}Y_1(b_1^S)\right),$$

$$\mu_t^B = 1-c_t^B h^{B\prime}\left(\frac{x^B}{\phi^B}Y_1(b_1^B)\right).$$

The social marginal rate of substitution is given by

$$-\frac{\lambda^S \phi^S \frac{dV^S}{db_1^S}}{\lambda^B \phi^B \frac{dV^B}{db_1^S}} = \frac{\lambda^S c_1^B}{\lambda^B c_1^S} \frac{1 + \mu_1^S \frac{x^S}{\phi^S} \frac{dY_1}{db_1^S}}{1 - \mu_1^B \frac{x^B}{\phi^S} \frac{dY_1}{db_1^S}}.$$

B.1.3 Monetary Policy Only

Equilibrium as a function of monetary policy. We use

$$Y_0 = \frac{1 - \beta_0}{\beta_0(1 - \beta_1)} \frac{Y_1(b_1^S)}{R_0},$$

$$(1 - \beta_0) \left(\frac{x^S}{\phi^S} Y_0 + \frac{1}{R_0} \frac{x^S}{\phi^S} Y_1(b_1^S) + b_0^S \right) = \frac{x^S}{\phi^S} Y_0 + b_0^S - \frac{b_1^S}{R_0},$$

We get $b_1^S(R_0)$ as the solution of a fixed-point equation:

$$b_1^S = \frac{\beta_1(1 - \beta_0)}{1 - \beta_1} \frac{x^S}{\phi^S} Y_1(b_1^S) + \beta_0 R_0 b_0^S,$$

with

$$\frac{db_1^S}{dR_0} = \frac{\beta_0 b_0^S}{1 - \frac{\beta_1(1-\beta_0)}{1-\beta_1} \frac{x^S}{\phi^S} \frac{dY_1}{db_1^S}}.$$

Plugging back into the equations above allows us to express the equilibrium as a function of R_0.

Optimal monetary policy. The problem for optimal monetary policy is

$$\max_{R_0} \Bigg\{ \lambda^S \phi^S \left[(1 - \beta_0) \left(\log(c_0^S(R_0)) - h^S(\frac{x^S}{\phi^S} Y_0(R_0)) \right) + \beta_0 V^S(b_1^S(R_0)) \right]$$
$$+ \lambda^B \phi^B \left[(1 - \beta_0) \left(\log(c_0^B(R_0)) - h^B(\frac{x^B}{\phi^B} Y_0(R_0)) \right) + \beta_0 V^B(b_1^S(R_0)) \right] \Bigg\}.$$

The first-order condition for optimality is

$$\frac{dY_0}{dR_0}\left(\frac{\lambda^S x^S}{c_0^S}+\frac{\lambda^B x^B}{c_0^B}\right)\left(\frac{\frac{\lambda^S x^S}{c_0^S}}{\frac{\lambda^S x^S}{c_0^S}+\frac{\lambda^B x^B}{c_0^B}}\mu_0^S+\frac{\frac{\lambda^B x^B}{c_0^B}}{\frac{\lambda^S x^S}{c_0^S}+\frac{\lambda^B x^B}{c_0^B}}\mu_0^B\right)$$
$$+\phi^S\left(\frac{\lambda^S}{c_0^S}-\frac{\lambda^B}{c_0^B}\right)\frac{b_1^S}{R_0^2}$$
$$+\frac{1}{R_0}\frac{db_1^S}{dR_0}\frac{dY_1}{db_1^S}\left(\frac{\lambda^S x^S}{c_0^S}+\frac{\lambda^B x^B}{c_0^B}\right)\left(\frac{\frac{\lambda^S x^S}{c_1^S}}{\frac{\lambda^S x^S}{c_1^S}+\frac{\lambda^B x^B}{c_1^B}}\mu_1^S+\frac{\frac{\lambda^B x^B}{c_1^B}}{\frac{\lambda^S x^S}{c_1^S}+\frac{\lambda^B x^B}{c_1^B}}\mu_1^B\right)=0,$$

where

$$\frac{\frac{\lambda^S x^S}{c_t^S}}{\frac{\lambda^S x^S}{c_t^S}+\frac{\lambda^B x^B}{c_t^B}}\mu_t^S+\frac{\frac{\lambda^B x^B}{c_t^B}}{\frac{\lambda^S x^S}{c_t^S}+\frac{\lambda^B x^B}{c_t^B}}\mu_t^B$$

is the correct notion of average labor wedge at date t use appropriate weights on the labor wedges of savers and borrowers. The three terms in the first-order condition for optimality capture the three margins that are traded off by monetary policy at date 0.

Assume that $\lambda^S/\lambda^B = c_0^S/c_0^B$. Then this simplifies to

$$\frac{dY_0}{dR_0}\left(x^S\mu_0^S+x^B\mu_0^B\right)+\frac{1}{R_0}\frac{db_1^S}{dR_0}\frac{dY_1}{db_1^S}\left(x^S\mu_1^S+x^B\mu_1^B\right)=0.$$

This expression is similar to the one in Section 3, and the results are almost identical to those in Proposition 1. The only subtlety is that we must now use the average labor wedge $x^S\mu_t^S+x^B\mu_t^B$ in period t to diagnose recessions, booms, and the stance of monetary policy at date 0.

B.1.4 Optimal Monetary and Macroprudential Policy

The planning problem for jointly optimal monetary and macroprudential policy is

$$\max_{\{Y_0,c_0^S,c_0^B,b_1^S\}}\left\{\lambda^S\phi^S\left[(1-\beta_0)\left(\log(c_0^S)-h(\frac{x^S}{\phi^S}Y_0)\right)+\beta_0V^S(b_1^S)\right]\right.$$
$$\left.+\lambda^B\phi^B\left[(1-\beta_0)\left(\log(c_0^B)--h(\frac{x^B}{\phi^B}Y_0)\right)+\beta_0V^B(b_1^S)\right]\right\},$$

subject to

$$\phi^S c_0^S+\phi^B c_0^B\leq Y_0.$$

The first-order conditions for optimality deliver

$$x^S \mu_0^S + x^B \mu_0^B = 0,$$

$$\frac{\lambda^S c_0^B}{\lambda^B c_0^S} = -\frac{\lambda^S \phi^S \frac{dV^S}{db_1^S}}{\lambda^B \phi^B \frac{dV^B}{db_1^S}} = \frac{\lambda^S c_1^B}{\lambda^B c_1^S} \frac{1 + \mu_1^S \frac{x^S}{\phi^S} \frac{dY_1}{db_1^S}}{1 - \mu_1^B \frac{x^B}{\phi^S} \frac{dY_1}{db_1^S}}.$$

This implies that the average labor wedge is zero at date 0, and that macroprudential taxes are required to offset the aggregate demand externality. The macroprudential taxes are given by

$$1 - \tau_0 = \frac{1 + \mu_1^S \frac{x^S}{\phi^S} \frac{dY_1}{db_1^S}}{1 - \mu_1^B \frac{x^B}{\phi^S} \frac{dY_1}{db_1^S}} \approx 1 + \frac{x^S \mu_1^S + x^B \mu_1^B}{\phi^S} \frac{dY_1}{db_1^S} + O(|\mu_1|^2).$$

The second part of this equation is a first-order approximation in the labor wedges at date 1. It amounts to a second-order approximation in the average labor wedge at date 1 if the deviation from efficient rationing at date 1 is of order 1 or more in the average labor wedge at date 1. The results for monetary policy are therefore identical to those in Proposition 2, and those for macroprudential policy are approximately identical. The only subtlety is that we must now use the average labor wedge $x^S \mu_t^S + x^B \mu_t^B$ in period t to diagnose recessions, booms, and the stance of monetary policy at date 0.

B.2 Extrapolative Expectations During the Boom

We assume that borrowers have extrapolative expectations during the boom between 0 and 1 regarding asset prices but not regarding the whole economy.

B.2.1 Monetary Policy Only

Equilibrium as a function of monetary policy. We use

$$Y_0 = \frac{1 - \beta_0}{\beta_0 R_0} \left(\frac{1 - \beta_1 + \beta_1 (1 - \rho)}{1 - \beta_1} Y_1(b_1^S) + \rho \frac{P_0^2(b_1^S, R_0)}{P_{-1}} \right),$$

$$(1 - \beta_0) \left(\frac{x^S}{\phi^S} Y_0 + \frac{1}{R_0} \frac{x^S}{\phi^S} Y_1(b_1^S) + b_0^S \right) = \frac{x^S}{\phi^S} Y_0 + b_0^S - \frac{b_1^S}{R_0}.$$

We get $b_1^S(R_0)$ as the solution of a fixed-point equation:

$$b_1^S = \beta_0 R_0 b_0^S + \frac{x^S}{\phi^S} (1 - \beta_0) \left(\frac{\beta_1}{1 - \beta_1} Y_1(b_1^S) + \rho \left(\frac{P_0^2(b_1^S, R_0)}{P_{-1}} - P_1(b_1^S) \right) \right),$$

with

$$\frac{db_1^S}{dR_0} = \frac{\beta_0 b_0^S + (1-\beta_0)\rho \frac{x^S}{\phi^S}\frac{2P_0}{P_{-1}}\frac{\partial P_0}{\partial R_0}}{1-(1-\beta_0)\left((1-\rho)\frac{x^S}{\phi^S}\frac{\beta_1}{1-\beta_1}\frac{dY_1}{db_1^S} + \rho\frac{x^S}{\phi^S}\frac{2P_0}{P_{-1}}\frac{\partial P_0}{\partial b_1^S}\right)}.$$

Plugging back into the equations above allows us to express the equilibrium as a function of R_0.

Optimal monetary policy. The planning problem for optimal monetary policy is

$$\max_{R_0}\left\{\lambda^S\phi^S\left[(1-\beta_0)\left(\log(c_0^S(R_0)) - h(\frac{x^S}{\phi^S}Y_0(R_0))\right) + \beta_0 V^S(b_1^S(R_0))\right]\right.$$
$$\left. +\lambda^B\phi^B\left[(1-\beta_0)\left(\log(c_0^B(R_0)) - h(\frac{x^B}{\phi^B}Y_0(R_0))\right) + \beta_0 V^B(b_1^S(R_0))\right]\right\}.$$

The first order condition for optimality is

$$\frac{dY_0}{dR_0}\left(\frac{\lambda^S x^S}{c_0^S} + \frac{\lambda^B x^B}{c_0^B}\right)\left(\frac{\frac{\lambda^S x^S}{c_0^S}}{\frac{\lambda^S x^S}{c_0^S} + \frac{\lambda^B x^B}{c_0^B}}\mu_0^S + \frac{\frac{\lambda^B x^B}{c_0^B}}{\frac{\lambda^S x^S}{c_0^S} + \frac{\lambda^B x^B}{c_0^B}}\mu_0^B\right)$$
$$+\phi^S\left(\frac{\lambda^S}{c_0^S} - \frac{\lambda^B}{c_0^B}\right)\frac{b_1^S}{R_0^2}$$
$$+\frac{1}{R_0}\frac{db_1^S}{dR_0}\left[\frac{dY_1}{db_1^S}\frac{\beta_0(1-\beta_1)R_0}{1-\beta_0}\left(\frac{\lambda^S x^S}{c_1^S} + \frac{\lambda^B x^B}{c_1^B}\right)\left(\frac{\frac{\lambda^S x^S}{c_1^S}}{\frac{\lambda^S x^S}{c_1^S} + \frac{\lambda^B x^B}{c_1^B}}\mu_1^S + \frac{\frac{\lambda^B x^B}{c_1^B}}{\frac{\lambda^S x^S}{c_1^S} + \frac{\lambda^B x^B}{c_1^B}}\mu_1^B\right)\right.$$
$$\left. +\phi^S\frac{\lambda^B}{c_0^B}\left(1 - \frac{c_1^{B,e}}{c_1^B}\right)\right] = 0,$$

and can be approximated as

$$\frac{dY_0}{dR_0}\left(\frac{\frac{\lambda^S x^S}{c_0^S}}{\frac{\lambda^S x^S}{c_0^S}+\frac{\lambda^B x^B}{c_0^B}}\mu_0^S+\frac{\frac{\lambda^B x^B}{c_0^B}}{\frac{\lambda^S x^S}{c_0^S}+\frac{\lambda^B x^B}{c_0^B}}\mu_0^B\right)$$
$$+\phi^S\frac{\frac{\lambda^S}{c_0^S}-\frac{\lambda^B}{c_0^B}}{\frac{\lambda^S x^S}{c_0^S}+\frac{\lambda^B x^B}{c_0^B}}\frac{b_1^S}{R_0^2}$$
$$+\phi^S\frac{1}{R_0}\frac{db_1^S}{dR_0}\left[\frac{dY_1}{db_1^S}\frac{\frac{\frac{\lambda^S x^S}{c_1^S}}{\frac{\lambda^S x^S}{c_1^S}+\frac{\lambda^B x^B}{c_1^B}}\mu_1^S+\frac{\frac{\lambda^B x^B}{c_1^B}}{\frac{\lambda^S x^S}{c_1^S}+\frac{\lambda^B x^B}{c_1^B}}\mu_1^B}{\phi^S}+\frac{\frac{\lambda^B}{c_0^B}}{\frac{\lambda^S x^S}{c_0^S}+\frac{\lambda^B x^B}{c_0^B}}\left(1-\frac{c_1^{B,e}}{c_1^B}\right)\right]=O(|\mu,(c_1^B-c_1^{B,e})/c_1^B|^2).$$

Assume that $\lambda^S/\lambda^B=c_0^S/c_0^B$. Then this simplifies to

$$\frac{dY_0}{dR_0}\left(x^S\mu_0^S+x^B\mu_0^B\right)+\phi^S\frac{1}{R_0}\frac{db_1^S}{dR_0}\left[\frac{c_1^B}{c_1^{B,e}}\left(\frac{dY_1}{db_1^S}\frac{x^S\mu_1^S+x^B\mu_1^B\frac{c_1^{B,e}}{c_1^B}}{\phi^S}+1\right)-1\right]\frac{c_1^{B,e}}{c_1^B}=0,$$

which can be approximated as

$$\frac{dY_0}{dR_0}\left(x^S\mu_0^S+x^B\mu_0^B\right)+\phi^S\frac{1}{R_0}\frac{db_1^S}{dR_0}\left[\frac{dY_1}{db_1^S}\frac{x^S\mu_1^S+x^B\mu_1^B}{\phi^S}+\frac{c_1^B-c_1^{B,e}}{c_1^B}\right]=O(|\mu,(c_1^B-c_1^{B,e})/c_1^B|^2).$$

This is a first-order approximation in the labor wedges at date 1 and in the belief wedge. The results for monetary policy are therefore approximately identical to those in Proposition 1. The only subtlety is that we must now use the average labor wedge $x^S\mu_t^S+x^B\mu_t^B$ in period t to diagnose recessions, booms, and the stance of monetary policy at date 0.

B.2.2 Monetary Policy and Macroprudential Policy

The planning problem for jointly optimal monetary and macroprudential policy is exactly the same as under rational expectations. The optimal allocation is the same. The instruments that implement the optimal allocation are not.

The first-order conditions for optimality delivers

$$x^S\mu_0^S+x^B\mu_0^B=0,$$

$$1-\tau_0 = \left(\frac{c_1^B}{c_1^{B,e}}\right) \frac{1+\mu_1^S \frac{x^S}{\phi^S}\frac{dY_1}{db_1^S}}{1-\mu_1^B \frac{x^B}{\phi^S}\frac{dY_1}{db_1^S}} \approx 1 + \frac{c_1^B - c_1^{B,e}}{c_1^B} + \frac{x^S\mu_1^S + x^B\mu_1^B}{\phi^S}\frac{dY_1}{db_1^S} + O(|\mu, (c_1^B - c_1^{B,e})/c_1^B|^2),$$

where the last equation is an approximation in the labor wedges at date 1 and in the belief wedge. The results for monetary policy are therefore identical to those in Proposition 4, and those for macroprudential policy are approximately identical. The only subtlety is that we must now use the average labor wedge $x^S\mu_t^S + x^B\mu_t^B$ in period t to diagnose recessions, booms, and the stance of monetary policy at date 0.

B.3 Extrapolative Expectations During the Bust

In this section, we assume that borrowers have extrapolative expectations during the bust between dates 1 and 2. They have rational expectations between dates 0 and 1. It does not matter whether savers have rational or extrapolative expectations. It also does not matter whether borrowers have extrapolative expectations only regarding asset prices or also regarding the whole economy. For simplicity only, we assume that savers have rational expectations, and that borrowers have extrapolative expectations only regarding asset prices. The analysis for the other variants is identical.

B.3.1 Main Changes to the Setup

We now have two state variables at date 1: b_1^S and P_0. The subjective probabilities $\pi_G(P_1, P_0)$ and $\pi_B(P_1, P_0)$ of the borrowers regarding the state of the world $\omega \in \{H, L\}$ at date 2 are defined by

$$\mathbb{E}[\frac{D_{2,\omega}}{P_1}] = (1-\rho)\bar{\mathbb{E}}[\frac{D_{2,\omega}}{P_1}] + \rho\frac{P_1}{P_0}$$

where $\bar{\pi}_G$ and $\bar{\pi}_B$ denote true probabilities and $\bar{\mathbb{E}}$ denotes expectation under the true probabilities.

The function $Y_1(b_1^S, P_0)$ is defined by the equation

$$Y_1 = (1-\beta_1)\left(\phi^S b_1^S + x^S Y_1\right) + \frac{1}{\frac{\beta_1}{1-\beta_1}\mathbb{E}\left[\frac{1}{D_2 - \beta_1(\phi^S b_1^S + x^S Y_1)}\right]}$$

where the expectations are computed under the subjective probabilities $\pi_G(Y_1\beta_1/(1-\beta_1), P_0)$ and $\pi_B(Y_1\beta_1/(1-\beta_1), P_0)$. From this we get

$$P_1(b_1^S, P_0) = \frac{\beta_1}{1-\beta_1} Y_1(b_1^S, P_0)$$

$$c_1^B(b_1^S, P_0) = (1-\beta_1)\left[-\frac{\phi^S}{\phi^B}b_1^S + \frac{1}{\phi^B}\left(x^B + \frac{\beta_1}{1-\beta_1}\right)Y_1(b_1^S, P_0)\right]$$

$$c_0^B(b_1^S, P_0) = \frac{(1-\beta_0)}{\beta_0(1-\beta_1)\frac{P_1(b_1^S,P_0)}{P_0}}c_1^B(b_1^S, P_0)$$

Close enough to rational expectations (when ρ is small enough) we have

$$\frac{\partial c_0^B}{\partial b_1^S} > 0, \quad \frac{\partial c_0^B}{\partial P_0} > 0,$$

$$\frac{\partial Y_1}{\partial b_1^S} < 0, \quad \frac{\partial Y_1}{\partial P_0} < 0.$$

In particular, we now have $\partial Y_1/\partial P_0 < 0$. An increase in the price of the risky asset at date 0 reduces the return on the risky asset between dates 0 and 1. This makes borrowers, who have extrapolative expectations, more pessimistic at date 1. This in turn reduces the price of the risky asset at date 1 and hence also aggregate demand and output.

The value functions encoding welfare in general equilibrium are given by

$$\begin{aligned}\bar{V}^S(b_1^S, P_0) = (1-\beta_1)&\left[\log\left((1-\beta_1)\left(b_1^S + \frac{x^S}{\phi^S}Y_1(b_1^S, P_0)\right)\right) - h\left(\frac{x^S}{\phi^S}Y_1(b_1^S, P_0)\right)\right] \\ &+ \beta_1 \log\left(\beta_1\left(b_1^S + \frac{x^S}{\phi^S}Y_1(b_1^S, P_0)\right)\right),\end{aligned}$$

$$\begin{aligned}\bar{V}^B(b_1^S, P_0) = (1-\beta_1)&\left[\log\left((1-\beta_1)\left(-\frac{\phi^S}{\phi^B}b_1^S + \frac{P_1(b_1^S, P_0)}{\phi^B} + \frac{x^B}{\phi^B}Y_1(b_1^S, P_0)\right)\right) - h\left(\frac{x^B}{\phi^B}Y_1(b_1^S, P_0)\right)\right] \\ &+ \beta_1\bar{\mathbb{E}}\left[\log\left(\frac{D_{2,\omega}}{\phi_B} - \frac{\phi^S}{\phi^B}\beta_1\left(b_1^S + \frac{x^S}{\phi^S}Y_1(b_1^S, P_0)\right)\right)\right].\end{aligned}$$

We have

$$\phi^S\frac{\partial \bar{V}^S}{\partial b_1^S} = \phi^S\frac{(1-\beta_1)}{c_1^S}\left(1 + \mu_1^S\frac{x^S}{\phi^S}\frac{\partial Y_1}{\partial b_1^S}\right),$$

$$\phi^S\frac{\partial \bar{V}^S}{\partial P_0} = \phi^S\frac{(1-\beta_1)}{c_1^S}\left(\mu_1^S\frac{x^S}{\phi^S}\frac{\partial Y_1}{\partial P_0}\right),$$

$$\phi^B\frac{\partial \bar{V}^B}{\partial b_1^S} = -\phi^S\frac{1-\beta_1}{c_1^B}\left[1 - \frac{x^B}{\phi^S}\mu_1^B\frac{\partial Y_1}{\partial b_1^S} + \beta_1\left(\frac{\bar{\mathbb{E}}\left[\frac{1}{c_2^B}\right]}{\mathbb{E}\left[\frac{1}{c_2^B}\right]} - 1\right)\left[1 + \frac{x^S}{\phi^S}\frac{\partial Y_1}{\partial b_1^S}\right]\right],$$

$$\phi^B \frac{\partial \bar{V}^B}{\partial P_0} = -\phi^S \frac{1-\beta_1}{c_1^B} \left[-\frac{x^B}{\phi^S} \mu_1^B \frac{\partial Y_1}{\partial P_0} + \beta_1 \left(\frac{\bar{\mathbb{E}}\left[\frac{1}{c_2^B}\right]}{\mathbb{E}\left[\frac{1}{c_2^B}\right]} - 1 \right) \frac{x^S}{\phi^S} \frac{\partial Y_1}{\partial P_0} \right].$$

There is no bias in beliefs in equilibrium despite extrapolative expectations

$$\frac{\bar{\mathbb{E}}\left[\frac{1}{c_2^B}\right]}{\mathbb{E}\left[\frac{1}{c_2^B}\right]} - 1 = 0.$$

At that point with no bias in beliefs, we have

$$\phi^S \frac{\partial \bar{V}^S}{\partial b_1^S} = \phi^S \frac{(1-\beta_1)}{c_1^S} \left(1 + \mu_1^S \frac{x^S}{\phi^S} \frac{\partial Y_1}{\partial b_1^S} \right),$$

$$\phi^S \frac{\partial \bar{V}^S}{\partial P_0} = \phi^S \frac{(1-\beta_1)}{c_1^S} \left(\mu_1^S \frac{x^S}{\phi^S} \frac{\partial Y_1}{\partial P_0} \right),$$

$$\phi^B \frac{\partial \bar{V}^B}{\partial b_1^S} = -\phi^S \frac{1-\beta_1}{c_1^B} \left(1 - \frac{x^B}{\phi^S} \mu_1^B \frac{\partial Y_1}{\partial b_1^S} \right),$$

$$\phi^B \frac{\partial \bar{V}^B}{\partial P_0} = -\phi^S \frac{1-\beta_1}{c_1^B} \left(-\frac{x^B}{\phi^S} \mu_1^B \frac{\partial Y_1}{\partial P_0} \right).$$

B.3.2 Monetary Policy

We have the following fixed-point equations

$$b_1^S = \frac{\beta_1(1-\beta_0)}{1-\beta_1} \frac{x^S}{\phi^S} Y_1(b_1^S, P_0) + \beta_0 R_0 b_0^S,$$

$$P_0 = \frac{P_1(b_1^S, P_0)}{R_0}.$$

These equations define functions $b_1^S(R_0)$ and $P_0(R_0)$. Close enough to rational expectations, we have

$$\frac{dP_0}{dR_0} < 0, \quad \frac{dY_0}{dR_0} < 0.$$

In addition, we have

$$\frac{db_1^S}{dR_0} = \frac{\beta_0 b_0^S + \frac{\beta_1(1-\beta_0)}{1-\beta_1} \frac{x^S}{\phi^S} \frac{\partial Y_1}{\partial P_0} \frac{dP_0}{dR_0}}{1 - \frac{\beta_1(1-\beta_0)}{1-\beta_1} \frac{x^S}{\phi^S} \frac{\partial Y_1}{\partial b_1^S}}.$$

There is a new channel through which toughening monetary policy at date 0 (increasing the interest rate R_0) increases debt at date 1: it reduces the price P_0 of the risky asset at date 0, makes borrowers more optimistic at date 1, stimulates the price of the risky asset at date 1, increases aggregate demand and output at date 1, mitigates the adverse effect on output at date 0, and makes borrowers relatively more eager to borrow than savers at date 0.

The planning problem for optimal monetary policy is

$$\max_{R_0} \Big\{ \lambda^S \phi^S \Big[(1-\beta_0) \Big(\log(c_0^S(R_0)) - h(\frac{x^S}{\phi^S} Y_0(R_0)) \Big) + \beta_0 \bar{V}^S(b_1^S(R_0), P_0(R_0)) \Big]$$
$$+ \lambda^B \phi^B \Big[(1-\beta_0) \Big(\log(c_0^B(R_0)) - h(\frac{x^B}{\phi^B} Y_0(R_0)) \Big) + \beta_0 \bar{V}^B(b_1^S(R_0), P_0(R_0)) \Big] \Big\}.$$

The first-order condition for optimality is

$$\frac{dY_0}{dR_0} \left(\frac{\lambda^S x^S}{c_0^S} + \frac{\lambda^B x^B}{c_0^B} \right) \left(\frac{\frac{\lambda^S x^S}{c_0^S}}{\frac{\lambda^S x^S}{c_0^S} + \frac{\lambda^B x^B}{c_0^B}} \mu_0^S + \frac{\frac{\lambda^B x^B}{c_0^B}}{\frac{\lambda^S x^S}{c_0^S} + \frac{\lambda^B x^B}{c_0^B}} \mu_0^B \right)$$
$$+ \phi^S \left(\frac{\lambda^S}{c_0^S} - \frac{\lambda^B}{c_0^B} \right) \frac{b_1^S}{R_0^2}$$
$$+ \left(\frac{1}{R_0} \frac{db_1^S}{dR_0} \frac{\partial Y_1}{\partial b_1^S} + \frac{1}{R_0} \frac{dP_0}{dR_0} \frac{\partial Y_1}{\partial P_0} \right) \left(\frac{\lambda^S x^S}{c_0^S} + \frac{\lambda^B x^B}{c_0^B} \right) \left(\frac{\frac{\lambda^S x^S}{c_1^S}}{\frac{\lambda^S x^S}{c_1^S} + \frac{\lambda^B x^B}{c_1^B}} \mu_1^S + \frac{\frac{\lambda^B x^B}{c_1^B}}{\frac{\lambda^S x^S}{c_1^S} + \frac{\lambda^B x^B}{c_1^B}} \mu_1^B \right) = 0.$$

Assume that $\lambda^S / \lambda^B = c_0^S / c_0^B$. Then this simplifies to

$$\frac{dY_0}{dR_0} \left(\frac{\frac{\lambda^S x^S}{c_0^S}}{\frac{\lambda^S x^S}{c_0^S} + \frac{\lambda^B x^B}{c_0^B}} \mu_0^S + \frac{\frac{\lambda^B x^B}{c_0^B}}{\frac{\lambda^S x^S}{c_0^S} + \frac{\lambda^B x^B}{c_0^B}} \mu_0^B \right)$$
$$+ \left(\frac{1}{R_0} \frac{db_1^S}{dR_0} \frac{\partial Y_1}{\partial b_1^S} + \frac{1}{R_0} \frac{dP_0}{dR_0} \frac{\partial Y_1}{\partial P_0} \right) \left(\frac{\frac{\lambda^S x^S}{c_1^S}}{\frac{\lambda^S x^S}{c_1^S} + \frac{\lambda^B x^B}{c_1^B}} \mu_1^S + \frac{\frac{\lambda^B x^B}{c_1^B}}{\frac{\lambda^S x^S}{c_1^S} + \frac{\lambda^B x^B}{c_1^B}} \mu_1^B \right) = 0$$

or

$$\frac{dY_0}{dR_0} \left(x^S \mu_0^S + x^B \mu_0^B \right) + \left(\frac{1}{R_0} \frac{db_1^S}{dR_0} \frac{\partial Y_1}{\partial b_1^S} + \frac{1}{R_0} \frac{dP_0}{dR_0} \frac{\partial Y_1}{\partial P_0} \right) \left(x^S \mu_1^S + x^B \mu_1^B \right) = 0.$$

Assume that there is a recession at date 1 with $x^S \mu_1^S + x^B \mu_1^B > 0$. Assume also that $b_0^S = 0$. Under rational expectations, we would have $db_1^S / dR_0 = 0$ and $\partial Y_1 / \partial P_0 = 0$, and so we would have perfect macroeconomic stabilization at date 0 with $x^S \mu_0^S + x^B \mu_0^B = 0$. With extrapolative expectations, but close enough to rational expectations, there are two opposing forces: on the one hand, tough monetary policy at date 0 now encourages borrowing with $db_1^S / dR_0 > 0$ (a leverage effect); on the other hand tough monetary policy at date 0 now makes agents more optimistic at date 1 with $\partial Y_1 / \partial P_0 < 0$ (a belief effect). The leverage effect pushes in the direction of a boom at date 0 ($x^S\mu_0^S + x^B\mu_0^B < 0$) and the belief effect in the direction of a recession at date 0 ($x^S\mu_0^S + x^B\mu_0^B > 0$). The belief effect is guaranteed to dominate if savers have little labor income (x^S is small).

B.3.3 Monetary Policy and Macroprudential Policy

The planning problem for jointly optimal monetary and macroprudential policies is

$$\max_{\{Y_0, c_0^S, c_0^B, b_1^S, P_0\}} \left\{ \lambda^S \phi^S \left[(1-\beta_0) \left(\log(c_0^S) - h(\frac{x^S}{\phi^S} Y_0) \right) + \beta_0 \bar{V}^S(b_1^S, P_0) \right] \right.$$
$$\left. + \lambda^B \phi^B \left[(1-\beta_0) \left(\log(c_0^B) - h(\frac{x^B}{\phi^B} Y_0) \right) + \beta_0 \bar{V}^B(b_1^S, P_0) \right] \right\},$$

s.t.

$$\phi^S c_0^S + \phi^B c_0^B \leq Y_0,$$
$$c_0^B = c_0^B(b_1^S, P_0).$$

We can rewrite the planning problem as

$$\max_{\{c_0^S, b_1^S, P_0\}} \left\{ \lambda^S \phi^S \left[(1-\beta_0) \left(\log(c_0^S) - h(\frac{x^S}{\phi^S} (\phi^S c_0^S + \phi^B c_0^B(b_1^S, P_0))) \right) + \beta_0 \bar{V}^S(b_1^S, P_0) \right] \right.$$
$$\left. + \lambda^B \phi^B \left[(1-\beta_0) \left(\log(c_0^B(b_1^S, P_0)) - h(\frac{x^B}{\phi^B} (\phi^S c_0^S + \phi^B c_0^B(b_1^S, P_0))) \right) + \beta_0 \bar{V}^B(b_1^S, P_0) \right] \right\}.$$

Assume from now on that there is no bias in beliefs at the optimum. Then the first-order conditions for optimality deliver

$$\lambda^S \frac{x^S}{c_0^S} \mu_0^S + \frac{\lambda^B x^B}{c_0^B} \mu_0^B = x^B \left(\frac{\lambda^B}{c_0^B} - \frac{\lambda^S}{c_0^S} \right) = \frac{x^B}{\phi^B} \frac{\beta_0(1-\beta_1)}{1-\beta_0} \frac{-\frac{\partial Y_1}{\partial P_0}}{\frac{\partial c_0^B}{\partial P_0}} \left[\frac{\lambda^S}{c_1^S} \mu_1^S x^S + \frac{\lambda^B}{c_1^B} x^B \mu_1^B \right],$$

$$1 - \tau_0 = \frac{1 + \frac{x^S \mu_1^S}{\phi^S} \left(\frac{\partial Y_1}{\partial b_1^S} + \frac{-\frac{\partial Y_1}{\partial P_0}}{\frac{\partial c_0^B}{\partial P_0}} \frac{\partial c_0^B}{\partial b_1^S} \right)}{1 - \frac{x^B \mu_1^B}{\phi^S} \left(\frac{\partial Y_1}{\partial b_1^S} + \frac{-\frac{\partial Y_1}{\partial P_0}}{\frac{\partial c_0^B}{\partial P_0}} \frac{\partial c_0^B}{\partial b_1^S} \right)} \frac{1 - \mu_0^B}{1 + \frac{x^S}{x^B} \mu_0^S}.$$

As long as there is a recession at date 1 with

$$\frac{\lambda^S}{c_1^S}\mu_1^S x^S + \frac{\lambda^B}{c_1^B} x^B \mu_1^B > 0,$$

then if we are close enough to rational expectations (when ρ is small enough), there is also a recession at date 0:

$$\lambda^S \frac{x^S}{c_0^S}\mu_0^S + \frac{\lambda^B x^B}{c_0^B}\mu_0^B > 0.$$

This continues to hold if the bias in beliefs is not too large at the optimum, which is in turn guaranteed if we are close enough to rational expectations. This means that close enough to rational expectations, even with optimal macroprudential policy, it is optimal to lean against the wind with monetary policy and to engineer a recession at date 0 in order to depress the asset price at date 0 and make agents more optimistic at date 1 by raising the realized return between 0 and 1.

We also get

$$\tau_0 = \frac{x^S \mu_1^S + x^B \mu_1^B}{\phi^S}\left(-\frac{\partial Y_1}{\partial b_1^S} - \frac{-\frac{\partial Y_1}{\partial P_0}}{\frac{\partial c_0^B}{\partial P_0}}\left(\frac{\partial c_0^B}{\partial b_1^S} - \frac{\phi^S}{\phi^B}\frac{1}{R_0}\right)\right) + O(|\mu_0|^2, |\mu_1|^2),$$

where

$$\frac{\partial c_1^B}{\partial b_1^S} - \frac{\phi^S}{\phi^B}\frac{b_1^S P_0}{P_1} \approx -\frac{\phi^S}{\phi^B}\frac{b_1^S P_0}{P_1}\frac{1}{Y_1}\frac{\partial Y_1}{\partial b_1^S} + O(|\mu_0|^2, |\mu_1|^2),$$

which is positive and implies that close enough to rational expectations, the macroprudential tax τ_0 is lower than it would be under rational expectations (given wedges). This is because apart from directly raising the cost of borrowing for borrowers at date 0, the macroprudential tax lowers the asset price, makes borrowers more pessimistic, and hence also indirectly reduces their desire to borrow at a given cost.