Aggregation in Heterogeneous-Firm Models

Theory and Measurement

Yann Koby — Brown

Christian K. Wolf — Chicago/MIT

NBER Monetary Economics, July 2020

Investment in General Equilibrium

- Old debate: does **lumpy** i shape the behavior of I?

  Caballero-Engel (1995), Khan-Thomas (2008), Bachmann et al. (2013), Winberry (2018), …

Investment in General Equilibrium

- Old debate: does **lumpy** i shape the behavior of I?

  Caballero-Engel (1995), Khan-Thomas (2008), Bachmann et al. (2013), Winberry (2018), …

  - **Yes in PE**: recession = fewer adjusters = less elastic I demand

Investment in General Equilibrium

- Old debate: does **lumpy** i shape the behavior of I?

  Caballero-Engel (1995), Khan-Thomas (2008), Bachmann et al. (2013), Winberry (2018), …

  - **Yes in PE**: recession = fewer adjusters = less elastic I demand
  - **Maybe in GE**: prices smooth out state dependence = **"GE aggregation"**

Investment in General Equilibrium

- Old debate: does **lumpy** *i* shape the behavior of *I*?

  Caballero-Engel (1995), Khan-Thomas (2008), Bachmann et al. (2013), Winberry (2018), …

  - **Yes in PE**: recession = fewer adjusters = less elastic *I* demand
  - **Maybe in GE**: prices smooth out state dependence = **"GE aggregation"**

- New questions, same tension: **financial frictions/uncertainty shocks**

  Khan-Thomas (2013), Shourideh-Zetlin-Jones (2017), Bloom et al. (2018), …

Investment in General Equilibrium

- Old debate: does **lumpy** *i* shape the behavior of *I*?

  Caballero-Engel (1995), Khan-Thomas (2008), Bachmann et al. (2013), Winberry (2018), …

  - **Yes in PE**: recession = fewer adjusters = less elastic *I* demand
  - **Maybe in GE**: prices smooth out state dependence = **"GE aggregation"**

- New questions, same tension: **financial frictions/uncertainty shocks**

  Khan-Thomas (2013), Shourideh-Zetlin-Jones (2017), Bloom et al. (2018), …

This paper: theory & measurement for strength of **GE aggregation**

Our Contributions

1. **Theory**: GE aggregation = **price-elastic investment**

Our Contributions

1. **Theory**: GE aggregation = **price-elastic investment**

    - Formal result: investment is highly elastic around fixed eq'm price path

Our Contributions

1. **Theory**: GE aggregation = **price-elastic investment**

   - Formal result: investment is highly elastic around fixed eq'm price path
   - Previous work disagreed because price elasticity wasn't targeted

     E.g.: $\boldsymbol{\beta \equiv \frac{\partial \log I}{\partial r}}$ *is* **-500%** *in Khan & Thomas (2008), vs.* **-10%** *in Winberry (2018)*

Our Contributions

1. **Theory**: GE aggregation = **price-elastic investment**
    - Formal result: investment is highly elastic around fixed eq'm price path
    - Previous work disagreed because price elasticity wasn't targeted

      E.g.: $\boldsymbol{\beta \equiv \frac{\partial \log I}{\partial r}}$ *is* **-500%** *in Khan & Thomas (2008), vs.* **-10%** *in Winberry (2018)*

2. **Measurement**: experimental evidence to learn about β

Our Contributions

1. **Theory**: GE aggregation = **price-elastic investment**
    - Formal result: investment is highly elastic around fixed eq'm price path
    - Previous work disagreed because price elasticity wasn't targeted

      E.g.: $\boldsymbol{\beta \equiv \frac{\partial \log I}{\partial r}}$ *is* **-500%** *in Khan & Thomas (2008), vs.* **-10%** *in Winberry (2018)*

2. **Measurement**: experimental evidence to learn about β
    - Estimate cross-sectional tax stimulus regressions *[à la Zwick-Mahon (2017)]*

$$\log(i_{jt}) \quad = \quad \underbrace{\beta_{ZM}}_{\approx -7\%} \times \text{cost of capital}_{jt} + \text{controls} + \text{error}$$

Our Contributions

1. **Theory**: GE aggregation = **price-elastic investment**
   - Formal result: investment is highly elastic around fixed eq'm price path
   - Previous work disagreed because price elasticity wasn't targeted

     E.g.: $\boldsymbol{\beta \equiv \frac{\partial \log I}{\partial r}}$ *is* **-500%** *in Khan & Thomas (2008), vs.* **-10%** *in Winberry (2018)*

2. **Measurement**: experimental evidence to learn about β
   - Estimate cross-sectional tax stimulus regressions *[à la Zwick-Mahon (2017)]*

$$\log(i_{jt}) \quad = \quad \underbrace{\beta_{ZM}}_{\approx -7\%} \times \text{cost of capital}_{jt} + \text{controls} + \text{error}$$

   - Interpretation: give sufficient conditions for $|\beta| \leq |\beta_{ZM}|$

Our Contributions

1. **Theory**: GE aggregation = **price-elastic investment**
   - Formal result: investment is highly elastic around fixed eq'm price path
   - Previous work disagreed because price elasticity wasn't targeted

     E.g.: $\boldsymbol{\beta \equiv \frac{\partial \log I}{\partial r}}$ *is* **-500%** *in Khan & Thomas (2008), vs.* **-10%** *in Winberry (2018)*

2. **Measurement**: experimental evidence to learn about β
   - Estimate cross-sectional tax stimulus regressions *[à la Zwick-Mahon (2017)]*

$$\log(i_{jt}) \quad = \quad \underbrace{\beta_{ZM}}_{\approx -7\%} \times \text{cost of capital}_{jt} + \text{controls} + \text{error}$$

   - Interpretation: give sufficient conditions for $|\beta| \leq |\beta_{ZM}|$

3. **Applications**: state dependence in monetary & fiscal policy transmission

Theory

Roadmap

- Today's focus: **lumpy investment**

Roadmap

- Today's focus: **lumpy investment**
  - Well-established: fraction of firms willing to adjust k is procyclical

    Caballero-Engel (1995), Khan-Thomas (2008), Winberry (2018), …
  - **Q**: Does this imply that the response of I to macro shocks is procyclical?

Roadmap

- Today's focus: **lumpy investment**
  - Well-established: fraction of firms willing to adjust k is procyclical

    Caballero-Engel (1995), Khan-Thomas (2008), Winberry (2018), …
  - **Q**: Does this imply that the response of I to macro shocks is procyclical?

$$\underbrace{\hspace{12cm}}_{\text{Where we're going: limit case where } \frac{dI}{d\text{shock}} \perp \text{ \# of adjusters}}$$

Roadmap

- Today's focus: **lumpy investment**
  - Well-established: fraction of firms willing to adjust k is procyclical
    Caballero-Engel (1995), Khan-Thomas (2008), Winberry (2018), …
  - **Q**: Does this imply that the response of I to macro shocks is procyclical?

Where we're going: limit case where $\frac{dI}{d\text{shock}} \perp$ # of adjusters

- At the end of the talk: extension to **financial frictions**
  - Similar result: limit case where $\frac{dI}{d\text{shock}} \perp$ # of borr.-constrained firms

Environment

- **Setting**: discrete-time, infinite-horizon, aggregate TFP shocks

Environment

- **Setting**: discrete-time, infinite-horizon, aggregate TFP shocks
- **Production block**: competitive intermediate goods producer $j \in [0, 1]$

$$\max \quad \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\prod_{s=0}^{t-1} \frac{1}{1 + r_s^b} \right) d_{jt} \right]$$

such that

$$\begin{aligned} d_{jt} &= x_t y_{jt} - w_t \ell_{jt} - q_t i_{jt} - \boldsymbol{\phi(k_{jt}, k_{jt-1})} \\ y_{jt} &= z_t e_{jt} (k_{jt-1}^{\alpha} \ell_{jt}^{1-\alpha})^{\nu} \\ i_{jt} &= k_{jt} - (1 - \delta) k_{jt-1} \end{aligned}$$

Environment

- **Setting**: discrete-time, infinite-horizon, aggregate TFP shocks
- **Production block**: competitive intermediate goods producer $j \in [0, 1]$

$$\max \quad \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\prod_{s=0}^{t-1} \frac{1}{1 + r_s^b} \right) d_{jt} \right]$$

such that

$$\begin{aligned} d_{jt} &= x_t y_{jt} - w_t \ell_{jt} - q_t i_{jt} - \boldsymbol{\phi(k_{jt}, k_{jt-1})} \\ y_{jt} &= z_t e_{jt} (k_{jt-1}^{\alpha} \ell_{jt}^{1-\alpha})^{\nu} \\ i_{jt} &= k_{jt} - (1 - \delta) k_{jt-1} \end{aligned}$$

Distribution μ_0 over
$s^f = (e, k) \in \mathcal{S}^f$

→ With **fixed costs**: shifts in μ_0 = changes in # of adjusters

Environment

- **Setting**: discrete-time, infinite-horizon, aggregate TFP shocks
- **Production block**: competitive intermediate goods producer $j \in [0, 1]$

$$\max \quad \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\prod_{s=0}^{t-1} \frac{1}{1 + r_s^b} \right) d_{jt} \right]$$

such that

$$\begin{aligned} d_{jt} &= x_t y_{jt} - w_t \ell_{jt} - q_t i_{jt} - \boldsymbol{\phi(k_{jt}, k_{jt-1})} \\ y_{jt} &= z_t e_{jt} (k_{jt-1}^{\alpha} \ell_{jt}^{1-\alpha})^{\nu} \\ i_{jt} &= k_{jt} - (1 - \delta) k_{jt-1} \end{aligned}$$

Distribution μ_0 over
$s^f = (e, k) \in \mathcal{S}^f$

$\rightarrow$ With **fixed costs**: shifts in μ_0 = changes in # of adjusters

- **Rest**: representative household, sticky prices & wages, Taylor rule, …

Smets-Wouters (2007), Justiniano-Primiceri-Tambalotti (2010), …

Experiment

Q: μ_0 is s.t. few firms adjust k. Does I respond less to z?

Experiment

Q: μ_0 is s.t. few firms adjust k. Does I respond less to z?

- Notation: Boldface is path, hat indicates IRF. Will characterize $\hat{\mathbf{I}}_{\mu_0}(\mathbf{z})$.

Experiment

Q: μ_0 is s.t. few firms adjust k. Does I respond less to z?

- Notation: Boldface is path, hat indicates IRF. Will characterize $\hat{\mathbf{I}}_{\mu_0}(\mathbf{z})$.

R1 **First-order perturbation** around perfect foresight transition path

$\rightarrow$ Equivalently: perfect foresight for $\boldsymbol{z}$ given arbitrary initial state μ_0

Experiment

Q: μ_0 is s.t. few firms adjust k. Does I respond less to z?

- Notation: Boldface is path, hat indicates IRF. Will characterize $\hat{\mathbf{I}}_{\mu_0}(\mathbf{z})$.

R1 **First-order perturbation** around perfect foresight transition path

$\rightarrow$ Equivalently: perfect foresight for $\boldsymbol{z}$ given arbitrary initial state μ_0

R2 Only for simplicity: eq'm = market clearing in **one market**

Experiment

Q: μ_0 is s.t. few firms adjust k. Does I respond less to z?

- Notation: Boldface is path, hat indicates IRF. Will characterize $\hat{\mathbf{I}}_{\mu_0}(\mathbf{z})$.

R1 **First-order perturbation** around perfect foresight transition path

$\rightarrow$ Equivalently: perfect foresight for $\boldsymbol{z}$ given arbitrary initial state μ_0

R2 Only for simplicity: eq'm = market clearing in **one market**

Assumption

▸ Nested Models

Let $\mathbf{p} = (\mathbf{r}, \mathbf{x}, \mathbf{w}, \mathbf{q})$ denote a price path. There exists a function $\mathcal{P}(\bullet)$, independent of the production block, s.t. an equilibrium is a path $\mathbf{C}$ with

$$C_t \quad = \quad Y_t(\mathbf{p}; \mathbf{z}) \; - \; I_t(\mathbf{p}; \mathbf{z}) \quad \equiv \quad C_t^s, \quad \text{for } t = 0, 1, 2, \ldots$$

where $\mathbf{p} = \mathcal{P}(\mathbf{C})$.

Exact Aggregation

R3 To build intuition: **reduced-form model of lumpy investment**

- $\rightarrow$ Special adjustment costs: fraction $\xi \in (0, 1)$ of firms has infinite adjustment costs, the rest zero
- $\rightarrow$ $1 - \xi$ = # of adjusters is reduced-form stand-in for changes in μ_0

Exact Aggregation

R3 To build intuition: **reduced-form model of lumpy investment**

- $\rightarrow$ Special adjustment costs: fraction $\xi \in (0, 1)$ of firms has infinite adjustment costs, the rest zero
- $\rightarrow$ $1 - \xi$ = # of adjusters is reduced-form stand-in for changes in μ_0

- **Q**: Does $\widehat{\mathbf{I}}_\xi(\mathbf{z})$ change with ξ?

Exact Aggregation

R3 To build intuition: **reduced-form model of lumpy investment**

- → Special adjustment costs: fraction $\xi \in (0,1)$ of firms has infinite adjustment costs, the rest zero
- → $1-\xi$ = # of adjusters is reduced-form stand-in for changes in μ_0

- **Q**: Does $\widehat{\mathbf{I}}_\xi(\mathbf{z})$ change with ξ?
  - **Yes in PE**: fewer adjusters = less investment demand

$$\widehat{\mathbf{I}}_\xi(\mathbf{p};\mathbf{z}) - \widehat{\mathbf{I}}_0(\mathbf{p};\mathbf{z}) \quad = \quad -\xi \;\times\; \widehat{\mathbf{I}}_0(\mathbf{p};\mathbf{z}), \quad \text{for any } (\mathbf{p},\mathbf{z})$$

Exact Aggregation

R3 To build intuition: **reduced-form model of lumpy investment**

- $\rightarrow$ Special adjustment costs: fraction $\xi \in (0, 1)$ of firms has infinite adjustment costs, the rest zero
- $\rightarrow$ $1 - \xi$ = # of adjusters is reduced-form stand-in for changes in μ_0

- **Q**: Does $\widehat{\mathbf{I}}_\xi(\mathbf{z})$ change with ξ?
  - **Yes in PE**: fewer adjusters = less investment demand

$$\widehat{\mathbf{I}}_\xi(\mathbf{p}; \mathbf{z}) - \widehat{\mathbf{I}}_0(\mathbf{p}; \mathbf{z}) \quad = \quad -\xi \ \times \ \widehat{\mathbf{I}}_0(\mathbf{p}; \mathbf{z}), \quad \text{for any } (\mathbf{p}, \mathbf{z})$$

  - **Not necessarily in GE**

Proposition

Impose R1 - R3, and let $\nu \rightarrow 1$ or $\bar{r} + \delta \rightarrow 0$. Then the equilibrium price paths $\mathbf{p}$ and the investment path $\mathbf{I}$ are independent of ξ. ▸ vs. House (2014)

Exact Aggregation

- **Q**: How does $\mathbf{p}$ respond to changes in ξ?

$$\mathbf{Y}(\mathcal{P}(\mathbf{C}); \mathbf{z}) - \mathbf{I}(\mathcal{P}(\mathbf{C}); \mathbf{z}) \; = \; \mathbf{C}$$

Exact Aggregation

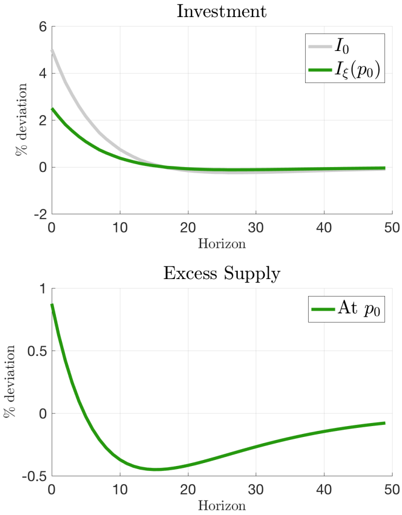
- **Q**: How does **p** respond to changes in ξ?

$$\mathbf{Y}(\mathcal{P}(\mathbf{C});\mathbf{z}) - \mathbf{I}(\mathcal{P}(\mathbf{C});\mathbf{z}) \;=\; \mathbf{C}$$

- Heuristic argument – pretend it's static:

$$\hat{\mathrm{p}}_\xi - \hat{\mathrm{p}}_0 = \underbrace{\frac{\mathcal{P}_C}{1 - C^s_p \cdot \mathcal{P}_C}}_{\text{price elasticities}} \times \underbrace{\left[\hat{C}^s_\xi(p_0;z) - \hat{C}^s_0(p_0;z)\right]}_{\text{net excess supply}}$$

Exact Aggregation



- **Q**: How does **p** respond to changes in ξ?

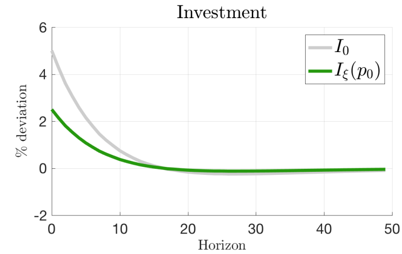
$$\mathbf{Y}(\mathcal{P}(\mathbf{C});\mathbf{z}) - \mathbf{I}(\mathcal{P}(\mathbf{C});\mathbf{z}) \;=\; \mathbf{C}$$

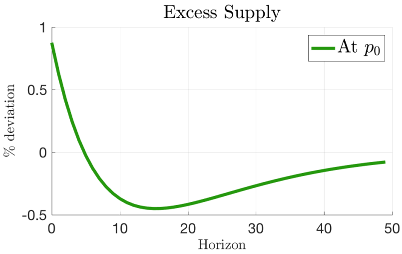
- Heuristic argument – pretend it's static:

$$\hat{\mathrm{p}}_\xi - \hat{\mathrm{p}}_0 = \underbrace{\frac{\mathcal{P}_C}{1 - C_p^s \cdot \mathcal{P}_C}}_{\text{price elasticities}} \times \underbrace{\left[\hat{C}_\xi^s(p_0; z) - \hat{C}_0^s(p_0; z)\right]}_{\text{net excess supply}}$$

  - Dynamic: **supply vector**

Exact Aggregation





- **Q**: How does $\mathbf{p}$ respond to changes in ξ?

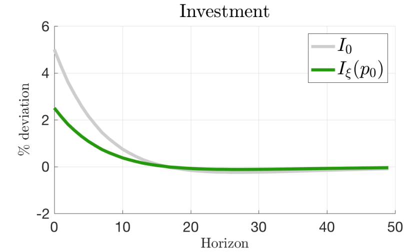
$$\mathbf{Y}(\mathcal{P}(\mathbf{C});\mathbf{z}) - \mathbf{I}(\mathcal{P}(\mathbf{C});\mathbf{z}) \;=\; \mathbf{C}$$

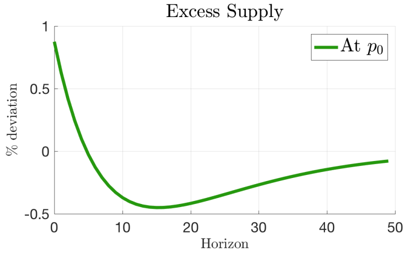
- Heuristic argument – pretend it's static:

$$\hat{\mathrm{p}}_\xi - \hat{\mathrm{p}}_0 = \underbrace{\frac{\mathcal{P}_C}{1 - C_p^s \cdot \mathcal{P}_C}}_{\text{price elasticities}} \times \underbrace{\left[\hat{C}_\xi^s(p_0;z) - \hat{C}_0^s(p_0;z)\right]}_{\text{net excess supply}}$$

  - Dynamic: **matrix $\mathcal{H}$** × **supply vector**

Exact Aggregation





- **Q**: How does **p** respond to changes in ξ?

$$\mathbf{Y}(\mathcal{P}(\mathbf{C});\mathbf{z}) - \mathbf{I}(\mathcal{P}(\mathbf{C});\mathbf{z}) \;=\; \mathbf{C}$$

- Heuristic argument – pretend it's static:

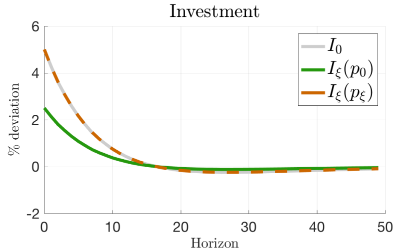
$$\hat{\mathrm{p}}_\xi - \hat{\mathrm{p}}_0 = \underbrace{\frac{\mathcal{P}_C}{1 - C_p^s \cdot \mathcal{P}_C}}_{\text{price elasticities}} \times \underbrace{\left[\hat{C}_\xi^s(p_0;z) - \hat{C}_0^s(p_0;z)\right]}_{\text{net excess supply}}$$

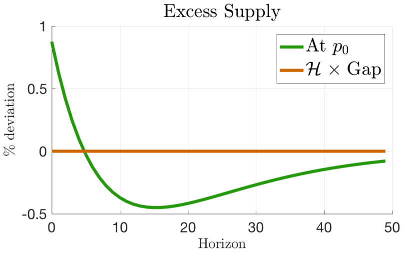
  - Dynamic: **matrix $\mathcal{H}$** $\times$ **supply vector**

- As $\nu \to 1$ or $\bar{r} + \delta \to 0$:

$$\frac{\partial \widehat{k}_{jt+1}}{\partial \widehat{q}_t} = -\frac{1-(1-\alpha)\nu}{1-\nu} \times \frac{1+\bar{r}}{\bar{r}+\delta} \to \infty$$

Exact Aggregation





- **Q**: How does $\mathbf{p}$ respond to changes in ξ?

$$\mathbf{Y}(\mathcal{P}(\mathbf{C});\mathbf{z}) - \mathbf{I}(\mathcal{P}(\mathbf{C});\mathbf{z}) \;=\; \mathbf{C}$$

- Heuristic argument – pretend it's static:

$$\hat{\mathrm{p}}_\xi - \hat{\mathrm{p}}_0 = \underbrace{\frac{\mathcal{P}_C}{1 - C_p^s \cdot \mathcal{P}_C}}_{\text{price elasticities}} \times \underbrace{\left[\hat{C}_\xi^s(p_0; z) - \hat{C}_0^s(p_0; z)\right]}_{\text{net excess supply}}$$

  - Dynamic: **matrix $\mathcal{H}$** × **supply vector**

- As $\nu \to 1$ or $\bar{r} + \delta \to 0$:

$$\underbrace{\frac{\partial \widehat{k}_{jt+1}}{\partial \widehat{q}_t} = -\frac{1-(1-\alpha)\nu}{1-\nu} \times \frac{1+\bar{r}}{\bar{r}+\delta} \to \infty}_{C_p^s \to \infty\text{: “shifting a flat } C^s\text{-curve”}}$$

Interpreting Previous Work

Same eq'm characterization with general fixed costs:

$$\hat{\mathbf{p}}_{\mu_0} - \hat{\mathbf{p}}_{\bar{\mu}_0} = \mathcal{H} \times \left[\hat{\mathbf{C}}^s_{\mu_0}(\mathbf{p}_{\bar{\mu}_0}; \mathbf{z}) - \hat{\mathbf{C}}^s_{\bar{\mu}_0}(\mathbf{p}_{\bar{\mu}_0}; \mathbf{z})\right]$$

Interpreting Previous Work

Same eq'm characterization with general fixed costs:

$$\hat{\mathbf{p}}_{\mu_0} - \hat{\mathbf{p}}_{\bar{\mu}_0} = \mathcal{H} \times \left[\hat{\mathbf{C}}^s_{\mu_0}(\mathbf{p}_{\bar{\mu}_0}; \mathbf{z}) - \hat{\mathbf{C}}^s_{\bar{\mu}_0}(\mathbf{p}_{\bar{\mu}_0}; \mathbf{z})\right]$$

GE adjustment: no direct estimation target = disagreement ▸ Details

Interpreting Previous Work

Same eq'm characterization with general fixed costs:

$$\hat{\mathbf{p}}_{\mu_0} - \hat{\mathbf{p}}_{\bar{\mu}_0} = \mathcal{H} \times \left[\hat{\mathbf{C}}^s_{\mu_0}(\mathbf{p}_{\bar{\mu}_0}; \mathbf{z}) - \hat{\mathbf{C}}^s_{\bar{\mu}_0}(\mathbf{p}_{\bar{\mu}_0}; \mathbf{z})\right]$$

GE adjustment: no direct estimation target = disagreement ▸ Details

1. **Khan & Thomas (2008)**: $\beta \equiv \frac{\partial \log I(\bar{p};\bar{\mu}_0)}{\partial q} \approx -500\%$

   → Focal point RBC: linear firm side (= firms adjust), small Δr and large smoothing

   → Thus: I is highly elastic around eq'm price path of rep.-firm economy

Interpreting Previous Work

Same eq'm characterization with general fixed costs:

$$\hat{\mathbf{p}}_{\mu_0} - \hat{\mathbf{p}}_{\bar{\mu}_0} = \mathcal{H} \times \left[\hat{\mathbf{C}}^s_{\mu_0}(\mathbf{p}_{\bar{\mu}_0}; \mathbf{z}) - \hat{\mathbf{C}}^s_{\bar{\mu}_0}(\mathbf{p}_{\bar{\mu}_0}; \mathbf{z})\right]$$

GE adjustment: no direct estimation target = disagreement ‣ Details

1. **Khan & Thomas (2008)**: $\beta \equiv \frac{\partial \log I(\bar{p};\bar{\mu}_0)}{\partial q} \approx -500\%$
   - $\rightarrow$ Focal point RBC: linear firm side (= firms adjust), small Δr and large smoothing
   - $\rightarrow$ Thus: I is highly elastic around eq'm price path of rep.-firm economy
2. **Winberry (2018)**: $\beta \equiv \frac{\partial \log I(\bar{p};\bar{\mu}_0)}{\partial q} \approx -10\%$
   - $\rightarrow$ 10% cyclical asymmetry needs 1% $\Delta r \rightarrow$ large ΔC demand $\rightarrow$ large Δr^{GE}

Interpreting Previous Work

Same eq'm characterization with general fixed costs:

$$\hat{\mathbf{p}}_{\mu_0} - \hat{\mathbf{p}}_{\bar{\mu}_0} = \mathcal{H} \times \left[\hat{\mathbf{C}}^s_{\mu_0}(\mathbf{p}_{\bar{\mu}_0};\mathbf{z}) - \hat{\mathbf{C}}^s_{\bar{\mu}_0}(\mathbf{p}_{\bar{\mu}_0};\mathbf{z})\right]$$

GE adjustment: no direct estimation target = disagreement ▸ Details

1. **Khan & Thomas (2008)**: $\beta \equiv \frac{\partial \log I(\bar{p};\bar{\mu}_0)}{\partial q} \approx -500\%$

   → Focal point RBC: linear firm side (= firms adjust), small Δr and large smoothing

   → Thus: I is highly elastic around eq'm price path of rep.-firm economy

2. **Winberry (2018)**: $\beta \equiv \frac{\partial \log I(\bar{p};\bar{\mu}_0)}{\partial q} \approx -10\%$

   → 10% cyclical asymmetry needs 1% Δr → large ΔC demand → large Δr^{GE}

Distinguish using response of r to z? Here instead: **measure β**!

Measurement

Measurement

How can we learn about β?

Measurement

How can we learn about β?

- Ideal candidate: **firm-level quasi-experiment**
    - Interpretation: β = slope of investment demand curve
    - Thus: time series variation is not useful, cross-sectional variation is

Measurement

How can we learn about β?

- Ideal candidate: **firm-level quasi-experiment**
  - Interpretation: β = slope of investment demand curve
  - Thus: time series variation is not useful, cross-sectional variation is
- Our approach: **bonus depreciation stimulus** Details
  - Policy: ability to temporarily write-off/tax-deduct investment at a faster rate
  - Research design: DiD using heterogeneity in treatment by δ_j *[Zwick-Mahon]*

$$\log(i_{jt}) = \alpha_j + \delta_t + \beta_{ZM} \times q_{jt}(\delta_j) + \text{error}$$

Measurement

How can we learn about β?

- Ideal candidate: **firm-level quasi-experiment**
  - Interpretation: β = slope of investment demand curve
  - Thus: time series variation is not useful, cross-sectional variation is
- Our approach: **bonus depreciation stimulus** ▸ Details
  - Policy: ability to temporarily write-off/tax-deduct investment at a faster rate
  - Research design: DiD using heterogeneity in treatment by δ_j *[Zwick-Mahon]*

$$\underbrace{\log(i_{jt}) = \alpha_j + \delta_t + \beta_{ZM} \times q_{jt}(\delta_j) + \text{error}}$$

$\boldsymbol{\beta_{ZM} \approx -7\%}$. What does that tell us?

Estimand Interpretation

$$\log(i_{jt}) = \alpha_j + \delta_t + \beta_{ZM} \times q_{jt}(\delta_j) + \text{error}$$

Estimand Interpretation

$$\log(i_{jt}) = \alpha_j + \delta_t + \beta_{ZM} \times q_{jt}(\delta_j) + \text{error}$$

Proposition

Extend the baseline model to allow for permanent heterogeneity in $\{\delta_j\}$. Let

$$\tilde{\beta} \quad \equiv \quad \int_{s:i_t(s)>0} \frac{\partial \log(i_t(s))}{\partial q_t} d\tilde{\mu}(s)$$

where q is the cost of capital and $\tilde{\mu}$ is the truncated firm state distribution.

Estimand Interpretation

$$\log(i_{jt}) = \alpha_j + \delta_t + \beta_{ZM} \times q_{jt}(\delta_j) + \text{error}$$

A1 Investment is **equally price-elastic** at all (adjusting) firms.

Proposition

Extend the baseline model to allow for permanent heterogeneity in $\{\delta_j\}$. Let

$$\tilde{\beta} \quad \equiv \quad \int_{s:i_t(s)>0} \frac{\partial \log(i_t(s))}{\partial q_t} d\tilde{\mu}(s)$$

where q is the cost of capital and $\tilde{\mu}$ is the truncated firm state distribution. Then, under A1 and to first order,

$$\beta_{ZM} \quad \overset{p}{\to} \quad \tilde{\beta}$$

Estimand Interpretation

$$\log(i_{jt}) = \alpha_j + \delta_t + \beta_{ZM} \times q_{jt}(\delta_j) + \text{error}$$

A1 Investment is equally price-elastic at all (adjusting) firms.

A2 All firms respond identically to the **movements in p** induced by the policy.

Proposition

Extend the baseline model to allow for permanent heterogeneity in $\{\delta_j\}$. Let

$$\tilde{\beta} \equiv \int_{s:i_t(s)>0} \frac{\partial \log(i_t(s))}{\partial q_t} d\tilde{\mu}(s)$$

where q is the cost of capital and $\tilde{\mu}$ is the truncated firm state distribution. Then, under A2 and to first order,

$$\beta_{ZM} \xrightarrow{p} \tilde{\beta} + \underbrace{\frac{Cov_{\tilde{\mu}(s)}\left(\left(\frac{\partial \log(i_t(s))}{\partial q_t} - \tilde{\beta}\right) q_t(s), q_t(s)\right)}{Var_{\tilde{\mu}(s)}(q_t(s))}}_{\text{selection effect}}$$

Estimand Interpretation

$$\log(i_{jt}) = \alpha_j + \delta_t + \beta_{ZM} \times q_{jt}(\delta_j) + \text{error}$$

A1 Investment is equally price-elastic at all (adjusting) firms.

A2 All firms respond identically to the movements in **p** induced by the policy.

Proposition

Extend the baseline model to allow for permanent heterogeneity in $\{\delta_j\}$. Let

$$\tilde{\beta} \equiv \int_{s:i_t(s)>0} \frac{\partial \log(i_t(s))}{\partial q_t} d\tilde{\mu}(s)$$

where q is the cost of capital and $\tilde{\mu}$ is the truncated firm state distribution. Then, to first order,

$$\beta_{ZM} \xrightarrow{p} \tilde{\beta} + \underbrace{\frac{Cov_{\tilde{\mu}(s)}\left(\left(\frac{\partial \log(i_t(s))}{\partial q_t} - \tilde{\beta}\right) q_t(s), q_t(s)\right)}{Var_{\tilde{\mu}(s)}(q_t(s))}}_{\text{selection effect}} + \underbrace{\frac{Cov_{\tilde{\mu}(s)}\left(\frac{\partial \log(i_t(s))}{\partial \mathrm{p}} \hat{\mathbf{p}}, q_t(s)\right)}{Var_{\tilde{\mu}(s)}(q_t(s))}}_{\text{heterogeneous GE exposure}}$$

Results

$$\log(i_{jt}) = \alpha_j + \delta_t + \beta_{ZM} \times q_{jt}(\delta_j) + \text{error}$$

- **Headline number**: $\beta_{ZM} \approx -7\%$ ▸ Details, Robustness & Extensions
  - Estimation details: "universe" (corporate tax return data), pool two bonus depreciation episodes, b_{jt} at 4-digit industry level
  - Extensions/robustness: Compustat, dynamics, GDP & trend interactions, extensive margin, b_{jt} at firm level …

Results

$$\log(i_{jt}) = \alpha_j + \delta_t + \beta_{ZM} \times q_{jt}(\delta_j) + \text{error}$$

- **Headline number**: $\beta_{ZM} \approx -7\%$ ▸ Details, Robustness & Extensions
  - Estimation details: "universe" (corporate tax return data), pool two bonus depreciation episodes, b_{jt} at 4-digit industry level
  - Extensions/robustness: Compustat, dynamics, GDP & trend interactions, extensive margin, b_{jt} at firm level …
- **Interpretation**: $|\beta| \leq |\beta_{ZM}|$

Results

$$\log(i_{jt}) = \alpha_j + \delta_t + \beta_{ZM} \times q_{jt}(\delta_j) + \text{error}$$

- **Headline number**: $\beta_{ZM} \approx -7\%$ ▸ Details, Robustness & Extensions
  - Estimation details: "universe" (corporate tax return data), pool two bonus depreciation episodes, b_{jt} at 4-digit industry level
  - Extensions/robustness: Compustat, dynamics, GDP & trend interactions, extensive margin, b_{jt} at firm level …
- **Interpretation**: $|\beta| \leq |\beta_{ZM}|$
  1. Back-of-the-envelope (Model + A1): $\beta = \beta_{ZM} \approx -7\%$

Results

$$\log(i_{jt}) = \alpha_j + \delta_t + \beta_{ZM} \times q_{jt}(\delta_j) + \text{error}$$

- **Headline number**: $\beta_{ZM} \approx -7\%$ ▸ Details, Robustness & Extensions
  - Estimation details: "universe" (corporate tax return data), pool two bonus depreciation episodes, b_{jt} at 4-digit industry level
  - Extensions/robustness: Compustat, dynamics, GDP & trend interactions, extensive margin, b_{jt} at firm level …
- **Interpretation**: $|\beta| \leq |\beta_{ZM}|$
  1. Back-of-the-envelope (Model + A1): $\beta = \beta_{ZM} \approx -7\%$
  2. Indirect inference (Model + $\approx$ A2): $\beta \approx -5\%$

     → Add β_{ZM} as estimation target ("identified moment") in rich het.-firm model with two depreciation types, persistent z shocks, aggregate effects, in recession, …

     → Upward bias due to selection effect, GE exposure effect is small

Applications

Monetary Policy: Pushing on a String

Q: Why does monetary policy seem to "push on a string" in recessions?

Monetary Policy: Pushing on a String

Q: Why does monetary policy seem to "push on a string" in recessions?

- Possible mechanism: procyclical price elasticity of investment demand

Monetary Policy: Pushing on a String

Q: Why does monetary policy seem to "push on a string" in recessions?

- Possible mechanism: procyclical price elasticity of investment demand
- Our approach: NK model + lumpy investment + β_{ZM} ▸ Details
  1. "**PE calibration**": $\mathbb{E}(i)$, $\sigma(i)$, spike rate, inaction rate
  2. "**GE calibration**": β_{ZM} plus standard non-production block

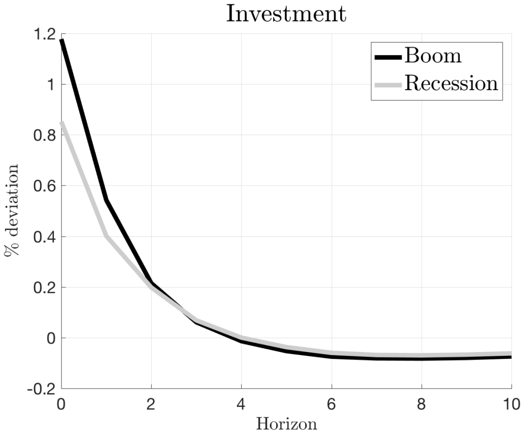
Monetary Policy: Pushing on a String

Q: Why does monetary policy seem to "push on a string" in recessions?

- Possible mechanism: procyclical price elasticity of investment demand
- Our approach: NK model + lumpy investment + β_{ZM} ▸ Details
  1. "**PE calibration**": $\mathbb{E}(i)$, $\sigma(i)$, spike rate, inaction rate
  2. "**GE calibration**": β_{ZM} plus standard non-production block
- Find: pushing-on-a-string in **PE** & **GE**
  - i is 70% more responsive **given prices**, and 40% more responsive in **GE**
  - Without β_{ZM} targeted: asymmetry disappears *[Smets-Wouters + Khan-Thomas]*

Monetary Policy: Pushing on a String

Q: Why does monetary policy seem to "push on a string" in recessions?



Fiscal Policy & Firm Cash Flow

- In paper: theory & measurement with **financial frictions** ▸ Details

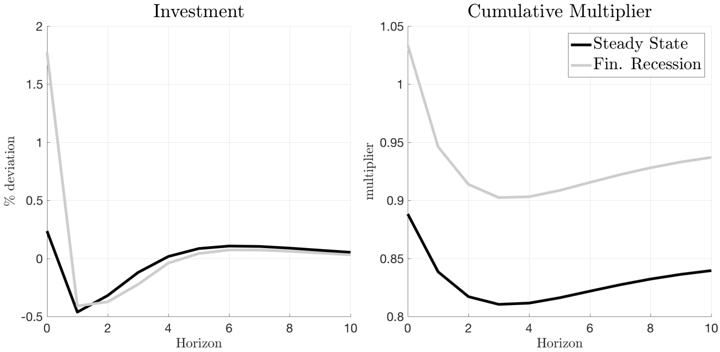
Fiscal Policy & Firm Cash Flow

- In paper: theory & measurement with **financial frictions** ▸ Details
- Experiment: fiscal stimulus with cash flow-sensitive investment ▸ Details

  1. "**PE calibration**": earnings-based borr. constraint, initial entrant size/debt
  2. "**GE calibration**": β_{ZM} plus standard non-production block

Fiscal Policy & Firm Cash Flow

- In paper: theory & measurement with **financial frictions** ▸ Details
- Experiment: fiscal stimulus with cash flow-sensitive investment ▸ Details
  1. "**PE calibration**": earnings-based borr. constraint, initial entrant size/debt
  2. "**GE calibration**": β_{ZM} plus standard non-production block



Conclusions

Conclusions

1. Investment price elasticities are central to **GE aggregation**

    - Applies to smoothing for lumpy investment/durables & financial frictions
    - Reduces disagreement in previous work to measurable "sufficient statistic"

      Khan-Thomas (2008), Bachmann-Caballero-Engel (2013), Winberry (2018)

Conclusions

1. Investment price elasticities are central to **GE aggregation**
   - Applies to smoothing for lumpy investment/durables & financial frictions
   - Reduces disagreement in previous work to measurable "sufficient statistic"

     Khan-Thomas (2008), Bachmann-Caballero-Engel (2013), Winberry (2018)

2. Preferred **direct measurement** suggests weak GE price effects

Conclusions

1. Investment price elasticities are central to **GE aggregation**
   - Applies to smoothing for lumpy investment/durables & financial frictions
   - Reduces disagreement in previous work to measurable "sufficient statistic"

     Khan-Thomas (2008), Bachmann-Caballero-Engel (2013), Winberry (2018)

2. Preferred **direct measurement** suggests weak GE price effects

3. Implications: μ_0 **matters** – but in which direction?
   - Pro- or counter-cyclical? lumpiness vs. cash-flow effects
   - Matters because investment takes center stage in (monetary) policy stimulus

     [e.g. Christiano-Eichenbaum-Evans, Kaplan-Moll-Violante, …]

Appendix

Model Closure

- Explicit closure: medium-scale NK-DSGE model

  close to Smets-Wouters (2007) and Justiniano-Primiceri-Tambalotti (2010)

- With mild additional restrictions this model satisfies R2:

Lemma

Suppose that:

1. *Labor disutility is linear.*
2. *The coefficient on output in then Taylor rule is* 0*.*
3. *There are no aggregate capital adjustment costs.*

Then, to first order, the full structural model satisfies R2. If prices and wages are flexible, then R2 is satisfied globally.

back

Relation to House (2014)

- Flat investment curve logic is related to House (2014)
  - He shows: in investment re-set model with $\delta \to 0$ investment timing is infinitely elastic w.r.t. q
  - Implies: in eq'm model of investment market distribution μ_0 is irrelevant
- How does our result generalize this?
  1. Rich GE model closure, rather than just investment market
  2. Aggregation not just for long-lived capital goods, also for linear revenue f'n
  3. Result is generic: infinite elasticity around rep.-firm eq'm price path, doesn't matter what friction delivers a gap given prices

back

General Equilibrium Adjustment $\mathcal{H}$

- $\mathcal{H}$ combines supply and demand price elasticities:

$$\mathcal{H} = \frac{\partial \mathcal{P}}{\partial \mathbf{C}} \times (I - \mathcal{G})^{-1}$$

where

$$\mathcal{G} \equiv \underbrace{\begin{pmatrix} \frac{\partial C^s}{\partial r} & \frac{\partial C^s}{\partial p^I} & \frac{\partial C^s}{\partial w} & \frac{\partial C^s}{\partial q} \end{pmatrix}}_{\text{Supply Elasticity}} \times \underbrace{\begin{pmatrix} \frac{\partial r}{\partial C} & \frac{\partial p^I}{\partial C} & \frac{\partial w}{\partial C} & \frac{\partial q}{\partial C} \end{pmatrix}}_{\text{Inverse Demand Elasticity}}$$

  - Note: unique left-inverse of $(I - \mathcal{G})$ is guaranteed if eq'm is unique

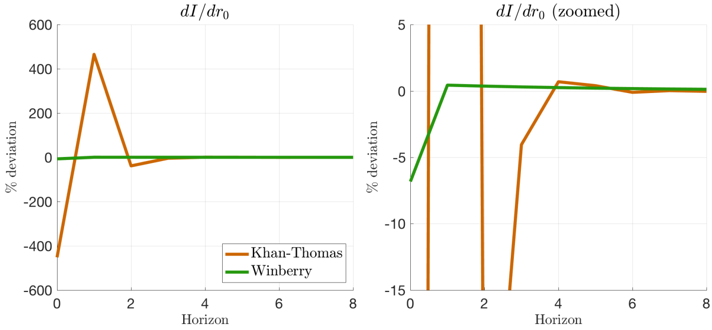
- R1-R3: for $\nu = 1$ or $\bar{r} + \delta = 0$, the map $\mathcal{H}$ is column rank-deficient, with

$$\left\{ \hat{\mathbf{C}}^s_\xi(\mathbf{p}_0; \mathbf{z}) - \hat{\mathbf{C}}^s_0(\mathbf{p}_0; \mathbf{z}) \in \text{null}(\mathcal{H}) \right\}$$

back

Khan-Thomas (2008) vs. Winberry (2018)

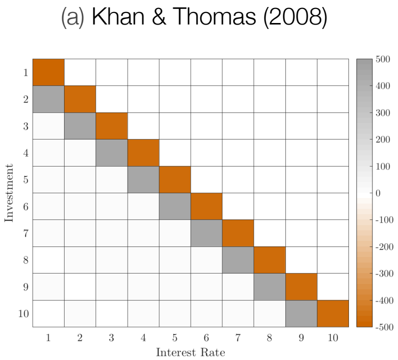
What do PE price elasticities look like in previous work?



back

Khan-Thomas (2008) vs. Winberry (2018)

The implied GE adjustment matrices look dramatically different:



▸ back

Standard Calibration Targets

- Investment lumpiness
  - All previous work matches $\mathbb{E}(i)$, $\sigma(i)$, spike rate, inaction rate
  - Implies: price elasticity $\perp$ lumpiness
- Aggregate prices
  - Winberry (2018): real rate is acyclical
  - Concerns
    1. Cyclicality conditional on z is ill-measured
    2. Theory: arbitrary rate cyclicality is consistent with aggregation
- Investment rate dispersion
  - Dispersed e + high elasticity $\Rightarrow$ dispersed i
  - Direct evidence on e suggests large dispersion $\Rightarrow$ need small elasticities

▸ back

Bonus Depreciation

- What is bonus depreciation?
  - In general: for every $ of investment reduce future tax liabilities
  - With bonus depreciation: tax reductions come earlier = PV benefit
- Computation of exposure term:

$$q_{jt}(\delta_j) = \sum_{t=0}^{\infty} \zeta^t \left(\prod_{q=0}^{\infty} \frac{1}{1 + r_{q-1}^b} \right) \tau_t^b(\delta_j)$$

- Formal equivalence to reduction in price of capital:

Lemma

The paths of all aggregates in response to an unexpected bonus depreciation shock with firm-specific schedules $\{\tau_{jt}^b\}_{t=0}^{\infty}$ are identical to response paths after a period-0 firm-specific investment subsidy shock with

$$\tau_{j0}^i = \tau_{j0}^b + \sum_{t=1}^{\infty} \zeta^t \left(\prod_{q=1}^{\infty} \frac{1}{1 + r_{q-1}^b} \right) \tau_{jt}^b$$

back

Estimation Details

- We extend the baseline analysis of Zwick & Mahon (2017):
  1. Compustat sample: larger firms, arguably less financially constrained
  2. Quarterly, dynamics: less time aggregation, learn about all entries of $\mathcal{H}$
  3. More controls: partial out heterogeneous exposure to aggregate conditions

Extension of Zwick & Mahon (2017)

Dependent Variable:	$\log(i_{j,t})$	$\log(i_{j,t+1})$	$\log(i_{j,t+2})$	$\log(i_{j,t+3})$	$\log(i_{j,t+4})$
$z_{n,t}$	1.64***	1.19***	0.78***	0.31	-0.12
	(0.28)	(0.28)	(0.29)	(0.29)	(0.30)
GDP Interaction	x	x	x	x	x
Trend Interaction	x	x	x	x	x
Firm & Time FEs	x	x	x	x	x
Observations	406,807	401,428	390,561	381,156	372,078
R-squared	0.85	0.85	0.85	0.86	0.86

back

Monetary Policy Application

- Standard NK parameterization for non-production (demand) block
  - → Robustness: habits, $\phi_y > 0$, non-linear labor disutility

DEMAND BLOCK PARAMETERIZATION

Parameter	Description	Value
β	Discount rate	1/1.04
h	Habit formation	0
γ	CRRA coefficient	1
φ	Frisch elasticity	∞
ϵ_p	Goods substitutability	10
θ_p	Price adjustment cost	40
ϵ_w	Wage substitutability	10
θ_w	Wage adjustment costs	100
κ	Aggregate K adjustment costs	0
ρ_{tr}	Taylor rule persistence	0.75
ϕ_π	Taylor rule inflation coefficient	1.5
ϕ_y	Taylor rule output coefficient	0

back

Monetary Policy Application

- Firm block: target **PE moments** + **GE price sensitivity**

Parameter Values

Parameter	Description	Value
Fixed Parameters		
$1-\xi$	Firm exit rate	0.065
δ	Depreciation rate	0.067
α	Capital share	0.310
ν	Returns to Scale	0.870
ρ	Productivity persistence	0.890
σ	Productivity dispersion	0.250
μ_0	Mean initial productivity	-0.375
σ_0	Initial productivity dispersion	0.330
Fitted Parameters		
κ	Quadratic adjustment costs	0.762
ϑ	Investment irreversibility	0.781
$\bar{\xi}$	Upper bound on fixed costs	0.450
a	Size of region without fixed costs	0.030
k_0	Capital of entrants	0.600

Targeted Moments

Target	Data	Model
Price Sensitivity		
Bonus depreciation estimand	2.890	2.984
Micro Investment		
Average investment rate	0.104	0.087
Std. of investment rates	0.160	0.147
Spike rate	0.144	0.108
Inaction rate	0.237	0.184
Employment Distribution		
Employment share of age-1 firms	0.016	0.028

back

Fiscal Policy Application

- Standard NK parameterization for non-production (demand) block
  - $\rightarrow$ Robustness: habits, $\phi_y > 0$, non-linear labor disutility

Demand Block Parameterization

Parameter	Description	Value
β	Discount rate	$1/1.04$
h	Habit formation	0
γ	CRRA coefficient	1
φ	Frisch elasticity	∞
ϵ_p	Goods substitutability	10
θ_p	Price adjustment cost	40
ϵ_w	Wage substitutability	10
θ_w	Wage adjustment costs	100
κ	Aggregate K adjustment costs	0
ρ_{tr}	Taylor rule persistence	0.75
ϕ_π	Taylor rule inflation coefficient	1.5
ϕ_y	Taylor rule output coefficient	0

back

Financial Frictions

- **Theory**
  - Allow for constraints on borrowing & dividend issue:

$$
\begin{aligned}
b_{jt} &\leq \Gamma(q_t k_{jt-1}, \pi_{jt}) \\
d_{jt} &\geq \underline{d}
\end{aligned}
$$

  - Aggregation theorem for fringe ξ of firms relying on retained earnings

- **Measurement**
  - Problem: $q_{jt}(\delta_j)$ ceases to be a sufficient statistic for stimulus policy
  - Approach: model simple form of bonus depreciation without additional state variable, then implement indirect inference

back

Fiscal Policy Application

- Firm block: target **PE moments** + **GE price sensitivity**

Parameter Values

Parameter	Description	Value
Fixed Parameters		
$\underline{d}$	Dividend constraint	0
Fitted Parameters		
κ	Quadratic adjustment costs	1.280
ϑ	Investment irreversibility	0.790
$\bar{\xi}$	Upper bound on fixed costs	0.00
a	Size of region without fixed costs	0.00
θ	Earnings-based borrowing constraint	3.000
k_0	Capital of entrants	0.420
b_0	Debt of entrants	0.180

Targeted Moments

Target	Data	Model
Price Sensitivity		
Bonus depreciation response	2.890	3.348
Micro Investment		
Average investment rate	0.104	0.136
Std. of investment rates	0.160	0.131
Spike rate	0.144	0.257
Inaction rate	0.237	0.205
Financial Frictions		
Earnings-based borrowing constraint	3.000	3.000
Entrants debt/output	1.280	1.501
Employment Distribution		
Employment share of age-1 firms	0.016	0.018

back