The Darwinian Returns to Scale

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July, 2020

Origins of Aggregate Increasing Returns to Scale

- Technical efficiency?
- Allocative efficiency?
- Unresolved theoretically and empirically.

Model Elements

- Monopolistic competition: Kimball demand.
- Heterogeneity: marginal costs, markups, pass-throughs.
- Technical increasing returns: fixed costs (entry, overhead).

Results

- Comparative statics in second best (welfare, output, TFP).
- Decomposition into technical and allocative efficiency.
- Non-parametric estimation.
- New “Matthew” Effect drives large gains in Alloc. Eff.
- Key: heterogeneity $\times$ inefficiency via Darwinian reallocations.

Households

- Mass L of identical households with unit labor supply.
- Kimball preferences over varieties of consumption goods:

$$\int_0^\infty \Upsilon\left(\frac{y_\theta}{Y}\right) dF(\theta) = 1.$$

- Maximize utility s.t. budget constraint (wage numeraire):

$$\max_{\{y_\theta\}} Y$$

s.t.

$$\int_0^\infty p_\theta y_\theta dF(\theta) = 1.$$

Producers

- Each variety supplied by single producer.
- Free entry with cost f_e (labor), type realization $\theta \sim g(\theta)$.
- Production with overhead cost f_o, marginal cost $1/A_\theta$ (labor).
- Maximize profits s.t. demand:

$$\max_{\{p_\theta, y_\theta\}} L(p_\theta y_\theta - \frac{1}{A_\theta} y_\theta) - f_o$$

s.t.

$$\frac{p_\theta}{P} = \Upsilon'(\frac{y_\theta}{Y}).$$

Demand Concepts

- Markups:

$$\mu\left(\frac{y}{Y}\right) = \frac{1}{1 - \frac{1}{\underbrace{\sigma\left(\frac{y}{Y}\right)}_{\text{demand elasticity}}}} \geq 1.$$

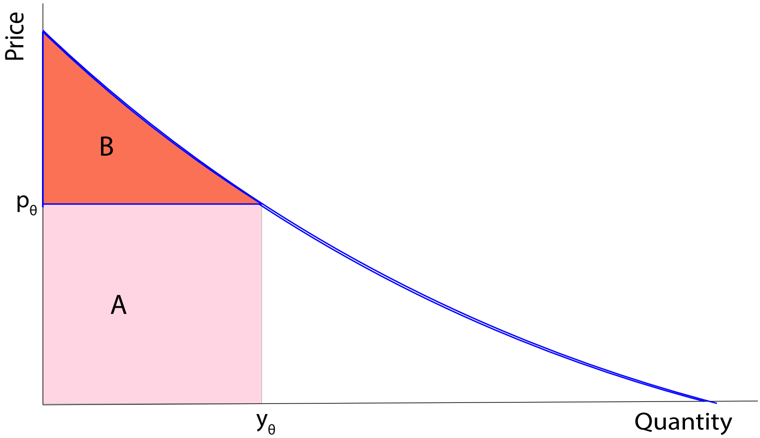
- Pass-throughs:

$$\rho\left(\frac{y}{Y}\right) = \frac{1}{1 + \frac{\frac{y}{Y}\mu'\left(\frac{y}{Y}\right)}{\mu\left(\frac{y}{Y}\right)}\sigma\left(\frac{y}{Y}\right)} \lesseqgtr 1.$$

- Infra-marginal surplus ratios (noting $\bar{\delta} = \mathbb{E}_\lambda[\delta_\theta]$):

$$\delta\left(\frac{y}{Y}\right) = \frac{\Upsilon\left(\frac{y}{Y}\right)}{\frac{y}{Y}\Upsilon'\left(\frac{y}{Y}\right)} \geq 1.$$

Infra-Marginal Surplus Ratio $\delta = \frac{A+B}{A}$



Equilibrium

- Households maximize utility.
- Firms maximize profits.
- Free entry and exit.
- Markets clear.

Technical and Allocative Efficiency

- Welfare function:
$$Y = \mathscr{Y}(L, \mathscr{X}).$$
where $\mathscr{X}$ is share of labor allocated to each type.

- Technical and allocative efficiency:
$$d \log Y = \underbrace{\frac{\partial \log \mathscr{Y}}{\partial \log L} d \log L}_{\text{technical efficiency}} + \underbrace{\frac{\partial \log \mathscr{Y}}{\partial \mathscr{X}} d \mathscr{X}}_{\text{allocative efficiency}} .$$

Social Inefficiency

- Three margins of efficiency: relative size, entry, selection.
- Excessive relative size θ' vs. θ iff:

$$\mu_{\theta'} < \mu_{\theta}.$$

- Excessive entry iff:

$$\mathbb{E}_{\lambda}[\delta_{\theta}] < \frac{1}{\mathbb{E}_{\lambda}[\frac{1}{\mu_{\theta}}]}.$$

- Excessive selection iff:

$$\delta_{\theta^*} > \mathbb{E}_{\lambda}[\delta_{\theta}].$$

Welfare

- Change in welfare per capita:

$$d \log Y = \underbrace{\left(\mathbb{E}_\lambda[\delta_\theta] - 1\right) d \log L}_{\text{technical efficiency}}$$

$$+ \underbrace{\frac{\xi^{\varepsilon} + \xi^{\theta^*} + \xi^{\mu}}{1 - \xi^{\varepsilon} - \xi^{\theta^*} - \xi^{\mu}} \left(\mathbb{E}_\lambda[\delta_\theta]\right) d \log L}_{\text{allocative efficiency}},$$

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$$+\underbrace{\frac{\xi^{\varepsilon} + \xi^{\theta^*} + \xi^{\mu}}{1 - \xi^{\varepsilon} - \xi^{\theta^*} - \xi^{\mu}}\Big(\mathbb{E}_\lambda[\delta_\theta]\Big)\, d\log L}_{\text{allocative efficiency}},$$

where

$$\textit{Entry}: \xi^{\varepsilon} = \Big(\mathbb{E}_\lambda[\delta_\theta] - 1\Big)\Big(\mathbb{E}_\lambda[\sigma_\theta] - \mathbb{E}_{\lambda(1-1/\mu)}[\sigma_\theta]\Big)\left(\mathbb{E}_\lambda\left[\frac{1}{\sigma_\theta}\right]\right),$$

$$\textit{Selection}: \xi^{\theta^*} = \Big(\mathbb{E}_\lambda[\delta_\theta] - \delta_{\theta^*}\Big)\left(\lambda_{\theta^*}\gamma_{\theta^*}\frac{\sigma_{\theta^*} - \mathbb{E}_{\lambda(1-1/\mu)}[\sigma_\theta]}{\sigma_{\theta^*} - 1}\right)\left(\mathbb{E}_\lambda\left[\frac{1}{\sigma_\theta}\right]\right),$$

$$\textit{Markups}: \xi^{\mu} = \left(\mathbb{E}_\lambda\left[(1-\rho_\theta)\left(1 - \frac{\mathbb{E}_\lambda[\delta_\theta] - 1}{\mu_\theta - 1}\right)\right]\right)\left(\mathbb{E}_\lambda\left[\frac{1}{\sigma_\theta}\right]\right).$$

Understanding ξ^{ε} via Demand Curve $\frac{p}{P} = \Upsilon'(\frac{y}{Y})$

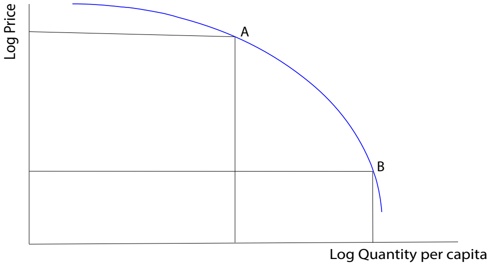

Figure: Reallocation effect due to increased entry (holding fixed markups and the selection cutoff).

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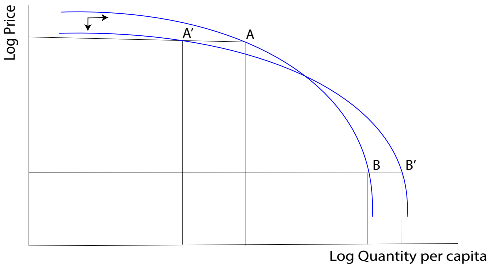


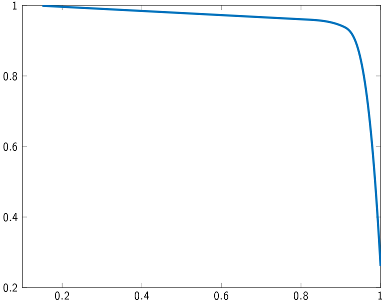
Figure: Reallocation effect due to increased entry (holding fixed markups and the selection cutoff).

- Matthew effect: $\xi^{\varepsilon} > 0$ (irrespective of shape of demand).
- Signs of ξ^{θ^*} and ξ^{μ} ambiguous (too much or too little selection and entry).

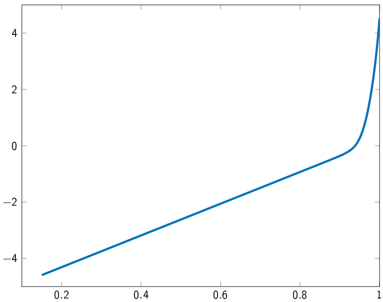
Non-Parametric Estimation

- Belgian data for manufacturing firms.
- Sales and pass-throughs by firm size for ProdCom sub-sample (price and quantity data) from Amiti et al. (19).
- Can back out primitives as solution to ODEs (up to $\bar{\delta}$ and $\bar{\mu}$).

Data



(a) Pass-through ρ_θ.

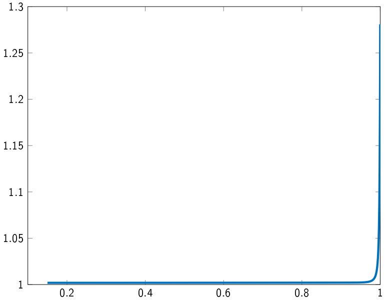


(b) Sales share density $\log \lambda_\theta$.

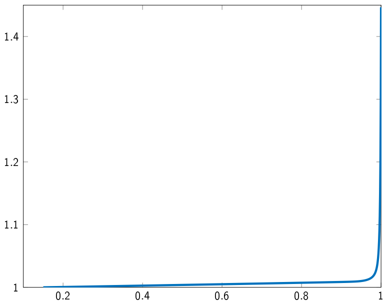
Postulates for Boundary Conditions

- Take one of two values for $\bar{\delta}$:
  - $\bar{\delta} = \bar{\mu}$ (efficient entry);
  - $\bar{\delta} = \delta_{\theta^*}$ (efficient selection).
- Take one of two values for $\bar{\mu}$
  - $\bar{\mu} = 1.045$ ($d \log Y / d \log L \approx 0.14$);
  - $\bar{\mu} = 1.09$ ($d \log Y / d \log L \approx 0.3$).

Estimates

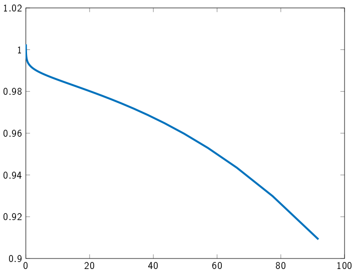


(a) Markup μ_θ ($\bar{\mu} = 1.045$)

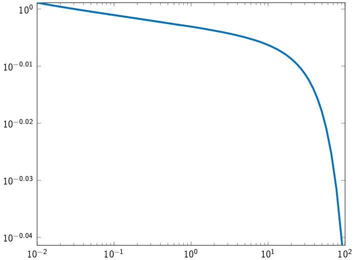


(b) Productivity $\log A_\theta$ ($\bar{\mu} = 1.045$)

Residual Demand Curve



(a) Residual demand curve (efficient entry, $\bar{\mu} = 1.045$).



(b) Log-log residual demand curve (efficient entry, $\bar{\mu} = 1.045$).

Counterfactual: 1% Population Shock

	$\bar{\mu} = 1.045$		$\bar{\mu} = 1.090$	
	$\bar{\delta} = \delta_{\theta^*}$	$\bar{\delta} = \bar{\mu}$	$\bar{\delta} = \delta_{\theta^*}$	$\bar{\delta} = \bar{\mu}$
Welfare	0.130	0.145	0.293	0.323
Technical efficiency	0.017	0.045	0.034	0.090
Allocative efficiency	0.114	0.100	0.260	0.233

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Welfare	0.130	0.145	0.293	0.323
Technical efficiency	0.017	0.045	0.034	0.090
Allocative efficiency	0.114	0.100	0.260	0.233
Entry	0.117	0.408	0.272	1.396
Exit	0.000	-0.251	0.000	-1.006
Markups	-0.004	-0.057	-0.012	-0.157
Real GDP per capita	0.024	0.024	0.051	0.052

Table: The elasticity of welfare and real GDP per capita to population with heterogeneous firms.

Counterfactual: 1% Population Shock (Homogenous Firms)

	$\bar{\mu} = 1.045$		$\bar{\mu} = 1.090$	
	$\bar{\delta} = \delta_{\theta^*}$	$\bar{\delta} = \bar{\mu}$	$\bar{\delta} = \delta_{\theta^*}$	$\bar{\delta} = \bar{\mu}$
Welfare	0.030	0.045	0.060	0.090
Technical efficiency	0.017	0.045	0.034	0.090
Allocative efficiency	0.013	0.000	0.026	0.000
Real GDP per capita	0.021	0.022	0.042	0.043

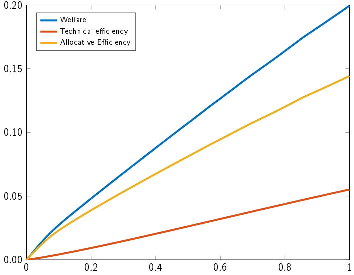
Table: The elasticity of welfare and real GDP per capita to population with homogenous firms.

Counterfactual: 50% Population Shock (Nonlinearities)

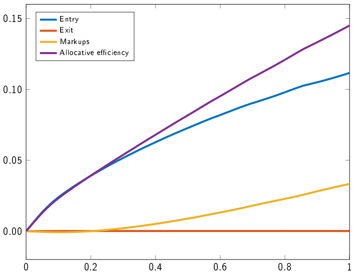
	$\bar{\mu} = 1.045$		$\bar{\mu} = 1.090$	
	$\bar{\delta} = \delta_{\theta^*}$	$\bar{\delta} = \bar{\mu}$	$\bar{\delta} = \delta_{\theta^*}$	$\bar{\delta} = \bar{\mu}$
Welfare	0.100	0.099	0.215	0.216
Technical efficiency	0.025	0.048	0.052	0.098
Allocative efficiency	0.075	0.051	0.162	0.117
Entry	0.066	0.107	0.145	0.272
Exit	0.000	-0.065	0.000	-0.176
Markups	0.008	0.008	0.017	0.021
Real GDP per capita	0.025	0.024	0.054	0.051

Table: The average elasticity of welfare and real GDP per capita to population with heterogeneous firms for a 50% population shock.

Counterfactual: 50% Shock (Nonlinearities)



(a) Welfare: technical and allocative efficiency as functions of $\log L$ (efficient selection, $\bar{\mu} = 1.09$).



(b) Allocative efficiency: entry, exit, and markups as functions of $\log L$ (efficient selection, $\bar{\mu} = 1.09$).

Conclusion: Summary

- Increasing returns to scale?
- Technical and allocative efficiency.
- Key: heterogeneity $\times$ inefficiency.
- Different for welfare and real output or TFP.

Back-Up Slides

Non-Parametric Estimation

- Inputs:
  - λ_θ, ρ_θ, M (data);
  - $\bar{\mu} = 1/[\mathbb{E}_\lambda[1/\mu_\theta]]$ and $\bar{\delta} = \mathbb{E}_\lambda[\delta_\theta]$ (postulates).

Non-Parametric Estimation (Key Equations)

- Changes in λ_θ with A_θ:

$$\frac{d\log\lambda_\theta}{d\theta} = \frac{\rho_\theta}{\mu_\theta - 1}\frac{d\log A_\theta}{d\theta}.$$

- Changes in μ_θ with A_θ:

$$\frac{d\log\mu_\theta}{d\theta} = (1-\rho_\theta)\frac{d\log A_\theta}{d\theta}.$$

Non-Parametric Estimation (Local)

- Recover μ_θ and A_θ by solving:

$$\frac{d\log\mu_\theta}{d\theta} = \frac{(\mu_\theta - 1)(1-\rho_\theta)}{\rho_\theta}\frac{d\log\lambda_\theta}{d\theta} \quad \text{s.t.} \quad \frac{1}{\mathbb{E}_\lambda[\frac{1}{\mu_\theta}]} = \bar{\mu},$$

$$\frac{d\log A_\theta}{d\theta} = \frac{\mu_\theta - 1}{\rho_\theta}\frac{d\log\lambda_\theta}{d\theta} \quad \text{s.t.} \quad A_{\theta^*} = 1.$$

- Recover δ_θ by solving:

$$\frac{d\log\delta_\theta}{d\theta} = \frac{\mu_\theta - \delta_\theta}{\delta_\theta}\frac{d\log\lambda_\theta}{d\theta} \quad \text{s.t.} \quad \mathbb{E}_\lambda[\delta_\theta] = \bar{\delta}.$$

Non-Parametric Estimation (Global)

- Recover Υ using:

$$\Upsilon(\frac{y}{Y}) = \frac{\delta_{\theta(y)}\lambda_{\theta(y)}}{\bar{\delta}M}.$$

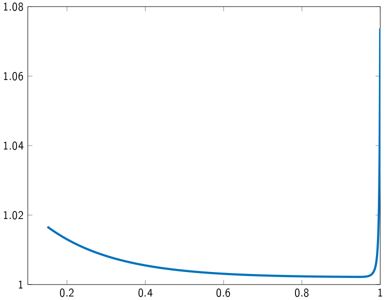
where $\theta(y)$ inverse of $y_\theta = (\lambda_\theta A_\theta)/(M\mu_\theta)$.

- Recover $f_e\Delta$ and f_o using:

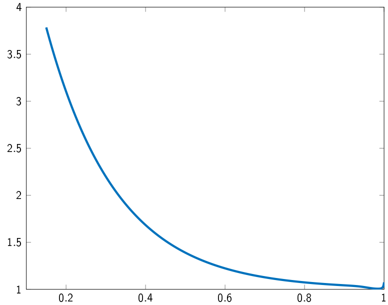
$$\frac{f_e\Delta}{L} + (1 - G(\theta^*))\frac{f_o}{L} = \frac{1}{M}\mathbb{E}\left[\lambda_\theta\left(1 - \frac{1}{\mu_\theta}\right)\right],$$

$$\frac{f_o}{L} = \frac{1}{M}\lambda_{\theta^*}\left(1 - \frac{1}{\mu_{\theta^*}}\right).$$

Estimates (Efficient Selection vs. Efficient Entry)



(a) Infra-marginal surplus ratio δ_θ (efficient selection, $\bar{\mu} = 1.045$).



(b) Infra-marginal surplus ratio δ_θ (efficient entry, $\bar{\mu} = 1.045$).

Welfare and Real Output

- Welfare per capita:

$$d\log Y = \left(\mathbb{E}_\lambda[\delta_\theta] - 1\right) d\log M$$
$$+ \left(\mathbb{E}_\lambda[\delta_\theta] - \delta_{\theta^*}\right) \lambda_{\theta^*} \frac{g(\theta^*)}{1 - G(\theta^*)} d\theta^* + \mathbb{E}_\lambda\left[d\log(\frac{A_\theta}{\mu_\theta})\right].$$

- Real output per capita (prices):

$$d\log Q^p = \mathbb{E}_\lambda\left[d\log(\frac{A_\theta}{\mu_\theta})\right].$$