Housing Tax Reform

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Housing is largest component of household consumption and wealth

Housing tax policy targets consumption and liquidity of housing

- consumption tax: property tax, mortgage interest deduction, ...
- transaction tax: capital gains tax, stamp duty

Question: How to efficiently design and reform housing tax policy?

Use an incomplete markets life cycle economy with housing

- 1. housing input in home production
- 2. illiquid due to adjustment costs
- 3. private, stochastic skill risk; elastic labor supply

- Measure current tax policy using tax records for the Netherlands
- **2** Study dynamic Mirrlees theory for efficient housing tax reform
- **③** Quantify theory for economy matched to the Netherlands

9 Measures of current housing consumption and transaction tax

- average owner's consumption subsidy of 8% (range from 20% to -5%)
- transaction tax of 6%

2 Theoretical motives to deviate from uniform commodity taxation

- tax housing when house complements leisure in home production
- subsidize and tax transactions to insure against adjustment costs

 Housing consumption tax should be similar to tax on other goods house is weak complement to leisure → housing consumption tax of 14%

\mathbf{Model}

Three key ingredients:

home technology

- home production preferences $u(c, d, \ell) = v(c) + \overbrace{g(d, \ell)}^{}$
- idiosyncratic skill shocks θ^t , labor supply $y = \theta(1 \ell)$
- own or rent decision driven by
 - tax treatment of owning versus renting
 - size restrictions: own if $d \ge \underline{d} \equiv \chi \underline{h}$ and rent if $d \le \underline{d}$
 - adjustment costs

Study optimality condition for housing services in two problems

• Positive economy of the Netherlands

• measure current effective tax policy

- 2 Mirrlees problem
 - characterize and quantify efficient tax policy

- savings in financial assets, house, mortgage, $s = a + p_H h m \ge 0$
- loan-to-value and income restrictions, $m \leq \kappa_t(h, y)$
- budget constraint

$$(1 + \tau_c)c + \Psi(d, d_-) + s' = y - T_t^y(\tilde{y}) + Ra - T^a(a) + (p_H' - \tau_p p_H - \delta)h - Rm$$

where,

• adjustment costs:
$$\Psi(d, d_{-}) = \overbrace{\Phi(d, d_{-})}^{\text{technology}} + \overbrace{T^{t}(d, d_{-})}^{\text{transaction tax}}$$

• taxable income: $\tilde{y} = y - rm + \tau_o p_H h$

• Household optimality condition:

${\rm consumption}\ {\rm tax}$

$$\underbrace{\frac{u_{d,t}}{u_{c,t}}}_{=p} = p \quad \overbrace{\frac{1+\tau_{di}}{1+\tau_c}}^{\frown}$$

• Household optimality condition:



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• Efficient optimality condition:

$$\frac{u_{d,t}}{u_{c,t}} = p \qquad \dots \qquad + \quad \Phi_{1,t} + \quad \dots \quad + \quad \beta \mathbb{E}_t \left(\Phi_{2,t+1} + \quad \dots \quad \right) \frac{u_{c,t+1}}{u_{c,t}}$$

• Household optimality condition:



• Efficient optimality condition:

$$\frac{u_{d,t}}{u_{c,t}} = p \qquad \dots \qquad + \quad \Phi_{1,t} + \quad \dots \quad + \quad \beta \mathbb{E}_t \left(\Phi_{2,t+1} + \quad \dots \quad \right) \frac{u_{c,t+1}}{u_{c,t}}$$

- 1. Measure current tax policy using tax records for the Netherlands
- 2. Study theory for efficient consumption and transaction tax
- 3. Quantify efficient consumption and transaction tax

Current Housing Tax Policy



- Administrative micro data from 2006 to 2014 on:
 - tax assessed property values
 - mortgage balance
 - who lives where
 - hours and earnings
 - marginal tax rates
- National accounts data on:
 - consumption shares

Used to measure current policy, calibrate wage process, preferences

Measures of housing consumption and transaction tax

O Effective tax rate on housing consumption for household i

$$\tau_{di} \equiv \left(\frac{\text{user cost under current policy}}{\text{user cost absent taxation}}\right)_i - 1$$

2 Effective tax rate on transactions

 $\tau_{ti}^{\text{buy}} \equiv T_1^t(d_i, d_-)$ for house you buy

 $\tau_{ti}^{\text{sell}} \equiv T_2^t(d_i, d_-)$ for house you sell

Later compare to efficient consumption and transaction tax rate

• Statutory tax rate when buying

$$\tau_{ti}^{\rm buy}=6\%$$

• Statutory tax rate when selling

$$\tau_{ti}^{\text{sell}} = 0\%$$

 $r \ + \ \hat{\delta} \ - \ \pi^H$

• opportunity cost of capital, r = 3.1%

average interest rate on mortgages

• depreciation rate of housing, $\hat{\delta} = 2.4\%$

depreciation of housing stock, capital accounts

• capital gain, $\pi^H = -2.8\%$

nominal house price inflation -0.7%, price inflation 2.1%

 \Longrightarrow 8.3%, or monthly rental value of 1,725 for 250K property

User cost for homeowner i



with $\tau_p = 0.1\%$, $\tau_o = 0.6\%$, and loan-to-value ratio $\lambda_i \equiv m_i/p_H h_i$

Use administrative data to measure:

- 1. property values
- 2. mortgage balances
- 3. marginal tax rates

User cost for homeowner i



with $\tau_p = 0.1\%$, $\tau_o = 0.6\%$, and loan-to-value ratio $\lambda_i \equiv m_i/p_H h_i$

Use administrative data to measure:

- 1. property values
- 2. mortgage balances
- 3. marginal tax rates

Then, construct $\tau_{di} \equiv \left(\frac{\text{user cost under current policy}}{\text{user cost absent taxation}}\right)_i - 1$

Histogram of owner's housing consumption tax



Histogram of owner's housing consumption tax



- Transaction tax rate on buyers 6%; on sellers 0%
- Average housing consumption subsidy of 8% (from 20% to -5%)

The model optimality condition for housing services:



Is this efficient? How to efficiently reform housing tax policy?

Reform Theory

So far, positive economy

- measurement of effective housing tax policy
- values under current policy for every household

Next, analyze efficient policy reform

- characterize efficient allocations and housing tax policy
- Pareto improvements using values under current policy

• allocation for household $i \equiv (\underline{j}, \theta^{t-1})$ is $x(i) \equiv \{x_{j+v}(\theta^{t+v})\}_{v=0}^{T-t}$ $x \equiv (c, d, y)$ birth year, private skill history

• set of households: all current $(0, \theta^{t-1})$ and future cohorts (j, θ_0)

• an allocation is feasible iff it is resource and incentive feasible

• allocation x is efficient iff there does not exist a feasible allocation \hat{x} where all households are better off with some strictly better off

• Formulate planning problem to characterize efficient allocations

• Exploit separability to solve household by household

- Solve component problem using a direct mechanism
 - Include only local downward incentive constraints

• Characterize efficient allocation, map to tax wedges

Given a history θ^{t-1}

Consumption wedge

$$\frac{u_d}{u_c} \equiv p(1 + \tau_d(\theta)) + \Phi_1 + \frac{1}{R} \sum \pi(\theta'|\theta) \Phi_2(d(\theta'), d)$$

2 Transaction wedges

$$\underbrace{\frac{u_d}{u_c}}_{risk-averse pricing} = p(1 + \tau_d(\theta)) + \Phi_1 + \underbrace{\beta \sum \pi \left(\theta' | \theta\right) \frac{u_c(\theta')}{u_c} \left(\Phi_2\left(d(\theta'), d\right) + \tau_t(\theta')\right)}_{risk-averse pricing}$$

Characterize, then compare to current housing tax policy

• $\tau_d(\theta) \ge 0$ iff housing and leisure are complements $g_{d\ell}(d, \ell) > 0$

- Prevent high type from mimicking low type
 - benefit of deviation is additional home production
 - depress housing to discourage deviation if complements

- Relax incentive constraint
 - provide additional insurance

- tax transactions when households sell their house in good states $u_c(c_-) \geq \beta R u_c(c(\theta))$
- precautionary downsizing due to adjustment cost in bad states
 - larger house increases exposure to future adjustment cost
 - with incomplete markets, households downsize to reduce exposure

- transaction tax insures households against adjustment costs
 - tax transactions in good times, subsidize transactions in bad times

$$\tau_t(\theta) = \Phi_2(d(\theta), d_-) \left(\underbrace{\frac{1}{\beta R} \frac{u_c}{u_c(\theta)}}_{\text{premium}} - \underbrace{1}_{\text{payout}}\right)$$

Efficient versus current policy

• From the planning problem

$$\frac{u_{d,t}}{u_{c,t}} = p\underbrace{\left(1 + \tau_d(\theta)\right)}_{\geq 1 \text{ iff } g_{d\ell} \geq 0} + \Phi_{1,t} + 0 + \beta \mathbb{E}_t \left(\Phi_{2,t+1} + \underbrace{\tau_t(\theta')}_{\leqslant 0}\right) \frac{u_{c,t+1}}{u_{c,t}}$$

Efficient versus current policy

• From the planning problem

$$\frac{u_{d,t}}{u_{c,t}} = p\underbrace{\left(1 + \tau_d(\theta)\right)}_{\geq 1 \text{ iff } g_{d\ell} \geq 0} + \Phi_{1,t} + 0 + \beta \mathbb{E}_t \left(\Phi_{2,t+1} + \underbrace{\tau_t(\theta')}_{\leqslant 0}\right) \frac{u_{c,t+1}}{u_{c,t}}$$

• From the model of the Netherlands

$$\frac{u_{d,t}}{u_{c,t}} = p \underbrace{\frac{1 + \tau_{di}}{1 + \tau_c}}_{\left[\frac{0.80}{1.13}, \frac{1.05}{1.13}\right]} + \Phi_{1,t} + \underbrace{\frac{\tau_{ti}^{\text{buy}}}{1 + \tau_c}}_{\frac{0.06}{1.13}} + \beta \mathbb{E}_t \left(\Phi_{2,t+1} + \underbrace{\frac{\tau_{ti}^{\text{sell}}}{1 + \tau_c}}_{0}\right) \frac{u_{c,t+1}}{u_{c,t}}$$

Efficient versus current policy

• From the planning problem

$$\frac{u_{d,t}}{u_{c,t}} = p\underbrace{\left(1 + \tau_d(\theta)\right)}_{\geq 1 \text{ iff } g_{d\ell} \geq 0} + \Phi_{1,t} + 0 + \beta \mathbb{E}_t \left(\Phi_{2,t+1} + \underbrace{\tau_t(\theta')}_{\leqslant 0}\right) \frac{u_{c,t+1}}{u_{c,t}}$$

• From the model of the Netherlands

$$\frac{u_{d,t}}{u_{c,t}} = p \underbrace{\frac{1 + \tau_{di}}{1 + \tau_c}}_{\begin{bmatrix} 0.80 \\ 1.13 \end{bmatrix}, \frac{1.05}{1.13}} + \Phi_{1,t} + \underbrace{\frac{\tau_{ti}^{\text{buy}}}{1 + \tau_c}}_{\frac{0.06}{1.13}} + \beta \mathbb{E}_t \left(\Phi_{2,t+1} + \underbrace{\frac{\tau_{ti}^{\text{sell}}}{1 + \tau_c}}_{0} \right) \frac{u_{c,t+1}}{u_{c,t}}$$

Takeaways:

- 1. current consumption subsidy can be efficient only if substitutes
- 2. current transaction tax is not efficient

Measurement

- transaction tax rate on buyers 6%; on sellers 0%
- average consumption subsidy of 8%~(20% subsidy to $5\%~{\rm tax})$

Theory

- subsidize and tax transactions to insure against adjustment costs
- tax housing when house complements leisure in home production

Quantify complementarity housing and leisure in home production

Quantitative Reform

Calibrate positive economy

- 1. Estimate skill process
- 2. Parameterize government policy
- 3. Parameterize technology
- 4. Calibrate preferences

- Do 1, 2, and 3 outside the model
- Use positive economy for 4
Households

•
$$u(c,d,\ell) = \gamma \log c + (1-\gamma) \log \left(\left(\omega d^{\frac{\sigma-1}{\sigma}} + (1-\omega) \ell^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \right)$$

housing services d and leisure ℓ complement iff $\sigma \leq 1$

• six types based on education level, differ in AR(1) skill process

Government

• collects taxes, provides pension benefits, regulates mortgages

Technology Φ

• 2% buyer's fee; 1.5% seller's fee

Today, transaction costs are inefficient in planner problem, $\Phi = 0$

	Value	Target	Data	Model
γ	0.343	Consumption to output ratio	0.64	0.66
ω	0.144	Housing share in consumption	0.17	0.16
σ	0.951	Covariance input and price ratio, $\hat{\beta}$	-0.43	-0.43

Identify elasticity σ by indirect inference from regression coefficient

$$\log\left(\frac{\ell}{d}\right)_i = \mathbb{C} + \beta \log\left(\frac{w}{p}\right)_i + \varepsilon_i$$

Current policy is not efficient



Simple Pareto improving reform

Use efficient reform to guide simple steady state policy reform holding government debt position constant by adjusting transfers

 $\frac{\text{increase to } 14\%}{\tau_{di} \propto \tau_{p} - \tau_{yi} r \lambda_{i} - \tau_{ai} (1 - \lambda_{i}) + \tau_{yi} \tau_{o}}$

- increase τ_p from 0.1% to 1.2% to move from -8% to 14%
- lower τ_o from 0.6% to 0.0% to ensure gain for high income groups

Δc	Δf_h	Welfare Gain by Education Group					
0.68	0.00	1.01	0.68	0.35	0.60	0.25	0.03

Conclusion

How to efficiently design and reform housing policy?

Theory

- tax housing services when housing services complement leisure
- tax and subsidize transactions to insure against adjustment costs

Quantitative

- $\bullet\,$ effective housing subsidy of 8% for average owner decreases in age
- $\bullet\,$ efficient housing tax of 14% almost constant over the life-cycle

Appendix

• housing (over the life cycle, user cost)

Laidler (1969); Aaron (1970); Poterba (1984); Gervais (2002); Fernández-Villaverde,

Krueger (2010); Sommer, Sullivan (2018); Kaplan, Mitman, Violante (2019).

• home production

Becker (1965); Gronau (1977); Greenwood, Hercowitz (1991); Benhabib, Rogerson, Wright (1991); Aguiar, Hurst (2005); Boerma, Karabarbounis (2019).

• public finance

Mirrlees (1971); Atkinson, Stiglitz (1976); Golosov, Kocherlakota, Tsyvinski (2003); Farhi, Werning (2013); Golosov, Troshkin, Tsyvinski (2016); Hosseini, Shourideh (2019).

• this paper: efficient tax reform for incomplete markets life cycle economy with illiquid housing capital and home production • time and expenditures produce goods

$$u(c, d, n_H, \ell) = v(c) + h(d, n_H, \ell)$$

• time constraint $\ell + n_M + n_H = 1$; effective labor supply $y = \theta n_M$

• household indirect utility given an allocation (c, d, y) and skills θ

$$\vartheta(c, d, y; \theta) = \max_{n_H \in [0, 1-n_M)} u(c, d, n_H, \ell) = v(c) + \tilde{h}(d, y)$$

Construction firm

- commits to build houses $Q_{j+1-\iota}$ for period $j+1-\iota$
- builds in period j, valued at p_{j+1}^H , using general good $(p_{j+1}^H = 1)$
- in first period, commits to deliver houses in period ι $p_j^H = 1 \text{ for } j > \iota$

Rental firm

•
$$p_r = \frac{1}{\chi} \Big(r(1 - \tau_f) + \tau_p + \delta - \pi^H \Big) p^H$$

- receive rent p_j per unit of housing services
- borrow at rate r to buy housing capital at p_i^H per unit
- incur maintenance cost δ , pay property tax τ_p
- sell housing capital at price p_{i+1}^H at the end of the period
- receive a subsidy on interest payments τ_f

$$p_{r,j} = \frac{1}{\chi} \Big(r(1 - \tau_f) + \tau_p + \hat{\delta} - \pi_{j+1}^H \Big) p_j^H$$

• housing services

 $D_j = \chi H_j$ services flow D proportional to housing stock H

- housing services
 - $D_j = \chi H_j$
- time to build $\iota \geq 1$

$$H_{j+1} = Q_{j+1-\iota} + H_j$$
 constructions Q planned in advance

- housing services
 - $D_j = \chi H_j$
- time to build $\iota \geq 1$

$$H_{j+1} = Q_{j+1-\iota} + H_j$$

housing supply perfectly inelastic in short run, perfectly elastic in long run

• general good

$$C_{j} + I_{j}^{K} + I_{j}^{H} + G_{j} + \Phi_{j} + B_{j+1} = F(K_{j}, Y_{j}) + RB_{j}$$

where $I_j^H = Q_{j+1-\iota} + \delta H_j$

Income and asset tax



• savings in financial assets, $s = a \ge 0$

• budget constraint

$$(1+\tau_c)c + p_rd + \Phi(d, d_-) + T^t(d, d_-) + s' = wy - T^y_t(\tilde{y}) + Ra - T^a(a)$$

where,

- rental price: p_r
- taxable income: $\tilde{y} = wy$

• largest house to rent, $d \leq \chi \underline{h}$

Equilibrium

Given public spending, construction plans, initial private savings, aggregate assets, an equilibrium is an allocation and prices so that:

- allocation solves household problems
- prices are consistent with firm optimization factor prices, rental prices, house prices
- goods and housing market clear
- government budget constraint is satisfied



$$c+T^c(c)+\Psi(d,d_-)+s'=wy-T^y(\tilde{y})+Ra+\big(p_H{'}-\delta\big)h-Rm$$

Home mortgage interest deduction is a subsidy because of a failure to

tax housing consumption

• Accrual system

$$p_i = r + \hat{\delta} - (1 - \tau_{\pi})\pi^H + \tau_p - \tau_{yi}r\lambda_i - \tau_{ai}(1 - \lambda_i) + \tau_{yi}\tau_o$$

• Realization system

 $T^{t}(d_{t}, d_{t-1}) \longrightarrow T^{t}(d_{t}, d_{t-1}, p_{j+1}^{H}, \overbrace{p_{a}^{H}}^{H})$

Incomplete markets



Complete markets

$$\frac{u_{d,t}}{u_{c,t}} = p \frac{1 + \tau_{di}}{1 + \tau_c} + \Phi_{1,t} + \frac{\tau_{ti}^{\text{buy}}}{1 + \tau_c} + \frac{1}{R} \mathbb{E}_t \left[\Phi_{2,t+1} + \frac{\tau_{ti}^{\text{sell}}}{1 + \tau_c} \right]$$

Housing consumption tax

Effective tax rate on housing consumption $\tau_{di} = p_i/p_n - 1$



(a) Ages 25-35

(b) Ages 50-65

Effective tax on housing consumption $\tau_{di} = p_r/p_n - 1 = -7.5\%$

$$p_r = \overbrace{r + \hat{\delta} - \pi^H}^{\text{baseline, } p^n} + \tau_p - \underbrace{\tau_f r}_{\text{financing subsidy}}$$

with property tax rate $\tau_p = 0.1\%$, and financing subsidy $\tau_f = 23.2\%$

Proof

Proposition. Allocation x with corresponding values $\mathcal{V}_j(x(i); \theta^{t-1})$ is efficient iff it solves the planner problem given $\mathcal{V}_j(x(i); \theta^{t-1})$ with a maximum of zero.

 \Rightarrow Suppose x does not solve the planner problem, let \hat{x} be a solution. Since x is feasible, \hat{x} generates excess resources. Construct \tilde{x} identical to \hat{x} but increase initial consumption (satisfying ICs). Allocation \tilde{x} Pareto dominates x, which is a contradiction. "resources are left on the table, hence households can be made better off"

 \Leftarrow Suppose x is not efficient, there exists a Pareto improving \hat{x} . Because \hat{x} is feasible and yields $\mathcal{V}_j(x(j, \theta^{t-1}); \theta^{t-1}), \hat{x}$ is a candidate solution to the planner problem. Construct \tilde{x} equal to \hat{x} but reduce initial consumption for *i* strictly better off under \hat{x} (satisfying ICs). \tilde{x} is feasible and increases excess resources, contradicting x solves the planner problem. "Pareto improvement is feasible, hence there must be excess resources" \Rightarrow Suppose \hat{x} , not x, solves the planner problem. Because x is feasible, \hat{x} generates excess resources. Construct \tilde{x} identical to \hat{x} but increase initial consumption (satisfying IC).

 \Leftarrow Suppose \hat{x} is a feasible Pareto improvement yielding values in excess of $\mathcal{V}_j(x(i); \theta^{t-1})$. Construct \tilde{x} equal to \hat{x} but reduce consumption for i strictly better off (satisfying IC).

$$\tau_{d}(\theta) = \left(g_{d}\left(d(\theta), 1 - y(\theta)/\theta^{+}\right) - g_{d}\left(d(\theta), 1 - y(\theta)/\theta\right)\right) \underbrace{q(\theta^{+})}_{\text{value of relaxing IC}} / (p_{j}\pi(\theta))$$

- prevent high type from mimicking low type benefit of deviation is additional home production depress housing to discourage deviation when complements
- value of relaxing incentive constraint, $q(\theta^+)$

$$q(\theta^+) = I(\theta) + \beta Rp \left(\pi_{\Sigma}(\theta) - \pi_{\Sigma}^+(\theta) \right) \frac{\tau_{y,t-1}}{\Delta g_y(d_{t-1}, y_{t-1}/\theta_{t-1}^+)}$$

$$\left(\text{Insurance value } I(\theta) = \sum_{s=i+1}^{N} \pi(\theta_s) \frac{1}{v_c(\theta_s)} - (1 - \pi_{\Sigma}(\theta)) \sum_{s=1}^{N} \pi(\theta_s) \frac{1}{v_c(\theta_s)}\right)$$

Labor and savings wedge

• labor wedge

$$\tau_y(\theta) = \left(g_y\left(d(\theta), 1 - y(\theta)/\theta^+\right) - g_y\left(d(\theta), 1 - y(\theta)/\theta\right)\right)q(\theta^+) / (p_j\pi(\theta))$$

• value of relaxing incentive constraint, $q(\theta^+)$

$$q(\theta^+) = I(\theta) + \beta Rp \left(\pi_{\Sigma}(\theta) - \pi_{\Sigma}^+(\theta) \right) \frac{\tau_{y,t-1}}{\Delta g_y(d_{t-1}, y_{t-1}/\theta_{t-1}^+)}$$

• savings wedge

$$\tau_s(\theta^t) = \frac{\left(\sum \pi(\theta_{t+1}|\theta_t) \left(v_c(c(\theta^{t+1}))\right)^{-1}\right)^{-1}}{\sum \pi(\theta_{t+1}|\theta_t) v_c(c(\theta^{t+1}))} - 1$$

(Rogerson (1985); Golosov, Troshkin, Tsyvinski (2016))

• Housing supply

fixed supply, land permit

• Preferences

home work and leisure, general, necessity, present bias, home productivity

• Frictions

limited commitment, production externality

• Political economy

bargaining

Expenditure share of housing



Household wealth, 2014

Composition of Assets and Debts (in thousands of 2014 euro)



Household Wealth in the United States

otivation (Netherland



Population, Employment, Hours, 2006–2014

Population in millions	
All ages	16.57
Ages 16 to 64	10.88
Population growth (%)	
All ages	0.35
Ages 16 to 64	0.05
Annual hours per worker	1,424
Annual hours per person	1,148

Lower (in euro)	Upper (in euro)	Worker (in $\%$)	Retiree (in $\%$)			
Labor Earnings						
	19,982	36.55	18.65			
$19,\!983$	33,791	40.80	22.90			
33,792	67,072	40.80	40.80			
67,073		52.00	52.00			
Assets						
	50,000	0.00	0.00			
50,000		30.00	30.00			

Loan-to-value



Wealth over the life cycle



Data (Polic

Policy in the United States, 2014–2018

Federal

٩	Personal tax receipts	1543 bln
•	Tax expenditures on housing	276 (18%)
	• Home mortgage interest deduction:	101 (#2)
	• Imputed rent	76 (#4)
	• Residential capital gains on home sales	46 (#9)
	• Deductibility of state and local property tax	$25 \ (\#16)$
	• Others: 28 (#19), (#25), (#37), (#46)	

State and Local

- Current tax receipts
- Property tax receipts

1660 bln 517* (31%)

Specification of home production technology

• General Beckerian framework with i = 1, ..., N commodities:

$$\begin{split} & \max_{\{c_i, n_i\}_{i=1}} \sum_{i=1} U_i(x_1, ..., x_N), \\ & x_i = F_i(c_i, n_i) \quad \forall i = 1, ..., N, \\ & \sum_{i=1} p_i c_i = wn, \text{ with } \sum_{i=1} n_i = 1. \end{split}$$

• My specification is a special case with N=2 commodities:

$$\max_{c,d,n,\ell} U_1(x_1) + U_2(x_2),$$

$$x_1 = F_1(c)$$
 and $x_2 = F_2(d, \ell)$,

c + pd = wn, with $\ell = 1 - n$.
Inelastic labor supply, u(c, d)

Gervais (2002), Yang (2008), Chambers, Garriga, Schlagenhauf (2009), Fernández-Villaverde, Krueger

(2011), Kaplan, Violante (2014), Berger, Vavra (2015), Favilukis, Ludvigson, van Nieuwerburgh (2016),

Sommer, Sullivan (2018), Garriga, Hedlund (2019), Garriga, Manuelli, Peralta-Alva (2019), Guren,

McKay, Nakamura, Steinsson (2019), Kaplan, Mitman, Violante (2019)

\implies Lump-sum taxes

Weakly separable, $u(g(c, d), \ell)$

Davis, Heathcote (2005), Favilukis, Mabille, van Nieuwerburgh (2019)

\implies Uniform commodity taxation

Home production, $u(c, g(d, \ell))$

Greenwood, Hercowitz (1991), Benhabib, Rogerson, Wright (1991)

- Housing in home production, $u(c, g(d, \ell))$
 - \implies tax housing when house complements leisure in home production
- Non-housing in home production, $u(d, g(c, \ell))$
 - \implies subsidize consumption when substitutes with leisure in home production
- Inelastic labor supply, u(c, d)
 - \implies Lump-sum taxes
- Weakly separable, $u(g(c,d), \ell)$
 - \implies Uniform commodity taxation

Housing consumption subsidized under current tax policy

• time and expenditures produce goods

$$u(c, d, \ell) = v(c) + g(d - \underline{d}, \ell)$$

• home production technology

$$g(d,\ell) = \mathcal{G}\left(\left(\omega(d-\underline{d})^{\frac{\sigma-1}{\sigma}} + (1-\omega)\,\ell^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}\right)$$

• results carry through

isomorphic problem by change of variables, $\hat{d} \equiv d - \underline{d}$

• housing tax

$$\tau_d(\theta) = \Delta u_d \left(c(\theta), d(\theta), 1 - y(\theta)/\theta^+ \right) \underbrace{q(\theta^+)}_{\text{value of relaxing IC}} / (p_j \pi(\theta))$$

• consumption tax

$$\tau_c(\theta) = \Delta u_c \left(c(\theta), d(\theta), 1 - y(\theta)/\theta^+ \right) q(\theta^+) / \pi(\theta)$$

• transaction tax

$$\tau_t(\theta) = \Phi_2(d(\theta), d_-) \left(\underbrace{\frac{1}{\beta R} \frac{u_c/(1 + \tau_c)}{u_c(\theta)/(1 + \tau_c(\theta))}}_{\text{premium}} - \underbrace{1}_{\text{payout}}\right)$$

If home productivity is perfectly correlated with market productivity

 $u(c,d,\ell) = v(c) + g(d,\theta\ell)$

• housing tax

$$\tau_d(\theta) = \Delta g_d \left(d(\theta), \theta^+ - y(\theta) \right) \underbrace{q(\theta^+)}_{\text{value of relaxing IC}} / (p_j \pi(\theta))$$

• transaction tax

$$\tau_t(\theta) = \Phi_2(d(\theta), d_-) \left(\underbrace{\frac{1}{\beta R} \frac{u_c}{u_c(\theta)}}_{-1} - \underbrace{1}_{-1}\right)$$

premium payout

Household bankruptcy



(a) The Netherlands

(b) United States

Engel curves



Calibration

• allocation for household
$$i \equiv (\underline{j, \theta^{t-1}})$$
 is $x(i) \equiv \{x_{j+v}(\theta^{t+v})\}_{v=0}^{T-t}$

birth year, private skill history

 $x\equiv (c,d,y)$

future generations

• set of households
$$\mathcal{I} \equiv \left\{ \underbrace{\{(0, \theta^{t-1})\}_{t=1}^T}, \overline{\{(j, \theta_0)\}_{j=1}^\infty} \right\}$$

current generations

• an allocation is feasible iff it is resource and incentive feasible

Solving Planner Problem

$$\max F(K_1, Y_1) + RB_1 - C_1 - I_1^K - I_1^H - G_1 - \Phi_1 - B_2$$

subject to

• resource feasible

$$F(K_j, Y_j) + RB_j = C_j + I_j^K + I_j^H + G_j + \Phi_j + B_{j+1} \qquad \forall j > 1$$
$$D_j = \chi H_j \qquad \forall j$$

• incentive feasible (truth-telling)

$$x(i) \in X_{IC}(i) \qquad \forall i$$

$$\mathcal{V}(i) \leq \mathcal{V}(x(i); i) \qquad \forall i$$

• No Ponzi condition

$$\lim_{J \to \infty} \frac{1}{R^{J-1}} \left(B_J + H_J + K_J \right) \ge 0$$

• No Arbitrage condition

$$F_K(K_j, Y_j) + (1 - \delta^K) = R \implies r + \delta^K = F_K(K_j, Y_j)$$

• Simplifying assumption

$$r+\delta^{H}=\chi$$

$$\max F(K_1, Y_1) + RB_1 - C_1 - I_1^K - G_1 - \Phi_1 - B_2$$

subject to

• resource feasible

$$F(K_j, Y_j) + RB_j = C_j + I_j^K + G_j + \Phi_j + B_{j+1} \qquad \forall j > 1$$
$$D_j = \chi \overline{H} \qquad \forall j$$

• incentive feasible

 $x(i) \in X_{IC}(i) \qquad \forall i \in \mathcal{I}$

$$\mathcal{V}(i) \le \mathcal{V}(x(i); i) \qquad \forall i \in \mathcal{I}$$

$$\max F(K_1, Y_1) + RB_1 - C_1 - I_1^K - I_1^H - G_1 - \Phi_1 - B_2$$

subject to

• resource feasible

$$F(K_j, Y_j) + RB_j = C_j + I_j^K + I_j^H + G_j + \Phi_j + B_{j+1} \qquad \forall j > 1$$
$$D_j = \chi H_j \qquad \forall j$$
$$\bar{L} \geq H_{j+1} - H_j \qquad \forall j$$

• incentive feasible

$$x(i) \in X_{IC}(i) \qquad \forall i \in \mathcal{I}$$

$$\mathcal{V}(i) \le \mathcal{V}(x(i); i) \qquad \qquad \forall i \in \mathcal{I}_{29}$$

The Lagrangian is linearly separable in x(i).

Given values $\mathcal{V}(\mathcal{I})$, solve:

$$\max \sum_{j=1}^{\infty} \frac{1}{R^{j-1}} \left(wY_j - C_j - D_j - \Phi_j - G_j \right) + R \left(K_1 + B_1 + H_1 \right)$$

subject to

• resource feasible

$$D_j = \chi H_j$$
 (p_j) $\forall j = 1, \dots, \iota$

• incentive feasible

 $x(i) \in X_{IC}(i) \qquad \forall i \in \mathcal{I}$

$$\mathcal{V}(i) \le \mathcal{V}_j(x(i); \theta^{t-1}) \qquad \forall i \in \mathcal{I}$$

Since the Lagrangian is linearly separable in x(i)

Given value $\mathcal{V}(i)$, solve:

$$\max \sum_{t,\theta^t} \pi(\theta^t) \Big(wy(\theta^t) - c(\theta^t) - p_j d(\theta^t) - \Phi\left(d(\theta^t), d(\theta^{t-1})\right) \Big) \Big/ R^{t-1}$$

subject to

• incentive feasible

$$x(i) \in X_{IC}(i)$$

$$\mathcal{V}(i) \le \mathcal{V}_j(x(i); \theta^{t-1})$$

- reporting strategy $\sigma \equiv \{\sigma_t(\theta^t)\}_{\Theta^t,t}$, with history $\sigma^t = (\sigma_1, \dots, \sigma_t)$
- corresponding allocation $x^{\sigma} \equiv \{x_t(\sigma^t(\theta^t))\}_{\Theta^t, t}$
- continuation utility given reporting strategy σ $V^{\sigma}(\theta^{t}) = u \left(x_{t}(\sigma^{t}(\theta^{t})); \theta_{t} \right) + \beta \sum \pi \left(\theta_{t+1} | \theta_{t} \right) V^{\sigma}(\theta^{t+1})$
- truthful reporting strategy, $\sigma_t(\theta^t) = \theta_t \ \forall \theta^t$, generating $V(\theta^t)$
- incentive compatibility, $X_{IC}(i)$

$$V(\theta^t) \ge V^{\sigma}(\theta^t) \qquad \qquad \forall \theta^t, \ \forall \sigma \in \Sigma$$

• continuation utility given one-shot deviation strategy σ^l

$$V^{\sigma^{l}}(\theta^{t}) = u\left(x_{t}(\theta^{t-1}, l); \theta_{t}\right) + \beta \sum \pi\left(\theta_{t+1} | \theta_{t}\right) V^{\sigma^{l}}(\theta^{t-1}, l, \theta_{t+1})$$

• incentive compatibility with one-shot deviations $(\forall \theta^t, \sigma^l)$

$$V(\theta^t) = \max_l V^{\sigma^l}(\theta^t)$$

= $\max_l u(x_t(\theta^{t-1}, l); \theta_t) + \beta \sum \pi (\theta_{t+1}|\theta_t) V(\theta^{t-1}, l, \theta_{t+1})$

• local downward incentive constraints, $X_{LD}(i)$

$$u(x_t(\theta^{t-1}, \theta_t); \theta_t) + \beta \sum \pi \left(\theta_{t+1} | \theta_t\right) V(\theta^{t-1}, \theta_t, \theta_{t+1})$$

$$\geq u(x_t(\theta^{t-1}, \theta_t^-); \theta_t) + \beta \sum \pi \left(\theta_{t+1} | \theta_t\right) V(\theta^{t-1}, \theta_t^-, \theta_{t+1}) \qquad \forall \theta^t$$

Since the Lagrangian is linearly separable in x(i)

Given value $\mathcal{V}(i)$, solve:

$$\max \sum_{t,\theta^t} \pi(\theta^t) \Big(wy(\theta^t) - c(\theta^t) - p_j d(\theta^t) - \Phi\left(d(\theta^t), d(\theta^{t-1})\right) \Big) \Big/ R^{t-1}$$

subject to

• incentive feasible

$$x(i) \in X_{LD}(i)$$

$$\mathcal{V}(i) \le \mathcal{V}_j(x(i); \theta^{t-1})$$

• continuation value

$$\mathcal{V}(\theta^t) \equiv \sum \pi(\theta_{t+1}|\theta_t) V(\theta^{t+1})$$

• threat value

$$\tilde{\mathcal{V}}(\theta^t) \equiv \sum \pi(\theta_{t+1}|\theta_t^+) V(\theta^{t+1})$$

continuation value given a one-time local deviation

• recursive local downward incentive constraints

$$u(x_t(\theta^{t-1}, \theta_t); \theta_t) + \beta \sum \pi \left(\theta_{t+1} | \theta_t\right) V(\theta^{t-1}, \theta_t, \theta_{t+1})$$

$$\geq u(x_t(\theta^{t-1}, \theta_t^-); \theta_t) + \beta \sum \pi \left(\theta_{t+1} | \theta_t\right) V(\theta^{t-1}, \theta_t^-, \theta_{t+1})$$

$$\forall \theta^t$$

• continuation value

$$\mathcal{V}(\theta^t) \equiv \sum \pi(\theta_{t+1}|\theta_t) V(\theta^{t+1})$$

• threat value

$$\tilde{\mathcal{V}}(\theta^t) \equiv \sum \pi(\theta_{t+1} | \theta_t^+) V(\theta^{t+1})$$

continuation value given a one-time local deviation

• recursive local downward incentive constraints

$$u\left(x_t(\theta^{t-1}, \theta_t); \theta_t\right) + \beta \mathcal{V}(\theta^{t-1}, \theta_t)$$

$$\geq u\left(x_t(\theta^{t-1}, \theta_t^-); \theta_t\right) + \beta \tilde{\mathcal{V}}(\theta^{t-1}, \theta_t^-)$$

Recursive Problem

Choose $(x_t(\theta), \mathcal{V}_t(\theta), \tilde{\mathcal{V}}_t(\theta))$ to solve $\Pi_t(\mathcal{V}, \tilde{\mathcal{V}}, d, \theta_-) = \max \sum \pi(\theta|\theta_-) \Big(wy_t(\theta) - c_t(\theta) - p_j d_t(\theta) - \Phi \left(d_t(\theta), d \right) + \Pi_{t+1}(\mathcal{V}_t(\theta), \tilde{\mathcal{V}}_t(\theta^+), d_t(\theta), \theta) / R \Big)$

subject to

• promise keeping

$$\mathcal{V} = \sum \pi(\theta|\theta_{-}) \left(u \left(x_t(\theta); \theta \right) + \beta \mathcal{V}_t(\theta) \right)$$

• threat keeping

$$\tilde{\mathcal{V}} = \sum \pi(\theta | \theta_{-}^{+}) \left(u \left(x_t(\theta); \theta \right) + \beta \mathcal{V}_t(\theta) \right)$$

• incentive constraints

$$u(x_t(\theta);\theta) + \beta \mathcal{V}_t(\theta) \ge u(x_t(\theta^-);\theta) + \beta \tilde{\mathcal{V}}_t(\theta) \qquad \forall \theta$$

Newton-Raphson algorithm

Given a state $(\nu, \mu, d, \theta_{-})$, 6N unknowns

- Guess $\{c_i\}_{N-1}, \{d_i\}_N$
- Optimality $\{c_i\}_N \Longrightarrow \{c_N, \{q_i\}_{N-1}\}$ exploits separability v(c)
- Optimality $\{\mathcal{V}_i\}_N, \{\tilde{\mathcal{V}}_i\}_{N-1} \Longrightarrow \{\nu_i\}_N, \{\mu_i\}_{N-1}$ imply continuation values
- Optimality y_N and incentive constraints $\Longrightarrow \{y_i\}_N$
- Residual equations: optimality $\{d_i\}_N, \{y_i\}_{N-1}$
- $\bullet\,$ Determine $\mathcal{V},\tilde{\mathcal{V}}$ using promise and threat-keeping condition

Parallelize

Given a history θ^t

• Labor wedge

$$-\frac{u_{y,t}(\theta)}{u_{c,t}(\theta)} \equiv w \frac{1-\tau_{yi}}{1+\tau_c}$$

• Savings wedge

$$u_{c,t}(\theta) \equiv \beta R \sum \pi \left(\theta'|\theta\right) \left(1 - \tau_{ai}/R\right) u_{c,t+1}(\theta')$$

• Housing wedge

$$\frac{u_{d,t}(\theta)}{u_{c,t}(\theta)} \equiv \frac{1+\tau_{di}}{1+\tau_c} + \frac{\Phi_1(\theta)}{1+\tau_c} + \frac{\tau_{ti}^1}{1+\tau_c} + \beta \sum \pi \left(\theta'|\theta\right) \left(\frac{\Phi_2(\theta')}{1+\tau_c} + \frac{\tau_{ti}^2}{1+\tau_c}\right) \frac{u_c(\theta')}{u_c}$$

- Planner's shadow price for housing services p_j
- Rental firm optimality

$$\hat{p}_j = \frac{1}{\chi} \left(r + \hat{\delta} - \hat{\pi}_{j+1}^H \right) \hat{p}_j^H$$

- Construction firm optimality implies $\hat{p}_j^H = 1 \ \forall j > \iota$
- Equate \hat{p}_j to p_j to obtain house price path

$$\hat{p}_{j}^{H} = 1 + \sum_{s=j}^{\iota} \frac{\phi_{s}}{R^{s-j+1}}$$

where ϕ_s is the multiplier on predetermined housing constraints

- Guess $\mathcal{V}(\mathcal{I})$
 - $\textcircled{0} Guess \{p_j\}$
 - **()** Solve a component planner for every *i* given $\mathcal{V}(i)$, giving x(i)
 - **②** Aggregate and evaluate housing services constraints
 - **3** Update $\{p_j\}$
 - **2** Aggregate x(i) and evaluate the objective function
- Update $\mathcal{V}(\mathcal{I})$

Two Stage Example

Component planner

Solve version with one cohort, two types in second period (θ_L, θ_H)

$$\max_{c,d,y} wy_0 - c_0 - pd_0 + \sum \pi_i \Big(wy_i - c_i - pd_i - \Phi(d_i, d_0) \Big) \Big/ R$$

subject to

 $u(c_0, d_0, y_0; \theta_0) + \beta (\pi_H u(c_H, d_H, y_H; \theta_H) + \pi_L u(c_L, d_L, y_L; \theta_L)) \geq \mathcal{V}$

 $u(c_H, d_H, y_H; \theta_H) \ge u(c_L, d_L, y_L; \theta_H)$

Characterize efficient distortions, analyze motives for taxation

• $\tau_{dL}^c \ge 0$ if and only if $\sigma \le 1$

• Prevent H from mimicking L

benefit of deviation is more home production depress d_L to discourage deviation

$$\left(\tau_{dL}^{c} = \left(g_{d}\left(d_{L}, 1 - y_{L}/\theta_{H}\right) - g_{d}\left(d_{L}, 1 - y_{L}/\theta_{L}\right)\right) \frac{\pi_{H}}{\pi_{L}p} \left(\frac{1}{v_{c}(c_{H})} - \sum \pi_{i} \frac{1}{v_{c}(c_{i})}\right)\right)$$

•
$$\tau_{di}^t \ge 0$$
 if and only if $v_c(c_0) \ge \beta R v_c(c_i)$

• Suppose $\sigma = 1$, then efficient to equate MRS and MRT

•
$$\tau_{di}^t = \Phi_2(d_i, d_0) \left(\frac{1}{\beta R} \frac{v_c(c_0)}{v_c(c_i)} - 1 \right)$$

- precautionary owner lives in smaller house due to concerns over selling fee in bad state
- implicitly subsidize through transaction tax

Disability Insurance

Component planner for disability insurance

Simplify to separable preferences, proportional adjustment costs Φ

$$\max_{c,d,y} y_0 - c_0 - d_0 + \sum \pi_i \left(y_i - c_i - d_i - \Phi d_0 \right) / R$$

subject to

 $u(c_{0}, d_{0}, y_{0}; \theta_{0}) + \beta(\pi_{H}u(c_{H}, d_{H}, y_{H}; \theta_{H}) + \pi_{L}u(c_{L}, d_{L}, 0)) \geq \mathcal{V}$

 $u(c_H, d_H, y_H; \theta_H) \ge u(c_L, d_L, 0)$

Characterize efficient distortions, study implementation with taxes

Implementation for disability insurance

The efficient allocation x is individually optimal given tax system \mathbf{T}

$$T(Rs_1, y_1, d_1) = \begin{cases} \tau_H^s Rs_1 + \tau_H^l + \tau_H^t d_1 & \text{if } y_1 > 0\\ \tau_L^s Rs_1 + \tau_L^l + \tau_L^t d_1 & \text{otherwise} \end{cases}$$

•
$$\tau_i^s = -\left(\frac{1}{\beta R}\frac{v_c(c_0)}{v_c(c_i)} - 1\right)$$

•
$$\tau_i^l = y_i - c_i - d_i + (1 - \tau_i^s)Rs_i - \tau_i^t d_i - \Phi d_i$$

•
$$\tau_i^t = \Phi\left(\frac{1}{\beta R}\frac{v_c(c_0)}{v_c(c_i)} - 1\right)$$

Why use the transaction tax ex-post?

• Alternative transaction tax

$$\frac{u_d}{u_c} \equiv 1 + \hat{\tau}_t + \beta \left[\pi_H \frac{u_c(c_H)}{u_c(c_0)} + \pi_L \frac{u_c(c_L)}{u_c(c_0)} \right] \Phi$$

• Double deviation

$$\frac{\Delta}{u_c} = 1 + \hat{\tau}_t + \beta \frac{u_c(c_L)}{u_c(c_0)} \Phi - \frac{u_d}{u_c} > 0$$

Report L in any case; downsize in period 1

How does tax policy relax the borrowing constraint?

- Savings s_1 increase one-for-one in endowment s_0
- Tax receipts in final period increase by Rs_1
- Borrowing constraint relaxed in Ricardian fashion
- Government debt increases to finance endowment

Component planner with limited commitment

Robustness

$$\max_{c,d,y} wy_0 - c_0 - pd_0 + \sum \pi_i \Big(wy_i - c_i - pd_i - \Phi(d_i, d_0) \Big) \Big/ R$$

subject to

 $u(c_0, d_0, y_0; \theta_0) + \beta (\pi_H u(c_H, d_H, y_H; \theta_H) + \pi_L u(c_L, d_L, y_L; \theta_L)) \geq \mathcal{V}$

 $u(c_H, d_H, y_H; \theta_H) \ge u(c_L, d_L, y_L; \theta_H)$

 $u(c_H, d_H, y_H; \theta_H) \ge \mathcal{V}_H$

 $u(c_L, d_L, y_L; \theta_L) \geq \mathcal{V}_L$

 \implies Housing consumption tax and transaction tax unchanged

Component planner with production externality (Robustness

Production externality, $F(K, Y) + \zeta D$

$$\max_{c,d,y} wy_0 - c_0 - (p - \zeta)d_0 + \sum \pi_i \Big(wy_i - c_i - (p - \zeta)d_i - \Phi(d_i, d_0) \Big) \Big/ R$$

subject to

 $u(c_0, d_0, y_0; \theta_0) + \beta(\pi_H u(c_H, d_H, y_H; \theta_H) + \pi_L u(c_L, d_L, y_L; \theta_L)) \ge \mathcal{V}$ $u(c_H, d_H, y_H; \theta_H) \ge u(c_L, d_L, y_L; \theta_H)$

 \implies Level shift in housing consumption tax

$$\tau_d(\theta) = -\zeta/p_j + \Delta g_d \left(c(\theta), d(\theta), 1 - y(\theta)/\theta^+ \right) q(\theta^+) / (p_j \pi(\theta))$$
Inverse Euler equation with present bias

• Inverse Euler equation (component problem is identical)

$$\frac{1}{v_c(c_0)} = \frac{1}{\beta R} \sum \pi_i \frac{1}{v_c(c_i)}$$

• Household Euler equation

$$v_c(c_0) = \beta R \, \delta \left(1 - \tau_s\right) \sum \pi_i v_c(c_i)$$

• Savings wedge

$$1 - \tau_s = \underbrace{\frac{1}{\delta}}_{\text{bias} > 1} \times \underbrace{\frac{1}{\sum \pi_i v_c(c_i) \sum \pi_i \frac{1}{v_c(c_i)}}}_{\text{Jensen's inequality} < 1}$$

Present bias is force towards subsidy, not differential subsidy

Nominal average mortgage rate



Ba

Average retirement age



Boerma and Heathcote (2019)

Mortgage regulation



Loan-to-Income and Loan-to-Value



Owner Quantitativ

Wage Dynamics

Wages

- Bin households in 6 groups based on their training
 - High school or vocational (Low)
 - University of applied sciences (Medium)
 - University (High)
- Construct hourly wage rate

 $W_{ijt} = A_j \exp\left(\tilde{w}_{ijt}\right)$

- A_j is time effect and \tilde{w}_{ijt} is individual specific wage
- Construct residual wage z_{ijt} from regression

$$\log W_{ijt} = \mathbf{A}_j + \mathbf{X}_{ijt} + z_{ijt}$$

Wage Profiles



Wage Dynamics

- Assume wage process is time-invariant
- Statistical model for wages

 $\log z_{it} = \log \theta_{it} + \varepsilon_{it}$ $\log \theta_{it} = \rho \log \theta_{it-1} + u_{it}$ $\varepsilon_{it} \sim \mathcal{N}(0, \sigma_{\varepsilon}^2)$ $u_{it} \sim \mathcal{N}(0, \sigma_u^2)$ $z_{i0} \sim \mathcal{N}(0, \sigma_{z_0}^2)$

with innovations (i) iid across individuals

 $\left(ii\right)$ orthogonal to one another

(iii) independent across time

Proof of Identification

1 Autoregressive coefficient

$$\rho = \frac{\operatorname{Cov}(\log z_{it}, \log z_{it-2})}{\operatorname{Cov}(\log z_{it-1}, \log z_{it-2})}$$

2 Variance of transitory innovation

$$\sigma_{\varepsilon}^2 = \operatorname{Var}(\log z_{it}) - \frac{1}{\rho} \operatorname{Cov}(\log z_{it+1}, \log z_{it})$$

③ Variance of initial shock

$$\sigma_{z0}^2 = \operatorname{Var}(\log z_{i0}) - \sigma_{\varepsilon}^2$$

④ Variance of persistent innovation

$$\sigma_u^2 = \operatorname{Var}(\log z_{it-1}) - \operatorname{Cov}(\log z_{it}, \log z_{it-2}) - \sigma_{\varepsilon}^2$$

Education	Persistence, ρ	Innovation, σ_u^2	
Low, Low	0.9542	0.0096	
Low, Medium	0.9660	0.0087	
Low, High	0.9673	0.0162	
Medium, Medium	0.9570	0.0099	
Medium, High	0.9616	0.0109	
High, High	0.9564	0.0172	

Parameter	Value	Data Target
T	53	Median life expectancy of 77
T_r	40	Median retirement age of 63
r	0.031	Mean interest rate on mortgage loans
α	0.439	Capital income share
δ^K	0.061	Depreciation rate of business capital
δ	0.024	Depreciation rate of residential structures
χ	0.055	Normalization of benchmark user cost, $r+\delta$
ι	2	Mean building time for new houses
ψ_b	0.020	Mean broker fee, buyers
ψ_s	0.015	Mean broker fee, sellers

CES specification



Back

• Gap

$$x_i \equiv \log\left(\frac{\ell_i}{d_i}\right) - \log\left(\frac{\ell_i^*}{d_i^*}\right)$$

where * denote optimality without taxes and transaction costs

• Choose parameter vector $\boldsymbol{\zeta}$ to solve

min
$$\int \left(\left(f_p^{\text{model}}(x;\zeta) - f^{\text{data}}(x) \right)^2 + \left(h^{\text{model}}(x;\zeta) - h^{\text{data}}(x) \right)^2 \right) \mathrm{d}x$$

with distribution f and hazard h

 \bullet Jointly estimate elasticity of substitution σ and moving costs Φ

(Berger, Vavra (2015))

Alternative approaches

Second Se

$$\Delta \log \left(\frac{\ell}{d}\right)_{it} = -\sigma \Delta \log \left(\frac{\hat{w}}{\hat{p}}\right)_{it}$$

where user cost \hat{p} strongly increased due to:

- increased standard deduction, cap on state and local deductions e.g. couple deduction from 13 to 24 thousand, cap of 10 thousand
- lowered cap on mortgage interest deduction maximum mortgage from 1 million to 750 thousand
- lowered income tax rates, and changed tax brackets
- **2** Identification from growth rates

$$\Delta \log \left(\frac{\ell}{d}\right)_{it} = -\sigma \Delta \log \left(\frac{w}{p}\right)_{it}$$

Use:

- $\bullet\,$ leisure hours per adult, $\ell\,$
- housing consumption controlling for household characteristics, *d* number of adults, number of children
- \bullet hours-weighted average wage rate, w

Households that are stable in:

- structure (singles, couples, ...)
- employment (single, dual earner) drop self-employed, institutionalized

Model validation



Model user cost



Dispersion consumption tax



Income tax



Baselin

Asset tax



Variational argument with separable preferences

- Incentive compatible variation (small δ)
 - $\uparrow d$ by $\varepsilon_d(\delta)$ to \uparrow housing utility by δ , $\varepsilon_d(\delta) = \delta/g_d(d)$
 - $\downarrow c$ by $\varepsilon_c(\delta)$ to \downarrow consumption utility by δ , $\varepsilon_c(\delta) = \delta / v_c(c)$
- Change in objective function

$$\Pi(\delta) = \frac{\delta}{v_c(c)} - \left(p + \Phi_1\right) \frac{\delta}{g_d(d)} - \frac{1}{R} \sum \pi\left(\theta'|\theta\right) \Phi_2\left(d(\theta'), d\right) \frac{\delta}{g_d(d)}$$

- At optimum, $\partial \Pi(\delta) / \partial \delta = 0$ $\frac{g_d(d)}{v_c(c)} = (p + \Phi_1) + \frac{1}{R} \sum \pi \left(\theta' | \theta\right) \Phi_2 \left(d(\theta'), d\right)$
- Align planner and private optimality condition $\frac{1}{R} \sum \pi \left(\theta'|\theta\right) \Phi_2\left(d(\theta'), d\right) = \beta \sum \pi \left(\theta'|\theta\right) \left(\Phi_2\left(d(\theta'), d\right) + \tau_d^t(\theta')\right) \frac{u_c(\theta')}{u_c}$

- tax savings in bad states $u_c(c_-) \leq \beta R u_c(c(\theta))$
- implementation of inverse Euler equation Kocherlakota (2005), Golosov, Tsyvinski (2006)

• discourage savings by increasing after-tax return risk in incomplete markets, households reduce savings to reduce exposure

$$\tau_s(\theta) = -\left(\frac{1}{\beta R}\frac{u_c}{u_c(\theta)} - 1\right)$$

Bargaining Solution

Axioms

- Monotonicity
- 2 Anonymity
- **3** Weak Pareto optimality
- **4** Invariant to additive utility transformations

A bargaining solution satisfies 1-4 iff it is the egalitarian solution.

Egalitarian solution

$$E(\underline{\mathcal{V}}, \mathbb{V}) \equiv \max\left\{ \mathcal{V} \in \mathbb{V} \mid \underbrace{\mathcal{V}_i - \underline{\mathcal{V}}_i = \mathcal{V}_j - \underline{\mathcal{V}}_j \ \forall \ (i, j) \in (1, \dots, N)}_{i \in \mathbb{V}} \right\}$$

Computational simplicity

Axioms

Monotonicity

If $\mathbb{V} \subset \mathbb{V}'$ and $\underline{\mathcal{V}} = \underline{\mathcal{V}}'$, then $\mathcal{F}(\underline{\mathcal{V}}, \mathbb{V}') \ge \mathcal{F}(\underline{\mathcal{V}}, \mathbb{V})$

- **②** Weak Pareto optimality If $\mathcal{V}' \gg \mathcal{F}(\underline{\mathcal{V}}, \mathbb{V})$, then $\mathcal{V}' \notin \mathbb{V}$
- 3 Anonymity

Let $\mathcal{P} : \mathbb{R}^N \to \mathbb{R}^N$ be a permutation operator. \mathcal{F} is anonymous if $\mathcal{P}(\mathcal{F}(\underline{\mathcal{V}}, \mathbb{V})) = \mathcal{F}(\mathcal{P}(\underline{\mathcal{V}}), \mathcal{P}(\mathbb{V}))$ for every $(\underline{\mathcal{V}}, \mathbb{V}) \in \mathcal{B}$

• Invariant to additive utility transformations For every $\xi \in \mathbb{R}^N$ and $(\underline{\mathcal{V}}, \mathbb{V}), \ \mathcal{F}(\underline{\mathcal{V}} + \xi, \mathbb{V} + \xi) = \xi + \mathcal{F}(\underline{\mathcal{V}}, \mathbb{V})$

Efficiency with Endogenous Prices

Wedges

If an allocation x does not satisfy

$$\frac{\tau_{l,t}(\theta)}{1-\tau_{l,t}(\theta)} = -\varepsilon_{l,t}(\theta)\frac{1-F^{t}(\theta|\theta_{-})}{\theta f^{t}(\theta|\theta_{-})} \left(\int_{\theta}^{\bar{\theta}} \frac{u_{c}(\theta)}{u_{c}(\hat{\theta})} \frac{f^{t}(\hat{\theta}|\theta_{-})}{1-F^{t}(\theta|\theta_{-})} \mathrm{d}\hat{\theta} - \int_{\underline{\theta}}^{\bar{\theta}} \frac{u_{c}(\theta)}{u_{c}(\hat{\theta})} f^{t}(\theta|\theta_{-}) \mathrm{d}\hat{\theta}\right)$$

$$+\beta R \frac{\tau_{y,t-1}}{1-\tau_{y,t-1}} \frac{\varepsilon_{l,t}(\theta)}{\varepsilon_{l,t-1}} \frac{u_c(\theta)}{u_{c,t-1}} \frac{\theta_{t-1}}{\theta} \frac{f^{t-1}(\theta_{t-1}|\theta_{t-2})}{f^t(\theta|\theta_{t-1})} \int_{\theta}^{\bar{\theta}} g^t(\hat{\theta}|\theta_-) \mathrm{d}\hat{\theta} ,$$

then it is not efficient.

$$\left(\varepsilon_{l,t}(\theta) \equiv 1 + 1/\gamma_{l,t}(\theta) \text{ , where } \gamma_{l,t}(\theta) = \frac{u_{ll,t}(\theta)l_t(\theta)}{u_{l,t}(\theta)}\right)$$

If an allocation x(i) solves the cost minimization problem given

 ν, then x(i) solves the welfare maximization problem when the
 resources are Π_j(x(i); θ^{t-1}). Maximum welfare is *ν*.

• If an allocation x(i) solves the welfare maximization problem given Π , then x(i) solves the cost minimization problem when required welfare is $\mathcal{V}_j(x(i); \theta^{t-1})$. Minimum cost is Π . Given i and resources $\Pi,$ the welfare maximization problem is:

$$\max_{x(i)} \sum_{v=0}^{T-t} \beta^v \int u\Big(c_{s+v}(\theta^{t+v}), d_{s+v}(\theta^{t+v}), y_{s+v}(\theta^{t+v}); \theta_{t+v}\Big) \mathrm{d}F^{t+v}\left(\theta^{t+v}|\theta^{t-1}\right)$$

subject to

 $\mathcal{C}_j(x(i); \theta^{t-1}) \le \Pi$

 $x(i) \in \mathcal{X}_{IC}(i)$

• Dual problem to cost minimization problem

Age	Household income (in thousand euro)					
	< 60	60-80	80-120	120 - 200	> 200	All
25-35	-9.8 4.3	-10.7 3.7	-11.6 3.3	-12.6 0.8	-14.1 0.0	-10.8 12.1
35-50	-7.1 10.4	-7.4 10.7	-8.3 15.0	-10.2 7.4	-11.4 1.2	-8.2 44.6
50-65	-5.1 7.5	-6.0 5.3	-6.8 8.7	-8.2 5.5	-10.0 0.9	-6.7 27.8
> 65	-3.6 10.8	-6.4 2.1	-7.2 1.8	-7.7 0.7	-9.5 0.1	-4.7 15.5
All	-6.1 33.0	-7.6 21.8	-8.1 28.8	-9.4 14.3	-11.0 2.1	-7.5 100.0

	< 40	40 - 75	75 - 125	125 - 250	> 250	All
25 - 35	-13.5	-14.5	-15.5	-17.0	-18.8	-14.6
35 - 50	-9.9	-10.2	-10.8	-12.6	-14.4	-10.9
50 - 65	-5.6	-7.0	-7.5	-8.3	-9.5	-7.2
> 65	-1.5	-2.2	-3.1	-3.8	-6.6	-2.0
All	-7.1	-9.5	-10.0	-10.8	-12.3	-8.9

	< 40	40 - 75	75 - 125	125 - 250	> 250	All
25 - 35	-0.1	-0.1	-0.1	-0.1	0.1	-0.1
35 - 50	-0.8	-1.0	-1.3	-1.8	-1.9	-1.2
50 - 65	-3.2	-2.9	-3.3	-4.2	-5.0	-3.5
> 65	-4.7	-7.2	-7.4	-7.4	-7.4	-5.6
All	-2.4	-1.8	-2.1	-2.9	-3.3	-2.4

	< 40	40 - 75	75 - 125	125 - 250	> 250	All
25 - 35	0.3	-6.0	-16.9	-23.2	-26.3	-9.1
35 - 50	1.9	-6.0	-15.4	-21.6	-21.6	-10.7
50 - 65	5.0	-6.0	-12.2	-18.5	-21.6	-9.1
> 65	12.9	1.9	-1.3	-12.2	-16.9	6.6
All	8.2	-4.4	-12.2	-20.1	-21.6	-6.0

Own calculation based on Poterba and Sinai (2008)

	< 40	40-75	75 - 125	125 - 250	> 250	All
25 - 35	6.4	6.0	5.3	4.9	4.7	5.8
35 - 50	6.5	6.0	5.4	5.0	5.0	5.7
50 - 65	6.7	6.0	5.6	5.2	5.0	5.8
> 65	7.2	6.5	6.3	5.6	5.3	6.8
All	6.9	6.4	5.6	5.1	5.0	6.0

Table 2 in Poterba and Sinai (2008)

$$p^n = r + \hat{\delta}^H - \pi^H = 6.0 + 2.5 - 2.1 = 6.4$$

Three life-cycle paths














Efficient housing wedge





Small variation in neighborhood house values

Taxatieverslag Woningen

Locatie woning

 Straatnaam
 Wezeboom

 Huisnummer
 8

 Postcode
 3755 WT

 Woonplaats
 Eemnes

WOZ-objectnummer

31700003060



Waardepeildatum	1 januari 2015	Toestandspeildatum	1 januari 2015	
Vastgestelde WOZ-waarde		€212.000	(waardepeildatum 1 januari 2015)	
Vorige Vastgestelde WOZ-waarde		€208.000	(waardepeildatum 1 januari 2014)	
Verandering van de WOZ-waarde		1,92 %		

Taxatieverslag Woningen

Locatie woning	
Straatnaam	Wezeboom
Huisnummer	8
Postcode	3755 WT
Woonplaats	Eemnes
WOZ-objectnummer	31700003060
Waardeneildatum	1 ianuari 2015



Waardepeildatum	1 januari 2015	Toestandspeildatum	1 januari 2015
Vastgestelde WOZ-waarde		€212.000	(waardepeildatum 1 januari 2015)
Vorige Vastgestelde WOZ-waarde		€208.000	(waardepeildatum 1 januari 2014)
Verandering van de WOZ-waarde		1,92 %	

(Back) Olde

Taxatieverslag Woningen

Locatie woning

Straatnaam	Wezeboom
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Verandering van de WOZ-waarde		1,92 %	

Homogeneity in Housing



Age	Household income (in thousand euro)					
	< 60	60-80	80-120	120-200	> 200	All
25 - 35	152.5	168.8	188.5	220.9	_	160.1
35 - 50	158.4	174.8	197.9	251.4	402.3	170.1
50 - 65	161.1	175.5	191.4	221.3	323.0	172.1
> 65	278.5	213.1	246.0	286.9	477.1	274.2
All	194.2	177.2	192.7	237.8	375.3	197.2

Age	Household income (in thousand euro)					
	< 60	60-80	80-120	120-200	> 200	All
25 - 35	167.1	185.1	210.8	256.5	321.6	190.6
35 - 50	212.7	223.5	255.5	324.9	433.4	255.3
50 - 65	233.6	245.8	269.4	325.7	425.0	274.4
> 65	255.0	314.0	348.9	395.4	507.2	285.7
All	223.0	229.5	260.0	325.4	431.4	255.4

Age	Household income (in thousand euro)					
	< 60	60-80	80-120	120-200	> 200	All
25 - 35	1.00	1.03	1.04	1.05	1.03	1.03
35 - 50	0.75	0.75	0.76	0.80	0.84	0.77
50 - 65	0.42	0.50	0.51	0.52	0.56	0.49
> 65	0.20	0.24	0.29	0.33	0.42	0.22
All	0.56	0.70	0.69	0.69	0.72	0.64

Given a history θ^t

• Housing

$$\frac{u_{d,t}(\theta)}{u_{c,t}(\theta)} \equiv p(1 + \tau_d(\theta)) + \Phi_1(\theta) + \frac{1}{R} \sum \pi\left(\theta'|\theta\right) \Phi_2(\theta')$$

• Labor

$$-\frac{u_{y,t}(\theta)}{u_{c,t}(\theta)} \equiv w(1 - \tau_y(\theta))$$

• Savings

$$u_{c,t}(\theta) \equiv \beta R(1 - \tau_s(\theta)) \sum \pi \left(\theta'|\theta\right) u_{c,t+1}(\theta')$$

Selling fee



Buying fee



Transaction tax



Transaction cost

