

Demographics, Wealth, and Global Imbalances in the Twenty-First Century

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Abstract

We use a shift-share approach to quantify the general equilibrium effects of population aging on wealth accumulation, real interest rates, and capital flows. Combining population projections with household survey data from the US and 24 other countries, we project the evolution of wealth-to-GDP ratios by changing the age distribution, holding life-cycle asset and income profiles constant. We find that this compositional effect of aging is large and heterogeneous across countries, ranging from 82 percentage points in Japan to 310 percentage points in India over the rest of the twenty-first century. In a general equilibrium overlapping generations model, our measure of composition provides a very good approximation to the evolution of the wealth-to-GDP ratio due to demographic change when interest rates remain constant. In an integrated world economy, aging generates large global imbalances in the twenty-first century, pushing net foreign asset positions to levels several times larger than those observed until today.

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1 Introduction

Three broad trends have dominated international macroeconomics in recent decades: rising wealth-to-output ratios around the world (e.g. [Piketty and Zucman 2014](#)), declining real rates of return on safe assets and also on total wealth¹ (e.g. [Laubach and Williams 2003](#), [King and Low 2014](#), [Jordà et al. 2019](#)), and rising "global imbalances", as measured by diverging international asset positions (e.g. [Caballero, Farhi and Gourinchas 2008](#), [Mendoza, Quadrini and Ríos-Rull 2009](#)). Figure 1 illustrates these well-known facts.

This paper is concerned with the causal effect of demographic change on these three trends. Our central objective is to forecast the likely evolution of wealth-output ratios, real returns on wealth, and global imbalances in the twenty-first century, given existing population projections such as those in panel A of figure 1. We also revisit the contribution of demographics to these trends in past decades. We answer both questions using a new approach, in the spirit of sufficient statistics, for quantifying the effect of demographic change.

We begin our analysis by studying the wealth-to-output ratio (W/Y). Based on balanced-growth considerations, we argue that a useful starting point is the *compositional effect*

$$\Delta_t^{comp} \equiv \frac{\sum_j \pi_{jt} a_{j0}}{\sum_j \pi_{jt} h_{j0}} - \frac{W_0}{Y_0}, \quad (1)$$

where π_{jt} is the fraction of individuals aged j at time t , and a_{j0} and h_{j0} are the initial age profiles of wealth and labor income, normalized so that $\Delta_0^{comp} = 0$. With aging, the compositional effect leads W/Y to increase over time, since older individuals have more assets, pushing up W , and work less, pushing down Y .

We combine population projections with household surveys from the United States and twenty-four other countries to calculate the compositional effect Δ^{comp} . Using 2016 as a base year, we find that Δ^{comp} is positive, very large, and heterogeneous across countries. By the end of the twenty-first century, Δ^{comp} ranges from 82 percentage points in Japan to 310 percentage points in India. These results reflect large differences in wealth and labor supply by age. For example, in the US, 70-year-olds are on average about four times wealthier than 40-year-olds, and earn about four times less in labor income. These large differences by age interact with large predicted changes in

¹Appendix A describes the construction of our total return series, depicted in figure 1, panel C. This series declines more since 1980 than the return in [Gomme, Ravikumar and Rupert \(2011\)](#) partly because it includes the returns on safe assets, but primarily because it is calculated as the ratio of income to wealth rather than the ratio of income to measured fixed assets. The very low safe returns in the 1950s are consistent with [Jordà, Knoll, Kuvshinov, Schularick and Taylor \(2019\)](#)

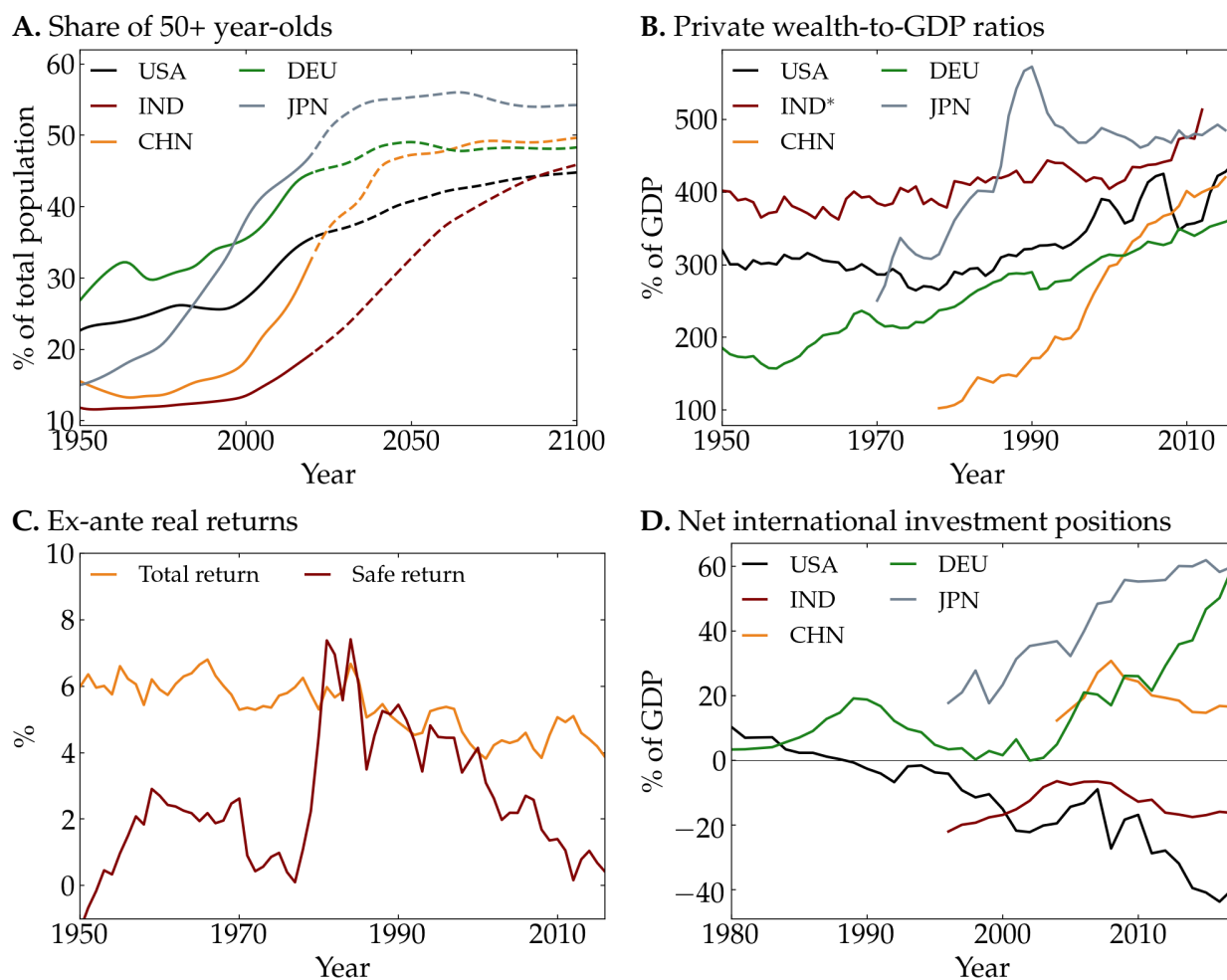


Figure 1: Demographics, wealth, interest rates and global imbalances

Notes: Panel A presents the share of 50+ year-olds from 1950 to 2100 as predicted by the 2019 UN World Population Prospects. Panel B present the evolution of wealth-to-GDP ratios until 2010 from the World Inequality Database (WID). The red line for India shows the national wealth-to-GDP ratio since the WID does not provide data on private wealth. Panel C presents a measure of the US total return on wealth (orange line) and of the US safe rate of return (red line). Details on the construction of the series are in appendix A. Panel D presents the Net International Investment Position taken from the IMF and normalized by GDP.

age structure to deliver large compositional effects on W/Y . However, the magnitudes of these compositional effects are heterogeneous across countries, since countries are at different stages of the demographic transition—as apparent from Figure 1, aging is mostly over in Japan, and only beginning in India. The largest compositional effects tend to be in countries, such as China and India, whose transitions still have a long way to go.

Our empirical results suggest that demographic change may be an important driver of the wealth-to-output ratio. From 1950 to 2016, for example, the compositional effect in the US has the same size as the rise in W/Y . This importance of demographics echoes [Piketty and Zucman](#)

(2014), who use a simple model of aggregate savings to argue that a decline in the population growth rate mechanically increases W/Y . Our analysis, in contrast, is explicit about population structure. We still find that a decline in the population growth rate increases W/Y , but that the effect is mediated through a changing age distribution and its interaction with wealth and labor income profiles, which we quantify in the data.

The full effect of demographics on W/Y includes both the compositional effect and "behavioral" effects from possible changes in wealth and labor supply profiles. Taking a stance on the behavioral effects requires a structural model. We set up a multi-country, heterogeneous agent general equilibrium model with overlapping generations, idiosyncratic income risk, intergenerational transmission of skills and bequests, and a social security system.

We first show that under certain conditions, there are no behavioral effects. In this case, Δ^{comp} is a sufficient statistic for forecasting W/Y : given a path for demographic change and the observed age profiles a_{j0} and h_{j0} , it does not matter whether these age profiles were generated by life-cycle motives, precautionary savings motives, or bequest motives, nor does it matter whether aggregate growth loads on time or cohort effects in agents' income. This is important, since there is a long standing debate in the literature regarding the sources of aggregate savings (e.g. [Kotlikoff and Summers 1981](#)), and we do not want our findings to be contingent on any specific assumption we make about, for example, the role of bequests in aggregate wealth accumulation.

However, the conditions for this sufficient statistic result are fairly stringent: they include a constant real interest rate, constant productivity growth, constant mortality rates by age, and a fiscal adjustment to demographics that loads entirely on reductions in government spending. We therefore use a calibrated version of our model to quantify the likely deviations from this result—that is, to gauge how large the behavioral effects plausibly are.

We first consider forces unrelated to interest rates: those that would arise in a small open economy with exogenous interest rates. Here, we find that behavioral effects are generally dwarfed by compositional effects. Different behavioral effects also go in different directions. Reductions in mortality and social security benefits, and increases in the ratio of bequest givers to bequest receivers, increase wealth accumulation. By contrast, increases in social security taxes reduce wealth accumulation. Across a range of calibrations, these effects tend to balance each other out, so that in the small open economy, the compositional effect ends up being a very good approximation for the full effect of demographic change on W/Y .

Next, we consider an integrated world economy, where interest rates are endogenous. The

interest rate response to demographic change is closely connected to the compositional effect, Δ^{comp} , and the behavioral effect that we quantified at constant interest rates, $\Delta^{beh|r}$. Indeed, the long-run change in the world real interest rate Δr is approximately

$$\Delta r \approx -\frac{\bar{\Delta}^{comp} + \bar{\Delta}^{beh|r}}{\bar{\epsilon}^{d,r} + \bar{\epsilon}^{s,r}}. \quad (2)$$

The numerator gives the output-weighted averages of Δ^{comp} and $\Delta^{beh|r}$ across countries. $\bar{\epsilon}^{d,r} + \bar{\epsilon}^{s,r}$ denotes the sum of the long-run worldwide sensitivities of wealth accumulation and asset supply to interest rate changes.

Since Δ_c^{comp} is large and positive in each country and $\Delta_c^{beh|r}$ tends to be small, and since $\bar{\epsilon}^{d,r} + \bar{\epsilon}^{s,r}$ is positive, equation (2) implies that demographic change always causes long-run interest rates to fall. However, as we discuss, there is considerable uncertainty on the level of the interest rate sensitivities $\bar{\epsilon}^{d,r}$ and $\bar{\epsilon}^{s,r}$. Thus, predictions for interest rates depend on deep parameters, such as the elasticity of intertemporal substitution, or the elasticity of substitution between capital and labor, that are difficult to pin down. Our predictions for the effect of demographics on interest rates ranges from a decline of 36 basis points to a decline of 100 basis points in 2100, for values of the elasticities that are plausible in light of the literature. This uncertainty around sensitivities also affects projections of W/Y . General equilibrium forces attenuate the rise in W/Y , but the exact quantity varies across calibrations.

Finally, we turn to global imbalances. There, in contrast to our interest rate and wealth predictions, our results are much more robust to values of interest rate sensitivities. Indeed, the change in the net foreign asset position NFA_c of country c relative to its GDP is approximately

$$\Delta \left(\frac{NFA_c}{Y_c} \right) \approx (\Delta_c^{comp} + \Delta_c^{beh|r}) - \left(\bar{\Delta}^{comp} + \bar{\Delta}^{beh|r} \right) + \left[\epsilon_c^{d,r} + \epsilon_c^{s,r} - \left(\bar{\epsilon}^{d,r} + \bar{\epsilon}^{s,r} \right) \right] \Delta r. \quad (3)$$

Equation (3) does not depend on the level of sensitivities, but on the difference of sensitivities across countries. While we allow for a number of country-specific parameters in our calibration, the differences in sensitivities are small across all our calibrations, and not correlated with the compositional effects. Combined with our finding of small behavioral effects at constant interest rates $\bar{\Delta}^{beh|r}$, this implies that a purely empirical object, the demeaned compositional effect $\Delta_c^{comp} - \bar{\Delta}^{comp}$, very accurately forecasts the general equilibrium effect of population aging on the net foreign asset positions of country c across a wide range of model specifications.

We validate the predictive power of the demeaned compositional effect using historical data on net foreign asset positions. We find that, over a 40 year window, our calculated compositional

effect is positively correlated with changes in the net foreign asset position. The predictiveness of demographics may be surprising, given that there are several well documented non-demographic forces, such as increasing financial development, that have clearly played a role in driving net foreign asset positions over this time period (Caballero et al. 2008, Mendoza et al. 2009). But the overall historical patterns are also consistent with an explanation of global imbalances driven by the timing of demographic change across countries, as also argued by Domeij and Flodén (2006), Krueger and Ludwig (2007), Backus, Cooley and Henriksen (2014), and Bárány, Coeurdacier and Guibaud (2019). For example, Japan has aged the most rapidly and has accumulated the largest NFA relative to its GDP.

Taken together, the conclusion from our general equilibrium forecasting exercise is that, over the course of the twenty-first century, demographics will (a) continue to increase wealth-to-GDP ratios by significant amounts, though the magnitude of this effect will be dampened by falling interest rates, (b) continue to exert downward pressure on interest rates, though the magnitudes could plausibly be between 40 basis points and 120 basis points, and (c) push global imbalances to very high levels by the end of the twenty-first century, driven by the future aging of China and India. In short, our results suggest that Bernanke (2005)'s "global savings glut" has just begun.

Our measure of compositional effects (1) is related to previous attempts at using measures of demographic composition to predict the effects of populating aging on aggregate wealth accumulation and interest rates. An early literature focused on the predictions of life-cycle hypothesis for the effect of changes in the population growth rate on the aggregate savings rates at constant interest rates (e.g. Modigliani 1986, Deaton and Paxson 1995). Closer to our shift-share exercise, one literature, following Mankiw and Weil (1989) and Poterba (2001), computes versions of the numerator in (1).² A separate literature, following Cutler, Poterba, Sheiner, Summers and Akerlof (1990) and the so-called demographic dividend literature (Bloom, Canning and Sevilla 2003), computes versions of the denominator in (1). We take the ratio between these and show that it is a key input into a fully specified general equilibrium model. We then compute updated versions of this measure for twenty-five countries, showing that it increases in all countries at a heterogeneous pace. Some of our findings echo those of this earlier literature. In particular, Poterba (2001) used his calculation of our numerator to argue that the "asset market meltdown hypothesis" that was

²There is also a tradition of computing shift-shares on savings rates (Summers and Carroll 1987, Auerbach and Kotlikoff 1990, Bosworth, Burtless and Sabelhaus 1991). This calculation is subject to substantial measurement error and may not give the correct sign of the effect on rates of return because it ignores the effect of the population growth rate on W/Y (see section 6.1).

popular at the time (the idea that the retirement of baby boomers would push up no interest rates) was unlikely to be right, since his measured showed no change in projected asset demand between 2020 and 2050. We echo this finding, but our updated calculations for the United States show that in fact projected asset demand will rise dramatically until the end of the 21st century. The declining denominator is also important and contributes a third of the overall increase in normalized asset demand.

The modern analysis of the causal effect of demographics relies on fully specified structural models. This tradition, which originated in [Auerbach and Kotlikoff \(1987\)](#), has tackled effects of demographics on aggregate wealth accumulation and pension reforms ([İmrohoroglu, İmrohoroglu and Joines 1995](#), [De Nardi, Imrohoroglu and Sargent 2001](#), [Kitao 2014](#)), international capital flows ([Henriksen 2002](#), [Börsch-Supan, Ludwig and Winter 2006](#), [Domeij and Flodén 2006](#), [Krueger and Ludwig 2007](#), [Backus et al. 2014](#), [Bárány et al. 2019](#)), and asset returns ([Abel 2003](#), [Geanakoplos, Magill and Quinzii 2004](#), [Carvalho, Ferrero and Nechio 2016](#), [Gagnon, Johannsen and Lopez-Salido 2019](#), [Eggertsson, Mehrotra and Robbins 2019](#), [Lisack, Sajedi and Thwaites 2017](#), [Jones 2018](#), [Papetti 2019](#), [Rachel and Summers 2019](#), [Kopecky and Taylor 2020](#)). We contribute to this literature by pointing out a key moment that drives the counterfactuals in these models and can be measured directly in the data. Our framework also helps reconcile the diverging conclusions that this literature has reached regarding the quantitative impact of aging on interest rates.³

Our paper bridges the gap between the reduced-form shift-share literature and the quantitative general equilibrium literature. In doing so, it relates to a literature on sufficient statistics, which is well developed in public finance ([Chetty 2009](#)) and trade ([Arkolakis, Costinot and Rodríguez-Clare 2012](#)) and is now gaining some traction in macroeconomics. This literature either focused on using cross-sectional information to capture the impact effect macroeconomic aggregates during transition dynamics ([Alvarez, Le Bihan and Lippi 2016](#), [Auclert 2019](#), [Berger, Guerrieri, Lorenzoni and Vavra 2018](#), [Auclert and Rognlie 2018](#), [Auclert, Rognlie and Straub 2018](#)). We show how to apply this methodology to forecast the macroeconomic effects of demographics both in the transition and in the long-run.

³For example, many papers in the literature calibrate their initial steady state to a stationary population, which by construction will not hit the compositional effect Δ_i^{comp} in the data. Papers also make different assumptions on parameters that ultimately drive $\varepsilon^{d,r}$ and $\varepsilon^{s,r}$. See section 6.4.

2 Compositional and behavioral effects of demographics

In this section, we study how demographic change affects the economy's wealth-to-output ratio, denoted W/Y . We derive an accounting identity that relates W/Y to the age structure of the population, the life-cycle profile of asset holdings, and the life-cycle profile of labor income. This accounting identity allows us to decompose changes in W/Y over time into a *compositional* effect, reflecting changes in the age structure for fixed life-cycle profiles, and a *behavioral* effect, reflecting changes in the life-cycle profiles themselves. We then calculate the compositional effect using data from 25 countries and show it to be large and heterogeneous. In later sections, we consider what this finding implies for the causal effect of demographic change on macroeconomic outcomes.

2.1 Decomposing the effects of demographics on W/Y

We consider an economy experiencing demographic change. N_{jt} denotes the number of agents of age j at time t , and $N_t = \sum_j N_{jt}$ denotes total population. We write aggregate wealth W_t and aggregate labor supply at time t as the sum of the contribution from each age group,

$$W_t = \sum N_{jt} A_{jt}, \quad (4)$$

$$L_t = \sum N_{jt} h_{jt}, \quad (5)$$

where A_{jt} and h_{jt} are the average asset holdings and effective labor supply per person of age j .

Let Y_t denote aggregate output at time t , and write Y_t/L_t for age-adjusted labor productivity. Balanced growth considerations suggest that wealth should grow at the same trend rate as Y_t/L_t .⁴ This motivates us to define *productivity-normalized assets* as

$$a_{jt} \equiv \frac{A_{jt}}{Y_t/L_t}, \quad (6)$$

Given this definition, we can rewrite aggregate wealth in (4) as

$$W_t = \frac{Y_t}{L_t} \sum_j N_{jt} a_{jt},$$

and therefore the wealth-to-GDP ratio, using (5), as

$$\frac{W_t}{Y_t} = \frac{\sum_j N_{jt} a_{jt}}{\sum_j N_{jt} h_{jt}} = \frac{\sum_j \pi_{jt} a_{jt}}{\sum_j \pi_{jt} h_{jt}}, \quad (7)$$

where $\pi_{jt} \equiv N_{jt}/N_t$ is the population share of age- j individuals.

⁴In section 3, we formalize the concept of *balanced growth by age*.

Equation (7) shows that there are three sources of variation in the aggregate wealth-to-GDP ratio over time: changes in the fraction of agents in each age group π_{jt} (a consequence of demographic change), changes in productivity-normalized assets of each age group a_{jt} , and changes in the labor supply of each age group h_{jt} .

We define the *compositional effect of aging* as the impact on W/Y from changing the age distribution under constant age profiles a_{jt} and h_{jt} of assets and labor supply:

$$\Delta_t^{comp} \equiv \frac{\sum_j \pi_{jt} a_{j0}}{\sum_j \pi_{jt} h_{j0}} - \frac{\sum_j \pi_{j0} a_{j0}}{\sum_j \pi_{j0} h_{j0}}. \quad (8)$$

Apart from demographic inputs, Δ_t^{comp} is only a function of a_{j0} and h_{j0} , which can be measured from cross-sectional average asset holdings and labor incomes by age, up to a normalization.⁵ Below, we combine demographic inputs with household income and wealth data from 25 countries to show that Δ_t^{comp} increases as the world ages, reflecting that old people hold more assets and work less.

The full change in W/Y reflects both the compositional effect and a *behavioral effect* coming from changes in a_{jt} and h_{jt} :

$$\underbrace{\frac{W_t}{Y_t} - \frac{W_0}{Y_0}}_{\equiv \Delta_t} = \underbrace{\frac{\sum_j \pi_{jt} a_{j0}}{\sum_j \pi_{jt} h_{j0}} - \frac{\sum_j \pi_{j0} a_{j0}}{\sum_j \pi_{j0} h_{j0}}}_{\Delta_t^{comp}} + \underbrace{\frac{\sum_j \pi_{jt} a_{jt}}{\sum_j \pi_{jt} h_{jt}} - \frac{\sum_j \pi_{j0} a_{j0}}{\sum_j \pi_{j0} h_{j0}}}_{\Delta_t^{beh}}. \quad (9)$$

To discipline the behavioral effect, a fully specified structural model is required, which is the topic of section 3 to 5. That analysis will ultimately confirm the important role of the compositional effect.

2.2 Compositional effects on W/Y in the United States

This section measures Δ^{comp} in the United States. The US. is an attractive test case for our methodology, since it has high quality data, is important for the world economy, and has experienced a strong rise in its wealth-to-GDP ratio. Section 2.3 repeats the exercise for 24 other countries.

Data. To measure the age distribution π_{jt} , we draw on the meticulous work of Gagnon et al. (2019), who combine information from a wide range of sources to model the US population from 1900 to 2100, including effects from births, deaths, and migration.⁶ Both their historical population

⁵Relative h_{j0} 's reflect relative labor incomes since L_t is a linear labor aggregator. The normalization does not affect the ratio, since scaling up the h_{j0} scales up the a_{j0} proportionally, by (5) and (6).

⁶See appendix B.1 for details about their procedure.

shares and their projections are very similar to those from the United Nations World Population Prospects. Later, we use other elements of their population model as inputs into our quantitative model.

To measure effective labor supply h_{j0} , we use data on the US from the Luxembourg Income Study (LIS).⁷ For each age group j , h_{j0} is proportional to average labor income per person, including monetary, non-monetary, and windfall labor income before taxes. The normalization $\sum_j \pi_{j0} h_{j0} = 1$ equates initial labor supply to the population.

To measure productivity-normalized asset holdings a_{j0} , we use data from the Survey of Consumer Finances (SCF) from 1989 through 2016. Our main measure of individual assets is the SCF net worth variable, to which we add an estimate of the funded part of defined benefit (DB) pensions. For this estimate, we use 37.5% of the value individual DB pensions provided by [Sabelhaus and Henriques Volz \(2019\)](#). This ensures that the overall amount of defined benefit pension plans in our data is consistent with the aggregate amount of non-federal funded defined benefit asset in the US economy. Assets are measured on the household level, which we allocate to individuals assuming that all assets are owned by the head of household.⁸ Hence, A_{j0} is the aggregate net worth including defined benefit wealth of all households with a head of age j , divided by the number of individuals of age j . To obtain a_{j0} , we normalize A_{j0} such that the aggregate coincides with the measured wealth-to-GDP ratio from the World Inequality Database (WID).⁹

Results. Figure 2 shows the evolution of the compositional effect with 2016 as a base year. Between 1950 and 2016, there is a positive compositional effect of 100 pp, similar in size to the entire observed increase in W/Y . Between now and the end of the 21st century, there is an additional compositional effect of 100 pp under the baseline fertility scenario. A higher fertility scenario pushes down the effect to around 50 pp, but a lower fertility scenario rises the effect to 200pp.¹⁰

The large compositional effect reflects how falling fertility and mortality rates lead to population aging, which, in turn, implies more people in ages with high asset holdings and low labor supply. Figure 3 illustrates this logic. The grey bars show the population distribution, starting

⁷For the US, the data in the LIS is based on the Annual Social and Economic Supplement of the Current Population Survey (CPS). We use the LIS instead of the CPS directly for comparability across countries.

⁸In appendix B.4, we also construct a compositional effect with all measurements on the household level. This involves calculating average household labor income by the age of household head, and constructing demographic projections for the age distribution of household heads. The results are very similar to the individual exercise.

⁹Figure 1 presents the evolution of wealth-to-GDP from the WID.

¹⁰We replicate the 2019 UN World Population Prospects fertility scenarios. The low fertility scenario reduces births per mother by 0.25 in 2020-2025, by 0.40 in 2025-2030 and by 0.50 for 2030-2100. The high fertility scenario has increases of the same size.

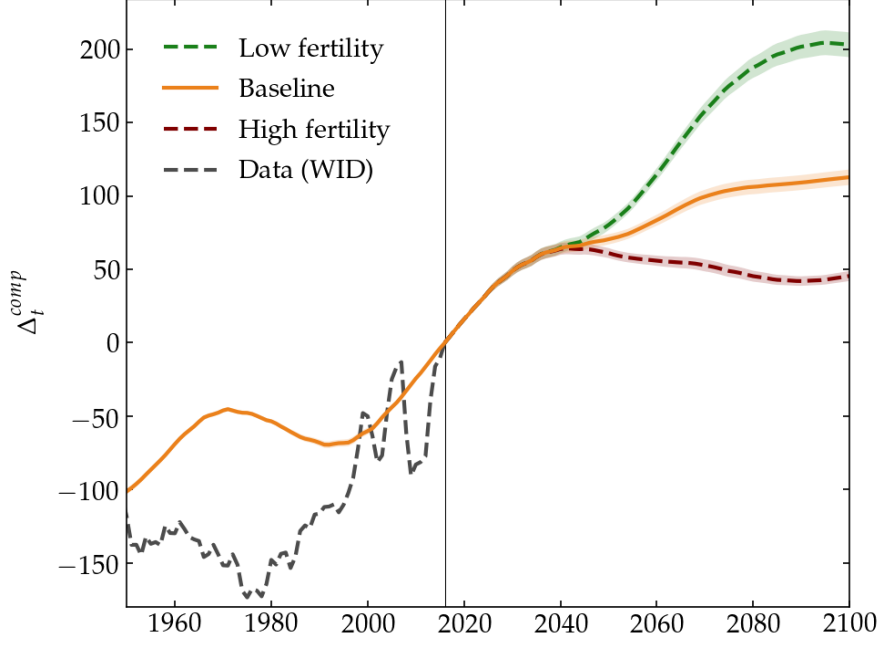


Figure 2: Compositional effects on W/Y

Notes: This figure depicts the evolution of the compositional term Δ_t^{comp} . The base year is 2016 (vertical line). The solid orange line corresponds to the baseline fertility, the dashed green and red lines consider the low and high fertility scenario as defined in the 2019 UN World Population Prospects. A bootstrapped 95% confidence interval is computed by resampling observations 10,000 times with replacement in both surveys. The dashed gray line presents the evolution of the Private Wealth over GDP ratio obtained from the World Inequality Database (WID).

young in 1950, gradually flattening over time. By 2100, it is almost flat until age 75, with a large number of very old people. Panel A superimposes the asset profile on the population distribution, and illustrate how aging moves people into high asset ages. Panel B superimposes the labor income profile on the population distribution, and illustrates how aging first increases labor supply as the baby boomers reach middle-age – the so-called *demographic dividend* (Bloom et al., 2003) – and later decreases labor supply as more people reach old age.

To formalize this logic, we do a first order approximation of (8), and write

$$\Delta_t^{comp} = \sum_j a_{j0} (\pi_{jt} - \pi_{j0}) - \left(\frac{W_0}{Y_0} \right) \sum_j h_{j0} (\pi_{jt} - \pi_{j0}) + \Delta_t^{\pi, err}, \quad (10)$$

where $\Delta_t^{\pi, err}$ is a second-order error. The first term captures the effect of having more people in high-asset ages, and the second term captures the effect of having more people in low labor-supply ages.

Figure 4 plots these two terms over time. Panel A shows the increase in $\Delta^{comp,a}$ as more people reach their peak asset age. Towards the end of the period, the trend flattens somewhat as more

people reach very old ages with lower asset holdings, but this effect is limited by the well-known slow decumulation of assets at the end of life (Mirrlees 1979, Hurd 1990).¹¹ Panel B shows a U-shaped pattern for $\Delta^{comp,h}$. Initially, the demographic dividend pushes up Y and down W/Y as more people reach their peak earning age. However, as others have documented (e.g. Cutler et al. 1990), this effect reverses course around 2010. Going forward, $\Delta^{comp,h}$ turns positive as the share of old-age people increases, and it contributes about one third of the full compositional effect between 2016 and 2100.

Robustness. Our overall message is robust to using different base years for the cross-sectional labor supply and asset holdings, or to using age profiles that result from a time-age-cohort decomposition. We consider ten asset surveys between 1989 and 2016, and twelve income surveys from 1976 to 2016. We also compute age effects in a time-age-cohort decomposition where growth loads on time effects, as per our model in the next section.¹² We then calculate Δ_t^{comp} for 143 combinations of asset and income profiles (see appendix B.3 for details). The results for 1950 to 2016 varies between 63 and 135 percentage points, and the results for 2016 to 2100 varies between 58 and 133 percentage points. Where there is some variation across specifications, the effect is always large and positive. This reflects the relative stability of asset and income age profiles over time.

2.3 Compositional effects on W/Y around the world

This section repeats the exercise for 24 other countries. To construct comparable age profiles of labor income and asset holdings, we collect and harmonize wealth and income surveys, and for population data and projections, we draw on the UN 2019 World Population Prospects (see appendix B.1 for a detailed description).

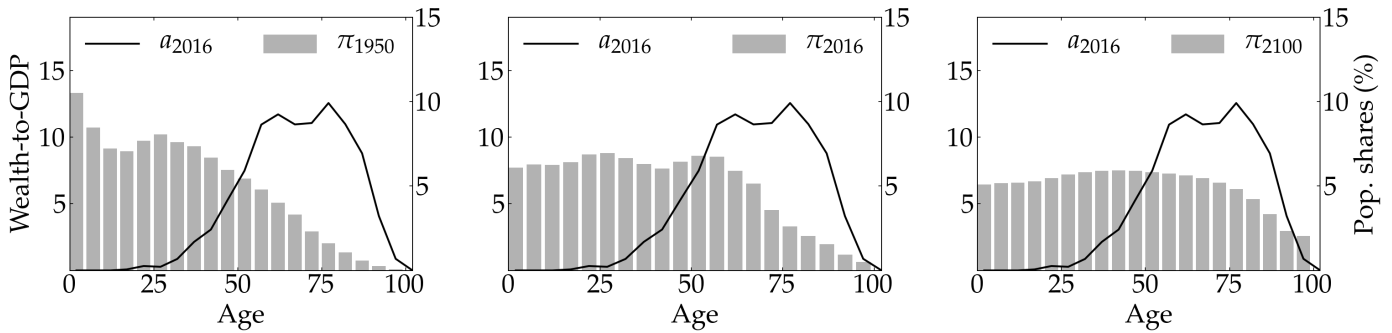
Figure 5 presents the results. Δ^{comp} has increased in virtually every country after 1950, and is projected to continue to increase during the rest of the century. The magnitudes are large, but also heterogeneous, ranging from Hungary at 35 percentage points to China and India at 281 and 307 percentage points respectively. The main heterogeneity is that developed countries tend to have smaller compositional effects than developing countries. This reflects that developing countries have more aging left in front of them, since their demographic transitions were later, and faster.¹³

¹¹For discussions about how to reconcile slow decumulation with standard life-cycle models, see for example, Ameriks and Zeldes (2004) and De Nardi, French and Jones (2010).

¹²This is similar in spirit to the Deaton (1997) normalization, but normalizes cohort effects rather than time effects to average to 0 and have no time trend.

¹³Appendix B.5 shows that this demographic difference is more important for heterogeneity than differences in labor

A. Changing population distributions over a fixed 2016 age-wealth profile



B. Changing population distributions over a fixed 2016 age-labor income profile

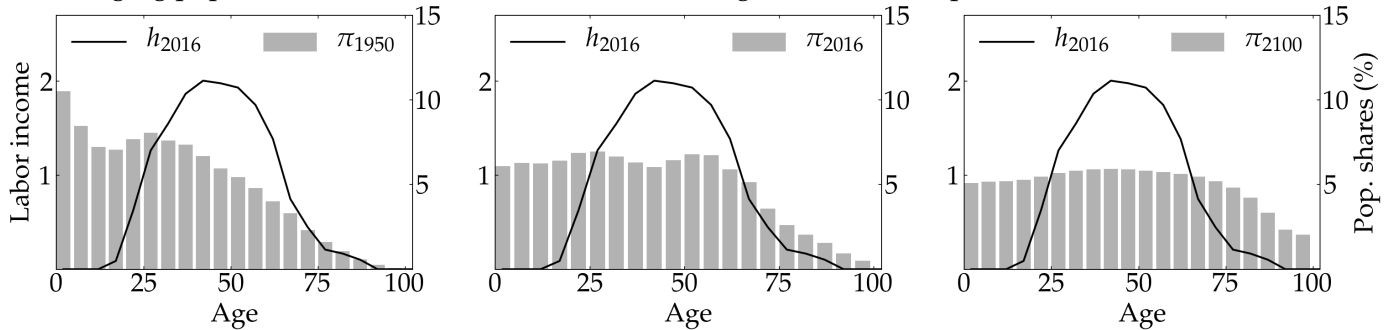


Figure 3: Age-wealth profiles, and age-labor income profiles, and population age distributions

Notes: Panel A presents the 2016 age-wealth profiles, and the 1950, 2016 and 2100 population distribution. Panel B presents the 2016 normalized age-labor income profiles, and the 1950, 2016 and 2100 population distribution.

Taking stock. The compositional effect of demographics on wealth-to-GDP has been quantitatively significant historically—mechanically accounting for the entire documented rise in the United States since the 1950s—and is projected to carry on at a similar pace for the rest of the twenty-first century. There are several reasons why this is important. First, while the demographic dividend is well studied, demographics have not been, so far, a point of major discussion behind the well documented increase in aggregate wealth. Second, these shifts are potentially large enough to significantly affect real interest rates going forward. However, projecting the effects on interest rates requires a model of behavioral effects. We turn to this exercise next.

3 A model of wealth accumulation and population aging

The prospective exercise we conducted in section 2.2 held normalized average life-cycle asset and income profiles fixed. In this section, we write down a quantitative model of wealth accumula-

income and asset profiles. In particular, heterogeneity virtually disappears if all countries are assumed to have US demographics, but is virtually unmoved if all countries are assumed to have US income and asset profiles.

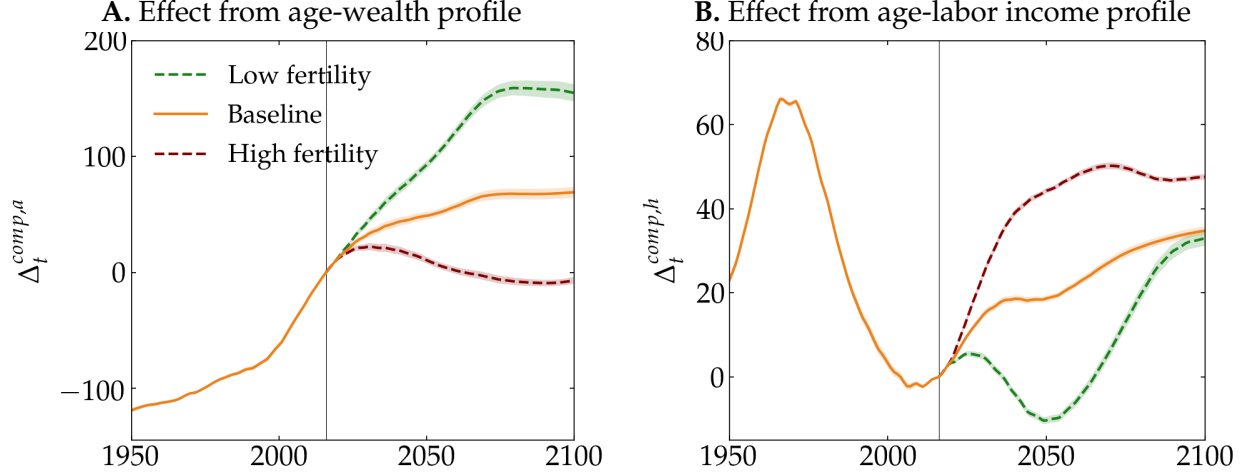


Figure 4: Effects of demographic composition on W, Y

Notes: This figure depicts the evolution of the components in the decomposition (38). Panel A presents the contribution from asset profiles $\Delta_t^{comp,a} = \sum_j \pi_{j0}(a_{jt} - a_{j0})$. Panel B presents the contribution from labor supply $\Delta_t^{comp,h} = -\frac{W_0}{Y_0} \sum_j \pi_{j0}(h_{jt} - h_{j0})$. In both graphs, the solid orange line corresponds to the baseline fertility scenario and the dashed green and red lines consider the low and high fertility scenario of the 2019 UN World Population Prospects. A bootstrapped 95% confidence interval is computed by resampling observations 10,000 times with replacement.

tion and population aging to study the conditions under which this assumption is justified. We show that there is a certain small-open economy special case of the model in which the aging transition in fact endogenously generates fixed normalized asset profiles. In this case, demographic projections and cross-sectional age profiles are *sufficient statistics* for the transitional dynamics of the wealth-to-GDP ratio, and Figure 2 gives the exact transitional dynamics for $\frac{W_t}{Y_t}$ in the transition. In later sections, we will study the model away from this special case, which will allow us to evaluate the likely strength of behavioral responses to aging, as well as the equilibrium effects of population aging on interest rates.

3.1 Model

We consider an overlapping generations (OLG) model with heterogeneous households. Time is discrete and runs from $t = 0$ to ∞ . There is a path of real interest rates r_t that may be exogenous (in the small open economy version of the model) or endogenous (in the integrated world economy version).

Demographics. The demographic parameters of the economy are a sequence of births N_{0t} , a sequence of age- and time-specific mortality rates ϕ_{jt} for individuals between age j and $j + 1$, a

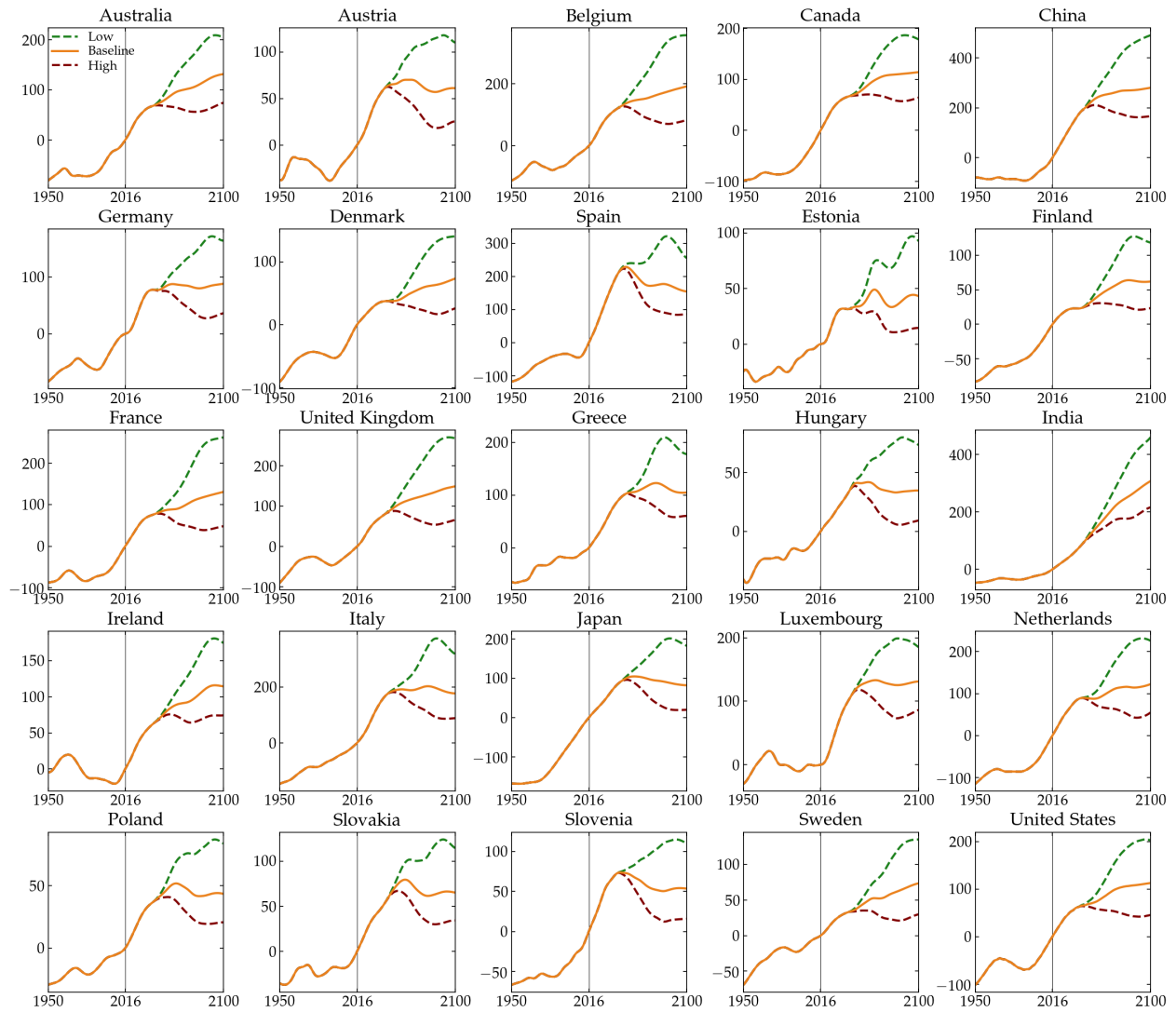


Figure 5: Compositional effects globally

Notes: This figure depicts the evolution of the compositional term Δ_t^{comp} across countries from 1950 to 2100. The base year is 2016 (vertical line). The solid orange line corresponds to the medium fertility scenario from the UN, the dashed green line to the low fertility scenario, and the dashed red line to the high fertility scenario.

sequence of net migration levels M_{jt} , as well as an initial number of agents by age N_{j0} . Given these parameters, the population for $t \geq 1$ evolves according to

$$N_{jt} = (N_{j-1,t-1} + M_{j-1,t-1}) \phi_{j-1,t-1}, \quad (11)$$

As in Section 2, we write $N_t \equiv \sum_j N_{jt}$ for the total population at time t , and $\pi_{jt} \equiv \frac{N_{jt}}{N_t}$ for the age distribution of the population.

Production. Output Y_t is produced using capital K_t and effective labor supply L_t . There is an aggregate production function F such that

$$Y_t = F(K_t, Z_t L_t).$$

L_t denotes the total number efficient units of labor

$$L_t = \sum_{j=0}^T N_{jt} h_{jt}, \quad (12)$$

where h_{jt} is the average labor supply per person of age j .

Households. The economy is populated by heterogeneous households, interpreted as single-parent families. Households are born at age 0, start their working life at age T^w with no assets, live at most until age T , and face a time-varying survival probability ϕ_{jt} between age j and $j+1$ at time t .

Starting at age T^w , households start working and making financial decisions. They face labor income risk driven by an arbitrary stochastic process s_j , such that their income at age j and time t when a household is in state s_j is

$$y_{jt}(s_j) = w_t \rho_{jt} \widetilde{h}_{jt} \ell(s_j). \quad (13)$$

Here, w_t is the wage per unit of effective labor supply, $\rho_{jt} \in [0, 1]$ is a parameter of the retirement system indicating the fraction of full efficiency-hours that households of age j can provide to the market at time t , and $\ell(s_j)$ is a labor supply shifter with cross-sectional mean of one at every age ($\mathbb{E}\ell(s_j) = 1$). Hence, the average labor supply per person of age j is equal to

$$h_{jt} \equiv \rho_{jt} \widetilde{h}_{jt}. \quad (14)$$

Households have life-cycle and self-insurance motives for saving. They can do so by investing in a safe asset a that delivers returns r_t at time t . They can also use this asset to self-insure against mortality risk; any assets left at the end of life are given as bequests, which agents also value in

utility.

Writing $s^j = (s_0, \dots, s_j)$ for the history of income shocks though age j , a household aged j in state (s^j, a) at time t solves the following utility maximization problem:

$$\begin{aligned} V_{jt}(s^j, a) &= \max_{c, a'} \psi_{jt} \frac{c^{1-\sigma}}{1-\sigma} + \Upsilon Z_t^{\nu-\sigma} (1 - \phi_{jt}) \frac{(a')^{1-\nu}}{1-\nu} + \beta \phi_{jt} \mathbb{E} [V_{j+1, t+1}(s^{j+1}, a') | s^j] \\ c + a' &= (1 - \tau_t^y) y_{jt}(s_j) + (1 + r_t) a + tr_{jt}(s^j) + b_{jt}^r(s^j) \\ a' &\geq -\bar{a} Z_t. \end{aligned} \quad (15)$$

The per-period utility term $\psi_{jt} c^{1-\sigma} / (1 - \sigma)$ represents the flow utility from consumption, with ψ_{jt} an age-dependent utility modifier reflecting changes in the number of children over the life cycle. In Appendix C.1, we show that if n_{jt} is the number of children currently living in the household, then a Barro-Becker-type formulation of utility delivers

$$\psi_{jt} = \left(1 + \lambda^{\frac{1}{\sigma}} n_{jt}^{\frac{\sigma+\varphi-1}{\sigma}} \right)^{\sigma}, \quad (16)$$

with $\lambda > 0$ and $\nu > 0$ are two parameters. The term $\Upsilon Z_t^{\nu-\sigma} (1 - \phi_{jt}) (a')^{1-\nu} / (1 - \nu)$ represents the utility from giving bequests a' . It is scaled by mortality risk $1 - \phi_{jt}$, since agents only give bequests if they die, and $\nu \geq \sigma$ captures potential non-homotheticities in bequests, which we introduce, as in De Nardi (2004), to capture the highly uneven distribution of bequests in the data. The scaling factor $Z_t^{\nu-\sigma}$ ensures balanced growth in spite of this non-homotheticity.

The budget constraint shows that households buy consumption goods and assets using labor income $y_{jt}(s_j)$ taxed at rate τ_t^y , asset income $(1 + r_t^a)a$, as well as history-contingent transfers $tr_{jt}(s^j)$ and bequests received $b_{jt}^r(s^j)$. The households also face a borrowing constraint, which grows with technology Z_t .

The household problem described above defines the optimal household behavior from an arbitrary starting point in terms of age, asset holdings, and a history of income shocks. As stated above, agents always start their working life with zero assets and may receive bequests at any age starting at T^w as a function of their history of shocks s^j . Hence, s^j encodes the entire state space of households at age j . Households of age $j > 0$ are drawn for some given initial distribution $G_{j0}(s^j)$, encoding their joint distribution of assets and age. We also assume that migrants of any age j are indistinguishable from the rest of the population, in that their state s^j is drawn from the same distribution as that of the general population of age j .

Government. The government sets the retirement policy ρ_{jt} , purchases G_t goods, and can finance itself using a risk-free bond. It faces the flow budget constraint

$$G_t + \sum_{j=0}^T N_{jt} \mathbb{E}[tr_{jt}(s^j)] + (1 + r_{t-1})B_{t-1} = \tau_t^y w_t \sum_{j=0}^T N_{jt} \mathbb{E}[y_{jt}(s^j)] + B_t,$$

where a positive B_t denotes government borrowing. In the aggregation, $\mathbb{E}x(s^j)$ denotes an expectation over the full set of histories s^j , and we use the fact that this fully encodes household states. A government policy is sustainable if it satisfies the flow budget constraint and the debt-to-output ratio $\frac{B_t}{Y_t}$ does not diverge.

Asset market. The assets in the economy consist of capital K_t , government bonds B_t , and foreign assets NFA_t . In equilibrium, the value of these assets equals the asset holdings of domestic residents plus the assets of migrants that are arriving in the next period, $A_t^{mig.net}$.¹⁴

$$K_t + B_t + NFA_t = A_t + A_t^{mig.net}, \quad (17)$$

Small open economy equilibrium. Given a sequence of interest rates $\{r_t\}$, a process for technology $\{Z_t\}$, labor supply $\{\widetilde{h}_{jt}\}$, and demographics $\{\pi_{jt}, \phi_{jt}, n_t\}$, a *small open economy equilibrium* consists of a set of prices and quantities (satisfying the initial conditions) such that firms and households optimize, the government policy is sustainable, and bequests received match bequests given at every date,

$$(1 + r_t^a) \sum_{j=1}^T N_{j-1,t-1} (1 - \phi_{j-1,t-1}) \mathbb{E}[a'_{j-1,t-1}(s^{j-1})] = \sum_{j=1}^T N_{j,t} \mathbb{E}[b_{jt}^r(s^j)]. \quad (18)$$

World-economy equilibrium. Consider a set of countries $c \in \mathcal{C}$. Given a country-specific process for technology $\{Z_{ct}\}$, labor supply $\{\widetilde{h}_{cjt}\}$, and demographics $\{\pi_{cjt}, \phi_{cjt}, n_{ct}\}$, a *world-economy equilibrium* is a small open economy equilibrium for each country in which the endogenous sequence of interest rates $\{r_t\}$ is such that the world's net foreign asset position is zero in every period,

$$\sum_{c \in \mathcal{C}} NFA_{ct} = 0.$$

¹⁴The interpretation is that incoming migrants arriving at time $t + 1$ choose to allocate their assets in the domestic market prior to their arrival. Similarly, migrants that are leaving between t and $t + 1$ allocate their assets abroad, which implies a negative contribution to $A_t^{mig.net}$.

3.2 Asset supply and demand effects of demographics

We now turn to a structural analysis of the causal effect of demographic change on macroeconomic outcomes. We build up from the compositional effects in each country towards an analysis of the integrated world economy. The structure of our argument is guided by a steady state asset supply and demand framework. As we will show, this framework is useful to understand macroeconomic dynamics along a demographic transition as well.

Consider a country c in a steady state, experiencing a constant real interest rate r , and constant demographics summarized by a parameter Θ .¹⁵ The country's net foreign asset position is the difference between household wealth W_c and asset supply A_c^s . Normalizing by GDP Y_c ,

$$\frac{NFA_c}{Y_c}(r, \Theta) \equiv \frac{W_c}{Y_c}(r, \Theta) - \frac{A_c^s}{Y_c}(r). \quad (19)$$

where steady state asset supply $A_c^s = K_c + B_c$ is the sum of capital and bonds, per equation (17). Note, importantly, that provided that the government fiscal rule maintains a constant ratio of B_c to Y_c in the long run, the ratio of asset supply to GDP is independent of demographics.¹⁶ By contrast, aggregate wealth (asset demand in this economy) depends on both demographics and interest rates.

Given a change in the demographic parameter $\Delta\Theta$, equations (7) and (19) imply a first order decomposition

$$\Delta \left(\frac{NFA_c}{Y_c} \right) \approx \Delta_c^{comp} + \Delta_c^{beh|r} + \epsilon_c^{d,r} \Delta r + \epsilon_c^{s,r} \Delta r. \quad (20)$$

The first term in this expression is the compositional effect Δ_c^{comp} from section 2. The second two terms split the overall behavioral effect Δ^{beh} into a term that is due to demographics at constant interest rates $\Delta^{beh|r}$, and a term that is due to interest rate changes at constant demographics $\epsilon_c^{d,r} \Delta r$. The fourth term, $\epsilon_c^{s,r} \Delta r$, captures the negative of the response of asset supply to interest rates (to ensure that $\epsilon^{s,r}$ is positive, we assume it is the negative asset supply sensitivity).

In an integrated world economy, the world NFA must be 0 both before and after the change in Θ : $\sum_c NFA_c = 0$. As appendix C.4 shows, this implies that to a first order approximation, equilibrium changes in the interest rate, world wealth, and net foreign asset positions are:

¹⁵Here, Θ represents exogenous fertility, mortality rates by age, as well as migration rates by age, and our steady state assumption requires a stationary population given those parameters.

¹⁶The level of asset supply, however, does depend on demographics. To see this, note that $K_t^s = k(r)Z_t L_t$, where $k(r)$ is the solution to $F_K(k(r), 1) = r + \delta$. This depends on effective labor supply L_t . By contrast, $\frac{K_t^s}{Y_t} = \frac{k(r)}{F(k(r), 1)}$, which only depends on r .

$$\Delta r = -\frac{1}{\bar{\epsilon}^{d,r} + \bar{\epsilon}^{s,r}} (\bar{\Delta}^{comp} + \bar{\Delta}^{beh|r}), \quad (21)$$

$$\Delta \left(\frac{W_{world}}{Y_{world}} \right) = \frac{\bar{\epsilon}^{s,r}}{\bar{\epsilon}^{d,r} + \bar{\epsilon}^{s,r}} (\bar{\Delta}^{comp} + \bar{\Delta}^{beh|r}), \quad (22)$$

$$\Delta \left(\frac{NFA_c}{Y_c} \right) = (\Delta_c^{comp} + \Delta_c^{beh|r}) - (\bar{\Delta}^{comp} + \bar{\Delta}^{beh|r}) + \left[\epsilon_c^{d,r} + \epsilon_c^{s,r} - (\bar{\epsilon}^{d,r} + \bar{\epsilon}^{s,r}) \right] \Delta r, \quad (23)$$

where we have defined bars to denote averages weighted by country output Y_c/Y .

The change in the interest rate is due to the world asset demand shift $\bar{\Delta}^{comp} + \bar{\Delta}^{beh|r}$, deflated by the sum of the world average asset demand and supply sensitivity $\bar{\epsilon}^{d,r} + \bar{\epsilon}^{s,r}$. By contrast, changes in net foreign asset positions are due to *differences* in asset demand shifts relative to the world average asset demand shift, and *differences* in sensitivities $\epsilon_c^d + \epsilon_c^s$ relative to the world average sensitivity.

Outline for the rest of the paper. To move from Δ_c^{comp} to general equilibrium, we need a model to obtain behavioral effects at constant interest rates, $\Delta^{beh|r}$, as well as the sensitivities of asset demand and supply to the interest rate $\epsilon^{d,r}$ and $\epsilon^{s,r}$. Section 3 sets up the model. Section 4 establishes that $\Delta^{beh|r}$ is small compared to Δ^{comp} . Section 5 shows that ϵ^d and ϵ^s are similar across countries, though their levels are subject to much uncertainty. This leads us to find that $\Delta_c^{comp} - \bar{\Delta}_c^{comp}$ is a robust predictor of NFA changes across countries, while the exact level of interest rates and changes in wealth-to-GDP ratios are more difficult to predict.

4 The small open economy

We use a calibrated version of our model to evaluate the importance of deviations from the sufficient statistic result. To obtain the parameters and initial conditions, we set up a steady state version of the model and calibrate it to 2016 US data. This exercise delivers an initial distribution. We then simulate the model from that starting point by feeding in population projections, solving for the path of the wealth-to-GDP ratio $\{W_t/Y_t\}_{t=2016}^{2300}$ under different scenarios in terms of demographics, policy, and labor supply. To evaluate the deviations from the sufficient statistic, we start with a scenario that satisfies the assumptions behind Proposition 1 exactly. Sequentially, we add more realism, and evaluate how much the result deviates from the predictions of our simple shift-share exercise.

Our main conclusion is that the sufficient statistic provides a good approximation for the full

effect of demographic change on W_t/Y_t . Even under relatively large variations of parameters, the full effect is in the range of 0.7–2 times that of the shift-share prediction. The shift-share works well because of the large difference in asset holdings across age groups: as is evident from figure 3, 70-year olds hold on average four times more assets than 40 year olds, and population aging in the data reshuffles mass from the latter to the former age groups. The magnitude of this change is sufficiently large that it generally overwhelms plausible economic changes in asset accumulation decisions themselves. In addition, the economic mechanisms we study do not all have the same effect on asset accumulation and tend to offset each other in our most preferred scenario.

Section 4.2 makes parametric restrictions on the model of Section 3 to allow for quantification. Section 4.3 uses a steady state version of our model in conjunction with 2016 data to calibrate the parameters and derive the initial conditions. Section 4.4 uses these results to study $\{W_t/Y_t\}$ in the non-stationary model.

4.1 Sufficient statistic result for W/Y in an open-economy transition

Under some conditions, there is an open-economy equilibrium where the compositional effect is a sufficient statistic for the full effect of demographics on the wealth-to-GDP ratio.

In terms of (9), this is equivalent to the behavioral term being zero, which obtains when the effective labor supply profiles and the asset decision functions (normalized by technology) are both constant over time. In addition to requiring a constant interest rate and technological growth rate, this naturally requires a fixed normalized labor supply $h_{jt} = \rho_j \tilde{h}_j$. In addition, we see from the household problem (15) that constant asset profiles also requires turning off the time-variation coming from child preferences ψ_{jt} , mortality ϕ_{jt} , taxes τ_t , interest rates r_t and transfers and bequests $tr_{jt}(s^t) + beq_{jt}^r(s^t)$.

We introduce the following five assumptions.

Assumption 1. *The exogenous life-cycle profiles of labor efficiency by age and the technology growth rate are constant: $\tilde{h}_{jt} = \tilde{h}_j, \forall j, \gamma_t = \gamma$.*

Assumption 2. *Mortality profiles are constant over time: $\phi_{jt} = \phi_j$, all j .*

Assumption 3. *Households' valuation of children's consumption is constant over time: $\psi_{jt} = \psi_j$, all j .*

Assumption 4. *Tax and retirement policy are constant over time: $\tau_t^y = \tau^y$ and $\rho_{jt} = \rho_j$, all j . Moreover, normalized income from transfers and bequests, $\frac{tr_{jt}(s^j) + b_{jt}^r(s^j)}{Z_t}$ is independent of t for any history s^j .*

Assumption 5. *The economy is small and open, with interest rates $r_t = r$ constant over time.*

Assumption 1 ensures both that the labor income profile is constant, and that there are no indirect effects on asset accumulation from a change in the timing of earnings. Assumptions 2-3 ensure that households do not change their asset accumulation decisions due to a lower mortality or having fewer children. Assumption 4 constrains government behavior by ensuring that household income net of taxes, transfers, and bequests is constant over time. A first implication of this assumption is that the government uses only variations in G_t to balance its budget. The second, and more technical, implication is that the government balances variations in bequests using transfers, since this is required to keep household income exactly constant over time (in practice, this effect turns out to be unimportant quantitatively). Last, assumption 5 ensures that households do not change asset accumulation decisions due to price effects.

We can prove the following proposition.

Proposition 1. *Under assumptions 1–5, there exists an initial distribution of assets such that the equilibrium wealth-to-output ratio evolves according to*

$$\frac{W_t}{Y_t} = \frac{\sum_j \pi_{jt} a_{j0}}{\sum_j \pi_{jt} h_{j0}}, \quad (24)$$

for all $t \geq 0$, where $a_{j0} = \frac{\mathbb{E}a_{j0}(s^j)}{Y_0/N_0}$ is average assets of household aged j normalized by time-0 GDP per capita, and $h_{j0} = \rho_j \tilde{h}_j$ is normalized effective labor supply by age. The same result obtains in a model where economic growth loads on cohort effects rather than on time effects.

Proposition 1 shows that assumptions 1–5 imply the existence of an equilibrium where the behavioral term in (9) is 0, and the full transition of W_t/Y_t is obtained by simply rolling forward population projections under fixed age profiles of assets and labor income. In this equilibrium, the simple projections from sections 2.2–2.3 coincide with the simulated transition from a fully specified structural model. To the best of our knowledge, this sufficient statistic result is the first of its kind for general equilibrium transitions in heterogeneous-agent models.

Mathematically, the result exploits that assumptions 1–5 imply a time-invariant normalized household problem. This means that any two individuals born after time 0 that have the same age j and shock history s^j will also have the same asset position normalized by technology, regardless of their date of birth. This implies a micro level balanced growth result for individuals born after time 0, where asset positions by state and age growing at a common rate γ . All changes in normalized macroeconomic aggregates only reflect a changing distribution across ages. If the economy starts

in the balanced growth distribution, it will stay there, and we obtain the equilibrium in Proposition 1.

Economically, the proposition highlights that many details of the household problem are irrelevant for the effect of demographics on W_t/Y_t . In the household problem, wealth profiles reflect some combination of life cycle motives, precautionary motives, and bequest motives. However, the proposition shows that if demographic change does not directly alter the incentive for asset accumulation, the only thing that matters for W_t/Y_t is the final age profile of assets. Changes in fertility and migration can leave micro behavior invariant, with aggregates only changing for compositional reasons. Moreover, the sufficiency of cross-sectional profiles holds regardless of whether growth effects load on cohort effects or on time effects. This is important because, while there is a lot of debate in the literature as to what generates the shape of existing savings profile, in particular the role of bequests (Kotlikoff and Summers 1981, Abel 2003) the result here does not require taking a stance on the underlying cause.

The assumptions underlying Proposition 1 are fairly strong. However, in conjunction with the decomposition result (9), the proposition provides a useful benchmark to explore more realistic models. From (9), we learn that the change in wealth-to-output ratios can be decomposed into a compositional and a behavioral effect. From Proposition 1, we further learn that any non-zero behavioral effects can be traced back to violations of assumptions 1–5. Hence, for any version of the model, we can analyze the changes in W_t/Y_t at exogenous interest rates as a combination of the compositional effects and various violations of assumptions 1–5. In the next section, we conduct such an analysis.

4.2 Parametric assumptions

The production function F has a constant elasticity of substitution η between capital and labor.

For the income process, we assume that households become economically active at age T^w , and that the labor supply shifter ℓ is the product of a fixed effect θ and a transitory component ϵ , both with mean 1. The transitory component follows a Markov chain over time, and the fixed effect follows a Markov chain across generations.

For bequests, we model partial intergenerational wealth persistence by assuming that all bequests from individuals of type θ are pooled and distributed across the types θ' of survivors in accordance with the intergenerational transmission of types. The distribution of bequests across different ages is regulated by a fixed weight F_j that determines the share of bequests going to age- j

individuals.

For the government policy, we assume that working age individuals face a constant tax rate $\tau_t^y = \tau_t^{ss} + (1 - \tau_t^{ss})\tau_t$, where τ_t is an income tax rate and τ_t^{ss} is a social security tax rate (which is not paid by the retirees). Retirement is phased in from age T_t^r with agents still working a fraction $\rho_t \in [0, 1]$ of full time hours

$$\rho_{jt} = 1_{j < T_t^r} + \rho_t 1_{j = T_t^r}.$$

The social security replacement rate d_t is applied to the current income of agents with the same fixed effect. Hence, transfers net of income taxes are $tr_{jt} = (1 - \rho_{jt})(1 - \tau_t)\bar{d}_t w_t \theta$.

In terms of fiscal rules, our main quantitative analysis updates the government policy dynamically to keep a constant the debt-to-output ratio. This is done using different mixes of adjustment margins. In the appendix, we explain how these mixes are parametrized, and we also discuss other fiscal rules that allows for changes in the debt to output ratio.

4.3 Steady state calibration for the US

We obtain the parameters and the initial conditions by calibrating a steady state version of our model to US data from 2016. The steady state setup is standard, with constant interest rates r and technology growth rates γ , constant government policies for tax rates τ, τ^{ss} , spending G/Y , retirement policy T^r, ρ , and replacement rates d , and constant mortality rates ϕ_j and birth rates $N_{0,t}/N_0$. Appendix D.4 summarizes the equations that characterize our steady state.

The only non-standard element is that we introduce a counterfactual flow of migrants to ensure a time-invariant population distribution at the 2016 level. The migration strategy is one way to address a generic problem in the calibration of steady state demographic models, which is that observed mortality and population shares might not be consistent with a stationary population distribution. This particular strategy is similar to the assumptions made in the [Penn Wharton Budget Model \(2019\)](#).

Calibration procedure. Table 1 provides a summary of our calibration parameters and sources.

As noted earlier, we obtain our demographic parameters from [Gagnon et al. \(2019\)](#). For the steady state, we use their data on 2016 population shares, population growth rates and mortality rates. The migrant shares by age are calculated to ensure a stable age distribution.

We calculate W by aggregating from the 2016 SCF (following equation (17)), we assume by convention that this records both domestic wealth A and net migrant wealth $A^{mig,net}$. Dividing

by 2016 GDP, we obtain $W/Y = 500\%$.

For asset supply, we assume that the net foreign asset position is zero $NFA = -43.6\%$ ¹⁷, and we set B/Y to its 2016 value of 74%.¹⁸ We back out the value of capital as a residual: $\frac{K}{Y} = 500\% - 74\% + 43.6\% = 469\%$.

To calibrate r , we note that, as per the classic test of dynamic inefficiency by [Abel, Mankiw, Summers and Zeckhauser \(1989\)](#), that the difference between r and the economy's growth rate g must equal the dividend yield on corporate assets, or

$$\frac{r - g}{1 + g} = \frac{1 - s^L - I/Y}{K/Y}, \quad (25)$$

where s^L is the labor compensation share. We assume that the labor share is $s^L = 0.62$ to reflect the 2016 labor share of factor income, excluding the self-employed.¹⁹ To obtain g , we use population growth together with the TFP growth rate $\gamma = 0.73\%$, its 2010-2017 average.²⁰ Given the investment-output ratio $\frac{I}{Y}$ of 17%²¹, and our calibration so far, we obtain $r = 6.1\%$. We believe this number is reasonable when interpreted as the total return on all assets.

On the production side, we calibrate δ such that $(r + \delta)\frac{K}{Y}$ equals the capital share of compensation. Furthermore, we assume a capital-labor elasticity of substitution of $\eta = 1$, which gives us a Cobb-Douglas production function where we calibrate the capital share as one minus the labor compensation share. In two alternative calibrations, we consider $\eta = 0.6$ from [Oberfield and Raval \(2019\)](#) and $\eta = 1.25$ from [Karabarounis and Neiman \(2014\)](#).

For the government, we set the retirement age at the Social Security full retirement age of $T^r = 65$, and make T^r a complete retirement by setting $\rho = 0$. We set the benefit level $\bar{d} = 69\%$ such that the share of GDP paid out as social security benefits, $\mu^{ret}\bar{d}s^L$, is equal to 6.94%,²² and we set $\tau^{ss} = 11.2\%$ to balance the social security system given this benefit level. Last, we set government expenditures over GDP to its 2016 level $\frac{G}{Y} = 17.6\%$ and pick $\tau = 33.8\%$ to balance the government flow budget constraint.

For the household problem, we calibrate the income process and the intergenerational transmission of skills externally. The logarithm of idiosyncratic labor productivity is assumed to follow

¹⁷From the IMF's IMF Net International Investment Positions database.

¹⁸From the Flow of Funds L.210, marketable treasuries (\$13.880T) divided by the 2016 US GDP.

¹⁹From National Income and Product Accounts (NIPA) Table 1.11, compensation of employees (52%) divided by total gross domestic income (100%) minus taxes on production and imports less subsidies (6.7%) and proprietors' income (7.6%).

²⁰Using TFP growth from [Fernald \(2014\)](#) and scaling accordingly.

²¹2016 Gross Private Domestic Investment from the US Bureau of Economic Analysis (BEA) divided by 2016 US GDP.

²²From the OECD statistics database [OECD \(2019\)](#).

an $AR(1)$ process with a persistence parameter of 0.91 and a variance of 0.92, as in [Auclert and Rognlie \(2018\)](#). For intergenerational transmission of skills, we also assume that the logarithm of the fixed effect follows an $AR(1)$ process with a persistence parameter of 0.677 and a variance of 0.61, as in [De Nardi \(2004\)](#). We calibrate the labor efficiency profiles \tilde{h}_j to be proportional to aggregate labor income per person in an age group, and normalized so that $L = \sum_j \pi_j \rho_j \tilde{h}_j = 1$, as in section 2. The distribution of bequest F_j is derived from the 2016 SCF by calculating the share of total bequests received that goes to each age group. The distribution of children n_j is taken from [Gagnon et al. \(2019\)](#), and we map this to age-dependent marginal utility modifiers ψ_j using equation (16). Last, we also set an exogenous zero borrowing limit, $\bar{a} = 0$, and an elasticity of intertemporal substitution (EIS) σ^{-1} of 1.²³

The rest of the preference parameters ($\beta, Y, \nu, \lambda, \varphi$) are calibrated to fit the 2016-2100 shift-share estimate from section 2, the age profile of consumption-over-GDP from the Consumer Expenditure Surveys (CEX), an aggregate bequests-to-GDP ratio of 5%²⁴ and the distribution of bequests received taken from Table 1 in [Hurd and Smith \(2002\)](#), subject to the constraint of fitting exactly the aggregate wealth-to-output ratio W/Y of 500%. The resulting parameters are in Table 1.

Calibration outcomes. Figure 6 presents the outcomes of our calibration. Panel A compares the resulting model age-wealth profile to the 2016 SCF profile used in section 2. In the calibration, we target the 2016-2100 empirical shift-share, but not the age-wealth profile directly. Panel B presents the model gross labor efficiency profile h_{jt} and the net-of-taxes profile where retirement is enforced at age 65, as well as the normalized empirical age-labor income profile taken from LIS. Note that since the empirical profile h_{jt} includes the retirement age of 65, we rescale it for $j \geq 65$.²⁵

4.4 Results

To evaluate the deviations from the sufficient statistic prediction, we solve for W_t/Y_t in a non-stationary version of the model for the time period 2016-2300 and report the transition outcomes until 2100. Holding preferences, production structure, and interest rates constant, we feed the model with different future scenarios for demographics and policies.

²³The value of σ^{-1} turns out to not be important for the open economy counterfactuals in this section. However, it is very important in the closed economy analysis. See the discussion in Section 5.

²⁴Intermediate value between the estimates of 8% from [Alvaredo, Garbinti and Piketty \(2017\)](#) and 2% from [Hendricks \(2001\)](#).

²⁵More precisely, for $j \geq 65$ we rescale using the ratio of the labor force participation rate at age 65 to its value at age j : $h_{jt} \frac{LFPR_{65}}{LFPR_j}$. See appendix figure figure D.4.

Table 1: Calibration parameters

Parameter	Description	Value	η	μ	Source
r	Real interest rate	3.9%			Equation (25) and $\frac{I}{Y}$
\underline{a}	Borrowing limit	0%			
<i>Demographics</i>					
T^w, T	Age structure	20, 100			
n	Population growth rate	0.78%			Gagnon et al. (2019)
π_j	Population distribution	Data			Gagnon et al. (2019)
ϕ_j	Survival probabilities	Data			Gagnon et al. (2019)
<i>Technology</i>					
γ	Technological growth rate	0.82%			Average 2010-17 (Fernald)
s^L	Labor share	0.688			2016 US Value (NIPA)
I/Y	Investment over GDP	22.1%			2016 US value (BEA)
K/Y	Capital over GDP	404%		232%	Fixed Assets Table 1.1
δ	Depreciation rate	4%		8%	Equation (62) and $\frac{K}{Y}$
μ	Gross markup rate	1		1.04	Equation (45) and $\frac{W}{Y}, \frac{NFA}{Y}, \frac{B}{Y}, \frac{K}{Y}$
η	K/L elasticity of substitution	1	0.6	1.25	Oberfield and Raval (2019) Karabarbounis and Neiman (2014)
<i>Government</i>					
T^r	Retirement age	65			
G/Y	Expenditures over GDP	17.7%			2016 US value (BEA)
B/Y	Debt over GDP	77%			2016 US value (FoF)
\bar{d}	Social security benefits	62.2%			Benefits 6.94% of GDP
τ^{ss}	Payroll tax	10.1%			Balanced SS system
τ	Total income tax	28.2%			Balanced total budget
<i>Other assets</i>					
W/Y	Total wealth over GDP	438%			2016 US value (SCF)
NFA/Y	Net foreign assets	-42.5%			2016 US value (IMF)
<i>Income process</i>					
σ_ϵ	Idiosyncratic std. dev.	0.92			Auclert and Rognlie (2018)
ρ_ϵ	Idiosyncratic persistence	0.91			Auclert and Rognlie (2018)
σ_θ	Intergenerational std. dev.	0.61			De Nardi (2004)
ρ_θ	Intergenerational persistence	0.677			De Nardi (2004)
<i>Preferences</i>					
σ^{-1}	EIS	1			Standard calibration
β	Subjective discount factor	0.965			See text
ν	Bequest curvature	0.61			See text
Y	Bequests scaling factor	3.01			See text
λ	Weight of children (factor)	0.07			See text
φ	Weight of children (exponent)	1.82			See text

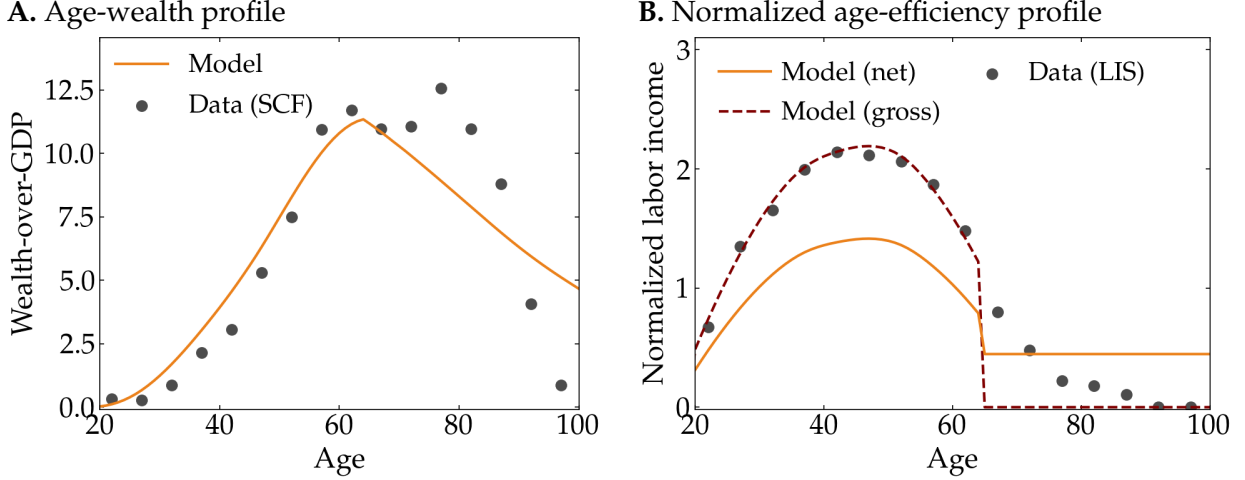


Figure 6: Calibration

Notes: Panel A presents the model age-wealth profile at our calibration (orange line) as well as the empirical age-wealth profile from the 2016 SCF used in section 2 (black dots). Panel B presents the empirical age-labor supply profile from LIS used in section 2 (black dots), as well as the model gross age-labor supply profile h_{jt} (dashed red line) and the net-of-taxes profile (orange line).

For future demographics, we use the population model and time-varying population shares π_{jt} from Gagnon et al. (2019). For future policies, we use our demographic projections to back out the social security financing shortfall, and consider various budgetary assumptions for how this shortfall will be covered. Our analysis starts from a sufficient statistic scenario where the assumptions of Proposition 1 are satisfied, and we cumulatively add more realism to this scenario.

Figure 7 plots the evolution of aggregate wealth and its decomposition under four different scenarios. Δ^{comp} and $\Delta^{comp,data}$ denote the compositional effect from the model and the data respectively, and they are by construction the same in all panels. Note that they are very similar, reflecting that our model replicates the initial wealth distribution well. Δ denotes the full model transition, and is the sum of the compositional term Δ^{comp} and the behavioral terms Δ^{beh} which reflect adjustments in asset and labor income profiles. In the appendix, we consider a large set of alternative variations of policies and parameters.

Panel A shows the results when the sufficient statistic result hold. In this case, the mortality rates and utility weights on children are constant over time, with net migration m_{jt} adjusting over time to fit the age projection π_{jt} . The government fixes taxes and social security system, and uses time varying transfers to adjust for changes in bequests received, with government consumption adjusting over time to keep the debt to output ratio constant (since the economy is open, the timing of this adjustment is irrelevant for household decisions and for W_t/Y_t). In line with Proposition 1,

the full counterfactual and the compositional effects coincide, and the contribution from changing age-wealth and age-labor supply profiles are both zero.

In panel B, we relax the assumptions that the utility weight on children is constant and that the government compensates for bequest variations. In this setup, future reductions in fertility rates increase wealth accumulation, since people have fewer children which reduce the incentive to consume early in life, and since people have fewer siblings, which increase the bequests received per person. These behavioral effects push up wealth an additional 14% of GDP by 2100.

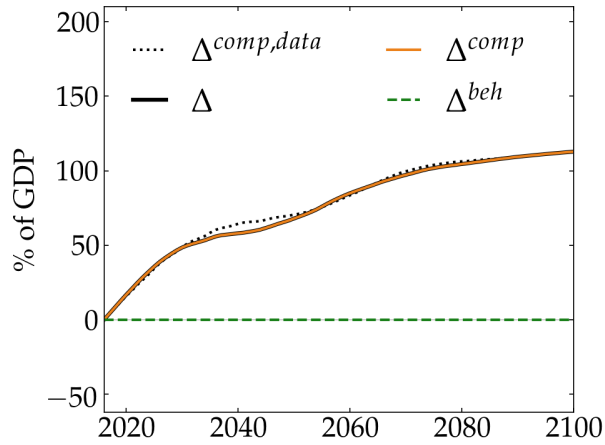
In panel C, we also relax the assumption that mortality rates are constant. The projected fall in mortality rates increases the incentive to save. The total behavioral effect is 16%, with mortality adding 2% to the effect in panel B.

Panel D is our baseline scenario where the government also reduces benefits and increases taxes in addition to reducing expenditures. In our preferred calibration, we assume that the government lets the retirement age increase by a month every year until reaching 70.²⁶ We assume that debt B/Y is constant and that for the remaining adjustment an equal amount of revenue/cost cutting comes from G/Y , τ^{ss} , τ , and \bar{d} . The behavioral effect from varying age-wealth profiles remains the same. However, the behavioral effect is counteracted by the increased labor supply coming from an increased retirement age, which reduces W_t/Y_t by 25 percentage points. Hence, the full behavioral effect is a decrease of 9 percentage points in W_t/Y_t in the baseline scenario.

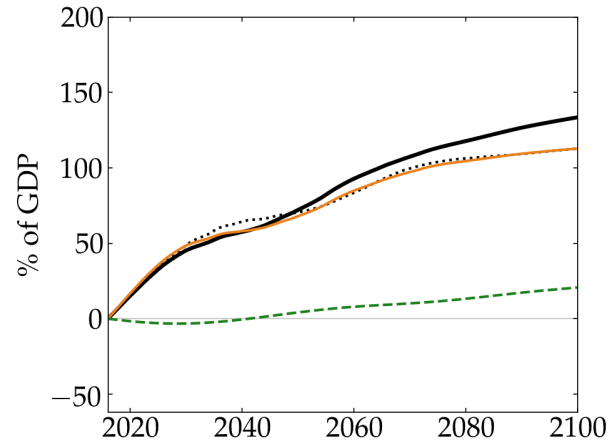
In the appendix D.7, we consider a much wider range of different policies and scenarios, for example introducing markups, shutting down migration, changing the intertemporal elasticity of substitution, and introducing actuarially fair annuities. The only variation that yields a substantial behavioral effect is if all fiscal adjustment goes through reducing social security benefits. This leads to a dramatic fall in social security benefits, and a 90 percentage point positive behavioral effect on asset accumulation. This finding echoes the findings in the pension reform literature that the type of fiscal adjustment matters for its macroeconomic effects (e.g. Feldstein 1974, Auerbach and Kotlikoff 1987, Kitao 2014), but also shows that compositional effects play a central role even when pension adjustments significantly alter wealth accumulation decisions.

²⁶This is in line with the CBO's main scenario (<https://www.cbo.gov/publication/54868>). In the appendix we also consider an increase of the retirement age to 80.

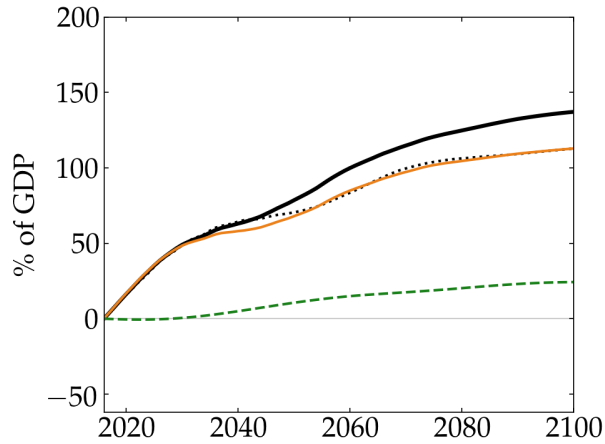
A. Sufficient statistic scenario



B. Adding time-varying bequests and children



C. Adding time-varying mortality



D. Baseline (adding taxes and benefits adjustment)

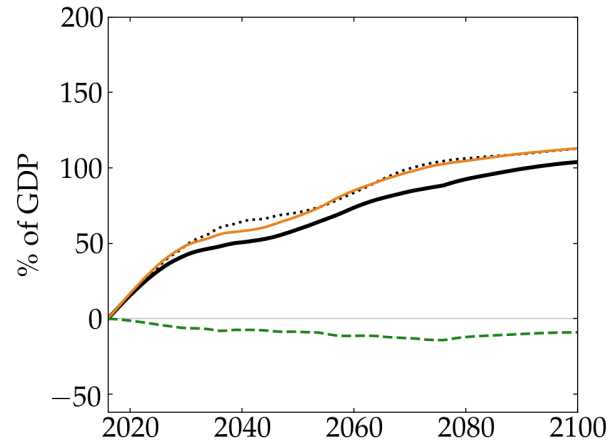


Figure 7: Evolution of W/Y and decomposition under different scenarios

Notes: This figure presents the change in aggregate wealth, Δ , between 2016 and 2100 (solid black line), as well as its decomposition into the compositional effect Δ^{comp} (solid orange line) and the behavioral effect Δ^{beh} (dashed green line). The black dotted line, $\Delta^{comp,data}$, is the shift-share from section 2. Panel A corresponds to the case where the sufficient statistic is exact, i.e. $\Delta = \Delta^{comp}$. In panel B, we let the amount of bequests received and the utility weight of children vary over time. In panel C, we also let mortality rates vary over time. Panel D presents our baseline scenario where the government keeps debt fixed and adjusts its four instruments equally, and the retirement age is increased by a month every year until 70.

4.5 Accounting for the historical evolution of W/Y in the United States

Armed with a calibrated model for the U.S. economy, we can consider the extent to which our model can explain the evolution of the wealth-to-GDP ratio since the 1950s. Recall from figure 2 that the compositional effect is able to account for the entire rise in W/Y . Of course, any historical exercise has to recognize that several other trends were happening at once over this time period. The widely talked about trends that come out of typical retrospective analyses of this period (eg. Farhi and Gourio 2018, Eggertsson et al. 2019) are: falling productivity growth, declining interest rates, and changing fiscal policy, in particular rising government debt.

In general, the calibration of the model to 1960 follows the one in section 4.3. We use the bequest curvature ν from this calibration and set λ to zero.²⁷ We then calibrate the discount factor β and the bequest factor ν to hit exactly the 1960 wealth-to-GDP ratio of from WID and the shift-share between 1950 and 2016. Appendix figure D.11 shows the paths for r_t , γ_t , B_t and G_t that are the basis of our exercise. Since we are interested in long-run trends we consider linear trends from 1960 to 2016. For the interest rate, we use the trend from our measure derived in appendix A. For government expenditures over GDP, debt over GDP, and technological growth, we use the same sources as in section 4.3, detrend the series and use a linear trend between 1960 and 2016.

Figure 8 displays our results. Given all these trends our model is in fact, able to account for overall increase in W/Y during this period, in spite of falling interest rates. At fixed demographics, the model would completely miss on the entirety of this increase, as the dashed line of panel A indicates. The reason for this fit is clear from Panel B: the fall in interest rates and the decline in productivity growth have roughly offsetting effects on wealth accumulation, so that the effect of demographics remains to "explain" the increase in W/Y (changing bond supply only plays a modest role). However, this exercise does not amount to fully accounting for the causal effect of demographics on the wealth-to-output ratio, since it is likely that part of the decline in the real interest rate was itself caused by demographic. To understand this connection, we need to turn to the determination of equilibrium world interest rates. We turn to this question next.

5 Global imbalances in the twenty-first century

Next, we calibrate the model separately for multiple countries, and endogenize the interest rate to clear the world asset market. We use the calibrated model to study how interest rates, wealth

²⁷As shown in the appendix, altruism towards children is not important quantitatively.

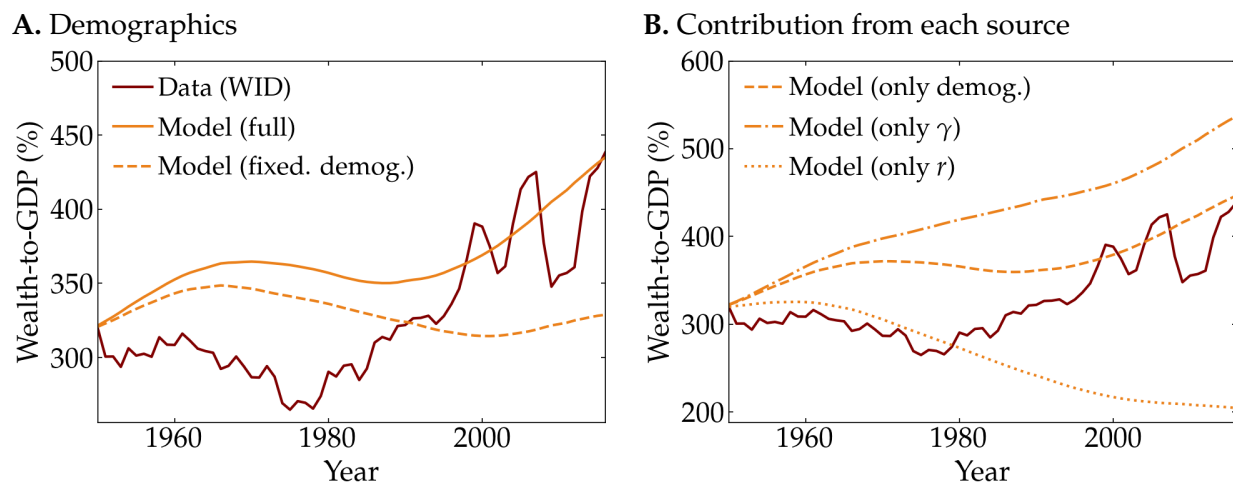


Figure 8: Historical experiment

Notes: In both panels, the solid red line depicts the wealth-to-GDP ratio from the World Inequality Database. Panel A presents the evolution of the wealth-to-GDP ratio from the model in the baseline scenario (solid orange line) and when mortality and fertility are fixed to their 1960 level (dashed orange line). Panel B presents the effect of holding all inputs fixed except demographics (solid orange line), debt-to-GDP (dashed line), technological growth (dot-dashed line), and the interest rate (dotted line).

levels, and global imbalances evolve in response to demographic changes.

5.1 Calibration

The model is calibrated to the 12 largest economies for which we have age-specific wealth and income data.²⁸ In general, the calibration procedure follows the one in Section 4.3. The key difference is that every country has its own discount factor β_c and bequest preference parameter Y_c . These are varied to fit W_c/Y_c and the compositional effects Δ_c^{comp} from Section 2.²⁹ For the government, we consider country-specific debt and expenditures-to-GDP ratios.³⁰ Benefits are calibrated using country-specific benefits-to-GDP ratios³¹, then the social security tax rate is adjusted to balance the social security system, and the total income tax rate is adjusted to balance the government flow budget constraint.

Table 2 shows the calibration results. The aggregate wealth-to-output ratios are fitted perfectly in every country, and the compositional effects are fitted perfectly in all but one country, where the constraint $Y \geq 0$ is binding.

²⁸ Among the 25 countries, we select the countries whose 2016 nominal GDP shares are above 1%.

²⁹ The compositional effect is targeted as an informative summary of the shape of the age-wealth profile.

³⁰ We take the expenditures-to-GDP ratios from the World Bank World Development Indicators, and the debt-to-GDP ratios from the IMF's World Economic Outlook.

³¹ From the OECD statistics database [OECD \(2019\)](#).

Table 2: World economy calibration

Country	Parameters		$\frac{W_c}{Y_c}$		Δ_c^{comp}		Other targets		
	β	Y	Model	Data	Model	Data	G/Y	B/Y	Ben./Y
AUS	0.99	0.78	5.09	5.09	1.32	1.32	0.19	0.40	0.09
CAN	0.96	2.34	4.63	4.63	1.14	1.14	0.21	0.92	0.05
CHN	0.95	4.63	4.20	4.20	2.81	2.81	0.14	0.44	0.04
DEU	0.95	3.41	3.64	3.64	0.89	0.89	0.20	0.69	0.10
ESP	1.00	0.00	5.33	5.33	1.64	1.55	0.19	0.99	0.11
FRA	0.98	1.68	4.85	4.85	1.31	1.31	0.24	0.98	0.14
GBR	0.97	2.15	5.35	5.35	1.49	1.49	0.19	0.88	0.07
IND	0.95	3.28	3.44	3.44	3.07	3.07	0.10	0.68	0.01
ITA	1.00	0.61	5.83	5.83	1.77	1.77	0.19	1.31	0.17
JPN	0.96	1.68	4.85	4.85	0.82	0.82	0.20	2.36	0.10
NLD	0.95	3.93	3.92	3.92	1.23	1.23	0.25	0.62	0.05
USA	0.97	1.82	4.38	4.38	1.13	1.13	0.18	0.77	0.07

Notes: This table presents the values of the subjective discount factor β and the bequest scaling factor Y calibrated to target the wealth-to-GDP ratio and the 2016-2100 shift-share from section 2. The last four columns present the values used for government expenditures, debt and total benefits paid all as a fraction of GDP.

5.2 Results

Figure 9 shows the results for interest rates, global wealth, and global imbalances. Qualitatively, interest rates fall, wealth levels rise, and rapidly aging countries improve their net foreign asset positions. However, the robustness of quantities differs across outcomes. The interest rate and wealth results vary considerably with parameter changes, whereas the results on net foreign asset positions are relatively robust.

In particular, for interest rates and wealth, panel A and C show that increasing the intertemporal elasticity of substitution from 0.5 to 2 reduces the interest rate decline from 100 basis points to 36 basis points, and reduces the increase in wealth from 53 percentage points to 16 percentage points. Panel B and D show that increasing the elasticity of substitution between capital and labor from 0.6 to 1.25 reduces the interest rate decline from 72 basis points to 60 basis points, while it pushes up the increase in wealth from 37 percentage points to 62 percentage points. For global imbalances, in contrast, panel E and F show that across all parameter variations, there is less than a 10 percentage point variation in the NFA increase of the most rapidly aging countries.

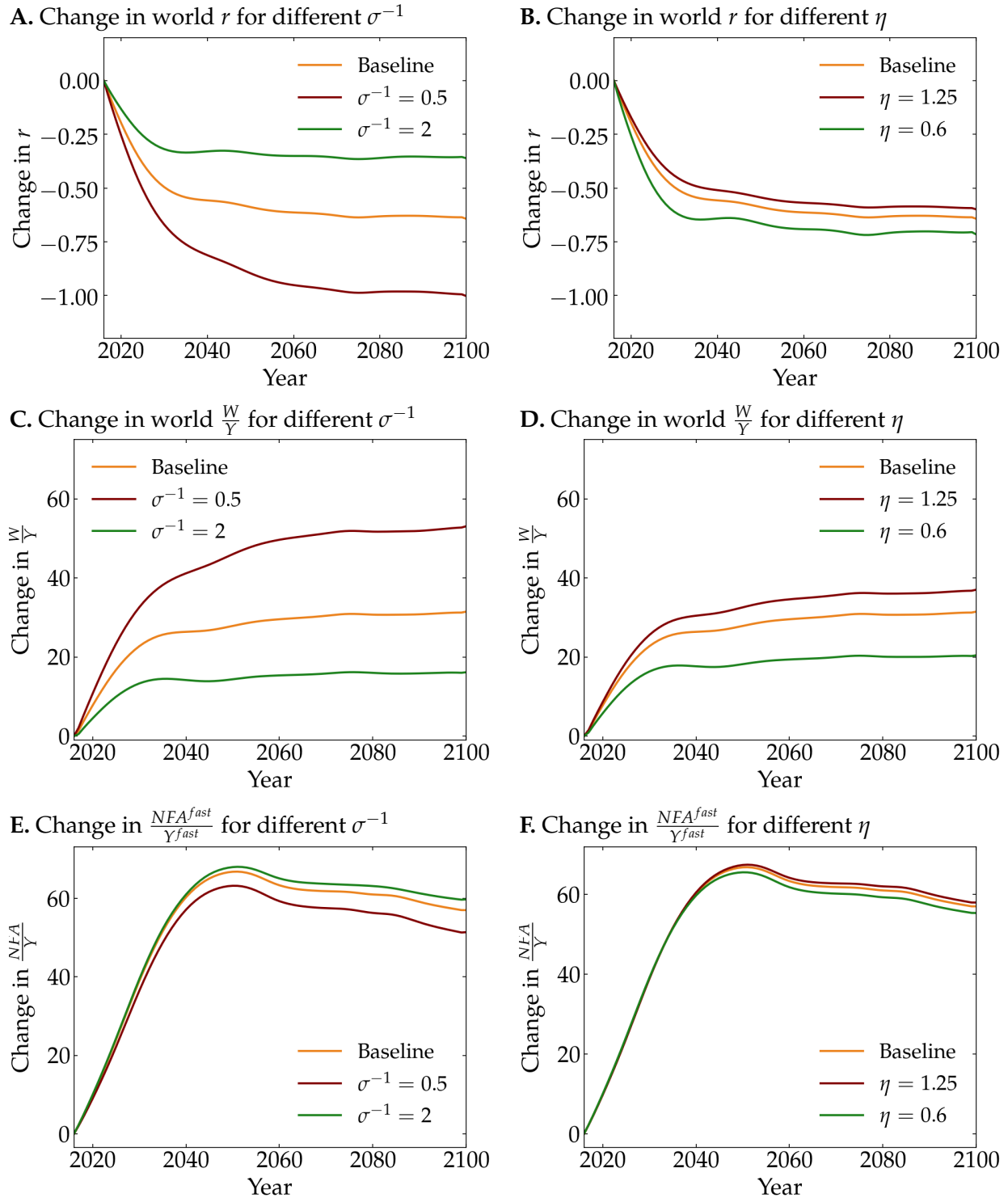


Figure 9: Change in the NFA for different σ^{-1} and η

Notes: This figure presents the change in NFA for fast aging countries (Australia, China, India, Italy, Spain), world wealth, and world interest rate between 2016 and 2100 from our model simulations. In panels A, C and E, the solid orange line correspond to our baseline case with an elasticity of substitution σ^{-1} of 2, the red line to the case $\sigma = 2$ and the green line to $\sigma = 0.5$. In panels B, D and F, the solid orange line correspond to our baseline case with a capital-labor elasticity of substitution η of 1, the red line to the case $\sigma^{-1} = 0.5$ and the green line to $\sigma^{-1} = 2$.

5.3 Asset supply and demand interpretation.

To interpret our findings, we use the asset supply and demand framework from section 3.2. For convenience, we reproduce its main equations (21)-(23)³²:

$$\Delta r = -\frac{1}{\bar{\epsilon}^{d,r} + \bar{\epsilon}^{s,r}}(\bar{\Delta}^{comp} + \bar{\Delta}^{beh|r}), \quad (26)$$

$$\Delta \left(\frac{W_{world}}{Y_{world}} \right) = \frac{\bar{\epsilon}^{s,r}}{\bar{\epsilon}^{d,r} + \bar{\epsilon}^{s,r}}(\bar{\Delta}^{comp} + \bar{\Delta}^{beh|r}), \quad (27)$$

$$\Delta \left(\frac{NFA_c}{Y_c} \right) = (\Delta_c^{comp} + \Delta_c^{beh|r}) - (\bar{\Delta}^{comp} + \bar{\Delta}^{beh|r}) + [\epsilon_c^{d,r} + \epsilon_c^{s,r} - (\bar{\epsilon}^{d,r} + \bar{\epsilon}^{s,r})] \Delta r. \quad (28)$$

While (26)-(28) are based on steady-state considerations, table 3 shows that they provide a very good approximation of the full general equilibrium transition, especially for interest rates.

Since Δ^{comp} is large and positive in every country and $\Delta_c^{beh|r}$ tends to be small, and since $\bar{\epsilon}^{d,r} + \bar{\epsilon}^{s,r}$ is positive, (26) and (27) explain why demographics cause interest rates to fall and wealth ratios to rise. This finding is robust to the details of the model specification, as long as the model correctly captures the compositional effect. However, the exact magnitudes depend on the interest rate sensitivities of asset demand and supply. These are largely orthogonal to the partial equilibrium shock from demography.³³ Instead, they depend on deep parameters that are difficult to pin down, such as the intertemporal elasticity of substitution and the elasticity of substitution between capital and labor. This explains the disparate findings in panels A-D in figure 9 (in section 6.4, we show that different sensitivities also help explain disparate findings in the literature on the interest rate effect of demographics).

For global imbalances, the logic is different. Equation (28) does not depend on the level of sensitivities, but on their difference across countries. While we allow for a number of country-specific parameters in our calibration, the differences in sensitivities are small across all our calibrations, and not correlated with the compositional effects. Combined with our finding of small behavioral effects at constant interest rates $\bar{\Delta}^{beh|r}$, this implies that a purely empirical object, the demeaned compositional effect $\Delta_c^{comp} - \bar{\Delta}^{comp}$, accurately forecasts the general equilibrium effect of population aging on the net foreign asset positions of country c . Panel A in figure 10 illustrates the close correspondence between the compositional effect and the full equilibrium change in the net

³²Recall that $\bar{\Delta}^{comp} + \bar{\Delta}^{beh|r}$ denotes the partial equilibrium changes in W/Y from demographics, and that $\bar{\epsilon}^{d,r} + \bar{\epsilon}^{s,r}$ denotes the sum of asset demand and supply sensitivities to change interest rates. As before, bars denote output-weighted averages.

³³In the appendix, this argument is formalized by showing that we can hit the age-wealth profiles regardless of the value of the intertemporal elasticity of substitution. Hence, σ is not identified by cross-sectional data. Clearly, cross-sectional data does not identify η either, since η can only be disciplined if there are changes in the cost of capital.

Table 3: Decomposition and interest rate effect

	Baseline	$\eta = 0.6$	$\eta = 1.25$	$\sigma^{-1} = 0.5$	$\sigma^{-1} = 2$
$\bar{\Delta}\pi$	159	158	158	147	159
$\bar{\epsilon}_d$	135	134	134	63	276
$\bar{\epsilon}_s$	65	41	86	69	69
<i>Predicted in steady state</i>					
$\Delta \frac{W^w}{Y^w}$	52	37	62	77	32
Δr	-0.80%	-0.90%	-0.72%	-1.12%	-0.46%
<i>Transitions</i>					
$\Delta \frac{W^w}{Y^w}$	31	20	37	53	16
Δr	-0.64%	-0.72%	-0.60%	-1.00%	-0.36%

Notes: This table presents implied changes in world wealth-to-GDP and world interest rates from equations (21) and (22). The first column corresponds to the baseline scenario, and the next four columns to the scenarios with different values of the capital-labor elasticity of substitution η or the intertemporal elasticity of substitution $\sigma^{-1}c$. The first line corresponds to the world-level shift-share $\bar{\Delta}^{comp}$. The next two lines present sensitivities of world asset demand and supply at our the 2016 calibration. We then report the values of the predicted change in world wealth and world interest rate implied by equations (21) and (22). The last two lines report the actual changes in wealth-to-output and in the interest rate from our transitional dynamics experiments.

foreign asset position.

5.4 Global imbalances: validation and projection

To validate the predictive power of the demeaned compositional effect, we use historical data on net foreign asset positions. Panel B in figure 10 shows that between 1970 and 2015, the calculated compositional effect is positively correlated with changes in net foreign asset positions. In particular, we note that Japan has aged most rapidly, and has also accumulated the biggest net foreign asset position.

We also look ahead and use our model to project the future development of global imbalances based on demographic developments. Figure 11 shows the results. We see that future demographics imply dramatic effects on net foreign asset positions relative to the historical experience. In particular, panel A shows that the aging of first China and then of India lead them to accumulate considerable net foreign asset positions. The relatively rapid aging of these large countries in turn implies that the positive asset positions of current large savers, such as Germany and Japan, reverse, while the US continues into more negative territory. Last, panel B shows that these predictions are little altered if the demeaned compositional effect is used in place of a full model solution.

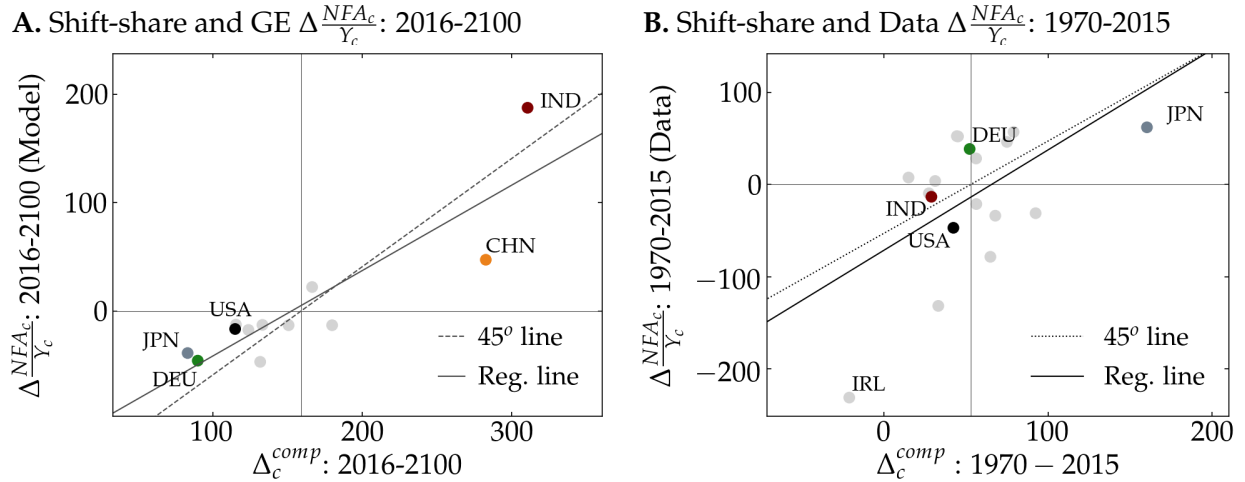


Figure 10: Change in the NFA

Notes: Panel A compares the shift-share between 2016 and 2100 (x-axis) to the actual change in the NFA in the model simulations (y-axis) for the 12 countries in our analysis. The vertical line indicates $\bar{\Delta}_c^{comp}$. Panel B compares the shift-share between 1970 and 2015 (x-axis) to the change in NFA from the updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007) (y-axis). In both graphs, the black dotted line is a 45° line with intercept $-\bar{\Delta}_c^{comp}$ and the vertical line indicates $\bar{\Delta}_c^{comp}$.

Panel B of figure 10 shows the relationship between historical compositional effects and historical changes in net foreign asset positions. The 45 degree line shows the demeaned compositional effect. Historically, most countries have had similar compositional effects. However, for Japan and Ireland that have substantial deviations, the net foreign asset positions change in the predicted direction, with Japan accumulating a large positive position, and Ireland a large negative position.³⁴ Overall, there is a positive correlation between compositional effects and the change in net foreign asset positions.

6 Discussion

6.1 Shift-share in flows instead of stocks

In our compositional analysis and calibration, we focus on wealth holdings by age. These are stock measures. A large literature has taken an alternative route, focusing instead on age-specific savings rates, i.e., on flow measures (Summers and Carroll 1987, Auerbach and Kotlikoff 1990, Bosworth et al. 1991).

Technically speaking, our approach could start from flows instead of from stocks. In analogy

³⁴In the case of Ireland, there is a worry that the negative NFA reflects tax arbitrage.

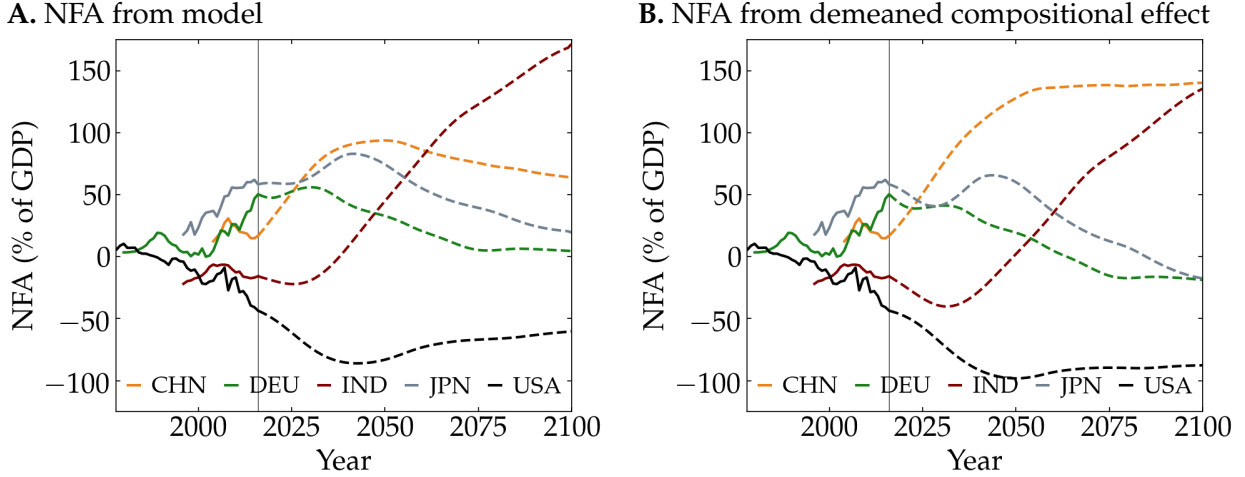


Figure 11: NFA over GDP

Notes: Panel A presents the empirical NFA-to-GDP ratio as presented in figure 1 until 2016, and from 2016 on the predicted evolution of this ratio according to our main model experiment. Panel B repeats the same data point until 2016 and then presents the country-specific demeaned shift-shares until 2100.

with (7), the net savings rate can be expressed in terms of the savings of different age groups

$$s = \frac{\sum_j \pi_{jt} s_{jt}}{\sum_j \pi_{jt} h_{jt}}, \quad (29)$$

where s_{jt} is net personal savings by age, normalized by labor productivity Y_t/L_t . Given (29), changes in the savings rate can be decomposed into compositional and behavioral effects, just as with W/Y in (9). Moreover, since the steady-state wealth-to-output ratio W/Y and the net savings rate s are related through

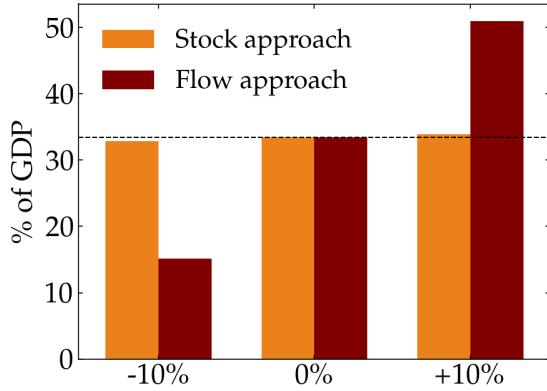
$$\frac{W}{Y} = \frac{s}{g/(1+g)}, \quad (30)$$

any steady-state statement about savings can be translated into a steady-state statement about wealth, and vice versa.

Despite the strong links between the two approaches, the stock approach is more attractive because it is less sensitive to measurement error than the flow approach. The flow approach relies on the measurement of personal savings, which is the difference between disposable income and consumption, two large and independently measured quantities. Small relative measurement errors in income and consumption translate into large relative errors in the measurements of savings. For example, in 2019, aggregate US personal savings were 6.1% of GDP – the difference between 76.7% of GDP in disposable personal income and 70.6% of GDP in personal outlays.³⁵ If actual income were only one percentage point higher, and actual consumption only one percent-

³⁵Table 1.1.5. and 2.1 in the 2019 full year national accounts.

A. Change in wealth-to-GDP



B. Change in savings-to-GDP

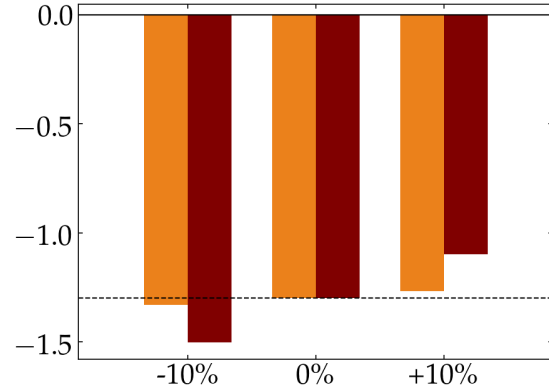


Figure 12: Effects of measurement error on wealth and savings shift-shares

age point lower, the actual net savings rate would be 8.1% instead of 6.1%. This shows that 1-2% of measurement error in income and consumption can translate to a 30% mismeasurement in the savings rate. With wealth measurement, there is no analogous amplification of error.

To explore the differential effect of mismeasurement, we do both a flow-based and a stock-based shift-share using model output data, and test the effect of introducing a small degree of mismeasurement in consumption and wealth profiles.³⁶ Figure 12 shows the results. We see that the two approaches are equivalent without measurement errors. However, when we introduce measurement error, there are barely any changes in the stock-based shift share, whereas the flow-based shift-share varies from 15% to 50% for the wealth-to-GDP measure, and -1.5 pp to -1.3 pp for savings. The falling growth rate of GDP ensures that W/Y increases even though the savings rate falls. This shows that, even if one had very accurate measures of the predicted savings rate, it would not be sufficient on its own to conclude anything about the pressure of demographics on asset demand and therefore equilibrium returns.

6.2 Multiple assets

Our main analysis has only one type of asset. However, our methodology extends to the case when there are multiple assets with different return profiles. For concreteness, we analyze a setting where there is one safe and one risky asset, that are in global supplies B^{world} and K^{world} respectively.

³⁶Formally, we hold aggregates constant, and tilt the wealth and consumption profile so that they go from 10% undermeasurement for young people to 10% overmeasurement for old people (and vice versa). See appendix F.1 for details.

This section summarizes the key conceptual points, while appendix F.2 provides more details.

To separate wealth accumulation from risk allocation, we define two new assets that span the original asset space. The first asset is a globally representative portfolio with weights $\left(\frac{B^{world}}{A^{world}}, \frac{K^{world}}{A^{world}}\right)$ on the safe and risky asset respectively. The second asset places weights $(1, -1)$ on two assets, that is, it is a zero priced asset consisting of a long position on the safe asset and a short position on the risky asset. The expected return of the first new asset is the average global return on wealth, and the expected return on the second new asset is the negative risk premium.³⁷ A portfolio with b risk-free assets and k risky assets can then be expressed as $b + k$ units of the first asset, and $b - (b + k)\frac{B^{world}}{K^{world}}$ units of the second asset. Note that the second asset can be interpreted as "excess demand for safety", since it captures the holdings of safe assets in excess of what would predicted based on total wealth and the global portfolio share of safe assets.

In figure 13, we use data from the 2016 SCF to calculate the excess demand for safety by age. Panel A shows the raw holdings of risky and safe assets, while panel B shows the resulting excess demand for safety. Panel C calculates the compositional effect of demographic change on aggregate excess safety demand

$$\Delta_t^{safe,comp} = \frac{\sum_j \pi_{jt} a_{j0}^{safe}}{\sum_j \pi_{jt} h_{j0}} - \frac{\sum_j \pi_{j0} a_{j0}^{safe}}{\sum_j \pi_{j0} h_{j0}} \quad a_{j0}^{safe} = b_{j,2016} - a_{j,2016} \frac{B^{world}}{A^{world}}, \quad (31)$$

obtained by rolling forward the age distribution across fixed profiles of labor supply and a_j^{safe} . From panel B, we see that households are net suppliers of safe assets at young ages, but that they grow to be net demanders as their mortgage loans shrink and their retirement accounts grow. From panel C, we see that compositional effects push up the US demand for safety, but only 12 percentage points of GDP (that is, the increase could be negated by an asset swap that replaced 12% of GDP in risky assets with 12% of GDP in government bonds).³⁸

To analyze general equilibrium outcomes, we note that the net foreign asset position by asset class is

$$\frac{NFA_c(r^w, r^{safe}, \Theta)}{Y_c} = \frac{A^d(r^w, r^{safe}, \Theta)}{Y_c} - \frac{A^s(r^w, r^{safe})}{Y_c}, \quad (32)$$

where NFA , A^d , and A^s are vectors of net foreign asset positions, household asset holdings, and

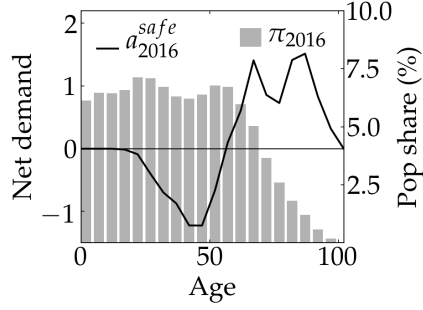
³⁷Here we assume that the differential returns of the assets reflect a utility from owning a safe asset, as in e.g. [Stein \(2012\)](#), but we conjecture that the same implications would follow from a model where the differential returns stem from differences in liquidity, as in e.g. [Kaplan, Moll and Violante \(2018\)](#). An interesting extension is to consider an explicitly risk-based model, in which case steady-state asset holdings and global imbalances should be interpreted as long-run averages.

³⁸However, note that in analogy with the single-asset case, (31) only gives the full counterfactual if excess safety demand profiles a_j^{safe} do not change over time. In general, we need a fully specified model to determine the behavioral effect $\Delta^{beh,safe}$.

A. Equity and risk-free profiles



B. Net safety demand



C. Shift-share

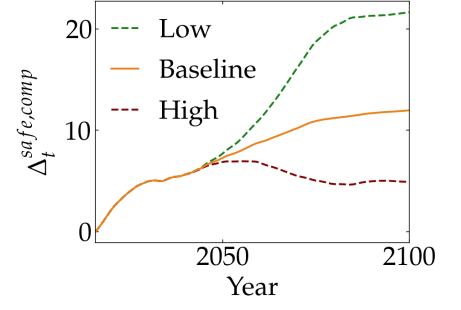


Figure 13: Excess demand for safety

Notes: Panel A depicts the age profiles of risk-free (red line) and risky holdings (orange line). Appendix F.2 details their construction. Panel B presents the net safety asset holdings computed as $a_j^{safe} = b_j - \frac{B^{world}}{A^{world}}(k_j + b_j)$ where we take $B^{world}/A^{world} = 18\%$ from our 2016 world economy calibration. Panel C reports the shift-share as defined in equation (31) between 2016 and 2100.

asset supplies respectively, with one element per asset class. The equilibrium is defined by a zero net foreign asset position for the world, and the first order change in asset levels, asset returns, and global imbalances satisfy multidimensional analogues of (21)-(23).

$$\Delta r = -(\bar{\mathcal{E}}^{d,r} + \bar{\mathcal{E}}^{s,r})^{-1}[\bar{\Delta}^{comp} + \bar{\Delta}^{beh|r}] \quad (33)$$

$$\Delta A_c^d = \Delta_c^{comp} + \Delta_c^{beh|r} + \mathcal{E}_c^{d,r} \Delta r \quad (34)$$

$$\Delta \left(\frac{NFA}{Y^c} \right) = (\Delta_c^{comp} + \Delta_c^{beh|r}) - (\bar{\Delta}^{comp} + \bar{\Delta}^{beh|r}) + [\mathcal{E}_c^{d,r} + \mathcal{E}_c^{s,r} - (\bar{\mathcal{E}}_c^{d,r} + \bar{\mathcal{E}}_c^{s,r})] \Delta r.. \quad (35)$$

On a conceptual level, the similarity between (21)-(23) and (33)-(35) highlight the similarity between the multi-asset and single-asset case. The left-hand sides, Δ_c^{comp} and $\Delta_c^{beh|r}$ from (21)-(23) all have vector analogues in (33)-(35). The sensitivities $\epsilon^{r,d}$ and $\epsilon^{r,s}$ from (21)-(23) correspond to $\mathcal{E}_c^{d,r}$ and $\mathcal{E}_c^{s,r}$, which are 2x2 matrices with both own-price and cross-price sensitivities of asset demand and supply.

In practice, the key challenge with a multi-asset analysis is to find a credible model of portfolio choice and asset creation that can discipline the behavioral effects $\Delta_c^{beh|r}$ and the sensitivity matrices $\mathcal{E}_c^{d,r}$ and $\mathcal{E}_c^{s,r}$. One recent example of work that relate to this question is that of [Kopecky and Taylor \(2020\)](#), which integrates life-cycle portfolio choice models with demographic change.

6.3 Housing

Housing accounts for a sizable fraction of household assets. For instance, in the United States in 2016, the gross value of owner-occupied housing was roughly 25% of aggregate household net

worth, and home equity was slightly above 15% of net worth.³⁹ Although our model of household wealth accumulation does not explicitly account for housing, our calibration includes housing in total assets, capital income, and consumption. Implicitly, this assumes that there is no difference between owner-occupied and rental housing, and that rental markets are frictionless. While this is clearly an approximation, we are still able to achieve a good fit to the age profile of household wealth accumulation, perhaps because households are able to separate their wealth accumulation and housing decisions by choosing the terms of their mortgages.

One important feature of housing is that it directly provides housing services as part of household consumption, rather than being combined with labor in an ordinary production function. If we assume that consumption is a Cobb-Douglas aggregate of housing and other goods, then the demand for housing is proportional to consumption rather than labor. This means that the housing component of aggregate asset supply $A^s/Y(r)$ includes the ratio of consumption to output, C/Y , which is affected directly by demographics. In principle, the impact of demographics on this ratio can be approximated by a ratio of two shift-shares (on consumption and labor), just like our compositional effect. Due to the measurement issues involved with life-cycle consumption, however, we do not attempt this (see section 6.1). The likely result of such an exercise would be to slightly shrink the magnitude of our effects, since the housing component of asset supply falls less than other components with aging.

6.4 Reconciling different findings on falling real rates

Our methodology can be used to interpret disparate findings on demographics and the falling real rate. For illustration, we use the asset supply and demand framework from section 3.2 to interpret the findings in Eggertsson et al. (2019) (EMR) and Gagnon et al. (2019) (GJLS), two recent papers that find very different effects of demographics on the real interest rate.

The results are displayed in table 4. The first line shows the different effects of demographics in the two models: between 1970 and 2015, Δr is -3.19 pp in EMR, but only -0.71 pp in GJLS. Δr can be decomposed into compositional effects, behavioral effects, and asset supply and demand sensitivities, using the steady-state approximation (21). We see that the approximation is good, and that the large interest rate effect in EMR reflects a low asset demand sensitivity, which is an order of magnitude smaller than that in GJLS, 13 versus 148. There are some additional differences between the papers in the behavioral effects and the asset supply sensitivities, but these are

³⁹See table B.101 from the Flow of Funds accounts, lines 4, 33, and 40.

Table 4: Decomposition of the change in r

	Eggertsson et al. (2019)	Gagnon et al. (2019)
Time-period	1970-2015	1970-2015
<i>GE transition</i>		
Δr^{GE}	−3.19%	−0.71%
<i>Steady-state approximation</i> $\Delta r^{SS} = \frac{-(\Delta^{comp} + \Delta^{beh r})}{\epsilon^d + \epsilon^s}$		
Δr^{SS}	−3.46%	−0.49%
Δ^{comp}	52 pp	51 pp
$\Delta^{beh r}$	23 pp	39 pp
ϵ^s	8	33
ϵ^d	13	148

dwarfed by the differences in asset demand sensitivities.⁴⁰

In depending strongly on the asset demand sensitivity, the literature on demographics and interest rates connects to the literature on wealth and capital taxation, where the asset demand sensitivity has been identified as a sufficient statistic for optimal taxation (Saez and Stantcheva 2018). The asset demand sensitivity has also been identified as important in regulating the effects of automation (Moll, Rachel and Restrepo, 2019). These literatures all share the empirical challenge that cross-sectional data alone cannot identify the asset demand sensitivity. In appendix E.1, we use the results in Jakobsen, Jakobsen, Kleven and Zucman (2020) on the accumulation effect of wealth taxes to develop one way to discipline the sensitivity, and we find a relatively high value. However, looking ahead, even more sharper estimates of the asset demand sensitivity would be very valuable.

6.5 Wealth-income ratios and inequality

As demonstrated in the previous sections, population aging will have very large implications for aggregate wealth. But how will it affect the distribution of wealth? In principle, since wealth is unequally distributed across ages, with young agents holding almost none of it, a rise in aggregate wealth can increase wealth inequality via a between-age effect (Deaton and Paxson 1995). This effect, in turn, could contribute to increasing overall inequality.⁴¹

To quantify the contribution of this effect, we perform a within-between decomposition of log wealth, historically in the data, and in our model until the end of the twenty first century. We

⁴⁰The importance of the asset demand sensitivity echoes our findings on the intertemporal elasticity of substitution in section 5.

⁴¹Antunes and Ercolani (2020) show that an general equilibrium OLG model with idiosyncratic risk can account for the change in between-age wealth inequality in the U.S.

express the variance of log wealth as

$$\sigma_{w,t}^2 = \sum_j \sum_s \pi_{jt} \cdot p_{sjt} (w_{sjt} - \bar{w}_{jt})^2 + \sum_j \pi_{jt} (\bar{w}_{jt} - \bar{w}_t)^2 \quad (36)$$

where w_{sjt} denotes the log-wealth of individual s who is j -years old in year t , \bar{w}_{jt} is average log-wealth by age in year t , and \bar{w}_t is average log wealth in year t . The first term corresponds to the *within-age* component, capturing variation within age groups, and the second to the *between-age* component, capturing variation across age groups.

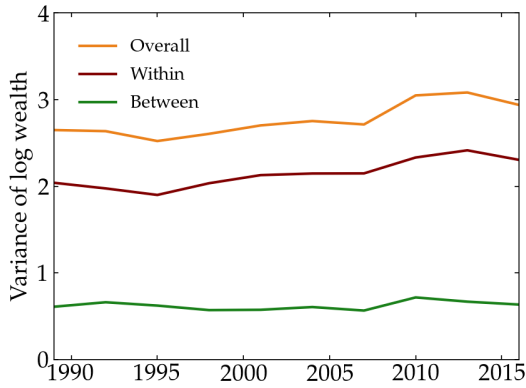
Panel A of Figure 14 presents the results of this decomposition using SCF data from 1989 to 2016. We treat the data exactly as we did to construct the age-wealth profiles in section 2. Two features of the data are salient. First, more than 75% of the overall variance is explained by the within-age component in any given year. Second, a modest increase of the overall variance between 1995 and 2016 was driven by an increase of the within-age component while the between-age component was relatively stable.

Panel B performs the same decomposition in our baseline scenario between 2016 and 2100. Since the population shares π_{jt} are the same, and because the model and the data age-wealth profiles are closely related (see figure 6), the between-age component is of similar magnitude in the model and in the data. The model generates a slightly lower within-component, resulting in a slightly lower overall variance of log wealth than in the data. The most remarkable feature of this graph is that both lines are almost completely flat for the entire time period. This indicates that aging has very little to do, if at all, with increasing wealth inequality. Alternative mechanisms that generate rises in inequality are needed, such as changes in income processes (Auclert and Rognlie 2018), technology (Straub 2017) or automation (Moll et al. 2019).

7 Conclusion

We use a sufficient statistic approach to quantify the effects of population aging on wealth accumulation, equilibrium interest rates, and capital flows. A simple calculation that rolls forward projected population distributions over fixed age profiles of assets and income constitutes a sufficient statistic for the transition dynamics of the wealth-to-GDP ratio in a special case of our model. Our sufficient statistic approach shows that the macroeconomic effect of aging on aggregate wealth accumulation is large and heterogeneous across countries. This calculation remains a very useful input into the calculation of equilibrium wealth and interest rates away from this special case as it

A. Data (SCF)



B. Model

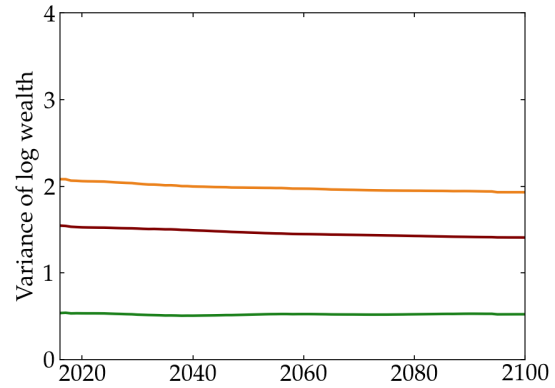


Figure 14: Variance of log wealth

Notes: Panel A presents the variance of the log of wealth-to-output calculated from the SCF for waves 1989 to 2016, and its decomposition into a *within* age groups component and a *between* component according to equation (36). To ensure comparison between the data and the model, we calculate wealth-to-output using the same procedure we used to calculate the age-wealth profiles in section 2. Panel B presents a similar variance decomposition exercise in the model for the baseline social security scenario. We discard all observations below 1% of average wealth both in the data and the model.

approximates closely the evolution of wealth-to-GDP due to demographic change at fixed interest rate. In an integrated economy, population aging will push the equilibrium rate of return down, and generate large global imbalances.

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Appendix to "Demographics, Wealth and Global Imbalances in the Twenty-First Century"

A Appendix to Section 1

The total return on wealth r_t for the US from 1950–2016 in panel C of figure 1 is constructed as follows. We take:

- Capital K_t as total private fixed assets at current cost from line 1 of Table 2.1 in the BEA's Fixed Assets Accounts (FA).
- Output Y_t as gross domestic product from line 1 of Table 1.1.5 in the BEA's National Income and Product Accounts (NIPA).
- Wealth W_t as "net private wealth" from the World Inequality Database (WID).
- Net foreign assets NFA_t as the net worth of the "rest of the world" sector from line 147 of Table S.9.a in the Integrated Macroeconomic Accounts (IMA).⁴²
- Government bonds B_t as gross federal debt held by the public, from the Economic Report of the President (accessed via fred at FYGFDPUB).
- The safe real interest rate r_t^{safe} as the 10-year constant maturity interest rate—from Federal Reserve release H.15 (accessed via fred at GS10), extended backward from 1953 to 1950 by splicing with the NBER macrohistory database's yield on long-term US bonds (accessed via fred at M1333BUSM156NNBR)—minus a slow-moving inflation trend, calculated as the trend component of annual HP-filtered inflation in the PCE deflator, with smoothing parameter $\lambda = 100$.
- Net capital income $(s_K Y - \delta K)_t$ as corporate profits plus net interest and miscellaneous payments of the corporate sector (sum of lines 7 and 8 in NIPA Table 1.13), plus imputed net capital income from the noncorporate business sector, under the assumption that the ratio of net capital income to net factor income (line 11 minus line 17) in the noncorporate business

⁴²This is very similar to the standard net international investment position computed by the BEA, but is chosen because it offers a longer time series.

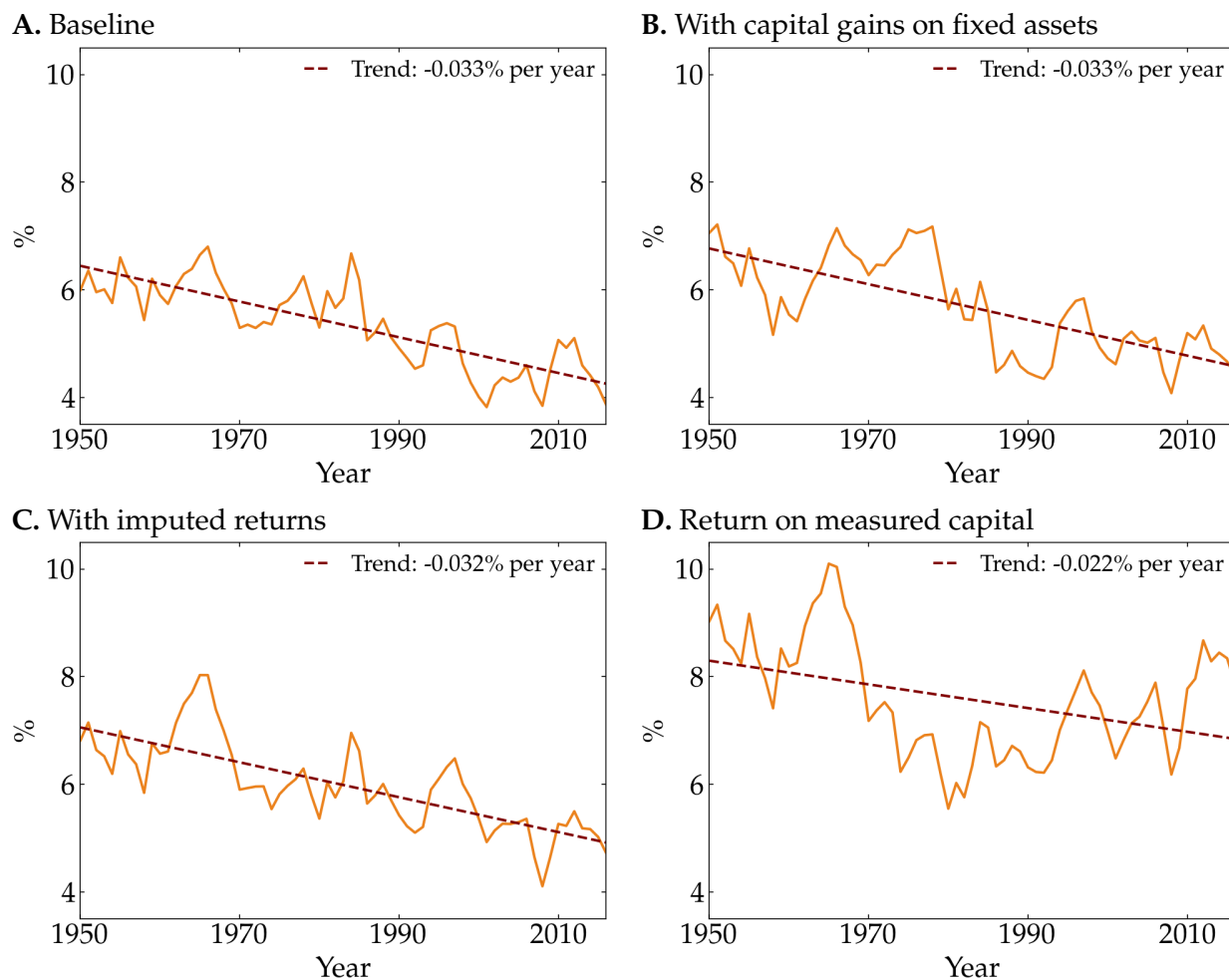


Figure A.1: Alternative ways of constructing total return on wealth in US

Notes: Panel A gives our baseline series for the total return on wealth in the US, as described in the text. Panel B adds capital gains on fixed assets, as measured in the fixed assets accounts. Panel C imputes an additional return on unmeasured wealth $W_t - K_t - B_t - NFA_t$ equal to trend growth. Panel D takes our baseline capital income series and divides it by capital measured in the fixed assets accounts.

sector is the same as the ratio of net capital income to net factor income (line 3 minus line 9) in the corporate sector.⁴³

We then calculate our baseline total return on wealth series as

$$r_t \equiv \frac{(s_K Y - \delta K)_t + r_t^{safe} B_t}{W_t - NFA_t} \quad (37)$$

i.e. as the ratio of net capital income plus real interest income on government debt to domestic assets. This calculation gives the total return on private wealth, excluding changes in asset valua-

⁴³This imputation is a common way of splitting mixed income within the noncorporate sector between labor and capital, used e.g. by Piketty and Zucman (2014).

tion, under the assumption that the average return on net foreign assets is the same as the average return on private wealth.⁴⁴

This baseline r_t and its trend are displayed in panel A of figure A.1. The other three panels provide alternative ways to calculate r_t .

Panel B adds a slow-moving trend of capital good inflation minus PCE inflation, which we denote by π_{Kt} :

$$r_t \equiv \frac{(s_K Y - \delta K)_t + r_t^{sae} B_t + \pi_{Kt} K_t}{W_t - NFA_t}$$

Average inflation of goods in the capital stock is inferred by taking the ratio of changes in the nominal stock (FA Table 2.1, line 1) and changes in the quantity index (FA Table 2.2, line 1), and as with PCE inflation above, we take the slow-moving trend component using the HP filter with $\lambda = 100$. This accounts for expected capital gains on fixed capital (assuming that the expectation follows the trend).

Panel C assumes that there is some unmeasured return on the portion of wealth $W_t - K_t - B_t - NFA_t$ that cannot be accounted for by capital, bonds, or net foreign assets, which it sets equal to the trend real GDP growth rate g_t :

$$r_t \equiv \frac{(s_K Y - \delta K)_t + r_t^{sae} B_t + g_t (W_t - K_t - B_t - NFA_t)}{W_t - NFA_t}$$

where g_t is again calculated using the HP filter with $\lambda = 100$. If $W_t - K_t - B_t - NFA_t$ is the capitalized value of pure rents in the economy, for instance, its value might be expected to grow in line with output.

Finally, panel D simply divides net capital income by the measured capital stock:

$$r_t \equiv \frac{(s_K Y - \delta K)_t}{K_t}$$

Note that despite these alternative constructions, the 1950–2016 trends in panels A, B, and C of figure A.1 are almost identical: -.033, -.033, and -.032 percentage points, respectively. All show a steady decline.

The return on capital in panel D, on the other hand, is quite different: it has a smaller long-term trend decline, of -.022 percentage points per year, and since roughly 1980 it actually displays a mild increase. This post-1980 pattern of a constant or increasing return on capital has been widely

⁴⁴This can be seen by rearranging (37) as $r_t = \frac{s_K Y - \delta K + r_t^{sae} B + r NFA}{W}$, which gives the total return r_t on private wealth if r_t equals the return on NFA_t . We take this route because data on capital income from foreign assets is not comparable to domestic data; for instance, the national accounts only measure dividend payments, not the total net capital income, on foreign equities (other than FDI) held in the US, and also only measure nominal rather than real interest payments on bonds. The trend in r_t , however, is not very sensitive to alternative assumptions on the average rate for NFA_t .

remarked upon in the literature—for instance, [Gomme et al. \(2011\)](#), [Farhi and Gourio \(2018\)](#), [Eggertsson, Robbins and Wold \(2018\)](#). The main source of the disparity between panels A–C and panel D is that the former divide by wealth, while the latter divides only by measured capital. Since our primary object of interest is wealth, we prefer the former convention. Another advantage of using wealth in the denominator is that capital may be imperfectly measured in the fixed assets accounts.

B Appendix to Section 2

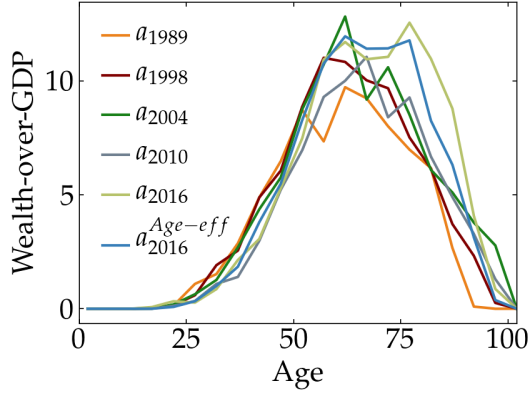
B.1 Data sources and construction of age-profiles

Demographics For the United States, we use data on survival probabilities, ϕ_{jt} , and population distributions, π_{jt} , from [Gagnon et al. \(2019\)](#). They start from a given demographic structure for the US population in 1899, and let population evolve according to deaths rates, net migration, and either births or fertility rates. They collect mortality rates by age and birth cohort from the US Social Security Administration’s publication “Life Tables for the United States Social Security Area 1900-2100”. Live births are provided by the National Center for Health Statistics (NCHS) from 1972 to 2014, for before 1972 they construct live births by adjusting for infant mortality the infant population estimates from the Census Bureau’s intercensal population estimates, and from 2014 on they rely on the UN World Population Prospects. Net migration is created to ensure that the size and composition of the total population closely matches the Census Bureau’s historical estimates and projections of population from 1900 to 2060. For after 2060 they assume that their estimates of net migration linearly converges to the estimates of the Census for 2060. They also derive a measure of the number of dependents per representative adult from data on live births per mother of a given age from the Center for Disease Control (CDC), adjusting for the median age difference between men and women at time of their first marriage.

For the rest of the world, we use population distributions from the 2019 UN World Population Prospects.⁴⁵ They provide cross-country measures of population by age groups and total population that we use to derive population shares between 1950 and 2100. From 2020 on, they consider three scenarios for fertility: the medium scenario, a low fertility scenario where fertility is 0.25 births per mother lower than the medium scenario for 2020-2025, 0.4 for 2025-2030, and 0.5 for 2030-2100, and a high fertility scenario where fertility is 0.25 births per mother higher than the

⁴⁵<https://population.un.org/wpp/>

A. Age-wealth profiles



B. Age-labor income profiles

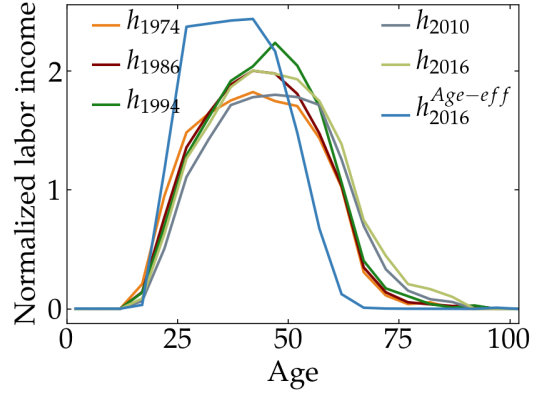


Figure B.1: US age-wealth and labor income profiles

Notes: This figure presents age-wealth and age-labor income profiles for the US for different years.

medium scenario for 2020-2025, 0.4 for 2025-2030, and 0.5 for 2030-2100.

The population distributions resulting from Gagnon et al. (2019)'s procedure are historically very close to the UN population distribution, and are in line with the UN's medium fertility scenario until 2100. We are also able to replicate the three UN fertility scenarios.

Age-labor supply profiles We measure the US age-labor supply profiles, h_{jt} , as

$$h_{jt} = \frac{h_{jt}^{USD}}{\sum_j \pi_{jt} h_{jt}^{USD}} \frac{l_t^{BLS}}{l_0^{BLS}}$$

where h_{jt}^{USD} is the average total labor income per person of age group j , including monetary, non-monetary and windfall labor income, obtained from the Luxembourg Income Study (LIS)⁴⁶, and $l_t^{BLS} = L_t^{BLS} / N_t$ is aggregate effective labor input obtained from the US Bureau of Labor Statistics⁴⁷ divided by total population. We use the LIS directly instead of the raw data since it provides harmonized labor income microdata for more than 50 countries.⁴⁸ This construction ensures that $\sum_j \pi_{jt} h_{jt} = 1$ in a base year which we take to be 2016 and follows the evolution of l_t^{BLS} for all other years. Some of the resulting US profiles are presented on Panel B of figure B.1. For all other countries, we only measure the profile in a baseline year.

We use the LIS to construct the base-year age-labor supply profiles for all the other coun-

⁴⁶<https://www.lisdatacenter.org/our-data/lis-database/>

⁴⁷https://www.bls.gov/mfp/special_requests/mfptablehis.xlsx

⁴⁸For the US, the LIS is based on the Annual Social and Economic Supplement of the Current Population Survey (CPS).

tries we consider. For Australia, it is based on the Survey of Income and Housing (SIH) and the Household Expenditure Survey (HES), for Austria on the Survey on Income and Living Conditions (SILC), for Canada on the Canadian Income Survey (CIS), for China on the Chinese Household Income Survey (CHIP), for Denmark on the Law Model (based on administrative records), for Estonia on the Estonian Social Survey (ESS) and the Survey on Income and Living Conditions (SILC), for Finland on the Income Distribution Survey (IDS) and the Survey on Income and Living Conditions (SILC), for France on the Household Budget Survey (BdF), for Germany on the German Socio-Economic Panel (GSOEP), for Greece on the Survey of Income and Living Conditions (SILC), for Hungary on the Tárki Household Monitor Survey, for India on the India Human Development Survey (IHDS), for Ireland on the Survey on Income and Living Conditions (SILC), for Italy on the Survey of Household Income and Wealth (SHIW), for Japan on the Japan Household Panel Survey (JHPS), for Luxembourg on the Socio-economic Panel “Living in Luxembourg” (PSELL III) and the Survey on Income and Living Conditions (SILC), for Netherlands on the Survey on Income and Living Conditions (SILC), for Norway on the Household Income Statistics, for Poland on the Household Budget Survey, for Slovakia on the Survey of Income and Living Conditions (SILC), for Slovenia on the Household Budget Survey (HBS), for Estonia on the Survey on Income and Living Conditions (SILC), for Sweden on the Household Income Survey (HINK/HEK), and for the United Kingdom on the Family Resources Survey (FRS).

Age-wealth profiles We measure individual-level US age-wealth profiles, a_{jt} , as

$$a_{jt} = W_{jt}^{hh} \frac{N_{jt}^{hh}}{N_{jt}^{ind}} \frac{1}{x_t},$$

where W_{jt}^{hh} is average nominal net worth by age of the household head in the Survey of Consumer Finances (SCF), N_{jt}^{hh} the number of households of the same age and N_{jt}^{ind} the number of individuals of the same age. The SCF expresses all values in 2016 USD using the “current methods” version of the consumer price index (CPI) for all urban consumers from the BLS. We use the same price index to obtain nominal measures of household wealth W_{jt}^{hh} in current USD.

Finally, the profile is normalized by a constant x_t which ensures that the resulting aggregate wealth-to-GDP ratio equals the measure from the World Inequality Database in every year.⁴⁹

Figure B.2 details the construction of the profile in 2016: Panel A presents average net worth by age of head of household from the SCF, Panel B presents the total number of household by

⁴⁹The WID series is presented in figure 1 and is taken from <https://wid.world/>.

age from the SCF and the total number of individuals by age from the UN, Panel C reported the ratio of total households over total individuals, and Panel D is the resulting age-wealth profile computed as the profile in Panel A multiplied by the ratio in Panel C and normalized such that it aggregates to the 2016 wealth-to-GDP ratio from WID. Age-wealth profiles for additional years are presented on Panel A of figure B.1.

We construct the age-wealth profiles for all other countries in a baseline year using a similar procedure. We obtain measures of total nominal wealth by age group at the household-level from survey data, use the total number of individuals by age from the UN World Population Prospects, and normalize the resulting profiles to ensure that the aggregates wealth-to-GDP ratios coincide with the WID data. We gather data from the Luxembourg Wealth Study (LWS)⁵⁰ for Australia in 2010, Canada in 2012, Germany in 2012, United Kingdom in 2011, Italy in 2010, and Sweden in 2005. For Australia the LWS is based on the Survey of Income and Housing (SIH) and the Household Expenditure Survey (HES), for Canada on the Survey of Financial Securities (SFS), for Germany on the German Socio-Economic Panel (GSOEP), for Italy on the Survey of Household Income and Wealth (SHIW), for Sweden on the Household Income Survey (HINK/HEK), and for United Kingdom on the Wealth and Assets Survey (WAS). We rescale the survey weights such that they sum up to the correct number of households according to, respectively, the Australian Bureau of Statistics, Statistics Canada, Statistisches Bundesamt, the Office for National Statistics, the Istituto Nazionale di Statistica, and the United Nations Economic Commission for Europe (UNECE). We use the Household Finance and Consumption Survey (HFCS)⁵¹ for Austria in 2010, Belgium in 2010, Estonia in 2014, Spain in 2010, Finland in 2010, France in 2010, Greece in 2010, Hungary in 2014, Ireland in 2014, Luxembourg in 2014, Netherlands in 2010, Poland in 2014, Slovenia in 2014, and Slovakia in 2014. For China, we rely on the 2013 China Household Finance Survey (CHFS).⁵² For India, we use the National Sample Survey (NSS).⁵³ For Japan, we construct a measure of total wealth by age of household head from Table 4 of the 2009 National Survey of Family Income and Expenditure (NFSIE) available on the online portal of Japanese Government Statistics⁵⁴. This table provides average net worth and total number of households by age groups for single person households and households with two or more members, which we aggregate to obtain total household net worth by age group. For Denmark, we use the 2014 table “Assets and

⁵⁰<https://www.lisdatacenter.org/our-data/lws-database/>

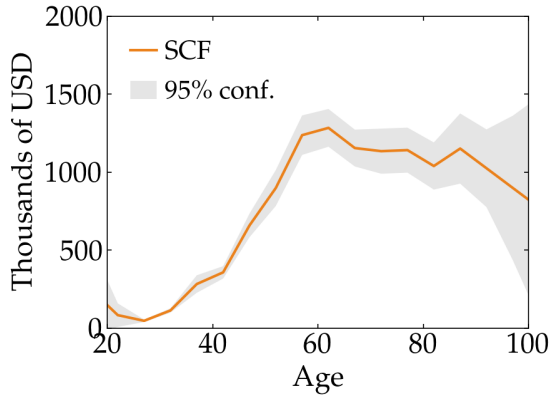
⁵¹https://www.ecb.europa.eu/stats/ecb_surveys/hfcs/

⁵²<http://www.chfsdata.org/>

⁵³<http://microdata.gov.in>

⁵⁴<https://www.e-stat.go.jp>

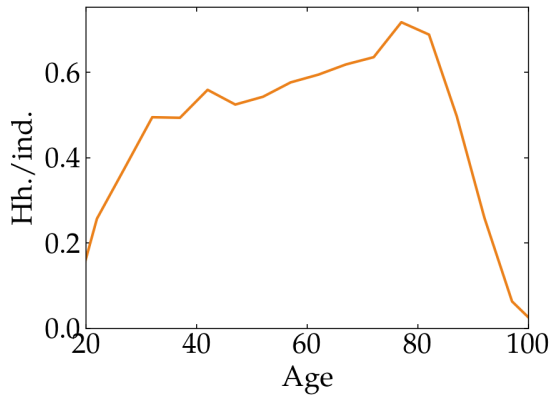
A. Average household net worth



B. Total households and individuals



C. Households over individuals



D. Wealth-to-GDP profile

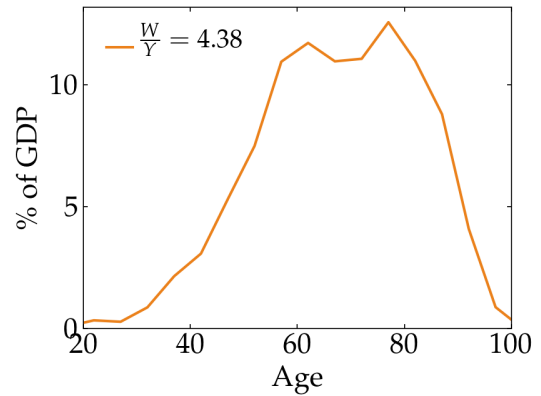


Figure B.2: Construction of the 2016 US age-wealth profile

Notes: This figure details the construction of the 2016 US age-wealth profile. Panel A presents average net worth by age of head of household from the 2016 SCF. The bootstrapped 95% confidence interval is computed by resampling observations 10,000 times with replacement. Panel B presents the total number of household by age from the 2016 SCF and the total number of individuals by age in 2016 from the UN. Panel C reported the ratio of total households over total individuals. Panel D is the resulting age-wealth profile computed as the profile in Panel A multiplied by the ratio in Panel C and normalized such that it aggregates to the 2016 W/Y ratio.

liabilities per person by type of components, sex, age and time” produced by Statistics Denmark that provides a measure of average net wealth per person by age group produced from tax data. For any country, when no data is available after a certain age, we assume that wealth linearly declines to zero in the age group 100+.

B.2 Historical decomposition of W/Y for the United States

In this section, we use equation (9) in order to interpret historical movements in the US wealth-to-GDP ratio. We further split behavioral effects Δ_t^{beh} into a first-order contribution from changing age and income profiles, as well as a residual, using:⁵⁵

$$\underbrace{\frac{W_t}{Y_t} - \frac{W_0}{Y_0}}_{\equiv \Delta_t} \simeq \underbrace{\frac{\sum_j \pi_{jt} a_{j0}}{\sum_j \pi_{jt} h_{j0}} - \frac{\sum_j \pi_{j0} a_{j0}}{\sum_j \pi_{j0} h_{j0}}}_{\Delta_t^{comp}} + \underbrace{\sum_j \pi_{j0} (a_{jt} - a_{j0})}_{\Delta_t^{beh,a}} - \underbrace{\left(\frac{W_0}{Y_0}\right) \sum_j \pi_{j0} (h_{jt} - h_{j0})}_{\Delta_t^{beh,h}} + \Delta_t^{err}, \quad (38)$$

Here, we study the period over which we have the best data on individual asset holdings from the Survey of Consumer Finances (SCF), which is 1989–2016.

Results. We find that the compositional effect of demographics accounts for about 75% of this increase, with the remainder accounted for by shifts in age-asset profiles (the contribution from labor supply is negative but modest). More generally, we find that demographics are an important force behind the trend rise in the wealth-to-GDP ratio, as it tends to push up W/Y by around 25 percentage points per decade. By contrast, the volatility in the wealth-to-GDP ratio is accounted for by shifts in age-asset profiles, which we interpret as reflecting medium-run fluctuations in asset prices.

Figure B.4 presents our demographic-based analysis of historical movements in the wealth-to-GDP ratio. The dark solid line of panel A displays W/Y in the data, which has risen from about 345% to 440% of GDP, with medium-run movements (a decline in the early 1990s, a rapid increase in the late 1990s, and another decline in the late 2000s) that broadly track changes in financial asset prices. The three additional lines display the three contributors to W_t/Y_t : each freezes two of π_{jt} , a_{jt} or h_{jt} at the base year of 2016, and lets the remaining term vary as in the data. The panel shows that demographics made a significant contribution to the long-run increase in wealth-to-GDP, pushing it up by around 30 percentage points per decade, so that its cumulative contribution

⁵⁵A derivation of equation (38) is provided at the end of this section.

is about half of the total increase in the data.

As regards shifts in age profiles, the dashed green line shows that the contribution of labor supply is modest, and with a small but negative effect over the period entire period. Indeed, as panel A.1 of figure 3 illustrates, the age-labor efficiency profiles that we construct have been mostly stable over this time period: their bell-shape, with a peak in the late 40s, has not changed much and their overall level has also been stable. Finally, the dashed red line shows that the medium-frequency movements in W/Y are due to changes in age-asset profiles. Panel A.2 of figure 3 shows that while the overall shape of age-asset profiles have remained stable over the period, with a slow rise until the late 60s followed by a limited decumulation pattern later on, their overall magnitude has shifted significantly over time with general asset valuations. This suggests an interpretation of these movements as reflecting movements in financial asset prices, although it is possible that they also reflect other behavioral responses to aging—a hypothesis we will further examine in section 3.

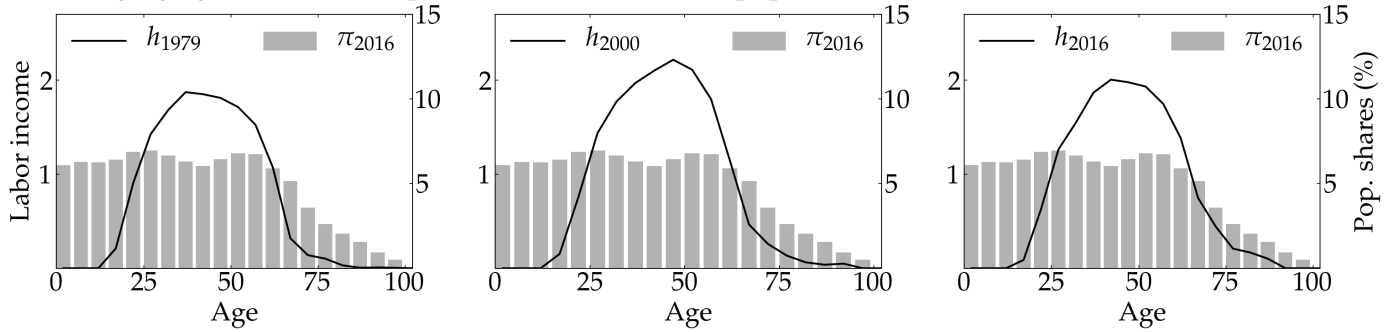
Panel B of figure B.4 computes the first-order approximation terms in equation 38, using 1989 as the baseline year. These tell the same story as panel A: out of the rise of 147% of GDP, 78 points came from demographics, 110 points from changes in asset profiles, and labor supply contributed -27 points (the first-order approximation error accounts from the remaining -14 points).

The fact that demographics makes such a significant contribution to historical changes in the wealth-to-GDP ratio is the main finding of this section.

Derivation of equation (38). Note that totally differentiating $\frac{\sum_j \pi_{j,t} a_{j0}}{\sum_j \pi_{j,t} h_{j0}}$ at time $t = 0$ with respect to time implies

$$\begin{aligned} d \left(\frac{\sum_j \pi_{j,t} a_{j0}}{\sum_j \pi_{j,t} h_{j0}} \right) &= \frac{(\sum_j \pi_{j0} h_{j0})(\sum_j d\pi_{j,t} a_{j0}) - (\sum_j d\pi_{j,t} h_{j0})(\sum_j \pi_{j0} a_{j0})}{(\sum_j \pi_{j0} h_{j0})^2} \\ &= \sum_j d\pi_{j,t} \left[a_{j0} - h_{j0} \left(\sum_{j'} \pi_{j'0} a_{j'0} \right) \right], \quad (*) \end{aligned}$$

A. Changing age-labor income profiles over a fixed 2016 population distribution



B. Changing age-wealth profiles over a fixed 2016 population distribution

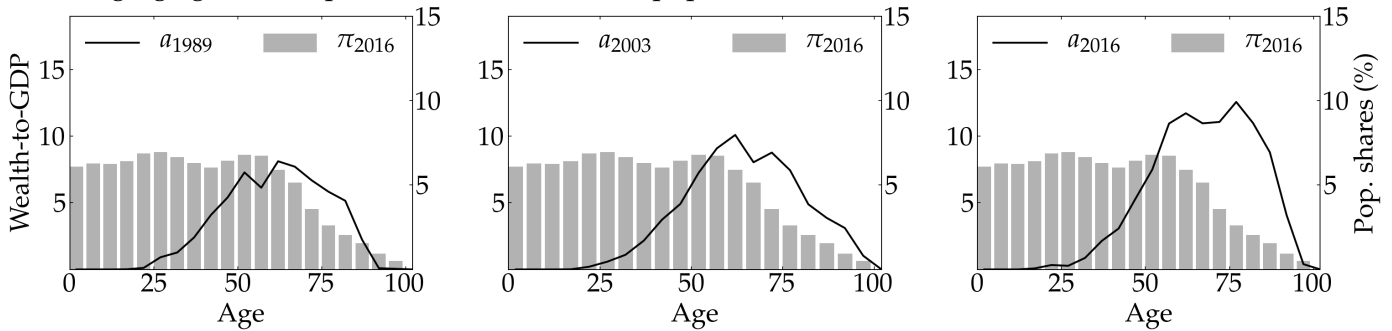
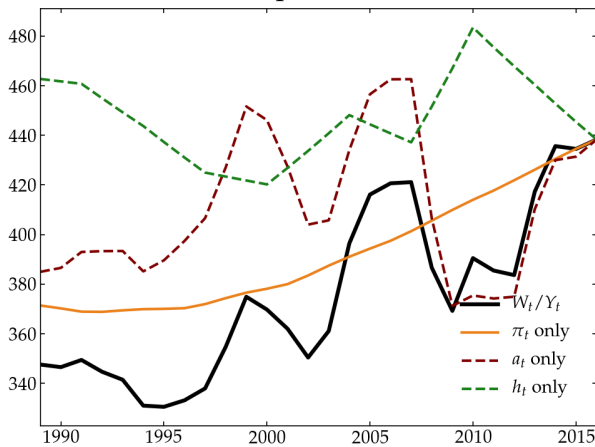


Figure B.3: Age-wealth profiles, and age-labor income profiles, and population age distributions

Notes: Panel A presents the 1979, 2000 and 2016 age-labor income profiles (solid black line, left-axis), and the 2016 population distribution (gray bars, right-axis). Panel B presents the 1989, 2003 and 2016 age-wealth profiles, and the 2016 population distribution.

A. W_t/Y_t and its components



B. Changes in W/Y , 1989-2016

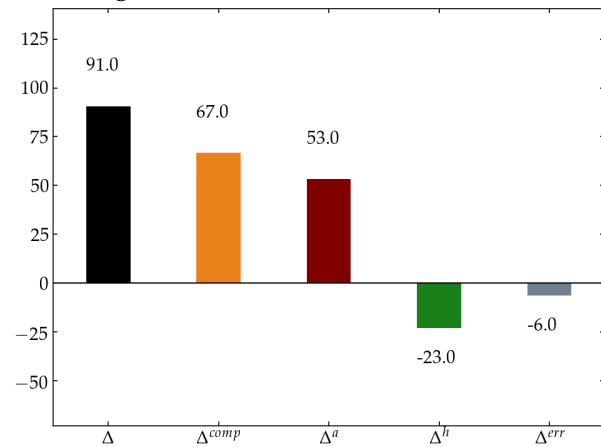


Figure B.4: Retrospective decomposition

Notes: Panel A presents W_t/Y_t (solid black line) as well as a level decomposition. The orange line is the contribution from demographics, $\frac{\sum_j \pi_{jt} a_{j,2016}}{\sum \pi_{jt} h_{j,2016}}$. The dashed red line is the contribution from asset profiles, $\frac{\sum_j \pi_{jt,2016} a_{jt}}{\sum \pi_{j,2016} h_{j,2016}}$. The dashed green line is the contribution from labor supply, $\frac{\sum_j \pi_{j,2016} a_{j,2016}}{\sum \pi_{j,2016} h_{j,2016}}$. Panel B presents all three terms in equation (38), together with the approximation error, when 1989 is used as the base year.

where the second line uses $\sum_j \pi_{j0} h_{j0} = 1$. Now, if we totally differentiate (7) with respect to time at $t = 0$, we obtain

$$\begin{aligned}
d\left(\frac{W_t}{Y_t}\right) &= \frac{(\sum_j \pi_{j,t} h_{j,0})(\sum_j d\pi_{j,t} a_{j,0} + \pi_{j,0} da_{j,t}) - (\sum_j d\pi_{j,t} h_{j,0} + \pi_{j,0} dh_{j,0})(\sum_j \pi_{j,0} a_{j,0})}{(\sum_j \pi_{j,0} h_{j,0})^2} \\
&= \sum_j d\pi_{j,t} \left[a_{j,0} - h_{j,0} \left(\sum_{j'} \pi_{j,0} a_{j,0} \right) \right] + \sum_j \pi_{j,0} da_{j,t} - \left(\sum_j \pi_{j,0} a_{j,0} \right) \left(\sum_j \pi_{j,0} dh_{j,0} \right) \\
&= d\left(\frac{\sum_j \pi_{j,t} a_{j,0}}{\sum_j \pi_{j,t} h_{j,0}}\right) + \sum_j \pi_{j,0} da_{j,t} - \frac{W_0}{Y_0} \left(\sum_j \pi_{j,0} dh_{j,0} \right).
\end{aligned}$$

The second line again uses $\sum_j \pi_{j,0} h_{j,0}$, and the third line uses (*) and that $\sum_j \pi_{j,0} a_{j,0} = \frac{W_0}{Y_0}$ given $\sum_j \pi_{j,0} h_{j,0} = 1$, and the definition of $a_{j,0}$. With this expression for $d\left(\frac{W_t}{Y_t}\right)$, equation (38) follows. \square

B.3 Robustness of Δ^{comp} exercise to choice of base profiles

Table B.1 explores the robustness of the shift-share to the choice of base age-profiles. Panel A considers the shift-share for the period 1950 to 2016, and panel B 2016 to 2100. In each panel, we use as base year every possible combination of survey years for the age-wealth profile a_j from the SCF (rows) and age-labor income profile h_j from LIS (columns). In the last row and column (Age eff.) we use the average age-effects. We estimate the age-effects from the SCF and LIS using the method proposed by Hall (1968) and Deaton (1997) to separate between the age effects, time effects, and cohort effects, imposing that all growth comes from time effects. The resulting profiles are presented in figure B.1. This table indicates that the shift-share lies between 64 and 135 pp for 1950-2016 and between 58 and 133 pp for 2016-2100. There are only small variations across columns, indicating that the choice of base year for the age-labor income profile is not important for the shift-share. The choice of base year for the age-wealth profile matter more. This follows from fluctuations in the aggregate wealth-to-GDP ratio as noted in section D.8 since the profiles are .

Table B.1: Sensitivity of the shift-share to base year

Panel A. Change in W/Y between 1950 and 2016													
a_j year	h_j year												Age eff.
	1974	1979	1986	1991	1994	1997	2000	2004	2007	2010	2013	2016	
1989	68	70	72	71	70	69	71	68	67	64	63	64	97
1992	73	75	77	77	75	74	77	73	72	69	69	70	103
1995	76	77	80	79	78	77	79	76	75	72	72	73	105
1998	82	84	87	87	85	84	86	83	82	79	78	79	115
2001	84	86	89	89	87	86	89	85	84	81	81	81	116
2004	89	91	94	94	92	91	94	91	90	86	86	87	122
2007	96	99	102	102	100	99	102	98	97	93	93	94	131
2010	84	86	88	88	87	86	89	86	85	82	82	83	112
2013	87	89	92	92	91	90	92	89	88	85	85	86	118
2016	102	104	108	108	106	106	108	105	104	101	101	101	135
Age eff.	98	101	104	104	102	101	104	101	100	97	96	97	132

Panel B. Change in W/Y between 2016 and 2100													
a_j year	h_j year												Age eff.
	1974	1979	1986	1991	1994	1997	2000	2004	2007	2010	2013	2016	
1989	63	64	65	65	65	65	66	63	62	61	59	58	75
1992	65	66	67	67	68	67	68	66	64	63	61	60	77
1995	71	73	74	74	75	74	75	73	71	70	68	67	84
1998	73	74	76	76	76	76	77	74	73	71	69	68	87
2001	78	79	81	81	81	81	82	79	78	76	74	73	92
2004	86	88	90	90	90	90	91	88	87	85	83	82	101
2007	92	94	96	97	97	97	98	95	93	91	89	88	109
2010	82	84	86	86	87	86	87	85	84	82	80	79	96
2013	91	93	95	95	96	95	96	94	92	91	89	88	106
2016	114	116	119	120	121	121	122	119	117	116	113	112	133
Age eff.	100	102	105	105	106	105	106	104	102	100	98	97	117

Notes: This table reports values of the shift-share for alternative base years of the age-wealth and the age-labor income profiles. Panel A considers 1950 to 2016, and panel B 2016 to 2100. Every column correspond to the the base year for the age-labor income profile, and each row for the age-wealth profile. The last line and column correspond to the case where the average age-effect is used.

B.4 Household level analysis

All our exercises are conducted at the individual level. To gauge the importance of the household vs individual distinction, we perform the shift-share analysis of section 2 at the household-level. To compute the age-wealth and labor income profiles, we adopt the same procedure detailed in section B.1. To convert US household-level population distributions to individual-level distributions, we use the distribution of age of household head by age of individual that we estimate from the PSID.

Figure B.5 presents the shift-share results at the household-level. We follow the same procedure as in section 2. For comparison, the graph also shows the individual-level shift-share from

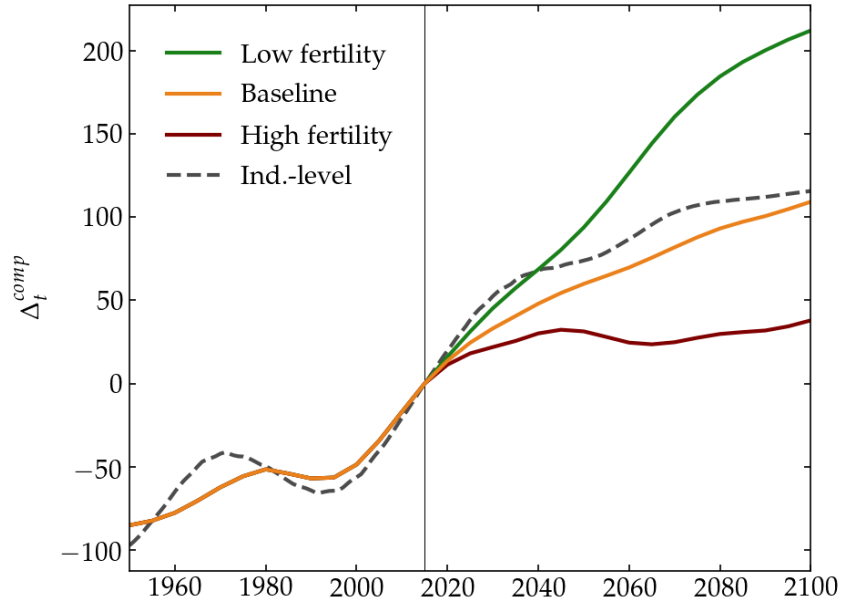


Figure B.5: Household-level shift-share on W/Y

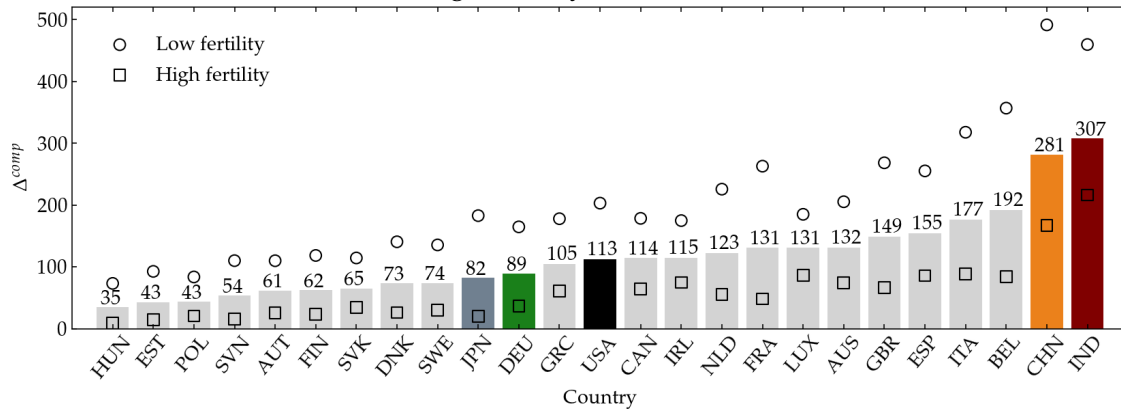
Notes: This figure depicts the evolution of the compositional term Δ_t^{comp} when the analysis is conducted at the household-level. The solid orange line corresponds to the medium fertility scenario from the UN, the dashed green line to the low fertility scenario, and the dashed red line to the high fertility scenario. The gray dashed line repeats the baseline individual-level shift-share reported in figure 2.

figure 2. Overall, the two shift-shares are very close.

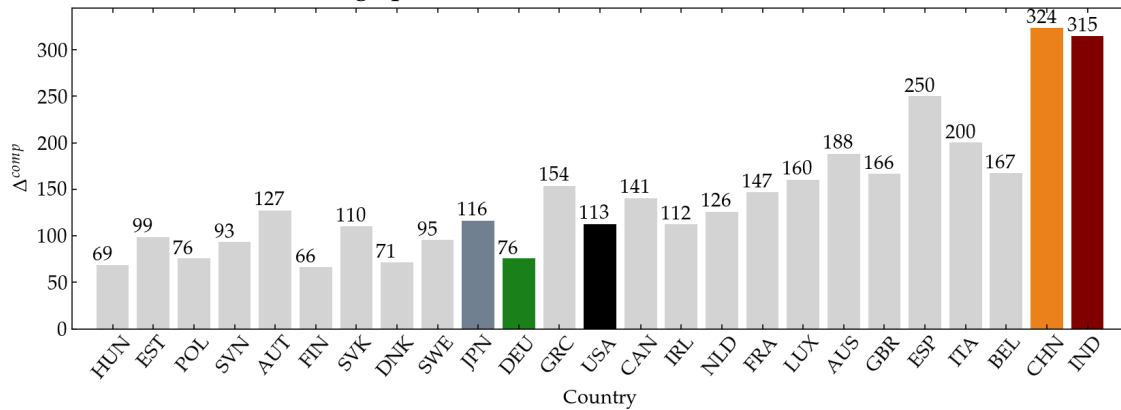
B.5 World exercise with US profiles and US demographics

Figure B.6 presents the change in W/Y between 2016 and 2100 from the shift-share and isolates the contributions from demographic forces and from the age-profiles. Panel A repeats the results from section 2, ranking countries from lowest to the highest shift-share. It also presents the results under the two UN fertility scenarios. To isolate the contribution from demographic forces, panel B performs a shift-share exercise where age-profiles in all countries are identical to the US profile. The age-wealth profiles are rescaled such that the 2016 aggregate W/Y is correct in all countries. To isolate the contribution from the profiles, panel C performs a shift-share exercise where population distributions of the US are used in every country. country-specific The age-wealth profiles are rescaled to ensure correct aggregation in 2016. Panels B and C show that both the shape of the profiles and the changes in population distributions matter to the shift-share, but that the demographic forces play a much more important role in generating shift-shares that are high and heterogeneous across countries.

A. Baseline shift-share and low/high fertility scenarios



B. Shift-share at common age profiles



C. Shift-share at common demographic change

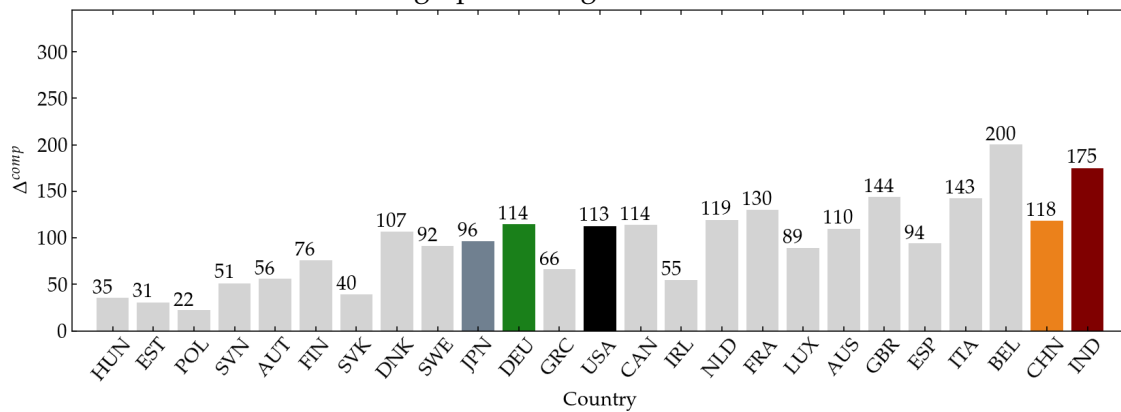


Figure B.6: Shift-share between 2016 and 2100

Notes: Panel A presents the change in W/Y between 2016 and 2100 from the shift-share as well as its value using the low fertility (circles) and high fertility (squares) scenarios. Panel B performs a shift-share exercise where age-profiles in all countries are identical to the US profiles. Panel C performs a shift-share exercise where population distributions of the US are used in every country. For both panels B and C, the profiles are rescaled to ensure correct aggregation in 2016.

C Appendix to Section 3

C.1 Barro-Becker microfoundation for utility modifier

The utility modifier ψ_j can be microfounded from the number of children n_j present in the household using a Barro-Becker formulation in which children enter the utility function of parents. Let $c = c^P + nc^C$ be the total consumption of parent and children, and assume that the flow utility function of a parent who weighs their children with weight λ and exponent φ is

$$\mathcal{U}(c^P, c^C) = u(c^P) + \lambda n^\varphi u(c^C),$$

since the solution to

$$\begin{aligned} W(c) = \max & \quad u(c^P) + \lambda n^\varphi u(c^C) \\ \text{s.t.} & \quad c^P + nc^C \leq c, \end{aligned}$$

involves

$$u'(c^P) = \lambda n^{\varphi-1} u'(c^C),$$

and $u'(c) = c^{-\sigma}$ we obtain a solution for consumption for parents and children of

$$c^P = \frac{1}{1 + \lambda^{\frac{1}{\sigma}} n^{\frac{\sigma+\varphi-1}{\sigma}}} c, \quad c^C = \frac{\lambda^{\frac{1}{\sigma}} n^{\frac{\varphi-1}{\sigma}}}{1 + \lambda^{\frac{1}{\sigma}} n^{\frac{\sigma+\varphi-1}{\sigma}}} c,$$

Hence, we have that the total value of having children is

$$W(c) = u(c^P) + \lambda n^\varphi u(c^C) = \frac{1 + \lambda^{\frac{1}{\sigma}} n^{\frac{\varphi-1+\sigma}{\sigma}}}{\left[1 + \lambda^{\frac{1}{\sigma}} n^{\frac{\sigma+\varphi-1}{\sigma}}\right]^{1-\sigma}} u(c) = \left(1 + \lambda^{\frac{1}{\sigma}} n^{\frac{\sigma+\varphi-1}{\sigma}}\right)^\sigma u(c).$$

In this formulation, $\psi_j = \left(1 + \lambda^{\frac{1}{\sigma}} n^{\frac{\sigma+\varphi-1}{\sigma}}\right)^\sigma$. In particular, for any $\lambda > 0$, children raise the marginal utility of consumption.

C.2 Adding markups

In the version with markups, we use a standard monopolistic competition microfoundation. Perfectly competitive final goods producers produce using a CES aggregate of intermediate inputs,

$$Y_t = \left(\int_0^1 y_t(\omega)^{\frac{1}{\mu_t}} d\omega \right)^{\mu_t},$$

where $\{\mu_t\}$ follows an exogenous path. Intermediate goods producers produce under monopolistic competition using capital and labor,

$$y_t(\omega) = F(\kappa_t(\omega), Z_t l_t(\omega)),$$

where $\kappa_t(\omega)$ and $l_t(\omega)$ denote, respectively, capital and effective labor hired by firm ω . Profit maximization subject to the demand curve induced by final goods producers leads to the first order conditions

$$\frac{1}{\mu_t} F_K \left(\frac{\kappa_t(\omega)}{Z_t l_t(\omega)}, 1 \right) = r_t + \delta, \quad \frac{1}{\mu_t} F_L \left(\frac{\kappa_t(\omega)}{Z_t l_t(\omega)}, 1 \right) = w_t.$$

Hence all firms have the same capital-labor ratio $k_t = \frac{K_t}{Z_t L_t} = \frac{\kappa_t(\omega)}{Z_t l_t(\omega)}$, which satisfies

$$F_K(k_t, 1) = \mu_t(r_t + \delta). \quad (39)$$

Hence $\{\mu_t\}$ is the exogenous path for markups.

Demographic decomposition of W/Y . Aggregate output can now be written as

$$Y_t = y(\mu_t, r_t) Z_t L_t = y(\mu_t, r_t) \cdot \left(Z_t N_t \sum_j \pi_{jt} h_{jt} \right), \quad (40)$$

where $y(\mu_t, r_t) \equiv F[k_t(\mu_t, r_t), 1]$, with $k_t(\mu_t, r_t)$ implicitly defined by (39), so that y depends only on μ_t and r_t . With our normalization $\sum_j \pi_{j0} h_{j0} = 1$, $y(\mu_0, r_0) Z_0$ is also equal to GDP per capita at time 0, Y_0/N_0 .

Asset market. With markups, the assets of households are invested in a mutual fund. At the end of period t , the mutual fund combines the assets of domestic residents, A_t , with the assets of migrants that are arriving in the next period, $A_t^{mig,net}$.⁵⁶ The funds are invested in firm shares with price p_t , government bonds B_t , and foreign assets NFA_t :

$$p_t + B_t + NFA_t = A_t + A_t^{mig,net}. \quad (41)$$

Since government bonds and foreign assets yield the same safe return r_{t-1} in period t , the gross return on the mutual fund r_t^a satisfies the valuation equation

$$(1 + r_t^a) \left[A_{t-1}^{mig,net} + A_{t-1} \right] = p_t + D_t + (1 + r_{t-1}) B_{t-1} + (1 + r_{t-1}) NFA_{t-1}. \quad (42)$$

⁵⁶The interpretation is still that incoming migrants arriving at time $t+1$ choose to allocate their assets in the domestic mutual fund in the period prior to their arrival. Similarly, migrants that are leaving between t and $t+1$ allocate their assets abroad, which implies a negative contribution to $A_t^{mig,net}$.

We assume that, in each period t , mutual fund managers maximize expected returns $\mathbb{E}_t[r_{t+1}^a]$. Optimal asset allocation then implies that expected returns are equalized across assets⁵⁷

$$1 + r_t = \mathbb{E}_t[1 + r_{t+1}^a] = \frac{\mathbb{E}_t[p_{t+1} + D_{t+1}]}{p_t}. \quad (43)$$

Equation (43) determines r_t^a and p_t as a function of safe interest rates and dividends for $t \geq 1$. In turn, the share price in the first period must satisfy

$$p_0 = \sum_{t=0}^{\infty} \frac{\mathbb{E}_0 D_{t+1}}{\prod_{s=0}^t (1 + r_s)}, \quad (44)$$

which determines r_0^a in conjunction with the initial mutual fund balance sheet and the valuation equation (42). We are now ready to define the three notions of equilibrium we will study in the paper.

Since all markups are capitalized, the steady state equality between asset supply and asset demand requires that aggregate wealth be equal to

$$\frac{W}{Y} \equiv \frac{A}{Y} + \frac{A^{mig,net}}{Y} = \frac{p_{\Pi}}{Y} + \frac{K}{Y} + \frac{B}{Y} + \frac{NFA}{Y}, \quad (45)$$

where $\frac{p_{\Pi}}{Y} = \frac{1}{\frac{1+r}{1+g}-1} \left(1 - \frac{1}{\mu}\right)$ is the capitalized value of extranormal profits.

C.3 Aggregation and Walras's law

To obtain the evolution of macroeconomic aggregates, we aggregate the solution to the household problem. For a given individual-level variable x , we write

$$X_t \equiv \sum_{j=0}^T N_{jt} \mathbb{E}[x_{jt}(s^j)]$$

for its macroeconomic counterpart. When the relevant quantity is the $x_{jt}(s^j)$ of individuals who survive until period $t + 1$, we discount the values with their survival probability, and write

$$X_t^{surv} \equiv \sum_{j=0}^T N_{jt} \phi_{jt} \mathbb{E}[x_{jt}(s^j)]$$

Given these definitions, we can aggregate up household budget constraints in (15) to obtain

$$C_t + A_t = (1 + r_t^a)(A_{t-1}^{surv} + A_{t-1}^{mig,net} + B_t^r) + w_t L_t - T_t, \quad (46)$$

⁵⁷Since there is no aggregate uncertainty, r_{t+1}^a , p_{t+1} , and D_{t+1} are constant random variables. We use a notation with \mathbb{E}_t to clarify that they are t -measurable.

where T_t is aggregate net taxes. In equilibrium, (18) implies that $A_{t-1}^{surv} + \frac{B_t^r}{1+r_t^d} = A_{t-1}$. We can then use equations (17), (42) and (46) to obtain

$$C_t + B_t + A_t^F = D_t + (1 + r_{t-1})B_{t-1} + (1 + r_{t-1})A_{t-1}^F + \omega_t L_t - T_t + A_t^{mig,net}$$

to obtain the evolution of the economy's net foreign asset position as:

$$C_t + G_t + I_t + A_t^F - A_{t-1}^F = Y_t + r_{t-1}A_{t-1}^F + A_t^{mig,net}. \quad (47)$$

The right-hand side of equation (47) is national income: aggregate domestic output adjusted by net income on foreign assets and net unilateral transfers from migrants $A_t^{mig,net}$.⁵⁸ An alternative way of formulating (47) is in terms of the evolution of the net foreign asset position.

$$A_t^F - A_{t-1}^F = (Y_t + r_{t-1}A_{t-1}^F + A_t^{mig,net}) - (C_t + I_t + G_t) \quad (48)$$

The right-hand side is national income (aggregate domestic output adjusted by net income on foreign assets and net unilateral transfers⁵⁹ from migrants $A_t^{mig,net}$) net of domestic absorption. In a closed-economy equilibrium, A_t^F is zero at all times and we obtain the standard goods market clearing condition, adjusted by exports coming from migrants:

$$C_t + I_t + G_t = Y_t + A_t^{mig,net}$$

In a world economy equilibrium, a similar equation holds at the world level and world net migrant transfers are zero,

$$\sum_{c \in \mathcal{C}} C_{ct} + \sum_{c \in \mathcal{C}} I_{ct} + \sum_{c \in \mathcal{C}} G_{ct} = \sum_{c \in \mathcal{C}} Y_{ct}$$

C.4 Derivation of equations (21)-(23).

The global equilibrium is defined by

$$\sum_c \omega_c \frac{NFA_c}{Y_c} = 0,$$

where $\omega_c = \frac{Y_c}{\sum_{c'} Y_{c'}}$. Totally differentiating (19) gives

$$\Delta \left(\frac{NFA_c}{Y_c} \right) \approx \frac{\partial \frac{W_c}{Y_c}}{\partial \Theta} \Delta \Theta + \frac{\partial \frac{W_c}{Y_c}}{\partial r} \Delta r - \frac{\partial \frac{A_c^s}{Y_c}}{\partial r} \Delta r,$$

⁵⁸Here, $A_t^{mig,net}$ shows up since arriving migrants augment the amount of goods available in the economy, and leaving migrants deplete the amount of goods available in the economy.

⁵⁹Here, $A_t^{mig,net}$ shows up since arriving migrants augment the amount of goods available in the economy, and leaving migrants deplete the amount of goods available in the economy.

where $\Delta\Theta$ is a change in the demographic parameter and Δr a change in the interest rate. We rewrite it as

$$\Delta \left(\frac{NFA_c}{Y_c} \right) \approx \Delta^{|r} \left(\frac{W_c}{Y_c} \right) + \epsilon_c^{d,r} \Delta r + \epsilon_c^{s,r} \Delta r,$$

where $\epsilon_c^{d,r} \equiv \frac{\partial \frac{W_c}{Y_c}}{\partial r}$ and $\epsilon_c^{s,r} \equiv -\frac{\partial \frac{A_c^s}{Y_c}}{\partial r}$. We further split the total change in wealth-to-GDP $\Delta^{|r} \left(\frac{W_c}{Y_c} \right)$ into the compositional effect Δ_c^{comp} and the behavioral effect $\Delta_c^{beh|r}$ to obtain

$$\Delta \left(\frac{NFA_c}{Y_c} \right) \approx \Delta_c^{comp} + \Delta_c^{beh|r} + \epsilon_c^{d,r} \Delta r + \epsilon_c^{s,r} \Delta r.$$

From the definition of a world equilibrium, it must be that

$$\sum_c \omega_c (\Delta_c^{comp} + \Delta_c^{beh|r} + \epsilon_c^{d,r} \Delta r + \epsilon_c^{s,r} \Delta r) \approx 0,$$

where $\omega_c \equiv \frac{Y_c}{\sum_c Y_c}$ are nominal GDP weights. This implies

$$\Delta r \approx \frac{\sum_c \omega_c (\Delta_c^{comp} + \Delta_c^{beh|r})}{\sum_c \omega_c (\epsilon_c^{d,r} + \epsilon_c^{s,r})} \equiv \frac{\bar{\Delta}^{comp} + \bar{\Delta}^{beh|r}}{\bar{\epsilon}^{d,r} + \bar{\epsilon}^{s,r}},$$

which gives us (21). We use bars to indicate GDP-weighted averages.

The change in the wealth-to-GDP is now given by

$$\begin{aligned} \Delta \left(\frac{W_c}{Y_c} \right) &\approx \Delta_c^{comp} + \Delta_c^{beh|r} + \epsilon_c^{d,r} \Delta r, \\ &= \Delta_c^{comp} + \Delta_c^{beh|r} - \frac{\epsilon_c^{d,r}}{\bar{\epsilon}_c^{d,r} + \bar{\epsilon}_c^{s,r}} (\bar{\Delta}_c^{comp} + \bar{\Delta}_c^{beh|r}), \end{aligned}$$

which gives (22).

The change in the NFA-to-GDP is now given by

$$\begin{aligned} \Delta \left(\frac{NFA_c}{Y_c} \right) &\approx \Delta_c^{comp} + \Delta_c^{beh|r} - \frac{\epsilon_c^{d,r} + \epsilon_c^{s,r}}{\bar{\epsilon}_c^{d,r} + \bar{\epsilon}_c^{s,r}} (\bar{\Delta}_c^{comp} + \bar{\Delta}_c^{beh|r}), \\ &= \Delta_c^{comp} + \Delta_c^{beh|r} - (\bar{\Delta}_c^{comp} + \bar{\Delta}_c^{beh|r}) + \Delta r \left[\epsilon_c^{d,r} + \epsilon_c^{s,r} - (\bar{\epsilon}_c^{d,r} + \bar{\epsilon}_c^{s,r}) \right], \end{aligned}$$

where the second line uses that $(\bar{\Delta}_c^{comp} + \bar{\Delta}_c^{beh|r}) = -(\bar{\epsilon}_c^{d,r} + \bar{\epsilon}_c^{s,r}) \Delta r$. This establishes (23).

D Appendix to section 4

D.1 Proof of Proposition 1

The conditions in assumptions 1-5 imply that w_t/Z_t is constant over time. Imposing these on the household problem normalized by technology Z_t , we find

$$\begin{aligned}\hat{V}_{jt}(\hat{a}, s^j) &= \max_{\hat{c}, \hat{a}'} \psi_j \frac{\hat{c}^{1-\sigma}}{1-\sigma} + Y(1-\phi_j) \frac{(\hat{a}')^{1-\nu}}{1-\nu} + \hat{\beta} \phi_j \mathbb{E} \left[\hat{V}_{j+1, t+1}(\hat{a}', s^{j+1}) | s^j \right] \\ \hat{c} + \hat{a}' &= \frac{1+r}{1+\gamma} \hat{a} + (1-\tau^y) \rho_j \hat{w} h_j l(s_j) + \hat{f} r_j(s^j) + \hat{b}_j^r(s^j) \\ \hat{a}' &\geq -\bar{a},\end{aligned}\tag{49}$$

where hats denote normalization by $Z_t = (1+\gamma)^t Z_0$, with the exception of \hat{V} , \hat{a} and $\hat{\beta}$.⁶⁰

Since the problem is time-invariant, it implies that the decision functions are also time-invariant. This means that two individuals that have the same initial normalized asset position, age, and shock history will have the same assets. In particular, since any individuals born after age-0 start their life with zero assets, we have that there exists a function $\hat{a}_j(s^j)$ such that

$$\hat{a}_{jt}(s^j) = \hat{a}_j(s^j) \quad \forall t \geq j,\tag{50}$$

where $t \geq j$ captures that an individual was born after time 0. If, furthermore, the initial distribution of assets is given by $\hat{a}_j(s^j)$, then (50) will be true for all $t \geq 0$.

If normalized assets are constant over time, we have that

$$\begin{aligned}\frac{W_t}{Y_t} &= \frac{\sum_{j, s^j} \Pi_j(s^j) N_{jt} a_{jt}(s^t)}{Z_t y(r) \sum_{j=1} \sum_j N_{jt} \mathbb{E} h_j} \\ &= \frac{\sum_{j=0, s^j}^T \Pi_j(s^j) N_{jt} \hat{a}_j(s^j) Z_t}{Z_t y(r) \sum_j N_{jt} \mathbb{E} h_{jt}} \\ &= \frac{\sum_{j=0} \pi_{jt} \mathbb{E} \hat{a}_j(s^j) / y(r)}{\sum_j \pi_{jt} h_{j0}},\end{aligned}$$

where Π is the measure over states s^j . Noting that $\mathbb{E} \hat{a}_j(s^j) / y(r)$ is average assets per individual, we recover the statement in the proposition. Once it has been established that the wealth process from the household side satisfies the right condition, the rest of the open-equilibrium features are standard.

⁶⁰Instead, these objects are defined as $\hat{V}_{jt} \equiv \frac{V_{jt}}{Z_t^{1-\sigma}}$, $\hat{a} \equiv \frac{a}{Z_{t-1}}$, and $\hat{\beta} \equiv \frac{\beta}{(1+g)^{1-\sigma}}$.

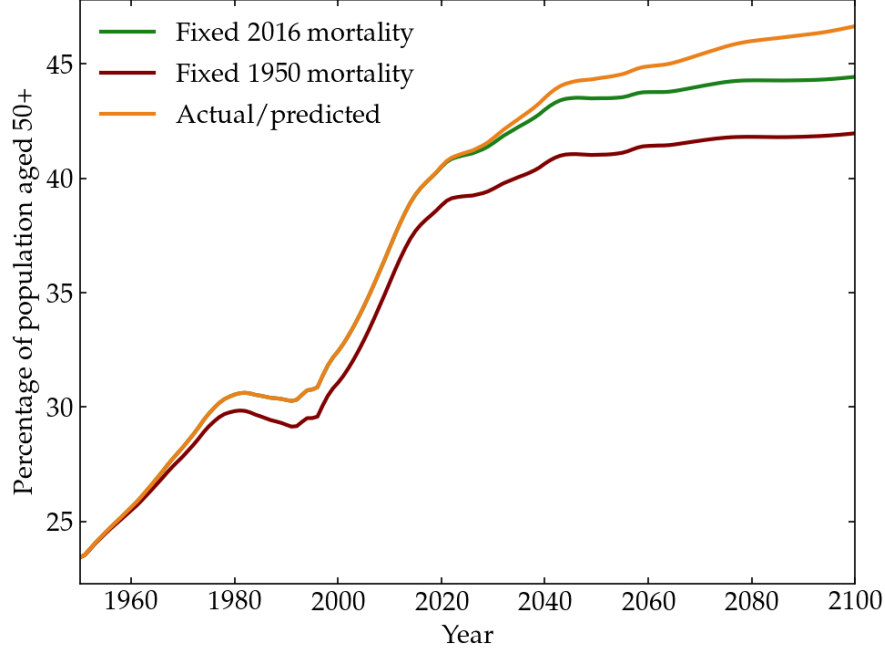


Figure D.1: Share of the US population above 50 years of age

Notes: This figure presents the share of the US population that is 50 years of age or more. The solid orange line presents the share when actual/predicted mortality rates are used. The green line presents the share when holding mortality rates fixed from 2016 on, and the red line from 1950 on. In all cases, migration is always set to zero.

D.2 Contribution of fertility to aging in the 21st century

Figure D.1 presents the simulated share of the US population above 50 years of age under different assumptions. To isolate the respective contributions of fertility and mortality, migration is always set to zero. The solid orange line presents the implied share when actual and predicted mortality rates are used. The green line presents the share when holding mortality rates fixed from 2016 on, and the red line from 1950 on. While mortality matters, this figure clearly indicates that fertility plays a dominant role in for demographic changes.

D.3 Parametric assumptions

Production function.

$$F[k_{t-1}(\omega), Z_t l_t(\omega)] = \left[\alpha k_{t-1}(\omega)^{\frac{\eta-1}{\eta}} + (1-\alpha) (Z_t l_t(\omega))^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \quad (51)$$

Income process. Households become economically active at age T^w . The income process s_t is modelled as having a fixed effect and a transitory component

$$s_j = (\theta, \epsilon_j).$$

The terms θ and ϵ_j are independent, and the labor supply shifter satisfies

$$l(s_j) = \theta(s_j)\epsilon(s_j),$$

where $\mathbb{E}\theta(s_j) = \mathbb{E}\epsilon(s_j) = 1$.⁶¹ Across generations, the fixed effect is governed by a Markov chain with a transition function $\Pi^\theta(\theta'|\theta)$, and a stationary distribution $\pi^\theta(\theta)$. The idiosyncratic component of skill ϵ_j follows a Markov chain $\Pi^\epsilon(\epsilon'|\epsilon)$, with $\mathbb{E}[\epsilon_j] = 1$ for every t , and has a stationary distribution $\pi^\epsilon(\epsilon)$.⁶²

Government policy. Taxes net of transfers satisfy

$$T_t = w_t N_t \left((\tau_t^{ss} + \tau_t(1 - \tau_t^{ss})) L_t - (1 - \tau_t) \bar{d}_t \mu_t^{ret} \right), \quad (52)$$

where $L_t \equiv \sum_{j < T_t^r} \pi_{jt} \widetilde{h}_{jt} + \rho_t \pi_{T_t^r t} \widetilde{h}_{T_t^r t}$ is effective labor supply and $\mu_t^{ret} \equiv (1 - \rho_t) \pi_{T_t^r t} + \sum_{j \geq T_t^r} \pi_{jt}$ is the effective fraction of retired agents in the population. Population aging reduces effective labor supply L_t and increases the share of retired workers μ_t^{ret} . At fixed tax policy, this lowers net tax revenues, and some adjustment of policy is needed at some point in time to ensure that the policy is sustainable. The mix of policy tools used to balance the budget can in principle have large consequences for aggregate saving behavior. To model this adjustment in a flexible way, we assume that the government sets a sequence of retirement ages T_t^r and adjusts its policy over time

⁶¹This implies $\mathbb{E}l(s_j) = 1$, in line with our earlier assumptions. Here, we use the notation $\theta(s_j)$ and $\epsilon(s_j)$ for the fixed effects and transient shocks associated with s_j . Going forward, we will drop the explicit dependence on s when there is no risk for ambiguity.

⁶²Note that our assumptions on production imply that households receive the benefits of technology growth during their lifetime. However, as long as technology growth is constant γ , it is not material whether the growth term loads on time or cohort effects, since we could reparametrize the model by writing

$$w_t l_{it} = \tilde{Z}_{t-j} \theta_i \tilde{h}_{jt} \epsilon_{it},$$

where \tilde{Z}_{t-j} is a cohort effect growing at rate g , and $\tilde{h}_{j,t} = h_{j,t}(1 + \gamma)^j$ is an age effect.

according to a fiscal rule

$$\tau_t^{ss} = \bar{\tau}^{ss} + \varphi^{ss} \left(\frac{B_t}{Y_t} - \bar{b} \right) \quad (53)$$

$$\tau_t = \bar{\tau} + \varphi^\tau \left(\frac{B_t}{Y_t} - \bar{b} \right) \quad (54)$$

$$\frac{G_t}{Y_t} = \frac{\bar{G}}{\bar{Y}} - \varphi^G \left(\frac{B_t}{Y_t} - \bar{b} \right) \quad (55)$$

$$\bar{d}_t = \bar{d} - \varphi^d \left(\frac{B_t}{Y_t} - \bar{b} \right), \quad (56)$$

where $\bar{b} \equiv (1 + g) \frac{B_{-1}}{Y_0}$ is the debt to GDP ratio in 2016. The absolute magnitude of parameters φ^x , for $x \in \{ss, \tau, G, d\}$ regulate the aggressiveness of the response policy to the level of debt. $\varphi^x = 0$ corresponds to the case where instrument x is fixed at its initial value forever, and $\varphi^x = \infty$ corresponds to the case where this tax instrument is adjusted to ensure that the debt-to-GDP ratio $\frac{B_t}{Y_t}$ remains at its initial value forever. Moreover, the relative magnitude of φ^x regulates the relative importance of each instrument in the overall adjustment.

Bequest distribution. The distribution of bequests is given by

$$b_{jt}^r(\theta) = \frac{(1 + r_t^a)}{N_{jt}} F_j \sum_{j'=1}^T N_{j'-1,t-1} (1 - \phi_{j'-1,t-1}) \sum_{\theta_-} \frac{\Pi^\theta(\theta|\theta_-) \pi^\theta(\theta_-)}{\pi^\theta(\theta)} \mathbb{E}[a'_{j'-1,t-1}(s^{j-1})|\theta_-], \quad (57)$$

where F_j is an exogenous and time-invariant distribution of bequests by age (with $\sum_j F_j = 1$), and $\pi^\theta(\theta)$ is the probability distribution across θ . The right-hand of equation (57) counts the total amount of bequests, appropriately augmented by the period- t return, left by the potential parents of children of type θ , takes a fraction F_j of this amount to be distributed to children of age j , and divides by the number of children aged j and type θ at date t , $N_{jt} \pi^\theta(\theta)$. An aging population alters the number of older agents that give bequests (the numerator of equation (57)) to the number of agents that receive bequests (the denominator of equation (57)), and this modifies the bequest that each child can receive, in turn potentially affecting saving behavior.

D.4 Steady state equations

Interest rates, markups, government policies, TFP growth rates, and labor supply profiles are all constant. Government policy consists of a constant tax rates τ , τ^{ss} , spending G , retirement policy T^r , ρ and replacement rate \bar{d} . The demography consists of constant mortality rates, a fixed age distribution, and a constant population growth rate

$$\phi_{jt} \equiv \phi_j, \quad \pi_{jt} \equiv \pi_j, \quad N_t = (1 + n)^t N_0,$$

as well as of net migration by age that "explains" why the population distribution remains constant over time

$$m_j \equiv \frac{M_{jt}^{mig,net}}{N_{jt}} = (1+n)\pi_j - \phi_{j-1}\pi_{j-1}, \quad (58)$$

In the steady state, we normalize all macroeconomic aggregates by $Z_t N_t$, and all individual-level variables by Z_t . We also define g as the overall growth rate of the economy

$$1+g \equiv (1+n)(1+\gamma).$$

We set steady state normalized GDP to 1:

$$\frac{Y_t}{Z_t N_t} = 1.$$

Investments are given by

$$\frac{I}{Y} = \frac{K}{Y} \left[1 - \frac{1-\delta}{1+g} \right]. \quad (59)$$

The profit, labor, and capital share are

$$\frac{\Pi}{Y} = 1 - \frac{1}{\mu'}, \quad (60)$$

$$\frac{wL}{Y} = \frac{1-\alpha}{\mu'}, \quad (61)$$

$$\frac{K/(1+g)}{Y} = \frac{\alpha}{(r+\delta)}. \quad (62)$$

Assets satisfy

$$\frac{A}{Y} + \frac{A^{mig,net}}{Y} = \frac{K}{Y} + \frac{B}{Y} + \frac{NFA}{Y},$$

both sides are equal to aggregate wealth $\frac{W}{Y}$. This is equation (45). The difference between r and g is

$$\frac{r-g}{1+g} = \frac{1-s^L - I/Y}{W^c/Y},$$

where the last equality uses the steady state equation for dividends. The aggregate resource constraint is

$$\frac{C}{Y} + \frac{I}{Y} + \frac{G}{Y} = 1 + \frac{NFA}{Y} \left(\frac{1+r}{1+g} - 1 \right) + \frac{A^{mig,net}}{Y}. \quad (63)$$

From (52) and using our normalization that $L = 1$, net taxes normalized by GDP are equal to

$$\frac{T}{Y} = s_L [\tau + (1-\tau)(\tau^{ss} - \mu^{ret} \bar{d})],$$

combining with the flow budget constraint $G + rB = T$, we obtain the government's steady state flow budget constraint as

$$\frac{T}{Y} = s_L [\tau + (1-\tau)(\tau^{ss} - \mu^{ret} \bar{d})] = \frac{G}{Y} + \left(\frac{1+r}{1+g} - 1 \right) \frac{B}{Y}.$$

The solution to the household problem implies stationary functions for consumption and assets as functions of the history of shocks (and normalized by technology). These connect to macroeconomic quantities via

$$\frac{C}{Y} = \sum_{j=0}^T \pi_j \mathbb{E}c_j, \quad \frac{A_j}{Y} = \mathbb{E}a_j, \quad \frac{A}{Y} = \sum_{j=0}^T \pi_j \frac{A_j}{Y}, \quad \frac{Beq}{Y} = \sum_{j=0}^T \pi_j (1 - \phi_j) \mathbb{E}a_j.$$

Finally, since we assume that steady state migrants have the same distribution of assets as regular households, we have

$$\frac{A^{mig,net}}{Y} = \sum_{j=0}^T \frac{m_{j+1}}{\pi_j} A_j, \quad (64)$$

where we recall that m_j is the number of migrants as a share of age group j at time t .

D.5 Calibration details

The baseline calibration procedure is explained in section 4.3. The baseline calibration considers no markups, ie $\mu = 1$. In an alternative calibration with markups, we assume that the capital-to-GDP ratio is at its 2016 value of $\frac{K}{Y} = 232\%$ ⁶³, and back out the out $\mu = 1.04$ from (45) to match the total value of corporate assets. In either case, we calibrate δ to ensure that the steady state version of the firm capital first-order condition (62) is satisfied given K/Y .

Figure D.2 presents the bequest distribution rule use to allocate an aggregate amount of bequests over age groups. This is computed from the SCF as inheritances received by 5-year age group over total inheritances received and interpolating between age groups. Figure D.3 presents the distribution of average number of dependent children per adult by age taken from Gagnon et al. (2019). Figure D.4 presents the adjustment of the age-efficiency profile for ages above 65. We scale it up for ages above 65 since the empirical profile is for a retirement age at this age. The profile used in all model experiments is the dashed red line.

Panel A of figure D.5 compares the distribution of bequests in the model to the empirical distribution in the data. We measure it as the value of bequests at certain percentiles divided by average bequests. We take the empirical distribution from Table 1 in Hurd and Smith (2002). The legend also reports the resulting model aggregate bequests-to-GDP ratio $\frac{B}{Y} = 4.5\%$. Panel B compares the model age-consumption profile to the empirical age-consumption profile from the 2010 CEX. The smoothed empirical profile is used in the calibration procedure.

⁶³From the Fixed Assets Table 1.1, private fixed asset \$43.48T divided by 2016 US GDP.

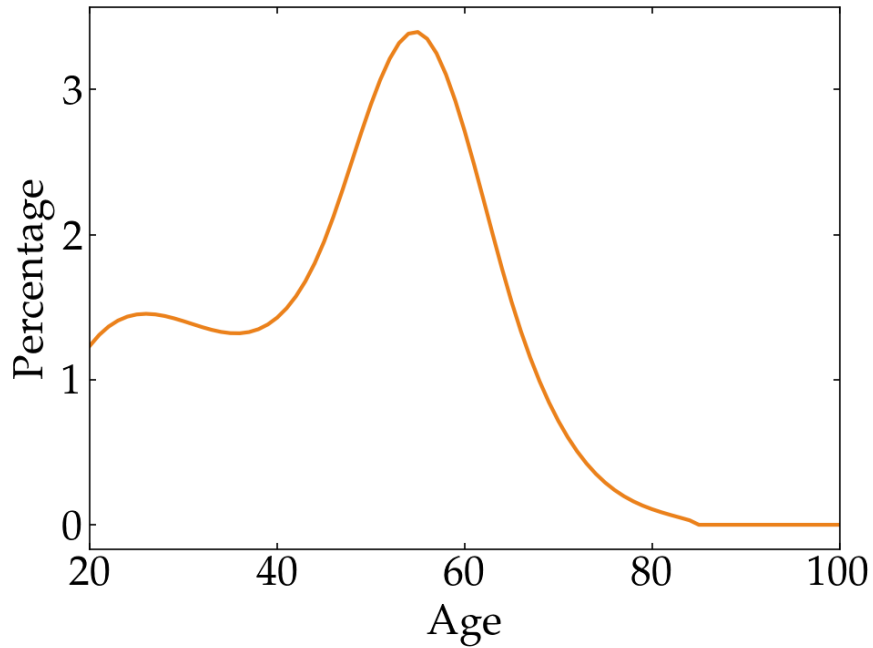


Figure D.2: Bequest distribution rule

Notes: This figure presents the bequest allocation rule used to allocate an aggregate amount of bequest across age groups. It is obtained from the SCF as inheritance received by 5-year age group over total inheritance received. We follow [Hendricks \(2001\)](#) and convert any amount received in the past to present value using a discount rate of 4% and deflating using a Consumer Price Index.

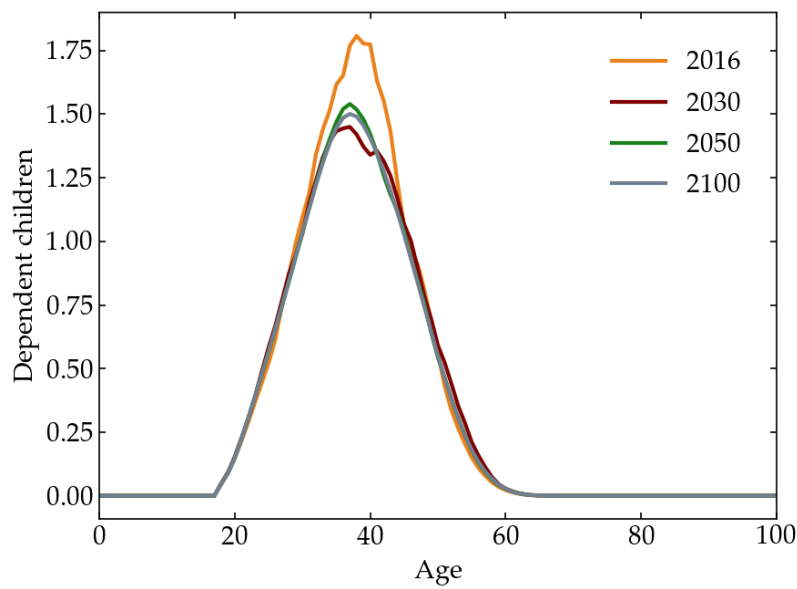


Figure D.3: Average number of dependent children by age

Notes: This figure presents the average number of dependent children per adult by age taken from [Gagnon et al. \(2019\)](#).

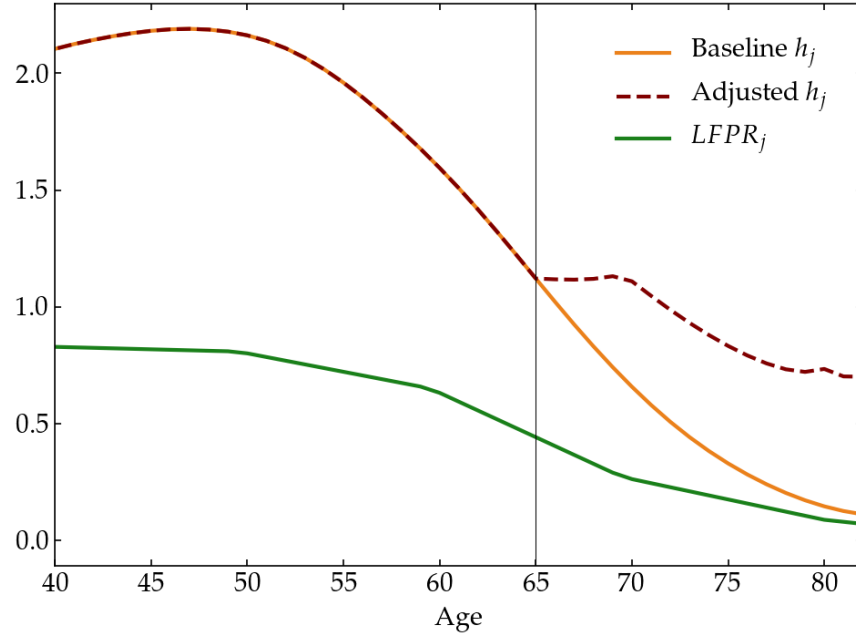
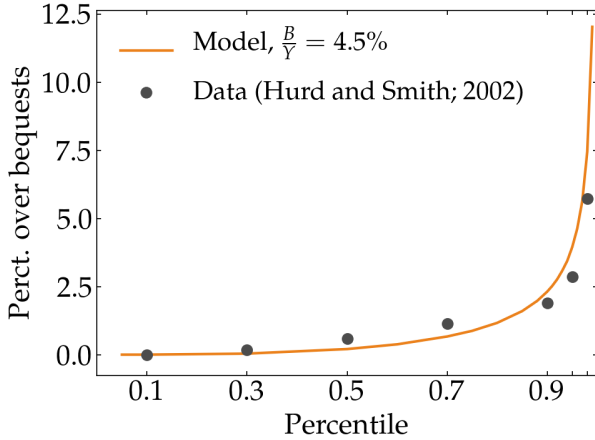


Figure D.4: Age-efficiency profile

Notes: This figure describes the scaling of the age-efficiency profile for ages above 65. The baseline efficiency profile h_j obtained from the normalized empirical profile from LIS is depicted by the solid orange line. Since the empirical profile is for a retirement age of 65, we rescale h_j by $LFPR_{65}/LFPR_j$ for $j \geq 65$. The LFPR by age is obtained from the BLS (<https://www.bls.gov/emp/tables/civilian-labor-force-participation-rate.htm>). The resulting profile that we use as a model input is the dashed red line.

A. Bequests distribution



B. Age-consumption profile

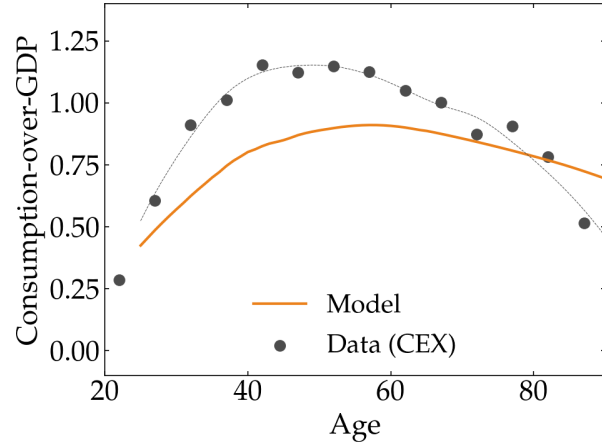


Figure D.5: Calibration

Notes: Panel A presents the model distribution of bequests (orange line) and the empirical one from [Hurd and Smith \(2002\)](#). We measure it from their Table 1 as the value of bequests at percentiles 0.1, 0.3, 0.5, 0.7, 0.9, 0.95, 0.98 divided by average bequests. The legend also reports the resulting bequests-to-GDP ratio in the model. Panel B presents the model age-consumption profile at our calibration (orange line) as well as the empirical age-consumption profile from the 2010 CEX. There are no observations after the 85 – 90 age bin. The dashed black line corresponds to the empirical profile smoothed using a convolution of polynomials that is used in the calibration procedure.

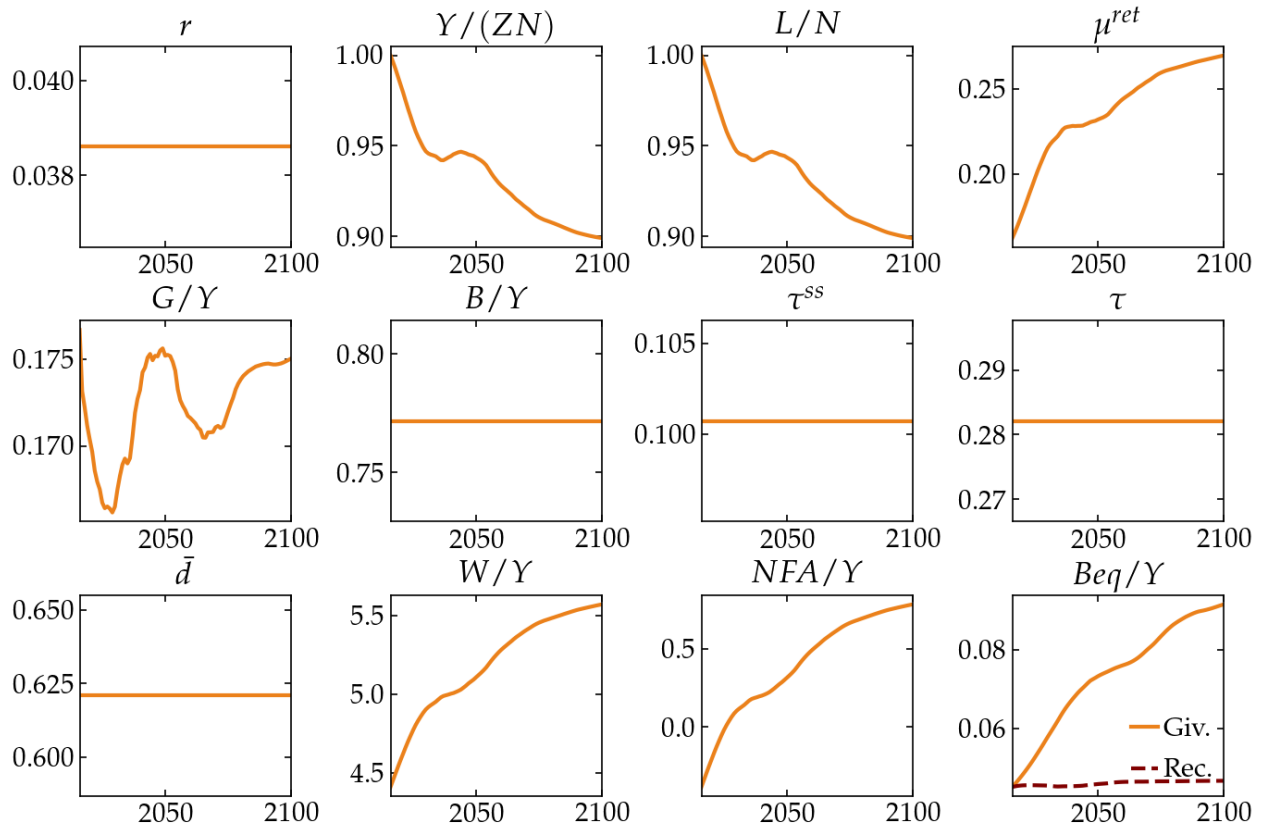


Figure D.6: Sufficient statistic scenario

Notes: This figure presents the transitional dynamics in the sufficient statistic scenario.

D.6 Transition dynamics

Figures D.6 to D.9 present the transitional dynamics for the main experiments of section 4.

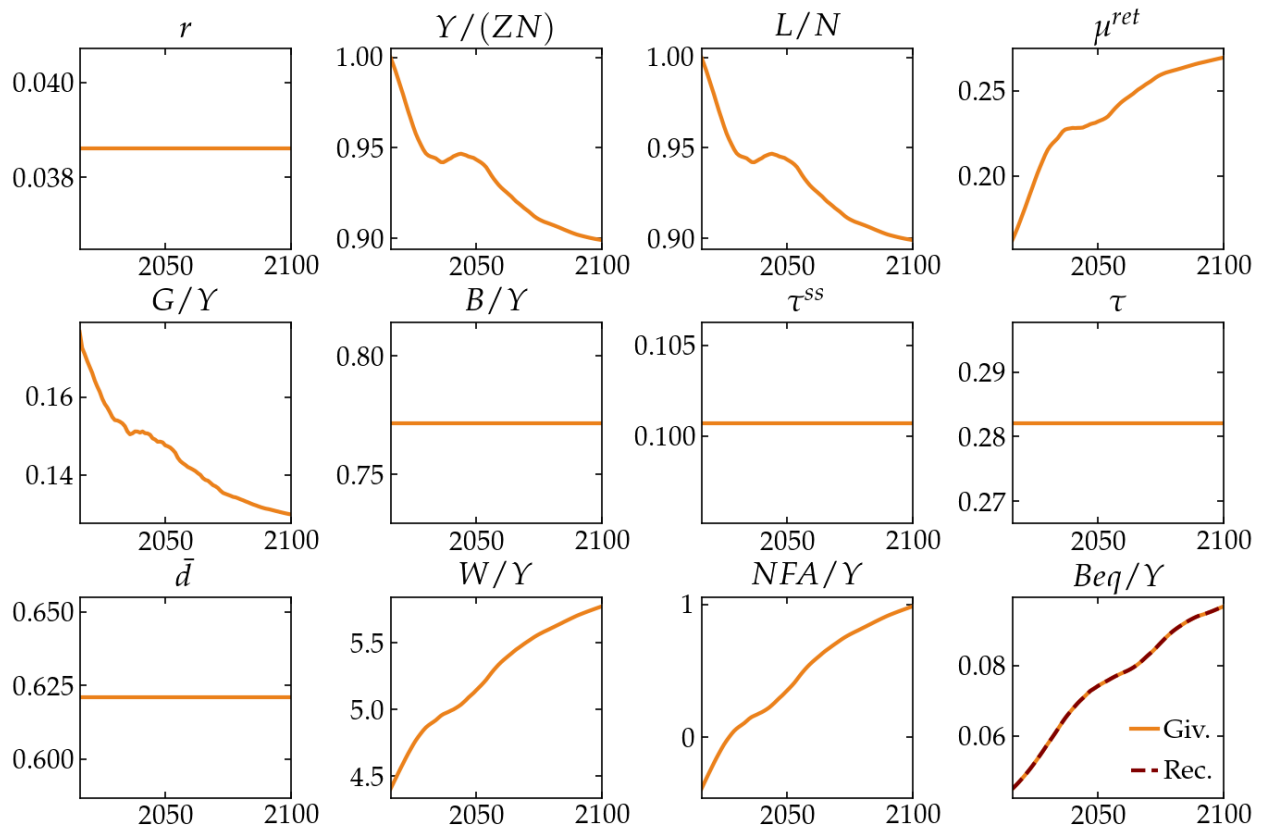


Figure D.7: Bequests and children

Notes: This figure presents the transitional dynamics in the scenario with time-varying altruism towards children and bequests received.

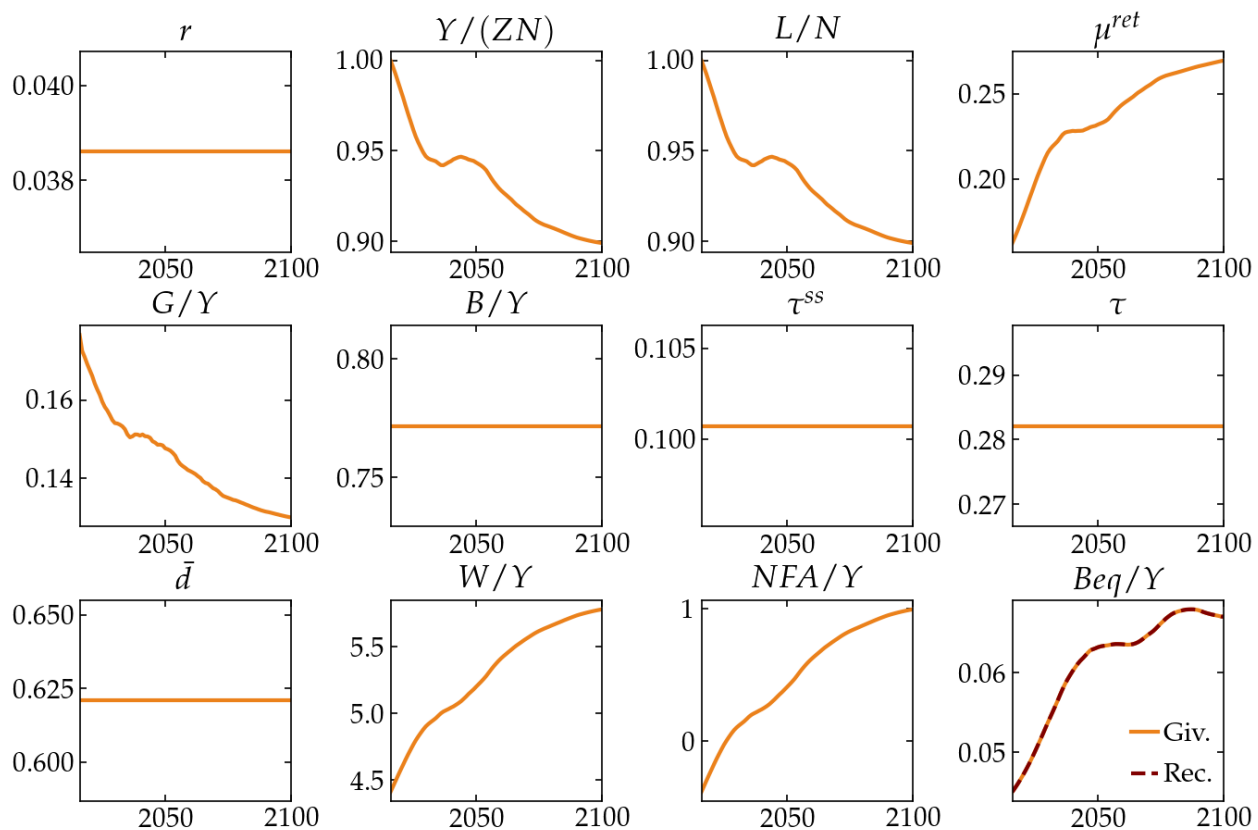


Figure D.8: Mortality

Notes: This figure presents the transitional dynamics in the scenario with time-varying mortality.

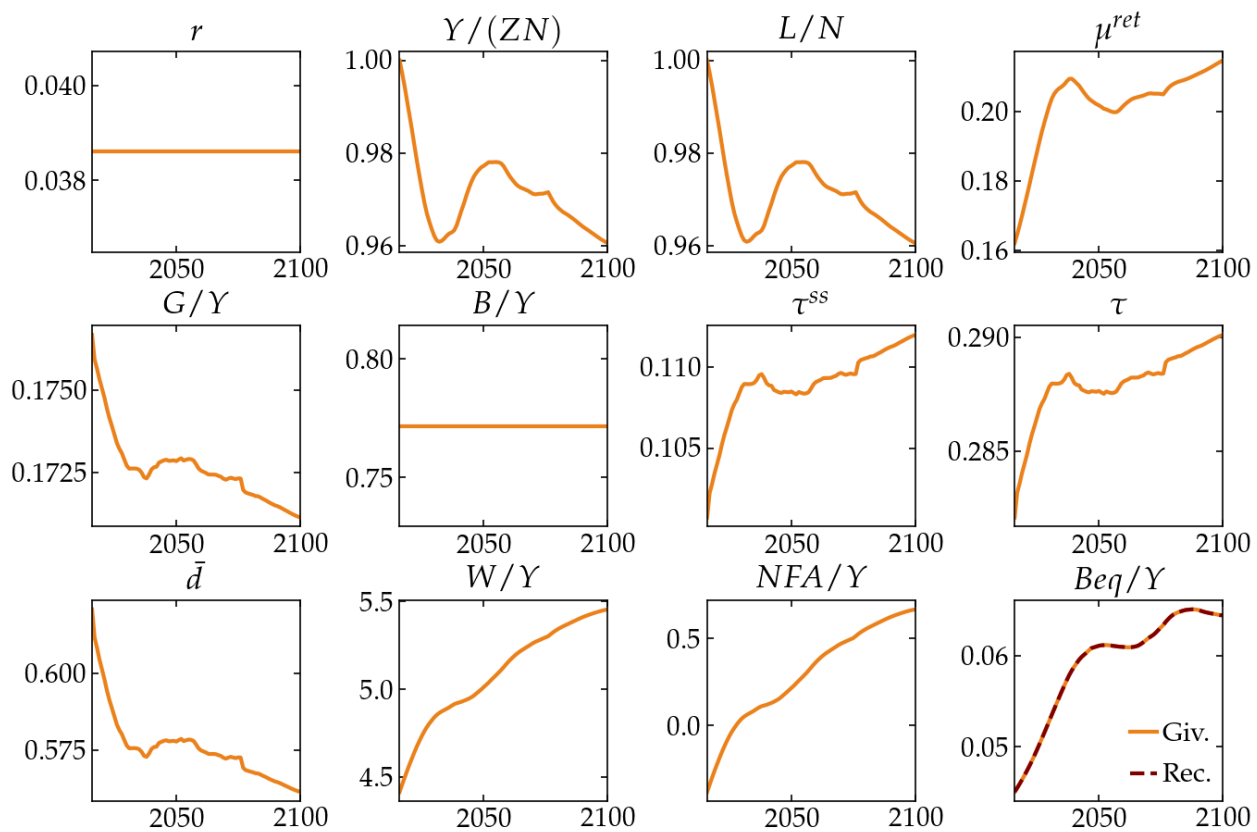


Figure D.9: Baseline

Notes: This figure presents the transitional dynamics in our baseline scenario that includes social security.

D.7 Additional results

Alternative methods of closing the shortfall. In the main results, we considered a setup where the social security shortfall was closed by a combination of tax increases, retirement age increases, and benefit reductions. Here, we explore how the method of closing the shortfall affects the result.

First, we consider extreme policies that load all adjustments on one variable, without changing the path of B/Y . That is, in every period, we solve for the value of a single adjustment variable that is required to balance the budget with a constant level of debt-to-GDP. In the context of the fiscal rule, this corresponds to setting all coefficients φ to zero except for one, which we take to infinity.

Panel A, B and C of figure D.10 shows the results when we load all adjustments on, respectively, the social security tax rate, the total income tax rate and the benefits. To better isolate the effect of these policies, we keep the retirement age fixed at 65.

We see that the method of adjustment has large impacts on the evolution of W_t/Y_t . For example, if all adjustments load on increasing the social security tax, agents' behavioral responses dampen the rise in wealth by 62% of GDP. This is because the rise in the social security tax erodes the net income agents received during their work life, hence they reduce their overall savings. In contrast, if all adjustments are loaded on lower benefits, the behavioral effect imply an additional 86 pp increase in wealth by 2100. In this case, the lower benefits reduce the net income agents received during retirement who then save more to compensate this loss. Increasing the total income tax has an homogeneous effect on the age-profile of net income. In this case, agents respond by saving less which dampens the rise in W/Y by 39 pp.

In panel D, we consider a large increase of the retirement age to age 80. To better isolate the effect of this policy the adjustment is done through adjustments in spending which increases by about 1% over the period in response to higher tax revenue.

Panel E considers the case of fiscal rules that let government debt rise over time. Formally, we choose φ^* and set

$$\varphi^G = \varphi^*, \quad \varphi^{ss} = \frac{\varphi^*}{s^L}, \quad \varphi^\tau = \frac{\varphi^*}{s^L(1 - \tau^{ss})}, \quad \varphi^{ss} = \frac{\varphi^*}{\mu_{ret}s^L}.$$

such that the debt-to-GDP ratio increases by about 50% of GDP by 2100. The adjustment in the retirement is assumed to be the same as in baseline scenario. To finance the flow cost of an increasing level of debt, the government adjusts its instruments by more than in the baseline scenario. In this case, this results in behavioral responses that are almost identical to the baseline scenario. We

conduct the same experiment with a 50% increase in B/Y and obtain similar results.

Annuities contracts. So far, we have assumed agents lack access annuity markets to insure against the uncertainty about the length of life. To explore the implications of this assumption for our results, we recalibrate a version of our model that includes annuity markets.

Concretely, the household problem (15) becomes

$$\begin{aligned}
V_{jt}(s^j, a) &= \max_{c, a'} \psi_{jt} \frac{c^{1-\sigma}}{1-\sigma} + YZ_t^{\nu-\sigma} (1 - \phi_{jt}) \frac{(b^s)^{1-\nu}}{1-\nu} + \beta \phi_{jt} \mathbb{E} \left[V_{j+1, t+1}(s^{j+1}, a') | s^j \right] \\
c + a' + (1 - \phi_{jt}) b^s &= (1 - \tau_t^y) y_{jt}(s_j) + \frac{1 + r_t^a}{\phi_{j-1, t-1}} a + tr_{jt}(s^j) + b_{jt}^r(s^j) \\
a' &\geq -\bar{a} Z_t.
\end{aligned} \tag{65}$$

where b^s denotes the planned amount of bequests given, and the ex-post interest rate is inflated by the inverse of the survival probability. Bequests are pooled and distributed in the same fashion as in our baseline scenario. Since the survival probability now enters the budget constraint but not the Euler equation, the presence of annuities changes how agents respond to changes mortality.

Panel F in figure D.10 plots the evolution of wealth and its decomposition in the calibration with annuity markets under the assumption of our baseline scenario for everything else. The presence of annuity markets dampens the behavioral response as agents decide to increase their savings by less.

Bequests per kid. Our baseline calibration assumes that agents get utility out of bequeathing a certain amount of wealth. We explore an alternative where agents instead care about the amount of bequests per kid. That is, utility out of bequests becomes

$$v(a') = \frac{\left(\frac{a'}{\chi}\right)^{1-\nu}}{1-\nu}.$$

where χ is the average number of children. Hence, if the average number of children decreases over time, agents get more utility out of bequeathing the same amount of wealth since it is divided by fewer children.

Panel G in figure D.10 plots the evolution of wealth and its decomposition in the calibration with utility of bequests per kid. Compared to the baseline scenario, behavioral responses are expected to drive up wealth by an additional 8% of GDP.

Changing migration. In 2016, the US had a population growth rate of 0.7%, and a net migration rate of 0.3%. Hence, migration is a major demographic force in the US that contributes to about half of the US population growth. In light of recent decreases in the US fertility rate (1.93 to 1.72 between 2010 and 2018⁶⁴), this share is likely to increase, and absent migration, the US is likely to face population decline in the long run.

Given the demographic importance of migration, we study how changes in migration flows affects our exercise. To do this, we consider two experiments. First, we consider where we shut off migration completely. Second, we consider the case where migrants have no asset when they arrive to contrast the baseline assumption that migrants have the same assets as residents. These exercises echo the analysis of [Storesletten \(2000\)](#) on the importance of migration in OLG models.

Panel H in figure [D.10](#) plots the evolution of wealth and its decomposition when we shut off migration. Wealth increases more rapidly than in the baseline scenario and by 18 more percentage points of GDP by 2100. This can be explained by the stronger compositional effect implied by a population that is aging more rapidly, and a lightly higher behavioral responses.

Panel I presents the case where migrants come in with no assets. Concretely, this reduces aggregate wealth in the economy in every period. However, since wealth holdings are only a few percentage points of GDP in the baseline scenario, there are no discernible differences with the baseline scenario.

Positive markups. In panel J, we consider our calibration with positive markups $\mu = 1.04$. The household side of the model is exactly identical to our preferred calibration. The only difference with the baseline scenario is the presence of a time-0 revaluation effect. It is induced by a drop in the initial ex-post interest rate r_0^a and generates an initial drop in wealth. Since the terminal steady state is identical to the one in the baseline scenario, it must be that the calculated behavioral term Δ^a is larger. Indeed, this term implies a 21 percentage points larger increase in wealth by 2100.

Alternative elasticity of intertemporal substitution (EIS). In our baseline calibration, we assume an intertemporal elasticity of substitution of $\sigma^{-1} = 1$. Here we investigate whether this assumption has sizable consequences for our results. To do so, we consider a higher value of $\sigma^{-1} = 0.5$ and a lower value of $\sigma^{-1} = 2$. In both cases, we recalibrate the five preferences parameters $(\beta, Y, \nu, \lambda, \varphi)$ to hit the same targets as in our baseline calibration. Panels K and L present

⁶⁴https://www.cdc.gov/nchs/data/nvsr/nvsr68/nvsr68_13-508.pdf

the results for these two alternative cases. For both cases, the outcomes are very close the baseline scenario. By construction, the shift-shares are very close since in all cases we calibrate the model to hit the empirical shift-share. More importantly, these graphs show the the magnitude of the behavioral responses do not depend on the size of the EIS.

The assumed EIS is unimportant for this exercise since we do an open economy exercise where interest rates are constant. Intuitively, changing the EIS simply implies that a different combination of parameters is used to explain the cross-sectional patterns in 2016. Since interest rates are constant over time, these differences in the parameter configuration gives similar predictions over time as long as interest rates stay constant (equivalently, the IES is not well-identified from a steady state analysis).

In contrast, if interest rates adjust down, parameter combinations with a high IES will see much lower wealth-to-output levels, and vice versa. Hence, even if the EIS is unimportant for the open economy response to aging, it will be crucial for the closed-economy response to aging.

Table D.1: Change in W/Y between 2016 and 2100: decomposition

	Δ	Δ^{comp}	$\Delta^{beh,a}$	$\Delta^{beh,h}$	Δ^{err}
Empirical exercise	1.13	1.13	-	-	-
Sufficient statistic scenario	1.13	1.13	-0.00	0.00	-0.00
+ Utility weight of children	1.15	1.13	0.02	0.00	0.00
+ Bequests	1.33	1.13	0.14	0.00	0.07
+ Mortality	1.37	1.13	0.16	0.00	0.08
+ Social security (baseline)	1.04	1.13	0.16	-0.25	0.01
<i>Alternative scenarios</i>					
Only social security tax	0.41	1.13	-0.62	0.00	-0.10
Only benefits	2.24	1.13	0.86	0.00	0.25
Only total income tax	0.69	1.13	-0.39	0.00	-0.05
Retirement	0.76	1.13	0.29	-0.50	-0.16
Fiscal rules	1.05	1.13	0.16	-0.25	0.01
Annuities	1.08	1.13	0.18	-0.25	0.02
Bequests per kid	1.18	1.13	0.24	-0.25	0.06
No migration	1.22	1.27	0.17	-0.25	0.03
Migrants with no assets	1.04	1.13	0.16	-0.25	0.01
Positive markups	1.04	1.07	0.37	-0.24	-0.16
Lower σ^{-1}	1.05	1.13	0.16	-0.25	0.01
Higher σ^{-1}	1.05	1.13	0.16	-0.25	0.01

Notes: This table summarizes the results of the decomposition for our model experiments. All numbers are computed according to equation (38). The first column (Δ) presents the total effect on aggregate wealth, the second column (Δ^{comp}) to the effect from shifting age-distributions, the third column (Δ^a) to the effect from shifting age-wealth profiles, the fourth column (Δ^h) to the effect from shifting age-labor supply profiles, and the last column (Δ^{err}) to the approximation error.

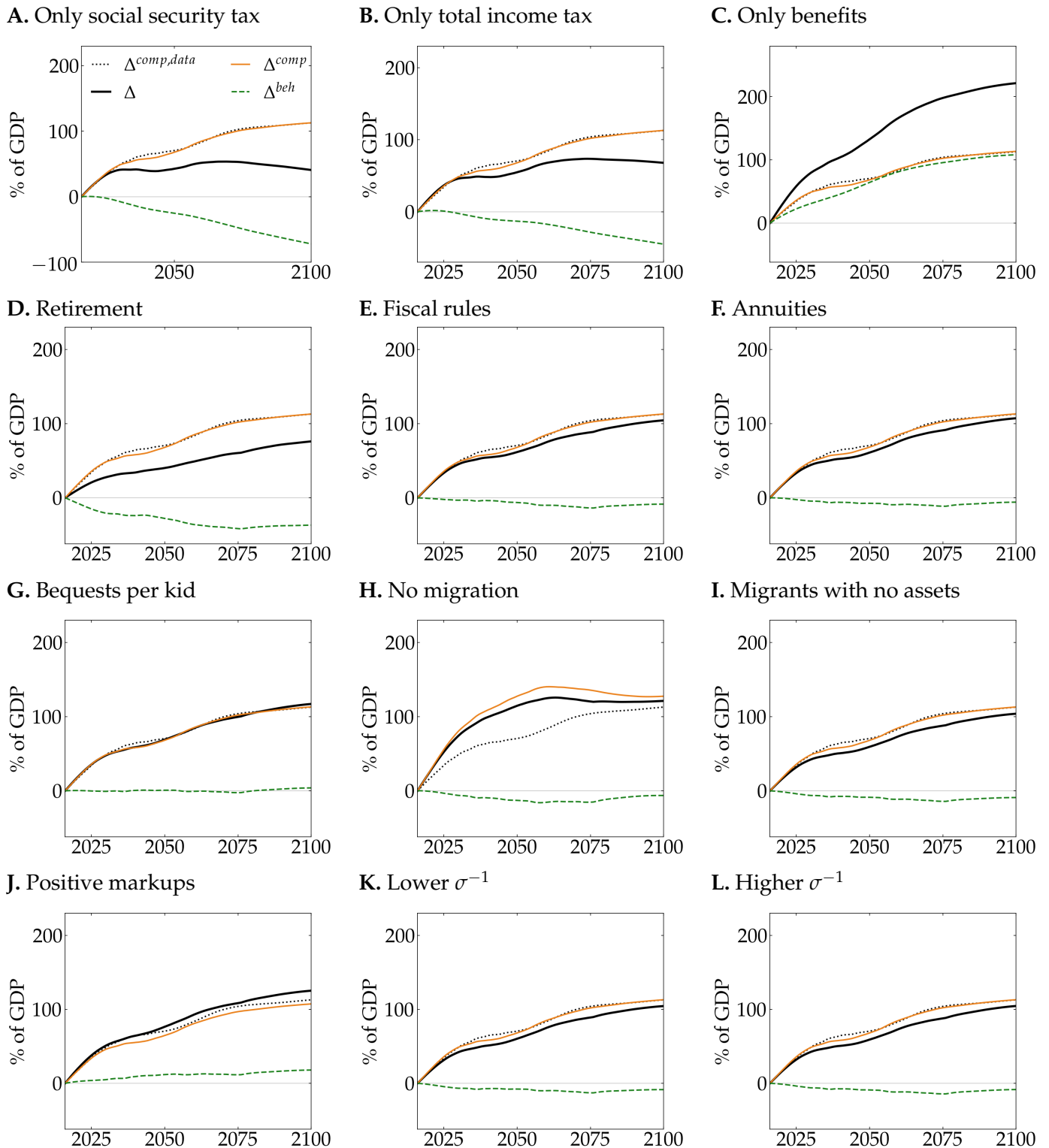


Figure D.10: Evolution of W_t/Y_t and decomposition under alternative scenarios

Notes: This figure presents the change in wealth (Δ) and its decomposition according to equation (38). Panel A to E present alternative government policy adjustments. In panel A, the retirement age remains fixed at 65 and the only instrument that is adjusted is the social security tax. Similarly, only the benefits and only the total income tax are adjusted in, respectively, panels B and C. In panel D, all instruments are adjusted according to the fiscal rules such that debt rises by about 50% of GDP by 2100. Panel E considers an increase in the retirement age to age 80. Panel F considers the case of annuities. In panel G, households get utility out of bequests per kid instead of total bequests. In panel H, we shut off migration completely. In panel I, we allow for migration but assume that migrants come in with no assets. Panel J presents the case with positive markups. Finally, panel K and L consider alternative values of σ^{-1} of, respectively, 0.5 and 2.

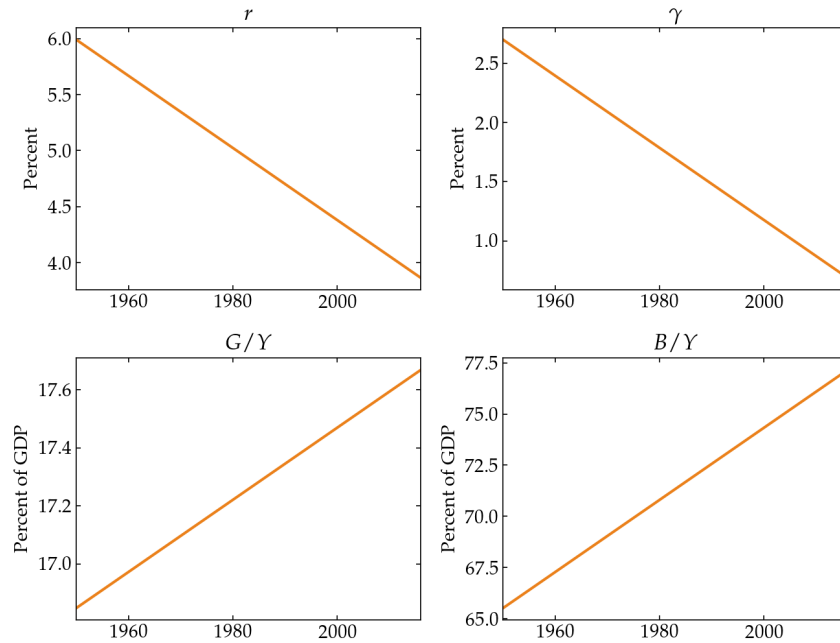


Figure D.11: Inputs for historical experiment

Notes: This figure presents the input paths used in the historical experiment: the interest rate r , the technological growth rate γ , the government expenditures-to-GDP ratio G/Y , and the government debt-to-GDP ratio B/Y . All inputs are assumed constant after 2016.

D.8 Historical exercise

Figure D.11 presents the inputs in the historical experiment conducted in section 4.5.

Figure D.12 repeats the baseline results but also shows the results under fixed mortality or fixed fertility.

E Appendix to section 5

E.1 The size of the asset demand sensitivity

In Section 4, we found that the EIS σ^{-1} was not well-identified from cross-sectional data on assets by age. At the same time, the EIS is an important determinant of asset demand sensitivity ε_d .

Here, we validate our choice of σ^{-1} against external evidence from [Jakobsen et al. \(2020\)](#). They study the effect of variation in post-tax returns on taxable wealth using evidence from a 1989 Danish wealth tax reform, which cut the marginal tax rate on wealth from 2.2% to 1% and doubled the exemption threshold for couples. They use this reform to perform two analyses. In the first analysis, the "Couples DD", they compare couples and singles with wealth levels between the old

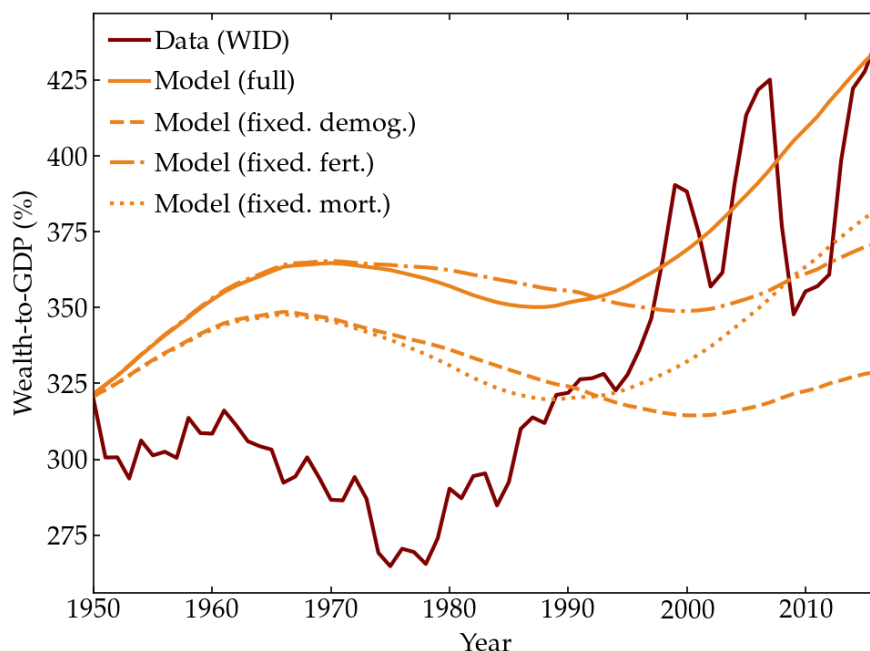


Figure D.12: Effects from mortality and fertility

Notes: The solid red line depicts the wealth-to-GDP ratio from the World Inequality Database. The solid orange line presents the evolution of the wealth-to-GDP ratio from the model in the baseline scenario. The dotted line considers fixed mortality from 1960 on, the dot-dashed line fixed fertility rates from 1960 on, and the dashed line fixed mortality and fertility. and when mortality and fertility are fixed to their 1960 level.

and new exemption thresholds. They exploit the fact that post-reform, a marginal tax rate gap of one percentage point opened up between the two groups. This experiment allows them to study the response in asset holdings of the moderately wealthy (approximately top 1%). In the second analysis, the "Ceiling DD", they use the fact that the Danish wealth tax system had a ceiling, so that the reduction in the marginal tax rate from 2.2% to 1% did not affect those above the ceiling. This implies a differential change of 1.2 percentage points in the returns of assets for those above and below the ceiling, and allows the authors to study the accumulation response among the very wealthy.

In Figure E.1, we introduce a wealth tax into our model and replicate the two model experiments from [Jakobsen et al. \(2020\)](#).⁶⁵ We follow their model exercises very closely. The left panel shows the results from the "Couple DD", which considers the effect of reducing the wealth tax from 2.2% to 1.2% for people in the top-1% wealth range in the initial steady state of our model. The lines show the dynamic response for different values of σ^{-1} , and the dots shows the quasi-

⁶⁵Formally, we recalibrate our model so that post-tax capital returns are the same as in our baseline calibration when the wealth tax is at the 2.2% pre-reform level.

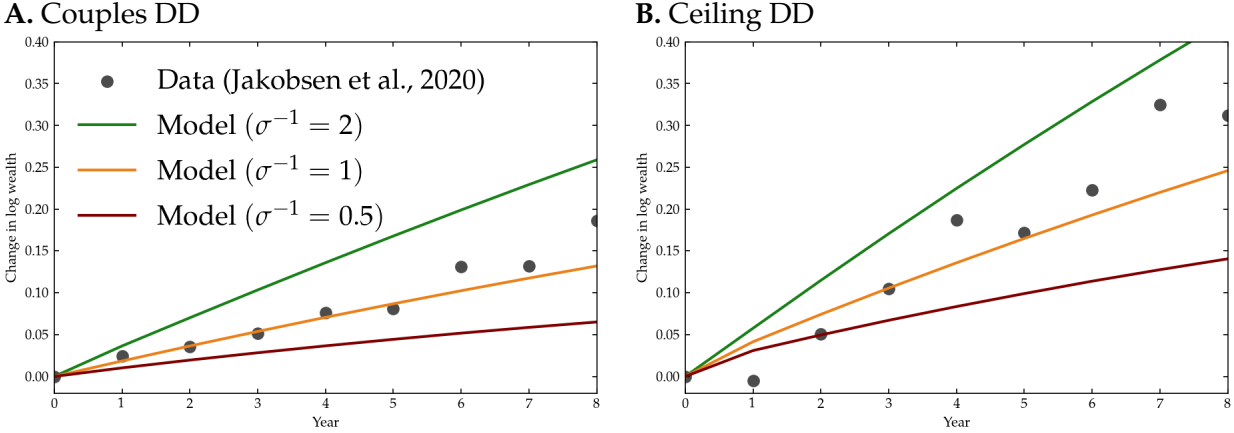


Figure E.1: Effects of a wealth tax cut and external evidence

Notes: This figure presents the effects of a cut in the wealth tax in our model and compares them to the evidence from [Jakobsen et al. \(2020\)](#). We replicate the model experiments they conduct in their section 5 and present in figures 9 and 10. Panel A replicates their first experiment. It analyzes the effects of lowering the wealth tax from 2.2% to 1.2% on the top 1%. This is comparable to their quasi-experimental "Couples DD" estimates. Our results for the second experiment are presented in panel B. It analyzes the effects of cutting the wealth tax by 1.56 percentage points on the top 0.3%. This is comparable to their quasi-experimental "Ceiling DD" estimates.

experimental estimates from the Danish experiment. In the right panel, we consider the "Ceiling DD" by analyzing the effect of reducing the wealth tax 1.56 percentage points on the 0.3% wealthiest households in our model.⁶⁶ Again, the points show the outcome of the Danish experiment.

From the results, we see that our model endogenously generates the steeper dynamic profile in the Ceiling DD versus the Couples DD. Compared to the exact outcomes of the Danish experiment, $\sigma^{-1} = 2$ and $\sigma^{-1} = 0.5$ imply, respectively, too high and too low sensitivities of wealth, while our main calibration with $\sigma = 1$ has a much better fit.

E.2 Demographic inputs

Figure [E.2](#) presents the observed population growth rate across countries between 2016 and 2100. Figure [E.3](#) presents the population distributions in 2016 and 2100 as well as the stationary distribution across countries.

E.3 Calibration outcomes

Figure [E.4](#) presents the calibrated age-wealth profiles across countries (solid orange lines) as well as the empirical age-wealth profiles (gray dots). Figure [E.5](#) presents the values of the country

⁶⁶The reduction is bigger than the 1.2 percentage point difference in tax cuts between people above and below the ceiling, because some people below the ceiling also were affected by the couple exemption.

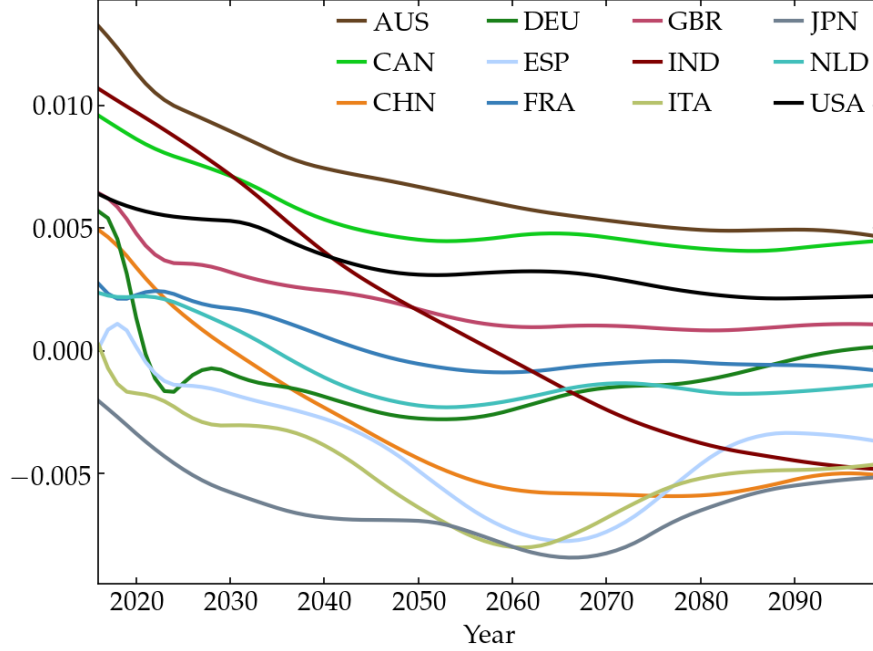


Figure E.2: Population growth rates

Notes: This figure presents the population growth rates between 2016 and 2100 for the 12 countries we consider.

specific sensitivities of net asset demand $\epsilon_c^d + \epsilon_c^s$ (y-axis) in our baseline calibration and compares it to the shift-shares (x-axis). The size of the dots denotes the 2016 nominal GDP shares. The horizontal line denotes the weighted average $\bar{\epsilon}^d + \bar{\epsilon}^s$.

F Appendix to section 6

F.1 Stock vs flow approach

This appendix details the measurement error exercise for the shift-share results. We define normalized personal savings is defined as

$$\begin{aligned}
 s_j &= a_j - \frac{a_{j-1}}{\phi_{j-1}(1+\gamma)} \\
 &= \frac{1+r}{1+\gamma} b_j^r + \frac{r}{1+\gamma} a_{j-1} + (1-\tau^y)y_j + tr_j - c_j - \frac{1-\phi_{j-1}}{\phi_{j-1}} \frac{a_{j-1}}{1+\gamma},
 \end{aligned}$$

from which we can obtain aggregate personal savings (29) that is always consistent with (30). This suggests a natural way to use a flow shift-share for the wealth-to-GDP ratio.

We start from a steady-state where population is at its stationary distribution and consider a decline in the population growth rate from n to $n/2$. This induces a new growth rate of the

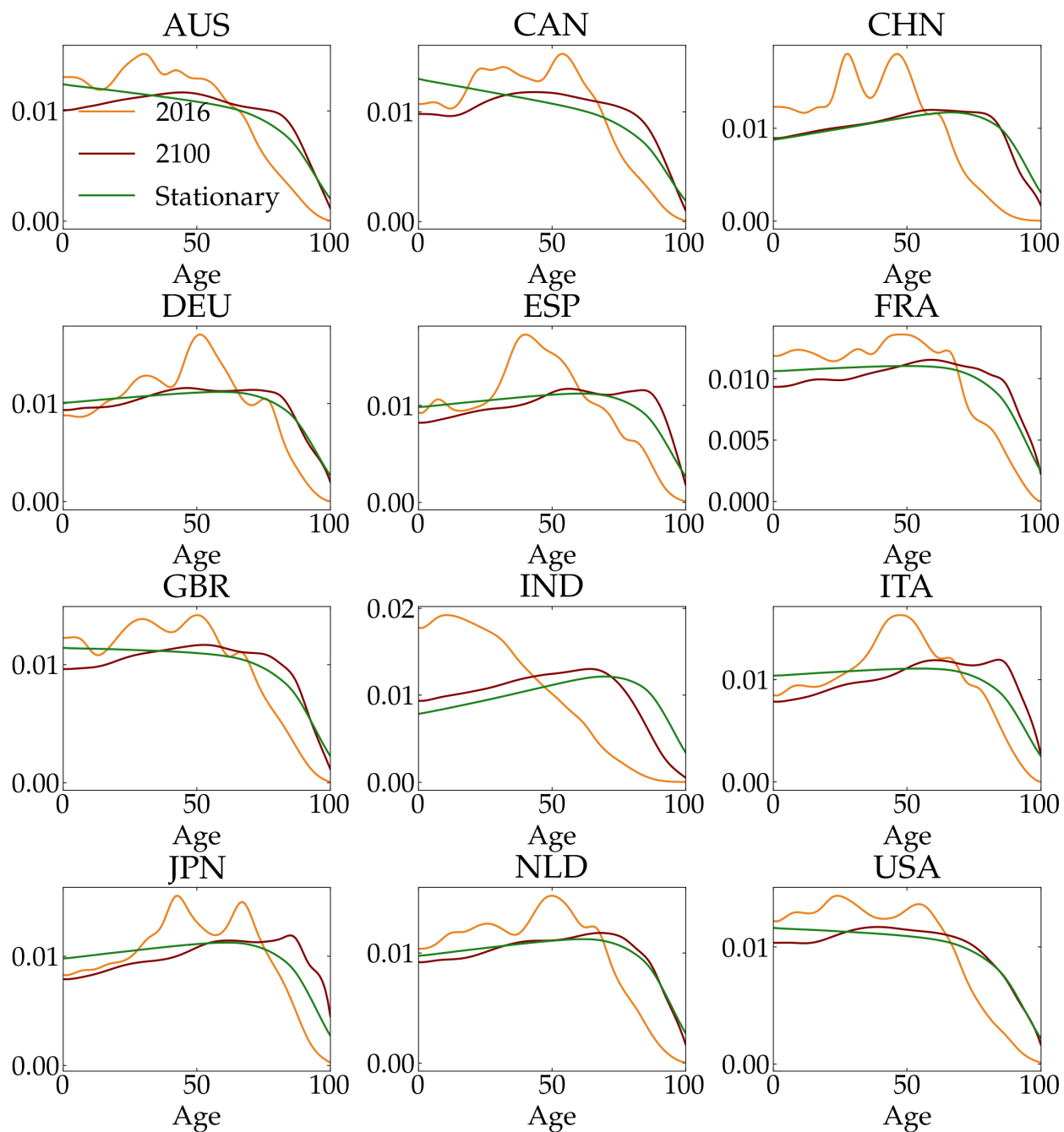


Figure E.3: Population distributions

Notes: This figure presents the 2016, 2100 and stationary population distributions for the 12 countries we consider.

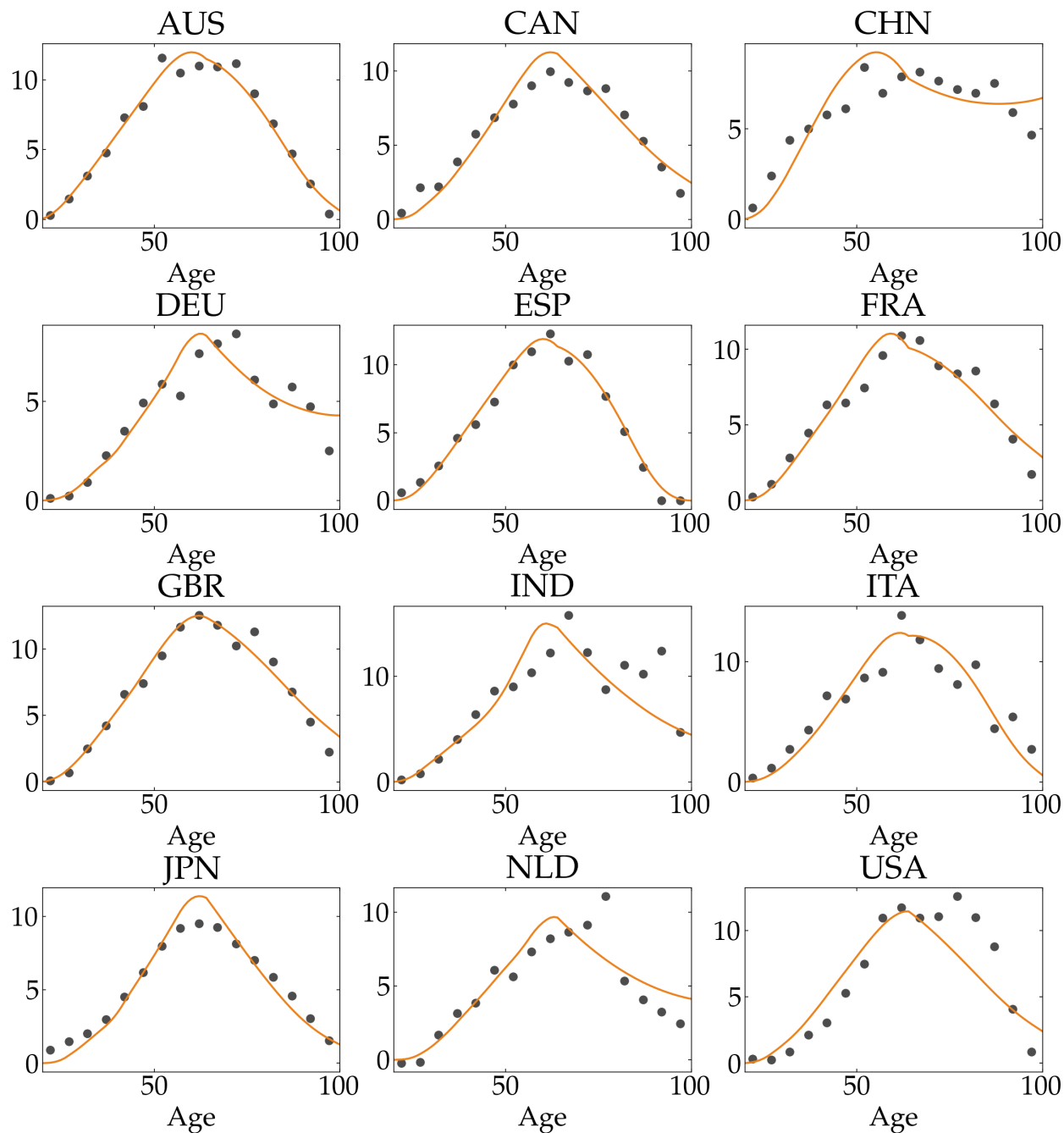


Figure E.4: Calibration outcomes

Notes: This figure presents the empirical age-wealth profiles (gray dots) and the calibrated model age-wealth profiles (orange line) for the 12 countries we consider.

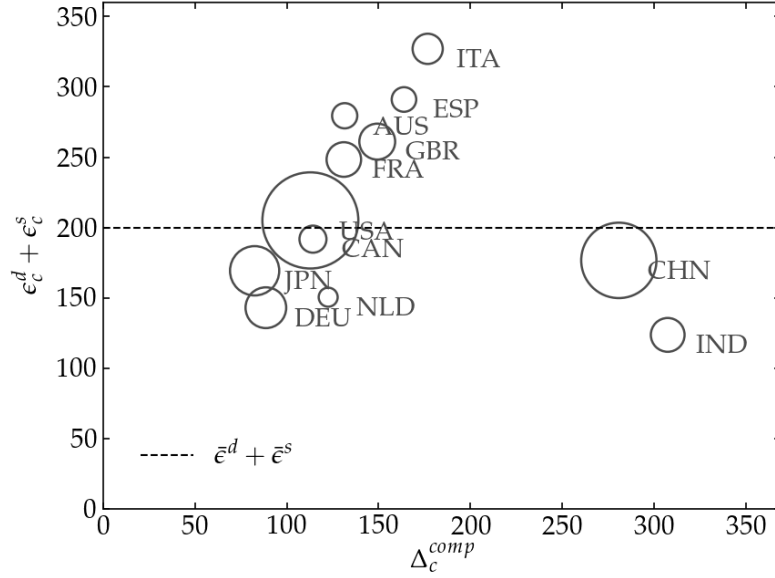


Figure E.5: Country specific sensitivities

Notes: This figure presents the values of the country specific sensitivities of net asset demand to the interest rate $\epsilon_c^d + \epsilon_c^s$ (y-axis) in our baseline calibration and compares it to the shift-shares (x-axis). The size of the dots denotes the 2016 nominal GDP shares. The horizontal line denotes the weighted average $\bar{\epsilon}^d + \bar{\epsilon}^s$.

economy g' and a new stationary population distribution π'_j . We compute the shift-shares for the savings rate and the wealth-to-GDP ratio from the stock approach as

$$\Delta \left(\frac{W}{Y} \right)^{stock} = \frac{\sum_j \pi'_j a_j}{\sum_j \pi'_j h_j} - \frac{\sum_j \pi_j a_j}{\sum_j \pi_j h_j'} \quad (66)$$

$$\Delta s^{stock} = \frac{g'}{1 + g'} \frac{\sum_j \pi'_j s_j}{\sum_j \pi'_j h_j} - \frac{g}{1 + g} \frac{\sum_j \pi_j s_j}{\sum_j \pi_j h_j'} \quad (67)$$

and from the flow approach as

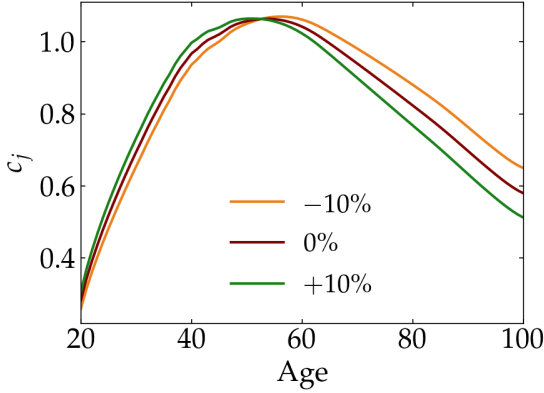
$$\Delta \left(\frac{W}{Y} \right)^{flow} = \frac{1 + g'}{g'} \frac{\sum_j \pi'_j s_j}{\sum_j \pi'_j h_j} - \frac{1 + g}{g} \frac{\sum_j \pi_j s_j}{\sum_j \pi_j h_j'} \quad (68)$$

$$\Delta s^{flow} = \frac{\sum_j \pi'_j s_j}{\sum_j \pi'_j h_j} - \frac{\sum_j \pi_j s_j}{\sum_j \pi_j h_j}. \quad (69)$$

Measurement error is captured by tilting the age-wealth and the age-consumption profiles in the following way. We tilt them linearly such that the value at age 20 is 10% lower and 10% higher at age 100, while ensuring that the aggregate value is the same. We also consider the opposite case.

Figure 12 reports the outcome of this exercise. Panel a presents the shift-share for the wealth-

A. Age-consumption profile



B. Age-wealth profile

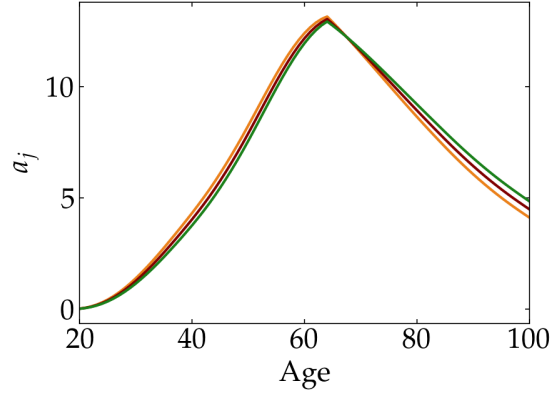


Figure F.1: Tilted age profiles

Notes: This figure shows the tilted age-consumption and age-wealth profiles used in section F.1. The orange profiles are tilted linearly such that consumption is 10% lower at age 20 and 10% higher at age 100, and wealth 10% higher at 20 and 10% lower at age 100. The green profiles are the opposite case.

to-GDP ratio. The orange bars denote the value of the stock shift-share (66) and the red bars of the flow shift-share (68). The black dotted line presents the value under no measurement error of 38pp. As expected, the stock and the flow shift-share coincide. However, the two approaches differ substantially when measurement error is introduced. Tilting the age-wealth profile in any direction affects the stock shift-share by less than 2pp. When the consumption profile is tilted towards younger ages, the shift-share drops by more than 50%, and when the profile is tilted towards older ages the effect on W/Y is overestimated by almost 50%. This indicates that the stock shift-share is much more robust to measurement error than the flow approach. This also holds when looking at the effect on savings-to-GDP as depicted in panel B of the same figure.

F.2 Multiple assets

As a reduced form of microfounding differential returns across assets, we assume that households derive utility from holding safe assets. To illustrate the logic, we use a simple setup where we abstract from time-varying returns, borrowing, income risk, and taxes and transfers. Households have access to safe assets b and risky assets k , and solve

$$V_j(b_j, k_j) = \max_{b_{j+1}, k_{j+1}} \frac{c_j^{1-\sigma} - 1}{1-\sigma} + \chi \frac{b_{j+1}^{1-\sigma} - 1}{1-\sigma} + \phi_j \beta V_{j+1}(b_{j+1}, k_{j+1})$$

subject to

$$c_j + b_{j+1} + k_{j+1} \leq y_j + (1 + r_b)b_j + (1 + r_k)k_j.$$

The term $\frac{b_{j+1}^{1-\sigma}-1}{1-\sigma}$ is the utility of holding safe assets, where the functional form is chosen to ensure a homothetic demand for safe assets. The first order conditions for k_{j+1} and b_{j+1} , together with the envelope theorem, imply

$$\begin{aligned} c_j^{-\sigma} &= \beta\phi_j(1+r_k)c_{j+1}^{-\sigma} \\ c_j^{-\sigma} &= b_{j+1}^{-\sigma} + \beta\phi_j(1+r_b)c_{j+1}^{-\sigma}, \end{aligned}$$

which implies

$$b_{j+1} = c_j \left(\frac{r_k - r_b}{1 + r_k} \right)^{-1/\sigma}.$$

This defines a demand for safe asset by age, and if safe assets are in positive net supply, we need $r_k > r_b$.

The formal definition of our reparametrized asset space is

$$\begin{aligned} \text{Asset 1:} & \quad \left(\frac{B^{\text{world}}}{A^{\text{world}}}, \frac{K^{\text{world}}}{A^{\text{world}}} \right) \\ \text{Asset 2:} & \quad (1, -1) \end{aligned}$$

The prices and returns of the assets are

$$\begin{pmatrix} p_w \\ p_s \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} r_w \\ r_s \end{pmatrix} = \begin{pmatrix} r_b \frac{B^{\text{world}}}{A^{\text{world}}} + r_k \frac{K^{\text{world}}}{A^{\text{world}}} y \\ -(r_k - r_b b) \end{pmatrix}. \quad (70)$$

In this basis, the asset vector of a household with b risk-free assets and k risky assets is

$$\left(a, a \left[\frac{b}{a} - \frac{B^{\text{world}}}{A^{\text{world}}} \right] \right), \quad (71)$$

where $a = b + k$.

To obtain the age profiles of risk-free and risky holdings from the 2016 SCF, we break down total net worth into its components and allocate each item to risky vs risk-free according to the percentages reported in table F.1.

We allocate liquid financial assets such as transaction accounts, certificates, or bonds as 100% risk-free. For other financial assets, the risky share is given by the leverage ratio of stocks multiplied by the portfolio share of equity. We consider an age-depend equity share x_j taken from [Meeuwis, Parker, Schoar and Simester \(2018\)](#) who observe portfolio holdings of millions of household using proprietary data. The equity share is constant at around 85% until age 45, declines linearly to 55% at age 65 and remains roughly constant at this level for older ages. We select a leverage of stocks of 85% to ensure that the aggregate ratio of risky assets to total assets from the

SCF is consistent with aggregate US data. For stocks we simply use the assumed leverage ratio. Non-financial assets are allocated as 100% risky, except for business assets for which we consider the leverage ratio of stocks. Finally, all debt is assumed to be 100% risk-free. We then follow the procedure of section 2 to obtain the normalized profiles of risk-free and risky holdings depicted in panel A of figure 13.

Last, to derive (33)-(35) we totally differentiate the vector based NFA to obtain

$$\Delta \left(\frac{NFA_c}{Y_c} \right) = \begin{pmatrix} \Delta_\theta A^{d,1} \\ \Delta_\theta A^{d,2} \end{pmatrix} + \mathcal{E}^{d,r} \Delta r + \mathcal{E}^{s,r} \Delta r, \quad (72)$$

where

$$\mathcal{E}^{d,r} = \begin{pmatrix} \frac{\partial A_c^{d,1}}{\partial r_1} & \frac{\partial A_c^{d,1}}{\partial r_2} \\ \frac{\partial A_c^{d,2}}{\partial r_1} & \frac{\partial A_c^{d,2}}{\partial r_2} \end{pmatrix}, \quad \mathcal{E}^{s,r} = - \begin{pmatrix} \frac{\partial A^{s,1}}{\partial r_1} & \frac{\partial A^{s,1}}{\partial r_2} \\ \frac{\partial A^{s,2}}{\partial r_1} & \frac{\partial A^{s,2}}{\partial r_2} \end{pmatrix}.$$

Using that $\Delta_\theta A_c^d = \Delta^{comp} + \Delta^{beh|r}$, we obtain

$$\Delta \left(\frac{NFA_c}{Y_c} \right) = \Delta_c^{comp} + \Delta^{beh|r} + \mathcal{E}^{d,r} \Delta r + \mathcal{E}^{s,r} \Delta r. \quad (73)$$

Doing a weighted sum with output weights Y_c and imposing equilibrium, we obtain

$$0 = \bar{\Delta}^{comp} + \bar{\Delta}^{beh|r} + (\bar{\mathcal{E}}^{d,r} - \bar{\mathcal{E}}^{s,r}) \Delta r \iff \Delta r = -(\bar{\mathcal{E}}^{d,r} + \bar{\mathcal{E}}^{s,r})^{-1} (\bar{\Delta}^{comp} + \bar{\Delta}^{beh|r}),$$

which is (33). Equation (34) is trivial. Last, substituting (33) into (73) implies (35).

Table F.1: Risk-free vs risky allocation of net worth

Description	% risky
<i>Financial Assets</i>	
All types of transactions accounts	0%
Certificates of deposit	0%
Directly held pooled investment funds	$x_j \cdot 85\%$
Directly held stocks	85%
Directly held bonds	0%
Quasi-liquid retirement assets	$x_j \cdot 85\%$
Savings bonds	0%
Cash value of whole life insurance	$x_j \cdot 85\%$
Other managed assets	$x_j \cdot 85\%$
Other financial assets	$x_j \cdot 85\%$
Funded defined benefits pensions assets	$x_j \cdot 85\%$
<i>Non-financial Assets</i>	
All vehicles	100%
Primary residence of household	100%
Other residential real estate	100%
Net equity in nonresidential real estate	100%
Business(es)	85%
Other non-financial assets	100%
<i>Debt</i>	
Debt held by household	0%