

# A Ramsey Theory of Financial Distortions

**Wei Cui** (UCL)    **Marco Bassetto** (Minneapolis Fed)<sup>1</sup>

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<sup>1</sup>This paper reflects the views of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

# Intertemporal Distortions

- **Capital taxes** are used throughout the world
- $r < g$  frequently for government bonds with shortage of these safe/liquid assets
- Low rates of return **in years following fiscal shocks**
  - ▶ Reinhart & Sbrancia (2015) (financial repression)
  - ▶ Berndt, Lustig, & Yeltekin (2012)

# Are capital taxes and low interest rates on debt optimal?

- Not in a Ramsey problem with a neoclassical growth economy.
  - ▶ with distortionary taxes on labor and capital.
- Not optimal to distort the intertemporal margin
  - ▶ Long run: neither capital tax nor low rates on government debt (Judd, 1985, Chamley, 1986). Rely on labor alone in the long run.
  - ▶ Fiscal shocks: absorbed by quick devaluation and **immediate** low return (Lucas - Stokey, 1983, and Siu, 2004,...);
  - ▶ or, by future taxes, smoothed by debt (Barro, 1979, and AMSS, 2002).
- Our view: **optimal policy with imperfect substitutes** between public and private liquidity
  - ▶ *Idiosyncratic investment risks + liquidity frictions in financing investment*
  - ▶ partially liquid private claims;
  - ▶ fully liquid government debt (but costly in terms of distortionary taxes)

# Optimal financial distortions: Results

- **Two-period** model - if public liquidity is insufficient, we have a trade-off
  - ▶ want to subsidize investment which is underprovided
  - ▶ but credit frictions make the capital supply inelastic; optimal to **tax** it when gov budget is tight
- **Infinite-horizon** model - provide as much public liquidity as possible:
  - ▶ either attain unconstrained solution (sufficient self-financing), standard Ramsey, zero capital taxes
  - ▶ or, get to the “top of the Laffer curve:”
    - ★ shortage of safe/liquid assets remains
    - ★ low interest rates
    - ★ positive capital taxes
- Our model can be related to broader financial distortion policies.
  - ▶ capital controls / banking regulation / collateralized borrowing requirements...

# Part 1: The two-period model

- Periods 1 and 2
- A continuum of families, with preferences:

$$\sum_{t=1}^2 \beta^{t-1} [u(c_t) - v((1-\chi)\ell_t)] \quad (1)$$

- ▶ only in period 1: workers (population  $1-\chi$ ) and entrepreneurs (population  $\chi$ )
  - ▶ reunite at the end of period 1 and period 2
- Firms
- Government (with legacy debt  $B_0$ )

# An entrepreneur of a family in period 1

- An entrepreneur with (exogenous) government debt  $b_0^e = B_0^e/\chi$  finances  $k_1^e$

$$k_1^e \leq b_0^e + q_1 s_1^e \quad (2)$$

- ...financial claims  $s_1^e$  sold at price  $q_1$ ;  $s_1^e$  less than  $\phi_1$  units of investment  $k_1^e$ :

$$s_1^e \leq \phi_1 k_1^e \quad (3)$$

# A worker of a family in period 1

- Each worker begins with government bonds  $b_0^w = B_0^w / (1 - \chi)$ 
  - ▶ works and earns  $w_1 \ell_1$ ;
  - ▶ buys government bonds and financial claims.
- At the end of period 1:
  - ▶ workers rejoin entrepreneurs and pool assets together

$$c_1 + B_1 + \chi(k_1^e - q_1 s_1^e) + (1 - \chi)q_1 s_1^w = (1 - \tau_1^l)w_1 \ell_1 (1 - \chi) + B_0 \quad (4)$$

- ▶ government bonds  $B_1 = (1 - \chi)b_1^w$ ; capital  $K_1 = \chi(k_1^e - s_1^e) + (1 - \chi)s_1^w$

## The family in period 2

- Capital return  $r_2$  and wage rate  $w_2$
- The budget constraint:

$$c_2 = (1 - \tau_2^k)r_2K_1 + (1 - \tau_2^l)w_2(1 - \chi)l_2 + R_1B_1 \quad (5)$$

- The HH maximizes utility in (1)
  - budget constraints in period 1 and period 2
  - financing constraints in period 1



# Firms

- Goods produced by competitive firms
- Period 1:  $w_1 = A$

$$Y_1 = AL_1$$

$L_1 = (1 - \chi)\ell_1$ : hours from workers

- Period 2:  $w_2 = F_L(K_1, L_2)$ ,  $r_2 = F_K(K_1, L_2)$ , and

$$Y_2 = F(K_1, L_2)$$

$L_2 = (1 - \chi)\ell_2$ : hours from workers

# Government

- Benevolent (same objective as households)
- Exogenous spending  $G_1$  and  $G_2$ 
  - ▶ Taxes labor at rate  $\tau_1^l$  and  $\tau_2^l$  and capital at rate  $\tau_2^k$
  - ▶ Issues bonds  $B_1$  in period 1 (interest rate rate  $R_1$ )

- Period 1:

$$G_1 + R_0 B_0 = B_1 + \tau_1^l w_1 L_1$$

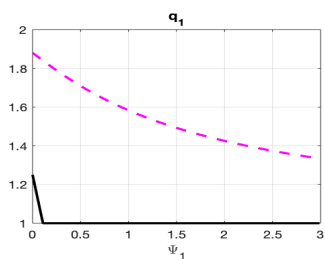
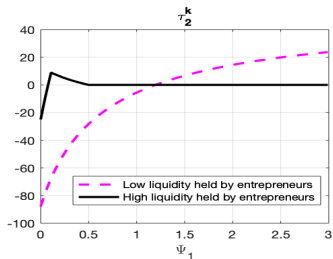
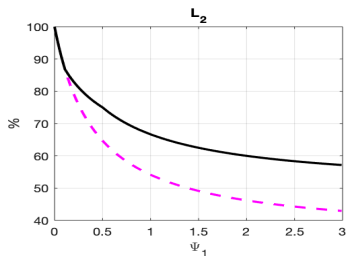
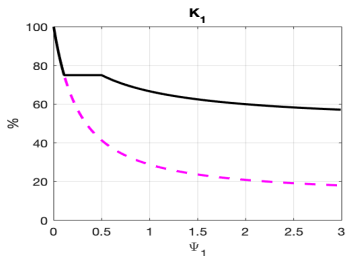
- Period 2:

$$G_2 + R_1 B_1 = \tau_2^k r_2 K_1 + \tau_2^l w_2 L_2$$

- Solution strategy:
  - ▶ Compute optimal competitive-equilibrium allocation (primal approach);
  - ▶ back out taxes and prices.

# To be or not to be (financing constrained)?

$\Psi_1$ : shadow cost of government revenue; quasi-linear utility example



## Part 2: The $\infty$ -horizon model

- Idiosyncratic investment risks **repeated many times**
- Financing constraints tied to endogenous asset liquidity and price
  - ▶  $(\phi_t, q_t)$  pairs determined through directed search
  - ▶ avoid kinks but does not matter for the fundamentals
  - ▶ other good co-movement properties between  $\phi_t$  and  $q_t$ ; see Cui & Radde (2016, 2019) and Cui (2016)
- The planner chooses  $\{C_t, L_t, K_t, B_t, \phi_t\}$  for  $t \geq 0$  given  $K_{-1}$  and  $B_{-1}$ 
  - ▶ deterministic model: as if with (aggregate) state-contingent bonds
- New results (compared to the 2-period model)
  - ▶ low rates and capital tax
  - ▶ may not be able to provide enough liquidity

# The “best” competitive equilibrium

- Private FOC for bonds:

$$1 = \frac{\beta u'(C_{t+1})}{u'(C_t)} R_{t+1} (1 + \chi \rho_{t+1})$$

- ▶  $u'(C_t)\chi\rho_t > 0$  measures tightness of financing constraint
  - ▶ reflects the **liquidity service** provided by government debt.
- Note: asset price  $q_t$  and  $\rho_t$  are functions of  $\phi_t$
- Public FOC for bonds:

$$\Psi_{t+1} = (1 + \chi\rho_{t+1})\Psi_t + \chi\gamma_{t+1}$$

$\Psi_t$ : shadow cost of government revenue (measures gov't budget tightness)

$\gamma_t$ : shadow cost of financing constraints

# Long-run result

## Proposition

If the economy converges to a steady state, there are two possibilities.

(1). Slack financing constraints in the limit, *capital tax*  $\tau^k = 0$ , and the interest rate  $R = 1/\beta$ . Tightness  $\Psi_t$  and converges to finite constant.

(2). Financing constraints bind in the limit, budget tightness  $\Psi_t \rightarrow \infty$ ,  $R < 1/\beta$ , and  $\tau^k \neq 0$ . Sufficient conditions for  $\tau^k > 0$ :

(1).  $u(c) = c$ ; (2).  $\beta$  close to 1



- *The key FOC for bonds shows  $\Psi_t$  is non-decreasing:*

$$\Psi_{t+1} = (1 + \chi\rho_{t+1})\Psi_t + \chi\gamma_{t+1}$$

- The growth rate of  $\Psi_t$  reflects supply of government debt. Make budget tighter and tighter; when to stop?
  - ▶ liquidity is satiated
  - ▶ top of the Laffer curve with low rate and capital tax

# Conclusion

- Financial distortions can be optimal
  - ▶ A new trade-off: the tightness of government budget and the tightness of financing constraint
- If provision of public liquidity cannot undo financing constraint, then it provides a reason to tax capital and run low interest rate:
  - ▶ a wedge between returns
  - ▶ distorting the inter-temporal margin (different from secular stagnation)
  - ▶ endogenous capital price important: bond in the utility / capital adjustment cost is not enough
- ... more to come with the  $\infty$ -horizon model
  - ▶ response to adverse MIT shocks: more debt issuance in the short run
  - ▶ aggregate risks can generate a finite  $\Psi$  and low rates: government debt not aggregate-state contingent as in AMSS (2002)

Thank You!



# Literature

- *Liquidity frictions*: Woodford (1990); Holmstrom - Tirole (1998); Kiyotaki - Moore (2012); Shi (2015); Cui - Radde (2016, 2019)...
- *Ramsey plans* under various asset market structures: Lucas - Stokey (1983); Chari - Kehoe (1999), Aiyagari et.al. (2002), Farhi (2010), Chien - Wen (2019)...
- *Optimal public supply of liquidity*: Angeletos et.al. (2013), Azzimonti - Yared (2017, 2019)...
- Our paper: why government will **distort the inter-temporal margin** with capital taxes/subsidies
  - ▶ government debt is chosen not to fully insure idiosyncratic risks

## Eqm conditions

- Labor supply: for  $t = 1, 2$

$$(1 - \tau_t^\ell) w_t u'(C_t) = v'(L_t) \quad (6)$$

- Demand for bonds

$$\frac{1}{R_1} = \frac{\beta u'(C_2)}{u'(C_1)} \quad (7)$$

- Demand for claims and supply for claims:

$$q_1 = \frac{\beta u'(C_2)}{u'(C_1)} (1 - \tau_2^k) r_2 \quad (8)$$

$$q_1 = \max \left\{ 1, \frac{K_1 - B_0^e}{\phi_1 K_1} \right\} \quad (9)$$

using  $(1 - \phi_1 q_1) K_1 \leq B_0^e$

- Feasibility

$$AL_1 = C_1 + K_1 + G_1 \quad (10)$$

$$F(K_1, L_2) = C_2 + G_2 \quad (11)$$

# The primal approach

- The planner maximizes welfare (1), subj. to
  - ▶ feasibility constraints (10) and (11);
  - ▶ implementability constraint (financing-constrained adjusted)
- Maximum level of investment when  $q_1 = 1$  (slack financing constraint)

$$K^* := \frac{B_0^e}{1 - \phi_1}$$

- The implementability constraint:

$$\begin{aligned} & \sum_{t=1}^2 \beta^{t-1} [u'(C_t)C_t - v'(L_t)L_t] - u'(C_1)B_0 \\ &= \begin{cases} 0 & \text{if } K_1 \leq K^* \\ \left(\frac{1}{\phi_1} - 1\right) u'(C_1)(K_1 - K^*) & \text{if } K_1 > K^* \end{cases} \end{aligned}$$

- What happen when the asset market is fully liquid ( $\phi_1 = 1$ )?

## An analytical case

- Production:  $F(K_1, L_2) = AK_1^\alpha L_2^{1-\alpha} + (1 - \delta)K_1$ . Preferences (no interest rate effect)

$$u(c, \ell) = c - \frac{\mu \ell^{1+\nu}}{1+\nu}$$

- $\Psi_1$ : multiplier w.r.t. the implementability constraint; measures legacy debt  $B_0$  (or, PV of gov spending)
- The planner's FOC:

$$\beta \left[ A\alpha \left( \frac{K_1}{L_2} \right)^{\alpha-1} + 1 - \delta \right] = 1 + \begin{cases} 0 & \text{if } K_1 \leq K^* \\ \frac{\Psi_1(\phi_1^{-1}-1)}{1+\Psi_1} & \text{if } K_1 > K^* \end{cases}$$

- Recall the HH FOC:

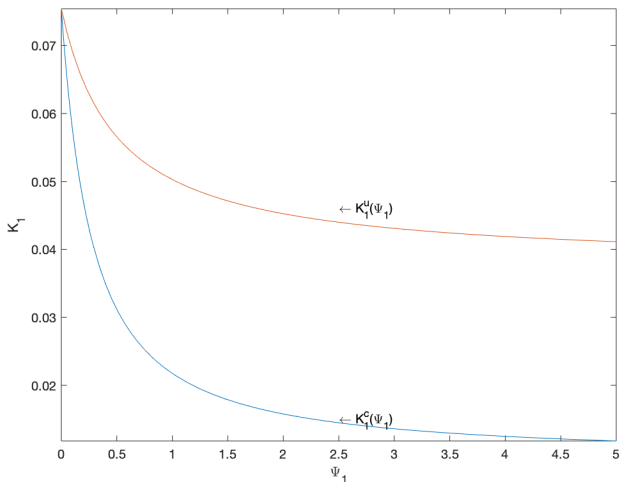
$$\beta(1 - \tau_k^2) \left[ A\alpha \left( \frac{K_1}{L_2} \right)^{\alpha-1} + 1 - \delta \right] = q_1$$

# Government financing and individual financing constraint

- If financing constraint is **slack**,  $\tau_k^2 = 0$ , independent of  $\Psi_1$  (e.g.,  $\phi_1 = 1$ )
  - ▶ private cost: 1; private rewards  $\beta r_2$
  - ▶ social cost:  $1 + \Psi_1$ ; the social reward:  $\beta r_2(1 + \Psi_1)$
  - ▶ reward / cost are the same (**only related to technologies**)
- If financing constraint is **binding**,  $\tau_k^2 \neq 0$ , interacting with  $\Psi_1$ 
  - ▶ the private return needs to be adjusted by financing constraint
- Subsidy initially when government financing is flexible, but tax later (the quasi-rent)
  - ▶ a higher  $q_1$  tightens the implementability constraint, raising  $\Psi_1$
  - ▶ thus, using more distortionary taxes, affecting investment and  $q_1$  again

# To be (financing constrained) or not to be

$$\beta = 0.96, \phi = 0.5, \delta = 0.95, \text{ and } A = 1$$



# Households

- Preferences:

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) - v((1-\chi)\ell_t)] \quad (12)$$

- $\forall t$ , liquidity held by entrepreneurs  $b_t^e = \chi B_t$
- The financing constraint

$$(1 - \phi_t q_t) k_t^e \leq R_t b_{t-1} + \phi_t q_t (1 - \delta) k_{t-1}$$

- The budget constraint

$$\begin{aligned} c_t + b_t + q_t^S k_t &= (1 - \tau_t^\ell) w_t (1 - \chi) \ell_t + R_t b_{t-1} + (1 - \tau_t^k) r_t k_{t-1} \\ &\quad + \left[ q_t^S - \chi \phi_t (q_t^S - q_t) \right] (1 - \delta) k_{t-1} \\ &\quad + \left[ q_t^S - 1 - \phi_t (q_t^S - q_t) \right] \chi k_t^e \end{aligned}$$

- $q_t^S \geq q_t$  because of financial intermediation

## Goods and Financial Firms (optional)

- $w_t$  and  $r_t$  are marginal products of labor and capital
- Competitive financial intermediaries: **cost is  $\eta(\phi_t)$**  per unit of capital intermediated

$$q_t^S - q_t = \eta(\phi_t) \quad (13)$$

- ▶ Search-and-matching to link  $\phi_t$  and  $q_t$
- ▶  $\eta(\phi_t)$  is increasing and convex w.r.t.  $\phi_t$
- Think about paying cost for each asset orders processed
  - ▶  $\phi_t$  is also the probability to fill sell orders



## Directed search / competitive search (optional)

- An entrepreneurs brings back:  $k_t^e - s_t^e = (1 - \phi_t)k_t^e$
- Therefore, the financing constraint (2) becomes

$$\underbrace{\frac{1 - \phi_t q_t}{1 - \phi_t}}_{\text{replacement cost}} (k_t^e - s_t^e) \leq B_0^e / \chi$$

- Replacement cost is similar to down-payment
- Financial intermediaries open sub-markets  $(\phi_t, q_t)$ ; search is directed with price posting

$$\min_{(\phi_t, q_t)} \frac{1 - \phi_t q_t}{1 - \phi_t}$$

s.t. the zero-profit (13).

- The solution is

$$q_t = 1 + (1 - \phi_t)\phi_t\eta'(\phi_t)$$

# Permanent fiscal expansion

$\beta = 0.96$ ,  $\delta = 0.1$ ,  $\alpha = 1/3$ ,  $\delta = 0.1$ ,  $\nu = 1$ ,  $\eta(\phi) = \eta_0\phi^2$ ,  $\mu = 1$   
 $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ ,  $\sigma = 0.1$ ;  $G/Y$  is 36.7%

**Table:** Steady state of the Ramsey allocation for different government expenditures

	$G = G^*$	$G = 1.02G^*$	$G = 1.04G^*$	$G = 1.06G^*$
Capital: $K$	100%	103.33%	106.78%	110.49 %
Capital tax: $\tau^k$	11.88%	11.40%	10.49%	9.15%
Labor tax: $\tau^\ell$	53.85%	53.84%	53.83%	53.77%
Interest rate:	4.17%	4.10%	3.90%	3.53%
Debt-to-output: $B/Y$	81.93%	66.20%	49.01%	29.49%
Asset Liquidity $\phi$	0	0.0660	0.1337	0.2064

# Permanent worsening financial conditions

Table: The long-run economies with different financial intermediation

	$\eta_0 = 0.2$	$\eta_0 = 0.4$	$\eta_0 = 0.8$	$\eta_0 = 1$
Capital: $K$	100%	99.94%	99.74%	99.50%
Capital tax: $\tau^k$	11.21%	10.81%	9.58%	8.38%
Labor tax: $\tau^\ell$	53.85%	53.84%	53.82%	53.79%
Interest rate:	4.07%	3.94%	3.56%	3.18%
Debt-to-output: $B/Y$	62.20%	60.99%	57.12%	53.10%
Asset Liquidity $\phi$	0.0821	0.0866	0.1006	0.1149

# Inter-temporal substitution (optional)

Table: The long-run economies with different intertemporal substitution

	$\sigma = 0$	$\sigma = 0.2$	$\sigma = 0.8$	$\sigma = 1.0$
Capital: $K$	100%	98.81%	85.46%	76.64%
Capital tax: $\tau^k$	11.48%	12.35%	17.53%	21.37%
Labor tax: $\tau^\ell$	50.00%	57.61%	77.75%	83.39%
Interest rate:	4.17%	4.17%	4.17%	4.17%
Debt-to-output: $B/Y$	84.32%	79.18%	58.73%	51.58%
G-to-output: $G/Y$	33.10%	39.99%	61.10%	68.42%
Asset Liquidity $\phi$	0	0	0	0