

# SONOMA: a Small Open ecoNOmy for MAcrofinance

M. M. Croce      M. R. Jahan-Parvar      S. Rosen\*

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## Abstract

We develop a small open production economy model in which external debt, corporate domestic debt, and risky equities coexist. Our economy features shocks to short- and long-run productivity, as well as shocks to both domestic credit conditions and global credit markets. We show that credit shocks are an important determinant of economic fluctuations in a model consistent with asset pricing facts. According to a novel empirical investigation from many small-but-developed countries, our setting features a powerful quantitative performance well-suited for future monetary and fiscal policy analysis.

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\*Max Croce is affiliated with Bocconi University, NBER, CEPR, IGIER, and the University of North Carolina at Chapel Hill, Kenan-Flagler Business School. Mohammad Jahan-Parvar is affiliated with the Federal Reserve Board. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. Samuel Rosen is affiliated with Temple University, Fox School of Business. We thank Jim Pagels and Antonio Giribaldi for excellent research assistance. We thank Gurdip Bakshi, Luca Guerrieri, and Vincenzo Quadrini for early feedback. We are grateful to our discussants, Aytex Malkhozov, Daniel Neuhann, Dejanir Silva, Andreas Stathopoulos, and Robert Richmond. We also thank participants at the BIS seminar sequence, the CFE Conference (2018), the SEA Annual Conference (2018), the Macro Asset Pricing Conference (FRB, 2019), the ITAM finance conference (2019), the Finance Cavalcade (2019), the EFA conference (2019), the NFA conference (2019), the MFA conference (2019), and the BOOTH-IMF conference.

# 1 Introduction

Over ten years after the onset of the Global Financial Crisis, many developed mid- and small-size countries are still grappling with its consequences. These countries face material economic challenges rooted in the disruption caused by the U.S. financial crisis and its reincarnation in sovereign default concerns in the EU. For countries like Portugal, Italy, Greece and Spain (PIGS), there is a general concern not only about their macroeconomic fundamentals, but also about their overall financial health. Their excessive government debt has spilled over to the private economy and has contributed to cause a relevant contraction in financing conditions both in their domestic and in the international capital markets. Given the interconnection between public finances, credit markets and investment opportunities, having at least a partial equilibrium setting in which to jointly study these dimensions is of first-order importance to guide the current policy debate.

In this paper, we propose a new production-based general equilibrium model that encompasses several strands of literature in international macroeconomic, finance, and asset pricing. Our goal is to propose a new setting in which we can reproduce qualitatively and quantitatively key properties of macroeconomic aggregates, external balances, and cost of capital. These variables are all crucial determinants of corporate investment, employment decisions, and ultimately much needed growth. We think of this setting as a novel platform to be used for complex policy analysis both with respect to conventional and unconventional monetary policy, and with respect to fiscal and trade policies.

We start from a benchmark small open economy model (see, among others, Mendoza (1991), Schmitt-Grohé and Uribe (2003), and Mendoza (2006)) and modify it in two dimensions. First, we introduce recursive preferences and productivity growth

news shocks in the spirit of the long-run risk literature (among others, see Bansal and Yaron (2004), Kaltenbrunner and Lochstoer (2010) and Croce (2014)). The macroeconomic literature has already established that growth news shocks are important sources of macroeconomic fluctuations. In finance, several theoretical and empirical studies show that growth news shocks are important determinants of both risk premia and, more broadly, the cost of capital for private firms. Recursive preferences are important so that news shocks are directly priced by investors. We consider this step as genuinely essential in order to properly address uncertainty about the long-term growth of economies in distress (among others, we think of the PIGS economies).

Second, we introduce financial frictions to have a dynamic tradeoff between equity and debt financing for private firms. In our model, a corporate tax shield on the cost of debt makes debt-financing more appealing than equity. On the other hand, an enforcement constraint limits debt-financing exactly as in Jermann and Quadrini (2012). The tightness of this constraint depends on exogenous shocks that we interpret as domestic credit shocks. Even though there is no default at the equilibrium, this stochastic financing constraint distorts the optimal demand for labor, i.e., it produces a wedge that causes a first-order departure from the frictionless first-best.

To these domestic credit shocks, we add shocks to the cost of external financing, i.e., to the cost of borrowing from global financial markets. Analyzing the role of these economic shocks is standard in the context of the small open economy literature. Nonetheless, our setting is novel because of the interplay of two elements: (i) with recursive preferences, persistent credit shocks are priced because of their effect on economic growth; and (ii) because of internal credit constraints, foreign liabilities are adjusted to facilitate both consumption smoothing and optimal rebalancing of the corporate capital structure.

In this environment, we document several findings. First, the model inherits the success of a standard macroeconomic model in moment-matching the relevant economic and financial quantities. At the equilibrium, the annualized equity premium is almost 7%, despite the ability of agents to hedge shocks through the labor, the domestic investment and the international borrowing margins. When we remove all of the financial frictions, the equity premium declines to about 2% per year. This effect is not just a mechanical outcome of the lack of financial leverage, since even the equilibrium return on assets declines substantially. In our model, credit shocks are disruptive not just for labor decisions, but also for the intertemporal decision of saving and hence investing.

When we compare our open economy to a closed one, we find two important results. First, domestic credit shocks are less disruptive in an open economy than in a closed economy because the household can borrow from abroad in order to recapitalize the domestic firm. That is, domestic corporate debt is reduced through issuance of equity ultimately financed by issuing more international debt. Second, shocks that tighten foreign credit availability, i.e., shocks that increase the cost of foreign liabilities, force the country to reduce foreign debt. In order to do so, the country needs to run substantial positive current account balances. This is accomplished by cutting down domestic consumption, increased working hours, and reducing domestic investment. The external shock, indeed, increases the cost of capital for the domestic firm across both equity and debt financing. Interestingly, when financial frictions are present this adjustment is less pronounced and more sluggish, reinforcing the idea that corporate frictions have a first-order role in macroeconomic dynamics even in small open economies.

We also present a novel empirical investigation that, although not complete yet, gives us three useful insights from a broad cross-section of small developed economies.

First, credit shocks are difficult to hedge as external interest rate shocks are correlated with adverse internal credit shocks. Second, external credit shocks are leading indicators of sluggish long-run growth. Third, external credit shocks feature a long positive tail which is correlated with a negative tail in expected long-run growth. More formally, the coskewness of the external cost of capital and local long-run growth is positive implying that severe external credit shocks are associated to strong expected growth declines. When we introduce these elements in our equilibrium model, the equity risk premium increases substantially, the equilibrium capital accumulation slows down and welfare declines by 16%. Overall, we see these results as confirming that financial frictions and shocks are a first-order concern for economic activity.

## 1.1 Related Literature

In this paper, we bring together several strands in the macroeconomics and finance literature. The paper builds on earlier works on implementing dynamic stochastic general equilibrium (DSGE) models, in their real business cycle form, in a small open economy setting. Influential earlier studies include Mendoza (1991), Schmitt-Grohé and Uribe (2003), and Mendoza (2006), among others. The literature on small open economies is generally focused on issues pertaining to developing or emerging markets, and in particular on problems with sudden stops or managing exchange rates. For example, refer to the voluminous literature on sudden stops or currency crises. Examples, among many others, include Mendoza and Smith (2006), Chari et al. (2005), Aghion et al. (2001), Mendoza and Uribe (2000). Other studies focus on sovereign default. Among them, we may mention Uribe and Yue (2006), Aguiar and Gopinath (2007), and Arellano (2008). In this study we focus on smaller developed countries. While they still qualify as small open economies, they typically do not face the same type of currency problems that emerging markets face.

A strand of literature embeds production-based asset pricing for small open economies. For example, Jahan-Parvar et al. (2013) is a small open economy production-based asset pricing model featuring Greenwood et al. (1988) preferences, which builds on earlier production-based studies such as Jermann (1998) and Boldrin et al. (2001).

Bansal and Yaron (2004) study long-run risks, i.e., persistent and predictable components in the first and second moment of consumption growth, in an asset pricing model with Epstein and Zin (1989) preferences. Their setting has since turned into a widely used approach to study the joint aggregate dynamics of the financial markets and the macroeconomy. Bansal, Gallant, and Tauchen (2007), Bansal, Kiku, and Yaron (2012, 2016), Schorfheide, Song, and Yaron (2018) explore the implications of this approach extensively in a closed endowment economy. A parallel literature embeds disaster risks, that is, the possibility of low probability but highly severe real-side events like a depression in asset pricing models. Examples include Rietz (1988), Barro (2006), Gabaix (2012), Barro and Jin (2011), Nakamura, Steinsson, Barro, and Ursúa (2013), Gourio (2012), Wachter (2013), and Horvath (2017). We focus on growth news shocks and abstract from disaster risk.

Kaltenbrunner and Lochstoer (2010) and Croce (2014) study long-run risks in one-country production-based asset pricing models. Colacito, Croce, Ho, and Howard (2018) explore long-run productivity risks in a general equilibrium production-based international setting. Our study differs from this work in many dimensions. First, we propose a flexible and tractable partial equilibrium approach. Second, none of these manuscripts has studied the interplay of corporate capital structure adjustments, labor and investment distortions, and external balances.

Our study is closely tied to the growing literature on financially constrained firms and the implications of credit shocks in a DSGE macroeconomic framework. As stated earlier, our work is closely related to Jermann and Quadrini (2012). Other

studies that explore credit constraints in a DSGE model include Gertler and Karadi (2011), Khan and Thomas (2013), Guerrieri and Lorenzoni (2017), and Guerrieri and Iacoviello (2017). In contrast to our setting, these models features closed economies.

Coeurdacier et al. (2015) considers financial frictions in a small open economy to assess three prominent global trends: a divergence in private saving rates between advanced and emerging economies, large net capital outflows from the latter, and a sustained decline in the world interest rate. We focus on smaller developed countries in which (i) shocks that determine long-run growth risk are directly priced, and (ii) the interplay of corporate financing decisions an external balances has a first-order impact on macroeconomic aggregates.

The remainder of this manuscript is organized as follows. We describe both our empirical methods and findings in the next section. We detail our model and calibration in sections 3 and 4. In section 5, we discuss our main results. Section 6 concludes.

## 2 Empirical Evidence

We start our analysis by investigating the empirical properties of the fundamental shocks that drive the economic dynamics in the SONOMA model. This step is based on methods that are familiar in the macro-finance literature. Thus, we proceed with our empirical estimations and postpone the description of our model to section 3.

Specifically, the SONOMA model features both short-run productivity growth shocks and long-run growth news shocks that we identify as in Croce (2014). The internal shocks are measured as in Jermann and Quadrini (2012) by mean of a leverage constraint. The external financing shocks are identified by estimating a specification of the costs of capital inspired by Schmitt-Grohé and Uribe (2003).

We focus on a cross section of Western European developed small open economies. We currently have a cross section of 10 countries such as Portugal, Italy, Spain, Finland, Sweden, Switzerland, Austria, Belgium, Denmark and the Netherlands. The data collection is ongoing and represents one of our main contributions since we plan on updating it regularly, extending our sample to other countries, and make it available to the public. For the sake of housekeeping, the data sources are detailed in a companion document (Croce et al. (2019), available here: <https://sites.google.com/view/mmcroce/wps>) that we update regularly together with the exhibits in this manuscript. The main sources that we use are reported in Appendix B.

## 2.1 Country-level Measurements

For each country in our sample, we compute essential aggregate variables needed for our analysis. In what follows, we focus on the series that are necessary in order to compute our fundamental shocks.

**Capital stock.** In each country  $j$ , we measure the capital stock as in Jermann and Quadrini (2012) by capitalizing quarterly investment net of depreciation,

$$K_{t+1}^j = K_t^j(1 - \delta_t^j) + I_t^j.$$

Our measure of investment is very broad. It includes investments across all sectors in the economy. Quarterly investment ( $I^j$ ) data are from the OECD dataset whereas the depreciation rate ( $\delta^j$ ) is from the Penn World Table (PWT).

For each country, we initialize the recursion above by making sure that (i) the initial and the final values of capital-to-output have the same value (as in Jermann



and Quadrini (2012)); and (ii) the average capital-to-output ratio is equal to that computed from the annual data from PWT.

**Productivity.** For each country  $j$ , we compute total factor productivity  $Z_t^j$  by postulating the Cobb-Douglas production function  $Y_t^j = Z_t^j K_t^{j\theta_j} N_t^{j1-\theta_j}$ , in which  $K_t^j$  measures beginning-of-the-quarter aggregate capital,  $N_t^j$  refers to total labor hours, and  $\theta_j$  is a moving average of the labor income share of GDP as reported in the PWT. Jermann and Quadrini (2012) recommend using business value added to measure  $Y_t^j$ . We believe that GDP is more appropriate measure in our case since capital stock and labor series are total measures, i.e., they include all sectors and economic activities. In some of countries in our sample non-profit and government sector activities are sizeable and should be accounted for in our empirical exercise.

**Internal credit conditions.** We take seriously the Jermann and Quadrini (2012) credit constraint and measure its tightness,  $\xi_t^j$ , directly from the following ratio

$$\xi_t^j = \frac{Y_t^j}{K_{t+1}^j - B_{t+1}^{j,end}},$$

where  $K_{t+1}^j$  measures the end-of-the-quarter capital stock, and  $B_{t+1}^{j,end}$  measures end-of-the-quarter corporate debt (source: BIS). In contrast to Jermann and Quadrini (2012), we do not focus on detrended variables, i.e., on deviations from a long-run trend. Given our attention to long-run dynamics and expectations, we extract our  $\xi_t^j$  series using raw aggregate variables in levels.

**External credit conditions.** We think of the external cost of capital in a small open economy,  $r_t^j$ , as the result of both exogenous external shocks and an endogenous

spread component that is related to external leverage, i.e., net foreign liabilities. We model the country-level spread,  $P$ , in the spirit of Schmitt-Grohé and Uribe (2003):

$$P_t^j \equiv P \left( \frac{X_t^j}{Y_t^j} \right) = p_2 e^{p_1 (X_t^j/Y_t^j - \overline{XY}^j)} - 1$$

where the left-hand side is measured by the difference between the real interest rate of country  $j$ ,  $r_t^j$ , and the real German rate, and  $(X/Y^j - \overline{XY}^j)$  is the demeaned net external debt measured using data from the IMF IIP/BOP database (see the methods in Lane and Milesi-Ferretti, 2007).

In order to estimate the parameters of this function by country, we regress the log-domestic spread on the log of our functional form

$$\ln(1 + P_t^j) = \ln(p_2^j) + p_1^j \left( X_t^j/Y_t^j - \overline{XY}^j \right) + \varepsilon_t^j \quad j = 1, 2, \dots, 10$$

Since  $\ln(1 + P_t^j) \approx P_t^j$ , we derive the exogenous component of the external cost of capital for each country,  $r_t^{j,w}$ , as follows,

$$r_t^{j,w} = r_t^j - \hat{P}_t^j,$$

and point out that this process embodies both changes to the world risk-free rate and changes to the external credit sentiment about country  $j$ .

The results of this estimation are reported in the top two panels of table 1. We find substantial heterogeneity across countries and for this reason we split our sample into two groups. In panel A, we group selected countries for which  $p_1^j$  is both positive and statistically significant, consistent with theory and with prior studies.

In panel B, on the other hand, we report the results for countries that feature either a null or negative connection between external debt and external financing

**Table 1: Interest Rates and External Debt Ratios**

| $\ln(1 + P_t^j) = \ln(p_2^j) + p_1^j (X_t^j/Y_{t-1}^j - \overline{XY}^j) + \varepsilon_t^j \quad = 1, 2, \dots$ |                    |                   |                    |                    |                   |                   |
|---|--------------------|-------------------|--------------------|--------------------|-------------------|-------------------|
| Panel A: Selected Countries   |                    |                   |                    |                    |                   |                   |
|   | Finland            | Italy             | Portugal           | Spain              | Sweden            | Switzerland       |
| $p_1^j$   | 0.02*<br>(0.02)    | 0.06***<br>(0.02) | 0.08***<br>(0.02)  | 0.04***<br>(0.01)  | 0.04**<br>(0.03)  | 0.03***<br>(0.01) |
| $\ln(p_2^j)$  | 0.31*<br>(0.23)    | 0.71***<br>(0.25) | 1.14**<br>(0.51)   | 0.26<br>(0.35)     | 0.76***<br>(0.27) | -0.35**<br>(0.21) |
| $R^2$   | 0.11               | 0.44              | 0.56               | 0.45               | 0.05              | 0.31              |
| $\overline{XY}^j$   | 25                 | 33                | 59                 | 49                 | 49                | -70               |
| $XY_{2017q4}^j$   | 43                 | 53                | 81                 | 79                 | 44                | -33               |
| $XY_{2017q4}^j - XY_{2010q1}^j$   | 23                 | 7                 | 2                  | -2                 | -4                | 30                |
| Panel B: Excluded Countries   |                    |                   |                    |                    |                   |                   |
|   | Austria            | Belgium           | Denmark            | Netherlands        |                   |                   |
| $p_1^j$   | -0.01<br>(0.01)    | -0.02*<br>(0.01)  | -0.05***<br>(0.01) | -0.01***<br>(0.00) |                   |                   |
| $\ln(p_2^j)$  | -0.09*<br>(0.06)   | -0.02<br>(0.13)   | -0.10<br>(0.11)    | 0.12**<br>(0.07)   |                   |                   |
| $R^2$   | 0.01               | 0.08              | 0.33               | 0.20               |                   |                   |
| $\overline{XY}^j$   | 22                 | -17               | 39                 | 93                 |                   |                   |
| $XY_{2017q4}^j$   | 23                 | 5                 | 17                 | 60                 |                   |                   |
| $XY_{2017q4}^j - XY_{2010q1}^j$   | 0                  | 42                | -34                | -53                |                   |                   |
| Panel C: Panel Regression   |                    |                   |                    |                    |                   |                   |
|   | Selected Countries |                   | Excluded Countries |                    | All Countries     |                   |
| $p_1$   | 0.05***<br>(0.01)  |                   | -0.02***<br>(0.01) |                    | 0.04***<br>(0.01) |                   |
| $\ln(p_2)$  | 0.46***<br>(0.15)  |                   | -0.03<br>(0.06)    |                    | 0.29***<br>(0.11) |                   |
| $R^2$   | 0.35               |                   | 0.13               |                    | 0.24              |                   |

*Notes:* Our sample starts in 1995 and ends in 2017. Standard errors in parentheses are Newey-West adjusted.

cost. Given that these countries feature either a very low outstanding external debt-to-output ratio or a recent history of strong external debt consolidation, we have good reasons to consider them not representative of the small open economies that we model in this manuscript. As a result, we denote them as ‘excluded countries’.

In panel C, we report the estimation results that we obtain by running a panel regression in which we impose that both  $p_1$  and  $p_2$  are common across several countries.

Consistent with prior studies, our estimate of  $p_1$  for our group of selected countries is 0.05 with a Student  $t$ -statistic of 5. We use this result when calibrating our model.

**Long-run productivity risk.** As in Croce (2014), we estimate the long-run component of productivity growth by forecasting the demeaned productivity growth rate  $(\Delta a_{t+1}^j - \mu_a^j)$  using the lagged price-dividend ratio,  $PD_t^j$ ,

$$\Delta a_{t+1}^j - \mu_a^j = x_t^j + \epsilon_{a,t+1}^j \quad (1)$$

$$x_t^j = b_x PD_t^j. \quad (2)$$

This is common in the long-run growth news shock literature, as equity valuation is a forward looking variable highly correlated with future expected growth.

## 2.2 VAR Analysis

As detailed in a companion document Croce et al. (2019), our data panel is unbalanced. This means that for some variables or countries, observations start at a later date than others. Thus, over our longest sample, we have data for a smaller set of countries. We overcome this problem by using GDP-weighted averages of our data across countries in each quarter. Our results hence could be viewed as applying to a representative developed small open economy or, equivalently, to a global component which is common to all the countries in our sample.

Given these GDP-weighted average time series, we estimate the following VAR(1):

$$Y_t = \Phi Y_{t-1} + \Sigma u_t \quad (3)$$

in which

$$Y_t = \begin{bmatrix} r_t^{w,avg} & \xi_t^{avg} & x_t^{avg} \end{bmatrix}, \quad (4)$$

where  $r_t^{w,avg}$ ,  $\xi_t^{avg}$ , and  $x_t^{avg}$ , denote the GDP-weighted averages of the exogenous component of the external interest rate, the internal credit tightness, and the long-run productivity component, respectively. In what follows, we adopt a lower-triangular Cholesky decomposition to orthogonalize our shocks.

**Co-movements.** Our VAR analysis highlights two very relevant results which are reported both in the top part of Table 2 and in the top panels of Figure 1. First, when we focus on our selected countries, credit shocks are not easy to hedge as internal and external shocks are contemporaneously correlated. When the external cost of capital increases unexpectedly, the  $\xi$  process declines, meaning that simultaneously the internal credit markets tighten. These dynamics reach their peak after about 8 quarters, and are present for both GDP-weighted and equal-weighted data.

Second, adverse world interest rate shocks are correlated with negative long-run growth shocks, meaning that tight credit conditions tend to be associated to periods of lower expected long-run growth. We find it reassuring that these adjustments are not statistically significant for our excluded countries (bottom panels of figure 1), as external credit shocks are expected be less relevant for them.

In our current dataset, we also found a significant positive contemporaneous effect of domestic credit shocks on expected productivity growth. In this study, we abstract away from this channel and focus on the role of external shocks.

**Co-skewness.** An analysis of the VAR shocks reveals an interesting pattern in our sample. Remember that, in our setting, positive external credit shocks and negative

**Table 2: Responses to an External Interest Rate ( $r^w$ ) Shock.**

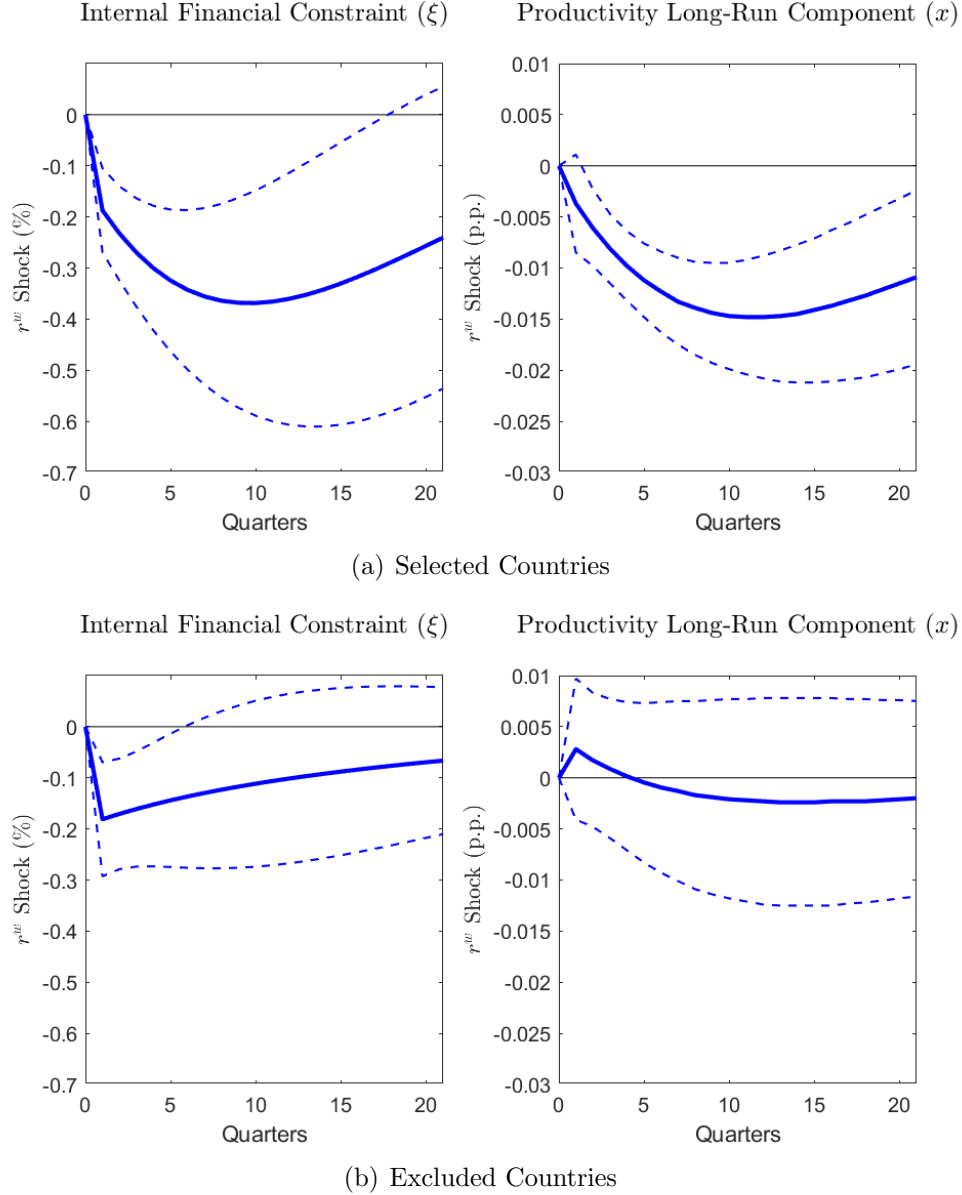
|  | Estimate | Confidence Interval |
|--|----------|---------------------|
| Panel A: Impulse Response Function               |          |                     |
| $\xi^{avg}$ Response to a $r^{w,avg}$ Shock (%)  |          |                     |
| GDP-Weighted Average                             | -0.368   | [-0.574, -0.163]    |
| Equal-Weighted Average                           | -0.305   | [-0.470, -0.140]    |
| $x^{avg}$ Response to a $r^{w,avg}$ Shock (p.p.) |          |                     |
| GDP-Weighted Average                             | -0.014   | [-0.019, -0.010]    |
| Equal-Weighted Average                           | -0.009   | [-0.014, -0.005]    |
| Panel B: Coskewness                              |          |                     |
| $CoSk(r^{w,avg}, x^{avg})$ (VAR)                 | 0.425    | [0.065, 0.785]      |
| $CoSk(r^{j,w}, x^j)$ (Unbalanced Panel Regr.)    | 0.370    | [0.181, 0.559]      |
| $CoSk(r^{j,w}, x^j)$ (Balanced GMM)              | 0.164    | [0.059, 0.269]      |

*Notes:* Panel A of this table shows the estimates of the responses of the internal financial constraint ( $\xi$ ) and the productivity long-run component ( $x$ ) to an  $r^w$  shock after eight quarters. All results are based on the VAR specified in equation (3)-(4). Our data sources are detailed in Appendix B and our sample starts in 1995:Q1 and ends in 2017:Q4. In panel B, ‘ $CoSk$ ’ refers to the coskewness between growth news shocks and shocks to external credit conditions. Numbers in square brackets represent the bottom- and top-decile of the confidence interval. Standard errors are HAC-adjusted.

long-run growth shocks are bad news for economic activity. In what follows, we assess whether downside long-run growth risk realizes simultaneously to downside external credit risk. We expect to observe a positive value. To test this hypothesis, we compute coskewness as in Harvey and Siddique (2000),

$$CosK(r^w, x) = \frac{E[(\epsilon_t^x)^2 \cdot \epsilon_t^{r^w}]}{Std(\epsilon_t^x) \cdot V(\epsilon_t^{r^w})}.$$

When using GDP-weighted data in the VAR setting discussed earlier, we find a positive co-skewness of about 0.42 (Table 2, panel B). As robustness checks, we also run our VAR at the country level in order to recover country-level shocks. We then



**Fig. 1. Impulse Responses in Data VAR using GDP-Weighted Averages.** This figure shows the response to an external credit shocks of domestic credit conditions ( $\xi$ ) and the long-run component of growth ( $x$ ). All results are based on the VAR specified in equations (3)-(4). Our sources are detailed in Appendix B and our quarterly sample starts in 1995:Q1 and ends in 2017:Q4. Confidence intervals are HAC-adjusted.

compute co-skewness both with an unbalanced panel regression and with a balanced 2-step GMM approach. The existence of positive coskewness is broadly confirmed also using country-level data in both VAR and GMM settings. This result is important

because it points to the existence of a common adverse tail shock in both external credit conditions and long-run productivity growth. To the best of our knowledge, this is the first study documenting this empirical fact.<sup>1</sup>

### 3 The Economy

In this section, we first embed recursive preferences (Kreps and Porteus 1978, Epstein and Zin 1989) and long-run risks (LRR) (Bansal and Yaron 2004) in the Jermann and Quadrini (2012) model. We then open the economy and expose it to exogenous financing shocks.

The Jermann and Quadrini (2012) economy consists of a representative household and a representative firm. The firms need to decide the optimal mix of equity- and corporate debt-financing. Firms issue debt because of the existence of a tax shield on corporate interests. A borrowing constraint limits the amount of corporate debt and introduces a wedge in labor market.

Our economy differs from the Jermann and Quadrini (2012)'s one in two dimensions. First, our household holds all domestic equity and corporate debt and can borrow from or lend to the rest of the world. Second, our government sector collects a corporate profit tax net of tax shield. This tax flow is rebated to the household as a lump-sum payment.

The model features four sources of risk. Similar to traditional real business cycle models, there are shocks to the level of the productivity growth rate. In addition, we follow Croce (2014) and introduce long-run productivity growth risk. As in Jermann and Quadrini (2012), there are financial shocks to the firm's borrowing collateral

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<sup>1</sup>Since we find no significant skewness in domestic credit shocks and no other statistically relevant results on coskewness across other shocks, in our model we treat all other processes as subject to symmetric innovations.



constraint. In addition, we consider shocks to the level of the external cost of financing (see, among others, Jahan-Parvar et al. 2013, Aguiar and Gopinath 2007, and Uribe and Yue 2006).

### 3.1 Household's problem

The representative household is endowed with Kreps and Porteus (1978) recursive preferences as specified in Epstein and Zin (1989), generally represented as

$$U_t = \left[ (1 - \beta) \tilde{C}(C_t, \ell_t)^{1 - \frac{1}{\psi}} + \beta (\mathbb{E}_t U_{t+1}^{1 - \gamma})^{\frac{1 - \frac{1}{\psi}}{1 - \gamma}} \right]^{\frac{1}{1 - \frac{1}{\psi}}}, \quad (5)$$

where  $\tilde{C}(C_t, \ell_t)$  is a consumption bundle defined over consumption ( $C_t$ ) and leisure ( $\ell_t$ ),  $\gamma$  is the coefficient of relative risk aversion, and  $\psi$  represents the elasticity of intertemporal substitution (IES). We define leisure as the portion of time not worked (the residual of labor supply) as  $1 \geq H_t^s + \ell_t$  where  $H_t^s$  is the supply of labor by the household. We define the consumption bundle as

$$\tilde{C}_t = \left( \tilde{w}_1 C_t^{1 - \frac{1}{f}} + \tilde{w}_2 (A_{t-1} \ell_t)^{1 - \frac{1}{f}} \right)^{\frac{1}{1 - \frac{1}{f}}}.$$

The definition of weights  $\tilde{w}_1$  and  $\tilde{w}_2$  is available in Appendix A.

The household maximizes lifetime utility subject to the following constraints by choosing the levels of consumption and leisure, and a portfolio of financial assets:

$$\begin{aligned} C_t^P &\leq w_t^P H_t + (S_t - S_{t+1}) V_t^{P,ex} + S_t \cdot d_t + (1 + r_t^D) D_t - D_{t+1} + X_{t+1} - (1 + r_t) X_t - T_t^H \\ 1 &\geq H_t^s + \ell_t \end{aligned}$$

where  $w_t^P$  represents wages earned;  $S_t, D_t$  and  $-X_t$  are equity, corporate debt, and net foreign assets held by the household at time  $t$ , respectively. The variation of the net foreign assets equals the country's current account. In this formulation,  $d_t, r_t^D$  and  $r_t$  are the dividend income per share, corporate bond interest rate, and the interest rate paid to foreign lenders. The ex-dividend share price is  $V_t^{P,ex}$ . The government makes a lump-sum tax transfer to the household equal to  $T_t^H$ .

Given this notation, the net equity payout (NEP) to the household is  $(S_t - S_{t+1})V_t^{P,ex} + S_t \cdot d_t$ , net corporate debt payout (NDP) is  $(1 + r_t^D)D_t - D_{t+1}$ , and changes in net foreign assets ( $\Delta NFA$ ) are  $-(X_{t+1} - (1 + r_t)X_t)$ . In this model, only households can borrow from abroad and they can use these resources for investment in domestic equities and corporate bonds.

Following Schmitt-Grohé and Uribe (2003), Jahan-Parvar et al. (2013), and a number of other studies in international finance, we posit that the interest paid to foreign lenders,  $r_t$ , is a function of the world financing rate,  $r_t^W$ , and a country spread,  $P_t$ . The country spread depends on the external debt position of the country. Explicitly, we define this interest rate and country spread as

$$r_t = r_t^w + P \left( \frac{X_t}{Y_t} \right) \quad (6)$$

$$P_t = p_2 e^{p_1 (X_t/Y_t - \overline{XY})}. \quad (7)$$

We follow Schmitt-Grohé and Uribe (2003) in our formulation of country spread  $P_t$ , where the size of this spread depends on the difference between the ratio of external debt to output and its steady-state value,  $\overline{XY}$ . Neumeyer and Perri (2005) argue that country spreads have both endogenous and exogenous components. In this study, the

spread is defined to be endogenous, but exogenous random factors can affect the baseline external financing rate ( $r_t^W$ ) as follows:

$$r_{t+1}^w = (1 - \rho_{rw}) \mu_{rw} + \rho_{rw} r_t^w + \sigma^{RW} \epsilon_{t+1}^{rw}. \quad (8)$$

These shocks must be interpreted in a broad way, as they capture both changes in the world risk-free rate and changes in the sentiment of external lenders toward our small open economy.

We note that our small open economy becomes a closed economy if we replace equation (6) with  $X_t = 0 \quad \forall t$ . Since this model does not admit a sovereign default, at equilibrium, we have  $r_t^D = r_t$ .

The stochastic discount factor (SDF) of the household is given as

$$M_{t+1} = \beta \left( \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\frac{1}{\psi}} \left( \frac{U_{t+1}}{E_t [U_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma} \frac{\partial \tilde{C}_{t+1} / \partial C_{t+1}}{\partial \tilde{C}_t / \partial C_t}. \quad (9)$$

Since the consumption bundle is a composite of both leisure and consumption, one needs to adjust the SDF so that it only reflects variations in consumption. The last term in equation (9) represents this adjustment. Since consumption is endogenous and the model features long-run risks (Section 3.2), the SDF features long-run risk aversion.

The optimal investment strategy in bonds implies that the interest rate equals the reciprocal of the conditional expectation of the SDF:

$$1 + r_t = \frac{1}{E_t [M_{t+1}]}.$$

All other first order conditions are standard in production-based models and are detailed in Appendix A.

### 3.2 Firm's problem

Our representative firm has a neoclassical production function

$$F_t = F(A_t, K_t, H_t) = K_t^{\alpha_p} (A_t H_t)^{1-\alpha_p},$$

where  $K_t$ ,  $H_t$  and  $A_t$  represent capital input, labor input, and the stochastic level of productivity, respectively. Following the typical timing convention,  $K_t$  is used at time  $t$  but predetermined at time  $t - 1$ , while the input of labor  $H_t$  is determined flexibly at time  $t$ . Capital evolves according to

$$K_{t+1} \leq (1 - \delta)K_t + I_t - \Phi\left(\frac{I_t}{K_t}\right) K_t, \quad (10)$$

$$\Phi\left(\frac{I_t}{K_t}\right) = \frac{\phi_1}{1 - \frac{1}{\phi_2}} \left(\frac{I_t}{K_t}\right)^{1 - \frac{1}{\phi_2}} + \phi_3, \quad (11)$$

where,  $\delta$  is the depreciation rate for capital.  $I_t$  is investment at time  $t$ .  $\Phi\left(\frac{I_t}{K_t}\right)$  is the capital adjustment cost defined as in Jermann (1998). The elasticity of adjustment costs is determined by  $\phi_2$ . As noted in Jermann (1998) and Boldrin et al. (2001), among others, capital adjustment costs improve the model's asset pricing properties.

As is common in the RBC literature, we have an exogenous productivity process. Following Croce (2014), we posit that the growth rate of productivity,  $\Delta a_t = \log(A_{t+1}/A_t)$ , features long-run risks. As shown by Croce (2014), LRR in productivity growth is tightly related to equity and interest rate fluctuations. In addition, the study shows that consumption, investment, and output all show a statistically

significant positive exposure to the long-run productivity. In this study, we assume that productivity growth follows

$$\Delta a_{t+1} = \mu_a + x_t + \exp \sigma_t^{srv} \epsilon_{t+1}^a, \quad (12)$$

$$x_{t+1} = \rho_x x_t + \exp \sigma_t^{lrv} \epsilon_{t+1}^x + \beta_{r,x} \sigma^{RW} \epsilon_t^{rw}, \quad (13)$$

where  $x_t$  is the long-run risk component, and the volatility of short- and long-run risks are represented by  $\sigma_t^{srv}$  and  $\sigma_t^{lrv}$ , respectively. These two processes may be time-invariant (Case I of Bansal and Yaron (2004)) or time-varying. In this study, we use the time-invariant volatility specification. Thus,  $\sigma_t^{srv} = \sigma_a$  and  $\sigma_t^{lrv} = \sigma_x$ . Long-run shocks alter productivity growth, and in turn propagate throughout the economy. The parameter  $\beta_{r,x}$  determines the contemporaneous impact of external shocks on the long-run component. Since in our empirical analysis we found that internal credit shocks have a statistically insignificant effect on long-run growth, we abstract away from this channel.

To finance its operations, the firm issues debt or equity. As mentioned earlier, in our study we assume that the firm cannot borrow from abroad directly. Rather, the household may use the foreign borrowings to invest in equity or debt. This convention enables us to replicate the Jermann and Quadrini (2012) when we impose  $X \equiv 0$  in order to look at the closed economy version of our model. Debt carries a tax advantage over equity, and is thus the preferred source of funding for the firm. Jermann and Quadrini (2012) and Hennessy and Whited (2005) maintain this assumption.

The firm's objective is maximizing its cum-dividend value, by hiring labor, investing and accumulating capital, and addressing its funding needs. The objective function of the firm, thus, is:

$$V_t^P = d_t + \mathbb{E}_t [M_{t+1}V_{t+1}^P], \quad (14)$$

where  $V_t^P$  is the value of the firm at time  $t$ ,  $d_t$  is the dividend payout, and  $M_t$  is the SDF.<sup>2</sup> This optimization is subject to the following constraints:

$$d_t \leq F(A_t, K_t, H_t) - w_t^P H_t^P - I_t - \chi(d_t) + D_{t+1} - D_t(1 + r_t^D) - T_t^c, \quad (15)$$

$$F_t \leq \xi_t(K_{t+1} - D_{t+1}). \quad (16)$$

Equation (15) represents the firm's budget constraint. Wages paid for labor input  $H_t^P$  are denoted as  $w^P$ . To impose some rigidity in substitution of funding sources for the firm, we introduce a dividend payout cost,  $\chi(d_t)$ . It is a simple way of modeling the speed with which firms can change the source of funds when the financial conditions change. Following Jermann and Quadrini (2012), we impose a quadratic payout cost

$$\begin{aligned} \chi(d_t) &= A_{t-1} \cdot \kappa \left( \frac{d_t}{MA_{t-1}} - \bar{d} \right)^2, \\ \log MA_t &= (1 - \theta)(\mu + \log MA_{t-1} - \Delta a_t), \end{aligned}$$

where  $\kappa \geq 0$ ,  $\bar{d}$  is the steady state payout target, and  $MA_t$  is a slow moving average of the stochastic productivity process that mimics a time-trend and enables us to have

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<sup>2</sup>The firm issues equity that only the household buys. Thus, the SDF of the household, as the owner, applies to the firm's problem.

balanced growth. The government collects a corporate revenue tax,  $T_t^c$ . This tax is defined as

$$T_t^c = \tau_F (F_t - w_t^P H_t^P) - D_t r_t^D \tau_F. \quad (17)$$

The corporate tax rate is represented by  $\tau_F$ . Here, the tax collected on corporate revenues ( $\tau_F (F_t - w_t^P H_t^P)$ ) is partially offset by the tax rebate on interest payments, ( $D_t r_t^D \tau_F$ ).

In equation (16), we impose a credit constraint as in Jermann and Quadrini (2012). The process  $\xi_t$  captures exogenous, country-specific, changes in internal credit conditions. We assume that domestic credit conditions evolve as follows:

$$\xi_{t+1} = (1 - \rho_\xi) \mu_\xi + \rho_\xi \xi_t + \epsilon_{t+1}^\xi + \beta_{r,\xi} \sigma^{RW} \epsilon_t^{rw}. \quad (18)$$

The rationale and specification of both credit constraint and credit shocks are the same as in Jermann and Quadrini (2012) and constitute “internal financial shocks” in this study. The parameter  $\beta_{r,\xi}$  determines the contemporaneous impact of external credit shocks on internal financial shocks.

By solving the optimization problem of the firm, we obtain the following optimality condition for investment:

$$1 = E_t [M_{t+1} R_{t+1}^K | Z]$$

where the return on capital is

$$R_{t+1}^K = \frac{q_{t+1}^d}{q_t^d} \left[ \frac{\left( (1 - \tau_F) - \tilde{\Lambda}_{CC,t+1} \right) F_{K,t+1} + q_{t+1}^K \left( 1 - \delta - \frac{\partial}{\partial K_{t+1}} \left( \Phi \left( \frac{I_{t+1}}{K_{t+1}} \right) K_{t+1} \right) \right)}{q_t^K - \tilde{\Lambda}_{CC,t} \xi_t} \right].$$

In the above expression,  $\tilde{\Lambda}_{CC,t}$  captures the marginal benefit of debt evaluated when the constraint is binding,

$$\tilde{\Lambda}_{CC,t} = \frac{1 - E \left[ M_{t+1} \left\{ \frac{q_{t+1}^d}{q_t^d} (1 + r_t^D (1 - \tau_F)) \right\} \mid Z \right]}{\xi_t},$$

whereas the prices  $q^K$  and  $q^d$  are defined as follows,

$$q_t^K = \frac{1}{1 - \Phi' \left( \frac{I_t}{K_t} \right)} \quad q_t^d = \frac{1}{1 + \chi' (d_t)},$$

and capture the marginal value of installed capital with either only physical adjustment costs or only equity issuance costs.<sup>3</sup> As in Jermann and Quadrini (2012), the financial constraint causes a wedge in the labor market,

$$w_t^P = \left( 1 - \frac{\tilde{\Lambda}_{CC,t}}{1 - \tau_F} \right) F_{H,t},$$

such that a tighter credit constraint (i.e.,  $\tilde{\Lambda}_{CC}$  increases) reduces labor demand.

In this class of models, there is a possibility that the credit constraint may not be binding at equilibrium. In our implementation, we observe negligible instances of such an event. For robustness, we have also solved a convexified version of the problem, i.e., we have considered the problem of the firm with convex distress costs. In Appendix C, we detail our solution of the convexified problem and show that our results are very similar regardless of whether we employ convex distress costs or assume a binding constraint.

Summarizing, there are four exogenous processes in the model: short-run productivity growth shocks, long-run growth news shocks, domestic credit shocks, and

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<sup>3</sup>We use  $\Phi'$  and  $\chi'$  to denote first-order derivatives.



external funding rate shocks. These shocks are assumed to be uncorrelated and serially independent.

### 3.3 Tax Policy and Market Clearing

Next, we turn to the government and then specify the equilibrium conditions.

**Government.** The government levies and collects a tax on firm's sales net of labor costs and interest on corporate debt, as shown in equation (17). The government then redistributes the collected taxes back to the household in a non-distortionary manner. The government runs a balanced budget, i.e., it does not incur deficits or debts. Since we abstract away from wasteful government expenditure, our government has a purely distributive role in the economy.

**Market Clearing.** The recursive competitive equilibrium in this economy requires that (i) the labor market clears, (ii) all equities and corporate bonds are held domestically by the household, and (iii) the goods market clears:

$$F_t = C_t + I_t + (1 + r_t)X_t - X_{t+1} + \chi(d_t).$$

## 4 Calibration

We report our benchmark calibration in table 3 and notice that our parameter values are broadly consistent with those reported in empirical studies of the Euro Area (among others, see Smets and Wouters 2003, Gerali et al. 2010, and Brinca et al. 2016).

Epstein and Zin (1989) preferences have three parameters: the coefficient of relative risk aversion  $\gamma$ , the subjective discount factor  $\beta$ , and the intertemporal elasticity of

substitution  $\psi$ . We choose these parameter values in the spirit of the long-run risk literature (see, among others, Bansal et al. 2012 and Croce 2014).

Since we use a CES aggregator to construct consumption bundle  $\tilde{C}_t$ , we need to calibrate three parameters: consumption coefficient  $\tilde{w}_1$ , leisure coefficient  $\tilde{w}_2$ , and elasticity of substitution coefficient  $f$ . Our choices of parameter are comparable to other studies in the international macroeconomics literature such as, for example, Zetlin-Jones and Shourideh (2017).

The countries in our cross section are all developed and share production characteristics very similar to those in the US. As a result, we choose values for capital share of output  $\alpha_p$  and capital depreciation rate  $\delta$  that are very close to values used in previous studies by Kaltenbrunner and Lochstoer (2010) and Croce (2014). We set the coefficient of the elasticity of adjustment cost  $\phi_2$  to 2 in order to match the volatility of investment growth relative to that of output growth. Our value is smaller than  $\phi_2 = 7$  used by Croce (2014), but it is larger than 0.7 used by Kaltenbrunner and Lochstoer (2010). The corporate tax rate changes across countries, but 0.35 is a good representative value.

We next discuss the parameter values chosen for our exogenous processes, which are based on our empirical investigation. The productivity parameters are consistent with the estimates from our VAR analysis. Our choice of parameter values for the internal financial constraint process is both consistent with our VAR analysis and with the figures chosen in Jermann and Quadrini (2012). The parameters determining the contemporaneous effect of external credit shock on domestic credit conditions and expected productivity growth are set to be consistent with our VAR results in Section 2.2. The parameters that determine the behavior of the country spread  $P_t$  are broadly consistent with the inference that we obtained focusing on the cross section of our

**Table 3: Benchmark Calibration**

| Preferences                              |                     |                 |
|--|---------------------|-----------------|
| Relative Risk Aversion                   | ( $\gamma$ )        | 10              |
| Intertemporal Elasticity of Substitution | ( $\psi$ )          | 2               |
| Subjective Discount Rate                 | ( $\beta$ )         | 0.99            |
| Consumption-Leisure Aggregator           |                     |                 |
| Consumption Coefficient                  | ( $\tilde{w}_1$ )   | 0.35            |
| Leisure Coefficient                      | ( $\tilde{w}_2$ )   | 0.65            |
| Elasticity of Substitution               | ( $f$ )             | 1               |
| Production                               |                     |                 |
| Capital Share                            | ( $\alpha_P$ )      | 0.36            |
| Capital Depreciation Rate                | ( $\delta$ )        | 0.10/4          |
| Capital Adjustment Cost Elasticity       | ( $\phi_2$ )        | 2               |
| Corporate Tax Rate                       | ( $\tau_F$ )        | 0.35            |
| Productivity Growth Rate                 |                     |                 |
| Average                                  | ( $\mu_a$ )         | 0.020/4         |
| Volatility of Short-Run Shock            | ( $\sigma_a$ )      | 0.046/2         |
| Persistence of Long-Run Component        | ( $\rho_x$ )        | 0.95            |
| Volatility of Long-Run Shock             | ( $\sigma_x$ )      | 0.10 $\sigma_a$ |
| Internal Financial Constraint            |                     |                 |
| Average                                  | ( $\mu_\xi$ )       | 0.24            |
| Persistence                              | ( $\rho_\xi$ )      | 0.97            |
| Volatility of Financial Shock            | ( $\sigma_\xi$ )    | 0.012/2         |
| Equity Adj. Cost                         | ( $\kappa$ )        | 0.146           |
| Smooth rescaling factor                  | ( $\theta$ )        | 0.02            |
| External Interest Rate ( $r^W$ )         |                     |                 |
| Average                                  | ( $\mu_{RW}$ )      | 0.011/4         |
| Persistence                              | ( $\rho_{RW}$ )     | 0.80            |
| Volatility of $r^W$ Shock                | ( $\sigma_{RW}$ )   | 0.010/2         |
| Country Spread ( $P_t$ )                 |                     |                 |
| Average External Debt Ratio              | ( $\overline{XY}$ ) | 0.60            |
| Interest Rate Cost Function Exponent     | ( $p_1$ )           | 8               |
| Interest Rate Cost Function Coefficient  | ( $p_2$ )           | 0.008           |

*Notes:* This table reports our benchmark quarterly calibration. The parameters determining the impact of external shocks on domestic shocks and long-run risk are  $\beta_{r,\xi} = -0.70$  and  $\beta_{r,x} = -0.2\%$ , respectively.

selected countries. The contemporaneous correlation between our credit and long-run productivity shocks are as negative as in the data.

## 5 Results

**Simulated moments.** We start our analysis of our SONOMA model by focusing on standard moments obtained from simulations. We report our moments of interest in table 4. The top portion of the table refers to average levels of macroeconomic variables. The middle portion refers to volatilities of macroeconomic aggregates. The bottom part of the table comprises asset pricing moments. The data column refers to average moments of interest across eleven European countries. Numbers in parenthesis are cross-country standard deviations for the moments of interest and do not include estimation uncertainty coming from the time series dimension. As a result, our assessment of the model performance is based on a higher bar as our ranges are tighter than they should be. Our data sources are detailed in Appendix B.

On the macroeconomic side, we point out several results. First, both the open and the closed economy version of our model perform well in explaining the dynamics of macro aggregates. In the open economy setting, we replicate a significant extent of volatility of the external liabilities. This is a relevant success because we abstract away from foreign equities, exchange rate and long-term bonds and hence we mute the valuation channel. Second, in our SONOMA, the correlation between consumption and investment growth is more moderate than in the closed economy setting. Most importantly, this moderate correlation is consistent with our data. Under our benchmark calibration investment is not volatile enough compared to the data because of the presence of strong adjustment costs. We do not consider this result as concerning because untabulated results suggest that we could easily reduce our adjustment costs, match investment volatility, and maintain a sizeable equity premium of at least 5% per year.

**Table 4: Performance of SONOMA Open/Closed**

|   | Data  |                 | Open  | Closed |
|---|-------|-----------------|-------|--------|
|   | Est.  | Range           |       |        |
| $E[C^P/Y^P]$ (%)                        | 68.74 | [63.18 , 74.07] | 79.60 | 79.55  |
| $E[I/Y^P]$ (%)                          | 27.88 | [26.93 , 29.60] | 20.54 | 20.47  |
| $E[X/Y^P]$ (%)                          | 50.68 | [32.68 , 65.46] | 44.93 | –      |
| $\sigma(\Delta i)/\sigma(\Delta y_p)$   | 2.12  | [1.76 , 2.22]   | 1.72  | 1.74   |
| $\sigma(\Delta c_p)/\sigma(\Delta y_p)$ | 0.72  | [0.69 , 0.99]   | 0.98  | 0.90   |
| $corr(\Delta i, \Delta c_p)$            | 0.42  | [0.22 , 0.62]   | 0.76  | 0.68   |
| $\sigma(\Delta(X/Y^P))$ (%)             | 3.12  | [2.02 , 4.29]   | 2.44  | –      |
| $E[R - R_W]$ (%)                        | 0.51  | [0.26 , 0.77]   | 0.70  | –      |
| $\sigma(R - R_W)$ (%)                   | 0.62  | [0.53 , 0.91]   | 0.56  | –      |
| $E[R_E - R]$ (%)                        | 8.07  | [5.80 , 11.11]  | 4.14  | 4.24   |
| $\sigma(R_E - R)$ (%)                   | 23.04 | [22.28 , 24.57] | 6.02  | 6.84   |
| $E[R_K - R]$ (%)                        | 4.98  | [1.99 , 6.94]   | 2.69  | 2.81   |
| $\sigma(R_K - R)$ (%)                   | 19.11 | [13.54 , 20.97] | 3.83  | 3.44   |
| $E[D/K]$ (%)                            | 22.44 | [14.86 , 33.71] | 39.56 | 40.00  |
| $\sigma(D/K)$ (%)                       | 2.61  | [0.99 , 4.08]   | 6.84  | 6.30   |

*Notes:* Both data and model moments are based on quarterly observations. Interest rate and return moments are annualized. Quarterly data used to compute moments are from 1995Q1–2017Q4. Data moment estimates are the median of the moment values computed for each country in our sample. The data moment range shows the second smallest and second largest values. The open economy calibration is the same as the closed economy calibration except for the addition of open economy parameters (see table 3). Both calibrations are chosen to produce the same average interest rate.  $R_K$  and  $R_E$  denote unlevered returns and equity returns, respectively.

On the asset pricing side, we note that our model has two limitations. First, equity returns are too smooth compared to the data. This is a very well-known problem that is common to many macro-finance models. Simultaneously, leverage is more volatile than in the data. Importantly, average leverage is not excessive and hence our equity risk premium is not driven by an implausible amount of financial leverage.

**Impulse responses.** In order to better understand these results, we study impulse responses to shocks both in our SONOMA setting and in its companion closed economy version. We depict the relevant responses in figure 2. The left-hand side panels refer to the closed economy setting and replicate the baseline results of Jermann and

Quadrini (2012): both adverse productivity and adverse credit shocks are contractionary.

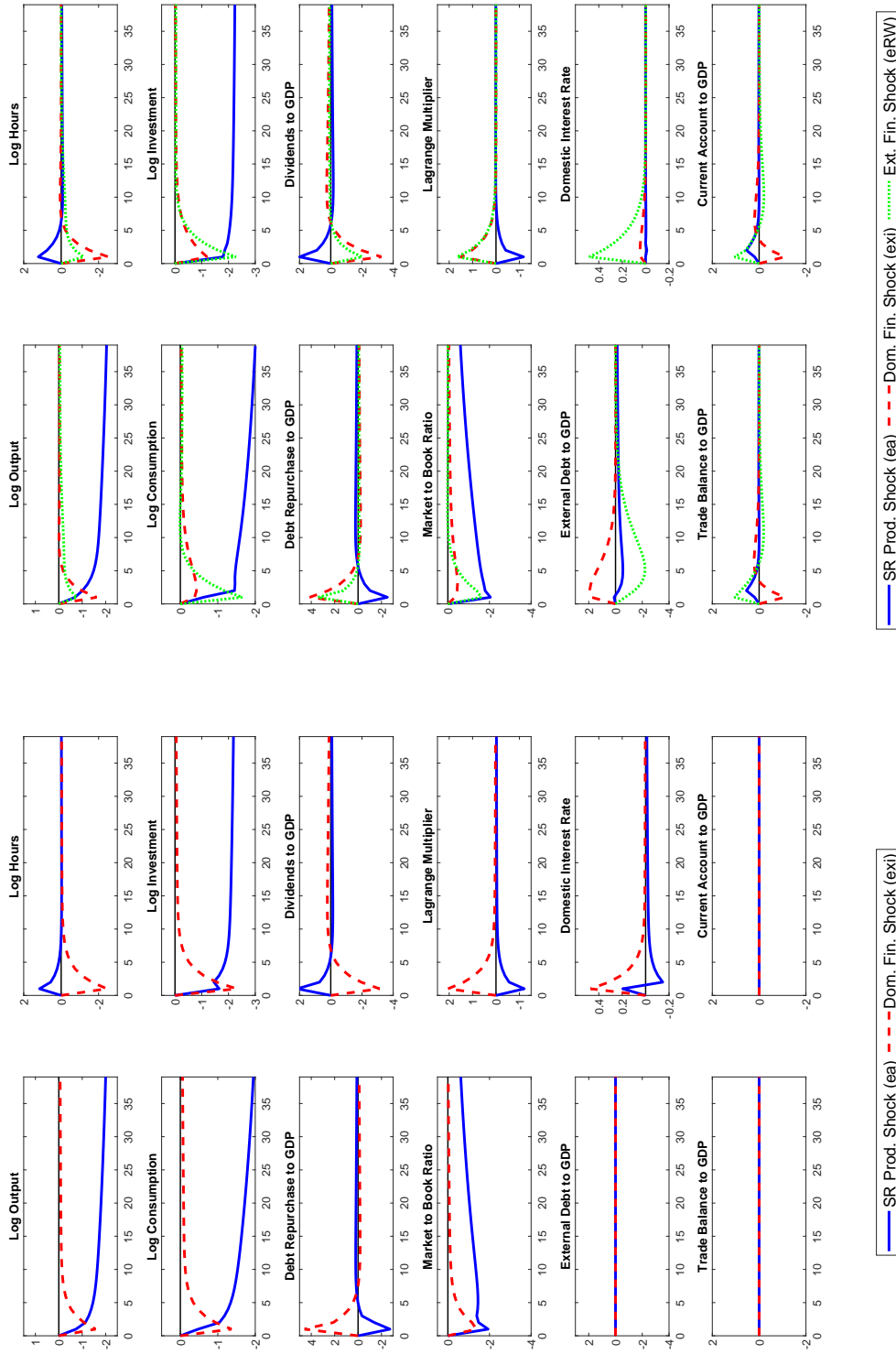
The right-hand side panels show the responses that we obtain in the open economy. The responses with respect to a negative productivity short-run shock is almost identical to the one obtained in closed economy. Here we just point out that a negative productivity shock reduces output and makes external imbalances-to-output more pronounced. As a result, the household finds it optimal to export more in order to reduce the external debt and hence the portion of its external financing costs stemming from the spread function.

Having access to global capital markets helps the firm in hedging domestic credit shocks. This is particularly visible in the corporate structure adjustments. When an adverse internal credit shock materializes, the firm reduces its debt and issues more equities that the household purchases thanks to external financing, i.e., foreign liabilities.

On the other hand, in SONOMA our household is subject to external credit shocks. When an adverse external shock materializes, several effects unfold. First, this shock is as contractionary as a domestic credit shock. In an attempt to reduce the external cost of borrowing, the country increases exports in order to reduce external liabilities. Higher exports are obtained both by reducing domestic expenditures ( $C + I$ ), and by working more.<sup>4</sup> This kind of labor response is due to the negative income effect generated by the external credit shock. We note that even though both external and internal credit shocks have contractionary effects on consumption and investment, they have opposite implications for net foreign liabilities. This result is important for the identification of domestic and external credit shocks.

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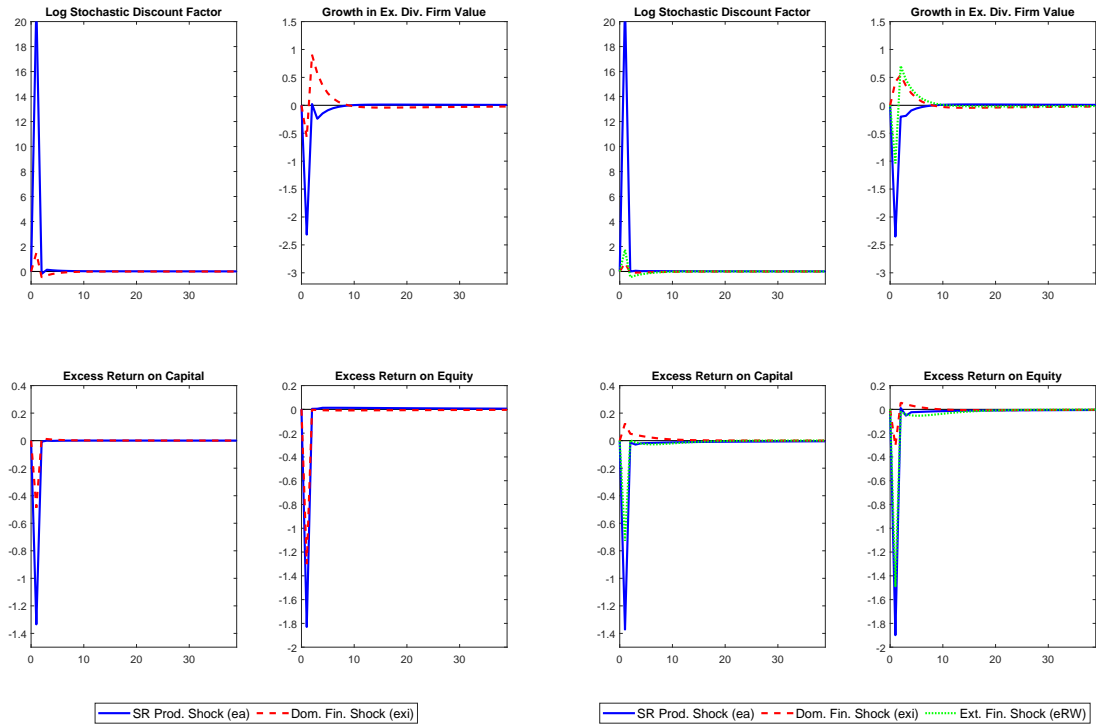
<sup>4</sup>The responses to a pure external shock can be obtained by setting  $\beta_{r,\xi} = \beta_{r,x} = 0$  and look very similar to the ones we depict. The only difference is that output increases slightly because the household works more (negative wealth effect).



(a) SONOMA Closed

(b) SONOMA

**Fig. 2. Impulse Responses in SONOMA vs SONOMA Closed.** This figure shows percentage deviations from steady state for variables expressed in logs (percentage point deviations from steady state for variables expressed in levels) in response to a one standard deviation adverse shock to short-run productivity, domestic credit conditions, and external cost of financing. Our benchmark calibration is reported in table 3. The left-hand side panels refer to the closed economy case (see Appendix A).



(a) SONOMA Closed

(b) SONOMA

**Fig. 3. Impulse Responses in SONOMA vs SONOMA Closed (II).** This figure shows percentage deviations from steady state for variables expressed in logs (percentage point deviations from steady state for variables expressed in levels) in response to a one standard deviation adverse shock to short-run productivity, domestic credit conditions, and external cost of financing. Our benchmark calibration is reported in table 3. The left-hand side panels refer to the closed economy case (see Appendix A).

At the corporate level, we notice that the firm finds it optimal to rebalance its capital structure by tilting it toward equity. This adjustment is optimal because, by no arbitrage, the external shock causes corporate debt to be relatively more expensive.

We now turn our attention toward financial variables and show key impulse response functions in figure 3. As before, the left-hand side panels refer to dynamics obtained in a closed economy. This specific case is interesting because Jermann and Quadrini (2012) have not addressed the implications of their setting for asset pricing.

Even though domestic financial shocks have a moderate impact of future utility and hence on the SDF, they have very severe negative effects on equity returns.



Specifically, domestic financial shocks produce a drop in excess returns as severe as the fall caused by negative productivity shocks. There is, however, an important difference that we point out: productivity shocks depress significantly the ex-dividend value of the firm, whereas domestic financial shocks cause de-leveraging and depress significantly the equity payout.

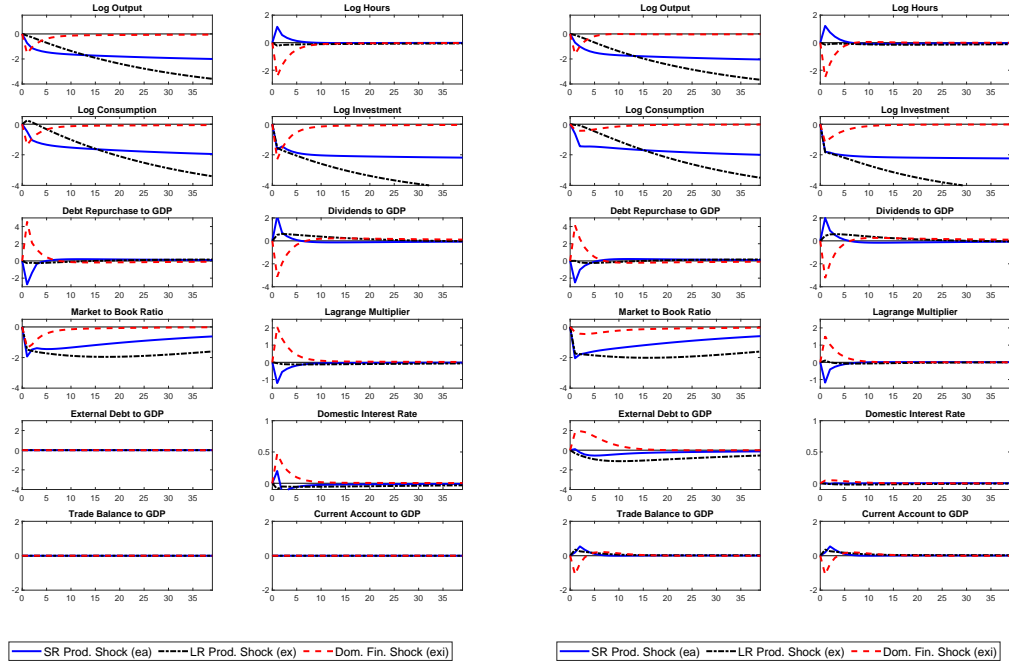
In SONOMA, the effect of a short-run productivity shock on the SDF and the excess returns is very similar to that documented in the closed economy. The effects of a domestic credit shock, however, become less severe as foreign liabilities can be used to substitute away corporate debt. Equivalently, the interest rate does not adjust as in the closed economy setting. If we account for both external and internal credit shocks, however, it is still true that a sizeable share of equity excess returns variance is explained by financial shocks.

In figure 4, we depict the responses of our variables of interest with respect to a negative long-run growth news shock. Like in the previous figures, we compare these responses to those generated from short-run productivity shocks and domestic credit shocks. We note a few interesting features. First, in our closed economy setting, long-run news shocks generate dynamics qualitatively similar to those in Croce (2014) despite the presence of financial frictions. Second, in our SONOMA setting, negative long-run news shocks generate an incentive to reduce external debt by exporting more. In anticipation of slow output growth, the representative agent finds it optimal to immediately reduce external debt in order to avoid an increase in the external cost of capital. Third, negative growth news shocks are associated with significant increases in marginal utility and severe negative returns. This dynamics explain a substantial part of our equity risk premium.

**Inspecting the mechanism.** In table 5, we examine the performance of SONOMA when we remove one or more of its salient elements. For example, in the second column we show how our results change if we adopt time-additive log preferences as in Jermann and Quadrini (2012). On the macroeconomic side, we observe minor changes (see also figure 5). On the asset pricing side, instead, we have a strong deterioration of the results. This should not be surprising given that with CRRA preferences, growth news shocks are not priced.

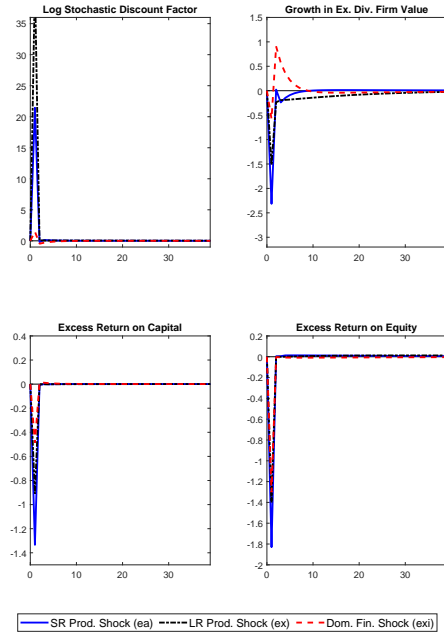
In the last three columns of table 5, we assess the role of financial shocks and frictions through different perspectives. Specifically, we first retain all features of our SONOMA economy, but zero out domestic credit shocks. In this case, the volatility of output immediately declines substantially, consistent with the fact that credit shocks have a first-order impact on economic activity. Because of the presence of shocks to the external borrowing rate, consumption continues to be quite volatile and becomes more volatile than output, a counterfactual result. Furthermore, the equity risk premium declines by about 11 basis points on an annual basis. This is an important result as it confirms that domestic credit shocks are relevant in explaining equity dynamics.

In the second to last column of table 5, we retain external borrowing shocks, but we completely eliminate the corporate tax advantage on debt and any other form of financial frictions. In this case, the firm is 100% equity financed and equity issuance is free, as in a frictionless neoclassical model. In this case, we see a very pronounced decline in equity returns relative to the benchmark SONOMA setting. Importantly, the decline is not just mechanically due to financial leverage. Focusing on unlevered returns, it is easy to see that they fall significantly as soon as financial frictions are removed. This result confirms that the financial frictions studied in this manuscript are very important for both production and risk in all segments of the capital markets.

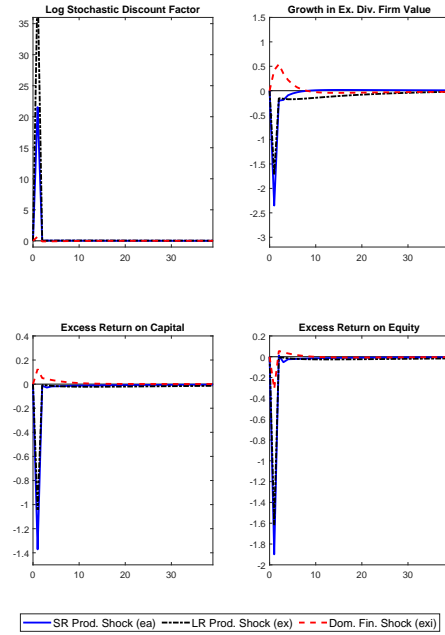


(a) SONOMA Closed

(b) SONOMA



(c) SONOMA Closed



(d) SONOMA

**Fig. 4. Impulse Responses in SONOMA vs SONOMA Closed (III).** This figure shows percentage deviations from steady state for in response to a one standard deviation adverse shock to short-run productivity, long-run productivity, and domestic credit conditions. Our benchmark calibration is reported in table 3. The left-hand side panels refer to the closed economy case (see Appendix A).

**Table 5: SONOMA - Inspecting the Mechanism**

|   | SONOMA | No EZ | No Fin. Factors  |                  |                   |
|---|--------|-------|------------------|------------------|-------------------|
|   |        |       | $\sigma_\xi = 0$ | $\kappa = D = 0$ | $\sigma_{RW} = 0$ |
| $E[C^P/Y^P]$ (%)                        | 79.60  | 78.55 | 79.55            | 79.31            | 79.55             |
| $E[I/Y^P]$ (%)                          | 20.54  | 21.24 | 20.57            | 20.73            | 20.53             |
| $E[X/Y^P]$ (%)                          | 44.93  | 55.66 | 45.94            | 42.99            | 46.61             |
| $\sigma(\Delta i)/\sigma(\Delta y_p)$   | 1.72   | 1.51  | 2.81             | 1.32             | 1.44              |
| $\sigma(\Delta c_p)/\sigma(\Delta y_p)$ | 0.98   | 1.09  | 1.67             | 0.75             | 0.61              |
| $corr(\Delta i, \Delta c_p)$            | 0.76   | 0.98  | 0.75             | 0.68             | 0.53              |
| $\sigma(\Delta(X/Y^P))$ (%)             | 2.44   | 2.22  | 1.47             | 4.04             | 1.98              |
| $E[R - R_W]$ (%)                        | 0.70   | 2.78  | 0.82             | 0.29             | 0.79              |
| $\sigma(R - R_W)$ (%)                   | 0.56   | 1.07  | 0.50             | 0.99             | 0.42              |
| $E[R_E - R]$ (%)                        | 4.14   | 0.52  | 4.03             | 2.29             | 4.05              |
| $\sigma(R_E - R)$ (%)                   | 6.02   | 5.20  | 5.90             | 3.64             | 5.08              |
| $E[R_K - R]$ (%)                        | 2.69   | 0.30  | 2.64             | 2.29             | 2.64              |
| $\sigma(R_K - R)$ (%)                   | 3.83   | 3.12  | 3.71             | 3.64             | 3.44              |
| $E[D/K]$ (%)                            | 39.56  | 41.15 | 39.86            | 0                | 39.84             |
| $\sigma(D/K)$ (%)                       | 6.84   | 7.01  | 5.05             | 0                | 6.51              |

*Notes:* The calibration for SONOMA is reported in table 3. “No EZ” refers to the CRRA preferences case ( $\gamma = 1/\psi = 1$ , as in Jermann and Quadrini (2012)). For the “No Fin. Factors” columns, we either remove the credit shocks by setting their standard deviation to zero or remove the presence of financial frictions.

We conclude our analysis by looking at the case in which only external credit shocks are removed. In this case, consumption becomes very smooth and external imbalances too stable.

**The role of co-skewness.** We modify our exogenous processes by adding a common tail event,  $J_t = j_0 e^{j_1 \cdot \epsilon_{j,t}} > 0$ , to both the external cost of capital and expected long-run growth as follows:

$$r_t^w = (1 - \rho_{rw})(\mu_{rw} - j_0) + \rho_{rw}r_{t-1}^w + \epsilon_{r,t} + (1 - \rho_{rw})J_t$$

$$x_t = (1 - \rho_x)j_0 + \rho_x x_{t-1} + \epsilon_{x,t} + s_{xr}\epsilon_{r,t} + s_{x\xi}\epsilon_{\xi,t} - (1 - \rho_x)J_t,$$

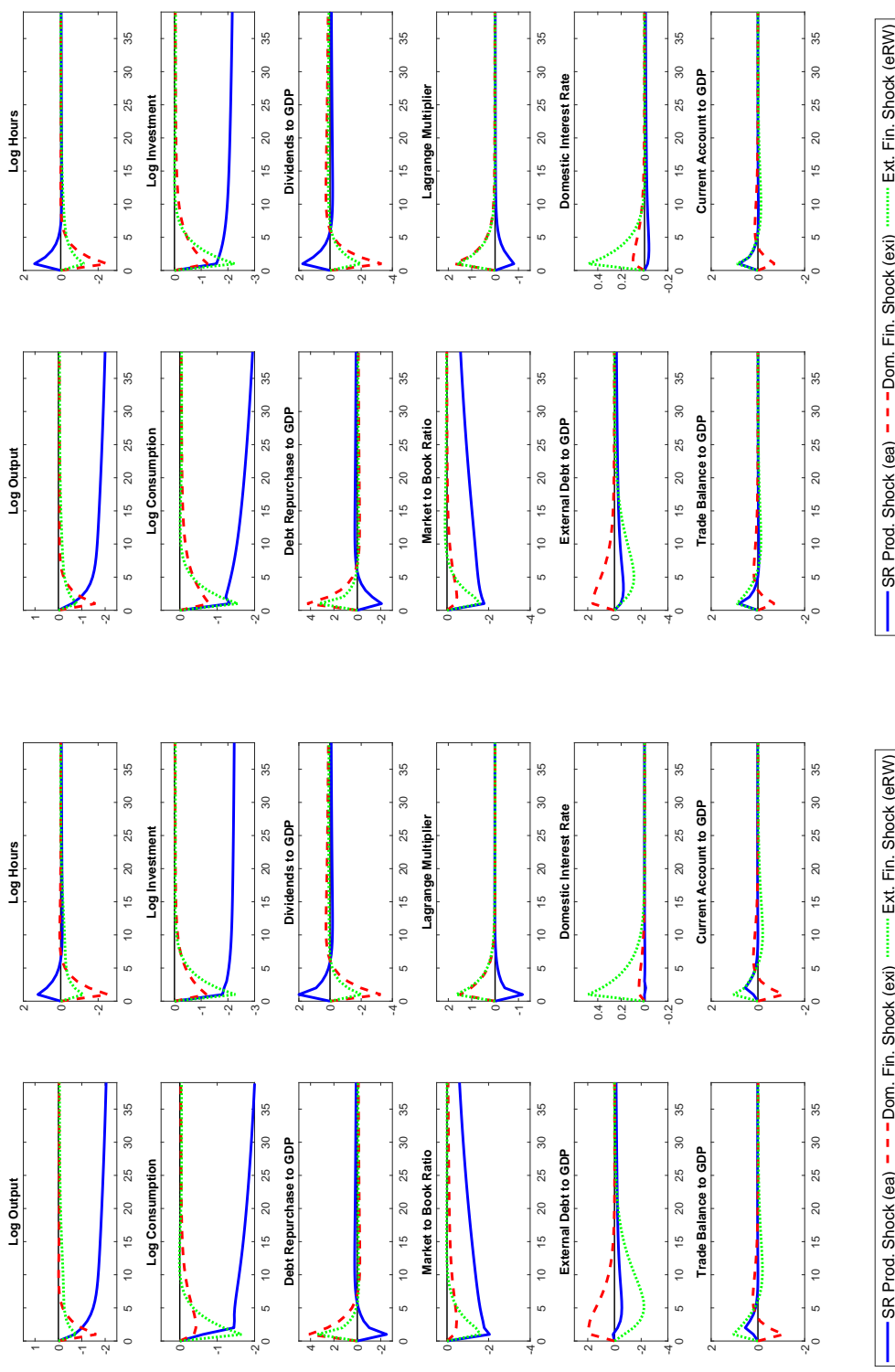
where  $\epsilon_{j,t} \sim N(0, 1)$ ,  $j_1 = 2.2$ , and  $j_0 = 2.e^{-5}$ . We set  $j_0$  to a very small number so that under our exponential functional form this term is on average very small and almost irrelevant. Simultaneously, we set  $j_1$  to a level that enables us to obtain a co-skewness of 0.37, that is, a value that is consistent with our empirical findings.

In small samples, most of our simulated moments change only slightly from the benchmark reported in table 4. There are, however, three notable departures. First, the introduction of this tail event makes the overall economy riskier. As a result the return on equity increases by 19 basis points on an annual basis. Second, this increase in the cost of capital reduces the steady state capital-to-productivity ratio by 0.9%. In order to assess the economic relevance of this decline, we compute the welfare loss induced by this tail shock and find a cost equal to 16% of life-time consumption. This number is due both to the substantial average decline in the level of the consumption profile and to the fact the our agent dislike adverse tail shocks. Our SONOMA model hence suggests that in small economies with financial frictions, credit shocks can have very strong detrimental effects because they are a leading indicator of a negative outlook about long-run growth.

## 6 Conclusions

We develop a small economy model in which external debt, corporate domestic debt and risky equities coexist. Our economy features shocks to short- and long- run productivity, as well as shocks to both domestic credit conditions (Jermann and Quadrini, 2012) and global credit markets. We show that credit shocks are an important determinant of economic fluctuations in a model consistent with asset pricing facts. The powerful quantitative performance of our setting makes it ideal for future monetary and fiscal policy analysis.

Current additional work in progress suggests that external credit shocks are very sizable when we focus on European small-but-developed economies (among others, the PIGS). Furthermore, credit shocks share a common negative jump component with long-run growth. That is, extremely negative external credit shocks tend to realize simultaneously to severe bad growth news shocks. These observations, along with the potential of this setting for conducting very rich policy analysis, make our SONOMA model very appealing for a broad set of future economic investigations.



(a) SONOMA ( $\gamma = 10, \psi = 2$ )

(b) SONOMA, NO EZ

**Fig. 5. Impulse Responses in SONOMA: EZ vs CRRA.** This figure shows percentage deviations from steady state for variables expressed in logs, and percentage point deviations from steady state for variables expressed in levels. The calibration for SONOMA is reported in table 3. “No EZ” refers to the CRRA preferences case ( $\gamma = 1/\psi = 1$ , as in Jermann and Quadrini (2012)).

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## Appendix A: Model Solution Details

In what follows, we report our derivations step by step so that our computational methods can be easily replicated. Our set of equilibrium equations is solved by high-order perturbations computed in dynare++.

### Handling Non-Stationary Utility in Dynare++

The stochastic discount factor (SDF) of the household is given as

$$\tilde{M}_{t+1} = \beta \left( \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\frac{1}{\psi}} \left( \frac{U_{t+1}}{E_t [U_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma} \frac{\partial \tilde{C}_{t+1} / \partial C_{t+1}^P}{\partial \tilde{C}_t / \partial C_t^P}$$

The term involving  $U_{t+1}$  can be manipulated into being in terms of  $\frac{U}{C}$  ratio and  $\frac{\tilde{C}_{t+1}}{\tilde{C}_t}$

$$\begin{aligned} \left( \frac{U_{t+1}}{E_t [U_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma} &= \left( \frac{\frac{1}{\tilde{C}_t} U_{t+1}}{\frac{1}{\tilde{C}_t} E_t [U_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma} \\ &= \left( \frac{\frac{1}{\tilde{C}_t} U_{t+1}}{E_t \left[ \left( \frac{1}{\tilde{C}_t} U_{t+1} \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma} \\ &= \left( \frac{\frac{1}{\tilde{C}_t} U_{t+1} \cdot \frac{\tilde{C}_{t+1}}{\tilde{C}_{t+1}}}{E_t \left[ \left( \frac{1}{\tilde{C}_t} U_{t+1} \cdot \frac{\tilde{C}_{t+1}}{\tilde{C}_{t+1}} \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma} \\ &= \left( \frac{\frac{U_{t+1}}{\tilde{C}_{t+1}} \cdot \frac{\tilde{C}_{t+1}}{\tilde{C}_t}}{E_t \left[ \left( \frac{U_{t+1}}{\tilde{C}_{t+1}} \cdot \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma} \end{aligned}$$

All of the variables in this expression are stationary and can be represented in Dynare++.

## Solving the Household Problem

The consolidated HH problem with Lagrange multipliers is

$$\begin{aligned}
 U(B, S, D, X, Z) = \max_{C^P, H^{P,s}, S', D', X'} W & \left( \tilde{C}(C^P, 1 - H^{P,s}), \{U(S', D', X', Z')\}_{Z'} \right) \\
 & + \Lambda_{BC} [w_P H^{P,s} - T_{LS} + S(V^{P,ex} + d) + (1 + r_{-1}^D) D + X'] \\
 & - \Lambda_{BC} [C^P + B' + S' V^{P,ex} + D' + (1 + r_{-1}) X]
 \end{aligned}$$

where we have substituted out leisure using the time constraint as  $\ell = 1 - H^{P,s}$ .

The FOCs are

$$\begin{aligned}
 0 &= W_1 \tilde{C}_1 - \Lambda_{BC} \\
 0 &= W_1 \tilde{C}_2 + \Lambda_{BC} w_P \\
 0 &= \sum_{z'} W_2' U_S' - \Lambda_{BC} V^{P,ex} \\
 0 &= \sum_{z'} W_2' U_D' - \Lambda_{BC} \\
 0 &= \sum_{z'} W_2' U_X' + \Lambda_{BC}
 \end{aligned}$$

The envelope conditions for  $S$ ,  $D$ , and  $X$  are

$$\begin{aligned}
 U_S &= \Lambda_{BC} (V^{P,ex} + d) \\
 U_D &= \Lambda_{BC} (1 + r_{-1}^D) \\
 U_X &= -\Lambda_{BC} (1 + r_{-1})
 \end{aligned}$$

Combine above together to get the following optimality conditions

$$\begin{aligned}
 w_P &= -\frac{\tilde{C}_2}{\tilde{C}_1} \\
 V^{P,ex} &= \sum_{z'} \frac{W_2' W_1' \tilde{C}_1'}{W_1 \tilde{C}_1} \left( (V^{P,ex})' + d' \right) \\
 1 &= \sum_{z'} \frac{W_2' W_1' \tilde{C}_1'}{W_1 \tilde{C}_1} (1 + r^D) \\
 1 &= \sum_{z'} \frac{W_2' W_1' \tilde{C}_1'}{W_1 \tilde{C}_1} (1 + r)
 \end{aligned}$$

We see that the interest rates on private debt ( $r^D$ ) and external debt ( $r$ ) must equal each other.

## Solving the Firm Problem

The firm's consolidated problem with Lagrange multipliers is

$$\begin{aligned}
V^P(K, D; Z) = & \max_{d, H^{P,d}, I, K', D'} d + \mathbb{E} [M'V(K', D'; Z') \mid Z] \\
& + \Lambda_{BC} \left[ (1 - \tau_F) \left( F(K, H^{P,d}, Z) - w_P H^{P,d} \right) - I - d - \chi(d) \right] \\
& + \Lambda_{BC} [D' - D(1 + r_{-1}^D(1 - \tau_F))] \\
& + \Lambda_K \left[ (1 - \delta)K + I - \Phi\left(\frac{I}{K}\right)K - K' \right] \\
& + \Lambda_{CC} \left[ \xi(K' - D') - F(K, H^{P,d}, Z) \right]
\end{aligned}$$

The FOCs are

$$\begin{aligned}
0 &= 1 - \Lambda_{BC} (1 + \chi'(d)) \\
0 &= \Lambda_{BC} (1 - \tau_F) (F_H - w_P) - \Lambda_{CC} F_H \\
0 &= -\Lambda_{BC} + \Lambda_K \left( 1 - \Phi' \left( \frac{I}{K} \right) \right) \\
0 &= \mathbb{E} [M'V'_K \mid Z] - \Lambda_K + \Lambda_{CC} \xi \\
0 &= \mathbb{E} [M'V'_D \mid Z] + \Lambda_{BC} - \Lambda_{CC} \xi
\end{aligned}$$

The envelope conditions are

$$\begin{aligned}
V_K &= \Lambda_{BC} (1 - \tau_F) F_K + \Lambda_K \left( 1 - \delta - \frac{\partial}{\partial K} \left( \Phi \left( \frac{I}{K} \right) K \right) \right) - \Lambda_{CC} F_K \\
V_D &= -\Lambda_{BC} (1 + r_{-1}^D (1 - \tau_F))
\end{aligned}$$

Combine together the FOCs and envelope conditions and then simplify to get the following optimality conditions

$$\begin{aligned}
w_P &= \left(1 - \frac{(1 + \chi'(d)) \Lambda_{CC}}{1 - \tau_F}\right) F_H \\
\frac{1}{(1 - \Phi'(\frac{I}{K}))} - (1 + \chi'(d)) \Lambda_{CC} \xi &= \mathbb{E} \left[ M' \left\{ \frac{1 + \chi'(d)}{1 + \chi'(d')} [(1 - \tau_F - (1 + \chi'(d')) \Lambda'_{CC}) F'_K] \right\} \mid Z \right] \\
&\quad + \mathbb{E} \left[ M' \left\{ \frac{1 + \chi'(d)}{1 + \chi'(d')} \left[ \frac{\left(1 - \delta - \frac{\partial}{\partial K'} \left( \Phi \left( \frac{I'}{K'} \right) K' \right) \right)}{(1 - \Phi'(\frac{I'}{K'}))} \right] \right\} \mid Z \right] \\
(1 + \chi'(d)) \Lambda_{CC} \xi &= 1 - \mathbb{E} \left[ M' \left\{ \frac{1 + \chi'(d)}{1 + \chi'(d')} (1 + r^D (1 - \tau_F)) \right\} \mid Z \right]
\end{aligned}$$

These equations can be rewritten and expanded as

$$\begin{aligned}
w_P &= \left(1 - \frac{\tilde{\Lambda}_{CC}}{1 - \tau_F}\right) F_H \\
1 &= \mathbb{E} \left[ M' R'_K \mid Z \right] \\
R'_K &= \frac{q'_d}{q_d} \left[ \frac{\left( (1 - \tau_F) - \tilde{\Lambda}_{CC} \right) F'_K + q'_K \left( 1 - \delta - \frac{\partial}{\partial K'} \left( \Phi \left( \frac{I'}{K'} \right) K' \right) \right)}{q_K - \tilde{\Lambda}_{CC} \xi} \right] \\
\tilde{\Lambda}_{CC} &= \frac{1 - \mathbb{E} \left[ M' \left\{ \frac{q'_d}{q_d} (1 + r^D (1 - \tau_F)) \right\} \mid Z \right]}{\xi} \\
q_K &= \frac{1}{1 - \Phi'(\frac{I}{K})} \\
q_d &= \frac{1}{1 + \chi'(d)}
\end{aligned}$$

where we have defined

$$\begin{aligned}
\tilde{\Lambda}_{CC} &\equiv (1 + \chi'(d)) \Lambda_{CC} \\
&\equiv \frac{\Lambda_{CC}}{q_d}
\end{aligned}$$

From this expression, the value of the exact LM is recovered as

$$\Lambda_{CC} = \frac{\tilde{\Lambda}_{CC}}{(1 + \chi'(d))}$$

or

$$\Lambda_{CC} = q_d \tilde{\Lambda}_{CC}$$



where  $\tilde{\Lambda}_{CC}$  is pinned down by the model equation above.

If the collateral constraint binds in equilibrium then  $\Lambda_{CC} > 0$  and

$$\xi (K' - D') = F(K, H^{P,d}, Z)$$

This constraint can be rearranged to define  $D'$  as

$$D' = K' - \frac{F(K, H^{P,d}, Z)}{\xi}$$

From this expression, we can see that  $\xi > 0$  must be set sufficiently high in order for  $D' > 0$  given a set of values for  $K'$  and  $F(K, H^{P,d}, Z)$ .

If  $\Lambda_{CC} = 0$ , then we would have

$$\begin{aligned} 1 &= \mathbb{E} \left[ M' \left\{ \frac{1 + \chi'(d)}{1 + \chi'(d')} (1 + r^D (1 - \tau_F)) \right\} \mid Z \right] \\ &= (1 + r^D (1 - \tau_F)) \mathbb{E} \left[ M' \left\{ \frac{1 + \chi'(d)}{1 + \chi'(d')} \right\} \mid Z \right] \end{aligned}$$

In this case,  $D$  is not pinned down by the collateral constraint (which may not binding because  $\Lambda_{CC} = 0$ ) but rather  $D$  is pinned down according to this above equation.

Note that if  $\chi'(d) = \chi'(d') = 0$  (e.g.,  $\kappa = 0$ ) then the third optimality condition above becomes

$$\Lambda_{CC} \xi = 1 - \mathbb{E} [M' \mid Z] (1 + r^D (1 - \tau_F))$$

From the HH optimality conditions, we know that

$$\mathbb{E} [M' \mid Z] = \frac{1}{1 + r^D}$$

so therefore if  $\tau_F > 0$  then

$$\mathbb{E} [M' \mid Z] (1 + r^D (1 - \tau_F)) < 1$$

meaning that  $\Lambda_{CC} > 0$  with certainty and the collateral constraint binds in equilibrium. As we saw above, it is possible for  $\Lambda_{CC} = 0$  in equilibrium when  $\chi'(d) \neq 0$  and  $\tau_F > 0$ . We will solve the model assuming that the collateral constraint binds and check to make sure that  $\Lambda_{CC}$  is positive in all simulations.

## Closing the Economy

There are only a few equations that need to be modified. First, we no longer need the following equation

$$r_t \neq r_t^w + P \left( \frac{X_t}{Y_t^P} \right)$$

The variable  $r$  will now only represent the domestic risk-free rate through the following existing equations

$$\begin{aligned} M' &\equiv \frac{W_2' W_1' \widehat{C}_1'}{W_1' \widehat{C}_1'} \\ 1 &= E [M' (1 + r^D)] \\ 1 &= E [M' (1 + r)] \end{aligned}$$

We need a new equation to replace the one we are dropping. To replace this equation, we can add

$$X_t = 0$$

This value will flow through to the market clearing for the private sector good

$$Y^P = C^P + I + (1 + r_{-1}) X - X' + \chi(d)$$

and to the interest rate cost function

$$P \left( \frac{X_t}{Y_t^P} \right) = p_2 \exp \left\{ p_1 \left( \frac{X_t}{Y_t^P} - \overline{XY} \right) \right\}$$

Note that  $P \left( \frac{X_t}{Y_t^P} \right)$  is now an extra model variable that does not impact the other equations.

The steady state computations require similar modifications, but they are not exactly the same. We currently back out  $\beta$  such order for

$$\begin{aligned} r_{ss} &= \frac{1}{M_{ss}} - 1 \\ r_{ss} &= r_{ss}^w + p_2 \end{aligned}$$

to both hold. The result is

$$\beta^* = \frac{e^{\mu_a/\psi}}{1 + r_{ss}^w + p_2}$$

We can continue to utilize this value for  $\beta$  especially to keep comparisons between the closed and open economy. The change we must make is to set  $\overline{XY} = 0$  and also

$$\frac{X_{ss}}{A_{t-1}} = 0$$

## Appendix B: Data Sources

In the table below, we summarize our main sources by data type. The sample for each data source and country is detailed in a companion document (Croce et al. (2019), available here: <https://sites.google.com/view/mmcroce/wps>) that we update regularly together with the exhibits in this manuscript.

**Table B1: Summary of Data Sources**

| <b>Panel A: List of Sources</b>                        |                |
|--|----------------|
| <b>Name</b>  | <b>Acronym</b> |
| Bank of International Settlements                      | BIS            |
| International Monetary Fund                            | IMF            |
| Organisation for Economic Co-Operation and Development | OECD           |
| Penn World Table                                       | PWT            |
| Ken French Data Library                                | KF             |
| Bloomberg  | BLOOM          |
| <b>Panel B: Data Sources</b>                           |                |
| <b>Variable</b>  | <b>Source</b>  |
| National aggregates (GDP, C, I)                        | OECD           |
| Depreciation   | PWT            |
| Labor hours  | OECD, PWT      |
| Private sector debt                                    | BIS            |
| Net external debt                                      | IMF            |
| Domestic interest rates                                | OECD           |
| Public equity data                                     | KF, BLOOM      |
| Inflation  | IMF            |
| Exchange rates   | IMF            |

*Notes:* this table summarizes our main data sources.

**National Aggregates.** Quarterly gross domestic product, investment, and consumption data are from the OECD. The data series are pulled from the statistics tool (<https://stats.oecd.org/>) as the full time series are considered estimates and therefore not posted on the main OECD website. Other comprehensive databases such as the IMF's *International Financial Statistics* database only include the recent period (e.g., from the mid 1990s for most European countries).

**Depreciation.** Annual average depreciation rates are from the PWT.

**Labor Income Share.** Annual labor income share values are from the PWT.

**Labor.** Quarterly and annual hours worked are computed from OECD labor data. for most of our countries from Haver Analytics. We compute quarterly (annual) hours worked as the product of quarterly (annual) number of persons employed by the average annual number of hours worked.

**Private Sector Debt.** Quarterly credit to non-financial corporations from all lenders is available from the BIS.

**Net External Debt.** Quarterly and annual asset and liability positions in US dollars are available from the IMF's *Balance of Payments and International Investment Position Statistics* database. We follow the same method as Lane and Milesi-Ferretti (2007) to compute net external debt from the reported asset and liability positions. While Lane and Milesi-Ferretti (2007) do make available their *External Wealth of Nations* dataset, they only provide annual data.

**Domestic Interest Rates.** Quarterly long-term interest rates are available from the OECD. We construct domestic interest rates as the sum of a benchmark large economy (e.g., Germany) plus the spread between the benchmark large economy and the small open economy.

**Public Equity Data.** Monthly equity returns with dividends ( $r^{cum}$ ) and without dividends ( $r^{ex}$ ) are available from the Ken French Data Library. We compute quarterly price-dividend ratios in three steps. First, we compute monthly price ( $p$ ) and dividend ( $d$ ) series using the recursion  $p_t = (1 + r_t^{ex})p_{t-1}$  and  $d_t = (1 + r_t^{cum})p_{t-1} - p_t$  after initializing  $p_0 = 1$ . Second, we compute monthly price-dividend ratios as  $PD_t = p_t / \sum_{\tau=t-11}^t d_\tau$ . Third and finally, we compute quarterly price-divided ratios as the average of the monthly values in a given quarter. For Portugal, the quarterly price-dividend series is from Bloomberg for the MSCI Portugal Index (*MXPT Index*).

**Inflation.** Quarterly consumer price indices for all items are available from the IMF's *International Financial Statistics* database. We use these series in two ways. First, we construct measures of quarterly inflation in order to convert nominal returns into real returns. Second, we deflate nominal quantities using the price index in order to compute real growth rates.

**Exchange Rates.** Quarterly exchange rates are available from the IMF's *International Financial Statistics* database. We use these data to compute gross domestic product figures in US dollars in order to compute net external debt ratios. The external debt data are only provided in US dollars.

**Table C1: Effect of Using Convexified Costs**

|   | SONOMA  |             | Closed SONOMA |             |
|---|---------|-------------|---------------|-------------|
|   | Binding | Convexified | Binding       | Convexified |
| $E[C^P/Y^P]$ (%)                        | 80.2    | 80.1        | 80.6          | 82.5        |
| $E[I/Y^P]$ (%)                          | 20.2    | 20.2        | 19.4          | 17.6        |
| $E[X/Y^P]$ (%)                          | 38.2    | 39.3        | 0             | 0           |
| $\sigma(\Delta i)/\sigma(\Delta y_p)$   | 1       | 1           | 1.4           | 1.4         |
| $\sigma(\Delta y_p)/\sigma(\Delta c_p)$ | 1.4     | 1.4         | 1.1           | 1           |
| $corr(\Delta i, \Delta c_p)$            | 0.7     | 0.7         | 0.8           | 0.8         |
| $\sigma(X/Y^P)$ (%)                     | 10.5    | 10.7        | 0             | 0           |
| $E[R - R_W]$ (%)                        | 0.17    | 0.14        | 0             | 0           |
| $\sigma(R - R_W)$ (%)                   | 0.49    | 0.54        | 0             | 0           |
| $E[R_{E,t} - R_{t-1}^W]$ (%)            | 1.63    | 1.63        | 1.5           | 1.31        |
| $\sigma(R_{E,t} - R_{t-1}^W)$ (%)       | 4.17    | 4.18        | 5.38          | 4.08        |
| $E[R_{K,t} - R_{t-1}^W]$ (%)            | 0.7     | 0.69        | 0.68          | 0.65        |
| $\sigma(R_{K,t} - R_{t-1}^W)$ (%)       | 2       | 2           | 1.86          | 1.87        |

*Notes:* The calibration for SONOMA (our benchmark open economy) is reported in table 3. The SONOMA Closed calibration is the same except that the open economy parameters are superfluous. See Appendix A for description of how we close an open economy.

## Appendix C: Convex Distress Cost

This section shows the results from using a convexified cost instead of the collateral constraint in benchmark calibrations. As we can see in table C1, our results are very similar regardless of whether we employ convex distress costs or assume a binding constraint. We also describe how the model equations are modified to include the convexified cost instead of the binding constraint.

When using a convexified cost, we no longer include the equation for the collateral constraint. This cost effectively represents the constraint. The firm's condensed problem is

$$V^P(K, D; Z) = \max_{d, H^{P,d}, I, K', D'} d + \mathbb{E} [M'V(K', D'; Z') | Z]$$

subject to

$$(1 - \tau_F) \left( F(K, H^{P,d}, Z) - w_P H^{P,d} \right) - I = d + \chi(d) + D(1 + r_{-1}^D(1 - \tau_F)) - D' + CC$$

$$K' = (1 - \delta)K + I - \Phi\left(\frac{I}{K}\right)K$$

where the constraint  $\xi_t (K' - D') \geq F(K, H^{P,d}, Z)$  is now captured by the convexified cost function  $CC$ . Following Croce et al. (2012), we use the following functional form

$$CC = A_{t-1} \times cc_1 \times e^{-cc_2[\xi_t(K'-D')-F(K,H^{P,d},Z)]}$$

Note how we still can have the financial shocks  $\xi_t$  and that it can be time-varying. In order to best represent an inequality, the parameter  $cc_1$  is set to be small and close to zero ( $cc_1 = 0.001$ ) and the parameter  $cc_2$  is large ( $cc_2 = 1000$ ).

The firm's consolidated problem with Lagrange multipliers is

$$\begin{aligned} V^P(K, D; Z) = & \max_{d, H^{P,d}, I, K', D'} d + \mathbb{E} [M'V(K', D'; Z') \mid Z] \\ & + \Lambda_{BC} \left[ (1 - \tau_F) \left( F(K, H^{P,d}, Z) - w_P H^{P,d} \right) \right] \\ & + \Lambda_{BC} \left[ -I - d - \chi(d) + D' - D(1 + r_{-1}^D(1 - \tau_F)) - CC \right] \\ & + \Lambda_K \left[ (1 - \delta)K + I - \Phi\left(\frac{I}{K}\right)K - K' \right] \end{aligned}$$

The FOCs are

$$\begin{aligned} 0 &= 1 - \Lambda_{BC} (1 + \chi'(d)) \\ 0 &= \Lambda_{BC} \left[ (1 - \tau_F) (F_H - w_P) - \frac{\partial CC}{\partial H^{P,d}} \right] \\ 0 &= -\Lambda_{BC} + \Lambda_K \left( 1 - \Phi' \left( \frac{I}{K} \right) \right) \\ 0 &= \mathbb{E} [M'V'_K \mid Z] - \Lambda_K - \Lambda_{BC} \frac{\partial CC}{\partial K'} \\ 0 &= \mathbb{E} [M'V'_D \mid Z] + \Lambda_{BC} \left( 1 - \frac{\partial CC}{\partial D'} \right) \end{aligned}$$

The envelope conditions are

$$\begin{aligned} V_K &= \Lambda_{BC} \left[ (1 - \tau_F) F_K - \frac{\partial CC}{\partial K} \right] + \Lambda_K \left( 1 - \delta - \frac{\partial}{\partial K} \left( \Phi \left( \frac{I}{K} \right) K \right) \right) \\ V_D &= -\Lambda_{BC} (1 + r_{-1}^D(1 - \tau_F)) \end{aligned}$$

Combine together the FOCs and envelope conditions to get the following optimality conditions

$$\begin{aligned}
\Lambda_{BC} &= \frac{1}{1 + \chi'(d)} \\
(1 - \tau_F)(F_H - w_P) &= \frac{\partial CC}{\partial H^{P,d}} \\
\Lambda_{BC} &= \Lambda_K \left( 1 - \Phi' \left( \frac{I}{K} \right) \right) \\
\Lambda_K + \Lambda_{BC} \frac{\partial CC}{\partial K'} &= \mathbb{E} \left[ M' \left\{ \Lambda'_{BC} \left[ (1 - \tau_F) F'_K - \left( \frac{\partial CC}{\partial K} \right)' \right] \right\} \mid Z \right] \\
&= +\mathbb{E} \left[ M' \left\{ \Lambda'_K \left( 1 - \delta - \frac{\partial}{\partial K'} \left( \Phi \left( \frac{I'}{K'} \right) K' \right) \right) \right\} \mid Z \right] \\
\Lambda_{BC} \left( 1 - \frac{\partial CC}{\partial D'} \right) &= \mathbb{E} \left[ M' \left( \Lambda'_{BC} (1 + r^D (1 - \tau_F)) \right) \mid Z \right]
\end{aligned}$$

Simplify and condense

$$\begin{aligned}
(1 - \tau_F)(F_H - w_P) &= \frac{\partial CC}{\partial H^{P,d}} \\
\left( \frac{1}{1 + \chi'(d)} \right) \left( \frac{1}{(1 - \Phi'(\frac{I}{K}))} + \frac{\partial CC}{\partial K'} \right) &= \mathbb{E} \left[ M' \left\{ \frac{1}{1 + \chi'(d')} \left[ (1 - \tau_F) F'_K - \left( \frac{\partial CC}{\partial K} \right)' \right] \right\} \mid Z \right] \\
&= +\mathbb{E} \left[ M' \left\{ \frac{\left( 1 - \delta - \frac{\partial}{\partial K'} \left( \Phi \left( \frac{I'}{K'} \right) K' \right) \right)}{(1 + \chi'(d')) (1 - \Phi'(\frac{I'}{K'}))} \right\} \mid Z \right] \\
1 - \frac{\partial CC}{\partial D'} &= (1 + r^D (1 - \tau_F)) \mathbb{E} \left[ M' \left( \frac{1 + \chi'(d)}{1 + \chi'(d')} \right) \mid Z \right]
\end{aligned}$$

These equations can be rewritten and expanded as

$$\begin{aligned}
\frac{\partial CC}{\partial H^{P,d}} &= (1 - \tau_F) (F_H - w_P) \\
1 &= \mathbb{E} \left[ M' R'_K \mid Z \right] \\
R'_K &= \left( \frac{q'_d}{q_d} \right) \left( \frac{\left[ (1 - \tau_F) F'_K - \left( \frac{\partial CC}{\partial K} \right)' \right] + q'_K \left( 1 - \delta - \frac{\partial}{\partial K'} \left( \Phi \left( \frac{I'}{K'} \right) K' \right) \right)}{q_K + \frac{\partial CC}{\partial K'}} \right) \\
\frac{\partial CC}{\partial D'} &= 1 - \mathbb{E} \left[ M' \left( \frac{q'_d}{q_d} \right) (1 + r^D (1 - \tau_F)) \mid Z \right] \\
q_K &= \frac{1}{1 - \Phi' \left( \frac{I}{K} \right)} \\
q_d &= \frac{1}{1 + \chi' (d)}
\end{aligned}$$

Using the functional form

$$CC = A_{t-1} \times cc_1 \times e^{-cc_2 [\xi_t (K' - D') - F(K, H^P, d, Z)]}$$

we have the following expressions for the derivatives

$$\begin{aligned}
\frac{\partial CC}{\partial H^{P,d}} &= CC \times cc_2 \times F_H \\
\left( \frac{\partial CC}{\partial K} \right)' &= CC' \times cc_2 \times F'_K \\
\frac{\partial CC}{\partial K'} &= CC \times (-cc_2) \times \xi_t \\
\frac{\partial CC}{\partial D'} &= CC \times cc_2 \times \xi_t
\end{aligned}$$

The only equations that were different under the Lagrange multiplier were

$$\begin{aligned}
w_P &= \left( 1 - \frac{\tilde{\Lambda}_{CC}}{1 - \tau_F} \right) F_H \\
R'_K &= \frac{q'_d}{q_d} \left[ \frac{\left( (1 - \tau_F) - \tilde{\Lambda}'_{CC} \right) F'_K + q'_K \left( 1 - \delta - \frac{\partial}{\partial K'} \left( \Phi \left( \frac{I'}{K'} \right) K' \right) \right)}{q_K - \tilde{\Lambda}_{CC} \xi} \right] \\
\tilde{\Lambda}_{CC} &= \frac{1 - \mathbb{E} \left[ M' \left\{ \frac{q'_d}{q_d} (1 + r^D (1 - \tau_F)) \right\} \mid Z \right]}{\xi}
\end{aligned}$$



The first equation that determines private sector wages can be rearranged as  $\tilde{\Lambda}_{CC}F_H = (1 - \tau_F)(F_H - w_P)$  meaning that

$$\tilde{\Lambda}_{CC}^{WageEqn} F_H \equiv \frac{\partial CC}{\partial H^{P,d}} = CC \times cc_2 \times F_H$$

In the equation for  $R'_K$  we find that

$$\tilde{\Lambda}_{CC}^{CapEqn} \xi \equiv (-1) \times \frac{\partial CC}{\partial K'} = CC \times cc_2 \times \xi$$

In the final equation for the level of debt we find that

$$\tilde{\Lambda}_{CC}^{DebtEqn} \xi \equiv \frac{\partial CC}{\partial D'} = CC \times cc_2 \times \xi$$

So, in the end, we have that the model equations need to be adjusted so that

$$\tilde{\Lambda}_{CC} = CC \times cc_2$$

and also we need to add the equation for the collateral constraint function itself.

## Appendix D: Replication of Jermann and Quadrini (2012)

In this section, we first show that we are able to replicate the results in Jermann and Quadrini (2012). Next, we show how these results change when modifying the parameter values to those in SONOMA.

Table D1 shows the calibration to replicate the benchmark model in Jermann and Quadrini (2012). These parameter values are the same as those in their paper with the following exceptions: we set  $(\sigma_a, \mu_\xi, \sigma_\xi) = (0.0045 \times 3/2, 0.35, 0.005)$  instead of  $(\sigma_a, \mu_\xi, \sigma_\xi) = (0.0045, 0.1634, 0.0098)$ . We choose these values to match their target steady state firm debt to quarterly GDP ratio of 3.36 and the quantitative magnitudes of the impulse response functions seen in Figure 6 of Jermann and Quadrini (2012). We cannot use the same exact values that they use because their production function  $(F(A_t, K_t, H_t) = A_t K_t^\theta H_t^{1-\theta})$  cannot be generalized to the case when  $a$  is non-stationary. In our model equations, we use a functional form that can handle the case when  $A$  and  $K$  are non-stationary:  $F(A_{t-1}, K_t, H_t) = K_t^\theta (A_{t-1} H_t^{1-\theta})$ . As a result, we need to modify slightly the shock sizes and average level of  $\xi$  to quantitatively match their output.

**Table D1: Calibration to Replicate Jermann and Quadrini (2012)**

| Preferences                              |                      |         |
|--|----------------------|---------|
| Relative Risk Aversion                   | $(\gamma)$           | 1.1     |
| Intertemporal Elasticity of Substitution | $(\psi)$             | 0.9     |
| Subjective Discount Rate                 | $(\beta)$            | 0.98    |
| Consumption-Leisure Aggregator           |                      |         |
| Consumption Coefficient                  | $(\tilde{\omega}_1)$ | 1       |
| Leisure Coefficient                      | $(\tilde{\omega}_2)$ | 1.88    |
| Elasticity of Substitution               | $(f)$                | 1       |
| Production                               |                      |         |
| Capital Share                            | $(\alpha_P)$         | 0.36    |
| Capital Depreciation Rate                | $(\delta)$           | 0.10/4  |
| Capital Adjustment Cost Elasticity       | $(\phi_2)$           | 1000    |
| Corporate Tax Rate                       | $(\tau_F)$           | 0.35    |
| Productivity Growth Rate                 |                      |         |
| Average                                  | $(\mu_a)$            | –       |
| Volatility of Short-Run Shock            | $(\sigma_a)$         | 0.027/4 |
| Persistence of Long-Run Component        | $(\rho_x)$           | –       |
| Volatility of Long-Run Shock             | $(\sigma_x)$         | –       |
| Internal Financial Constraint            |                      |         |
| Average                                  | $(\mu_\xi)$          | 0.35    |
| Persistence                              | $(\rho_\xi)$         | 0.97    |
| Volatility of Financial Shock            | $(\sigma_\xi)$       | 0.02/4  |
| Equity Adj. Cost                         | $(\kappa)$           | 0.146   |
| Smooth rescaling factor                  | $(\theta)$           | 0.02    |
| External Interest Rate ( $r^W$ )         |                      |         |
| Average                                  | $(\mu_{RW})$         | –       |
| Persistence                              | $(\rho_{RW})$        | –       |
| Volatility of $r^W$ Shock                | $(\sigma_{RW})$      | –       |
| External Debt and Domestic Interest Rate |                      |         |
| Average External Debt Ratio              | $(\overline{XY})$    | –       |
| Interest Rate Cost Function Exponent     | $(p_1)$              | –       |
| Interest Rate Cost Function Coefficient  | $(p_2)$              | –       |

*Notes:* Parameter values to replicate the benchmark model in Jermann and Quadrini (2012).

In Figure D1, we show impulse responses that confirm our replication effort. The left panel can be compared against Figure 6 of Jermann and Quadrini (2012) and the right panel can be compared against their Figure 7. The key panel in the case with adjustment costs is the Market to Book Ratio. The response to a financial shock switches sign compared to the benchmark.

**Table D2: From Closed Replication to Closed SONOMA**

|   | JQ (w. CAC) | Stronger CAC,<br>Prod. LRR | SONOMA<br>Closed |
|---|-------------|----------------------------|------------------|
| $E[C^P/Y^P]$ (%)                        | 84.5        | 79                         | 79.6             |
| $E[I/Y^P]$ (%)                          | 15.5        | 21                         | 20.5             |
| $E[X/Y^P]$ (%)                          | –           | –                          | –                |
| $\sigma(\Delta i)/\sigma(\Delta y_p)$   | 2.5         | 1.1                        | 1.7              |
| $\sigma(\Delta c_p)/\sigma(\Delta y_p)$ | 0.7         | 1.0                        | 0.9              |
| $corr(\Delta i, \Delta c_p)$            | 1.0         | 1.0                        | 0.7              |
| $\sigma(X/Y^P)$ (%)                     | –           | –                          | –                |
| $E[R - R_W]$ (%)                        | –           | –                          | –                |
| $\sigma(R - R_W)$ (%)                   | –           | –                          | –                |
| $E[R_E - R]$ (%)                        | 0.08        | 0.12                       | 4.24             |
| $\sigma(R_E - R)$ (%)                   | 1.12        | 1.96                       | 6.84             |
| $E[R_K - R]$ (%)                        | 0.04        | 0.04                       | 2.81             |
| $\sigma(R_K - R)$ (%)                   | 0.44        | 0.62                       | 3.44             |
| $E[D/K]$ (%)                            | 53.8        | 59.9                       | 40.0             |
| $\sigma(D/K)$ (%)                       | 2.3         | 2.2                        | 6.3              |

*Notes:* “JQ (w. CAC)” is our Jermann and Quadrini (2012) replication calibration (table D1) with capital adjustment costs set to  $\phi_2 = 5$ . “Stronger CAC, Prod. LRR” means that we further tighten capital adjustment costs to the level in SONOMA and also that we add long-run risk. The SONOMA Closed calibration is the same as the benchmark open economy calibration (table 3) except that the open economy parameters are superfluous.

In table D2, we show how key simulated moments in the model change as we move towards the closed version of our benchmark SONOMA calibration. By comparing the moments across columns, we see that there are three key outcomes that are different in SONOMA closed. First, the correlation between investment growth and consumption growth becomes smaller than one. Second, average excess returns (i.e., the equity premium) increase significantly as well as the volatility of excess returns. This result was to be expected given that SONOMA closed features EZ preferences, higher risk aversion, and an intertemporal elasticity of substitution greater than one. Third and finally, we see the volatility of the firm debt to capital ratio jumps meaning that the firm is more actively changing its debt position.

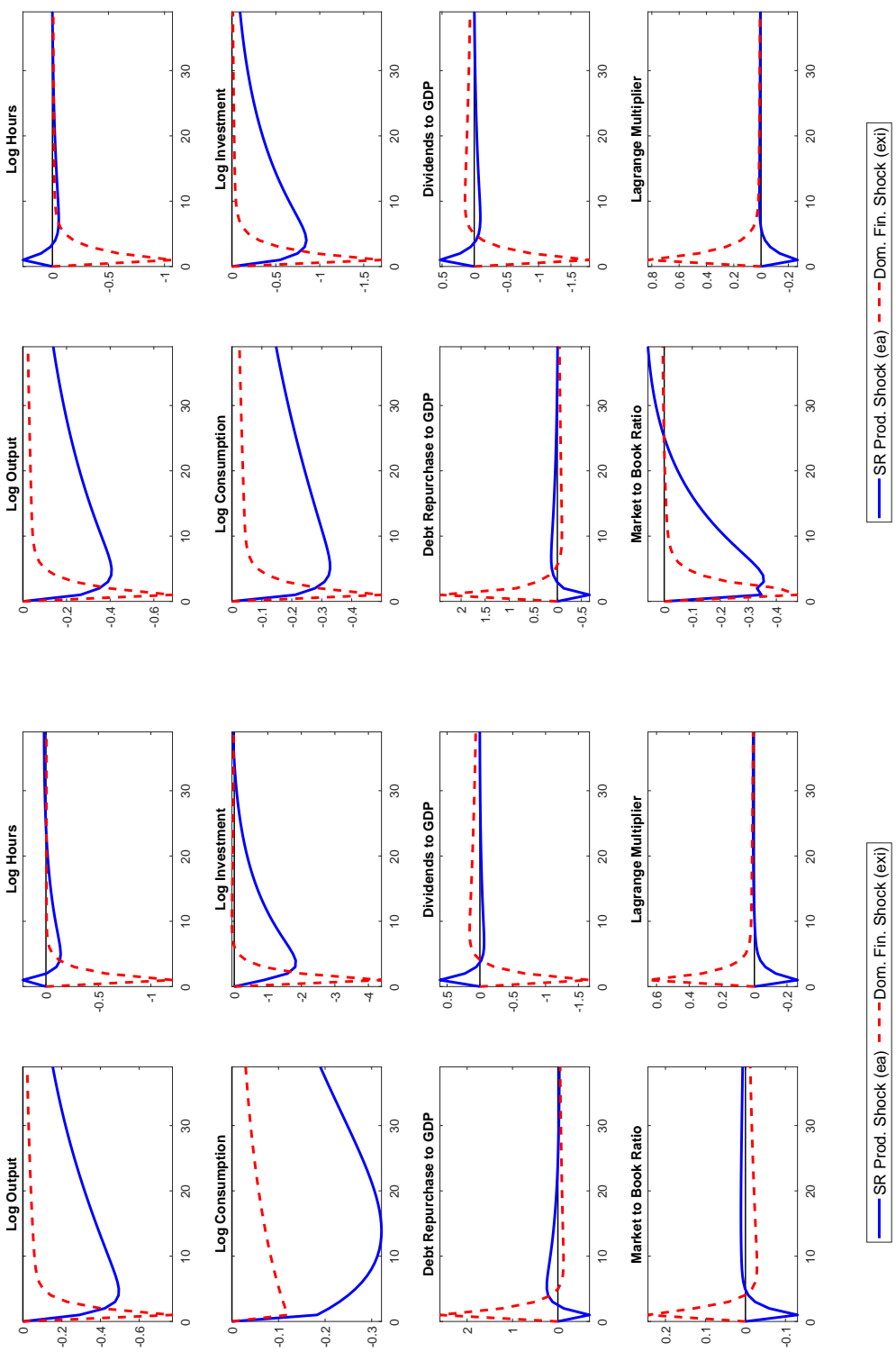
In table D3, we show how key simulated moments in the model change as we move towards our benchmark SONOMA calibration. In the first column, we see that investment is quite volatile given that the benchmark calibration in Jermann and Quadrini (2012) does not have capital adjustment costs. In the remaining columns, we find a similar outcome to table D2 as described above. The difference in the open economy setting is that the volatility of external debt to GDP also increases significantly in SONOMA. This outcome

**Table D3: From Open Replication to SONOMA**

|   | Open JQ | w. CAC | Stronger CAC,<br>Prod. LRR | SONOMA |
|---|---------|--------|----------------------------|--------|
| $E[C^P/Y^P]$ (%)                        | 81.5    | 81.4   | 78.1                       | 79.6   |
| $E[I/Y^P]$ (%)                          | 17.2    | 18.1   | 21.7                       | 20.5   |
| $E[X/Y^P]$ (%)                          | 45.6    | 48.5   | 48.3                       | 44.9   |
| $\sigma(\Delta i)/\sigma(\Delta y_p)$   | 46.7    | 4.0    | 1.4                        | 1.7    |
| $\sigma(\Delta c_p)/\sigma(\Delta y_p)$ | 0.3     | 1.1    | 1.2                        | 1.0    |
| $corr(\Delta i, \Delta c_p)$            | 0.7     | 1.0    | 1.0                        | 0.8    |
| $\sigma(X/Y^P)$ (%)                     | 11.5    | 5.8    | 5.8                        | 9.8    |
| $E[R - R_W]$ (%)                        | 3.04    | 3.00   | 3.00                       | 0.70   |
| $\sigma(R - R_W)$ (%)                   | 1.52    | 0.80   | 0.78                       | 0.56   |
| $E[R_E - R]$ (%)                        | 0.16    | 0.28   | 0.32                       | 4.14   |
| $\sigma(R_E - R)$ (%)                   | 0.68    | 3.16   | 2.84                       | 6.02   |
| $E[R_K - R]$ (%)                        | 0.08    | 0.08   | 0.12                       | 2.69   |
| $\sigma(R_K - R)$ (%)                   | 0.80    | 1.32   | 1.52                       | 3.83   |
| $E[D/K]$ (%)                            | 42.6    | 42.6   | 43.4                       | 39.6   |
| $\sigma(D/K)$ (%)                       | 2.2     | 2.0    | 2.1                        | 6.8    |

*Notes:* Open JQ is an open version of our Jermann and Quadrini (2012) replication (calibration in table D1) with small open economy parameters as in SONOMA (table 3). “w. CAC” means that we add capital adjustment costs by setting  $\phi_2 = 5$ . “Stronger CAC, Prod. LRR” means that we further tighten capital adjustment costs to the level in SONOMA and also that we add long-run risk.

means that the household is more actively changing its external borrowing position to smooth consumption and in response to the domestic firm borrowing demand.



(a) Benchmark

(b) Benchmark with Capital Adjustment Costs

**Fig. D1. Replication of Impulse Responses.** This figure shows percentage deviations from steady state for variables expressed in logs and percentage point deviations from steady state for variables expressed in levels. “Benchmark” refers to the benchmark calibration in Jermann and Quadrini (2012) (see table D1). To add capital adjustment costs parameter as  $\phi_2 = 5$ .