Should monetary policy care about redistribution? Optimal fiscal and monetary policy with heterogeneous agents

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July 11, 2020

Abstract

We derive optimal monetary and fiscal policies in an heterogeneous-agent economy with nominal frictions and aggregate shocks, when a rich set of fiscal tools is available. This enables us to investigate the redistributive role of optimal monetary policy. We determine the optimal dynamics of nominal interest rate, capital and labor taxes, transfers and public debt. The role of monetary policy is shown to depend on the fiscal tools that are available. When taxes on capital and labor are available, then there is no redistributive role for monetary policy. When fiscal tools are incomplete, we show quantitatively that optimal inflation volatility is very small: redistribution is mostly a matter of fiscal policy. We provide analytical and numerical results thanks to a truncated representation and an extensive use of the Lagrangian approach that enables to derive optimal Ramsey policies in this heterogeneous-agent setting.

Keywords: Heterogeneous agents, optimal Ramsey policies, monetary policy, fiscal policy.

JEL codes: D31, E52, D52, E21.

*We would like to thank seminar participants at T2M Paris-Dauphine Conference, at the 3rd Conference on Financial Markets and Macroeconomic Performance in Frankfurt, Goethe University and at UCL.
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1 Introduction

Monetary policy generates redistributive effects through various channels that have been studied in a vast empirical and theoretical literature, reviewed below. However, it is not clear how these channels *should* change the conduct of monetary policy. It might be possible that monetary policy should take into account these effects to improve welfare, and thus participate in a function usually devoted to fiscal policy. On the contrary, monetary policy could only focus on monetary goals and let fiscal tools either dampen or strengthen the redistributive effects of monetary policy. To distinguish between these two claims, one must solve for optimal fiscal and monetary policy in a realistic environment, where heterogeneity among agents generates a concern for redistribution.

The goal of this paper is to investigate the redistributive role of monetary policy. To do so, we study the joint optimal fiscal and monetary policy in an economy featuring heterogeneous agents and aggregate shocks, when a relevant set of fiscal instruments is available. Following the so-called Bewley (1980) literature, we assume incomplete insurance markets for idiosyncratic risks to be the main source of agents heterogeneity. This framework is known to be general enough to generate realistic income and wealth distributions. We further add nominal frictions, modeled as costly price adjustments. This environment has been named HANK following the seminal paper of Kaplan et al. (2018). Thanks to some methodological contribution explained below, we derive optimal monetary and fiscal policies with commitment and with four fiscal instruments: a linear tax on capital income, a linear tax on labor income, lump-sum transfer and a riskless one-period public debt.

First and foremost, we show that an economy, in which all these instruments are available, constitutes a meaningful benchmark. Indeed, when the government can levy resources through both capital and labor taxes, the redistributive effects of monetary policy are shown to be absent, after both a technology and a public spending shock. In this case, monetary policy solely aims at ensuring price stability in each period – as in any representative agent economy – and to let fiscal policy alone deal with redistribution. In this sense, there is a perfect dichotomy between the objectives of monetary and fiscal policies in such an economy. The redistributive role of monetary policy only stems from missing fiscal instruments or, more precisely, from non-optimally time-varying fiscal instruments.

Second, we characterize optimal fiscal policy in this environment with heterogeneous agents and aggregate shocks. In particular, we compute the optimal dynamics of capital tax and public debt which are shown to be very different from those in a complete-market economy.

Third, we extend this analysis by considering various assumptions regarding the availability of fiscal instruments. We assume that the capital tax is not time-varying, which leaves a potential role for monetary policy to affect inflation and ex-post real interest rate. It turns out that in a quantitatively relevant economy, inflation volatility remains very small, even though not null. As a consequence, inflation apprears to optimally have a very minor role for redistributive purposes.

These effects are identified at a theoretical level, thanks to two methodological contributions. The first one is the use of a truncated representation of incomplete insurance market economies that we apply here to a monetary economy. This theory of the truncation is presented in LeGrand and Ragot (2020). The basic idea of the theory is to design a partial insurance mechanism guaranteeing that heterogeneity only depends on a finite but arbitrarily large number, denoted N, of past consecutive realizations of idiosyncratic risk. As a theoretical outcome, agents having the same idiosyncratic risk history for the previous N periods choose the same consumption and hold the same wealth. The full-fledged Bewley economy corresponds to the case where $N = \infty$, which means no partial insurance mechanism. The representative agent is also mapped into this setup and corresponds to the case where $N = 0$, where there is full insurance among agents. The gain of the truncated representation is that the equilibrium features a finite – though possibly arbitrarily large – number of heterogeneous agents. This allows us to use the same tools as in representative agent models. Second, we show that the Lagrangian approach, used in Marcet and Marimon (2019), is particularly well-suited for monetary economies. This allows us to derive first-order conditions and obtain simple intuition about optimal monetary and fiscal policies.

The effects are also quantified in realistically calibrated economies and the following conclusions can be drawn. First, market incompleteness affects the optimal behavior of fiscal instruments. Whereas the capital tax is very volatile in the complete-market economy, it is two orders of magnitude less volatile in the incomplete-market setup. Indeed, due market incompleteness, savings are also held for self-insurance motives, which makes capital taxation more costly. Second, the implication of a less volatile capital tax is a more volatile public debt. Adding market incompleteness increases the counter-cyclicality of public debt after technology shocks, compares to the complete market case. Third, the labor tax remains almost constant in the IM economy, as this tax distorts the labor supply and directly affects output and household revenues. Finally, we find that monetary policy is mostly concerned by price stability and has little role to play for redistribution, even in absence of time-varying capital tax.

Related literature. This paper is related to the literature on monetary policy with nominal frictions and heterogeneous agents. This is a vast literature (including the seminal work of Bewley, 1980, 1983). The more recent literature, in which our work is embedded, focuses on sticky prices as the main friction. For instance, McKay et al. (2016), Gornemann et al. (2016), and Kaplan et al. (2018) study the transmission channels of monetary policy in this setup. McKay and Reis (2016) investigate the interaction between monetary and fiscal policies. Auclert (2019) analyzes the transmission channels of monetary policy with heterogeneous agents. Regarding normative issues, Nuño and Moll (2018) use a continuous-time approach and mean-field games to characterize optimal monetary policies for economies without aggregate shocks. They do not consider additional fiscal tools (or public debt neither). As a consequence, their results can be consider as an upper bound of the redistributive objective of monetary policy.

Second, the paper is related to the literature studying optimal fiscal policy with distorting tools and the interactions between monetary and fiscal policy. This literature is developed under the assumption of complete-market for idiosyncratic risk (Chari and Kehoe, 1999; Aiyagari et al., 2002; Farhi, 2010; Bhandari et al., 2017a, among others). In this literature, our paper is close to Correia et al. (2008) and Correia et al. (2013) who derive equivalence results in monetary economies with optimal fiscal policies.

Finally the closest paper to our contribution is Bhandari et al. (2020). They derive optimal fiscal and monetary policy when the government has access to non-distorting taxes, labor-income tax and public debt. They find an important redistributive role for inflation when capital taxes are constant. The difference with our results comes from the different modeling strategy. Indeed, they consider an economy without capital and focus on the optimal policy in the transition from an initial distribution of wealth, where credit constraints are not occasionally binding. Thanks to our truncation theory we consider optimal policy in an economy with capital and occasionally-binding credit constraints, around a steady-state with a consistent distribution of wealth. The introduction of capital and the perturbation around an optimal steady state may explain the difference between results.

2 The environment

Time is discrete, indexed by $t \geqslant 0$. The economy is populated by a continuum of agents of size 1, distributed on a segment J following a non-atomic measure ℓ: $J(\ell) = 1$. Following Green (1994), we assume that the law of large number holds.

2.1 Risk

The only aggregate shock affects technology level in the economy. We denote this risk by $(z_t)_{t\geq 0}$ and we assume that is follows an AR(1) process: $z_t = \rho_z z_{t-1} + u_t^z$ with $\rho_z > 0$ the persistence parameter and the shock u_t^z being a white noise with a normal distribution $\mathcal{N}(0, \sigma_z^2)$. The economy-wide productivity, denoted $(Z_t)_{t\geq 0}$ is assumed to relate to z_t through the following functional form: $Z_t = Z_0 e^{z_t}$.

In addition of this aggregate shock, agents face an uninsurable idiosyncratic labor productivity shock $y_t \in \mathcal{Y}$. The set $\mathcal{Y}$ contains only distrnct values and a larger value for the realization of y_t means a higher productivity. An agent i can adjust her labor supply, denoted by l_t^i and earns the before-tax hourly wage $\tilde{w}_t$ (that depends on the aggregate shock). Therefore, her total before-tax wage amounts to $y_t^i \tilde{w}_t l_t^i$. We assume that the productivity process is a first-order Markov chain with constant transition probabilities. We denote by $\Pi_{yy'}$ the probability to switch from productivity y in one period to productivity y' in the following period. The share of agents with productivity y, denoted by S_y, is constant and equal to: $S_y = \sum_{\tilde{y}} \Pi_{\tilde{y}y} S_y$ for all $y \in \mathcal{Y}$. Finally, an history of productivity shock up to date t is denoted by $y^{i,t} = \{y_0^i, \ldots, y_t^i\}$. Using transition probabilities, we compute the measure $\mu_t : \mathcal{Y}^{t+1} \to [0,1]$, such that $\mu_t(y^t)$ represents the measure of agents with history y^t in period t.

2.2 Preferences

In each period, the economy has two goods: a consumption good and labor. Households are expected utility maximizers and rank streams of consumption $(c_t)_{t\geq 0}$ and of labor $(l_t)_{t\geq 0}$ according to a time-separable intertemporal utility function equal to $\sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$, where $\beta \in (0,1)$ is a constant discount factor and $U(c,l)$ is an instantaneous utility function. As is standard in this class of models, we focus on the case where U is a Greenwood-Hercowitz-Huffman (GHH) utility function, exhibiting no wealth effect for the labor supply. For any consumption c and labor supply l, the instantaneous utility $U(c,l)$ can be expressed as:

$$U(c,l) = u\left(c - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}\right),$$

where $\varphi > 0$ is the Frisch elasticity of labor supply, $\chi > 0$ scales labor disutility, and $u : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously derivable, increasing, and concave, with $u'(0) = \infty$.

2.3 Production

The consumption good Y_t is produced by a unique profit-maximizing representative firm that combines intermediate goods $\left(y_t^f(j)\right)_j$ from different sectors indexed by $j \in [0,1]$ using a standard Dixit-Stiglitz aggregator. Denoting by ε the elasticity of substitution for the goods belonging to the different sectors, we obtain that the production Y_t can be expressed using a CES aggregation of individual productions:

$$Y_t = \left[\int_0^1 y_t^f(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

For any intermediate good $j \in [0,1]$, the production $y_t^f(j)$ is realized by a monopolistic firm. The profit maximization for the firm producing the final good implies that its demand for the intermediate good is:

$$y_t^f(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon} Y_t,$$

where P_t is the price of the consumption good. The zero profit condition of the firm producing the final good implies that the price P_t can be expressed as:

$$P_t = \left(\int_0^1 p_t(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}.$$

Intermediary firms are endowed with a Cobb-Douglas production technology and use labor and capital as a production factors. The production technology involves that $\tilde{l}_t(j)$ units of labor and $\tilde{k}_t(j)$ units of capital are transformed into $Z_t\tilde{k}_t(j)^\alpha \tilde{l}_t(j)^{1-\alpha}$ units of intermediate good. At the equilibrium, this production will exactly cover the demand $y_t^f(j)$ for the good j, that will sold with the real price $p_t(j)/P_t$. We denote as $\tilde{w}_t$ the real before-tax wage and $\tilde{r}_t^K$ the real before-tax net interest rate on capital – that are both identical for all firms. The capital depreciation is denoted $\delta > 0$. Since intermediate firms have market power and internalize it, the firm's objective is to minimize production cost, including capital depreciation, subject to producing the demand $y_t(j)$. The cost function $C(j)$ of firm j can be therefore be expressed as:

$$C(j) = \min_{\tilde{l}_t(j),\tilde{k}_t(j)} \left\{ (\tilde{r}_t^K + \delta)\tilde{k}_t(j) + \tilde{w}_t\tilde{l}_t(j) \left| y_t^f(j) = Z_t\tilde{k}_t(j)^\alpha \tilde{l}_t(j)^{1-\alpha} \right. \right\}.$$

Denoting $\zeta_t(j)$ the Lagrange multiplier of the production constraint, first-order conditions imply:

$$\tilde{r}_t^K + \delta = \zeta_t(j)\alpha \frac{y_t^f(j)}{\tilde{k}_t(j)} \text{ and } \tilde{w}_t = \zeta_t(j)(1-\alpha)\frac{y_t^f(j)}{\tilde{l}_t(j)}. \tag{1}$$

At the optimum, the production constraint yields therefore a common value for all $\zeta_t(j)$ among all firms j. We denote by ζ_t this common value, which can be expressed as:

$$\zeta_t = \frac{1}{Z_t}\left(\frac{\tilde{r}_t^K + \delta}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha}. \tag{2}$$

The firm j's cost becomes then $C(j) = \zeta_t y_t^f(j)$, which is linear in the demand $y_t^f(j)$. Following the literature, we furthermore assume the presence of a subsidy τ^Y on the total cost, that will compensate for steady-state distortions, such that the total cost supported by firm's j is $\zeta_t y_t^f(j)(1-\tau^Y)$. Furthermore, integrating factor price equations (1) over all firms, lead to characterize total capital K_{t-1} and total labor supply L_t:

$$K_{t-1} = \frac{1}{Z_t}\left(\frac{\tilde{r}_t^K + \delta}{\alpha}\right)^{\alpha-1} \left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha} Y_t \text{ and } L_t = \frac{1}{Z_t}\left(\frac{\tilde{r}_t^K + \delta}{\alpha}\right)^{\alpha} \left(\frac{\tilde{w}_t}{1-\alpha}\right)^{-\alpha} Y_t, \tag{3}$$

where Y_t is the total production, which can also be written under the standard form as:

$$Y_t = Z_t K_{t-1}^\alpha L_t^{1-\alpha}. \tag{4}$$

Finally, in this set-up, the usual factor prices relationships do not hold but we still have:

$$\frac{K_{t-1}}{L_t} = \frac{\alpha}{1-\alpha}\frac{\tilde{w}_t}{\tilde{r}_t^K + \delta}. \tag{5}$$

In a real setup (featuring $\zeta_t = 1$ for all t), equations (2) and (5) are equivalent to the standard definitions of factor prices: $\tilde{r}_t + \delta = \alpha Z_t \left(\frac{K_{t-1}}{L_t}\right)^{\alpha-1}$ and $\tilde{w}_t = (1-\alpha)Z_t\left(\frac{K_{t-1}}{L_t}\right)^\alpha$.

In addition to the production cost, intermediate firms face a quadratic price adjustment cost *à la* Rotemberg (1982) when setting their price in the period. The price adjustment cost is proportional to the magnitude of the firm's relative price change, which is in other words the magnitude of the inflation in firm's price. More formally, the adjustment cost can be expressed as $\frac{\kappa}{2}\left(\frac{p_t(j)}{p_{t-1}(j)}-1\right)^2 Y_t$, where $\kappa \geq 0$ is a scaling factor. We can thus deduce the real profit, denoted $\Omega_t(j)$, at date t of firm j:

$$\Omega_t(j) = \left(\frac{p_t(j)}{P_t} - \left(\frac{\tilde{r}_t+\delta}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha}\frac{1-\tau^Y}{Z_t}\right)\left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon} Y_t - \frac{\kappa}{2}\left(\frac{p_t(j)}{p_{t-1}(j)}-1\right)^2 Y_t - t_t^Y.$$

where t_t^Y is a lump-sum tax financing the subsidy τ^Y. Computing the firm j's intertemporal profit requires to define the firm's pricing kernel. In a heterogeneous agent economy, there is no obvious choice for the pricing kernel. We discuss the reasons and the several options below. For the moment, we assume that the firm's j pricing kernel is independent of its type and we denote the pricing kernel at date t by $\frac{M_t}{M_0}$. With this notation, the firm j's program consisting in choosing the price schedule $(p_t(j))_{t\geq 0}$ maximizing the intertemporal profit at date 0, can be expressed as follows:

$$\begin{aligned}\max_{(p_t(j))_{t\geq 0}} \mathbb{E}_0\Bigg[\sum_{t=0}^{\infty}\beta^t\frac{M_t}{M_0}\Bigg(\left(\frac{p_t(j)}{P_t} - \left(\frac{\tilde{r}_t+\delta}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha}\frac{1-\tau^Y}{Z_t}\right)\left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon} Y_t \\ -\frac{\kappa}{2}\left(\frac{p_t(j)}{p_{t-1}(j)}-1\right)^2 Y_t - t_t^Y\Bigg)\Bigg].\end{aligned} \tag{6}$$

Observing that the program (6) is independent of the firm type j, we deduce that in the symmetric equilibrium, all firms will charge the same price: $p_t(j) = P_t$ for all $j \in [0,1]$. Denoting the gross inflation rate as $\Pi_t = \frac{P_{t+1}}{P_t}$, we deduce the following first-order condition for the firm's program:

$$\Pi_t(\Pi_t - 1) = \frac{\varepsilon}{\kappa}\left(\zeta_t(1-\tau_t^Y) - \left(1-\frac{1}{\varepsilon}\right)\right) + \beta\mathbb{E}_t\Pi_{t+1}(\Pi_{t+1}-1)\frac{Y_{t+1}}{Y_t}\frac{M_{t+1}}{M_t},$$

which the equation characterizing the Phillips curve in our environment. We set $\tau^Y = \frac{1}{\varepsilon}$ to obtain an efficient steady-state. The real profit becomes then:

$$\Omega_t = \left(1 - \zeta_t - \frac{\kappa}{2}\pi_t^2\right) Y_t, \tag{7}$$

where $\pi_t = \Pi_t - 1$ is the inflation rate. The Phillips curve becomes:

$$\Pi_t(\Pi_t - 1) = \frac{\varepsilon - 1}{\kappa}(\zeta_t - 1) + \beta\mathbb{E}_t\Pi_{t+1}(\Pi_{t+1}-1)\frac{Y_{t+1}}{Y_t}\frac{M_{t+1}}{M_t}, \tag{8}$$

Choosing the pricing kernel. As explained above, in a heterogeneous agent economy, there is no straightforward choice for the firm's pricing kernel. In a representative agent economy, the unique agent's is necessarily the firm's owner and there is no possible dispute about the firm's pricing kernel, which has to be the representative agent's pricing kernel. The choice that we make is to assume that the firm pricing kernel is defined based on the average marginal utility among agents. We provide a formulation in equation (22) below in the Ramsey program. The definition of the pricing kernel as a minor effect in the quantitative outcome of the model.

2.4 Assets

Agents have the possibility to trade two assets. The first one is public nominal debt, whose size is denoted by B_t at date t. Public debt is issued by the government and is assumed to be exempt of default risk. The nominal debt pays off a nominal gross interest rate that is pre-determined. In other words, the interest rate between dates $t-1$ and t is known at $t-1$. We denote this (gross and before-tax) interest rate by $\tilde{R}^{B,N}_{t-1}$. The associated real before-tax (gross) interest rate for public debt is $\tilde{R}^{B,N}_{t-1}/\Pi_t$, where Π_t is the gross inflation rate. Note that due to inflation, this ex-post real rate is not pre-determined anymore. The second asset is capital shares, whose pays off a (net and before-tax) real interest rate $\tilde{r}^K$ – as introduced above.

We assume that the whole public debt and the whole capital are held by a risk-neutral fund and that agents can trade shares of this fund. The interest rate paid by this fund to agents is denoted by $\tilde{r}$.[1] The three interest rates, for public debt, capital and fund, are connected by two different relationships. The first one reflects the non-profit condition of the fund. We denote by A_t the total asset amount in the economy, equal to the sum of public debt and capital, which verifies $A_t = K_t + B_t$. Since the fund holds all the public debt and the capital and sell shares, its non-profit condition implies:

$$\tilde{r}_t = \frac{\tilde{r}^K_t K_{t-1} + \left(\frac{\tilde{R}^{B,N}_{t-1}}{\Pi_t} - 1\right) B_{t-1}}{A_{t-1}}. \tag{9}$$

The second relationship is the no-arbitrage condition between public debt holdings and capital shares. This condition states that one unit of consumption invested in each of the two assets should yield the same expected return. Formally, this condition can be written as:

$$\mathbb{E}_t \left[\frac{\tilde{R}^{B,N}_t}{\Pi_{t+1}}\right] = \mathbb{E}_t \left[1 + \tilde{r}^K_{t+1}\right]. \tag{10}$$

Because of the fund intermediation, households make no actual portfolio choice and we will denote by $a_{t,i}$ their holdings in fund claims. Agents face borrowing constraints, and their fund holdings must be higher than $-\bar{a} \leq 0$. Alternatively, this constraint states that agents cannot borrow more than the amount $\overline{a}$. In the rest of the paper, we will focus on the case where the

[1]The main advantage of this mechanism is that it allows us to have two different asset classes, without portfolio choice.

credit limit is above the steady-state natural borrowing limit.[2]

2.5 Government, fiscal tools and monetary policy

In each period t, the government has to finance an exogenous and possibly stochastic public good expenditure $G_t \equiv G_t(z_t)$, as well as lump-sum transfers T_t, which will be optimally chosen. The latter transfers can be thought of as social transfers, which can contribute to generate progressivity in the overall tax system. Indeed, Dyrda and Pedroni (2018) have shown that such transfers are needed to properly replicate the US fiscal system. The government has several tools for financing the expenditure. First, the government can levy two distorting taxes. A first tax τ_t^K is based on payoffs of all interest-rate bearing assets. The second tax τ_t^L concerns labor income. Second, in addition to these distorting taxes on households, the government can also tax the firms' profits. Finally, besides taxation, the government can also issue a one-period public nominal bond, that is assumed to be riskless. To sum it up, fiscal policy is characterized by four instruments $\left(\tau_t^L, \tau_t^K, T_t, B_t\right)_{t=0,\ldots,\infty}$ for an exogenous public spending stream $(G_t)_{t=0,\ldots,\infty}$.

To simplify notation, after-tax quantities are denoted without a tilde. The real after tax wage w_t, as well as the real after-tax interest rates r_t, r_t^K, and $R_t^{B,N}$ (for the fund, the capital and public debt, respectively) can therefore be expressed as follows:

$$w_t = (1-\tau_t^L)\tilde{w}_t, \quad r_t = (1-\tau_t^K)\tilde{r}_t, \tag{11}$$

$$r_t^K = (1-\tau_t^K)\tilde{r}_t^K, \quad \frac{R_t^{B,N}}{\Pi_t} - 1 = (1-\tau_t^K)(\frac{\tilde{R}_t^{B,N}}{\Pi_t} - 1). \tag{12}$$

Taxes on asset-bearing assets are identical for all asset classes and they are levied on real returns.

Regarding firm taxation, we assume that the government fully taxes profits. This solution greatly simplifies the question of the distribution of firm profits among the population of heterogeneous agents. As labor and capital taxes are distorting, this profit policy avoids further distortion. We can now express the governmental budget constraints at date t:

$$\begin{aligned} G_t + \frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t} B_{t-1} + T_t \leq \tau_t^L \tilde{w}_t L_t + \tau_t^K \left(\tilde{r}_t^K K_{t-1} + \left(\frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t} - 1\right) B_{t-1}\right) \\ + \left(1 - \zeta_t - \frac{\kappa}{2}\pi_t^2\right) Y_t + B_t. \end{aligned}$$

The government uses its financial resources, made of labor and asset taxes, firm profits and public debt issuance, to finance public good, lump-sum transfers and debt repayment.

We now simplify the expression of the government budget constraint, following Chamley (1986). Using the relationship (4) stating that $\zeta_t Y_t = (\tilde{r}_t^K + \delta)K_{t-1} + \tilde{w}_t L_t$, as well as the definition

[2] Aiyagari (1994) discusses the relevant values of $\bar{a}$, called the natural borrowing limit in an economy without aggregate shocks. Shin (2006) provides a similar discussion in presence of aggregate shocks. A standard value in the literature is $\bar{a} = 0$, which ensures that consumption remains positive in all states of the world.

of post-tax rates in equations (11) and (12), the governmental budget constraint becomes:

$$G_t + B_{t-1} + r_t (B_{t-1} + K_{t-1}) + w_t L_t + T_t = B_t + \left(1 - \frac{\kappa}{2}\pi_t^2\right) Y_t - \delta K_{t-1}. \tag{13}$$

Monetary policy consists in choosing the nominal interest rate $\tilde{R}_{t-1}^{B,N}$ on public debt (between $t-1$ and t), as well as the inflation rate π_t. The choice of optimal monetary-fiscal policy is thus the choice of the path of the instruments $\left(\tau_t^L, \tau_t^K, T_t, B_t, \tilde{R}_t^{B,N}, \pi_t\right)_{t=0,\ldots,\infty}$.

2.6 Agents' program and resource constraints

We consider an agent $i \in \mathcal{I}$. Her savings pay off the post-tax real interest rate r_t, but must remain greater than an exogenous threshold denoted $-\bar{a} \leq 0$. Formally, the agent's program can be expressed, for a given initial endowment a_{-1}^i as:

$$\max_{\{c_t^i, l_t^i, a_t^i\}_{t=0}^\infty} \mathbb{E}_0 \sum_{t=0}^\infty \beta^t u\left(c_t^i - \chi^{-1}\frac{l_t^{i,1+1/\varphi}}{1+1/\varphi}\right) \tag{14}$$

$$c_t^i + a_t^i = (1+r_t)a_{t-1}^i + w_t y_t^i l_t^i + T_t, \tag{15}$$

$$a_t^i \geq -\bar{a},\ c_t^i > 0,\ l_t^i > 0. \tag{16}$$

where $\mathbb{E}_0$ an expectation operator. At date 0, the agent decides her consumption $(c_t^i)_{t\geq 0}$, her labor supply $(l_t^i)_{t\geq 0}$, and her saving plans $(a_t^i)_{t\geq 0}$ that maximize her intertemporal utility of equation (14), subject to a budget constraint (15) and the previous borrowing limit (16).

The first-order conditions corresponding to the agent's program (14)–(16) can be expressed as – by using the properties of the GHH utility function:

$$u'(c_t^i - \chi^{-1}\frac{l_t^{i,1+1/\varphi}}{1+1/\varphi}) = \beta\mathbb{E}_t\left[(1+r_{t+1})u'(c_{t+1}^i - \chi^{-1}\frac{l_{t+1}^{i,1+1/\varphi}}{1+1/\varphi})\right] + \nu_t^i, \tag{17}$$

$$l_t^{i,1/\varphi} = \chi w_t y_t^i, \tag{18}$$

where the Lagrange multiplier of the credit constraint of agent i has been denoted $\beta^t\nu_t^i$. Of course, this Lagrange multiplier is null when the agent is not credit-constrained.

We now express the economy-wide constraints. First, the clearing of financial market, labor market and goods market can be expressed respectively as:

$$\int_i a_t^i \ell(di) = A_t = B_t + K_t, \quad \int_i y_t^i l_t^i \ell(di) = L_t, \tag{19}$$

$$\int_i c_t^i \ell(di) + G_t + K_t = Y_t + K_{t-1} - \delta K_{t-1}, \tag{20}$$

We can now formulate our equilibrium definition.

Definition 1 (Sequential equilibrium) *A sequential competitive equilibrium is a collection of individual allocations* $(c_t^i, l_t^i, a_t^i)_{t\geq 0, i\in\mathcal{I}}$*, of aggregate quantities* $(K_t, L_t, Y_t)_{t\geq 0}$*, of price processes* $(w_t, r_t, r_t^K, R_t^{B,N}, \tilde{w}_t, \tilde{r}_t, \tilde{r}_t^K, \tilde{R}_t^{B,N})_{t\geq 0}$*, of fiscal policies* $(\tau_t^K, \tau_t^L, B_t, T_t)_{t\geq 0}$*, and of monetary policies* $(\Pi_t)_{t\geq 0}$ *such that, for an initial wealth distribution* $(a_{-1}^i)_{i\in\mathcal{I}}$*, and for initial values of capital stock* $K_{-1} = \int_i a_{-1}^i \ell(di)$*, of public debt* B_{-1} *and of the aggregate shock* z_{-1}*, we have:*

1. *given prices,* $(c_t^i, l_t^i, a_t^i)_{t\geq 0, i\in\mathcal{I}}$ *solve the agent's optimization program in equations (14)–(16);*

2. *financial, labor, and goods markets clear at all dates: for any* $t \geq 0$*, equations (19) and (20) hold;*

3. *the government budget is balanced at all dates: equation (13) holds for all* $t \geq 0$*;*

4. *factor prices* $(w_t, r_t, r_t^K, R_t^{B,N}, \tilde{w}_t, \tilde{r}_t, \tilde{r}_t^K, \tilde{R}_t^{B,N})_{t\geq 0}$ *are consistent with condition (5), restrictions (9) and (10), as well as with post-tax definitions (11) and (12);*

5. *the inflation path* $(\Pi_t)_{t\geq 0}$ *is consistent with the dynamics of the Phillips curve: at any date* $t \geq 0$*, equation (8) holds.*

The goal of this paper is to determine the optimal fiscal policy that generates the sequential equilibrium-maximizing aggregate welfare, using a utilitarian welfare criteria. This is a difficult question, as the policy is composed of five instruments $\left(\tau_t^L, \tau_t^K, T_t, B_t, \tilde{R}_t^{B,N}\right)$ whichaffects the saving decisions and the labor supplies of all agents, the capital stock, and the price dynamics. We propose a solution that involves three steps. First, we derive the solution of the Ramsey program in the general case, so as to obtain a general characterization of the role of monetary policy (Section 3). Second, we introduce the truncation theory in the space of idiosyncratic histories of LeGrand and Ragot (2019), that enables to provide finite-state representation of the Ramsey solution (Section 4). Finally, we use this finite state representation to quantitatively solve the Ramsey problem (Section 5).

3 Optimal fiscal and monetary policies

3.1 The Ramsey problem

We now solve the Ramsey problem in our incomplete-market economy with aggregate shocks. The Ramsey problem requires the government to jointly choose fiscal and monetary policies that maximize aggregate welfare. This fiscal policy consists of a path for transfers, labor and capital taxes, as well as a path of public debt, while the monetary policy consists of an inflation path. Interestingly, monetary policy has to balance the cost of output destruction (through price adjustment cost) and nominal debt monetization (as well as a more indirect role on mark-ups).

The aggregate welfare is an additive criterion that depends on the weights, denoted $(\omega_t^i)_{t\geq 0}^{i\in\mathcal{I}}$, on each agent. Weights are normalized such that the total weight of the population is one at each date: $\int_i \omega_t^i \ell(di) = 1$. Loosely speaking, these weights represent the relative importance of each agent in the planner's objective. Formally, the aggregate welfare criterion can be expressed as follows:

$$\sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i U(c_t^i, l_t^i) \ell(di). \tag{21}$$

Those weights will be calibrated in our quantitative exercise of Section 5, so as to match the US fiscal and monetary policies at the steady-state. We also assume that the pricing kernel M_t is set consistently with the aggregate welfare criterion and that the pricing kernel is the average "weighted" marginal utility:

$$M_t = \int_i \omega_t^i U_c(c_t^i, l_t^i) \ell(di). \tag{22}$$

We choose this pricing kernel to avoid any inefficiency in the financial sector, that the planner would like to correct. It appears that the choice of another pricing kernel has minor quantitative effects. The government has to select the competitive equilibrium associated to the highest welfare subject to a constraint of balanced governmental budget. We can formalize the Ramsey program as follows:

$$\max_{\left(w_t, r_t, \tilde{w}_t, \tilde{r}_t^K, \tilde{R}_t^{B,N}, \tau_t^K, \tau_t^L, B_t, K_t, L_t, \Pi_t, (a_t^i, c_t^i, l_t^i)_i\right)_{t\geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i U(c_t^i, l_t^i) \ell(di) \right], \tag{23}$$

$$G_t + B_{t-1} + r_t \left(B_{t-1} + K_{t-1}\right) + w_t L_t + T_t = B_t + \left(1 - \frac{\kappa}{2} \pi_t^2\right) Y_t - \delta K_{t-1}. \tag{24}$$

for all $i \in \mathcal{I}$:

$$a_t^i + c_t^i = (1 + r_t) a_{t-1}^i + w_t y_t^i l_t^i + T_t, \tag{25}$$

$$a_t^i \geq -\bar{a}, \tag{26}$$

$$U_c(c_t^i, l_t^i) = \beta \mathbb{E}_t \left[U_c(c_{t+1}^i, l_{t+1}^i)(1 + r_{t+1}) \right] + \nu_t^i, \tag{27}$$

$$l_t^{i,1/\varphi} = \chi w_t y_t^i, \tag{28}$$

$$\Pi_t(\Pi_t - 1) = \frac{\varepsilon - 1}{\kappa} \left(\zeta_t - 1\right) + \beta \mathbb{E}_t \Pi_{t+1}(\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \frac{M_{t+1}}{M_t}, \tag{29}$$

$$K_t + B_t = \int_i a_t^i \ell(di), \; L_t = \int_i y_t^i l_t^i \ell\left(di\right), \tag{30}$$

$$r_t = (1 - \tau_t^K) \frac{\tilde{r}_t^K K_{t-1} + \left(\frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t} - 1 \right) B_{t-1}}{A_{t-1}} \tag{31}$$

and subject to several other constraints (that are not reported here for space constraints): the definition (2) of ζ_t, the one (4) of Y_t, the ones (11) and (12) of after-tax wage w_t, the no-arbitrage constraint (10), the relationship (5) between factor prices, the pricing kernel definition (22), and the positivity of labor and consumption choices, and initial conditions. The constraints in the Ramsey program include: the governmental and individual budget constraints (24) and (25), individual credit constraint (26), Euler equations for consumption and labor (27) and (28), the Phillips curve (29), market clearing conditions for financial and labor markets (30), and the zero profit condition for the fund (31). It is noteworthy that we have modified the zero-profit condition (9) to express it as a function of r_t, $\tilde{R}_t^{B,N}$, and $\tilde{r}_t^K$. This enables us to drop $\tilde{r}_t$, r_t^K, and $R_t^{B,N}$ from the planner's program since they do not play any role.

The Ramsey program can be reformulated by integrating in the objective function the individual Euler equations (27) for consumption as well as the equation for the Phillips curve. The strategy is to use the methodology of Marcet and Marimon (2019) applied to incomplete market environments. The general methodology is developed in LeGrand and Ragot (2020), where we discuss more generally the use of the Lagrangian approach in an environment featuring incomplete markets and occasionally binding credit constraints (see also Section 4 for a discussion). We denote by $\beta^t \lambda_t^i \omega_t^i$ the Lagrange multiplier of the Euler equation of agent i at date t. Similarly, we denote by $\beta^t \gamma_t$ the Lagrange multiplier of the equation (29) of the Phillips curve. With this notation, the objective of the Ramsey program (23) becomes (see Appendix B for further detail about the analytical derivation):

$$\begin{aligned} J =& \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i U(c_t^i, l_t^i) \ell(di) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \left(\omega_t^i \lambda_t^i - (1+r_t) \lambda_{t-1}^i \omega_{t-1}^i \right) U_c(c_t^i, l_t^i) \ell(di) \\ & - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left((\gamma_t - \gamma_{t-1}) \Pi_t \left(\Pi_t - 1 \right) - \frac{\varepsilon - 1}{\kappa} \gamma_t \left(\zeta_t - 1 \right) \right) Y_t M_t. \end{aligned} \tag{32}$$

With this notation, the Ramsey program (23)–(30) can now be expressed as:

$$\max_{\left(w_t, r_t, \tilde{R}_t^{B,N}, \tilde{w}_t, \tilde{r}_t^K, \tau_t^K, \tau_t^L, B_t, T_t, K_t, L_t, \Pi_t, (a_t^i, c_t^i, l_t^i)_i \right)_{t \geq 0}} J,$$

subject to the same set of constraints, except the individual Euler equations for consumption (27) and the Phillips curve (29). Marcet and Marimon (2019) rely on a similar transformation for individual Euler equations. We here use the same methodology to cope with the Phillips curve. The main idea of this representation is that minimizing the cost of the constraints is now an objective.

It is worth mentioning a new difference between real and monetary frameworks. As is standard in optimal fiscal policy literature and following Chamley (1986), the Ramsey problem in a real framework can be written in post-tax prices r_t, r_t^K, $R_t^{B,N}$, and w_t. To derive fiscal policy, the literature usually solves for the post-tax allocation and then derive the value of the taxes comparing post-tax prices and marginal productivities. This methodology cannot be applied in a monetary framework when the Phillips curve is a binding constraint (i.e., $\gamma_t \neq 0$), because the before-tax price now enters into the objective of the planner.

Finally, for simplifying result interpretation, it is useful to introduce a new concept, that we call the *social valuation of liquidity for agent i* denoted by ψ_t^i and formally defined as:

$$\begin{aligned}\psi_t^i \equiv \omega_t^i U_c(c_t^i, l_t^i) - U_{cc}(c_t^i, l_t^i) \left(\lambda_t^i - (1+r_t)\lambda_{t-1}^i\right) \\ - \left((\gamma_t - \gamma_{t-1}) \Pi_t (\Pi_t - 1) - \frac{\varepsilon - 1}{\kappa} \gamma_t (\zeta_t - 1)\right) Y_t \omega_t^i U_{cc}(c_t^i, l_t^i).\end{aligned} \tag{33}$$

The valuation ψ_t^i measures the benefit – from the planer's perspective – of transferring an extra unit of consumption to agent i, valued with weight ω_t^i. As can be seen in equation (33), this valuation consists of three terms. The first one is the marginal utility of consumption $\omega_t^i U_c(c_t^i, l_t^i)$, which can be seen as the private valuation of liquidity for agent i. The second and third terms can been seen as the internalization, by the planner, of the economy-wide externalities of this extra consumption unit. More precisely, the second term in (33) takes into consideration the impact of the extra unit consumption on saving incentives from periods $t-1$ to t and from periods t to $t+1$. An extra consumption unit makes the agent more willing to smooth out her consumption between periods t and $t+1$ and thus makes her Euler equation more "binding". This more "binding"constraint reduces the utility by the algebraic quantity $U_{cc}(c_t^i, l_t^i)\lambda_t^i$, where λ_t^i is the Lagrange multiplier of the agent's Euler equation at date t. The extra consumption unit at t also makes the agent less willing to smooth her consumption between periods $t-1$ and t and therefore "relaxes" the constraint of date $t-1$. This is reflected in the quantity λ_{t-1}^i.

These two first effects are present both in a real and a nominal frameworks. This is not the case of the third effect that is specific to the nominal framework and vanishes in the real one (when $\Pi_t = \zeta = 1$ for all t). This effect, due to the third and last term of equation (33), reflects how the extra consumption unit affects the pricing kernel of the agent and thereby the valuation of monopoly profits.

In addition to ψ_t^i, another key quantity is the Lagrange multiplier, μ_t, on the governmental budget constraint. The quantity μ_t represents the marginal cost at period t of transferring one extra unit of consumption to households. Therefore, the quantity $\psi_t^i - \mu_t$ can be interpreted as the "net" valuation of liquidity: this is from the planner's perspective, the benefit of transferring one extra unit of consumption to agent i, net of the governmental cost. We thus define:

$$\hat{\psi}_t^i = \psi_t^i - \mu_t. \tag{34}$$

3.2 Understanding the role of monetary policy: A decomposition

The previous environment is very general and various effects are at stake. To better understand the mechanisms, we decompose the investigation in the analysis of four different economies, which differ according to the tools available to the planner. We consider:

1. the real economy with both time-varying capital and labor taxes, without monetary frictions ($\kappa = 0$);

2. the monetary economy, with nominal frictions and time-varying capital and labor taxes;

3. the economy without time-varying capital taxes and with only time-varying labor taxes;

4. the economy without time-varying nominal interest rate and time-varying taxes..

The roadmap is the following. The first economy will serve as a benchmark. Our main result will consist in showing that the allocations of the real economy can be replicated with the full set of fiscal instruments (second economy). The following two economies (economy 3 and 4) allow us to identify the distortions implied by a fixed capital tax and a fixed nominal interest rate, respectively.

3.2.1 The benchmark: The real economy case

To understand the impact of nominal frictions on our results, we compare a monetary economy with frictions to a frictionless economy. We define the real-economy allocation as a flexible-price economy, where the government can choose in each period capital and labor taxes, public debt, and transfers so as to optimize the aggregate welfare. More formally, the real economy allocation is the solution of the following program:

$$\max_{\left(w_t, r_t, B_t, K_t, L_t, T_t, (a_t^i, c_t^i, l_t^i)_i\right)} J, \tag{35}$$

with $\kappa = 0$ for all t (real economy, which means no Phillips curve), $\zeta_t = 1$ (because no price-setting inefficiency), $\Pi_t = 1$ (no inflation), and subject to budget constraints (24) and (25), Euler equations (27) and (28), and aggregation equations (30), and the relationships between r_t, r_t^K, (equations (9) and (10) for post-tax interest rates). The before-tax rates $\tilde{w}_t$ and $\tilde{r}_t^K$ can then be deduced from equations (2) and (5) – with $\zeta_t = 1$. Taxes τ^L and τ^K are then obtained from the relationships between pre-tax and post-tax rates (11) and (12). Finally, the nominal rate $\tilde{R}_t^{B,N}$ can be deduced from relationship (31).

A solution to Ramsey program is characterized by five first-order conditions. The first one, with respect to individual savings can be written as:

$$\hat{\psi}_t^i = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \hat{\psi}_t^i \right], \tag{36}$$

for non-constrained agents i. Constrained agents i face $\lambda_t^i = 0$. Equation (36) states that the net social value of liquidity should be smoothed out over time. It can be interpreted as a Euler-like equation for the planner and generalizes the standard individual Euler equation by taking into account the externalities of saving choices on interest rate.

The second first-order condition concerns the role of public debt and is written as follows:

$$\mu_t = \beta \mathbb{E}_t \left[\mu_{t+1} \left(1 + \tilde{r}_{t+1}^K \right) \right]. \tag{37}$$

We recall that μ_t is the Lagrange multiplier on the governmental budget constraint – and that therefore represents the shadow cost of one unit of consumption for the planner. Equation (37) therefore states that the shadow cost of resources should be smoothed out through time. Again, as equation (36), equation (37) can be interpreted as a Euler-like equation, with one difference: the interest rate is the before-tax interest rate on capital ($\tilde{r}^K$) instead of the post-tax interest rate of the fund (r) in equation (36).

The third first-order condition deals with the post-tax interest rate r_t and can be written as:

$$\int_i \hat{\psi}_t^i a_{t-1}^i \ell(di) = -\int_i \lambda_{t-1}^i U_c(c_t^i, l_t^i)\ell(di). \tag{38}$$

In absence of any side effect, the planner would chose the interest rate so as to set to zero the aggregate net value of liquidity among all agents – weighted by agents' asset holdings, such that $\int_i \hat{\psi}_t^i a_{t-1}^i \ell(di) = 0$, or equivalently to equalize social liquidity valuation to its cost: $\int_i \omega_t^i \psi_t^i a_{t-1}^i \ell(di) = \mu_t \int_i \omega_t^i a_{t-1}^i \ell(di)$. However, this is not possible since the planner needs to account for the side effect of r_t on the savings incentives, through the Euler equation. This effect is proportional to the shadow cost of the Euler equation. Note that the sign of this shadow cost depends on the planner's perception of the savings quantity in the economy. It is positive when the planner perceives excess savings in the economy, and negative the other way around (see LeGrand and Ragot, 2020 for a lengthier discussion). In consequence, for instance, when there is an excess quantity of savings in the economy, the total net valuation of liquidity is negative.

The fourth condition regarding the post-tax wage rate w_t is:

$$\int_i \hat{\psi}_t^i y_t^i l_t^i \ell(di) = \varphi \mu_t \left(L_t - (1-\alpha)\frac{Y_t}{w_t} \right).$$

Similarly to equation (38), in absence of side effect for the wage rate, the planner would like to set the aggregate net liquidity value – weighted by individual labor supply in efficient terms – to zero. However, this is not possible, since planner has also to take into account the distortions implied by wage variations on total labor supply and relatedly on output. These distortions are proportional to the labor elasticity φ and vanish when labor supply is inelastic ($\varphi = 0$).

Finally, the fifth condition regrading the lump-sum transfer T_t is:

$$\int_i \hat{\psi}_t^i \ell(di) = 0. \tag{39}$$

Since there is no distortions implied by the lump-sum transfer, it is set such that the redistributive effect is null.

Characterizing the optimal fiscal policy in an heterogeneous-agent framework is already a difficult task. We are aware of very few papers doing so in absence of aggregate shocks. This is the case of Açikgöz (2015), that is further refined in Açikgöz et al. (2018), who relies on a Lagrangian approach to compute the planner's first-order conditions and uses a numerical approximation to compute Lagrange multipliers. Another approach is the one of Dyrda and Pedroni (2018) and Chang et al. (2018), who maximize the aggregate welfare through an extensive search over all possible instrument values. This solution is computionnally very intensive, which limits further developments. Nuño and Moll (2018) analyze social optima in a continuous-time framework. To the best of our knowledge, Bhandari et al. (2020) is the only paper deriving optimal Ramsey policy in a heterogeneous-agent framework with aggregate shocks. If their solution technique can handle large aggregate shocks, their approach must feature credit constraints that are either always binding or never binding. Our solution – that we describe in detail in Section 4 – can account for occasionally binding credit constraints, which may be the relevant case in some environments.

3.2.2 An irrelevance result with incomplete markets: The monetary economy with a full set of fiscal tools

We now turn to the monetary set-up where the full set of fiscal instruments is available. We also state our main theoretical result showing the irrelevance of monetary tools. The real allocation can be recovered in a monetary economy when the full set of fiscal instruments is available.

As a preliminary remark, observe that our monetary economy features two market imperfections. The first imperfection is the imperfect competition between firms that yields a price markup ζ_t strictly above one. The second imperfection is the Rotemberg inefficiency that prevents firms from setting at no cost their price. The two imperfections are complements. Indeed, in absence of Rotemberg inefficiency (i.e., $\kappa = 0$), firm's profit maximization yields $\zeta_t = 1$ and the markup inefficiency vanishes, as can be seen from the Phillips curve in equation (8). Similarly, we observe from the Phillips curve that if $\zeta_t = 1$, we can set $\Pi_t = 1$ and thereby avoid any cost related to Rotemberg inefficiency. The objective of the planner's – in a monetary setup – therefore includes minimizing the impact of these two inefficiencies.

We first solve for the optimal monetary and fiscal policies when the government has access to a full set of fiscal tools. This program can be written as:

$$\max_{\left(w_t, r_t, \tilde{w}_t, \tilde{r}_t^K, B_t, T_t, K_t, L_t, \Pi_t, (a_t^i, c_t^i, l_t^i)_i\right)_{t \geq 0}} J,$$

subject to the same equations as in the real economy case (without $\kappa \neq 0$), as well as the Phillips curve (29) and the factor price equations (2) and (5). The first observation is that we have dropped the taxes from the Ramsey program since, as in the real-economy case, they do not play a direct role and can be substituted by post-tax rates r_t, r_t^K and w_t. The second observation is that the pre-tax nominal rate $\tilde{R}_t^{B,N}$, along with constraints (9) and (10), are also dropped since they do not play any role. The second and more important observation is that, as in the real economy, the before-tax rates $\tilde{w}_t$ and $\tilde{r}_t^K$ only play a role in the markup coefficient ζ_t of equation (2) and in the factor price equation (5). The planner has thus two independent instruments ($\tilde{r}_t^K$ or $\tilde{w}_t$ on one side and Π_t on the other side) to address the two monetary frictions of the economy. The planner can thus set $\tilde{r}_t^K$ or $\tilde{w}_t$ such that the markup inefficiency vanishes (i.e., such that $\zeta_t = 1$). The gross inflation rate can then be set to 1 at all dates, so as to neutralize the Rotemberg inefficiency. The program, with the full set of tools, can be expressed as $\max_{\left(w_t, r_t, B_t, K_t, L_t, (a_t^i, c_t^i, l_t^i)_i\right)_{t\geq 0}} J$, subject to exactly the same constraints as in the real economy case (because $\Pi_t = \zeta_t = 1$ at all dates). This thus leads the same allocation as in the real economy. We summarize this first result in the following proposition.

Proposition 1 (An irrevelance result) *When both labor and capital taxes are available, the government exactly reproduces the real-economy allocation.*

3.2.3 The economy without time-varying capital taxes

We now turn to our main variation, in which we assume that the planner cannot vary the capital tax, but only the labor tax. The capital tax is constant and fixed at its optimal steady-state τ_{SS}^K to avoid steady-state distortions. The problem of the planner can now be written as:

$$\max_{\left(w_t, r_t, \tilde{R}_t^{B,N}, \tilde{w}_t, \tilde{r}_t^K, \tau_t^L, T_t, B_t, K_t, L_t, \Pi_t, (a_t^i, c_t^i, l_t^i)_i\right)_{t\geq 0}} J,$$

subject to same equations as in case with the full set of instruments (Section 3.2.2), as well as to the additional constraint $\tau_t^K = \tau_{SS}^K$, reflecting the new steady state value of the capital tax. The main difference with the previous case is that we cannot choose $\tilde{w}_t$ and $\tilde{r}_t^K$ to set $\zeta_t = 1$ and fully offset markup inefficiency. Indeed, because of the fixed capital tax rate and the factor price relationship (5), the pre-tax rate $\tilde{r}_t^K$ affects both the markup ζ_t and the fund interest rate r_t. Nominal inefficiencies cannot be removed, and the inflation rate cannot consequently set be set to 1. We derive the first-order conditions in Appendix D.

Compared to the full case, we have two additional Lagrange multiplier: one, denoted by Υ_t on the no-arbitrage condition (10), and another one, denoted by Γ_t on the zero-profit condition (31) of the fund. We also have three additional first-order conditions related to nominal rate $\tilde{R}_t^{B,N}$, pre-tax capital rate $\tilde{r}_t^K$, and inflation rate Π_t. First, the condition with respect to $\tilde{R}_t^{B,N}$ is:

$$\left(1 - \tau^K\right) \mathbb{E}_t \left[\frac{\Gamma_{t+1}}{\Pi_{t+1}}\right] B_t = \Upsilon_t \mathbb{E}_t \left[\frac{1}{\Pi_{t+1}}\right], \tag{40}$$

which reflects the connection between the two constraints involving $\tilde{R}_t^{B,N}$. The first-order condition with respect to $\tilde{r}_t^K$is:

$$\Upsilon_{t-1} + \Gamma_t(1 - \tau_{SS}^K)\left(A_{t-1} - B_{t-1}\right) = \frac{\varepsilon - 1}{\alpha\kappa}\gamma_t K_{t-1} M_t,$$

where the left-hand side accounts for the effect of $\tilde{r}_t^K$ on nominal interest rate $\tilde{R}_t^{B,N}$, and on the fund rate r_t, while the right-hand side reflects the effects of $\tilde{r}_t^K$ on the markup ζ_t. This is exactly what was explained before: due to fixed capital tax, nominal inefficiencies cannot be addressed independently of saving incentives.

The third and last additional first-order condition concerns the inflation rate:

$$\mu_t \kappa \left(\Pi_t - 1\right) = -\left(\gamma_t - \gamma_{t-1}\right)\left(2\Pi_t - 1\right) M_t + \left(\Gamma_t \left(1 - \tau_{SS}^K\right) B_{t-1} - \Upsilon_{t-1}\right) \frac{\tilde{R}_{t-1}^{B,N}}{Y_t \Pi_t^2}, \tag{41}$$

where we recall that M_t is the pricing kernel. The left side captures the cost of an increase in inflation in terms of output destruction. The right hand side is the benefits of an increase in current's inflation in terms of Phillips curve's relaxation and of nominal interest rate variation.

Second, the choice of individual savings a_t^i yields:

$$\begin{aligned} \hat{\psi}_t^i = & \overbrace{\beta \mathbb{E}_t \left[(1 + r_{t+1})\hat{\psi}_{t+1}^i\right]}^{=\text{extended real effect}} \qquad (42) \\ & + \underbrace{\beta \mathbb{E}_t \left[\Gamma_{t+1} \left(r_{t+1} - \left(1 - \tau_{SS}^K\right)\left(\frac{\tilde{R}_t^{B,N}}{\Pi_{t+1}} - 1\right)\right)\right]}_{=\text{wedge rate effect}}. \end{aligned}$$

which is similar to the real case, except that the net social value of liquidity $\hat{\psi}_t^i$ now includes a term related to the variation of the pricing kernel due to one extra unit of consumption. This is why we refer to this term as an "extended real effect". There is a second difference related to the wedge in interest rate: the (pre-tax) rate on agents' saving is $\tilde{r}_t$, while the one on public debt, and thus the cost of planner's funding, is $\frac{\tilde{R}_t^{B,N}}{\Pi_{t+1}}$. We refer to this as a "wedge rate effect".

Third, the choice of public debt yields the following first-order condition:

$$\begin{aligned} \mu_t = & \overbrace{\beta \mathbb{E}_t \left[\mu_{t+1}\left(1 - \delta + \zeta_{t+1}^{-1}(\tilde{r}_{t+1} + \delta)\left(1 - \frac{\kappa}{2}\left(\Pi_{t+1} - 1\right)^2\right)\right)\right]}^{=\text{extended real effect}} \qquad (43) \\ & \underbrace{- \alpha\beta \mathbb{E}_t \left[\left(\left(\gamma_{t+1} - \gamma_t\right)\Pi_{t+1}\left(\Pi_{t+1} - 1\right) + \frac{\varepsilon - 1}{\kappa}\gamma_{t+1}\right)\frac{Y_{t+1}}{K_t} M_{t+1}\right]}_{=\text{Phillips curve effect}} \\ & + \underbrace{\frac{\varepsilon - 1}{\kappa}\beta \mathbb{E}_t \left[\gamma_{t+1}\frac{\zeta_{t+1} Y_{t+1}}{K_t} M_{t+1}\right]}_{=\text{markup effect}} \\ & + \underbrace{\beta\left(1 - \tau_{SS}^K\right)\mathbb{E}_t \left[\Gamma_{t+1}\left(\frac{\tilde{R}_t^{B,N}}{\Pi_{t+1}} - 1 - \tilde{r}_{t+1}^K\right)\right]}_{=\text{wedge rate effect}} \qquad (44) \end{aligned}$$

The first line states that public debt enables the planner to smooth out the cost of liquidity for the government. This is similar to the real economy case, except that the smoothing also needs to account for nominal effects, and in particular the presence of the markup and of the price adjustment cost, as well as the wedge between the capital rate and the public debt rate. When monetary imperfections go away ($\zeta_t = 1$ and $\Pi_t = 1$), we exactly fall back on the smoothing effect of the real economy (equation (37)), up to the wedge rate effect. The second line is related to the impact of public debt on the Phillips curve through capital crowding out and output mitigation. The third line comes from effect on public debt on the markup inefficiency – through capital crowding-out and interest rate. Finally, the fourth line is related to the wedge rate effect, stemming from the difference in rates between public debt and capital.

The first-order condition related to the interest rate r_t is:

$$\int_i \hat{\psi}_t^i a_{t-1}^i \ell(di) = -\overbrace{\int_i \lambda_{t-1}^i U_c(c_t^i, l_t^i)\ell(di)}^{=\text{extended real effect}} - \underbrace{\frac{\varepsilon - 1}{\kappa}\gamma_t \frac{Y_t \zeta_t}{r_t} M_t}_{=\text{markup effect}} + \overbrace{\Gamma_t A_{t-1}}^{=\text{rate setting}}. \tag{45}$$

We can parallel the interpretation of the real case. In the monetary setting, a second side-effect is present and is related to the impact of r_t on price mark-up. Due to the lack of appropriate instruments, the planner has to set the post-tax interest rate r_t to manage at the same time the aggregate net benefit of liquidity and mitigates the consequences of the markup inefficiency. The constraint related to the relationship between the rates of teh fund, of public debt and capital is also present.

Finally, equation (39) for T_t being unchanged, the choice of w_t yields the following FOC:

$$\begin{aligned}\int_i \omega_t^i \hat{\psi}_t^i y_t^i l_t^i \ell(di) = &\overbrace{\varphi \mu_t \left(L_t - (1-\alpha)\varphi \frac{Y_t}{w_t} \left(1 - \frac{\kappa}{2}(\Pi_t - 1)^2\right)\right)}^{=\text{extended real effect}} \\ &+ \underbrace{\frac{(1-\alpha)\varphi}{w_t}\left((\gamma_t - \gamma_{t-1})\Pi_t(\Pi_t - 1) + \frac{\varepsilon - 1}{\kappa}\gamma_t \right) Y_t M_t}_{=\text{Phillips curve effect}}.\end{aligned} \tag{46}$$

The interpretation of the FOC in equation (46) is similar to the one in equation (46). Due to missing instruments, setting the post-tax wage rate must also account for the effect related to nominal rigidities through the Phillips curve – in addition to the effect on labor supply and output already present in the real case. Note that when labor is inelastic ($\varphi = 0$), these two effects are absent, since labor supply – and thereby output – remain unaffected by wage variations. All in all, the planner sets the post-tax wage to pursue both a real and a nominal objective: manage aggregate net benefit of liquidity (while internalizing the possible effects on aggregate labor supply) and mitigate the nominal inefficiencies.

These characterizations will be used in the simulation exercises below. We here summarize these findings in the following proposition.

Proposition 2 *In the economy with constant capital taxes,*

1. *the Rotemberg inefficiency is solely – though imperfectly – addressed by the inflation rate;*

2. *the markup inefficiency is jointly – though imperfectly – addressed by the interest and wage rates, as well as the public debt;*
3. *once we account for the role of liquidity in pricing kernel for firms, the evolution of the net liquidity benefit still follows, as in the real case, an Euler equation.*

4 Projecting the model

We use the method detailed in LeGrand and Ragot (2020) to provide a finite state representation of the model and of the Ramsey program. The general idea of the method relies on the "truncation" of idiosyncratic histories. More precisely, we construct a representation of the model in which agents having the same history over last N periods (where N is a fixed horizon) are represented by the same history, having the same wealth and the same consumption-saving decisions. The corresponding truncated model features a consistent finite state space representation of Bewley models. One of the crucial features of our approach is that the truncated model is compatible with standard expected-utility maximization. More precisely the allocations of the truncated histories can be computed as the result of an almost-standard competitive equilibrium. To achieve this, we need to introduce so-called preference shifters that are multiplicative factors on utility functions. These shifters are easy to identify and can be computed such that the steady-state allocation of the truncated model are consistent with the one of the full-fledged model, and in particular both feature the same aggregate quantities and the same prices. The main benefit of our approach is that it enables us to use the tools of Marcet and Marimon (2019) to solve a Ramsey program in presence of aggregate shocks (with possibly a large number of planner's instruments). We now detail the truncation method in our monetary setup.

4.1 The truncation setup

Let $N > 0$ be a truncation length. A truncated history is a vector $y^N = (y_{-N+1}, \ldots, y_0) \in \mathcal{Y}^N$ (where $\mathcal{Y}$ is the finite set of possible idiosyncratic realizations) and there are therefore Y^N possible histories (where $Y = \text{Card}(\mathcal{Y})$). We present the truncated economy as an island-metaphor. We consider that there are Y^N different islands, corresponding to specific truncated histories. An agent with history y^N at date t will be located on the corresponding island y^N and when her N-length history becomes $\hat{y}^N$ in period $t+1$, she will move to island $\hat{y}^N$. An island-planner can freely transfer resources within – but not across – islands. As a consequence, all inhabitants of the same island are endowed with the same beginning-of-period wealth and the same allocation. For agents on island y^N, we denote the per capita consumption level c_{t,y^N}, the labor supply l_{t,y^N}, and the savings a_{t,y^N}. The beginning-of-period wealth, denoted by $\tilde{a}_{t,y^N}$ is equal to:

$$\tilde{a}_{t,y^N} = \sum_{\hat{y}^N \in \mathcal{Y}^N} \frac{S_{t-1,\hat{y}^N}}{S_{t,y^N}} \Pi_{t,\hat{y}^N,y^N} a_{t-1,\hat{y}^N}, \tag{47}$$

and reflects the fact that the wealth of all agents of island y^N are pooled together when they arrive from different islands. The quantity $\Pi_{t,\hat{y}^N,y^N}$ is the probability to transit from island $\hat{y}^N$ at $t-1$ to island y^N at t, and S_{t,y^N} is the size of island y^N at date t, which is defined by the recursion: $S_{t,y^N} = \sum_{\hat{y}^N \in \mathcal{Y}^N} S_{t-1,\hat{y}^N} \Pi_{t,\hat{y}^N,y^N}$.

We further assume that agents face island-specific preference shifters, denoted by ξ_{y^N}, that multiply their utility function (see Section 4.2for a discussion about the role and the calibration of these parameters). The island-planner's program can be expressed as:

$$\max_{(c_{t,y^N}, l_{t,y^N}, a_{t,y^N}, \tilde{a}_{t,y^N})_{t\geq 0, y^N \in \mathcal{Y}^N}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \xi_{y^N} U(c_{t,y^N}, l_{t,y^N}), \tag{48}$$

$$a_{t,y^N} + c_{t,y^N} = w_t y_0^N l_{t,y^N} + (1+r_t)\tilde{a}_{t,y^N} + T_t, \text{ for } y^N \in \mathcal{Y}^N, \tag{49}$$

$$c_{t,y^N},\, l_{t,y^N} \geq 0, a_{t,y^N} \geq -\bar{a}, \text{ for } y^N \in \mathcal{Y}^N, \tag{50}$$

where y_0^N is the current productivity status. Denoting by $\beta^t \nu_{t,y^N}$ the Lagrange multiplier of the credit constraint, the Euler equations are:

$$\xi_{y^N} U_c(c_{t,y^N}, l_{t,y^N}) = \beta \mathbb{E}_t \left[\sum_{\tilde{y}^N \succeq y^N} \Pi_{t+1,y^N,\tilde{y}^N} \xi_{\tilde{y}^N} U_c(c_{t+1,\tilde{y}^N}, l_{t+1,\tilde{y}^N})(1+r_{t+1}) \right] + \nu_{t,y^N}, \tag{51}$$

$$l_{t,y^N}^{1/\varphi} = \chi w_t y_0^N, \tag{52}$$

$$\nu_{t,y^N}(a_{t,y^N} + \bar{a}) = 0 \text{ and } \nu_{t,y^N} \geq 0, \tag{53}$$

which are very close to the individual Euler equations (17) and (18). The main difference is the ξs that we explain below. It is worth noting that Euler equations (51) and (52) could be derived in a decentralized set-up featuring a well-chosen lump-sum transfer scheme. For the sake of simplicity, we only presented here the island metaphor and see LeGrand and Ragot (2020, Section 3.2) for a detailed presentation of the decentralization mechanism.

We assume that in the island economy, the production and governmental sectors are unchanged, compared to full-fledged economy of Section 2. We can then define market clearing conditions, for labor supply and asset holdings, as follows:

$$L_t = \sum_{y^N \in \mathcal{Y}^N} y_0^N S_{t,y^N} l_{t,y^N}, \text{ and } B_t + K_t = \sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} a_{t,y^N}. \tag{54}$$

We can then define a truncated equilibrium as follows.

Definition 2 (Truncated equilibrium) *A truncated equilibrium is a collection of individual* *allocations* $(c_t^i, l_t^i, a_t^i)_{t\geq 0, i\in\mathcal{I}}$*, of aggregate quantities* $(K_t, L_t, Y_t)_{t\geq 0}$*, of the set of price processes* $(w_t, r_t, r_t^K, R_t^{B,N}, \tilde{w}_t, \tilde{r}_t, \tilde{r}_t^K, \tilde{R}_t^{B,N})_{t\geq 0}$*, of fiscal policies* $(\tau_t^K, \tau_t^L, B_t, T_t)_{t\geq 0}$*, and of monetary policies* $(\Pi_t)_{t\geq 0}$ *such that, for an initial wealth distribution* $(a_{-1}^i)_{i\in\mathcal{I}}$*, and for initial values of capital stock* $K_{-1} = \int_i a_{-1}^i \ell(di)$*, of public debt* B_{-1} *and of the aggregate shock* z_{-1}*, we have:*

1. *given prices,* $\left(c_{t,y^N}, l_{t,y^N}, a_{t,y^N}\right)_{t\geq 0}^{y^N\in\mathcal{Y}^N}$ *solve the Euler equations (51) and (52);*
2. *financial, labor, and goods markets clear at all dates: for any* $t \geq 0$*, equations (54) hold;*
3. *the government budget is balanced at all dates: equation (13) holds for all* $t \geq 0$*;*
4. *factor prices* $(w_t, r_t, r_t^K, R_t^{B,N}, \tilde{w}_t, \tilde{r}_t, \tilde{r}_t^K, \tilde{R}_t^{B,N})_{t\geq 0}$ *are consistent with condition (5), restrictions (9) and (10), as well as with post-tax definitions (11) and (12);*
5. *the inflation path* $(\Pi_t)_{t\geq 0}$ *is consistent with the dynamics of the Phillips curve: at any date* $t \geq 0$*, equation (8) holds.*

4.2 Constructing an approximated economy

We now show how to construct the preference shifters ξs such that the truncated equilibrium is consistent with the equilibrium of the full-fledged economy. As a preliminary step, we formally define the *truncation* of an allocation. Consider the sequential representation of a general Bewley model. In this economy, individual choices depend on the whole history of idiosyncratic and aggregate shocks, y^t and z^t respectively. For a generic variable, $X_t(y^t, z^t)$, its truncation N-period history y^N, denoted by X_{t,y^N} is formally defined as:

$$X_{t,y^N} = \frac{\sum_{y^t\in\mathcal{Y}^t|(y_{t-N+1}^t,\ldots,y_t^t)=y^N} X_t(y^t, z^t)\mu_t(y^t)}{S_{t,y^N}}, \tag{55}$$

where we recall that $\mu_t(y^t)$ is the measure of agents with history y^t. The truncation X_{t,y^N} is equal to the average value of the variable X among the population of agents experiencing history y^N over the last N periods in the full-fledged Bewley model.

The steady state. The first step is that the ξs can be constructed such that at the steady state, allocations of the truncated model are truncations of allocations of the full-fledged model. We repeat and adapt Proposition 2 of LeGrand and Ragot (2020).

Proposition 3 (Constructing the ξs) *The preference shifters* $(\xi_{y^N})_{y^N\in\mathcal{Y}^N}$ *can be computed at the steady state, such that the truncation – following equation (55) – of the steady-state equilibrium allocations of the full-fledged model (Definition 1) is an equilibrium allocation in the truncated model (Definition 2).*

The logic of Proposition 3 is as follows. The first step is to compute the policy functions in the full-fledged model at steady state. These allocations can then be integrated using formula (55) to compute truncated allocations and characterized credit-constrained truncated histories. Euler equations (51) can then be inverted to compute the ξs. From an calculus perspective, the computation of the ξs only elementary matrix calculus. A noteworthy side-effect of the construction of Proposition 3 is that the prices and aggregate quantities are identical in both the truncated and full-fledged equilibria.

Why are the ξs needed for? Because there is a non-degenerate distribution of agents with the same truncated history in the full-fledged Bewley model, truncated allocations are not consistent with history-level Euler equations. More precisely, truncating Euler equations does not yield valid Euler equations for truncated allocations (except in the particular case when marginal utilities are linear). The ξs precisely aim to reconcile Euler equations and truncated allocations can be seen as the result of a competitive equilibrium (provided that agents are endowed with preference shifters). Their role is therefore to (partly) recover the within-truncated-history heterogeneity that has been removed by the truncation operation – that assigns the same truncated history and thus the same allocation to actually different agents.

Finally, we can state a convergence result regarding the truncated economy. When the length N becomes infinitely large, the truncated allocations converge to the allocations of the full-fledged model at the steady state, and the preference shifters (ξs) converge to 1.

With aggregate shocks. To simulate the model in presence of aggregate shocks, we make two additional assumptions: (i) the ξs remain unchanged and equal to their steady-state value; (ii) the set of credit-constrained agents remains unchanged compared to the steady-state. With these two assumptions, we can simulate the model in presence of aggregate shocks using perturbation techniques, and therefore by using tried-and-tested tools, such as Dynare (Adjemian et al., 2011). As in the steady state, we can prove a convergence result showing that in presence of aggregate shocks, truncated allocations converge to allocations of the full-fledged model (as long as allocations are computed using a perturbation approach).

A couple of remarks regarding assumptions are in order. The first assumption (regarding ξs) implies that the within-truncated-history heterogeneity remains constant, but importantly is not discarded. The second assumption (regarding credit-constrained histories) is not imposed by our truncation technique but by the perturbation method that we use. Should we use another resolution method, this assumption could be dropped out.

4.3 Ramsey program

The construction of the truncated economy provides a setup in which Ramsey policies can be computed. Indeed, solving Ramsey program in heterogeneous agents economies (with or without aggregate shocks) is a difficult task as it involves the joint distribution of two additional infinite-dimensional state variables, which are in our case, agents' wealth and Lagrange multipliers (on Euler equations). The truncated economy enables us to circumvent these difficulties and to take advantage of a finite-state economy to solve the Ramsey program using the approach of Marcet and Marimon (2019).

Formally, if we denote by $(\omega_{y^N})_{y^N \in \mathcal{Y}^N}$ the Pareto weights associated to history y^N, the Ramsey program in the truncated economy can be expressed as follows.

$$\max_{(w_t, r_t, \tilde{r}_t^K, \tilde{R}_t^{B,N}, \tau_t^K, \tau_t^L, B_t, T_t, K_t, L_t, \Pi_t, (a_{t,y^N}, c_{t,y^N}, l_{t,y^N})_{y^N})_{t \geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \omega_{y^N} \xi_{y^N} U(c_{t,y^N}, l_{t,y^N}) \right], \quad (56)$$

subject to truncated Euler equations (51) and (52), truncated budget constraint (49), truncated credit-constraint (50), truncated aggregation equations (54), as well as equations that were already present in the full-fledged Ramsey program: the governmental budget constraint (24), the Phillips curve (29), the definition (2) of ζ_t, the one (4) of Y_t, the ones (11) and (12) of after-tax rates r_t, r_t^K, $R_t^{B,N}$ and w_t, the zero profit condition for the fund (9), the no-arbitrage constraint (10), and the relationship (5) between factor prices. The only difference is that truncated pricing kernel is now $M_t = \sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \xi_{y^N} \omega_{y^N} U_c(c_{t,y^N}, l_{t,y^N})$.

As we did in Section 3.1 and in equation (32), it is possible to use the tools of Marcet and Marimon (2019) to rewrite the Ramsey program. Two points are worth mentioning regarding the application of the tools Marcet and Marimon (2019) in our set-up. First, the truncation adds no complexity to the formulation of the planner's objective. Second, the application of Marcet and Marimon (2019) to models with occasionally binding credit constraints can raise concerns due to the Slater (1950) condition that may not be fulfilled. This condition requires the existence of an interior solution of the primal problem. We tackle this difficulty by showing that the first-order conditions of our Ramsey problem can be seen as the limit of first-order conditions of a Ramsey program with penalty functions (instead of credit constraints), when the concavity of these penalty function become infinitely high. Details can be found in Appendix.

First-order conditions can similarly be derived as in the general case and we obviously have the same irrelevance results. The first-order conditions in the three cases (real economy, no time-varying capital tax and fixed nominal interest rate) can be found in Appendix.

A final aspect regarding the truncated Ramsey program is that it solutions can be shown to converge to the solutions of the full-fledged Ramsey program (if they exist), when the truncation length N becomes infinitely long. See LeGrand and Ragot (2020, Proposition 5). This convergence property is the parallel of the convergence result regarding allocations of the competitive equilibrium. However, it is noteworthy that this result requires the existence of an upper bound (that be arbitrarily large) on agents' saving choices. As in the previous case, the convergence result is valid for the steady state and in presence of aggregate shocks (as long as the solutions are computed using the perturbation methods).

5 Quantitative assessment

As we are mainly concerned about the time-varying behavior of policy instruments in a quantitatively relevant environment, we use the following strategy.

1. We calibrate a Bewley model with an empirically relevant fiscal system, monetary parameters and idiosyncratic income process. We check that both aggregate quantities and the wealth distribution across agents are close to their empirical counterparts.

2. We project the model and derive the preference shifters ξs to make the projected model consistent with the full-fledged Bewley model.[3]

3. We estimate the Pareto weights $(\omega_{y^N})_{y^N}$ such that the actual US fiscal system is the optimal fiscal system for the planner in the economy without aggregate shocks. We thus follow the methodology of the inverse optimal taxation literature, which estimates social welfare functions that are consistent with observed fiscal systems (see Bargain and Keane, 2010; Bourguignon and Amadeo, 2015; Heathcote and Tsujiyama, 2017; Chang et al., 2018, among others). This strategy ensures that we investigate the dynamics around a quantitatively relevant steady state.[4]

4. We simulate the optimal allocation after a technology shock, in an economy model where the planner has access to a full set of fiscal instruments. We compare this allocation to the one generated by a complete market economy to identify the role of market incompleteness. We already know from the theoretical analysis of Section 3 that inflation is constant in these economies.

5. We simulate the economy where the capital tax is constant, and fixed at its steady state value, but where the planner has access to other fiscal tools and to monetary policy. We then investigate the residual role of monetary policy.

5.1 The calibration

Preferences. The period is a quarter. The discount factor is $\beta = 0.99$ and the period utility function $\log(c - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi})$. The Frisch elasticity of labor supply is set to $\varphi = 0.5$, which is the value recommended by Chetty et al. (2011) for the intensive margin in heterogeneous agent models. The scaling parameter is set to $\chi = 0.068$, which implies normalizing the aggregate labor supply, defined in (19), to 1/3.

[3] We have simulated the truncated model with aggregate shock, to compare our solution method to the histogram method developed by Rios-Rull (1999), Reiter (2009), and Young (2010), as well as to the global method of Boppart et al. (2018) and Auclert et al. (2019). The three methods are shown to deliver very similar quantitative results. See also LeGrand and Ragot (2020) for the same exercise in a different environment.

[4] Estimated Pareto weights provide insightful information regarding social preferences. We do not pursue this investigation further in the current paper, and we leave it for future work.

Technology and TFP shock. The production function is Cobb-Douglas: $Y = ZK^{\alpha}L^{1-\alpha}$. The capital share is set to $\alpha = 36\%$ and the depreciation rate to $\delta = 2.5\%$, as in Krueger et al. (2018) among others. The TFP process is a standard AR(1) process, with $Z_t = \exp(z_t)$ and:

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z, \tag{57}$$

where $\varepsilon_t^z \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_z^2)$. We use the standard values $\rho_z = 0.95$ and $\sigma_z = 0.31\%$ to obtain a deviation of the TFP shock z_t equal to 1% at a quarterly frequency (see Den Haan, 2010 for instance).

Idiosyncratic risk. We use a standard productivity process:

$$\log y_t = \rho_y \log y_{t-1} + \varepsilon_t^y,$$

with $\varepsilon_t^y \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_y^2)$. We calibrate choose a persistence of the productivity process $\rho_y = 0.99$ and a standard deviation of $\sigma_y = 0.14$. These values are in line with empirical estimates.[5] This process generates a realistic empirical pattern for wealth. First, the Gini coefficient of the wealth distribution amounts to 0.77, in line with the data (see below). Second, the model implies an average wealth-to-GDP ratio of 11.8 and an average capital-to-GDP ratio of 2.5. These two values are in line with their empirical counterparts.

Finally, the Rouwenhorst (1995) procedure is used to discretize the productivity process into 5 idiosyncratic states with a constant transition matrix.

Taxes and government budget constraint. Fiscal parameters are calibrated based the computations of Trabandt and Uhlig (2011), who use the methodology of Mendoza et al. (1994) on public finance data prior to 2008. This approach consists in computing a linear tax on capital and on labor, as well as lump-sum transfers that are consistent with the governmental budget constraint. Their estimations for the US in 2007 yield a capital tax (including both personal and corporate taxes) of 36%, a labor tax of 28% and lump-sum transfers equal to 8% of the GDP. This affine structure (lump sum transfers and linear marginal tax rates) is often used in the literature because it enables to properly reproduce the amount of redistribution in the US, as shown for instance by Bhandari et al. (2017b), Heathcote and Tsujiyama (2017), and Dyrda and Pedroni (2018).

This fiscal system generates two untargeted outcome. First, it implies a public debt-to-GDP ratio equal to 60%, which is very close to 63%, which is the value estimated by Trabandt and Uhlig (2011). Second, it also implies a public spending-to-GDP ratio equal to 12.4%. This value is a little bit low compared to postwar values but consistent with other quantitative investigations of the US tax system (Bhandari et al., 2017b).

[5] For instance, Krueger et al. (2018) estimate a more general process with an additional transitory process. The implied AR(1) process generates a standard deviation equal to $\sigma^y = 0.13$.

Monetary parameters. The monetary friction is captured by two parameters. The first one is the elasticity of substitution across goods ε. The second one is the price adjustment cost κ, in the Rotemberg representation. We follow the literature and set $\varepsilon = 6$ and $\kappa = 100$ (see Bilbiie and Ragot, 2017 for a discussion and references). As will be clear below, the quantitative results do not crucially depend on the specific choice of these parameters.

Table 1 provides a summary of the model parameters.

Parameter	Description	Value
	Preference and technology	
β	Discount factor	0.99
α	Capital share	0.36
δ	Depreciation rate	0.025
$\bar{a}$	Credit limit	0
χ	Scaling param. labor supply	0.068
φ	Frisch elasticity labor supply	0.5
	Shock process	
ρ_z	Autocorrelation TFP	0.95
σ_z	Standard deviation TFP shock	0.31%
ρ_y	Autocorrelation idio. income	0.99
σ_y	Standard dev. idio. income	14%
	Tax system	
τ^K	Capital tax	36%
τ^L	Labor tax	28%
T	Transfer over GDP	8%
B/Y	Public debt over yearly GDP	60%
G/Y	Public spending over yearly GDP	12.4%
	Monetary parameters	
κ	Price adjustment cost	100
ε	Elasticity of sub.	6

Table 1: Parameter values in the baseline calibration. See text for descriptions and targets.

5.2 Steady-state equilibrium distribution

We first simulate a Bewley model (i.e., without aggregate shocks). In Table 2, we report the wealth distribution generated by the model and compare it to the empirical distribution. We compute a number of standard statistics – listed in the first column – including the quartiles, the Gini coefficient, and 95-100 intercentiles.

	Data		Model
Wealth statistics	PSID, 06	SCF, 07	
Q1	-0.9	-0.2	0.0
Q2	0.8	1.2	0.1
Q3	4.4	4.6	3.5
Q4	13.0	11.9	15.1
Q5	82.7	82.5	81.3
Top 5%	36.5	36.4	37.8
Top 1%	30.9	33.5	10.7
Gini	0.77	0.78	0.77

Table 2: Wealth distribution in the data and in the model.

The empirical wealth distribution, reported in the second and third columns of Table 2, is computed using two sources, the PSID for the year 2006 and the SCF for the year 2007. The fourth column reports the wealth distribution generated by our model. Overall, the distribution of wealth generated by the model is quite similar for the two replacement rate values and is close to the data. In particular, the model does a good job in matching the wealth distribution with a high Gini of 0.77. The concentration of wealth at the top 1% of the distribution is higher in the data than in the model. It is known that additional model features must be introduced to match the high wealth inequality in the US, such as heterogeneous discount rates, as in Krusell and Smith (1998), or entrepreneurship, as in Quadrini (1999).

5.3 Truncated model

We now construct the truncated model. We use a truncation length of $N = 5$. This implies that we consider $5^5 = 3125$ different truncated histories.

Computing the ξs. For each history, we compute $(\xi_{y^N})_{y^N}$ such the truncated allocations are equal to the truncation of allocations in the Bewley model (see Section 4 for further details). More precisely, we compute the policy functions in the Bewley model and integrate them to deduce the allocations for truncated histories (savings (a_{y^N}), consumption (c_{y^N}) and labor supply (l_{y^N})). We then inverse the projected Euler equations (51) to compute the (ξ_{y^N}) that are compatible with the truncated allocations. As we show in Appendix, this computation pins down to simple matrix algebra.

We constrain the average value of (ξ_{y^N}) to be equal to 1 and our computation implies that theirs standard deviation across histories is $std\left(\xi_{y^N}\right) = 0.13$.

Estimating Pareto weights. We estimate the value of Pareto weights, such that the first-order conditions of the planner are fulfilled at the steady-state for the actual US tax system, characterized in Section 5.1. However, the problem is underidentified and there is no uniqueness of such Pareto weights. To circumvent this difficulty, we choose among the admissible weights, that minimize the distance to the equi-weighted case (where every truncated history has the same weight). Formally, the weights solve $\min_{(\omega_{y^N})} ||(\omega_{y^N})_{y^N} - (1/Y^N)_{y^N}||_2$ subject to $\sum_{y^N} \omega_{y^N} = 1$ and such that planner's first-order conditions hold.[6]

Figure 1 plots the value of the estimated Pareto weights as a function of the current productivity level. Productivity levels stem from the Rouwenhorst discretization of the income process. For each of the five productivity levels, there are also 5^4 different Pareto weights, due to the truncation with $N = 5$. Figure 1 exactly reports the average Pareto weight (over the 5^4 histories as a function of the current productivity level. Note that weights are increasing with productivity. For agents with the lowest productivity level, the weights are equal to 0.6, while they more than twice larger for agents with the highest productivity level. We do not report it here, but we find a similar relationship between Pareto weight and current weight: The higher the wealth, the higher the weight.

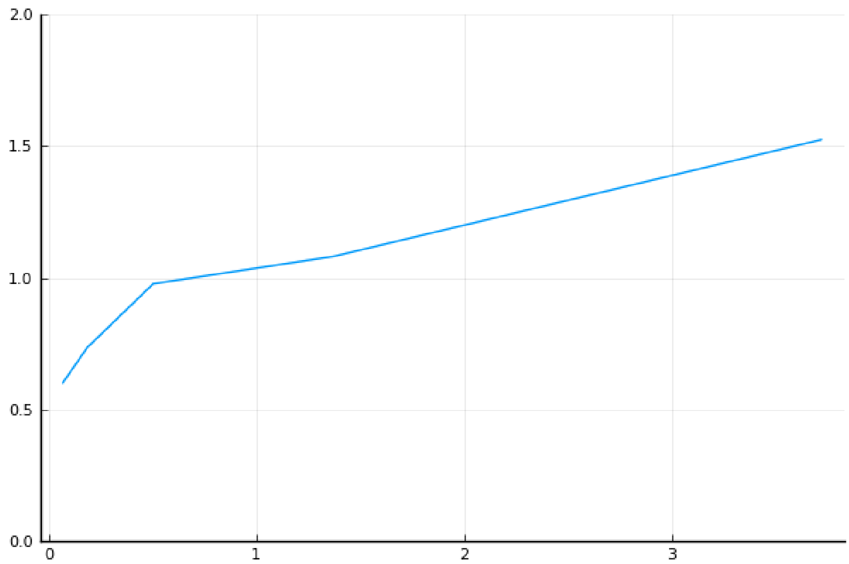

Figure 1: Estimated weights as a function of productivity.

The computation of Pareto weights concludes the calibration of the projected model, that is by construction consistent with the underlying Bewley allocations and with the optimality of the actual fiscal system.

[6]In the previous expression, $||\cdot||_2$ denotes the Euclidean norm, $(\omega_{y^N})_{y^N}$ is the vector of Pareto weights, and $(1/Y^N)_{y^N}$ the vector corresponding to equal weighting.

5.4 The complete-market economy benchmark

We solve for the allocation in the complete market economy (henceforth, CM for complete-market and IM for incomplete market) taht will be used as a benchmark. In the complete market case (or truncation with $N = 1$), the economy is represented by a representative agent, a family head, who decides the aggregate allocation. The family head then allocates the aggregate consumption across family members according to Pareto weights. Importantly these weights have no effect on the dynamics of the economy. This environment is studied in a vast literature (Chari and Kehoe, 1999; Aiyagari et al., 2002; Farhi, 2010; Bhandari et al., 2017a, among others). Our calibration of the CM economy relies on the same parameters as those of Table 1, with one exception. The parameter χ is set such that the labor supply is 1/3, as in the IM economy. Otherwise the steady-state zero labor tax in CM economy modifies labor supply. Another consequence of the recalibrated χ is that the steady-state GDP is the same in both the CM and IM economies, which ease the comparison between the two economies.

The main feature of the steady-state allocation in the CM economy is that the government ends up holding a negative debt (i.e., it holds a part of the capital stock) to finance public consumption out of interest payment, and thereby avoids the costs implied by distortionary taxation. At the steady state, taxes on capital and labor are null: $\tau^L = \tau^K = 0$, and the steady state allocation is the first-best allocation. This economy is only of interest only for the variations of the policy instruments along the business cycle.

5.5 Dynamic of the fiscal system with complete set of instruments

We now present the dynamics of the fiscal system after a technology shock in the IM economy. We solve the model when the planner has the full set of fiscal instruments (τ^K, τ^L, T, and B). We proved in Section 3 that inflation does not play any role in this case. To understand the effect of heterogeneity, we compare the IRFs in the IM economy with those of the CM economy. Results are plotted in Figure 2. Each panel of Figure 2 reports the proportional change for the relevant variable, in percentage points, except for tax rates, for which teh absolute variation is reported. For instance, Panel 1 reports a persistent fall in TFP for 100 periods, after a fall of 1% on impact.

Overall, the comparison of the dynamics of aggregate quantities (Consumption, Panel 2; Capital, Panel 3; GDP, Panel 4) shows that the two economies exhibit very similar behavior along those dimensions. The main difference between the two economies concern fiscal instruments. The capital tax is very volatile in the CM economy, which is a standard result in this literature. The planner uses the capital tax to front-load all adjustments, such that the public debt jumps on a path consistent with zero tax on both capital and labor.[7]As the public debt is negative, the decrease in public debt means that the planner actually further accumulates assets to pay for

[7]

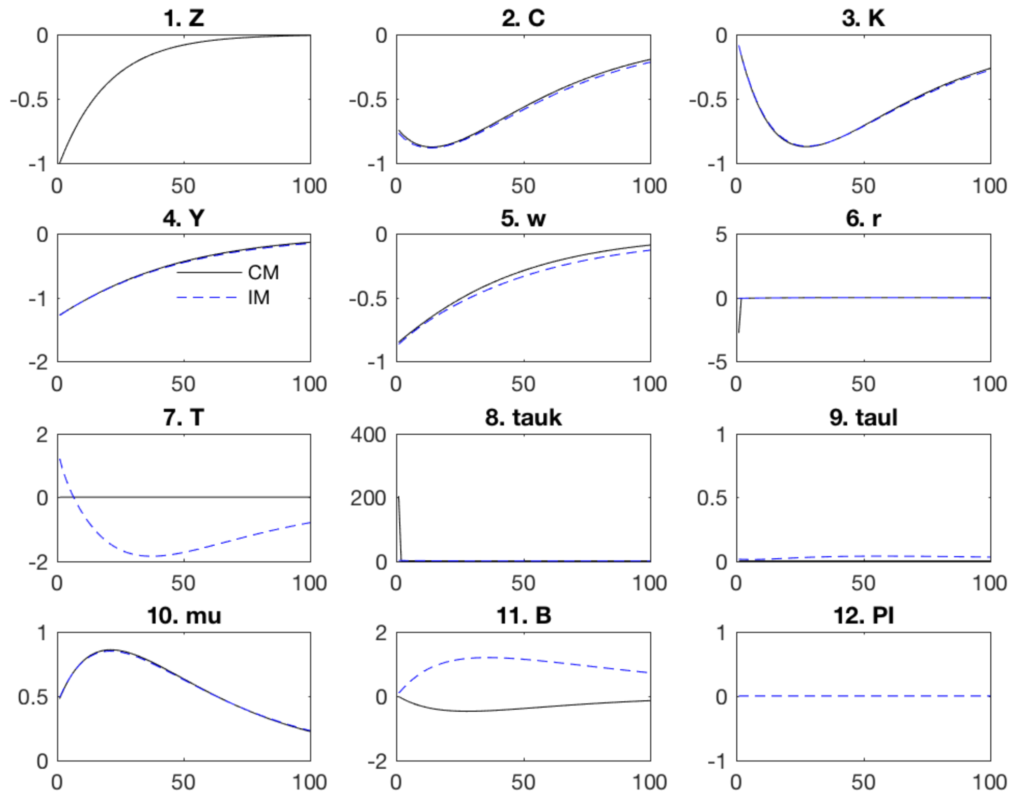

Figure 2: Comparison between the complete-market economy (black solid line) and the incomplete-market economy with all instruments (blue dashed line).

public spendings, because it needs to compensate for lower governmental revenues (due not the negative shock). This very high capital tax volatility is known to be an unappealing feature of the CM economy.

In the IM economy, the fiscal policy differs sharply. First, the capital tax is much less volatile. It is actually 100 times less volatile. This result shows that incomplete markets contribute to solve the capital tax volatility puzzle. Indeed, due to precautionary saving motive, it is very costly for the planner to raise very sharply the capital tax and thereby wipe out agents' savings. A consequence of this very moderate increase in the capital tax is that public debt increases (and public debt is actual debt in the IM economy) to smooth out the impact of the negative TFP shock on public finance. The governmental budget adjustment is actually performed by a moderate increase in the labor tax (around 0.1%) for quite a long period. Finally, inflation is constant over the dynamics, as expected.

From this experiment, we conclude that; compared to CM, the IM economy implies a sharp reduction in the volatility of the capital tax, as well as a countercyclical public debt.

5.6 Dynamics of the fiscal system without time-varying capital tax

We now turn to the dynamics of the IM economy, when the capital tax is not time-varying and is set to its steady-state value. IRFs are reported in Figure 3 and compared to those of the IM economy with the full set of instruments. As in Figure 2, the dynamics of aggregate quantities

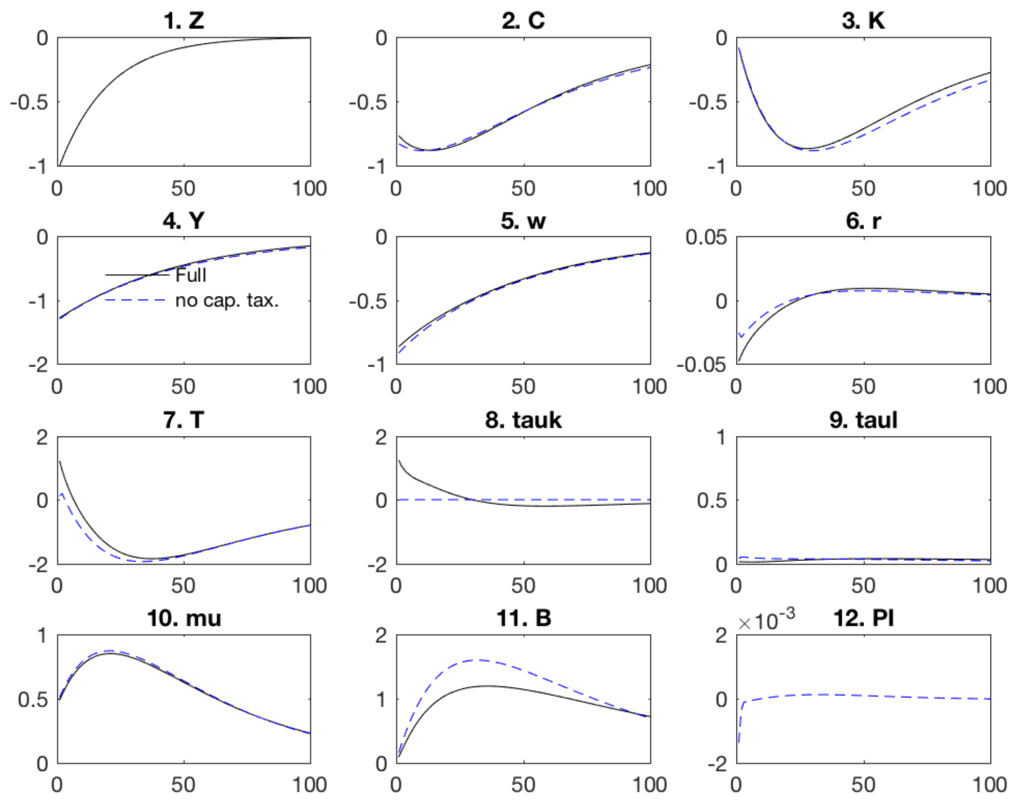

Figure 3: Comparison between the IM economy with all instruments (black solid line) and the IM economy with a fixed capital tax (blue dashed line).

(aggregate consumption, capital stock and GDP, in panels 2, 3 and 4, respectively) are a very similar pattern in the two economies. Keeping a constant capital tax generates a public debt, that is more countercylical and slightly higher labor tax. Indeed, due to the distortions implied by the labor tax, its increase remains very limited and smoothed out through time, which is permitted by a higher public debt. Regarding inflation (Panel 12), it can be observed that it barely moves, even when the capital tax is constant. Although inflation could be a partial substitute to the missing capital tax, the implied distortions (on savings and output destruction) are too high for the planner to actively rely on inflation.

		CM	Full	No cap.tax
C	Mean	0.7543	0.7542	0.7542
	Std	0.0259	0.0266	0.0269
K	Mean	11.0557	11.0536	11.0535
	Std	0.0268	0.0270	0.0288
Y	Mean	1.1760	1.1759	1.1759
	Std	0.0264	0.0268	0.0274
L	Mean	0.3334	0.3334	0.3334
	Std	0.0088	0.0094	0.0098
τ^K	Mean	0.0009	0.3600	0.3600
	Std	0.8855	0.0145	0.0000
τ^L	Mean	0.0000	0.2800	0.2800
	Std	0.0000	0.0016	0.0015
B	Mean	-10.9327	2.8435	2.8424
	Std	0.0146	0.0462	0.0541
T	Mean	0.0000	0.0941	0.0941
	Std	0.0000	0.0610	0.0637
π	Mean	0.0000	0.0000	0.0000
	Std	0.0000	0.0000	0.0007
Correlations				
$corr(\tau^K, Y)$		-0.2085	-0.4868	0.0000
$corr(\tau^L, Y)$		0.9273	-0.6374	-0.9249
$corr(B, Y)$		-0.8349	-0.7592	-0.8291
$corr(T, Y)$		0.0000	0.6523	0.7796
$corr(C, Y)$		0.9673	0.9691	0.9755
$corr(Y, Y_{-1})$		0.9776	0.9781	0.9785
$corr(B, B_{-1})$		0.9992	0.9996	0.9994

Table 3: First- and second-order moments for key variables, in the three economies (CM: complete market; Full: IM with full fiscal set; No cap. tax: IM with fixed capital tax). See text for details.

5.7 Second-order moments

We report in Table 3 the unconditional first- and second-order moments for several key variables, in the three economies (CM, IM with a full set of instruments and IM with a fixed capital tax). For each variable, reported in the first column, we report the steady-state value (mean) and the normalized standard deviation (standard deviation divided by the mean), except for taxes for which the standard deviation is reported. The second part of the table reports correlations.

First, we can observe that Table 3 confirms the IRFs regarding aggregate variables. Their mean and standard deviations are very close in the three economies. Second, the main difference, as for IRFs again, lies in the volatility of taxes and of the behavior of public debt that significantly differs among the three economies. The volatility of capital tax decreases in the two IM economies, compared to CM. The labor tax volatility remains low in the three economies, even though it becomes more countercyclical in the absence of capital tax. When capital tax is not available, the public debt becomes more volatile and more countercyclical (as can be seen with the correlation with output).

6 Conclusion

We derive optimal fiscal-monetary policy in an economy with incomplete insurance markets, nominal frictions, and aggregate shocks. We find that market-incompleteness considerably reduces the volatility of capital tax, but increases the counter-cyclicality of public debt after technology shocks. We find that at the optimum, monetary policy has little role for redistribution, even when the capital tax remains fixed along the business cycle. Although monetary policy could be a partial substitute for capital tax, the planner chooses to not use it.

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Appendix

A Ramsey program

We provide below a detailed expression of the Ramsey program.

$$\max_{\left(w_t, r_t, \tilde{w}_t, \tilde{r}_t^K, \tilde{R}_t^{B,N}, \tau_t^K, \tau_t^L, B_t, T_t, K_t, L_t, \Pi_t, (a_t^i, c_t^i, l_t^i)_i\right)_{t\geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega^i U(c_t^i, l_t^i)\ell(di)\right],$$

subject to individual constraints (for all $i \in \mathcal{I}$):

$$\begin{aligned}
a_t^i + c_t^i &= (1 + r_t)a_{t-1}^i + w_t y_t^i l_t^i + T_t, \\
a_t^i &\geq -\bar{a}, \\
U_c(c_t^i, l_t^i) &= \beta \mathbb{E}_t \left[U_c(c_{t+1}^i, l_{t+1}^i)(1 + r_{t+1})\right] + \nu_t^i, \\
\nu_t^i(a_t^i + \bar{a}) &= 0 \text{ and } \nu_t^i \geq 0, \\
l_t^{i,1/\varphi} &= \chi w_t y_t^i, \\
c_t^i,\, l_t^i \geq 0,\, a_t^i &\geq -\bar{a},
\end{aligned}$$

subject to aggregate constraints:

$$\begin{aligned}
G_t + B_{t-1} &= B_t + \left(1 - \frac{\kappa}{2}\pi_t^2\right) Y_t - \delta K_{t-1}, \\
\Pi_t(\Pi_t - 1) &= \frac{\varepsilon - 1}{\kappa} (\zeta_t - 1) + \beta \mathbb{E}_t \Pi_{t+1}(\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \frac{M_{t+1}}{M_t}, \\
K_t + B_t &= \int_i a_t^i \ell(di),\ L_t = \int_i y_t^i l_t^i \ell\,(di)\,,\ Y_t = Z_t K_{t-1}^{\alpha} L_t^{1-\alpha},
\end{aligned}$$

subject to interest rate definitions:

$$\begin{aligned}
r_t &= (1 - \tau_t^K) \frac{\tilde{r}_t^K K_{t-1} + \left(\frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t} - 1\right) B_{t-1}}{A_{t-1}}, \\
w_t &= (1 - \tau_t^L)\tilde{w}_t, \\
\mathbb{E}_t \left[\frac{\tilde{R}_t^{B,N}}{\Pi_{t+1}}\right] &= \mathbb{E}_t \left[1 + \tilde{r}_{t+1}^K\right], \\
\zeta_t &= \frac{1}{Z_t} \left(\frac{\tilde{r}_t^K + \delta}{\alpha}\right)^{\alpha} \left(\frac{\tilde{w}_t}{1 - \alpha}\right)^{1-\alpha}, \\
\frac{K_{t-1}}{L_t} &= \frac{\alpha}{1 - \alpha} \frac{\tilde{w}_t}{\tilde{r}_t^K + \delta},
\end{aligned}$$

and finally subject to given initial conditions.

B Transforming the Ramsey program

Denote $\beta^t \omega_t^i \lambda_t^i$ the Lagrange multiplier of the Euler equation for agent i at date t. Denote $\beta^t \gamma_t$ the Lagrange multiplier of the Phillips curve at date t.

The objective of the Ramsey program can be rewritten as:

$$
\begin{aligned}
J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i U(c_t^i, l_t^i) \ell(di) \\
- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \lambda_t^i \omega_t^i \left(U_c(c_t^i, l_t^i) - \nu_t^i - \beta \mathbb{E}_t \left[U_c(c_{t+1}^i, l_{t+1}^i)(1 + r_{t+1}) \right] \right) \ell(di) \\
- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_t \left(\Pi_t (\Pi_t - 1) Y_t M_t - \frac{\varepsilon - 1}{\kappa} (\zeta_t - 1) Y_t M_t - \beta \mathbb{E}_t [\Pi_{t+1} (\Pi_{t+1} - 1) Y_{t+1} M_{t+1}] \right).
\end{aligned}
$$

With $\lambda_t^i \nu_t^i = 0$, we obtain after some manipulations the following expression for the objective of the Ramsey program:

$$
\begin{aligned}
J = & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i U(c_t^i, l_t^i) \ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \lambda_t^i \omega_t^i U_c(c_t^i, l_t^i) \ell(di) + \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t (1 + r_t) \int_i \lambda_{t-1}^i \omega_{t-1}^i U_c(c_t^i, l_t^i) \ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_t (\Pi_t (\Pi_t - 1) Y_t M_t) \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^{t+1} \gamma_t [\Pi_{t+1} (\Pi_{t+1} - 1) Y_{t+1} M_{t+1}] \\
& + \frac{\varepsilon - 1}{\kappa} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_t (\zeta_t - 1) Y_t M_t.
\end{aligned}
$$

or using $\gamma_{-1} = 0$:

$$
\begin{aligned}
J = & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i U(c_t^i, l_t^i) \ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \lambda_t^i \omega_t^i U_c(c_t^i, l_t^i) \ell(di) + \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t (1 + r_t) \int_i \lambda_{t-1}^i \omega_{t-1}^i U_c(c_t^i, l_t^i) \ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_t \Pi_t (\Pi_t - 1) Y_t M_t \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{t-1} \Pi_t (\Pi_t - 1) Y_t M_t \\
& + \frac{\varepsilon - 1}{\kappa} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_t (\zeta_t - 1) Y_t M_t.
\end{aligned}
$$

We finally obtain:

$$J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i U(c_t^i, l_t^i) \ell(di) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \left(\omega_t^i \lambda_t^i - (1+r_t) \lambda_{t-1}^i \omega_{t-1}^i \right) U_c(c_t^i, l_t^i) \ell(di) \tag{58}$$
$$- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left((\gamma_t - \gamma_{t-1}) \Pi_t (\Pi_t - 1) - \frac{\varepsilon - 1}{\kappa} \gamma_t (\zeta_t - 1) \right) Y_t M_t.$$

C First-order conditions for the real economy

The planner's program becomes $\max_{\left(w_t, r_t, B_t, K_t, L_t, T_t, (a_t^i, c_t^i, l_t^i)_i\right)} J$ with $\gamma_t = 0$ and $\Pi_t = 1$ for all t (because of a real economy), subject to:

$$\begin{aligned}
l_t^i &= \left(\chi w_t y_t^i \right)^{\varphi}, \\
c_t^i &= \left(\chi w_t y_t^i \right)^{\varphi+1} + (1+r_t) a_{t-1}^i - a_t^i, \\
B_t + Y_t - \delta K_{t-1} &= G_t + B_{t-1} + r_t (B_{t-1} + K_{t-1}) + w_t L_t + T_t, \\
L_t &= \int_i y_t^i l_t^i \ell(di), \\
Y_t &= Z_t \left(\int_i a_{t-1}^i \ell(di) - B_{t-1} \right)^{\alpha} L_t^{1-\alpha}.
\end{aligned}$$

We denote by $\beta^t \mu_t$ the Lagrange multiplier on the governmental budget constraint.

Derivative with respect to w_t: the labor tax. We have:

$$\begin{aligned}
0 = &\int_i \omega_t^i \left(U_c(c_t^i, l_t^i) \frac{\partial c_t^i}{\partial w_t} + U_l(c_t^i, l_t^i) \frac{\partial l_t^i}{\partial w_t} \right) \ell(di) \\
&- \int_i \left(\lambda_t^i - (1+r_t) \lambda_{t-1}^i \right) \left(U_{cc}(c_t^i, l_t^i) \frac{\partial c_t^i}{\partial w_t} + U_{cl}(c_t^i, l_t^i) \frac{\partial l_t^i}{\partial w_t} \right) \ell(di) \\
&+ \mu_t \left(\frac{\partial Y_t}{\partial w_t} - L_t - w_t \frac{\partial L_t}{\partial w_t} \right).
\end{aligned}$$

We have $U_l(c_t^i, l_t^i) = -w_t y_t^i U_{c,t}^i$and $U_c(c_t^i, l_t^i) \frac{\partial c_t^i}{\partial w_t} + U_l(c_t^i, l_t^i) \frac{\partial l_t^i}{\partial w_t} = U_{c,t}^i \left(\frac{\partial c_t^i}{\partial w_t} - w_t y_t^i \frac{\partial l_t^i}{\partial w_t} \right) = U_{c,t}^i w_t^{\varphi} (y_t^i)^{\varphi+1}$. Similarly, $U_{cc}(c_t^i, l_t^i) \frac{\partial c_t^i}{\partial w_t} + U_{cl}(c_t^i, l_t^i) \frac{\partial l_t^i}{\partial w_t} = U_{cc,t}^i w_t^{\varphi} (y_t^i)^{\varphi+1}$. Furthermore:

$$\begin{aligned}
\frac{\partial L_t}{\partial w_t} &= \int_i y_t^i \frac{\partial l_t^i}{\partial w_t} \ell(di) = \varphi \int_i y_t^i \frac{l_t^i}{w_t} \ell(di) = \frac{\varphi}{w_t} L_t, \\
\frac{\partial Y_t}{\partial w_t} &= (1-\alpha) \frac{\varphi}{w_t} Y_t.
\end{aligned}$$

Using the definition of $\hat{\psi}^i$ in (34), we obtain:

$$\mu_t \varphi \left(L_t - (1-\alpha)\frac{Y_t}{w_t} \right) = \int_i \hat{\psi}_t^i y_t^i l_t^i \ell(di).$$

Derivative with respect to r_t: the capital tax. Using $\frac{\partial c_t^i}{\partial r_t} = a_{t-1}^i$ and $\frac{\partial l_t^i}{\partial r_t} = 0$, we have:

$$\begin{aligned} 0 = & \int_i \left(\omega_t^i U_{c,t}^i - \left(\lambda_t^i - (1+r_t)\lambda_{t-1}^i \right) U_{cc,t}^i \right) a_{t-1}^i \ell(di) \\ & + \int_i \lambda_{t-1}^i U_c(c_t^i, l_t^i) \ell(di) - \mu_t (K_{t-1} + B_{t-1}). \end{aligned}$$

We therefore obtain:

$$\int_i \lambda_{t-1}^i U_c(c_t^i, l_t^i) \ell(di) + \int_i \hat{\psi}_t^i a_{t-1}^i \ell(di) = 0. \tag{59}$$

Derivative with respect to B_t: the public debt. We obtain:

$$0 = \mu_t - \beta \mathbb{E}_t \left[\mu_{t+1}(1-\delta) \right] + \beta \mathbb{E}_t \mu_{t+1} \frac{\partial Y_{t+1}}{\partial B_t}. \tag{60}$$

Note that from (3), we have: $\frac{K_{t-1}}{L_t} = \frac{\alpha}{1-\alpha} \frac{\tilde{w}_t}{\tilde{r}_t^K + \delta}$ and $1 = \frac{1}{Z_t} \left(\frac{\tilde{r}_t^K + \delta}{\alpha} \right)^{\alpha} \left(\frac{\tilde{w}_t}{1-\alpha} \right)^{1-\alpha}$

$$\begin{aligned} \frac{\partial Y_{t+1}}{\partial B_t} & = -\alpha Z_{t+1} \left(\frac{L_{t+1}}{K_t} \right)^{1-\alpha}, \\ & = -\alpha \left(\frac{\tilde{r}_{t+1}^K + \delta}{\alpha} \right)^{\alpha} \left(\frac{\tilde{w}_{t+1}}{1-\alpha} \right)^{1-\alpha} \left(\frac{\tilde{r}_{t+1}^K + \delta}{\alpha} \right)^{1-\alpha} \left(\frac{\tilde{w}_{t+1}}{1-\alpha} \right)^{-1+\alpha}, \\ & = -(\tilde{r}_{t+1}^K + \delta). \end{aligned}$$

and we obtain $0 = \mu_t - \beta \mathbb{E}_t \left[\mu_{t+1}(1 - \delta + \tilde{r}_{t+1}^K + \delta) \right]$, or:

$$\mu_t = \beta \mathbb{E}_t \left[\mu_{t+1} \left(1 + \tilde{r}_{t+1}^K \right) \right]. \tag{61}$$

Derivative with respect to a_t^i: the net saving of consumers. We have, since $\frac{\partial l_t^i}{\partial a_t^i} = 0$:

$$\begin{aligned} 0 = & \int_i \omega_t^i \left(U_c(c_t^i, l_t^i) \frac{\partial c_t^i}{\partial a_t^i} \right) \ell(di) - \int_i \left(\lambda_t^i - (1+r_t)\lambda_{t-1}^i \right) \left(U_{cc}(c_t^i, l_t^i) \frac{\partial c_t^i}{\partial a_t^i} \right) \ell(di) \\ & + \beta \mathbb{E}_t \int_i \omega_t^i U_c(c_{t+1}^i, l_{t+1}^i) \frac{\partial c_{t+1}^i}{\partial a_t^i} \ell(di) - \int_i \left(\lambda_t^i - (1+r_t)\lambda_{t-1}^i \right) U_{cc}(c_{t+1}^i, l_{t+1}^i) \frac{\partial c_{t+1}^i}{\partial a_t^i} \ell(di) \\ & + \beta \mathbb{E}_t \left[\mu_{t+1} \left(\frac{\partial Y_{t+1}}{\partial a_t^i} - r_{t+1} - \delta \right) \right]. \end{aligned}$$

This yields using $\frac{\partial c_t^i}{\partial a_t^i} = -1$, $\frac{\partial c_{t+1}^i}{\partial a_t^i} = 1 + r_{t+1}$, and $\frac{\partial Y_{t+1}}{\partial a_t^i} = \tilde{r}_{t+1}^K + \delta$:

$$\psi_t^i = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \psi_{t+1}^i \right] + \beta \mathbb{E}_t \left[\mu_{t+1} \left(\tilde{r}_{t+1}^K - r_{t+1} \right) \right].$$

Observing that $\mu_t = \beta \mathbb{E}_t \left[\mu_{t+1} \left(1 + \tilde{r}_{t+1}^K \right) \right]$ (equation (61)), we obtain by difference using the notation $\hat{\psi}^i$:

$$\hat{\psi}_t^i = \mathbb{E}_t \left[(1 + r_{t+1}) \hat{\psi}_{t+1}^i \right]. \tag{62}$$

Derivative wrt T_t.

$$\int_i \hat{\psi}_t^i \ell(di) = 0. \tag{63}$$

D First-order conditions for the economy with fixed capital taxes

The planner's objective is:

$$\begin{aligned}
J = {} & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i U(c_t^i, l_t^i) \ell(di) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \left(\lambda_t^i - (1 + r_t) \lambda_{t-1}^i \right) U_c(c_t^i, l_t^i) \ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\gamma_t - \gamma_{t-1} \right) \Pi_t \left(\Pi_t - 1 \right) Y_t \int_i \omega_t^i U_{c,t}^i \ell(di) \\
& + \frac{\varepsilon - 1}{\kappa} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_t \left(\zeta_t - 1 \right) Y_t \int_i \omega_t^i U_{c,t}^i \ell(di) \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left(B_t + \left(1 - \frac{\kappa}{2} \left(\Pi_t - 1 \right)^2 \right) Y_t - \delta \left(\int_i a_{t-1}^i \ell(di) - B_{t-1} \right) \right) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left(G_t + B_{t-1} + r_t \int_i a_{t-1}^i \ell(di) + w_t L_t + T_t \right) \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Gamma_t \left(\left(r_t - \left(1 - \tau_{SS}^K \right) \tilde{r}_t^K \right) A_{t-1} - \left(1 - \tau_{SS}^K \right) \left(\frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t} - 1 - \tilde{r}_t^K \right) B_{t-1} \right) \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Upsilon_{t-1} \left(\frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t} - (1 + \tilde{r}_t^K) \right).
\end{aligned}$$

Derivative with respect to $\tilde{R}_t^{B,N}$.

$$\left(1 - \tau^K \right) \mathbb{E}_t \left[\frac{\Gamma_{t+1}}{\Pi_{t+1}} \right] B_t = \Upsilon_t \mathbb{E}_t \left[\frac{1}{\Pi_{t+1}} \right]. \tag{64}$$

Derivative with respect to $\tilde{r}_t^K$. We have $\zeta_t = \frac{1}{\alpha Z_t}\left(\tilde{r}_t^K + \delta\right)\left(\frac{\int_i a_{t-1}^i \ell(di) - B_{t-1}}{L_t}\right)^{1-\alpha}$ and:

$$Y_t \frac{\partial \zeta_t}{\partial \tilde{r}_t^K} = \frac{1}{\alpha Z_t}\left(\frac{K_{t-1}}{L_t}\right)^{1-\alpha} Z_t K_t^{\alpha} L_t^{1-\alpha} = \frac{K_{t-1}}{\alpha}.$$

We obtain:

$$\Upsilon_{t-1} + \Gamma_t(1-\tau_{SS}^K)\left(A_{t-1} - B_{t-1}\right) = \frac{\varepsilon - 1}{\alpha\kappa}\gamma_t K_{t-1} M_t.$$

Derivative with respect to Π_t.

$$\begin{aligned} 0 = {} & \mu_t \kappa \left(\Pi_t - 1\right) + \left(\gamma_t - \gamma_{t-1}\right)\left(2\Pi_t - 1\right) M_t \\ & + \left(\Gamma_t \left(1 - \tau^K\right) B_{t-1} - \Upsilon_{t-1}\right) \frac{\tilde{R}_{t-1}^{B,N}}{Y_t \Pi_t^2}. \end{aligned} \tag{65}$$

At the steady state, $\Pi^{SS} = 1$ (indeed, note that the last term is zero because of (64)).

Derivative with respect to r_t. We obtain:

$$\begin{aligned} \mu_t \int_i \omega_t^i a_{t-1}^i \ell(di) = {} & \int_i \psi_t^i a_{t-1}^i \ell(di) + \int_i \lambda_{t-1}^i \omega_{t-1}^i U_c(c_t^i, l_t^i)\ell(di) && (66) \\ & - \left(\left(\gamma_t - \gamma_{t-1}\right)\Pi_t\left(\Pi_t - 1\right) - \frac{\varepsilon - 1}{\kappa}\gamma_t\left(\zeta_t - 1\right)\right) Y_t \int_i \omega_t^i U_{cc,t}^i a_{t-1}^i \ell(di) \\ & + \Gamma_t A_{t-1}, && (67) \end{aligned}$$

or

$$0 = \int_i \hat{\psi}_t^i a_{t-1}^i \ell(di) + \int_i \lambda_{t-1}^i \omega_{t-1}^i U_{c,t}^i \ell(di) + \Gamma_t A_{t-1}. \tag{68}$$

Derivative with respect to w_t. Recall that:

$$\begin{aligned} \frac{\partial L_t}{\partial w_t} &= \int_i y_t^i \frac{\partial l_t^i}{\partial w_t}\ell(di) = \varphi \int_i y_t^i \frac{l_t^i}{w_t}\ell(di) = \frac{\varphi}{w_t} L_t, \\ \frac{\partial Y_t}{\partial w_t} &= (1-\alpha)\frac{\varphi}{w_t} Y_t, \\ U_{cc}(c_t^i, l_t^i)\frac{\partial c_t^i}{\partial w_t} + U_{cl}(c_t^i, l_t^i)\frac{\partial l_t^i}{\partial w_t} &= U_{cc,t}^i y_t^i l_t^i. \end{aligned}$$

Furthermore, note that:

$$\begin{aligned}
\zeta_t &= \frac{1}{\alpha Z_t}\left(\tilde{r}_t^K + \delta\right)\left(\frac{\int_i a_{t-1}^i \ell(di) - B_{t-1}}{L_t}\right)^{1-\alpha} \\
\frac{\partial \zeta_t}{\partial w_t} &= \frac{1}{\alpha Z_t}\left(\tilde{r}_t^K + \delta\right)\left(\frac{\int_i a_{t-1}^i \ell(di) - B_{t-1}}{L_t}\right)^{1-\alpha} \frac{-(1-\alpha)}{L_t}\frac{\varphi}{w_t}L_t \\
&= -(1-\alpha)\zeta_t\frac{\varphi}{w_t} \\
Y_t\frac{\partial \zeta_t}{\partial w_t} &= -\varphi\frac{1-\alpha}{w_t}Y_t\zeta_t
\end{aligned}$$

We deduce:

$$\begin{aligned}
&\mu_t\left((1+\varphi)L_t - (1-\alpha)\varphi\frac{Y_t}{w_t}\left(1-\frac{\kappa}{2}\left(\Pi_t-1\right)^2\right)\right) = \int_i \psi_t^i y_t^i l_t^i \ell(di), \\
&-\left(\left(\gamma_t - \gamma_{t-1}\right)\Pi_t\left(\Pi_t - 1\right) - \frac{\varepsilon-1}{\kappa}\gamma_t\left(\zeta_t - 1\right)\right)Y_t\int_i \omega_t^i U_{cc,t}^i y_t^i l_t^i \ell(di) \\
&-(1-\alpha)\varphi\left(\left(\gamma_t - \gamma_{t-1}\right)\Pi_t\left(\Pi_t - 1\right) - \frac{\varepsilon-1}{\kappa}\gamma_t\left(\zeta_t - 1\right)\right)\frac{Y_t}{w_t}\int_i \omega_t^i U_{c,t}^i \ell(di) \\
&-\frac{\varepsilon-1}{\kappa}(1-\alpha)\varphi\gamma_t\zeta_t\frac{Y_t}{w_t}\int_i \omega_t^i U_{c,t}^i \ell(di),
\end{aligned}$$

or using $\tilde{\psi}$:

$$\begin{aligned}
&\mu_t\varphi\left(L_t - (1-\alpha)\frac{Y_t}{w_t}\left(1-\frac{\kappa}{2}\left(\Pi_t-1\right)^2\right)\right) = \int_i (\tilde{\psi}_t^i - \mu_t) y_t^i l_t^i \ell(di), \\
&-\frac{(1-\alpha)\varphi}{w_t}\left(\left(\gamma_t - \gamma_{t-1}\right)\Pi_t\left(\Pi_t - 1\right) + \frac{\varepsilon-1}{\kappa}\gamma_t\right)Y_t\int_i \omega_t^i U_{c,t}^i \ell(di).
\end{aligned} \tag{69}$$

Derivative with respect to B_t: the public debt. Note that we have:

$$\frac{\partial Y_{t+1}}{\partial B_t} = -\alpha Z_{t+1}\left(\int_i a_t^i \ell(di) - B_t\right)^{\alpha-1} L_{t+1}^{1-\alpha} = -\alpha\frac{Y_{t+1}}{K_t} = -\zeta_{t+1}^{-1}(\tilde{r}_{t+1}^K + \delta).$$

We also have:

$$\zeta_t = \frac{1}{\alpha Z_t}\left(\tilde{r}_{t+1}^K + \delta\right)\left(\frac{\int_i a_{t-1}^i \ell(di) - B_{t-1}}{L_t}\right)^{1-\alpha} = \frac{K_{t-1}}{\alpha Y_t}\left(\tilde{r}_{t+1}^K + \delta\right),$$

and

$$\begin{aligned}
\frac{\partial \zeta_{t+1}}{\partial B_t} &= -(1-\alpha)\frac{\zeta_{t+1}}{K_t}, \\
Y_{t+1}\frac{\partial \zeta_{t+1}}{\partial B_t} &= -(1-\alpha)\zeta_{t+1}\frac{Y_{t+1}}{K_t} = -\frac{1-\alpha}{\alpha}(\tilde{r}_{t+1}^K + \delta).
\end{aligned}$$

We obtain:

$$\mu_t = \beta\mathbb{E}_t\left[\mu_{t+1}\left(1-\delta+\zeta_{t+1}^{-1}(\tilde{r}_{t+1}^K+\delta)\left(1-\frac{\kappa}{2}\left(\Pi_{t+1}-1\right)^2\right)\right)\right] \tag{70}$$
$$-\alpha\beta\mathbb{E}_t\left[\left(\left(\gamma_{t+1}-\gamma_t\right)\Pi_{t+1}\left(\Pi_{t+1}-1\right)+\frac{\varepsilon-1}{\kappa}\gamma_{t+1}\right)\frac{Y_{t+1}}{K_t}M_{t+1}\right]$$
$$+\frac{\varepsilon-1}{\kappa}\beta\mathbb{E}_t\left[\gamma_{t+1}\frac{\zeta_{t+1}Y_{t+1}}{K_t}M_{t+1}\right]$$
$$+\beta\left(1-\tau^K\right)\mathbb{E}_t\left[\Gamma_{t+1}\left(\frac{\tilde{R}_t^{B,N}}{\Pi_{t+1}}-1-\tilde{r}_{t+1}^K\right)\right].$$

Derivative with respect to a_t^i: the net saving of consumers . We have, using $\frac{\partial c_t^i}{\partial a_t^i}=-1$, $\frac{\partial c_{t+1}^i}{\partial a_t^i}=1+r_{t+1}$, $\frac{\partial Y_{t+1}}{\partial a_t^i}=\alpha\frac{Y_{t+1}}{K_t}=\zeta_{t+1}^{-1}(\tilde{r}_{t+1}^K+\delta)$, and $\frac{\partial \zeta_{t+1}}{\partial a_t^i}Y_{t+1}=\frac{1-\alpha}{\alpha}(\tilde{r}_{t+1}^K+\delta)$:

$$\tilde{\psi}_t^i = \beta\mathbb{E}_t\left[(1+r_{t+1})(\tilde{\psi}_{t+1}^i-\mu_{t+1})\right]+\beta\mathbb{E}_t\left[\mu_{t+1}\left(1-\delta+\zeta_{t+1}^{-1}(\tilde{r}_{t+1}^K+\delta)\left(1-\frac{\kappa}{2}\left(\Pi_{t+1}-1\right)^2\right)\right)\right]$$
$$-\alpha\beta\mathbb{E}_t\left[\left(\left(\gamma_{t+1}-\gamma_t\right)\Pi_{t+1}\left(\Pi_{t+1}-1\right)-\frac{\varepsilon-1}{\kappa}\gamma_{t+1}\left(\zeta_{t+1}-1\right)\right)\frac{Y_{t+1}}{K_t}M_{t+1}\right]$$
$$+(1-\alpha)\frac{\varepsilon-1}{\kappa}\beta\mathbb{E}_t\left[\gamma_{t+1}\frac{\zeta_{t+1}Y_{t+1}}{K_t}M_{t+1}\right]$$
$$+\beta\mathbb{E}_t\left[\Gamma_{t+1}\left(r_{t+1}-\left(1-\tau^K\right)\tilde{r}_{t+1}^K\right)\right]$$

By difference with (70):

$$\hat{\psi}_t^i = \beta\mathbb{E}_t\left[(1+r_{t+1})\hat{\psi}_{t+1}^i\right]+\beta\mathbb{E}_t\left[\Gamma_{t+1}\left(r_{t+1}-\left(1-\tau^K\right)\left(\frac{\tilde{R}_t^{B,N}}{\Pi_{t+1}}-1\right)\right)\right]. \tag{71}$$

Derivative wrt T_t.

$$\int_i \hat{\psi}_t^i \ell(di) = 0. \tag{72}$$

E Projected model

We derive first-order conditions of the planer for projected economies.

E.1 Program in the economy with full set of fiscal tools

E.1.1 Program formulation

We omit the monetary policy tool $\tilde{R}_t^{B,N}$ as it is redundant. We directly use the equilibrium condition $\pi_t = 0$. The program is the following:

$$\max_{\left((a_{t,y^N},c_{t,y^N},l_{t,y^N})_{y^N\in\mathcal{Y}^N},w_t,r_t,B_t\right)_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{y^N\in\mathcal{Y}} \Bigg[S_{t,y^N} \Bigg(\omega_{y^N}\xi_{y^N} U(c_{t,y^N}, l_{t,y^N}) \\ - \left(\lambda_{t,y^N} - \tilde{\lambda}_{t,y^N}(1+r_t)\right) \xi_{y^N} U_c(c_{t,y^N}, l_{t,y^N}) \Bigg) \Bigg],$$

subject to:

$$G_t + B_{t-1} + r_t\left(B_{t-1} + K_{t-1}\right) + w_t L_t + T_t = B_t + K_{t-1}^{\alpha} L_t^{1-\alpha} - \delta K_{t-1},$$

$$\tilde{\lambda}_{t,y^N} = \frac{\sum_{\tilde{y}^N\in\mathcal{Y}^N} S_{t-1,\tilde{y}^N}\lambda_{t-1,\tilde{y}^N}\Pi_{t,\tilde{y}^N,y^N}}{S_{t,y^N}}, \tag{73}$$

$$c_{t,y^N} + a_{t,y^N} = w_t l_{t,y^N} y_{y^N} + (1+r_t)\,\tilde{a}_{t,y^N} + T_t, \tag{74}$$

$$\tilde{a}_{t,y^N} = \sum_{\tilde{y}^N\in\mathcal{Y}^N} \Pi_{\tilde{y}^N y^N,t} \frac{S_{t-1,\tilde{y}^N}}{S_{t,y^N}} a_{t-1,\tilde{y}^N}, \tag{75}$$

$$l_{t,y^N} = \left(\chi y_{y^N} w_t\right)^{\phi}. \tag{76}$$

E.1.2 First-order conditions

Define the net social value of liquidity as:

$$\hat{\psi}_{t,y^N} = \omega_{y^N}\xi_{y^N} U_c(c_{t,y^N}, l_{t,y^N}) - \left(\lambda_{t,y^N} - \tilde{\lambda}_{t,y^N}(1+r_t)\right)\xi_{y^N} U_{cc}(c_{t,y^N}, l_{t,y^N}) - \mu_t.$$

The first-order conditions on a_{t,y^N} is:

$$\begin{cases} \hat{\psi}_{t,y^N} & = \beta\mathbb{E}_t\left[(1+r_{t+1})\sum_{\tilde{y}^N\in\mathcal{Y}^N} \Pi_{t,y^N\tilde{y}^N}\hat{\psi}_{t+1,\tilde{y}^N}\right] \text{ if } \nu_{y^N} = 0, \\ \lambda_{t,y^N} & = 0 \text{ if } \nu_{y^N} > 0, \end{cases}$$

and the first-order conditions for w_t, r_t, B and T_t are, respectively:

$$\begin{aligned}
\sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \hat{\psi}_{t,y^N} y_{y^N} l_{t,y^N} &= \varphi \mu_t \left(L_t - (1-\alpha) \frac{Y_t}{w_t} \right), \\
\sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \hat{\psi}_{t,y^N} \tilde{a}_{t,y^N} &= - \sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \tilde{\lambda}_{t,y^N} \xi_{y^N} U_c(c_{t,y^N}, l_{t,y^N}), \\
\mu_t &= \beta \mathbb{E}_t \left[\mu_{t+1} (1 + \tilde{r}^K_{t+1}) \right], \\
\sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \hat{\psi}_{t,y^N} &= 0.
\end{aligned}$$

E.2 Program in the economy without time-varying capital tax

The net social value of liquidity is $\hat{\psi}_{t,y^N} = \psi_{t,y^N} - \mu_t$, where:

$$\begin{aligned}
\psi_{t,y^N} = {} & \omega_{y^N} \xi_{y^N} U_c(c_{t,y^N}, l_{t,y^N}) - \left(\lambda_{t,y^N} - \tilde{\lambda}_{t,y^N} (1 + r_t) \right) \xi_{y^N} U_{cc}(c_{t,y^N}, l_{t,y^N}) \\
& - \left((\gamma_t - \gamma_{t-1}) \Pi_t (\Pi_t - 1) - \frac{\varepsilon - 1}{\kappa} \gamma_t (\zeta_t - 1) \right) Y_t \omega_{y^N} \xi_{y^N} U_{cc}(c_{t,y^N}, l_{t,y^N}).
\end{aligned}$$

E.2.1 Program formulation

The program is:

$$\begin{aligned}
\max_{\left((a_{t,y^N}, c_{t,y^N}, l_{t,y^N})_{y^N \in \mathcal{Y}^N}, w_t, r_t, B_t \right)_{t \geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{y^N \in \mathcal{Y}} \Bigg[& S_{t,y^N} \Big(\omega_{y^N} \xi_{y^N} U(c_{t,y^N}, l_{t,y^N}) \\
& - \left(\lambda_{t,y^N} - \tilde{\lambda}_{t,y^N} (1 + r_t) \right) \xi_{y^N} U_c \left(c_{t,y^N}, l_{t,y^N} \right) \Big) \Bigg],
\end{aligned}$$

subject to truncated-history constraints:

$$\begin{aligned}
c_{t,y^N} + a_{t,y^N} &= w_t (\chi y_0^N w_t)^{\phi} y_{y^N} + (1 + r_t) \tilde{a}_{t,y^N} + T_t, \\
\tilde{a}_{t,y^N} &= \sum_{\tilde{y}^N \in \mathcal{Y}^N} \Pi_{t, \tilde{y}^N y^N} \frac{S_{t-1,\tilde{y}^N}}{S_{t,y^N}} a_{t-1,\tilde{y}^N},
\end{aligned}$$

to aggregate constraints:

$$\begin{aligned}
G_t + r_t (B_{t-1} + K_{t-1}) + w_t L_t &= B_t - B_{t-1} - T_t + \left(1 - \kappa \frac{\pi_t^2}{2} \right) K_{t-1}^{\alpha} L_t^{1-\alpha} - \delta K_{t-1}, \\
\Pi_t (\Pi_t - 1) &= \frac{\varepsilon - 1}{\kappa} (\zeta_t - 1) + \beta \mathbb{E}_t \Pi_{t+1} (\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \frac{M_{t+1}}{M_t}, \\
\zeta_t &= \frac{1}{\alpha Z_t} \left(\tilde{r}^K_t + \delta \right) \left(\frac{K_{t-1}}{L_t} \right)^{1-\alpha}, \\
A_t = K_t + B_t &= \sum_{y^N \in \mathcal{Y}} S_{t,y^N} a_{t,y^N}, \; L_t = \sum_{y^N \in \mathcal{Y}} S_{t,y^N} y_0^N l_{t,y^N},
\end{aligned}$$

and to interest rate constraints:

$$\left(r_t - \left(1-\tau^K\right)\tilde{r}_t^K\right)A_{t-1} = \left(1-\tau^K\right)\left(\frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t} - 1 - \tilde{r}_t^K\right)B_{t-1}, \tag{77}$$

$$\mathbb{E}_t\left[\frac{R_t^{B,N}}{\Pi_{t+1}}\right] = \mathbb{E}_t\left[1+\tilde{r}_{t+1}^K\right]. \tag{78}$$

E.2.2 First-order conditions

We denote by $\beta^t\Gamma_t$ and $\beta^t\Upsilon_t$ the Lagrange multipliers on (77) and (78), respectively. We define the pricing kernel M_t as follows:

$$M_t = \sum_{y^N\in\mathcal{Y}^N} S_{t,y^N}\xi_{y^N}\omega_{y^N}U_c(c_{t,y^N}, l_{t,y^N}).$$

Derivative with respect to $\tilde{R}_t^{B,N}$.

$$\left(1-\tau_{SS}^K\right)\mathbb{E}_t\left[\frac{\Gamma_{t+1}}{\Pi_{t+1}}\right]B_t = \Upsilon_t\mathbb{E}_t\left[\frac{1}{\Pi_{t+1}}\right]. \tag{79}$$

Derivative with respect to $\tilde{r}_t^K$.

$$\Upsilon_{t-1} + \Gamma_t(1-\tau_{SS}^K)\left(A_{t-1}-B_{t-1}\right) = \frac{\varepsilon-1}{\alpha\kappa}\gamma_t K_{t-1}M_t.$$

Derivative with respect to Π_t.

$$\begin{aligned}0 &= \mu_t\kappa\left(\Pi_t-1\right) + \left(\gamma_t-\gamma_{t-1}\right)\left(2\Pi_t-1\right)M_t \\ &\quad + \left(\Gamma_t\left(1-\tau^K\right)B_{t-1} - \Upsilon_{t-1}\right)\frac{\tilde{R}_{t-1}^{B,N}}{Y_t\Pi_t^2}.\end{aligned} \tag{80}$$

Derivative wrt r_t.

$$\sum_{y^N\in\mathcal{Y}^N} S_{t,y^N}\hat{\psi}_{t,y^N}\tilde{a}_{t,y^N} = -\sum_{y^N\in\mathcal{Y}^N} S_{t,y^N}\tilde{\lambda}_{t,y^N}\xi_{y^N}U_c(c_{t,y^N}, l_{t,y^N}) - \Gamma_t A_{t-1}.$$

Derivative wrt w_t.

$$\begin{aligned}\mu_t\varphi\left(L_t - (1-\alpha)\frac{Y_t}{w_t}\left(1-\frac{\kappa}{2}\left(\Pi_t-1\right)^2\right)\right) &= \sum_{y^N\in\mathcal{Y}^N} S_{t,y^N}\hat{\psi}_{t,y^N}y_0^N l_{t,y^N} \\ -\frac{(1-\alpha)\varphi}{w_t}\left(\left(\gamma_t-\gamma_{t-1}\right)\Pi_t\left(\Pi_t-1\right) + \frac{\varepsilon-1}{\kappa}\gamma_t\right)Y_tM_t.\end{aligned}$$

Derivative wrt B_t.

$$\mu_t = \beta \mathbb{E}_t \left[\mu_{t+1} \left(1 - \delta + \zeta_{t+1}^{-1} (\tilde{r}_{t+1}^K + \delta) \left(1 - \frac{\kappa}{2} \left(\Pi_{t+1} - 1 \right)^2 \right) \right) \right]$$
$$- \alpha \beta \mathbb{E}_t \left[\left(\left(\gamma_{t+1} - \gamma_t \right) \Pi_{t+1} \left(\Pi_{t+1} - 1 \right) + \frac{\varepsilon - 1}{\kappa} \gamma_{t+1} \right) \frac{Y_{t+1}}{K_t} M_{t+1} \right]$$
$$+ \frac{\varepsilon - 1}{\kappa} \beta \mathbb{E}_t \left[\gamma_{t+1} \frac{\zeta_{t+1} Y_{t+1}}{K_t} M_{t+1} \right]$$
$$+ \beta \left(1 - \tau_{SS}^K \right) \mathbb{E}_t \left[\Gamma_{t+1} \left(\frac{\tilde{R}_t^{B,N}}{\Pi_{t+1}} - 1 - \tilde{r}_{t+1}^K \right) \right].$$

Derivative wrt a_{t,y^N}.

$$\hat{\psi}_{t,y^N} = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \sum_{\tilde{y}^N \in \mathcal{Y}^N} \Pi_{t,y^N \tilde{y}^N} \hat{\psi}_{t+1,\tilde{y}^N} \right] \tag{81}$$
$$+ \beta \mathbb{E}_t \left[\Gamma_{t+1} \left(r_{t+1} - \left(1 - \tau_{SS}^K \right) \left(\frac{\tilde{R}_t^{B,N}}{\Pi_{t+1}} - 1 \right) \right) \right].$$

Derivative wrt T_t.

$$\sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \hat{\psi}_{t,y^N} = 0. \tag{82}$$

E.3 Program in the economy without time-varying labor tax

We keep the same notation as in the no-capital tax case of Section E.2. The program is also very similar to the one of Section E.2, except that the capital tax is time-varying, and the labor tax is not. We have a new constraint:

$$\alpha \frac{w_t}{1 - \tau_{SS}^L} L_t = (1 - \alpha) K_{t-1} (\tilde{r}_t^K + \delta),$$

while the constraints on r_t ($\left(r_t - \left(1 - \tau_{SS}^K \right) \tilde{r}_t^K \right) A_{t-1} = \left(1 - \tau_t^K \right) \left(\frac{\tilde{R}_{t-1}^{B,N}}{\Pi_t} - 1 - \tilde{r}_t^K \right) B_{t-1}$) and on $\tilde{R}_t^{B,N}$ ($\mathbb{E}_t \frac{\tilde{R}_t^{B,N}}{\Pi_{t+1}} = \mathbb{E}_t [1 + \tilde{r}_{t+1}^K]$) are not binding anymore.

Derivative with respect to $\tilde{r}_t^K$.

$$\Gamma_t^L = \frac{\varepsilon - 1}{\alpha (1 - \alpha) \kappa} \gamma_t M_t.$$

Derivative with respect to Π_t.

$$0 = \mu_t \kappa \left(\Pi_t - 1 \right) + \left(\gamma_t - \gamma_{t-1} \right) \left(2 \Pi_t - 1 \right) M_t. \tag{83}$$

Derivative with respect to r_t.

$$\sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \hat{\psi}_{t,y^N} \tilde{a}_{t,y^N} = - \sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \tilde{\lambda}_{t,y^N} \xi_{y^N} U_c(c_{t,y^N}, l_{t,y^N}).$$

Derivative with respect to w_t.

$$\begin{aligned}
&\mu_t \varphi \left(L_t - (1-\alpha) \frac{Y_t}{w_t} \left(1 - \frac{\kappa}{2} (\Pi_t - 1)^2 \right) \right) = \sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \hat{\psi}_{t,y^N} y_0^N l_{t,y^N} \\
&- \frac{(1-\alpha)\varphi}{w_t} \left((\gamma_t - \gamma_{t-1}) \Pi_t (\Pi_t - 1) + \frac{\varepsilon - 1}{\kappa} \gamma_t \right) Y_t M_t \\
&+ \Gamma_t^L \frac{\alpha}{1 - \tau_{SS}^L} (1+\varphi) L_t.
\end{aligned} \tag{84}$$

Derivative with respect to B_t: the public debt.

$$\begin{aligned}
\mu_t = {} & \beta \mathbb{E}_t \left[\mu_{t+1} \left(1 - \delta + \zeta_{t+1}^{-1} (\tilde{r}_{t+1}^K + \delta) \left(1 - \frac{\kappa}{2} (\Pi_{t+1} - 1)^2 \right) \right) \right] \\
& - \alpha \beta \mathbb{E}_t \left[\left((\gamma_{t+1} - \gamma_t) \Pi_{t+1} (\Pi_{t+1} - 1) + \frac{\varepsilon - 1}{\kappa} \gamma_{t+1} \right) \frac{Y_{t+1}}{K_t} M_{t+1} \right] && (85) \\
& + \frac{\varepsilon - 1}{\kappa} \beta \mathbb{E}_t \left[\gamma_{t+1} \frac{\zeta_{t+1} Y_{t+1}}{K_t} M_{t+1} \right] \\
& - \beta (1-\alpha) \mathbb{E}_t \left[\Gamma_{t+1}^L (\tilde{r}_{t+1}^K + \delta) \right]. && (86)
\end{aligned}$$

Derivative with respect to a_t^i: the net saving of consumers .

$$\hat{\psi}_{t,y^N} = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \sum_{\tilde{y}^N \in \mathcal{Y}^N} \Pi_{t,y^N \tilde{y}^N} \hat{\psi}_{t+1,\tilde{y}^N} \right]. \tag{87}$$

Derivative wrt T_t.

$$\sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \hat{\psi}_{t,y^N} = 0. \tag{88}$$

F Matrix representation at the steady state to compute ξ and ω

Before turning to the matrix representation, we introduce the following notation:

$\circ$ is the Hadamard product, $\otimes$ is the Kronecker product, $\times$ is the usual matrix product.

For any vector V, we denote by $diag\,(V)$ the diagonal matrix with V on the diagonal.

All previous equations now have to be stacked such that the economy can be written using a matrix notation at the steady state.

This represenation provides an efficient way to compute the relevant ξ and ω.

Indeed, a history y^N can be seen as an N-length numeric vector $\{y_{-N+1}, \ldots, y_0\}$, where $y_k = 1, \ldots, Y$ denotes her productivity level. The number of histories is $N_{tot} = Y^N$. We can identify each history by the integer $k_{y^N} = 1, \ldots, N_{tot}$:

$$k_{y^N} = \sum_{k=0}^{N-1} N_{tot}^{-N+1-k} (y_k - 1) + 1, \tag{89}$$

which corresponds to an enumeration in base Y.

Let $\mathbf{U_c}$, $\boldsymbol{\xi}$ and $\boldsymbol{\xi}^u$ be the N_{tot}-vectors of end-of-period marginal utilities, and preference shifters. Define as $\mathbf{I}$ the identity matrix and $\mathbf{\Pi} = (\mathbf{\Pi}_{kk'})_{k,k'=1,\ldots,N_{tot}}$ as the transition matrix from history k to history k'.

F.1 Computing the ξs

Let $\mathbf{S}$ be the N_{tot}-vector of steady-state history sizes. Similarly, let $\mathbf{a}$, $\mathbf{c}$, $\boldsymbol{\ell}$, $\boldsymbol{\nu}$, $\mathbf{U_c}$, $\mathbf{U_{cc}}$ be the N_{tot}-vectors of end-of-period wealth, consumption, labor supply, Lagrange multipliers, marginal utilities, and derivatives of the marginal utility, respectively. These vectors are known from the steady-state equilibrium of the Bewley model. Each element is defined as the truncation of the relevant variable computed using equation (55). We also define:

$$\mathbf{W} = w \otimes \begin{bmatrix} y_1 \\ \vdots \\ y_Y \end{bmatrix} \otimes 1_B, \; \mathbf{L} = \begin{bmatrix} y_1 \\ \vdots \\ y_Y \end{bmatrix} \otimes 1_B.$$

Where 1_B is a vector of 1 of length Y^{N-1}. Let $\mathbb{P}$ be the diagonal matrix having 1 on the diagonal at y^N if and only if the history y^N is not credit constrained (i.e., $\nu_{y^N} = 0$), and 0 otherwise. Similarly, define $\mathbb{P}^c = \mathbf{I} - \mathbb{P}$, where $\mathbf{I}$ is the $(N_{tot} \times N_{tot})$-identity matrix. Let Π be the transition matrix across histories. In the steady state:

$$\mathbf{S} = \Pi \mathbf{S}, \tag{90}$$
$$\mathbf{S} \circ \mathbf{c} + \mathbf{S} \circ \mathbf{a} = (1+r)\Pi \left(\mathbf{S} \circ \mathbf{a}\right) + \left(\mathbf{S} \circ \mathbf{W} \circ \boldsymbol{\ell}\right),$$
$$\mathbb{P}^c \mathbf{a} = -\bar{a} 1_{N_{tot} \times 1},$$

The value of the ξ can then easily be found from the allocation of the Bewley model. Indeed, the vector $\boldsymbol{\xi}$ solves:

$$\boldsymbol{\xi} = \left[\mathbb{P}\left(diag\left(u'\left(\mathbf{c}\right)\right) - \beta(1+r)\Pi \times diag\left(u'\left(\mathbf{c}\right)\right)\right) + \mathbb{P}^c\right]^{-1} \boldsymbol{\nu}, \tag{91}$$

F.2 Finding the Pareto weights ω

The first-order conditions of the planner at the steady state are:

$$\begin{aligned}
\mathbb{P}\hat{\psi} &= \beta\mathbb{P}\Pi^{\top}\hat{\psi}\left(1+r\right), \\
\mathbb{P}^{c}\lambda^{p} &= 0, \\
\left(\mathbf{y}\circ\mathbf{l}\right)'\times\left(\mathbf{S}\circ\hat{\psi}\right) &= \mu L\left(1+\varphi\frac{w-F_L}{w}\right), \\
1_{YL}^{\intercal}\times\left(\mathbf{S}\circ\hat{\psi}\circ\tilde{a}\right) &= 1_{YL}^{\intercal}\times\left(\mathbf{S}\circ\tilde{\lambda}\circ\xi_h\circ\mathbf{U}_c\left(\mathbf{c},\mathbf{l}\right)\right), \\
1 &= \beta(F_K+1), \\
\mathbf{S}\circ\hat{\psi} &= 0.
\end{aligned}$$

After some algebra, one can express the relationship that the Pareto weights ω must fulfill for the observed allocation to be a solution of the previous first-order conditions of the planner. One finds that the Pareto weight must satisfy two relationships:

$$\mathbf{H}^{\mathbf{1}}\omega = 0 \text{ and } \mathbf{H}^{2}\omega = 0,$$

where $\mathbf{H}^1 = \tilde{\mathbf{H}}^1\mathbf{D}\left(\mathbf{S}\circ\xi\circ\mathbf{U}_c(\mathbf{c},\mathbf{l}_h)\right)$, $\mathbf{H}^2 = \tilde{\mathbf{H}}^2\mathbf{D}\left(\mathbf{S}\circ\xi\circ\mathbf{U}_c(\mathbf{c},\mathbf{l}_h)\right)$ and:

$$\begin{aligned}
\tilde{\mathbf{H}}^1 &\equiv 1_{YL}^{\intercal}\times\left[D\left(\tilde{a}\right)\mathbf{N}+\mathbf{D}\left(\xi_h\circ\mathbf{U}_c\left(\mathbf{c},\mathbf{l}\right)\right)\Pi\mathbf{J}\right]-A\mathbf{Q}, \\
\mathbf{Q} &\equiv \left(\frac{\mathbf{y}\circ\mathbf{l}}{L\left(1+\varphi\frac{w-F_L}{w}\right)}\right)'\times\mathbf{N}, \\
\mathbf{N} &\equiv \mathbf{Id}_{\mathbf{YN}}-\mathbf{B}\left(\mathbb{P}^c+\mathbb{P}\mathbf{M}\mathbf{B}\right)^{-1}\mathbb{P}\mathbf{M}, \\
\mathbf{J} &\equiv -\left(\mathbb{P}^c+\mathbb{P}\mathbf{M}\mathbf{B}\right)^{-1}\mathbb{P}\mathbf{M},
\end{aligned}$$

and

$$\begin{aligned}
\mathbf{M} &\equiv \mathbf{Id}_{\mathbf{YN}}-\beta\left(1+r\right)\Pi^{S}+\beta\frac{r-F_K}{L\left(1+\varphi\frac{w-F_L}{w}\right)}\mathbf{S}\left(\mathbf{y}\circ\mathbf{l}\right)', \\
\mathbf{\Pi}^{S} &\equiv \mathbf{D}\mathbf{S}\Pi^{\top}D^{-1}\mathbf{S}, \\
\mathbf{B} &\equiv \mathbf{D}\left(\xi_h\circ\mathbf{U}_{cc}\left(\mathbf{c},\mathbf{l}\right)\right)\left(\left(1+r\right)\Pi-\mathbf{Id}_{YL}\right), \\
\tilde{\mathbf{H}}^2 &\equiv 1_{YL}^{\intercal}\times\mathbf{N}-\mathbf{Q}.
\end{aligned}$$

The estimation of Pareto weights is thus given by:

$$\begin{aligned}
&\min_{\omega}\left\|\omega-\mathbf{1}\right\|, \\
&\mathbf{H}^{\mathbf{1}}\omega = 0 \text{ and } \mathbf{H}^{2}\omega = 0,
\end{aligned}$$

which can be found by simple linear algebra. Once the steady state is found, perturbation methods can be used.