Entry versus Rents

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Aggregating Shocks

- How to aggregate shocks?
- Efficient economy, no entry (Hulten, 1978):

$$\frac{\mathrm{d}\log Y}{\mathrm{d}\log A_k} = \frac{sales_k}{GDP}.$$

- What happens with entry?
- General result: $\mathrm{d}\log Y$ depends on changes in rents *and* quasi-rents.
- Quantitatively, the entry margin is very powerful.
  (e.g. doubles losses from misallocation)

Goal

- General theory of aggregation with entry.
- General class of models: IO network, structure of entry, elasticities, Ricardian rents, monopoly rents, and increasing or decreasing internal or external returns to scale.
- Characterize comparative statics.
- First and second-best policy and associated gains.

Agenda

Agenda

Framework

Marginal-Cost Pricing Benchmark

Inefficient Model

Policy

Framework: Producers and Entrants

- Entrants $j \in E$ pay entry cost and draw technology and markup according to $\zeta(i,j)$.
- Entrants decide whether or not to pay overhead costs.
- Producer ω of type i makes

$$y_i(\omega) = f_i\left(\{x_{ij}(\omega)\}_{j\in N}, A_i\right),$$

- Industry aggregator over producers of type i:

$$Y_i = F_i\left(\int y_i(\omega) d\omega\right).$$

Framework: Household

- Representative household maximizes homothetic aggregator

$$Y = D(C_1, \ldots, C_N)$$

subject to

$$\sum_i P_i C_i \leq \text{net rents},$$

net rents are Ricardian and monopoly rents net of entry costs. Primary factor payments are pure net Ricardian rents.

- Focus on Walrasian equilibrium.

- Nests: Hopenhayn (1992), Melitz (2003), Romer (1990), Grossman and Helpman (1991), as well as Baqaee (2018)/Baqaee and Farhi (2019).

Agenda

Marginal-Cost Pricing Equilibrium

Theorem (First Welfare Theorem)

The marginal-cost pricing equilibrium is Pareto-efficient.

- Normative implication: optimal policy can be achieved by enforcing marginal-cost pricing.
- Also useful for positive questions:
  - straightforward comparative statics à la Hulten;
  - straightforward aggregation à la Domar.

Examples of Perfectly Competitive Comparative Statics

- With A_i Hicks-neutral shifter to variable production of producer i:

$$\frac{\mathrm{d}\log Y}{\mathrm{d}\log A_i} = \frac{\text{sales of type } i}{GDP}.$$

- With z_i Hicks-neutral shifter to overhead cost of producer i:

$$\frac{\mathrm{d}\log Y}{\mathrm{d}\log z_i} = \frac{\text{overhead cost of producer } i}{GDP}.$$

- With z_j Hicks-neutral shifter to entry cost of entrant j:

$$\frac{\mathrm{d}\log Y}{\mathrm{d}\log z_j} = \frac{\text{entry cost of entrant } j}{GDP}.$$

Agenda

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Inefficient Model

- We introduce markup μ_i and output tax/wedge μ_i^Y.
- Assume

$$q_i = A_i f_i \left(\{x_{ij}\}_{j \in N} \right), \quad \text{with } f_i \text{ CRS},$$
$$y_i = q_i^{1-\varepsilon_i},$$
$$Y_i = A_i (M_i y_i)^{\frac{1}{\gamma_i}}.$$

- Hopenhayn/DRS Benchmark $\gamma = 1$;
- Dixit-Stiglitz/IRS/CES Benchmark is $\gamma = 1 - \varepsilon$.

Inefficient Model Preliminaries

Define

$$\tilde{\zeta}(i,j) = \text{Prob}\,(\,\text{Entrant } j \,|\, \text{Product } i\,)\,,$$

$$\lambda_{\pi_k} = \frac{profits_k}{GDP}.$$

Lemma

In equilibrium,

$$\mathrm{d}\log M = \tilde{\zeta}'(\tilde{\zeta}\lambda_\pi\tilde{\zeta}')^{-1}\left(\tilde{\zeta}\lambda_\pi\,\mathrm{d}\log\lambda_\pi\right)$$

Δentry is projection of Δrents on entry conditions.

Domar Weights

- Using input-output table, we can define

$$\lambda_i^B = \text{Backward Domar weight},$$

measure of importance as consumer of inputs.

$$\lambda_i^F = \text{Forward Domar weight},$$

measure of importance as supplier of inputs.

- When marginal cost pricing,

$$\lambda_i^F = \lambda_i^B = sales_i / GDP.$$

Comparative Statics when Inefficient

Proposition (Productivity Shocks)

In response to a perturbation $\mathrm{d}\log A$*:*

$$\begin{aligned}
\mathrm{d}\log Y = {} & \lambda^F \,\mathrm{d}\log A \\
& - \lambda^F \cdot \left(1 - \frac{1-\varepsilon}{\gamma}\right)\left(\mathrm{d}\log \lambda_\pi - \mathrm{d}\log \hat{\lambda}_\pi\right) \\
& + \lambda^F \cdot \left(\frac{1}{\gamma} - 1\right)\mathrm{d}\log \hat{\lambda}_\pi,
\end{aligned}$$

- “Hulten” term.
- Deviation from “Hulten” term.
- Paper has formulas for $d\log \lambda_\pi$ as function of primitives.

Reallocation

- Let $\mathscr{X}(\mu, \mu^Y, A)$ be allocation of resources across all uses.
- Output in any feasible allocation is $Y(A, \mathscr{X})$, so

$$\mathrm{d}\log Y = \underbrace{\frac{\partial \log Y}{\partial \log A}\mathrm{d}\log A}_{\Delta \text{Pure Technology}} + \underbrace{\frac{\partial \log Y}{\partial \mathscr{X}}\mathrm{d}\mathscr{X}}_{\Delta \text{Reallocation}}.$$

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Pure technology is the “Hulten” term:

$$\frac{\partial \log Y}{\partial \log A}\mathrm{d}\log A = \lambda^F \cdot \mathrm{d}\log A,$$

Reallocation

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  Pure technology is the "Hulten" term:

$$\frac{\partial \log Y}{\partial \log A}\mathrm{d}\log A = \lambda^F \cdot \mathrm{d}\log A,$$

  and reallocation effects are the rest.
- In marginal-cost pricing equilibrium, reallocation effects are zero.

Hopenhayn-style: Residual (Pure Rent) Matters

For Hopenhayn-style models ($\gamma = 1$):

$$\mathrm{d}\log Y = \lambda^F \cdot d\log A - \lambda^F \cdot \varepsilon \cdot \left(d\log \lambda_\pi - d\log \hat{\lambda}_\pi \right).$$

- The second-term captures how the price of quasi-fixed-factors is changing in equilibrium.
- If the allocation of resources improves, then fixed-factors become less scarce and their price declines.

Dixit-Stiglitz-style: Projection (Quasi-rent) Matters

For Dixit-Stiglitz-style models $(1-\varepsilon_i=\gamma_i)$:

$$\mathrm{d}\log Y=\lambda^F\cdot\mathrm{d}\log A+\lambda^F\cdot\left(\frac{1}{\gamma}-1\right)\cdot d\log\hat{\lambda}_\pi.$$

- Projections of rents on entry key suff. stat.
- If entry/quasi-rents increase, then by increasing marginal returns, productivity shocks are magnified.

Agenda

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Inefficient Model

Policy

First-Best Policy

Theorem

The optimal allocation is attained when $\mu_i = \mu_i^Y = 1$ for every $i \in N$.

- Optimal policy is “network” blind.

Second-Best Policy

- Consider marginal intervention around the decentralized equilibrium (markup regulation or entry subsidy).
- Consider the social bang for marginal buck.
- Revives Hirschman's argument that policy encourage forward and backward linkages.

Second-Best Competition Policy

- For example, at CES markups:

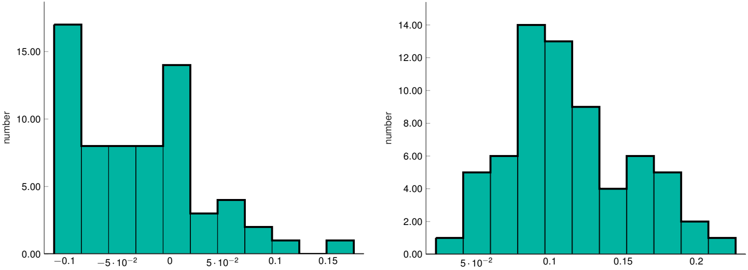
$$\frac{\mathrm{d}\log Y}{\mathrm{d}\log \mu_i} = \sum_{j \in N} \left(\frac{1}{\gamma_j} - 1 \right) \lambda_j^F \frac{\mathrm{d}\log \hat{\lambda}_j^B}{\mathrm{d}\log \mu_i}.$$

  Maximize a forward-weighted-sum of backward linkages.
  Intuitively, should try to boost sales over GDP.

- For Cobb-Douglas Input-Output Network:
  - Reduce markups of sectors that have complex supply chains.
  - Subsidize entry of sectors that have complex demand chains.

General Takeaway

- Example with CES markups and Input-Output:



(a) Entry subsidies for CES markups

(b) Markup reduction for CES markups

- Biggest entry bank-for-buck: Oil, mining, forestry.
- Smallest entry bank-for-buck: nursing, social assistance.
- Biggest markup bang-for-buck: Motor vehicles, metals, plastics.
- Smallest markup bang-for-buck: Housing, legal, oil, forestry.

Gains from Optimal Policy

Theorem

At the efficient point, to a second order, the losses are

$$d^2 \log Y = \sum_{i \in N} \frac{1}{2} \lambda_i d \log Y_i d \log \mu_i \mu_i^Y$$
$$- \sum_{j \in E} \sum_{i \in N} \frac{1}{2} \frac{\lambda_i \tilde{\zeta}_{ij}}{\gamma_i} d \log M_j^E d \log \mu_i .$$

- Distance to efficient frontier as sum of Harberger triangles.
- Paper: formulas in terms of primitives.

Examples

- One sector CES model without entry

$$d^2 \log Y = -\frac{1}{2}\theta\, Var_\lambda \left(d \log \mu^2\right).$$

- One sector CES model with undirected entry

$$d^2 \log Y = -\frac{1}{2}\theta\, E_\lambda \left(d \log \mu^2\right).$$

- Both the level and the dispersion matter.

Application: Markups in US

- Suppose firm-level markups are only distortions.
- Assign Compustat firms to industries.
- Estimate markups using production function estimation.
- Elasticities of substitution: across industries in consumption 0.9; between value-added and intermediates 0.5; across intermediates in production 0.01; between labor and capital 1; within industries 8.

Gains from Industrial/Competition Policy

IRS, $\gamma = 1 - \varepsilon = 0.75$	No Entry	Free Entry
Level only	4.6%	17%
Dispersion only	22%	23%
Benchmark	19%	32%
DRS, $\gamma = 1, 1 - \varepsilon = 0.75$		
Level only	0.8%	9.5%
Dispersion only	9.2%	9.2%
Benchmark	9.6%	19%

- Structure of entry matters.
- Entry and variable production networks matter.

Comparative Statics

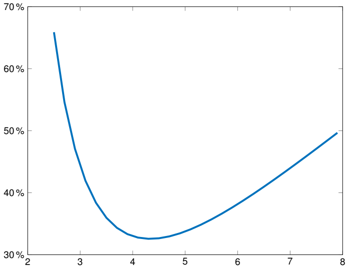


Figure: Efficiency losses for the benchmark IRS model as function of elasticity of substitution

- Unlike classical misallocation, losses are non-monotone in the elasticity of substitution.

Conclusion

- Aggregation with scale.
- Propagation of shocks with entry.
- Normative and positive implications.

Determination of Profit Shares

Proposition (Profit Shares)

Assuming $\lambda_\pi \neq 0$*, in response to a perturbation* $(\mathrm{d}\log A, \mathrm{d}\log\mu)$

$$\mathrm{d}\log\lambda_\pi = \mathrm{d}\log\lambda + \frac{(1-\varepsilon)/\mu}{1-(1-\varepsilon)/\mu}\mathrm{d}\log\mu.$$

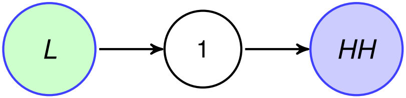
Special Case: Decreasing-returns and Entry

- Suppose $\varepsilon_i \in (0,1]$ and $\gamma_i = 1$ and ζ lower rank than N. Then

$$\mathrm{d}\log Y = \tilde{\lambda}_i \left(\mathrm{d}\log A_i - \frac{1-\varepsilon_i}{1-\frac{1-\varepsilon_i}{\mu_i}} \left(1 - \frac{1}{\mu_i}\right) \mathrm{d}\log \mu_i \right) - \sum_{k \in N} \tilde{\lambda}_k \left(\varepsilon_k \left(d\log \lambda_{\pi_k} - d\log \hat{\lambda}_{\pi_k} \right)\right).$$

- A big residual means a big rent. When production hits diminishing returns, it reduces output.

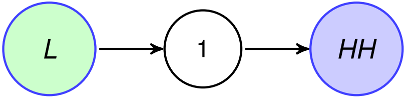
Example: One Sector Model



- Let $\zeta = 0$ (no entry)

$$\mathrm{d}\log Y = \frac{1}{\gamma}\mathrm{d}\log A + 0\,\mathrm{d}\log \mu.$$

Example: One Sector Model

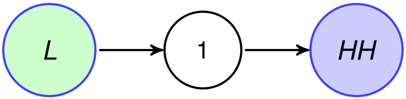


- Suppose $\zeta \neq 0$ (entry) but $\gamma = 1$ (no diminishing marginal cost):

$$\mathrm{d}\log Y = \tilde{\lambda}\, d\log A - \tilde{\lambda}\,\frac{(1-\varepsilon)}{1-(1-\varepsilon)/\mu}\,(1-1/\mu)\, d\log\mu$$

- Elasticity of substitution is irrelevant.

Example: One Sector Model

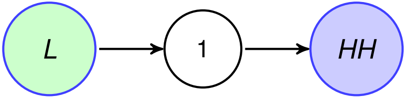


- Suppose $\zeta \neq 0$ and $\gamma \neq 1$, but entry cost paid in units of labor

$$d\log Y = \frac{d\log A}{\gamma} - \frac{1-\varepsilon}{\gamma}\frac{1-1/\mu}{1-(1-\varepsilon)/\mu}d\log\mu.$$

- Similar to the case without diminishing marginal cost.

Example: One Sector Model



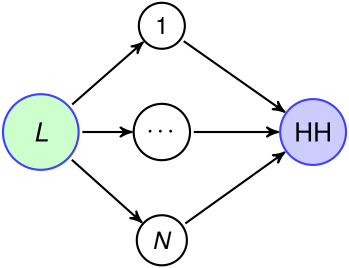
- Suppose $\zeta \neq 0$ and $\gamma \neq 1$, and allow entry cost to be paid partly in units of output:

$$\mathrm{d}\log Y = \frac{\tilde{\lambda}/\gamma}{1-\tilde{\lambda}\left(\frac{1}{\gamma}-1\right)(\lambda-1)(\theta-1)\left(1-\Omega^{E}\right)} d\log A$$

$$+\tilde{\lambda}\frac{\left(\left(\frac{1}{\gamma}-1\right)\left((\lambda-1)\left(\frac{1}{\mu-1}\right)+\frac{(1-\varepsilon)/\mu}{1-(1-\varepsilon)/\mu}\right)-\frac{(1-\varepsilon)/\mu}{1-(1-\varepsilon)/\mu}\left(\frac{\mu}{\gamma}-1\right)\right)}{1-\tilde{\lambda}\left(\frac{1}{\gamma}-1\right)(\lambda-1)(\theta-1)\left(1-\Omega^{E}\right)} d\log\mu.$$

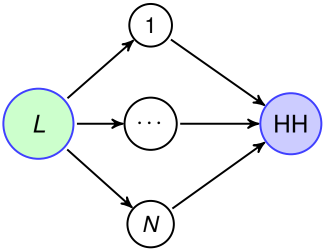
- Multiplier that depends on elasticity of substitution θ and share of output in entry cost Ω^{E}.

Example: Multisector Model



- Elasticity of substitution across sectors is θ_0.
- Consider CES specification inside sectors $1 - 1/\theta_i = 1 - \varepsilon_i = \gamma_i \in (0, 1)$.

Example: Multisector Model



- With no entry $\zeta = 0$,

$$\mathrm{d}\log Y = \lambda_k(\mathrm{d}\log A_k - \mathrm{d}\log \mu_k) - \mathrm{d}\log \lambda_L.$$

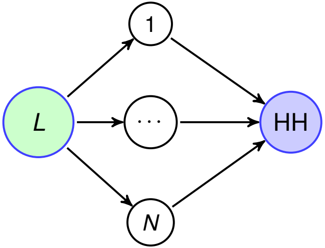
- For a productivity shock,

$$\frac{\mathrm{d}\log Y}{\mathrm{d}\log A_k} = \lambda_k \,\mathrm{d}\log A_k - (1-\theta_0)\lambda_k \left(\frac{1/\mu_k}{\sum_{j\in N} \lambda_j/\mu_j} - 1 \right),$$

- For markup shocks,

$$\frac{\mathrm{d}\log Y}{\mathrm{d}\log \mu_k} = \theta_0 \lambda_k \left(\frac{1/\mu_k}{\sum_{j\in N} \lambda_j/\mu_j} - 1 \right).$$

Example: Multisector Model



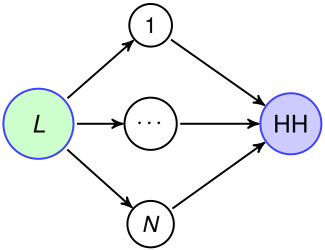
- Suppose single entrant type, then

$$d\log Y = -\lambda_k \left(d\log \mu_k - d\log A_k\right) + \sum_{j\in N} \left(\frac{\lambda_j}{\theta_j - 1}\right) d\log \hat{\lambda}_{\pi_j}.$$

or,

$$d\log Y = -\lambda_k \left(d\log \mu_k / A_k\right) + \left(\sum_{j\in N} \frac{\lambda_j}{\theta_j - 1}\right) \left(\sum_{i\in N} \frac{\lambda_{\pi_k}}{\sum_{l\in N} \lambda_{\pi_l}} \lambda_{\pi_k} \frac{d\log \mu_k}{\mu_k - 1}\right)$$
$$+ \left(\sum_{j\in N} \frac{\lambda_j}{\theta_j - 1}\right) \frac{1}{\sum_j \lambda_{\pi_j}} \left(\left[\lambda_{\pi_k} - \left(\sum_{j\neq k} \lambda_{\pi_j}\right) \frac{\lambda_k}{1-\lambda_k}\right]\right) d\log \lambda_k.$$

Example: Multisector Model



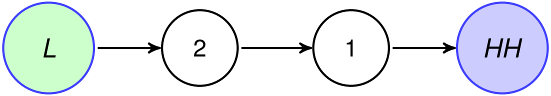
- Suppose entrant for each sector, then

$$\mathrm{d}\log Y = -\lambda_k \left(\mathrm{d}\log \mu_k - \mathrm{d}\log A_k\right) + \sum_{j \in N} \left(\frac{\lambda_j}{\theta_j - 1}\right) \mathrm{d}\log \lambda_{\pi_j}.$$

or,

$$\mathrm{d}\log Y = -\lambda_k \left(\mathrm{d}\log \mu_k - \mathrm{d}\log A_k\right) + \left(\frac{\lambda_k}{\theta_j - 1}\right) \frac{\mathrm{d}\log \mu_k}{\mu_k - 1}$$
$$+ \lambda_k \left(\frac{1}{\theta_k - 1} - \sum_{j \neq k} \left(\frac{\lambda_j / \sum_{l \neq k} \lambda_l}{\theta_j - 1}\right)\right) \mathrm{d}\log \lambda_k.$$

Example: Vertical Economy with $\gamma = 1$



- Suppose no entry $\zeta = 0$ and $\gamma = 1$, then

$$\mathrm{d}\log Y = \mathrm{d}\log A_1 + (1 - \varepsilon_1)\,\mathrm{d}\log A_2.$$

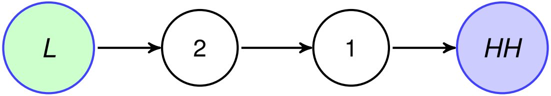
- Suppose entry only downstream (upstream perfect competition CRS), then

$$d\log Y = d\log A_1 + (1-\varepsilon_1) d\log A_2 - \frac{(1-\varepsilon_1)/\mu_1}{1-(1-\varepsilon_1)/\mu_1}(\mu_1 - 1)\, d\log \mu_1.$$

- Suppose entry upstream and downstream

$$d\log Y = d\log A_1 + (1-\varepsilon_1) d\log A_2 - \frac{(1-\varepsilon_1)/\mu_1}{1-(1-\varepsilon_1)/\mu_1}(\mu_1 - 1)\, d\log \mu_1$$
$$-(1-\varepsilon_1)\frac{(1-\varepsilon_2)/\mu_2}{1-(1-\varepsilon_2)/\mu_2}(\mu_2 - 1)\, d\log \mu_2.$$

Example: Vertical Economy with $1-\varepsilon=\gamma$



- Suppose no entry $\zeta=0$ and $\gamma=1-\varepsilon$

$$\mathrm{d}\log Y=\mathrm{d}\log A_1+\mathrm{d}\log A_2,$$

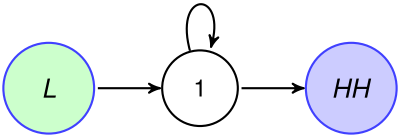
- Suppose entry only downstream (upstream competitive CRS)

$$\mathrm{d}\log Y=d\log A_1+d\log A_2+\left(\frac{1}{(\varepsilon_1-1)(\mu_1-1)}-1\right)\mathrm{d}\log\mu_1.$$

- Suppose entry upstream and downstream

$$d\log Y=d\log A_1+d\log A_2+\left(\frac{1}{(\varepsilon_2-1)(\mu_2-1)}-1\right)d\log\mu_2$$
$$+\left(\frac{1}{(\varepsilon_1-1)(\mu_1-1)}-\frac{1}{(\varepsilon_2-1)}-1\right)d\log\mu_1.$$

Example: Round-about Economy

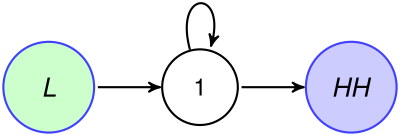


- No entry $\zeta = 0$, with constant external returns and internal returns $\gamma = 1 - \varepsilon = 1$.

$$
\begin{aligned}
d\log Y = & \tilde{\lambda}\left(d\log A - (1-\varepsilon)d\log\mu\right) \\
& - \tilde{\lambda}\varepsilon d\log\lambda - \tilde{\lambda}(1-\varepsilon)(1-\tilde{\Omega})d\log\lambda_l, \\
& = \tilde{\lambda}d\log A - \left(\lambda - \tilde{\lambda}\right)(\theta - 1)d\log A + \theta(\lambda - \tilde{\lambda})d\log\mu,
\end{aligned}
$$

- Generally, θ is an important parameter.
- Intuitively, markups affect output by distorting allocation of resources between intermediates and consumption.

Example: Round-about Economy



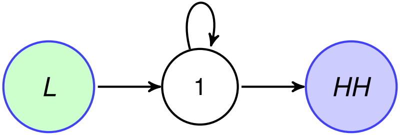
- With entry $\zeta \neq 0$, with constant external returns $\gamma = 1$ and arbitrary internal marginal returns.

$$d\log Y = \tilde{\lambda}\left(d\log A - \frac{(1-\varepsilon)/\mu}{1-(1-\varepsilon)/\mu}(\mu-1)d\log\mu\right).$$

- Elasticity of substitution θ disappears.

- Intuitively, markups no longer affect allocation between materials and final uses, instead, they distort scale of each firm.

Example: Round-about Economy

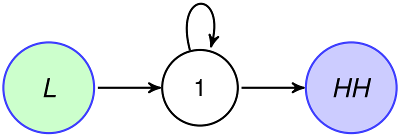


- Without entry $\zeta = 0$, with increasing external returns and decreasing internal marginal returns $\gamma = 1 - \varepsilon$.

$$\mathrm{d}\log Y = -\theta(\tilde{\lambda}_1 - 1)(1 - \mu^{-1})\lambda_1 d\log\mu$$
$$+ \left(\tilde{\lambda}_1 - (\tilde{\lambda}_1 - 1)\lambda_1(\mu^{-1} - 1)(\theta - 1)\right) \frac{d\log A}{1 - \varepsilon}.$$

- Similar to case without increasing external economies.

Example: Round-about Economy



- With entry $\zeta \neq 0$, with increasing external returns and decreasing internal marginal returns $\gamma = 1 - \varepsilon$.

$$
\begin{aligned}
d\log Y = {} & \tilde{\lambda}\left(\frac{d\log A}{1-\varepsilon} - d\log\mu\right) + \tilde{\lambda}\frac{\varepsilon}{1-\varepsilon}\frac{(1-\varepsilon)/\mu}{1-(1-\varepsilon)/\mu}d\log\mu \\
& + \tilde{\lambda}\frac{\varepsilon}{1-\varepsilon}\Bigg(\frac{\lambda\tilde{\Omega}(1-\theta)}{1-\lambda\tilde{\Omega}(\theta-1)\frac{\varepsilon}{1-\varepsilon}}\Bigg(\bigg(\frac{\mu-1}{\varepsilon+\mu-1}d\log\mu - \frac{1}{1-\varepsilon}d\log A\bigg)
\end{aligned}
$$

- Elasticity of substitution is very important — if $\theta > 1$, then a positive shock increases intermediate input usage, which increases entry, which reduces the price of output, which induces additional entry, positive feedback loop.

Example: Projections vs. Residuals with Directed Entry

- Multiple sectors, heterog. producers, variable production and entry in labor, directed entry.
- Hopenhayn-style model:

$$d \log Y = \mathbb{E}_\lambda \left(d \log A \right).$$

- Dixit-Stiglitz-style-model:

$$\mathrm{d} \log Y = \mathbb{E}_\lambda \left(\mathrm{d} \log A \right) + \mathbb{E}_\lambda \left(\left(\frac{1}{\gamma_i} - 1 \right) \mathrm{d} \log \lambda_i \right).$$

Forward Propagation

To solve model fully, we need forward and backward propagation:

Proposition

Changes in prices on response to shocks $d\log A$*:*

$$
\begin{aligned}
d\log P = & \Psi^F d\log A + \Psi^F \left(1 - \frac{1-\varepsilon}{\gamma}\right)\left(d\log \lambda_\pi - d\log \hat{\lambda}_\pi\right) \\
& + \lambda^F \left(1 - \frac{1}{\gamma}\right) d\log \hat{\lambda}_\pi .
\end{aligned}
$$

- Forward-linkages equations.

Backward Propagation

Assume nested-CES with θ_m as elasticity of substitution for nest m (easy to generalize).

Proposition

Changes in shares in response to shocks $\mathrm{d}\log A$:

$$\mathrm{d}\log\lambda_i = -\sum_{m\in N\cup E}\lambda_m\mu_m^{-1}(\theta_m - 1)Cov_m\left(\mathrm{d}\log P, \frac{\Psi^B_{(i)}}{\lambda_i}\right).$$

- Backward-linkages equations.
- Forward and backward propagation together pin down everything.

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