Motivation

- Different monetary theories emphasize different roles of money and equilibrium equations
  - Fiscal Theory of the Price Level (FTPL):
    - broad money (including nom. bonds) as a store of value
    - value of government debt given by discounted stream of future primary surpluses
      \[
      \frac{B_t + M_t}{P_t} = \mathbb{E}_t \left[ \int_t^{\infty} \frac{\xi_s}{\xi_t} (T_s - G_s) \, ds \right] + \mathbb{E}_t \left[ \int_t^{\infty} \frac{\xi_s}{\xi_t} \Delta i_s \frac{M_s}{P_s} \, ds \right]
      \]
  - The Japan critique:
    - Broader question: can a country permanently run primary deficits?
Deriving the Key Equation of the FTPL

- Nominal government flow budget constraint

\[
(\mu^B_t B_t + \mu^M_t M_t + P_t T_t) \ dt = (i_t B_t + i^m_t M_t + P_t G_t) \ dt
\]

- Multiply by nominal SDF $\xi_t/P_t$, integrate from $t$ to $T$, and take expectations and limit $T \to \infty$
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  \]
  - Multiply by nominal SDF \( \xi_t / P_t \), integrate from \( t \) to \( T \), and take expectations and limit \( T \to \infty \)
  \[\Rightarrow\] General form of the key equation of the FTPL
  ("government debt valuation equation", "intertemporal government budget constraint")
  \[
  \frac{B_t + M_t}{P_t} = E_t \left[ \int_t^\infty \frac{\xi_s}{\xi_t} (T_s - G_s) \, ds \right] + E_t \left[ \int_t^\infty \frac{\xi_s}{\xi_t} (i_s - i^m_s) \frac{M_s}{P_s} \, ds \right] + \lim_{T \to \infty} E_t \left[ \frac{\xi_T}{\xi_t} \frac{B_T + M_T}{P_T} \right]
  \]
  - Bubble term?
    - in literature: invoke private-sector transversality condition to conclude \( E_t \left[ \frac{\xi_T}{\xi_t} \frac{B_T + M_T}{P_T} \right] \to 0 \)
    - this paper: environments in which the previous argument fails
When Can a Bubble Exist?

- Assume stationary debt-to-GDP ratio and no aggregate risk
  \[
  \frac{B_{T}+M_{T}}{P_{T}} = \frac{B_{t}+M_{t} e^{g(T-t)}}{P_{t}}
  \]
  \[
  \frac{\xi_{T}}{\xi_{t}} \propto e^{-r_{f}(T-t)}
  \]

- Then \( \mathbb{E}_{t} \left[ \frac{\xi_{T} B_{T}+M_{T}}{\xi_{t} P_{T}} \right] \rightarrow 0 \iff r_{f} > g \)
  
  thus: bubble can exist \( \iff r_{f} \leq g \)

  more generally: \( r^{b} \leq g \) with \( r^{b} \) = risk-adjusted discount rate for gov. debt
3 Forms of Seigniorage

\[
\frac{B_t + M_t}{P_t} = \mathbb{E}_t \left[ \int_t^\infty \frac{\xi_s}{\xi_t} (T_s - G_s) \, ds \right] + \mathbb{E}_t \left[ \int_t^\infty \frac{\xi_s (i_s - i^m_s)}{\xi_t} \frac{M_s}{P_s} \, ds \right] + \lim_{T \to \infty} \mathbb{E}_t \left[ \frac{\xi_T}{\xi_t} \frac{B_T + M_T}{P_T} \right]
\]

1. **Surprise devaluation**
   - nonrational expectations
   - likely small (Hilscher, Reis, Raviv 2014)

2. **Exploiting liquidity benefits of “narrow” cash**
   - only for “narrow” cash that provides medium-of-exchange services
   - Reis (2019): flow $\approx 0.36\%$ of GDP, PV $< 30\%$ of GDP

3. **Mining the fiscal bubble**
   - bubble is a fiscal resource that can be “mined”
   - ever-expanding Ponzi scheme generates a steady revenue flow for the government
1. Revisiting the Key Equation of the FTPL

2. Example with a Bubble: Model with Idiosyncratic Return Risk
   - Model Environment & Steady State
   - Transversality Condition and Bubble Existence
   - “Mining the Bubble”
   - Price Level Determination (Uniqueness)

3. Conclusion
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Continuous time, infinite horizon, 1 consumption good, 1 capital good
Model Environment

- Continuous time, infinite horizon, 1 consumption good, 1 capital good
- Continuum of household-entrepreneurs (index by $i$), $i$’s preferences
  \[
  \mathbb{E} \left[ \int_0^\infty e^{-\rho t} \log c_t^i dt \right]
  \]
Model Environment

- Continuous time, infinite horizon, 1 consumption good, 1 capital good
- Continuum of household-entrepreneurs (index by $i$), $i$’s preferences
  \[ \mathbb{E} \left[ \int_0^\infty e^{-\rho t} \log c^i_t \, dt \right] \]
- Agent $i$ manages one firm operating capital $k^i_t$
  - output $y^i_t = ak^i_t$, capital investment $\iota^i_t k^i_t$
  - capital evolution (absent market transactions):
    \[ \frac{dk^i_t}{k^i_t} = \left( \Phi(\iota^i_t) - \delta \right) dt + \sigma d\tilde{Z}^i_t \]
    ($\tilde{Z}^i_i$ idiosyncratic Brownian motion)
- Friction: agents cannot trade idiosyncratic risk (only physical capital and bonds)
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- Government:
  - proportional output tax $\tau_t$
  - nominal bonds:
    - nominal aggregate supply: $dB_t / B_t = \mu^B_t dt$
    - pays (floating) nominal interest $i_t$
  - policy choices: $\tau_t$, $\mu^B_t$, $i_t$ s.t. flow budget constraint $\left( \mu^B_t - i_t \right) B_t + \mathcal{P} \tau_t a K_t = 0$
- Aggregate resource constraint: $C_t + \nu_t K_t = a K_t$

\[=:\bar{\mu}^B_t \]
Agent Problem

Agent $i$'s problem: choose consumption $c^i$, investment $\iota^i$, bond portfolio weight $\theta^i$ to maximize

$$E \left[ \int_0^\infty e^{-\rho t} \log c^i_t \, dt \right]$$

subject to

- net worth evolution

$$\frac{dn^i_t}{n^i_t} = -c^i_t/n^i_t \, dt + \theta^i_t \, dr^B_t + (1 - \theta^i_t) \, dr^{K,i}_t (\iota^i_t)$$

- return processes $dr^{K,i}_t (\iota^i_t)$, $dr^B_t$
Stationary Equilibria

- Assume constant policies $\tilde{\mu}^B$, $\tau$
  (index policies by $\tilde{\mu}^B$, $\tau$ implied by gov. budget constraint)

- Two steady states

<table>
<thead>
<tr>
<th>non-monetary</th>
<th>monetary</th>
</tr>
</thead>
<tbody>
<tr>
<td>gov. bonds worthless exists for all $\tilde{\mu}^B$</td>
<td>gov. bonds have pos. value exists only if $\tilde{\sigma} \geq \sqrt{\rho + \tilde{\mu}^B}$</td>
</tr>
</tbody>
</table>
$r^f$ versus $g$ for Different Policies (Monetary Steady State)
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Government debt is a bubble: provides risk-free store of value

Bonds allow for self-insurance through trading

\[ d\tilde{Z}_t^i < 0 \Rightarrow \text{buy capital, sell bonds} \]
\[ d\tilde{Z}_t^i > 0 \Rightarrow \text{sell capital, buy bonds} \]

\[ \Rightarrow \text{lowers volatility of total wealth } n_t^i, \text{ but increases volatility of bond wealth } n_{t}^{b,i} := \theta_t^i n_t^i. \]
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Why does the transversality condition (TVC) not rule out the bubble?

- TVC for bond wealth: \( \lim_{T \to \infty} \mathbb{E}[\xi^i_T n_T^{b,i}] = 0 \)
- Effective discount rate in TVC = discount rate for stochastic bond portfolio \( n_t^{b,i} \)
  \[ = \text{risk-free rate } r^f + (\text{risk premium for idiosyncratic } n_t^{b,i}-\text{fluctuations}) \]
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  $= \text{risk-free rate } r^f + (\text{risk premium for idiosyncratic } n^{b,i}-\text{fluctuations})$
- Discount rate for individual bond = discount rate for aggregate bond stock $\int n^{b,i} \, di$
  $= \text{risk-free rate } r^f$
- Risk premium: (self-insurance) service flow from re trading bonds (like a convenience yield)

More general point: beneficial equilibrium trades are essential feature of (rational) bubbles
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Primary surplus $sK_t = \tau aK_t$

Debt valuation equation ($K_0 \equiv 1$):

$$\frac{B_0}{P_0} = \lim_{T \to \infty} \left( \int_0^T e^{-(r^f - g)t} s dt + e^{-(r^f - g)T} \frac{B_0}{P_0} \right)$$

Risk-free rate $r^f = g - \tilde{\mu}^B$

\[\begin{array}{ccc}
s > 0 & s = 0 & s < 0 \\
\hline
r^f > g & r^f = g & r^f < g \\
PVS > 0 & PVS = 0 & PVS < 0 \\
\text{no bubble} & \text{bubble} > 0 & \text{bubble} > 0
\end{array}\]
In all three cases, the bubble – or its mere possibility – grants government some leeway:

- **s < 0**: perpetual deficits are funded out of the bubble, never have to raise taxes (“bubble mining”)
- **s = 0**: government debt enjoys positive value despite zero surpluses (debt “backed” by the bubble)
- **s > 0**: no equilibrium bubble, yet possibility of bubble makes debt more sustainable
  - unexpected (persistent) drop in surpluses below zero
  - ⇒ bubble emerges instead of collapse of the value of debt
Bubble Mining Laffer Curve

see Brunnermeier, Merkel, Sannikov (2020): “The Limits of Modern Monetary Theory”
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In particular equilibrium:

- with a bubble, FTPL equation is no longer a one-to-one relationship between PV of surpluses and price level $P_t$
- FTPL equation alone no longer determines $P_t$
  ... because the size of the bubble is not determined (by that equation)
- economic mechanism emphasized by FTPL still valid:
  - government debt generates a wealth effect, goods market clearing “determines” the price level
  - given the price level, the FTPL equation determines the size of the bubble
    (i.e. how much of government debt is net wealth)

Multiple equilibria (FTPL as a selection device):

- off-equilibrium fiscal backing is sufficient
- but requires credibility and fiscal capacity to promise off-equilibrium surpluses
  (otherwise: vulnerability to bubble crashes)
FTPL: Resolving Equilibrium Multiplicity

- If $\tau > 0$ along equilibrium path:
  - standard FTPL argument applies: unique $P_t$ consistent with equilibrium, if surpluses ($\tau_s$) do not react (too strongly) to the price level
  - but then $r^f > g$ and there is no bubble in equilibrium

- Resolving multiplicity with an equilibrium bubble:
  - more challenging: continuum of bubble values consistent with the same surplus path
    $\Rightarrow$ exogenous surplus sequence insufficient for uniqueness
  - contingent policy can select the bubble equilibrium
    - primary deficits on the equilibrium path (bubble mining)
    - switch to $\tau > 0$ if inflation breaks out

- Difference to Bassetto, Cui (2018):
  (their conclusion: “the FTPL breaks down in [a dynamically inefficient] OLG economy”)
  - contingent policy versus constant taxes
  - government lending to private sector
Integrate the “missing” bubble term into the FTPL

3 forms of seigniorage including “mining the bubble”

Model with idiosyncratic risk and \( r^f \leq g \)
  - bubble can exist despite transversality condition
  - bond trading allows self-insurance

Price level determination
  - goods market clearing condition (through bubble wealth effect)
  - uniqueness: off-equilibrium tax backing