Housing Tax Reform

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Motivation

Housing is largest component of household consumption and wealth

Housing tax policy targets consumption and liquidity of housing

- consumption tax: property tax, mortgage interest deduction, …
- transaction tax: capital gains tax, stamp duty

Question: How to efficiently design and reform housing tax policy?
What I do

Use an incomplete markets life cycle economy with housing

1. housing input in home production
2. illiquid due to adjustment costs
3. private, stochastic skill risk; elastic labor supply

1. Measure current tax policy using tax records for the Netherlands
2. Study dynamic Mirrlees theory for efficient housing tax reform
3. Quantify theory for economy matched to the Netherlands
What I find

1. Measures of current housing consumption and transaction tax
   - average owner’s consumption *subsidy* of 8% (range from 20% to −5%)
   - transaction tax of 6%

2. Theoretical motives to deviate from uniform commodity taxation
   - tax housing when house complements leisure in home production
   - subsidize and tax transactions to insure against adjustment costs

3. Housing consumption tax should be similar to tax on other goods
   - house is weak complement to leisure → housing consumption *tax* of 14%
Model
Standard life cycle model

Three key ingredients:

- home production preferences $u(c, d, \ell) = v(c) + g(d, \ell)$
- idiosyncratic skill shocks $\theta_t$, labor supply $y = \theta(1 - \ell)$
- own or rent decision driven by
  - tax treatment of owning versus renting
  - size restrictions: own if $d \geq d \equiv \chi h$ and rent if $d \leq d$
  - adjustment costs
Plan for today

Study optimality condition for housing services in two problems

1. Positive economy of the Netherlands
   - measure current effective tax policy

2. Mirrlees problem
   - characterize and quantify efficient tax policy
- savings in financial assets, house, mortgage, \( s = a + p_H h - m \geq 0 \)

- loan-to-value and income restrictions, \( m \leq \kappa_t(h, y) \)

- budget constraint

\[
(1+\tau_c)c + \Psi(d, d_-) + s' = y - T_t^y(\tilde{y}) + Ra - T^a(a) + (p_{H'} - \tau_p p_H - \delta) h - Rm
\]

where,

- adjustment costs: \( \Psi(d, d_-) = \Phi(d, d_-) + T^t_t(d, d_-) \)

- taxable income: \( \tilde{y} = y - rm + \tau_o p_H h \)
Household optimality condition:

\[
\frac{u_{d,t}}{u_{c,t}} = p \cdot \frac{1 + \tau_{di}}{1 + \tau_c}
\]
Household optimality condition:

\[
\frac{u_{d,t}}{u_{c,t}} = p \left( 1 + \tau_{di} \right) + \Phi_{1,t} + \frac{\tau_{buy}}{1 + \tau_c}
\]
Household optimality condition:

\[
\frac{u_{d,t}}{u_{c,t}} = p \left( \frac{1 + \tau_{di}}{1 + \tau_{c}} \right) + \Phi_{1,t} + \frac{\tau_{buy}}{1 + \tau_{c}} + \beta \mathbb{E}_t \left( \Phi_{2,t+1} + \frac{\tau_{sell}}{1 + \tau_{c}} \right) \frac{u_{c,t+1}}{u_{c,t}}
\]
- Household optimality condition:

\[
\frac{u_{d,t}}{u_{c,t}} = p \left( \frac{1 + \tau_{di}}{1 + \tau_c} \right) + \Phi_{1,t} + \frac{\tau_{ti}}{1 + \tau_c} + \beta E_t \left( \Phi_{2,t+1} + \frac{\tau_{sell}}{1 + \tau_c} \right) \frac{u_{c,t+1}}{u_{c,t}}
\]

- Efficient optimality condition:

\[
\frac{u_{d,t}}{u_{c,t}} = p \left( \ldots \right) + \Phi_{1,t} + \left( \ldots \right) + \beta E_t \left( \Phi_{2,t+1} + \left( \ldots \right) \right) \frac{u_{c,t+1}}{u_{c,t}}
\]
Household optimality condition:

\[
\frac{u_{d,t}}{u_{c,t}} = p \frac{1 + \tau_{di}}{1 + \tau_c} + \Phi_{1,t} + \frac{\tau_{buy}}{1 + \tau_c} + \beta E_t \left( \Phi_{2,t+1} + \frac{\tau_{sell}}{1 + \tau_c} \right) \frac{u_{c,t+1}}{u_{c,t}}
\]

Efficient optimality condition:

\[
\frac{u_{d,t}}{u_{c,t}} = p \ldots + \Phi_{1,t} + \ldots + \beta E_t \left( \Phi_{2,t+1} + \ldots \right) \frac{u_{c,t+1}}{u_{c,t}}
\]

1. Measure current tax policy using tax records for the Netherlands
2. Study theory for efficient consumption and transaction tax
3. Quantify efficient consumption and transaction tax
Current Housing Tax Policy
Administrative micro data from 2006 to 2014 on:

- tax assessed property values
- mortgage balance
- who lives where
- hours and earnings
- marginal tax rates

National accounts data on:

- consumption shares

Used to measure current policy, calibrate wage process, preferences
Measures of housing consumption and transaction tax

1. Effective tax rate on housing consumption for household $i$

$$\tau_{di} \equiv \left( \frac{\text{user cost under current policy}}{\text{user cost absent taxation}} \right)_i - 1$$

2. Effective tax rate on transactions

$$\tau_{t_i}^{\text{buy}} \equiv T_1^t(d, d_-) \quad \text{for house you buy}$$

$$\tau_{t_i}^{\text{sell}} \equiv T_2^t(d, d_-) \quad \text{for house you sell}$$

Later compare to efficient consumption and transaction tax rate
Transaction tax

- Statutory tax rate when buying

\[ \tau_{ti}^{\text{buy}} = 6\% \]

- Statutory tax rate when selling

\[ \tau_{ti}^{\text{sell}} = 0\% \]
User cost absent taxation

$$r + \hat{\delta} - \pi^H$$

- **opportunity cost of capital**, $r = 3.1\%$
  - average interest rate on mortgages

- **depreciation rate of housing**, $\hat{\delta} = 2.4\%$
  - depreciation of housing stock, capital accounts

- **capital gain**, $\pi^H = -2.8\%$
  - nominal house price inflation $-0.7\%$, price inflation $2.1\%$

$\implies 8.3\%$, or monthly rental value of $1,725$ for $250K$ property
User cost for homeowner $i$

$$r + \hat{\delta} - \pi^H + \tau_p - \tau_{yi} r \lambda_i - \tau_{ai} (1 - \lambda_i) + \tau_{yi} \tau_o$$

baseline, $uc_n$  
property tax  
mortgage interest deduction  
exclusion from asset income tax  
imputed rent tax

with $\tau_p = 0.1\%$, $\tau_o = 0.6\%$, and loan-to-value ratio $\lambda_i \equiv m_i / p_H h_i$

Use administrative data to measure:

1. property values
2. mortgage balances
3. marginal tax rates
User cost for homeowner \( i \)

\[
\begin{align*}
\hat{r} + \hat{\delta} - \pi^H + \tau_p - \tau_y i r \lambda_i - \tau_{ai}(1 - \lambda_i) + \tau_y i \tau_o
\end{align*}
\]

baseline, \( \text{uc}_n \)  
mortgage interest deduction  
imputed rent tax  
property tax  
exclusion from asset income tax

with \( \tau_p = 0.1\% \), \( \tau_o = 0.6\% \), and loan-to-value ratio \( \lambda_i \equiv m_i/p_H h_i \)

Use administrative data to measure:

1. property values
2. mortgage balances
3. marginal tax rates

Then, construct \( \tau_{di} \equiv \left( \frac{\text{user cost under current policy}}{\text{user cost absent taxation}} \right)_i - 1 \)
Histogram of owner’s housing consumption tax

By age

Average subsidy of 8%
Histogram of owner’s housing consumption tax

By age

Average subsidy of 8%

young, high income

old, low income
Recap measurement

- Transaction tax rate on buyers 6%; on sellers 0%
- Average housing consumption subsidy of 8% (from 20% to −5%)

The model optimality condition for housing services:

\[
\frac{u_{d,t}}{u_{c,t}} = p \frac{1 + \tau_{di}}{1 + \tau_c} + \Phi_{1,t} + \frac{\tau_{bi}}{1 + \tau_c} + \beta \mathbb{E}_t \left[ \left( \Phi_{2,t+1} + \frac{\tau_{si}}{1 + \tau_c} \right) \frac{u_{c,t+1}}{u_{c,t}} \right]
\]

Is this efficient? How to efficiently reform housing tax policy?
Reform Theory
Efficient reform

So far, positive economy

- measurement of effective housing tax policy

- values under current policy for every household

Next, analyze efficient policy reform

- characterize efficient allocations and housing tax policy

- Pareto improvements using values under current policy
allocation for household $i \equiv (j, \theta^{t-1})$ is $x(i) \equiv \{x_{j+v}(\theta^{t+v})\}_{v=0}^{T-t}$

$x \equiv (c, d, y)$

set of households: all current $(0, \theta^{t-1})$ and future cohorts $(j, \theta_0)$

an allocation is feasible iff it is resource and incentive feasible

allocation $x$ is efficient iff there does not exist a feasible allocation $\hat{x}$ where all households are better off with some strictly better off
Efficient reform in practice

- Formulate planning problem to characterize efficient allocations
- Exploit separability to solve household by household
- Solve component problem using a direct mechanism
  - Include only local downward incentive constraints
- Characterize efficient allocation, map to tax wedges
Housing wedge definitions

Given a history $\theta^{t-1}$

1. Consumption wedge

$$\frac{u_d}{u_c} \equiv p(1 + \tau_d(\theta)) + \Phi_1 + \frac{1}{R} \sum \pi(\theta' | \theta) \Phi_2(d(\theta'), d)$$

2. Transaction wedges

$$\frac{u_d}{u_c} = p(1 + \tau_d(\theta)) + \Phi_1 + \beta \sum \pi(\theta' | \theta) \frac{u_c(\theta')}{u_c} \left( \Phi_2(d(\theta'), d) + \tau_t(\theta') \right)$$

Characterize, then compare to current housing tax policy
Housing consumption wedge

- \( \tau_d(\theta) \geq 0 \) iff housing and leisure are complements, \( g_{d\ell}(d, \ell) > 0 \)

- Prevent high type from mimicking low type
  - benefit of deviation is additional home production
  - depress housing to discourage deviation if complements

- Relax incentive constraint
  - provide additional insurance
Housing transaction wedge

- tax transactions when households sell their house in good states
  \[ u_c(c_-) \geq \beta R u_c(c(\theta)) \]

- precautionary downsizing due to adjustment cost in bad states
  - larger house increases exposure to future adjustment cost
  - with incomplete markets, households downsize to reduce exposure

- transaction tax insures households against adjustment costs
  - tax transactions in good times, subsidize transactions in bad times

\[ \tau_t(\theta) = \Phi_2(d(\theta), d_-) \left( \frac{1}{\beta R u_c(\theta)} - 1 \right) \]

premum payout
Efficient versus current policy

- From the planning problem

\[
\frac{u_{d,t}}{u_{c,t}} = p \left(1 + \tau_d(\theta)\right) + \Phi_{1,t} + 0 + \beta \mathbb{E}_t \left(\Phi_{2,t+1} + \tau_t(\theta')\right) \frac{u_{c,t+1}}{u_{c,t}}
\]

\[\geq 1 \text{ iff } g_{d\ell} \geq 0\]

Takeaways:
1. current consumption subsidy can be efficient only if substitutes
2. current transaction tax is not efficient
Efficient versus current policy

From the planning problem

\[
\frac{u_{d,t}}{u_{c,t}} = p \left(1 + \tau_d(\theta)\right) + \Phi_{1,t} + 0 + \beta E_t \left(\Phi_{2,t+1} + \tau_t(\theta')\right) \frac{u_{c,t+1}}{u_{c,t}}
\]

≥1 iff \(g_{d\epsilon} \geq 0\)

From the model of the Netherlands

\[
\frac{u_{d,t}}{u_{c,t}} = p \frac{1 + \tau_{di}}{1 + \tau_c} + \Phi_{1,t} + \frac{\tau_{buy}}{1 + \tau_c} + \beta E_t \left(\Phi_{2,t+1} + \frac{\tau_{sell}}{1 + \tau_c}\right) \frac{u_{c,t+1}}{u_{c,t}}
\]

\[\begin{bmatrix}
0.80 & 1.05 \\
1.13 & 1.13
\end{bmatrix}\]

\[\begin{bmatrix}
0.06 \\
1.13
\end{bmatrix}\]

Takeaways:
1. current consumption subsidy can be efficient only if substitutes
2. current transaction tax is not efficient
Efficient versus current policy

- From the planning problem

\[
\frac{u_{d,t}}{u_{c,t}} = p \left( 1 + \tau_d(\theta) \right) + \Phi_{1,t} + 0 + \beta \mathbb{E}_t \left( \Phi_{2,t+1} + \tau_t(\theta') \right) \frac{u_{c,t+1}}{u_{c,t}} \geq 1 \text{ iff } g_{d\ell} \geq 0
\]

- From the model of the Netherlands

\[
\frac{u_{d,t}}{u_{c,t}} = p \left( \frac{1 + \tau_{di}}{1 + \tau_c} \right) + \Phi_{1,t} + \frac{\tau_{buy}}{1 + \tau_c} + \beta \mathbb{E}_t \left( \Phi_{2,t+1} + \frac{\tau_{sell}}{1 + \tau_c} \right) \frac{u_{c,t+1}}{u_{c,t}} \prod_{0.06, 1.13}^{0.80, 1.05, 1.13}
\]

Takeaways:

1. current consumption subsidy can be efficient only if substitutes
2. current transaction tax is not efficient
Recap theory

Measurement

- transaction tax rate on buyers 6%; on sellers 0%
- average consumption subsidy of 8% (20% subsidy to 5% tax)

Theory

- subsidize and tax transactions to insure against adjustment costs
- tax housing when house complements leisure in home production

Quantify complementarity housing and leisure in home production
Quantitative Reform
Calibrate positive economy

1. Estimate skill process

2. Parameterize government policy

3. Parameterize technology

4. Calibrate preferences

- Do 1, 2, and 3 outside the model
- Use positive economy for 4
Households

- $u(c, d, \ell) = \gamma \log c + (1 - \gamma) \log \left( \left( \omega d^{\frac{\sigma - 1}{\sigma}} + (1 - \omega) \ell^{\frac{\sigma - 1}{\sigma}} \right)^{\frac{\sigma}{\sigma - 1}} \right)$

  housing services $d$ and leisure $\ell$ complement iff $\sigma \leq 1$

- six types based on education level, differ in AR(1) skill process

Government

- collects taxes, provides pension benefits, regulates mortgages

Technology $\Phi$

- 2% buyer’s fee; 1.5% seller’s fee

Today, transaction costs are inefficient in planner problem, $\Phi = 0$
Identify elasticity $\sigma$ by indirect inference from regression coefficient

$$\log \left( \frac{\ell}{d} \right)_i = \mathbb{C} + \beta \log \left( \frac{w}{p} \right)_i + \varepsilon_i$$
Current policy is not efficient

Dispersion
Labor
Savings
\( \sigma \)
Paths

Housing consumption tax

-15
-10
-5
0
5
10
15

25
35
45
55
65
75

current

efficient
Simple Pareto improving reform

Use efficient reform to guide simple steady state policy reform holding government debt position constant by adjusting transfers

\[ \tau_{di} \propto \tau_p - \tau_{yi}r\lambda_i - \tau_{ai}(1 - \lambda_i) + \tau_{yi}\tau_o \]

- increase \( \tau_p \) from 0.1% to 1.2% to move from \(-8\%\) to 14%

- lower \( \tau_o \) from 0.6% to 0.0% to ensure gain for high income groups

<table>
<thead>
<tr>
<th>( \Delta c )</th>
<th>( \Delta f_h )</th>
<th>Welfare Gain by Education Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.68</td>
<td>0.00</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.68 0.35 0.60 0.25 0.03</td>
</tr>
</tbody>
</table>
Conclusion
Conclusion

How to efficiently design and reform housing policy?

Theory

- tax housing services when housing services complement leisure
- tax and subsidize transactions to insure against adjustment costs

Quantitative

- effective housing *subsidy* of 8% for average owner decreases in age
- efficient housing *tax* of 14% almost constant over the life-cycle
Appendix
Related literature

- housing (over the life cycle, user cost)
  
  Laidler (1969); Aaron (1970); Poterba (1984); Gervais (2002); Fernández-Villaverde, 
  Krueger (2010); Sommer, Sullivan (2018); Kaplan, Mitman, Violante (2019).

- home production
  
  Becker (1965); Gronau (1977); Greenwood, Hercowitz (1991); Benhabib, Rogerson, Wright 
  (1991); Aguiar, Hurst (2005); Boerma, Karabarbounis (2019).

- public finance
  
  Mirrlees (1971); Atkinson, Stiglitz (1976); Golosov, Kocherlakota, Tsyvinski (2003); Farhi, 
  Werning (2013); Golosov, Troshkin, Tsyvinski (2016); Hosseini, Shourideh (2019).

- this paper: efficient tax reform for incomplete markets life cycle economy 
  with illiquid housing capital and home production
Home production preferences

- time and expenditures produce goods

\[ u(c, d, n_H, \ell) = v(c) + h(d, n_H, \ell) \]

- time constraint \( \ell + n_M + n_H = 1 \); effective labor supply \( y = \theta n_M \)

- household indirect utility given an allocation \( (c, d, y) \) and skills \( \theta \)

\[ \vartheta(c, d, y; \theta) = \max_{n_H \in [0, 1-n_M]} u(c, d, n_H, \ell) = v(c) + \tilde{h}(d, y) \]
Firms’ problem

Construction firm

- commits to build houses $Q_{j+1-\iota}$ for period $j + 1 - \iota$

- builds in period $j$, valued at $p_{j+1}^H$, using general good ($p_{j+1}^H = 1$)

- in first period, commits to deliver houses in period $\iota$

$$p_j^H = 1 \text{ for } j > \iota$$

Rental firm

$$p_r = \frac{1}{\chi} \left( r(1 - \tau_f) + \tau_p + \delta - \pi^H \right) p^H$$
Rental firm

- receive rent $p_j$ per unit of housing services
- borrow at rate $r$ to buy housing capital at $p^H_j$ per unit
- incur maintenance cost $\delta$, pay property tax $\tau_p$
- sell housing capital at price $p^H_{j+1}$ at the end of the period
- receive a subsidy on interest payments $\tau_f$

$$p_{r,j} = \frac{1}{\chi} \left( r(1 - \tau_f) + \tau_p + \hat{\delta} - \pi^H_{j+1} \right) p^H_j$$
housing services

\[ D_j = \chi H_j \] 

services flow \( D \) proportional to housing stock \( H \)
Technology

- housing services

\[ D_j = \chi H_j \]

- time to build \( \iota \geq 1 \)

\[ H_{j+1} = Q_{j+1-\iota} + H_j \]

constructions \( Q \) planned in advance
Technology

- housing services

\[ D_j = \chi H_j \]

- time to build \( \iota \geq 1 \)

\[ H_{j+1} = Q_{j+1-\iota} + H_j \]

housing supply perfectly inelastic in short run, perfectly elastic in long run

- general good

\[ C_j + I_j^K + I_j^H + G_j + \Phi_j + B_{j+1} = F(K_j, Y_j) + RB_j \]

where \( I_j^H = Q_{j+1-\iota} + \delta H_j \)
Income and asset tax

**Income Taxes (in 000's), T^y**
- **Working Age**
- **Retirees**

**Financial Asset Tax (in 000's), T^a**
- **Working Age**

![Graph showing income and asset tax](image-url)
Renter’s constraints

- savings in financial assets, \( s = a \geq 0 \)

- budget constraint

\[
(1 + \tau_c)c + p_r d + \Phi(d, d_-) + T^t(d, d_-) + s' = wy - T^y_t(\tilde{y}) + Ra - T^a(a)
\]

where,

- rental price: \( p_r \)

- taxable income: \( \tilde{y} = wy \)

- largest house to rent, \( d \leq \chi h \)
Equilibrium

Given public spending, construction plans, initial private savings, aggregate assets, an equilibrium is an allocation and prices so that:

- allocation solves household problems

- prices are consistent with firm optimization
  factor prices, rental prices, house prices

- goods and housing market clear

- government budget constraint is satisfied
Homeowner as a firm

mortgage interest deduction

- taxable income \( \tilde{y} = wy + (p_n - r \lambda_i - r(1 - \lambda_i) - \hat{\delta} - \pi^H) p^H h \)

implies zero subsidy (=0)

\[
c + T^c(c) + \Psi(d, d_-) + s' = wy - T^y(\tilde{y}) + Ra + (p^H' - \delta) h - Rm
\]

Home mortgage interest deduction is a subsidy because of a failure to tax housing consumption
Capital gains tax

- Accrual system

  \[ p_i = r + \hat{\delta} - (1 - \tau_\pi) \pi^H + \tau_p - \tau_y r \lambda_i - \tau_{ai}(1 - \lambda_i) + \tau_y \tau_o \]

- Realization system

  \[ T^t(d_t, d_{t-1}) \rightarrow T^t(d_t, d_{t-1}, p_{j+1}^H, \hat{p}_a^H) \]

  acquisition price
Incomplete markets

\[
\frac{u_{d,t}}{u_{c,t}} = p \frac{1 + \tau_{di}}{1 + \tau_c} + \Phi_{1,t} + \tau_{ti} \frac{\text{buy}}{1 + \tau_c} + \beta \mathbb{E}_t \left[ \left( \Phi_{2,t+1} + \tau_{ti} \frac{\text{sell}}{1 + \tau_c} \right) \frac{u_{c,t+1}}{u_{c,t}} \right]
\]

Complete markets

\[
\frac{u_{d,t}}{u_{c,t}} = p \frac{1 + \tau_{di}}{1 + \tau_c} + \Phi_{1,t} + \tau_{ti} \frac{\text{buy}}{1 + \tau_c} + \frac{1}{\mathbb{E}_t} \left[ \Phi_{2,t+1} + \tau_{ti} \frac{\text{sell}}{1 + \tau_c} \right]
\]
Housing consumption tax

Effective tax rate on housing consumption $\tau_{di} = p_i/p_n - 1$

(a) Ages 25–35

(b) Ages 50–65
User cost for renters $i$

Effective tax on housing consumption $\tau_{di} = \frac{p_r}{p_n} - 1 = -7.5\%$

$\underbrace{p_n}_{\text{baseline}}$

$p_r = r + \hat{\delta} - \pi^H + \tau_p - \tau_f r$

financing subsidy

with property tax rate $\tau_p = 0.1\%$, and financing subsidy $\tau_f = 23.2\%$
Proposition. Allocation $x$ with corresponding values $V_j(x(i); \theta^{t-1})$ is efficient iff it solves the planner problem given $V_j(x(i); \theta^{t-1})$ with a maximum of zero.

⇒ Suppose $x$ does not solve the planner problem, let $\hat{x}$ be a solution. Since $x$ is feasible, $\hat{x}$ generates excess resources. Construct $\tilde{x}$ identical to $\hat{x}$ but increase initial consumption (satisfying ICs). Allocation $\tilde{x}$ Pareto dominates $x$, which is a contradiction.

“resources are left on the table, hence households can be made better off”

⇐ Suppose $x$ is not efficient, there exists a Pareto improving $\hat{x}$. Because $\hat{x}$ is feasible and yields $V_j(x(j, \theta^{t-1}); \theta^{t-1})$, $\hat{x}$ is a candidate solution to the planner problem. Construct $\tilde{x}$ equal to $\hat{x}$ but reduce initial consumption for $i$ strictly better off under $\hat{x}$ (satisfying ICs). $\tilde{x}$ is feasible and increases excess resources, contradicting $x$ solves the planner problem.

“Pareto improvement is feasible, hence there must be excess resources”
⇒ Suppose \( \hat{x} \), not \( x \), solves the planner problem. Because \( x \) is feasible, \( \hat{x} \) generates excess resources. Construct \( \tilde{x} \) identical to \( \hat{x} \) but increase initial consumption (satisfying IC).

⇐ Suppose \( \hat{x} \) is a feasible Pareto improvement yielding values in excess of \( \mathcal{V}_j(x(i); \theta^{t-1}) \). Construct \( \tilde{x} \) equal to \( \hat{x} \) but reduce consumption for \( i \) strictly better off (satisfying IC).
Housing consumption tax

\[ \tau_d(\theta) = \left( g_d(d(\theta), 1 - y(\theta)/\theta^+) - g_d(d(\theta), 1 - y(\theta)/\theta) \right) \frac{q(\theta^+)}{(p_j \pi(\theta))} \]

- prevent high type from mimicking low type
  - benefit of deviation is additional home production
  - depress housing to discourage deviation when complements

- value of relaxing incentive constraint, \( q(\theta^+) \)

\[ q(\theta^+) = I(\theta) + \beta R_p \left( \pi_{\Sigma}(\theta) - \pi_{\Sigma}^+(\theta) \right) \frac{\tau_{y,t-1}}{\Delta g_y(d_{t-1}, y_{t-1}/\theta_{t-1}^+)} \]

\[ I(\theta) = \sum_{s=i+1}^{N} \pi(\theta_s) \frac{1}{v_c(\theta_s)} - (1 - \pi_{\Sigma}(\theta)) \sum_{s=1}^{N} \pi(\theta_s) \frac{1}{v_c(\theta_s)} \]

(Insurance value)
Labor and savings wedge

- labor wedge

\[ \tau_y(\theta) = \left( g_y(d(\theta), 1 - y(\theta)/\theta^+) - g_y(d(\theta), 1 - y(\theta)/\theta) \right) q(\theta^+)/ (p_j \pi(\theta)) \]

- value of relaxing incentive constraint, \( q(\theta^+) \)

\[ q(\theta^+) = I(\theta) + \beta Rp \left( \pi_{\Sigma}(\theta) - \pi_{\Sigma}^+(\theta) \right) \frac{\tau_y(t-1)}{\Delta g_y(d_{t-1}, y_{t-1}/\theta_{t-1}^+)} \]

- savings wedge

\[ \tau_s(\theta^t) = \frac{\left( \sum \pi(\theta_{t+1}|\theta_t)(v_c(c(\theta^{t+1})))^{-1} \right)^{-1}}{\sum \pi(\theta_{t+1}|\theta_t)v_c(c(\theta^{t+1}))} - 1 \]

(Rogerson (1985); Golosov, Troshkin, Tsyvinski (2016))
Robustness and extensions

- Housing supply
  - fixed supply, land permit

- Preferences
  - home work and leisure, general, necessity, present bias, home productivity

- Frictions
  - limited commitment, production externality

- Political economy
  - bargaining
Expenditure share of housing

- Motivation
- Data

### Table

<table>
<thead>
<tr>
<th>Year</th>
<th>Consumption</th>
<th>Nondurables</th>
</tr>
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<tbody>
<tr>
<td>1995</td>
<td>0.100</td>
<td>0.150</td>
</tr>
<tr>
<td>2000</td>
<td>0.200</td>
<td>0.250</td>
</tr>
<tr>
<td>2005</td>
<td>0.180</td>
<td>0.230</td>
</tr>
<tr>
<td>2010</td>
<td>0.210</td>
<td>0.240</td>
</tr>
<tr>
<td>2015</td>
<td>0.220</td>
<td>0.250</td>
</tr>
</tbody>
</table>
Household Wealth in the United States

Composition of Assets and Debt
(In thousands of 2014 United States dollars)

- **Residential**
- **Primary Residence**
- **Business**
- **Financial**
- **Other Debt**
- **Home-Ownership**
## Population, Employment, Hours, 2006–2014

<table>
<thead>
<tr>
<th>Population in millions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>All ages</td>
<td>16.57</td>
</tr>
<tr>
<td>Ages 16 to 64</td>
<td>10.88</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Population growth (%)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>All ages</td>
<td>0.35</td>
</tr>
<tr>
<td>Ages 16 to 64</td>
<td>0.05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Annual hours per worker</th>
<th>1,424</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual hours per person</td>
<td>1,148</td>
</tr>
<tr>
<td>Labor Earnings</td>
<td>Worker (in %)</td>
</tr>
<tr>
<td>------------------------</td>
<td>---------------</td>
</tr>
<tr>
<td>Lower (in euro)</td>
<td>Upper (in euro)</td>
</tr>
<tr>
<td>19,982</td>
<td>36.55</td>
</tr>
<tr>
<td>19,983</td>
<td>33,791</td>
</tr>
<tr>
<td>33,792</td>
<td>67,072</td>
</tr>
<tr>
<td>67,073</td>
<td>52.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Assets</th>
<th>Worker (in %)</th>
<th>Retiree (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower (in euro)</td>
<td>Upper (in euro)</td>
<td></td>
</tr>
<tr>
<td>50,000</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>50,000</td>
<td>30.00</td>
<td>30.00</td>
</tr>
</tbody>
</table>
Loan-to-value Policy

<table>
<thead>
<tr>
<th>Year</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>1.00</td>
</tr>
<tr>
<td>2013</td>
<td>1.00</td>
</tr>
<tr>
<td>2014</td>
<td>1.00</td>
</tr>
<tr>
<td>2015</td>
<td>1.00</td>
</tr>
<tr>
<td>2016</td>
<td>1.00</td>
</tr>
<tr>
<td>2017</td>
<td>1.00</td>
</tr>
<tr>
<td>2018</td>
<td>1.00</td>
</tr>
<tr>
<td>2019</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Wealth over the life cycle

- **Net Wealth**
- **Debt**
- **Real Estate**

Age categorizations:
- 25-34
- 35-44
- 45-54
- 55-64
- 65-74
- 75-84

Wealth (in thousands)
Federal

- **Personal tax receipts**: 1543 bln
- **Tax expenditures on housing**: 276 (18%)
  - Home mortgage interest deduction: 101 (#2)
  - Imputed rent: 76 (#4)
  - Residential capital gains on home sales: 46 (#9)
  - Deductibility of state and local property tax: 25 (#16)
  - Others: 28 (#19), (#25), (#37), (#46)

State and Local

- **Current tax receipts**: 1660 bln
- **Property tax receipts**: 517* (31%)
General Beckerian framework with $i = 1, ..., N$ commodities:

$$\max \{c_i, n_i\}_{i=1} \sum_{i=1} U_i(x_1, ..., x_N),$$

$$x_i = F_i(c_i, n_i) \quad \forall i = 1, ..., N,$$

$$\sum_{i=1} p_i c_i = wn, \text{ with } \sum_{i=1} n_i = 1.$$

My specification is a special case with $N = 2$ commodities:

$$\max_{c, d, n, \ell} U_1(x_1) + U_2(x_2),$$

$$x_1 = F_1(c) \text{ and } x_2 = F_2(d, \ell),$$

$$c + pd = wn, \text{ with } \ell = 1 - n.$$
Preferences in housing literature

Inelastic labor supply, $u(c, d)$


$\Rightarrow$ Lump-sum taxes

Weakly separable, $u(g(c, d), \ell)$

Davis, Heathcote (2005), Favilukis, Mabille, van Nieuwerburgh (2019)

$\Rightarrow$ Uniform commodity taxation

Home production, $u(c, g(d, \ell))$

Greenwood, Hercowitz (1991), Benhabib, Rogerson, Wright (1991)
Alternative preferences

- Housing in home production, $u(c, g(d, \ell))$
  $\Rightarrow$ tax housing when house complements leisure in home production

- Non-housing in home production, $u(d, g(c, \ell))$
  $\Rightarrow$ subsidize consumption when substitutes with leisure in home production

- Inelastic labor supply, $u(c, d)$
  $\Rightarrow$ Lump-sum taxes

- Weakly separable, $u(g(c, d), \ell)$
  $\Rightarrow$ Uniform commodity taxation

Housing consumption subsidized under current tax policy
time and expenditures produce goods

\[ u(c, d, \ell) = v(c) + g(d - d, \ell) \]

home production technology

\[ g(d, \ell) = G \left( (\omega (d - d)^{\frac{\sigma - 1}{\sigma}} + (1 - \omega) \ell^{\frac{\sigma - 1}{\sigma}}) \frac{\sigma}{\sigma - 1} \right) \]

results carry through

isomorphic problem by change of variables, \( \hat{d} \equiv d - d \)
General preferences

- housing tax

\[ \tau_d(\theta) = \Delta u_d \left( c(\theta), d(\theta), 1 - y(\theta)/\theta^+ \right) \left( q(\theta^+) / (p_j \pi(\theta)) \right) \]

value of relaxing IC

- consumption tax

\[ \tau_c(\theta) = \Delta u_c \left( c(\theta), d(\theta), 1 - y(\theta)/\theta^+ \right) q(\theta^+) / \pi(\theta) \]

- transaction tax

\[ \tau_t(\theta) = \Phi_2 \left( d(\theta), d_- \right) \left( \frac{1}{\beta R} \frac{u_c/(1 + \tau_c)}{u_c(\theta)/(1 + \tau_c(\theta))} - 1 \right) \]

premium payout
Home productivity differences

If home productivity is perfectly correlated with market productivity

\[ u(c, d, \ell) = v(c) + g(d, \theta \ell) \]

- housing tax

\[ \tau_d(\theta) = \Delta g_d (d(\theta), \theta^+ - y(\theta)) \frac{q(\theta^+)}{(p_j \pi(\theta))} \]

value of relaxing IC

- transaction tax

\[ \tau_t(\theta) = \Phi_2(d(\theta), d_-) \left( \frac{1}{\beta R} \frac{u_c}{u_c(\theta)} - 1 \right) \]

premium payout
Household bankruptcy

(a) The Netherlands

(b) United States
Engel curves
• allocation for household $i \equiv (j, \theta^{t-1})$ is $x(i) \equiv \{x_{j+v}(\theta^{t+v})\}_{v=0}^{T-t}$

  $x \equiv (c, d, y)$

  birth year, private skill history

• set of households $\mathcal{I} \equiv \left\{ \{(0, \theta_{0})\}_{t=1}^{T}, \{(j, \theta_{0})\}_{j=1}^{\infty} \right\}$

  current generations

  future generations

• an allocation is feasible iff it is resource and incentive feasible
Solving Planner Problem
max $F(K_1, Y_1) + RB_1 - C_1 - I^K_1 - I^H_1 - G_1 - \Phi_1 - B_2$

subject to

- resource feasible

$$F(K_j, Y_j) + RB_j = C_j + I^K_j + I^H_j + G_j + \Phi_j + B_{j+1} \quad \forall j > 1$$

$$D_j = \chi H_j \quad \forall j$$

- incentive feasible (truth-telling)

$$x(i) \in X_{IC}(i) \quad \forall i$$

- promise keeping

$$\nu(i) \leq \nu(x(i); i) \quad \forall i$$
No Ponzi, No Arbitrage

- No Ponzi condition

\[
\lim_{J \to \infty} \frac{1}{R^{j-1}} (B_J + H_J + K_J) \geq 0
\]

- No Arbitrage condition

\[
F_K(K_j, Y_j) + (1 - \delta^K) = R \quad \implies \quad r + \delta^K = F_K(K_j, Y_j)
\]

- Simplifying assumption

\[
r + \delta^H = \chi
\]
Fixed supply

\[
\max F(K_1, Y_1) + RB_1 - C_1 - I^K_1 - G_1 - \Phi_1 - B_2
\]

subject to

- resource feasible
  \[
  F(K_j, Y_j) + RB_j = C_j + I^K_j + G_j + \Phi_j + B_{j+1} \quad \forall j > 1
  \]
  \[
  D_j = \chi \bar{H} \quad \forall j
  \]

- incentive feasible
  \[
  x(i) \in X_{IC}(i) \quad \forall i \in \mathcal{I}
  \]

- promise keeping
  \[
  \mathcal{V}(i) \leq \mathcal{V}(x(i); i) \quad \forall i \in \mathcal{I}
  \]
max $F(K_1, Y_1) + RB_1 - C_1 - I^K_1 - I^H_1 - G_1 - \Phi_1 - B_2$

subject to

- resource feasible
  
  $F(K_j, Y_j) + RB_j = C_j + I^K_j + I^H_j + G_j + \Phi_j + B_{j+1} \quad \forall j > 1$
  
  $D_j = \chi H_j \quad \forall j$
  
  $\bar{L} \geq H_{j+1} - H_j \quad \forall j$

- incentive feasible
  
  $x(i) \in X_{IC}(i) \quad \forall i \in \mathcal{I}$

- promise keeping
  
  $\forall(i) \leq \forall(x(i); i) \quad \forall i \in \mathcal{I}$
The Lagrangian is linearly separable in $x(i)$.

Given values $\mathcal{V}(\mathcal{I})$, solve:

$$\max \sum_{j=1}^{\infty} \frac{1}{R_{j-1}} \left( wY_j - C_j - D_j - \Phi_j - G_j \right) + R \left( K_1 + B_1 + H_1 \right)$$

subject to

- resource feasible
  $$D_j = \chi H_j \quad (p_j) \quad \forall j = 1, \ldots, \iota$$

- incentive feasible
  $$x(i) \in X_{IC}(i) \quad \forall i \in \mathcal{I}$$

- promise keeping
  $$\mathcal{V}(i) \leq \mathcal{V}_j(x(i); \theta^{t-1}) \quad \forall i \in \mathcal{I}$$
Component Planner Problem

Since the Lagrangian is linearly separable in $x(i)$

Given value $\mathcal{V}(i)$, solve:

$$\max \sum_{t, \theta^t} \pi(\theta^t) \left( w y(\theta^t) - c(\theta^t) - p_j d(\theta^t) - \Phi(d(\theta^t), d(\theta^{t-1})) \right) / R^{t-1}$$

subject to

- incentive feasible
  $$x(i) \in X_{IC}(i)$$

- promise keeping
  $$\mathcal{V}(i) \leq \mathcal{V}_j(x(i); \theta^{t-1})$$
reporting strategy $\sigma \equiv \{\sigma_t(\theta^t)\}_\Theta^{t,t}$, with history $\sigma^t = (\sigma_1, \ldots, \sigma_t)$

corresponding allocation $x^\sigma \equiv \{x_t(\sigma^t(\theta^t))\}_\Theta^{t,t}$

continuation utility given reporting strategy $\sigma$

$$V^\sigma(\theta^t) = u(x_t(\sigma^t(\theta^t)); \theta_t) + \beta \sum \pi(\theta_{t+1}|\theta_t) V^\sigma(\theta^{t+1})$$

truthful reporting strategy, $\sigma_t(\theta^t) = \theta_t \ \forall \theta^t$, generating $V(\theta^t)$

incentive compatibility, $X_{IC}(i)$

$$V(\theta^t) \geq V^\sigma(\theta^t) \quad \forall \theta^t, \ \forall \sigma \in \Sigma$$
continuation utility given one-shot deviation strategy \( \sigma^l \)

\[
V^{\sigma^l}(\theta^t) = u(x_t(\theta^{t-1}, l); \theta_t) + \beta \sum \pi(\theta_{t+1}|\theta_t) V^{\sigma^l}(\theta^{t-1}, l, \theta_{t+1})
\]

incentive compatibility with one-shot deviations (\( \forall \theta^t, \sigma^l \))

\[
V(\theta^t) = \max_l V^{\sigma^l}(\theta^t)
\]

\[
= \max_l u(x_t(\theta^{t-1}, l); \theta_t) + \beta \sum \pi(\theta_{t+1}|\theta_t) V(\theta^{t-1}, l, \theta_{t+1})
\]

local downward incentive constraints, \( X_{LD}(i) \)

\[
u(x_t(\theta^{t-1}, \theta_t^i); \theta_t) + \beta \sum \pi(\theta_{t+1}|\theta_t) V(\theta^{t-1}, \theta_t^i, \theta_{t+1})
\]

\[
\geq u(x_t(\theta^{t-1}, \theta_t^-); \theta_t) + \beta \sum \pi(\theta_{t+1}|\theta_t) V(\theta^{t-1}, \theta_t^-, \theta_{t+1})
\]

\( \forall \theta^t \)
Since the Lagrangian is linearly separable in $x(i)$

Given value $V(i)$, solve:

$$\max \sum_{t, \theta^t} \pi(\theta^t) \left( wy(\theta^t) - c(\theta^t) - p_j d(\theta^t) - \Phi(d(\theta^t), d(\theta^{t-1})) \right) / R^{t-1}$$

subject to

- incentive feasible
  $$x(i) \in X_{LD}(i)$$

- promise keeping
  $$V(i) \leq V_j(x(i); \theta^{t-1})$$
Recursive Problem: States and Incentive Constraints

- continuation value

\[ \mathcal{V}(\theta^t) \equiv \sum \pi(\theta_{t+1} | \theta_t) V(\theta^{t+1}) \]

- threat value

\[ \tilde{\mathcal{V}}(\theta^t) \equiv \sum \pi(\theta_{t+1} | \theta^+_t) V(\theta^{t+1}) \]

continuation value given a one-time local deviation

- recursive local downward incentive constraints

\[ u(x_t(\theta^{t-1}, \theta_t); \theta_t) + \beta \sum \pi(\theta_{t+1} | \theta_t) V(\theta^{t-1}, \theta_t, \theta_{t+1}) \]
\[ \geq u(x_t(\theta^{t-1}, \theta^-_t); \theta_t) + \beta \sum \pi(\theta_{t+1} | \theta_t) V(\theta^{t-1}, \theta^-_t, \theta_{t+1}) \quad \forall \theta^t \]
Recursive Problem: States and Incentive Constraints

- **continuation value**
  \[ \mathcal{V}(\theta^t) \equiv \sum \pi(\theta_{t+1}|\theta_t) V(\theta^{t+1}) \]

- **threat value**
  \[ \tilde{\mathcal{V}}(\theta^t) \equiv \sum \pi(\theta_{t+1}|\theta^+_t) V(\theta^{t+1}) \]

continuation value given a one-time local deviation

- **recursive local downward incentive constraints**
  \[ u \left( x_t(\theta^{t-1}, \theta_t); \theta_t \right) + \beta \mathcal{V}(\theta^{t-1}, \theta_t) \]
  \[ \geq u \left( x_t(\theta^{t-1}, \theta_t^-); \theta_t \right) + \beta \tilde{\mathcal{V}}(\theta^{t-1}, \theta_t^-) \quad \forall \theta^t \]
Choose \((x_t(\theta), \nu_t(\theta), \tilde{\nu}_t(\theta))\) to solve

\[
\Pi_t(\nu, \tilde{\nu}, d, \theta_-) = \max \sum \pi(\theta|\theta_-) \left( wy_t(\theta) - c_t(\theta) - p_j d_t(\theta) - \Phi(d_t(\theta), d) \right.
\]
\[
+ \Pi_{t+1}(\nu_t(\theta), \tilde{\nu}_t(\theta^+), d_t(\theta), \theta) / R \right)
\]

subject to

- promise keeping

\[
\nu = \sum \pi(\theta|\theta_-) \left( u(x_t(\theta); \theta) + \beta \nu_t(\theta) \right)
\]

- threat keeping

\[
\tilde{\nu} = \sum \pi(\theta|\theta^+) \left( u(x_t(\theta); \theta) + \beta \nu_t(\theta) \right)
\]

- incentive constraints

\[
u(x_t(\theta); \theta) + \beta \nu_t(\theta) \geq u(x_t(\theta^-); \theta) + \beta \tilde{\nu}_t(\theta) \quad \forall \theta
\]
Newton-Raphson algorithm

Given a state \((\nu, \mu, d, \theta_\cdot)\), \(6N\) unknowns

- Guess \(\{c_i\}_{N-1}, \{d_i\}_N\)

- Optimality \(\{c_i\}_N \implies \{c_N, \{q_i\}_N\}_{N-1}\)
  exploits separability \(v(c)\)

- Optimality \(\{V_i\}_N, \{\tilde{V}_i\}_{N-1} \implies \{\nu_i\}_N, \{\mu_i\}_{N-1}\)
  imply continuation values

- Optimality \(y_N\) and incentive constraints \(\implies \{y_i\}_N\)

- Residual equations: optimality \(\{d_i\}_N, \{y_i\}_{N-1}\)

- Determine \(V, \tilde{V}\) using promise and threat-keeping condition

Parallelize
Given a history $\theta^t$

- Labor wedge
  
  $$- \frac{u_{y,t}(\theta)}{u_{c,t}(\theta)} \equiv w \frac{1-\tau y_i}{1+\tau_c}$$

- Savings wedge
  
  $$u_{c,t}(\theta) \equiv \beta R \sum \pi (\theta' | \theta) \left( 1 - \frac{\tau a_i}{R} \right) u_{c,t+1}(\theta')$$

- Housing wedge
  
  $$\frac{u_{d,t}(\theta)}{u_{c,t}(\theta)} \equiv \frac{1+\tau_{di}}{1+\tau_c} + \frac{\Phi_1(\theta)}{1+\tau_c} + \frac{\tau_{ti}}{1+\tau_c} + \beta \sum \pi (\theta' | \theta) \left( \frac{\Phi_2(\theta')}{1+\tau_c} + \frac{\tau_{ti}^2}{1+\tau_c} \right) \frac{u_c(\theta')}{u_c}$$
House price to decentralize

- Planner’s shadow price for housing services $p_j$

- Rental firm optimality

$$\hat{p}_j = \frac{1}{\chi} (r + \hat{\delta} - \hat{\pi}_{j+1}^H) \hat{p}_j^H$$

- Construction firm optimality implies $\hat{p}_j^H = 1 \ \forall j > \iota$

- Equate $\hat{p}_j$ to $p_j$ to obtain house price path

$$\hat{p}_j^H = 1 + \sum_{s=j}^{\iota} \frac{\phi_s}{R^{s-j+1}}$$

where $\phi_s$ is the multiplier on predetermined housing constraints
Algorithm

- Guess $\mathcal{V}(\mathcal{I})$

1. Guess $\{p_j\}$

   1. Solve a component planner for every $i$ given $\mathcal{V}(i)$, giving $x(i)$

   2. Aggregate and evaluate housing services constraints

   3. Update $\{p_j\}$

2. Aggregate $x(i)$ and evaluate the objective function

- Update $\mathcal{V}(\mathcal{I})$
Two Stage Example
Solve version with one cohort, two types in second period ($\theta_L, \theta_H$)

$$\max_{c,d,y} \quad wy_0 - c_0 - pd_0 + \sum \pi_i (wy_i - c_i - pd_i - \Phi(d_i, d_0)) / R$$

subject to

$$u(c_0, d_0, y_0; \theta_0) + \beta (\pi_H u(c_H, d_H, y_H; \theta_H) + \pi_L u(c_L, d_L, y_L; \theta_L) ) \geq V$$

$$u(c_H, d_H, y_H; \theta_H) \geq u(c_L, d_L, y_L; \theta_H)$$

Characterize efficient distortions, analyze motives for taxation
Efficient housing consumption tax, $\tau_d^c$

- $\tau_{dL}^c \geq 0$ if and only if $\sigma \leq 1$

- Prevent $H$ from mimicking $L$

  benefit of deviation is more home production
depress $d_L$ to discourage deviation

\[
\left(\tau_{dL}^c = (g_d (d_L, 1 - y_L/\theta_H) - g_d (d_L, 1 - y_L/\theta_L)) \frac{\pi_H}{\pi_{LP}} \left(\frac{1}{v_c(c_H)} - \sum \pi_i \frac{1}{v_c(c_i)}\right)\right)
\]
Efficient transaction tax

- $\tau_{di}^t \geq 0$ if and only if $v_c(c_0) \geq \beta R v_c(c_i)$

- Suppose $\sigma = 1$, then efficient to equate MRS and MRT

- $\tau_{di}^t = \Phi_2(d_i, d_0) \left( \frac{1}{\beta R v_c(c_i)} - 1 \right)$

  - precautionary owner lives in smaller house due to concerns over selling fee in bad state

  - implicitly subsidize through transaction tax
Disability Insurance
Component planner for disability insurance

Simplify to separable preferences, proportional adjustment costs $\Phi$

$$\max_{c,d,y} \left( y_0 - c_0 - d_0 + \sum \pi_i(y_i - c_i - d_i - \Phi d_0) \right) / R$$

subject to

$$u(c_0, d_0, y_0; \theta_0) + \beta (\pi_H u(c_H, d_H, y_H; \theta_H) + \pi_L u(c_L, d_L, 0)) \geq V$$

$$u(c_H, d_H, y_H; \theta_H) \geq u(c_L, d_L, 0)$$

Characterize efficient distortions, study implementation with taxes
Implementation for disability insurance

The efficient allocation $x$ is individually optimal given tax system $T$

$$T(Rs_1, y_1, d_1) = \begin{cases} 
\tau_H^s Rs_1 + \tau_H^l + \tau_H^t d_1 & \text{if } y_1 > 0 \\
\tau_L^s Rs_1 + \tau_L^l + \tau_L^t d_1 & \text{otherwise} 
\end{cases}$$

- $\tau_i^s = - \left( \frac{1}{\beta R} \frac{v_c(c_0)}{v_c(c_i)} - 1 \right)$
- $\tau_i^l = y_i - c_i - d_i + (1 - \tau_i^s) Rs_i - \tau_i^t d_i - \Phi d_i$
- $\tau_i^t = \Phi \left( \frac{1}{\beta R} \frac{v_c(c_0)}{v_c(c_i)} - 1 \right)$
Double deviation

Why use the transaction tax ex-post?

- **Alternative transaction tax**

  \[
  \frac{u_d}{u_c} \equiv 1 + \hat{\tau}_t + \beta \left[ \pi_H \frac{u_c(c_H)}{u_c(c_0)} + \pi_L \frac{u_c(c_L)}{u_c(c_0)} \right] \Phi
  \]

- **Double deviation**

  \[
  \frac{\Delta}{u_c} = 1 + \hat{\tau}_t + \beta \frac{u_c(c_L)}{u_c(c_0)} \Phi - \frac{u_d}{u_c} > 0
  \]

  Report $L$ in any case; downsize in period 1
How does tax policy relax the borrowing constraint?

- Savings $s_1$ increase one-for-one in endowment $s_0$
- Tax receipts in final period increase by $Rs_1$
- Borrowing constraint relaxed in Ricardian fashion
- Government debt increases to finance endowment
Component planner with limited commitment

\[
\max_{c,d,y} \quad wy_0 - c_0 - pd_0 + \sum \pi_i \left( wy_i - c_i - pd_i - \Phi(d_i, d_0) \right) / R
\]

subject to

\[
u(c_0, d_0, y_0; \theta_0) + \beta (\pi_H u(c_H, d_H, y_H; \theta_H) + \pi_L u(c_L, d_L, y_L; \theta_L)) \geq V
\]

\[
u(c_H, d_H, y_H; \theta_H) \geq u(c_L, d_L, y_L; \theta_H)
\]

\[
u(c_H, d_H, y_H; \theta_H) \geq \nu_H
\]

\[
u(c_L, d_L, y_L; \theta_L) \geq \nu_L
\]

\[\Rightarrow\quad \text{Housing consumption tax and transaction tax unchanged}\]
Component planner with production externality

Production externality, $F(K, Y) + \zeta D$

$$\max_{c,d,y} \quad wy_0 - c_0 - (p - \zeta)d_0 + \sum \pi_i \left( wy_i - c_i - (p - \zeta)d_i - \Phi(d_i, d_0) \right) / R$$

subject to

$$u(c_0, d_0, y_0; \theta_0) + \beta(\pi_H u(c_H, d_H, y_H; \theta_H) + \pi_L u(c_L, d_L, y_L; \theta_L)) \geq V$$

$$u(c_H, d_H, y_H; \theta_H) \geq u(c_L, d_L, y_L; \theta_H)$$

\[ \implies \] Level shift in housing consumption tax

$$\tau_d(\theta) = -\zeta / p_j + \Delta g_d \left( c(\theta), d(\theta), 1 - y(\theta)/\theta^+ \right) q(\theta^+) / (p_j \pi(\theta))$$
Inverse Euler equation with present bias

- Inverse Euler equation (component problem is identical)

\[
\frac{1}{\nu_c(c_0)} = \frac{1}{\beta R} \sum_i \pi_i \frac{1}{\nu_c(c_i)}
\]

- Household Euler equation

\[
\nu_c(c_0) = \beta R \delta (1 - \tau_s) \sum_i \pi_i \nu_c(c_i)
\]

- Savings wedge

\[
1 - \tau_s = \frac{1}{\delta} \times \frac{1}{\sum \pi_i \nu_c(c_i) \sum \pi_i \frac{1}{\nu_c(c_i)}}
\]

bias > 1  Jensen’s inequality < 1

Present bias is force towards subsidy, not differential subsidy
Nominal average mortgage rate

Average Mortgage Rate


4.00 4.25 4.50 4.75 5.00
Average retirement age

Boerma and Heathcote (2019)
Mortgage regulation

- Mortgage Limit (in 1000s)
  - Working Age
  - Retirees

- Taxable Income (in 1000s)
  - Working Age
  - Retirees
Loan-to-Income and Loan-to-Value

![Graphs showing the relationship between Mortgage Limit and Taxable Income, and Mortgage Structure and Age.]
Wage Dynamics
Bin households in 6 groups based on their training

- High school or vocational (Low)
- University of applied sciences (Medium)
- University (High)

Construct hourly wage rate

$$W_{ijt} = A_j \exp (\tilde{w}_{ijt})$$

$A_j$ is time effect and $\tilde{w}_{ijt}$ is individual specific wage

Construct residual wage $z_{ijt}$ from regression

$$\log W_{ijt} = A_j + X_{ijt} + z_{ijt}$$
Wage Profiles

Quantitative Model

Hourly wage vs. Age graph showing different wage profiles for different age groups: LL, LM, LH, MM, MH, HH.
Wage Dynamics

- Assume wage process is time-invariant

- Statistical model for wages

  \[ \log z_{it} = \log \theta_{it} + \varepsilon_{it} \]
  \[ \log \theta_{it} = \rho \log \theta_{it-1} + u_{it} \]

  \[ \varepsilon_{it} \sim \mathcal{N}(0, \sigma_{\varepsilon}^2) \]
  \[ u_{it} \sim \mathcal{N}(0, \sigma_u^2) \]
  \[ z_{i0} \sim \mathcal{N}(0, \sigma_{z0}^2) \]

  with innovations (i) iid across individuals

  (ii) orthogonal to one another

  (iii) independent across time
Proof of Identification

1. Autoregressive coefficient

\[ \rho = \frac{\text{Cov}(\log z_{it}, \log z_{it-2})}{\text{Cov}(\log z_{it-1}, \log z_{it-2})} \]

2. Variance of transitory innovation

\[ \sigma_{\varepsilon}^2 = \text{Var}(\log z_{it}) - \frac{1}{\rho} \text{Cov}(\log z_{it+1}, \log z_{it}) \]

3. Variance of initial shock

\[ \sigma_{z0}^2 = \text{Var}(\log z_{i0}) - \sigma_{\varepsilon}^2 \]

4. Variance of persistent innovation

\[ \sigma_u^2 = \text{Var}(\log z_{it-1}) - \text{Cov}(\log z_{it}, \log z_{it-2}) - \sigma_{\varepsilon}^2 \]
<table>
<thead>
<tr>
<th>Education</th>
<th>Persistence, $\rho$</th>
<th>Innovation, $\sigma_u^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low, Low</td>
<td>0.9542</td>
<td>0.0096</td>
</tr>
<tr>
<td>Low, Medium</td>
<td>0.9660</td>
<td>0.0087</td>
</tr>
<tr>
<td>Low, High</td>
<td>0.9673</td>
<td>0.0162</td>
</tr>
<tr>
<td>Medium, Medium</td>
<td>0.9570</td>
<td>0.0099</td>
</tr>
<tr>
<td>Medium, High</td>
<td>0.9616</td>
<td>0.0109</td>
</tr>
<tr>
<td>High, High</td>
<td>0.9564</td>
<td>0.0172</td>
</tr>
</tbody>
</table>
## Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Data Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>53</td>
<td>Median life expectancy of 77</td>
</tr>
<tr>
<td>$T_r$</td>
<td>40</td>
<td>Median retirement age of 63</td>
</tr>
<tr>
<td>$r$</td>
<td>0.031</td>
<td>Mean interest rate on mortgage loans</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.439</td>
<td>Capital income share</td>
</tr>
<tr>
<td>$\delta^K$</td>
<td>0.061</td>
<td>Depreciation rate of business capital</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.024</td>
<td>Depreciation rate of residential structures</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.055</td>
<td>Normalization of benchmark user cost, $r + \delta$</td>
</tr>
<tr>
<td>$\iota$</td>
<td>2</td>
<td>Mean building time for new houses</td>
</tr>
<tr>
<td>$\psi_b$</td>
<td>0.020</td>
<td>Mean broker fee, buyers</td>
</tr>
<tr>
<td>$\psi_s$</td>
<td>0.015</td>
<td>Mean broker fee, sellers</td>
</tr>
</tbody>
</table>
CES specification

![Graph showing the CES specification with leisure per Housing Services (logs) on the y-axis and Real Wage (logs) on the x-axis. The graph displays a downward sloping curve indicating a negative relationship between the two variables.](image)
Gap-based estimator

- Gap

\[ x_i \equiv \log \left( \frac{l_i}{d_i} \right) - \log \left( \frac{l_i^*}{d_i^*} \right) \]

where * denote optimality without taxes and transaction costs

- Choose parameter vector \( \zeta \) to solve

\[
\min \int \left( \left( f_p^{\text{model}}(x; \zeta) - f^{\text{data}}(x) \right)^2 + \left( h^{\text{model}}(x; \zeta) - h^{\text{data}}(x) \right)^2 \right) dx
\]

with distribution \( f \) and hazard \( h \)

- Jointly estimate elasticity of substitution \( \sigma \) and moving costs \( \Phi \)

(Berger, Vavra (2015))
Alternative approaches

1. Exogenous variation due to 2017 TCJA with PSID 2017, 2019

\[ \Delta \log \left( \frac{\ell}{d} \right)_{it} = -\sigma \Delta \log \left( \frac{\hat{w}}{\hat{p}} \right)_{it} \]

where user cost \( \hat{p} \) strongly increased due to:

- increased standard deduction, cap on state and local deductions
  e.g. couple deduction from 13 to 24 thousand, cap of 10 thousand

- lowered cap on mortgage interest deduction
  maximum mortgage from 1 million to 750 thousand

- lowered income tax rates, and changed tax brackets

2. Identification from growth rates

\[ \Delta \log \left( \frac{\ell}{d} \right)_{it} = -\sigma \Delta \log \left( \frac{w}{p} \right)_{it} \]
Use:

- leisure hours per adult, \( l \)
- housing consumption controlling for household characteristics, \( d \)
  - number of adults, number of children
- hours-weighted average wage rate, \( w \)

Households that are stable in:

- structure (singles, couples, ...) 
- employment (single, dual earner)
  - drop self-employed, institutionalized
Model validation
Dispersion consumption tax
Asset tax
Variational argument with separable preferences

- Incentive compatible variation (small $\delta$)
  
  $\uparrow d$ by $\varepsilon_d(\delta)$ to $\uparrow$ housing utility by $\delta$, $\varepsilon_d(\delta) = \delta/g_d(d)$
  
  $\downarrow c$ by $\varepsilon_c(\delta)$ to $\downarrow$ consumption utility by $\delta$, $\varepsilon_c(\delta) = \delta/v_c(c)$

- Change in objective function
  
  $\Pi(\delta) = \frac{\delta}{v_c(c)} - (p + \Phi_1) \frac{\delta}{g_d(d)} - \frac{1}{R} \sum \pi(\theta'|\theta) \Phi_2(d(\theta'), d) \frac{\delta}{g_d(d)}$

- At optimum, $\partial \Pi(\delta)/\partial \delta = 0$
  
  $\frac{g_d(d)}{v_c(c)} = (p + \Phi_1) + \frac{1}{R} \sum \pi(\theta'|\theta) \Phi_2(d(\theta'), d)$

- Align planner and private optimality condition
  
  $\frac{1}{R} \sum \pi(\theta'|\theta) \Phi_2(d(\theta'), d) = \beta \sum \pi(\theta'|\theta) (\Phi_2(d(\theta'), d) + \tau^t_d(\theta') \frac{u_c(\theta')}{u_c})$
Savings tax

- tax savings in bad states
  \[ u_c(c_{-}) \leq \beta R u_c(c(\theta)) \]

- implementation of inverse Euler equation
  Kocherlakota (2005), Golosov, Tsyvinski (2006)

- discourage savings by increasing after-tax return risk
  in incomplete markets, households reduce savings to reduce exposure

\[ \tau_s(\theta) = - \left( \frac{1}{\beta R u_c(\theta)} - 1 \right) \]
Bargaining Solution
Egalitarian Solution

Axioms

1. Monotonicity
2. Anonymity
3. Weak Pareto optimality
4. Invariant to additive utility transformations

A bargaining solution satisfies 1–4 iff it is the egalitarian solution.

Egalitarian solution

\[ E(\mathcal{V}, \mathcal{V}) \equiv \max \left\{ \mathcal{V} \in \mathcal{V} \mid \mathcal{V}_i - \mathcal{V}_i = \mathcal{V}_j - \mathcal{V}_j \ \forall \ (i,j) \in (1, \ldots, N) \right\} \]
Axioms

1. **Monotonicity**
   
   If $\mathcal{V} \subset \mathcal{V}'$ and $\mathcal{V} = \mathcal{V}'$, then $F(\mathcal{V}, \mathcal{V}') \geq F(\mathcal{V}, \mathcal{V})$

2. **Weak Pareto optimality**
   
   If $\mathcal{V}' \gg F(\mathcal{V}, \mathcal{V})$, then $\mathcal{V}' \not\in \mathcal{V}$

3. **Anonymity**
   
   Let $\mathcal{P} : \mathbb{R}^N \to \mathbb{R}^N$ be a permutation operator. $F$ is anonymous if $\mathcal{P}(F(\mathcal{V}, \mathcal{V})) = F(\mathcal{P}(\mathcal{V}), \mathcal{P}(\mathcal{V}))$ for every $(\mathcal{V}, \mathcal{V}) \in \mathcal{B}$

4. **Invariant to additive utility transformations**
   
   For every $\xi \in \mathbb{R}^N$ and $(\mathcal{V}, \mathcal{V})$, $F(\mathcal{V} + \xi, \mathcal{V} + \xi) = \xi + F(\mathcal{V}, \mathcal{V})$
Efficiency with Endogenous Prices
Efficiency Test

If an allocation \( x \) does not satisfy

\[
\frac{\tau_{l,t}(\theta)}{1 - \tau_{l,t}(\theta)} = -\varepsilon_{l,t}(\theta) \cdot \frac{1 - F^t(\theta|\theta_-)}{\theta f^t(\theta|\theta_-)} \left( \int_{\theta}^{\bar{\theta}} \frac{u_c(\theta)}{u_c(\hat{\theta})} \frac{f^t(\hat{\theta}|\theta_-)}{1 - F^t(\theta|\theta_-)} \, d\hat{\theta} - \int_{\tilde{\theta}}^{\bar{\theta}} \frac{u_c(\theta)}{u_c(\hat{\theta})} f^t(\theta|\theta_-) \, d\hat{\theta} \right)
\]

\[
+ \beta R \cdot \frac{\tau_{y,t-1}}{1 - \tau_{y,t-1}} \cdot \frac{\varepsilon_{l,t}(\theta)}{\varepsilon_{l,t}(\theta_-)} \cdot \frac{u_c(\theta)}{u_{c,t-1}} \cdot \frac{\theta_{t-1}}{\theta} \cdot \frac{f^{t-1}(\theta_{t-1}|\theta_{t-2})}{f^t(\theta|\theta_{t-1})} \int_{\theta}^{\bar{\theta}} g^t(\hat{\theta}|\theta_-) \, d\hat{\theta},
\]

then it is not efficient.

\[
\left( \varepsilon_{l,t}(\theta) \equiv 1 + 1/\gamma_{l,t}(\theta) \right), \text{ where } \gamma_{l,t}(\theta) = \frac{u_{ll,t}(\theta)l_t(\theta)}{u_{l,t}(\theta)}
\]
If an allocation $x(i)$ solves the cost minimization problem given $\mathcal{V}$, then $x(i)$ solves the welfare maximization problem when the resources are $\Pi_j(x(i); \theta^{t-1})$. Maximum welfare is $\mathcal{V}$.

If an allocation $x(i)$ solves the welfare maximization problem given $\Pi$, then $x(i)$ solves the cost minimization problem when required welfare is $\mathcal{V}_j(x(i); \theta^{t-1})$. Minimum cost is $\Pi$. 
Given $i$ and resources $\Pi$, the welfare maximization problem is:

$$\max_{x(i)} \sum_{v=0}^{T-t} \beta^v \int u\left(c_{s+v}(\theta^{t+v}), d_{s+v}(\theta^{t+v}), y_{s+v}(\theta^{t+v}); \theta_{t+v}\right) dF^{t+v} \left(\theta^{t+v} | \theta^{t-1}\right)$$

subject to

$$C_j(x(i); \theta^{t-1}) \leq \Pi$$

$$x(i) \in X_{IC}(i)$$

- Dual problem to cost minimization problem
## Owner’s consumption tax by income, age

<table>
<thead>
<tr>
<th>Age</th>
<th>Household income (in thousand euro)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&lt; 60</td>
</tr>
<tr>
<td>25–35</td>
<td>-9.8</td>
</tr>
<tr>
<td></td>
<td>4.3</td>
</tr>
<tr>
<td>35–50</td>
<td>-7.1</td>
</tr>
<tr>
<td></td>
<td>10.4</td>
</tr>
<tr>
<td>50–65</td>
<td>-5.1</td>
</tr>
<tr>
<td></td>
<td>7.5</td>
</tr>
<tr>
<td>&gt; 65</td>
<td>-3.6</td>
</tr>
<tr>
<td></td>
<td>10.8</td>
</tr>
<tr>
<td>All</td>
<td>-6.1</td>
</tr>
<tr>
<td></td>
<td>33.0</td>
</tr>
</tbody>
</table>


### Home mortgage interest deduction

<table>
<thead>
<tr>
<th></th>
<th>&lt; 40</th>
<th>40–75</th>
<th>75–125</th>
<th>125–250</th>
<th>&gt; 250</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>25–35</td>
<td>-13.5</td>
<td>-14.5</td>
<td>-15.5</td>
<td>-17.0</td>
<td>-18.8</td>
<td>-14.6</td>
</tr>
<tr>
<td>35–50</td>
<td>-9.9</td>
<td>-10.2</td>
<td>-10.8</td>
<td>-12.6</td>
<td>-14.4</td>
<td>-10.9</td>
</tr>
<tr>
<td>50–65</td>
<td>-5.6</td>
<td>-7.0</td>
<td>-7.5</td>
<td>-8.3</td>
<td>-9.5</td>
<td>-7.2</td>
</tr>
<tr>
<td>&gt; 65</td>
<td>-1.5</td>
<td>-2.2</td>
<td>-3.1</td>
<td>-3.8</td>
<td>-6.6</td>
<td>-2.0</td>
</tr>
<tr>
<td>All</td>
<td>-7.1</td>
<td>-9.5</td>
<td>-10.0</td>
<td>-10.8</td>
<td>-12.3</td>
<td>-8.9</td>
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</table>
### Exemption from asset income taxation

<table>
<thead>
<tr>
<th></th>
<th>&lt; 40</th>
<th>40–75</th>
<th>75–125</th>
<th>125–250</th>
<th>&gt; 250</th>
<th>All</th>
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</thead>
<tbody>
<tr>
<td>25–35</td>
<td>-0.1</td>
<td>-0.1</td>
<td>-0.1</td>
<td>-0.1</td>
<td>0.1</td>
<td>-0.1</td>
</tr>
<tr>
<td>35–50</td>
<td>-0.8</td>
<td>-1.0</td>
<td>-1.3</td>
<td>-1.8</td>
<td>-1.9</td>
<td>-1.2</td>
</tr>
<tr>
<td>50–65</td>
<td>-3.2</td>
<td>-2.9</td>
<td>-3.3</td>
<td>-4.2</td>
<td>-5.0</td>
<td>-3.5</td>
</tr>
<tr>
<td>&gt; 65</td>
<td>-4.7</td>
<td>-7.2</td>
<td>-7.4</td>
<td>-7.4</td>
<td>-7.4</td>
<td>-5.6</td>
</tr>
<tr>
<td>All</td>
<td>-2.4</td>
<td>-1.8</td>
<td>-2.1</td>
<td>-2.9</td>
<td>-3.3</td>
<td>-2.4</td>
</tr>
</tbody>
</table>
### Homeowner subsidy in the United States

<table>
<thead>
<tr>
<th></th>
<th>&lt; 40</th>
<th>40–75</th>
<th>75–125</th>
<th>125–250</th>
<th>&gt; 250</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>25–35</td>
<td>0.3</td>
<td>-6.0</td>
<td>-16.9</td>
<td>-23.2</td>
<td>-26.3</td>
<td>-9.1</td>
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<tr>
<td>35–50</td>
<td>1.9</td>
<td>-6.0</td>
<td>-15.4</td>
<td>-21.6</td>
<td>-21.6</td>
<td>-10.7</td>
</tr>
<tr>
<td>50–65</td>
<td>5.0</td>
<td>-6.0</td>
<td>-12.2</td>
<td>-18.5</td>
<td>-21.6</td>
<td>-9.1</td>
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<tr>
<td>&gt; 65</td>
<td>12.9</td>
<td>1.9</td>
<td>-1.3</td>
<td>-12.2</td>
<td>-16.9</td>
<td>6.6</td>
</tr>
<tr>
<td>All</td>
<td>8.2</td>
<td>-4.4</td>
<td>-12.2</td>
<td>-20.1</td>
<td>-21.6</td>
<td>-6.0</td>
</tr>
</tbody>
</table>

Own calculation based on Poterba and Sinai (2008)
## User Cost in the United States

<table>
<thead>
<tr>
<th></th>
<th>&lt; 40</th>
<th>40–75</th>
<th>75–125</th>
<th>125–250</th>
<th>&gt; 250</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>25–35</td>
<td>6.4</td>
<td>6.0</td>
<td>5.3</td>
<td>4.9</td>
<td>4.7</td>
<td>5.8</td>
</tr>
<tr>
<td>35–50</td>
<td>6.5</td>
<td>6.0</td>
<td>5.4</td>
<td>5.0</td>
<td>5.0</td>
<td>5.7</td>
</tr>
<tr>
<td>50–65</td>
<td>6.7</td>
<td>6.0</td>
<td>5.6</td>
<td>5.2</td>
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<td>5.8</td>
</tr>
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<td>6.5</td>
<td>6.3</td>
<td>5.6</td>
<td>5.3</td>
<td>6.8</td>
</tr>
<tr>
<td>All</td>
<td>6.9</td>
<td>6.4</td>
<td>5.6</td>
<td>5.1</td>
<td>5.0</td>
<td>6.0</td>
</tr>
</tbody>
</table>

Table 2 in Poterba and Sinai (2008)

\[ p^n = r + \hat{\delta}^H - \pi^H = 6.0 + 2.5 - 2.1 = 6.4 \]
Three life-cycle paths

- Productivity
- Effective Labor
- Consumption
- Housing Services
Three life-cycle paths

Productivity

Effective Labor

Consumption

Housing Services
Three life-cycle paths

- **Productivity**: Graph showing productivity per hour by age.
- **Effective Labor**: Graph showing effective labor by age.
- **Consumption**: Graph showing consumption by age.
- **Housing Services**: Graph showing housing services by age.
Three life-cycle paths

Productivity

Effective Labor

Consumption

Housing Services

Age

Age

Age

Age
Three life-cycle paths

- **Productivity**
  - Y-axis: per hour
  - X-axis: Age

- **Effective Labor**
  - Y-axis: thousands
  - X-axis: Age

- **Consumption**
  - Y-axis: thousands
  - X-axis: Age

- **Housing Services**
  - Y-axis: thousands
  - X-axis: Age
Three life-cycle paths

- **Productivity**
- **Effective Labor**
- **Consumption**
- **Housing Services**
Three life-cycle paths

- **Productivity**
  - Graph showing productivity per hour across different ages.

- **Effective Labor**
  - Graph showing effective labor in thousands across different ages.

- **Consumption**
  - Graph showing consumption in thousands across different ages.

- **Housing Services**
  - Graph showing housing services in thousands across different ages.
Efficient housing wedge

(a) complements, $\sigma = 2/3$

(b) substitutes, $\sigma = 2$
Netherlands

Small variation in neighborhood house values
# Taxatieverslag Woningen

<table>
<thead>
<tr>
<th>Locatie woning</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Straatnaam</td>
<td>Wezeboom</td>
</tr>
<tr>
<td>Huisnummer</td>
<td>8</td>
</tr>
<tr>
<td>Postcode</td>
<td>3755 WT</td>
</tr>
<tr>
<td>Woonplaats</td>
<td>Eemnes</td>
</tr>
<tr>
<td>WOZ-objectnummer</td>
<td>31700003060</td>
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</table>

<table>
<thead>
<tr>
<th>Waardepeildatum</th>
<th>1 januari 2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toestandspeildatum</td>
<td>1 januari 2015</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Vastgestelde WOZ-waarde</th>
<th>€212.000</th>
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</thead>
<tbody>
<tr>
<td>Vorige Vastgestelde WOZ-waarde</td>
<td>€208.000</td>
</tr>
<tr>
<td>Verandering van de WOZ-waarde</td>
<td>1,92 %</td>
</tr>
</tbody>
</table>

(waardepeildatum 1 januari 2015)
(waardepeildatum 1 januari 2014)
## Taxatieverslag Woningen

### Locatie woning

<table>
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</tr>
</tbody>
</table>

| WOZ-objectnummer | 31700003060     |

### Waardepeildatum

| Datum          | 1 januari 2015 |

### Vastgestelde WOZ-waarde

| Datum          | €212.000       |

| Datum          | (waardepeildatum 1 januari 2015) |

| Datum          | €208.000       |

| Datum          | (waardepeildatum 1 januari 2014) |

### Verandering van de WOZ-waarde

| Verandering | 1,92 %         |
## Taxatieverslag Woningen

<table>
<thead>
<tr>
<th>Locatie woning</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Straatnaam</td>
<td>Wezeboom</td>
</tr>
<tr>
<td>Huisnummer</td>
<td>8</td>
</tr>
<tr>
<td>Postcode</td>
<td>3755 WT</td>
</tr>
<tr>
<td>Woonplaats</td>
<td>Eemnes</td>
</tr>
<tr>
<td>WOZ-objectnummer</td>
<td>31700003060</td>
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</table>

<table>
<thead>
<tr>
<th>Waardepeildatum</th>
<th>1 januari 2015</th>
</tr>
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<tbody>
<tr>
<td>Toestandspeildatum</td>
<td>1 januari 2015</td>
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<table>
<thead>
<tr>
<th>Vastgestelde WOZ-waarde</th>
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</thead>
<tbody>
<tr>
<td>€212.000 (waardepeildatum 1 januari 2015)</td>
<td></td>
</tr>
<tr>
<td>€208.000 (waardepeildatum 1 januari 2014)</td>
<td></td>
</tr>
</tbody>
</table>

| Verandering van de WOZ-waarde | 1,92 % |
# Rental property value by income, age

<table>
<thead>
<tr>
<th>Age</th>
<th>&lt; 60</th>
<th>60–80</th>
<th>80–120</th>
<th>120–200</th>
<th>&gt; 200</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>25–35</td>
<td>152.5</td>
<td>168.8</td>
<td>188.5</td>
<td>220.9</td>
<td>–</td>
<td>160.1</td>
</tr>
<tr>
<td>35–50</td>
<td>158.4</td>
<td>174.8</td>
<td>197.9</td>
<td>251.4</td>
<td>402.3</td>
<td>170.1</td>
</tr>
<tr>
<td>50–65</td>
<td>161.1</td>
<td>175.5</td>
<td>191.4</td>
<td>221.3</td>
<td>323.0</td>
<td>172.1</td>
</tr>
<tr>
<td>&gt; 65</td>
<td>278.5</td>
<td>213.1</td>
<td>246.0</td>
<td>286.9</td>
<td>477.1</td>
<td>274.2</td>
</tr>
<tr>
<td>All</td>
<td>194.2</td>
<td>177.2</td>
<td>192.7</td>
<td>237.8</td>
<td>375.3</td>
<td>197.2</td>
</tr>
</tbody>
</table>
# Owner property value by income, age

<table>
<thead>
<tr>
<th>Age</th>
<th>&lt; 60</th>
<th>60–80</th>
<th>80–120</th>
<th>120–200</th>
<th>&gt; 200</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>25–35</td>
<td>167.1</td>
<td>185.1</td>
<td>210.8</td>
<td>256.5</td>
<td>321.6</td>
<td>190.6</td>
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<tr>
<td>35–50</td>
<td>212.7</td>
<td>223.5</td>
<td>255.5</td>
<td>324.9</td>
<td>433.4</td>
<td>255.3</td>
</tr>
<tr>
<td>50–65</td>
<td>233.6</td>
<td>245.8</td>
<td>269.4</td>
<td>325.7</td>
<td>425.0</td>
<td>274.4</td>
</tr>
<tr>
<td>&gt; 65</td>
<td>255.0</td>
<td>314.0</td>
<td>348.9</td>
<td>395.4</td>
<td>507.2</td>
<td>285.7</td>
</tr>
<tr>
<td>All</td>
<td>223.0</td>
<td>229.5</td>
<td>260.0</td>
<td>325.4</td>
<td>431.4</td>
<td>255.4</td>
</tr>
</tbody>
</table>
## Owner loan-to-value by income, age

<table>
<thead>
<tr>
<th>Age</th>
<th>&lt; 60</th>
<th>60–80</th>
<th>80–120</th>
<th>120–200</th>
<th>&gt; 200</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>25–35</td>
<td>1.00</td>
<td>1.03</td>
<td>1.04</td>
<td>1.05</td>
<td>1.03</td>
<td>1.03</td>
</tr>
<tr>
<td>35–50</td>
<td>0.75</td>
<td>0.75</td>
<td>0.76</td>
<td>0.80</td>
<td>0.84</td>
<td>0.77</td>
</tr>
<tr>
<td>50–65</td>
<td>0.42</td>
<td>0.50</td>
<td>0.51</td>
<td>0.52</td>
<td>0.56</td>
<td>0.49</td>
</tr>
<tr>
<td>&gt; 65</td>
<td>0.20</td>
<td>0.24</td>
<td>0.29</td>
<td>0.33</td>
<td>0.42</td>
<td>0.22</td>
</tr>
<tr>
<td>All</td>
<td>0.56</td>
<td>0.70</td>
<td>0.69</td>
<td>0.69</td>
<td>0.72</td>
<td>0.64</td>
</tr>
</tbody>
</table>
Wedge definitions

Given a history $\theta^t$

- **Housing**

  $$\frac{u_{d,t}(\theta)}{u_{c,t}(\theta)} \equiv p(1 + \tau_d(\theta)) + \Phi_1(\theta) + \frac{1}{R} \sum \pi(\theta'|\theta)\Phi_2(\theta')$$

- **Labor**

  $$-\frac{u_{y,t}(\theta)}{u_{c,t}(\theta)} \equiv w(1 - \tau_y(\theta))$$

- **Savings**

  $$u_{c,t}(\theta) \equiv \beta R(1 - \tau_s(\theta)) \sum \pi(\theta'|\theta) u_{c,t+1}(\theta')$$
Buying fee

Housing wedge definitions

Homeowner problem