# The Fiscal Theory of the Price Level with a Bubble

Markus Brunnermeier
Princeton University

Sebastian Merkel
Princeton University

Yuliy Sannikov
Stanford University

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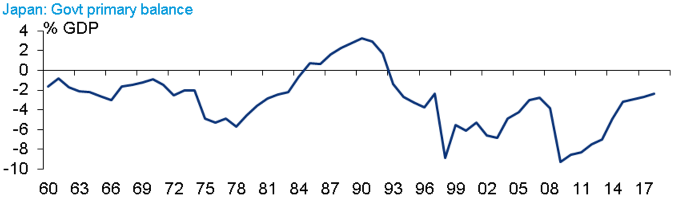
# Motivation

- Different monetary theories emphasize different roles of money and equilibrium equations
- Fiscal Theory of the Price Level (FTPL):
  - broad money (including nom. bonds) as a store of value
  - value of government debt given by discounted stream of future primary surpluses

$$\frac{\mathcal{B}_t + \mathcal{M}_t}{\mathcal{P}_t} = \mathbb{E}_t\left[\int_t^\infty \frac{\xi_s}{\xi_t}\left(T_s - G_s\right) ds\right] \quad \left[+ \quad \mathbb{E}_t\left[\int_t^\infty \frac{\xi_s}{\xi_t} \Delta i_s \frac{\mathcal{M}_s}{\mathcal{P}_s} ds\right]\right]$$

- The Japan critique:




- Broader question: can a country permanently run primary deficits?

# Deriving the Key Equation of the FTPL

- Nominal government flow budget constraint

$$\left(\mu_t^{\mathcal{B}}\mathcal{B}_t + \mu_t^{\mathcal{M}}\mathcal{M}_t + \mathcal{P}_t T_t\right) dt = \left(i_t\mathcal{B}_t + i_t^m\mathcal{M}_t + \mathcal{P}_t G_t\right) dt$$

- Multiply by nominal SDF $\xi_t/\mathcal{P}_t$, integrate from $t$ to $T$, and take expectations and limit $T \to \infty$

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$\Rightarrow$ General form of the key equation of the FTPL
("government debt valuation equation", "intertemporal government budget constraint")

$$\frac{\mathcal{B}_t + \mathcal{M}_t}{\mathcal{P}_t} = \underbrace{\mathbb{E}_t\left[\int_t^\infty \frac{\xi_s}{\xi_t}\left(T_s - G_s\right) ds\right]}_{\text{PV of primary surpluses}} + \underbrace{\mathbb{E}_t\left[\int_t^\infty \frac{\xi_s}{\xi_t}(i_s - i_s^m)\frac{\mathcal{M}_s}{\mathcal{P}_s} ds\right]}_{\text{PV of future transaction services}} + \underbrace{\lim_{T\to\infty}\mathbb{E}_t\left[\frac{\xi_T}{\xi_t}\frac{\mathcal{B}_T + \mathcal{M}_T}{\mathcal{P}_T}\right]}_{\text{bubble}}$$

- Bubble term?
  - in literature: invoke private-sector transversality condition to conclude $\mathbb{E}_t\left[\frac{\xi_T}{\xi_t}\frac{\mathcal{B}_T+\mathcal{M}_T}{\mathcal{P}_T}\right] \to 0$
  - this paper: environments in which the previous argument fails

# When Can a Bubble Exist?

- Assume stationary debt-to-GDP ratio and no aggregate risk
  - $\frac{\mathcal{B}_T+\mathcal{M}_T}{\mathcal{P}_T} = \frac{\mathcal{B}_t+\mathcal{M}_t}{\mathcal{P}_t} e^{g(T-t)}$
  - $\frac{\xi_T}{\xi_t} \propto e^{-r^f(T-t)}$
- Then $\mathbb{E}_t\left[\frac{\xi_T}{\xi_t}\frac{\mathcal{B}_T+\mathcal{M}_T}{\mathcal{P}_T}\right] \to 0 \Leftrightarrow r^f > g$
  - thus: bubble can exist $\Leftrightarrow r^f \leq g$
  - more generally: $r^b \leq g$ with $r^b =$ risk-adjusted discount rate for gov. debt

# 3 Forms of Seigniorage

$$\frac{\mathcal{B}_t + \mathcal{M}_t}{\mathcal{P}_t} = \mathbb{E}_t\left[\int_t^\infty \frac{\xi_s}{\xi_t}\left(T_s - G_s\right) ds\right] + \mathbb{E}_t\left[\int_t^\infty \frac{\xi_s}{\xi_t}(i_s - i_s^m)\frac{\mathcal{M}_s}{\mathcal{P}_s} ds\right] + \lim_{T\to\infty} \mathbb{E}_t\left[\frac{\xi_T}{\xi_t}\frac{\mathcal{B}_T + \mathcal{M}_T}{\mathcal{P}_T}\right]$$

1. Surprise devaluation
   - nonrational expectations
   - likely small (Hilscher, Reis, Raviv 2014)
2. Exploiting liquidity benefits of “narrow” cash
   - only for “narrow” cash that provides medium-of-exchange services
   - Reis (2019): flow $\approx 0.36\%$ of GDP, PV $< 30\%$ of GDP
3. Mining the fiscal bubble
   - bubble is a fiscal resource that can be “mined”
   - ever-expanding Ponzi scheme generates a steady revenue flow for the government

# Outline

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$$\mathbb{E}\left[\int_0^\infty e^{-\rho t} \log c_t^i dt\right]$$
- Agent $i$ manages one firm operating capital $k_t^i$
  - output $y_t^i = ak_t^i$, capital investment $\iota_t^i k_t^i$
  - capital evolution (absent market transactions): $dk_t^i / k_t^i = \left(\Phi(\iota_t^i) - \delta\right) dt + \tilde{\sigma} d\tilde{Z}_t^i$
    ($\tilde{Z}^i$ idiosyncratic Brownian motion)
- Friction: agents cannot trade idiosyncratic risk (only physical capital and bonds)

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- Government:
    - proportional output tax $\tau_t$
    - nominal bonds:
        - nominal aggregate supply: $d\mathcal{B}_t / \mathcal{B}_t = \mu_t^{\mathcal{B}} dt$
        - pays (floating) nominal interest $i_t$
    - policy choices: $\tau_t$, $\mu_t^{\mathcal{B}}$, $i_t$ s.t. flow budget constraint $\underbrace{\left(\mu_t^{\mathcal{B}} - i_t\right)}_{=:\breve{\mu}_t^{\mathcal{B}}} \mathcal{B}_t + \mathcal{P}_t \tau_t a K_t = 0$
- Aggregate resource constraint: $C_t + \iota_t K_t = a K_t$

# Agent Problem

Agent $i$'s problem: choose consumption $c^i$, investment $\iota^i$, bond portfolio weight $\theta^i$ to maximize

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t}\log c_t^i dt\right]$$

subject to

- net worth evolution

$$dn_t^i/n_t^i = -c_t^i/n_t^i dt + \theta_t^i dr_t^{\mathcal{B}} + \left(1-\theta_t^i\right) dr_t^{K,i}\left(\iota_t^i\right)$$

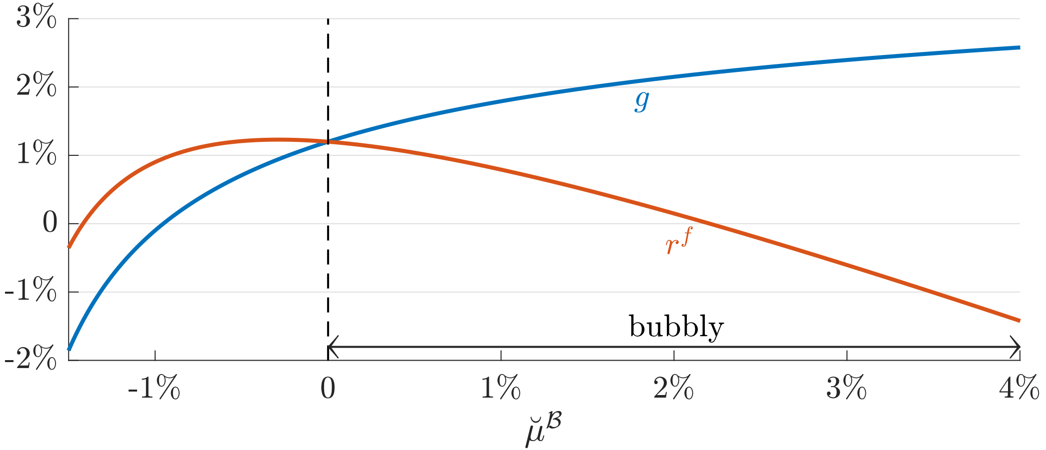
- return processes $dr_t^{K,i}\left(\iota_t^i\right)$, $dr_t^{\mathcal{B}}$

# Stationary Equilibria

- Assume constant policies $\breve{\mu}^{\mathcal{B}}$, $\tau$
  (index policies by $\breve{\mu}^{\mathcal{B}}$, $\tau$ implied by gov. budget constraint)
- Two steady states

| non-monetary | monetary |
|---|---|
| gov. bonds worthless | gov. bonds have pos. value |
| exists for all $\breve{\mu}^{\mathcal{B}}$ | exists only if $\tilde{\sigma} \geq \sqrt{\rho + \breve{\mu}^{\mathcal{B}}}$ |

# $r^f$ versus $g$ for Different Policies (Monetary Steady State)



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# Bubble and Transversality

- Government debt is a bubble: provides risk-free store of value
- Bonds allow for self-insurance through trading
  - $d\tilde{Z}_t^i < 0 \Rightarrow$ buy capital, sell bonds
  - $d\tilde{Z}_t^i > 0 \Rightarrow$ sell capital, buy bonds

$\Rightarrow$ lowers volatility of total wealth $n_t^i$, but increases volatility of bond wealth $n_t^{b,i} := \theta_t^i n_t^i$

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- Why does the transversality condition (TVC) not rule out the bubble?
  - TVC for bond wealth: $\lim_{T\to\infty} \mathbb{E}[\xi_T^i n_T^{b,i}] = 0$
  - effective discount rate in TVC $=$ discount rate for stochastic bond portfolio $n^{b,i}$

    $=$ risk-free rate $r^f$ $+$ (risk premium for idiosyncratic $n^{b,i}$-fluctuations)

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  - discount rate for individual bond $=$ discount rate for aggregate bond stock $\int n^{b,i} di$

    $=$ risk-free rate $r^f$
  - risk premium: (self-insurance) service flow from retrading bonds (like a convenience yield)
- More general point: beneficial equilibrium trades are essential feature of (rational) bubbles

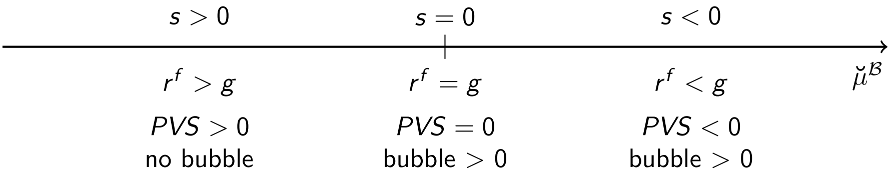
# Outline

# FTPL Equation with a Bubble

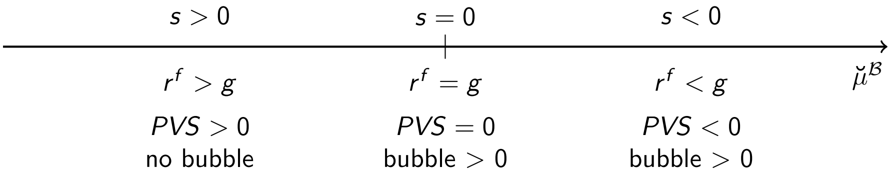
- Primary surplus $sK_t = \tau a K_t$
- Debt valuation equation ($K_0 \equiv 1$):

$$\frac{\mathcal{B}_0}{\mathcal{P}_0} = \lim_{T\to\infty} \left( \underbrace{\int_0^T e^{-(r^f - g)t} s dt}_{=:PVS_{0,T}} + e^{-(r^f - g)T} \frac{\mathcal{B}_0}{\mathcal{P}_0} \right)$$

- risk-free rate $r^f = g - \breve{\mu}^{\mathcal{B}}$



# “Mining the Fiscal Bubble”

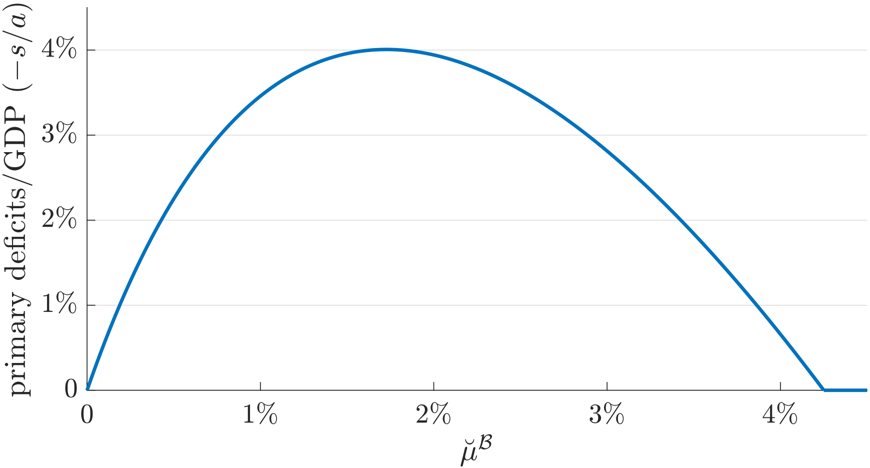



In all three cases, the bubble – or its mere possibility – grants government some leeway:

- $s < 0$: perpetual deficits are funded out of the bubble, never have to raise taxes (“bubble mining”)
- $s = 0$: government debt enjoys positive value despite zero surpluses (debt “backed” by the bubble)
- $s > 0$: no equilibrium bubble, yet possibility of bubble makes debt more sustainable

  unexpected (persistent) drop in surpluses below zero
  $\Rightarrow$ bubble emerges instead of collapse of the value of debt

# Bubble Mining Laffer Curve




see Brunnermeier, Merkel, Sannikov (2020): “The Limits of Modern Monetary Theory”

# Outline

# Determination of Price Level

1. In particular equilibrium:
   - with a bubble, FTPL equation is no longer a one-to-one relationship between PV of surpluses and price level $\mathcal{P}_t$

   $\Rightarrow$ FTPL equation alone no longer determines $\mathcal{P}_t$
   ... because the size of the bubble is not determined (by that equation)
   - economic mechanism emphasized by FTPL still valid:
     - government debt generates a wealth effect, goods market clearing "determines" the price level
     - given the price level, the FTPL equation determines the size of the bubble (i.e. how much of government debt is net wealth)
2. Multiple equilibria (FTPL as a selection device):
   - off-equilibrium fiscal backing is sufficient
   - but requires credibility and fiscal capacity to promise off-equilibrium surpluses (otherwise: vulnerability to bubble crashes)

# FTPL: Resolving Equilibrium Multiplicity

- If $\tau > 0$ along equilibrium path:
  - standard FTPL argument applies: unique $\mathcal{P}_t$ consistent with equilibrium, if surpluses ($\tau_s$) do not react (too strongly) to the price level
  - but then $r^f > g$ and there is no bubble in equilibrium
- Resolving multiplicity with an equilibrium bubble:
  - more challenging: continuum of bubble values consistent with the same surplus path
    $\Rightarrow$ exogenous surplus sequence insufficient for uniqueness
  - contingent policy can select the bubble equilibrium
    - primary deficits on the equilibrium path (bubble mining)
    - switch to $\tau > 0$ if inflation breaks out
- Difference to Bassetto, Cui (2018):
  (their conclusion: "the FTPL breaks down in [a dynamically inefficient] OLG economy")
  - contingent policy versus constant taxes
  - government lending to private sector

# Conclusion

- Integrate the "missing" bubble term into the FTPL
- 3 forms of seigniorage including "mining the bubble"
- Model with idiosyncratic risk and $r^f \leq g$
    - bubble can exist despite transversality condition
    - bond trading allows self-insurance
- Price level determination
    - goods market clearing condition (through bubble wealth effect)
    - uniqueness: off-equilibrium tax backing