The Optimal Taxation of Lotteries: Who P(l)ays and Who Wins?

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Acknowledgments

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Views here are our own.

Lottery consumption in the U.S.

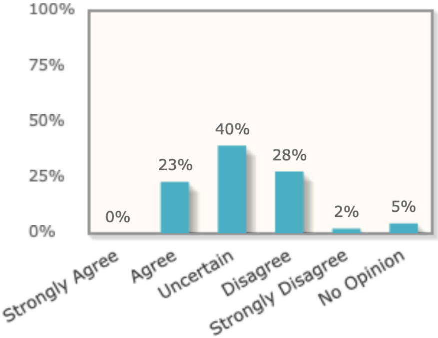
- Americans spend over $70 billion each year on state-run lotteries.
  - Over $600 per household.
  - More than on music, sports events, movie tickets, and video games combined.
- Lotteries are administered by 44 state governments.
  - Over $30 billion annually in public funds.
  - More revenue than federal gasoline tax or estate tax.
- Not unique to U.S., of course
  - E.g., National Lottery in the U.K.

A motivating question

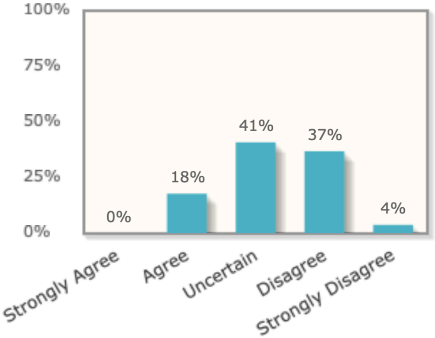
State-run Lotteries

Taking into account the revenues, consumer surplus, purchasing patterns by income, and possible consumer biases, state-run lotteries (such as Powerball and scratch-off games) increase social welfare.

Responses



Responses weighted by each expert's confidence



Are state-run lotteries welfare-enhancing?

Our view

This is fundamentally a question of optimal taxation.

- Lotteries are a heavily taxed product.
  - Implicit + explicit taxes over 50%.
- Distributional concerns
  - Regressive tax on low-income, low-education consumers?
- Behavioral biases
  - Gambling considered a classic “sin good”
  - Misperception? Overoptimism? Self-control problems?

This project

Part 1: Model of Optimal Lottery Taxation

- New sufficient statistics formula for optimal lottery attributes.

Part 2: Empirical evidence

- New large-scale survey of lottery demand and behavioral biases.
- Present descriptive evidence on key parameters that govern optimal policy.

Part 3: Calibration and welfare estimation

- Add structure to study non-local reforms.
- Address policy questions: Are lotteries welfare enhancing? What is optimal tax treatment?

Model

Conceptual framework

- Many challenges normatively evaluating lottery consumption.
  - How to reconcile with expected utility theory and risk aversion?
  - What does it mean to “consume” a lottery ticket?
- Our perspective: a lottery is simply a good with a set of **attributes**:
  - vector of potential **winnings** w_k with **probabilities** π_k, and other attributes of game design
- Basic idea: consumer *i*’s utility from a lottery is

$$U_i = \sum_k \Phi_i(\pi_k) u_i(w_k)$$

  - Consumers apply **decision weights** to potential outcomes; may differ from π_k.
  - Difference may be normatively valid (e.g., anticipatory utility, Caplin Leahy 2001) or driven by behavioral biases (e.g., perceptual distortion, Woodford 2012)

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- Question: how to regulate price *and attributes* $(w_k, \pi_k, \ldots)$ optimally?

Intuition for regulating attributes

- Suppose a “sin good” $\boldsymbol{s}$ has (continuous) attribute $\boldsymbol{a}$ which affects its appeal
  - Examples: cigarette nicotine content, gas-mileage in cars, lottery prizes.
- Like a tax, changing $\boldsymbol{a}$ may affect demand $\Rightarrow$ direct corrective effect.
- Unlike a tax, $\Delta\boldsymbol{a}$ may also change bias cost for inframarginal consumers.
  - Intuition: even if raising nicotine content reduces cigarette demand, may not be good policy...
- We formalize this to characterize optimal *attribute* regulation.

Model setup

- **Consumers**
  - Heterogeneous income-earning ability, preferences; types indexed by i.
  - Numeraire consumption $c(i)$.
  - Discrete choice: share $s(i)$ of i-types choose to purchase lottery on occasion t.
  - Money-metric bias $\gamma(i)$: "price reduction that would cause debiased i to buy $s(i)$."
- **Policymaker**
  - Inequality averse, with welfare weights $g(i)$.
  - Sells lottery tickets at price p, and sets *attributes* (prizes, probabilities, advertising, ...)
- **Today's application**: attribute of interest is lottery expected value, $a := \sum_k \pi_k w_k$
  - Government revenue $= (p - a) \cdot \bar{s} \Rightarrow$ resembles a tax of $p - a$, though a may affect bias.
  - Key new statistics: $\kappa(i) =$ i's average WTP for Δa; $\rho(i) =$ bias in average WTP for Δa.

Optimal prices and attributes

- Optimal p^*: increases with **corrective motive**, decreases with **redistributive motive** (see also: Allcott, Lockwood, Taubinsky 2019):

$$p^* - a = \bar{\gamma}(1+\sigma) - \frac{Cov\left[g(i), s(i)\right]}{\bar{s}\bar{\zeta}_p}$$

$\sigma = Cov\left[g(i), \frac{\gamma(i)}{\bar{\gamma}} \frac{\zeta_p(i)}{\bar{\zeta}_p} \frac{s(i)}{\bar{s}}\right]$: bias correction progressivity

$\zeta_p(i) = \frac{d \ln s(i)}{dp}$: semi-elasticity of demand with respect to price (avg: $\bar{\zeta}_p$)

- Optimal a^*, given price:

$$p - a^* = \bar{\gamma}(1+\sigma_a) - \frac{\mathbb{E}\left[g(i)\left(\kappa(i) - \rho(i)\right) - s(i)\right]}{\bar{\zeta}_a \bar{s}}$$

$\kappa(i)$: i's WTP for Δa; $\rho(i)$: how much of that WTP is due to bias?

$\zeta_a(i) = \frac{d \ln s(i)}{da}$: semi-elasticity of demand with respect to a (avg: $\bar{\zeta}^c$)

- If income effects, use s_{pref}, κ_{pref}: from preference heterogeneity (vs. causal income effects).

Empirical agenda

Optimal lottery regulation formula

$$p - a^* = \bar{\gamma}(1 + \sigma_a) - \frac{\mathbb{E}\left[g(i)\left(\kappa(i) - \rho(i)\right) - s(i)\right]}{\bar{\zeta}_a \bar{s}}$$

Empirical estimation

Formula motivates empirical questions of interest:

1. $s(i)$: What is profile of lottery spending across income distribution?
2. $\gamma(i)$: What is money-metric bias in lottery consumption, across incomes?
3. $\bar{\zeta}_p$: What is price elasticity of lottery demand?
4. $\bar{\zeta}_1$: What is elasticity of lottery demand with respect to jackpots?
5. $\bar{\zeta}_{2+}$: What is elasticity of lottery demand with respect to smaller prizes?

Then: use these moments to calibrate $\sum_k \Phi_i(\pi_k) u_i(w_k)$, then compute welfare, optimal policy.

Empirical Evidence

Data: combine three sources

1. New large representative survey

AmeriSpeak panel: $\sim$2,800 respondents; balanced demographics

2. La Fleur's sales data

Lottery ticket sales by week $\times$ state $\times$ game since 1994

3. Prize and probability data

Collected from lottery rules, prizes scraped from online "are your numbers lucky?" tools

Road map: empirics

1. **What is profile of lottery spending across income distribution?** $s(i)$
2. What is quantity effect of bias in lottery consumption, across incomes? $\gamma(i)\zeta_p(i)$
3. What is elasticity of lottery demand with respect to jackpots? $\bar{\zeta}_1$
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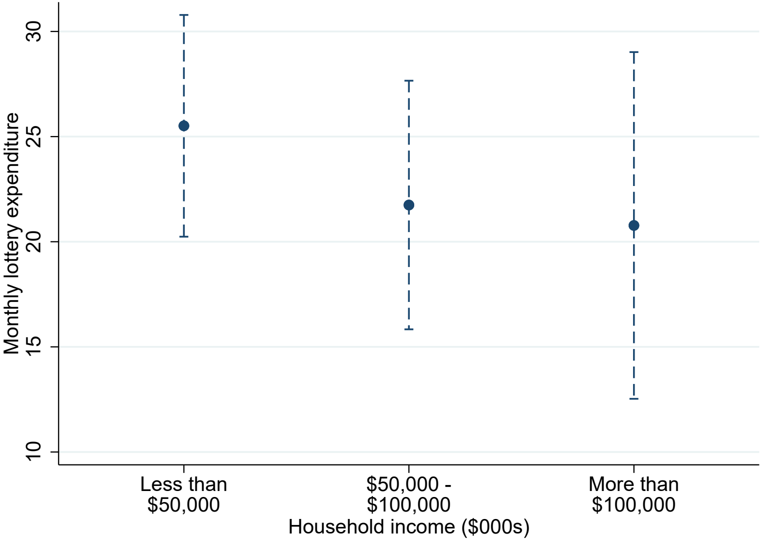
Key statistic $s(i)$: lottery spending across incomes



- Spending declines modestly as income rises.
- Wide confidence intervals due to skewness: top 10% of spenders account for 71% of spending.
- Consistent with 1998 NORC survey of gambling consumption.

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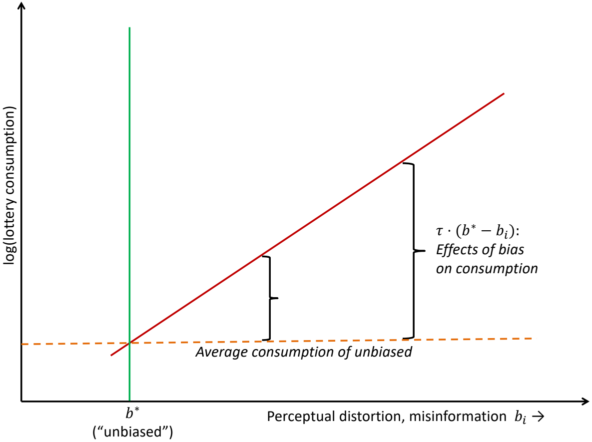
Quantifying bias: conceptual framework



Define:

- $\boldsymbol{b}_i$: bias proxy
- $\boldsymbol{b}^*$: value for "normative" consumer (e.g., well-informed)

Estimate relationship between consumption and bias (controlling for prefs, demographics):

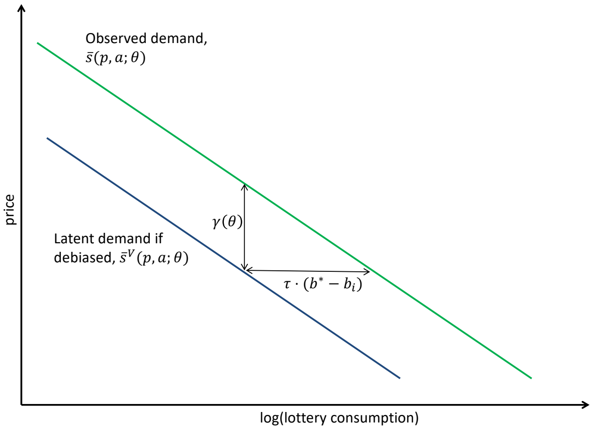
$$\ln(s_i + 1) = \boldsymbol{\tau} \boldsymbol{b}_i + \beta^a \boldsymbol{a}_i + \beta^x \boldsymbol{x}_i + \varepsilon_i$$

Predict debiased consumption s_i^V:

$$\ln(\hat{s}_i^V + 1) = \boldsymbol{\tau} \boldsymbol{b}^* + \beta^a \boldsymbol{a}_i + \beta^x \boldsymbol{x}_i + \varepsilon_i$$

Key assumption: $\boldsymbol{b}_i \perp \varepsilon_i | (\boldsymbol{a}_i, \boldsymbol{x}_i)$

Quantifying bias: conceptual framework



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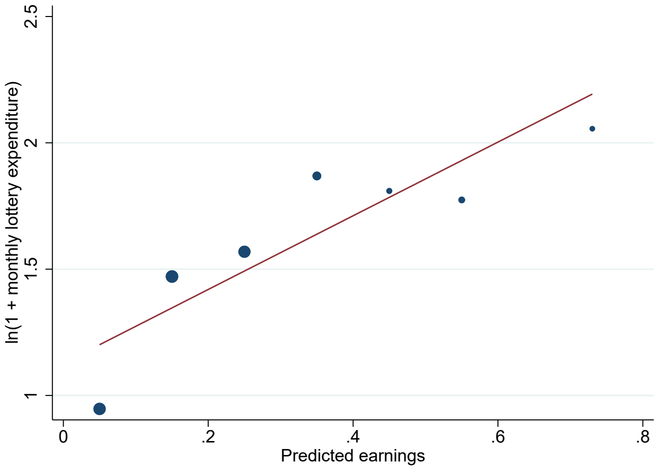
Qty effect of bias: $\ln(s_i) - \ln(s_i^V) \approx \gamma(i)\zeta_p(i)$

Survey questions to assess bias

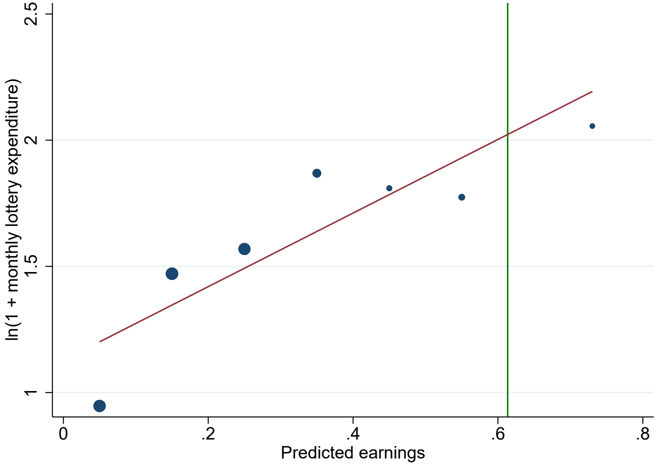
- **Expected returns**: *What percent of the total spending on lottery tickets do you think is given out in prizes?*
- **Self-control**: *Do you feel you should play the lottery less/same/more than you do now?*
- **Financial literacy**: share of correct answers to set of standard financial literacy questions
- **Statistical mistakes**: gambler's fallacy, law of small numbers, expected value calculation
- **Overconfidence**: *"For every $1000 you spend, how much do you think you would win back in prizes, on average?"* vs. *"How much would average player win back?"*
- **Predicted life satisfaction**: *How much do you think $100k more in winnings raised reported well-being?*

Lottery expenditures across perceived returns to lottery



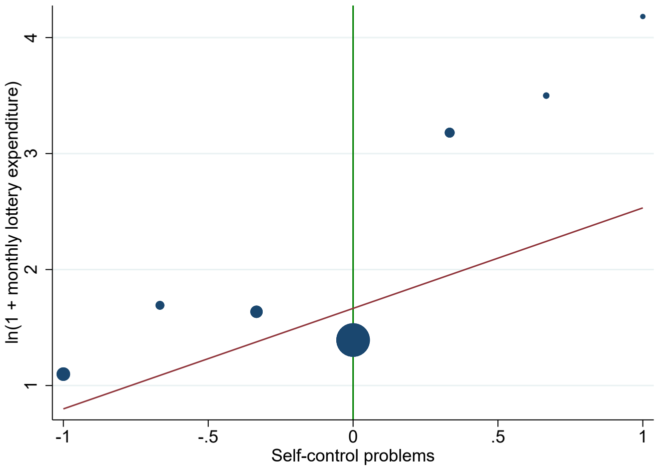
- Plot expenditures across bias proxy.

Lottery expenditures across perceived returns to lottery



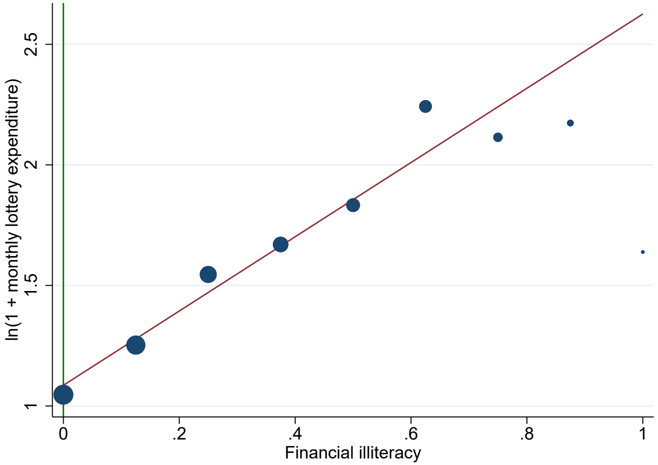
- Plot expenditures across bias proxy.
- Green line indicates "normative" (unbiased) response.
- On average people substantially *underestimate* payout: unlikely source of overconsumption bias. (See also Clotfelter & Cook 1999)

Lottery expenditures by self-control problems



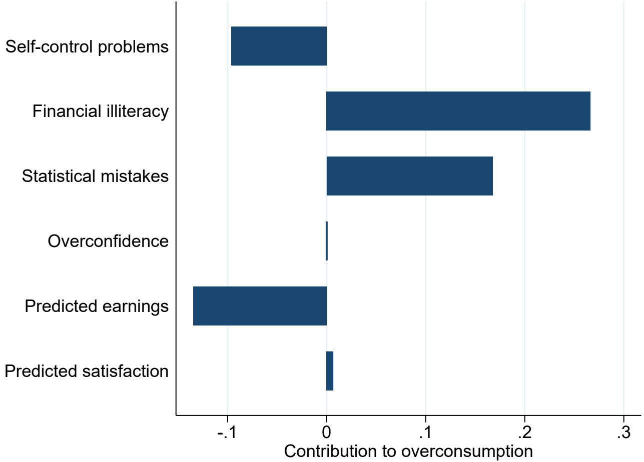
- Most respondents report little self control problems. (Contrast: soda consumption.)
- Little scope for driving substantial consumption bias.

Lottery expenditures by financial illiteracy



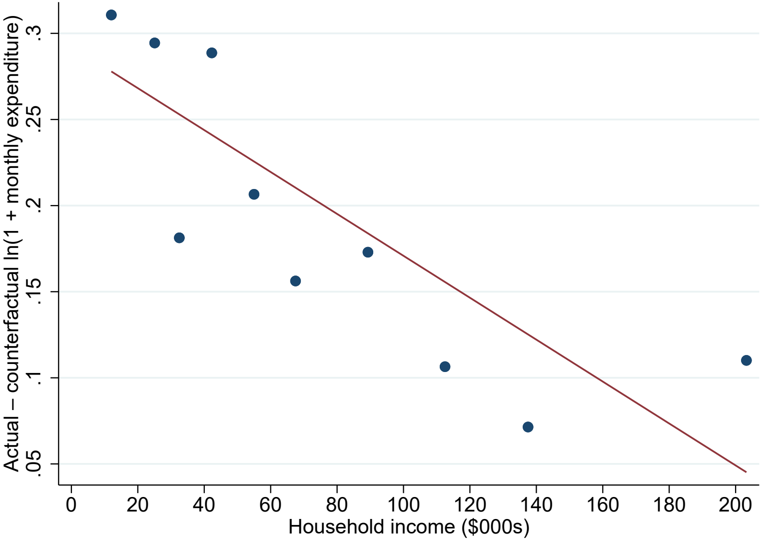
- Robust relationship, quantitatively important.
- Substantial heterogeneity in population.

Biases contributing to overconsumption



- Compute counterfactual spending for each consumer if they were unbiased on each dimension.
- Financial illiteracy and statistical mistakes are primary drivers.

Key statistic $\gamma(i)\zeta_p(i)$: quantity effect of bias



- On average, 18% of lottery spending attributable to bias.
- Declines across incomes.
- $\sim$ half as big as for soda (Allcott, Lockwood, Taubinsky 2019)

Road map: empirics

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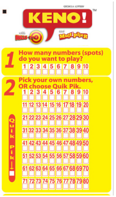
Background on lotteries



"Lotto" style games

- Mega Millions, Powerball, many other state lotteries.
- Player picks a set of numbers.
- Prize drawings held daily or (bi-)weekly.
- Parimutuel jackpot pool: accumulates until won.
- Tickets typically cost $1 or $2

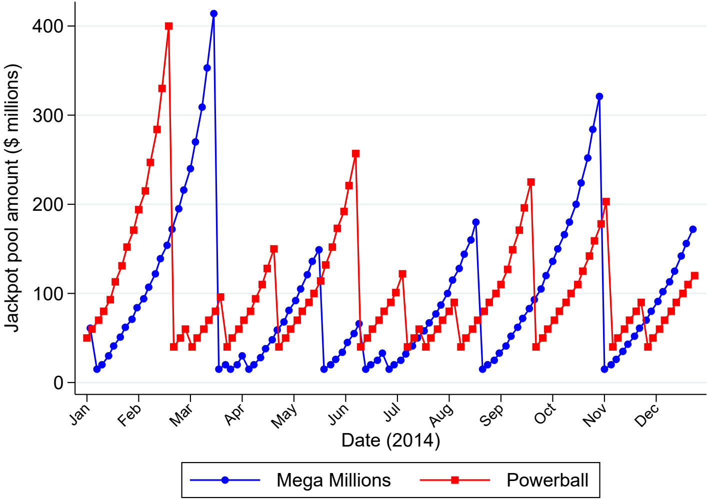
Instant games

- "Scratch tickets"
- Tickets typically cost $1 to $20

Other games

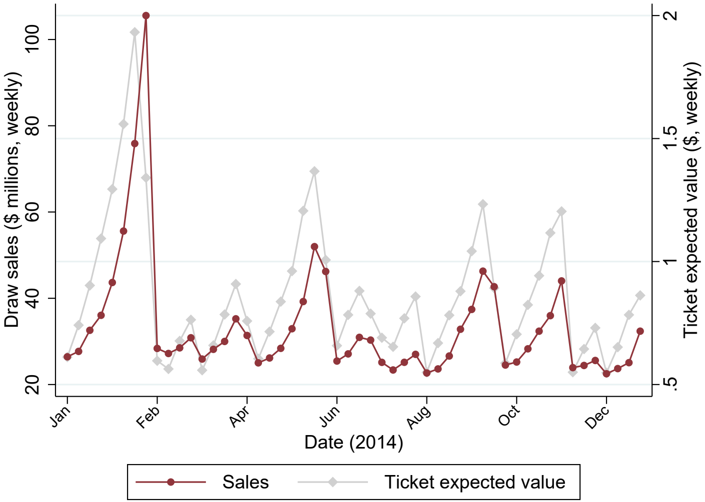
- Video lottery terminals, Keno

Large variation in lotto jackpots over time



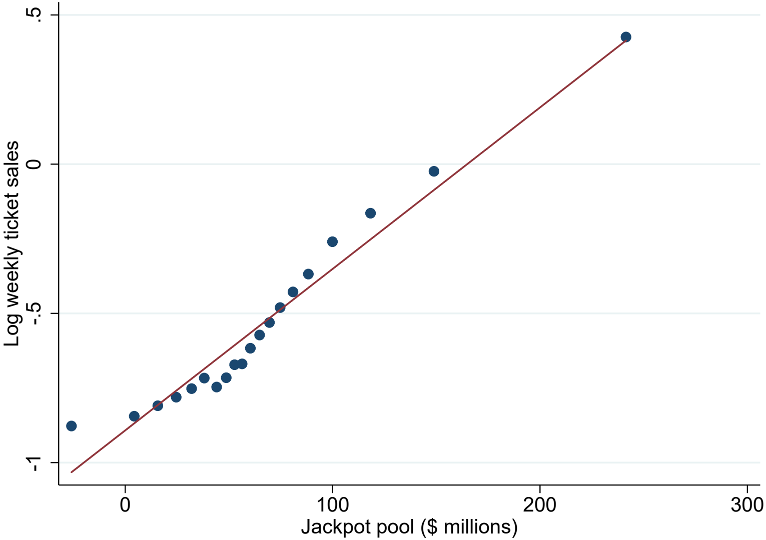
- Here: jackpots from 2014.
- Jackpot starts at “reset value.”
- If not won, a predetermined share of revenues are added to the prize pool and it rolls over to the next drawing.
- If won, split equally between all winners.

Sales covary with jackpot



- Powerball sales and ticket expected value over time, 2014.
- Expected value varies from ~$0.50 to ~$2 depending on jackpot. (Ticket price is $2.)

Sales covary with jackpot



- Strong positive relationship. (Absorbing game-state-structure FEs.)
- But: simultaneity bias $\Rightarrow$ period t demand shock affects jackpot size.
- Strategy: exploit randomness in lotto drawing to construct instrument for jackpot.

Key statistic $\bar{\zeta}_1$: semi-elasticity of demand with respect to jackpot

	(1) IV	(2) IV	(3) OLS
Jackpot expected value	0.7930*** (0.0875)	0.7986*** (0.0832)	0.9058*** (0.0755)
Lags in H	4	2	0
Quadratic terms in H	Yes	No	No
R^2	0.71	0.67	0.60
Observations	59,789	59,960	60,128

$$\ln s_{jt} = \zeta \pi_j w_{jt} + f(H_{jt-1}) + \xi_j + \eta_{T(t)} + \epsilon_{jt}$$

- Jackpot expected value $\pi_j w_{jt}$, instrumenting for w_{jt} with forecast update based on random rollover realization.
- Fixed effects for game-state-structure, quarter-of-sample; flexible controls for history H_{jt-1}(lags, quadratic terms).
- No measurable substitution across time or across games. [Details]
- Point estimate for $\bar{\zeta}_1$: 1 cent increase in jackpot EV raises sales by 0.79%.

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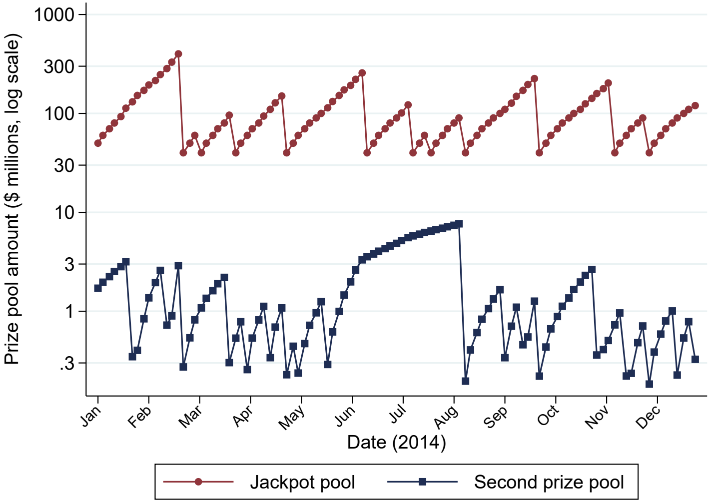
4. **What is elasticity of lottery demand with respect to smaller prizes?** $\bar{\zeta}_{2+}$
5. What is price elasticity of lottery demand? $\bar{\zeta}_p$

Elasticity with respect to sub-jackpot prizes

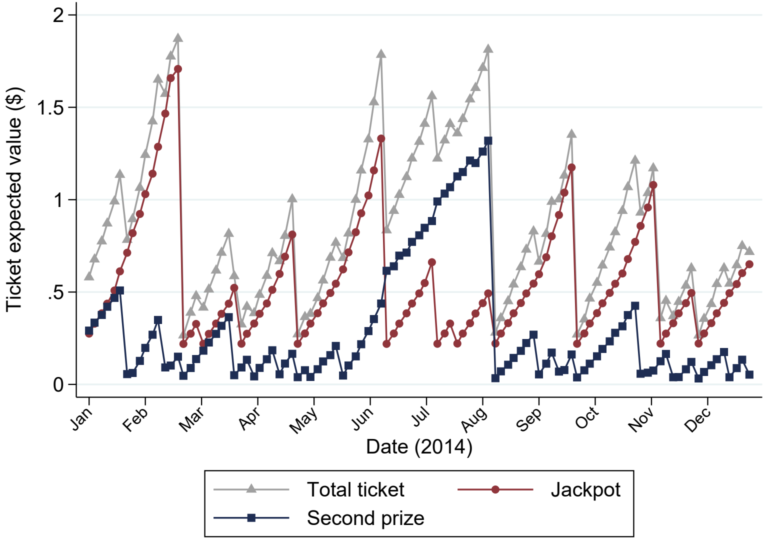
- Challenge: most lotto games vary jackpots over time, but other prizes fixed.
- Strategy: exploit unusual legal rule in California
  - *all* lottery prize levels vary randomly, independently.

In California: jackpot and 2nd prize pools vary independently



- Example: Powerball jackpot and 2nd prize pools in 2014.
- 3rd+ prizes virtually always won, but 2nd prize often rolls over.

Expected value of jackpot prize and 2nd prize



- Total ticket expected value is sum of EV of jackpot and other prizes.
- June – July: ticket EV mostly from large 2nd prize pool.

Key statistic $\bar{\zeta}_2$: semi-elasticity with respect to sub-jackpot prizes

	(1) IV	(2) IV	(3) OLS
Jackpot expected value	0.7743***	0.8120***	0.9802***
	(0.0343)	(0.0367)	(0.0265)
2nd prize expected value	0.0712	-0.1245	-0.1610***
	(0.1226)	(0.0875)	(0.0519)
Lags included in H	4	2	0
H includes quadratic terms	Yes	No	No
R^2	0.74	0.70	0.62
Observations	3,101	3,110	3,201

Includes FEs for game-state-structure, day-of-week, quarter-of-sample

$$\ln s_{jt} = \zeta_1 x_{j1t} + \zeta_2 x_{j2t} + f(H_{jt-1}) + \xi_j + \eta_{T(t)} + \phi_{d(t)} + \epsilon_{jt}$$

- Prize EV x_{jkt} instrumented with prize forecast.
- Point estimate: 1 cent increase in 2nd prize EV raises sales by 0.071%.
- Caveat: variation in 2nd prize may be less salient. (Endogenous to advertising?)

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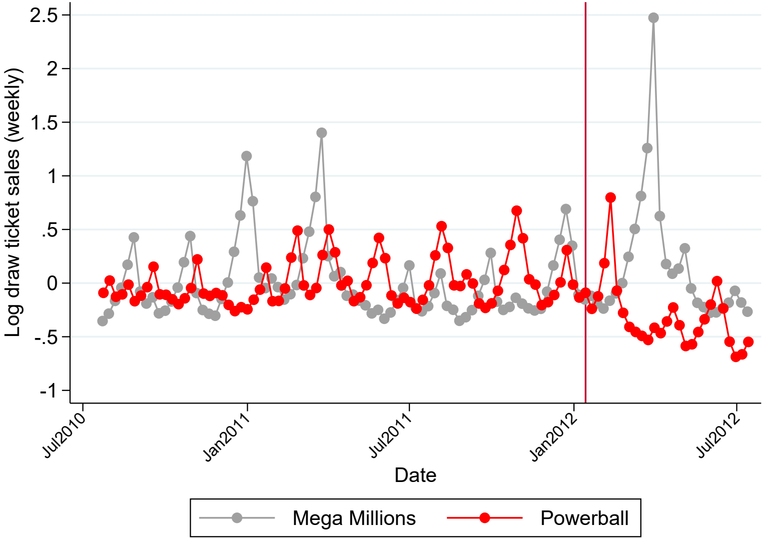
4. What is the elasticity of lottery demand with respect to smaller prizes? $\bar{\zeta}_{2+}$
5. **What is the price elasticity of lottery demand?** $\bar{\zeta}_p$

Price elasticity: estimation strategy

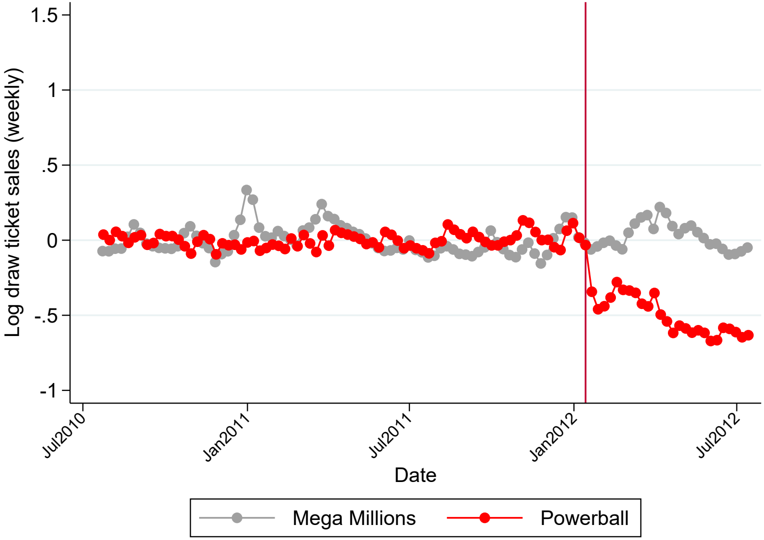
- Challenge: unlike prizes, prices (and probabilities) generally constant over time.
- Two key exceptions:
    - January 2012: Powerball ticket price increased \$1 $\rightarrow$ \$2
    - October 2017: Mega Millions ticket price increased \$1 $\rightarrow$ \$2

Powerball price change 2012



- Powerball price increased $1 → $2 in January 2012.

Powerball price change 2012



- Powerball price increased \$1 $\rightarrow$ \$2 in January 2012.
- Control for jackpot using jackpot forecast IV.

Key statistic $\bar{\zeta}_p$: semi-elasticity of lottery demand with respect to price

	(1) Pooled	(2) Pooled	(3) Powerball	(4) Mega Millions
Price	-0.5583***	-0.5356***	-0.6031***	-0.5079***
	(0.0660)	(0.0624)	(0.1023)	(0.0652)
Jackpot pool	0.0040***		0.0059***	0.0032***
	(0.0003)		(0.0006)	(0.0003)
Jackpot expected value		0.9657***		
		(0.0696)		
Observations	416	416	208	208

$$\ln s_{jt} = -\zeta_p p_{jt} + \zeta_1 \pi_{j1t} w_{j1t} + f(H_{jt-1}) + \hat{\zeta}_2 EV_{jt}^{2+} + \xi_j + \phi_{d(t)} + \epsilon_{jt}$$

- Instrument for jackpot w_{j1t} using jackpot forecast IV.
- Control for minor changes in sub-jackpot prize expected value using estimated semi-elasticity $\hat{\zeta}_2$.
- Point estimate: 1 cent rise in price reduces sales by -0.558%.

Remark: $\bar{\zeta}_1 > |\bar{\zeta}_p| > \bar{\zeta}_2$ informs choice of probability weighting function

- $\bar{\zeta}_1 \gg \bar{\zeta}_2$ is inconsistent with "standard" probability weighting functions used in prospect theory and cumulative prospect theory
- Note: incentivized experiments (and KT '79 surveys) don't study magnitudes in this range
  - Preliminary hypothesis: standard probability weighting functions do not extend to the small probabilities / large prizes we have here
- Ranking *is* consistent with probability weighting fn in Chateauneuf, Eichberger and Grant (2007)
  - Most weight given to highest prize and lowest prize
  - We use this specification in calibrations to follow

Calibration

Structural model

Individual utility

Consumer i's utility from buying lottery L, with price p and {prizes, probabilities} = $\{w_k, \pi_k\}_{k=1}^{K}$:

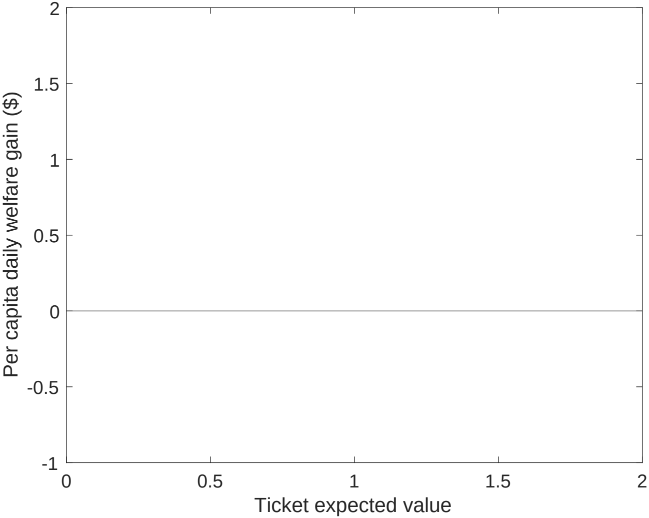
$$U_i(L) = c_i - p + \sum_{k=1}^{K} \underbrace{\Phi_i(\pi_k)}_{\text{decision wts}} u_i(w_k) + \epsilon_{it}$$

$$V_i(L) = U_i(L) - \underbrace{\sum_k \chi_i \left(\Phi_i(\pi_k) - \pi_k\right) u_i(w_k)}_{\text{bias}=\gamma_i}$$

Calibration assumptions

- CRRA utility over wealth (baseline = log).
- Chateauneuf et al. weighting function.
- Representative lottery: Mega Millions, \$300 million jackpot. Overhead costs = \$0.20/ticket.
- Discretized income grid, welfare weights declining with income ($g_i \propto 1/c_i$)
- Random taste shock $\epsilon_{it} = \xi + \alpha\varepsilon_{it}$ iid logit. (Model selects $\xi < 0$, "hassle costs")
- Income tax rate on winnings: 40%. Overhead costs = \$0.20/ticket.

Are lotteries welfare enhancing? Welfare gains across expected value



- Hold ticket price fixed at status quo ($2).
- Scale all prizes up/down to change expected value (status quo: $0.74).

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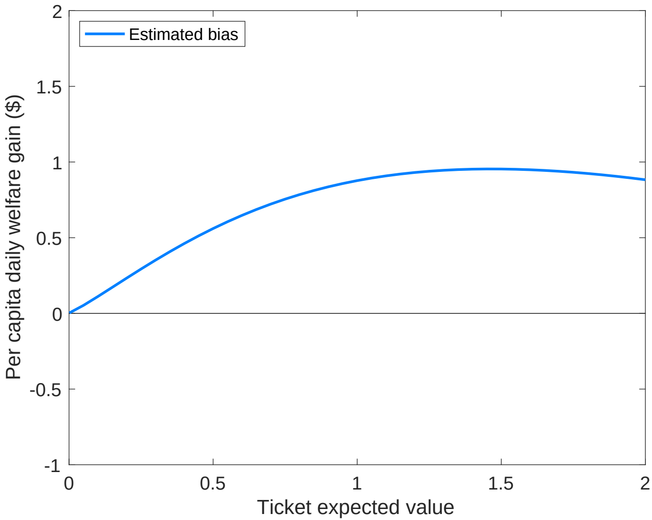

- Hold ticket price fixed at status quo ($2).
- Scale all prizes up/down to change expected value (status quo: $0.74).
- In baseline, optimal EV is higher than status quo (lower than price)

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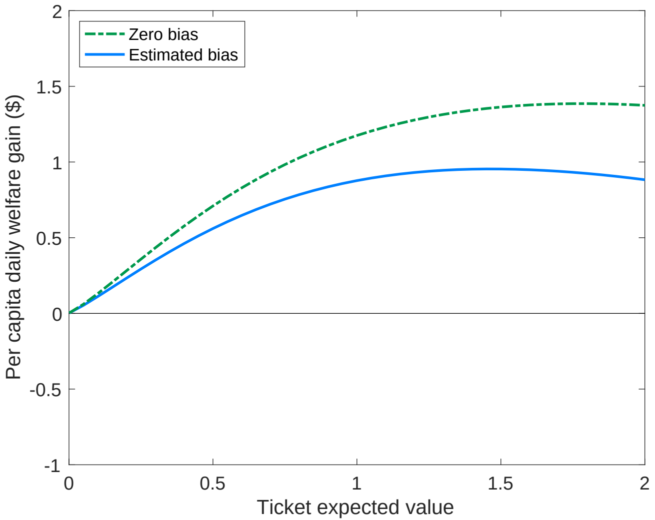

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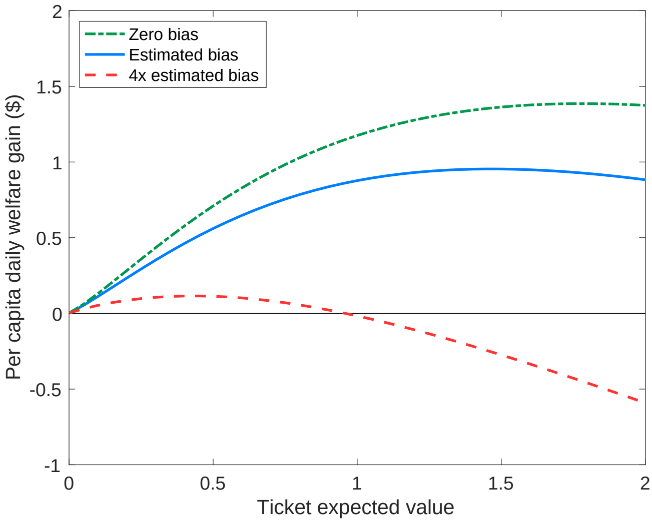
- In baseline, optimal EV is higher than status quo (lower than price)
- Absent bias, price $\approx$ marginal cost (EV + overhead); no corrective implicit tax.

Are lotteries welfare enhancing? Welfare gains across expected value



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- Optimal expected value falls as bias grows larger.

Optimal lottery structure (preliminary)

- Price: \$2.48 (compare to \$2)
- Expected value of prize payout: \$1.67 (compare to \$0.74)
- Implicit tax rate: 25% (compare to 53%)

Conclusion

Recap

1. Derivation of new “optimal regulation” formula and application to lotteries.
   - Extends behavioral public finance policies to non-price attributes.

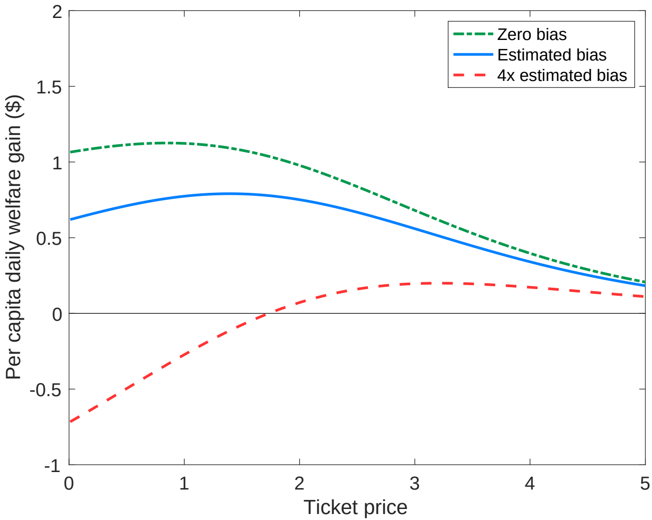
2. New descriptive evidence on lottery consumption, behavioral biases, and elasticities.
   - Consumption mildly declining with income.
   - Modest share of consumption explained by bias.

3. Calibrated model to explore welfare and policy counterfactuals.
   - Lotteries likely raise welfare on average.
   - Could be improved by reducing implicit tax rate.

Thank you!

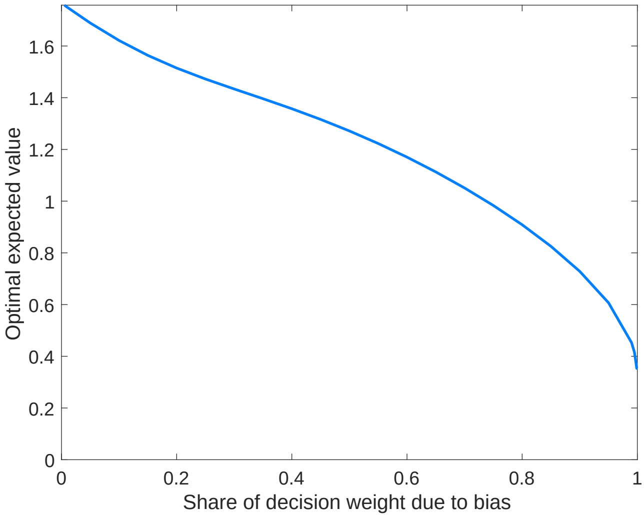
Appendix

Are lotteries welfare enhancing? Welfare gains across price



- Welfare gain across p (fixing w_k, π_k)
- If unbiased, $p^* \approx$ marginal cost (no implicit tax)
- With estimated bias: $p^* > MC$
- Large bias: low prices are welfare-reducing.

How optimal lottery structure depends on bias



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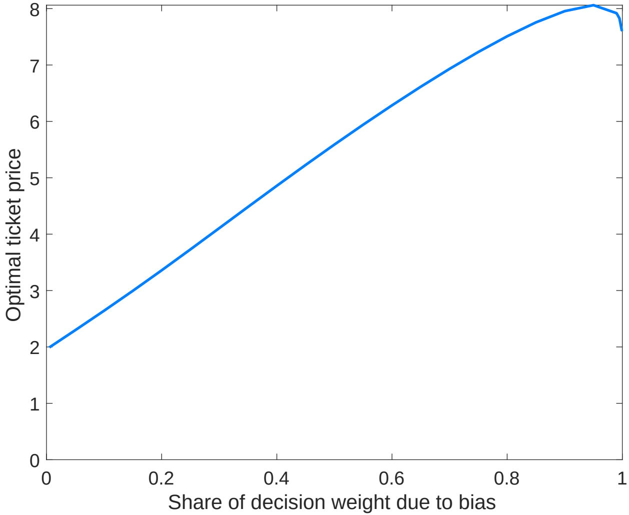
How optimal lottery structure depends on bias



- Optimal expected value falls as bias grows larger.
- Corrective implicit tax also rises with bias, making price large.

Substitution across time

	(1)	(2)	(3)	(4)	(5)
Jackpot expected value (t)	0.8975***	0.8944***	0.8805***	0.9263***	0.7930***
	(0.0462)	(0.0445)	(0.0473)	(0.0436)	(0.0875)
Jackpot expected value (t-1)	0.1061***	0.0934***	0.1454***	-0.0504	
	(0.0167)	(0.0192)	(0.0330)	(0.0880)	
Jackpot expected value (t-2)	-0.0165	0.0397*	-0.1341		
	(0.0196)	(0.0228)	(0.0905)		
Jackpot expected value (t-3)	0.0528**	-0.1145			
	(0.0213)	(0.0866)			
Jackpot expected value (t-4)	-0.1211				
	(0.0822)				
Observations	59,421	59,513	59,605	59,697	59,789
Akaike Information Criterion	-8,044.68	-8,113.91	-8,553.20	-9,153.55	-13,925.26
Bayesian Information Criterion	-7,891.81	-7,961.01	-8,409.27	-9,045.59	-13,817.28

- Lagged jackpots (instrumented) do not crowd out current demand.
- AIC/BIC minimized with no lags.

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Substitution across games

	(1) Own game sales	(2) All other games sales	(3) Other lotto games sales	(4) Instant games sales
Jackpot expected value	1.8833***	0.0887	0.0578	0.0452
	(0.3422)	(0.1655)	(0.1447)	(0.0598)
Observations	58,756	58,756	58,756	58,756

- Outcome: total sales of game type in each column.
- Higher jackpot (instrumented) raises own-game sales; does not reduce other games' sales.

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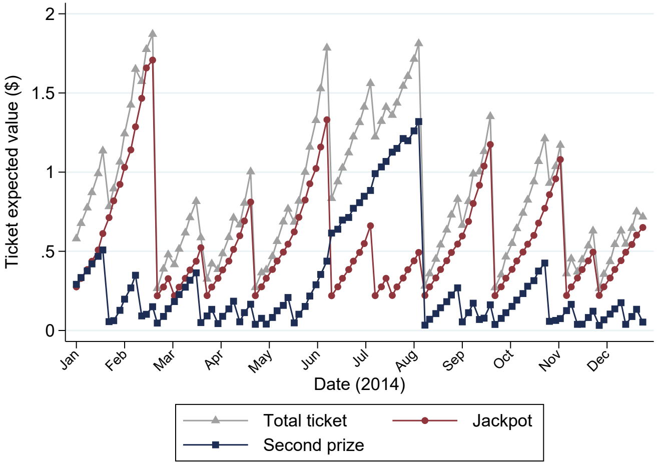
Instrument construction: sub-jackpot prizes

Prize expected value: $x_{jkt} := \pi_{jk}\left(w_{jkt}(1-\pi_{jk})^{s_{jkt}-1} + \frac{w_{jkt}}{2}\pi_{jk}(1-\pi_{jk})^{s_{jkt}-2}(s_{jkt}-1) + \ldots\right)$



- Probability π_{jk} of winning; $s_{jkt}-1$ others to potentially split prize k
- Prize w_{jkt} **if unshared**
- Prize $\frac{w_{jkt}}{2}$ **if split 2 ways**, ...
- etc.

Instrument construction: sub-jackpot prizes

Regression equation

$$\ln s_{jt} = \zeta_1 x_{j1t} + \zeta_2 x_{j2t} + f(H_{jt-1}) + \xi_j + \eta_{T(t)} + \phi_{d(t)} + \epsilon_{jt}$$

j: game-structure, t: index of drawing date
s_{jt}: tickets sold
x_{jkt} := expected value of prize level k
ξ_j, $\eta_{T(t)}$, $\phi_{d(t)}$: fixed effects for game-state-structure, quarter of sample, day of week

Instrument construction

$$z_{jkt} = \begin{cases} \pi_{jk} \bar{w}_{jk} \left((1-\pi_{jk})^{\hat{s}_{jkt}-1} + \frac{\pi_{jk}}{2}(1-\pi_{jk})^{\hat{s}_{jkt}-2}(\hat{s}_{jkt}-1) \right) & \text{if } r_{jkt-1} = 0 \\ \pi_{jk} \left(w_{jkt-1} + \kappa_{jk} p_j \hat{s}_{jkt} \right) \left((1-\pi_{jk})^{\hat{s}_{jkt}-1} + \frac{\pi_{jk}}{2}(1-\pi_{jk})^{\hat{s}_{jkt}-2}(\hat{s}_{jkt}-1) \right) & \text{if } r_{jkt-1} = 1 \end{cases}$$

- $\hat{s}_{jkt}(\vec{r}_{jt-1}, H_{jt-1})$: flexible best-predictor of s_{jkt} (tickets with which prize k risks being split), based on history H_{jt-1}, and prize rollover vector $\vec{r}_{jt-1} = (r_{j1t-1}, r_{j2t-1})$.
- Accounts for risk of splitting prize (more important for 2nd prize than jackpot)
- Improves conditional prize forecast by predicting sales from H_{jt-1} (important when jackpot moves sales affecting smaller prizes

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