Dynamic Oligopoly and Price Stickiness

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Imperfect Competition
Imperfect Competition

- Monopolistic competition: continuum of firms (Dixit-Stiglitz)
  - simple and tractable
  - reigns supreme: trade, macro, growth, …
Imperfect Competition

- **Monopolistic competition**: continuum of firms (Dixit-Stiglitz)
  - simple and tractable
  - reigns supreme: trade, macro, growth, ...

- **Oligopoly**: finite number of firms
  - more realistic and complicated
  - extensive IO literature
  - “rise in market power”: markups, concentration, superstar firms, ...

**Q**: Oligopoly important for macro?
This Paper

- Standard macro model…
  - representative agent, infinite horizon
  - consumption, labor and money
  - nominal rigidities a la Calvo
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- Here:
  - oligopoly with any $n$ firms
  - general demand structure (e.g. Kimball, not just CES)
This Paper

- Standard macro model...
  - representative agent, infinite horizon
  - consumption, labor and money
  - nominal rigidities a la Calvo
- Here:
  - oligopoly with any $n$ firms
  - general demand structure (e.g. Kimball, not just CES)
- Results
  1. Sufficient statistics for M shocks
  2. Calibration and counterfactuals
  3. Inspecting the mechanism
  4. Phillips Curve
Literature

- Mongey (2016)

- Rotemberg-Saloner (1986), Rotemberg-Woodford (1992)


Setup

- Households: consumption, labor, money
- Firms: continuum of sectors $s$...
  - $n_s$ firms within sector $s$
  - Calvo price rigidity: constant probability of price change $\lambda_s$

- Equilibrium concepts for oligopoly game...
  - Markov: dominant equilibrium concept in IO
\[
\int_0^\infty e^{-\rho t} U \left( C(t), L(t), \frac{M(t)}{P(t)} \right) dt
\]

\[C(t) = G(\{C_s(t)\}_s)\]

\[C_s(t) = H(c_{s,1}(t), c_{s,2}(t), \ldots, c_{s,n}(t))\]
\[ \int_0^\infty e^{-\rho t} U \left( C(t), L(t), \frac{M(t)}{P(t)} \right) dt \]

\[ \int_0^\infty e^{-\rho t} \left( \log(C(t)) - L(t) + \log\left( \frac{M(t)}{P(t)} \right) \right) dt \]  

(Golosov-Lucas)

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\[ C_s(t) = H(c_{s,1}(t), c_{s,2}(t), \ldots, c_{s,n}(t)) \]

\[ \Rightarrow C(t) = \exp \int_0^1 \log C_s(t) ds \]

\[ \frac{1}{n_s} \sum_{j=1}^{n_s} \Psi \left( \frac{c_{i,s}}{C_s} \right) = 1 \]  

(Kimball)
\[
\int_0^\infty e^{-\rho t} U \left( C(t), L(t), \frac{M(t)}{P(t)} \right) dt
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\[
P(t)C(t) + \dot{B}(t) + \dot{M}(t) = W(t)L(t) + \tilde{\Pi}(t) + T(t) + r(t)B(t)
\]

(Kimball)
\[
\int_0^\infty e^{-\rho t} \left( C(t), L(t), \frac{M(t)}{P(t)} \right) dt
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(Golosov-Lucas)

\[
C(t) = G\{C_s(t)\}_s
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\[
C_s(t) = H(c_{s,1}(t), c_{s,2}(t), \ldots, c_{s,n}(t)) \rightarrow C(t) = \exp \int_0^1 \log C_s(t) ds
\]

(Kimball)

\[
P(t)C(t) + \dot{B}(t) + \dot{M}(t) = W(t)L(t) + \tilde{\Pi}(t) + T(t) + r(t)B(t)
\]

\[
c_{i,s}(t) = l_{i,s}(t)
\]

\[
\mathfrak{E}_0 \int_0^\infty e^{-\int_0^t r(s) ds} \tilde{\Pi}^{i,s}(t) dt
\]

\[
\tilde{\Pi}^{i,s}(t) = c_{i,s}(t) \left( p_{i,s}(t) - W(t) \right)
\]

\[
c_{i,s}(t) = C(t)P(t) D^{i,s}(p_s(t))
\]
\[
\int_0^\infty e^{-\rho t} \mathcal{U} \left( C(t), L(t), \frac{M(t)}{P(t)} \right) dt = \int_0^\infty e^{-\rho t} \left( \log(C(t)) - L(t) + \log\left( \frac{M(t)}{P(t)} \right) \right) dt
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\[C(t) = G(\{C_s(t)\}_s)\]

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\[\Rightarrow C(t) = \exp \int_0^1 \log C_s(t) ds\]

\[
P(t)C(t) + \dot{B}(t) + \dot{M}(t) = \mathcal{W}(t)L(t) + \widetilde{\Pi}(t) + T(t) + r(t)B(t)
\]

\[c_{i,s}(t) = l_{i,s}(t)\]

\[
\mathbb{E}_0 \int_0^\infty e^{-\int_0^t r(s) ds} \widetilde{\Pi}^{i,s}(t) dt
\]

\[
\widetilde{\Pi}^{i,s}(t) = c_{i,s}(t) \left( p_{i,s}(t) - \mathcal{W}(t) \right)
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\[c_{i,s}(t) = C(t)P(t) D^{i,s}(p_s(t))
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\[p_{1,s}, p_{2,s}, \ldots, p_{n,s}\]
\[ \int_0^\infty e^{-\rho t} U \left( C(t), L(t), \frac{M(t)}{P(t)} \right) dt \]

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\[ \mathbb{E} \int_0^\infty e^{-\int_0^t r(s) ds} \tilde{\Pi}_{i,s}^i(t) dt \]

\[ \tilde{\Pi}_{i,s}^i(t) = c_{i,s}(t) \left( p_{i,s}(t) - W(t) \right) \]

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\[ \int_0^\infty e^{-\rho t} U \left( C(t), L(t), \frac{M(t)}{P(t)} \right) dt \quad \int_0^\infty e^{-\rho t} \left( \log(C(t)) - L(t) + \log\left( \frac{M(t)}{P(t)} \right) \right) dt \]

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Calvo pricing
Poisson arrival
Reset strategy
\[ p_{i,t}^* = g_{i,s}(p_{-i,s}; t) \]
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Calvo pricing
Poisson arrival

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\rightarrow \frac{1}{n_s} \sum_{j=1}^{n_s} \Psi \left( \frac{c_{i,s}}{C_s} \right) = 1
\]

(Golosov-Lucas)

(Kimball)

Reset strategy

\[
p_{i,t}^* = g^{i,s}(p_{-i,s}; t)
\]

\{p_{j,s}\}_j \neq i
Steady State

- Constant $C, L, M, P, W, r$
Steady State

- Constant $C, L, M, P, W, r$
- Household and market clearing

\[ C = L \]

\[ \frac{U_C}{P} = \frac{U_L}{W} = \frac{U_m}{rP} \]

\[ r = \rho \]
Steady State

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    C = L \\
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    r = \rho
    \]
- firms
  \[
  (\rho + n\lambda)V(p) = D^i(p)(p_i - W) + \lambda \sum_{j=1}^{n} V(g^j(p_{-j}), p_{-j})
  \]
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  \[ g(p_{-i}) \in \text{arg max}_{p_i} V(p_i, p_{-i}) \]
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  \]

- steady state price vector $P = g(P, P, \ldots, P)$
\[ p^i = g(p_{-i}) \]
The diagram illustrates a relationship between two variables, $p_i$ and $p_{-i}$, with the equation $p^i = g(p_{-i})$. The point $P$ lies on the orange line connecting the axes, indicating a specific relationship at that point.
\[ p^i = g(p_{-i}) \]
\[ p^{Nash} < P \]

\[ p^i = g(p_{-i}) \]
1. Sufficient Statistics
Money Shock

- Starting at steady state...
Money Shock

- Starting at steady state...
- ...unanticipated permanent shock to money...
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Money Shock

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Money Shock

- Starting at steady state...
- ...unanticipated permanent shock to money...

\[ P(t)C(t) = M(t) \]

**Proposition. (Aggregation)**

\[
\log \frac{P(t)}{P^*} \approx \log \frac{P(0)}{P^*} e^{-\lambda(1-\sum_n (n-1)\beta_n \omega_n)t}
\]

\[
\beta_n \equiv \frac{\partial}{\partial p_j} g(P^*)
\]

extensions: heterogeneous \( \lambda \), productivity, costs
Money Shock

- Starting at steady state...
- ...unanticipated permanent shock to money...

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extensions: heterogeneous \( \lambda \), productivity, costs
$p_{Nash} < P$

$p^i = g(p_{-i})$

steeper policy $\rightarrow$ slower convergence
Monopolistic vs Duopoly with CES

\[ W(t) \]
Monopolistic vs Duopoly with CES

$W(t)$

Firm i
Monopolistic vs Duopoly with CES
Monopolistic vs Duopoly with CES
Monopolistic vs Duopoly with CES
Monopolistic vs Duopoly with CES

W(t)

Firm i

Firm j

P(t)

W(t)

Firm 1
Monopolistic vs Duopoly with CES
Monopolistic vs Duopoly with CES

W(t)

Firm i

Firm j

P(t)

W(t)

Firm 1

Firm 2

Firm 1
Monopolistic vs Duopoly with CES
Sufficient Statistic

**Proposition.**

\[(n - 1)\beta_n = \frac{p + \lambda}{\lambda} \cdot \frac{n - 1}{n - 2} + \frac{\epsilon_i^2 - 1}{\epsilon_i^2 - \frac{\mu}{\mu - 1}}\]

\[
\mu = \frac{p}{W}
\]

\[
\epsilon_i^i = -\frac{\partial \log D^i}{\partial \log p_i}
\]
Proposition.

\[(n - 1) \beta_n = \frac{\rho + \lambda}{\lambda} \frac{n - 1}{n - 2 + \frac{\epsilon^i - 1}{\epsilon^i - \frac{\mu}{\mu - 1}}} \]

\[\mu = \frac{P}{W}\]
\[\epsilon^i = -\frac{\partial \log D^i}{\partial \log p_i}\]

- Intuition... (reverse causality \(\beta \rightarrow \mu\))
  - Nash markup \(\Longleftrightarrow \beta = 0\)
  - higher markup \(\Longleftrightarrow\) rivals mimic my price (high \(\beta\))
**Sufficient Statistic**

**Proposition.**

\[(n - 1)\beta_n = \frac{\rho + \lambda}{\lambda} \frac{n - 1}{n - 2 + \frac{\varepsilon^i_i - 1}{\varepsilon^i_i - \frac{\mu}{\mu - 1}}}\]

\[\mu = \frac{P}{W}\]

\[\varepsilon^i_i = -\frac{\partial \log D^i}{\partial \log p^i}\]

- Intuition… (reverse causality \(\beta \rightarrow \mu\))
  - Nash markup \(\iff\) \(\beta = 0\)
  - higher markup \(\iff\) rivals mimic my price (high \(\beta\))

- **Very few statistics needed!**
  - markup observable? maybe
  - elasticity observable? maybe
\[ p^i = g(p_{-i}) \]
$p_i = g(p_{-i})$
2. Counterfactuals
Under monopolistic competition

\[ \frac{1}{n} \sum \Psi \left( \frac{c_i}{C} \right) = 1 \]

\[ \Psi'(x) = \frac{\eta - 1}{\eta} \exp \left( \frac{1 - x^{\theta/\eta}}{\theta} \right) \]  \hspace{1cm} \text{(Klenow-Willis)}

- Under monopolistic competition
  - \rightarrow \text{elasticity } \eta
  - \rightarrow \text{superelasticity } \theta

- Oligopoly: elasticities also depend on \( n \)
Kimball Demand

Half-life

$\theta = 0$ (CES)
Kimball Demand

Half-life

\[ \theta = 15 \]
\[ \theta = 10 \]
\[ \theta = 5 \]
\[ \theta = 0 \text{ (CES)} \]
Kimball Demand

- Low $\theta$ similar to CES: slowest convergence at $n=2$
- But with high enough $\theta$, fastest convergence at $n=2$!
- Duopoly is knife-edge: half-life stuck at CES level…
  … in contrast: $n \geq 3$ arbitrarily large as $\theta$ increases
Pass-Through

- Amiti-Itskhoki-Konings 19: own cost pass-through
  - high for small firms
  - low for large firms
  - consistent with CES Cournot but not Bertrand

- Depart from CES to match
  \[ \text{pass-through} = f(\text{market share}) \]
  in dynamic (Bertrand) model
- National HHI 0.05 to 0.1 (e.g., Gutierrez-Philippon): MP 15% stronger
- Local HHI 0.15 to 0.05 (Rossi-Hansberg, Sarte, Trachter): MP 25% weaker
3. Inspecting the mechanism
Inspecting Mechanism

\[ W(t) \]

Firm 1

Firm 2

\[ P(t) \]
Inspecting Mechanism

- Two effects with finite $n$...
  - **feedback**: firm $i$ cares about others’ prices
  - **strategic**: firm $i$ can affect others’ prices

\[ W(t) \]

\[ P(t) \]

Firm 1

Firm 2
Inspecting Mechanism

- Two effects with finite $n$...
  - **feedback**: firm $i$ cares about others’ prices
  - **strategic**: firm $i$ can affect others’ prices

- Feedback effect with $n = \infty$
  - inputs from other firms
  - Kimball (1995) demand
Inspecting Mechanism

- Compare MPE with \( n \) firms to

  \textbf{as if} monopolistic market

- \( n = \infty \) and modified Kimball preferences to match elasticities
- \( \Leftrightarrow \) equilibrium if \( n \) firms ignore how they affect rivals’ pricing \( \Rightarrow \) “non-strategic” model
Small strategic effects

\[ \frac{h_l}{\bar{h}_l} \]

- \( \theta = 10 \)
- \( \theta = 0 \) (CES)
- AIK
4. Phillips Curve
Phillips Curve

- Generalize preferences and allow arbitrary paths of
  - Interest rate shocks
  - Real shocks

- Monopolistic NKPC
  - First order ODE
  - Inflation only depends on future MC
  - Kimball $\Leftrightarrow$ less frequent adjustment (lower $\lambda$)

- Oligopolistic NKPC
  - Higher order ODE: inflation persistence
  - Not just MC: demand, interest rates
  - Not equivalent to lower $\lambda$
Phillips Curve

- Standard NKPC
  \[ \dot{\pi} = 0.05\pi - 1.05mc \]

- Oligopoly: Example with \( n = 3 \)
  - MPE
    \[ \dot{\pi} = 0.07\pi - 0.28mc \]
    \[ +1.31\ddot{\pi} + 0.45mc + 0.03(r - \rho) \]
  - Non-strategic (= monopolistic Kimball)
    \[ \dot{\pi} = 0.05\pi - 0.27mc \]
3-Eq Oligopoly NK

- Combine with Euler equation
  \[ \dot{c} = \sigma^{-1} (r - \pi - \rho - \epsilon^r) \]
- Taylor rule
  \[ r = \rho + \phi \pi + \epsilon^m \]
- AR(1) \( \epsilon^r, \epsilon^m \) shocks

<table>
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<tr>
<th>( n )</th>
<th>Model</th>
<th>( \sigma(\pi) )</th>
<th>( \sigma(c) )</th>
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<td>( \theta = 0 ) (CES)</td>
<td>( \epsilon^r )</td>
<td>( \epsilon^m )</td>
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<td>( \infty )</td>
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<td>2.0%</td>
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<td>10</td>
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<tr>
<td></td>
<td>Non-strategic</td>
<td>2.7%</td>
<td>3.3%</td>
</tr>
</tbody>
</table>
Conclusions

- Monopolistic competition used pervasively

- Our paper: oligopoly…
  1. sufficient statistics for micro to macro
  2. calibration: concentration amplifies non-neutrality
  3. for simple shocks: mostly driven by implied demand shape, rather than strategic interactions
  4. more differences with Phillips curve and general shocks