Rents and Intangible Capital: A Q+ Framework

Nicolas Crouzet\textsuperscript{1}  Janice Eberly\textsuperscript{2}

\textsuperscript{1}Kellogg and Chicago Fed
\textsuperscript{2}Kellogg and NBER

The views expressed here are the authors’ own and do not represent those of the Federal Reserve Bank of Chicago or the Federal Reserve system.
Question

Why is PPE investment weak?
Question

Why is PPE investment weak?
PPE investment is weak
Question

Why is PPE investment weak?

- ... *despite* high returns?
PPE investment is weak despite high returns

![Graph showing the ratio of profits to capital (Π / K)](image_url)

- **Π / K** (Compustat, aggregate)
- **Π / K** (BEA)

*Sectoral data*
Question

Why is PPE investment weak?

- ... *despite* high returns?
Question

Why is PPE investment weak?

- ... *despite* high returns?

Explanation 1: economic rents (Barkai, 2017; Gutierrez and Philippon, 2018)
Why is PPE investment weak?

- ... **despite** high returns?

**Explanation 1:** **Economic rents**

- rents reduce the incentive to increase scale at the margin

(Barkai, 2017; Gutierrez and Philippon, 2018)
Why is PPE investment weak?

- ... *despite* high returns?

**Explanation 1:** *economic rents*  
(Barkai, 2017; Gutierrez and Philippon, 2018)

- rents reduce the incentive to increase scale at the margin

**Explanation 2:** *intangibles*  
(Crouzet and Eberly, 2018, 2019)
The growing importance of intangible capital

K₁ = PPE and K₂ = R&D capital.
Why is PPE investment weak?

- ... despite high returns?

Explanation 1: economic rents (Barkai, 2017; Gutierrez and Philippon, 2018)

- rents reduce the incentive to increase scale at the margin

Explanation 2: intangibles (Crouzet and Eberly, 2018, 2019)

- omitting intangibles biases upward measured returns
Why is PPE investment weak?

- ... *despite* high returns?

**Explanation 1:** economic rents  
(Barkai, 2017; Gutierrez and Philippon, 2018)

- rents reduce the incentive to increase scale at the margin

**Explanation 2:** intangibles  
(Crouzet and Eberly, 2018, 2019)

- omitting intangibles biases upward measured returns
This paper

What we do:
- Neo-classical investment model with rents + intangibles
- Quantify in aggregate and sectoral data

What we find:
1. "Investment gap" ≡ \( Q - q = \text{Rents} \rightarrow \text{physical capital} + \text{Omitted capital effect} \times (\text{Rents} \rightarrow \text{intangibles}) \)
2. Aggregate data: \( \text{intan} = 1/3 \text{ to } 2/3 \text{ of gap} \)
3. Sectoral data: no common trends; \( \text{intan} > 2/3 \text{ of gap} \text{ in high-growth sectors} \)
This paper

What we do:

- Neo-classical investment model with rents + intangibles
- Quantify in aggregate and sectoral data

What we find:

1. "Investment gap" \(= Q - q\) = Rents \(\rightarrow\) physical capital + Omitted capital effect + (Rents \(\rightarrow\) intangibles) \times (Omitted capital effect)

2. Aggregate data: intan = 1/3 to 2/3 of gap

3. Sectoral data: no common trends; intan > 2/3 of gap in high-growth sectors
This paper

What we do:
- Neo-classical investment model with rents + intangibles
- Quantify in aggregate and sectoral data

What we find:

1. "Investment gap" ≡ $Q - q = \text{Rents} \to \text{physical capital} + \text{Omitted capital effect} + (\text{Rents} \to \text{intangibles}) \times (\text{Omitted capital effect})$

2. Aggregate data: intan = 1/3 to 2/3 of gap

3. Sectoral data: no common trends; intan > 2/3 of gap in high-growth sectors
This paper

What we do:

- Neo-classical investment model with rents + intangibles


- Quantify in aggregate and sectoral data

What we find:

1. "Investment gap" ≡ $Q - q$
This paper

What we do:
- Neo-classical investment model with rents + intangibles
- Quantify in aggregate and sectoral data

What we find:
1. "Investment gap" $\equiv Q - q = \text{Rents} \rightarrow \text{physical capital}$
   + Omitted capital effect
   + $(\text{Rents} \rightarrow \text{intangibles}) \times (\text{Omitted capital effect})$
This paper

What we do:
- Neo-classical investment model with rents + intangibles
  
- Quantify in aggregate and sectoral data

What we find:
1. "Investment gap" $\equiv Q - q = \text{Rents} \rightarrow \text{physical capital}$
   + Omitted capital effect
   + (Rents $\rightarrow$ intangibles) $\times$ (Omitted capital effect)

2. Aggregate data:
This paper

What we do:

- Neo-classical investment model with rents + intangibles


- Quantify in aggregate and sectoral data

What we find:

1. “Investment gap” \( \equiv Q - q \) = Rents → physical capital

   + Omitted capital effect

   + (Rents → intangibles) \times (Omitted capital effect)

2. Aggregate data: intan = 1/3 to 2/3 of gap
This paper

What we do:

- Neo-classical investment model with rents + intangibles
  

- Quantify in aggregate and sectoral data

What we find:

1. “Investment gap” $\equiv Q - q = \text{Rents} \rightarrow \text{physical capital}$
   + Omitted capital effect
   + $(\text{Rents} \rightarrow \text{intangibles}) \times (\text{Omitted capital effect})$

2. Aggregate data: intan = $1/3$ to $2/3$ of gap

3. Sectoral data:
This paper

What we do:
- Neo-classical investment model with rents + intangibles
- Quantify in aggregate and sectoral data

What we find:
1. "Investment gap" $\equiv Q - q = \text{Rents} \rightarrow \text{physical capital} + \text{Omitted capital effect} + (\text{Rents} \rightarrow \text{intangibles}) \times (\text{Omitted capital effect})$

2. Aggregate data: intan = 1/3 to 2/3 of gap

3. Sectoral data: no common trends; intan > 2/3 of gap in high-growth sectors
1. Theory
A (fairly) general $Q$-theory model
A (fairly) general $Q$-theory model

$$V^c_t(K_t) = \max_{K_{t+1}} \Pi_t(K_t) - \tilde{\Phi}_t(K_t, K_{t+1}) + \mathbb{E}_t \left[ M_{t,t+1} V^c_{t+1}(K_{t+1}) \right]$$

s.t. $K_t = \left\{ K_{n,t} \right\}_{n=1}^2$, $K_t = F_t(K_t)$
A (fairly) general $Q$-theory model

$$V^c_t(K_t) = \max_{K_{t+1}} \Pi_t(K_t) - \tilde{\Phi}_t(K_t, K_{t+1}) + \mathbb{E}_t \left[ M_{t,t+1} V^c_{t+1}(K_{t+1}) \right]$$

s.t. $K_t = \left\{ K_{n,t} \right\}_{n=1}^2$, $K_t = F_t(K_t)$

- $F_t(.)$ is homogeneous of degree 1
A (fairly) general $Q$-theory model

$$V_c^t(K_t) = \max_{K_{t+1}} \Pi_t(K_t) - \tilde{\Phi}_t(K_t, K_{t+1}) + \mathbb{E}_t \left[ M_{t,t+1} V_{t+1}^c(K_{t+1}) \right]$$

s.t. $K_t = \left\{ K_{n,t} \right\}_{n=1}^2$, $K_t = F_t(K_t)$

- $F_t(.)$ is homogeneous of degree 1
- $\Pi_t(.)$ is homogeneous of degree $\frac{1}{\mu}$, $\mu \geq 1$
A (fairly) general $Q$-theory model

\[
V_c^t(K_t) = \max_{K_{t+1}} \Pi_t(K_t) - \tilde{\Phi}_t(K_t, K_{t+1}) + \mathbb{E}_t \left[ M_{t,t+1} V_c^{t+1}(K_{t+1}) \right]
\]

s.t. \( K_t = \left\{ K_{n,t} \right\}_{n=1}^2, \quad K_t = F_t(K_t) \)

- \( F_t(.) \) is homogeneous of degree 1
- \( \Pi_t(.) \) is homogeneous of degree \( \frac{1}{\mu}, \quad \mu \geq 1 \)
- Total investment costs \( \tilde{\Phi}_t(.,.) \) satisfy:

\[
\tilde{\Phi}_t(K_t, K_{t+1}) = \sum_{n=1}^{2} \Phi_{n,t} \left( \frac{K_{n,t+1}}{K_{n,t}} \right) K_{n,t}, \quad \Phi_{n,t} \text{ increasing and convex.}
\]
A (fairly) general $Q$-theory model

$$V^c_t(K_t) = \max_{K_{t+1}} \Pi_t(K_t) - \tilde{\Phi}_t(K_t, K_{t+1}) + \mathbb{E}_t \left[ M_{t,t+1} V^c_{t+1}(K_{t+1}) \right]$$

s.t. $K_t = \left\{ K_{n,t} \right\}_{n=1}^2$, $K_t = F_t(K_t)$

- $F_t(.)$ is homogeneous of degree 1
- $\Pi_t(.)$ is homogeneous of degree $\frac{1}{\mu}$, $\mu \geq 1$
- Total investment costs $\tilde{\Phi}_t(.,.)$ satisfy:

$$\tilde{\Phi}_t(K_t, K_{t+1}) = \sum_{n=1}^2 \Phi_{n,t} \left( \frac{K_{n,t+1}}{K_{n,t}} \right) K_{n,t}, \quad \Phi_{n,t} \text{ increasing and convex.}$$
The value of the firm

### Lemma

\[
V^e_t = q_{1,t}K_{1,t+1} + q_{2,t}K_{2,t+1} + \sum_{n=1}^{2} \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k}(\mu - 1)\Pi_{n,t+k}K_{n,t+k}]
\]

\[
V^e_t = \mathbb{E}_t [M_{t,t+1}V^c_{t+1}], \quad q_{n,t} \equiv \frac{\partial V^e_t}{\partial K_{n,t+1}}, \quad \Pi_{n,t} \equiv \frac{\partial \Pi_t}{\partial K_t} \frac{\partial K_t}{\partial K_{n,t}}.
\]
The value of the firm

**Lemma**

\[ V_t^e = q_{1,t}K_{1,t+1} + q_{2,t}K_{2,t+1} + \sum_{n=1}^{2} \sum_{k \geq 1} \mathbb{E}_t \left[ M_{t,t+k}(\mu - 1)\Pi_{n,t+k}K_{n,t+k} \right] \]

\[ V_t^e = \mathbb{E}_t \left[ M_{t,t+1}V_{t+1}^c \right] , \quad q_{n,t} \equiv \frac{\partial V_t^e}{\partial K_{n,t+1}} , \quad \Pi_{n,t} \equiv \frac{\partial \Pi_t}{\partial K_t} \frac{\partial K_t}{\partial K_{n,t}}. \]

- \( \mu = 1 \): \( V_t^e = q_{1,t}K_{1,t+1} + q_{2,t}K_{2,t+1} \)  
  (Hayashi and Inoue, 1991)
The value of the firm

Lemma

\[
V_t^e = q_{1,t}K_{1,t+1} + q_{2,t}K_{2,t+1} + \sum_{n=1}^{2} \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k}(\mu - 1)\Pi_{n,t+k}K_{n,t+k}]
\]

\[
V_t^e = \mathbb{E}_t [M_{t,t+1}V_{t+1}^c], \quad q_{n,t} \equiv \frac{\partial V_t^e}{\partial K_{n,t+1}}, \quad \Pi_{n,t} \equiv \frac{\partial \Pi_t}{\partial K_t} \frac{\partial K_t}{\partial K_{n,t}}.
\]

- \( \mu = 1 \): \( V_t^e = q_{1,t}K_{1,t+1} + q_{2,t}K_{2,t+1} \) (Hayashi and Inoue, 1991)

- \( \mu > 1 \): \( V_t^e = q_{1,t}K_{1,t+1} + q_{2,t}K_{2,t+1} + \text{rents} \) (Lindenberg and Ross, 1981)
The value of the firm

Lemma

\[ V_t^e = q_{1,t}K_{1,t+1} + q_{2,t}K_{2,t+1} + \sum_{n=1}^{2} \sum_{k \geq 1} E_t \left[ M_{t,t+k} (\mu - 1) \Pi_{n,t+k} K_{n,t+k} \right] \]

\[ V_t^e = E_t \left[ M_{t,t+1} V^c_{t+1} \right], \quad q_{n,t} \equiv \frac{\partial V_t^e}{\partial K_{n,t+1}}, \quad \Pi_{n,t} \equiv \frac{\partial \Pi_t}{\partial K_t} \frac{\partial K_t}{\partial K_{n,t}}. \]

- \( \mu = 1 \):
  \[ V_t^e = q_{1,t}K_{1,t+1} + q_{2,t}K_{2,t+1} \]  
  (Hayashi and Inoue, 1991)

- \( \mu > 1 \):
  \[ V_t^e = q_{1,t}K_{1,t+1} + q_{2,t}K_{2,t+1} + \text{rents} \]  
  (Lindenberg and Ross, 1981)

\[ (\mu - 1)\Pi_{n,t} = \left( \frac{\Pi_t}{K_t} - \frac{\partial \Pi_t}{\partial K_t} \right) \times \frac{\partial K_t}{\partial K_{n,t}} = \text{flow value of rents from } K_n \]

gap btw. average and marginal product
The investment gap

$Q_{1,t} - q_{1,t}$
The investment gap

\[ Q_{1,t} - q_{1,t} = 0 \]

No intan + no rents \((\mu = 1)\): no investment gap, \( Q_{1,t} = q_{1,t} \) (Hayashi, 1982)
The investment gap

\[ Q_{1,t} - q_{1,t} = \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k}(\mu - 1) \Pi_{1,t+k}(1 + g_{1,t+k})] \]

No intan + rents ($\mu > 1$): $Q_{1,t} > q_{1,t}$ due to rents

(Lindenberg and Ross, 1981)
The investment gap

\[ Q_{1,t} - q_{1,t} = q_{2,t} \times \frac{K_{2,t+1}}{K_{1,t+1}} \]

Intan + no rents (\( \mu = 1 \)): \( Q_{1,t} > q_{1,t} \) due to omitted intangibles (Hayashi and Inoue, 1991)
The investment gap

\[ Q_{1,t} - q_{1,t} = \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k}(\mu - 1) \Pi_{1,t+k}(1 + g_{1,t+k})] + q_{2,t} \times \frac{K_{2,t+1}}{K_{1,t+1}} \]

Intan + rents (\( \mu > 1 \)):
The investment gap

\[ Q_{1,t} - q_{1,t} = \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k}(\mu - 1) \Pi_{1,t+k}(1 + g_{1,t+k})] \]

\[ + q_{2,t} \times \frac{K_{2,t+1}}{K_{1,t+1}} \]

\[ + \sum_{k \geq 1} \mathbb{E}_t [M_{t,t+k}(\mu - 1) \Pi_{2,t+k}(1 + g_{2,t+k})] \times \frac{K_{2,t+1}}{K_{1,t+1}} \]

Intan + rents (\(\mu > 1\)): additional term: **omitted intangibles \times rents**
The investment gap

\[ Q_{1,t} - q_{1,t} = \sum_{k \geq 1} \mathbb{E}_{t} [M_{t,t+k}(\mu - 1) \Pi_{1,t+k}(1 + g_{1,t+k})] \]

\[ + q_{2,t} \times \frac{K_{2,t+1}}{K_{1,t+1}} \]

\[ + \sum_{k \geq 1} \mathbb{E}_{t} [M_{t,t+k}(\mu - 1) \Pi_{2,t+k}(1 + g_{2,t+k})] \times \frac{K_{2,t+1}}{K_{1,t+1}} \]

Balanced growth: productivity grows at rate \( g \); constant discount rate \( r \)
The investment gap

\[ Q_1 - q_1 = \frac{(\mu - 1)R_1}{r - g} \] (rents → physical capital)

\[ + q_2 \times \frac{K_{2,t+1}}{K_{1,t+1}} \] (intangibles)

\[ + q_2 \frac{(\mu - 1)R_2}{r - g} \times \frac{K_{2,t+1}}{K_{1,t+1}} \] (rents → intangibles)

**Balanced growth:** \[ R_n \equiv r + \delta_n + \gamma_n gr, \quad n = 1, 2 \]
2. Measurement: aggregate data
Constructing the investment gap

\[ Q_1 - q_1 = \frac{\mu - 1}{r - g} R_1 + q_2 S + \frac{\mu - 1}{r - g} R_2 S \]

Scope: non-financial corporate business (NFCB) sector, 1947-2017

What data moments do we need to construct this decomposition?
Constructing the investment gap

\[ Q_1 - q_1 = \frac{\mu - 1}{r - g} R_1 + q_2 S + \frac{\mu - 1}{r - g} R_2 S \]

Scope: non-financial corporate business (NFCB) sector, 1947-2017

What data moments do we need to construct this decomposition?

\{S, \}

Ratio of intangible to physical capital

\[ S = \frac{K_{2,t+1}}{K_{1,t+1}} \]

BEA — \( K_{2,t+1} = \) only R&D capital
Constructing the investment gap

\[ Q_1 - q_1 = \frac{\mu - 1}{r - g} R_1 + q_2 S + \frac{\mu - 1}{r - g} R_2 S \]

Scope: non-financial corporate business (NFCB) sector, 1947-2017

What data moments do we need to construct this decomposition?

\{ S, ROA_1, \}

Rents parameter \( \mu \)

BEA — \( \Pi_t \) = operating surplus

\[ \mu = \frac{ROA_1}{R_1 + SR_2} \]

\[ ROA_1 = \frac{\Pi_t}{K_{1,t}} \]
Constructing the investment gap

\[ Q_1 - q_1 = \frac{\mu - 1}{r - g} R_1 + q_2 S + \frac{\mu - 1}{r - g} R_2 S \]

Scope: non-financial corporate business (NFCB) sector, 1947-2017

What data moments do we need to construct this decomposition?

\{S, ROA_1, i_1, i_2, \}

User costs \( R_1, R_2 \)

BEA — \( i_n \) = gross investment rate

\[ R_n = r + \delta_n \]

\[ = r - g + g + \delta_n = r - g + i_n \]
Constructing the investment gap

\[ Q_1 - q_1 = \frac{\mu - 1}{r - g} R_1 + q_2 S + \frac{\mu - 1}{r - g} R_2 S \]

Scope: non-financial corporate business (NFCB) sector, 1947-2017

What data moments do we need to construct this decomposition?

\{S, ROA_1, i_1, i_2, Q_1\}

Gordon growth term \( r - g \):

\[ r - g = \frac{ROA_1 - (i_1 + Si_2)}{Q_1} \]

Flow of Funds — \( Q_1 = V_t/K_{1,t+1}, V_t = \text{m.v. of debt + equity} \)
Constructing the investment gap

\[ Q_1 - q_1 = \frac{\mu - 1}{r - g} R_1 + q_2 S + \frac{\mu - 1}{r - g} R_2 S \]

Scope: non-financial corporate business (NFCB) sector, 1947-2017

What data moments do we need to construct this decomposition?

\( \{S, ROA_1, i_1, i_2, Q_1\} \)

Gordon growth term \( r - g \):

Flow of Funds — \( Q_1 = V_t/K_{1,t+1}, V_t = \text{m.v. of debt + equity} \)

\[ r - g = \frac{ROA_1 - (i_1 + Si_2)}{Q_1} \]

No adjustment costs: \( q_1 = q_2 = 1 \); otherwise, \( q_1 = 1 + \gamma_1 g, q_2 = 1 + \gamma_2 g \).
The investment gap in the non-financial sector

(adj. costs = 0)
The investment gap in the non-financial sector

(adj. costs = 0)
The investment gap in the non-financial sector

(\text{adj. costs} = 0)
The investment gap in the non-financial sector

(Adj. costs = 0)

Rents attributable to physical capital
Intangibles
Rents attributable to intangibles
Total

% of Total

- 77%
- 23%
- 77%
- 23%
- 64%
- 10%
- 26%

Adjustment costs
Underlying structural changes

Cobb-Douglas intangible share

\[ K_t = K_{1,t}^{1-\eta} K_{2,t}^\eta \]

Rents/v.a.

\[ s = (1 - WL/PY)(1 - \frac{1}{\mu}) \]
Underlying structural changes

Cobb-Douglas intan share

\[ K_t = K_{1,t}^{1-\eta} K_{2,t}^{\eta} \]

Rents/v.a.

\[ s = (1 - WL/PY) \left(1 - \frac{1}{\mu}\right) \]

\[ \Delta s = 0.07 \]

<table>
<thead>
<tr>
<th>Year</th>
<th>Rents 1985 → 2015 (v.a.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>-5 → 7.5</td>
</tr>
<tr>
<td>1990</td>
<td>0 → 13</td>
</tr>
<tr>
<td>2000</td>
<td>17 → 38</td>
</tr>
<tr>
<td>2015</td>
<td>26 → 57</td>
</tr>
<tr>
<td>2020</td>
<td>1 → 8</td>
</tr>
</tbody>
</table>

This paper (intan=R&D)

Underlying structural changes

Cobb-Douglas intangibles share: $K_t = K_{1,t}^{1-\eta} K_{2,t}^\eta$

Rents/v.a.: $s = (1 - WL/PY)(1 - \frac{1}{\mu})$

- Mild discount rate decline (7.9% → 5.6%), consistent with small rise in risk premia

Caballero, Gourinchas and Farhi (2017), Farhi and Gourio (2018)
Underlying structural changes

Cobb-Douglas intan share \[ K_t = K_{1,t}^{1-\eta} K_{2,t}^\eta \]

Rents/v.a. \[ s = (1 - WL/PY)(1 - \frac{1}{\mu}) \]

- Mild discount rate decline (7.9% → 5.6%), consistent with small rise in risk premia

Caballero, Gourinchas and Farhi (2017), Farhi and Gourio (2018)
3. Measurement: firm-level data
The investment gap: publicly traded firms

(intan = R&D)
The investment gap: publicly traded firms

(intan = R&D)

Rents attributable to physical capital
Intangibles
Rents attributable to intangibles
Total

Structural changes

Rents attributable to physical capital
Intangibles
Rents attributable to intangibles
Total


64%
15%
21%

21%
15%
64%
The investment gap: publicly traded firms

(intan = R&D + org. cap.)

Rents attributable to physical capital
Intangibles
Rents attributable to intangibles
Total

Structural changes
The investment gap across sectors (intan = R&D)

**Consumer**

**High-tech**

**Healthcare**

**Manufacturing**

- Rents attributable to physical capital
- Intangibles
- Rents attributable to intangibles
- Total

Rents vs. intangibles by sector

Consumer sector
Key take-aways
Key take-aways

1. General decomposition of the investment gap:

\[ Q_1 - q_1 = \text{Rents } \rightarrow \text{physical capital} \]

\[ + \text{ Omitted capital effect} \]

\[ + (\text{Rents } \rightarrow \text{intangibles}) \times (\text{Omitted capital effect}) \]

2. Aggregate: intan is \( \frac{1}{3} \) to \( \frac{2}{3} \) of the gap; \( \Delta s = 0.07 \) instead of 0.12

3. Sectoral: heterogeneous trends; intan is \( > \frac{2}{3} \) of the gap in Health, Tech
Additional slides
Alternative measures of $Q$

- Netting only financial assets identified as liquid in the Flow of Funds (baseline)
- Netting out all financial assets reported in the Flow of Funds (Hall, 2001)
The investment gap with the alternative Q measure (Hall, 2001)
Implied rents with expanded measures of intangibles

Robustness
The investment gap in the non-financial sector

(\text{adj. costs} > 0)

\gamma_1 = 3, \quad \gamma_2 = 12 \quad \text{(Belo et al., 2019)}
User costs

\[ R_n = r + \delta_n + \gamma_n rg \]
Related literature

1. Aggregate implications of rising rents:

   This paper: investment-$Q$; new approach for estimating of rents; sectoral heterogeneity

2. Q theory and firm value:

   This paper: general decomposition of $Q - q$, including market power
PPE investment is weak: sectoral data

Consumer

High-tech

Healthcare

Manufacturing

$\frac{I_t}{K_1}$ (Compustat, aggregate)  $\frac{I_t}{K_1}$ (BEA)
PPE investment is weak despite high returns: sectoral data

[Graph showing PPE investment trends for consumer, high-tech, healthcare, and manufacturing sectors from 1985 to 2015.]

Aggregate data

$\Pi / K_1$ (Compustat, aggregate)
Investment is weak relative to $Q$

\[
i_{j,t} = \alpha_j + \gamma_t + \delta Q_{j,t} + \beta CF_{j,t} + \epsilon_{j,t}
\]
Investment is weak relative to $Q$: sectoral data

\[ i_{j,t} = \alpha_j + \gamma_t + \delta Q_{j,t} + \beta CF_{j,t} + \epsilon_{j,t} \]
The growing importance of intangibles: sectoral data

Aggregate data

\[ K_1 = \text{PPE and } K_2 = \text{R&D capital.} \]
How general is this model?

- No restrictions on exogenous shifters to $\Pi_t$, $F_t$, and $\Phi_{n,t}$

- Particular cases of this framework:
  

- What about labor?
  
  The model can accommodate any flexible input: $\mu = \frac{\tilde{\mu} - \alpha}{1 - \alpha}$

- Which cases does this model not fit?
  
  - Non-homogeneous and/or non-smooth adjustment costs
  - Endogenous markups
  - Financial frictions
The investment gap in the general case

The first-order condition for investment is:

\[ g_{n,t+1} = \Psi_{n,t}(q_{n,t} - 1) \]

where:

\[ \Psi_{n,t}(y) = \left( \Phi'_{n,t} \right)^{-1} (1 + y) - 1. \]

Since \( \Phi_{n,t} \) is convex, \( \Psi_{n,t} \) is strictly increasing. Therefore:

\[ g_{n,t+1} = \Psi_{n,t}(q_{n,t} - 1) \]

\[ = \Psi_{n,t}(Q_{n,t} - 1 - G_{n,t}) \]

\[ < \Psi_{n,t}(Q_{n,t} - 1) \quad \text{iff} \quad G_{n,t} > 0 \]
Total $Q$

Define the total investment rate as:

$$i_{t}^{(tot)} = \frac{\sum_{n=1}^{N} I_{n,t}}{\sum_{n=1}^{N} K_{n,t}} = \sum_{n=1}^{N} \omega_{n,t} i_{n,t}.$$ 

In the quadratic adj. cost case:

$$i_{t}^{(tot)} = \tilde{\delta}_{t} + \sum_{n=1}^{N} \frac{\omega_{n,t}}{\gamma_{n}} (q_{n,t} - 1), \quad \tilde{\delta}_{t} = \sum_{n=1}^{N} \omega_{n,t} \delta_{n}.$$ 

Let $Q_{t}^{(tot)} \equiv \frac{V_{t}^{c}}{\sum_{n=1}^{N} K_{n,t+1}}$. Then:

$$i_{t}^{(tot)} = \tilde{\delta}_{t} + \frac{1}{\gamma} \left( Q_{t}^{(tot)} - 1 \right)$$

if and only if $\mu = 1$, and:

- $\gamma_{n} = \gamma$ for all $n$;
- or, $q_{n,t} = q_{t}$ for all $n$. 

The investment gap
Stochastic growth

Suppose $A_t$ follows the “regime-switching process”:

\[
\frac{A_{t+1}}{A_t} = 1 + g_t = \begin{cases} 
1 + g_{t-1} & \text{w.p. } (1 - \lambda) \\
1 + \tilde{g} & \text{w.p. } \lambda
\end{cases}, \quad \tilde{g} \sim F(\cdot).
\]

Then:

\[
G_{1,t} = \frac{(\mu - 1)}{r - \nu(g_t)} R_1 \quad (\text{Rents } \to \text{ physical capital})
\]

\[
+ S \quad (\text{Ommitted capital effect})
\]

\[
+ \frac{(\mu - 1)}{r - \nu(g_t)} R_2 S \quad (\text{Rents } \to \text{ intangibles})
\]

where:

\[
\frac{1}{r - \nu(g_t)} = \frac{1}{r - \mathbb{E}(\tilde{g})} \left(1 + \frac{g_t - \mathbb{E}(\tilde{g})}{1 + r}\right) \quad \text{if } \lambda = 1.
\]
Stochastic growth

The expression for $\nu(.)$ is:

$$\nu(g_t) = g_t + \lambda(1 + g_t) \frac{(r - g_t)\zeta^* - (1 + r)}{(1 + r) + \lambda(1 + g_t)\zeta^*}$$

where $\zeta^*$ is a constant that only depends on $F(.), \lambda$ and $r$. 

Analytical example
A microfoundation for Example 1 (1/2)

Representative household:

\[ U_t = \max \frac{C_t^{1-\sigma}}{1-\sigma} + \beta U_{t+1}, \]
\[ (1) \]

implying \( M_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma}. \)

Final goods producer

\[ Y_t = \left( \int_0^1 Y_{j,t}^{\frac{1}{\mu}} \, dj \right)^{\tilde{\mu}}, \quad \tilde{\mu} > 1. \]
\[ (2) \]

Intermediate goods producer: \( Y_{j,t} = Z_{j,t} K_{j,t}^{\alpha} L_{j,t}^{1-\alpha} \), implying the profit function:

\[ \Pi_{j,t} = A_{j,t}^{\mu - 1} K_{j,t}^{\mu}, \]
\[ \mu = 1 + \frac{\tilde{\mu} - 1}{\alpha}, \]
\[ A_{j,t} = (\alpha + \tilde{\mu} - 1)^{1+\frac{\alpha}{\mu-1}} \tilde{\mu} - \frac{\tilde{\mu}^{1-\alpha}}{\mu-1} (1 - \alpha) \frac{1-\alpha}{\mu-1} D_t W_t^{-\frac{1-\alpha}{\mu-1}} Z_{j,t}^{\frac{1}{\mu-1}}, \]
\[ D_t \equiv P_t^{\tilde{\mu}-1} Y_t. \]
A microfoundation for Example 1 (2/2)

Rest of the solution to the problem is:

\[ P_{j,t} = \tilde{\mu}MC_{j,t} \]

\[ L_{j,t} = \left( \frac{(1 - \alpha)MC_{j,t}Z_j}{W_t} \right)^{\frac{1}{\alpha}} K_{j,t} \]

\[ MC_{j,t} = (1 - \alpha) - \frac{(1 - \alpha)(\tilde{\mu} - 1)}{\mu - 1 + \alpha} \tilde{\mu} - \frac{\alpha\tilde{\mu}}{\mu - 1 + \alpha} \frac{\alpha(\tilde{\mu} - 1)}{\alpha + \mu - 1} \frac{W_t}{W_t} \frac{(1 - \alpha)(\tilde{\mu} - 1)}{\mu - 1 + \alpha} Z_j - \frac{\tilde{\mu} - 1}{\mu - 1 + \alpha} \frac{1}{K_{j,t}} - \frac{(\tilde{\mu} - 1)\alpha}{\mu - 1 + \alpha}. \]

This implies:

\[ LS_{j,t} \equiv \frac{W_tL_{j,t}}{P_{j,t}Y_t} = \frac{1 - \alpha}{\tilde{\mu}}. \]

We have:

\[ \tilde{\mu} = \alpha(\mu - 1) + 1 = (1 - \tilde{\mu}LS_{j,t})(\mu - 1) + 1, \]

and so, solving for \( \tilde{\mu} \):

\[ \tilde{\mu} = \frac{\mu}{\mu LS_{j,t} + (1 - LS_{j,t})}. \]
The investment gap in the non-financial sector (adj. costs > 0)

\[ \gamma_1 = 3, \quad \gamma_2 = 12 \]  (Belo et al., 2019)
User costs

\[ R_n = r + \delta_n + \gamma_n rg \]

Physical capital

Intanglible capital

- zero adjustment costs
- positive adjustment costs
Counterfactual: intan share $\eta$ with no change in rents

$S_t^{cf} = (K_{2,t}/K_{1,t})^{cf}: 9\% \rightarrow 39\%, \text{ vs. } 9\% \rightarrow 17\%$ in the R&D data
Robustness

Adjustment costs $\gamma_1 \in [0, 10]$ and $\gamma_2 \in [0, 20]$.

Adjustment costs $\gamma_1 = \gamma_2 = 0$: lowest contribution of intangibles to $Q_1 - q_1$; highest rents.

Alternative measure of net claims on NFCB sector using net NFCB claims lower $Q_1$; lower rents; same contribution of intangible to $Q_1 - q_1$.

Match PD ratio $= (r - g) - 1$ instead of $Q_1$; matching PD ratio larger investment gap, particularly 1965-1975; same contribution of intangibles; higher rents.

Implications for the labor share: implied labor share $0.69 \rightarrow 0.64$, but earlier than in the data.
Robustness

- Adjustment costs $\gamma_1 \in [0, 10]$ and $\gamma_2 \in [0, 20]$
Robustness

- Adjustment costs \( \gamma_1 \in [0, 10] \) and \( \gamma_2 \in [0, 20] \)

\[ \gamma_1 = \gamma_2 = 0: \text{ lowest contribution of intan to } Q_1 - q_1; \text{ highest rents} \]
Robustness

- Adjustment costs $\gamma_1 \in [0, 10]$ and $\gamma_2 \in [0, 20]$

  $\gamma_1 = \gamma_2 = 0$: lowest contribution of intan to $Q_1 - q_1$; highest rents

- Alternative measure of net claims on NFCB sector
Robustness

- Adjustment costs $\gamma_1 \in [0, 10]$ and $\gamma_2 \in [0, 20]$

  $\gamma_1 = \gamma_2 = 0$: lowest contribution of intan to $Q_1 - q_1$; highest rents

- Alternative measure of net claims on NFCB sector

  lower $Q_1$;
Robustness

- Adjustment costs $\gamma_1 \in [0, 10]$ and $\gamma_2 \in [0, 20]$

  $\gamma_1 = \gamma_2 = 0$: lowest contribution of intan to $Q_1 - q_1$; highest rents

- Alternative measure of net claims on NFCB sector

  lower $Q_1$; lower rents; same contribution of intan to $Q_1 - q_1$
Robustness

- Adjustment costs $\gamma_1 \in [0, 10]$ and $\gamma_2 \in [0, 20]$

  $\gamma_1 = \gamma_2 = 0$: lowest contribution of intan to $Q_1 - q_1$; highest rents

- Alternative measure of net claims on NFCB sector

  lower $Q_1$; lower rents; same contribution of intan to $Q_1 - q_1$

- Match PD ratio $= (r - g)^{-1}$ instead of $Q_1$
Robustness

- Adjustment costs $\gamma_1 \in [0, 10]$ and $\gamma_2 \in [0, 20]$

  $\gamma_1 = \gamma_2 = 0$: lowest contribution of intan to $Q_1 - q_1$; highest rents

- Alternative measure of net claims on NFCB sector

  lower $Q_1$; lower rents; same contribution of intan to $Q_1 - q_1$

- Match PD ratio $= (r - g)^{-1}$ instead of $Q_1$

  larger investment gap, particularly 1965-1975;
Robustness

- Adjustment costs $\gamma_1 \in [0, 10]$ and $\gamma_2 \in [0, 20]$

  $\gamma_1 = \gamma_2 = 0$: lowest contribution of intan to $Q_1 - q_1$; highest rents

- Alternative measure of net claims on NFCB sector

  lower $Q_1$; lower rents; same contribution of intan to $Q_1 - q_1$

- Match PD ratio $= (r - g)^{-1}$ instead of $Q_1$

  larger investment gap, particularly 1965-1975; same contribution of intan; higher rents
Robustness

- Adjustment costs $\gamma_1 \in [0, 10]$ and $\gamma_2 \in [0, 20]$

  $\gamma_1 = \gamma_2 = 0$: lowest contribution of intan to $Q_1 - q_1$; highest rents

- Alternative measure of net claims on NFCB sector

  lower $Q_1$; lower rents; same contribution of intan to $Q_1 - q_1$

- Match PD ratio $= (r - g)^{-1}$ instead of $Q_1$

  larger investment gap, particularly 1965-1975; same contribution of intan; higher rents

- Implications for the labor share
Robustness

- Adjustment costs $\gamma_1 \in [0, 10]$ and $\gamma_2 \in [0, 20]$

  $\gamma_1 = \gamma_2 = 0$: lowest contribution of intan to $Q_1 - q_1$; highest rents

- Alternative measure of net claims on NFCB sector

  lower $Q_1$; lower rents; same contribution of intan to $Q_1 - q_1$

- Match PD ratio $=$ $\left(r - g\right)^{-1}$ instead of $Q_1$

  larger investment gap, particularly 1965-1975; same contribution of intan; higher rents

- Implications for the labor share

  implied labor share $0.69 \rightarrow 0.64$, but earlier than in the data
1985-2015 change in $Q_{1-q1}$

2015 contribution of intangibles to $Q_{1-q1}$

2015 intangible share

2015 rents as a fraction of value added

Robustness
Netting out all financial assets (Hall, 2001)
Matching the PD ratio

Robustness
Implications for the labor share (1/2)

Value of $1-\alpha$ implied by the model when matching the labor share

Robustness
Implications for the labor share (2/2)

Value of the labor share implied by the model when setting $1-\alpha = 0.7$
Returns to physical capital, $\text{ROA}_t$

Physical investment rate, $i_t$

Intangible investment rate, $i_t$

Ratio of intangible to physical capital, $S$

Average Tobin’s $Q$ of physical capital, $Q_t$

Growth rate of total capital stock, $g$

--- Compustat NF, intangibles = R&D
--- Compustat NF, intan = R&D + organization capital
--- NFCB

Data sources
Intangible share

Rents as a fraction of value added

User cost of physical capital

User cost of intangible capital

Intangibles = R&D
Intangibles = R&D + org. cap.
Consumer sector

- Consumer sector, intangibles = R&D
- Consumer sector, intangibles = R&D + organization capital

Sectoral heterogeneity
High-tech sector

- High-tech sector, intangibles = R&D
- High-tech sector, intangibles = R&D + organization capital
Healthcare sector

Returns to physical capital, $\text{ROA}_1$

Physical investment rate, $i_1$

Intangible investment rate, $i_2$

Ratio of intangible to physical capital, $S$

Average Tobin's Q of physical capital, $Q_1$

Growth rate of total capital stock, $g$

--- Healthcare sector, intangibles = R&D

--- Healthcare sector, intangibles = R&D + organization capital

Sectoral heterogeneity
Returns to physical capital, ROA

Physical investment rate, i

Intangible investment rate, i

Ratio of intangible to physical capital, S

Average Tobin's Q of physical capital, Q

Growth rate of total capital stock, g

Manufacturing sector, intangibles = R&D

Manufacturing sector, intangibles = R&D + organization capital
The consumer sector: intangibles or rents?

- organization capital: no discernible trend, but high level
- still, including organization capital $\Rightarrow$ smaller markup trend after 1985
## Rents vs. intangibles by sector

<table>
<thead>
<tr>
<th>Sector</th>
<th>Intan share ((\eta; 2015))</th>
<th>Rents/v.a. ((s; 2015))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer</td>
<td>0.11</td>
<td>0.14</td>
</tr>
<tr>
<td>High-tech</td>
<td>0.39</td>
<td>0.13</td>
</tr>
<tr>
<td>Healthcare</td>
<td>0.57</td>
<td>0.12</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.12</td>
<td>0.02</td>
</tr>
</tbody>
</table>

- Intangibles = R&D
## Rents vs. intangibles by sector

<table>
<thead>
<tr>
<th></th>
<th>Consumer</th>
<th>High-tech</th>
<th>Healthcare</th>
<th>Manufacturing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intan share</td>
<td>0.63</td>
<td>0.56</td>
<td>0.69</td>
<td>0.30</td>
</tr>
<tr>
<td>( \eta; 2015 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rents/v.a.</td>
<td>0.03</td>
<td>0.09</td>
<td>0.07</td>
<td>0.02</td>
</tr>
<tr>
<td>( s; 2015 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Intangibles = R&D + org. cap.