The Optimal Taxation of Lotteries: Who P(l)ays and Who Wins?

Hunt Allcott (NYU, Microsoft Research)
Benjamin Lockwood (Penn Wharton)
Dmitry Taubinsky (UC Berkeley)
Acknowledgments

Thanks for financial support

- Alfred P. Sloan Foundation
- Time-sharing Experiments for the Social Sciences (TESS)

Views here are our own.
Lottery consumption in the U.S.

- Americans spend over $70 billion each year on state-run lotteries.
  - Over $600 per household.
  - More than on music, sports events, movie tickets, and video games combined.

- Lotteries are administered by 44 state governments.
  - Over $30 billion annually in public funds.
  - More revenue than federal gasoline tax or estate tax.

- Not unique to U.S., of course
  - E.g., National Lottery in the U.K.
A motivating question

State-run Lotteries

Taking into account the revenues, consumer surplus, purchasing patterns by income, and possible consumer biases, state-run lotteries (such as Powerball and scratch-off games) increase social welfare.
Are state-run lotteries welfare-enhancing?

Our view
This is fundamentally a question of optimal taxation.

- Lotteries are a heavily taxed product.
  - Implicit + explicit taxes over 50%.

- Distributional concerns
  - Regressive tax on low-income, low-education consumers?

- Behavioral biases
  - Gambling considered a classic “sin good”
  - Misperception? Overoptimism? Self-control problems?
Part 1: Model of Optimal Lottery Taxation

- New sufficient statistics formula for optimal lottery attributes.

Part 2: Empirical evidence

- New large-scale survey of lottery demand and behavioral biases.
- Present descriptive evidence on key parameters that govern optimal policy.

Part 3: Calibration and welfare estimation

- Add structure to study non-local reforms.
- Address policy questions: Are lotteries welfare enhancing? What is optimal tax treatment?
Model
Conceptual framework

- Many challenges normatively evaluating lottery consumption.
  - How to reconcile with expected utility theory and risk aversion?
  - What does it mean to “consume” a lottery ticket?
- Our perspective: a lottery is simply a good with a set of attributes:
  - vector of potential winnings $w_k$ with probabilities $\pi_k$, and other attributes of game design
- Basic idea: consumer $i$’s utility from a lottery is

$$U_i = \sum_k \Phi_i(\pi_k) u_i(w_k)$$

- Consumers apply decision weights to potential outcomes; may differ from $\pi_k$.
- Difference may be normatively valid (e.g., anticipatory utility, Caplin Leahy 2001) or driven by behavioral biases (e.g., perceptual distortion, Woodford 2012)
Conceptual framework

- Many challenges normatively evaluating lottery consumption.
  - How to reconcile with expected utility theory and risk aversion?
  - What does it mean to “consume” a lottery ticket?

- Our perspective: a lottery is simply a good with a set of attributes:
  - vector of potential winnings \( w_k \) with probabilities \( \pi_k \), and other attributes of game design

- Basic idea: consumer \( i \)’s utility from a lottery is

\[
U_i = \sum_k \Phi_i(\pi_k)u_i(w_k)
\]

- Consumers apply decision weights to potential outcomes; may differ from \( \pi_k \).
- Difference may be normatively valid (e.g., anticipatory utility, Caplin Leahy 2001) or driven by behavioral biases (e.g., perceptual distortion, Woodford 2012)

- Question: how to regulate price and attributes \((w_k, \pi_k, ...)\) optimally?
Intuition for regulating attributes

- Suppose a “sin good” $s$ has (continuous) attribute $a$ which affects its appeal
  - Examples: cigarette nicotine content, gas-mileage in cars, lottery prizes.
- Like a tax, changing $a$ may affect demand $\Rightarrow$ direct corrective effect.
- Unlike a tax, $\Delta a$ may also change bias cost for inframarginal consumers.
  - Intuition: even if raising nicotine content reduces cigarette demand, may not be good policy...
- We formalize this to characterize optimal attribute regulation.
Model setup

- **Consumers**
  - Heterogeneous income-earning ability, preferences; types indexed by \( i \).
  - Numeraire consumption \( c(i) \).
  - Discrete choice: share \( s(i) \) of \( i \)-types choose to purchase lottery on occasion \( t \).
  - Money-metric bias \( \gamma(i) \): “price reduction that would cause debiased \( i \) to buy \( s(i) \).”

- **Policymaker**
  - Inequality averse, with welfare weights \( g(i) \).
  - Sells lottery tickets at price \( p \), and sets *attributes* (prizes, probabilities, advertising, ...)

- **Today’s application**: attribute of interest is lottery expected value, \( a := \sum_k \pi_k w_k \)
  - Government revenue \( = (p - a) \cdot \bar{s} \) resembles a tax of \( p - a \), though \( a \) may affect bias.
  - Key new statistics: \( \kappa(i) = i \)'s average WTP for \( \Delta a \); \( \rho(i) = \text{bias in average WTP for } \Delta a \).
Optimal prices and attributes

- Optimal $p^*$: increases with **corrective motive**, decreases with **redistributive motive** (see also: Allcott, Lockwood, Taubinsky 2019):

  $$p^* - a = \bar{\gamma}(1 + \sigma) - \frac{{\text{Cov}} [g(i), s(i)]}{\bar{s}\zeta_p}$$

  $$\sigma = \text{Cov} \left[ g(i), \frac{\gamma(i)}{\bar{\gamma}} \frac{\zeta_p(i)}{\zeta_p(i)} \frac{s(i)}{\bar{s}} \right]: \text{bias correction progressivity}$$

  $$\zeta_p(i) = \frac{d\ln s(i)}{dp}: \text{semi-elasticity of demand with respect to price (avg: } \bar{\zeta}_p)$$

- Optimal $a^*$, given price:

  $$p - a^* = \bar{\gamma}(1 + \sigma_a) - \frac{\mathbb{E} [g(i)(\kappa(i) - \rho(i)) - s(i)]}{\zeta_a \bar{s}}$$

  $\kappa(i)$: $i$'s WTP for $\Delta a$; $\rho(i)$: how much of that WTP is due to bias?

  $$\zeta_a(i) = \frac{d\ln s(i)}{da}: \text{semi-elasticity of demand with respect to } a \text{ (avg: } \bar{\zeta}_c)$$

- If income effects, use $s_{pref}$, $\kappa_{pref}$: from preference heterogeneity (vs. causal income effects).
Empirical agenda

Optimal lottery regulation formula

\[ p - a^* = \bar{\gamma}(1 + \sigma_a) - \frac{\mathbb{E}[g(i)(\kappa(i) - \rho(i)) - s(i)]}{\bar{\zeta}_a \bar{S}} \]

Empirical estimation

Formula motivates empirical questions of interest:

1. \(s(i)\): What is profile of lottery spending across income distribution?
2. \(\gamma(i)\): What is money-metric bias in lottery consumption, across incomes?
3. \(\bar{\zeta}_p\): What is price elasticity of lottery demand?
4. \(\bar{\zeta}_1\): What is elasticity of lottery demand with respect to jackpots?
5. \(\bar{\zeta}_{2+}\): What is elasticity of lottery demand with respect to smaller prizes?

Then: use these moments to calibrate \(\sum_k \Phi_i(\pi_k) u_i(w_k)\), then compute welfare, optimal policy.
Empirical Evidence
1. New large representative survey
AmeriSpeak panel: ~2,800 respondents; balanced demographics

2. La Fleur’s sales data
Lottery ticket sales by week × state × game since 1994

3. Prize and probability data
Collected from lottery rules, prizes scraped from online “are your numbers lucky?” tools
1. What is profile of lottery spending across income distribution? $s(i)$
2. What is quantity effect of bias in lottery consumption, across incomes? $\gamma(i)\zeta_p(i)$
3. What is elasticity of lottery demand with respect to jackpots? $\tilde{\zeta}_1$
4. What is elasticity of lottery demand with respect to smaller prizes? $\tilde{\zeta}_{2+}$
5. What is price elasticity of lottery demand? $\zeta_p$
Key statistic $s(i)$: lottery spending across incomes

- Spending declines modestly as income rises.
- Wide confidence intervals due to skewness: top 10% of spenders account for 71% of spending.
- Consistent with 1998 NORC survey of gambling consumption.
1. What is profile of lottery spending across income distribution? \( s(i) \)

2. **What is quantity effect of bias in lottery consumption, across incomes?** \( \gamma(i) \zeta_p(i) \)

3. What is elasticity of lottery demand with respect to jackpots? \( \tilde{\zeta}_1 \)

4. What is elasticity of lottery demand with respect to smaller prizes? \( \tilde{\zeta}_{2^+} \)

5. What is price elasticity of lottery demand? \( \tilde{\zeta}_p \)
Define:

- $b_i$: bias proxy
- $b^*$: value for “normative” consumer (e.g., well-informed)

**Estimate** relationship between consumption and bias (controlling for prefs, demographics):

$$\ln(s_i + 1) = \tau b_i + \beta^a a_i + \beta^x x_i + \varepsilon_i$$

**Predict** debiased consumption $s_i^V$:

$$\ln(s_i^V + 1) = \tau b^* + \beta^a a_i + \beta^x x_i + \varepsilon_i$$

Key assumption: $b_i \perp \varepsilon_i | (a_i, x_i)$
Define:

- $b_i$: bias proxy
- $b^*$: value for “normative” consumer (e.g., well-informed)

Estimate relationship between consumption and bias (controlling forprefs, demographics):

$$\ln(s_i + 1) = \tau b_i + \beta^a a_i + \beta^x x_i + \varepsilon_i$$

Predict debiased consumption $s_i^V$:

$$\ln(\hat{s}_i^V + 1) = \tau b^* + \beta^a a_i + \beta^x x_i + \varepsilon_i$$

Key assumption: $b_i \perp \varepsilon_i | (a_i, x_i)$

Qty effect of bias: $\ln(s_i) - \ln(s_i^V) \approx \gamma(i) \zeta_p(i)$
Survey questions to assess bias

- **Expected returns**: What percent of the total spending on lottery tickets do you think is given out in prizes?
- **Self-control**: Do you feel you should play the lottery less/same/more than you do now?
- **Financial literacy**: share of correct answers to set of standard financial literacy questions
- **Statistical mistakes**: gambler’s fallacy, law of small numbers, expected value calculation
- **Overconfidence**: “For every $1000 you spend, how much do you think you would win back in prizes, on average?” vs. “How much would average player win back?”
- **Predicted life satisfaction**: How much do you think $100k more in winnings raised reported well-being?
Lottery expenditures across perceived returns to lottery

\[ \ln(1 + \text{monthly lottery expenditure}) \]

Predicted earnings

- Plot expenditures across bias proxy.

• Green line indicates "normative" (unbiased) response.

• On average people substantially underestimate payout: unlikely source of overconsumption bias. (See also Clotfelter & Cook 1999)
Lottery expenditures across perceived returns to lottery

- Plot expenditures across bias proxy.
- Green line indicates “normative” (unbiased) response.
- On average people substantially underestimate payout: unlikely source of overconsumption bias. (See also Clotfelter & Cook 1999)
Lottery expenditures by self-control problems

- Most respondents report little self-control problems. (Contrast: soda consumption.)
- Little scope for driving substantial consumption bias.
Lottery expenditures by financial illiteracy

- Robust relationship, quantitatively important.
- Substantial heterogeneity in population.
Biases contributing to overconsumption

- Overconfidence
- Statistical mistakes
- Financial illiteracy
- Self-control problems
- Predicted earnings
- Predicted satisfaction

- Compute counterfactual spending for each consumer if they were unbiased on each dimension.
- Financial illiteracy and statistical mistakes are primary drivers.
Key statistic $\gamma(i)\zeta_p(i)$: quantity effect of bias

- On average, 18% of lottery spending attributable to bias.
- Declines across incomes.
- $\sim$ half as big as for soda (Allcott, Lockwood, Taubinsky 2019)
Road map: empirics

1. What is profile of lottery spending across income distribution? $s(i)$
2. What is quantity effect of bias in lottery consumption, across incomes? $\gamma(i)\zeta_p(i)$
3. **What is elasticity of lottery demand with respect to jackpots?** $\tilde{\zeta}_1$
4. What is elasticity of lottery demand with respect to smaller prizes? $\tilde{\zeta}_{2+}$
5. What is price elasticity of lottery demand? $\zeta_p$
Background on lotteries

“Lotto” style games
- Mega Millions, Powerball, many other state lotteries.
- Player picks a set of numbers.
- Prize drawings held daily or (bi-)weekly.
- Parimutuel jackpot pool: accumulates until won.
- Tickets typically cost $1 or $2

Instant games
- “Scratch tickets”
- Tickets typically cost $1 to $20

Other games
- Video lottery terminals, Keno
Large variation in lotto jackpots over time

- Here: jackpots from 2014.
- Jackpot starts at “reset value.”
- If not won, a predetermined share of revenues are added to the prize pool and it rolls over to the next drawing.
- If won, split equally between all winners.
Sales covary with jackpot

- Powerball sales and ticket expected value over time, 2014.
- Expected value varies from ~$0.50 to ~$2 depending on jackpot. (Ticket price is $2.)
Sales covary with jackpot

- Strong positive relationship. (Absorbing game-state-structure FEs.)

- But: simultaneity bias ⇒ period $t$ demand shock affects jackpot size.

- Strategy: exploit randomness in lotto drawing to construct instrument for jackpot.
Key statistic $\bar{\zeta}_1$: semi-elasticity of demand with respect to jackpot

<table>
<thead>
<tr>
<th></th>
<th>(1) IV</th>
<th>(2) IV</th>
<th>(3) OLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jackpot expected value</td>
<td>0.7930*** (0.0875)</td>
<td>0.7986*** (0.0832)</td>
<td>0.9058*** (0.0755)</td>
</tr>
<tr>
<td>Lags in H</td>
<td>4</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Quadratic terms in H</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.71</td>
<td>0.67</td>
<td>0.60</td>
</tr>
<tr>
<td>Observations</td>
<td>59,789</td>
<td>59,960</td>
<td>60,128</td>
</tr>
</tbody>
</table>

\[
\ln s_{jt} = \zeta \pi_j w_{jt} + f(H_{jt-1}) + \xi_j + \eta_T(t) + \epsilon_{jt}
\]

- Jackpot expected value $\pi_j w_{jt}$, instrumenting for $w_{jt}$ with forecast update based on random rollover realization.
- Fixed effects for game-state-structure, quarter-of-sample; flexible controls for history $H_{jt-1}$ (lags, quadratic terms).
- No measurable substitution across time or across games. [Details]
- Point estimate for $\bar{\zeta}_1$: 1 cent increase in jackpot EV raises sales by 0.79%.
1. What is profile of lottery spending across income distribution? $s(i)$
2. What is quantity effect of bias in lottery consumption, across incomes? $\gamma(i)\zeta_p(i)$
3. What is elasticity of lottery demand with respect to jackpots? $\tilde{\zeta}_1$
4. **What is elasticity of lottery demand with respect to smaller prizes?** $\tilde{\zeta}_{2+}$
5. What is price elasticity of lottery demand? $\zeta_p$
Elasticity with respect to sub-jackpot prizes

- Challenge: most lotto games vary jackpots over time, but other prizes fixed.

- Strategy: exploit unusual legal rule in California
  - *all* lottery prize levels vary randomly, independently.
In California: jackpot and 2nd prize pools vary independently

- Example: Powerball jackpot and 2nd prize pools in 2014.
- 3rd+ prizes virtually always won, but 2nd prize often rolls over.
Expected value of jackpot prize and 2nd prize

- Total ticket expected value is sum of EV of jackpot and other prizes.
- June – July: ticket EV mostly from large 2nd prize pool.
### Key statistic $\zeta_2^*$: semi-elasticity with respect to sub-jackpot prizes

<table>
<thead>
<tr>
<th></th>
<th>(1) IV</th>
<th>(2) IV</th>
<th>(3) OLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jackpot expected value</td>
<td>0.7743***</td>
<td>0.8120***</td>
<td>0.9802***</td>
</tr>
<tr>
<td></td>
<td>(0.0343)</td>
<td>(0.0367)</td>
<td>(0.0265)</td>
</tr>
<tr>
<td>2nd prize expected value</td>
<td>0.0712</td>
<td>-0.1245</td>
<td>-0.1610***</td>
</tr>
<tr>
<td></td>
<td>(0.1226)</td>
<td>(0.0875)</td>
<td>(0.0519)</td>
</tr>
<tr>
<td>Lags included in H</td>
<td>4</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>H includes quadratic terms</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.74</td>
<td>0.70</td>
<td>0.62</td>
</tr>
<tr>
<td>Observations</td>
<td>3,101</td>
<td>3,110</td>
<td>3,201</td>
</tr>
</tbody>
</table>

Includes FEs for game-state-structure, day-of-week, quarter-of-sample

$$\ln s_{jt} = \zeta_1 x_{1jt} + \zeta_2 x_{2jt} + f(H_{jt-1}) + \xi_j + \eta T(t) + \phi d(t) + \epsilon_{jt}$$

- Prize EV $x_{jkt}$ instrumented with prize forecast.
- Point estimate: 1 cent increase in 2nd prize EV raises sales by 0.071%.
- Caveat: variation in 2nd prize may be less salient. (Endogenous to advertising?)
1. What is the profile of lottery spending across income distribution? $s(i)$
2. What is quantity effect of bias in lottery consumption, across incomes? $\gamma(i)\zeta_p(i)$
3. What is the elasticity of lottery demand with respect to jackpots? $\zeta_1$
4. What is the elasticity of lottery demand with respect to smaller prizes? $\zeta_{2+}$
5. **What is the price elasticity of lottery demand?** $\zeta_p$
• Challenge: unlike prizes, prices (and probabilities) generally constant over time.

• Two key exceptions:
  • January 2012: Powerball ticket price increased $1 → $2
  • October 2017: Mega Millions ticket price increased $1 → $2
- Powerball price increased $1 → $2 in January 2012.

<table>
<thead>
<tr>
<th>Date</th>
<th>Mega Millions</th>
<th>Powerball</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jul2010</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan2011</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jul2011</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan2012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jul2012</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The graph shows the log draw ticket sales (weekly) for Mega Millions and Powerball from July 2010 to July 2012. The price change is reflected in the graph by the increase in sales for the Powerball ticket from January to January 2012.
Powerball price change 2012

- Powerball price increased $1 → $2 in January 2012.
- Control for jackpot using jackpot forecast IV.
Key statistic $\bar{\zeta}_p$: semi-elasticity of lottery demand with respect to price

<table>
<thead>
<tr>
<th></th>
<th>(1) Pooled</th>
<th>(2) Pooled</th>
<th>(3) Powerball</th>
<th>(4) Mega Millions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>-0.5583***</td>
<td>-0.5356***</td>
<td>-0.6031***</td>
<td>-0.5079***</td>
</tr>
<tr>
<td></td>
<td>(0.0660)</td>
<td>(0.0624)</td>
<td>(0.1023)</td>
<td>(0.0652)</td>
</tr>
<tr>
<td>Jackpot pool</td>
<td>0.0040***</td>
<td>0.0059***</td>
<td>0.0032***</td>
<td>0.0032***</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0006)</td>
<td>(0.0003)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>Jackpot expected value</td>
<td>0.9657***</td>
<td>0.9657***</td>
<td>0.9657***</td>
<td>0.9657***</td>
</tr>
<tr>
<td></td>
<td>(0.0696)</td>
<td>(0.0696)</td>
<td>(0.0696)</td>
<td>(0.0696)</td>
</tr>
<tr>
<td>Observations</td>
<td>416</td>
<td>416</td>
<td>208</td>
<td>208</td>
</tr>
</tbody>
</table>

- Instrument for jackpot $w_{j1t}$ using jackpot forecast IV.
- Control for minor changes in sub-jackpot prize expected value using estimated semi-elasticity $\hat{\zeta}_2$.
- Point estimate: 1 cent rise in price reduces sales by $-0.558\%$.

$$\ln s_{jt} = -\zeta_p p_{jt} + \zeta_1 \pi_{j1t} w_{j1t} + f(H_{jt-1}) + \hat{\zeta}_2 EV^{2+}_{jt} + \xi_j + \phi_d(t) + \epsilon_{jt}$$
Remark: $\tilde{\zeta}_1 > |\tilde{\zeta}_p| > \tilde{\zeta}_2$ informs choice of probability weighting function

- $\tilde{\zeta}_1 \gg \tilde{\zeta}_2$ is inconsistent with “standard” probability weighting functions used in prospect theory and cumulative prospect theory

- Note: incentivized experiments (and KT ’79 surveys) don’t study magnitudes in this range
  - Preliminary hypothesis: standard probability weighting functions do not extend to the small probabilities / large prizes we have here

- Ranking is consistent with probability weighting fn in Chateauneuf, Eichberger and Grant (2007)
  - Most weight given to highest prize and lowest prize
  - We use this specification in calibrations to follow
Calibration
Individual utility

Consumer $i$’s utility from buying lottery $L$, with price $p$ and \{prizes, probabilities\} = \{w_k, \pi_k\}_{k=1}^K:

$$U_i(L) = c_i - p + \sum_{k=1}^{K} \Phi_i(\pi_k) \ u_i(w_k) + \epsilon_{it}$$

$$V_i(L) = U_i(L) - \sum_{k} \chi_i \left( \Phi_i(\pi_k) - \pi_k \right) u_i(w_k)$$

Calibration assumptions

- CRRA utility over wealth (baseline = log).
- Chateauneuf et al. weighting function.
- Representative lottery: Mega Millions, $300$ million jackpot. Overhead costs = $0.20/ticket.
- Discretized income grid, welfare weights declining with income ($g_i \propto 1/c_i$)
- Random taste shock $\epsilon_{it} = \xi + \alpha\epsilon_{it}$ iid logit. (Model selects $\xi < 0$, “hassle costs”)
- Income tax rate on winnings: 40%. Overhead costs = $0.20/ticket.$
Are lotteries welfare enhancing? Welfare gains across expected value

- Hold ticket price fixed at status quo ($2).
- Scale all prizes up/down to change expected value (status quo: $0.74).
Are lotteries welfare enhancing? Welfare gains across expected value

- Hold ticket price fixed at status quo ($2).
- Scale all prizes up/down to change expected value (status quo: $0.74).
- In baseline, optimal EV is higher than status quo (lower than price)
Are lotteries welfare enhancing? Welfare gains across expected value

- Hold ticket price fixed at status quo ($2).
- Scale all prizes up/down to change expected value (status quo: $0.74).
- In baseline, optimal EV is higher than status quo (lower than price).
- Absent bias, price ≈ marginal cost (EV + overhead); no corrective implicit tax.
Are lotteries welfare enhancing? Welfare gains across expected value

- Hold ticket price fixed at status quo ($2).
- Scale all prizes up/down to change expected value (status quo: $0.74).
- In baseline, optimal EV is higher than status quo (lower than price).
- Absent bias, price ≈ marginal cost (EV + overhead); no corrective implicit tax.
- Optimal expected value falls as bias grows larger.
Optimal lottery structure (preliminary)

- Price: $2.48 (compare to $2)
- Expected value of prize payout: $1.67 (compare to $0.74)
- Implicit tax rate: 25% (compare to 53%)
Recap

1. Derivation of new “optimal regulation” formula and application to lotteries.
   - Extends behavioral public finance policies to non-price attributes.

2. New descriptive evidence on lottery consumption, behavioral biases, and elasticities.
   - Consumption mildly declining with income.
   - Modest share of consumption explained by bias.

3. Calibrated model to explore welfare and policy counterfactuals.
   - Lotteries likely raise welfare on average.
   - Could be improved by reducing implicit tax rate.
Thank you!
Appendix
Are lotteries welfare enhancing? Welfare gains across price

- Welfare gain across $p$ (fixing $w_k$, $\pi_k$)
- If unbiased, $p^* \approx$ marginal cost (no implicit tax)
- With estimated bias: $p^* > MC$
- Large bias: low prices are welfare-reducing.
How optimal lottery structure depends on bias

- Optimal expected value falls as bias grows larger.

Graph showing the relationship between share of decision weight due to bias and optimal expected value.
Optimal lottery structure depends on bias

- Optimal expected value falls as bias grows larger.
- Corrective implicit tax also rises with bias, making price large.
## Substitution across time

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jackpot expected value (t)</td>
<td>0.8975***</td>
<td>0.8944***</td>
<td>0.8805***</td>
<td>0.9263***</td>
<td>0.7930***</td>
</tr>
<tr>
<td></td>
<td>(0.0462)</td>
<td>(0.0445)</td>
<td>(0.0473)</td>
<td>(0.0436)</td>
<td>(0.0875)</td>
</tr>
<tr>
<td>Jackpot expected value (t-1)</td>
<td>0.1061***</td>
<td>0.0934***</td>
<td>0.1454***</td>
<td>-0.0504</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0167)</td>
<td>(0.0192)</td>
<td>(0.0330)</td>
<td>(0.0880)</td>
<td></td>
</tr>
<tr>
<td>Jackpot expected value (t-2)</td>
<td>-0.0165</td>
<td>0.0397*</td>
<td>-0.1341</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0196)</td>
<td>(0.0228)</td>
<td>(0.0905)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jackpot expected value (t-3)</td>
<td>0.0528**</td>
<td>-0.1145</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0213)</td>
<td>(0.0866)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jackpot expected value (t-4)</td>
<td>-0.1211</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0822)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>59,421</td>
<td>59,513</td>
<td>59,605</td>
<td>59,697</td>
<td>59,789</td>
</tr>
<tr>
<td>Akaike Information Criterion</td>
<td>-8,044.68</td>
<td>-8,113.91</td>
<td>-8,553.20</td>
<td>-9,153.55</td>
<td>-13,925.26</td>
</tr>
<tr>
<td>Bayesian Information Criterion</td>
<td>-7,891.81</td>
<td>-7,961.01</td>
<td>-8,409.27</td>
<td>-9,045.59</td>
<td>-13,817.28</td>
</tr>
</tbody>
</table>

- Lagged jackpots (instrumented) do not crowd out current demand.
- AIC/BIC minimized with no lags.
## Substitution across games

<table>
<thead>
<tr>
<th></th>
<th>(1) Own game sales</th>
<th>(2) All other games sales</th>
<th>(3) Other lotto games sales</th>
<th>(4) Instant games sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jackpot expected value</td>
<td>1.8833***</td>
<td>0.0887</td>
<td>0.0578</td>
<td>0.0452</td>
</tr>
<tr>
<td></td>
<td>(0.3422)</td>
<td>(0.1655)</td>
<td>(0.1447)</td>
<td>(0.0598)</td>
</tr>
<tr>
<td>Observations</td>
<td>58,756</td>
<td>58,756</td>
<td>58,756</td>
<td>58,756</td>
</tr>
</tbody>
</table>

- Outcome: total sales of game type in each column.
- Higher jackpot (instrumented) raises own-game sales; does not reduce other games’ sales.
Prize expected value: $x_{jkt} := \pi_{jk} \left( w_{jkt} (1 - \pi_{jk})^{s_{jkt} - 1} + \frac{w_{jkt}}{2} \pi_{jk} (1 - \pi_{jk})^{s_{jkt} - 2} (s_{jkt} - 1) + \ldots \right)$

- Probability $\pi_{jk}$ of winning; $s_{jkt} - 1$ others to potentially split prize $k$
- Prize $w_{jkt}$ if unshared
- Prize $\frac{w_{jkt}}{2}$ if split 2 ways, ...
- etc.
Regression equation

\[ \ln s_{jt} = \zeta_1 x_{j1t} + \zeta_2 x_{j2t} + f(H_{jt-1}) + \xi_j + \eta T(t) + \phi_d(t) + \epsilon_{jt} \]

- \( j \): game-structure, \( t \): index of drawing date
- \( s_{jt} \): tickets sold
- \( x_{jkt} := \) expected value of prize level \( k \)
- \( \xi_j, \eta T(t), \phi_d(t) \): fixed effects for game-state-structure, quarter of sample, day of week

Instrument construction

\[ Z_{jkt} = \begin{cases} 
\pi_{jk} \tilde{W}_{jk} \left( (1 - \pi_{jk}) \hat{s}_{jkt}^{-1} + \frac{\pi_{jk}}{2} (1 - \pi_{jk}) \hat{s}_{jkt}^{-2} (\hat{s}_{jkt} - 1) \right) & \text{if } r_{jkt-1} = 0 \\
\pi_{jk} (w_{jkt-1} + \kappa_{jk} \rho_j \hat{s}_{jkt}) \left( (1 - \pi_{jk}) \hat{s}_{jkt}^{-1} + \frac{\pi_{jk}}{2} (1 - \pi_{jk}) \hat{s}_{jkt}^{-2} (\hat{s}_{jkt} - 1) \right) & \text{if } r_{jkt-1} = 1 
\end{cases} \]

- \( \hat{s}_{jkt}(\tilde{r}_{jkt-1}, H_{jt-1}) \): flexible best-predictor of \( s_{jkt} \) (tickets with which prize \( k \) risks being split), based on history \( H_{jt-1} \), and prize rollover vector \( \tilde{r}_{jkt-1} = (r_{j1t-1}, r_{j2t-1}) \).
- Accounts for risk of splitting prize (more important for 2nd prize than jackpot)
- Improves conditional prize forecast by predicting sales from \( H_{jt-1} \) (important when jackpot moves sales affecting smaller prizes)