Aggregation in Heterogeneous-Firm Models
Theory and Measurement

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Investment in General Equilibrium

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**This paper**: theory & measurement for strength of GE aggregation
Our Contributions

1. **Theory**: GE aggregation = *price-elastic investment*

   - Formal result: investment is highly elastic around fixed eq'm price path
   - Previous work disagreed because price elasticity wasn’t targeted
   - E.g.: \[
   \frac{\partial \log I}{\partial \log r} = -500\% \text{ in Khan & Thomas (2008), vs. } -10\% \text{ in Winberry (2018)}
   \]

2. **Measurement**: experimental evidence to learn about
   - Estimate cross-sectional tax stimulus regressions \[\log(i_{jt}) = ZM|\{z\}_{7\%} + \text{cost of capital}_{jt} + \text{controls} + \text{error}\]
   - Interpretation: give sufficient conditions for

3. **Applications**: state dependence in monetary & fiscal policy transmission
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   - Estimate cross-sectional tax stimulus regressions \([\text{à la} \ Zwick-Mahon (2017)]\)
     
     \[
     \log(i_{jt}) = \beta_{ZM} \times \text{cost of capital}_{jt} + \text{controls} + \text{error}
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     \( \approx -7\% \)
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Theory
• Today’s focus: *lumpy investment*
Roadmap

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  ○ Well-established: fraction of firms willing to adjust $k$ is procyclical
    
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  ○ **Q**: Does this imply that the response of $I$ to macro shocks is procyclical?
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  Where we’re going: limit case where $\frac{dI}{d\text{shock}} \perp \# \text{ of adjusters}$

• At the end of the talk: extension to **financial frictions**
  
  ○ Similar result: limit case where $\frac{dI}{d\text{shock}} \perp \# \text{ of borr.-constrained firms}$
Environment

- **Setting**: discrete-time, infinite-horizon, aggregate TFP shocks
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- **Production block**: competitive intermediate goods producer $j \in [0, 1]$

$$\max \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \left( \prod_{s=0}^{t-1} \frac{1}{1 + r_s^b} \right) d_{jt} \right]$$

such that

$$d_{jt} = x_t y_{jt} - w_t l_{jt} - q_t i_{jt} - \phi(k_{jt}, k_{jt-1})$$

$$y_{jt} = z_t e_{jt} (k_{jt-1}^{\alpha} l_{jt}^{1-\alpha})^\nu$$

$$i_{jt} = k_{jt} - (1 - \delta) k_{jt-1}$$
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→ With **fixed costs**: shifts in \( \mu_0 = \) changes in # of adjusters

- **Rest**: representative household, sticky prices & wages, Taylor rule, …

*Smets-Wouters (2007), Justiniano-Primiceri-Tambalotti (2010), …*
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R1 **First-order perturbation** around perfect foresight transition path

→ Equivalently: perfect foresight for $z$ given arbitrary initial state $\mu_0$
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**Assumption**

Let $\mathbf{p} = (r, x, w, q)$ denote a price path. There exists a function $\mathcal{P}(\bullet)$, independent of the production block, s.t. an equilibrium is a path $\mathbf{C}$ with

$$C_t = Y_t(p; z) - l_t(p; z) \equiv C_t^s, \quad \text{for } t = 0, 1, 2, \ldots$$

where $\mathbf{p} = \mathcal{P}($C$)$. 

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**Nested Models**
Exact Aggregation

R3 To build intuition: **reduced-form model of lumpy investment**

→ Special adjustment costs: fraction $\xi \in (0, 1)$ of firms has infinite adjustment costs, the rest zero

→ $1 - \xi = \# \text{ of adjusters is reduced-form stand-in for changes in } \mu_0$
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• **Q:** Does $\hat{I}_\xi(z)$ change with $\xi$?
  
  ○ **Yes in PE:** fewer adjusters = less investment demand

  \[
  \hat{I}_\xi(p; z) - \hat{I}_0(p; z) = -\xi \times \hat{I}_0(p; z), \quad \text{for any } (p, z)
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**Q:** Does $\hat{l}_\xi(z)$ change with $\xi$?

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$$\hat{l}_\xi(p; z) - \hat{l}_0(p; z) = -\xi \times \hat{l}_0(p; z), \text{ for any } (p, z)$$

○ **Not necessarily in GE**

**Proposition**

Impose R1 - R3, and let $\nu \to 1$ or $\bar{r} + \delta \to 0$. Then the equilibrium price paths $p$ and the investment path $l$ are independent of $\xi$.  ▶ vs. House (2014)
• **Q:** How does \( p \) respond to changes in \( \xi \)?

\[
Y(\mathcal{P}(C); z) - I(\mathcal{P}(C); z) = C
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• Heuristic argument – pretend it’s static:

\[
\hat{p}_\xi - \hat{p}_0 = \frac{P_C}{1 - C^s_p \cdot P_C} \times \left[ \hat{C}^s_{\xi}(p_0; z) - \hat{C}^s_0(p_0; z) \right]
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- net excess supply
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- Dynamic: supply vector

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\text{price elasticities}
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- As \( \nu \to 1 \) or \( \bar{r} + \delta \to 0 \):

\[
\frac{\partial \tilde{k}_{jt+1}}{\partial \tilde{q}_t} = -\frac{1 - (1 - \alpha)\nu}{1 - \nu} \times \frac{1 + \bar{r}}{\bar{r} + \delta} \to \infty
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  $C^s_\rho \to \infty$: “shifting a flat $C^s$-curve”
Interpreting Previous Work

Same eq’n characterization with general fixed costs:

\[ \hat{p}_{\mu_0} - \hat{p}_{\bar{\mu}_0} = \mathcal{H} \times [\hat{C}^s_{\mu_0}(p_{\bar{\mu}_0}; z) - \hat{C}^s_{\bar{\mu}_0}(p_{\bar{\mu}_0}; z)] \]
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   \( \rightarrow \) Focal point RBC: linear firm side (= firms adjust), small \( \Delta r \) and large smoothing

   \( \rightarrow \) Thus: \( I \) is highly elastic around eq’m price path of rep.-firm economy
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   - 10% cyclical asymmetry needs 1% \( \Delta r \rightarrow \) large \( \Delta C \) demand \( \rightarrow \) large \( \Delta r^{GE} \)
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Distinguish using response of \( r \) to \( z \)? Here instead: **measure \beta**!
Measurement
How can we learn about $\beta$?
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• Ideal candidate: firm-level quasi-experiment
  ○ Interpretation: $\beta = \text{slope of investment demand curve}$
  ○ Thus: time series variation is not useful, cross-sectional variation is
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• Our approach: **bonus depreciation stimulus**
  
  ◦ Policy: ability to temporarily write-off/tax-deduct investment at a faster rate
  ◦ Research design: DiD using heterogeneity in treatment by $\delta_j$ [Zwick-Mahon]

$$\log(i_{jt}) = \alpha_j + \delta_t + \beta_{ZM} \times q_{jt}(\delta_j) + \text{error}$$
How can we learn about $\beta$?

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$\beta_{ZM} \approx -7\%$. What does that tell us?
Estimand Interpretation

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Proposition

Extend the baseline model to allow for permanent heterogeneity in \( \{\delta_j\} \). Let

\[ \hat{\beta} \equiv \int_{s : i_t(s) > 0} \frac{\partial \log(i_t(s))}{\partial q_t} d\tilde{\mu}(s) \]

where \( q \) is the cost of capital and \( \tilde{\mu} \) is the truncated firm state distribution.
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A1 Investment is **equally price-elastic** at all (adjusting) firms.

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*where \( q \) is the cost of capital and \( \tilde{\mu} \) is the truncated firm state distribution. Then, under A1 and to first order,*

\[ \beta_{ZM} \xrightarrow{P} \tilde{\beta} \]
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A1 Investment is equally price-elastic at all (adjusting) firms.

A2 All firms respond identically to the movements in \(p\) induced by the policy.

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where \(q\) is the cost of capital and \(\tilde{\mu}\) is the truncated firm state distribution. Then, under A2 and to first order,

\[ \beta_{ZM} \xrightarrow{p} \tilde{\beta} + \frac{\text{Cov}_{\tilde{\mu}(s)} \left( \left( \frac{\partial \log(i_t(s))}{\partial q_t} - \tilde{\beta} \right) q_t(s), q_t(s) \right)}{\text{Var}_{\tilde{\mu}(s)}(q_t(s))} \]

selection effect
\[
\log(i_{jt}) = \alpha_j + \delta_t + \beta_{ZM} \times q_{jt}(\delta_j) + \text{error}
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A2. All firms respond identically to the movements in \(p\) induced by the policy.

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where \(q\) is the cost of capital and \(\tilde{\mu}\) is the truncated firm state distribution.

Then, to first order,

\[
\beta_{ZM} \xrightarrow{p} \tilde{\beta} + \underbrace{\text{Cov}_{\tilde{\mu}(s)} \left( \left( \frac{\partial \log(i_t(s))}{\partial q_t} - \tilde{\beta} \right) q_t(s), q_t(s) \right)}_{\text{selection effect}} + \underbrace{\text{Cov}_{\tilde{\mu}(s)} \left( \frac{\partial \log(i_t(s))}{\partial p} \hat{p}, q_t(s) \right)}_{\text{heterogeneous GE exposure}} + \frac{\text{Var}_{\tilde{\mu}(s)}(q_t(s))}{\text{Var}_{\tilde{\mu}(s)}(q_t(s))}
\]
Results

\[ \log(i_{jt}) = \alpha_j + \delta_t + \beta_{ZM} \times q_{jt}(\delta_j) + \text{error} \]

- **Headline number**: \( \beta_{ZM} \approx -7\% \)

  - Estimation details: “universe” (corporate tax return data), pool two bonus depreciation episodes, \( b_{jt} \) at 4-digit industry level
  - Extensions/robustness: Compustat, dynamics, GDP & trend interactions, extensive margin, \( b_{jt} \) at firm level …
Results

\[ \log(i_{jt}) = \alpha_j + \delta_t + \beta_{ZM} \times q_{jt}(\delta_j) + \text{error} \]

- **Headline number**: \( \beta_{ZM} \approx -7\% \) ▶ Details, Robustness & Extensions
  - Estimation details: “universe” (corporate tax return data), pool two bonus depreciation episodes, \( b_{jt} \) at 4-digit industry level
  - Extensions/robustness: Compustat, dynamics, GDP & trend interactions, extensive margin, \( b_{jt} \) at firm level …

- **Interpretation**: \(|\beta| \leq |\beta_{ZM}|\)
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- **Interpretation:** $|\beta| \leq |\beta_{ZM}|$
  
  1. Back-of-the-envelope (Model + A1): $\beta = \beta_{ZM} \approx -7\%$
  
  2. Indirect inference (Model + $\approx$ A2): $\beta \approx -5\%$

  → Add $\beta_{ZM}$ as estimation target (“identified moment”) in rich het.-firm model with two depreciation types, persistent $z$ shocks, aggregate effects, in recession, …

  → Upward bias due to selection effect, GE exposure effect is small
Applications
Q: Why does monetary policy seem to “push on a string” in recessions?
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• Possible mechanism: procyclical price elasticity of investment demand
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- Our approach: NK model + lumpy investment + $\beta_{ZM}$

1. “PE calibration”: $E(i)$, $\sigma(i)$, spike rate, inaction rate
2. “GE calibration”: $\beta_{ZM}$ plus standard non-production block
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1. “PE calibration”: $E(i)$, $\sigma(i)$, spike rate, inaction rate
2. “GE calibration”: $\beta_{ZM}$ plus standard non-production block

• Find: pushing-on-a-string in PE & GE
  - $i$ is 70% more responsive given prices, and 40% more responsive in GE
  - Without $\beta_{ZM}$ targeted: asymmetry disappears [Smets-Wouters + Khan-Thomas]
Q: Why does monetary policy seem to “push on a string” in recessions?
• In paper: theory & measurement with financial frictions ▸ Details
Fiscal Policy & Firm Cash Flow

• In paper: theory & measurement with financial frictions

• Experiment: fiscal stimulus with cash flow-sensitive investment

  1. “PE calibration”: earnings-based borr. constraint, initial entrant size/debt
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1. Investment price elasticities are central to **GE aggregation**
   
   ○ Applies to smoothing for lumpy investment/durables & financial frictions
   
   ○ Reduces disagreement in previous work to measurable “sufficient statistic”

   *Khan-Thomas (2008), Bachmann-Caballero-Engel (2013), Winberry (2018)*
Conclusions

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Conclusions

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   - Reduces disagreement in previous work to measurable “sufficient statistic”
     

2. Preferred **direct measurement** suggests weak GE price effects

3. Implications: $\mu_0$ **matters** – but in which direction?
   - Pro- or counter-cyclical? lumpiness vs. cash-flow effects
   - Matters because investment takes center stage in (monetary) policy stimulus
     
     [e.g. Christiano-Eichenbaum-Evans, Kaplan-Moll-Violante, …]
Appendix
Model Closure

• Explicit closure: medium-scale NK-DSGE model
  
  close to Smets-Wouters (2007) and Justiniano-Primiceri-Tambalotti (2010)

• With mild additional restrictions this model satisfies R2:

**Lemma**

Suppose that:

1. Labor disutility is linear.
2. The coefficient on output in the Taylor rule is 0.
3. There are no aggregate capital adjustment costs.

Then, to first order, the full structural model satisfies R2. If prices and wages are flexible, then R2 is satisfied globally.
• Flat investment curve logic is related to House (2014)
  ○ He shows: in investment re-set model with $\delta \rightarrow 0$ investment timing is infinitely elastic w.r.t. $q$
  ○ Implies: in eq’m model of investment market distribution $\mu_0$ is irrelevant

• How does our result generalize this?
  1. Rich GE model closure, rather than just investment market
  2. Aggregation not just for long-lived capital goods, also for linear revenue f’n
  3. Result is generic: infinite elasticity around rep.-firm eq’m price path, doesn’t matter what friction delivers a gap given prices
General Equilibrium Adjustment $\mathcal{H}$

- $\mathcal{H}$ combines supply and demand price elasticities:

$$\mathcal{H} = \frac{\partial \mathcal{P}}{\partial \mathcal{C}} \times (I - \mathcal{G})^{-1}$$

where

$$\mathcal{G} = \mathcal{G} = \left( \begin{array}{cccc} \frac{\partial C_s}{\partial r} & \frac{\partial C_s}{\partial p^I} & \frac{\partial C_s}{\partial w} & \frac{\partial C_s}{\partial q} \end{array} \right) \times \left( \begin{array}{cccc} \frac{\partial r}{\partial \mathcal{C}} & \frac{\partial p^I}{\partial \mathcal{C}} & \frac{\partial w}{\partial \mathcal{C}} & \frac{\partial q}{\partial \mathcal{C}} \end{array} \right)$$

Supply Elasticity \hspace{2cm} Inverse Demand Elasticity

- Note: unique left-inverse of $(I - \mathcal{G})$ is guaranteed if eq’m is unique

- R1-R3: for $\nu = 1$ or $\bar{r} + \delta = 0$, the map $\mathcal{H}$ is column rank-deficient, with

$$\{ \hat{\mathbf{C}}^s_\xi(p_0; z) - \hat{\mathbf{C}}^0_0(p_0; z) \in \text{null}(\mathcal{H}) \}$$
What do PE price elasticities look like in previous work?

![Graph showing comparison between Khan-Thomas (2008) and Winberry (2018) for PE price elasticities over time.](image)
The implied GE adjustment matrices look dramatically different:

(a) Khan & Thomas (2008)  
(b) Winberry (2018)
Standard Calibration Targets

• Investment lumpiness
  - All previous work matches $\mathbb{E}(i)$, $\sigma(i)$, spike rate, inaction rate
  - Implies: price elasticity $\perp$ lumpiness

• Aggregate prices
  - Winberry (2018): real rate is acyclical
  - Concerns
    1. Cyclicality conditional on $z$ is ill-measured
    2. Theory: arbitrary rate cyclicality is consistent with aggregation

• Investment rate dispersion
  - Dispersed $e$ + high elasticity $\Rightarrow$ dispersed $i$
  - Direct evidence on $e$ suggests large dispersion $\Rightarrow$ need small elasticities
Bonus Depreciation

- What is bonus depreciation?
  - In general: for every $ of investment reduce future tax liabilities
  - With bonus depreciation: tax reductions come earlier = PV benefit

- Computation of exposure term:

  \[ q_{jt}(\delta_j) = \sum_{t=0}^{\infty} \zeta^t \left( \prod_{q=0}^{\infty} \frac{1}{1 + r^b_{q-1}} \right) \tau^b_t(\delta_j) \]

- Formal equivalence to reduction in price of capital:

  Lemma

  The paths of all aggregates in response to an unexpected bonus depreciation shock with firm-specific schedules \( \{ \tau^b_{jt} \}_{t=0}^{\infty} \) are identical to response paths after a period-0 firm-specific investment subsidy shock with

  \[ \tau^i_{j0} = \tau^b_{j0} + \sum_{t=1}^{\infty} \zeta^t \left( \prod_{q=1}^{\infty} \frac{1}{1 + r^b_{q-1}} \right) \tau^b_{jt} \]
Estimation Details

• We extend the baseline analysis of Zwick & Mahon (2017):
  1. Compustat sample: larger firms, arguably less financially constrained
  2. Quarterly, dynamics: less time aggregation, learn about all entries of $\mathcal{H}$
  3. More controls: partial out heterogeneous exposure to aggregate conditions

| Dependent Variable: | $\log(i_{j,t})$ | $\log(i_{j,t+1})$ | $\log(i_{j,t+2})$ | $\log(i_{j,t+3})$ | $\log(i_{j,t+4})$
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<td>$z_{n,t}$</td>
<td>1.64***</td>
<td>1.19***</td>
<td>0.78***</td>
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<td></td>
<td>(0.28)</td>
<td>(0.28)</td>
<td>(0.29)</td>
<td>(0.29)</td>
<td>(0.30)</td>
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<td>Trend Interaction</td>
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<tr>
<td>Firm &amp; Time FE</td>
<td>x</td>
<td>x</td>
<td>x</td>
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<td>Observations</td>
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<td>401,428</td>
<td>390,561</td>
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<td>R-squared</td>
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Monetary Policy Application

• Standard NK parameterization for non-production (demand) block
  → Robustness: habits, $\phi_y > 0$, non-linear labor disutility

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tr>
<td>$\beta$</td>
<td>Discount rate</td>
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<tr>
<td>$h$</td>
<td>Habit formation</td>
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<td>$\gamma$</td>
<td>CRRA coefficient</td>
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<td>$\varphi$</td>
<td>Frisch elasticity</td>
<td>$\infty$</td>
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<tr>
<td>$\epsilon_p$</td>
<td>Goods substitutability</td>
<td>10</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>Price adjustment cost</td>
<td>40</td>
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<td>$\epsilon_w$</td>
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<tr>
<td>$\kappa$</td>
<td>Aggregate $K$ adjustment costs</td>
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<td>$\rho_{tr}$</td>
<td>Taylor rule persistence</td>
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<td>$\phi_{\pi}$</td>
<td>Taylor rule inflation coefficient</td>
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<tr>
<td>$\phi_y$</td>
<td>Taylor rule output coefficient</td>
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Monetary Policy Application

- Firm block: target **PE moments** + **GE price sensitivity**

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<tbody>
<tr>
<td>$1 - \xi$</td>
<td>Firm exit rate</td>
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<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
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<tr>
<td>$\alpha$</td>
<td>Capital share</td>
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<td>$\nu$</td>
<td>Returns to Scale</td>
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<tr>
<td>$\rho$</td>
<td>Productivity persistence</td>
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<td>$\sigma$</td>
<td>Productivity dispersion</td>
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<td>$\mu_0$</td>
<td>Mean initial productivity</td>
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<td>Initial productivity dispersion</td>
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### Targeted Moments

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<tr>
<td><strong>Price Sensitivity</strong></td>
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<tr>
<td>Bonus depreciation estimand</td>
<td>2.890</td>
<td>2.984</td>
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<tr>
<td><strong>Micro Investment</strong></td>
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<tr>
<td>Average investment rate</td>
<td>0.104</td>
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<td>Std. of investment rates</td>
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<tr>
<td>Spike rate</td>
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<td>0.108</td>
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<tr>
<td>Inaction rate</td>
<td>0.237</td>
<td>0.184</td>
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<tr>
<td><strong>Employment Distribution</strong></td>
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<tr>
<td>Employment share of age-1 firms</td>
<td>0.016</td>
<td>0.028</td>
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Financial Frictions

• Theory

  ○ Allow for constraints on borrowing & dividend issue:

    \[ b_{jt} \leq \Gamma(q_t k_{jt-1}, \pi_{jt}) \]
    \[ d_{jt} \geq d \]

  ○ Aggregation theorem for fringe \( \xi \) of firms relying on retained earnings

• Measurement

  ○ Problem: \( q_{jt}(\delta_j) \) ceases to be a sufficient statistic for stimulus policy

  ○ Approach: model simple form of bonus depreciation without additional state variable, then implement indirect inference
Fiscal Policy Application

- Firm block: target **PE moments** + **GE price sensitivity**

### Targeted Moments

<table>
<thead>
<tr>
<th>Parameter Values</th>
<th>Target</th>
<th>Data</th>
<th>Model</th>
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<tbody>
<tr>
<td><strong>Fixed Parameters</strong></td>
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<tr>
<td>$d$</td>
<td>Dividend constraint</td>
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<td><strong>Fitted Parameters</strong></td>
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<td>$\bar{\xi}$</td>
<td>Upper bound on fixed costs</td>
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<td>Size of region without fixed costs</td>
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<td>$\theta$</td>
<td>Earnings-based borrowing constraint</td>
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