Sorting Out the Real Effects of Credit Supply *

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Abstract

We document that banks which cut lending the most during the Great Recession were lending to the riskiest firms. Motivated by this evidence, we build a competitive matching model of bank-firm relationship, in which firms with riskier projects borrow from the banks with lower holding costs or higher ability to securitize. A firm's ability to borrow depends on the entire distribution of bank holding cost. We derive and estimate a simple measure of this distribution using loan rate and credit ratings data. We conclude that its upward shift or a credit supply effect explains over half of the decline in aggregate loans during the Great Recession.

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1. Introduction

A large macro-finance literature seeks to quantify the credit supply effects of bank lending for business cycle outcomes (Bernanke & Blinder (1988), Bernanke & Gertler (1989), Kashyap *et al.* (1993), Khwaja & Mian (2008), Ivashina & Scharfstein (2010), Chodorow-Reich (2013), Amiti & Weinstein (2018)). The main empirical approach in this literature is to regress the output of firms on the health of the banks they are borrowing from.

There are two difficulties with this approach. First, it requires identifying variations in bank health uncorrelated with firm riskiness. This is sometimes hard to justify: during the Great Recession, measures of bank health were systematically correlated with measures of firm riskiness as we demonstrate below. Second, this approach remains silent on how to relate the cross-sectional effect of bank health to its aggregate effect on total lending (see, in particular, Jiménez *et al.* (2019) and Herreno (2019)).

We propose a structural approach that overcomes these two difficulties. We present a sorting model in which firms with the riskiest projects borrow from the banks with the lowest holding costs or the highest ability to securitize. In the model, a firm's ability to borrow depends on the entire distribution of banks' holding costs, not just on the holding cost of the bank it is currently borrowing from (as is typically assumed in the literature). We use the model to disentangle the effect of bank holding costs and firm riskiness on aggregate lending.

We start by providing two sets of findings that point to the importance of sorting in credit markets and the large aggregate loan fluctuations for risky firms. Our empirics in categorizing what constitutes a hit bank follow the protocol in the literature (Ivashina & Scharfstein (2010), Chodorow-Reich (2013)). We use corporate loan data from Dealscan and define hit banks in three ways: (1) low lending growth over the 2006-2008 period, (2) low Lehman distance defined as fraction of a bank's syndication portfolio where Lehman Brothers had no lead role, and (3) low deposit to asset ratios. We then measure the downside risk of firms using three metrics: (1) borrower loan spread or interest rates, (2) the yields of the public debt for firms that issue in both public and syndicated private corporate loan markets, and (3) borrower leverage.

First, firms with greater downside risk are much more likely to be in the loan portfolios of hit banks. This correlation is robust across the downside risk measures used. The economic magnitudes are large. When we use borrower loan or bond spread as our measure of firm downside risk, we get a decrease of bank lending growth by around 0.70 standard deviation with a t-statistic of around 4. A one unit increase in the average borrower leverage is associated with a decrease of bank lending growth by 3.7 standard deviation with a t-statistic of 3.32.

Our findings differ from Schwert (2018), where there is if anything a negative correlation between borrower risk measured using firm leverage and bank size. The difference is due to our samples — he excludes investments banks while we keep them. We retain corporate loans from investment banks such as Lehman, Goldman or Bear Stearns since they made significant amounts of corporate loans during our sample period which they ultimately sold as securitized products (Shivdasani & Wang (2011)). That is, we document that banks which cut lending during the Great Recession of 2008 made and securitized loans to risky firms.

Second, firms with greater downside risk cut their CAPX more over the period of 2006-2008 than less risky firms—that is downside risk firms are more cyclical. This is true even controlling for bank fixed effects. A one percentage point (p.p) increase in loan spread (or in bond spread for public firms) is associated with a 5% decrease in the CAPX growth and t-statistics of around 3. This is consistent with extant evidence that firms with greater downside risk are more sensitive to business cycles even outside of the Great Recession (Philippon (2009), Greenwood & Hanson (2013)).

Guided by this evidence, we develop a competitive matching model of a credit market where banks only differ in their holding costs of loans and firms only differ in their risks. For instance, bank heterogeneity can be interpreted as quality differences in bank health, risk management or securitization programs.[1] Given our evidence that a firm's first and second lenders are highly correlated in terms of attributes, we assume one-to-one matching and abstract away from a risky firm borrowing from multiple banks of the same ability. Our focus is on the main forces of underlying firm risk and differential bank holding costs.[2]

We first establish a sufficient condition for sorting: the cross partial derivative of the bank holding cost function with respect to bank type and firm risk is less than zero. The safest firms with the best credit ratings benefit least from bank talent. They match with the least talented banks (i.e. those with highest holding costs) and get the lowest interest rates and all the surplus in the relationship.

In contrast, the riskiest firms benefit the most from bank talent. So they match with the banks with the lowest holding costs and those talented banks get all of the surplus in the match. The interest rate that the best banks charge is dependent on competitive supply (i.e. heterogeneity in banking talent or holding costs) and demand (i.e. heterogeneity in firm credit risk) across the entire credit market. For the best banks, the more scarce their talent, the more surplus they can extract through higher loan payoffs.

The equilibrium condition that pins down the cutoff type (i.e., the aggregate loan supply) also depends on the aggregate distribution of banks' holding costs. We establish that this condition can be interpreted from the viewpoint of a social planner: The benefit of including one additional firm is the NPV of the project net the holding cost valued at the most talented bank. The cost, however, is moving other firms to less talented banks. When the talent becomes scarce, the cost increases and thus less firms can borrow. Through this channel, a firm's ability to borrow thus depends on the distribution of bank

[1] For example, talented banks securitize and offer lower interest rates than traditional banks during the years before the Great Recession of 2008 (Nadauld & Weisbach (2012)).

[2] We can think of the model as describing the matching of risky firms to banks that can securitize. Firms might borrow from multiple banks of roughly the same ability to securitize for various other reasons.

holding cost, which is in contrast to the traditional view underlying the panel regression approach.

To estimate the distribution of bank holding costs which is key to calculating the real effects of credit supply, we need to make a functional form assumption regarding holding cost. We make a separability assumption in this paper, which yields the simplest estimation procedure. But our model can be estimated under general functional forms, which we leave to future research.

Under the assumption that the bank's holding cost function is separable in bank health or talent and firm default probability, we can derive a measure of bank holding cost by looking at the gradient of loan payoffs and default probability with respect to the ranking of firm default probability. Consider two firms with default probability rank i and $i+1$ where i is the riskier firm. The difference in the loan payoff for firm i and $i+1$ divided by the change in the respective default probabilities measures bank holding cost. A key prediction of our sorting model is that firms lending to the riskiest firms should have the lowest holding costs — hence bank holding costs should be monotonically falling with firm credit risk.

In our empirics to infer this bank holding cost distribution, we focus on our subsample of firms that issue both public debt and that borrow from banks. We use the credit rating of the firm's last issued debt as our proxy for firm credit risk. We can then calculate our measure of bank holding cost using the historical default probabilities associated with these credit ratings. We estimate this cross-sectional distribution for the 2005-2007 period and the 2008-2010 period. We verify the prediction that bank holding costs falls as firm credit risk rises.

We can also compare the cross-sectional distribution of holding costs before and during the Great Recession. A credit supply effect in our model then is simply an upward shift in bank holding costs during the Great Recession of 2008. We indeed find that there were fewer banks that had low holding costs in the crisis period. Moreover, even for the less talented banks their holding costs roughly doubled. We provide bootstrap standard errors and discuss some robustness calculations regarding these estimates.

We then take these estimates to calculate a counterfactual — holding fixed the pre-crisis firm default rates, how much can the change in distribution of bank holding costs account for the drop in the loans supplied to risky firms? Our estimates point to a sizeable credit supply effect. We find that the upward shift in the bank holding cost distribution accounts for 61.3% of the observed drop in aggregate number of loans during the Great Recession. Even though the credit risk of firms went up significantly during the crisis, credit supply nonetheless contributed significantly to the drop off in aggregate number of loans because it was the banks best at securitization and that could absorb the riskiest loans pre-crisis that could no longer perform this role post-crisis.

For simplicity, we have assumed that loan size is the same across firms. As such, we have focused on how the fraction of firms in the economy that get a loan depends on the distribution of bank holding cost and distribution of firm risk. Our model can be extended to also allow loan size to vary across firms by modeling heterogeneous expected project value — our analysis goes through except that one must also condition on loan size.

Finally, we then compare our new approach to the panel regression approach in the literature using risk controls or firm fixed effects. Our approaches are by and large complementary. Existing empirical tests assume that a firm's borrowing decision depends only on the health of the banks it is borrowing from and there is a random sorting aspect of firm-bank matches. In contrast, our competitive sorting model, similar to matching models for CEO labor or underwriters (Terviö (2008), Gabaix & Landier (2008), Chang & Hong (2019)), shows that a firm's borrowing decision depends on the health distribution of all banks in the economy. That is, current panel regression estimates establish that there are potential credit supply effects as opposed to what we do, which is to quantify how much of the drop in aggregate loan supply in risky firms is accounted for by credit supply effects.

Moreover, our sorting model can potentially also guide panel regression approach empirics. To the extent there is measurement error regarding firm risk, both credit risk and the hit status of a bank can be informative about loan outcomes.[3] The panel regression approach addresses this issue by comparing estimates obtained with and without firm fixed effects. The fact that tests in the literature find similar estimates with and without firm fixed effects is often interpreted as no sorting on unobservables. However, our analysis suggests that this conclusion is not generally true.

Our analysis follows earlier work providing evidence for the validity of positive assortative matching models in credit markets (Chen & Song (2013), Schwert (2018)). We differ in terms of our focus on leveraging such models to separate credit demand versus supply effects, particularly in regards to examining macroeconomic implications tied to the Great Recession. Moreover, our analysis differs in assuming that only firm credit risk is observable and that bank health or talent is unobservable, i.e. it need not be related to size. We instead back out bank health or talent using competitive sorting and cost function restrictions. Future work can consider other measures of firm risk and potentially even loosen the assumption of firm risk observability (Chiappori & Salanié (2016)).

Our analysis is also related recent papers on the origins of the Great Recession. Justiniano *et al.* (2019) point to the importance of lending standards as opposed to collateral requirements for the housing boom. Our approach to disentangle credit supply from demand shocks might also be applied in the context of mortgages. Martinez-Miera & Repullo (2017) point to the the role of low interest rates in creating financial fragility. These lower

[3]Indeed, the banking literature has uncovered a number of factors such as size (Berger *et al.* (2005)), distance between borrowers and lenders (Petersen & Rajan (2002)), specialized knowledge of export markets (Paravisini *et al.* (2014)), and friendships between bank and firm CEOs Engelberg *et al.* (2012) that drive sorting, consistent with potential measurement error issues with credit risk.

interest rates, we suggest, arise from the expansion of securitized products and sorting in credit markets.

2. Data

Dealscan. Our principal source of data is the Thomson Reuters Dealscan database. Dealscan collects loan-level information on syndicated loans from Securities and Exchange Commission filings, company statements, and media reports. The data include the names of the borrowers and lenders, the purpose of the loan, the role of each bank in the agreement (underwriter, agent, adviser, etc...), and the interest rate of the loan. Following the literature, we exclude loans to financial companies (SIC between 6000 and 6999) from the sample. Because we focus on the Financial Crisis or Great Recession period around 2008-2009, we only keep borrowers that obtained a loan between 2004 and August 2008 or that obtained a loan prior to 2004 that matured after October 2007.

Bank Characteristics. We are interested in bank characteristics that are related to the health of the bank during the financial crisis. These three bank characteristics are constructed following Chodorow-Reich (2013). The first measure is simply the bank *lending growth* ΔL_{it} during the financial crisis. It is defined as

$$\Delta L_{it} = \frac{L_{\text{crisis}}}{L_{\text{normal}}},$$

where L_{crisis} denotes the total amount of loans originated in the 9-month period from October 2008 to June 2009, and L_{normal} denotes the average of the the total amount of loans originated from October 2005 to June 2006 and October 2006 to June 2007. That is, we are measuring the abnormal loan growth over the financial crisis period by normalizing with loan growth over the prior years during the same October to June window.

The second bank characteristic is the *Lehman distance*. Following Ivashina & Scharfstein (2010) and Chodorow-Reich (2013), it is given by the fraction of a bank's syndication portfolio where Lehman Brothers had no lead role. This characteristic is directly related to the exposure of the bank to the financial crisis: because firms in a relationship with Lehman drew down their lines more than other firms, this decreased the liquidity of all the banks in the same syndicate as Lehman. The third bank characteristic is the ratio of *bank deposit to asset*. This characteristic is traditionally seen as a measure of bank safety. To facilitate ease of interpretation, all bank characteristics are normalized to have a standard deviation of one across the sample.

Borrower Characteristics. We are interested in borrower characteristics that are associated with downside risk. We propose three different measures. The first measure is the average of all-in-drawn *loan spread*, which is the borrower last loan's credit spread over LIBOR plus annual fees to the lenders.

The second measure is the *bond spread*, which is the average spread of public bonds issued by the borrower measured in January 2007 before the crisis. To obtain this borrower level characteristic, we first look for bonds issued by the borrower in the Fixed Income Securities Database (FISD).[4] We then obtain the spread of each bond in January 2007 from the Lehman Corporate Bond Data. We obtain a borrower level spread by averaging the spread of all outstanding bonds in January 2007, weighted by their face value. Because this measure is only available for firms that issue public bonds, this only covers 30% of the initial sample of all borrowers.

The last measure is the *market leverage* of each firm in January 2007. To obtain it, we match borrowers to Compustat using the DealScan-Compustat Link from Chava & Roberts (2008). We define leverage as the ratio of market value of asset to market value

[4] We match Dealscan with FISD by using the first 6 numbers of the CUSIP, keeping only matches with similar names.

of equity.[5]

3. Sorting and Aggregate Loan Fluctuations

In this section, we first show that these downside risk firms are much more likely to be in the portfolios of hit banks — where a hit bank is variously defined using the measures of lending growth during the financial crisis, distance from Lehman in terms of co-syndication and deposit to asset ratio. Hit banks, which were typically investment banks, securitized much of their corporate loans. Second, we then present evidence that our three measures of firm downside risk are negatively related to firm performance measured using CAPX during the financial crisis period. That is, these two sets of findings are consistent with sorting in credit markets being a first-order concern when evaluating the causal inference of bank capital shocks for firm output.

3.1. Bank Health and Firm Risk

We are interested in measuring the systematic relationship between a borrower exposure to downside risk and the health of their bank during the financial crisis. To do this, we estimate by OLS the following model:

$$Y_i = \alpha + \beta \overline{X}_i + \epsilon_{it},$$

where i denotes a bank, Y_i is a measure of the change in bank health during the financial crisis, and $\overline{X}_i$ reports the average of the borrowers' characteristics for all syndicated loans in which the bank is a lead lender.

Panel A of Table 1 reports the result of regressing bank lending growth during the

[5] The market value of asset is constructed as the market value of equity plus debt in current liabilities and long term debt.

crisis on our three measures of borrower downside risk. We find that all three measures of borrower downside risk negatively predict corresponding bank lending growth. The banks that did the worst during the crisis were systematically lending to firms with higher loan spread, higher bond spread, and higher leverage.

Using borrower loan spread, we find that a one p.p. increase in the average borrower loan spread is associated with a decrease of bank lending growth by 0.65 standard deviation. This is a not only a large economic effect but also highly statistically significant with a t-statistic of 3.25. The R^2 from the regression is 19%. When we use borrower bond spread as our measure of firm downside risk, we get a decrease of bank lending growth by 0.70 standard deviation with a t-statistic of 4.21. Furthermore, a one unit increase in the average borrower leverage is associated with a decrease of bank lending growth by 3.7 standard deviation and t-statistic of 3.32. Again, regardless of the measure of borrower downside risk we use, we find significant sorting.

To visualize this sorting result, Figure 1 displays a scatter-plot between bank lending growth from 2006 to 2009 on the y-axis and borrower loan spread on the x-axis. The larger circles indicate the number of loans from the bank and hence reflect the size of the bank. Investment banks such as Lehman Brothers, Bear Stearns, and Merrill Lynch all drastically decreased lending during the period and were also matched with borrowers with high spreads, i..e high downside risk firms. Notice that the banks at the extremes of this sorting are investment banks. Banking studies in the literature often drop these investment banks from their analyses but it is clear in this setting that they are economically relevant players in the corporate loans market.

Panel B of Table 1 reports the result of regressing bank Lehman distance on the three measures of borrower characteristics. We find that all three measures of borrower downside risk are negatively related to Lehman distance. The correlation is actually even stronger quantitatively than when it comes to predicting bank lending growth in Panel

A. Using borrower loan spread, a one p.p. increase in the average borrower loan spread is associated with a decrease of Lehman distance by more than 1.37 standard deviations. The t-statistic if 3.39. The R^2 is now 37%. Using borrower bond spread, the economic magnitude with minus 1.18 standard deviations with a t-statistic of 3.34. A one unit increase in the average borrower leverage is associated with a decrease of bank lending growth by 8.33 standard deviation. The t-statistic is 3.99 and the R^2 is 52%.

Panel C of Table 1 reports the result of regressing bank deposit on the three measures of firm downside risk. The results are very similar to the Lehman distance results in Panel B. Quantitatively, a one p.p. increase in the average borrower loan spread is associated with a decrease of bank deposits by 1.24 standard deviations with a t-statistic of 5.3 and an R^2 of 48%. Using the borrower loan spread, the coefficient is -1.06 with a t-statistic of 4.07. A one unit increase in the average borrower leverage is associated with a decrease of bank lending growth by 6.15 standard deviations. The t-statistic is 6.55 with an R^2 of 47%.

Overall, these results suggests a strong positive assortative matching between banks and borrowers. Firms that are more exposed to downside risk tend to be matched with banks that were particularly hit during the financial crisis. This matching matters to understand the dispersion of bank level outcome during recessions. Banks that were more exposed to the Financial Crisis were also matched to firms that did particularly badly during this period. In this world, it is hard to differentiate between the "bank" effect and the "firm" effect in explaining the negative real effects during the Financial Crisis associated with risky firms borrowing from hit banks.

3.2. Heterogeneity in Underwriting Securitized Products

Many investment banks such as Lehman, Goldman or Bear Stearns, despite being considerably smaller in deposit size than traditional banks, were also leaders in the securitization of risky corporate loans, such as those for leveraged buyouts (Shivdasani & Wang (2011)). Marketing documents for Lehman during this period pointed to all the awards that they had recognizing their special ability to assess and package the riskiest loans compared to other banks. This expertise consists of having large distribution networks or marketing ability to sell their loans at favorable prices, i.e. with less underpricing.

This ability recalls the initial public offering (IPO) underpricing literature. This literature also focused on the ability of certain banks to underwrite IPOs with lower underpricing (Carter & Manaster (1990), Ritter & Welch (2002)) through prestige, certification effects, or marketing ability. As a result, there is naturally positive assortative sorting of the risky IPOs to be underwritten by banks that have more prestige and marketing ability (Fernando *et al.* (2005), Akkus *et al.* (2016)).

3.3. Downside Risk and CAPX

We next show that our three firm downside risk measures are negatively related to firm performance during the financial crisis—that is our measures of downside risk are correlated with cyclicality or sensitivity to economic downturns. To show this, we run regressions of the form

$$\Delta CAPX_i = \alpha + \beta X_i + \epsilon_i,$$

where CAPX from Compustat measures the growth of investment between 2006 and 2009, and X_i denotes one of the three borrower characteristics. We run these specifications with and without fixed effects.

Table 2 reports the results. In column (1), we find that borrower loan spread from the Dealsacan universe is negatively correlated with borrower CAPX growth from 2006 to 2009. A a one percentage point (p.p) increase in loan spread (or in bond spread for public firms) is associated with a 5% decrease in the CAPX growth. In column (2), we add bank fixed effects. That is for the firms in the portfolio of the same bank, we find that borrower loan spread predicts a lower CAPX growth over the financial crisis period. Interestingly, the coefficients of interest are virtually identical in columns (1) and (2) and both have t-statistics around 3.

In columns (3) and (4), our independent variable if interest if borrow bond spread from their public debt. Notice that the number of observations is around 500 in columns (3) and (4) compared to columns (1) and (2). As we mentioned earlier, the reason for difference in sample sizes across the two measures is that not all firms issue public debt, which is a prerequisite for having our second measure Nonetheless, this subsample is interesting since it provides a different measure of borrower downside risk. We get virtually identical point estimates in column (3) without bank fixed effects and in column (4) with bank fixed effects as in columns (1) and (2) when we had the entire Dealscan sample. In column (3), the coefficient of interest is -0.049 with a t-statistic of -2.8. In column (4), we have a coefficient of -0.048 with a t-statistic of -2.6 when we introduce bank fixed effects. Not only are these two coefficient similar with each other, they are also similar to those obtained in columns (1) and (2).

In columns (5) and (6), we find that a one standard deviation increase in market leverage is associated with a 1.3% decrease in CAPX growth when there are no bank fixed effects and 1.8% decrease in CAPX growth when there are bank fixed effects. The sample here is almost similar to Dealscan absent data lost in the merger between Dealscan and Compustat. The t-statistics in columns (5) and (6) are comparable to those in the earlier columns. Hence, regardless of our measure of borrower downside risk, we get robustly similar results that risky borrowers are more procyclical than less risky ones.

The findings in Table 2 can to a large extent be anticipated from findings in earlier papers. Risky firms should be more procyclical than less risky firms as in a Q-theory set-up proposed in Philippon (2009). Moreover, Greenwood & Hanson (2013) find that aggregate corporate debt issuance is positively correlated with aggregate credit risk appetite. They find that credit risk appetite was extremely high before the financial crisis and fell substantially during the financial crisis. While these findings might to a large extent be anticipated, they are nonetheless important for the macro-finance literature on credit supply effects to the extent borrower risk characteristics are correlated with whether or not a bank is hit during the financial crisis.

4. Credit Supply Effects in a Sorting Framework

In this section, we develop a competitive matching model of a credit market where banks differ in their costs of holding risks. This model allows us to examine the conditions under which there is assortative matching between firms and banks. Moreover, because we explicitly model the matching decision between firms and banks, this also allows us to examine the full effect of a credit shock, including the reallocation of firms to different banks (i.e. through the extensive margin). In this set up, we stress that a firm borrowing decision does not only depend on the health of the banks it is borrowing from, but also on the health distribution of all the banks in the economy.

We first establish the conditions under which firms with greater downside risk borrow from banks that are better at absorbing risks. For instance, banks that are better at securitization and can sell loans for lower underpricing would effectively incur lower holding costs. More generally, one can interpret more talented banks as those that are better at risk management or in better health. The goal of our model is to offer guidance on how to account for the role of credit supply effects on aggregate loan fluctuations, particularly to risky firms.

Credit Market. We consider a competitive matching model of a credit market with heterogeneous firms and banks. Firms own the project but do not have capital. Banks are endowed with one unit of capital, and they can either offer funding to a firm or earn a risk-free gross return $1 + r_f$.

There is a continuum of firms. Each firm has one project that requires 1 unit investment. The project of firm $i \in [0, N]$ succeeds with probability $1 - \delta[i]$ and yields return $y_H[i]$. It fails with probability $\delta[i]$ and yields return $y_L[i]$. A high i denotes a firm with higher probability of default. That is, firms are ranked by their default probability $\delta'[i] \geq 0$. We make the following two assumptions:

Assumption A1. $y_H[i] \geq 1 + r_f > y_L[i]$, i.e. the payoff is large enough to pay back the loan at the risk free rate only if the project succeeds;

Assumption A2. The NPV of the project, $NPV[i] \equiv (1-\delta[i])y_H[i]+\delta[i]y_L[i]-(1+r_f) \equiv y$, is constant across i.

That is, firms with higher i are more likely to default.

There is also a continuum of heterogeneous risk-neutral banks, indexed by $j \in [0, N]$, who differ in their ability of holding risks. For instance, one can interpret banks that are better at securitization would effectively incur lower holding costs. More generally, one can interpret more talented banks as those that are better at risk management.

Specifically, let $C(i, j)$ denote the cost of holding risk for bank j when lending to the firm i. We assume that $C_1(i, j) \geq 0$: fixing any bank j, the cost is higher if lending to riskier firms (i.e., the ones with higher default probability). We also assume that $C_2(i, j) \leq 0$: a bank with a higher j is better at holding risk in the sense that it has a lower holding cost given any firm i.[6]

[6] A bank holding cost function that satisfies this condition comes from interpreting the cost as the underpricing that the market requires from a bank when they sell the securitized loans to the market due to informational frictions (Rock (1986)).

For simplicity, we assume that each firm can only at most borrow from one bank. That is, any individual firm i can be matched with a bank j or remains autarky. Within a match, bank and firm agree on a debt contract, which specifies a promised repayment d. We assume that the project is pledgeable and thus when the firm defaults, the bank obtains the value of the project. Banks thus understand that they will receive $\min\{d, y_L[i]\}$. Banks' payoff when lending to firm i with repayment d thus yields,

$$w(i, j|d) = (1 - \delta[i])d + \delta[i]y_L[i] - C(i, j) - (1 + r_f).$$

The firm's payoff, on the other hand, yields

$$u(i, j|d) = (1 - \delta[i])(y_H[i] - d).$$

While a firm's payoff does not directly depend on its matching bank b, different banks can charge different interest rates in equilibrium, and thus banks' abilities can indirectly affect firms' payoffs.

The joint surplus between a matching pair is given by:

$$s(i, j) \equiv w(i, j|d) + u(i, j|d) = y - C(i, j),$$

which is simply the NPV of the project minus its holding cost and it is independent of the repayment d. In other words, the repayment d only affects how these two agents split the surplus, where a higher repayment d means a lower surplus to the firm.

Our equilibrium concept follows the standard assignment model. The bank's decision can be rewritten as choosing the firm optimally while taking the equilibrium utility $U[i]$ as given:

$$W(j) = \max_i \{y - C(i, j) - U[i]\}. \tag{1}$$

Fixing any given level of firm payoff, lending to a riskier firm means a higher holding cost. Hence all banks prefer to match with safer firms. As all banks compete for those firms, in equilibrium, these firms must obtain higher payoffs. That is, safer firms must receive a better loan term in the sense that, $U[i]$ must decrease in i.

In equilibrium, all banks thus trade off between a riskier firm vs. a higher rate. Banks lending to the riskier firms are thus compensated by getting a higher payment. The matching outcome is then determined by which bank is more willing to absorb firm risk.

Lemma 1. *If $C_{12}(i, j) < 0$, a more talented bank is matched with a riskier firm.*

When $C_{12}(i, j) < 0$, it means that a bank's ability matters more for a riskier firm in the sense that it can reduce the firm's borrowing cost more. Thus, it is relatively cheap for banks with higher ability to take on the riskier loans. Formally, it implies that there is a complementarity between firm riskiness and bank ability in the surplus function $s_{12}(i, j) = -C_{12}(i, j^*(i)) > 0$, which thus explains Lemma 1.

Given the sorting outcome i, j the firms' utility $U[i]$ is pinned down competitively in equilibrium so that all banks are matched with the correct firms. That is,

$$U'[i] = -C_1(i, j). \tag{2}$$

That is, as standard in the matching model, the marginal payoff increases for firm i is given by his contribution to the surplus within the match given his optimal assignment $j^*(i)$.

Lemma 2. *When $C_{12}(i, j) < 0$, the equilibrium consists of a unique cutoff type $i^* \in [0, N]$ such that (1) for all $i \leq i^*$, his matching bank is given by $j^*(i) = N - (i^* - i)$ and (2) $U'[i]$ solves Equation 2 with $U[i^*] = 0$.*

Notice that the marginal risky cut-off firm i^* is matched with the best bank $j^*(i^*) = N$, while the safest firm $i = 0$ is matched with the marginal bank $N - i^*$.

4.1. Implications for Aggregate Loan Risk Capacity

In a model without sorting, one might think a firm's ability of getting a loan only depends on the health of his bank. This is no longer true when there is sorting among firms and banks. In particular, as shown in Lemma 2, the marginal firms (i.e., the riskiest firm conditional on getting a loan) must be matched with the most talented bank. Thus, intuitively, adding one more firm into the market means that all existing firms must be reallocated to a less talented bank.

Formally, the risk capacity of the banking sector (i.e., the aggregate loan amount) is measured by the cutoff type. We now elaborate on how the marginal type is determined. According to Lemma 2, given $U'[i]$ and the boundary condition $U[i^*] = 0$, we thus have

$$0 = U[0] - \int_0^{i^*} C_1(i, j^*(i))di. \tag{3}$$

Given that the safest type receives all the surplus $U[0] = y - C(0, N - i^*)$, Equation (3) can then be rewritten as

$$y = C(0, N - i^*) + \int_0^{i^*} C_1(i, j^*(i))di, \tag{4}$$

where the RHS of the equation can be interpreted as the aggregate cost of holding risks, given the sorting in the market.

Similarly, the same condition can be seen from the view point of banks. Applying the envelope theorem to Equation 1, the marginal increase in the payoff of bank j is pinned down by $W'(j) = -C_2(\imath^*(j), j)$, where $\imath^*(j)$ is the inverse of $j^*(i)$, representing the firm that bank j is matched with. The payoff to the best bank relative the marginal bank is given by

$$W(N) - W(N - i^*) = -\int_{N-i^*}^{N} C_2(\imath^*(j'), j')dj', \tag{5}$$

which highlights that his payoff depends on the ability of other banks below him, where the RHS of this equation goes to zero when all agents have the same ability of holding risks (i.e., $C_2(i,j) \to 0$). According to Lemma 2, the payoff of the marginal bank must be zero, $W(N-i^*) = 0$, as he is matched to the best firm, who must obtain all the surplus. The best bank $W(N)$, on the other hand, is matched to the riskiest firm and must obtain all the surplus. Thus, Equation (5) can be expressed as

$$W^*(N) = y - C(i^*, N) = -\int_{N-i^*}^{N} C_2(\imath^*(j'), j')dj' > 0, \tag{6}$$

which uniquely pins down the marginal types i^*.

One can show that, with integration by parts, Equations (4) and (6) are equivalent. This is because that, due to market clearing, the effect of adding one additional firm is the same as adding one additional bank.

Both equations highlight that the compensation to the best bank and the utility to the safest firm are pinned down by their competing banks and firms. Intuitively, if other banks become better at taking risks and/or other firms become safer, then the marginal types i^* go up and thus lower $W[N]$ and $U[0]$.

As in the standard matching model, the equilibrium is necessarily efficient. Indeed, Equation (6) can be interpreted from the social planner's view point. The LHS of Equation (6) can be understood as the benefit of adding the marginal firm, which is the net NPV of that firm, taking into account the holding cost valued at the best bank.

The RHS of Equation (6), on the other hand, captures the distributional effect that the holding costs have increased for the existing firms' loans, as each of them is now borrowing from a slightly worse bank, captured by $C_2(i,j) < 0$.

The marginal type is thus pinned down so that the benefit equals the cost. Note that, exactly because of the negative sorting, the market would not lend to some firms even though they have positive surplus. To see this, given that Equation 6 is strictly larger than zero, it means that firms that are slightly above the marginal type must have positive surplus ($y - C(i^* + \epsilon, N) > 0$). They are nevertheless excluded. This is in contrast to the positive sorting case, where the marginal type is pinned down so that their joint surplus is zero.

Given that the aggregate distribution matters for the aggregate loan supply, the following lemma below formalizes how it is affected by banks' holding cost (supply effect) and by firms' characteristics (demand effect).

Talent scarcity. Let $C(i, j) = c(\delta[i], \kappa[j])$, where the holding cost $c(\delta, \kappa)$ increase with firms' default probability and banks' costs of holding risks denoted by κ. That is, $c_\delta > 0$, $c_\kappa > 0$, $c_{\delta\kappa} > 0$, and $\kappa'[j] < 0$. We define talents become more scarce when $\kappa'[j]$ becomes steeper (less talented banks) while the best bank's ability $\kappa[N]$ remains the same.[7]

Lemma 3. *The total loan supply decreases for more scarce talents, riskier firms ($\tilde{\gamma}[i] \geq \gamma[i]$), and lower y.*

4.2. Model-Based Decomposition

We now use our model to decompose these two effects and to estimate banks' risk-bearing capacity. In general, the holding cost of a bank and their ability of holding risks are not observable. Nevertheless, we demonstrate below that, using the equilibrium condition in the sorting model, one can infer bank's ability by using the information from interest rates and firms' default probability.

Let $L[i] \equiv y - U[i]$ denotes the loan payoff of the bank that lends to firm i. Such a payoff is observable and can be measured by $L[i] = (1 - \delta[i])D^*[i] + \delta[i]y_L[i]$, where $D^*[i]$

[7]By definition, $\kappa[N] = \kappa[j] + \int_j^N \kappa'[j']dj'$. Hence, a more scarce talents means that a distribution under which $\kappa[j]$ is weakly higher.

is the repayment and $y_L[i]$ is the recover value. Equation (2) can thus be rewritten as

$$\frac{L'[i]}{\delta'[i]} = c_\delta(\delta[i], \kappa[j^*(i)]), \tag{7}$$

which increases with matching banks' holding cost, $\kappa[j^*(i)]$, conditional on firm's characteristic $\delta[i]$. That is, assuming the cost function, one can use the estimate above to infer the matching banks' holding cost $\kappa[j^*(i)]$.

4.3. Counterfactual Exercise

As shown in Lemma 3, the total loan supply depends on firm risks (i.e., the distribution of firms $\delta[i]$ and their the NPV of the project y), as well as the banks' ability to hold risks (i.e, the distribution of bank $\kappa[i]$). In other words, aggregate loan volume depends on both credit demand and supply effects.

We now use our estimates to answer how much of the drop in loan volume during the crisis is driven by the change in credit supply (i.e., change in banks' ability). That is, we ask what is the counterfactual loan supply during the crisis if firms' risk characteristics remain the same.

Formally, let $t \in \{c, 0\}$ denote the crisis and non-crisis periods and $i^*(\delta_t[i], y_t, \kappa_t[j])$ denote the marginal type given the characteristics of banks $\kappa_t[j]$ and firms $(\delta_t[i], y_t)$ at period $t \in \{c, 0\}$. The counterfactual cutoff type is then calculated assuming the firm distribution remains the same as before the crisis $(\delta_0[i], y_0)$ while the banks ability are changed to $\kappa_c[j]$, which is denoted by $i^*(\delta_0[i], y_0, \kappa_c[i])$.

According to Equation 6, $i^*(\delta_0[i], y_0, \kappa_c[i])$ is then given by the variable i^* that solves the following equation:

$$y_0 = c(\delta_0[0], \kappa_c[N - i^*]) + \int_0^{i^*} c_\delta(\delta_0[i], \kappa_c[N - (i^* - i)])\delta_0'[i]di \tag{8}$$

where $i_0^* \equiv i^*(\delta_0[i], y_0, \kappa_0[i])$ is the cutoff type during the non-crisis period.

Measure of the Credit Supply Effect. Based on our model, we thus formally define the credit supply effect as

$$\phi^S \equiv \frac{i^*(\delta_0[i], y_0, \kappa_0[i]) - i^*(\delta_0[i], y_0, \kappa_c[i])}{i^*(\delta_0[i], y_0, \kappa_0[i]) - i^*(\delta_c[i], y_c, \kappa_c[i])},$$

where the denominator represents the change in the loan volume during the crisis, where both firms characteristics and banks wealth have changed. The numerator, however, represents the change in the aggregate loan if firms' characteristics remain the same. That is, by definition, if all firms' characteristics indeed remain the same (i.e., $\delta_0[i] = \delta_c[i]$, and $y_0 = y_c$), then $\phi^S = 1$. That is, the change in the credit supply are purely driven by banks' wealth.

Similarly, one can define the demand effect as the change in the loan supply by consider the counterfactual where banks' characteristics remain the same

$$\phi^D \equiv \frac{i^*(\delta_0[i], y_0, \kappa_0[i]) - i^*(\delta_c[i], y_c, \kappa_0[i])}{i^*(\delta_0[i], y_0, \kappa_0[i]) - i^*(\delta_c[i], y_c, \kappa_c[i])},$$

and thus $\phi^D = 1$ if all banks indeed remain the same.

5. Estimates

In this section, we estimate the holding costs of individual banks and then calculate the credit supply effect, i.e. the counterfactual exercise, given these estimates.

Holding Cost Function of Individual Banks. We now focus on a separable cost function, to illustrate how to use our estimate to derive a simple measure of bank health and conduct a counterfactual calculation. The same procedure can nevertheless be applied to different forms of the cost function.

Assumption A3. Cost function is separable: $C(i, j) = \delta[i]\kappa[j]$.

With this assumption, Equation (2) is thus reduced to

$$\frac{L'[i]}{\delta'[i]} = \kappa[j^*(i)] = \kappa([N - (i^* - i)]). \tag{9}$$

That is, the ratio of change in loan payoff and the change in default probability of firm i captures the holding cost of its matching bank $\kappa[j^*(i)]$ with the optimal assignment $j^*(i) = N - (i^* - i)$. Note that, under this simplified Assumption A3, right hand side only depends on bank's characteristics $\kappa[j]$ but not firm's characteristics $\delta[i]$. This is however not generally true under other specified cost functions. Nevertheless, one can still back out $\kappa[j]$, given that $\delta[i]$ is observable.

Recall that i^* is the default probability of the cut-off firm. So $i^* - i$ close to zero correspond to risky firms close to the cut-off, while large values of $i^* - i$ correspond to safer firms. The sorting implies that the most risky firm (i.e., $(i^* - i) \to 0$) is always matched with the best bank, who has the lowest cost $\kappa[N]$; the safest firm (i.e., $(i^* - i) \to i^*$, on the other hand, is matched to the worst bank conditioned on that bank being active with $\kappa[N - i^*]$.

More generally, we thus expect that safer firms (firms with higher $i^* - i$) are matched with banks with higher costs. That is, $\kappa[N - (i^* - i)]$ is increasing in $i^* - i$ when there is heterogeneity in banks' ability. In the case when banks are homogeneous, we would then expect the ratio is then constant across all ranking.

Estimates of Banks' Holding Cost Distribution for Pre-Crisis and Crisis Periods. We now calculate banks' holding cost distributions for two time periods: a pre-crisis span from 2005-2007 and a crisis span of 2008-2010. In the pre-crisis period, we have roughly 4000 firms with loans from Dealscan. In the crisis period, we have roughly 2500 firms. We take each firm's Moody's credit rating and use the average historical default rate associated with that rating as our proxy for $\delta[i]$. As our proxy for $L[i]$, we take this to be $(1-\delta[i])$ times the credit spread over Libor for the firm loan. We assume that $y_L[i] = 0$ in this calculation. That is, we assume that heterogeneity in recovery rate is small across firms relative to the credit spread, which is a reasonable assumption.

In Figure 2, we plot on the x-axis $i^* - i$, firm loan ranks based on default probability. We have $i^*_{2005} = 4000$ loans in the pre-crisis period. We use 4000 to normalize the x-axis. On the y-axis is the default probability. Firms with $i^* - i$ near zero are riskier, while firms with $i^* - i$ closer to one are safer. We construct four bins for the 2005-2007 with around 1000 observations per bin. We can see that default probabilities are near zero for firms closer to 1 but rise considerably as we get closer to 0. For firms near the cut-off i^*, the default probability is around .18.

For the 2008-2010 sample, we also create four bins similar to the pre-crisis period.[8] There are roughly 600 observations per bin. The default probabilities for the safest firms (i.e. those close to 1) are still close to zero. But default probabilities rise quickly and are higher than pre-crisis levels. This reflects downgrades in credit ratings during this period for a large set of firms. Moreover, there are fewer risky firms with loans in the crisis period as witnessed by the red line's observations being concentrated at lower values of i.

In Figure 3, we again plot on the x-axis $i^* - i$, firm loan ranks based on default probability. On the y-axis is the expected payoff of the loan $L[i]$. We can see that banks charge much higher interest rates during the crisis period compared to the pre-crisis period.

In Figure 4, we plot bank holding costs $\kappa[j]$ as a function of j/j^*_{2005}. For both the pre-crisis and post-crisis, we see that the curves are upward sloping consistent with het-

[8]Whether it is 2005-2007 or 2008-2010, we are still normalizing the x-axis by 4000.

erogeneity in bank talent and assortative matching. The banks that make loans to the riskiest firms, i.e. those near 0 in the x-axis, have the lowest cost. The banks that make loans to the safer firms, i.e. those near one in the x-axis, have higher cost. Since fewer loans are made, it also in our matching framework means fewer banks are making loans. That is, we find that there were fewer banks that had low holding costs in the crisis period.

For instance, take a bank situated at j/j^*_{2005} of .2. Pre-crisis, a bank at this rank had a holding cost close to zero but after the crisis, a bank at this rank had a holding cost of around 0.25 per unit of default.

Robustness. Because our sorting model is parsimonious, there is a certain number of assumptions that we need to make when mapping the model to the data. We consider some alternative estimation strategies.

First, we consider imperfect matching. In the model, the interest rate is a monotonic function of a firm's probability of default. That is, whether one ranks firms by their probability of defaults or by their interest rates should give exactly the same result. To quantify how close the data is to our stylized model, we report the rank-correlation between a firm's probability of default and a firm's interest rate in the period 2005-2007 in Table 3.[9] The stylized model gives a correlation of one. In the data, however, we obtain a correlation around 60%, which reflects the fact that the distributions are close, but not exactly similar.

This result suggests that there are other determinants of the interest rate beyond the probability of default. To handle this, we average the bank-firm matches into five groups determined by the quantiles of the probability of default. After doing this step, there is a near-perfect rank-correlation between the average probability of default within a group and the average interest rate.

[9] This is also called the Spearman correlation.

Second, we consider one-to-many matching. In the model, each bank only lends to one firm. In reality, however, banks lend to multiple firms (i.e., in the data, each bank j lends to multiple firms i). To map the model to the data, in the main text, we have considered each bank as a group of sub-banks, each lending to only one firm. An alternative method would be to consider all firms borrowing from the same bank as one firm, that is, to average the interest rate and the probability of default of all firms borrowing from the same bank. We report the results of this method in Figure 5, which are qualitatively similar to our earlier Figures 2-4.

Estimate of the Credit Supply Effect. Under our Assumption A3, Equation (8) then becomes

$$y_0 = \delta_0[0]\kappa_c[N - i^*] + \int_0^{i^*} \delta_0'[i]\kappa_c[N - (i^* - i)]di \tag{10}$$

Note that, to calculate the counterfactual $i^*(\delta_0[i], y_0, \kappa_c[i])$ that solves Equation (10), one would need to know (1) the NPV of projects y_0 and (2) the whole distribution of $\kappa_c[j]$ and $\delta_0[i]$, both of which might not be observable from the data.

The first difficulty can be resolved by using the fact that the loan supply under the non-crisis period, denoted by $i_0^* \equiv i^*(\delta_0[i], y_0, \kappa_0[i])$, solves

$$y_0 = \delta_0[0]\kappa_0[N - i_0^*] + \int_0^{i_0^*} \delta_0'[i]\kappa_0[N - (i_0^* - i)]di,$$

where RHS can be estimated from the data.

To address the second difficulty that we only observe the characteristics of active banks and firms, we then *extrapolate* these values based on what we observe in the data and use it to calculate $i^*(\delta_0[i], y_0, \kappa_c[i])$. Specifically, based on what we observe in Figure 4, we assume that banks' holding cost at period t is characterized by the following: there is a positive measure of talented banks, denoted by n_t^b with approximately zero holding cost (i.e., we set $\kappa[j] = 0$ for banks for $j \geq N - n_t^b$). Banks that are lower ranked have positive holding cost and is linear decreasing in j where $\kappa_t'[j] = -h_t^b$. That is, banks' distribution can be summarized by two parameters (h_t^b, n_t^b), with

$$\kappa_t[j] = \begin{matrix} h_t^b(N - n_t^b - j) & \forall j \in [0, N - n_t^b] \\ 0 & \forall j \in [N - n_t^b, N] \end{matrix} .$$

Similarly, based on Figure 2, the distribution of firms' default probability is summarized a n_t^f measure of safe firms who has zero default probability and for firms who ranking $i \geq n_t^f$, we assume that the change in default probability in linear increasing in its ranking (i.e., $\delta_t'[i] = h_t^f > 0$). Thus, the distribution of firms are parameterized by (h_t^f, n_t^f), which yields

$$\delta_t[i] = \begin{matrix} 0 & \forall i \in [0, n_t^f] \\ h_t^f(i - n_t^f) & \forall i \in [n_t^f, N] \end{matrix} .$$

Given this simple functional form, the cutoff type, according to Equation 6, is thus given by

$$\int_{n_t^f}^{i^*} h_t^f \kappa_t[N - (i^* - i)] di = y_t.$$

Using the fact that $\kappa_t[j]$ is only positive for $j < N - (i^* - i)$, this expression can be further simplified as

$$\int_{n_t^f}^{i^* - n_t^b} h_t^f \kappa_t[N - (i^* - i)] di = \frac{h_t^f h_t^b}{2} \left(i^* - n_t^b - n_t^f \right)^2 = y_t. \tag{11}$$

First of all, one can see from Equation (11) that, as we expect, fixing firms' characteristics (y_t, h_t^f, n_t^f), total loan supply i^* increases when banks are better at absorbing risks (i.e., either a higher measure of very talented banks n_t^b or a lower h_t^b.) Similarly, fixing banks' characteristics (h_t^b, n_t^b), total loan supply increases when the project is more profitable or when firms become safer, captured by a higher measure of safe firms n_t^f or a lower h_t^f.

Moreover, given the estimates characteristics $\{h_t^x, n_t^x\}$ for banks and firms $x \in \{f, b\}$ for both period $t \in \{c, 0\}$, and the total loan supply during the normal time i_0^*, the counterfactual loan supply $i^*(\delta_0[i], y_0, \kappa_c[i])$ thus solves

$$h_c^b \left(i^* - n_c^b + n_0^f \right)^2 = y_0 = h_0^b \left(i_0^* - n_0^b + n_0^f \right)^2. \tag{12}$$

Given that the value of $(h_c^b, n_c^b, h_0^b, n_0^b, n_0^f, i_0^*)$ can be calibrated from the data, i^* can thus be calculated without observing y_0.

Based on Figure 4, the slope remains the same for both periods $h_c^b = h_0^b$; however, the measure of very talented banks drops from 0.38*4K before crisis to 0.15*4K (i.e., $n_0^b = 1.52K$ and $n_c^b = 0.6K$). This thus shows that $i^* = i_0^* - n_0^b + n_c^b = 3.08K$, where $i_0^* = 4K$. In other words, under the simple specification $h_c^b = h_0^b$, the change in the loan supply is the change of measure of talented banks.

Based on our estimate above, we conclude that $\phi^S = \frac{i_0^* - i^*}{i_0^* - i_c^*} = \frac{4000-3080}{4000-2500} = 0.613$. In other words, a sizeable portion of the decline in aggregate loan volume is due to credit supply effects. One can calculate the demand supply in a similar way. Under particular example, one can show that $\phi^D = 0.386$.[10]This is because that, under this particular specification, the conditions is linear and thus $\phi^D + \phi^S = 1$.

6. Relating to Panel Regression Approach

As we suggested in the Introduction, our approach and the panel regression approach are largely complementary. Recall that the panel regression approach in the literature

[10]The counterfactual loan supply $i^*(y_c, \delta_c, \kappa_0)$ when banks' ability remain the same solves $i^* - n_0^b + n_c^f = y_c = i_c^* - n_c^b + n_c^f$, which gives $i^* = 2.5 + 1.52 - 0.6 = 3.42$. Thus, $\phi^D = \frac{4-3.42}{4-2.5} = 0.3867$.

uses risk controls or firm fixed effects to identify a credit supply effect. The literature implicitly recognizes that hit banks are more likely to make loans to risky firms due to the sorting mechanism modeled in our paper. But to the extent there is also random matching between banks and firms in the data, firm risk and hit bank status need not be perfectly collinear. As a result, it is possible to run an OLS regression with the dependent variable being whether a firm's loan is cut and have both firm risk and whether the firm's bank is hit by a capital shock as independent variables of interest. The coefficient on hit bank status identifies a credit supply effect, assuming that the residual variation of hit bank status and firm risk is driven by random matching.

Our approaches are complementary for two reasons. First, this panel regression approach does not directly address why risky firms' loans are cut during economic downturns. It essentially assumes that treatment effects are the same across the firm risk distribution. But of course as our model and empirics show, this need not be the case. Second, existing empirical tests assume that a firm's borrowing decision depends only on the health of the banks it is borrowing from. Our model points to general equilibrium effects — a shock to a firm's loan depends not just on the health of its lender but the health of the entire bank distribution due to our sorting model.

It is this second point from our analysis that can potentially also guide panel regression approach empirics. Suppose that there is measurement error regarding firm risk. Then both credit risk and the hit status of a bank can be informative about loan outcomes even if sorting is entirely driven by credit risk and bank talent as we have modeled it. The panel regression approach addresses this issue in the following manner.

Consider the regression of firm lending growth on bank health at the level of a firm-bank relationship. This regression can be estimated with and without fixed effects for the set of firms that borrow from multiple banks. Researchers tend to interpret the difference between the OLS estimate in this regression, $\hat{\beta}^{\mathrm{OLS}}$, and the fixed effect estimate, $\hat{\beta}^{\mathrm{FE}}$, as a measure of a less than random matching between banks and firms, i.e. a correlation between bank health and unobserved firm productivity.

Following the setups in Khwaja & Mian (2008), Jiménez *et al.* (2019), and Amiti & Weinstein (2018), we assume that the set of banks the firm is borrowing from is fixed over time. The change in lending from bank i to firm j can be written as:

$$\Delta \ln L_{ij} = \underbrace{-\chi \Delta \ln r_i}_{\text{Bank Component}} + \underbrace{\alpha \Delta \ln A_j + \mu \Delta \ln \overline{r}_j}_{\text{Firm Component}} \tag{13}$$

In particular the lending growth from bank i to firm j is additively separable into a bank component and a firm component. This equation is consistent with the reduced form equations used in the empirical banking literature. The bank component simply reflects that a firm j borrows less from bank i when the rate offered is higher, i.e. a bank that is hit and effectively charges higher interest rates will then see less loan growth for firm j.

The firm component reflects two pieces, only one of which has been recognized in the literature. The first is the change in TFP or risk piece denoted by $\alpha \Delta \ln A_j$, which has been well-studied by researchers. Critically, however, the firm fixed effect does not just depend on the firm change in TFP: it also depends on the change in the firm cost of capital which has been ignored in the literature (i.e. the $\Delta \ln \overline{r}_j$ term).

There is generically a common component in the shock to bank health as we have seen in our empirical analysis where the holding cost rises uniformly across the pre-crisis and crisis periods. As such, there is also a *cross-elasticity term.* Because we think of loans as easily substitutable (at least among the set of banks from which the firm borrows from), we expect that $\mu > 0$ — so that when other banks are hit, the firm borrows more from bank i.

Because of this cross-elasticity or general equilibrium term, we argue in this section that the difference between $\hat{\beta}^{\text{OLS}}$ and $\hat{\beta}^{\text{FE}}$ does not necessarily reflect the degree of sorting between banks and firms. This happens because a firm's loan demand to one bank may depend on the health of the other banks in the economy as emphasized by our model and analysis.

The difference between OLS and FE estimates Let δ_i denote the change in health for bank i, normalized to have a standard deviation of one. Regressing $\Delta \ln L_{ij}$ on the change in bank health δ_i with firm fixed effects gives

$$\beta^{FE} = \frac{\text{Cov}(-\chi(\Delta \ln r_i - \Delta \ln \overline{r}_j), \delta_i - \overline{\delta}_j)}{\text{Var}(\delta_i - \overline{\delta}_j)} \tag{14}$$

The firm fixed effect estimate depends on the elasticity of substitution with respect to loans from different banks, where $\overline{\delta}_j$ is the average bank health change within firm j, normalized to have a standard deviation of one.

Regressing $\Delta \ln L_{ij}$ on the change in bank health δ_i without firm fixed effects gives[11]:

$$\beta^{\text{OLS}} = \beta^{\text{FE}} + \underbrace{-\mu \frac{\text{Cov}(-\Delta \ln \overline{r}_j, \delta_i)}{\text{Var}(\delta_i)}}_{\text{cross elasticity term}} + \underbrace{\alpha \frac{\text{Cov}(\Delta \ln A_j, \delta_i)}{\text{Var}(\delta_i)}}_{\text{sorting term}}. \tag{15}$$

The difference between β^{OLS} and β^{FE} is the sum of two distinct terms.

There is a *sorting term*, depends on the covariance between a bank liquidity shock and a firm productivity shock. It measures the extent to which the matching between firms

[11]There are two additional terms in this derivation that roughly cancel out and which we omit for simplicity:

$$\beta^{\text{OLS}} = \beta^{\text{FE}} + \underbrace{-\mu \frac{\text{Cov}(-\Delta \ln \overline{r}_j, \delta_i)}{\text{Var}(\delta_i)}}_{\text{cross elasticity term}} + \underbrace{\alpha \frac{\text{Cov}(\Delta \ln A_j, \delta_i)}{\text{Var}(\delta_i)}}_{\text{sorting term}}$$
$$+ \frac{\text{Cov}(-\chi \Delta \ln r_i, \delta_i)}{\text{Var}(\delta_i)} - \frac{\text{Cov}(-\chi(\Delta \ln r_i - \Delta \ln \overline{r}_j), \delta_i - \overline{\delta}_j)}{\text{Var}(\delta_i - \overline{\delta}_j)}$$

and banks is non-random. The term is positive if banks with higher health are matched with firms with higher productivity growth as is the case in our model. This term appears in empirical banking literature.

There is also a *cross-elasticity term*. Because we think of loans as easily substitutable (at least among the set of banks from which the firm borrows from), that is $\mu > 0$, we expect this term to be negative. This term is negative if a decrease in one lender health tends to push firms to borrow more from other banks.[12]

In conclusion, the difference between the FE and OLS estimate measures sorting only when $\mu = 0$, i.e. when the cross price elasticity of loans across different banks is zero. This is the implicit assumption in the banking literature that justifies this test, starting with Khwaja & Mian (2008).[13] However, our model and empirical analysis suggests that the fact that researchers find similar estimates with versus without firm fixed effects actually implies significant sorting according to our analysis.

7. Conclusion

We show in this paper that the bank loans for risky firms are more likely to be syndicated by banks with a greater propensity to securitize. This sorting affects causal inference of bank credit supply effects for firm output which is not easily addressed with standard

[12]The presence of this cross elasticity term is related to Marshall-Hicks formula, which relates the price-elasticity of input i to the elasticity of substitution between different factors:

$$\frac{\partial \ln L_{ij}}{\partial \ln r_i} = \chi + \mu s_i$$

where s_i denotes the cost share of input i.

[13]For instance, Khwaja & Mian (2008) write "Thus the difference between the OLS estimate $\hat{\beta}^{\mathrm{OLS}}$ and the FE estimate $\hat{\beta}^{\mathrm{FE}}$ provide a direct test of how (...) the bank liquidity shock is correlated with (...) the firm productivity shock.". Similarly, Chodorow-Reich (2013) write "Under certain assumptions, the difference in the point estimates between regressions including and excluding the fixed effects captures the amount of bias induced by not-as-good-as random matching of borrowers and lenders. Specifically, the true model of bank lending must be additively separable over bank health and firm characteristics.". See also Jiménez *et al.* (2017), Jiménez *et al.* (2019), Schnabl (2012) for similar reasonings.

reduced-form approaches for identification. We show that this sorting arises naturally in a competitive matching model of a credit market where talented banks (i.e. those with lower holding costs) can offer lower interest rates to risky firms. We then use our model to assess the importance of credit supply (heterogeneity in bank talent) from demand (firm riskiness) effects. We implement our approach using available data on loan interest rates and historical default rates from credit ratings.

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Table 1: Sorting on Observables

	β	t-stat	R^2	N
	(1)	(2)	(3)	(4)
Panel A: Bank Lending Growth 06-09				
Borrower Loan Spread	-0.65^{***}	3.25	0.19	43
Borrower Bond Spread	-0.70^{***}	4.21	0.21	38
Borrower Leverage	-3.65^{***}	3.32	0.24	43
Panel B: Bank Lehman Distance				
Borrower Loan Spread	-1.37^{***}	3.39	0.37	42
Borrower Bond Spread	-1.18^{***}	3.34	0.24	37
Borrower Leverage	-8.33^{***}	3.99	0.52	42
Panel C: Bank Deposit				
Borrower Loan Spread	-1.24^{***}	5.30	0.48	43
Borrower Bond Spread	-1.06^{***}	4.07	0.32	38
Borrower Leverage	-6.15^{***}	6.55	0.47	43

Notes: This table estimates the model

$$Y_i = \alpha + \beta \overline{X}_i + \epsilon_i,$$

where i denotes a bank, Y_i is alternatively the bank lending growth from 2006-2009 (Panel A), bank Lehman distance (Panel B), bank deposit (Panel C). $\overline{X}_i$ denotes the average observable of borrowers from bank i in 2004-2006. It is alternatively the borrower loan spread, bond spread, and market leverage. Estimation with WLS — estimates in column (1) and t-statistics in column (2).

Table 2: Characteristics Related to Downside Risk Predict CAPX Growth during the Financial Crisis

	Borrower CAPX Growth 06-09					
	(1)	(2)	(3)	(4)	(5)	(6)
Borrower Loan Spread	-.05***	-.05***				
	(-3)	(-2.7)				
Borrower Bond Spread			-.049***	-.048***		
			(-2.8)	(-2.6)		
Borrower Leverage					-.13**	-.18***
					(-2.3)	(-3.1)
Bank FE	No	Yes	No	Yes	No	Yes
R^2	.0087	.04	.024	.098	.01	.042
N	1913	1912	599	592	1709	1708

Notes: This table estimates the model

$$\Delta CAPX_i = \alpha + \beta X_i + \epsilon_i,$$

where i denotes a firm, X_i is alternatively its loan spread in columns (1) and (2), bond spread in columns (3) and (4), and market leverage in columns (5) and (6). The specification is estimated using bank fixed effects in columns (2), (4), and (6). Estimation with OLS. t-statistics in parenthesis.

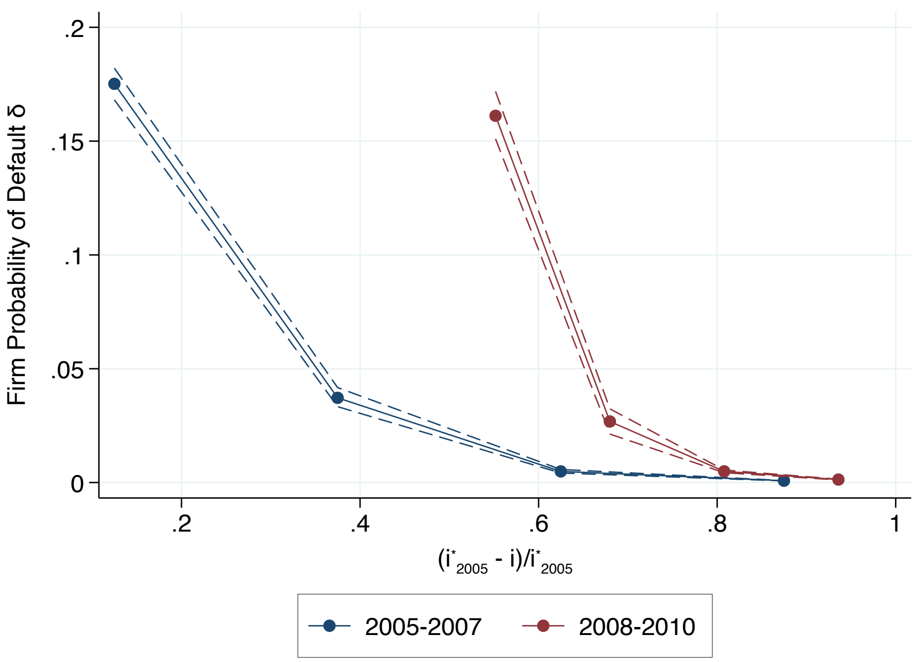
Table 3: Spearman Correlation Probability of Default and Interest Rate in 2005-2007

	Rank Probability of Default
	(1)
Rank Interest Rate	0.60***
	(0.01)

Figure 1: Bank Lending Growth 2006-2009 and Corporate Borrower Loan Spread

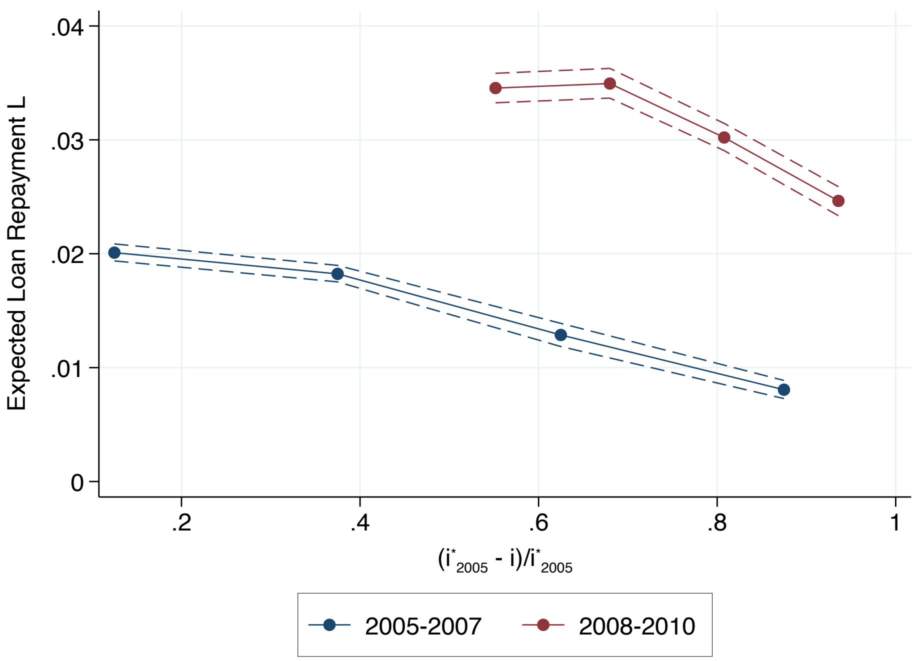
Notes: This figure reports plots for the pre-crisis (2005-2007) and the crisis (2008-2010) periods the bank holding cost $\kappa = \frac{L'}{\delta'}$) by firm credit rating rankings. On the axis is ranking of the banks j normalized by the 2005 j^*. Banks closer to 0 in the x-axis are the low holding cost or more talented banks at securitization.

Figure 2: Firm Probability of Default by Credit Rating Rankings



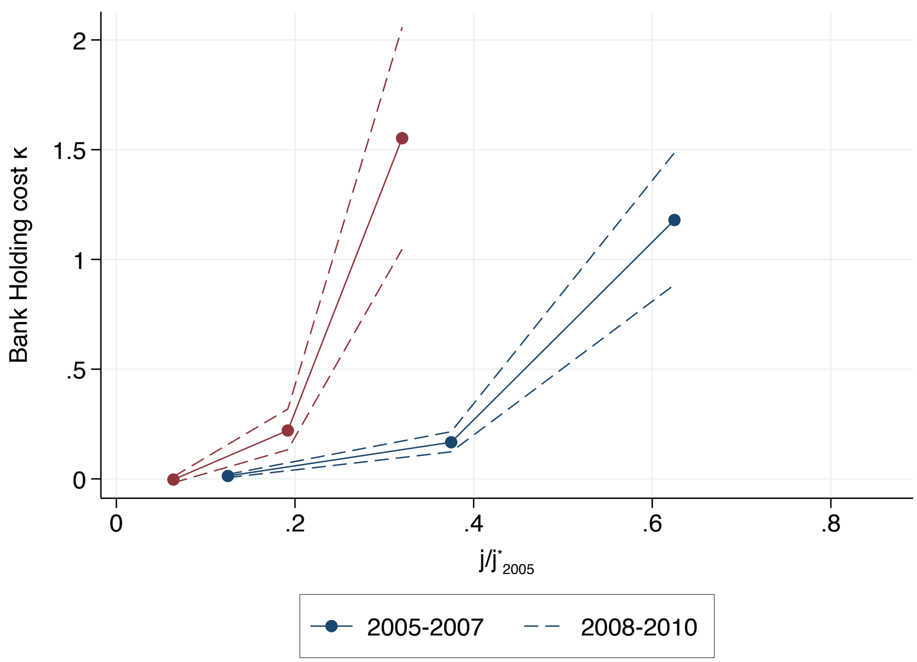
Notes: This figure plots for the pre-crisis (2005-2007) and the crisis (2008-2010) periods historical firm probability of default δ based on Moody's credit ratings from the firm's most recent public debt issuance. The pre-crisis period has roughly 4000 firms placed into four bins based on their credit rating rankings. The crisis-period has around 2500 firms placed into four bins based on their credit rating rankings. The ranks are all normalized by 4000 the number of firms in the pre-crisis period. Firms near zero in the x-axis are the risky firms, while firms near 1 are the safe firms. Dotted lines represent bootstrap standard error bands.

Figure 3: Expected Loan Repayment by Firm Credit Rating Rankings



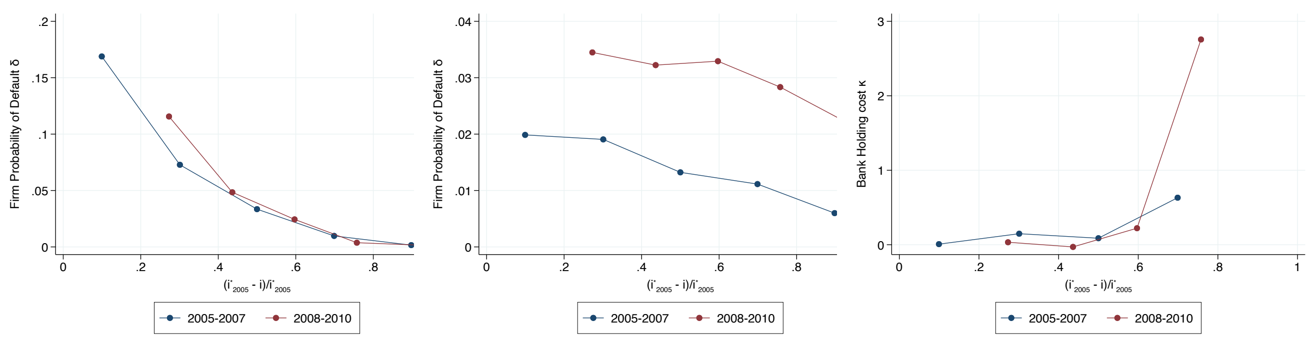
Notes: This figure plots for the pre-crisis (2005-2007) and the crisis (2008-2010) periods the expected loan repayment L by firm credit rating rankings. The pre-crisis period has roughly 4000 firms placed into eight bins based on their credit rating rankings. The crisis-period has around 2500 firms placed into six bins based on their credit rating rankings. The ranks are normalized by 4000 the number of firms in the pre-crisis period. Firms near zero in the x-axis are the risky firms, while firms near 1 are the safe firms. Dotted lines represent bootstrap standard error bands

Figure 4: Bank Holding Cost by Firm Credit Rating Rankings



Notes: This figure plots for the pre-crisis (2005-2007) and the crisis (2008-2010) periods the bank holding cost ($\kappa = \frac{L'}{\delta'}$) by bank rankings. On the axis is ranking of the banks j normalized by the 2005 j^*. Banks closer to 0 on the x-axis are banks making loans to riskiest firms and are more talented at securitization.

Figure 5: Averaging δ and r within each bank



Notes: This figure is similar to Figures 2-4 except that we implement an alternative method where we consider all firms borrowing from the same bank as one firm, that is, to average the interest rate and the probability of default of all firms borrowing from the same bank.