Search Complementarities, Aggregate Fluctuations, and Fiscal Policy

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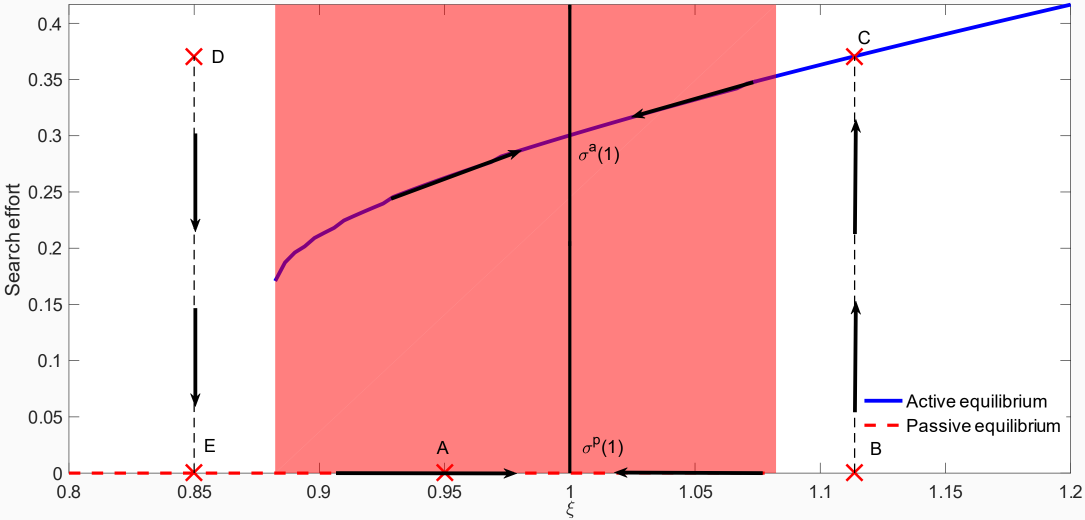
Our starting point

- Search often involves two parties.
- Two-sided searches can generate a strategic complementarity: If the probability of a match is supermodular on search effort, an increase in the search effort by one party might lead to a rise in the search effort by the other party.
- Under certain conditions, this strategic complementarity begets multiple Nash equilibria.
- Strategic complementarities interplay with exogenous shocks to fundamentals and economic policy.
- Why? Multiplicity of Nash equilibria might lead to lasting changes in economic activity without large and persistent exogenous shocks.
- To study this interplay, we build a quantitative business cycle model, calibrate it to U.S. data, and explore its properties.

Our environment

- Firms must form joint ventures to produce, as in the linkages of a modern economy with complex production networks.
- Number of joint ventures depends on fundamentals *and* on search effort of potential partners.
- Exogenous shocks to productivity and discount factor alter profitability of matches.
- Different regions.
- History dependance: when two equilibria coexist, economy stays at the current equilibrium $\Rightarrow$ high persistence.

Phase diagram for search effort



Results, I

- Sufficiently large (but empirically relevant) movements in the discount factor or technology may trigger switch of static equilibrium.
- Nonlinear properties of the model different from standard models:
    1. Bimodal and asymmetric distributions of ergodic variables even with Gaussian shocks.
    2. Strong autocorrelation of variables without shock persistence.
    3. Endogenous movements in labor productivity.
    4. Realistic volatility of unemployment.
- Small shocks fail to move the system away from original equilibrium and we have similar properties than in standard models.

Results, II

- Aggregate volatility critical for the selection of static equilibria and persistence of fluctuations.

    1. Low volatility environment: reduces probability of switching equilibrium, and increases the duration of the equilibrium.

    2. A large shock during a period of low volatilities moves the system to a new, long-lasting spell of low output: the long recovery from the Great Recession is what you should expect given the Great Moderation.

- Fiscal policy markedly different from standard models:

    1. Powerful in the passive equilibrium, but only if stimulus is large enough.

    2. Weak with large crowding-out effect otherwise.

    3. Thus, fiscal multiplier is strongly state-dependent.

Relation to the literature

- Seminal models with strategic complementarities: Diamond (1982), Cooper and John (1988).
- Strategic complementarities for business cycle fluctuations:
  - Sterk (2016), Taschereau-Dumouchel and Schaal (2015), Eeckhout and Lindenlaud (2015).
  - Huo and Rios-Rull (2013), Kaplan and Mezio (2016).
- Large fiscal policy intervention during downturns:
  - Murphy et al. (1989), Fernández-Villaverde et al. (2015), Ghassibe and Zanetti (2019).

Our contribution: develop a quantitative model with strategic complementarity, exogenous shocks to fundamentals, and a standard calibration. First to consider effect of volatility for the length of output activity spells and their switches.

A simple model with search complementarities

- The economy is composed of a continuum of islands of unit measure.
- Two risk-neutral firms populate each island.
- Firms are in two separate locations within the island, and they must meet to engage in production.
    - No meeting: zero output.
    - Meeting: jointly produce 2 units of output that they split evenly.
- At the end of each period, the match is dissolved, and each firm moves to a new, separate location to search in the next period *ex novo*.

Matching probabilities

- The probability of meeting depends on the search effort of each firm.
- For a search effort $\sigma_1 \in [0,1]$ of firm 1 and a search effort $\sigma_2 \in [0,1]$ of firm 2, the matching probability is:

$$\pi(\sigma_1, \sigma_2) = \frac{1 + \sigma_1 + \sigma_2 + \sigma_1\sigma_2}{4}$$

- Thus, matching probability of $1/4$ when $\sigma_1 = \sigma_2 = 0$, 1 when $\sigma_1 = \sigma_2 = 1$, and probabilities between $1/4$ and 1 in the intermediate cases.
- For an $\alpha \in [0,1)$, the cost of search effort for firm $i \in \{1,2\}$ is:

$$c(\sigma_i) = \frac{1+\alpha}{4}\sigma_i + \frac{\sigma_i^3}{3}$$

Nash equilibria

- The expected profit function of firm 1 is:

$$J(\sigma_1, \overline{\sigma}_2) = \frac{1 + \sigma_1 + \overline{\sigma}_2 + \sigma_1 \overline{\sigma}_2}{4} - \frac{1+\alpha}{4}\sigma_1 - \frac{\sigma_i^3}{3}$$

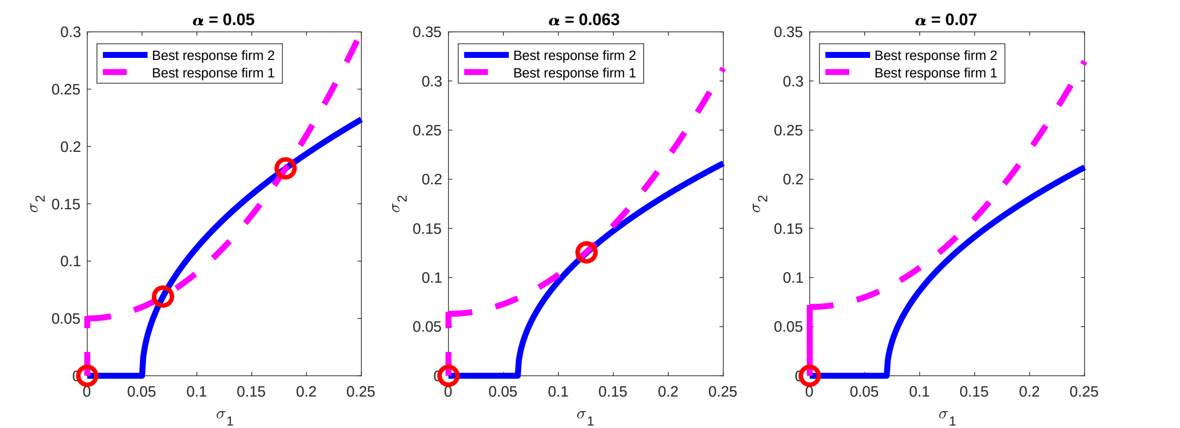
- Maximizing $J(\sigma_1, \overline{\sigma}_2)$ w.r.t. σ_1, we get the best response $\Pi(\sigma_2)$ for firm 1:

$$\sigma_1^* = \begin{cases} 0 & \text{if } \sigma_2 \leq \alpha \\ \frac{1}{2}\sqrt{\sigma_2 - \alpha} & \text{if } \sigma_2 > \alpha \end{cases}$$

- Analogous expression for the best response $\Pi(\sigma_1)$ of firm 2.
- Role of cost parameter:
  1. Values of $\alpha < 0$: unique Nash equilibrium with positive search effort.
  2. Values of $\alpha \geq 1$: unique Nash equilibrium with zero search effort.
  3. Values of $\alpha \in [0, 1)$: we can have multiple Nash equilibria.

Illustration

Figure 1: Three cases of cost parameter α



Stochastic shocks

- Firms produce $2z_t$, where z_t is a productivity shock in period t.
- The new expected profit function of firm 1 is:

$$J\left(\sigma_{1,t}, \overline{\sigma}_{2,t}, z_t\right) = z_t \frac{1 + \sigma_{1,t} + \overline{\sigma}_{2,t} + \sigma_{1,t}\overline{\sigma}_{2,t}}{4} - \frac{1+\alpha}{4}\sigma_{1,t} - \frac{\sigma_{1,t}^3}{3}$$

- The best response $\Pi\left(\sigma_{2,t}, z_t\right)$ for firm 1 is:

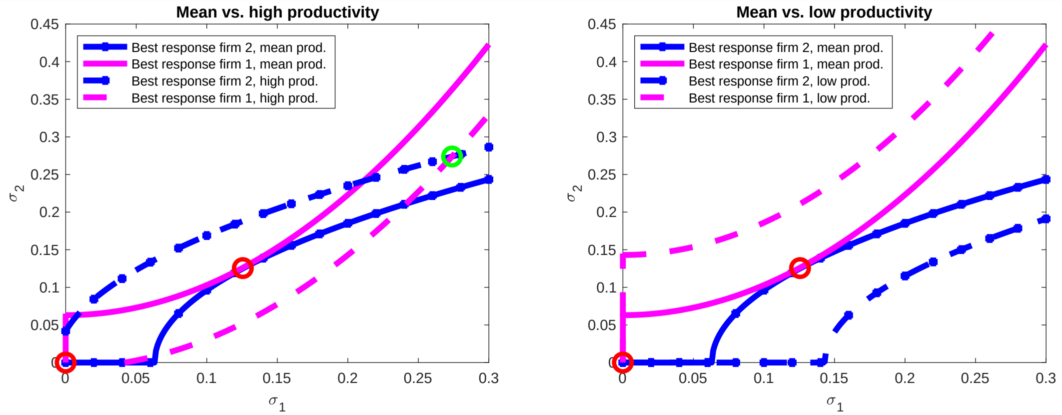
$$\sigma_{1,t}^* = \begin{cases} 0 & \text{if } z_t\left(1+\overline{\sigma}_{2,t}\right) \leq (1+\alpha) \\ \frac{1}{2}\sqrt{z_t\left(1+\sigma_{2,t}\right) - (1+\alpha)} & \text{if } z_t\left(1+\overline{\sigma}_{2,t}\right) > (1+\alpha) \end{cases}$$

  and best response $\Pi\left(\sigma_{1,t}, z_t\right)$ for firm 2 similarly determined.

- Selection of equilibria: history dependence.

Illustration

Figure 2: Changing productivity z_t



$\alpha = 0.063$ and z_t follows a Markov chain with support $\{0.93, 1, 1.07\}$.

The search complementarity "multiplier"

- Consumption usually moves more than productivity.
- Amplification mechanism comes from search complementarities: search efforts by firm 1 triggers search efforts by firm 2.

Table 1: Multiplier

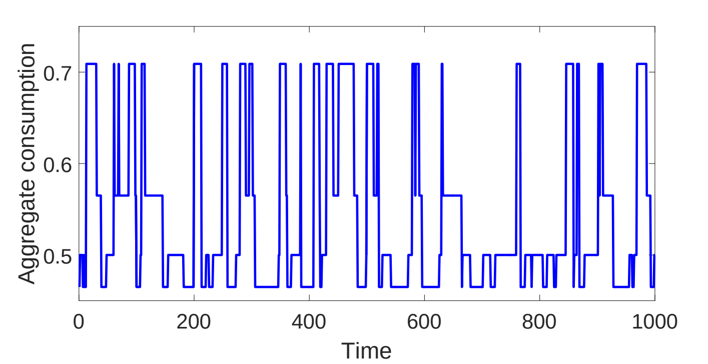
Productivity shock	$\left\lvert \frac{\Delta c_t / c_t}{\Delta z_t / z_t} \right\rvert$
$z_{low} \rightarrow z_{mean,passive}$	1
$z_{low} \rightarrow z_{high}$	3.485
$z_{mean,passive} \rightarrow z_{high}$	5.969
$z_{mean,active} \rightarrow z_{high}$	3.627
$z_{high} \rightarrow z_{low}$	4.009
$z_{high} \rightarrow z_{mean,active}$	3.095

Persistence

- Standard business cycle transition matrix Π with symmetry and medium persistence:

$$\Pi = \begin{pmatrix} 0.90 & 0.08 & 0.02 \\ 0.05 & 0.90 & 0.05 \\ 0.02 & 0.08 & 0.90 \end{pmatrix}$$

- Simulation of aggregate consumption:



Takeaways from the simple model

- The simple model illustrates four points:

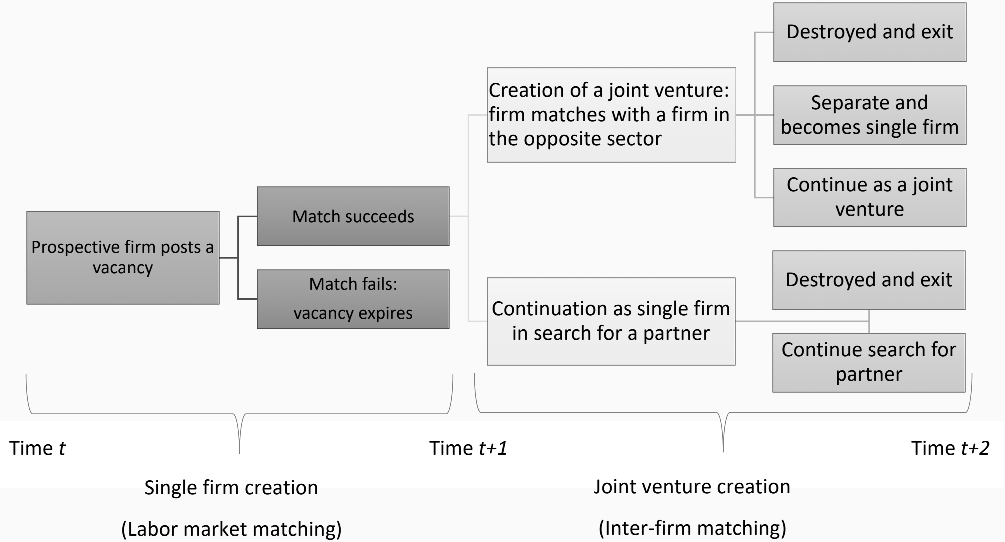
  1. Search complementarities create multiple static Nash equilibria.
  2. Interaction of search complementarities with stochastic shocks amplifies the impact of the shocks on endogenous variables.
  3. The effects of shocks are history-dependent: the multiplier of consumption to a productivity shock is a highly non-linear function of the state of the economy.
  4. History dependence enhances the persistence of aggregate variables.

- We now develop a fully-fledged quantitative business cycle model with search complementaries.

Overview of the model

- Search and matching model where time is discrete and infinite.
- Continuum of risk-neutral households of size 1. Households discount the future by $\beta\xi_t$ per period.
- Two sectors: final goods (F) and intermediate (I) goods sector.
- Single firm creation: firm and worker establish job relation in a standard DMP labor market.
- Joint venture creation: a single final-goods firm must form a joint venture with an intermediate-goods firm to manufacture together.
- Search effort is important for the probability of matching.

Timeline



Prospective firm posts a vacancy
Match succeeds
Match fails: vacancy expires
Creation of a joint venture: firm matches with a firm in the opposite sector
Continuation as single firm in search for a partner
Destroyed and exit
Separate and becomes single firm
Continue as a joint venture
Destroyed and exit
Continue search for partner
Time t
Time t+1
Time t+2
Single firm creation
(Labor market matching)
Joint venture creation
(Inter-firm matching)

Joint venture creation

- I uses a fixed unit of labor to produce: $y_{I,t} = z_t$.
- F uses a fixed unit of labor and $y_{I,t}$ to produce: $y_{F,t} = y_{I,t}$.
- Search effort:

$$\eta_i = \underbrace{\psi}_{\text{fixed}} + \underbrace{\widetilde{\sigma}_{i,t}^{0.5}}_{\text{variable}}$$

- Search cost:

$$c\left(\widetilde{\sigma}_{i,t}\right) = c_0 \widetilde{\sigma}_{i,t}^{0.5} + c_1 \frac{\widetilde{\sigma}_{i,t}^{(1+\nu)/2}}{1+\nu}$$

- The **inter-firm** matching probability (under symmetry: $\widetilde{n}_F = \widetilde{n}_I$) is:

$$\pi_{F,t} = \pi_{I,t} = \phi + \eta_{F,t}\eta_{I,t} = \phi + \left(\psi + \widetilde{\sigma}_{F,t}^{0.5}\right)\left(\psi + \widetilde{\sigma}_{I,t}^{0.5}\right)$$

Value functions of firms

- Single firms:

$$\widetilde{J}_{i,t|\iota_t} = \max_{\sigma_{i,t}\geq 0}\left\{-\widetilde{w}_{i,t} - c(\widetilde{\sigma}_{i,t}) + \beta\left(1-\delta\right)\xi_t\mathbb{E}_t\left[\pi_{i,t}J_{i,t+1} + \left(1-\pi_{i,t}\right)\widetilde{J}_{i,t+1} \mid \iota_t\right]\right\}$$

- Value function of firms in a joint-venture:

$$J_{I,t|\iota_t} = z_t p_t - w_{I,t} + \beta\xi_t\mathbb{E}_t\left\{(1-\delta-\widetilde{\delta})J_{I,t+1} + \widetilde{\delta}\widetilde{J}_{I,t+1} \mid \iota_t\right\}$$

$$J_{F,t|\iota_t} = z_t(1-p_t) - w_{F,t} + \beta\xi_t\mathbb{E}_t\left\{(1-\delta-\widetilde{\delta})J_{F,t+1} + \widetilde{\delta}\widetilde{J}_{F,t+1} \mid \iota_t\right\}$$

1. TFP shock: z_t, AR(1), Gaussian innovations.
2. Discount rate shock: ξ_t, Gaussian i.i.d.

- Analogous value functions of households and new firms.
- Nash bargaining rules determine wages and intermediate good price.

Optimal search effort and DSS

- If interior solution, the optimal search effort for firm i is:

$$c_0 + c_1 \widetilde{\sigma}_{i,t}^{0.5\nu} = \widetilde{\beta}\xi_t \underbrace{\left(\psi + \widetilde{\sigma}_{j,t}^{0.5}\right)}_{\text{search effort in sector } j} \underbrace{\mathbb{E}_t\left(J_{i,t+1} - \widetilde{J}_{i,t+1} | \iota_t\right)}_{\text{expected capital gain}}$$

- Otherwise:

$$c_0 \geq \widetilde{\beta}\xi_t \underbrace{\left(\psi + \widetilde{\sigma}_{j,t}^{0.5}\right)}_{\text{search effort in sector } j} \underbrace{\mathbb{E}_t\left(J_{i,t+1} - \widetilde{J}_{i,t+1} | \iota_t\right)}_{\text{expected capital gain}}$$

- The model has two stable deterministic steady states (DSS).

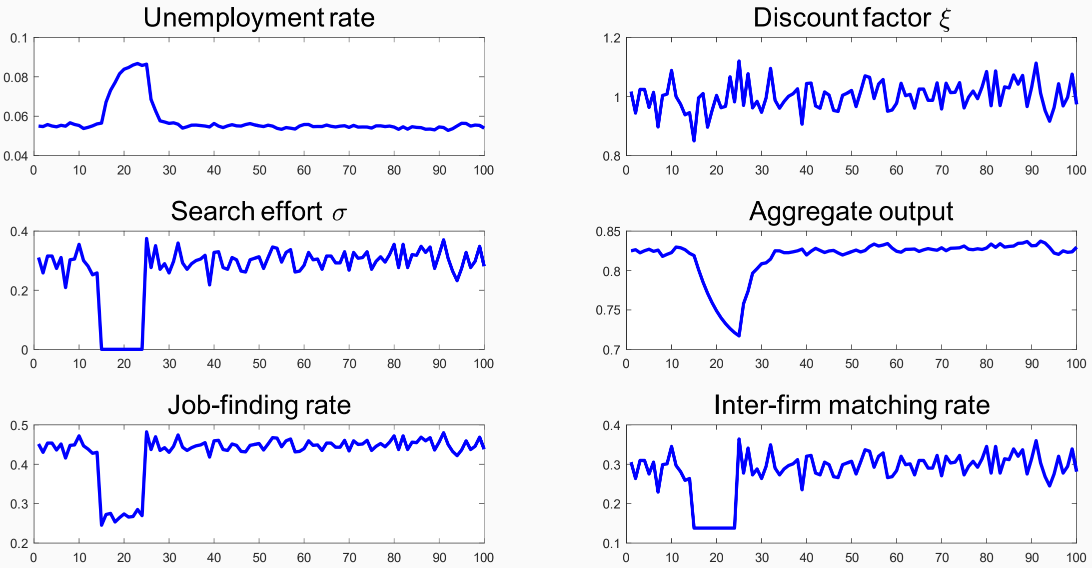
Quantitative analysis

- Monthly calibration.
- Computation: standard dynamic programming with variation of Krusell-Smith's algorithm.
- Simulate the model (3 million months) and generate quarterly data.
- Focus on discount factor shocks.
- Technology shocks change equilibria if permanent.

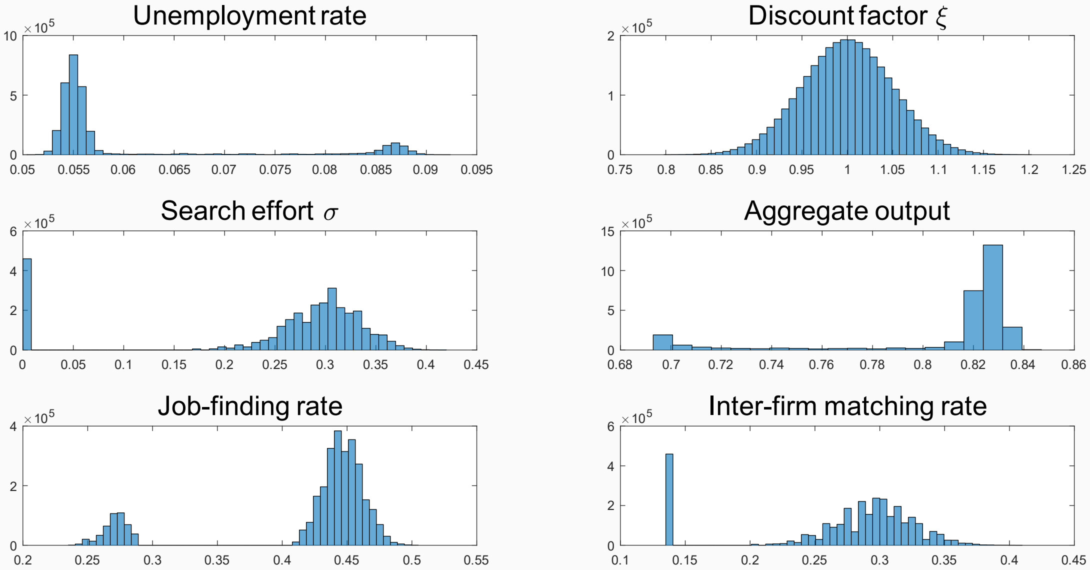
Calibration

Parameter	Value	Source or Target
β	0.996	5% annual risk-free rate
α	0.4	Shimer (2005)
τ	0.4	Hosios condition
χ	0.28	0.45 monthly job-finding rate
κ	1.25	Den Haan et al. (2000)
h	0.3	Thomas and Zanetti (2009)
$\widetilde{\tau}$	0.5	Sectoral symmetry
δ	0.027	5.5% unemployment rate in active DSS
$\widetilde{\delta}$	0.017	5 years duration of joint venture
ϕ	0.135	22% rate of idleness in recessions
ψ	0.114	Condition for multiplicity and 15% recession periods
c_0	0.33	Condition for multiplicity and 15% recession periods
c_1	5	12% rate of idleness in booms
ν	2	Ensure concavity of best response function
σ_ξ	0.05	Justiniano et al. (2008)
ρ_z	$0.95^{1/3}$	BLS
σ_z	0.008	BLS

Simulated variables



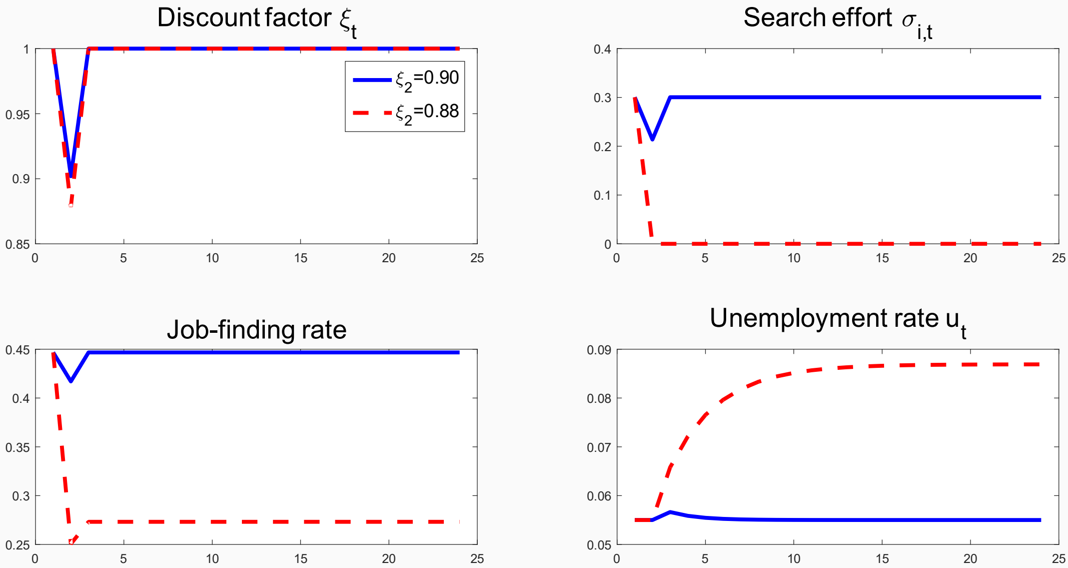
Ergodic distributions



Unconditional second moments

		u	v	v/u	lp	ξ
(a) Quarterly U.S. data, 1951-2016						
Autocorrelation coefficient		0.95	0.95	0.95	0.90	–
Standard deviation		0.20	0.21	0.40	0.02	–
	u	1	-0.92	-0.98	-0.25	–
Correlation matrix	v		1	0.98	0.29	–
	v/u			1	0.27	–
	lp				1	–
(b) Benchmark model						
Autocorrelation coefficient		0.82	0.55	0.71	0.88	0
Standard deviation		0.10	0.21	0.28	0.02	0.03
	u	1	-0.71	-0.85	-0.94	-0.06
Correlation matrix	v		1	0.97	0.54	0.39
	v/u			1	0.72	0.30
	lp				1	0.00
	ξ					1
(c) Model without search complementarities						
Autocorrelation coefficient		0.06	-0.27	-0.08	1	0
Standard deviation		0.02	0.04	0.05	0	0.03
	u	1	-0.27	-0.56	0	-0.56
Correlation matrix	v		1	0.95	0	0.95
	v/u			1	0	1.00
	lp				1	0
	ξ					1

GIRFs to a negative discount factor shock



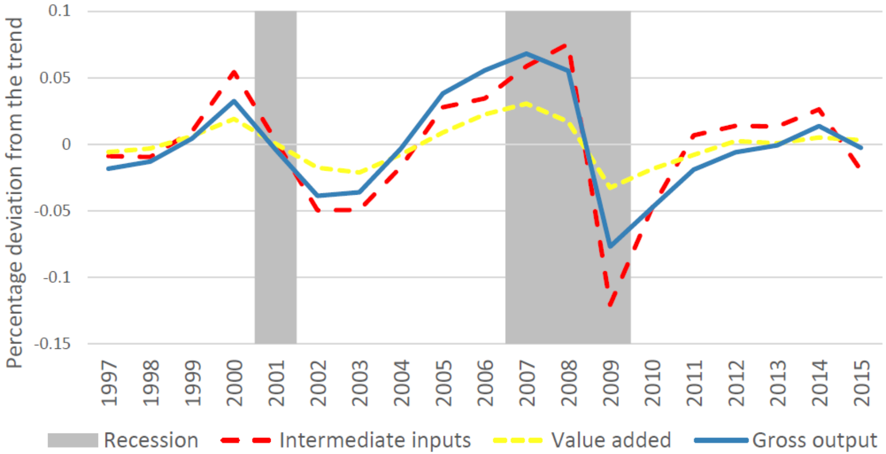
Evidence

- The central mechanism builds on two legs:
    1. The existence of search complementarities among firms that lead to a joint movement of output and intermediate inputs.
    2. Relevance of the discount factor for business cycle fluctuations and important for changes in real activity.
- The data supports these mechanisms:
    - Increases in search effort are correlated.
    - Fluctuations in intermediate input account for almost 2/3 of fluctuations in gross industry output.
    - Discount factors series volatile over the business cycle. Large decline during Great Moderation.

Table 2: Search efforts are positively correlated between connected industries

	(1)	(2)	(3)	(4)
Measure of search efforts	Search-related employment		Signaling cost	
	Level	Residual	Level	Residual
$\sigma_{i,t}^{connect}$	0.45***	0.16*	1.04***	2.39***
	(0.08)	(0.09)	(0.16)	(0.08)
Time FE	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes
R^2	0.16	0.08	0.32	0.29
Observations	15×47	14×47	21×66	20×66

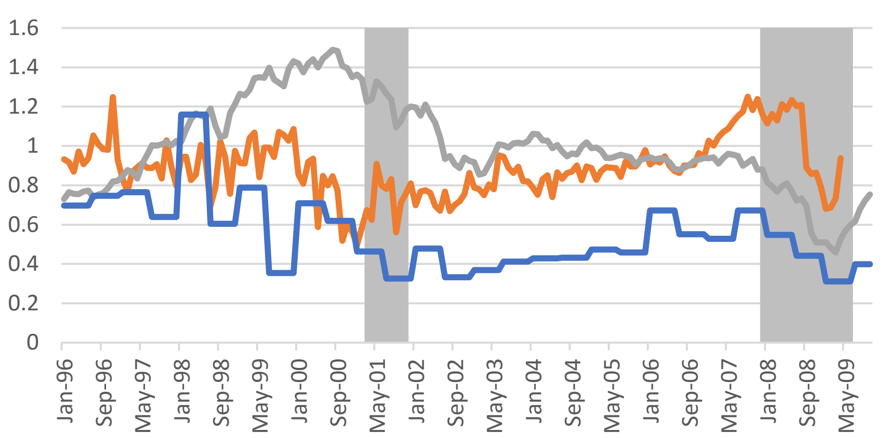
Cyclical role of intermediate goods for changes in production



- Bureau of Economic Analysis: $O = VA + II$.
- Fluctuations in intermediate input $\frac{\text{Cov}(II, VA+II)}{\text{Var}(VA+II)} = 0.71$

Relevance of the discount factor

Figure 3: Alternative measures of the discount factor



Dividend strip (grey line), the price-to-dividend ratio (orange line), and the Livingston Survey (blue line).

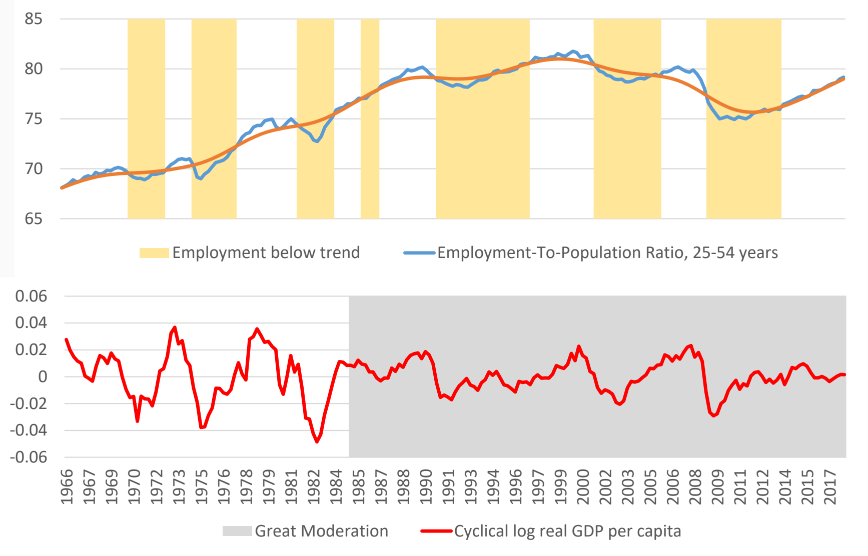
Volatility of shocks

Table 3: Variance of shocks and duration of equilibria

	(a) $\sigma_\xi = 0.04$		(b) $\sigma_\xi = 0.07$	
Fraction of periods at pass. equi.	0.11		0.27	
Average n. of quarters at pass. equi.	11		3.4	
Transition matrix				
	Active	Passive	Active	Passive
Active	0.98	0.02	0.89	0.11
Passive	0.09	0.91	0.29	0.71

- A reduction in volatility increases the duration of each equilibrium.
- If a sufficiently large shock hits, the system switches equilibrium and stays there for a long time.
- This mechanism links Great Moderation to Great Recession.

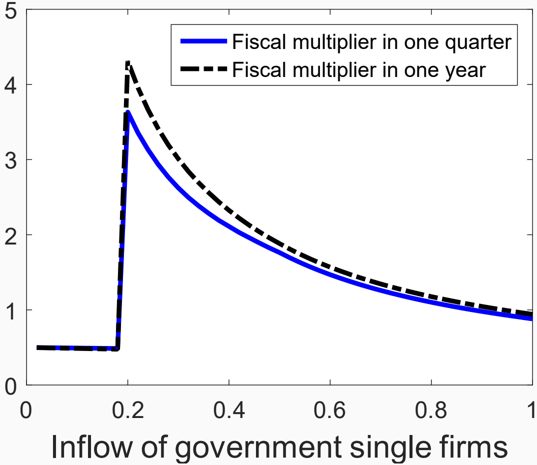
Lower volatility of fundamentals associated with more prolonged spells



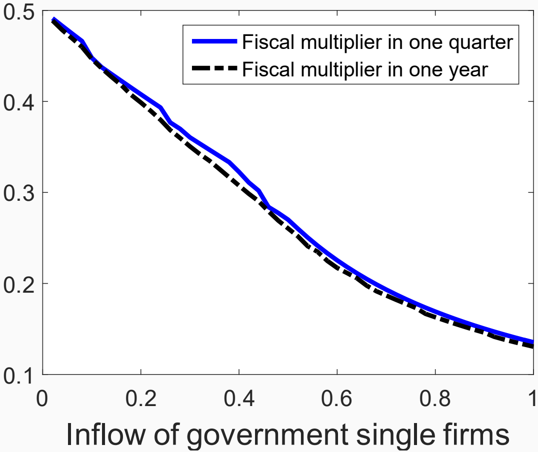
How does fiscal policy work with search complementarities?

- Fiscal policy: exogenous increase in the number of single firms in the final-goods sector.
- The effect of fiscal policy state dependent.
    1. Sufficiently strong fiscal stimulus moves the system from passive to active equilibrium.
    2. Fiscal policy not powerful in the active equilibrium.
- Thus, fiscal multiplier.
    1. (Potentially) large in the passive equilibrium, small in the active equilibrium.
    2. Non-linear and state dependent.

Fiscal multipliers



(a) Passive Equilibrium



(b) Active Equilibrium

Conclusion

- Search complementarities have broad implications for business cycle fluctuations and economic policy.
- Complementarities generate two static equilibria: an active one with large economic activity and a passive one with low economic activity:
  1. Dynamic properties of our economy are unlike those of standard models.
  2. Volatility of shocks determines equilibrium duration.
  3. Effect of fiscal policy is non-linear and state-dependent.
- Additional research:
  1. Embed strategic complementarities in richer models of the business cycle.
  2. Role of agent heterogeneity.