

Misallocative Growth

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Abstract

This paper develops a Schumpeterian model of firm and worker dynamics to assess how growth and labor market dynamics interact. Creative-destructive firm entry and exit result in workers being misallocated in the labor market. Labor misallocation, in turn, incentivizes further firm creation by lowering individuals' opportunity cost of starting firms and the cost of hiring labor to operate new firms. Unique Swedish matched employer-employee data indicate that this novel interaction between growth and labor market dynamics is quantitatively important. Applied to the labor market consequences of aging, aging accounts for over half of substantial secular declines in firm creation and worker mobility in Sweden since 1986, primarily through equilibrium forces. Yet growth rises through the 1990s as the baby boomers reallocate toward productive firms.

Keywords: Innovation; Spinoff; New firm growth; Employment; Demographics

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1 Introduction

According to [Schumpeter \(1934\)](#), people must be "thrown out of employment, before the ground is clear and the way paved for new achievement of the kind which has created modern civilization." A long subsequent literature has argued along Schumpeter's lines that growth is associated with unemployment or, more broadly, workers being misallocated in the labor market ([Aghion and Howitt, 1994](#); [Mortensen and Pissarides, 1998](#); [Hornstein et al., 2007](#); [Aghion et al., 2016](#)).¹

This paper argues that labor misallocation may, in turn, encourage firm creation and hence speed up creative-destructive growth, setting the stage for a potentially powerful dynamic interaction between growth and labor market dynamics. Two observations lead me to formulate this hypothesis. First, as most firm founders were previously employed somewhere else, labor misallocation may lower the opportunity cost of starting a firm. Second, entrepreneurs rank recruiting as the main barrier to growth, such that labor misallocation may encourage firm creation by making it easier to hire.

To assess the dynamic interaction between growth and labor market dynamics, I start by developing a Schumpeterian theory of firm and worker dynamics. Technology is embodied in firms, such that growth entails replacing old with new firms. As unemployed and employed, an individual searches for jobs and business ideas. Through job-to-job mobility, she gradually moves toward more productive firms ([Burdett and Mortensen, 1998](#)). This raises her opportunity cost of doing something else, but also possibly her ability to come up with new better business ideas. As a business owner, she chooses how many workers to try to hire. Entry of new entrepreneurs bids up prices of factors of production such that incumbent entrepreneurs eventually find it optimal to exit and return to unemployment.

The theory highlights that, on the one hand, less firm entry leads to slower technological obsolescence. A lower rate of obsolescence, on the other hand, affords individuals more time to reallocate across firms before those are replaced. On the balanced growth path, employment shifts toward more productive firms. This further discourages firm creation by raising the opportunity cost of starting a firm and the cost of hiring workers. This novel force is conceptually distinct from the opposing *duration* and *capitalization* effects emphasized by the earlier literature on growth and labor market dynamics ([Aghion and Howitt, 1994](#)), and arises because I allow for on-the-job search across incumbent firms.

At a conceptual level, this interaction between growth and labor market dynamics is instead reminiscent of a literature on vintage capital and technological "lock-in." That literature has argued that as agents accumulate knowledge about existing vintages of capital, it discourages innovation in new vintages ([Chari and Hopenhayn, 1991](#); [Jovanovic and Nyarko, 1996](#); [Redding, 2002](#); [Atkeson and Kehoe,](#)

¹By *labor misallocation* this paper refers to a scenario in which labor could be reallocated so as to increase output.

2007). The same fundamental force is at work here, with "knowledge about existing vintages" taking the form of improvements in individuals' ability to produce using existing technologies through worker mobility across incumbent firms. The search-theoretic approach, however, has the key advantage that it allows me to discipline the theory using rich data on labor market flows. Moreover, the current framework expands on previous works by allowing a better ability to produce using existing technologies to also facilitate the development of new better ideas. In fact, key for the strength of the dynamic interaction between growth and labor market dynamics is the scope for worker reallocation to raise productivity in existing technologies relative to the ability to come up with new better technologies.

The second task for this paper is to quantify the dynamic interaction between growth and labor market dynamics. To do so, however, requires a unique data set. To that end, I combine several Swedish administrative data sources into a unique matched employer employee (MEE) data set with the ability to observe (revenue) productivity of every firm, connect firms to the workers they hire, and link firms to their founders and founders to their previous labor market histories. I structurally estimate 18 parameters based on 43 moments in these unique data. The model fits the data well and reproduces several key correlations in the data as non-targeted outcomes, providing confidence in the theory.

My estimates reveal substantial scope for worker reallocation to increase individuals' productivity in existing technologies and reduce the probability of starting a new firm (and switching employer). In particular, mobility toward more productive firms accounts for over 60 percent of large life-cycle declines in the probability of starting a firm. In contrast, I estimate that such mobility only modestly improves individuals' ability to come up with better new technologies. As a consequence, I infer significant scope for labor misallocation to increase firm creation and growth.

Furthermore, I estimate that creative-destructive entry and exit account for over half of overall growth. This is despite the fact that new firms are low productive, likely to fail and small. The reason is that entrants are still somewhat more productive than the exiting firms they replace, and entry and exit rates are high. Moreover, despite the importance of creative-destructive entry and exit, I show that recent attempts to decompose the sources of growth fail to detect this (Garcia-Macia et al., 2016). The reason is labor market frictions, which prevent such growth from taking a stark, bang-bang form as in Klette and Kortum (2004). Hence, I conclude that a large share of growth is creative-destructive in nature.

The final task for this paper is to use the estimated framework to understand and quantify the impact of aging, motivated by rapid aging in Sweden since 1986 that coincide with declines in firm creation and worker mobility. These trends largely mirror patterns in the US over the same period (Davis and Haltiwanger, 2014). In response to aging, firm creation and worker mobility decline due to a mechanical composition effect, as mature individuals are less likely to start a firm and switch employer since they

have found a good job. Moreover, as in [Shimer \(2001\)](#), a larger share of well-matched mature workers discourages firm and job creation by driving up the cost of hiring. Finally, the effects of aging are amplified by the dynamic interaction between growth and labor market dynamics highlighted above.

Quantitatively, aging accounts for over half of the declines in firm creation and worker mobility since 1986, with a key role played by the dynamic interaction between growth and labor market dynamics. This mirrors recent findings in the trade literature that accounting for growth effects may lead to substantially larger effects of opening up to trade ([Sampson, 2016](#)). As in the data, job reallocation falls, despite no change in the volatility of firm-level shocks. Instead, firms adjust employment less in response to shocks. The reason is that hiring is harder in a mature labor market, discouraging firms that receive positive shocks from expanding, while firms that receive negative shocks do not see their workers poached away as quickly since other firms create fewer jobs. The fall in the rate of obsolescence associated with aging leads a few incumbent firms to become, in a relative sense, very productive. Income inequality increases, as does the productivity gap between entrant and incumbent firms, in line with Swedish trends. Finally, while the rate of innovation falls secularly as the labor force matures, the level of output rises as the baby boomers gradually find good jobs. In fact, the level effect is so strong that measured growth is high throughout the 1990s. Starting in the 2000s, the growth effect overtakes the level effect, such that measured growth declines. This mirrors the Swedish (and US) experience over this period.

Related literature. This paper contributes to three literatures. First, I expand in three dimensions on a literature on knowledge diffusion and growth, including [Luttmer \(2007, 2010, 2011, 2012\)](#), [Alvarez et al. \(2008\)](#), [Lucas and Moll \(2014\)](#), and [Perla and Tonetti \(2014\)](#). First, I introduce labor market frictions, which highlights an interaction between growth and labor market dynamics. Second, I allow for a role for *specific* knowledge spillovers from the founder's prior employer, over and above *general* knowledge spillovers from the entire distribution of incumbent technologies. Third, these earlier works are primarily theoretical in nature, while I confront the theory with rich MEE data. This is in the spirit of recent work on firm dynamics and growth, including [Lentz and Mortensen \(2008\)](#), [Acemoglu et al. \(2018\)](#) and [Akcigit and Kerr \(2018\)](#).² Relative to these rich models of firm dynamics, my framework simplifies in some dimensions to offer a richer, microfounded model of the joint dynamics of firms and workers.

Second, the focus on the joint dynamics of firms and workers relates to a recent literature that includes [Elsby and Michaels \(2013\)](#), [Kaas and Kircher \(2015\)](#), [Coles and Mortensen \(2016\)](#), [Schaal \(2017\)](#), [Bilal et al. \(2019\)](#) and [Elsby and Gottfries \(2019\)](#). I follow the lead of [Coles and Mortensen \(2016\)](#), [Lise and](#)

²Building on the canonical work of [Klette and Kortum \(2004\)](#), who are the first to confront a version of the creative-destructive theory in [Aghion and Howitt \(1992\)](#) and [Grossman and Helpman \(1991\)](#) with firm-level data.

Robin (2017), Fajgelbaum (2019) and Gouin-Bonenfant (2020) in assuming constant returns to scale in production in order to focus on two key innovations relative to these earlier works. First, I incorporate an entrepreneurial choice in this type of framework, encouraged by Coles and Mortensen (2016) suggestion that this would be "both realistic and worth pursuing." Second, I introduce endogenous growth through technology diffusion. The theory builds on Engbom (2019), but is significantly richer including allowing for specific knowledge spillovers. I also relate to a few papers on employee spin-offs, including Anton and Yao (2002), Silveira and Wright (2010) and Chatterjee and Rossi-Hansberg (2012).³ These papers analyze in detail why ideas are difficult to sell, focusing on the role of private information. I abstract from such considerations to assume that private information is so severe that ideas cannot be traded.

Third, the application to the effects of aging connects to a vast literature on the impact of demographic change on various macroeconomic outcomes. Most closely related are several recent papers that study the impact of labor supply growth on firm dynamics, including Karahan et al. (2016), Hopenhayn et al. (2018) and Peters and Walsh (2019). Instead of studying changes in labor supply growth—a measure of the *quantity* of labor—I focus on changes in the age composition of the labor force, a measure of the *quality* of the labor force. Two reasons lead me to focus on the age composition as distinct from labor supply growth. First, Sweden has not seen a secular change in the growth rate of labor supply over this period, but significant aging due to a temporary spike and subsequent decline in labor supply growth (see Appendix A). Second, while it is well-known that labor supply growth impacts growth, much less is known about the impact of the age composition. Yet several proposed policies to deal with demographic change such as a gradual raising of the retirement age would lead to aging of the labor force *and* an increase in labor supply growth. Hence, a separate assessment is required of the impact of aging and labor supply growth. Liang et al. (2018) show that entrepreneurship entry is lower at all ages in mature economies using a cross-section of countries, and rationalize this in a partial equilibrium model.

Closely related, a literature studies declining labor market dynamics in the US—see Davis and Haltiwanger (2014) for a summary. In addition to the papers cited above that focus on the impact of changes in labor supply growth, other papers include Mercan (2017), Salgado (2017), Bornstein (2018), Akcigit and Ates (2019a,b), Jiang and Sohail (2019), and Pries and Rogerson (2019). While the literature on US business dynamism is quickly growing, evidence from other countries is close to non-existent.⁴ Moreover, apart from Akcigit and Ates (2019a,b), these papers focus on some aspects of the declines in US labor market dynamics in isolation, whereas I propose a single factor behind all of these trends.

³Holmes and Schmitz (1990) focus on the related issue of business transfers. The model in this paper could be easily adjusted such that entrepreneurs sell their businesses soon after entry, without much effect on any of the results in this paper (an earlier iteration of this paper solved such a version of the model).

⁴Bijnens and Konings (2017) document large declines in firm dynamics in Belgium starting in the late 1980s.

2 The Economy

This section outlines a rich equilibrium model of life-cycle careers with two main features. The first is a firm dynamics model of technology diffusion in the spirit of [Luttmer \(2007\)](#) and the second a job ladder model of individual labor market dynamics ([Burdett and Mortensen, 1998](#)).

2.1 Environment

Time is infinite and continuous. I abstract from aggregate shocks and focus the exposition on an economy on its long run, balanced growth path (BGP), and later turn to transitional dynamics.

Demographics. The economy consists of a unit mass of ex-ante identical, *perpetual youth* individuals who permanently exit the labor force at Poisson rate κ ([Blanchard, 1985](#); [Yaari, 1965](#)). At that point, they are replaced by their offspring. Offspring enter the labor market as unemployed, unless their parent was an entrepreneur in which case they take over their firm.

Endowments and preferences. Individuals are endowed with L units of land and a unit of time. Time may be indivisibly allocated toward not working (unemployed), working for someone else (worker) or running an own firm (entrepreneur). As unemployed, an individual enjoys consumption-equivalent flow value of leisure $B(t)$, as employed she gets flow wage $W(t)$, and as an entrepreneur she enjoys the flow profits of her firm $P(t)$ as well as flow utility $K(t)$ from being her own boss.⁵ Individuals have risk-neutral, dynastic preferences with discount rate $\bar{\rho}$ over the consumption-equivalent flow of a single output good $C(\tau) = B(\tau) + W(\tau) + P(\tau) + K(\tau)$,

$$U(t) = \int_t^{\infty} C(\tau) e^{-\bar{\rho}(\tau-t)} d\tau$$

The market for the final good is competitive and its price serves as the numeraire.

Technology. Entrepreneurs own and run firms. Firms produce a single final good using only labor n ,

$$Y = e^Z n$$

where the idiosyncratic productivity Z evolves according to a geometric Brownian motion

$$dZ = \mu dt + \sigma dW(t)$$

⁵The utility value of being one's own boss allows for entrepreneurship for other reasons than solely profits, which the literature has argued is an important feature of the data ([Hurst and Pugsley, 2011](#)).

where μ is an exogenous drift, σ an exogenous intensity of shocks, and $W(t)$ is the standard Brownian motion. I discuss in greater detail below how the number of workers, n , evolves.

In order to remain active, a firm must rent a unit of land. If the firm shuts down, the idea is permanently lost and the founder returns to unemployment. As in [Hopenhayn \(1992\)](#), the fixed cost gives rise to firm exit. Land is rented in a competitive market at price $R(t)$.

The labor market. Workers and firms search at random in a common labor market. The unemployed search with unit intensity, while the employed search with exogenous relative intensity ϕ . Entrepreneurs may not simultaneously search for jobs. The employed become unemployed either because they choose to quit, their employer lays them off or for exogenous reasons at rate δ .

To hire workers, an entrepreneur has to spend resources $C_v(v, t)$ advertising v jobs. $C_v(v, t)$ is strictly convex in v and in units of the final good. The equilibrium rate at which searching individuals contact potential jobs per unit of search efficiency, p , as well as the equilibrium rate at which an open job contacts a worker, q , are given by a Cobb-Douglas matching function,

$$p = \frac{\chi V^\theta S^{1-\theta}}{S}, \quad q = \frac{\chi V^\theta S^{1-\theta}}{V} \quad (1)$$

where V is the total number of vacancies created and S the efficiency mass of searching workers.

When an unemployed worker meets with a firm, the firm and the worker engage in alternating offers bargaining such that the worker receives a share β of the marginal surplus. When an employed worker gets poached by another firm, bargaining takes place as in [Dey and Flinn \(2005\)](#) and [Cahuc et al. \(2006\)](#). This bargaining protocol has become a benchmark in the literature as it is both tractable, has theoretically appealing properties and has been shown to match well many features of the data. The poaching and incumbent firm first Bertrand compete for the worker, after which the worker and the winning firm alternating offers bargain such that the worker receives a share β of the differential marginal surplus between the two firms. I assume that outside options are verifiable and that there are no contracting frictions preventing renegotiation. As a consequence, in instances when the firm would rather layoff the worker or the worker would rather quit the match at the agreed upon wage, renegotiation takes place to preserve the match if there are gains from doing so. In addition to being tractable, this assumption has the appealing, Coasian feature of bilateral efficiency, i.e. matches maximize joint surplus ([Coase, 1960](#)).

Entrepreneurship. Unemployed and employed individuals also chose how much effort to devote to searching for business ideas. As in [Chatterjee and Rossi-Hansberg \(2012\)](#), entrepreneurs may not simul-

taneously search for new ideas.⁶ In return for effort $C_e(s, t)$, an individual receives ideas at rate s . The cost is strictly convex in s and in terms of the final good. If she gets an idea she would like to pursue, she has to quit her job in case she is employed. At that point, she realizes the initial productivity of the idea, Z_0 , which may depend on the productivity of her current employer, Z , as well as the entire distribution of technologies in use at time t , $X(t)$,

$$Z_0 \sim Y\left(Z, X(t)\right)$$

This captures the notion that potential founders get ideas both through observations off the job of other ideas in current use and by working with the particular technology used by their current employer. I refer to the former as *general* and the latter as *specific* knowledge spillovers. Individuals hence have some information about the quality of their future idea based on the productivity of their current employer and the overall distribution of productivity of incumbent firms at time t .⁷ Yet there is also initial uncertainty about the viability of the project. The latter is motivated by the high exit rates of young firms in the data, which suggests substantial initial uncertainty.

If the individual pursues the idea, she becomes the owner of an incumbent firm, enjoying the profits that the firm makes plus the utility value of being one's own boss. She hires workers and produces as specified above, and optimally decides when it is no longer worth paying the rental cost of land. At that point, she exits entrepreneurship and returns to unemployment.

2.2 Recursive formulation

I focus the exposition on a *balanced growth path* (BGP), in which relevant variables and all quantiles of the productivity and employment distributions grow at the same, constant rate M . That is, the distributions evolve as "traveling waves," shifting out over time while preserving their shape. In order to ensure that a BGP exists, I assume that the flow value of leisure, $\bar{Z}(t)$, $B(t) = \tilde{b}e^{\bar{Z}(t)}$, the value of being one's own boss, $K(t) = \tilde{k}e^{\bar{Z}(t)}$, the cost of searching, $C_e(s, t) = \tilde{c}_e(s)e^{\bar{Z}(t)}$ and the cost of vacancies, $C_v(v, t) = \tilde{c}_v(v)e^{\bar{Z}(t)}$, are all proportional to average productivity at time t . This captures the view that as the opportunity cost

⁶The drift of incumbent firm productivity, μ , could be interpreted as incumbent entrepreneurs improving their productivity through search, but I take it as exogenous and hence unaffected by aging. It would be very interesting to endogenize incumbent innovation. I note, though, that my estimates in the next section imply that most growth stems from the entry/exit process.

⁷An alternative interpretation of the patterns in Section ?? is that potential founders have some information about the productivity of the new idea, and that individuals working at more productive firms are more selective in terms of what ideas they pursue. In its simplest form, however, such a hypothesis would predict an increasing left-censoring of the productivity distribution of new firms. In the data, the entire distribution of new productivity shift to the right with the productivity of the current employer (see Appendix A). More work is needed on this, though. Nevertheless, I believe that many of the predictions of this theory would carry through also under such a selection based story.

of time rises with output, so do these costs.

Instead of analyzing the growing, non-stationary economy, it is convenient to study a transformed, stationary version of the model. To that end, let z denote a firm's productivity relative to the exit threshold at time t , $\underline{Z}(t)$: $z = Z - \underline{Z}(t)$.⁸ On a BGP, the exit threshold must grow at the rate of the economy, $d\underline{Z}(t) = M = \mu + m$, where μ is the exogenous drift of productivity of incumbent firms and m is the endogenous *rate of obsolescence* to be characterized further below,

$$dz = d\left(Z - \underline{Z}(t)\right) = \mu dt + \sigma dW(t) - Mdt = -mdt + \sigma dW(t)$$

Furthermore, denote by $b = \tilde{b}e^{\bar{Z}(t)-Z(t)}$, $k = \tilde{k}e^{\bar{Z}(t)-Z(t)}$, $c_e(s) = \tilde{c}_e(s)e^{\bar{Z}(t)-Z(t)}$ and $c_v(v) = \tilde{c}_v(v)e^{\bar{Z}(t)-Z(t)}$. Finally, let $\rho = \tilde{\rho} - M$ be the transformed discount rate on the BGP.⁹

In order to make analytical progress, I abstract for now from specific knowledge spillovers from the prior employer and assume that entrants make small, stochastic improvements on the ideas of firms that are just about to exit, following [Luttmer \(2012\)](#). In the transformed, stationary economy denominated in the exit threshold, this is equivalent to assuming that entrant firms start with productivity $\varepsilon \sim \Gamma$.

Even though the production technology is constant returns, the cost of land implies increasing returns to scale, necessitating keeping the current size of the firm as a state variable and stipulating a multilateral bargaining protocol. In particular, there may be instances when it is jointly optimal for a coalition of multiple workers and the entrepreneur to keep the firm in business, but this requires renegotiation of current employment contracts. A multilateral bargaining protocol is needed to deal with such cases. In order to simplify further, I assume that the flow value of leisure, b , is sufficiently high that workers prefer to quit a firm before the entrepreneur wants to exit (equation (7) makes this precise). I view this as workers abandoning a "sinking ship" prior to the firm exiting, supported by the empirical observation that job-to-job separations are the main channel through which contracting firms shed workers. Then,

Proposition 1. *Suppose that production is constant returns to scale and the flow value of leisure, b , is sufficiently*

⁸Normalizing to the least productive firm at time t turns out to be a particularly useful normalization, but it has no economic significance (I could, for instance, have equally well normalized to the mean of productivity at time t).

⁹To see where the change in discount rate comes from, consider a simplified recursion for the value of an incumbent match in terms of non-normalized productivity, $Y(Z, t)$, in which the only action is the drift in productivity, $\tilde{\rho}Y(Z, t) = e^Z + \mu \frac{\partial Y(Z, t)}{\partial Z} + \frac{\partial Y(Z, t)}{\partial t}$. Define $z(Z, t) = Z - Mt$ and $Y(Z, t) = J(z)e^{Mt}$. By a change of variables,

$$\begin{aligned} \tilde{\rho}J(z)e^{Mt} &= e^{z+Mt} + \mu \frac{\partial J(z(Z, t))e^{Mt}}{\partial Z} + \frac{\partial J(z(Z, t))e^{Mt}}{\partial t} = e^{z+Mt} + \mu \frac{\partial J(z)}{\partial z} \frac{\partial z}{\partial Z} e^{Mt} + \frac{\partial J(z)}{\partial z} \frac{\partial z}{\partial t} e^{Mt} + J(z)Me^{Mt} \\ &= e^{z+Mt} + \mu \frac{\partial J(z)}{\partial z} e^{Mt} + \frac{\partial J(z)}{\partial z} (-M)e^{Mt} + J(z)Me^{Mt} \end{aligned}$$

Dividing by e^{Mt} and rearranging gives $(\tilde{\rho} - M)J(z) = \rho J(z) = e^z - mJ'(z)$. Adding the other terms is straightforward.

high. Then the surplus value of the firm, $V(z, n) - (n + 1)U$, can be written as the surplus value of incumbent matches, $n(J(z) - U)$, plus the surplus value of the entrepreneur, $O(z) = V(z, 0) - U$,

$$V(z, n) - (n + 1)U = nJ(z) + O(z)$$

Proof. All proofs are in Appendix B. □

That is, the size of the firm does not impact the behavior of its constituent matches. Moreover, the need for a multilateral bargaining protocol is avoided, since at the point of exit the firm consists only of its owner. Under this assumption, it suffices to solve for the value of unemployment, U , the surplus value of a match, $J(z)$, and the surplus value of an entrepreneur, $O(z)$, in order to characterize the equilibrium. Moreover, neither is a function of the current size of the firm.

The value of unemployment U satisfies the Hamilton-Jacobi-Bellman (HJB) equation,

$$\rho U = b + p\beta \int_0^\infty x d\mathcal{F}(x) + \max_s \left(s \left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} O(\varepsilon) d\Gamma(\varepsilon) \right)^+ - c_e(s) \right) \quad (2)$$

where $\mathcal{F}(x)$ is the vacancy-weighted distribution of match surplus. The unemployed worker enjoys flow value of leisure b . She encounters potential job offers at rate p drawn from distribution \mathcal{F} . If she accepts the job offer, she gets a share β of the surplus from the match. Finally, she chooses how much to search for business opportunities subject to the cost of searching. I henceforth assume that the cost of search takes the iso-elastic form $c_e(s) = c_s^{\eta_e} s^{1+\eta_e} / (1 + \eta_e)$, such that the optimal search intensity of the unemployed, $s(u)$, is given by $(c_s s(u))^{\eta_e} = \left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} O(\varepsilon) d\Gamma(\varepsilon) \right)^+$.

The value of a match. The surplus value of a match solves the stopping time problem,

$$\begin{aligned} \rho J(z) &= e^z - (\delta + \kappa)J(z) + \phi p\beta \int_{J(z)}^\infty (x - J(z)) d\mathcal{F}(x) + \max_s \left(s \left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} O(\varepsilon) d\Gamma(\varepsilon) - J(z) \right)^+ - c_s^{\eta_e} \frac{s^{1+\eta_e}}{1 + \eta_e} \right) \\ &- mJ'(z) + \frac{\sigma^2}{2} J''(z) - b - p\beta \int_0^\infty x d\mathcal{F}(x) - \max_s \left(s \left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} O(\varepsilon) d\Gamma(\varepsilon) \right)^+ - c_s^{\eta_e} \frac{s^{1+\eta_e}}{1 + \eta_e} \right) \end{aligned} \quad (3)$$

subject to value matching and smooth pasting, $J(\underline{z}^w) = 0$ and $J'(\underline{z}^w) = 0$. The match produces e^z and terminates at rate $\delta + \kappa$, in which case the worker/worker's offspring becomes unemployed. The worker encounters other job opportunities at rate ϕp , accepts them if they are more valuable, and gets a share β of the incremental surplus.¹⁰ She chooses how hard to search for entrepreneurship opportunities. Produc-

¹⁰While the entrepreneur in a sense loses value when the worker moves on to a better job, the worker gets fully compensated for this loss by the new firm under the stipulated bargaining protocol. From the ex-ante perspective of the joint surplus, the

tivity falls behind at rate of obsolescence m and is subject to shocks with intensity σ . By being employed, the worker forsakes the value of leisure and the option to search for jobs and business opportunities from unemployment. Optimal search $s(z)$ is given by

$$s(z) = \frac{1}{c_s} \left(\left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} O(\varepsilon) d\Gamma(\varepsilon) - J(z) \right)^+ \right)^{\frac{1}{\eta_e}} \quad (4)$$

Proposition 2. *Abstract from drift of and shocks to productivity, $m = \sigma = 0$, and assume that parameter values are such that an individual who receives an entrepreneurship opportunity always seizes it ($\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} O(\varepsilon) d\Gamma(\varepsilon) \geq J(z)$).¹¹ Then the surplus value of a match $J(z)$ is the unique solution to the first-order ordinary differential equation*

$$J'(z) = \frac{e^z}{\rho + \delta + \kappa + \phi p \beta (1 - \mathcal{F}(J(z))) + s(z)} \quad (5)$$

subject to initial value $J(\underline{z}^w) = 0$, where the reservation threshold \underline{z}^w is given by

$$e^{\underline{z}^w} = b + p\beta(1 - \phi) \int_0^{\infty} 1 - \mathcal{F}(x) dx \quad (6)$$

The following corollaries are immediate,

Corollary 1. *The job ladder is a ranking of firms in marginal product z such that workers accept outside offers from firms with higher marginal product z and rejects offers from firms with lower marginal product*

$$\mathcal{F}(J(z)) = F(z), \quad \text{and} \quad \mathcal{G}(J(z)) = G(z)$$

where F is the vacancy-weighted distribution of firms and $G(z)$ the employment-weighted distribution.

Corollary 2. *The probability that a worker moves to another firm declines in the marginal product of her current firm, $jj'(z) = \frac{d}{dz}(\phi p(1 - F(z))) < 0$.*

As an individual climbs the job ladder, her opportunity cost of moving to another employer rises. Hence, she is less likely to make a subsequent job to job move.

loss to the entrepreneur cancels out with the gain to the worker, such that neither shows up in (3).

¹¹With a non-zero drift, equation (3) becomes a second-order ordinary differential equation in $J(z)$, which can only be solved analytically in some special cases. If individuals sometimes prefer to not seize an entrepreneurship opportunity, the max in equation (3) makes the problem non-differentiable. I find numerically that the intuition from the special case analyzed here carries through to the more general case estimated in the next section.

Corollary 3. *A worker's optimal search intensity for business opportunities, $s(z)$, decreases in the marginal product of her current firm, $s'(z) < 0$.*

If an individual encounters another opportunity, the net gain from seizing it is lower when she is currently in a better job. Anticipating this, she searches less.

The boundary condition (6) and the fact that $e^z = 1$ since the transformed economy is denominated in the exit threshold \underline{z} , implies that the condition on the flow value of leisure b that guarantees that workers want to quit before entrepreneurs want to exit, $\underline{z}^w \geq \underline{z}$, is

$$b \geq 1 - p\beta(1 - \phi) \int_0^\infty 1 - \mathcal{F}(x) dx \quad (7)$$

In the special case of a zero worker bargaining power, this reduces to requiring that $b \geq 1$.¹²

While age is not a state, it is nevertheless useful to briefly discuss how individual dynamics evolve with age. Denote by $G_a(z)$ the distribution of age a workers over productivity, by u_a the number of unemployed workers of age a , and by e_a the number of employed workers of age a (see Appendix B for details). The rate at which individuals of age a start firms, E_a , and switch employer, JJ_a , are given by

$$E_a = \frac{u_a}{u_a + e_a} s(u) + \frac{e_a}{u_a + e_a} \int_{\underline{z}^w}^\infty s(z) dG_a(z) \quad JJ_a = \phi p \int_{\underline{z}^w}^\infty (1 - F(z)) dG_a(z)$$

As highlighted by corollaries 2–3, employees of more productive firms are less likely to start a firm and switch employer. As individuals gradually move up the job ladder with age and unemployment falls with age, firm creation and worker mobility falls with age.

The value of a firm. I henceforth assume that the cost of vacancies takes the iso-elastic form $c_v(v) = c_v^{\eta_v} v^{1+\eta_v} / (1 + \eta_v)$. The surplus value of an entrepreneur solves the stopping time problem,

$$\begin{aligned} \rho O(z) &= e^z + k - r + \max_v \left\{ qv(1 - \beta) \left(\frac{u}{S} J(z)^+ + \frac{\phi e}{S} \int_0^{J(z)} J(z) - x d\mathcal{G}(x) \right) - c_v^{\eta_v} \frac{v^{1+\eta_v}}{1 + \eta_v} \right\} \\ &- mO'(z) + \frac{\sigma^2}{2} O''(z) - b - p\beta \int_0^\infty x d\mathcal{F}(x) - \max_s \left(s \left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} O(\varepsilon) d\Gamma(\varepsilon) \right)^+ - c_s^{\eta_e} \frac{s^{1+\eta_e}}{1 + \eta_e} \right) \end{aligned} \quad (8)$$

subject to $O(\underline{z}) = 0$ and $O'(\underline{z}) = 0$. The entrepreneur enjoys production e^z and the value of being her own boss k , but has to pay the rental price of land r . She benefits from the option of hiring more workers

¹²More generally, numerical methods are required to ensure that condition (7) holds, as both the job finding rate p and the vacancy-weighted distribution of match surplus \mathcal{F} are endogenous objects, and hence in general a function of b . That is, (7) only implicitly defines the constraint on b as a function of underlying structural parameters when $\beta > 0$.

into the firm since she gets a share of the surplus from additional hires. Productivity falls behind at rate of obsolescence m and is subject to shocks with intensity σ . The entrepreneur forsakes the value of leisure and the option of searching for jobs and other business opportunities.

Proposition 3. *Abstracting from drifts of and shocks to productivity, $m = \sigma = 0$, the equilibrium surplus value of an entrepreneur is given by*

$$\rho O(z) = e^z - 1 + \frac{1}{c_v} \frac{\eta_v}{1 + \eta_v} \left(q(1 - \beta) \left(\frac{u}{S} J(z)^+ + \frac{\phi e}{S} \int_0^{J(z)} \mathcal{G}(x) dx \right) \right)^{\frac{1 + \eta_v}{\eta_v}} \quad (9)$$

for $z \geq \underline{z}$, where $\underline{z} = 0$, and the rental rate of land satisfies

$$r = 1 - b + k - p\beta \int_0^\infty 1 - \mathcal{F}(x) dx - \frac{1}{c_s} \frac{\eta_e}{1 + \eta_e} \left(\left(\int_\varepsilon^{\bar{\varepsilon}} O(\varepsilon) d\Gamma(\varepsilon) \right)^+ \right)^{\frac{1 + \eta_e}{\eta_e}} \quad (10)$$

As the surplus value of a match, $J(z)$, is increasing in productivity z , so is the surplus value of an entrepreneur (9). Not only does the entrepreneur produce more output herself when productivity is higher, it also increases the gain to the entrepreneur from hiring workers, as these workers only have to be compensated with a fraction of their contribution to the firm.

The optimal vacancy policy solves the first-order condition

$$v(z) = \frac{1}{c_v} \left(q(1 - \beta) \left(\frac{u}{S} J(z)^+ + \frac{\phi e}{S} \int_0^{J(z)} J(z) - x d\mathcal{G}(x) \right) \right)^{\frac{1}{\eta_v}} \quad (11)$$

Given a vacancy policy $v(z)$, a firm's employment evolves for $z > \underline{z}^w$ according to,¹³

$$\frac{dn(n, z)}{n} = - \left(\delta + \kappa + s(z) + \phi p(1 - F(z)) \right) + \frac{qv(z)}{n} \left(\frac{u}{S} + \frac{\phi e}{S} G(z) \right) \quad (12)$$

where the offer distribution F is determined by the vacancy decisions (11) of individual firms,

$$F(z) = \frac{L}{V} \int_{\underline{z}}^z v(z') x(z') dz', \quad V = L \int_{\underline{z}}^\infty v(z) x(z) dz \quad (13)$$

¹³The separation rate (the first term in (12)) is independent of size, whereas the hiring rate (the second term) declines in size. Consequently, absent shocks a firm eventually attains optimal size, $n^*(z) = qv(z) \left(\frac{u}{S} + \frac{\phi e}{S} G(z) \right) / (\delta + \kappa + \phi p(1 - F(z)))$, which increases in productivity. Optimal size is hence determined by the labor market, in line with survey responses of Swedish firms that finding suitable labor is the number one obstacle to firm growth (in contrast to, for instance, perceiving a lack of demand at the going price) (Tillväxtverket, 2017). At face value, this suggests that a micro-level understanding of the labor market is important for understanding firm dynamics.

and $x(z)$ is the distribution of firms. The firm loses workers due to separations to unemployment and up the ladder, death and entrepreneurship. The firm meets potential hires at rate $qv(z)$, who accept the offer if they are unemployed (u/S) or employed at a less productive firm ($\phi e/SG(z)$).

Lemma 1. *Low productive firms post fewer vacancies and hire disproportionately from unemployment.*

In the data, new firms tend to be low productive. Hence, they disproportionately hire from unemployment. As the unemployed are more likely to be young, new entrepreneurs rely heavily on young workers in order to grow.

2.3 Aggregate entry, growth and inequality

The aggregate entry rate equals the entry probability of unemployed individuals times their population share, plus the entry probability of employed individuals times their population share,

$$E = \frac{1}{L} \left(us(u) + e \int_{\underline{z}^w}^{\infty} s(z) dG(z) \right) \quad (14)$$

divided by the number of firms, L , to express entry as a fraction of firms. Note that the price of land adjusts such that the mass of firms equals the amount of land in the economy. The distribution of individuals over the job ladder $g(z)$ is given by the Kolmogorov Forward Equation (KFE),

$$0 = mg'(z) + \frac{\sigma^2}{2} g''(z) - \left(\kappa + \delta + \phi p(1 - F(z)) + s(z) \right) g(z) + pf(z) \left(\frac{u}{E} + \phi G(z) \right) \quad (15)$$

subject to the density being zero at the boundary since workers exit there, g integrating to one,

$$g(\underline{z}^w) = 0, \quad \int_{\underline{z}^w}^{\infty} g(z) dz = 1 \quad (16)$$

and unemployment satisfying,

$$0 = -\left(\kappa + p + s(u) \right) u + \left(\delta + \frac{\sigma^2}{2} g'(\underline{z}^w) \right) (1 - u - L) + EL + \kappa(1 - L) \quad (17)$$

The first two terms in (15) are due to the drift of and shocks to productivity;¹⁴ the third due to outflows to death, unemployment, other firms and entrepreneurship; and the fourth due to inflows from unemployment and firms below in the job ladder. The first term in (17) is due to outflows to death, employment and entrepreneurship; the second is inflows from employment; the third is inflows from entrepreneur-

¹⁴The drift of productivity is $-m$, i.e. the first term in the KFE is $-(-m)$.

ship (the BGP entry rate E equals the exit rate); and the fourth is inflows of new individuals, who start as unemployed unless their parent was an entrepreneur.¹⁵

Given an entry rate E , the stationary distribution of firms over productivity, $x(z)$, is given by,

$$0 = mx'(z) + \frac{\sigma^2}{2}x''(z) + E\gamma(z) \quad (18)$$

$$x(\underline{z}) = 0, \quad \int_{\underline{z}}^{\infty} x(z)dz = 1, \quad \text{and} \quad E = \frac{\sigma^2}{2}x'(\underline{z}) \quad (19)$$

Productivity falls behind the market at the rate of obsolescence m , is subject to shocks with intensity σ , and new entrants enter at rate E according to the innovation distribution γ . The first boundary condition in (19) requires that the density is zero at the exit threshold, since firms exit when they hit it. The second condition imposes that x is a density. The final condition can be seen by integrating the KFE (18) from \underline{z} to infinity and imposing the first and second conditions.¹⁶ Given an aggregate entry rate (14), the second-order differential equation (18) subject to the three conditions (19) determine the distribution, x , and the rate of obsolescence, m .

Proposition 4. *Suppose that the innovation distribution Γ is Pareto distributed with shape ζ . Then the stationary distribution of firms $x(z)$ is given by,¹⁷*

$$x(z) = \frac{E}{-m + \frac{\sigma^2}{2}\zeta} \left(e^{\frac{2(-m)}{\sigma^2}z} - e^{-\zeta z} \right) \quad (20)$$

¹⁵To understand where equation (17) comes from, first integrate the KFE (15) from \underline{z}^w to infinity. Then use the boundary condition (16) (which in particular implies that $\lim_{x \rightarrow \infty} g(x) = 0$ and $\lim_{x \rightarrow \infty} g'(x) = 0$) to arrive at,

$$0 = -\frac{\sigma^2}{2}g'(0) - (\kappa + \delta) - \phi p \int_{\underline{z}^w}^{\infty} 1 - F(x)g(x)dx - \int_{\underline{z}^w}^{\infty} s(x)g(x)dx + pu/E + \phi p \int_{\underline{z}^w}^{\infty} f(x)G(x)dx$$

Integrating the third term by parts (noting that $G(\underline{z}^w) = 0$, $\lim_{z \rightarrow \infty} 1 - F(z) = 0$ and $\lim_{z \rightarrow \infty} G(z) = 1$), multiplying by $E = 1 - u - L$, and cancelling terms,

$$0 = \frac{\sigma^2}{2}g'(0)(1 - u - L) + (\kappa + \delta)(1 - u - L) + (1 - u - L) \int_{\underline{z}^w}^{\infty} s(x)g(x)dx - pu$$

Collecting terms and noticing that total inflows to entrepreneurship equals $EL = us(u) + (1 - u - L) \int_{\underline{z}^w}^{\infty} s(x)g(x)dx$,

$$0 = \left(\delta + \frac{\sigma^2}{2}g'(0) \right) (1 - u - L) - (\kappa + p + s(u))u + \kappa(1 - L) + EL$$

¹⁶Specifically, $0 = m \int_{\underline{z}}^{\infty} x'(z)dz + \frac{\sigma^2}{2} \int_{\underline{z}}^{\infty} x''(z)dz + E \int_{\underline{z}}^{\infty} \gamma(z)dz = -\frac{\sigma^2}{2}x'(\underline{z}) + E$ since the second condition requires that $\lim_{z \rightarrow \infty} x(z) = 0$ and $\lim_{z \rightarrow \infty} x'(z) = 0$, the first requires that $x(\underline{z}) = 0$, and $\gamma(z)$ must integrate to one.

¹⁷Parameter restrictions have to be imposed to ensure that a BGP equilibrium exists. In particular, the resulting growth rate cannot be too large or utility is not bounded. Moreover, it cannot be too small because no stationary distribution of firms may exist. I assume throughout that these parameter restrictions hold.

and the rate of obsolescence m is given by,

$$m = \frac{E}{\zeta} \quad (21)$$

Corollary 4. *An increase in aggregate entry E increases the rate of obsolescence and growth, m .*

Entrants build on incumbents to grow the economy, such the more entry is associated with faster growth. Faster growth implies that prices of factors of production grow faster on the BGP. Because incumbent firms' productivity do not grow at the same rate as the overall economy—growth is *embodied* in firms—greater entry leads incumbent firms to become obsolete at a faster rate.

Define log productivity z as following a power law if constants $a, \zeta^* > 0$ exist such that $Pr(Z > z) = ae^{-\zeta^*z}$. Log productivity z follows an asymptotic power law if constants $a, \zeta^* > 0$ exist such that $Pr(Z > z) \sim ae^{-\zeta^*z}$ as $z \rightarrow \infty$, where for any $f, g, f(z) \sim g(z)$ means $\lim_{z \rightarrow \infty} f(z)/g(z) = 1$.

Proposition 5. *The endogenous stationary distribution of productivity z follows an asymptotic power law with tail parameter ζ^* given by*

$$\zeta^* = \begin{cases} \zeta & \text{if } \frac{(\sigma\zeta)^2}{2} < E \\ \frac{2E}{\sigma^2\zeta} & \text{if } \frac{(\sigma\zeta)^2}{2} > E \end{cases}$$

Corollary 5. *If the initial entry rate is sufficiently low, $\frac{(\sigma\zeta)^2}{2} > E$, a decline in entry increases inequality.*

Creative destruction is a force that holds some firms back from becoming in a relative sense very productive. If the entry rate is high, $E > (\sigma\zeta)^2/2$, this force is so strong that the right tail of the productivity distribution is entirely driven by the tail of the entry distribution. It is as though firms fall behind the market so fast that no firm manages to move into the right tail of the distribution after entry. If, on the other hand, the entry rate is lower than this (the empirically relevant case), the tail of the productivity distribution is driven by endogenous productivity dynamics.¹⁸ In this case, lower entry is associated with greater tail dispersion in productivity across firms.

2.4 Equilibrium and discussion

Definition 1. *A stationary equilibrium consists of value functions U, J , and O ; decision rules of individuals $\{\underline{z}^w, s(u), s(z)\}$ and firms $\{\underline{z}, v(z)\}$; finding rates p and q , an offer distribution F , an aggregate mass of vacancies V , a rental rate of land r , an aggregate entry rate E , and a rate of obsolescence m ; and a distribution of workers $\{G, u\}$ and a distribution of firms x , such that*

¹⁸The distribution is not well defined in the limiting case $2E/\sigma^2 = \zeta^2$.

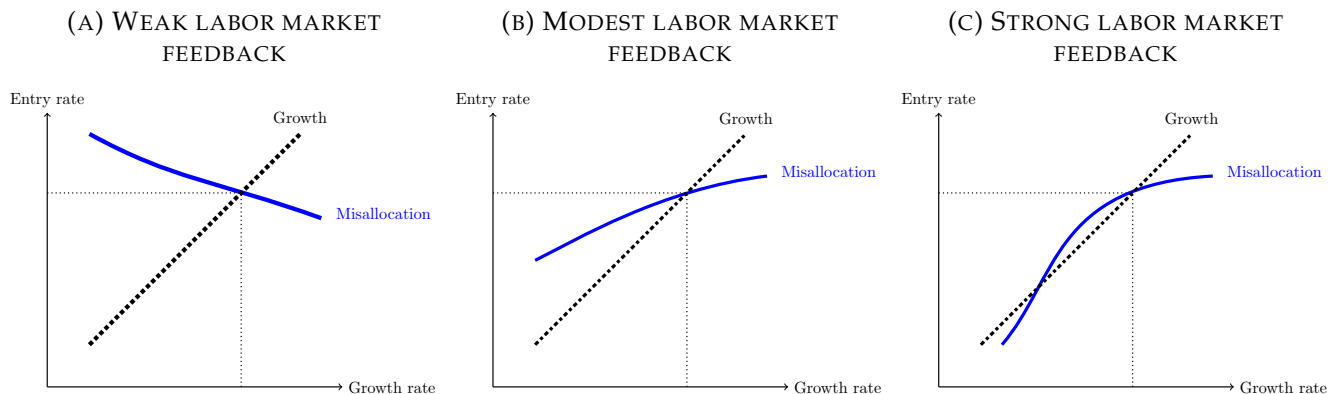
1. The value function U and search intensity $s(u)$ solve (2); the value function J and reservation policy \underline{z}^w solve the stopping time problem (3), and the search intensity $s(z)$ is given by (4); and the value function O and the exit threshold \underline{z} solve the stopping time problem (8), and the vacancy policy $v(z)$ is given by (11);
2. The finding rates p and q are given by (1), the offer distribution F and aggregate number of vacancies V by (13), the price of land r by (10), and the aggregate entry rate by (14);
3. The distribution of workers $\{G, u\}$ is given by (15)–(17), and the distribution of firms x and the rate of obsolescence m by (20)–(21).

Figure 1 illustrates how the equilibrium is determined in entry-growth space. The *growth* curve plots the growth-entry relationship (21)—more entry speeds up the selection process of entry and exit, leading to faster growth. The *misallocation* curve graphs the entry-growth relationship (14), arising from individuals’ optimizing behavior given a rate of obsolescence. A higher rate of obsolescence discourages entry by reducing the expected duration of a firm. Absent the richer model of the labor market here, this would be the key equilibrating force. In the current environment, on the other hand, a higher growth rate also results in greater labor misallocation, which encourages entry by lowering the opportunity cost of entry and the cost of hiring. If these forces are strong enough to outweigh the duration effect, the *misallocation* curve is also upward-sloping. The economy may in fact display multiple stationary equilibria. In one equilibrium, growth is high and the labor market mismatched. This encourages firm creation, rationalizing the high growth rate. Another equilibrium features low growth and a well-matched labor market. Key is how misallocative growth is and how much labor misallocation encourages entry. The latter is a function of how much scope there is for labor reallocation to increase individuals’ productivity in existing technologies versus ability to come up with new better technologies, as well as the elasticity of entry to the opportunity cost of entry and the cost of hiring.

Appendix B outlines the algorithm I use to solve and estimate the model based on insights in Figure 1. Before I bring the model to the data, I briefly reflect on some of the assumptions made.

The market for ideas. The model imposes a market failure by assuming that ideas cannot be traded, i.e. only the individual who receives an idea may pursue it. A particular concern is the possibility that potential founders employed in productive firms may be able to implement their idea within their existing firms, for instance by selling it to the firm. Theories of spin-offs, however, predict that the best ideas are spun off (Chatterjee and Rossi-Hansberg, 2012). Hence, if this were the case, one might have expected an increasing left-truncation of the productivity distribution of new firms by productivity of the

FIGURE 1. EQUILIBRIUM DETERMINATION



Note: Growth: Growth rate resulting from a particular entry (21). Misallocation: Entry rate given by individual's optimizing behavior given a growth rate (14). Weak labor market feedback: Duration effect outweighs misallocation effect. Modest labor market feedback: Misallocation effect modestly outweighs duration effect. Strong labor market feedback: Misallocation effect strongly outweighs duration effect. Source: Model.

prior employer. In the data, the entire distribution of new productivity shifts to the right (see Appendix A). While more work is needed on this, at face value it speaks against such a story.

Relatedly, I assume that individuals who exit entrepreneurship have to restart at the bottom of the job ladder. This is consistent with the eight log point average decline in earnings of founders in their first job post entrepreneurship relative to what they earned in their last job prior to entrepreneurship, conditional on (time-varying) observables, which is similar to the average loss associated with a spell of unemployment.¹⁹ I also note that the next section allows for a role also for heterogeneity in human capital, such that not all productivity and wage dispersion is due to search. Hence, restarting at the bottom should be interpreted in a residual sense.

The sources of growth. While I allow for incumbent innovation (the drift term μ), it is treated as exogenous. I focus on the entry and exit margin, since a vast literature has highlighted the critical contribution of entry and exit of firms to economic growth, going back to Schumpeter (1942). Moreover, I estimate that over half of growth stems from creative-destructive entry and exit. Hence, I argue that it is first-order to endogenize this process.

Other factors behind entrepreneurship. This paper focuses on providing a detailed theoretical and empirical understanding of the role of labor market dynamics in firm creation and subsequent success. This follows the recent entrepreneurship literature, which has forcefully argued that "a major problem for entrepreneurship and organization theorists has been the pervasive belief that the explanation for

¹⁹These findings are broadly in line with evidence from the PSID for the US (Bruce and Schuetze, 2004).

entrepreneurial achievements must be sought in personal traits [...] personal traits, taken out of context, simply do not explain very much" (Aldrich, 1999). Instead, that literature has emphasized the need to understand the role of the sociological context in the decision to start a firm and its subsequent success. This paper offers a step in that direction.

Nevertheless, in particular macroeconomics has a long history of viewing credit constraints as a key determinant of entrepreneurship (even though the empirical support for this is at best mixed—see for instance Hurst and Lusardi, 2004). In my estimation in the next section, I exploit the richness of my data to isolate the patterns of entry and subsequent performance that are linked to labor market events, as opposed to permanent individual characteristics or life-cycle factors. Nevertheless, this paper offers only a first attempt at linking firm creation and subsequent success to labor market dynamics. While it would be a difficult undertaking to incorporate a financial constraint in a theoretically and empirically appealing way in a bargaining model such as the current one, it would be very interesting to try.

Fixed amount of land. Computational reasons lead me to assume a fixed amount of land, as this avoids having to solve also for the mass of firms. With respect to the effect of policy or demographic change, however, I believe that this is of second-order concern. First, average firm size has barely changed over this period in Sweden. Second, if I allowed land to be adjusted subject to some elasticity, the natural moment to inform this elasticity would be changes in average firm size over time. As average firm size has barely moved, this would presumably lead me to infer that land creation must be quite inelastic. Third, under the estimated parameters, most of the return from exiting entrepreneurship comes from the option to search as unemployed, not from avoiding the rental rate of land. Hence quantitatively, the rental rate of land plays a second-order role. At a conceptual level, this assumption implies that I follow the *quality ladder* literature on economic growth (Aghion and Howitt, 1992; Grossman and Helpman, 1991), in the sense that there is a fixed set of "production lines" that can be innovated on. In their rich estimation exercise, Garcia-Macia et al. (2016) allow for multiple sources of growth and find that most US growth over the past 30+ years has taken the form of quality improvements.

3 Estimation

This section structurally estimates the model by simulated method of moments.

3.1 Empirical extensions

To allow the model to speak to the richness of the data, I first extend the framework in three directions.

Experience. First, to avoid overstating the role of search in life-cycle dynamics, I allow also for an independent role for experience. In particular, I introduce human capital, which I assume can take one of N_h values,²⁰ is drawn at labor market entry from distribution $\mathcal{H} \sim \log \mathcal{N}(0, \sigma_h)$, and accumulates to the next level at rate ζ , regardless of current employment status. Hence, human capital is much like experience and I refer to them interchangeably. It is general, in the sense that it can be used at all firms and production is linear in efficiency units of labor \tilde{n}_j , $y_j = e^{z_j} \tilde{n}_j$. Combined with the assumptions below that labor market flow parameters differ by human capital groups and human capital is allowed to influence the productivity of new firms, this also effectively allows for ex ante heterogeneity in the propensity to start a firm and the ability to start a successful firm.

Individuals with different experience search in separate markets and firms chose how many vacancies to post in each market, subject to the same iso-elastic vacancy cost function market by market. This limits the congestion exerted by a larger share of mature, well-matched workers. I let search frictions/costs of hiring differ across markets, $\chi(h) = \chi + \chi_h h$. Absent vacancy data, the intercept c_0 and matching efficiency χ are isomorphic, so I set $c_0 = 1$. I also let the cost of searching for business ideas vary with current productivity and experience, $c_e(z, s) = e^z c_e^0 (1 + c_e^1 h)$. Without the dependence on current productivity, the model cannot match the magnitude of the decline in the probability of starting a firm with the productivity of the current employer in the data. Finally, to match the decline in the EU hazard with the productivity of the firm, I let the separation rate vary with experience as well as with the productivity of the firm, $\delta(z) = (\delta_0 - \delta_1 h - \delta_2 (z - \bar{z}^w))^+$. One microfoundation is match-idiosyncratic shocks that lead matches close to the separation threshold to be more likely to break up.²¹

Hence, all parameters governing labor market flows are allowed to vary by experience groups. This lets the data determine the role of search in careers. In contrast to the rich model of the labor market, however, this paper is less ambitious in terms of understanding the underlying reasons for why flow parameters differ by experience. While I estimate that search is a key determinant of life-cycle dynamics of firm creation and worker mobility—motivating my focus—it would nevertheless be interesting to further analyze the experience component in future work.

A richer model of knowledge spillovers. Second, I assume that initial productivity, Z_0 , is related to the mean of the productivity distribution of incumbent firms, \bar{Z} , following [Sampson \(2016\)](#), as well as

²⁰In practice, I set $N_h = 2$, which appears to be sufficient to capture the forces at hand.

²¹An earlier version of this paper micro-founded this outcome along these lines following [Pries and Rogerson \(2005\)](#). It found, however, that learning was essentially unaffected with respect to the main counterfactual experiment considered by this paper—aging. Hence, to keep things manageable I impose this parametrically.

productivity of the prior employer Z and the (general) experience of the founder, h ,

$$Z_0 \sim \log \mathcal{N}\left(\alpha_0 \bar{Z} + \alpha_1 Z + \alpha_2 h, \sigma_\epsilon\right)$$

where $Z = 0$ denotes the case when the founder is previously unemployed.

Labor supply growth. Third, in order to later be able to vary the age composition without impacting the amount of time an individual expects to remain in the market, I introduce population growth.²² In particular, Appendix B shows how the model can be extended to incorporate population growth at rate λ such that neither the level nor the growth rate of labor supply directly impacts the economy.²³ This leaves all the value functions in the previous section the same to only change the KFE characterizing the distribution of employment (15), which becomes

$$\lambda g(z) = mg'(z) + \frac{\sigma^2}{2} g''(z) - \left(\kappa + \delta + \phi p(1 - F(z)) + s(z)\right) g(z) + pf(z) \left(\frac{u}{E} + \phi G(z)\right)$$

With population growth, the density must grow at the rate of population at any point of the distribution on the BGP. The boundary conditions are the same as before, apart from

$$\lambda u = -\left(\kappa + p + s(u)\right)u + \left(\delta + \frac{\sigma^2}{2} g'(\underline{z}^w)\right) (1 - u - L) + EL + \left(\kappa + \lambda\right)(1 - L)$$

As for employment, the number of unemployed must grow at the rate of population on the BGP. Moreover, with population growth there is an inflow to unemployment of $\kappa + \lambda$ individuals at any point in time. Henceforth, I refer to a decline in the growth rate of labor supply, λ , as *aging*, since it shifts the age distribution toward mature individuals.

3.2 Data and constructing the moments

In order to follow a large number of individuals as they move across employers and start firms, the performance of these start-up firms and who they hire, I combine several administrative data sources from Sweden. The resulting data set contains information on all workers and firms at a monthly frequency

²²That is, while I could make the population older by varying the rate at which "dynasties" are reborn, κ , it has the drawback that it simultaneously changes the amount of time an individual expects to remain in the market. Introducing labor supply growth allows me to separate these two distinct objects.

²³That is, the economy features no scale effects in either the number of workers or their growth rate. In that sense, a change in labor supply growth is isomorphic to an unexpected change in the rate at which individuals are replaced by their offspring, κ , where by unexpected I mean that it does not impact individuals' expected rate of discount. This would age the population without impacting the size or growth rate of the labor force, and have an identical effect to a change in the growth rate of labor supply, λ , in the current framework.

from 1985 to 2015, including revenue productivity of all firms regardless of size and sector from 1997 and onwards. In the interest of space, Appendix A contains more details.

I start by trying to isolate empirically the role of labor market dynamics in firm creation (and worker mobility). While these exercises are structurally misspecified, they provide useful auxiliary moments for the structural estimation. I regress the monthly probability of starting a firm (switching employer), $prob_{it}$, on a rich set of observable and unobservable controls, X_{it} , including gender, education, age, employment duration and percentile of person fixed effects in pay,²⁴ and a fixed effect for the current employer, $F_{j(i,t)}$,

$$\mathbf{1}[prob_{it}] = \beta X_{it} + F_{j(i,t)} + \varepsilon_{it} \quad (22)$$

I subsequently relate the estimated firm-fixed effects $\hat{F}_{j(i,t)}$ to characteristics of the employer.

To isolate the role of prior labor market experiences of the founder in the future success of the firm, I regress various post-entry outcomes of a new firm f founded by individual $i(f)$ in year t at some point τ years after inception, $out_{ft+\tau}$, on individual controls for the founder, $F_{i(f)t}$; controls for the sector, location and year of foundation of the new firm, N_f ; and controls for the founder's prior employer, $P_{j(i(f))t}$,

$$out_{ft+\tau} = F_{i(f)t} + N_f + P_{j(i(f))t} + \varepsilon_{it+\tau} \quad (23)$$

I include in $F_{i(f)t}$ gender, age (5 groups), education (5 groups) and decile of AKM person fixed effect, and in $P_{j(i(f))t}$ sector, location, size, firm age and decile in the firm productivity distribution. In case of multiple founders, I use mean values (modal in case of education, sector and location), with additional controls for the number of founders. I also exclude a very few number of firms with more than 10 founders. The post-entry outcomes I consider include the survival rate, the size and the productivity of new firms, measured at $\tau = \{1, 5, 10\}$ years after entry.

Figure 2 correlates the estimated firm-fixed effects from regression (22) for firm creation (left) and worker mobility (right) with firm productivity. Conditional on a rich set of controls, individuals employed at more productive firms are significantly less likely to quit to start a new firm or switch employer. This is almost entirely a within-sector phenomenon.²⁵

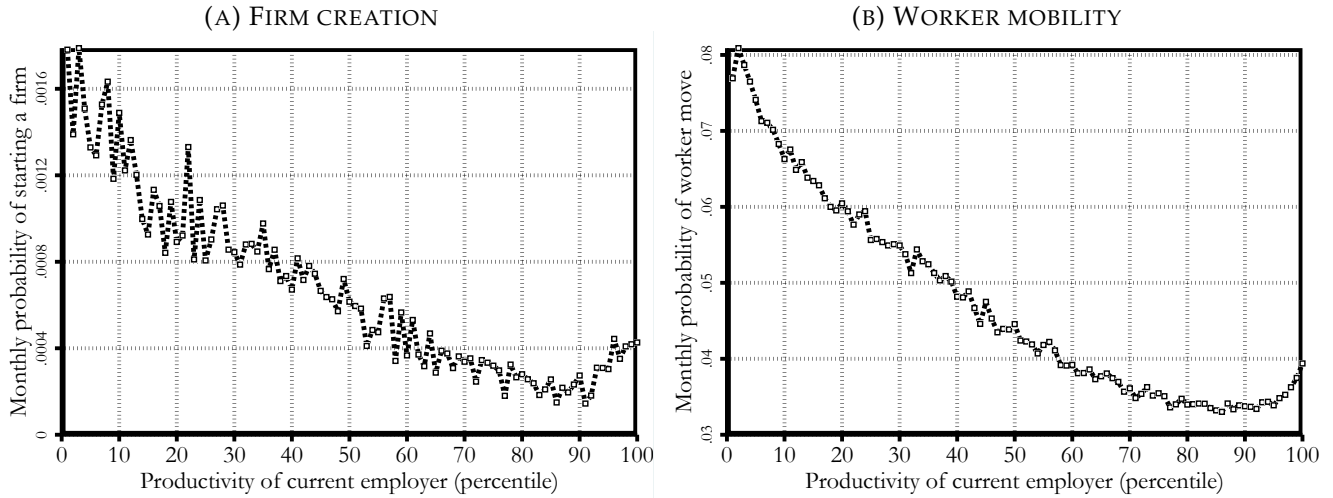
Figure 3 plots various (residualized) post-entry outcomes one and five years after entry by the founder's prior employer's decile in the firm productivity distribution. There is no systematic relationship be-

²⁴Based on a two-way fixed effects regression of log monthly earnings on individual fixed effects, firm-year fixed effects and age over the pooled 1985–2015 period. Age is restricted to be zero after age 50 to avoid collinearity and the sample includes only employee spells. See Engbom and Moser (2020) for more details.

²⁵Consistent with these patterns, a literature documents that individuals at smaller and younger firms are more likely to start a firm (Blanchflower and Meyer, 1994). I find, however, that this is not primarily accounted for by firm size or firm age per se, but the fact that small and young firms are low productive.

tween the productivity of the founder’s prior employer and the survival rate of new firms or initial size. Founders who were previously employed at more productive firms tend to start more productive and faster growing firms. Appendix A presents regression tables.

FIGURE 2. PROBABILITY OF STARTING A FIRM/SWITCHING EMPLOYER RELATIVE TO PRODUCTIVITY OF THE CURRENT EMPLOYER



Note: Firm creation: Share of employment in month t who is an owner-operator of a new firm in month $t + 1$. New firms are firm less than or equal to two years old, and include incorporated and unincorporated businesses. Worker mobility: Share of employment in month t who is employed by a different employer in month $t + 1$, including non-employment. Estimated firm fixed effect based on (22) relative to employment-unweighted percentile of firm level TFP, obtained from a regression of log value added per worker on log assets and log labor separately by sector-year. Private sector employment age 18–63. Source: RAMS, LISA and FEK 2011–2015.

FIGURE 3. POST-ENTRY OUTCOMES OF NEW FIRMS RELATIVE TO THE PRODUCTIVITY OF THE FOUNDER’S PRIOR EMPLOYER



Note: New firm outcomes a years after entry by employment-weighted decile of productivity of founder’s prior employer. Survival rate: Share of new firms that survive at least until age a . Size: Log number of employees. Productivity: Firm-level TFP from a regression of log value added per worker on log assets and log labor separately by sector-year. All outcomes are residual controlling for gender, age, education and decile of AKM individual fixed effect in pay for the founder; sector and location of the new firm; and sector, location, size and age of the previous employer, and evaluated at the mean value for these covariates. Mean/modal values are used in case of multiple founders, with additional controls for the number of founders. Sample excludes a very few number of firms with more than 10 founders. New firms include incorporated and unincorporated private businesses. Private sector employment age 18–63 at time of firm inception. Source: RAMS, LISA and FEK 1997–2015.

3.3 Structural estimation

I estimate the model by simulated method of moments targeting 2011–2015. I focus on these years because data on incorporated owner-workers are not available prior to 2011 at a monthly frequency. I solve the model in continuous time on a discretized grid for productivity and human capital. The estimation procedure requires a very large number of evaluations of the model, which makes simulating the model computationally infeasible. To avoid this, I derive the targeted moments directly in the continuous time model, bypassing the need for simulation. Having estimated the model, I simulate a weekly model of the model in order to compute a few (non-targeted) moments that are difficult to derive KFEs for.

TABLE 1. EXTERNALLY CALIBRATED PARAMETERS

Parameter	Value	Target
ρ Discount rate	0.0041	Annual real interest rate of 5%
M Overall growth rate	0.0017	Growth in GDP per capita of 2%
θ Elasticity of matching function	0.5	Petrongolo and Pissarides (2001)
β Bargaining power	0.3	Bagger et al. (2014)
κ Exit rate	0.0042	Average duration of careers of 40 years
λ Population growth rate	0.0007	Share of labor force aged 45+
L Mass of land	1/12	Average firm size

Note: When applicable, the frequency is monthly.

External. I start by pre-setting seven parameters in Table 1. The discount rate is set to five percent and the growth rate to two percent annually. While total growth is fixed, I estimate internally the importance of entry and exit relative to incumbent innovation. I set the elasticity of matches to vacancies to 0.5, a standard value. As noted by Bagger et al. (2014), it is difficult to pin down β in the data, and hence I opt to pre-set this parameter to their estimated value in a similar framework. The exit rate of individuals is set to target an average duration of careers of 40 years and the population growth rate to target the share of mature labor force participants. Finally, the mass of land targets average firm size.

Internal. That leaves 18 parameters to estimate together with normalized values for the flow value of leisure and of being one’s own boss,

$$\mathbf{p} = \left\{ \sigma, \eta_v, d, \alpha_0, \alpha_1, \alpha_2, \sigma_\varepsilon, \chi_1, \chi_h, \phi, \delta_0, \delta_1, \delta_2, \eta_e, c_e^0, c_e^1, \sigma_h, \xi, b_1, b_2, k \right\}$$

I set up a wide grid of potential parameter values that efficiently span this 18-dimensional space (using what are known as Sobol sequences).²⁶ For each parameter vector on this grid, I solve the model and

²⁶I thank Simon Mongey for teaching me this.

record model moments. I choose the parameter vector \mathbf{p} that minimizes the weighted sum of squared percentage deviations between 43 moments in the model and the data,

$$\mathbf{p} = \arg \min \sum_{m=1}^{43} w_m \left(\frac{data_m - model_m(\mathbf{p})}{data_m} \right)^2$$

I assign each moment the same weight apart from the relative productivity of entrant and exiting firms, which I give five times the weight of the other moments since they particularly inform growth due to firm selection. This algorithm has the advantage that it bypasses local minima and non-convergent parameter values and it stores outcomes for a large set of alternative parameter vectors, which can be used to learn about identification.²⁷

While identification is joint, it is useful to provide a heuristic discussion of what moment particularly informs what parameter. The standard deviation of productivity shocks, σ , is informed by TFP dispersion across firms. The larger this is, the greater is productivity dispersion. As all productivity shocks in the model are permanent, I target the first autocovariance as an estimate of the variance of permanent productivity in the data.²⁸ I allow for exogenous firm death at rate d , which is informed by the average productivity of exiting relative to all firms. If this is higher, exiting firms are on average more productive. The curvature of the hiring cost η_v is informed by the employment share of firms with 500 or more employees. If η_v is lower, it is cheaper for productive firms to scale up vacancy creation, leading to a more skewed size distribution.

General knowledge spillovers, α_0 , are informed by the TFP gap between entrant and incumbent firms. If this is lower, entrants start further from incumbents. Specific knowledge spillovers, α_1 , are informed by the relative productivity of firms five years after entry by productivity of the founder's prior employer. If this is larger, individuals currently working in more productive firms start more successful firms. The extent to which experience affects the ability to innovate, α_2 , is informed by the relative productivity of new firms five years after entry by age of the founder. Dispersion in initial productivity, σ_ϵ , is informed by TFP dispersion among new firms.

Matching efficiency, χ , is informed by the unemployment rate; relative search efficiency of the employed, ϕ , by the JJ rate; and the intercept in the separation rate, δ_0 , by the EU rate.²⁹ The extent to which matching efficiency and the separation rate change with experience, χ_h and δ_1 , are informed by the behavior of the JJ and EU rates, respectively, late in life. Because individuals move up the job ladder

²⁷The disadvantage is that it takes a couple of weeks to estimate the model running on 600 cores in parallel.

²⁸If z is subject to permanent shocks with intensity σ_z and i.i.d. shocks with intensity σ_ϵ , then $Cov(z_t, z_{t-1}) = \sigma_z^2$.

²⁹It is not clear conceptually whether one would like to target the EU or EU+EN rate. The data, however, only allow me to compute flows in and out of employment, and hence the EU rate always refers to the EU+EN rate.

relatively fast, the distribution of employment has roughly converged to its ergodic distribution at age 40. Hence, late-in-life dynamics are primarily driven by factors other than the job ladder. The decline in the separation rate with productivity, δ_2 , is informed by the overall decline in the EU rate over the life-cycle, which increases in δ_2 .

I set the rate of obsolescence m to target the overall entry rate. Of course, m is an endogenous outcome, but the entrepreneurship search cost, c_e^0 , is a free parameter. If m is higher, exit is higher which requires higher entry in equilibrium and hence a lower c_e^0 to rationalize a higher entry rate. The extent to which the cost of searching increases with experience, c_e^1 , is informed by the decline in entry late in life, for the same reason as the behavior of worker mobility late in life is informative of χ_h and δ_1 . The curvature, η_e , is informed by the overall decline in the probability of entry over the life-cycle, because if it is lower, search is more elastic and the decline is larger.

The rate of human capital accumulation, ζ , is informed by life-cycle wage growth, while overall dispersion in wages informs the initial dispersion in human capital, σ_h . I let the flow value of leisure b vary freely by experience group such that all workers want to quit just before entrepreneurs want to exit (in practice one grid point above entrepreneurs' exit threshold). While hypothetically one could estimate b to target the share of one-man firms in the data, in practice identification off this moment is weak. The flow value of being one's own boss k only impacts the price of land in equilibrium (recall (9)–(10)), so I set it such that the price of land is positive.

3.4 Parameter estimates and model fit

Table 2 shows the estimated parameter values. General knowledge diffusion is such that a new firm started by a low-experienced individual previously employed by the least productive incumbent firm on average starts with 65 percent of the average productivity of incumbent firms. Specific knowledge diffusion is positive, implying the founders previously working for more productive firms start more productive firms. General experience reduces the productivity of new firms, which could be interpreted as individuals becoming worse at innovating with age.

Search efficiency of the employed, ϕ , is high, for three reasons. First, the JJ rate is high relative to the UE rate in Sweden. Second, the "slippery" nature of the job ladder ($\delta_2 > 0$) isolates workers who have reached the top of the ladder from job loss. These workers stay at the top for long, rejecting many offers in the meantime. Third, I allow for an independent role for experience in reducing mobility ($\chi_h < 0$). Since the employed are also more experienced (and hence less mobile regardless of employment status), not controlling for the confounding correlation between experience and employment status would lead

to a downwardly biased estimate of ϕ . Because of the high estimated value for ϕ , the estimated flow value of leisure, b , is reasonably high, corresponding to roughly 20 percent of average output. The cost of searching for business opportunities increases with experience, $c_e^1 > 0$, which may be interpreted as evidence of increased risk aversion.

TABLE 2. INTERNALLY ESTIMATED PARAMETERS

	Parameter	Value	Moment	Data	Model
σ	St.d. of productivity shocks	0.035	St.d. of permanent TFP	0.421	0.466
η_v	Vacancy cost, curvature	58.728	Share of employment, firms ≥ 500 empl.	0.248	0.331
d	Exogenous exit rate	0.003	Mean TFP of exiting	-0.279	-0.283
α_0	Diffusion, general spillover	0.653	Mean TFP of entrant	-0.199	-0.199
α_1	Diffusion, specific spillover	0.016	New prod. by decile of prior prod.	See Figure 4	
α_2	Diffusion, human capital spillover	-0.922	New prod. by founder age	See Figure 4	
σ_e	Diffusion, i.i.d. dispersion	1.054	St.d. of TFP of entrant	0.403	0.321
χ_1	Search, matching efficiency = h_1	0.886	Unemployment rate	0.070	0.082
χ_2	Search, matching efficiency = h_2	0.211	Life-cycle JJ	See Figure 4	
ϕ	Search, relative efficiency of empl.	1.080	JJ rate	0.018	0.018
δ_0	Separation rate, intercept	0.337	EU rate	0.010	0.015
δ_1	Separation rate, human capital	0.103	EU, late in life	See Figure 4	
δ_2	Separation rate, productivity	0.300	EU, life-cycle decline	See Figure 4	
η_e	Entrepreneurial search, curvature	3.464	Entry, life-cycle decline	See Figure 4	
c_e^1	Entrepreneurial search, human capital	15.510	Entry, late in life	See Figure 4	
σ_h	Human capital, initial dispersion	0.098	St.d. of wages	0.604	0.715
ξ	Human capital, rate of accumulation	0.002	Wage, age 40–45 to age 25–29	0.374	0.380
m	Rate of obsolescence	0.001	Entry rate	0.041	0.032
b_1	Flow value of leisure, human capital = h_1	3.304	Normalization		
b_2	Flow value of leisure, human capital = h_2	3.968	Normalization		
k	Flow value of being own boss	20.000	Normalization		

Note: When applicable, the frequency is monthly. Estimates are for two experience groups, $N_i = 2$, such that χ_i and b_i is the matching efficiency/flow value of leisure for experience group i . Source: Model, RAMS, LISA and FEK 2011–2015.

Table 2 and Figure 4 illustrate that the model fits the data well in most of the targeted dimensions.³⁰ For instance, the model does an excellent job at matching life-cycle dynamics, including the unemployment rate which is not targeted (I target the overall unemployment rate and the separation and JJ rates by age, but not the UE rate).³¹ The moments the model primarily struggles with are the dispersion in

³⁰The entry profile differs somewhat from that documented by [Azoulay et al. \(2018\)](#) for the US. Apart from the fact that these are two different countries, five reasons may account for this. First, these authors use firm-level data to estimate the age of founders based on the set of start-up firms. Because this uses as denominator the set of start-up firms, not the set of potential founders, there is in general no reason to expect these two profiles to be the same (unless the underlying population age distribution is uniform, but it is not). Second, because of data limitations, their data records a firm as a start-up if and when it hires a worker, such that differences in the probability of hiring at least one worker conditional on starting a firm will appear as differences in the probability of starting a firm. Third, to the extent that it takes new firms time before they hire their first worker, founders will appear older in their data. Fourth, they are forced to impute who is the founder of a firm for a large class of firms in the US. Fifth, their data largely overlap with the Great Recession in the US, which may be a non-representative time period (in fact, the theory in this paper would seem to predict a relative increase in entry among mature workers in response to a large layoff shock, although it abstracts from many factors that may be first-order for a business cycle analysis and hence I interpret this prediction cautiously). In any case, the start-up profile I document for Sweden using the population of potential founders aligns closely with findings for the US in, for instance, [Dillon and Stanton \(2017\)](#) and [Liang et al. \(2018\)](#).

³¹For completeness, I start at age 20 although it is common for young Swedes to move in and out of part-time work and school. As the model abstracts from such considerations, I believe that little weight should be placed on the model-data comparison prior to age 25. It is not possible in the data to exclude in a time-consistent manner those not in the labor force. I assume that individuals enter the labor force once and for all at random between age 20–34 such that the labor force participation rate by age matches that in a separate survey data conducted by Statistics Sweden (the EU-LFS). I assume that individuals who

productivity among entrant firms and the productivity of new firms founded by the oldest age group. In the data, productivity dispersion among young firms is only modestly smaller than overall dispersion, whereas the model predicts a fanning out of productivity with firm age. If transitory shocks are more pronounced for young firms, that could account for this. Because dispersion in initial productivity, overall productivity and wages are positively related, the estimation trades these off by somewhat overstating the dispersion in overall productivity and wages. The reason the model fails to capture the low productivity of firms run by the oldest age group of founders may be driven by institutions specific to Sweden that push mature individuals to keep running low productive firms.³² In any case, few founders are above age 55, so from a macroeconomic perspective I believe that this is of less concern.

3.5 Validation and implications

To illustrate the workings of the model, I now use the theory in correspondence with the data to address three questions. First, what is the quantitative role of the job ladder in life-cycle careers? Second, who do new firms hire in order to grow? Third, what is the importance of creative-destructive entry and exit in growth? These questions are interesting in their own right. Moreover, as none of them is directly targeted in estimation, they provide important checks on the model. Finally, they are closely related to the focus of this paper: the interaction between growth and labor market dynamics. The first two exercises speak to how labor market dynamics impact firm creation and hence growth, while the latter relates to how growth affects labor market dynamics. Appendix C provides additional results.

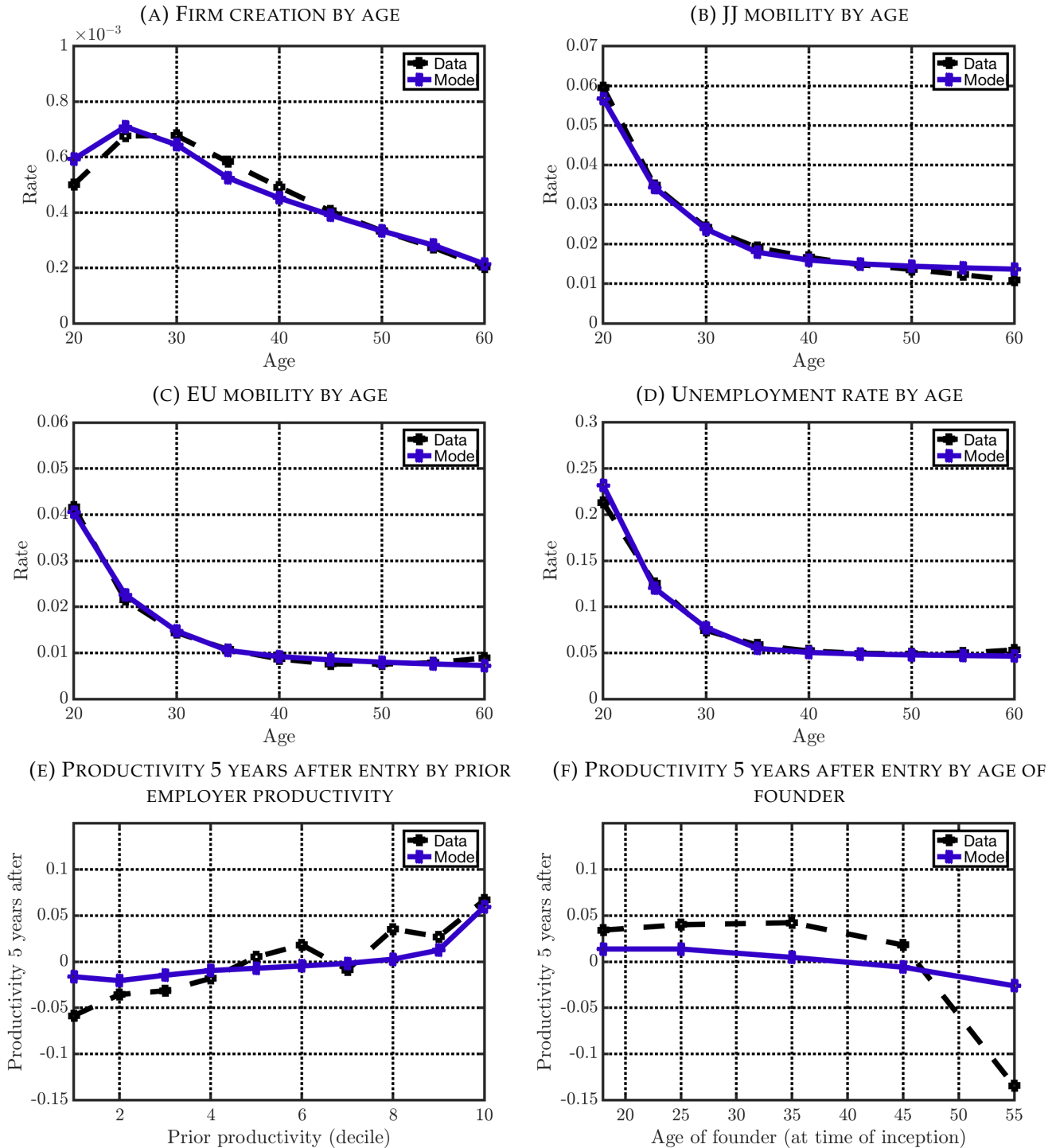
The role of the job ladder in life-cycle careers. Figure 5 reproduces regression (22) on model-generated data. Controlling for age and other confounding factors, individuals currently working for more productive employers are significantly less likely to quit their job to start a firm or switch employer, as their opportunity cost of doing so is greater. The model somewhat understates the decline in the probability of starting a firm with productivity of the current employer, but broadly matches the empirical patterns.

The *experience* term in Figure 6 plots the estimated age coefficients \hat{A} from regression (22) and the *search* term plots the difference between the raw age-conditional mean entry/mobility rate and the *experience* term, $\overline{prob}_a - \hat{A}$. I interpret the former to be the component of life-cycle dynamics that is unrelated to labor market events, while the latter is that accounted for by mobility up the job ladder (the great ma-

have not entered that labor force may not search for entrepreneurship opportunities, which accounts for the initial increase in the probability of starting a firm with age. When I consider the impact of aging, I assume that the age at which individuals enter the labor force remains the same.

³²Employee relationships are strongly discouraged (or outright prohibited) after age 65 (68), which may induce retired people to run unproductive firms in lack of other alternatives.

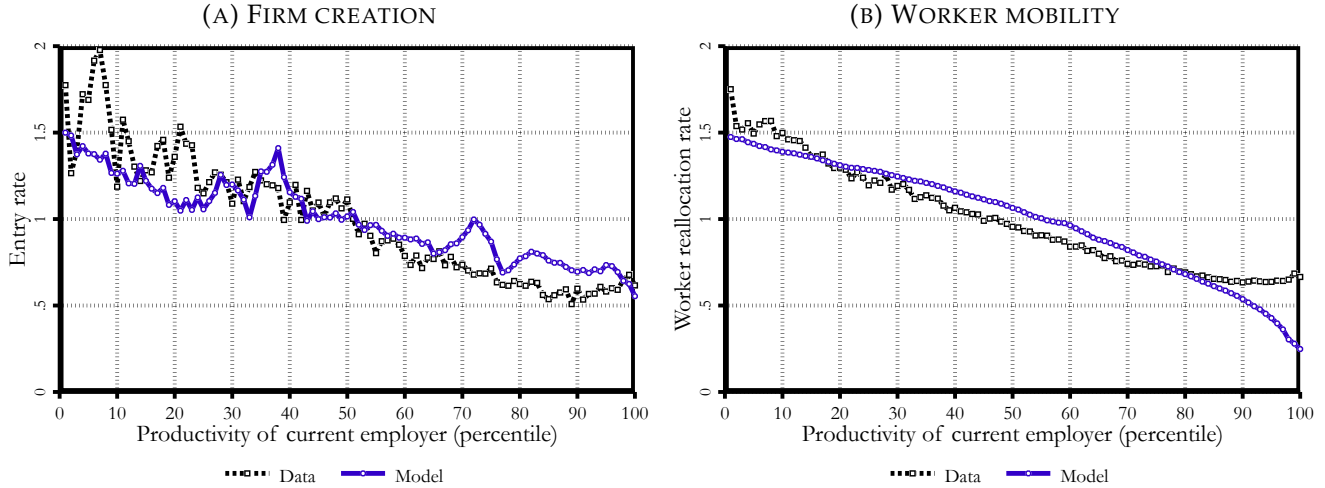
FIGURE 4. MODEL FIT



Note: Firm creation: Share of individuals in month t who is an owner-operator of a new firm in month $t + 1$, where a new firm is defined as a firm less than or equal to two years old. JJ mobility: Share of employment in month t who is employed at a different main employer in month $t + 1$. EU mobility: Share of employment in month t who is non-employed in month $t + 1$. Unemployment rate: standard ILO definition in the data. Productivity 5 years after entry: TFP of new firms by TFP of the founder's prior employer/age of the founder, where TFP is obtained as the residual from a regression of log value added on log assets and log employees separately by sector-year. Source: Model, RAMS, LISA, FEK and EU-LFS 2011–2015.

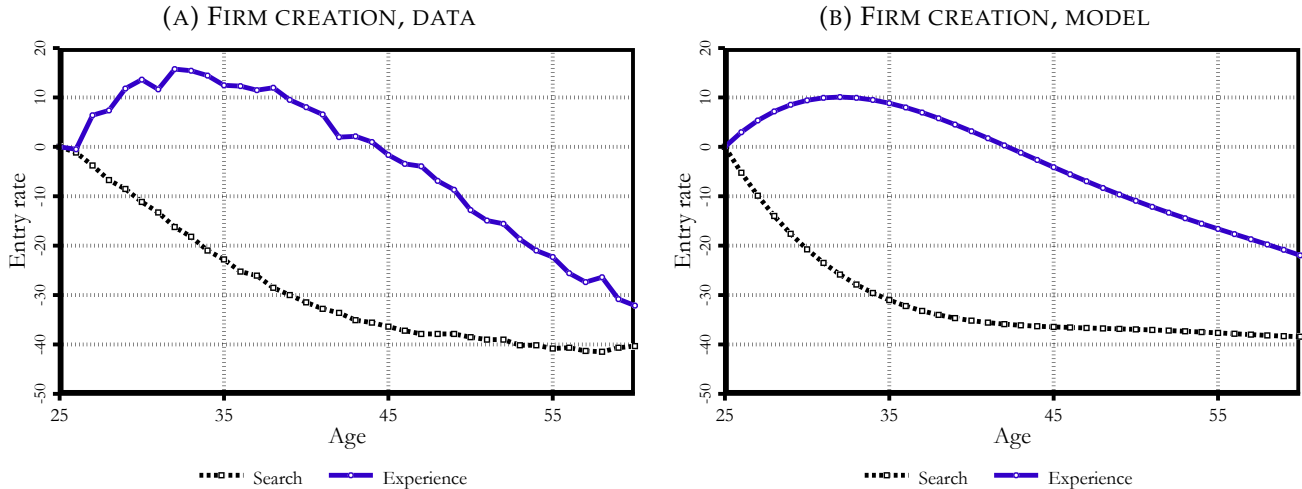
jectory is due to across-firm mobility). The model matches well the empirical decomposition of life-cycle firm creation, somewhat understating the *experience* term.

FIGURE 5. FIRM CREATION AND WORKER MOBILITY BY PRODUCTIVITY



Note: Firm creation: Share of individuals in month t who is an owner-operator of a new firm in month $t + 1$. Worker mobility: Share of employment in month t who is employed at a different main employer in month $t + 1$, including non-employment. Firm fixed effect from regression (22) for firm creation and worker mobility against firm-level labor productivity. Labor productivity is log value added per worker controlling for sector-year. Source: Simulated weekly version of the model for 25,000 firms, discarding an initial burn-in period and aggregating to monthly data, RAMS, LISA and FEK 2011–2015.

FIGURE 6. DECOMPOSITION OF LIFE-CYCLE MOBILITY



Note: Firm creation: Share of individuals in month t who is an owner-operator of a new firm in month $t + 1$. Experience: Estimated age effects from regression (22), \hat{A} . Search: Difference between the raw age-conditional mean and the estimated age effects, \hat{A} . Both are expressed in percent relative to the raw hazard at age 25. Source: Simulated weekly version of the model for 25,000 firms, discarding an initial burn-in period and aggregating to monthly data, RAMS, LISA and FEK 2011–2015.

The exercises above are useful to validate the model, but they are misspecified from the perspective of the theory. For instance, because young workers with little human capital tend to work for low z

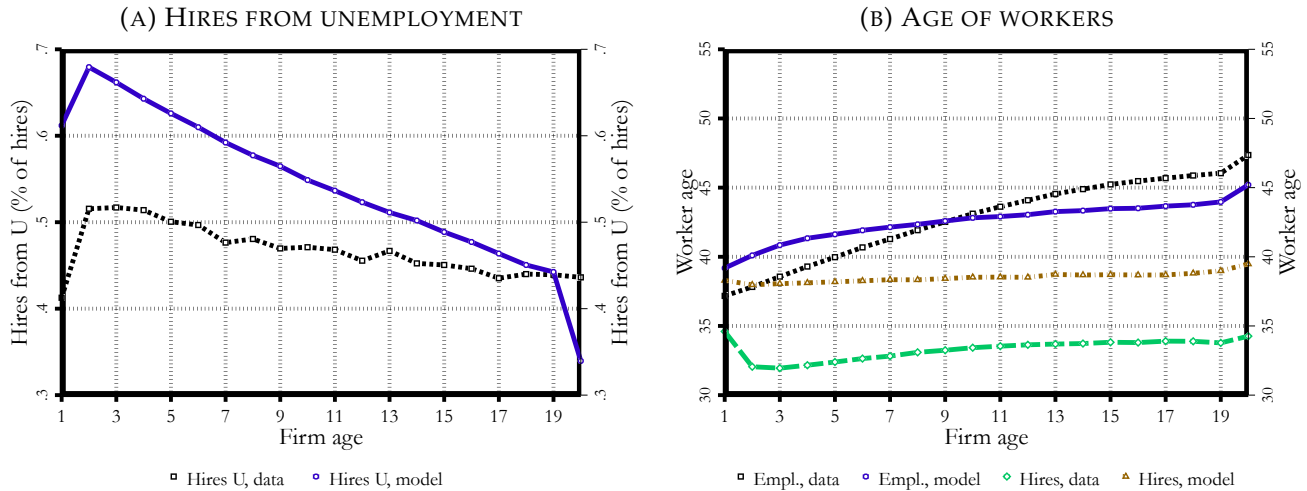
firms, value added dispersion in Figure 28 is larger than dispersion in z . Table 3 instead provides a structural assessment of the role of the job ladder in life-cycle careers. The first column summarizes average growth in z and h over the life-cycle. Recall that output is log additive in z and h . Human capital accumulation is the most important source of life-cycle productivity growth. Nevertheless, there is also significant scope for individuals to become more productive through JJ mobility. The last two columns show the component of entry/mobility accounted for by z/h . The share due to z is constructed by counter-factually setting human capital accumulation to zero, $\xi = 0$, holding all decision rules and equilibrium objects fixed at their estimated equilibrium values. The share due to h is the difference between the total and that due to z . Mobility up the job ladder plays a key role in the life-cycle decline in entry and JJ mobility. This suggests significant scope for labor misallocation to impact firm creation.

TABLE 3. THE ROLE OF JOB SHOPPING AND EXPERIENCE IN LIFE-CYCLE CAREERS

	% change, age 25–60	% of entry, age 25–60	% of JJ, age 25–60
z	0.370	0.629	0.931
h	0.603	0.371	0.069

Note: Change: Average change in $\log z/\log h$. Share of entry: Share of decline in entry accounted for by z/h . % of JJ: Share of decline in JJ rate accounted for by z/h . Decomposition by z : No experience accumulation, $\xi = 0$, holding decision rules and equilibrium objects fixed at their estimated values. Decomposition by h : difference between total and decomposition by z . Source: Model.

FIGURE 7. YOUNG FIRM DYNAMICS

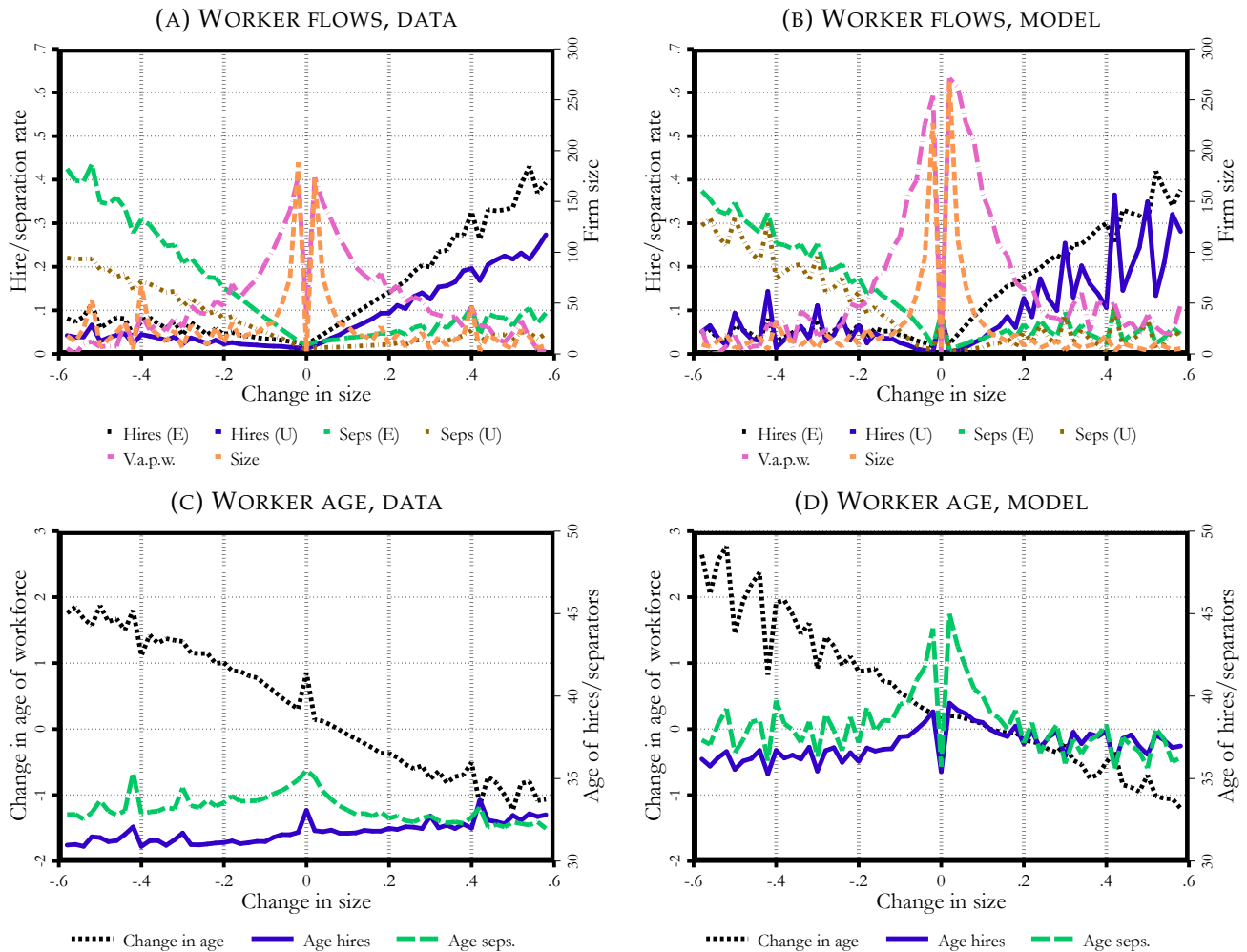


Note: Hires from unemployment: Share of hires from unemployment divided by total hires, where the former includes the unemployed, the NILF, and public sector workers (it is not possible to differentiate between the unemployed and NILF in the data; excluding public sector workers makes little difference to the patterns). Classification of worker flows is based on underlying monthly data to limit time aggregation bias. Empls/hires: Average age of employees/hires. Source: Simulated weekly version of the model for 25,000 firms, discarding an initial burn-in period and aggregating to monthly data, RAMS, LISA and FEK 2011–2015.

How do new firms grow? The left panel of Figure 7 highlights that young firms hire primarily from unemployment. The reason is that they are low productive and hence at the bottom of the job ladder

(recall Lemma 1). The model overstates somewhat the empirical patterns, which could be related to issues correctly measuring firm age in the data.³³ The right panel illustrates that older firms employ older workers in both the model and the data. Older firms also tend to hire older workers, although the gradient is weaker. Both observations mirror patterns for the US (Ouimet and Zarutskie, 2014). The theory provides an account of this that is not based on particular assumptions regarding the production function. Older firms and mature workers are both higher up the job ladder. As a consequence, mature workers are less likely to accept a job offer from a young firm.

FIGURE 8. THE NATURE OF FIRM GROWTH



Note: Hires/Seps (E/U): Sum of hires/separators from/to other employers/unemployment in quarter by average quarterly employment as well as average log value added per worker (normalized) and average firm size. Classification of worker flows is based on underlying monthly data to limit time aggregation bias. Change in age: Within-firm change in average age of the workforce. Age hires/seps: Average age of hires/separators as a function of annual employment growth. Source: Simulated weekly version of the model for 25,000 firms, discarding an initial burn-in period and aggregating to monthly data, RAMS, LISA and FEK 2011–2015.

³³Employer IDs change for reasons other than true firm births in the data, including M&A activity. While I try to clean for such events using worker flows, this approach is unlikely to fully correct for this (see Appendix A).

The top panels of Figure 8 plot worker flows by origin and destination as well as labor productivity by quarterly firm growth.³⁴ Firms primarily grow and shrink through poaching. Surprisingly, fast-growing firms are not at the top of the job ladder. Instead, high value added firms are already large and grow slow. The bottom panels plot the within-firm change in the average age of the firm’s workforce as a function of firm growth, illustrating that the workforces of expanding firms become significantly younger. This is not because they hire disproportionately young workers. Rather, hires are always young and expanding firms hire more. In fact, rapidly expanding firms hire somewhat *older* workers than rapidly contracting firms. The reason is that they are more productive and hence higher up the job ladder. I conclude that misallocated (typically young) workers play a key role in facilitating firm growth.

How creative-destructive is growth? Creative-destruction by entrants accounts for 54 percent of growth in the estimated model, with the remainder due to incumbent own innovation, as summarized by Table 4. I find such an important role for creative-destructive entry and exit despite the fact that entrants are much less productive than incumbents. The reason is that entrants are still more productive than the exiting firms they replace, and entry and exit rates are high.

TABLE 4. GROWTH DECOMPOSITION

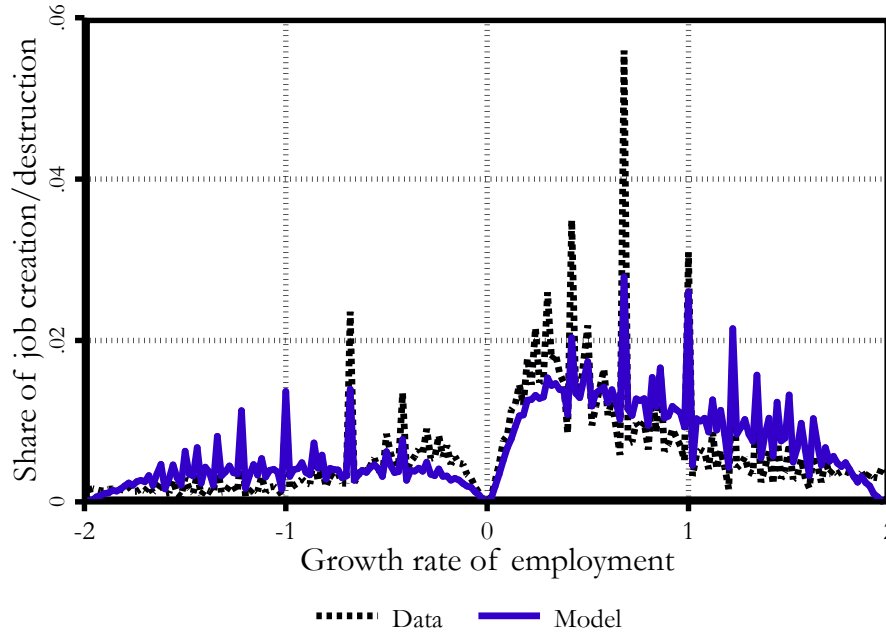
Entry/exit	Incumbent
1.07%	0.93%

Note: Entry/exit: Annual growth rate due to creative-destructive entry and exit, m . Incumbent: Annual growth rate due to incumbent own innovation, μ . *Source:* Model.

Nevertheless, the conclusion that creative-destruction by entrants is a key component of growth contrasts with recent work by Garcia-Macia et al. (2016), who argue that the distribution of employment growth rates suggests that this is a relatively minor source of growth. In particular, they note that in canonical models such as Klette and Kortum (2004), such growth predicts a lot of mass in the tails of the employment growth distribution. In contrast, the empirical distribution of job creation and destruction—replicated on Swedish data in Figure 9—displays little of this pattern (the Swedish patterns closely resemble those in the US). Nevertheless, the model closely matches the empirical patterns. The reason is that labor market frictions result in employment changes taking a less extreme form than what is predicted by these models. I conclude that despite the fact that entrants are low productive and the distribution of employment growth rates does not have much mass in the tails, creative-destruction through entry and exit plays a critical role in growth. Hence, a large share of growth is misallocative.

³⁴Where worker flows are classified based on monthly data such that a hire is poached if she was employed in the previous month by a different employer. This limits time aggregation bias relative to using quarterly data.

FIGURE 9. THE DISTRIBUTION OF 5-YEAR JOB CREATION/DESTRUCTION, MODEL VS DATA



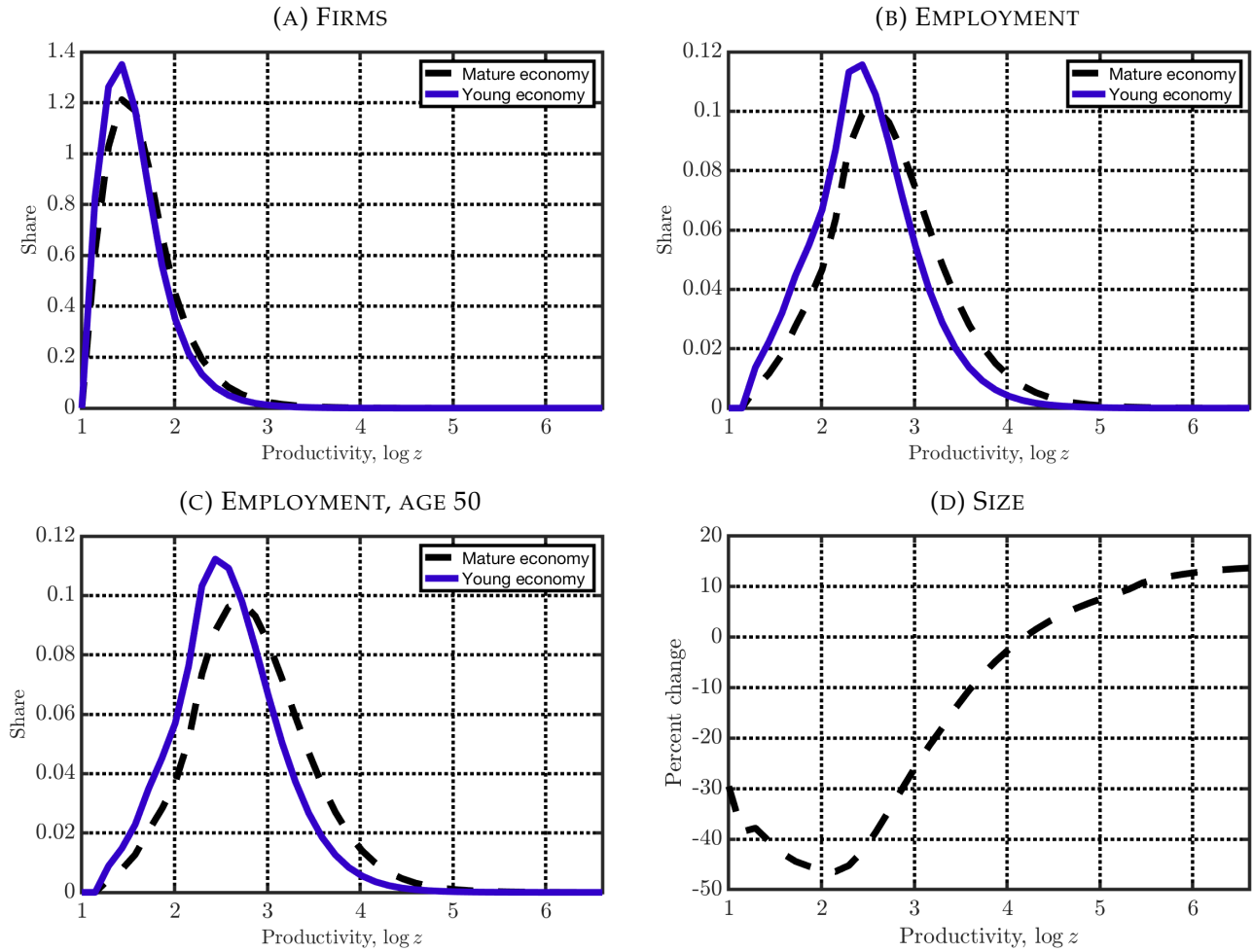
Note: Employment growth is the five-year difference in firm size divided by average firm size in the two years. Y-axis plots the share of total five-year job creation/destruction accounted for by firms with a particular five-year employment growth. Entrants and exiting firms are excluded following Garcia-Macia et al. (2016). *Source:* Simulated weekly version of the model for 25,000 firms, discarding an initial burn-in period and aggregating to monthly data, RAMS, LISA and FEK 2011–2015.

4 The impact of aging

My main application of the estimated framework is to understand and quantify the impact of aging, motivated by declines in firm creation and worker mobility in Sweden since 1986 that mirror trends in the US (Davis and Haltiwanger, 2014) (see Appendix A for details). To build intuition, I start with a comparative static, BGP exercise, and subsequently turn to a perfect foresight transition experiment. Both exercises isolate the impact of aging by holding fixed all parameters at their estimated values and changing only the growth rate of labor supply λ to match trends in the share of mature labor force participants in Sweden over the past 30 years.³⁵ Recall that this only impacts the economy through the change in the age composition, not the growth rate of labor supply.

³⁵I allow also the flow value of leisure to adjust such that individuals want to exit at the second grid point for productivity as in the estimated model. This ensures that Proposition 1 remains valid. I note two things with respect to this approach. First, it leads me to adjust upward somewhat the flow value of leisure in the younger economy, which dampens incentives for firms to create jobs and for entrepreneurs to enter relative to keeping b fixed. As such, this approach understates the equilibrium effect of aging, i.e. my estimates are a lower bound. Second, from a quantitative perspective, the required adjustment to b is small. Nevertheless, it turns out to be computationally critical to ensure convergence in the transition experiment, because without it the reservation threshold tends to move up and down marginally across iterations such that the algorithm fails to converge. As the model is estimated to fit 2011–2015, the BGP comparison runs "in reverse," i.e. I reduce the share of mature labor force participants by eight percentage points to match that in 1986.

FIGURE 10. BGP IMPACT OF AGING ON FIRM AND EMPLOYMENT DISTRIBUTIONS, MODEL



Note: Top left: Distribution of firms over log productivity. Top right: Distribution of workers over log productivity. Bottom left: Distribution of workers age 50 over log productivity. Bottom right: Percent change in firm size by log productivity. Source: Model.

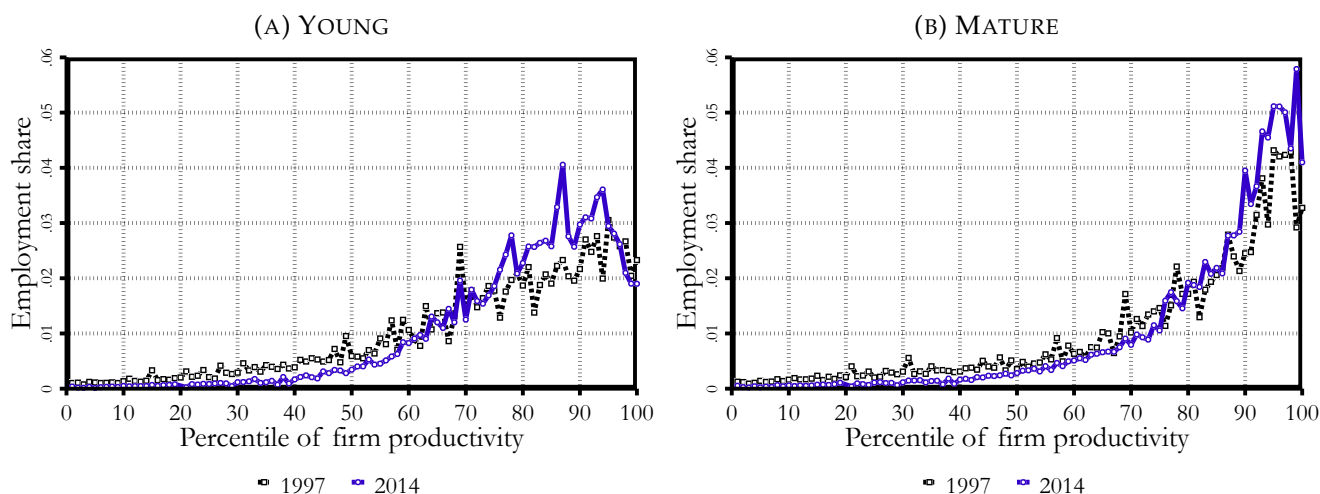
4.1 A BGP comparison

Figure 10 illustrates the BGP impact of aging on the distribution of firms and employment. The entry rate falls, which lowers the rate of obsolescence. This results in a more dispersed firm productivity distribution (top left). Employment gravitates up the job ladder (top right). This is partly mechanical as mature individuals are higher up the job ladder. But individuals also shift up the job ladder conditional on age, as the lower rate of obsolescence gives them more time to reallocate across incumbent firms before those firms are replaced by new entrants (bottom left). Finally, the bottom right panel illustrates that aging benefits in a relative sense productive firms, which grow larger. This insight is similar to Moscarini and Postel-Vinay (2018), who argue that firms at the bottom of the job ladder benefit in a relative sense from the easier hiring environment in recessions, because they on net lose workers through

poaching. In a similar spirit, low productive firms at the bottom of the ladder disproportionately hurt from the harder hiring environment in a mature economy. In addition, the lower rate of obsolescence isolates high-productive firms, which remain so for longer and hence grow larger.

Is there empirical support for the key prediction that the employment distribution has shifted up the job ladder as Sweden has aged, also conditional on individual age? Figure 11 plots the distribution of workers by age over the job ladder in 1997 and 2015 in the data. Unfortunately, the data only contain value added for a sample of firms prior to 1997, so I cannot reliably go back before that for these distributional exercises.³⁶ Note that the identity of firms and workers in a given bin may have changed. Since 1997, workers have shifted up the job ladder toward, in a relative sense, more productive firms, also conditional on age. This is exactly as predicted by the theory.

FIGURE 11. CHANGE IN EMPLOYMENT DISTRIBUTION OVER JOB LADDER, DATA



Note: Distribution of workers by age group over firms by percentile of employment-unweighted value added per worker in 1997 and 2015. Young: Age 18–24. Mature: Age 45–54. Age groups in between are convex combinations and omitted for brevity. Source: RAMS, LISA and FEK 1997–2015.

Table 5 summarizes the BGP effect of aging. Job reallocation is 13 percent lower in the mature economy relative to 17 percent lower in Sweden over this period (for comparison, the job reallocation rate fell by 25 percent in the US over the same period; in both the US and Sweden, controlling for sector exacerbates these declines). In both the model and the data, this is not driven by a lower volatility of firm level productivity shocks. Instead, firms adjust employment less in response to shocks, as highlighted by the decline in the elasticity of firm growth to productivity shocks (Decker et al., 2017, document a similar pattern in the US). The reason is that hiring is harder in a mature labor market. As a consequence, firms that receive positive productivity shocks grow slower, while firms that receive negative shocks do

³⁶Attempts to do so reveal an even more pronounced pattern, but I do not trust it due to non-random selection.

not contract as quickly, since their workers are not poached away as fast as before.³⁷ The lower rate of obsolescence leads a few incumbent firms to become in a relative sense very productive. Productivity dispersion across firms increases, as does the productivity gap between entrants and incumbents. That is, whereas other authors have interpreted the greater gap between "frontier" and other firms over this period in several countries as evidence of weaker knowledge spillovers (Akçigit and Ates, 2019b), here it arises endogenously in response to aging. The greater productivity dispersion is in turn reflected in greater earnings inequality. All of these patterns are consistent with trends in Sweden over this period.

TABLE 5. IMPACT OF AGING ON AGGREGATE OUTCOMES ACROSS BGP

	1986–1990		2011–2015		change		% change	
	Data	Model	Data	Model	Data	Model	Data	Model
<i>Panel A: Firm dynamics</i>								
Job reallocation	0.154	0.118	0.128	0.103	-0.026	-0.015	-17.1	-12.7
Elasticity of Δn to ΔTFP	0.145	2.712	0.102	2.522	-0.043	-0.190	-29.8	-7.0
St.d. of TFP	0.354	0.440	0.421	0.466	0.067	0.026		
Mean TFP of entrants	-0.070	-0.182	-0.199	-0.199	-0.129	-0.017		
<i>Panel B: Individual dynamics</i>								
Entry rate (monthly x 100)	0.054	0.044	0.041	0.032	-0.012	-0.008	-24.4	-27.3
JJ mobility rate	0.023	0.024	0.018	0.018	-0.005	-0.005	-23.4	-22.2
St.d. of earnings	0.456	0.632	0.604	0.715	0.148	0.083		
<i>Panel C: Aggregate outcomes</i>								
Annual growth rate	0.026	0.018	0.020	0.017	-0.006	-0.001		
Labor share	0.734	0.560	0.732	0.538	-0.002	-0.022		

Note: Impact of aging across BGP in response to a permanent change in the growth rate of labor supply from 0.0007 to 0.0013. Job reallocation: Quarterly sum of job creation and job destruction divided by average employment in the two quarters. Elasticity of change in employment to change in TFP: 5-year change in employment of incumbent firms relative to the 5-year change in TFP. Mean TFP of entrants: Employment-unweighted average productivity of new firms in the year relative to all firms. TFP in the data is from a regression of log value added on log employment and log assets, separately by sector-year. TFP in the model is total firm output divided by total firm employment. Entry rate: Share of employment in month t that is an operator-owner of a new firm in month $t + 1$, spliced back in time using the employment-unweighted firm entry rate due to data constraints. JJ mobility rate: Share of employment in month t that is working for a different employer in month $t + 1$. St.d. of earnings: St.d. of log monthly earnings from main employer in month. Labor share: total earnings divided by total output (value added in data). Source: Simulated weekly version of the model for 25,000 firms, discarding an initial burn-in period and aggregating to monthly data (firm dynamics and labor share); non-simulated model (individual dynamics and growth rate); RAMS, LISA, FEK and OECD 1986–2015.

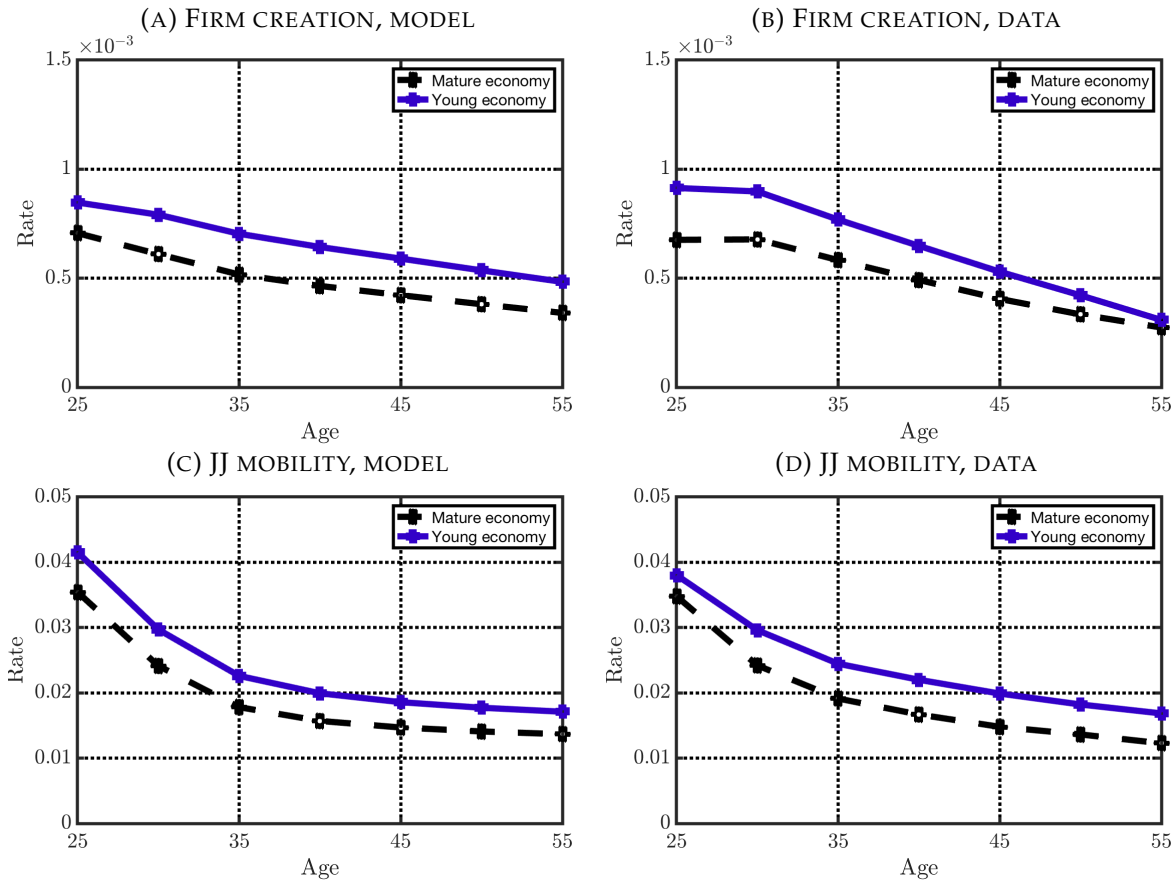
Finally, the labor share falls by a tiny amount, as a result of partly offsetting forces. On the one hand, conditional on productivity, firms pay more as workers have better outside options. On the other hand, workers are higher up the job ladder, and firms at the top pay less as a fraction of output. Moreover, the tail of the productivity distribution is fatter. This further reduces the labor share, because highly productive firms pay little relative to output due to less competition for workers at the top of the ladder. The latter is the mechanism emphasized by Gouin-Bonenfant (2020), but here the increase in productivity dispersion arises endogenously. Combined, these forces generate a tiny fall in the labor share.

³⁷While the magnitude of the elasticity is much larger in the model than the data, all productivity changes in the model are permanent and measured without error. Adjusting for such factors would presumably reduce the elasticity.

4.2 Composition and equilibrium effects

Figure 12 illustrates that age-conditional declines in the probability of starting a firm and switching employer play a key role behind the overall declines. These arise in equilibrium as hiring is harder in a mature economy, discouraging potential entrepreneurs from entering and firms from creating jobs. Moreover, the lower rate of obsolescence implies that workers are better matched to existing firms also conditional on age. Because of the lower rate of obsolescence, workers who have reached the top of the job ladder remain there for longer. Consequently, aging generates the largest relative declines late in life. In contrast, firm creation in the data display a proportional decline. One possible reason is that mature women have seen a relative increase in firm creation over time, countering the trend toward lower entry rates (see Appendix A). This may, in turn, be driven by forces outside the model, such as changes in social norms, but a more careful assessment of this is beyond the scope of this paper.

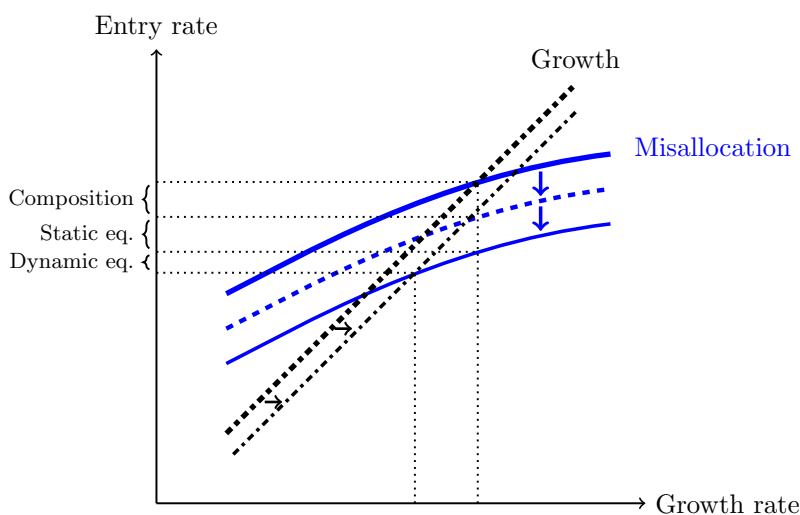
FIGURE 12. LIFE-CYCLE DYNAMICS ACROSS BGPs, MODEL VERSUS DATA



Note: BGP impact of a change in λ from 0.0007 to 0.0013. Firm creation: Share of employment in month t who is an owner-operator of a new firm in month $t + 1$, where a new firm is defined as a firm less than or equal to two years old. JJ mobility: Share of employment in month t who is employed at a different main employer in month $t + 1$. Young economy: 1986 (1993 for firm creation due to data constraints). Mature economy: 2015. Source: Model, RAMS, LISA and FEK 1986–2015.

While Figure 12 is useful to match up with the data, age is not a state according to the theory (experience and productivity are the states in the model). Hence, I now instead turn to a structural decomposition of the effects of aging. Figure 13 illustrates how aging impacts the equilibrium. Aging shifts the *misallocation* curve down through two channels. First, since mature individuals have had more time to climb the job ladder, a larger share of mature individuals reduces entry holding decision rules and equilibrium objects fixed through a *composition effect*. Second, potential entrants face a harder hiring environment, which discourages entry conditional on a potential entrant's state (holding fixed the rate of obsolescence). I refer to this as a *static equilibrium effect*. Aging also impacts the *growth* curve. Individuals employed by more productive employers are better able to innovate and mature individuals are higher up the job ladder. Consequently, a given entry rate generates a higher growth rate in a mature economy.³⁸ That is, aging shifts the *growth* curve to the right. The outward shift in the *growth* curve may in fact outweigh the downward shift in the *misallocation* curve, such that aging increases entry and growth. I refer to the incremental effect of aging as the rate of obsolescence adjusts as a *dynamic equilibrium effect*.

FIGURE 13. GROWTH AND LABOR MISALLOCATION



Note: Growth: Growth rate resulting from a particular entry (21). Misallocation: Entry rate given by individual's optimizing behavior given a growth rate. Composition effect: Impact of changing growth rate of labor supply λ holding all decision rules and equilibrium outcomes fixed. Static equilibrium effect: Impact of changing growth rate of labor supply letting all decision rules and equilibrium outcomes but the growth rate adjust. Dynamic equilibrium effect: Impact of changing growth rate of labor supply letting all decision rules and equilibrium outcomes including the growth rate adjust. Source: Model.

Table 6 shows that equilibrium effects account for a majority of the predicted decline in entry. Dynamic effects are the most important, mirroring findings in, for instance, the trade literature that accounting for growth effects may lead to substantially larger effects of opening up to trade (Sampson, 2016).

³⁸This is partly countered by the fact that experience reduces individuals' ability to innovate. Quantitatively, however, the specific spillovers from the prior employer are more important.

The decomposition highlights five factors that lead me to infer large effects of aging. First, the *misallocation* curve is positive and relatively steep. The reason is that a large share of growth is misallocative, misallocated individuals are estimated to be significantly more likely to start a firm and young firms hire primarily misallocated workers. Consequently, by increasing labor misallocation, a higher rate of obsolescence encourages firm creation. Second, the *growth* curve is relatively flat. The reason is modest specific knowledge spillovers. A higher entry rate increases growth and labor misallocation. If being misallocated, in turn, significantly reduces the ability to come up with good ideas, a given increase in the entry rate would generate a smaller increase in growth, i.e. the *growth* curve would be steep.

Third, mature individuals are significantly less likely to start a firm, such that the downward shift in the *misallocation* curve due to the *composition effect* is relatively large. Fourth, young firms depend heavily on young, poorly matched workers to grow, such that fewer young workers significantly disincentivize firm creation. That is, the downward shift in the *misallocation* curve due to the *static equilibrium effect* is relatively large. Fifth, the outward shift in the *growth* curve is modest, because specific knowledge spillovers are estimated to be weak.

TABLE 6. DECOMPOSING THE EFFECTS OF AGING ON FIRM CREATION

Composition	Static equilibrium	Dynamic equilibrium
23.8	9.5	66.7

Note: Composition effect: BGP impact of changing the growth rate of labor supply, λ , from 0.0007 to 0.0013 holding all decision rules and equilibrium objects fixed. Static equilibrium effect: Impact of changing the growth rate of labor supply letting decision rules and all equilibrium outcomes apart from the growth rate adjust. Dynamic equilibrium effect: Impact of changing the growth rate of labor supply letting all decision rules and equilibrium outcomes including the growth rate adjust. *Source:* Model.

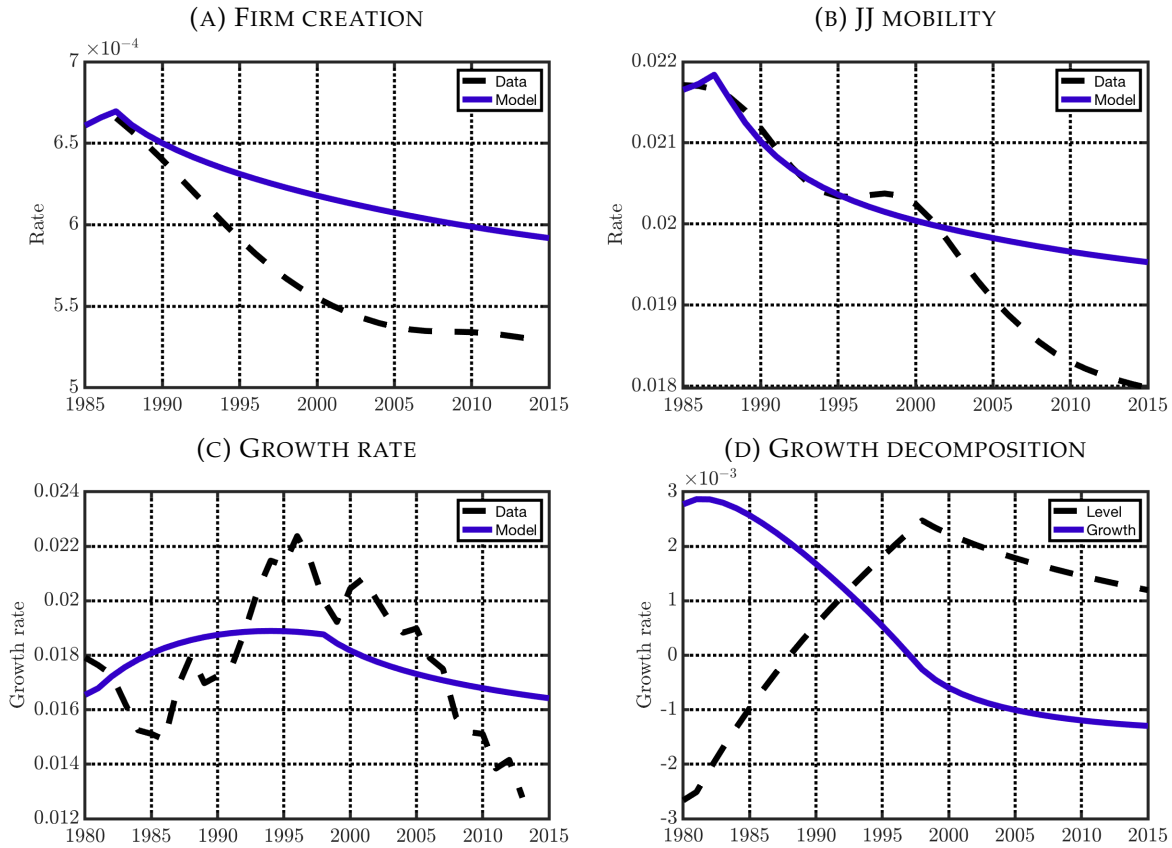
4.3 The baby boom

I finally turn to analyzing the transitory impact of aging. I assume that starting on the estimated BGP in 1945, individuals realize that starting in 1960, the growth rate of labor supply will evolve so as to match the share of mature individuals in the labor force over the 1960–2050 period (using OECD forecasts in the later years). After that, the growth rate of labor supply returns to steady-state and the share of mature individuals converges back to its original value (in line with OECD forecasts). Solving a random search model like the current one out of steady-state is notoriously hard, but I make progress by working in continuous time.³⁹ I use a large shooting algorithm that guesses a path for the distribution of individuals over employment status, human capital and productivity at discretized points, as well as a path for the growth rate. I solve the value functions backward in time under the guessed path for the distributions,

³⁹I am particularly grateful to Adrien Bilal for thoroughly teaching me the advantages of working in continuous time.

update the path for the distributions forward, etc, until convergence.

FIGURE 14. IMPACT OF BABY BOOM ON GROWTH AND LABOR MARKET DYNAMICS



Note: Growth rate: Annual growth rate of real output per hour (data)/output per employed worker (model). Growth: Growth of exit threshold (normalized to zero in 2000 to help readability). Level: Change in output due to shifts in the distribution of employment relative to the exit threshold as well as shifts in the stock of human capital relative to the BGP. 20-year moving average in both model and data. Source: Model, RAMS, LISA, FEK and OECD.

The top panels of Figure 14 show that aging results in a decline in firm creation and worker mobility of over 10 percent since 1986, accounting for over half of the empirical declines.⁴⁰ The bottom panels plot the transitory impact of aging on growth. Aging accounts for a non-trivial share of the hump-shaped pattern of productivity growth over this period, with high growth in the 1990s and early 2000s (a similar pattern is evident in the US). This non-monotonic behavior of the growth rate is due to competing level and growth effects. The growth effect—defined as the growth rate of the exit threshold, $\underline{Z}(t)$ —monotonically declines as the labor force matures. The level effect—the change in output resulting from distributional shifts relative to the exit threshold as well as differences in human capital relative to the

⁴⁰In contrast to the relatively large effects on firm and worker dynamics, the transitory effects on the productivity distribution are much slower to materialize, driven by slow convergence of the right tail. Aging accounts for only 10 percent of the increase in productivity dispersion and 25 percent of the larger productivity gap between entrants and incumbents over this period. Recent suggestions in the literature for how to generate faster convergence may lead to greater effects of aging over the transition path, but an assessment of this is beyond the scope of this paper.

BGP level—rises up to 2000 as the baby boomers climb the job ladder and accumulate human capital.

5 Conclusion

This paper develops a microfounded model of growth and labor market dynamics. Creative-destructive growth causes labor misallocation. Labor misallocation, in turn, increases firm creation and growth by lowering the opportunity cost of entry and the cost of hiring labor. Unique Swedish matched employer-employee data indicate significant scope for reallocation across incumbent firms to reduce the probability of starting a firm and switching employer, while only modestly improving the ability to come up with good ideas. Moreover, a large share of growth is creative-destructive driven by entry and exit. This implies significant amplification from the interaction between growth and labor market dynamics. Applied to quantify the labor market consequences of aging, aging accounts for over half of substantial secular declines in firm creation and worker mobility in Sweden since 1986, primarily through equilibrium forces. It also leads to greater earnings dispersion, a larger productivity gap between entrant and incumbent firms, and a weaker firm employment response to shocks. Nevertheless, despite a secular decline in the rate of innovation, growth is high through the 1990s as the baby boomers find good jobs.

Many questions related to the interaction between growth and the labor market remain unanswered, including the design of optimal policy when innovation causes some workers to become misallocated. A proper answer to this question would require risk aversion and imperfect financial markets. It would be interesting to extend the current framework to address such issues. It would also be interesting to apply the framework to study the labor market impact of policies such as the one-child policy in China, as well as other policies that discourage or encourage fertility.

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A Data

This section discusses the data in more detail, provides further empirical results and overviews Swedish labor market trends over the past 30 years.

A.1 Data

My analysis exploits three administrative data sets. The *Longitudinell integrationsdatabas för sjukförsäkrings- och arbetsmarknadsstudier* (LISA) contains demographic information on the entire Swedish population age 16 years and older, including gender, year of birth, highest educational attainment, and year of highest degree. The *Registerbaserad arbetsmarknadsstatistik* (RAMS) contains all employment spells of all individuals during a year, including start and end month of the spell, average monthly earnings, an employer ID, and type of employment (employee, unincorporated self-employed and incorporated self-employed). Earnings are gross. The *Företagens ekonomi* (FEK) contains income and balance sheet information on all firms in Sweden. Since 1997, these data cover, with a few exceptions, the universe of firms. I obtain data on sales, costs of intermediaries and assets, from which I construct value added per worker and a revenue-based measure of TFP. Data also exist at the establishment level, but are not as exhaustive as those on firms. For that reason, the income and balance sheet measures refer to firms throughout this paper.

A key issue in many administrative data is that establishment identifiers change between years for reasons such as ownership changes, changes to incorporation status, etc. I construct consistent establishment and firm IDs over time based on worker flows. In particular, if more than 50 percent of workers at establishment A in year t constitute more than 50 percent of workers at establishment B in year $t + 1$, then I call these two establishments the same establishment.⁴¹ I create consistent establishment IDs based on such flows. I subsequently aggregate up establishments to firms using a consistent within-year link between establishments and firms.

Apart from the rich set of variables on individuals and firms in these data, a unique feature is the ability to identify owner-workers of firms and link them to their prior work experiences, the performance of their new firms, and post-exit outcomes. Such links are possible for all unincorporated businesses, and for incorporated businesses in which four or fewer individuals jointly control at least half of the voting shares, where any immediate family members count as one unit. Because immediate family members count as one and the definition of "immediate" is broad, including grand parents, parents, spouses,

⁴¹This approach only makes sense for establishments with a sufficient number of workers. I impose a minimum five number of workers.

children, children's spouses, siblings, siblings' spouses, and siblings' children, the actual number of owners of incorporated businesses with identifiable founders is large.⁴² Hence, I am not able to link a firm to a founder if it is incorporated and it has more than four, non-related founders in the first year of operation.

A.2 Sample and variable construction

I primarily focus on people aged 25–60. A few observations are excluded because they lack valid education. I standardize degree classifications to be consistent over time into seven education groups and sector classifications to be consistent over time into nine one-digit sectors. The main analysis includes all private sector firms, regardless of size. To improve comparability, I compute most firm dynamics measures at the level of the establishment to be consistent with the US Business Dynamics Statistics. Results are similar at the level of firms, though.

Income and balance sheet data are only available for a sample of larger firms back to 1985, with expanding coverage of smaller firms over time (coverage is essentially complete since 1997). Productivity measures are at the level of the firm and available at an annual frequency. I convert them to real values using the national CPI. I compute value added per worker as total sales minus the total cost of intermediaries divided by total employment, and residualize it by regressing it on sector-year dummies. I construct TFP by regressing log value added on log assets and log employment separately by sector and year, and compute TFP as the residual from this regression. Note that these are all measures of revenue productivity.

I construct three data sets based on these data: a monthly-individual level data set, an annual individual level data set, and a quarterly firm-level data set. The first uses the employment-spell level data to define an employment spell as active in the month based on the reported start and end months. Because start and end dates are missing for self-employment spells, such spells are assumed to last throughout the year. In case an individual has multiple active spells in a month, I focus on the employment-spell earnings the highest income. An individual makes an employment to non-employment (EN) transition in month m if she is employed in month m but non-employed in month $m + 1$. She makes a non-employment to employment (NE) transition if she is non-employed in month m and employed in month $m + 1$. Finally, she makes a JJ transition if she is employed in month m and employed but at a different main establishment in month $m + 1$.

The second data set is an annual individual-level data set that contains employment status measured

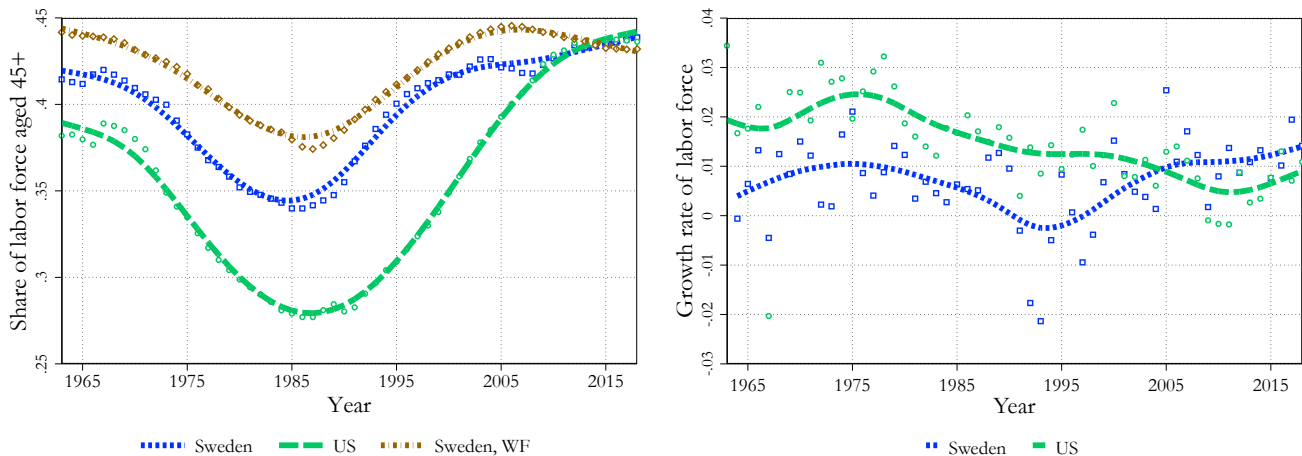
⁴²Children include adopted children and spouses include partnerships and people living together, that have children together or have previously been married.

in November each year (based on highest income in that month), which I use to define entrepreneurship entry. I cannot use the monthly individual level data set for this, because it does not allow me to identify owner-operators of incorporated firms prior to 2011. Since 2011, however, I can construct a monthly level indicator for entrepreneurship entry.

A.3 Labor Market Trends

Ageing and labor supply growth. Figure 15 plots the share of mature workers in the Swedish labor force over the past 40 years as well as the growth rate of labor supply. Sweden has aged significantly over this period, although much less than the US. This is not primarily driven by differential trends in labor force participation rates by age.

FIGURE 15. AGING AND LABOR SUPPLY GROWTH



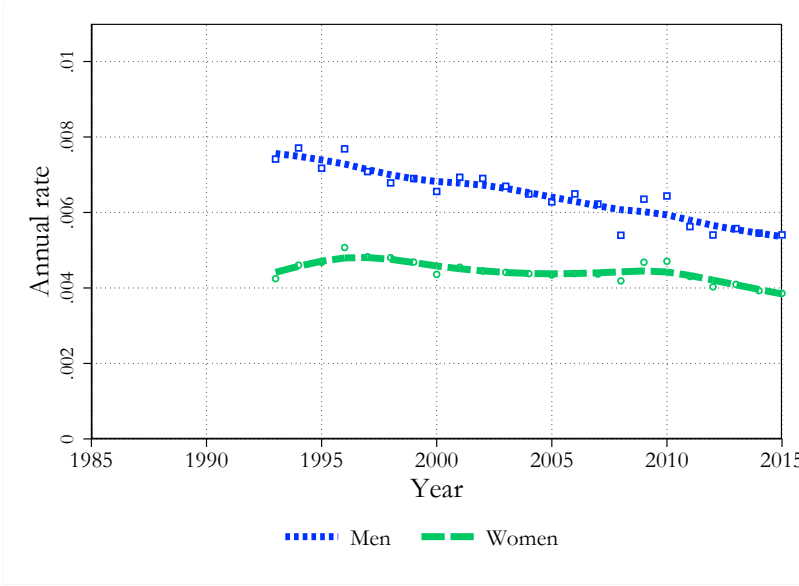
Note: Annual growth rate of labor force. Source: OECD.

Entry dynamics. Figure 16 plots the probability of starting a firm by gender. In the early years of the sample, men were almost twice as likely to start a firm. While the gender gap in the probability of starting a firm has subsequently fallen, it remains non-trivial. As the Swedish private sector labor force has become (modestly) more female over time, the change in composition has exerted a negative impact on firm creation. The predicted decline, however, is rather small. In other words, both men and women have seen a declines in the probability of starting a firm, although the magnitude of the fall is more pronounced for men.

Figure 17 splits men and women into further subgroups based on age. In a relative sense, the probability of starting a firm differs the most between men and women among mature workers. Moreover, while young men and women have seen a similar decline in the probability of starting a firm over time,

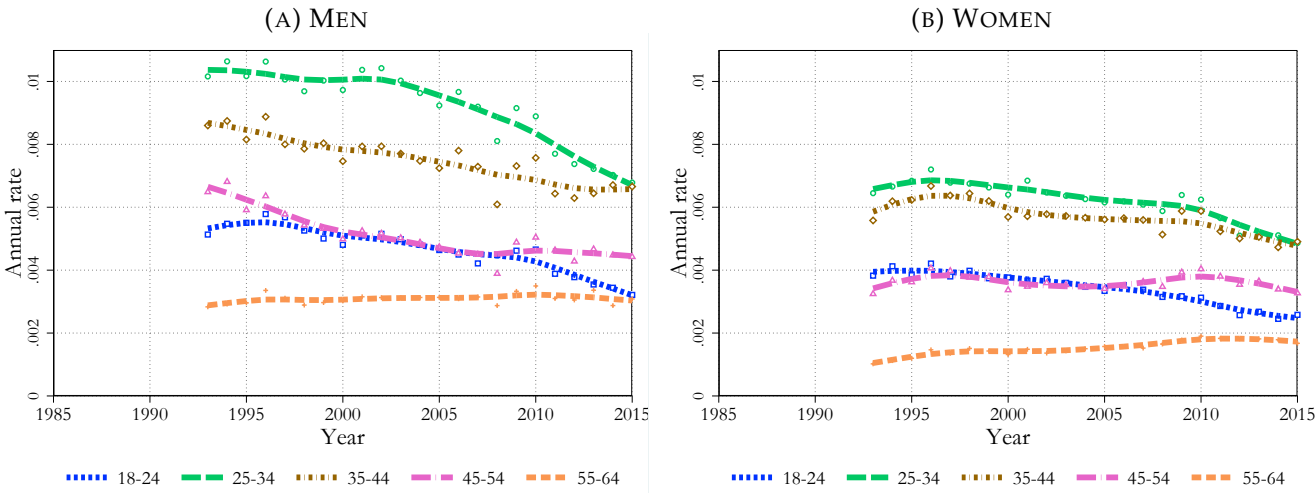
mature women have seen an increase in their probability of starting a firm. One interpretation is that the data contain a strong cohort effect for women, with younger cohorts of women being much more like men in terms of their probability of starting a firm.

FIGURE 16. ANNUAL PROBABILITY OF STARTING A FIRM, BY GENDER



Note: Share of employment in year t who is an owner-operator of a new firm in year $t + 1$. New firms are firm less than or equal to two years old, and include incorporated and unincorporated businesses. Source: RAMS, LOUISE and FEK 1993–2014.

FIGURE 17. ANNUAL PROBABILITY OF STARTING A FIRM, BY GENDER AND AGE

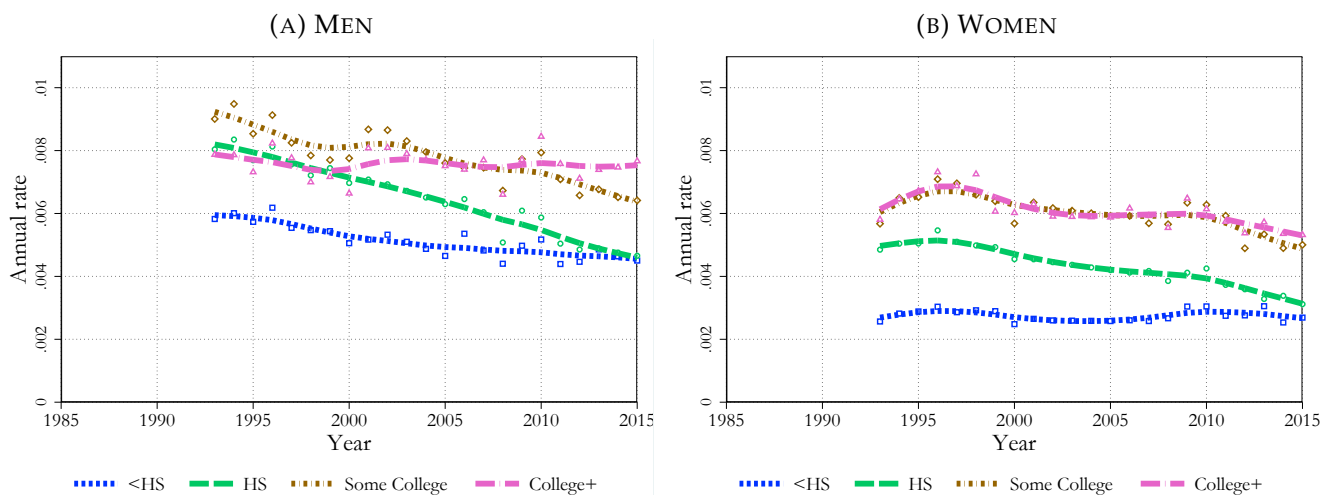


Note: Share of employment in year t who is an owner-operator of a new firm in year $t + 1$. New firms are firm less than or equal to two years old, and include incorporated and unincorporated businesses. Source: RAMS, LOUISE and FEK 1993–2014.

Figure 18 splits men and women into subgroups based on education. In contrast to findings in the US (Salgado, 2017; Jiang and Sohail, 2019), the probability of starting a firm has not fallen disproportionately

among high-skilled. Moreover, the entry rates they document are significantly higher than what I find in my administrative data and entry rates in US firm-level administrative data sets. To see that, denote by N the population size, by E the employment rate, by F the number of firms in the economy, by x the individual-level entry rate and by X the firm-level entry rate, then $x * N = X * F \iff X = x * 1/(E/N) * E/F$. Average firm size is around 20 in the US (12 in Sweden) and the employment to population rate is around 0.8. Their roughly 3 percent annual entry rate at the individual level would hence imply an annual firm entry rate of roughly 75 percent, which is an order of magnitude larger than the entry rate in for instance the Business Statistics Dynamics. While the latter covers only firms that hire at least one worker, the discrepancy is so large that it raises questions about what the individual survey level data sets actually capture. In contrast, in my administrative level data sets the individual level entry rate is much lower, and by construction consistent with the firm level entry rate.

FIGURE 18. ANNUAL PROBABILITY OF STARTING A FIRM, BY GENDER AND EDUCATION



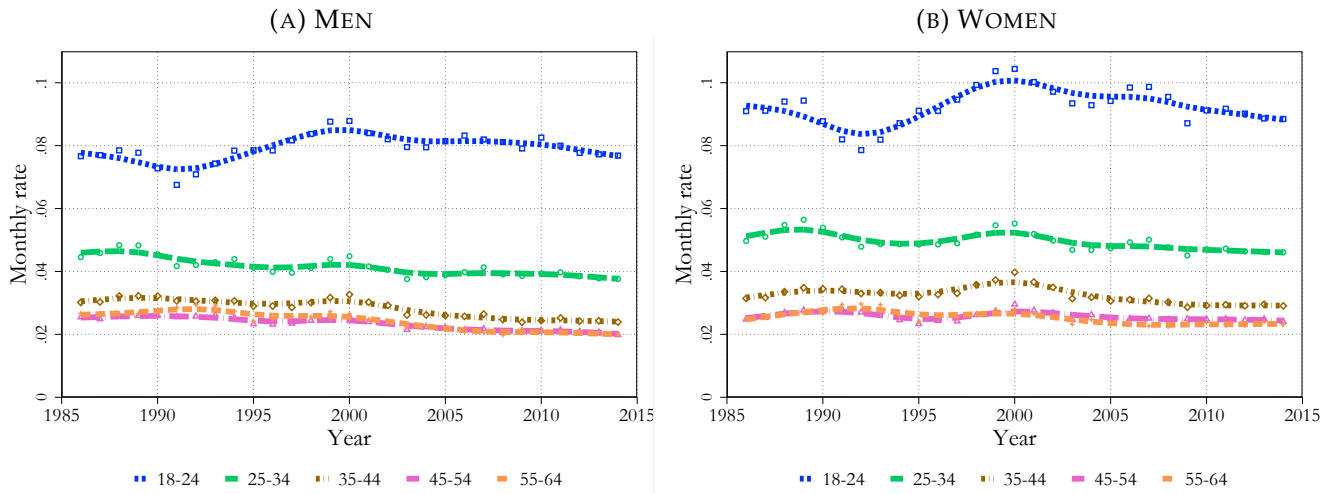
Note: Share of employment in year t who is an owner-operator of a new firm in year $t + 1$. New firms are firm less than or equal to two years old, and include incorporated and unincorporated businesses. Source: RAMS, LOUISE and FEK 1993–2014.

Worker dynamics. Figure 19 plots worker reallocation by age groups over time. With the exception of the youngest age group, there is a gradual decline in worker reallocation over time.

Figure 20 plots worker reallocation by education groups over time. Three observations are in place. First, relative to the large differences across age groups above and sectors below, differences across education groups in worker mobility are relatively minor. Second, Sweden has seen increases in educational attainment over this period, but because the relationship between education level and worker mobility is non-monotone, such shifts combine to actually predict a modest increase in worker mobility. The predicted increase is small, though. Third, with the exception of the least educated group—those who did

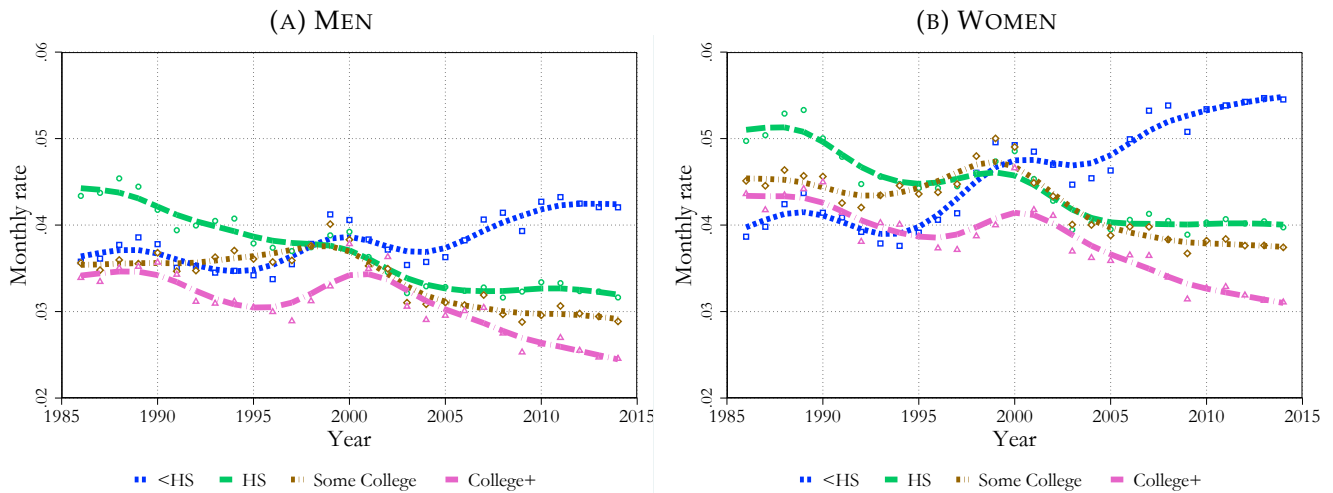
not complete high-school—all education groups have seen declines in worker mobility over this period.

FIGURE 19. MONTHLY PROBABILITY OF SWITCHING EMPLOYER, BY GENDER AND AGE



Note: Share of employment in month t who is not employed with the same employer in month $t + 1$. Source: RAMS, LOUISE and FEK 1993–2014.

FIGURE 20. MONTHLY PROBABILITY OF SWITCHING EMPLOYER, BY GENDER AND EDUCATION

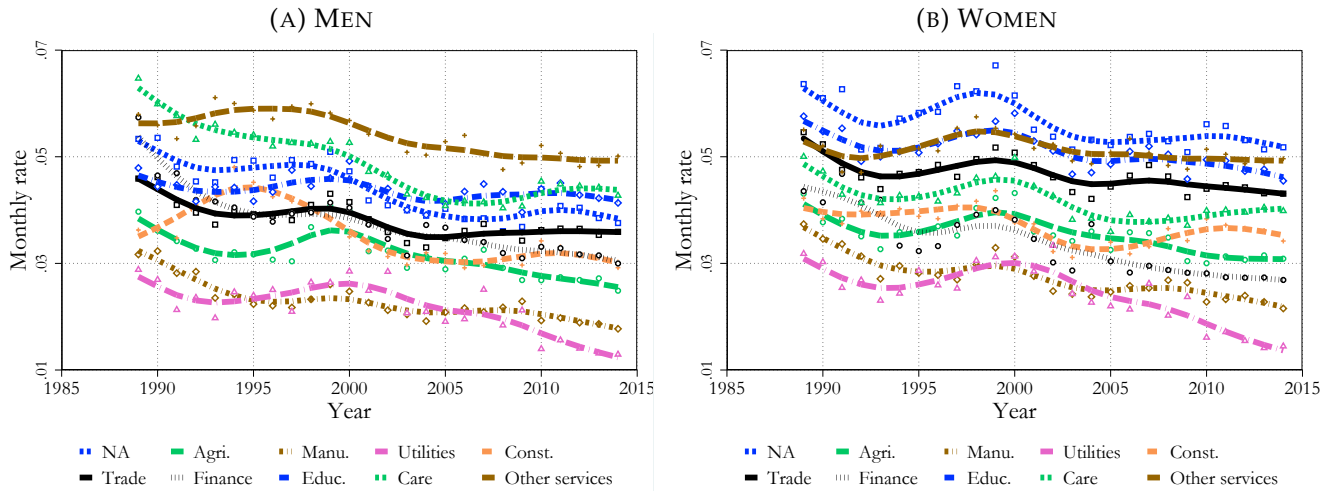


Note: Share of employment in month t who is not employed with the same employer in month $t + 1$. Source: RAMS, LOUISE and FEK 1993–2014.

Figure 21 plots worker reallocation by one-digit sector over time. Four observations are in place. First, there are significant differences across sectors in terms of the level of worker mobility. Second, differences in what sectors men and women work in account for some of the overall differences in worker mobility, but differences remain also within sectors. Third, all major sectors have seen declines in worker mobility over this period. The fact that a large share of the trends are shared across sectors argues against, for instance, trade or technological advancements as the primary factors behind the aggregate declines,

as one may have expected those to have very differential effect across sectors. Fourth, because Swedish employment has shifted out of manufacturing and into services over this period, accounting for sectoral shifts increases the declines in worker mobility.

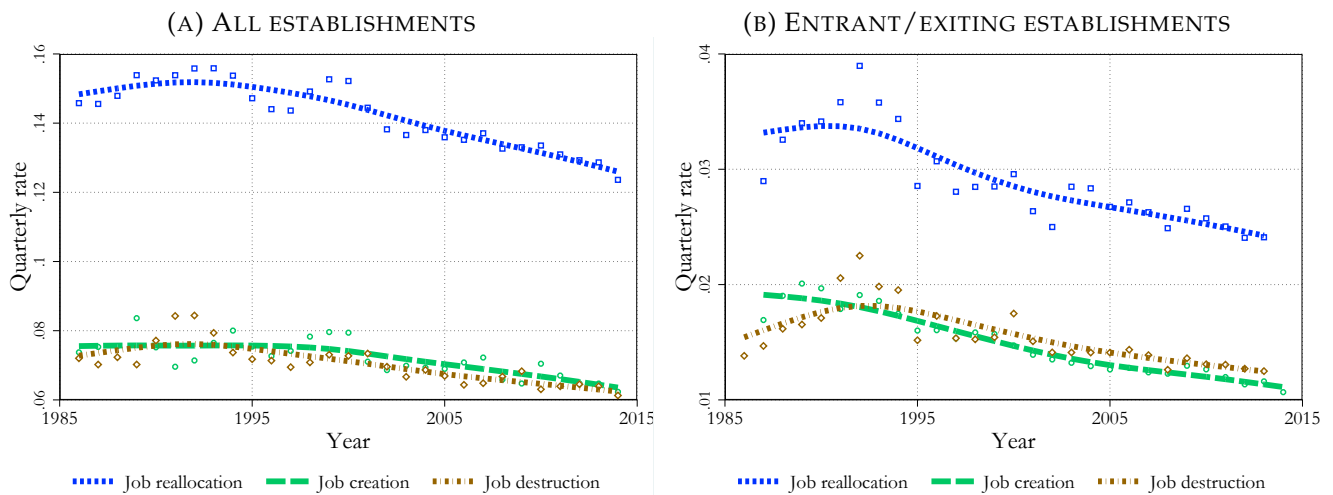
FIGURE 21. MONTHLY PROBABILITY OF SWITCHING EMPLOYER, BY GENDER AND ONE-DIGIT SECTOR



Note: Share of employment in month t who is not employed with the same employer in month $t + 1$. Source: RAMS, LOUISE and FEK 1993–2014.

Firm dynamics. Figure 22 plots overall job reallocation, as well as job creation, job destruction, job creation by entrants and job destruction by exiting firms.

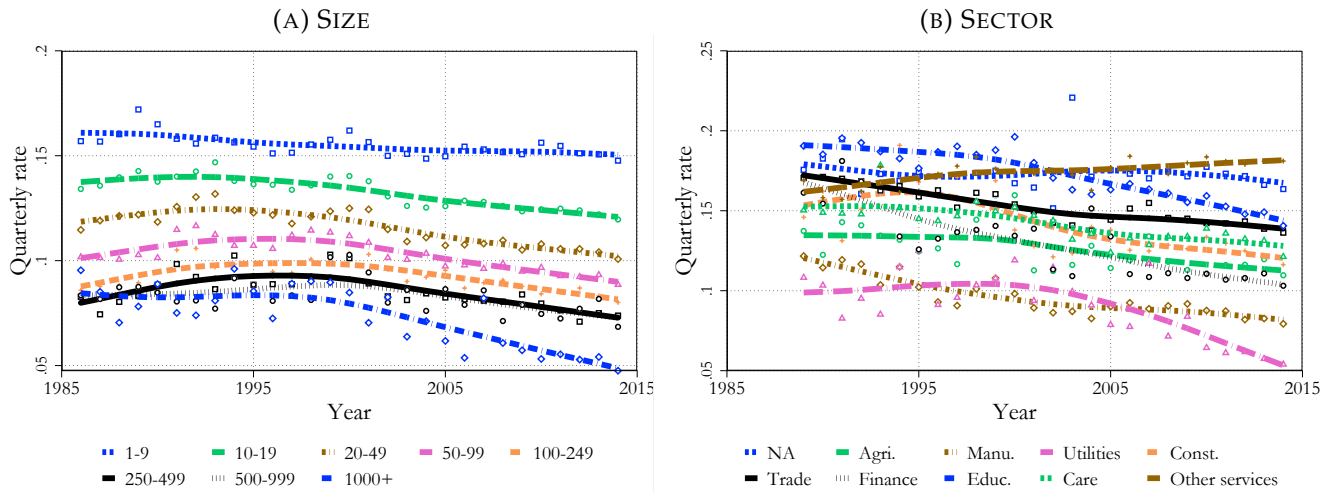
FIGURE 22. QUARTERLY JOB REALLOCATION RATE



Note: Sum of quarterly job creation and destruction divided by average quarterly employment. Job creation (destruction) is the change in employment among firms that grow (shrink) between quarter t and $t + 1$. Average quarterly employment is average employment between quarters t and $t + 1$. Source: RAMS, LOUISE and FEK 1993–2014.

Figure 23 plots the quarterly job reallocation rate by firm size groups as well as one-digit sector. The aggregate declines in job reallocation are not accounted for by reallocation of employment across firm size groups or sectors. In fact, accounting for changes in sectoral composition further increases the aggregate declines in job reallocation.

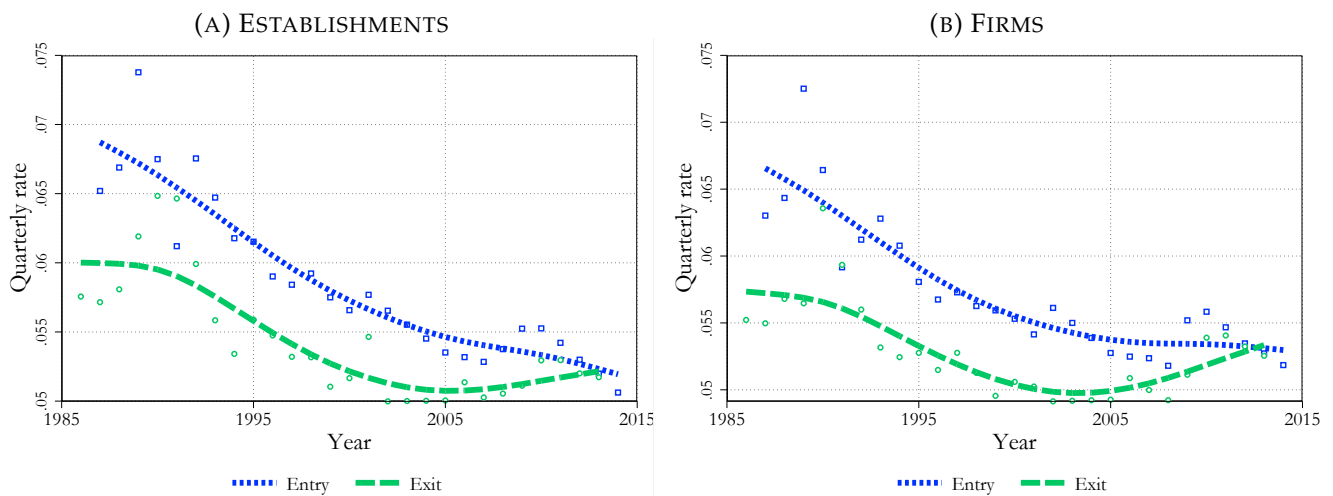
FIGURE 23. QUARTERLY JOB REALLOCATION RATE BY SIZE AND SECTOR



Note: Sum of quarterly job creation and destruction divided by average quarterly employment. Job creation (destruction) is the change in employment among firms that grow (shrink) between quarter t and $t + 1$. Average quarterly employment is average employment between quarters t and $t + 1$. Source: RAMS, LOUISE and FEK 1993–2014.

Figure 24 plots entry and exit rates of establishments and firms, highlighting a secular decline in both, although it is more pronounced for entry.

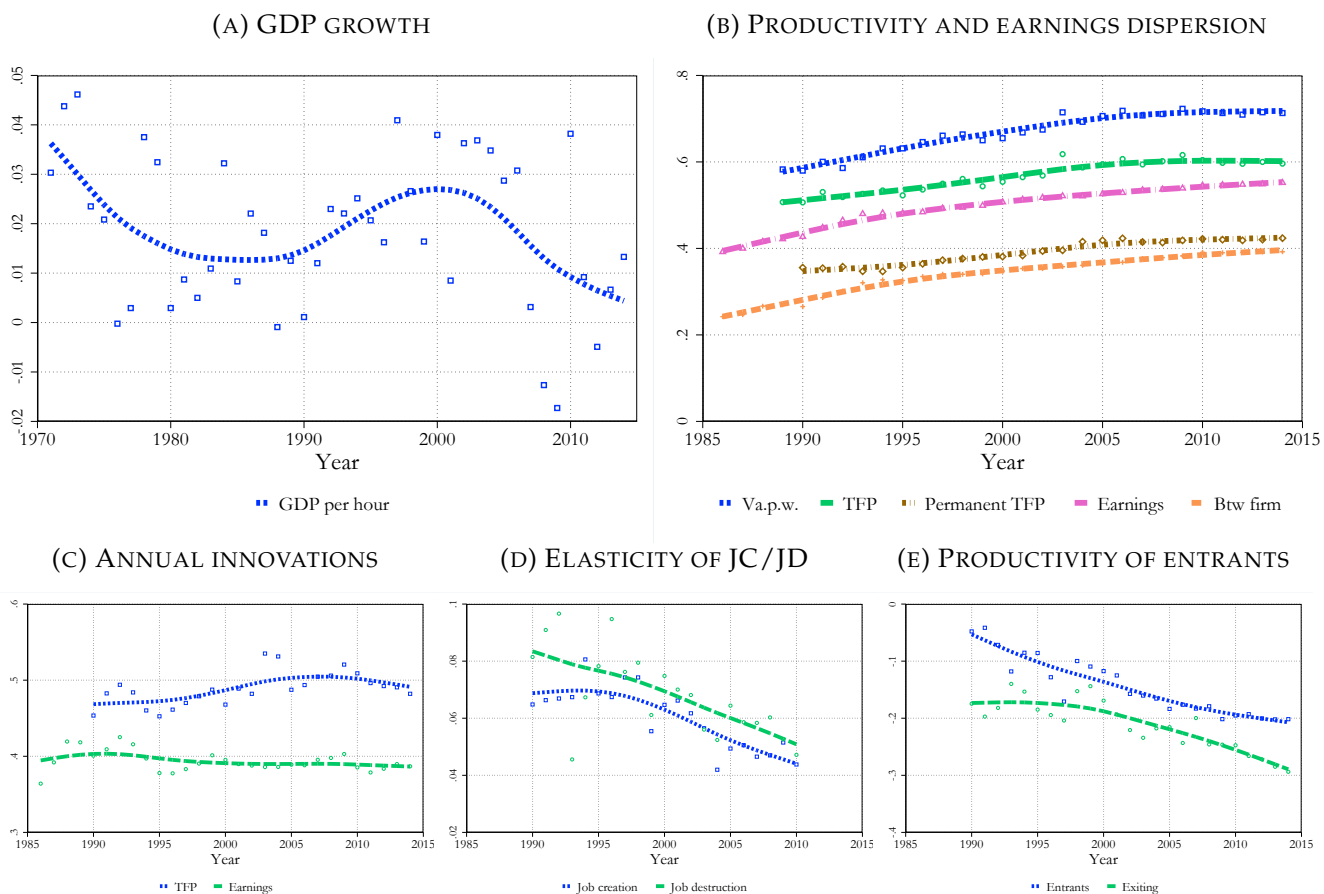
FIGURE 24. ENTRY AND EXIT RATES



Note: Exit rate: Share of establishments (firms) with positive employment in quarter t that have zero employment in quarter $t + 1$; Entry rate: Share of establishments (firms) with positive employment in quarter $t + 1$ that have zero employment in quarter t . Source: RAMS, LOUISE and FEK 1993–2014.

Other outcomes. Figure 25 plots several productivity outcomes over this period. The top left panel illustrates that the aggregate growth rate was low in the late 1970s and early 1980s, was high in the 1990s and has subsequently fallen. The top right panel plots various measures of productivity and earnings dispersion over this period, highlighting a trend toward greater dispersion across firms and workers. The bottom left panel graphs the volatility of annual innovations to productivity and earnings, indicating that the declines in employment adjustment is not driven by lower dispersion of shocks. The bottom middle panel plots the elasticity of 5-year job creation and job destruction to 5-year changes in firm level TFP (for visual ease, it plots minus the elasticity of job destruction), suggesting that the declines in employment adjustment are instead accounted for by firms responding less to shocks. The bottom right panel shows that entrants (and exiting firms) have fallen increasingly behind incumbent firms over this period in terms of productivity.

FIGURE 25. PRODUCTIVITY OUTCOMES



Note: Left: Hire and separation rates by origin and destination as a function of quarterly firm employment growth. Right: Within-firm change in average age of a firm's workforce by annual change in firm employment, $(n_{t+1} - n_t) / (\frac{1}{2}(n_{t+1} + n_t))$. Source: RAMS, LOUISE and FEK 2011–2015.

A.4 Link between prior employer and new firm

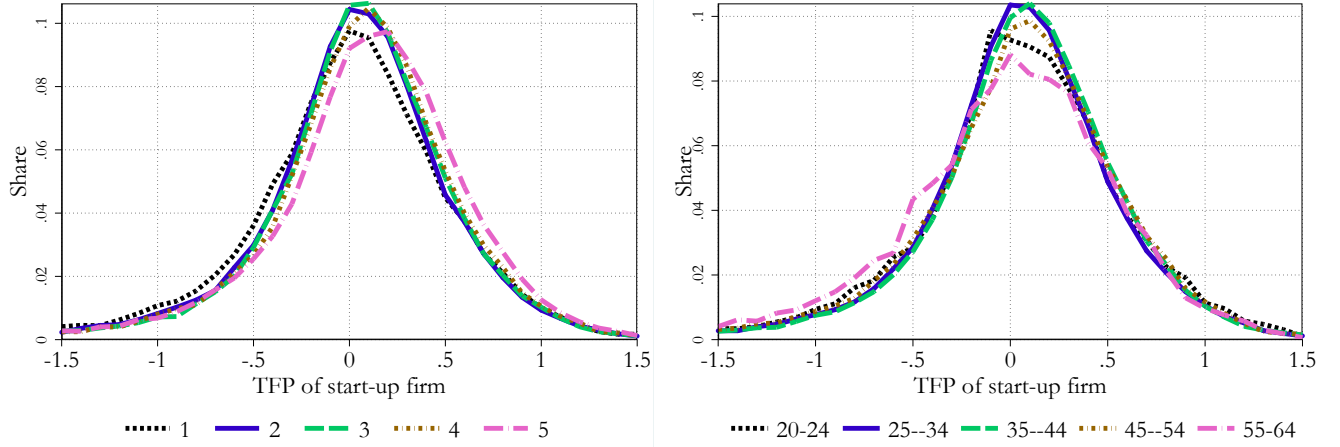
TABLE 7. CONDITIONAL CORRELATION BETWEEN PRODUCTIVITY OF PRIOR EMPLOYER AND NEW FIRM

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		Explanatory variable: TFP								
		Dependent variable: TFP					Dependent variable: Size			
		Age 2	Age 5	Age 10	All ages	$N \geq 10$	Age 2	Age 5	Age 10	All ages
Founder age	25	-0.019 (0.019)	-0.003 (0.022)	0.011 (0.038)	0.001 (0.012)	0.020 (0.016)	0.061** (0.012)	0.065** (0.017)	0.042 (0.030)	0.057** (0.012)
	35	0.007 (0.020)	0.016 (0.022)	0.009 (0.039)	0.019 (0.012)	0.033* (0.016)	0.046** (0.012)	0.033 (0.018)	-0.014 (0.031)	0.027* (0.013)
	45	-0.002 (0.021)	-0.002 (0.024)	-0.053 (0.042)	-0.004 (0.013)	0.008 (0.017)	-0.010 (0.014)	-0.070** (0.019)	-0.136** (0.033)	-0.047** (0.014)
	55	-0.082** (0.026)	-0.154** (0.034)	-0.195** (0.071)	-0.090** (0.017)	-0.069** (0.022)	-0.158** (0.015)	-0.216** (0.023)	0.195 (0.102)	-0.157** (0.017)
	1	-0.101** (0.018)	-0.074** (0.021)	-0.038 (0.037)	-0.088** (0.012)	-0.071** (0.017)	-0.029* (0.013)	-0.046** (0.018)	-0.023 (0.032)	-0.025 (0.013)
Prior firm TFP	2	-0.054** (0.018)	-0.040* (0.020)	-0.016 (0.039)	-0.044** (0.012)	-0.044* (0.017)	-0.009 (0.013)	-0.002 (0.018)	-0.001 (0.033)	-0.002 (0.014)
	3	-0.015 (0.017)	-0.041* (0.020)	0.040 (0.037)	-0.022* (0.011)	-0.031* (0.014)	-0.001 (0.013)	-0.012 (0.018)	-0.008 (0.033)	-0.002 (0.014)
	4	-0.036* (0.017)	-0.010 (0.019)	0.014 (0.035)	-0.019 (0.011)	-0.022 (0.013)	-0.005 (0.013)	0.018 (0.019)	0.067 (0.035)	0.019 (0.014)
	5	-0.014 (0.017)	-0.004 (0.019)	0.036 (0.038)	-0.009 (0.011)	-0.012 (0.013)	-0.001 (0.013)	0.013 (0.018)	-0.003 (0.033)	0.007 (0.014)
	7	-0.026 (0.017)	-0.013 (0.020)	0.054 (0.036)	-0.003 (0.011)	-0.005 (0.013)	-0.019 (0.013)	-0.025 (0.018)	-0.018 (0.032)	-0.017 (0.013)
	8	0.027 (0.017)	0.031 (0.019)	0.027 (0.036)	0.019 (0.011)	0.018 (0.013)	0.007 (0.013)	0.016 (0.018)	0.034 (0.033)	0.015 (0.014)
	9	0.038* (0.016)	0.014 (0.020)	0.075* (0.036)	0.036** (0.010)	0.035** (0.012)	0.010 (0.013)	0.023 (0.018)	0.058 (0.033)	0.025 (0.014)
	10	0.076** (0.017)	0.039* (0.019)	0.063 (0.036)	0.057** (0.011)	0.051** (0.013)	0.008 (0.013)	0.007 (0.018)	0.029 (0.032)	0.012 (0.013)
	N	91,568	63,216	20,984	712,971	439,052	226,656	143,253	52,130	1,752,357

Note: Firm-level TFP is the residual from a regression of value added on log number of employees and log assets, estimated separately by one-digit sector-year. "Prior firm TFP" refers to the founder's prior employer's decile in the (employment-unweighted) TFP distribution. "Age a" conditions on the start-up firm being age a ; "All ages" and " $N \geq 10$ " pool all observations with age controls. " $N \geq 10$ " conditions on the prior employer having at least 10 workers. All columns control for gender and education level of founder (7 groups); sector (10 groups), size and age of prior employer (linear in each); number of founders (linear); and year of inception, sector and location of start-up firm. All firms founded between 1997 and 2011 with an identified founder. Standard errors are clustered at the firm level. Source: RAMS, LOUISE and FEK 1996–2015.

Figure 26 plots the residual distribution of TFP of new firms by the productivity quintile of the founder's prior employer (left) and the founder's age at time of inception of the firm (right).

FIGURE 26. PRODUCTIVITY OF THE NEW FIRM



Note: Firm-level TFP is the residual from a regression of value added on log number of employees and log assets, estimated separately by one-digit sector-year. "Prior firm TFP" refers to the founder's prior employer's quintile in the (employment-unweighted) TFP distribution. Residual TFP of new firms controlling for gender and education level of founder (7 groups); sector (10 groups), size and age of prior employer (linear in each); number of founders (linear); and year of inception, sector and location of start-up firm. All firms founded between 1997 and 2011 with an identified founder. *Source*: RAMS, LOUISE and FEK 1996–2015.

B Model

B.1 HJB equations

This subsection provides the recursions for the value of non-employment and a firm in the general case with decreasing returns to scale. It follows closely recent work by [Bilal et al. \(2019\)](#).

The value of non-employment. Denote by U the value of non-employment to an individual, by $V(z, n)$ the joint value of a firm to its owner and n workers, and by $V_n(z, n)$ the marginal joint value of the firm to its owner and workers. The value of non-employment satisfies the HJB

$$\rho U = b + p\beta \int_{\underline{z}}^{\infty} (V_n(z, n) - U)^+ d\tilde{F}(z, n) + \max_s \left(s \left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} V(\varepsilon, 0) d\Gamma(\varepsilon) - U \right)^+ - c_e(s) \right) \quad (24)$$

where $x^+ = \max\{x, 0\}$. The individual enjoys flow value b . She encounters job opportunities at rate p drawn from offer distribution \tilde{F} , which she accepts if it is sufficiently good and gets a slice β of the surplus. She searches for entrepreneurship opportunities at rate s , and seizes an opportunity if the expected value of entry is greater than the value of remaining non-employed. In that case, she innovates an idiosyncratic amount ε on the least productive idea in the economy (which has a relative (log) productivity

of $z = 0$). Note that death does not enter the value since her preferences are dynastic and her offspring enters as non-employed.

The value of a firm. The joint value of a firm $V(z, n)$ satisfies the recursion⁴³

$$\begin{aligned}
\rho V(z, n) = \max_v \{ & e^z (n+1)^\alpha + k - r - c(v) \\
& + n(\delta + \kappa)(U - V_n(z, n)) \\
& + n\phi p \beta \int (V_n(z', n') - V_n(z, n))^+ d\tilde{F}(z', n') \\
& + n \max_s \left(s \left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} V(\varepsilon, 0) d\Gamma(\varepsilon) - V_n(z, n) \right)^+ - c_e(s) \right) \\
& + qv(1 - \beta) \left(\frac{u}{S} (V_n(z, n) - U)^+ + \frac{s}{S} \int (V_n(z, n) - V_n(z', n'))^+ d\tilde{G}(z', n') \right) \} \\
& - mV_z(z, n) + \frac{\sigma^2}{2} V_{zz}(z, n)
\end{aligned} \tag{25}$$

subject to

$$V(z, n) \geq (n+1)U, \quad V_n(z, n) \geq U$$

I discuss each term in sequence. The firm produces output $e^z(n+1)^\alpha$, the entrepreneur enjoys value of being her own boss k , has to pay the cost of land $r = R(t)e^{-Z(t)}$ and the cost of trying to recruit more workers, $c(v)$. Its incumbent workers separate at rate δ and permanently exit at rate κ , and there are n incumbent workers. A worker that separates to non-employment or permanently exits gets the value of non-employment U (recall the dynastic preferences). At the same time, the incumbent firm loses the marginal contribution of that worker to the firm, $V_n(z, n)$. From the perspective of the ex ante joint value of the firm and its workers, such a transition results in a net value change of $U - V_n(z, n)$. At rate ϕp , an incumbent worker gets an outside offer, drawn at random from the offer distribution $\tilde{F}(z, n)$. If the new firm has a higher valuation of the worker's services—captured by the marginal value of the worker to the new firm—the worker moves to the new employer. The worker gets compensated with the full marginal value at her current firm plus a share β of the differential surplus. From the perspective of the ex ante joint value of the incumbent firm and its workers, this results in a net value

⁴³For a derivation of this joint value formulation from its primitive components, see [Bilal et al. \(2019\)](#).

gain of $\beta(V_n(z', n') - V_n(z, n))$.⁴⁴

In return for paying the search cost $c_e(s)$, an incumbent worker gets an opportunity to start her own firm at rate s . As noted above, outside options are verifiable, including entrepreneurship opportunities, and a firm may offer an incumbent worker who is considering quitting to start her own firm compensation to remain with the firm. This ensures that all decisions serve to maximize the joint value of the firm and its workers, i.e. the worker accepts it if it provides a higher value than the marginal value in her current firm.

In return for paying the vacancy cost $c(v)$, the firm gets the chance to meet with new potential hires at rate q . A fraction u/S of such potential hires are non-employed, where u is the mass of non-employed workers and S is the search-efficiency weighted mass of workers. The firm successfully recruits them if the marginal value of a worker is higher in the firm than in non-employment. The firm retains a share $1 - \beta$ of the marginal surplus. With probability s/S , the potential hire is employed, distributed according to $\tilde{G}(z', n')$, where s is the search-efficiency weighted mass of employed workers (hence $S = u + s$). The firm successfully recruits the worker if the marginal value of the worker in the firm is greater than at the competing firm, and it gets a slice $1 - \beta$ of the differential value. Separations are chosen optimally, hence marginal value must always be greater than the value of non-employment.

B.2 Proofs

Proposition 1. Set $\alpha = 1$ and guess that the value of a firm can be written as the sum of the values of its matches plus the value of recruiting more workers to the firm, $V(z, n) = nJ(z) + O(z)$, such that $V_n(z, n) = J(z)$. Applying the guess, the right-hand side of the HJB (25) can be rewritten as

⁴⁴In particular, the incumbent worker gets $V_n(z, n) + \beta(V_n(z', n') - V_n(z, n))$ at the new firm, while the incumbent firm loses $V_n(z, n)$. Hence the change in joint value to the incumbent firm and its workers (which includes the incumbent worker), is $\beta(V_n(z', n') - V_n(z, n))$. See [Bilal et al. \(2019\)](#) for a more detailed discussion and derivation.

$$\begin{aligned}
\rho V(z, n) &= \max_v \left\{ e^z + k - r - c(v) \right. \\
&\quad + qv(1 - \beta) \left(\frac{u}{S} (J(z) - U)^+ + \frac{s}{S} \int (J(z) - J(z'))^+ dG(z') \right) \\
&\quad + \mu O'(z) + \frac{\sigma^2}{2} O''(z) \left. \right\} \\
&\quad + n \left\{ e^z \right. \\
&\quad \quad + \delta(U - J(z)) \\
&\quad \quad + \phi p \beta \int (J(z') - J(z))^+ dF(z') \\
&\quad \quad + \pi \left(\int O(\varepsilon) d\Gamma(\varepsilon) - J(z) \right)^+ \\
&\quad \quad \left. + \mu J'(z) + \frac{\sigma^2}{2} J''(z) \right\} \\
&= \rho O(z) + n \rho J(z)
\end{aligned}$$

subject to $O(z) + nJ(z) \geq (n + 1)U$ and $J(z) \geq U$. This problem is separable up to the boundary conditions. Denote by \underline{z}^w the reservation threshold of workers, $J(\underline{z}^w) = U$, and by \underline{z} the reservation threshold of the entrepreneur, $O(\underline{z}) + n(\underline{z})J(\underline{z}) \geq (n(\underline{z}) + 1)U$. Firm size is a function of productivity, since workers can always quit. Under the assumption that the value of being one's own boss is sufficiently high, the entrepreneur prefers to stay in business longer than workers prefer to be employed, $\underline{z} < \underline{z}^w$. As a consequence, $J(\underline{z}^w) = U > J(\underline{z})$ and $n(\underline{z}) = 0$. Hence, workers' optimal reservation threshold is not affected by that of the entrepreneur, while the entrepreneur only worries about herself at the relevant points for exit, $O(\underline{z}) = U$.

Proposition 2. Substituting optimal search intensity in the HJB for the value of a match (3), rearranging and integrating by parts, the value of a match can be written as

$$\begin{aligned}
(\rho + \delta + \kappa)J(z) &= e^z - b - mJ'(z) + \frac{\sigma^2}{2}J''(z) \\
&- p\beta \int_0^\infty 1 - \mathcal{F}(x)dx - \frac{1}{c_s} \frac{\eta_s}{1 + \eta_s} \left(\left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} O(\varepsilon)d\Gamma(\varepsilon) \right)^+ \right)^{\frac{1+\eta_s}{\eta_s}} \\
&+ \phi p\beta \int_{J(z)}^\infty 1 - \mathcal{F}(x)dx + \frac{1}{c_s} \frac{\eta_s}{1 + \eta_s} \left(\left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} O(\varepsilon)d\Gamma(\varepsilon) - J(z) \right)^+ \right)^{\frac{1+\eta_s}{\eta_s}}
\end{aligned} \tag{26}$$

subject to the initial value conditions that $J(\underline{z}^w) = 0$ and $J'(\underline{z}^w) = 0$. Suppose that the drift and intensity of shocks are zero, $m = \sigma = 0$, and that workers always search for business opportunities. Differentiating both sides of (26) with respect to z ,

$$(\rho + \delta + \kappa)J'(z) = e^z - \phi p\beta(1 - \mathcal{F}(J(z)))J'(z) - \frac{1}{c_s} \left(\left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} O(\varepsilon)d\Gamma(\varepsilon) - J(z) \right)^+ \right)^{\frac{1}{\eta_s}} J'(z)$$

Noting that optimal search intensity (4) is

$$s(z) = \frac{1}{c_s} \left(\left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} O(\varepsilon)d\Gamma(\varepsilon) - J(z) \right)^+ \right)^{\frac{1}{\eta_s}}$$

and rearranging,

$$J'(z) = \frac{e^z}{\rho + \delta + \kappa + \phi p\beta(1 - \mathcal{F}(J(z))) + s(z)}$$

The right-hand side is strictly positive and finite for any finite value for z . Hence, the solution is unique.

Corollaries 1–3. The right hand side of (5) is strictly positive. Hence, the surplus of a match increases strictly in productivity z . Since workers switch firms whenever the value of the new match is higher, it follows that they move whenever productivity is higher.

The higher the value of a worker's current match, the less likely is it that she will meet a firm that

offers higher value. Since the value of the match is strictly increasing in productivity, the higher the productivity of her current match, the less likely it is that she will meet a firm with higher productivity.

Finally, differentiating optimal search intensity (4) with respect to z , it declines in productivity z since $J'(z) > 0$.

Proposition 3. Imposing the optimal choice of vacancies (11) and the optimal choice of search (4) in (8) and integrating by parts, the surplus value of the entrepreneur (8) can be rewritten as

$$\begin{aligned} \rho O(z) &= e^z + k - b - r - mO'(z) + \frac{\sigma^2}{2}O''(z) \\ &- p\beta \int_0^\infty 1 - \mathcal{F}(x)dx - \frac{1}{c_s} \frac{\eta_s}{1 + \eta_s} \left(\left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} O(\varepsilon)d\Gamma(\varepsilon) \right)^+ \right)^{\frac{1+\eta_s}{\eta_s}} \\ &+ \frac{1}{c_v} \frac{\eta_v}{1 + \eta_v} \left(q(1 - \beta) \left(\frac{u}{S}J(z)^+ + \frac{\phi e}{S} \int_0^{J(z)} \mathcal{G}(x)dx \right) \right)^{\frac{1+\eta_v}{\eta_v}} \end{aligned} \quad (27)$$

In equilibrium, the price of land must be such that an entrepreneur prefers to exit at $\underline{z} = 0$. Imposing this in (27), noting that $J(\underline{z}) = 0$ since by assumption parameter values are such that workers separate before firms do, $\underline{z}^w > \underline{z}$, and match value is strictly increasing in productivity, the equilibrium price of land must be

$$r = 1 - b + k - p\beta \int_0^\infty 1 - F(x)dx - \left(\frac{1}{c_s} \right)^{\frac{1}{\eta_s}} \frac{\eta_s}{1 + \eta_s} \left(\left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} O(\varepsilon)d\Gamma(\varepsilon) \right)^+ \right)^{\frac{1+\eta_s}{\eta_s}} \quad (28)$$

where $O'(\underline{z}) = 0$ by the smooth pasting condition for optimal exit.

Substituting the equilibrium price of land (28) back into the surplus value of an entrepreneur (27),

$$\rho O(z) = e^z - 1 - mO'(z) + \frac{\sigma^2}{2}O''(z) + \left(\frac{1}{c_v} \right)^{\frac{1}{\eta_v}} \frac{\eta_v}{1 + \eta_v} \left(q(1 - \beta) \left(\frac{u}{S}J(z)^+ + \frac{e}{S} \int_0^{J(z)} \mathcal{G}(x)dx \right) \right)^{\frac{1+\eta_v}{\eta_v}}$$

Setting $m = \sigma = 0$ delivers the proposition.

Lemma 1. Integrating (11) by parts and then differentiating (assuming that $J(z)^+ = J(z)$), the derivative is

$$v'(z) = \left(\frac{1}{c_v}\right)^{\eta_v} v(z)^{1-\eta_v} (q(1-\beta)) \left(\frac{u}{S} + \frac{\phi e}{S} G(J(z))\right) J'(z) > 0$$

The share of hires from unemployment equals

$$share(z) = \frac{\frac{qv(z)}{n} \frac{u}{S}}{\frac{qv(z)}{n} \left(\frac{u}{S} + \frac{\phi e}{S} G(z)\right)} = \frac{1}{1 + \frac{\phi e}{u} G(z)}$$

which is declining in z since G is a cdf.

Proposition 4. Suppose that the productivity distribution satisfies the postulated solution,

$$x(z) = \frac{E}{-m + \frac{\sigma^2}{2}\zeta} \left(e^{\frac{2(-m)}{\sigma^2}z} - e^{-\zeta z} \right)$$

such that

$$\begin{aligned} x'(z) &= \frac{E}{-m + \frac{\sigma^2}{2}\zeta} \left(-\frac{2m}{\sigma^2} e^{\frac{2(-m)}{\sigma^2}z} + \zeta e^{-\zeta z} \right) \\ x''(z) &= \frac{E}{-m + \frac{\sigma^2}{2}\zeta} \left(\left(\frac{2m}{\sigma^2}\right)^2 e^{\frac{2(-m)}{\sigma^2}z} - \zeta^2 e^{-\zeta z} \right) \end{aligned}$$

This satisfies the boundary conditions that $x(0) = 0$,

$$\begin{aligned} \int_0^{\infty} \frac{E}{-m + \frac{\sigma^2}{2}\zeta} \left(e^{\frac{2(-m)}{\sigma^2}z} - e^{-\zeta z} \right) dz &= \frac{E}{-m + \frac{\sigma^2}{2}\zeta} \left(\frac{1}{\frac{2m}{\sigma^2}} - \frac{1}{\zeta} \right) \\ &= \frac{E}{-\frac{E}{\zeta} + \frac{\sigma^2}{2}\zeta} \left(\frac{\sigma^2}{2\frac{E}{\zeta}} - \frac{1}{\zeta} \right) \\ &= \frac{1}{-\frac{E}{\zeta} + \frac{\sigma^2}{2}\zeta} \left(-\frac{E}{\zeta} + \frac{\sigma^2\zeta}{2} \right) \\ &= 1 \end{aligned}$$

where I used the fact that $m = \frac{E}{\zeta}$ to substitute for m , and

$$\frac{\sigma^2}{2} x'(0) = \frac{\sigma^2}{2} \frac{E}{-m + \frac{\sigma^2}{2} \zeta} \left(-\frac{2m}{\sigma^2} + \zeta \right) = \frac{E}{-\frac{2m}{\sigma^2} + \zeta} \left(-\frac{2m}{\sigma^2} + \zeta \right) = E$$

Finally, it satisfies the KFE (18)

$$0 = m \frac{E}{-m + \frac{\sigma^2}{2} \zeta} \left(-\frac{2m}{\sigma^2} e^{\frac{2(-m)}{\sigma^2} z} + \zeta e^{-\zeta z} \right) + \frac{\sigma^2}{2} \frac{E}{-m + \frac{\sigma^2}{2} \zeta} \left(\left(\frac{2m}{\sigma^2} \right)^2 e^{\frac{2(-m)}{\sigma^2} z} - \zeta^2 e^{-\zeta z} \right) + E \zeta e^{-\zeta z}$$

which can be confirmed by cancelling terms.

Proposition 5. Integrating the stationary solution for productivity 4, the counter cumulative cdf of z is

$$Pr(Z > z) = \frac{1}{-\frac{1}{\zeta} + \frac{\sigma^2 \zeta}{2E}} \left(\frac{\sigma^2}{2\frac{E}{\zeta}} e^{-\frac{2E}{\sigma^2 \zeta} z} - \frac{1}{\zeta} e^{-\zeta z} \right)$$

Suppose first that $\frac{1}{\zeta} < \frac{\sigma^2 \zeta}{2E} \iff \frac{2E}{\sigma^2 \zeta} < \zeta$ such that the term outside the parenthesis is positive and the first term in the parenthesis decays at a slower rate in the tail than the second term. Then the limiting behavior of the distribution is driven entirely by the first term in the parenthesis,

$$\lim_{z \rightarrow \infty} \left(\frac{\sigma^2}{2\frac{E}{\zeta}} e^{-\frac{2E}{\sigma^2 \zeta} z} - \frac{1}{\zeta} e^{-\zeta z} \right) = \lim_{z \rightarrow \infty} e^{-\frac{2E}{\sigma^2 \zeta} z} \left(\frac{\sigma^2}{2\frac{E}{\zeta}} - \frac{1}{\zeta} e^{-(\zeta - \frac{2E}{\sigma^2 \zeta}) z} \right) = \lim_{z \rightarrow \infty} e^{-\frac{2E}{\sigma^2 \zeta} z} \frac{\sigma^2}{2\frac{E}{\zeta}}$$

since the second term vanishes as $z \rightarrow \infty$. Hence, z follows a power law with tail parameter $\frac{2E}{\sigma^2 \zeta}$. A symmetric argument in the case of $\frac{1}{\zeta} > \frac{\sigma^2 \zeta}{2E} \iff \frac{2E}{\sigma^2 \zeta} > \zeta$ implies that z follows a power law also in this case but with tail parameter ζ . In the first case, an increase in the entry rate increases the tail parameter, which means that the tail becomes thinner.

B.3 The evolution of the employment distribution with age

Denote by $h(z, a)$ the joint distribution of individuals over productivity and age. For $z > \underline{z}^w$ and $a > 0$, it evolves according to the partial differential equation

$$\begin{aligned} \frac{\partial h(z, a)}{\partial a} &= m \frac{\partial h(z, a)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 h(z, a)}{\partial z^2} \\ &- \left(\kappa + \delta + \phi p(1 - F(z)) + s(z) \right) h(z, a) \\ &+ pf(z) \left(\frac{u(a)}{E} + \phi \int_{\underline{z}^w}^z h(z, a) dz \right) \end{aligned} \quad (29)$$

where $E = 1 - \int u(a) da$ is the total mass of employed individuals. The first two terms describe changes due to the drift and shocks to productivity. The third term consists of outflows due to death, exogenous separations to non-employment, flows up the job ladder, and entry to entrepreneurship. The fourth term is due to inflows from non-employment and firms below in the job ladder. The Kolmogorov Forward Equation (KFE) (29) is subject to the density being zero along the exit boundary and the distribution integrating to one,

$$0 = h(\underline{z}^w, a), \quad 1 = \int_{\underline{z}^w}^{\infty} h(z, a) dz da \quad (30)$$

Finally, the number of non-employed of age a is given by

$$u'(a) = -(\kappa + p + s(u))u(a) + \left(\delta + \frac{\sigma^2}{2} \frac{\partial h(\underline{z}^w, a)}{\partial z} \right) e(a) + X(a) \quad (31)$$

where $e(a) = m(a) - u(a) - l(a)$ is the number of employed workers of age a , $m(a) = \kappa e^{-\kappa a}$ is the total number of individuals of age a , $l(a)$ the number of entrepreneurs of age a , and $X(a)$ the flow of individuals of age a exiting entrepreneurship. The initial value for (31) is $u(0) = \kappa(1 - l)$. The age-conditional non-employment rate is $u_a = u(a)/m(a)$ and the age-conditional cdf of employment $G_a(z)$ is

$$G_a(z) = \frac{1}{e(a)} \int_{\underline{z}^w}^z h(a, \tilde{z}) d\tilde{z}$$

B.4 Algorithm

I numerically solve the model in continuous time on a discretized grid for productivity, z , and human capital h . It starts by guessing initial value functions of the unemployed, U , the match, J , and the entrepreneur, O , as well as distributions g and x and aggregate states p and q . I fix the growth rate ex ante to a pre-determined value, and adjust the cost of entry so as to recover the pre-set growth rate as an equilibrium outcome. The reason is that the algorithm is more stable holding the growth rate fixed and updating the cost of entry, than vice versa. The algorithm consists of the following steps:

1. Given distributions and aggregate states, update the value functions and decision rules;
 - 1.1 Within each value function loop, solve for the cost of entry that generates an entry rate that in turn matches the pre-set growth rate;
 - 1.2 Update the flow value of leisure to ensure that workers of each human capital group want to separate voluntarily to unemployment at the second grid point for productivity;
2. Given updated value functions and decision rules, update the distributions and aggregate states;
3. If the value functions and distributions in the current iteration are sufficiently close to those in the previous iteration, stop. Otherwise return to 1.

B.5 Introducing population growth

This subsection illustrates how the model can be extended to incorporate population growth without featuring scale effects. To that end, suppose that when an individual dies, she is replaced with two offspring with some probability λ . That is, the size of the population grows at rate λ .⁴⁵ Suppose furthermore that having two children gives the parent no more joy than having one child, such that the chance of having two children does not directly impact her utility function. Finally, if a dying entrepreneur has two offspring, assume that one of them gets to take over the existing firm while the other gets to start a new firm with the same technology using a newly available plot of land. That is, also land grows at rate λ .⁴⁶ Under these assumptions, the rate of growth of the population only impacts the economy through its effect on the distribution of individuals over the job ladder. That is, the surplus value of the match and the entrepreneur continue to solve (3) and (8) and the KFE for the distribution of productivity (18) is unaffected, such that the only change is to the KFE for the employment distribution (15)–(16).

⁴⁵The particular restriction to one or two offspring implies that the population growth rate is limited to two, but in practice this constraint never binds.

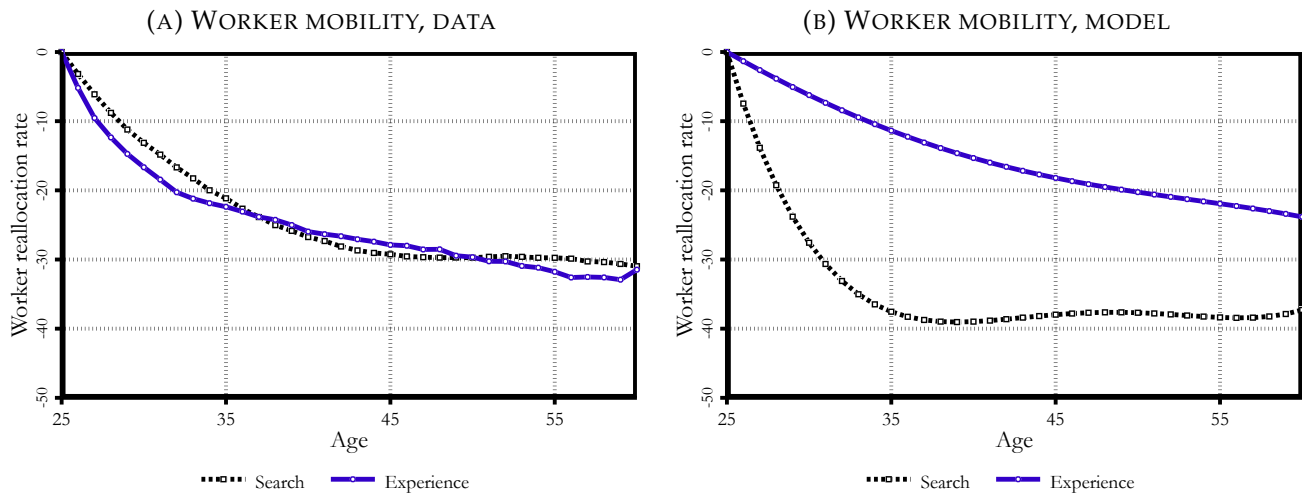
⁴⁶An alternative assumption would be to relabel the fixed factor land as managerial capital, and assume that managerial capital grows at the rate of the population.

C Estimation

This section contains some additional validation exercises.

The empirical role of search vs experience for life-cycle worker mobility. Figure 27 plots the decomposition of worker mobility into an *experience* term and a *search* term based on regression (22). The model overstates the speed and magnitude of the decline in worker mobility due to search and understates that due to experience. Nevertheless, as this decomposition is not targeted, the model does a decent job at reproducing the empirical patterns.

FIGURE 27. DECOMPOSITION OF LIFE-CYCLE MOBILITY

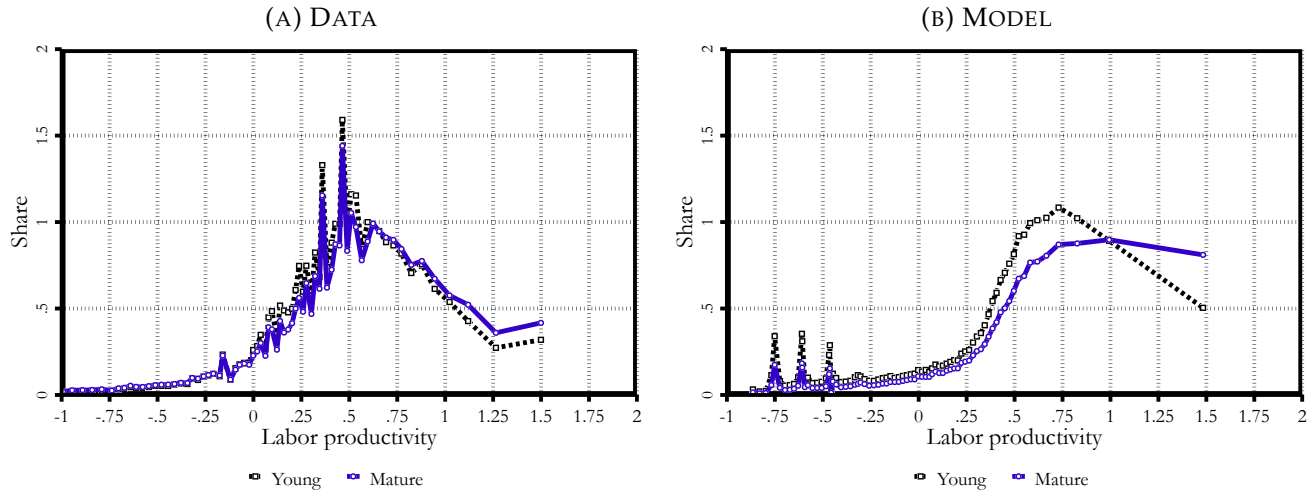


Note: Worker mobility: Share of employment in month t who is employed at a different main employer in month $t + 1$, including non-employment. Experience: Estimated age effects from regression (22), \hat{A} . Search: Difference between the raw age-conditional mean and the estimated age effects, \hat{A} . Both are expressed in percent relative to the raw hazard at age 25. Source: Simulated weekly version of the model for 25,000 firms, discarding an initial burn-in period and aggregating to monthly data, RAMS, LISA and FEK 2011–2015.

Mobility toward more productive firms Figure 28 plots the distribution of young and mature workers over labor productivity in the data and model, computed as firm value added (output in the model) divided by employment.⁴⁷ The model overstates somewhat the extent to which mature workers work for more productive firms. Productivity in the data, however, likely contains measurement error and transitory shocks, which would bias any differences to zero.

⁴⁷In both the model and the data, the other age groups are a convex combination and omitted for brevity. The coarseness of the graph results from minimum sample size restrictions imposed due to confidentiality restrictions; for comparability, model moments are constructed identically to the data.

FIGURE 28. DISTRIBUTION OF YOUNG AND MATURE WORKERS OVER THE JOB LADDER



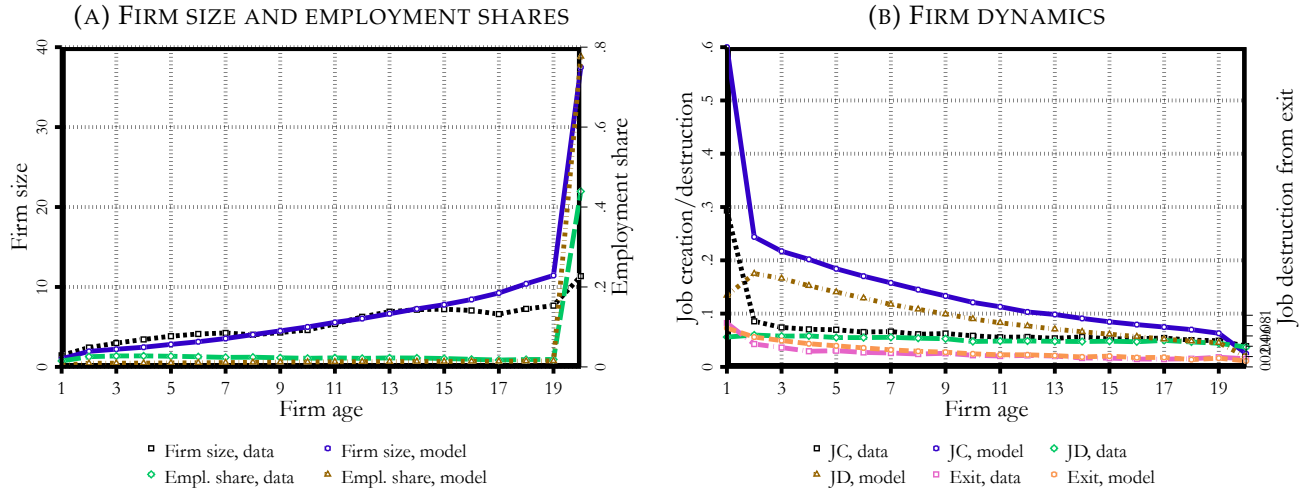
Note: Distribution of workers by age over firms by value added per worker. Young = 18–34 and Mature = 45–54 (the other age groups are convex combinations and excluded for readability). *Source:* Simulated weekly version of the model for 25,000 firms, discarding an initial burn-in period and aggregating to monthly data, RAMS, LISA and FEK 2011–2015.

Additional firm dynamics. The left panel of Figure 29 plots average firm size and employment shares by firm age. The model matches well average firm size apart from the very oldest age group of firms, which are too large. It also overstates the share of employment at the oldest firms.

As illustrated by the right panel, the model provides a decent fit to broader firm dynamics, given that none of these moments is directly targeted. Young firms are likely to exit, but conditional on remaining they grow fast. They have high job creation and destruction rates, despite the fact that firms are subject to the same proportional productivity shocks.⁴⁸ One reason is that older firms are larger, and the vacancy cost does not scale in size. Another reason is that older firms are higher up the job ladder. A given magnitude shock moves a firm fewer rungs in the ladder at the top, because firms are (endogenously) more spread out there. Hiring and separations depend on a firm’s rank in the job ladder, such that high productive firms response less to shocks.

⁴⁸Measurement error in firm age could again be why young firms are somewhat too dynamic in the model.

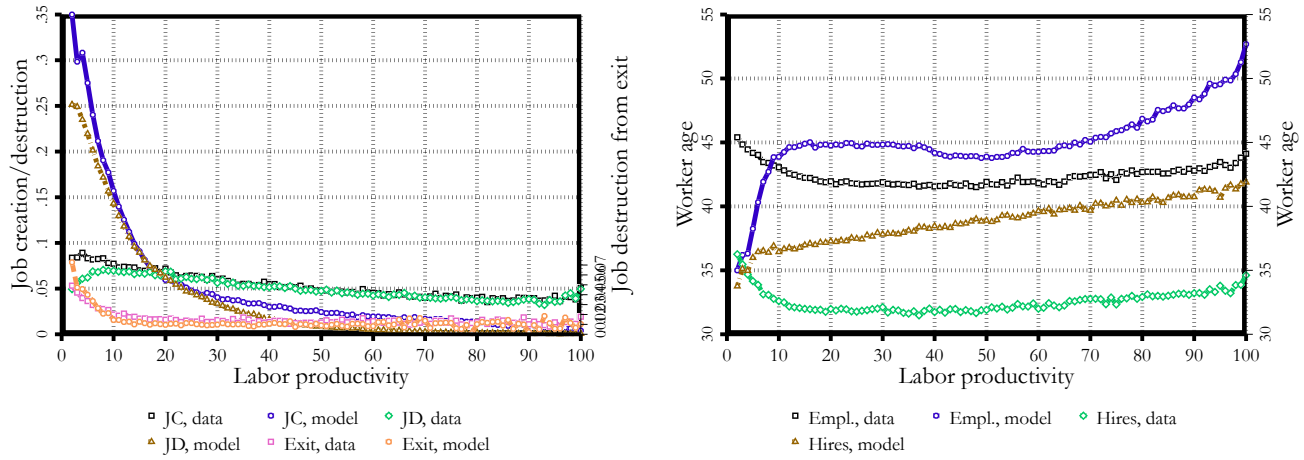
FIGURE 29. ADDITIONAL FIRM DYNAMICS



Note: Firm size: Total number of workers at firms of age a divided by total number of firms of age a . Employment share: Total number of workers at firms of age a divided by total number of workers. Job creation/destruction rate: Sum of jobs created/destroyed in the quarter divided by average employment. Job destruction due to exit: Sum of jobs destroyed at exiting firms divided by average employment. Source: Simulated weekly version of the model for 25,000 firms, discarding an initial burn-in period and aggregating to monthly data, RAMS, LISA and FEK 2011–2015.

Firm dynamics and worker age by productivity. Figure 30 plots firm dynamics and the age of employees and hires by firm productivity. The model overstates the decline in firm dynamics with productivity, but productivity in the data likely contains measurement error and/or transitory fluctuations that the model abstracts from. It also overstates somewhat the extent to which more productive firms hire and employ older workers.

FIGURE 30. FIRM DYNAMICS AND AGE OF WORKERS BY PRODUCTIVITY, MODEL VERSUS DATA



Note: Net poaching: Difference between hires from and separations to other employers in a month divided by average employment in the month. Hires from unemployment: Share of hires from unemployment divided by total hires, where the former includes the unemployed, the NILF, and public sector workers (it is not possible to differentiate between the unemployed and NILF in the data; excluding public sector workers makes little difference to the patterns). Labor productivity: Employment-weighted percentile of log value added per worker controlling for sector-year. The bottom 10 percent contain little employment and are excluded to improve readability. Job creation/destruction rate: Sum of jobs created/destroyed in the quarter divided by average employment. Job destruction due to exit: Sum of jobs destroyed at exiting firms divided by average employment. *Source:* Simulated weekly version of the model for 25,000 firms, discarding an initial burn-in period and aggregating to monthly data, RAMS, LISA and FEK 2011–2015.