Currency Choice in Contracts*

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Abstract

We study the interaction between the currency choice of private domestic contracts and optimal monetary policy. The optimal currency choice depends on the price risk of each currency, as well as the covariance of its price and the relative consumption needs of the agents signing the contract. When a larger share of contracts are denominated in local currency, the government can use inflation more effectively to either redistribute resources or reduce default costs, which, in turn, makes local currency more attractive for private contracts. When governments lack commitment, competitive equilibria can be constrained inefficient, thus providing a reason to regulate the currency choice of private contracts. We show that both the equilibrium use of local currency and the implications for regulation depend on the level of domestic policy risk. We also use our model to explain the wide use of the dollar in international trade contracts.

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1 Introduction

One of the central roles of currency is to serve as a unit of account in private credit contracts. While in most countries this role is exclusively fulfilled by the local currency, several countries also rely on a foreign currency (for example, the U.S. dollar) to denominate domestic contracts. The coexistence of multiple currencies is especially relevant in emerging economies, which are often subject to a greater degree of policy instability. In this paper, we address two related questions on the role of currencies as units of account. First, what determines the currency choice of credit contracts among private agents? Second, how do these individual currency choices affect the government's conduct of monetary policy?

To answer these questions, we study a general equilibrium model in which agents choose the currency in which to denominate contracts, and the government chooses the inflation rate. These contracts involve the provision of a good in exchange for a future payment denominated in some currency. The optimal choice of currency depends on the price risk of each currency, as well as how this price covaries with the relative consumption needs of the agents signing the contract. The price of the local currency is chosen ex-post by a benevolent government and depends on the use of local currency in private contracts. A key feature of this model is the complementarity between the actions of private agents and those of the government. When a larger share of private contracts is denominated in local currency, the government can use inflation to either redistribute resources more effectively or reduce default costs, which, in turn, makes local currency more attractive as a unit of account for private contracts. The government is also subject to exogenous policy risk, which affects the price risk of local currency and reduces the attractiveness of denominating contracts in local currency. We show that the set of equilibria depends crucially on the level of policy risk, and multiple equilibria can emerge. We also ask whether competitive equilibria are efficient and argue that there might be a role for regulation to encourage private agents to denominate contracts exclusively in local currency. This might help explain policy initiatives in many emerging economies aimed at discouraging or prohibiting the use of foreign currency in domestic contracts.

At the core of our theory is the debt-deflation channel studied by Fisher (1933) and the ability of governments to use monetary policy as a tool for redistribution in certain states of the world. Indeed, history offers examples during which inflation was used to reduce the real value of debt obligations of private agents. A notable example is the experience of the US during the Great Depression, when the continuous decline of commodity prices posed challenges to the highly indebted farm sector. In response to the situation, the Farm Relief Act enacted by Roosevelt paved the way for the abandonment of the gold standard and an increase in inflation. According to Edwards (2018): "This was what the president was after: higher prices that would increase farmers' incomes and would reduce the burden of their debts in real terms."

We begin our analysis by characterizing the optimal bilateral contract. Our framework nests a variety of contracts in which unit of account considerations are present, including debt contracts and trade credit contracts. Buyers and sellers sign contracts to exploit gains from trade of a special good. Contracts stipulate the amount of a special good that is provided at the date the contract is signed, in exchange for an amount of local and/or foreign currency to be paid in the future. Currencies serve only as units of account, since the actual payment in the future is made in terms of a numeraire good. The price levels of both currencies (measured in terms of the numeraire good) are stochastic and unknown at the time contracts are signed. Agents also receive taste shocks after signing contracts, which affect their marginal utility of consuming the numeraire good. This increases the desirability of currencies whose price covaries with these shocks. On the other hand, due to a constraint which requires that payments be feasible in all states of the world, currencies with higher price risk are less desirable. The optimal currency choice features a trade-off between these two forces.

In the model, the price of foreign currency is exogenous, while the price of local currency is chosen by a benevolent government that lacks commitment. The government's optimal choice of inflation trades off the benefits of either redistributing resources more effectively or reducing default costs with the costs of deviating from a target. In the baseline model, the benefits of using inflation are to redistribute resources given differences in taste shocks of buyers and sellers. We also study a model with costly default in which inflation can help reduce default costs and show that it maps into our baseline setup. The optimal inflation choice redistributes resources between agents in an ex-post efficient way. For example, when buyers have a high marginal utility (relative to sellers), the government chooses higher inflation to lower the real burden of payments. The degree of redistribution that takes place depends positively on the use of local currency in private contracts. The government's inflation choices also depend on the costs of deviating from a target which is stochastic, unknown at the time when contracts are signed, and independent of currency choice. We refer to fluctuations in the target as policy risk.

We fully characterize the set of equilibria for different levels of policy risk. This exercise is motivated by the positive relationship between domestic dollarization and measures of policy risk across countries. One measure of policy risk is the volatility of government expenditures.[1] As we show in Figure 4, domestic dollarization is positively correlated with the volatility of government expenditures across countries.[2] For example,

[1] In Appendix C.1, we show how policy risk can be mapped into this measure.

[2] Other measures of policy risk include institutional factors (Nicolo et al. (2003), Rennhack and Nozaki (2006)). Measures of policy stability, such as the implementation of inflation targeting, have also been

the U.S., Germany, and Japan rely exclusively on their local currency as a unit of account in domestic contracts, while countries in Latin America and Eastern Europe tend to partially or fully rely on foreign currency as a unit of account. Consistent with this, we find that for low levels of policy risk there is a unique equilibrium in which all contracts are denominated in local currency, while for high levels of this risk, all contracts are denominated in foreign currency. For intermediate levels of policy risk there are three equilibria: two of which involve the exclusive use of either the local or foreign currency, and a third interior one in which both local and foreign currencies are used. We then use a global games refinement to uniquely select an equilibrium for any level of policy risk and find that there is a unique cutoff below which all contracts are denominated in local currency and above which all contracts are denominated in foreign currency.

Both recently and historically, many countries have introduced policy initiatives which either encourage or discourage the use of foreign currency as a unit of account. On the one hand, there have been policy initiatives in a large number of emerging market economies that discourage the use of foreign currency as a unit of account. Two such examples are Brazil and Colombia, which prohibit the denomination of bank deposit and loan contracts in foreign currency. Other similar examples of recent initiatives include policies in Hungary and Poland, which either heavily regulated or forced conversion of foreign currency housing loans to domestic currency. On the other hand, two decades ago Ecuador and El Salvador fully dollarized their domestic economies.

Our paper can help rationalize the prevalence of such policy initiatives. We study the problem of a social planner subject to the same constraints as private agents. We find that the optimal allocation is characterized by a cutoff in policy risk below which all contracts are denominated in local currency and above which all contracts are denominated in foreign currency. Consequently, for low levels of policy risk, equilibria with foreign currency use are dominated by one in which there is full use of local currency, and for high levels of policy risk, equilibria with local currency use are dominated by one in which there is full use of foreign currency. There are two sources of inefficiency in private currency choices. First, since an individual agent is of measure zero, it does not affect the policy choice of the government; thus, if all agents are denominating contracts in foreign currency, an individual agent will not be incentivized to denominate contracts in local currency, even if it is socially efficient to do so. Second, private agents do not internalize the inflation costs associated with deviating from the target. While the first source leads to less use of local currency than is optimal, the second source has the opposite effect. We characterize

shown to decrease domestic dollarization (Lin and Ye (2013)). Note that we focus on one measure of domestic dollarization of contracts, namely, the share of dollar-denominated bank deposit contracts. Similar patterns are observed if we focus on other measures, including the share of dollar-denominated bank loan contracts.

which of these inefficiencies dominates as a function of the degree of policy risk. When we compare the planner's cutoff of policy risk to the unique cutoff selected by the global games refinement, we find that the former is strictly lower than the latter. This implies that there is a region of inefficiency in which the unique competitive equilibrium calls for less local currency use than the planner's solution.

We then use our model to study a variety of applications and extensions. First, we study a model with default in which the role of policy is to reduce the costs associated with default. We show that there exist processes for the taste shocks so that the equilibrium outcomes in the taste shock model are identical to that in the model with default. Thus, we can apply the results from the baseline to the default model. Moreover, this shows that the taste shock model is quite general and can be used to study other interesting environments. Another takeaway from this application is that observed inflation policy need not always reflect a redistributive motive. Indeed, during normal times, when there is no risk of default, inflation is set at its target. However, in times of crises, when default imposes large social costs, the government chooses inflation to reduce the burden of default and redistribute.

Second, we extend our model to study currency choice in international trade contracts. Gopinath (2016) documents that the U.S. dollar is widely used as a unit of account in international trade contracts. In particular, countries such as Japan have low inflation risk and low domestic dollarization, and yet have a significant fraction of their international trade contracts denominated in dollars. Moreover, as we document in Figure 5, the share of import contracts denominated in U.S. dollars exceeds the share of dollar-denominated domestic financial contracts in most countries. This suggests that the use of the dollar is more prevalent in international contracts than in domestic ones. Motivated by this, we study a two-country model in which buyers (respectively, sellers) in one country trade with sellers (respectively, buyers) from another symmetric country, and contracts can be set in three possible currencies: the currencies of either country and a foreign currency (which in this case stands for the U.S. dollar). Our model can rationalize the large use of the dollar in international contracts relative to domestic contracts. We show that there exist levels of policy risk such that a full local currency equilibrium exists for domestic contracts, but not for international contracts. In particular, in this range of policy risk agents strictly prefer to denominate international contracts in foreign currency (the dollar), while they prefer to denominate domestic contracts in local currency if all other agents do so. The reason is that the benefit for an agent to denominate contracts in the local currency of its trading partner is lower if the partner is from a different country. This is because the government has incentives to respond only to the taste shocks of its own citizens and not to those of other countries' citizens. In contrast, for domestic contracts, the government responds to the taste shocks of both partners involved, thus raising the benefit of denominating contracts in the local currency.

Finally, we use our model to shed light on the observed hysteresis in the share of foreign currency–denominated contracts. This pattern is most striking in many Latin American economies that still exhibit high levels of financial dollarization in spite of continued success in controlling inflation and inflation risk in the last decade (Ize and Levy-Yeyati (2003)). To address this empirical pattern, we enhance our baseline model by endowing buyers with claims on local and foreign currency that, as we show, can arise endogenously as a consequence of trading within a credit chain. In this model, currency choice exhibits hysteresis because there are benefits of matching the currency of denomination of new contracts to the outstanding claims that back the buyers' future payments. We illustrate this by showing that even if policy risk gets arbitrarily small, in equilibrium, foreign currency will still be used as a unit of account. The reason is that it is optimal to match the currency of older contracts and only de-dollarize the claims that are backed with future income.

Related Literature There is a large literature that studies the use of currencies for a variety of purposes. Our paper is related to a literature that studies the choice of currency denomination of debt contracts (see Ize and Levy-Yeyati (2003), Caballero and Krishnamurthy (2003), Schneider and Tornell (2004), Doepke and Schneider (2017), and Bocola and Lorenzoni (2019), among others).[3] These papers abstract from the interaction between private currency choices and monetary policy, which is the central focus of our paper. Our contracting framework builds on Doepke and Schneider (2017), who study the determination of a unit of account in the presence of exogenous price risk. Our environment is different in two key ways. First, our framework features a trade-off between price risk and the insurance properties of each currency, which is absent in their paper. Second, and more importantly, we model the optimal conduct of monetary policy, thus endogenizing both the price risk and insurance benefits, and focus on the interaction between private choices of the unit of account and government's policy choices. These two differences are fundamental to the characterization of equilibria and the implications for optimal regulation of currency choices.

Our paper contributes to the literature that studies the interaction between currency choice and policy. First, there are papers in which both the currency and policy choices are made by governments. Ottonello and Perez (2019), Du et al. (2019), and Engel and Park (2019) study the interaction between monetary policy and the currency denomina-

[3]Currency choice has also been studied in the context of denomination of prices (see, for example, Devereux and Engel (2003), Bacchetta and Van Wincoop (2005), Engel (2006), Goldberg and Tille (2008), Gopinath et al. (2010), Corsetti et al. (2015), Gopinath et al. (2018), and Drenik and Perez (2019)), and means of payment (see Matsuyama et al. (1993) and Uribe (1997)).

tion of sovereign debt. Neumeyer (1998), Alesina and Barro (2002), Arellano and Heathcote (2010), and Chari et al. (2019) study the trade-offs associated with forming currency unions or dollarizing the economy. In contrast to these papers, our paper focuses on currency choices of private agents and how they interact with the policy choices of the government.

Second, there is a set of papers that study the interaction between the currency choices of private agents and monetary policy. Svensson (1989), Chang and Velasco (2006), and Devereux and Sutherland (2008) analyze the optimal portfolio choice when there are nominal assets, for different monetary policy rules. Rappoport (2009) studies a model of currency choice in corporate debt to rationalize the prevalence of hysteresis in domestic dollarization. Fanelli (2019) studies the interaction between private debt choices and exchange rate policies when governments can commit. We contribute to this literature in two key dimensions. First, we study a model in which governments choose monetary policy without commitment. The lack of commitment can give rise to equilibrium multiplicity and inefficiency. Crucially, we show that both the equilibrium set and the existence and type of inefficiencies depend on the level of policy risk. In this sense our results can rationalize the cross-country heterogeneity in the use of dollars in domestic contracts, and shed light on the current debates surrounding the regulation of domestic dollarization in various countries. Second, our general framework allows us to study a variety of interesting applications including the role of inflation to mitigate default costs, international trade contracts, and hysteresis in dollarization.

Finally, our paper contributes to a growing literature on the global role of the dollar (see, for example, Maggiori (2017), Farhi and Maggiori (2017), Gopinath and Stein (2019), Chahrour and Valchev (2019), Maggiori et al. (2019), Mukhin (2019), and Eren and Malamud (2019)). Gopinath and Stein (2019) emphasize a complementarity between the use of the dollar for invoicing in international trade and the aggregate demand for dollar-safe assets. We propose a complementary view to theirs, which relies on the interaction between private currency choices and governments' policy choices. Our theory can help account for the relatively high use of the dollar in countries with greater policy risk, as well as the greater use of this currency in international contracts relative to domestic ones.

The rest of the paper is organized as follows. Section 2 presents the model, characterizes the equilibrium, and analyzes the constrained efficient allocation of the economy. In section 3, we study a variety of applications of our baseline model, including strategic default (subsection 3.1), international trade contracts (subsection 3.2), and an analysis of the observed hysteresis in the currency of contracts (subsection 3.3). We present our conclusions in section 4.

2 Model

In this section, we develop a model to study the interaction between the currency choice of private contracts and optimal monetary policy. First, we describe the competitive equilibrium keeping the government's policies fixed in order to highlight the trade-offs private agents face when choosing the currency of denomination of contracts. In the following subsections, we characterize the full equilibrium with endogenous government policy.

2.1 General Environment

There are two periods, $t = 1, 2$. The domestic economy is populated by two types of agents: citizens and a government. Citizens are further divided into sellers and buyers, with a unit measure of each.

Buyers have preferences over consumption of a special good produced by sellers in period 1. Buyers and sellers also value the consumption of a numeraire good, which takes place in period 2. The preferences of the representative seller are given by

$$u_s = -x + \mathbb{E}\left[\theta_s c_s\right],$$

where x is the special good produced by the seller, c_s is the seller's consumption of the numeraire good, and θ_s is a taste shock which measures the seller's marginal utility of consuming the numeraire good. The preferences of the representative buyer are given by

$$u_b = (1+\lambda)\, x + \mathbb{E}\left[\theta_b c_b\right],$$

where $1+\lambda$ is the valuation of the special good provided by the seller, c_b is the buyer's consumption of the numeraire good, and θ_b is the buyer's taste shock.[4] The parameter $\lambda \geqslant 0$ governs the gains of trading the special good between sellers and buyers. We assume that θ_s is drawn from a distribution with mean $\mathbb{E}\left[\theta_s\right]$ and support $\left[\underline{\theta}_s, \overline{\theta}_s\right]$ and that θ_b is drawn from a distribution with mean $\mathbb{E}\left[\theta_b\right]$ and support $\left[\underline{\theta}_b, \overline{\theta}_b\right]$. We make no explicit assumption about the correlation between θ_s and θ_b. The fact that θ_s and θ_b are unknown in period 1 introduces uncertainty in the relative marginal utilities of the numeraire good and gives rise to gains from making relative consumption state-contingent. A high (respectively, low) value of θ_b relative to θ_s makes consumption of buyers, relative to sellers, more (respectively, less) desirable. As we will see, these taste shocks are a stylized way of generating the value of having a flexible government policy. The differ-

[4]Note that θ_s and θ_b are shocks to the *representative* buyers and sellers, respectively. Since preferences are linear, and there is a continuum of agents, these shocks correspond to the aggregate component of individual shocks.

ences in θ_s and θ_b can capture any reason for why it is socially and privately desirable to shift resources between different groups of citizens in the population. For example, as we show in Section 3.1, a model with default and outside option shocks maps directly into our environment. Finally, buyers and sellers are endowed with $y > 0$ units of the numeraire good in period 2.

The timing of the model is as follows:

1. In period 1, sellers produce a special good for buyers in exchange for the promise of a payment in period 2.

2. In period 2, taste shocks θ_s and θ_b are realized, the domestic government chooses its policy consisting of the aggregate price level, all signed contracts are executed, and consumption of the numeraire good takes place.

Next, we formally define a contract and discuss its properties.

2.2 Bilateral Contracts

A contract between a buyer and a seller consists of the provision of the special good (from the seller to the buyer) in exchange for the promise of future payment (from the buyer to the seller). We impose three important assumptions on the contracting environment. The first is that payments are non-contingent and, in particular, cannot depend on the realization of the state (θ_s, θ_b). The second is that payments can be made only in two possible "units of account", which we call *currencies*. We denote the two possible currencies by l (local) and f (foreign). A payment b_l in currency l yields $b_l \phi_l$ units of the domestic numeraire good in period 2, while a payment b_f in currency f yields $b_f \phi_f$ units of the domestic numeraire good in period 2. Here, ϕ_l and ϕ_f denote the price of the local and foreign currencies in terms of the numeraire good, respectively. In general, ϕ_l and ϕ_f are random variables from the perspective of private agents that are unknown at the time the contract is signed. The third assumption is that default costs are sufficiently high so that contracts must be *default-free*. In other words, actual payments must equal promised payments in all states of the world. We relax this assumption in Section 3.1.

Formally, a bilateral contract signed is a tuple (x, b_l, b_f), where x indicates the units of the special good provided to the buyer, and (b_l, b_f) are the units of local and foreign currency promised to be paid to the seller at date 2, respectively. The assumption that contracts must be default-free, along with a non-negativity constraint on the buyer's consumption implies that contracts must satisfy the following payments feasibility constraint

$$b_l \phi_l + b_f \phi_f \leqslant y \;\forall (\phi_l, \phi_f) \in \Phi, \tag{1}$$

where $\Phi \subset \mathbb{R}^2_+$ is the compact set of possible price realizations. This inequality states that for all possible price realizations, the promised repayment must not exceed the income of the buyer. Citizens are exposed to risk from uncertainty about aggregate prices. Note that a low (respectively, high) value of ϕ_c indicates a high (respectively, low) level of domestic inflation in currency c. The price of local currency ϕ_l is endogenous and citizens take it as given. In particular, from the perspective of citizens, the price level ϕ_l is a random variable with support $\left[\underline{\phi}_l, \overline{\phi}_l\right]$. In equilibrium, the distribution of ϕ_l and the bounds of the support, $\underline{\phi}_l$ and $\overline{\phi}_l$, depend on the choices made by the government. On the other hand, the price of foreign currency ϕ_f is exogenous, stochastic with support $\left[\underline{\phi}_f, \overline{\phi}_f\right]$, and independent from the other random variables. We associate the foreign currency with relatively stable currencies such as the dollar or the euro, and interpret the risk in ϕ_f as real exchange rate risk.[5]

Without loss of generality, we assume that in each bilateral meeting the buyer makes a take-it-or-leave-it offer to the seller. The seller is willing to participate in the contract as long as the expected value of the repayment covers the cost of providing the special good

$$-x + \mathbb{E}\left[\theta_s\left(b_l\phi_l + b_f\phi_f\right)\right] \geqslant 0, \tag{2}$$

where we normalize the seller's outside option to zero. Thus, the optimal contract for the buyer solves

$$\max_{x,b_l,b_f} (1+\lambda)x - \mathbb{E}\left[\theta_b\left(b_l\phi_l + b_f\phi_f\right)\right] \tag{3}$$

subject to (1), (2), and the non-negativity constraints $b_l, b_f \geqslant 0$.[6]

In order to characterize the solution to problem (3), we make the following assumption guaranteeing that buyers and sellers find it worthwhile to sign contracts.

Assumption 1. *Assume that*

$$(1+\lambda)\,\mathbb{E}\left[\theta_s\right] - \mathbb{E}\left[\theta_b\right] > 0.$$

It is worth noting that our setup nests two types of contracts in which currency choice

[5]In Appendix C.1 we show how risk in ϕ_f can arise in a model with tradable and non-tradable goods and shocks to the relative demand of these goods. It is also worth noting that while we do not explicitly allow for hedging against foreign currency price movements, this is implicitly captured by properties of the distribution of ϕ_f. We make no assumptions about this distribution. In particular, the case in which ϕ_f is deterministic can be interpreted as a situation in which private agents can completely insure the risks of denominating contracts in foreign currency, or alternatively, contracts are denominated in the numeraire good.

[6]The restriction of payments being non-negative is not crucial. In Section 3.3 we show that under a tighter parametric condition, even if we allow buyers to promise negative payments (i.e. payments from the seller to the buyer) in a certain currency, these will not be part of the optimal contract.

is important. The first is a trade-credit contract, in which gains from trade only arise from the static exchange of the special good in period 1. This corresponds to the case in which $\mathbb{E}[\theta_b] = \mathbb{E}[\theta_s]$ and $\lambda > 0$. The second is a standard debt contract, in which gains from trade only arise from the intertemporal exchange of goods. In particular, one can interpret differences in expected taste shocks between buyers and sellers as heterogeneity in discount factors. Therefore, if $\mathbb{E}[\theta_b] < \mathbb{E}[\theta_s]$ and $\lambda = 0$, there are no static gains from trading but agent b is relatively more impatient than agent s and thus would like to borrow in period 1. Under this interpretation, x corresponds to the amount borrowed by agent b. Consequently, the labels of "special" and "numeraire" goods are merely used to distinguish goods traded in periods 1 and 2, respectively. Similarly, the labels of "buyers" and "sellers" are interchangeable with "borrowers" and "lenders", respectively.[7]

2.3 Competitive Equilibrium Given Government Policy

We now characterize the optimal bilateral contract between a seller and a buyer, taking the distribution of ϕ_l and ϕ_f as given. Since preferences are linear, Assumption 1 implies that there are positive gains from trading as much of the special good x as possible. The limit on how much x can be traded is given by the fact that buyers need to be able to pay for that good in the following period. This implies that the feasibility constraint (1) will always be binding. Additionally, the state for which this constraint will bind is the one in which inflation $\frac{1}{\phi_c}$ in both currencies is at its lowest possible realization (i.e., $\phi_l = \overline{\phi}_l$ and $\phi_f = \overline{\phi}_f$). If we substitute the participation and feasibility constraints into the objective, the derivative with respect to b_l is proportional to

$$\underbrace{\mathbb{E}\left[(\theta_s(1+\lambda)-\theta_b)\frac{\phi_l}{\overline{\phi}_l}\right]}_{\text{Marginal benefit of local currency }(M_l)} - \underbrace{\mathbb{E}\left[(\theta_s(1+\lambda)-\theta_b)\frac{\phi_f}{\overline{\phi}_f}\right]}_{\text{Marginal benefit of foreign currency }(M_f)}.$$

The expression above represents the difference between the marginal benefit of setting the contract in local currency (M_l) and the marginal benefit of setting it in foreign currency (M_f). Since the objective is linear, these objects are constant and independent of the choice of b_l. The optimal contract calls for using the currency that has the largest marginal benefit. When the marginal benefit is the same in both currencies, any combination of local and foreign currency is optimal. The following proposition formalizes this result.

[7]These broad classes of agents can have different interpretations depending on the particular application. For example, in the context of the US Great Depression discussed in the introduction, buyers would refer to the farmers who required debt to finance production, and sellers to their creditors. In other relevant applications, buyers would refer to firms taking on debt, or banks taking deposits, and sellers would refer to households. In the case of international trade contracts, analyzed in Section 3.2, buyers would refer to importers that make purchases with trade credit from exporters (sellers).

Proposition 1. *Suppose that Assumption 1 holds. In the optimal bilateral contract, the amount of special good is given by* $x = \mathbb{E}\left[\theta_s\left(b_l\phi_l + b_f\phi_f\right)\right]$*, while the payments satisfy*

1. *If* $M_l < M_f$*, then* $b_l = 0$ *and* $b_f = \frac{y}{\overline{\phi}_f}$
2. *If* $M_l = M_f$*, then* $b_l = \gamma\frac{y}{\overline{\phi}_l}$ *and* $b_f = (1-\gamma)\frac{y}{\overline{\phi}_f}$ *for any* $\gamma \in [0,1]$*.*
3. *If* $M_l > M_f$*, then* $b_l = \frac{y}{\overline{\phi}_l}$ *and* $b_f = 0$*.*

All proofs are included in the Appendix. To understand the marginal benefit of denominating the contract in a currency c, we can rewrite it as

$$M_c \equiv \left[(1+\lambda)\,\mathbb{E}\left[\theta_s\right] - \mathbb{E}\left[\theta_b\right]\right]\frac{\mathbb{E}\left[\phi_c\right]}{\overline{\phi}_c} + \operatorname{cov}\left(\left(\theta_s\left(1+\lambda\right) - \theta_b\right), \frac{\phi_c}{\overline{\phi}_c}\right) \tag{4}$$

for $c = l, f$. The marginal benefit of each currency has two components: a price risk term and a covariance term. The ratio $\frac{\mathbb{E}[\phi_c]}{\overline{\phi}_c}$ denotes the price risk of denominating contracts in currency c. A higher (respectively, lower) value of $\frac{\mathbb{E}[\phi_c]}{\overline{\phi}_c}$ represents a lower (respectively, higher) risk of indexing contracts in currency c. Note that it is the maximal value $\overline{\phi}_c$ that determines price risk due to the assumption that payments must be feasible in all states of the world, in particular in the state with the highest value of currency c in terms of the numeraire good. The second term is the covariance of relative taste shocks and currency prices. The marginal benefit of denominating the contract in foreign currency is exogenous and given only by the price risk term, since the covariance term is zero given our assumption of independence between ϕ_f and the shocks θ_b and θ_s.

To understand the results in Proposition 1, suppose first that θ_b and θ_s are deterministic. Then, the optimal currency choice is determined exclusively by comparing the price risk in both currencies, $\frac{\mathbb{E}[\phi_l]}{\overline{\phi}_l} - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}$. In this case, choosing the currency with the lowest price risk maximizes the gains from trade, as it allows buyers to promise sellers larger payments in expected value that can be made in all states. In contrast, suppose that the taste parameters are stochastic. Now the optimal currency choice also depends on the covariance between prices in local currency and marginal utilities (taste shocks). For example, if ϕ_l is high in the states in which the seller values consumption relatively more than the buyer (high θ_s relative to θ_b), denominating the contract in local currency is more attractive. As we will see in the next section, a benevolent government will choose ϕ_l so that this covariance term is positive. Finally, the optimal choice of x can be computed directly from the participation constraint (2).

At this point, it is worth describing the differences between our results and those in Doepke and Schneider (2017), who also study the determination of the optimal unit of account. First, Doepke and Schneider (2017) only focus on differences in price risk. Therefore, the trade-off between relative price risk and covariance benefits, characterized in our

Proposition 1, is absent in their paper. Second, as we describe in the next section, local currency prices are determined by a government, which in turn generates complementarities between private and government actions. These two differences are fundamental to the characterization of equilibria and the implications for optimal regulation of private currency choices.

2.4 Government

We consider a utilitarian government that controls monetary policy and chooses the price level of the domestic economy ϕ_l in the second period to maximize the sum of the utilities of buyers and sellers net of the losses associated with inflation, captured by $l(\phi_l)$. We assume that $l(\phi_l) = \frac{\psi}{2}\left(\phi_l - \hat{\phi}\right)^2$, where $\hat{\phi}$ denotes the price level targeted by the government in the absence of redistributional concerns. The target $\hat{\phi}$ is a random variable realized in period 2 and, thus, stochastic at the time at which contracts are signed. We assume that $\hat{\phi}$ has bounded support $\left[\underline{\hat{\phi}}, \overline{\hat{\phi}}\right]$. Similar to our definition of price risk, we refer to $\frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}}$ as *policy risk*. As before, a higher (respectively, lower) value of $\frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}}$ represents a lower (respectively, higher) policy risk. In Appendix C.2, we provide a microfoundation of the inflation loss function from the Ramsey problem of a government facing spending shocks that raises revenue through a combination of distortionary taxation and seigniorage. The target $\hat{\phi}$ denotes the optimal level of inflation for a given spending level. Thus, the loss function captures the costs of deviating from the optimal policy.[8]

An important assumption implied by the timing above is that the government lacks commitment. This choice is motivated by the fact that in reality governments find it hard to commit to state contingent policies. This is particularly true in emerging economies which tend to display higher levels of domestic dollarization. In Appendix C.4 we describe the problem with commitment. We show that in this case, the equilibrium is efficient. As we will see, this is in sharp contrast to equilibria without commitment.

Without commitment, the problem of the government is given by

$$\max_{\phi_l} \left[\theta_b C_b + \theta_s C_s\right] - l(\phi_l),$$

[8]Note that in this case, the use of inflation to collect seigniorage relies on the use of local currency as a means of payment, but not on the aggregate promised payments denominated in local currency, B_l. One can think of other channels through which the use of local currency in domestic contracts may affect the losses associated with inflation (for example, if the use of local currency as unit of account in credit contracts is complementary to its use as means of payments). Our model can incorporate such cases if, for example, the loss function takes the form $l(\phi_l) = \frac{(\psi + f(B_l))}{2}\left(\phi_l - \hat{\phi}\right)^2$, given some function $f(B_l)$. As long as $f'(B_l)$ is not too large, one can show that the main trade-offs that characterize the set of competitive equilibria in the baseline are still present in this model.

where

$$C_b = y - \phi_l B_l - \phi_f B_f \tag{5}$$

is the aggregate consumption of buyers, B_l and B_f are the aggregate promised payments denominated in local and foreign currency, respectively, and

$$C_s = y + \phi_l B_l + \phi_f B_f \tag{6}$$

is the aggregate consumption of sellers.[9]

Given the functional form of $l(\cdot)$, the solution to the government's problem is[10]

$$\phi_l = \hat{\phi} + \frac{1}{\psi}(\theta_s - \theta_b) B_l. \tag{7}$$

The optimal choice of inflation redistributes resources between sellers and buyers in an efficient way. When buyers have a high marginal utility (relative to sellers), the government chooses a higher inflation (lower ϕ_l) to lower the burden of debt payments by the buyer and redistribute resources from sellers to buyers. The opposite occurs when sellers have a high marginal utility relative to buyers. In this model, the choice of monetary policy is governed by redistributional concerns. In Section 3.1, we study a model with costly default in which the role of monetary policy is to reduce default costs. We show that such a model maps directly into this baseline environment.[11]

The government's choice of inflation affects the marginal benefit of setting contracts in local currency (M_l) (defined in equation (4)) in the first period. On the one hand, the redistribution that the government attains using monetary policy induces a positive covariance between relative marginal utilities and prices in local currency, thereby increasing the marginal benefit of the local currency. The higher the use of local currency, B_l, the higher the endogenous positive covariance for local currency. In this sense, the government's conduct of monetary policy helps make nominal contracts state-contingent in a desirable way. On the other hand, by reacting to taste shocks, the government also affects the price risk of local currency. Recall that we defined the price risk of lo-

[9] Recall that we imposed a non-negativity constraint on the buyer's consumption in the contracting problem, which is not imposed in the government's problem. This is not a concern since this constraint will never be violated in equilibrium as private agents will always choose contracts that respect it. However, including this constraint in the government's problem can give rise to additional peculiar equilibria in which the government's choice of inflation is driven purely by the need to satisfy the non-negativity constraint of private agents. We abstract from such equilibria.

[10] Note that the price level is always positive if $\underline{\hat{\phi}}$ is large enough.

[11] Another relevant margin for the choice of inflation is the collection of seigniorage revenues to finance the provision of public goods. As we show in Appendix C.2, this motive is captured by $\hat{\phi}$, which is the optimal inflation level in the absence of redistributional concerns. In particular, $\hat{\phi}$ determines the seigniorage revenues for the government as a function of (stochastic) spending needs. Note that in this case, the use of inflation to collect seigniorage relies on the use of local currency as a means of payment, but not on the aggregate promised payments denominated in local currency, B_l.

cal currency as the ratio $\frac{\mathbb{E}[\phi_l]}{\overline{\phi}_l}$. Given the optimal choice of ϕ_l, we have that $\mathbb{E}[\phi_l] = \mathbb{E}\left[\hat{\phi}\right] + \frac{1}{\psi}\left(\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]\right) B_l$ and the maximal value of ϕ_l is given by

$$\overline{\phi}_l = \overline{\hat{\phi}} + \frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right) B_l. \tag{8}$$

The higher the use of local currency B_l, the higher $\overline{\phi}_l$, which in turn can lead to a lower $\frac{\mathbb{E}[\phi_l]}{\overline{\phi}_l}$ (or a higher price risk of local currency). Throughout our baseline analysis we make the following parametric assumption.

Assumption 2. *Assume that*

$$\frac{1}{2} var\left(\theta_s - \theta_b\right) + \lambda\left[var\left(\theta_s\right) - cov\left(\theta_s, \theta_b\right)\right] \geqslant \kappa_1$$

where κ_1 is a constant depending on the model parameters defined in (15) in the Appendix.

As mentioned previously, introducing taste shocks is a simple way of generating value for flexibility in monetary policy. Thus, the variance of the relative taste shocks captures the importance of flexibility. Assumption 2 ensures that the value of flexibility is sufficiently large. To understand this assumption, it is instructive to consider the case in which θ_s and θ_b are independent and identically distributed with $\text{var}(\theta_s) = \text{var}(\theta_b) = \text{var}(\theta)$ and $\mathbb{E}[\theta_s] = \mathbb{E}[\theta_b] = 1$. Then this assumption reduces to

$$\text{var}(\theta) \geqslant \frac{\lambda}{1+\lambda}\left(\overline{\theta} - \underline{\theta}\right),$$

which is satisfied if the variance is large enough. If Assumption 2 is violated, the covariance benefits arising from denominating contracts in local currency are relatively small. As a result, currency choices in contracts are primarily governed by price risk. We study the effect of relaxing this assumption in Appendix C.3.

Denote $M_l(B_l)$ as the marginal benefit of denominating contracts in local currency (defined in equation (4)), once we substitute in the optimal choice of ϕ_l made by the government. Assumption 2 also guarantees that $M_l(B_l)$ is increasing in B_l. In particular, it guarantees that the positive effect of higher B_l on the covariance term more than offsets the effect of higher B_l on the price risk of local currency. Therefore, under this assumption, the benefit of denominating contracts in local currency is increasing in B_l, thus generating complementarities in currency choices.

Given this, we can now define a competitive equilibrium for this economy.

Definition 1. A competitive equilibrium is an allocation for private citizens (x, b_l, b_f), aggregate promised payments (B_l, B_f), and an inflation choice of the government ϕ_l such that: 1. Given ϕ_l, the private allocation solves the contracting problem defined in (3), 2. Given B_l, ϕ_l satisfies (7), and 3. Aggregate choices coincide with private ones, $b_l = B_l$ and $b_f = B_f$.

2.5 Equilibrium Characterization

We now provide a characterization of the set of competitive equilibria. The main objective of this exercise is to understand how the set of equilibria changes as we vary the level of policy risk. As we will show, for low levels of this risk, there is a unique equilibrium in which all contracts are denominated in local currency. For intermediate levels of this risk, there are three equilibria: two in which all contracts are completely denominated in either local or foreign currency, and an interior equilibrium. Finally, for high enough levels of policy risk, there is a unique equilibrium in which all contracts are denominated in foreign currency. In the next subsection, we will use a global games refinement to select a unique equilibrium given a level of policy risk.

To vary policy risk, we fix $\overline{\hat{\phi}}$ and vary $\mathbb{E}\left[\hat{\phi}\right]$. In particular, a higher value of $\mathbb{E}\left[\hat{\phi}\right]$ denotes a lower level of policy risk. The set of equilibria is characterized in the following proposition.

Proposition 2. *Suppose that Assumptions 1 and 2 hold. Then, there exist thresholds* $\mu_1 = \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}$ *and* $\mu_2 < \mu_1$ *such that:*

1. *If* $\frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}} > \mu_1$*, there exists a unique equilibrium in which* $B_l = \frac{y}{\overline{\phi}_l^*}$ *where* $\overline{\phi}_l^*$ *is the positive solution to*
$$\overline{\phi}_l^* = \overline{\hat{\phi}} + \frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right)\frac{y}{\overline{\phi}_l^*}.$$

2. *If* $\mu_2 < \frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}} \leqslant \mu_1$*, there exist three equilibria:* $B_l = \frac{y}{\overline{\phi}_l^*}$*,* $B_l = 0$*, and* $B_l \in \left(0, \frac{y}{\overline{\phi}_l^*}\right)$.

3. *If* $\frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}} \leqslant \mu_2$*, there exists a unique equilibrium in which* $B_l = 0$.

The threshold μ_2 depends on parameters and is defined in equation (18) in the Appendix. Figure 1 presents a graphical depiction of the set of equilibria when $\mathbb{E}[\theta_s] = \mathbb{E}[\theta_b] = 1$. The blue line is the average promised payment denominated in local currency B_l for a given government policy and, thus, for a given M_l. When $M_l > M_f$ private agents denominate contracts in local currency, and when $M_l < M_f$ they denominate them in foreign currency. The red lines depict the marginal benefit of local currency as a function of B_l for different values of policy risk. All lines are increasing since our assumption implies $M_l(B_l)$ is increasing. To understand the role of policy risk in the determination of equilibria it is useful to analyze how policy risk affects the marginal benefit of local

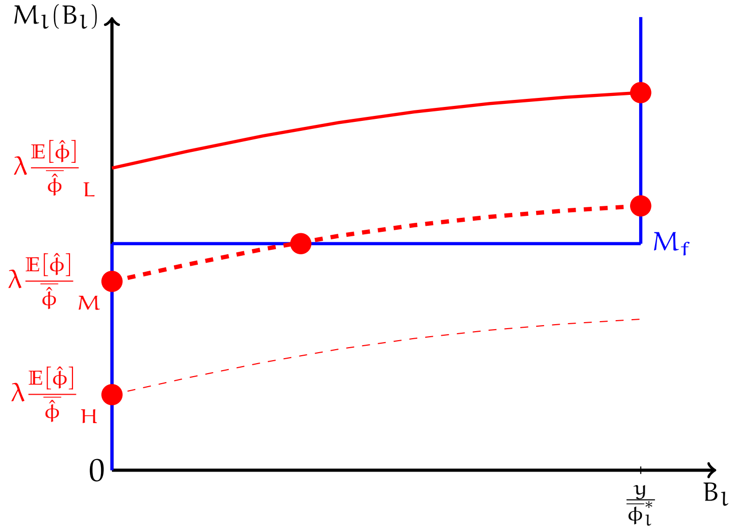

Figure 1: Characterization of competitive equilibrium

currency. Note that when there are no contracts in local currency, the optimal inflation choice is equal to the target, and the marginal benefit of the local currency is determined only by policy risk, i.e., $M_l(0) = \lambda\frac{\mathbb{E}[\hat{\phi}]}{\bar{\hat{\phi}}}$. As we increase policy risk (decrease the ratio), the marginal benefit of local currency decreases for all possible values of B_l. When policy risk is lower than the price risk of foreign currency (case 1), the unique equilibrium uses only local currency, as shown at the intersection of the red and blue solid lines. This is because even when no contracts are set in local currency, it is still worthwhile to denominate contracts in this currency if the price risk is low enough. As more contracts are signed in local currency, its attractiveness increases as the government endogenously uses inflation to redistribute resources more effectively.

When policy risk is intermediate (case 2) we have multiple equilibria. Multiplicity arises due to the complementarities between private and government actions. As more contracts are set in local currency, the government uses inflation to provide more insurance through better redistribution. One of the equilibria involves full use of foreign currency. If all private contracts are set in foreign currency, there are no incentives for the government to use inflation in order to redistribute resources. Therefore, the marginal benefit of local currency is given only by policy risk, which in this region is higher than the price risk of foreign currency. Another equilibrium involves full use of local currency. If all private contracts are denominated in local currency, then the government is incentivized to use inflation to redistribute resources efficiently, and this makes local currency more attractive than foreign currency. Finally, there is a third interior equilibrium in which the level of B_l is such that the marginal benefits of local and foreign currencies are equal. In the figure, the three equilibria correspond to the three intersections of the blue and the middle red dashed line.

Finally, when policy risk is high enough (case 3) the unique equilibrium involves full use of foreign currency. This equilibrium exists because the marginal benefit of local currency is completely determined by policy risk when all contracts are set in foreign currency, and policy risk is larger than the price risk of foreign currency. The equilibrium is unique because even if all contracts are set in local currency, the government's use of inflation to redistribute resources does not compensate for the high level of policy risk. In the figure, this case corresponds to the intersection of the lowermost red dashed line with the blue line.

This characterization helps rationalize observed differences in the use of foreign currency as a unit of account across countries. In particular, it offers a rationalization for why countries with low levels of policy risk, such as the U.S., Germany, and Japan, rely exclusively on their local currency as a unit of account in domestic contracts. In contrast, countries with high policy risk, such as those in Latin America and Eastern Europe, tend to partially or fully rely on foreign currency as a unit of account.

2.6 Equilibrium Selection

We now consider a global games refinement to uniquely select an equilibrium of the model above. This is useful as it allows for sharper predictions of model behavior as well as a cleaner comparison with the constrained efficient allocation in the next subsection.

We consider a variant of the model described above in which we relax the assumption that all fundamentals are common knowledge. In particular, we assume that in period 1, all buyer-seller pairs receive a noisy signal of local policy risk, $\xi \equiv \mathbb{E}\left[\hat{\phi}\right]$.[12] Private agents have a common uniform prior over ξ with support $\left[\underline{\xi}, \overline{\xi}\right]$. Let i index each buyer-seller pair with $i \in [0, 1]$. Then, pair i receives a signal

$$\hat{\xi}_i = \xi + \varepsilon_i,$$

where $\varepsilon_i \sim U\left[-\eta, \eta\right]$ is uniformly distributed and is independent across all i. We assume that the support of $\hat{\phi}$ is common knowledge across all agents with

$$\underline{\hat{\phi}} \leqslant \underline{\xi} < \overline{\xi} \leqslant \overline{\hat{\phi}}.$$

The following proposition characterizes the set of competitive equilibria in this environment with information asymmetries, and shows that there is a unique equilibrium, which satisfies a simple cutoff property.

[12]We assume that $\overline{\hat{\phi}}$ is common knowledge and, hence, a signal of $\mathbb{E}\left[\hat{\phi}\right]$ constitutes a signal of policy risk $\frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}}$.

Proposition 3. *Fix some $\xi \in \left(\underline{\xi}, \overline{\xi}\right)$. Then, for η small enough, the essentially unique strategy surviving iterated deletion of strictly dominated strategies satisfies:*

$$b_l(\xi) = \begin{cases} 0 & \xi < \xi^* \\ \frac{y}{\overline{\phi}_l^{**}} & \xi > \xi^* \end{cases},$$

where $\mu_1 > \mu_{GG} \equiv \frac{\xi^*}{\overline{\hat{\phi}}} > \mu_2$ and ξ^* and $\overline{\phi}_l^{**}$ are constants defined in (20) and (19), respectively.

As in the global games literature (see, for example, Morris and Shin (2001)), the introduction of dispersed signals introduces uncertainty about the agents' actions, and therefore, attenuates the source of strategic interaction. In this case, the uncertainty causes agents to perceive a lower aggregate B_l and, thus, anticipate lower insurance benefits from the government's monetary policy. This attenuates the complementarities and yields a unique equilibrium. The unique equilibrium satisfies a cutoff property: if policy risk is large (i.e., $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} < \mu_{GG}$), there is a unique equilibrium in which all contracts are denominated in foreign currency, while if policy risk is small (i.e., $\mathbb{E}\left[\hat{\phi}\right]/\overline{\hat{\phi}} > \mu_{GG}$), there is a unique equilibrium in which all contracts are denominated in local currency. Thus, economies with higher policy risk are more likely to use foreign currency to denominate contracts. The cutoff is in the region in which there are multiple equilibria in the economy with full information. Therefore, the global games perturbation selects one of the extreme equilibria (either full use of foreign or local currency) as the unique equilibrium in this range of policy risk. Figure 2 illustrates the set of equilibria, with and without the global games refinement, as a function of policy risk.

2.7 Constrained Efficiency

We now consider the problem of a social planner who chooses allocations subject to the same constraints that private agents face and the same choice of monetary policy made by the government in the second period. The utilitarian social planner solves

$$\max_{C_s, C_b, B_l, B_f, \phi_l} \mathbb{E}\left(-x + \theta_s C_s + (1+\lambda)x + \theta_b C_b - \psi l(\phi_l)\right)$$

subject to the definitions of C_b and C_s in (5) and (6), respectively, the participation constraint of the seller (2), the feasibility constraint (1), and the best responses of the government (7), and (8). Note that we assumed that the participation constraint of the buyer is slack and we check that it is satisfied ex-post.

Analogously to the competitive equilibrium, the following proposition characterizes the solution to the planner's problem for different values of policy risk, and shows that the efficient allocation involves the full use of foreign currency when policy risk is high, and the full use of local currency when policy risk is low.

Proposition 4. *Suppose that Assumptions 1 and 2 hold. Then, there exists a threshold μ_{SP}, with $\mu_2 < \mu_{SP} < \mu_1$, such that:*

1. *If $\frac{\mathbb{E}[\hat{\phi}]}{\bar{\hat{\phi}}} \geqslant \mu_{SP}$, then the solution to the social planner's problem is $B_l^{sp} = \frac{y}{\overline{\phi}_l^*}$, where $\overline{\phi}_l^*$ was defined in Proposition 2.*

2. *If $\frac{\mathbb{E}[\hat{\phi}]}{\bar{\hat{\phi}}} \leqslant \mu_{SP}$, then the solution to the social planner's problem is $B_l^{sp} = 0$.*

The proof follows from the observation that Assumption 2 implies that the social planner's problem is strictly convex. As a result, computing the solution of this problem involves comparing the values of the objective at end-points. The relative value of these end-points depends on whether policy risk is high or low. Intuitively, a low policy risk increases the value of the full local currency equilibrium relative to the full foreign currency one, while a high policy risk does the opposite.

This result also shows that an interior equilibrium can never be efficient. In particular, for policy risk within the range (μ_{sp}, μ_1), the full local currency equilibrium dominates the interior and full foreign currency equilibria, while for policy risk within (μ_2, μ_{sp}) the full foreign currency equilibrium dominates the other two equilibria. In contrast, if policy risk is either very low or very high, the unique competitive equilibrium (full local in the former, full foreign in the latter) is constrained efficient.

To understand why $\mu_2 < \mu_{SP} < \mu_1$, suppose first that policy risk equals $\mu_1 = \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}$. At this point, the price risk of the local and foreign currency is identical if all contracts are denominated in foreign currency. However, denominating contracts in local currency carries the additional insurance benefit because prices covary with the relative consumption needs of agents. Note that denominating contracts in local currency also carries an additional cost associated with the increase in price risk, but Assumption 2 implies that the net benefit is positive. Consequently, we show that from the planner's perspective, the full local currency allocation dominates the full foreign currency one even after netting out the costs associated with deviating from the inflation target. Therefore, it must be that $\mu_{SP} < \mu_1$. To see why an equilibrium with foreign currency use can exist in the region (μ_{sp}, μ_1), note that since private agents are infinitesimal, their actions do not affect the policy choices of the government. As a result, if all agents denominate contracts in foreign currency, a particular buyer-seller pair has no incentive to denominate contracts in local currency since it is associated with higher price risk and no insurance (since $B_l = 0$).

Next, suppose that policy risk equals μ_2. At this point, if all agents are denominating contracts in local currency, the private marginal benefits of denominating contracts in either currency is identical. However, there is an additional cost associated with having the price level deviate from its target which is internalized only by the planner. As a result, the planner strictly prefers to denominate all contracts in foreign currency, which implies that $\mu_{SP} > \mu_2$. However, for policy risk in the range (μ_2, μ_{sp}) an equilibrium with local currency use can exist because the private marginal benefit of denominating contracts in local currency is larger than that of denominating contracts in foreign currency if all agents denominate contracts in local currency. In particular, private agents do not internalize these inflation costs.

The combination of the equilibrium characterization and the above result helps rationalize some of the policies described in the introduction. Consider a country with very low policy risk. The model predicts that contracts signed within the country will be denominated in local currency and it is efficient to do so. For slightly higher levels of policy risk, equilibria in which contracts are denominated in foreign currency exist but are inefficient. Optimal regulation should prescribe limits on the use of foreign currency to denominate contracts. This might help explain the prevalence of policies in a variety of Latin American countries, including Brazil and Colombia that forced de-dollarization of contracts by restricting the denomination of bank deposits or loans in foreign currency. In contrast, for high enough levels of policy risk, optimal regulation should encourage and incentivize the use of foreign currency. Examples of these types of policies are the forced dollarization adopted by Ecuador in the year 2000.

The global games approach described in the previous section allows for a cleaner comparison between the equilibrium and the efficient allocation. The following proposition shows that the only type of inefficiency that survives the global games selection is the one in which there is excessive use of foreign currency to denominate private contracts.

Proposition 5. *Suppose that Assumptions 1 and 2 hold and $\overline{\hat{\phi}} > 1$. Then $\mu_{sp} < \mu_{GG}$ so that for policy risk in the interval (μ_{SP}, μ_{GG}), all equilibrium contracts are denominated in foreign currency, while the constrained-efficient allocation calls for all contracts to be denominated in local currency. For all other values of policy risk the equilibrium is constrained-efficient.*

Recall that μ_{GG} is the cutoff threshold selected by the global games approach above which all equilibrium contracts are denominated in local currency and below which all equilibrium contracts are denominated in foreign currency. The proposition states that once we restrict attention to the equilibrium that survives the global games perturbation, there is an interval of policy risk in which the equilibrium allocation involves the full use of foreign currency, while the efficient allocation involves the full use of local currency.[13] This implies that the region of policy risk in which the equilibrium use of

[13]The additional (mild) assumption that $\overline{\hat{\phi}} > 1$ is needed to ensure that the indirect effect of policy risk on the minimum level of inflation $\overline{\phi}$ is weaker than its direct effect on the choice of B_l.

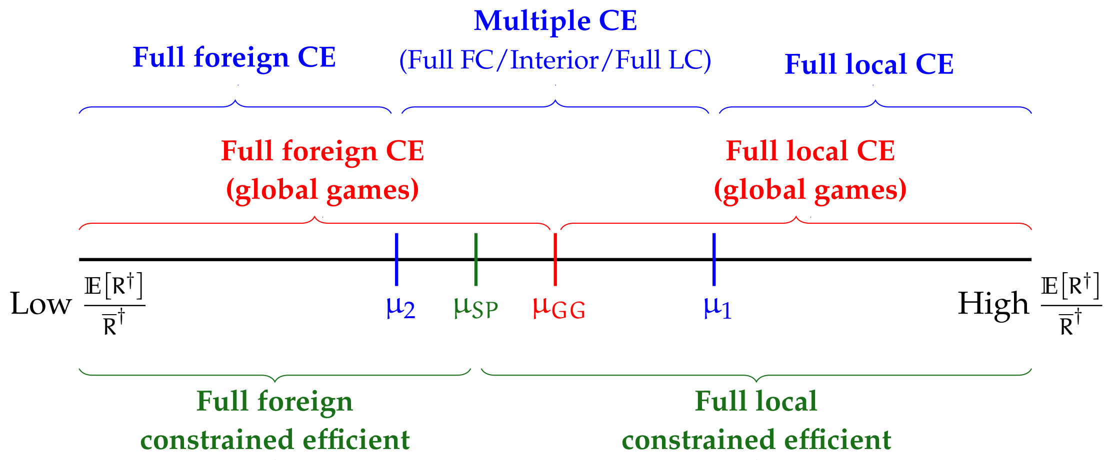

Figure 2: Equilibrium set and constrained efficient allocations for different levels of policy risk

foreign currency is low relative to the efficient allocation is not robust to an informational perturbation. Recall that this region exists because private agents do not internalize the inflation costs associated with deviating from the target. In particular, even though the private benefits of denominating contracts in local currency might be strictly larger than doing so in foreign currency, the presence of inflation costs imply that it is not socially optimal to denominate them in local currency.

The main difference in the global games equilibrium is that the perceived private benefits (from the insurance channel) of denominating contracts in local currency are lower, which reduces this region of inefficiency. The reason for this is that, since preferences are linear and signals heterogeneous, agents believe that the equilibrium level of B_l is strictly *lower* than in the full local currency competitive equilibrium and the constrained efficient allocation (when denominating contracts in local currency is efficient). This implies that in the region in which local currency use was inefficient, private agents are less likely to denominate contracts in local currency in the global games equilibrium. In particular, we show that this reduction in net private benefits of local currency is larger than the inflation costs not internalized by private agents. As a result, private agents denominate contracts less in local currency than is socially optimal. Figure 2 summarizes the set of equilibria and constrained efficient allocations for all possible values of policy risk.

3 Applications and Extensions

In this section, we extend the model to study three applications of the theory. In Section 3.1, we study a model in which the role of monetary policy is to reduce default costs. In Section 3.2, we introduce international trade into our model and study how the equilibrium use of foreign currency changes. Finally, in Section 3.3 we show how our model can generate the observed hysteresis in the use of foreign currency.

3.1 A Model with Strategic Default

In this section, we introduce a model with strategic default in which the role of policy is to minimize the costs associated with default, and show that this model maps directly into the baseline setup. We do this to argue that the environment with taste shocks described before is quite flexible and encompasses other interesting environments. Moreover, this analysis shows that the main results continue to hold even if we allow private agents to introduce some degree of state-contingency in contracts.

Consider a model similar to the baseline in which buyers and sellers are no longer subject to explicit taste shocks. However, there is still uncertainty about price risk. We allow buyers to fully default on their obligations in period 2 and suffer a cost proportional to the level of defaulted debt. In particular, a buyer defaulting on payments (b_l, b_f) obtains a utility of

$$y - \chi\left(\phi_l b_l + \phi_f b_f\right),$$

where $\chi\left(\phi_f b_f + \phi_l b_l\right)$ is the utility cost of default, which depends on the *level* of defaulted debt. This implies that a buyer who defaults on a larger stock of debt suffers a higher cost. One interpretation of this cost is that if there is exclusion after default, the exclusion time depends positively on the level of defaulted debt (see Kirpalani (2016), who shows the optimality of such punishments in a model with endowment risk, and Cruces and Trebesch (2013), who document in the sovereign default data that higher haircuts are associated with longer periods of exclusion). Assume that χ is a random variable with cdf $F_\chi(\cdot)$ and bounded support $\chi \in \left[\underline{\chi}, \overline{\chi}\right]$, with $\underline{\chi} < 1 < \overline{\chi}$. Therefore, since the value of not defaulting is $y - \left(\phi_l b_l + \phi_f b_f\right)$, the buyer defaults if $\chi < 1$.

The contracting problem is

$$\max_{x, b_l \geqslant 0, b_f \geqslant 0} (1+\lambda)\, x + \mathbb{E}\left[\left(y - \phi_l b_l - \phi_f b_f\right)\mathbb{I}_{\chi \geqslant 1} + \left(y - \chi\left(\phi_l b_l + \phi_f b_f\right)\right)\mathbb{I}_{\chi < 1}\right]$$

subject to the participation constraint of the buyer,

$$-x + \mathbb{E}\left[\left(\phi_l b_l + \phi_f b_f\right)\mathbb{I}_{\chi \geqslant 1}\right] \geqslant 0,$$

and the non-negativity constraint on buyer's consumption.[14] Next, we consider the government's problem. Clearly, if $\chi > 1$, then the government's optimal choice involves setting $\phi_l = \hat{\phi}$ since buyers repay their debt and there is no motive to deviate from the

[14]Note that given our default specification, the non-negativity constraint on the buyer's consumption is equivalent to imposing the payments feasibility constraint in the states in which the buyer chooses to repay.

inflation target. If instead $\chi < 1$, then the government's problem is

$$\max_{\phi_l} -\chi\left(\phi_l B_l + \phi_f B_f\right) - l\left(\phi_l\right),$$

which, given the functional form of the loss function, implies that the optimal choice of ϕ_l satisfies

$$\phi_l = \hat{\phi} - \frac{1}{\psi}\chi B_l.$$

In this case, the government optimally chooses to increase inflation more than its target to lower the burden of default for buyers. The higher the use of local currency in contracts B_l, the higher is the optimal inflation (lower ϕ_l) chosen by the government. Also, note that here the lowest choice of inflation is

$$\overline{\phi}_l = \overline{\hat{\phi}}.$$

Next, we show how this setup can be mapped into the model described in the previous section. Define

$$\theta_s = \begin{cases} 0 & \text{if } \chi < 1 \\ 1 & \text{if } \chi \geqslant 1 \end{cases} \tag{9}$$

and

$$\theta_b = \begin{cases} \chi & \text{if } \chi < 1 \\ 1 & \text{if } \chi \geqslant 1. \end{cases} \tag{10}$$

Given this mapping, we have that

$$\left(1+\lambda\right)\theta_s - \theta_b = \begin{cases} -\chi & \text{if } \chi < 1 \\ \lambda & \text{if } \chi \geqslant 1. \end{cases}$$

The next proposition shows that the set of equilibrium outcomes of the taste-shock model with the above processes is identical to the equilibrium outcomes of the default model. Moreover, we show that the implied taste-shock environment satisfies Assumptions 1 and 2. Therefore, we can apply all the previous results to the model with default.

Proposition 6. *The set of equilibrium outcomes of the taste-shock model implied by (9) and (10) is identical to that of the model with default. Suppose further that*

$$\lambda\left(1 - F_\chi\left(1\right)\right) - F_\chi\left(1\right)\mathbb{E}\left[\chi \mid \chi \leqslant 1\right] > 0.$$

Then, the taste-shock model implied by (9) and (10) satisfies Assumptions 1 and 2.

To understand the above result, notice that since the seller gets nothing in the default state, its payoffs are identical to the taste-shock model in which $\theta_s = 0$. The buyer's payoffs in the default state are identical (up to a constant) to the taste-shock model in which $\theta_b = \chi < 1$. We can similarly construct values for the taste shocks so that the payoffs of both buyer and seller coincide in the no-default states as well. Consequently, the proposition implies that the equilibrium characterization and the efficiency results are identical to those found in the baseline model. In particular, the optimal currency choice trades off price risk and the covariance benefits. The latter arises here from the reduction in default costs when inflation is high. Moreover, the model features complementarities in private and government's actions: the larger B_l is, the greater the incentive to use policy to reduce default costs. Finally, note that the assumption in the proposition is the analogue to Assumption 1 and states that the expected gains from trade are larger than the expected costs of default.

Another important takeaway from this model is that that observed inflation policy need not always reflect a redistributive motive. Indeed, during normal times, when there is no risk of default (when $\chi > 1$), inflation is set at its target ($\phi_l = \hat{\phi}$). However, in times of crises, when there is default ($\chi < 1$), the government chooses inflation to reduce the burden of default and redistribute from sellers to buyers.

In this model with default, the incentives of the government to increase inflation arise from a desire to lower the costs of default, since this cost is increasing in the real burden of payments. We argue that the main insights of our baseline model are not specific to this particular setup of the model with default, and are also present in a model in which the government has incentives to increase inflation to reduce the likelihood of default. In particular, in Appendix C.5, we study a model in which the costs of defaulting are stochastic and independent of promised payments. Under this specification, the decision of the buyer to default on the contract depends on the real burden of promised payments, and the government optimally chooses a higher inflation than its target to prevent default in some states of the world. We show that the complementarities between private and government's actions are also present in this model. A larger denomination of payments in local currency makes the government use inflation more actively to prevent default, which in turn makes the use of local currency more attractive.

3.2 Contracts in International Trade

One of the facts mentioned in the introduction is that there is extensive use of the U.S. dollar as a unit of account in international trade contracts. Trade involving countries with seemingly low policy risk is often invoiced in dollars. For example, Japan has low levels of policy risk and domestic dollarization, and yet has a significant fraction of trade contracts denominated in dollars. In this section, we study an extension of our baseline model with international trade that helps rationalize these facts. We incorporate international trade in our model by studying an economy in which agents from one country trade with agents from another country, and contracts can be set in any of the currencies of the involved countries or in a third, external currency. Our main result in this section shows that contracts between agents located in different countries are more likely to use foreign currency as compared with contracts signed by agents in the same country.

The extended setup of the model is as follows. There are two countries, denoted i and j, which are symmetric. In each country there is a continuum of buyers and sellers of equal size. Within each country, the taste shocks of buyers and sellers are distributed in an identical fashion to the baseline model. In addition, we assume that these shocks are independent across countries. A contract between a buyer and a seller consists of the provision of a special good in exchange for the promise of future payment. The first difference with the baseline model is that buyers in one country trade with sellers in the other country. The second difference is that we allow contracts to be set in three possible units of account: currencies from country i and j, and the foreign currency f. The price levels of currencies i and j (denoted by ϕ_i and ϕ_j, respectively) are chosen by the governments of each country, whereas the price of foreign currency is exogenous.

Let x_i be the amount of special good provided by a seller from country j to a buyer of country i[15] and b_{ic} be the promised payment of buyer from country i in currency c. The optimal private contract between a buyer of country i and a seller of country j solves

$$\max_{x_i, b_{ii} \geqslant 0, b_{ij} \geqslant 0, b_{if} \geqslant 0} (1+\lambda) x_i - \mathbb{E}\left[\theta_{ib}\left(\phi_i b_{ii} + \phi_j b_{ij} + \phi_f b_{if}\right)\right]$$

subject to the participation constraint

$$-x_i + \mathbb{E}\left[\theta_{js}\left(\phi_i b_{ii} + \phi_j b_{ij} + \phi_f b_{if}\right)\right] \geqslant 0,$$

and the feasibility constraint

$$\phi_i b_{ii} + \phi_j b_{ij} + \phi_f b_{if} \leqslant y, \tag{11}$$

for all possible price realizations, where θ_{ib} and θ_{js} denote the taste shocks of the buyer from country i and the seller from country j, respectively. The solution to this problem is characterized in Lemma 1 in the Appendix, and is similar to Proposition 1. Taking prices as given, agents write contracts using the currency that has the largest marginal benefit, allowing for combinations of two or three currencies whenever the buyer is indifferent.

Next, we revisit the government's problem. There are two utilitarian governments that

[15]Note that we have suppressed the dependency on j, since knowing that the buyer is located in country i implies that the seller is from country j.

control monetary policy and choose the price level of the local currencies in countries i and j. We assume that both countries have the same level of policy risk, $\frac{\mathbb{E}[\hat{\phi}_i]}{\bar{\hat{\phi}}_i} = \frac{\mathbb{E}[\hat{\phi}_j]}{\bar{\hat{\phi}}_j}$. This allows us to compare the equilibrium outcomes of the two-country model with those of the baseline model. Denote by B_{ic} the aggregate promised payments in currency c of buyers of country i to sellers of country j. The problem of the government in country i is given by

$$\max_{\phi_i} \theta_{ib} C_{ib} + \theta_{is} C_{is} - l(\phi_i),$$

where the aggregate consumption of buyers is given by

$$C_{ib} = y - \phi_i B_{ii} - \phi_j B_{ij} - \phi_f B_{if}, \tag{12}$$

and that of sellers is given by

$$C_{is} = y + \phi_i B_{ji} + \phi_j B_{jj} + \phi_f B_{jf}.$$

Given our functional form assumption for the inflation loss function, the solution to the problem of the government in country i is

$$\phi_i = \hat{\phi}_i + \frac{1}{\psi} \left(\theta_{is} B_{ji} - \theta_{ib} B_{ii} \right), \tag{13}$$

and the largest feasible price level the government can implement is

$$\overline{\phi}_i = \hat{\phi}_i + \frac{1}{\psi} \left(\overline{\theta}_s B_{ji} - \underline{\theta}_b B_{ii} \right).$$

The problem of the government in country j is symmetric. We restrict attention to symmetric equilibria in which all international trade contracts are set in the same currency, i.e., $B_{jc} = B_{ic} \equiv B_c$ for all c. Note that we restrict attention only to symmetric international contracts and not necessarily symmetric governments' inflation choices. In Appendix C.7, we relax this assumption and also consider asymmetric equilibria. We now define a competitive equilibrium with international trade.

Definition 2. A symmetric competitive equilibrium is an allocation for private citizens (x, b_i, b_j, b_f), aggregate promised payments (B_i, B_j, B_f), and inflation choices for governments ϕ_i and ϕ_j such that: 1. Given ϕ_i and ϕ_j , the private allocations solve the contracting problem defined in (11), 2. Given B_b and B_s, ϕ_i and ϕ_j satisfy (13), and 3. Aggregate choices are consistent with private ones, $b_k = B_k$ for $k \in \{i, j, f\}$.

In the following proposition, we argue that the use of foreign currency is more likely in the economy with international contracts than in the baseline economy with domestic contracts. Recall that μ_2 is the threshold, defined in the previous section, such that if policy risk is large (i.e., $\frac{\mathbb{E}[\hat{\phi}]}{\bar{\hat{\phi}}} < \mu_2$), then there is a unique equilibrium in which only foreign currency is used as a unit of account.

Proposition 7. *Suppose that Assumption 2 holds. Then, there exists a threshold μ_2^I such that, if $\frac{\mathbb{E}[\hat{\phi}_i]}{\bar{\hat{\phi}}_i} = \frac{\mathbb{E}[\hat{\phi}_j]}{\bar{\hat{\phi}}_j} \leqslant \mu_2^I$, there exists a unique symmetric equilibrium in which $B_i = B_j = 0$. Furthermore, $\mu_2^I > \mu_2$.*

The threshold μ_2^I depends on parameters and is defined in (23) in the Appendix. As in the baseline model, there exists a threshold μ_2^I such that if policy risk in country i and j is larger than this threshold, the unique equilibrium displays the use of foreign currency as the sole unit of account. However, the most important result of this proposition is that $\mu_2^I > \mu_2$; that is, the threshold obtained in the two-country model is larger than the one found in the baseline model. This implies that for levels of policy risk such that $\mu_2^I > \frac{\mathbb{E}[\hat{\phi}_i]}{\bar{\hat{\phi}}_i} = \frac{\mathbb{E}[\hat{\phi}_j]}{\bar{\hat{\phi}}_j} > \mu_2$, there exists a unique foreign currency equilibrium in the model with international trade, while there can exist equilibria with local currency in the model with only domestic contracts. This result suggests that we are more likely to observe international trade contracts denominated in foreign currency than domestic contracts denominated in such currency.

The intuition behind this result is that in the case of international contracts, each government finds it optimal to use inflation to respond only to taste shocks of its own citizens and not to those of the other country's citizens. That is, governments do not react to taste shocks of foreign buyers or sellers, which implies that the covariance term in equation (4) is lower for a given aggregate exposure to the local currency. This, in turn, lowers the marginal benefit of using local currencies of either country, and makes the foreign currency more attractive for private contracts. This is shown in Figure 3, which illustrates the set of equilibria in the economies with domestic and international contracts, for a level of policy risk in between μ_2 and μ_2^I, when $\mathbb{E}[\theta_s] = \mathbb{E}[\theta_b] = 1$. As before, the blue line corresponds to the aggregate promised payment denominated in currency i, B_i, for a given government policy and, thus, for a given M_i. The red line depicts the marginal benefit of currency i as a function of B_i in the economy with domestic contracts, whereas the dotted black line corresponds to the marginal benefit of currency i as a function of B_i in the economy with international contracts. The lower covariance term implies a lower slope of the black dotted line, which eliminates the equilibrium with full use of local currency.

While the proposition focuses on symmetric equilibria, in Appendix C.7 we argue that the uniqueness result generalizes to all equilibria under a slightly stronger parametric assumption.

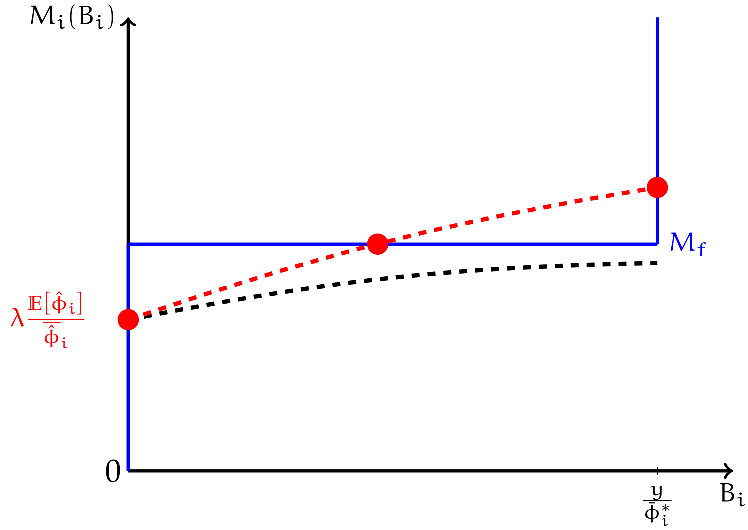

Figure 3: Comparing currency choice in international versus domestic contracts

3.3 Hysteresis

As discussed in the introduction, a distinctive feature among many Latin American countries is the hysteresis of dollarization even after inflation risk stabilized. The baseline model suggests that the set of equilibria can change dramatically for small changes in policy risk around the thresholds, which might seem to be at odds with this observation. However, the above analysis ignores the fact that citizens might be part of credit chains and thus might also have endowments of obligations in both currencies. Here, we present a simple extension in which the buyer is endowed with claims $(\hat{b}_f, \hat{b}_l)$ payable to the buyer in the second period. In Appendix C.6, we present a model of a credit chain in which these endowments arise endogenously as a consequence of trading within a credit chain. In this extended setup, the optimal contract solves

$$\max_{b_l, b_f} (1+\lambda)\, x + \mathbb{E}\left[\theta_b \left(y - \left(\phi_l \left(b_l - \hat{b}_l\right) + \phi_f \left(b_f - \hat{b}_f\right)\right)\right)\right]$$

subject to (2) and the feasibility constraint

$$\phi_l \left(b_l - \hat{b}_l\right) + \phi_f \left(b_f - \hat{b}_f\right) \leqslant y, \forall \left(\phi_l, \phi_f\right).$$

Assumption 3. *Assume that*

$$var\left(\theta_s - \theta_b\right) + \lambda\left[var\left(\theta_s\right) - cov\left(\theta_s, \theta_b\right)\right] < \kappa_2,$$

where κ_2 depends on model parameters and is defined in (24).

This assumption requires an upper bound on the variances. The term κ_2 contains a free parameter, $\underline{\phi}_f$, which can be made arbitrarily small in order to satisfy this restriction and Assumption 2. The following proposition shows that hysteresis can be rationalized with our extended model.

Proposition 8. *Under Assumption 3,* $b_f \geqslant \hat{b}_f$ *and* $b_l \geqslant \hat{b}_l$.

The proposition says that even if policy risk is small, the optimal contract will still use a combination of foreign and local currency to denominate contracts. In particular, the optimal contract will feature currency matching of stocks, but flows will be denominated in the currency with the largest marginal benefit. Given the presence of positive gains of trade, buyers will become net debtors in one currency by setting either $b_f \geqslant \hat{b}_f$ or $b_l \geqslant \hat{b}_l$ to obtain additional x, and pay for it with its endowment of goods y. What this proposition shows is that becoming a net creditor in any currency (i.e., setting $b_c < \hat{b_c}$) is not optimal since it exposes the buyer to an additional source of price risk, which in turn reduces how much the buyer can credibly promise to repay in all states of the world.

To illustrate this result, suppose that θ_s and θ_b are deterministic. Then, we know from previous results that the optimal currency choice only involves comparing price risk across currencies. Notice that with existing obligations, the price level that makes the feasibility constraint bind will now depend on whether $b_c \leqslant \hat{b}_c$ or $b_c > \hat{b}_c$. In the former case, the buyer is a net creditor in currency c and higher inflation in currency c is worse for the buyer. Therefore, the relevant price is $\underline{\phi}_c$. In the latter case, the buyer is a net debtor and the relevant price is $\overline{\phi}_c$. The difference in price risk is

$$\frac{\mathbb{E}\left[\phi_l\right]}{\tilde{\phi}_l} - \frac{\mathbb{E}\left[\phi_f\right]}{\tilde{\phi}_f},$$

where $\tilde{\phi}_c \in \left\{\underline{\phi}_c, \overline{\phi}_c\right\}$. Suppose that $b_f < \hat{b}_f$, which implies $b_l > \hat{b}_l$ to satisfy the feasibility constraint with equality. Then, the difference in price risk is

$$\frac{\mathbb{E}\left[\phi_l\right]}{\overline{\phi}_l} - \frac{\mathbb{E}\left[\phi_f\right]}{\underline{\phi}_f} < 0,$$

which implies that $b_f < \hat{b}_f$ can never be part of an equilibrium contract. A similar argument holds for the local currency. This suggests that currency mismatch is costly and tightens the feasibility constraint. As a result, the optimal contract currency matches stocks (i.e., promises to pay at least the endowment of each currency, $\hat{b}_c$) and denominates flows in the currency with the largest marginal benefit. The proof in the Appendix shows that the above argument generalizes as long as the variance of the taste shocks is not too large. If the variance is very large, then it might be optimal to cannibalize the stocks of foreign currency. This is a situation in which the insurance benefit of the local currency outweighs the price risk consideration behind the provision of the special good.

Finally, this result also illustrates why in the baseline model agents would not choose negative promised payments, even if allowed. To see this, note that the contract above is the same contract as in the baseline model without the non-negativity constraints, if we redefine promised payments as $\tilde{b}_c = b_c - \hat{b}_c$. Therefore, under Assumption 3, the optimal contract involves $\tilde{b}_c \geqslant 0$.

4 Conclusion

This paper develops a framework to study the optimal choice of currency in the denomination of private contracts in general equilibrium. There are two key channels that determine the optimal currency choice. The first is policy risk stemming from the government's ex-post desire to change the price level, which in turn affects the price risk of denominating contracts in local currency. The second is the covariance between the relative marginal utilities of the agents signing the contract and the price level. The latter channel generates a complementarity between the actions of private agents and those of the government. We show that our model can help explain the cross-country differences in the use of the U.S. dollar to denominate domestic contracts as well as rationalize policy measures aimed at limiting the use of the dollar.

One advantage of our framework is that its analytical tractability implies that it can be used to study a variety of interesting applications. For example, while we focus on static contracts in our model, it would be interesting to study the interaction between currency choice in long-term contracts and policy. In addition, one could embed this framework in a New Keynesian framework to study the effect of nominal rigidities on the currency choice of contracts. We leave these extensions for future work.

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Appendix: For Online Publication

A Useful Constants

For the proofs it will be useful to define the following constants, including the term κ_1 in Assumption 2. Define

$$\tilde{\kappa}_1 \equiv \left[(1+\lambda)\,\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]\right] \left(\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f} \left(\overline{\theta}_s - \underline{\theta}_b\right) - \left(\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]\right) \right), \tag{14}$$

$$\kappa_1 \equiv \left[(1+\lambda)\,\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]\right] \left(\left(\overline{\theta}_s - \underline{\theta}_b\right) - \left(\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]\right) \right) + \frac{1}{2} \left(\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]\right)^2, \tag{15}$$

and

$$\tilde{\kappa}_2 \equiv \mathrm{var}\left(\theta_s - \theta_b\right) + \lambda \left[\mathrm{var}\left(\theta_s\right) - \mathrm{cov}\left(\theta_s, \theta_b\right)\right]. \tag{16}$$

B Omitted Proofs

Proof of Proposition 1

First note that the participation constraint of problem (3) is binding in the optimum. To see this, suppose it is not binding. Then, increasing x by a little and leaving all remaining variables unchanged is feasible and implies a strictly higher objective function. This implies that at the optimum the participation constraint is binding. Solving for x using the participation constraint yields the first result of the proposition. Once we substitute the optimal value of x in the problem we obtain the following re-formulated problem:

$$\max_{b_l \geqslant 0, b_f \geqslant 0} \mathbb{E}\left[\left((1+\lambda)\,\theta_s - \theta_b\right)\left(\phi_l b_l + \phi_f b_f\right)\right]$$

subject to the feasibility constraint

$$\overline{\phi}_l b_l + \overline{\phi}_f b_f \leqslant y.$$

Solving for b_f using the feasibility constraint and substituting in the objective problem yields the following problem:

$$\max_{b_l \in \left[0, \frac{y}{\overline{\phi}_l}\right]} \mathbb{E}\left[\left((1+\lambda)\,\theta_s - \theta_b\right)\left(\phi_l b_l + \frac{\phi_f}{\overline{\phi}_f}\left(y - \overline{\phi}_l b_l\right)\right)\right]. \tag{17}$$

The objective is linear in b_l and the derivative with respect to b_l is $\mathbb{E}\left[\left(\theta_s\,(1+\lambda) - \theta_b\right)\left(\phi_l - \frac{\phi_f}{\overline{\phi}_f}\overline{\phi}_l\right)\right]$. Therefore, the solution is $b_l = \frac{y}{\overline{\phi}_l}$ when the derivative is positive, $b_l = 0$ when the derivative is negative, and any $b_l \in \left[0, \frac{y}{\overline{\phi}_l}\right]$ when the derivative is zero. Q.E.D.

Proof of Proposition 2

The following definitions will be useful for this proof. Define

$$\mathcal{H}(B) \equiv (1+\lambda)\, M_2(B) - M_1(B),$$

where

$$\begin{aligned}
M_2(B) &\equiv \mathbb{E}\left[\theta_s\left(\phi_l(B) - \frac{\phi_f}{\overline{\phi}_f}\overline{\phi}_l(B)\right)\right] \\
&= \mathbb{E}(\theta_s)\,\overline{\hat{\phi}}\left(\frac{\mathbb{E}(\hat{\phi})}{\overline{\hat{\phi}}} - \frac{\mathbb{E}(\phi_f)}{\overline{\phi}_f}\right) \\
&+ \frac{1}{\psi}\left(\mathrm{var}(\theta_s) - \frac{\mathbb{E}(\phi_f)}{\overline{\phi}_f}\mathbb{E}(\theta_s)\left(\overline{\theta}_s - \underline{\theta}_b\right) - \mathrm{cov}(\theta_s, \theta_b) + \mathbb{E}(\theta_s)\left(\mathbb{E}(\theta_s) - \mathbb{E}(\theta_b)\right)\right) B_l
\end{aligned}$$

and

$$M_1(B) \equiv \mathbb{E}\left[\theta_b\left(\phi_l(B) - \frac{\phi_f}{\overline{\phi}_f}\overline{\phi}_l(B)\right)\right]$$
$$= \mathbb{E}(\theta_b)\,\overline{\hat{\phi}}\left(\frac{\mathbb{E}(\hat{\phi})}{\overline{\hat{\phi}}} - \frac{\mathbb{E}(\phi_f)}{\overline{\phi}_f}\right)$$
$$- \frac{1}{\psi}\left(\mathrm{var}(\theta_b) + \frac{\mathbb{E}(\phi_f)}{\overline{\phi}_f}\mathbb{E}(\theta_b)\left(\overline{\theta}_s - \underline{\theta}_b\right) - \mathrm{cov}(\theta_s, \theta_b) - \mathbb{E}(\theta_s)\,\mathbb{E}(\theta_b) + \mathbb{E}(\theta_b)^2\right) B_l,$$

where we have used the best response of the government

$$\phi_l(B) = \hat{\phi} + \frac{1}{\psi}(\theta_b - \theta_s)\, B_l.$$

It will also be useful to compute

$$M_1'(B) = -\frac{1}{\psi}\left[\mathrm{var}(\theta_b) + \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\mathbb{E}[\theta_b]\left(\overline{\theta}_s - \underline{\theta}_b\right) - \mathrm{cov}(\theta_s, \theta_b) - \mathbb{E}[\theta_b]\left[\mathbb{E}(\theta_s) - \mathbb{E}[\theta_b]\right]\right]$$

and

$$M_2'(B) = \frac{1}{\psi}\left(\mathrm{var}(\theta_s) - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\mathbb{E}[\theta_s]\left(\overline{\theta}_s - \underline{\theta}_b\right) - \mathrm{cov}(\theta_s, \theta_b) + \mathbb{E}[\theta_s]\left[\mathbb{E}(\theta_s) - \mathbb{E}[\theta_b]\right]\right).$$

The function $\mathcal{H}(B)$ is useful for characterizing the set of equilibria in this model. This function is obtained by taking the first order condition of (17) with respect to b_l and substituting in the government's best response. There are three types of equilibria that can exist. First, an equilibrium with $B_l = 0$ exists if and only if $\mathcal{H}(0) \leqslant 0$. Next, an equilibrium in which $B_f = 0$ can exist if and only if $\mathcal{H}\left(\frac{y}{\overline{\phi}_l^*}\right) \geqslant 0$, where $\frac{y}{\overline{\phi}_l^*}$ corresponds to the maximal feasible value of B_l, and $\overline{\phi}_l^*$ solves

$$\overline{\phi}_l^* = \overline{\hat{\phi}} + \frac{1}{\psi}\left(\overline{\theta} - \underline{\theta}\right)\frac{y}{\overline{\phi}_l^*}$$

or

$$\overline{\phi}_l^* = \frac{\overline{\hat{\phi}} + \sqrt{\left(\overline{\hat{\phi}}\right)^2 + 4\frac{y}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right)}}{2}.$$

Finally, an interior equilibrium exists if and only if there exists some $B_l \in \left(0, \frac{y}{\overline{\phi}_l^*}\right)$ such that $\mathcal{H}(B_l) = 0$.

Define $\mu_1 \equiv \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}$. We will show that if $\frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}} - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f} > 0$, then there is a unique equilibrium in which $B_f = 0$ and $B_l = \frac{y}{\overline{\phi}_l^*}$. To see that an equilibrium with $B_l = 0$ cannot exist, notice that

$$\mathcal{H}(0) = [(1+\lambda)\,\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]]\,\bar{\hat{\phi}}\left(\frac{\mathbb{E}[\hat{\phi}]}{\bar{\hat{\phi}}} - \frac{\mathbb{E}[\phi_f]}{\bar{\phi}_f}\right) > 0.$$

To show that a unique equilibrium with $B_l = \frac{y}{\bar{\phi}_l^*}$ exists, it is sufficient to show that $\mathcal{H}'(B) \geqslant 0$ for all $B \in \left[0, \frac{y}{\bar{\phi}_l^*}\right]$. We have

$$\mathcal{H}'(B) = (1+\lambda)\,M_2'(B) - M_1'(B) = \frac{1}{\psi}[\tilde{\kappa}_2 - \tilde{\kappa}_1] > 0,$$

where $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$ were defined in (14) and (16) respectively, and it is positive as a consequence of Assumption 2.

Next, define

$$\mu_2 \equiv \frac{\mathbb{E}[\phi_f]}{\bar{\phi}_f} - \frac{1}{\psi}\frac{y}{\bar{\hat{\phi}}\bar{\phi}_l^*}\frac{(\tilde{\kappa}_2 - \tilde{\kappa}_1)}{[(1+\lambda)\,\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]]}. \tag{18}$$

Notice that Assumption 2 implies that $\mu_2 < \mu_1$. We show that for $\frac{\mathbb{E}[\hat{\phi}]}{\bar{\hat{\phi}}} \in (\mu_2, \mu_1]$, there exist three equilibria. First, we show an equilibrium exists in which $B_l = 0$. We know from above that for this equilibrium to exist it must be that $\mathcal{H}(0) \leqslant 0$. Using the expressions we derived earlier,

$$\mathcal{H}(0) = [(1+\lambda)\,\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]]\,\bar{\hat{\phi}}\left[\frac{\mathbb{E}[\hat{\phi}]}{\bar{\hat{\phi}}} - \frac{\mathbb{E}[\phi_f]}{\bar{\phi}_f}\right] \leqslant 0,$$

which follows from the case we are considering and Assumption 1. Next, we want to show that there exists an interior equilibrium, i.e. there exists a B such that $\mathcal{H}(B) = 0$. Since we established earlier that $\mathcal{H}'(B) > 0$, there must exist a unique B_l^* such that $\mathcal{H}(B_l^*) = 0$. The value B_l^* is

$$B_l^* = \frac{\psi\,[(1+\lambda)\,\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]]\,\bar{\hat{\phi}}\left[\frac{\mathbb{E}[\phi_f]}{\bar{\phi}_f} - \frac{\mathbb{E}[\hat{\phi}]}{\bar{\hat{\phi}}}\right]}{[\tilde{\kappa}_2 - \tilde{\kappa}_1]}.$$

For this to be strictly interior, a necessary and sufficient condition is

$$B_l^* < \frac{y}{\bar{\phi}_l^*},$$

or

$$\frac{\mathbb{E}[\hat{\phi}]}{\bar{\hat{\phi}}} > \mu_2.$$

Finally, since $\mathcal{H}'(B) > 0$, it follows that if there is an interior equilibrium, there also must exist an equilibrium with full use of local currency, since it must be that $\mathcal{H}\left(\frac{y}{\phi_l^*}\right) > 0$.

Finally, assume that $\frac{\mathbb{E}[\hat{\phi}]}{\hat{\phi}} \leqslant \mu_2$. Given the above analyses, it is straightforward to see that in this case there is a unique equilibrium in which $B_l = 0$. In particular, in this interval, it must be that $\mathcal{H}(B) \leqslant 0$ for all B. Q.E.D.

Proof of Proposition 3

The proof proceeds as follows. We first conjecture that the best response of private agents takes a simple cutoff structure. In particular, we show that if all other agents are playing this cutoff strategy an individual buyer-seller pair also finds it optimal to do so. Finally, we show that such a cutoff strategy is the unique strategy surviving iterated deletion of strictly dominated strategies and characterize it.

We conjecture that the best response takes the following cutoff structure

$$b_l = \begin{cases} 0 & \xi < \xi^* \\ \frac{y}{\phi_l} & \xi > \xi^* \end{cases}.$$

Suppose that a buyer-seller pair, receiving signal $\hat{\xi}$, believes that all other private agents are following this cutoff strategy. We want to show that the best response of this buyer-seller pair is also a cutoff strategy.Given the signal realization $\hat{\xi}$, this pair believes that the aggregate level of B_l is given by

$$B_l\left(\hat{\xi}\right) = \left[1 - H\left(\xi^* \mid \hat{\xi}\right)\right] \frac{y}{\phi_l}$$

where

$$H\left(\xi^* \mid \hat{\xi}\right) \equiv \Pr\left(\xi_j \leqslant \xi^* \mid \hat{\xi}\right)$$

which is the fraction of agents receiving a signal lower than ξ^* conditional on receiving a signal $\hat{\xi}$. Note that

$$\begin{aligned} H\left(\xi^* \mid \hat{\xi}\right) &= \Pr\left(\varepsilon_j \leqslant \xi^* - \hat{\xi} + \varepsilon_i \mid \hat{\xi}\right) \\ &= \int_{\varepsilon_i} \Pr\left\{\varepsilon_j \leqslant \xi^* - \hat{\xi} + \varepsilon_i \mid \hat{\xi}, \varepsilon_i\right\} \Pr\left(\varepsilon_i \mid \hat{\xi}\right) d\varepsilon_i \\ &= \int_{\varepsilon_i} \Pr\left\{\varepsilon_j \leqslant \xi^* - \hat{\xi} + \varepsilon_i \mid \hat{\xi}, \varepsilon_i\right\} \frac{\Pr\left(\xi = \hat{\xi} - \varepsilon_i\right) \Pr\left(\varepsilon_i\right)}{\int \Pr\left(\xi = \hat{\xi} - \hat{\varepsilon}\right) \Pr\left(\hat{\varepsilon}\right) d\hat{\varepsilon}} d\varepsilon_i \\ &= \int_{\varepsilon_i} \frac{\xi^* - \hat{\xi} + \varepsilon_i + \eta}{2\eta} \frac{1}{2\eta} d\varepsilon_i, \end{aligned}$$

so that $H\left(\xi^{*} \mid \hat{\xi}\right)$ is strictly decreasing in $\hat{\xi}$. For future use, it will be useful to note that

$$H\left(\xi^{*} \mid \xi^{*}\right)=\int_{\varepsilon_i}\left[\frac{\varepsilon_i}{2\eta}+\frac{1}{2}\right]\frac{1}{2\eta}d\varepsilon_i=\frac{1}{2}.$$

Given $B_l\left(\hat{\xi}\right)$ and the government's best response, we compute the maximal local currency price as follows

$$\overline{\phi}_l\left(\hat{\xi}\right)=\frac{\overline{\hat{\phi}}+\sqrt{\overline{\hat{\phi}}^{2}+4\frac{1}{\psi}\left(\overline{\theta}_s-\underline{\theta}_b\right)H\left(\xi^{*}\mid\hat{\xi}\right)y}}{2}.$$

Therefore, given signal realization $\hat{\xi}$, the first order condition of the contracting problem with respect to b_l is

$$\left[(1+\lambda)\,\mathbb{E}\left(\theta_s\right)-\mathbb{E}\left(\theta_b\right)\right]\overline{\hat{\phi}}\left(\frac{E\left[\xi\mid\hat{\xi}\right]}{\overline{\hat{\phi}}}-\frac{\mathbb{E}\left(\phi_f\right)}{\overline{\phi}_f}\right)+\frac{1}{\psi}\left[\tilde{\kappa}_2-\tilde{\kappa}_1\right]B_l\left(\hat{\xi}\right).$$

Thus the optimal choice of b_l satisfies $b_l=0$ if the above expression is negative, and satisfies $b_l=\frac{y}{\phi_l}$ if the above expression is positive.

To characterize the optimal currency choice we need to compute $E\left[\xi\mid\hat{\xi}\right]$. Fix any $\xi\in\left(\underline{\xi},\overline{\xi}\right)$, then for η small enough, we have that

$$\bar{\xi}-2\eta>\xi>\underline{\xi}+2\eta.$$

Therefore,

$$\hat{\xi}-\eta=\xi+\varepsilon_i-\eta\geqslant\xi-2\eta>\underline{\xi}$$

and

$$\hat{\xi}+\eta=\xi+\varepsilon_i+\eta\leqslant\xi+2\eta<\overline{\xi}.$$

Then,

$$E\left[\xi\mid\hat{\xi}\right]=\int_{\hat{\xi}-\eta}^{\hat{\xi}+\eta}\xi\Pr\left(\varepsilon=\hat{\xi}-\xi\right)d\xi=\frac{1}{2\eta}\frac{\left(\hat{\xi}+\eta\right)^{2}-\left(\hat{\xi}-\eta\right)^{2}}{2}=\hat{\xi}.$$

Given these computations, define $x\left(\hat{\xi}\right)$ to be the value of x that solves

$$\left[(1+\lambda)\,\mathbb{E}\left(\theta_s\right)-\mathbb{E}\left(\theta_b\right)\right]\overline{\hat{\phi}}\left(\frac{x}{\overline{\hat{\phi}}}-\frac{\mathbb{E}\left(\phi_f\right)}{\overline{\phi}_f}\right)+\frac{1}{\psi}\left[\tilde{\kappa}_2-\tilde{\kappa}_1\right]B_l\left(\hat{\xi}\right)=0,$$

where $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$ are defined in (14) and (16), respectively. Notice that if there exists a fixed point $x\left(\xi^{*}\right)=\xi^{*}$ of the above equation, and the above equation is strictly increasing in $\hat{\xi}$, then the private best response also follows a cutoff strategy with threshold ξ^{*}. In particular, we will show that the cutoff strategy characterized by ξ^{*} is the unique strategy surviving iterated deletion of strictly dominated strategies. To do this, we first show that $x\left(\hat{\xi}\right)$ is strictly decreasing. To show this is, due to Assumptions 1 and 2, it suffices to show that $B_l'(\xi) > 0$. We have

$$\begin{aligned}
B_l'\left(\hat{\xi}\right) &= -\frac{H_{\hat{\xi}}\left(\xi^* \mid \hat{\xi}\right) y}{\overline{\phi}_l\left(\hat{\xi}_f\right)} \\
&\quad - \frac{H\left(\xi^* \mid \hat{\xi}\right) y}{\left[\overline{\phi}_l\left(\hat{\xi}_f\right)\right]^2}\frac{1}{2}\left(\left(\overline{\hat{\phi}}^2 + 4\frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right) H\left(\xi^* \mid \hat{\xi}\right) y\right)^{-\frac{1}{2}} \frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right)\left(-H_{\hat{\xi}}\left(\xi^* \mid \hat{\xi}\right)\right)\right) \\
&= -\frac{H_{\hat{\xi}}\left(\xi^* \mid \hat{\xi}\right) y}{\overline{\phi}_l\left(\hat{\xi}_f\right)}\left[1 - \frac{H\left(\xi^* \mid \hat{\xi}\right) y}{\overline{\phi}_l\left(\hat{\xi}_f\right)}\frac{1}{2}\left(\left(\overline{\hat{\phi}}^2 + 4\frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right) H\left(\xi^* \mid \hat{\xi}\right) y\right)^{-\frac{1}{2}} \frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right)\right)\right].
\end{aligned}$$

Let us consider the term in square brackets. Since we have already established that $H_{\hat{\xi}}\left(\xi^* \mid \hat{\xi}\right) < 0$, we want to show that

$$1 - H\left(\xi^* \mid \hat{\xi}\right)\frac{y}{\overline{\phi}_l\left(\hat{\xi}_f\right)}\frac{1}{2}\left(\left(\overline{\hat{\phi}}^2 + 4\frac{1}{2\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right) H\left(\xi^* \mid \hat{\xi}\right) y\right)^{-\frac{1}{2}} \frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right)\right) > 0.$$

A sufficient condition for this is

$$\begin{aligned}
&\sqrt{\overline{\hat{\phi}}^2 + 4\frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right) H\left(\xi^* \mid \hat{\xi}\right) y} \\
>&H\left(\xi^* \mid \hat{\xi}\right) y\left(\left(\overline{\hat{\phi}}^2 + 4\frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right) H\left(\xi^* \mid \hat{\xi}\right) y\right)^{-\frac{1}{2}} \frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right)\right)
\end{aligned}$$

or

$$\overline{\hat{\phi}}^2 + \frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right) H\left(\xi^* \mid \hat{\xi}\right) y > 0,$$

which is true. Therefore, $B_l'\left(\hat{\xi}\right) > 0$ and so $x\left(\hat{\xi}\right)$ is strictly decreasing.

We will next show that if a strategy b_l survives n rounds of iterated deletion of strictly dominated strategies, then

$$b_l(\xi) = \begin{cases} 0 & \xi < x^{n-1}(\mu_2) \\ \frac{y}{\overline{\phi}_l} & \xi > x^{n-1}(\mu_1) \end{cases},$$

where μ_1 and μ_2 were defined in Proposition 2. It is easy to see that this claim is true for $n = 1$ since

$$b_l(\xi) = \begin{cases} 0 & \xi < x^0(\mu_2) = \mu_2 \\ \frac{y}{\overline{\phi}_l} & \xi > x^0(\mu_1) = \mu_1 \end{cases},$$

which follows from the definitions of μ_1 and μ_2. Now suppose the claim is true for some $n > 1$. Then, if a particular buyer-seller pair knew that all other pairs choose $b_l = 0$ if $\xi < x^{n-1}(\mu_2)$ and $b_l = \frac{y}{\overline{\phi}_l}$ if $\xi > x^{n-1}(\mu_1)$, its best response would be to choose $b_l = 0$ if the signal was below $x\left(x^{n-1}(\mu_2)\right)$. Since $x(\cdot)$ is strictly decreasing, it has a unique fixed point satisfying

$$[(1+\lambda)\mathbb{E}(\theta_s) - \mathbb{E}(\theta_b)]\,\overline{\hat{\phi}}\left(\frac{\xi^*}{\overline{\hat{\phi}}} - \frac{\mathbb{E}(\phi_f)}{\overline{\phi}_f}\right) + \frac{1}{\psi}[\tilde{\kappa}_2 - \tilde{\kappa}_1]\, B_l(\xi^*) = 0$$

and $x^n(\mu_2) \to \xi^*$ as $n \to \infty$. An identical argument holds for $x^n(\mu_1)$.

We can now solve for the fixed point

$$[(1+\lambda)\mathbb{E}(\theta_s) - \mathbb{E}(\theta_b)]\,\overline{\hat{\phi}}\left(\frac{\xi^*}{\overline{\hat{\phi}}} - \frac{\mathbb{E}(\phi_f)}{\overline{\phi}_f}\right) + \frac{1}{\psi}[\tilde{\kappa}_2 - \tilde{\kappa}_1]\frac{1}{2}\frac{y}{\overline{\phi}_l^{**}} = 0,$$

where

$$\overline{\phi}_l^{**} \equiv \frac{\overline{\hat{\phi}} + \sqrt{\overline{\hat{\phi}}^2 + 4\frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right)\frac{1}{2}y}}{2}. \tag{19}$$

Therefore,

$$\xi^* = \overline{\hat{\phi}}\frac{\mathbb{E}(\phi_f)}{\overline{\phi}_f} - \frac{1}{2}\frac{1}{\psi}\frac{(\tilde{\kappa}_2 - \tilde{\kappa}_1)}{[(1+\lambda)\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]]}\frac{y}{\overline{\phi}_l^{**}}. \tag{20}$$

Finally, we have

$$\frac{\xi^*}{\overline{\hat{\phi}}} - \mu_2 = \frac{(\tilde{\kappa}_2 - \tilde{\kappa}_1)}{[(1+\lambda)\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]]}\frac{1}{\psi}\frac{y}{\overline{\hat{\phi}}}\left[\frac{1}{\overline{\phi}_l^{*}} - \frac{1}{2\overline{\phi}_l^{**}}\right] > 0$$

since

$$2\overline{\phi}_l^{**} - \overline{\phi}_l^{*} = \frac{1}{2}\left[\overline{\hat{\phi}} + 2\sqrt{\left(\overline{\hat{\phi}}\right)^2 + 2\frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right)y} - \sqrt{\left(\overline{\hat{\phi}}\right)^2 + 4\frac{y}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right)}\right] > 0,$$

which completes the proof. Q.E.D.

Proof of Proposition 4

Given that both the participation constraint and the feasibility constraint will bind, we can write the planner's problem a

$$\max_{B_l}\left(\mathbb{E}\left([(1+\lambda)\theta_s-\theta_b]\left(\left(\phi_l-\frac{\phi_f}{\overline{\phi}_f}\overline{\phi}_l\right)B_l+\frac{\phi_f}{\overline{\phi}_f}y\right)\right)+2y\right)-l(\phi_l),$$

subject to (7), and (8). Given our previous definitions, it will be useful to define the planning problem as follows:

$$SP(B)\equiv\max_{B}[(1+\lambda)M_2(B)B-M_1(B)B-\mathbb{E}l(\phi_l(B))]+\tilde{y},$$

where $\tilde{y}\equiv\left(\mathbb{E}[\theta_s]+\mathbb{E}[\theta_b]+[(1+\lambda)\mathbb{E}[\theta_s]-\mathbb{E}[\theta_b]]\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right)y$, subject to

$$\phi_l(B)=\hat{\phi}+\frac{1}{\psi}(\theta_s-\theta_b)B.$$

The first order condition is

$$\begin{aligned}SP'(B)&=\left[(1+\lambda)\left[M_2(B)+M_2'(B)B\right]-M_1(B)-M_1'(B)B-\mathbb{E}l'(\phi_l(B))\phi_l'(B)\right]\\&=[(1+\lambda)M_2(B)-M_1(B)+\Delta(B)B],\end{aligned}$$

where we have used the definition of $l(\phi)$ and

$$\Delta(B)\equiv(1+\lambda)M_2'(B)-M_1'(B)-\mathbb{E}(\theta_s-\theta_b)\phi_l'(B).$$

Next, let us check the second order condition of the planner's problem. First, we have

$$\Delta'(B)=(1+\lambda)M_2''(B)-M_1''(B)-\mathbb{E}(\theta_s-\theta_b)\phi_l''(B)=0,$$

which implies that

$$\begin{aligned}SP''(B)=&(1+\lambda)M_2'(B)-M_1'(B)+\Delta(B)\\&=2(1+\lambda)M_2'(B)-2M_1'(B)-\mathbb{E}(\theta_s-\theta_b)\phi_l'(B)\\&=\frac{2}{\psi}\left(\frac{1}{2}\mathrm{var}(\theta_s-\theta_b)+\lambda[\mathrm{var}(\theta_s)-\mathrm{cov}(\theta_s,\theta_b)]-\tilde{\kappa}_1-\frac{1}{2}(\mathbb{E}[\theta_s]-\mathbb{E}[\theta_b])^2\right)\\&>0,\end{aligned}$$

where $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$ were defined in (14) and (16), respectively, and where the last inequality follows from Assumption 2. Therefore, the planner's problem is strictly convex, which implies that computing the solution involves comparing the value of the objective at end points $B_l = 0$ and $B_l = \frac{y}{\overline{\phi}_l^*}$. Note that the maximal feasible level of B_l depends only on parameters and thus is identical across both the competitive equilibrium and the planner's problem.

Define

$$\mu_{SP} \equiv \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f} - \frac{1}{\psi}\frac{y}{\overline{\hat{\phi}}\overline{\phi}_l^*}\frac{(\tilde{\kappa}_2 - \tilde{\kappa}_1) - \frac{1}{2}\mathbb{E}\left((\theta_s - \theta_b)^2\right)}{[(1+\lambda)\,\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]]}.$$

We have

$$SP(0) = \tilde{y}$$

and

$$SP\left(\frac{y}{\overline{\phi}_l^*}\right) = \tilde{y} + (1+\lambda)\,M_2\left(\frac{y}{\overline{\phi}_l^*}\right)\frac{y}{\overline{\phi}_l^*} - M_1\left(\frac{y}{\overline{\phi}_l^*}\right)\frac{y}{\overline{\phi}_l^*} - \frac{\psi}{2}\mathbb{E}\left(\frac{1}{\psi}(\theta_s - \theta_b)\frac{y}{\overline{\phi}_l^*}\right)^2.$$

Thus, to compare the above two terms, we need to compute the sign of

$$\begin{aligned}
&(1+\lambda)\,M_2\left(\frac{y}{\overline{\phi}_l^*}\right)\frac{y}{\overline{\phi}_l^*} - M_1\left(\frac{y}{\overline{\phi}_l^*}\right)\frac{y}{\overline{\phi}_l^*} - \frac{\psi}{2}\mathbb{E}\left(\frac{1}{\psi}(\theta_s - \theta_b)\frac{y}{\overline{\phi}_l^*}\right)^2 \\
=&\left(\frac{\mathbb{E}\left[\hat{\phi}\right]}{\overline{\hat{\phi}}} - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right) + \frac{1}{\psi}\frac{y}{\overline{\hat{\phi}}\overline{\phi}_l^*}\frac{(\tilde{\kappa}_2 - \tilde{\kappa}_1) - \frac{1}{2}\mathbb{E}\left((\theta_s - \theta_b)^2\right)}{[(1+\lambda)\,\mathbb{E}[\theta_s] - \mathbb{E}[\theta_b]]},
\end{aligned}$$

which immediately implies the result given threshold μ_{SP}. Let us now check that the participation constraint of the buyer is satisfied. The buyer's payoff is

$$\begin{aligned}
&(1+\lambda)\,x + y - \mathbb{E}\theta_s[\phi_l B_l + \phi_f B_f] \\
=&y + \mathbb{E}[(1+\lambda)\,\theta_b - \theta_s][\phi_l B_l + \phi_f B_f] \\
\geqslant&y + \mathbb{E}[(1+\lambda)\,\theta_b - \theta_s]\,\mathbb{E}[\phi_f]\frac{y}{\overline{\phi}_f} \\
>&0,
\end{aligned}$$

where the inequality in the third line follows from the fact that it is always feasible to denominate contracts exclusively in foreign currency. This implies that the participation constraint is satisfied. Finally, it is easy to see that $\mu_{SP} < \mu_1$ and a simple computation implies that

$$\mu_{SP}-\mu_2=\frac{1}{2\psi}\frac{\mathbb{E}\left((\theta_s-\theta_b)^2\right)}{((1+\lambda)\,\mathbb{E}\,(\theta_s)-\mathbb{E}\,(\theta_b))}\frac{y}{\overline{\phi}_l^*}>0,$$

which proves that $\mu_2<\mu_{SP}<\mu_1$. Q.E.D.

Proof of Proposition 5

Given the definitions of ξ^* and μ_{SP}, we have

$$\begin{aligned}
\frac{\xi^*}{\overline{\hat{\phi}}}-\mu_{SP}=&\frac{(\tilde{\kappa}_2-\tilde{\kappa}_1)}{[(1+\lambda)\,\mathbb{E}\,[\theta_s]-\mathbb{E}\,[\theta_b]]}\frac{1}{\psi}\frac{y}{\overline{\hat{\phi}}}\left[\frac{1}{\overline{\phi}_l^*}-\frac{1}{2}\frac{1}{\overline{\phi}_l^{**}}\right]-\frac{1}{\psi}\frac{y}{\overline{\hat{\phi}}\overline{\phi}_l^*}\left(\frac{\mathbb{E}\left((\theta_s-\theta_b)^2\right)}{2\,((1+\lambda)\,\mathbb{E}\,(\theta_s)-\mathbb{E}\,(\theta_b))}\right)\\
>&\frac{(\tilde{\kappa}_2-\tilde{\kappa}_1)}{[(1+\lambda)\,\mathbb{E}\,[\theta_s]-\mathbb{E}\,[\theta_b]]}\frac{1}{\psi}\frac{y}{\overline{\hat{\phi}}}-\frac{1}{\psi}\frac{y}{\overline{\hat{\phi}}\overline{\phi}_l^*}\left(\frac{\mathbb{E}\left((\theta_s-\theta_b)^2\right)}{2\,((1+\lambda)\,\mathbb{E}\,(\theta_s)-\mathbb{E}\,(\theta_b))}\right)\\
\geqslant&\frac{(\tilde{\kappa}_2-\tilde{\kappa}_1)-\frac{1}{2}\mathbb{E}\left((\theta_s-\theta_b)^2\right)}{[(1+\lambda)\,\mathbb{E}\,[\theta_s]-\mathbb{E}\,[\theta_b]]}\frac{1}{\psi}\frac{y}{\overline{\hat{\phi}}}\\
>&0,
\end{aligned}$$

where the first inequality follows from Assumption 2 and $2\overline{\phi}_l^{**}-\overline{\phi}_l^*>0$, the second inequality follows from the fact that $\overline{\phi}_l^*>1$, which is true since

$$1-\overline{\hat{\phi}}<\frac{y}{\psi}\left(\overline{\theta}_s-\underline{\theta}_b\right),$$

and finally the third inequality follows from Assumption 2. Q.E.D.

Proof of Proposition 6

Before proving the proposition it will be useful to compute the following:

$$\mathbb{E}\,[\theta_s]=\int_{\underline{\chi}}^{1}0dF_\chi\,(\chi)+\int_{1}^{\bar{\chi}}1dF_\chi\,(\chi)=(1-F\,(1))$$

$$\text{var}\,(\theta_s)=(1-F\,(1))\,F\,(1)$$

$$\mathbb{E}\,[\theta_b]=\int_{\underline{\chi}}^{1}\chi dF\,(\chi)+\int_{1}^{\bar{\chi}}1dF\,(\chi)=F\,(1)\,\mathbb{E}\,[\chi\mid\chi\leqslant 1]+(1-F\,(1))$$

$$\mathbb{E}\left[\theta_b\theta_s\right] = \int_1^{\bar{\chi}} 1 dF(\chi) = (1 - F(1))$$

$$\mathbb{E}\left[\theta_s - \theta_b\right] = -\int_{\underline{\chi}}^1 \chi dF(\chi) = -F(1)\,\mathbb{E}\left[\chi \mid \chi \leqslant 1\right]$$

$$\mathbb{E}\left[(\theta_s - \theta_b)^2\right] = \int_{\underline{\chi}}^1 \chi^2 dF(\chi) = F(1)\,\mathbb{E}\left[\chi^2 \mid \chi \leqslant 1\right]$$

$$\operatorname{var}(\theta_s - \theta_b) = F(1)\,\mathbb{E}\left[\chi^2 \mid \chi \leqslant 1\right] - \left(F(1)\,\mathbb{E}\left[\chi \mid \chi \leqslant 1\right]\right)^2$$

$$\operatorname{cov}(\theta_s, \theta_b) = \mathbb{E}\left[\theta_b\theta_s\right] - \mathbb{E}\left[\theta_b\right]\mathbb{E}\left[\theta_s\right] = (1 - F(1))\,F(1)\,(1 - \mathbb{E}\left[\chi \mid \chi \leqslant 1\right])$$

Proof of Proposition 6.

Consider the contracting problem for the model with default. First notice that the non-negativity constraint on consumption is equivalent to imposing the payments feasibility constraint in those states of the world in which the buyer repays. Substituting the participation and payments feasibility constraint (in the no-default states) into the objective yields

$$\max_{b_l \in \left[0, \frac{y}{\overline{\phi}_l}\right]} \lambda \mathbb{E}\left(\phi_l b_l + \frac{\phi_f}{\overline{\phi}_f}\left(y - \overline{\phi}_l b_l\right)\right)\mathbb{I}_{\chi \geqslant 1} + \mathbb{E}\left[y - \chi\left(\phi_l b_l + \frac{\phi_f}{\overline{\phi}_f}\left(y - \overline{\phi}_l b_l\right)\right)\mathbb{I}_{\chi<1}\right].$$

Next, consider the problem with taste shocks in (17). Using (9) and (10), the problem becomes

$$\max_{b_l \in \left[0, \frac{y}{\overline{\phi}_l}\right]} \lambda \mathbb{E}\left(\phi_l b_l + \frac{\phi_f}{\overline{\phi}_f}\left(y - \overline{\phi}_l b_l\right)\right)\mathbb{I}_{\chi \geqslant 1} + \mathbb{E}\left[\chi y - \chi\left(\phi_l b_l + \frac{\phi_f}{\overline{\phi}_f}\left(y - \overline{\phi}_l b_l\right)\right)\mathbb{I}_{\chi<1}\right],$$

so that the two problems only differ by a constant. Thus, they have the same solution. It is also easy to see that the problems for the government coincide. Thus, the set of equilibrium outcomes is identical.

Next, we show that the implied taste shock model satisfies Assumptions 1 and 2. Using the calculations prior to this proof we have

$$\begin{aligned}(1+\lambda)\,\mathbb{E}\left[\theta_s\right] - \mathbb{E}\left[\theta_b\right] &= (1+\lambda)\left(1 - F_\chi(1)\right) - F_\chi(1)\,\mathbb{E}\left[\chi \mid \chi \leqslant 1\right] - \left(1 - F_\chi(1)\right)\\ &= \lambda\left(1 - F_\chi(1)\right) - F_\chi(1)\,\mathbb{E}\left[\chi \mid \chi \leqslant 1\right],\end{aligned}$$

which is strictly positive as a consequence of the assumption in the proposition. Next, let us verify that Assumption 2 is satisfied. Notice that the term κ can be written as

$$\begin{aligned}\kappa &= \left[(1+\lambda)\,\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right]\left(\overline{\phi}_l'\left(B_l\right)-\left(\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)\right)-\frac{1}{2}\left(\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)^2\\ &= -\left[(1+\lambda)\,\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right]\left(\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)-\frac{1}{2}\left(\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)^2\end{aligned}$$

since $\overline{\phi}_l'\left(B_l\right)=0$. We have

$$\begin{aligned}&\frac{1}{2}\mathrm{var}\left(\theta_s-\theta_b\right)+\lambda\left[\mathrm{var}\left(\theta_s\right)-\mathrm{cov}\left(\theta_s,\theta_b\right)\right]\\ =&\frac{1}{2}\left[F(1)\,\mathbb{E}\left[\chi^2\mid\chi\leqslant 1\right]-\left(F(1)\,\mathbb{E}\left[\chi\mid\chi\leqslant 1\right]\right)^2\right]+\lambda\left(1-F(1)\right)F(1)\left[\mathbb{E}\left[\chi\mid\chi\leqslant 1\right]\right]\end{aligned}$$

and

$$\begin{aligned}&-\left[(1+\lambda)\,\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right]\left(\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)+\frac{1}{2}\left(\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)^2\\ =&\left[\lambda\left(1-F(1)\right)-F(1)\,\mathbb{E}\left[\chi\mid\chi\leqslant 1\right]\right]F(1)\,\mathbb{E}\left[\chi\mid\chi\leqslant 1\right]+\frac{1}{2}\left(F(1)\,\mathbb{E}\left[\chi\mid\chi\leqslant 1\right]\right)^2.\end{aligned}$$

Therefore,

$$\begin{aligned}&\frac{1}{2}\mathrm{var}\left(\theta_s-\theta_b\right)+\lambda\left[\mathrm{var}\left(\theta_s\right)-\mathrm{cov}\left(\theta_s,\theta_b\right)\right]\\ &+\left[(1+\lambda)\,\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right]\left(\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)-\frac{1}{2}\left(\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)^2\\ =&\frac{1}{2}F(1)\,\mathbb{E}\left[\chi^2\mid\chi\leqslant 1\right]\\ >&0,\end{aligned}$$

which proves the result.

Proof of Proposition 7

The proof of this proposition requires the following lemma.

Lemma 1. *In the optimal bilateral contract, the amount of special good is given by*

$$x_i=\mathbb{E}\left[\theta_{js}\left(\phi_i b_{ii}+\phi_j b_{ij}+\phi_f b_{if}\right)\right]$$

Additionally, for any currency c*, the optimal payments are given by* $b_{ic}=\gamma_c\frac{y}{\phi_c}$ *with* $\gamma_c\in[0,1]$*,*

$\sum_{k=i,j,f} \gamma_k = 1$, *and* $\gamma_c = 0$ *if*

$$\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right)-\theta_{ib}\right)\left(\frac{\phi_c}{\overline{\phi}_c}\right)\right] < \max_{k=i,j,f} \mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right)-\theta_{ib}\right)\left(\frac{\phi_k}{\overline{\phi}_k}\right)\right].$$

Proof. We can use the same argument used in the baseline model to show that the participation constraint of problem (11) is binding at the optimum. We then solve for x using the participation constraint. Once we substitute x in the problem we obtain the following re-formulated problem:

$$\max_{b_i \geqslant 0, b_j \geqslant 0, b_f \geqslant 0} \mathbb{E}\left[\left((1+\lambda)\,{}_j\theta_s - {}_i\theta_b\right)\left(\phi_i b_{ii} + \phi_j b_{ij} + \phi_f b_{if}\right)\right]$$

subject to the feasibility constraint

$$\overline{\phi}_i b_{ii} + \overline{\phi}_j b_{ij} + \overline{\phi}_f b_{if} \leqslant y.$$

This is a linear problem whose solution involves corners. We solve this by supposing $b_c = 0$ and then the problem is the same as (3), which we solve using proposition (1). We do this for $c = i, j, f$ and then compare the objective function in each of the three cases. Comparing the values yields the results stated in the proposition. Q.E.D.

Proof of Proposition 7. We restrict attention to symmetric equilibria in which $B_{jc} = B_{ic} \equiv B_c$ for $c = i, j, f$. The proof of the proposition proceeds in two steps. First, we compute a threshold for policy risk below which there is an equilibrium in which $B_i = 0$, $B_j = 0$ and $B_f = \frac{y}{\overline{\phi}_f}$. We next find the threshold below which the equilibrium is unique.

In order for $B_i = 0$, $B_j = 0$, and $B_f = \frac{y}{\overline{\phi}_f}$ to be an equilibrium, the marginal value of signing the contract in currency f has to be larger than the marginal values of doing it in currency i and j:

$$\frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right)-\theta_{ib}\right)\phi_f\right]}{\overline{\phi}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right)-\theta_{ib}\right)\phi_i\right]}{\overline{\phi}_i} \tag{21}$$

and

$$\frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right)-\theta_{ib}\right)\phi_f\right]}{\overline{\phi}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right)-\theta_{ib}\right)\phi_j\right]}{\overline{\phi}_j}. \tag{22}$$

These conditions ensure that contracts between buyers from country i and sellers from country j are set in currency f. We also need conditions for which contracts between buyers from country j and sellers from country i are set in currency f, but these are equivalent to the previous ones given the symmetry across countries. After substituting in the governments' best responses and evaluating these expressions at $B_i = 0$, $B_j = 0$ and $B_f = \frac{y}{\overline{\phi}_f}$, these optimality conditions simplify to $\mu_1 = \frac{\mathbb{E}(\phi_f)}{\overline{\phi}_f} > \frac{\mathbb{E}(\hat{\phi}_i)}{\overline{\hat{\phi}}_i} = \frac{\mathbb{E}(\hat{\phi}_j)}{\overline{\hat{\phi}}_j}$. These are identical to the conditions obtained in the baseline model.

Now we show the conditions under which this equilibrium is unique in the set of symmetric equilibria. For this to be a unique equilibrium, it must also be true that the above inequalities hold for all prices ϕ_i consistent with $B_i \in \left[0, \frac{y}{\overline{\phi}_i^*}\right]$. Note that imposing symmetry in the currency choices of international contracts yields the following optimal choice of inflation for the government of country i

$$\phi_i = \hat{\phi}_i + \frac{1}{\psi}\left(\theta_{is} - \theta_{ib}\right) B_i.$$

Additionally, the minimum level of inflation (maximum level of ϕ) is the same as in the baseline economy: $\overline{\phi}_i = \overline{\hat{\phi}}_i + \frac{1}{\psi}\left(\overline{\theta} - \underline{\theta}\right) B_i$. We obtain symmetric expressions for ϕ_j. Replacing the government's choice of inflation in inequality (21) yields

$$\frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right) - \theta_{ib}\right)\phi_f\right]}{\overline{\phi}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right) - \theta_{ib}\right)\left(\hat{\phi}_i + \frac{1}{\psi}\left(\theta_{is} - \theta_{ib}\right) B_i\right)\right]}{\overline{\hat{\phi}}_i + \frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right) B_i}$$

or equivalently

$$\left(\left(1+\lambda\right)\mathbb{E}\left[\theta_s\right] - \mathbb{E}\left[\theta_b\right]\right)\overline{\hat{\phi}}_i\left(\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f} - \frac{\mathbb{E}\left[\hat{\phi}_i\right]}{\overline{\hat{\phi}}_i}\right) > \frac{1}{\psi}\left(\mathrm{var}\left(\theta_b\right) - \mathrm{cov}\left(\theta_b, \theta_s\right) - \tilde{\kappa}_1\right) B_i.$$

To check if this inequality holds for all B_i we need to sign the expression in parentheses on the right side of the above expression. If it is negative then we know this holds for all B_i since $\frac{\mathbb{E}[\hat{\phi}_i]}{\overline{\hat{\phi}}_i} < \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}$. If it is positive, a necessary condition to have a unique foreign currency equilibrium is

$$\frac{\mathbb{E}\left[\hat{\phi}_i\right]}{\overline{\hat{\phi}}_i} < \frac{\mathbb{E}\left(\phi_f\right)}{\overline{\phi}_f} - \frac{1}{\psi\overline{\hat{\phi}}_i}\left(\frac{\mathrm{var}\left(\theta_b\right) - \mathrm{cov}\left(\theta_b, \theta_s\right) - \tilde{\kappa}_1}{\left(\left(1+\lambda\right)\mathbb{E}\left(\theta_s\right) - \mathbb{E}\left(\theta_b\right)\right)}\right)\frac{y}{\overline{\phi}_i^*}$$

Replacing the government's choice of inflation in inequality (22) yields

$$\frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right) - \theta_{ib}\right)\phi_f\right]}{\overline{\phi}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right) - \theta_{ib}\right)\left(\hat{\phi}_i + \frac{1}{\psi}\left(\theta_{js} - \theta_{jb}\right) B_j\right)\right]}{\overline{\hat{\phi}}_j + \frac{1}{\psi}\left(\overline{\theta}_s - \underline{\theta}_b\right) B_j}$$

or

$$\left(\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}-\frac{\mathbb{E}\left[\hat{\phi}_j\right]}{\overline{\hat{\phi}}_j}\right)>\frac{1}{\psi\overline{\hat{\phi}}_j}\left(\frac{(1+\lambda)\left(\operatorname{var}\left(\theta_s\right)-\operatorname{cov}\left(\theta_b,\theta_s\right)\right)-\tilde{\kappa}_1}{\left(\mathbb{E}\left[\theta_s\right](1+\lambda)-\mathbb{E}\left[\theta_b\right]\right)}\right)B_j.$$

As before, we need to sign the expression on the right hand side. If it is negative then we know this holds for all B_j . If it is positive then we need

$$\frac{\mathbb{E}\left[\hat{\phi}_j\right]}{\overline{\hat{\phi}}_j}<\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}-\frac{1}{\psi\overline{\hat{\phi}}_j}\left(\frac{(1+\lambda)\left(\operatorname{var}\left(\theta_s\right)-\operatorname{cov}\left(\theta_b,\theta_s\right)\right)-\tilde{\kappa}_1}{\left(\mathbb{E}\left[\theta_s\right](1+\lambda)-\mathbb{E}\left[\theta_b\right]\right)}\right)\frac{y}{\overline{\phi}_j^*}.$$

Given assumptions, we have $\overline{\phi}_i^*=\overline{\phi}_j^*=\overline{\phi}_l^*$. Since both inequalities need to hold simultaneously, the cutoff value of policy risk below which the equilibrium with $B_b=0$, $B_s=0$ and $B_f=\frac{y}{\phi_f}$ is the unique symmetric equilibrium is given by

$$\begin{aligned}\mu_2^I=\min\Bigg\{&\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f},\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}-\frac{1}{\psi\overline{\hat{\phi}}}\left(\frac{\operatorname{var}\left[\theta_b\right]-\operatorname{cov}\left(\theta_b,\theta_s\right)-\tilde{\kappa}_1}{\left((1+\lambda)\,\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)}\right)\frac{y}{\overline{\phi}_l^*},\\&\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}-\frac{1}{\psi\overline{\hat{\phi}}}\left(\frac{(1+\lambda)\left(\operatorname{var}\left(\theta_s\right)-\operatorname{cov}\left(\theta_b,\theta_s\right)\right)-\tilde{\kappa}_1}{\left((1+\lambda)\,\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)}\right)\frac{y}{\overline{\phi}_l^*}\Bigg\}\end{aligned}\tag{23}$$

Recall that

$$\mu_2=\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}-\frac{1}{\psi\overline{\hat{\phi}}}\left[\frac{\operatorname{var}\left(\theta_s-\theta_b\right)+\lambda\left[\operatorname{var}\left(\theta_s\right)-\operatorname{cov}\left(\theta_s,\theta_b\right)\right]-\tilde{\kappa}_1}{\left[(1+\lambda)\,\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right]}\right]\frac{y}{\overline{\phi}_l^*}.$$

It is easy to see that $\mu_2^I>\mu_2$. Q.E.D.

Proof of Proposition 8

As before, we can substitute the participation and feasibility constraint to write the contracting problem as

$$\max_{b_l}(1+\lambda)\left(\mathbb{E}\theta_s\left(\left(\phi_l-\frac{\phi_f}{\tilde{\phi}_f}\tilde{\phi}_l\right)\left(b_l-\hat{b}_l\right)+\frac{\phi_f}{\tilde{\phi}_f}y\right)\right)-\mathbb{E}\theta_b\left(\left(\phi_l-\frac{\phi_f}{\tilde{R}_f}\tilde{\phi}_l\right)\left(b_l-\hat{b}_l\right)+\frac{\phi_f}{\tilde{\phi}_f}y\right)$$

where $\tilde{\phi}=\left\{\overline{\phi},\underline{\phi}\right\}$ depending on whether $b\geqslant\hat{b}$. The first order condition is

$$(1+\lambda)\,\mathbb{E}\left[\theta_s\left(\phi_l-\frac{\phi_f}{\tilde{\phi}_f}\tilde{\phi}_l\right)\right]-\mathbb{E}\left[\theta_b\left(\phi_l-\frac{\phi_f}{\tilde{\phi}_f}\tilde{\phi}_l\right)\right]\geqslant 0$$

First, suppose that $b_l < \hat{b}_l$. Then, after replacing the government's optimal inflation, the first order condition is (recall that $\tilde{\kappa}_2$ is defined in (16)),

$$
\begin{aligned}
&\underline{\hat{\phi}}\left[(1+\lambda)\,\mathbb{E}\left[\theta_s\left(\frac{\hat{\phi}+\frac{1}{\psi}(\theta_s-\theta_b)\,B_l}{\underline{\hat{\phi}}}-\frac{\phi_f}{\overline{\phi}_f}\right)\right]-\mathbb{E}\left[\theta_b\left(\frac{\hat{\phi}+\frac{1}{\psi}(\theta_s-\theta_b)\,B_l}{\underline{\hat{\phi}}}-\frac{\phi_f}{\overline{\phi}_f}\right)\right]\right]\\
=&\underline{\hat{\phi}}\left[\left[(1+\lambda)\,\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right]\left(\frac{\mathbb{E}\left[\hat{\phi}\right]}{\underline{\hat{\phi}}}-\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\right)+\frac{1}{\psi}\left(\frac{\left(\tilde{\kappa}_2+\left[(1+\lambda)\,\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right]\left(\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)\right)}{\underline{\hat{\phi}}}\right)B_l\right]\\
>&0
\end{aligned}
$$

so that $b_l < \hat{b}_l$ can never be part of an equilibrium.

Define

$$
\begin{aligned}
\kappa_2 \equiv{}& \left[(1+\lambda)\,\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right]\left[\frac{\mathbb{E}\left[\phi_f\right]}{\underline{\phi}_f}\left(\overline{\theta}_s-\underline{\theta}_b\right)-\left(\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)\right]\\
&+\left((1+\lambda)\,\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)\frac{\overline{\hat{\phi}}\,\overline{\phi}_l^*}{y}\left[\frac{\mathbb{E}\left[\phi_f\right]}{\underline{\phi}_f}-\frac{\mathbb{E}\left[\hat{\phi}\right]}{\overline{\hat{\phi}}}\right]
\end{aligned}
\tag{24}
$$

Now, suppose that $b_f < \hat{b}_f$. Then, the first order condition is

$$
\begin{aligned}
&\overline{\phi}_l\left[(1+\lambda)\,\mathbb{E}\left[\theta_s\left(\frac{\hat{\phi}+\frac{1}{\psi}(\theta_s-\theta_b)\,B_l}{\overline{\hat{\phi}}+\frac{1}{\psi}\left(\overline{\theta}_s-\underline{\theta}_b\right)B_l}-\frac{\phi_f}{\underline{\phi}_f}\right)\right]-\mathbb{E}\left[\theta_b\left(\frac{\hat{\phi}+\frac{1}{\psi}(\theta_s-\theta_b)\,B_l}{\overline{\hat{\phi}}+\frac{1}{\psi}\left(\overline{\theta}_s-\underline{\theta}_b\right)B_l}-\frac{\phi_f}{\underline{\phi}_f}\right)\right]\right]\\
=&\frac{1}{\psi}\left(\tilde{\kappa}_2-\left[(1+\lambda)\,\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right]\left[\frac{\mathbb{E}\left[\phi_f\right]}{\underline{\phi}_f}\left(\overline{\theta}_s-\underline{\theta}_b\right)-\left(\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)\right]\right)B_l\\
&-\left((1+\lambda)\,\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)\overline{\hat{\phi}}\left[\frac{\mathbb{E}\left[\phi_f\right]}{\underline{\phi}_f}-\frac{\mathbb{E}\left[\hat{\phi}\right]}{\overline{\hat{\phi}}}\right]\\
\leqslant&\frac{1}{\psi}\left(\tilde{\kappa}_2-\left[(1+\lambda)\,\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right]\left[\frac{\mathbb{E}\left[\phi_f\right]}{\underline{\phi}_f}\left(\overline{\theta}_s-\underline{\theta}_b\right)-\left(\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)\right]\right)\frac{y}{\overline{\phi}_l^*}\\
&-\left((1+\lambda)\,\mathbb{E}\left[\theta_s\right]-\mathbb{E}\left[\theta_b\right]\right)\overline{\hat{\phi}}\left[\frac{\mathbb{E}\left[\phi_f\right]}{\underline{\phi}_f}-\frac{\mathbb{E}\left[\hat{\phi}\right]}{\overline{\hat{\phi}}}\right]\\
<&0
\end{aligned}
$$

where the last inequality follows from Assumption 3. Q.E.D.

C Additional Results and Extensions

C.1 TNT Model with Endogenous Real Exchange Rate Risk

This section shows that the presence of the exogenous risk of the price of foreign currency $\frac{\mathbb{E}[\phi_f]}{\phi_f}$ can arise in an extension of our model with tradable and non-tradable goods and shocks to the relative demand of these goods in the domestic economy. Suppose the numeraire good in our model is a composite of tradable and non-tradable goods, $c = c_T^{\alpha} c_N^{1-\alpha}$, where c_T (respectively, c_N) is the domestic consumption of tradables (respectively, non-tradables), and α is a stochastic parameter that captures shocks to the relative demand of these goods. The equivalent good in the foreign country is given by $c^* = \left(c_T^*\right)^{\alpha^*} \left(c_N^*\right)^{1-\alpha^*}$. We assume that α^* is deterministic. We also normalize the endowments $y_T = y_N = y_T^* = y_N^* = y$. Consistent with our baseline model, we denote the price of the local (respectively, foreign) currency in terms of the domestic composite by ϕ_l (respectively, ϕ_f). Additionally, we normalize the price of the foreign currency in terms of the foreign composite good to 1. The exchange rate e is defined as the price of the local currency in terms of the foreign currency. Let p_T denote the price of the tradable goods in the domestic economy in terms of the local currency and p_T^* denote the price of the tradable goods in the foreign economy in terms of the foreign currency.

Given the Cobb-Douglas structure, p_T and p_T^* are given by

$$p_T = \frac{1}{\phi_l} \alpha \left(\frac{c_N}{c_T} \right)^{1-\alpha} \text{ and } p_T^* = \alpha^* \left(\frac{c_N^*}{c_T^*} \right)^{1-\alpha^*}.$$

In this model, the law of one price for tradable goods holds. Market clearing in all goods implies that the exchange rate e is given by

$$e = \frac{p_T}{p_T^*} = \frac{\alpha}{\alpha^*} \frac{1}{\phi_l}.$$

Therefore,

$$\phi_f = e\phi_l = \frac{\alpha}{\alpha^*}.$$

In this model we can generate fluctuations in the real exchange rate (the price of the foreign currency in terms of the domestic composite good, ϕ_f) by assuming a stochastic process for α.

C.2 Microfoundation of Inflation Loss Function

Consider an extension of the baseline model in which, in addition to buyers and sellers, there are households. Households derive utility from the consumption of the numeraire good c_h, a cash good x, and disutility from exerting labor n in the second period captured by the utility function

$$U_h = \omega x + c_h - \frac{n^2}{2},$$

where $\omega > 1$. Households are endowed with money claims on the government m and need to pay ad-valorem taxes τ on labor income. The households' budget constraint is given by

$$c_h + p_x x = w(1-\tau)n + m\phi_l, \tag{25}$$

where p_x is the price of the cash good, w is the real wage, and ϕ_l is the price of money, all expressed in terms of the numeraire good. Households also face a cash-in-advance constraint which requires that the numeraire good needs to be purchased with money holdings

$$p_x x \leqslant m\phi_l. \tag{26}$$

The government uses taxes to finance government expenditure g (expressed in terms of the numeraire good) and repay money claims. Government expenditures are unknown in the initial period and drawn from a distribution with bounded support $\left[\underline{g}, \overline{g}\right]$. The government budget constraint is given by

$$g + m\phi_l = w\tau n. \tag{27}$$

Finally, we assume that both the numeraire good and the cash good can be produced with a linear technology that uses labor $n = c_h + g + x$. Free entry of firms implies that $w = p_x = 1$. The problem of the household is to maximize U_h subject to (25) and (26). We conjecture (and verify later) that the cash-in-advance constraint binds and thus the solution to the household problem is $n = (1-\tau)$, $c_h = (1-\tau)^2$, and $x = m\phi_l$. Define the target tax rate $\hat{\tau}$ and level of inflation $\hat{\phi}$ as the tax rate and the level of inflation that maximize the household's utility, subject to the allocations defined above and (27). The target tax rate is given by $\hat{\tau} = \frac{\omega-1}{2\omega-1}$. The target level of inflation is given by

$$\hat{\phi} = \frac{\hat{\tau}(1-\hat{\tau}) - g}{m}. \tag{28}$$

Note that the target tax rate is independent of g, which implies that shocks to government spending are absorbed with seigniorage. In order to guarantee that $\hat{\phi} \geqslant 0$, we assume that $\overline{g} < \hat{\tau}(1-\hat{\tau})$. Finally, notice that since $\hat{\tau} \leqslant 1$ and households strictly prefer the cash good to the numeraire good ($\omega > 1$), the cash-in-advance constraint always binds.

If we compute a second order approximation to the household's utility around the target government policies, we obtain

$$U_h = const - \frac{(2\omega - 1)\,2m^2}{(1-2\hat{\tau})^2}\left(\phi_l - \hat{\phi}\right)^2,$$

where $const \equiv \omega\left(\hat{\tau}(1-\hat{\tau}) - g\right) + \frac{(1-\hat{\tau})^2}{2}$. It follows that the loss function in the baseline model maps into a second order approximation of the household's utility. In particular, the inflation cost parameter is given by $\psi \equiv 2\frac{(2\omega-1)2m^2}{(1-2\hat{\tau})^2} > 0$ and $\hat{\phi}$ is given by (28), which implies that shocks to the inflation target can be microfounded by shocks to government expenditure. This interpretation of policy risk, together with the results on the characterization of the set of competitive equilibria, can shed light into why countries with more volatile government expenditures tend to have more domestic dollarization, as shown in Figure 4.

C.3 Relaxing Assumption 2

Recall that Assumption 2 ensures that the value of flexibility in monetary policy is sufficiently high. In this section, we consider an example of when the assumption is violated. Assume that θ_s and θ_b are independent and identically distributed with $\text{var}(\theta_s) = \text{var}(\theta_b) = \text{var}(\theta)$ and $\mathbb{E}[\theta_s] = \mathbb{E}[\theta_b] = 1$. In this case Assumption 2 is equivalent to

$$\text{var}(\theta) \geqslant \frac{\lambda}{1+\lambda}\left(\overline{\theta} - \underline{\theta}\right)$$

To understand what would happen if this were not true, we consider the extreme case in which there is no value to flexibility, i.e. $\text{var}(\theta) = 0$.

Assumption 4. *Assume that* $\text{var}(\theta) = 0$, $\theta_b > \theta_s$, *and* $(1+\lambda)\,\theta_s > \theta_b$.

We show that the main insights regarding the equilibrium characterization in the baseline model (Proposition 2) carry over when Assumption 2 is relaxed. In particular, we show that there is a region of high policy risk in which the unique equilibrium involves the full use of the foreign currency, and a region of low policy risk in which the unique equilibrium features positive use of local currency. We formalize this result in the following proposition.Suppose that Assumption 4 holds. If $\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f} \geqslant \frac{\mathbb{E}[\hat{\phi}]}{\bar{\hat{\phi}}}$, there exists a unique equilibrium in which $B_l = 0$. If $\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f} < \frac{\mathbb{E}[\hat{\phi}]}{\bar{\hat{\phi}}}$, there exists a unique equilibrium with $B_l > 0$.

Proof. The bilateral contracting and government's problem is identical to the baseline model. In particular, the policy functions for the government is given by (7) and (8). Suppose first that $\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f} \geqslant \frac{\mathbb{E}[\hat{\phi}]}{\bar{\hat{\phi}}}$. Then, there exists an equilibrium in which $B_l = 0$. To see that this a unique equilibrium, notice that price risk $\frac{\mathbb{E}[\phi_l]}{\overline{\phi}_l} = \frac{\mathbb{E}[\hat{\phi}]+\frac{1}{\psi}(\theta_s-\theta_b)B_l}{\overline{\hat{\phi}}+\frac{1}{\psi}(\theta_s-\theta_b)B_l}$ is decreasing in B_l. Next, suppose that $\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f} < \frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}}$. Then, it must be that $B_l > 0$, and the fact that $\frac{\mathbb{E}[\hat{\phi}]+\frac{1}{\psi}(\theta_s-\theta_b)B_l}{\overline{\hat{\phi}}+\frac{1}{\psi}(\theta_s-\theta_b)B_l}$ is decreasing in B_l implies that the equilibrium is unique. If B_l is interior then the unique equilibrium is the solution to the following system of equations

$$\phi_l = \hat{\phi} + \frac{1}{\psi}(\theta_s - \theta_b) B_l,$$

$$\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f} = \frac{\mathbb{E}[\phi_l]}{\overline{\phi}_l},$$

and

$$\overline{\phi}_l = \overline{\hat{\phi}} + \frac{1}{\psi}(\theta_s - \theta_b) B_l.$$

We can combine these equations to get

$$B_l = \frac{\overline{\hat{\phi}}\left(\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f} - \frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}}\right)}{\left[1 - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right]\frac{1}{\psi}(\theta_s - \theta_b)}.$$

Therefore,

$$\overline{\phi}_l = \overline{\hat{\phi}}\left(1 + \frac{\left(\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f} - \frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}}\right)}{\left[1 - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right]}\right).$$

To check that the solution is interior we need to check that $B_l \leqslant \frac{y}{\overline{\phi}_l}$ or

$$\frac{\overline{\hat{\phi}}\left(\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f} - \frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}}\right)}{\left[1 - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right]\frac{1}{\psi}(\theta_s - \theta_b)} \leqslant \frac{y\left[1 - \frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right]}{\overline{\hat{\phi}}\left(1 - \frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}}\right)},$$

otherwise the unique solution is $B_l = \frac{y}{\overline{\phi}_l}$. Q.E.D.

Next, we study the social planner's problem. The planning problem is identical to the baseline environment.

$$\max_{B_l, B_f} \mathbb{E}\left[\left[(1+\lambda)\theta_s - \theta_b\right](\phi_l B_l + \phi_f B_f) - l(\phi_l)\right]$$

subject to (1), (7), and (8).

Proposition 9. *Suppose that Assumption 4 holds. If* $\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f} \geqslant \frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}}$*, the competitive equilibrium* *is efficient and* $B_l^{sp}=0$*. If* $\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}<\frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}}$*, and the solution to the planning problem is interior, the competitive equilibrium is inefficient and* $B_l^{sp}<B_l^{ce}$*.*

Proof. The first order condition of the planning problem is

$$[(1+\lambda)\theta_s-\theta_b]\left(\overline{\hat{\phi}}\left[\frac{\mathbb{E}\left[\hat{\phi}\right]}{\overline{\hat{\phi}}}-\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\right]+\frac{1}{\psi}\left(\theta_s-\theta_b\right)B_l\left[1-\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\right]\right)-\frac{1}{\psi}\left(\theta_s-\theta_b\right)^2B_l.$$

If $\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\geqslant\frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}}$, the expression above is negative and thus $B_l^{sp}=0$. If $\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}<\frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}}$, then whenever the solution is interior it satisfies

$$B_l^{sp}=\frac{\overline{\hat{\phi}}\left[\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}-\frac{\mathbb{E}[\hat{\phi}]}{\overline{\hat{\phi}}}\right]}{\left(\left[1-\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right]-\frac{(\theta_s-\theta_b)}{[(1+\lambda)\theta_s-\theta_b]}\right)\frac{1}{\psi}\left(\theta_s-\theta_b\right)}.$$

Therefore,

$$\frac{B_l^{sp}}{B_l^{ce}}=\frac{\left[1-\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right]}{\left(\left[1-\frac{\mathbb{E}[\phi_f]}{\overline{\phi}_f}\right]-\frac{(\theta_s-\theta_b)}{[(1+\lambda)\theta_s-\theta_b]}\right)}<1,$$

which follows from Assumption 4. Q.E.D.

The key implication of relaxing Assumption 2 is that, in general, since there are no benefits of having a flexible monetary policy, the efficient allocation will always prescribe greater use of the foreign currency. In contrast, if Assumption 2 holds, then we show that private agents can underestimate the insurance channel and thus we can have situations in which the efficient allocation calls for greater use of local currency.

C.4 Model with Commitment

In this section we describe the problem in which the government can commit to state contingent policies and show that the solution is constrained efficient. Let $\sigma=\left(\hat{\phi},\phi_f,\theta_s,\theta_b\right)$ denote the state of the world after shocks are realized in period 2. We consider a government who first chooses a state contingent policy $\phi\left(\sigma\right)$ and then private agents make their decisions. Finally, the state of the world is realized and policy $\phi\left(\sigma\right)$ is implemented.

The government's problem entails choosing $\phi\left(\sigma\right)$ to maximize the sum of ex-ante utilities of buyers and sellers net of the inflation cost, subject to the equations that characterize the optimal private contracts. Notice that the solution to the private contracting problem is identical to that characterized in Proposition 1. Thus, the government's problem is

$$\max_{\phi_l(\sigma), C_s(\sigma), C_b(\sigma), B_l, B_f, x} (1+\lambda)\, x + \mathbb{E}\left[\theta_b C_b(\sigma)\right] - x + \mathbb{E}\left[\theta_s C_s(\sigma)\right] - \frac{1}{2}\psi \mathbb{E}\left(\phi_l - \hat{\phi}\right)^2$$

subject to

$$C_s(\sigma) = y + \phi_f B_f + \phi_l(\sigma) B_l$$

$$C_b(\sigma) = y - \phi_f B_f - \phi_l(\sigma) B_l$$

$$B_l = \begin{cases} \frac{y}{\overline{\phi}_l} & M_l > M_f \\ 0 & M_l < M_f \\ \gamma \frac{y}{\overline{\phi}_l}, \gamma \in [0,1] & M_l = M_f \end{cases}$$

$$B_f = \begin{cases} 0 & M_l > M_f \\ \frac{y}{\phi_f} & M_l < M_f \\ (1-\gamma)\frac{y}{\phi_f} & M_l = M_f \end{cases}$$

$$-x + \mathbb{E}\left[\theta_s C_s(\sigma)\right] = 0$$

$$\overline{\phi}_l = \max_\sigma \phi_l(\sigma)$$

where M_l and M_f are defined prior to Proposition 1. Note that we impose that the participation constraint for the seller always holds with equality since that is always true at the contracting stage. We can rewrite the above problem as

$$\max_{\phi_l(\sigma), B_l, B_f} \mathbb{E}\left(\left[(1+\lambda)\,\theta_s - \theta_b\right]\left[\phi_f B_f + \phi_l(\sigma) B_l\right]\right) - \frac{1}{2}\psi \mathbb{E}\left(\phi_l - \hat{\phi}\right)^2$$

subject to

$$B_l = \begin{cases} \frac{y}{\overline{\phi}_l} & M_l > M_f \\ 0 & M_l < M_f \\ \gamma \frac{y}{\overline{\phi}_l}, \gamma \in [0,1] & M_l = M_f \end{cases} \tag{29}$$

$$B_f = \begin{cases} 0 & M_l > M_f \\ \frac{y}{\phi_f} & M_l < M_f \\ (1-\gamma)\frac{y}{\phi_f} & M_l = M_f \end{cases} \tag{30}$$

$$\overline{\phi}_l = \max_\sigma \phi_l(\sigma).$$

Now, we lay out the social planner's problem when the government can commit. In this problem, the planner chooses the terms of the contracts, as well as a state contingent inflation policy. The constrained efficient allocation is given by the solution to the following problem:

$$\max_{\phi_l(\sigma), C_s(\sigma), C_b(\sigma), B_l, B_f, x} (1+\lambda)\, x + \mathbb{E}\left[\theta_b C_b\left(\sigma\right)\right] - x + \mathbb{E}\left[\theta_s C_s\left(\sigma\right)\right] - \frac{1}{2}\psi \mathbb{E}\left(\phi_l\left(\sigma\right) - \hat{\phi}\right)^2$$

subject to

$$C_s\left(\sigma\right) = y + \phi_f B_f + \phi_l\left(\sigma\right) B_l$$

$$C_b\left(\sigma\right) = y - \phi_f B_f - \phi_l\left(\sigma\right) B_l$$

$$C_b\left(\sigma\right) \geqslant 0, \ \forall \sigma$$

$$-x + \mathbb{E}\left[\theta_s C_s\left(\sigma\right)\right] \geqslant 0$$

$$B_l, B_f \geqslant 0$$

where the third constraint is the payments feasibility constraint. Thus, this problem can be written as

$$\max_{\phi_l(\sigma), B_l, B_f} \mathbb{E}\left(\left[(1+\lambda)\, \theta_s - \theta_b\right]\left[\phi_f B_f + \phi_l\left(\sigma\right) B_l\right]\right) - \frac{1}{2}\psi \mathbb{E}\left(\phi_l\left(\sigma\right) - \hat{\phi}\right)^2$$

subject to

$$y - \phi\left(\sigma\right) B_l - \phi_f B_f \geqslant 0, \ \forall \sigma \tag{31}$$

$$B_l, B_f \geqslant 0$$

We now show that the solution to the competitive equilibrium with commitment and the constrained efficiency problem coincide.

Proposition 10. *Suppose the government can commit. Then, competitive equilibria are constrained efficient.*

Proof. Define the following maximization problem

$$P(\phi(\sigma)) \equiv \max_{B_l \geqslant 0, B_f \geqslant 0} \mathbb{E}\left(\left[(1+\lambda)\, \theta_s - \theta_b\right]\left[\phi_f B_f + \phi_l\left(\sigma\right) B_l\right]\right)$$

subject to

$$y - \phi\left(\sigma\right) B_l - \phi_f B_f \geqslant 0, \ \forall \sigma.$$

The constrained efficient problem can be expressed as

$$\max_{\phi(\sigma)} P(\phi(\sigma)) - \frac{1}{2}\psi \mathbb{E}\left(\phi_l\left(\sigma\right) - \hat{\phi}\right)^2$$

Notice that $P(\phi(\sigma))$ corresponds to the contracting problem of private agents. The solution is characterized in Proposition 1, and is given by (29) and (30). Therefore, it follows that the two problems are identical. $\square$

Given the above result, we can now characterize the solution with commitment by working the planning problem.

$$\max_{B_l, \phi(\sigma)} \mathbb{E}\left(\left[(1+\lambda)\,\theta_s - \theta_b\right] \frac{\phi_f}{\overline{\phi}_f}\right) y + \mathbb{E}\left(\left[(1+\lambda)\,\theta_s - \theta_b\right] \overline{\phi}_l \left(\frac{\phi_l(\sigma)}{\overline{\phi}_l} - \frac{\phi_f}{\overline{\phi}_f}\right) B_l\right)$$
$$-\frac{1}{2}\psi \mathbb{E}\left(\phi_l(\sigma) - \hat{\phi}\right)^2$$

From the above problem we see that either $B_l = 0$ or $B_l = y/\overline{\phi}_l$. Suppose the latter is true. Then the problem is

$$\max_{\phi(\sigma)} \mathbb{E}\left(\left[(1+\lambda)\,\theta_s - \theta_b\right] \left(\frac{\phi_l(\sigma)}{\overline{\phi}_l} - \frac{\phi_f}{\overline{\phi}_f}\right) y\right) - \frac{1}{2}\psi \mathbb{E}\left(\phi_l(\sigma) - \hat{\phi}\right)^2$$

Let $\Sigma^{\max} = \left\{\sigma \mid \phi_l(\sigma) = \overline{\phi}_l = \max_\sigma \phi_l(\sigma)\right\}$. Then the above can be written as

$$\max_{\phi(\sigma)} y \left[\int_{\sigma \notin \Sigma^{\max}} \left(\left[(1+\lambda)\,\theta_s - \theta_b\right] \frac{\phi_l(\sigma)}{\overline{\phi}}\right) dF(\sigma) + \int_{\Sigma^{\max}} \left[(1+\lambda)\,\theta_s - \theta_b\right] dF(\sigma)\right]$$
$$-\frac{1}{2}\psi \left[\int_{\sigma \notin \Sigma^{\max}} \left(\phi_l(\sigma) - \hat{\phi}\right)^2 dF(\sigma) + \int_{\Sigma^{\max}} \left(\overline{\phi}_l - \hat{\phi}\right)^2 dF(\sigma)\right]$$

The first order conditions are: if $\sigma \notin \Sigma^{\max}$

$$\left[(1+\lambda)\,\theta_s - \theta_b\right] \frac{y}{\overline{\phi}_l} - \psi\left(\phi_l(\sigma) - \hat{\phi}\right) = 0$$

or

$$\phi_l(\sigma) = \hat{\phi} + \frac{y}{\overline{\phi}_l \psi}\left[(1+\lambda)\,\theta_s - \theta_b\right] \tag{32}$$

and if $\sigma \in \Sigma^{\max}$,

$$-\int_{\sigma \notin \Sigma^{\max}} y\left[(1+\lambda)\,\theta_s - \theta_b\right] \frac{\phi_l(\sigma)}{\overline{\phi}_l^2} dF(\sigma) - \psi \int_{\Sigma^{\max}} \left(\overline{\phi}_l - \hat{\phi}\right) dF(\sigma) = 0. \tag{33}$$

If the problem is concave, then the solution to these two sets of equations characterize the allocation with commitment. It is instructive to compare this with no-commitment case.

$$\phi_l^{nc} = \hat{\phi} + (\theta_s - \theta_b)\frac{y}{\psi \overline{\phi}_l^{nc}},$$

$$\overline{\phi}_l^{nc} = \overline{\hat{\phi}} + (\theta_s - \theta_b)\,\frac{y}{\psi\overline{\phi}_l^{nc}}.$$

In contrast to the case without commitment, the government with commitment internalizes the effect of inflation on private contracts. In particular, the government realizes that higher covariance between prices and the taste shocks of the seller, as well as lower price risk can increase the amount of special good provided. From (32), we see that the government takes into account the former effect, via the $\lambda\theta_s$ term. Also, from the first term in (33) we see that the government takes into account the latter effect since an increase in $\overline{\phi}_l$ increases the price risk.

C.5 Model with Fixed Cost of Default

In this section, we consider an alternative specification of the default model in which there is a fixed cost of default. We show that this model behaves in a similar fashion to the baseline model in that there are complementarities between the actions of private agents and those of the government. In particular, like in the baseline, denominating bilateral contracts in local currency becomes more attractive the higher the aggregate stock of contracts in local currency.

We assume that there is a *fixed* cost χ that the buyer must pay whenever it defaults on its contract in period 2. For simplicity, we shut down taste shocks, i.e., $\theta_s = \theta_b = 1$, and exogenous price risk in both currencies, i.e., $\phi_f = \hat{\phi} = 1$. The latter allows us to describe these complementarities in a more transparent way. We assume that the default cost is the same for all agents and is stochastic with $\chi = \chi_H$ with probability p_H and $\chi = \chi_L < \chi_H$ with probability $1 - p_H$. We will refer to the former case as the "high" state and the latter as the "low" state. We also assume that $\chi_H < y$ so that we can ignore the payment feasibility constraint. After the buyer defaults, the seller receives nothing and the buyer's utility is $y - \chi$. Therefore, the buyer defaults if

$$\chi < \phi_f b_f + \phi_l b_l.$$

This implies that the buyer defaults when the cost of repayment is larger than the cost of default.

The problem for the government is similar to that in the baseline except that now the choice of monetary policy trades off the cost associated with default with the inflation costs associated with deviating from the target. In states in which buyers repay if the government chooses the inflation target, the optimal choice of the government is indeed the inflation target. In states in which buyers would default if the government chooses the inflation target, i.e., $\hat{\phi}B_l + \phi_f B_f > \chi$, the government finds it optimal to use inflation to reduce the real burden of debt if

$$\chi > \frac{\psi}{2}\left(\frac{\chi - \phi_f B_f}{B_l} - \hat{\phi}\right)^2.$$

The left hand side denotes the loss associated with allowing agents to default and following the target, while the right hand side denotes the loss associated with deviating from the target in order to prevent default.

The next proposition shows that under a parametric assumption that bounds the above cost of deviating from the inflation target, there exists a full local currency equilibrium whose associated aggregate welfare is strictly larger than that associated with any equilibrium with full foreign currency.

Proposition 11. *Suppose that*

$$\frac{\psi}{2}\left(\frac{\chi_L}{\chi_H} - 1\right)^2 < \min\left\{\lambda \frac{p_H}{(1-p_H)}(\chi_H - \chi_L), \chi_L\right\}.$$

Then, there exists an equilibrium in which all contracts are denominated in foreign currency and another in which all contracts are denominated in local currency. Moreover, aggregate welfare, defined as the sum of the utilities of buyers and sellers, net of inflation costs, is higher in the latter equilibrium.

Proof. We first show that an equilibrium with full foreign currency use exists. There are two cases to consider. Either contracts allow for default in the low state or they require repayment in all states. In the former, $B_f = \frac{\chi_H}{\phi_f} = \chi_H$, and so the welfare associated with the contract is

$$y + \lambda p_H \chi_H - (1 - p_H)\chi_L.$$

In the latter, $B_f = \frac{\chi_L}{\phi_f} = \chi_L$, and the welfare is

$$y + \lambda \chi_L.$$

Therefore, the payoffs associated with the foreign currency equilibrium are

$$\max\{y + \lambda p_H \chi_H - (1 - p_H)\chi_L, y + \lambda \chi_L\}.$$

Since the government always sets inflation equal to its target if all contracts are denominated in foreign currency, these payoffs also coincide with aggregate welfare. To verify that this is indeed an equilibrium, note that if all agents are denominating contracts in foreign currency ($B_l = 0$), then agents are indifferent between denominating in either the foreign or the local currency since the government always chooses $\phi_l = \hat{\phi}$ and $\phi_f = \hat{\phi}$.

Thus, the full foreign currency equilibrium exists.

We now show that under the assumption in the proposition, an equilibrium with full local currency use exists. We conjecture that the allocation is given by $B_l = \frac{\chi_H}{\hat{\phi}} = \chi_H$, $\phi_l = \frac{\chi_L}{B_l}$ and $\phi_H = \hat{\phi} = 1$. We now show that the conjectured allocation is indeed an equilibrium. First, consider the government's problem. As mentioned above, the government chooses to deviate from the inflation target and reduce the real value of payments in the low state if

$$\chi_L > \frac{\psi}{2}\left(\frac{\chi_L}{B_l} - \hat{\phi}\right)^2.$$

It is easy to verify that the assumption in the proposition implies the above inequality under the conjectured allocation. In the high state, since there is no default, the government optimally chooses $\phi_H = \hat{\phi}$. Next, consider the contracting problem. Note that given the inflation policy, the maximum level of promised payments that avoids default is the same in both states, i.e., $\frac{\chi_L}{\phi_L} = \frac{\chi_H}{\hat{\phi}}$. Choosing $b_L < \frac{\chi_H}{\hat{\phi}}$ is suboptimal since increasing it slightly can induce larger gains of trade without defaulting. Similarly, choosing $b_L > \frac{\chi_H}{\hat{\phi}}$ induces default in both states. Therefore, the optimal level of promised payments is $b_l = \frac{\chi_H}{\phi_H}$, and the buyer's utility is given by $y + \lambda\left(p_H\chi_H + (1-p_H)\chi_L\right)$. This payoff is larger than the payoff of using foreign currency, which implies that denominating contracts in local currency is preferred to denominating them in foreign currency. Following a similar argument, one can show that contracts that involve positive promised payments in both currencies are dominated by the one with $b_f = 0$ and $b_l = \frac{\chi_H}{\phi_H}$. This confirms our conjecture. Moreover, the aggregate welfare (which includes expected inflation losses) is given by

$$y + \lambda\left(p_H\chi_H + (1-p_H)\chi_L\right) - (1-p_H)\frac{\psi}{2}\left(\frac{\chi_L}{B_l} - \hat{\phi}\right)^2,$$

which is larger than the aggregate welfare under the equilibrium with full use of foreign currency, under the parametric assumption. □

The intuition for the proposition is as follows. Inflation is valuable for buyers and sellers ex-post since it allows for greater state contingency in contracts and hence implies a larger provision of the special good in period 1. Moreover, buyers no longer have to pay the default cost. But in order for the government to be willing to use inflation to avoid default, it must be that the costs of deviating from the target are not too large. This is guaranteed by the assumption in the proposition. Moreover, note that the government never deviates from the target if $B_l = 0$. Thus, the model has similar complementarities to the baseline, i.e., as more private agents denominate contracts in local currency, the larger the incentives of the government to deviate from its target and use inflation to avoid full default. Therefore, the incentive for a buyer-seller pair to denominate contracts in local currency is larger if more agents are doing the same.

Finally, note that there are other competitive equilibria in which, for example, private agents might denominate contracts in both foreign and local currencies. However, by similar arguments, one can show that the full local currency equilibrium described above implies higher welfare than these equilibria as well.

C.6 Model of a Credit Chain

We now present a simple credit chain model that endogenizes the stocks of foreign and local currency in Section 3.3.

Suppose that citizens are further divided into one of I sub-types $\mathcal{I} \in \{1, 2, ..., I\}$ with a continuum of each. A citizen of type i has preferences over a special good produced by type $i+1$ and produces a special good valued by type $i-1$. All types also value the consumption of the numeraire good, which takes place at the end of period 2. Preferences for the representative citizen type i are given by

$$u_i = (1+\lambda)\ {}_{i}x_{i+1} - {}_{i-1}x_i + \mathbb{E}\left[\theta_i c_i\right],$$

where ${}_{i}x_{i+1}$ is the special good produced by a citizen of type $i+1$ for a citizen of type i and ${}_{i-1}x_i$ is the special good produced by a citizen of type i for a citizen of type $i-1$. We assume that ${}_{0}x_1 = {}_{I}x_{I+1} = 0$ so that type 1 does not produce a special good for any other type and type I does not consume a special good. We assume that $\theta_i \in \left[\underline{\theta}, \overline{\theta}\right]$ is independent across sub-types and that $\mathbb{E}\left[\theta_i\right] = 1$.

The timing of the model is as follows:

1. The first period $t = 1$ is divided into $I - 1$ sub-periods in which trade takes place sequentially:
   (a) In sub-period 1, citizens of type 2 produce a special good for citizens of type 1 in exchange for the promise of payment in period 2.
   (b) Similarly, in sub-period i, citizens of type $i+1$ produce a special good for citizens of type i in exchange for the promise of payment in period 2.
2. The second period $t = 2$ is divided into three sub-periods:
   (a) In sub-period 1, the taste shocks θ_i are realized for all citizens.
   (b) In sub-period 2, the type of the domestic government is realized and it chooses its policy, which is the aggregate price level.
   (c) In sub-period 3, all signed contracts are executed in the order in which they were signed and, finally, consumption of the composite good takes place.

Assume that all citizens are endowed with y units of the numeraire good. The definition of a bilateral contract between i and $i+1$ is identical to Section 3.3. Note that in this contract $i+1$ is the seller and i is the buyer. Given the structure of the credit chain, $(\hat{b}_f, \hat{b}_l)$ is the promised payment to type i from types $i-1$.

We can then use Propositions 2 and 8 to characterize the bilateral contract.

Proposition 12. *In the optimal bilateral contract, the amount of special good is given by* $x = \mathbb{E}\left[\theta_s\left(b_l\phi_l + b_f\phi_f\right)\right]$, *while the payments satisfy*

1. *If* $\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{\phi_l}{\phi_l}\right] < \mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{\phi_f}{\phi_f}\right]$, *then* $b_l = \hat{b}_l$ *and* $b_f = \hat{b}_f + \frac{y}{\phi_f}$.
2. *If* $\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{\phi_l}{\phi_l}\right] = \mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{\phi_f}{\phi_f}\right]$, *then* $b_l = \hat{b}_l + \gamma\frac{y}{\phi_l}$ *and* $b_f = \hat{b}_f + (1-\gamma)\frac{y}{\phi_f}$ *for any* $\gamma \in [0,1]$.
3. *If* $\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{\phi_l}{\phi_l}\right] > \mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{\phi_f}{\phi_f}\right]$, *then* $b_l = \hat{b}_l + \frac{y}{\phi_l}$ *and* $b_f = \hat{b}_f$.

The result follows immediately from Propositions 2 and 8. In particular, the optimal contract will feature currency matching of stocks and will denominate the flows in the currency with the largest marginal benefit.

C.7 Model with International Trade Contracts: Generalized Result

This section shows that the result in Proposition 7 can be generalized to show uniqueness among the entire set of equilibria, and not just symmetric equilibria, under a parametric assumption. For simplicity, we assume that θ_s and θ_b are independent and identically distributed with $\text{var}(\theta_s) = \text{var}(\theta_b) = \text{var}(\theta)$ and $\mathbb{E}[\theta_s] = \mathbb{E}[\theta_b] = 1$. The argument for the more general case is identical to the one below except that the parametric assumption will be different.

Assumption 5. *Assume that*

$$var(\theta) \geqslant \lambda \geqslant \left(\overline{\theta} - \underline{\theta}\right).$$

We can now prove a generalization of Proposition 7.

Proposition 13. *Under Assumption 5, there exists a threshold* μ_2^I *such that, if* $\frac{\mathbb{E}[\hat{\phi}_i]}{\bar{\hat{\phi}}_i} = \frac{\mathbb{E}[\hat{\phi}_j]}{\bar{\hat{\phi}}_j} \leqslant \mu_2^I$, *there exists a unique equilibrium in which* $B_{ii} = B_{ji} = B_{ij} = B_{jj} = 0$. *Furthermore,* $\mu_2^I > \mu_2$.

Proof. The proof is symmetric to that of Proposition 7. First, we show the existence of an equilibrium with $B_{ii} = B_{ji} = 0$, $B_{ij} = B_{jj} = 0$ and $B_{if} = B_{jf} = \frac{y}{\phi_f}$. Second, we show this equilibrium is unique.

In order for the above allocation to be part of an equilibrium, the marginal value of signing the contract in currency f has to be larger than the marginal values of doing it in currency i and j:

$$\frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right)-\theta_{ib}\right)\phi_f\right]}{\overline{\phi}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right)-\theta_{ib}\right)\phi_i\right]}{\overline{\phi}_i} \tag{34}$$

and

$$\frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right)-\theta_{ib}\right)\phi_f\right]}{\overline{\phi}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right)-\theta_{ib}\right)\phi_j\right]}{\overline{\phi}_j}. \tag{35}$$

These conditions ensure that contracts between buyers from country i and sellers from country j are set in currency f. We also need conditions for which contracts between buyers from country j and sellers of country i are set in currency f, but these are identical to the ones in the proof of Proposition 7. After substituting in the governments' best responses and evaluating these expressions at $B_{ii} = B_{ji} = 0$, $B_{ij} = B_{jj} = 0$ and $B_{if} = B_{jf} = \frac{y}{\phi_f}$, these optimality conditions simplify to $\mu_1 = \frac{\mathbb{E}(\phi_f)}{\overline{\phi}_f} > \frac{\mathbb{E}(\hat{\phi}_i)}{\overline{\hat{\phi}}_i} = \frac{\mathbb{E}(\hat{\phi}_j)}{\overline{\hat{\phi}}_j}$. These are identical to the conditions obtained in the baseline model.

Now we show the conditions under which this equilibrium is unique. For this to be a unique equilibrium, it must also be true that the previous inequalities hold for prices ϕ_i consistent with all possible $B_{ii}, B_{ji} \in \left[0, \frac{y}{\overline{\phi}_i^*}\right]$. The optimal choice of inflation for the government of country i is given by

$$\phi_i = \hat{\phi}_i + \frac{1}{\psi}\left(\theta_{is}B_{ji} - \theta_{ib}B_{ii}\right).$$

Additionally, the minimum level of inflation (maximum level of ϕ) is the same as in the baseline economy: $\overline{\phi}_i = \overline{\hat{\phi}}_i + \frac{1}{\psi}\left(\bar{\theta}B_{ji} - \underline{\theta}B_{ii}\right)$. We obtain symmetric expressions for ϕ_j. Replacing the government's choice of inflation in inequality (34) yields

$$\frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right)-\theta_{ib}\right)\phi_f\right]}{\overline{\phi}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right)-\theta_{ib}\right)\left(\hat{\phi}_i + \frac{1}{\psi}\left(\theta_{is}B_{ji} - \theta_{ib}B_{ii}\right)\right)\right]}{\overline{\hat{\phi}}_i + \frac{1}{\psi}\left(\bar{\theta}B_{ji} - \underline{\theta}B_{ii}\right)}$$

or equivalently

$$\overline{\hat{\phi}}_i\left(\frac{\mathbb{E}\left[\hat{\phi}_i\right]}{\overline{\hat{\phi}}_i} - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\right) + \frac{1}{\psi}\left(\left[1 - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\bar{\theta}\right]B_{ji} + \left[\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\underline{\theta} + \frac{\operatorname{var}(\theta)}{\lambda} - 1\right]B_{ii}\right) < 0. \tag{36}$$

Similarly, replacing the government's choice of inflation in inequality (35) yields

$$\frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right)-\theta_{ib}\right)\phi_f\right]}{\overline{\phi}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right)-\theta_{ib}\right)\left(\hat{\phi}_j+\frac{1}{\psi}\left(\theta_{js}B_{ij}-\theta_{jb}B_{jj}\right)\right)\right]}{\overline{\hat{\phi}}_j+\frac{1}{\psi}\left(\bar{\theta}B_{ij}-\underline{\theta}B_{jj}\right)}$$

or equivalently

$$\overline{\hat{\phi}}_j\left(\frac{\mathbb{E}\left[\hat{\phi}_j\right]}{\overline{\hat{\phi}}_j}-\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\right)+\frac{1}{\psi}\left[\left(\frac{(1+\lambda)}{\lambda}\mathrm{var}\left(\theta\right)+1-\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\bar{\theta}\right)B_{ij}+\left(\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\underline{\theta}-1\right)B_{jj}\right]<0. \tag{37}$$

Inequalities (36) and (37) should hold for any feasible $\mathbf{B}\equiv\left\{B_{ii},B_{ij},B_{ji},B_{jj}\right\}$. Since both inequalities are linear in $\mathbf{B}$, it suffices to show that they hold for all combinations of extreme values. The extreme values are computed by solving a non-linear equation for the maximum values of ϕ_i and ϕ_j. We start with inequality (36). We first check the case in which $B_{ji}=0$ and $B_{ii}=\frac{y}{\overline{\phi}_1^*}$. Here $\overline{\phi}_1^*$ solves $\phi_{I1}^*=\overline{\hat{\phi}}_i-\frac{1}{\psi}\underline{\theta}\frac{y}{\phi_{I1}^*}$. We take the largest root of this equation which is given by $\overline{\phi}_1^*=\frac{\overline{\hat{\phi}}_i+\sqrt{\left(\overline{\hat{\phi}}_i\right)^2-4\frac{1}{\psi}\underline{\theta}y}}{2}$. Substituting these values in (36) yields the following inequality

$$\overline{\hat{\phi}}_i\left(\frac{\mathbb{E}\left[\hat{\phi}_i\right]}{\overline{\hat{\phi}}_i}-\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\right)+\frac{1}{\psi}\left[\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\underline{\theta}+\frac{\mathrm{var}\left(\theta\right)}{\lambda}-1\right]\frac{y}{\overline{\phi}_1^*}<0. \tag{38}$$

Second we check the other case in which $B_{ji}=\frac{y}{\overline{\phi}_2^*}$ and $B_{ii}=0$. Here $\overline{\phi}_2^*$ is the largest root that solves $\phi_{I2}^*=\overline{\hat{\phi}}_i+\frac{1}{\psi}\bar{\theta}\frac{y}{\phi_{I2}^*}$, which is given by $\overline{\phi}_2^*=\frac{\overline{\hat{\phi}}_i+\sqrt{\left(\overline{\hat{\phi}}_i\right)^2+4\frac{1}{\psi}\bar{\theta}y}}{2}$. Substituting these values in in (36) yields the following inequality

$$\overline{\hat{\phi}}_i\left(\frac{\mathbb{E}\left[\hat{\phi}_i\right]}{\overline{\hat{\phi}}_i}-\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\right)+\frac{1}{\psi}\left[1-\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\bar{\theta}\right]\frac{y}{\overline{\phi}_2^*}<0. \tag{39}$$

Finally we also check the case in which both B_{ji},B_{ii} are at their maximum values. In this case $B_{ji}=B_{ii}=\frac{y}{\overline{\phi}_l^*}$,where $\overline{\phi}_l^*$ is defined as in the baseline model. Substituting these values in in (36) yields the following inequality

$$\overline{\hat{\phi}}_i\left(\frac{\mathbb{E}\left[\hat{\phi}_i\right]}{\overline{\hat{\phi}}_i}-\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\right)+\frac{1}{\psi}\left(\frac{\mathrm{var}\left(\theta\right)}{\lambda}-\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\left(\bar{\theta}-\underline{\theta}\right)\right)\frac{y}{\overline{\phi}_l^*}<0 \tag{40}$$

We follow a symmetric approach with inequality (37). We first check the case in which

$B_{ij} = 0$ and $B_{jj} = \frac{y}{\overline{\phi}_1^*}$. Substituting these values in (37) yields the following inequality

$$\overline{\hat{\phi}_j}\left(\frac{\mathbb{E}\left[\hat{\phi}_j\right]}{\overline{\hat{\phi}_j}} - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\right) + \frac{1}{\psi}\left(\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\underline{\theta} - 1\right)\frac{y}{\overline{\phi}_1^*} < 0. \tag{41}$$

Second we check the other case in which $B_{ij} = \frac{y}{\overline{\phi}_2^*}$ and $B_{jj} = 0$. Substituting these values into (37) yields the following inequality

$$\overline{\hat{\phi}_j}\left(\frac{\mathbb{E}\left[\hat{\phi}_j\right]}{\overline{\hat{\phi}_j}} - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\right) + \frac{1}{\psi}\left(\frac{(1+\lambda)}{\lambda}\operatorname{var}(\theta) + 1 - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\bar{\theta}\right)\frac{y}{\overline{\phi}_2^*} < 0. \tag{42}$$

Finally we also check the case in which both B_{jj}, B_{ij} are at their maximum values. In this case $B_{jj} = B_{ij} = \frac{y}{\overline{\phi}_l^*}$,where $\overline{\phi}_l^*$ is defined as in the baseline model. Substituting these values in (37) yields the following inequality

$$\overline{\hat{\phi}_j}\left(\frac{\mathbb{E}\left[\hat{\phi}_j\right]}{\overline{\hat{\phi}_j}} - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\right) + \frac{1}{\psi}\left(\frac{(1+\lambda)}{\lambda}\operatorname{var}(\theta) - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\left(\bar{\theta} - \underline{\theta}\right)\right)\frac{y}{\overline{\phi}_l^*} < 0. \tag{43}$$

Now we need to show that inequalities (38) - (43) are satisfied for values of policy risk such that $\frac{\mathbb{E}[\hat{\phi}_i]}{\overline{\hat{\phi}_i}} = \frac{\mathbb{E}[\hat{\phi}_j]}{\overline{\hat{\phi}_j}} \leqslant \mu_2$. First note that (41) always holds since the second term is negative. Additionally, if (42) holds then (39) is also satisfied. Finally, if (43) holds then (40) is also satisfied. This leaves us with (38), (42) and (43). It is worth noting that $\overline{\phi}_2^* > \overline{\phi}_l^* > \overline{\phi}_1^*$. Also recall that $\frac{\mathbb{E}[\hat{\phi}_i]}{\overline{\hat{\phi}_i}} = \frac{\mathbb{E}[\hat{\phi}_j]}{\overline{\hat{\phi}_j}} \leqslant \mu_2$ implies that

$$\overline{\hat{\phi}_j}\left(\frac{\mathbb{E}\left[\hat{\phi}_j\right]}{\overline{\hat{\phi}_j}} - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\right) + \frac{1}{\psi}\left(\frac{(2+\lambda)}{\lambda}\operatorname{var}(\theta) - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\left(\bar{\theta} - \underline{\theta}\right)\right)\frac{y}{\overline{\phi}_l^*} < 0. \tag{44}$$

It then follows that if $\frac{\mathbb{E}[\hat{\phi}_i]}{\overline{\hat{\phi}_i}} = \frac{\mathbb{E}[\hat{\phi}_j]}{\overline{\hat{\phi}_j}} \leqslant \mu_2$ (or equivalently if (44) holds), then (43) is satisfied. Additionally, note that if we use the assumption that $\operatorname{var}(\theta) > \lambda$ then (44) implies (42). Finally, we show that (44) implies (38). To show this we must have that

$$\left(\overline{\hat{\phi}}_i + \sqrt{\overline{\hat{\phi}}_i^2 - 4\frac{1}{\psi}\underline{\theta}y}\right)\left(\frac{(2+\lambda)}{\lambda}\operatorname{var}(\theta) - \frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\left(\bar{\theta} - \underline{\theta}\right)\right) >$$
$$\left(\overline{\hat{\phi}}_i + \sqrt{\overline{\hat{\phi}}_i^2 + 4\frac{1}{\psi}\left(\bar{\theta} - \underline{\theta}\right)y}\right)\left(\frac{\operatorname{var}(\theta) - \lambda}{\lambda} + \underline{\theta}\frac{\mathbb{E}\left(\phi_f\right)}{\overline{\phi}_f}\right).$$

This can be rewritten as

$$\begin{aligned}
&\frac{\operatorname{var}(\theta)}{\lambda}\left[\left(\overline{\hat{\phi}}_i+\sqrt{\overline{\hat{\phi}}_i^2-4\frac{1}{\psi}\underline{\theta}y}\right)(2+\lambda)-\left(\overline{\hat{\phi}}_i+\sqrt{\overline{\hat{\phi}}_i^2+4\frac{1}{\psi}\left(\overline{\theta}-\underline{\theta}\right)y}\right)\right]\\
-&\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\left[\left(\overline{\hat{\phi}}_i+\sqrt{\overline{\hat{\phi}}_i^2-4\frac{1}{\psi}\underline{\theta}y}\right)\left(\overline{\theta}-\underline{\theta}\right)\right]\\
&+\left(\overline{\hat{\phi}}_i+\sqrt{\overline{\hat{\phi}}_i^2+4\frac{1}{\psi}\left(\overline{\theta}-\underline{\theta}\right)y}\right)\left[1-\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\underline{\theta}\right]>0. \qquad (45)
\end{aligned}$$

First consider the term

$$\begin{aligned}
&\left(\overline{\hat{\phi}}_i+\sqrt{\overline{\hat{\phi}}_i^2-4\frac{1}{\psi}\underline{\theta}y}\right)(2+\lambda)-\left(\overline{\hat{\phi}}_i+\sqrt{\overline{\hat{\phi}}_i^2+4\frac{1}{\psi}\left(\overline{\theta}-\underline{\theta}\right)y}\right)\\
\geqslant&\left(\overline{\hat{\phi}}_i+\sqrt{\overline{\hat{\phi}}_i^2-4\frac{1}{\psi}\underline{\theta}y}\right)(2+\lambda)-\left(\overline{\hat{\phi}}_i+\sqrt{\overline{\hat{\phi}}_i^2-4\frac{1}{\psi}\underline{\theta}y}+\sqrt{4\frac{1}{\psi}\overline{\theta}y}\right)\\
=&(1+\lambda)\,\overline{\hat{\phi}}_i-\sqrt{4\frac{1}{\psi}\overline{\theta}y}+(1+\lambda)\sqrt{\overline{\hat{\phi}}_i^2-4\frac{1}{\psi}\underline{\theta}y}\\
\geqslant&0
\end{aligned}$$

if $\overline{\hat{\phi}}-\sqrt{4\frac{1}{\psi}\overline{\theta}y}\geqslant 0$, which is a condition we need for $\overline{\phi}_1^*$ to be well-defined. Given this, the first two lines of (45) are greater than

$$\frac{\mathbb{E}\left[\phi_f\right]}{\overline{\phi}_f}\left(\left(\lambda-\left(\overline{\theta}-\underline{\theta}\right)\right)\overline{\hat{\phi}}+\overline{\hat{\phi}}-\sqrt{4\frac{1}{\psi}\overline{\theta}y}+\left(1+\lambda-\left(\overline{\theta}-\underline{\theta}\right)\right)\sqrt{\overline{\hat{\phi}}^2-4\frac{1}{\psi}\underline{\theta}y}\right),$$

which is positive if $\lambda\geqslant\left(\overline{\theta}-\underline{\theta}\right)$. Hence, we showed that (38) - (43) are satisfied for values of policy risk such that $\frac{\mathbb{E}[\hat{\phi}_i]}{\overline{\hat{\phi}}_i}=\frac{\mathbb{E}[\hat{\phi}_j]}{\overline{\hat{\phi}}_j}\leqslant\mu_2$. Finally, the cutoff value μ_2^I is defined as the smallest cutoff value such that (38) - (43) are satisfied. Q.E.D.

D Figures

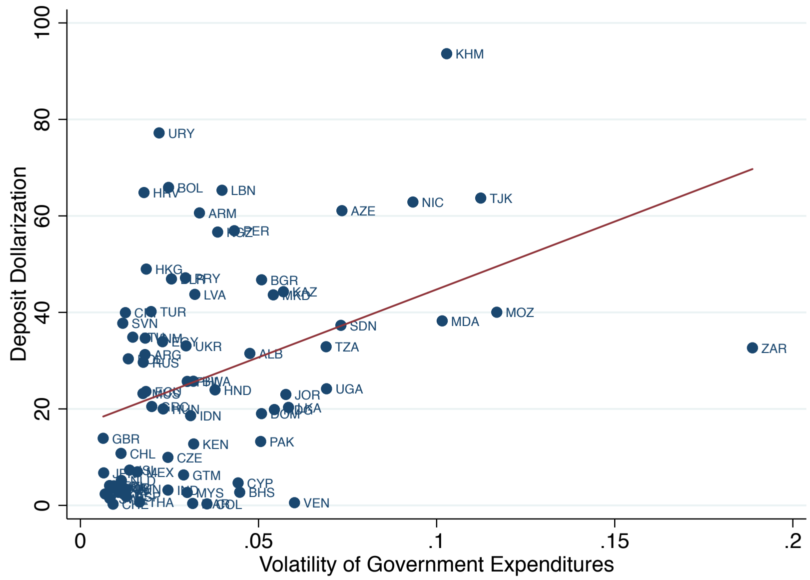

Figure 4: Financial Dollarization and Fiscal Policy Risk

Notes: Deposit dollarization is measured as the share of bank deposits denominated in US dollars. The horizontal axis shows the volatility of government expenditures across countries. The sample period is 1980-2017. Volatility is computed as the standard deviation of the cyclical component of real government expenditures.

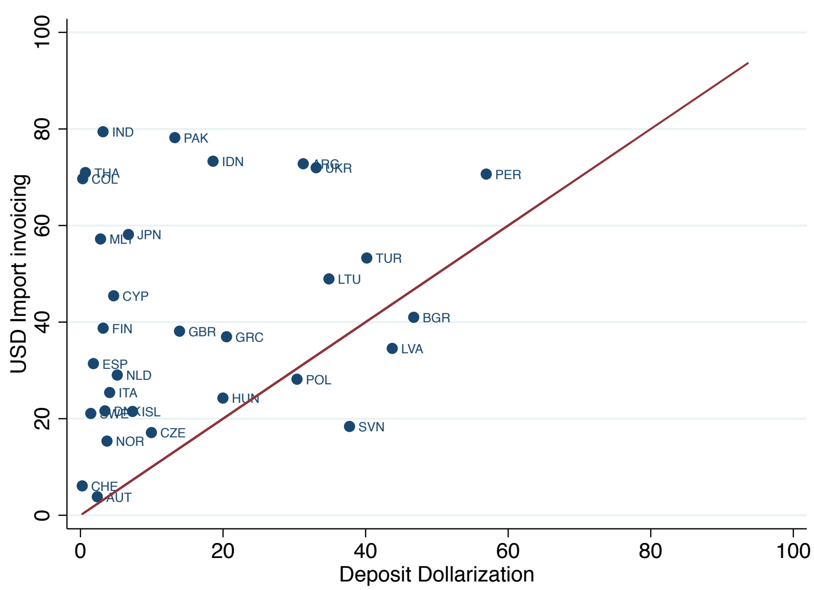

Figure 5: Financial versus International Trade Dollarization

Notes: Financial dollarization is measured as the share of bank deposits denominated in US dollars. The source of this data is Levy-Yeyati (2006). Inflation volatility is measured as the standard deviation of annual inflation for the period 1980-2017. The source of this data is IFS. Trade Dollarization is computed as the share of imports, from destinations other than the US, invoiced in US dollars. The source of this data is Gopinath (2016).