Revisiting Taste Change in Cost-of-Living Measurement*

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Abstract

This paper derives conditional cost-of-living indexes (COLI) for the Constant Elasticity of Substitution model in the presence of taste change. Recent proposals to incorporate changing tastes reflect a different conceptual target (an unconditional COLI) from a consumer price index (a conditional COLI), and a strong implicit assumption (cardinal utility). Using Nielsen retail scanner data for food and beverage products, I find that tastes can dominate prices in unconditional COLI estimates, while they have smaller impacts on conditional COLI. Using CPI data, I find that category-level tastes have a relatively minor average effect on an all-items price index.

Keywords: Cost of living index; price index; taste change

JEL Codes: C43, D12, E31

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1 Introduction

The target of a consumer price index (CPI) is typically a theoretical construct known as a cost-of-living index (COLI), which measures the proportional expenditure change required for a consumer to be indifferent between two price situations, such as periods of time (Pollak, 1989). The consumer models underlying COLI formulas are often specified with constant preferences or tastes between periods. As many have observed (e.g., Heien and Dunn (1985)), this is unrealistic empirically. In light of this, the conceptual target of a CPI is commonly (though not universally) agreed to be a conditional COLI, which aims to isolate the effect of price change by holding constant non-price factors including tastes (ILO, 2004). For example, the Tornqvist formula used by the Bureau of Labor Statistics (BLS) for the U.S. Chained Consumer Price Index for All Urban Consumers (C-CPI-U) approximates the COLI that conditions on the set of average tastes between those pertaining to the index's reference and comparison periods (Caves, Christensen, and Diewert, 1982).

This paper shows that for the Constant Elasticity of Substitution (CES) model, variants of the formula proposed by Lloyd (1975) and Moulton (1996) are exact for COLI that condition on either the reference or comparison period tastes.[1] I also show that an average of these indexes is "flexible" in the sense of Diewert (1976), and approximates a COLI that conditions on an intermediate level of tastes. Using retail scanner data for food and beverage products, I estimate that COLI conditioning on comparison period tastes exceed those conditioning on reference period tastes by an average of 0.5 to 2.9 percentage points per year, depending on the category, with COLI conditioning on intermediate tastes falling roughly in the middle. Conditional COLI that fix individual product tastes at either the reference or comparison period levels (rather than averages) are infeasible within current data constraints at the BLS. However, I find that CPI aggregates that similarly account for category-level tastes are affected relatively little by the choice of taste vector.

[1] Per Diewert (1976), a price index formula is exact if it equals a ratio of unit expenditure functions for a given set of preferences.

This paper also aims to clarify issues raised by the recent literature concerning taste change. Several papers have advocated constructing a COLI by evaluating each of its constituent expenditure functions at period-specific taste vectors. In particular, Redding and Weinstein (2020) (henceforth RW) argue that assuming constant tastes causes a positive "taste-shock bias" in COLI estimates.[2] Using the CES model and household scanner data, they estimate this bias to be around 0.4 percentage points per year on average, which would place it among the largest sources of bias in the U.S. CPI (Moulton, 2018). However, a careful definition of a conditional COLI does not actually assume tastes are constant (F. M. Fisher and Shell, 1972). The researcher needs to choose a taste vector to condition on, but indexes like the aforementioned Tornqvist implicitly make a reasonable choice. In contrast, proposals like RW's target a different theoretical concept—the unconditional COLI. Furthermore, incorporating preference change implicitly treats utility as cardinal. If treating utility as ordinal (which more common in economics), there is no way to interpret a variable-taste price index as "a money metric measure of being equally well off in two periods" (Balk, 1989).[3] My empirical analysis suggests estimated taste effects dominate those of prices when using RW's method, turning low or moderate conditional COLI increases into unconditional COLI declines. To interpret differences as evidence of taste shock bias, however, requires a strong assumption about the underlying utility function and ignores significant differences in the intended scope of the indexes being compared.

2 Existing literature

The economic approach to consumer price indexes, dating to Konüs (1924), is based on the expenditure function of an optimizing agent. The final version of the C-CPI-U, for example, uses the Tornqvist formula for upper-level aggregation (Cage, Greenlees, and Jackman,

[2] Among others, Hottman and Monarch (2018), Zadrozny (2019), and Ueda, K. Watanabe, and T. Watanabe (2019) also explore indexes of this type.

[3] F. M. Fisher and Shell (1972) argue that holding well-being constant with changing preferences amounts to an "arbitrary intertemporal weighting of utilities."

2003). The Tornqvist is an example of a "superlative" index (Diewert, 1976), meaning it approximates an arbitrary expenditure function. The seminal work of F. M. Fisher and Shell (1972) analyzes conditional and unconditional COLIs in an environment with changing preferences. The appropriate target for a consumer price index is generally considered to be a conditional COLI (National Research Council, 2002; ILO, 2004), which isolates the effect of changing prices by holding preferences (or anything else affecting well-being) fixed. By referencing a specific indifference surface, a conditional COLI requires only the ordinal properties of utility functions.

An unconditional COLI, on the other hand aims to track changes in expenditure whether driven by price change or preference change. As a consequence, the index may increase or decrease even if prices are constant. Unconditional COLI seek to answer interesting questions, and may be more comprehensive as *cost-of-living* concepts.[4] However, they do not contain any additional information on *prices* than what is already conveyed by a conditional COLI. Furthermore, when preferences change, an unconditional COLI must reference a cardinal utility level in order to compare expenditures across varying indifference maps.[5] Allowing pure taste change effects, therefore, requires a stronger assumption about preferences than is needed for a conditional COLI. Interested in the unconditional COLI concept, Balk (1989) proposes an index that attempts to hold constant some notion of well-being without fixing the cardinal utility level. The method tracks the change in expenditure required to reach an indifference surface that passes through a fixed bundle. Gábor-Tóth and Vermeulen (2018) apply this method to European scanner data and find the average annual contribution of taste change to be -1.1 percentage points.

As noted by F. M. Fisher and Shell (1972), deriving the relationship between preference change and either type of COLI is difficult without either assuming a specific parameteriza-

[4]National Research Council (2002) gives several situations where a conditional COLI may be inadequate, including medical products whose quality is difficult to separate from general health status, and regional comparisons where fixing weather conditions may make little sense.

[5]For this reason, unconditional COLI are sometimes called "cardinal," whereas conditional COLI are sometimes called "ordinal" (Muellbauer, 1975).

tion (allowing changes on a small subset of items only), or restricting attention to a particular utility function (as this paper does). Tastes do not pose much of a measurement challenge when the objective is a conditional COLI, however, even when the index formula does not appear to explicitly account for tastes. Caves, Christensen, and Diewert (1982), Diewert (2001), and Feenstra and Reinsdorf (2007) provide conditions under which the Tornqvist, Fisher, and Sato-Vartia price indexes, respectively, are exact for or approximate COLI that condition on some notion of average tastes. Section 3 discusses these results further, while Section 4 complements them by showing that variants of the Lloyd-Moulton index are also exact for conditional COLI in the CES case.

In order to isolate the issue of preference change, I focus my empirical analysis on matched-model indexes, i.e., those defined over a fixed set of specific product varieties with constant tangible attributes. RW's taste-shock bias is defined in association with a matched-model index. This is not to suggest that improvements to a matched model index should not be pursued for reasons of representativity. For instance, product turnover can cause matched model indexes to miss initial price declines for new items (Feenstra, 1994), as well as selection bias in the set of matched items (Pakes, 2003). Appendix B shows how with product turnover, it is possible to bound a conditional COLI in the CES case by applying Feenstra (1994). Preference change is also fundamentally different from quality change, though the two may have similar effects on relative demand. Price change for the matched model is measurable without quality adjustment, since the set of items and their associated bundles of attributes are constant by definition. Though the two issues are similar mechanically (F. M. Fisher and Shell, 1972), taste-shock bias should not be confused with quality bias. If item definitions are not constant, then whether or not demand shifts are attributed to quality changes or taste changes can have large effects on index estimation (Nevo, 2003).

3 Cost-of-living theory

A cost-of-living index is a ratio of two expenditure functions. It is helpful in this case to first specify a set of preferences rather than jump straight to a utility function. Consider an ordinal preference relation, denoted $\succeq$, on a commodity space $\mathcal{Q} \subseteq \mathbb{R}^N$, which is made up of bundles $\boldsymbol{q}$. We assume:

Assumption 3.1 *The representative consumer's preference relation $\succeq$ is i) rational (complete and transitive), ii) continuous, iii) convex, and iv) monotone.*

Assumption 3.1 is sufficient for the existence of a utility function, $u : \mathcal{Q} \to \mathbb{R}$ which represents $\succeq$, in the sense that we have $\boldsymbol{q} \succeq \boldsymbol{q}' \Leftrightarrow u(\boldsymbol{q}; \succeq) \geq u(\boldsymbol{q}'; \succeq)$ (Mas-Colell, Whinston, Green, et al., 1995). Due to the ordinal nature of preferences, the function u, is not unique. Any positive monotone transformation of u will also represent $\succeq$. Let $\boldsymbol{p}$ denote a vector of prices. We then assume:

Assumption 3.2 *Facing prices $\boldsymbol{p}$, the agent chooses $\boldsymbol{q}$ to maximize utility subject to a budget constraint, or equivalently, to minimize expenditure subject to a utility constraint.*

Let $\boldsymbol{h}(\boldsymbol{p}, \bar{u}; \succeq) = \underset{\boldsymbol{q}}{\operatorname{argmin}}\ \boldsymbol{p} \cdot \boldsymbol{q}$ s.t. $u(\boldsymbol{q}; \succeq) \geq \bar{u}$ denote the Hicksian demand function, which represents the quantities that minimize expenditure. The expenditure function is given as $C(\boldsymbol{p}, \bar{u}; \succeq) = \boldsymbol{p} \cdot \boldsymbol{h}(\boldsymbol{p}, \bar{u}; \succeq)$.

3.1 Conditional COLI

A conditional or ordinal COLI is defined as the minimum expenditure required for an agent to be indifferent between two price situations. I label the reference situation 0 and the comparison situation 1. This paper focuses on intertemporal comparisons, but the general theory accommodates other possibilities (e.g., regional comparisons).

Definition 3.1 *(F. M. Fisher and Shell, 1972; Pollak, 1989) The class of conditional cost-*

of-living indexes is given by:

$$\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1, \bar{u}; \succeq) = \frac{C(\boldsymbol{p}_1, \bar{u}; \succeq)}{C(\boldsymbol{p}_0, \bar{u}; \succeq)}, \tag{1}$$

for a given $\bar{u}$ and $\succeq$.

The combination of $\succeq$ (preference relation) and $\bar{u}$ (location) determine the specific indifference surface on which Φ is based. Two immediate candidates for preferences to plug in are $\succeq_0$ and $\succeq_1$, corresponding to the reference and comparison periods, respectively. It is worth emphasizing that the oft-cited bounding results for the Laspeyres and Paasche indexes are one-way only, i.e., the Laspeyres is an upper bound for $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1, \bar{u}_0; \succeq_0)$, and the Paasche is a lower bound for $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1, \bar{u}_1; \succeq_1)$. F. M. Fisher and Shell (1972) argue that from the standpoint of intertemporal compensation, the most interesting COLI is

$$\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1, \bar{u}_1^*; \boldsymbol{\varphi}_1) = \frac{C(\boldsymbol{p}_1, \bar{u}_1^*; \boldsymbol{\varphi}_1)}{C(\boldsymbol{p}_0, \bar{u}_1^*; \boldsymbol{\varphi}_1)}, \tag{2}$$

where $\bar{u}_1^*$ is the hypothetical utility that the consumer would receive facing the period 0 budget constraint with period 1 preferences. F. M. Fisher and Shell (1972) and others argue that a COLI based on $\succeq_1$ is more relevant for public policy than one based on the obsolete preferences $\succeq_0$, but Pollak (1989) notes that in principle, $\succeq$ need not be linked to either the reference or comparison situations. Indeed, two of the parameter-free price indexes discussed in the next subsection are exact for COLI based on average indifference surfaces.

3.2 Parameter-free COLI

Under Assumption 3.2, the observed market expenditures $\boldsymbol{p}_0 \cdot \boldsymbol{q}_0$ and $\boldsymbol{p}_1 \cdot \boldsymbol{q}_1$ equal the expenditure levels $C(\boldsymbol{p}_0, \bar{u}_0, \succeq_0)$ and $C(\boldsymbol{p}_1, \bar{u}_1, \succeq_1)$, respectively. Since, Eq. 1 holds the indifference surface fixed, however, estimation generally requires knowledge of the expenditure function for the given set of preferences.

Nevertheless, some well-known price index formulas are exact for conditional COLI, precluding any need for structural estimation. These indexes and their components are defined below. Let i index items or varieties, and denote the set of items as $\mathcal{I}$, which has dimension N.

Definition 3.2 *The Fisher price index*

$$P_F(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1) = \sqrt{P_L P_P}, \tag{3}$$

where $P_L(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1) = \frac{\sum_{i\in\mathcal{I}} p_{i1}q_{i0}}{\sum_{i\in\mathcal{I}} p_{i0}q_{i0}}$ *is the Laspeyres index, and* $P_P(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1) = \frac{\sum_{i\in\mathcal{I}} p_{i1}q_{i1}}{\sum_{i\in\mathcal{I}} p_{i0}q_{i1}}$ *is the Paasche index.*

Definition 3.3 *The Tornqvist price index*

$$P_T(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1) = \prod_{i\in\mathcal{I}} \left(\frac{p_{i1}}{p_{i0}}\right)^{0.5(s_{i0}+s_{i1})}, \tag{4}$$

where $s_{it} = \frac{p_{it}q_{it}}{\sum_{j\in\mathcal{I}} p_{jt}q_{jt}}, t = 0, 1.$

Definition 3.4 *The Sato-Vartia price index*

$$P_{SV}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1) = \prod_{i\in\mathcal{I}} \left(\frac{p_{i1}}{p_{i0}}\right)^{w_i}, \tag{5}$$

where $w_i = \left[\frac{s_{i1}-s_{i0}}{\ln s_{i1} - \ln s_{i0}}\right] / \left[\sum_{k\in\mathcal{I}} \frac{s_{k1}-s_{k0}}{\ln s_{k1} - \ln s_{k0}}\right].$

Suppose tastes (or environmental variables, as referred to in Diewert, 2001) are represented by the vector $\boldsymbol{\varphi}$, as will be the case in the following section on CES preferences. Diewert (2001) showed that there exists a u^* and $\boldsymbol{\varphi}^*$ such that $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1, u^*; \boldsymbol{\varphi}^*)$ is bounded by the Laspeyres and Paasche indexes, where $\bar{u}_0 \leq u^* \leq \bar{u}_1$ and $\varphi_{i0} \leq \varphi_i^* \leq \varphi_{i1}$, $i = 1, \ldots, N$. If the Laspeyres and Paasche are close numerically, a symmetric average like the Fisher index approximates this COLI. In addition, under the assumption that the expenditure function is translog, Caves, Christensen, and Diewert (1982) showed that the Tornqvist price index is exact for the geometric average of the COLI based on period 0 preferences and the COLI based on period 1 preferences. Due to translog functional form, this is equivalent to the COLI evaluated at the geometric averages of the taste parameters and utilities, respectively. The Tornqvist index is also attractive because the translog expenditure function approximates arbitrary expenditure functions to the second order. Finally, the Sato-Vartia index is exact for the CES COLI that conditions on intermediate levels of the tastes (Feenstra and Reinsdorf, 2007). Each of these results relates to an indifference surface that is, loosely speaking, an average of the base and current period indifference surfaces. Of course, the measurement of substitution effects (responses to relative price change), may change depending on which indifference surface the COLI is based, and so interpretations should be made carefully.

3.3 Unconditional COLI

An unconditional or cardinal COLI measures the change in expenditure required for the consumer to achieve the same utility level in the comparison period as they experienced in the reference period.

Definition 3.5 *(Muellbauer, 1975) The class of cardinal or unconditional COLI is given by:*

$$\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1, \bar{u}; \succeq_0, \succeq_1) = \frac{C(\boldsymbol{p}_1, \bar{u}; \succeq_1)}{C(\boldsymbol{p}_0, \bar{u}; \succeq_0)}, \tag{6}$$

for some $\bar{u}$. RW and others estimate this ratio when $C(\boldsymbol{p}, \bar{u}; \succeq)$ is the CES unit expenditure function, but they derive it for other models as well. Unless preferences are constant, the associated quantities $\boldsymbol{h}(\boldsymbol{p}_0, \bar{u}; \succeq_0)$ and $\boldsymbol{h}(\boldsymbol{p}_1, \bar{u}; \succeq_1)$ do not lie on the same indifference surface, even though both are labeled $\bar{u}$. Therefore, the expenditure comparison is only meaningful if the utility levels can be compared. This amounts to starting from the following instead of Assumption 3.1.

Assumption 3.3 *The utility function $u(\boldsymbol{q};\succeq)$ is a cardinal measure of the representative consumer's well-being.*

The following decomposition of an unconditional COLI is illustrative of the difference in intended scope.

$$\ln \Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1, \bar{u}; \succeq_0, \succeq_1) = \ln \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1, \bar{u}; \succeq_1) + \ln \left[\frac{C(\boldsymbol{p}_0, \bar{u}; \succeq_1)}{C(\boldsymbol{p}_0, \bar{u}; \succeq_0)}\right] \tag{7}$$

Eq. 7 decomposes the unconditional COLI into two parts; a price effect equal to a conditional COLI, and a pure taste change effect $C(\boldsymbol{p}_0, \bar{u}; \succeq_1)/C(\boldsymbol{p}_0, \bar{u}; \succeq_0)$. It is straightforward to compare the unconditional COLI with other conditional COLI in a similar fashion. Equation 7 describes the sense in which Φ_U is "unconditional" in that the last term aims to capture the impact of factors other than prices (National Research Council, 2002). It is also apparent that the contribution of price change is completely captured by the ordinal index, and that the pure taste change component is what depends on cardinal utility.

4 CES Preferences

Section 3 described a few conditional COLI that can be estimated with prices and quantities only. In general, however, estimating a COLI requires specifying and estimating a model of preferences. For comparability to other studies, I focus on the CES model for the rest of this paper. The CES model is a workhorse for its tractability, though it implies significant restrictions on price and income elasticities. Appendix D derives similar results for the homothetic translog expenditure function, which is more flexible, but requires estimating many more parameters. Specification error is a potential issue for an unconditional COLI, as well as COLI that condition on a specific period's tastes, as these depend on the model's ability to separate price responses from preference shifts (Martin, 2019).

We now assume:

Assumption 4.1 *The representative agent's expenditure function has the form:*

$$C(\boldsymbol{p}, \bar{u}; \boldsymbol{\varphi}) = \bar{u} \left[\sum_{i \in \mathcal{I}} \left(\frac{p_i}{\varphi_i} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{8}$$

For the purposes of a COLI, we take $\bar{u} = 1$ without further loss of generality (preferences are homothetic) and suppress the argument from further notation. The parameter $\sigma \neq 1$ is the elasticity of substitution, which we assume is constant over time, and so the notation now refers to preferences through the vector of demand shifters $\boldsymbol{\varphi}$. The agent's optimal expenditure shares are given by

$$s_i(\boldsymbol{p}; \boldsymbol{\varphi}) = \frac{p_i h_i(\boldsymbol{p}; \boldsymbol{\varphi})}{\sum_{j \in \mathcal{I}} p_j h_j(\boldsymbol{p}; \boldsymbol{\varphi})} = \frac{(p_i/\varphi_i)^{1-\sigma}}{\sum_{j \in \mathcal{I}} (p_j/\varphi_j)^{1-\sigma}} = \frac{(p_i/\varphi_i)^{1-\sigma}}{[C(\boldsymbol{p}; \boldsymbol{\varphi})]^{1-\sigma}}, \; i = 1, \ldots, N. \tag{9}$$

Under Assumptions 3.2 and 4.1, the observed expenditure shares $s_{it} = \frac{p_{it} q_{it}}{\sum_{j \in \mathcal{I}} p_{jt} q_{jt}}$ equal the optimal expenditure shares $s_i(\boldsymbol{p}_t; \boldsymbol{\varphi}_t)$. The indexes in the following subsections make use of this equation to estimate conditional and unconditional COLI.

Eq. 10 shows that under Assumption 4.1, the log expenditure share of item i in period t can be decomposed into its log price, the log expenditure function, and the log of the taste parameters.

$$\ln s_{it} = (1 - \sigma) \ln p_{it} + (\sigma - 1) \ln [c(\boldsymbol{p}_t; \boldsymbol{\varphi}_t)] + (\sigma - 1) \ln \varphi_{it} \tag{10}$$

As RW note, the taste parameters provide a source of idiosyncratic error which is necessary for empirical analysis.

4.1 Exact Price Indexes for CES Preferences

As previously mentioned, the index proposed by Sato (1976) and Vartia (1976) (see Eq. 5) is exact for the CES COLI that conditions on an intermediate taste vector $\bar{\boldsymbol{\varphi}}$ (Feenstra and Reinsdorf, 2007).[6] The salient question then is how do price comparisons using $\bar{\boldsymbol{\varphi}}$ compare to

[6]Each element $\bar{\varphi}_i$ of $\bar{\boldsymbol{\varphi}}$ lies between $\varphi_{i0}/\prod_{i \in \mathcal{I}} \varphi_{i0}^{w_i}$ and $\varphi_{i1}/\prod_{i \in \mathcal{I}} \varphi_{i1}^{w_i}$.

price comparisons using $\boldsymbol{\varphi}_0$, $\boldsymbol{\varphi}_1$ or some other tastes? Exact price indexes for reference period or current period tastes already exist for the CES model, though to my knowledge, their interpretation as such is novel. Lloyd (1975) and Moulton (1996) developed the following price index in the setting of constant tastes.

Definition 4.1 *Lloyd-Moulton Index*

$$P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = \left\{ \sum_{i \in \mathcal{I}} s_{i0} \left(\frac{p_{i1}}{p_{i0}} \right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}} \tag{11}$$

Similarly, the time-antithesis (I. Fisher, 1922), or "backwards" version of the Lloyd-Moulton index can be formed.

Definition 4.2 *Backwards Lloyd-Moulton Index*

$$P_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = \left\{ \sum_{i \in \mathcal{I}} s_{i1} \left(\frac{p_{i0}}{p_{i1}} \right)^{1-\sigma} \right\}^{\frac{-1}{1-\sigma}} \tag{12}$$

The Lloyd-Moulton and Backwards Lloyd-Moulton are exact for the COLI that condition on reference period tastes and comparison period tastes, respectively. To see this, start with Eq. 4.1 for $P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma)$. Use the right hand side of Eq. 9 to substitute for s_{i0}, re-arrange, and use Eq. 8. We then have

$$\begin{aligned}
P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) &= \left\{ \sum_{i \in \mathcal{I}} s_{i0} \left(\frac{p_{i1}}{p_{i0}} \right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}} \\
&= \left\{ \sum_{i \in \mathcal{I}} \frac{(p_{i0}/\varphi_{i0})^{1-\sigma}}{[C(\boldsymbol{p}_0, \boldsymbol{\varphi}_0)]^{1-\sigma}} \left(\frac{p_{i1}}{p_{i0}} \right)^{1-\sigma} \right\}^{\frac{1}{1-\sigma}} \\
&= \frac{\left\{ \sum_{i \in \mathcal{I}} (p_{i1}/\varphi_{i0})^{1-\sigma} \right\}^{\frac{1}{1-\sigma}}}{C(\boldsymbol{p}_0, \boldsymbol{\varphi}_0)} \\
&= \frac{C(\boldsymbol{p}_1, \boldsymbol{\varphi}_0)}{C(\boldsymbol{p}_0, \boldsymbol{\varphi}_0)} \\
&= \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0)
\end{aligned}$$

The case of $P_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma)$ is very similar. The following summarizes the result.

Proposition 1 *Under Assumption 4.1,*
$P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0)$, *and* $P_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1)$.

This result implies an additional superlative index may be of interest in the context of changing tastes. Consider the geometric mean of the Lloyd-Moulton indexes, denoted P_{LMM}. It has the form:

$$\begin{aligned} P_{LMM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) &= \left[P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) P_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma)\right]^{\frac{1}{2}} \\ &= \left[\frac{\sum_{i \in \mathcal{I}} s_{i0} \left(\frac{p_{i1}}{p_{i0}}\right)^{1-\sigma}}{\sum_{i \in \mathcal{I}} s_{i1} \left(\frac{p_{i0}}{p_{i1}}\right)^{1-\sigma}}\right]^{\frac{1}{2(1-\sigma)}} \end{aligned} \tag{13}$$

Eq. 13 shows P_{LMM} is, in fact, the Quadratic Mean of Order r price index, where $r = 2(1-\sigma)$ Diewert (1976). This implies the following result.

Proposition 2 *Under assumption 4.1, the Quadratic Mean of Order r price index is exact for the geometric mean of two CES conditional COLI,* $[\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0)\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1)]^{\frac{1}{2}}$, *where* $r = 2(1 - \sigma)$.

This is important because superlative indexes like this one have been shown to approximate each other to the second order (Diewert, 1978). This implies P_{LMM} should be somewhat robust to errors in estimation of σ or departures from CES functional form. Additionally, the availability of both P_{LMM} and P_{SV} for the CES case offers an interesting potential contrast. One averages COLI evaluated at different tastes, while the other is a COLI evaluated at an average of the tastes. A priori, we would not necessarily expect them to give identical answers, though their estimates in Section 5 are very similar.

4.2 RW's CES Common Varieties Index

RW propose a price index to target the CES unconditional COLI. A practical challenge concerns the scale of tastes. Given knowledge of σ, Eq. 9 implies that observed expenditure shares and prices identify φ_{it} up to a time-varying scale factor. The CES conditional COLI are invariant to the scale of the tastes, but the unconditional is not. RW address this issue by normalizing the φ_{it} to have a constant geometric mean over time, in effect allowing only relative taste changes. Although CES expenditure shares do not depend on the scale of tastes, expenditure levels do, and so this normalization is not free. Changes in the scale of tastes between periods affect the magnitude and possibly the direction of the unconditional COLI, an observation made by Kurtzon (2019).

Using the normalization, RW derive the following estimator for $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1)$:

Definition 4.3 *RW's CES Common Varieties Index (CCV)* [7]

$$P_{CCV}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma) = \exp\left[\frac{1}{N}\sum_{i=1}^{N} \ln\left(\frac{p_{i1}}{p_{i0}}\right) + \frac{1}{\sigma - 1}\frac{1}{N}\sum_{i=1}^{N} \ln\left(\frac{s_{i1}}{s_{i0}}\right)\right], \tag{14}$$

The time-constant scale factor precludes some potentially interesting situations. First, it prohibits systematic increases or decreases in the agent's "efficiency" as a producer of utility (Muellbauer, 1975). This rules out, among other phenomena, the "hedonic treadmill" hypothesis discussed in National Research Council (2002), whereby the agent needs to consume higher quantities over time to remain as well-off. Such general trends in well-being would clearly affect $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1)$ conditional on a set of relative taste changes. Second, the normalization of the unweighted geometric mean is just one of an infinite number of equivalent normalizations, on which the observed expenditure and price data bear no informational content (Kurtzon, 2019). Appendix C discusses these issues further and shows how estimating P_{CCV} over subsets of $\mathcal{I}$ implicitly changes the normalization.

[7] RW's proposed CES Unified Price Index consists of the CCV plus a product turnover adjustment in the style of Feenstra (1994).

Table 1 summarizes the price index formulas discussed in this and the previous section. The following two sections compare them empirically, first using retail scanner data, and then using CPI elementary item-area indexes.

5 Application to Retail Scanner Data

5.1 Data and model estimation

I estimate quarterly price indexes for food and beverage product categories using the CES model introduced in Section 4 and Scantrack, a point-of-sale scanner dataset from The Nielsen Company.[8] The data are similar in scope to Nielsen's household panel, which RW use. The Neilsen retail scanner data has been proposed for use in the CPI by Ehrlich et al. (2019), and similar data is used by the Australian Bureau of Statistics to estimate some food components of its CPI. The data cover the fourth quarter of 2005 through the second quarter of 2010, and include expenditures and quantities for roughly 600,000 universal product codes (UPC) sold by participating grocery, drug, and mass merchandise store chains.[9] Because items are defined by UPC, their characteristics and quality are arguably constant over time (Broda and Weinstein, 2010; Redding and Weinstein, 2020). UPCs are classified according to a structure defined by Nielsen. For instance, UPC 003800040500 is described as "Kellogg's Eggo Round Chocolate Chip 10 count." It belongs the brand module "Kellogg's Eggo," product module "Frozen Waffles/Pancakes/French Toast," product group "Breakfast Foods

[8] I use data for food and beverage products only, though Scantrack data also covers general merchandise, personal care, and other non-food grocery items sold in grocery and drug stores. Scantrack expenditures on nonfood goods equal only about 19% and 12% of comparable Consumer Expenditure Survey and Personal Consumption Expenditure estimates, respectively, suggesting the majority of consumption on these products originates from non-covered retailers (Bureau of Labor Statistics, 2019; Bureau of Economic Analysis, 2019). Furthermore, the degree to which the simple CES model is a suitable approximation for the data may differ between food and nonfood categories. The model assumes no dynamic behavior, i.e., stockpiling or durable goods, and expenditure on a nonfood product (e.g., "Kitchen gadgets") may be a relatively poor proxy for consumption of that product, even at a quarterly frequency.

[9] According to a Nielsen representative, the sample covers 90% of such retail chains and is weighted to be nationally representative. Potential selection bias is a limitation of this and other studies using convenience samples of transactions.

- Frozen," and department "Frozen Foods." Like RW, I calculate quarterly expenditure shares (within product group) and unit value prices by UPC, treating the continental United States as one market. I then winsorize by dropping items whose change in price or value were in the top or bottom one percentile for a given quarter.

Table 2 describes some basic attributes of the dataset. Just over 54% of food and beverage expenditures are from the Dry Grocery department, comprising about two-thirds of the total number of UPCs. Dairy (15%) and Frozen Foods (11%) are the next largest departments by expenditure. Use of these data for consumer price indexes treats retail sales as proxies for consumer expenditures, but they also include purchases by non-households. Total food and beverage expenditures in Scantrack exceed the BLS's Consumer Expenditure Survey (CE) estimates over the same time period by about 66%, while they exceed the Bureau of Economic Analysis's Personal Consumption Expenditure (PCE) estimates by about 9% (Bureau of Labor Statistics, 2019; Bureau of Economic Analysis, 2019).[10]

For each product group, I calculate a series of indexes of the form $P(\boldsymbol{p}_{t-4}, \boldsymbol{p}_t, \boldsymbol{q}_{t-4}, \boldsymbol{q}_t)$, where $P()$ is one of the formulas given in Section 3 or 4. The index for quarter t uses the same quarter in the year prior as its base period, so index values reflect year-over-year price changes. Table 3 presents summary statistics for the four-quarter price relatives $p_{it}/p_{i,t-4}$ pooled over the sample period. Average price relatives exceed one for all departments, ranging from 1.021 for Alcoholic Beverages to 1.037 for Dairy. The distributions are quite dispersed however, with standard deviations within departments ranging from 0.106 for Packaged Meat to 0.165 for Fresh Produce. Relatives are positively skewed in all departments, as one might expect if prices follow an upward trend over time. Compared to a normal distribution (which has kurtosis equal to 3), the distributions of price relatives have higher kurtosis, which indicates thicker tails.

Estimation of the substitution elasticities follows the "double-differencing" method of Feenstra (1994), using panel variation in prices and expenditure shares. This method as-

[10]CE and PCE cover slightly different target populations and rely on different survey methods. See Passero, Garner, and McCully (2014) for a discussion.

sumes $\sigma > 1$, which is reasonable for indexes over similar product varieties. I follow the weighting and estimation procedure of Broda and Weinstein (2010), though as in RW, I do not distinguish between within-brand and across-brand substitutions. Start with Eq. 10 and difference over time and with respect to a reference variety k, which is chosen to be the variety with the largest average market share. Denote the doubled-difference of variable x_{it} as $\Delta^k x_{it} = (x_{it} - x_{i,t-1}) - (x_{kt} - x_{k,t-1})$. For varieties $i = 1, \ldots, N$ and time periods $t = 1, \ldots, T$, the double-differenced demand equation is:

$$\Delta^k \ln s_{it} = -(\sigma - 1)\Delta^k \ln p_{it} + \Delta^k \ln u_{it}, \tag{15}$$

where $\Delta^k \ln u_{it}$ is the double-differenced error. The inverse supply equation is derived by assuming each variety is produced by a distinct firm in monopolistic competition, leading to a pricing equation that is linear in log-expenditure share, with slope depending on the inverse supply elasticity parameter ω. The double-differenced inverse supply equation is then:

$$\Delta^k \ln p_{it} = \frac{\omega}{1+\omega}\Delta^k \ln s_{it} + \Delta^k \ln v_{it}, \tag{16}$$

where $\Delta^k \ln v_{it}$ is the double-differenced supply error. We assume that the double-differenced demand and supply errors are drawn from stationary distributions, have variances that differ by product variety, and are uncorrelated with each other. The parameters can then be estimated using Generalized Method of Moments (Hansen, 1982) based on the moment conditions

$$E\left[\left(\Delta^k \ln p_{it}\right)^2 - \theta_1 \left(\Delta^k \ln s_{it}\right)^2 - \theta_2 \Delta^k \ln p_{it} \Delta^k \ln s_{it}\right] = 0, i = 1, \ldots, N, \tag{17}$$

where $\theta_1 = \frac{\omega}{(1+\omega)(\sigma-1)}$ and $\theta_2 = \frac{\omega(\sigma-2)-1}{(1+\omega)(\sigma-1)}$. As in Feenstra (1994), Eq. 17 can be written as a regression of time averaged variables and estimated using weighted nonlinear least squares.[11]

[11] From Broda and Weinstein (2006), I include the time average of $\left(q_{it}^{-1} + q_{i,t-1}^{-1}\right)$ as additional regressor to control for measurement error introduced by aggregating transaction prices into quarterly unit values. This

When analytical estimates are outside of theoretical bounds (e.g., $\sigma < 1$), parameters are estimated by a grid search over the parameter space.[12]

Two product groups in the Dairy department have too few varieties for estimation and are dropped from the analysis. Of the remaining, the procedure yields 55 analytical and 15 grid-searched estimates, the summary of which is presented in Table 4. The overall median elasticity is 4.32, which is lower than what RW found using the Feenstra method and Homescan data (6.48), but reasonable given data and time period differences.[13] There is heterogeneity in estimates by product group, as the interquartile range is nearly three.

5.2 Results

As described in the previous subsection, I calculate series of four-quarter CES price indexes for 70 food and beverage product groups in the Scantrack dataset. For ease of presentation, figures and tables show statistics that are weighted by comparison-period expenditure shares. For example, for a set of product groups $\mathcal{G}$ indexed by g, Figures 1 and 2 plot averages of the form $\sum_{g\in\mathcal{G}} s_{gt} P(\boldsymbol{p}_{g,t-4}, \boldsymbol{p}_{gt}, \boldsymbol{q}_{g,t-4}, \boldsymbol{q}_{gt})$, where $s_{gt} = \sum_{i\in\mathcal{I}_{gt}} p_{it}q_{it} / \sum_{g\in\mathcal{G}} \sum_{i\in\mathcal{I}_{gt}} p_{it}q_{it}$, $\mathcal{I}_{gt}$ is the set of items available in product group g in period t, $P()$ refers to one of the CES-based price index formulas, and price and quantity vectors are specific to g. Figure 1 shows averages across all product groups, while Figure 2 and Table 5 break these out by department. Table 6 gives the percentiles of the distributions of differences between price indexes, while Table 7 presents average differences by department.

As discussed in Sections 3 and 4, these indexes are derived from the same CES model with time-varying taste parameters. Differences between them reflect differing theoretical objectives as opposed to biases stemming from improper modeling assumptions. Compar-

means a product group must have at least four varieties for estimation.

[12] Following the code used for Broda and Weinstein (2010), the grid search, e.g., searches for the value of $\sigma \in [1.04, 50.5]$, at 4% increments, that minimizes the sample objective function.

[13] Previously, Kurtzon (2016) found estimated elasticities from Scantrack to be lower than those estimated using Homescan. The Feenstra method is based on large-T asymptotic arguments, so estimates may have finite-sample bias from the relatively sample period. See Soderbery (2010) and Soderbery (2015). As discussed in Section 4, P_{LMM} is robust to small changes in σ used, and comparisons among P_{CCV}, P_{LM}, and P_{BLM} are qualitatively similar when using alternative values of σ (e.g. setting all equal to 6.48).

isons among P_{LM}, P_{BLM}, P_{LMM}, and P_{SV} shed light on the degree to which the estimate of pure price change is affected by the choice of conditioning taste vector. A difference such as $P_{CCV} - P_{SV}$ estimates the partial effect of tastes on the unconditional COLI (like in Eq. 7), provided one treats utility as cardinal.

The Scantrack data show that the contributions of tastes dominate those of prices in determining P_{CCV}, driving it to be negative in most quarters. From Figure 1, the conditional COLI estimates tend to be positive, with average four-quarter percent changes ranging from 1.31 for P_{LM} to 3.61 for P_{BLM}. In contrast, P_{CCV} implies unconditional cost-of-living declines between 0.29 and 6.65 percentage points per year from late 2006 to mid 2010, implying a strong negative contribution from taste changes. Similar to RW, I find P_{CCV} tends to imply substantially lower inflation than P_{SV}, though I find a larger average difference of 5.64 percentage points in magnitude. In more than 75% of observations, the difference is greater than 2 percentage points.[14] Figure 1 shows P_{CCV} also tends to be much lower than the other conditional COLI estimates, though Figure 2 and Table 7 suggest considerable heterogeneity by department. The average $P_{SV} - P_{CCV}$ spread ranges from 1.51 percentage points for Dairy products to 11.59 percentage points for Frozen Foods. For the latter category, P_{CCV} estimates an average annual unconditional COLI decline of 9.65%, while the conditional COLI estimates range from 1.4% to 2.4% on average. With the exception of Dairy, the gap between the conditional COLI estimates and P_{CCV} persists over time.

Turning to the conditional COLI, the Scantrack estimates indicate that the choice of taste vector can impact the measurement of price change, but to a smaller degree than tastes affect the unconditional COLI. From Table 6, P_{BLM} exceeds P_{LM} by 2.3 percentage points on average and by 1.21 percentage points at the median. Looking across departments

[14] My results are most comparable to Redding and Weinstein (2018), which uses the same definition of common varieties and finds average differences between P_{SV} and P_{CCV} on the order of 2-4 percentage points. Redding and Weinstein (2020) finds the same sign for $P_{CGG} - P_{SV}$, but an average magnitude of only 0.4 percentage points. However, the basket of varieties is limited to items that have lifespans of at least six years and are not within three quarters of their introduction or exit in the reference or comparison quarter. I also find smaller $P_{CGG} - P_{SV}$ differences when I similarly limit the basket. Appendix C shows also that restricting the set of varieties has a large effect on the result by implicitly changing the normalization imposed on the CES taste parameters.

(Table 7), the average difference ranges from 0.54 for Packaged Meat, to 2.92 for Dry Grocery. As Figures 1 and 2 illustrate, the average $P_{BLM} - P_{LM}$ gap tends to be smaller than the gap between P_{CCV} and any one of the other indexes. Conditioning on an intermediate level of tastes, P_{SV} in most cases is very close to P_{LMM}, the geometric average of reference-taste and comparison-taste indexes. Overall, they differ by 0.24 percentage points on average, and 0.03 percentage points at median, with average differences by department spanning -0.01 (Fresh Meat) to 0.425 (Dry Grocery) percentage points. Conditioning on current preferences (as F. M. Fisher and Shell (1972) prefer), P_{BLM} implies that consumers, on average, needed to increase expenditure by 3.61% to be indifferent between current and year-ago food and beverage prices. This is 0.93 percentage points greater than if measured using intermediate tastes (P_{SV}).[15]

As discussed in Section 3, it is known in the literature that neither functional form assumptions nor structural parameter estimates are required to estimate conditional COLI based on intermediate tastes. In fact, the literature finds that even the Sato-Vartia formula, while not technically superlative, approximates superlatives like the Fisher and Tornqvist (Diewert, 1978). Appendix A shows this holds for the Scantrack data as well. In contrast, functional form does matter when estimating either the impact of tastes on a conditional COLI (e.g., $P_{BLM} - P_{LM}$) or the pure taste effect on the unconditional COLI, $P_{CCV} - P_{SV}$. The reason is P_{CCV}, P_{LM}, and P_{BLM} essentially recover the taste parameters as residuals in the CES expenditure share equation. Quantification of taste impacts may therefore be sensitive to model fit. In fact, Martin (2019) uses a simulation study that suggests a neglected nesting structure causes negative biases in P_{CCV} and P_{LM}, positive bias in P_{BLM}, and negligible bias in P_{SV}, leading to positive bias in the taste change indicators $P_{CCV} - P_{SV}$ and $P_{BLM} - P_{LM}$.

[15]Across the non-food products, where the data and model are perhaps less representative, differences among the CES indexes tend to be quite large. They imply department average $P_{SV} - P_{CCV}$ spreads of up to 18 percentage points and $P_{BLM} - P_{LM}$ spreads up to 43 percentage points. Results are available from the author by request.

6 Application to CPI aggregation

The previous section, while making use of detailed transactions data, applies only to food and beverage products consumed at home, which constitute less than 10% of CPI-eligible expenditures (Bureau of Labor Statistics, 2020). To the extent possible, I now use CPI data to estimate what role changing tastes play in the calculation of price indexes over a broad consumption basket. Subject to the limitation described below, I find that year-over-year differences in CES indexes tend to be smaller and less persistent than those found in Section 5.

The basic unit of this analysis is the monthly elementary item-area index (e.g., Mens Suits in Pittsburgh), which is considerably more aggregated than the UPC-level data employed in Section 5. Such indexes are the inputs to both the headline Consumer Price Index for Urban Consumers (CPI-U, which uses the Lowe formula for aggregation) and the C-CPI-U (which uses the Tornqvist). Currently, the BLS calculates elementary indexes for 243 item categories in 32 areas, for a total of 7,776 item-area indexes, though these dimensions have changed over time. Similar to Section 5, I consider an annual frequency of taste change by estimating a series of direct indexes where the base period for each is the same month during the prior year. For comparison with BLS methodology, I also include the Tornqvist index, P_T.[16]

A limitation of this analysis is that the elementary indexes are fixed. Weights for elementary indexes are available at a lag of up to four years, so indexes like P_T, P_{LM} and P_{BLM} are infeasible. The BLS uses either a weighted geometric mean or a modified Laspeyres formula (Bureau of Labor Statistics, 2018). Therefore, this paper's exercise is only informative about category-level tastes (e.g., for ground beef versus chicken) as opposed to variety-level tastes (e.g., for 85% ground beef versus 90% ground beef). Furthermore, the Lloyd-Moulton

[16] In contrast, the published C-CPI-U is a series of one-month indexes multiplied together. When calculating monthly chained versions of the CES indexes, I find monthly percent change differences to be quite small, but levels can drift apart somewhat over time for some values of σ. Results are available from the author upon request.

indexes require the elasticity of substitution σ. Estimation in the style of Feenstra (1994) requires $\sigma > 1$, which is not realistic for this application because it implies all varieties (or aggregates) are substitutes. Additionally, previous analysis of CPI elementary indexes, such as Klick (2018), have estimated elasticities less than one. RW also assume $\sigma > 1$, and for this reason, I do not present any estimates of P_{CCV} using CPI indexes. Given the importance of σ in the CES model's ability to separate price-related substitutions from taste-related substitutions, I present conditional COLI estimates for four different elasticities (0.6, 0.7, 0.8, and 0.9), and leave further inquiry into the correct choice of σ to future research.[17]

6.1 Results

Table 8 presents average 12-month percent changes of P_T, P_{SV}, P_{LM}, P_{BLM}, and P_{LMM} over the period from December 2000 to December 2017. Table 9 gives the average differences between pairs of indexes.[18] Figures 3 to 5 plot these indexes separately for the different values of σ chosen. For readability, the graphs have been split into three time periods.

While the official C-CPI-U is a series of chained month-over-month indexes, these results suggest that an alternative accounting for preferences would have a relatively modest average effect on year-over-year measurements. The Tornqvist, SV and LMM indexes tend to be very close on average, differing by less 0.03 percentage points in magnitude, again reflecting how functional form is less important when conditioning on an intermediate taste level. In fact, average differences among all of the indexes tend to be less than one tenth of one percentage point. The only exception is when $\sigma = 0.9$, P_{BLM} exceeds P_{LM} by 0.16 percentage points on average. Taking current preferences (i.e., P_{BLM}) as the most relevant reference point, then P_T overstates this conditional COLI by 0.057 percentage points (3.1%) under the assumption

[17] The initial and interim C-CPI-U use $\sigma = 0.6$, based on pooled, biennial regressions of logged, differenced shares on logged, differenced elementary indexes, in the style of Feenstra and Reinsdorf (2007). The goal in that case, however, is to predict the final value of the Tornqvist formula once updated expenditures are available, rather than estimating a true CES COLI.

[18] Item structure changes in 2008, 2010, and 2013 reduce the number of overlapping item-areas for those years by 0.47%, 2.84%, and 14.81%, respectively. The averages in Table 9 are qualitatively the same when excluding these years. Results available from the author upon request. Indexes ending in 2018 were not calculated due to implementation of a new CPI area sample.

that $\sigma = 0.6$, while it understates it by 0.079 percentage points (4.0%) under the assumption that $\sigma = 0.09$. Overall, compared to individual variety tastes, the effect of category-level tastes appears relatively small.

Figures 3 to 5 reveal that despite the indexes tending to give similar answers on average, short term divergences can occur. From November 2008 to September 2009, for example, with σ set to 0.9, P_{BLM} exceeds P_{LM} by an average of 0.8 percentage points each month, with individual month differences ranging from 0.34 to 1.22 percentage points. In other periods, however, differences are quite small. For instance, the average difference from January to November 2007 is only 0.01 percentage point, with individual month's differences ranging from -0.10 to 0.09 percentage points. This is different from Section 5, where differences between P_{BLM} and P_{LM} indexes using Scantrack are found to be persistent over time.

As noted before, each formula uses the same elementary indexes, and so the most can be said is that tastes for broader item categories have relatively little impact on the the conditional COLI. Section 5 suggests a larger role of tastes at the individual item level, however.

7 Conclusion

The criticism that traditional price indexes assume constant preferences is not quite correct. It is true that the Fisher, Tornqvist, Sato-Vartia, and Quadratic Mean of Order r indexes are exact in models with constant tastes, but even when tastes change, these still estimate interesting conditional COLI. Furthermore, the pure taste change effects RW and others aim to capture are arguably out of scope for a consumer price index, and are measurable only under a very strong assumption about utility. This paper's empirical analysis suggests the relative contribution of prices to such a cardinal index can be swamped by taste change effects, as RW's CCV index tends to imply cost-of-living deflation even as traditional price indexes show low-to-moderate inflation. If there is interest in a COLI that conditions on a specific period's taste vector, then this paper provides two novel possibilities for the CES case. Current data constraints for the CPI imply the BLS could only account for category-level tastes, which appear to have a relatively small impact on year-over-year inflation. Improvements to the simple CES model are likely possible, and so future research should include more general demand models to more precisely separate taste changes from price-related substitutions.

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Tables

Table 1: Summary of Price Index Properties

Index	Formula	Model	Tastes	Struct. parameters?
Fisher	$\left(\frac{\sum_i p_{i1} q_{i0}}{\sum_i p_{i0} q_{i0}} \frac{\sum_i p_{i1} q_{i1}}{\sum_i p_{i0} q_{i1}}\right)^{\frac{1}{2}}$	Un-specif.	Fixed, intermed.	No
Tornqvist	$\prod_i \left(\frac{p_{i1}}{p_{i0}}\right)^{.5(s_{i0}+s_{i1})}$	Translog	Fixed, geomean	No
Sato-Vartia	$\prod_i \left(\frac{p_{i1}}{p_{i0}}\right)^{w_i}$	CES	Fixed, intermed.	No
CCV	$\prod_i \left(\frac{p_{i1}}{p_{i0}}\right)^{\frac{1}{N}} \times \prod_i \left(\frac{s_{i1}}{s_{i0}}\right)^{\frac{1}{N(\sigma-1)}}$	CES	Vary, normalized	Yes
Lloyd-Moulton	$\left\{\sum_i s_{i0} \left(\frac{p_{i1}}{p_{i0}}\right)^{1-\sigma}\right\}^{\frac{1}{1-\sigma}}$	CES	Fixed, reference	Yes
Backwards Lloyd-Moulton	$\left\{\sum_i s_{i1} \left(\frac{p_{i0}}{p_{i1}}\right)^{1-\sigma}\right\}^{\frac{-1}{1-\sigma}}$	CES	Fixed, comparison	Yes

Table 2: Scantrack Food and Beverage Departments

Department	# PG	# UPC	Exp. Share
Alcoholic Beverages	4	46,656	0.073
Dairy	12	46,686	0.153
Deli	1	22,061	0.022
Dry Grocery	40	412,319	0.541
Fresh Meat	1	1,934	0.006
Fresh Produce	1	20,244	0.052
Frozen Foods	12	64,635	0.115
Packaged Meat	1	18,401	0.039
All	*72*	*632,936*	*1.000*

Note: Based on data provided by The Nielsen Company (U.S.), LLC.

Table 3: Summary Statistics for $p_{it}/p_{i,t-4}$ by Department

	Obs	Mean	StDev	Skew	Kurt	Min	Max
Alcoholic Beverages	343,007	1.021	0.118	0.415	7.163	0.270	2.098
Dairy	383,519	1.037	0.137	0.908	6.577	0.469	2.256
Deli	128,081	1.025	0.117	0.322	6.578	0.500	1.635
Dry Grocery	2,941,985	1.036	0.141	0.777	9.977	0.210	2.782
Fresh Meat	12,162	1.028	0.117	0.767	6.633	0.557	1.759
Fresh Produce	113,895	1.033	0.165	0.844	6.490	0.458	1.992
Frozen Foods	454,187	1.027	0.125	0.253	6.360	0.281	1.807
Packaged Meat	148,512	1.025	0.106	0.516	5.623	0.625	1.618
All	*4,525,348*	*1.033*	*0.136*	*0.74*	*9.254*	*0.21*	*2.782*

Note: Based on data provided by The Nielsen Company (U.S.), LLC.

Table 4: Summary of Elasticity of Substitution Estimates by Department

	# Prod. Gr.	P25	Med	P75
Alcoholic Beverages	4	5.96	7.06	8.63
Dairy	10	3.31	3.65	4.05
Deli	1	3.96	3.96	3.96
Dry Grocery	40	3.85	4.68	6.50
Fresh Meat	1	3.37	3.37	3.37
Fresh Produce	1	2.94	2.94	2.94
Frozen Foods	12	3.31	3.94	6.33
Packaged Meat	1	3.12	3.12	3.12
All	*70*	*3.39*	*4.32*	*6.29*

Note: Based on data provided by The Nielsen Company (U.S.), LLC.

Table 5: Averages of CES Indexes by Department (percent change)

	CCV	SV	LMM	LM	BLM
Alcoholic Beverages	0.0845	1.8585	1.8283	1.3839	2.2751
Dairy	1.3019	2.8165	2.7756	1.5423	4.0415
Deli	−2.4059	1.0556	1.0079	0.3502	1.6703
Dry Grocery	−3.4241	3.2381	2.8131	1.3882	4.3103
Fresh Meat	−2.0699	2.2678	2.2774	1.6551	2.9040
Fresh Produce	−2.0258	1.1468	1.1583	0.1446	2.1833
Frozen Foods	−9.6467	1.9408	1.8948	1.4150	2.3794
Packaged Meat	−1.0369	1.1688	1.1596	0.8894	1.4307
All	*−2.9549*	*2.6810*	*2.4373*	*1.3093*	*3.6107*

Note: Based on data provided by The Nielsen Company (U.S.), LLC. Statistics are averages of product group-level indexes weighted by the product group's share of expenditure in the comparison period. CCV refers to RW's CES Common Varieties Index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM.

Table 6: Summary of CES Index Differences (percentage points)

	SV – CCV	SV – LMM	SV – BLM	BLM – LM
P5	−1.3107	−0.1472	−3.2547	0.1450
P10	0.2764	−0.0748	−2.0360	0.3242
P25	2.1239	−0.0104	−1.1504	0.6187
Median	4.9995	0.0297	−0.5556	1.2058
P75	8.5198	0.1095	−0.2796	2.4390
P90	11.9109	0.3279	−0.1494	4.6537
P95	15.3193	1.0874	−0.0862	8.6224
Mean	5.6359	0.2437	−0.9297	2.3014

Note: Based on data provided by The Nielsen Company (U.S.), LLC. Statistics are over product group-level indexes weighted by the product group's share of expenditure in the comparison period. CCV refers to RW's CES Common Varieties Index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM.

Table 7: Mean CES Index Differences by Department (percentage points)

	SV – CCV	SV – LMM	SV – BLM	BLM – LM
Alcoholic Beverages	1.7740	0.0302	−0.4167	0.8912
Dairy	1.5146	0.0409	−1.2250	2.4992
Deli	3.4615	0.0477	−0.6147	1.3201
Dry Grocery	6.6622	0.4250	−1.0722	2.9221
Fresh Meat	4.3377	−0.0096	−0.6363	1.2490
Fresh Produce	3.1727	−0.0114	−1.0365	2.0388
Frozen Foods	11.5876	0.0460	−0.4386	0.9644
Packaged Meat	2.2057	0.0092	−0.2619	0.5413
All	*5.6359*	*0.2437*	*−0.9297*	*2.3014*

Note: Based on data provided by The Nielsen Company (U.S.), LLC. Statistics are average differences between product group-level indexes weighted by the product group's share of expenditure in the comparison period. CCV refers to RW's CES Common Varieties Index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM.

Table 8: CPI: Mean of 12-mo. Indexes over 2000m12-2017m12 (perc. points)

σ	Torn.	SV	LMM	LM0	LM1
0.6	1.8834	1.8999	1.8676	1.9090	1.8263
0.7	1.8834	1.8999	1.8773	1.8733	1.8813
0.8	1.8834	1.8999	1.8815	1.8388	1.9243
0.9	1.8834	1.8999	1.8832	1.8045	1.9620

Note: Torn refers to the Tornqvist index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM.

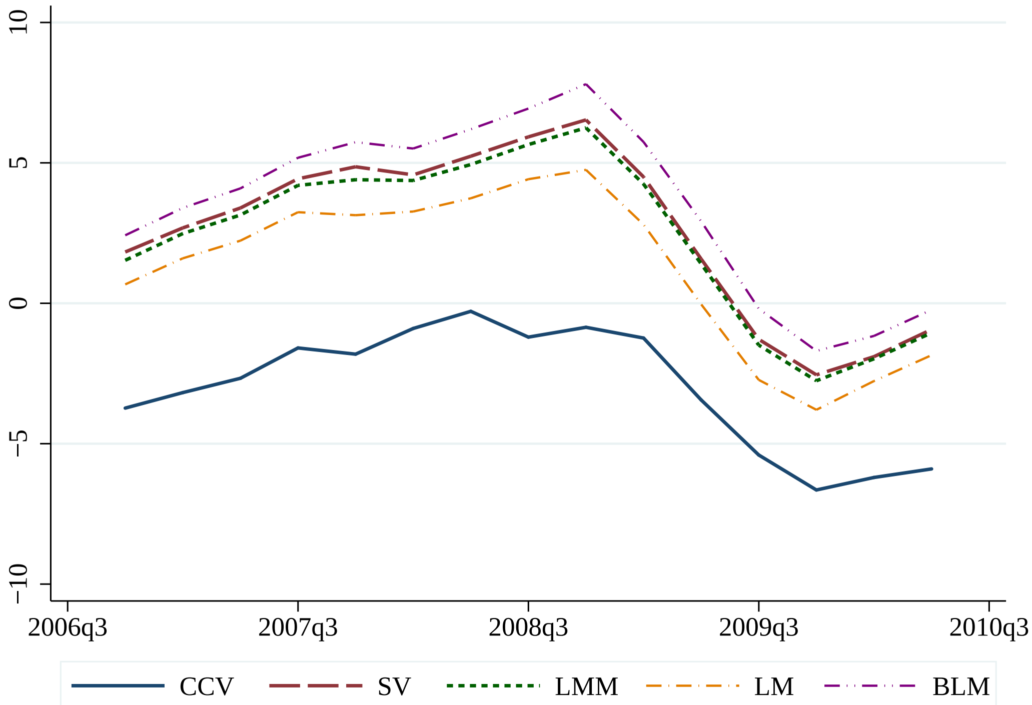
Table 9: CPI: Mean Differences in 12-mo. Indexes over 2000m12-2017m12 (perc. points)

σ	Torn. − SV	SV − LMM	SV − LM	SV − BLM	BLM − LM
0.6	−0.0165	0.0323	−0.0090	0.0736	−0.0827
0.7	−0.0165	0.0227	0.0266	0.0186	0.0080
0.8	−0.0165	0.0184	0.0611	−0.0244	0.0856
0.9	−0.0165	0.0168	0.0954	−0.0621	0.1575

Note: Torn refers to the Tornqvist index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM.

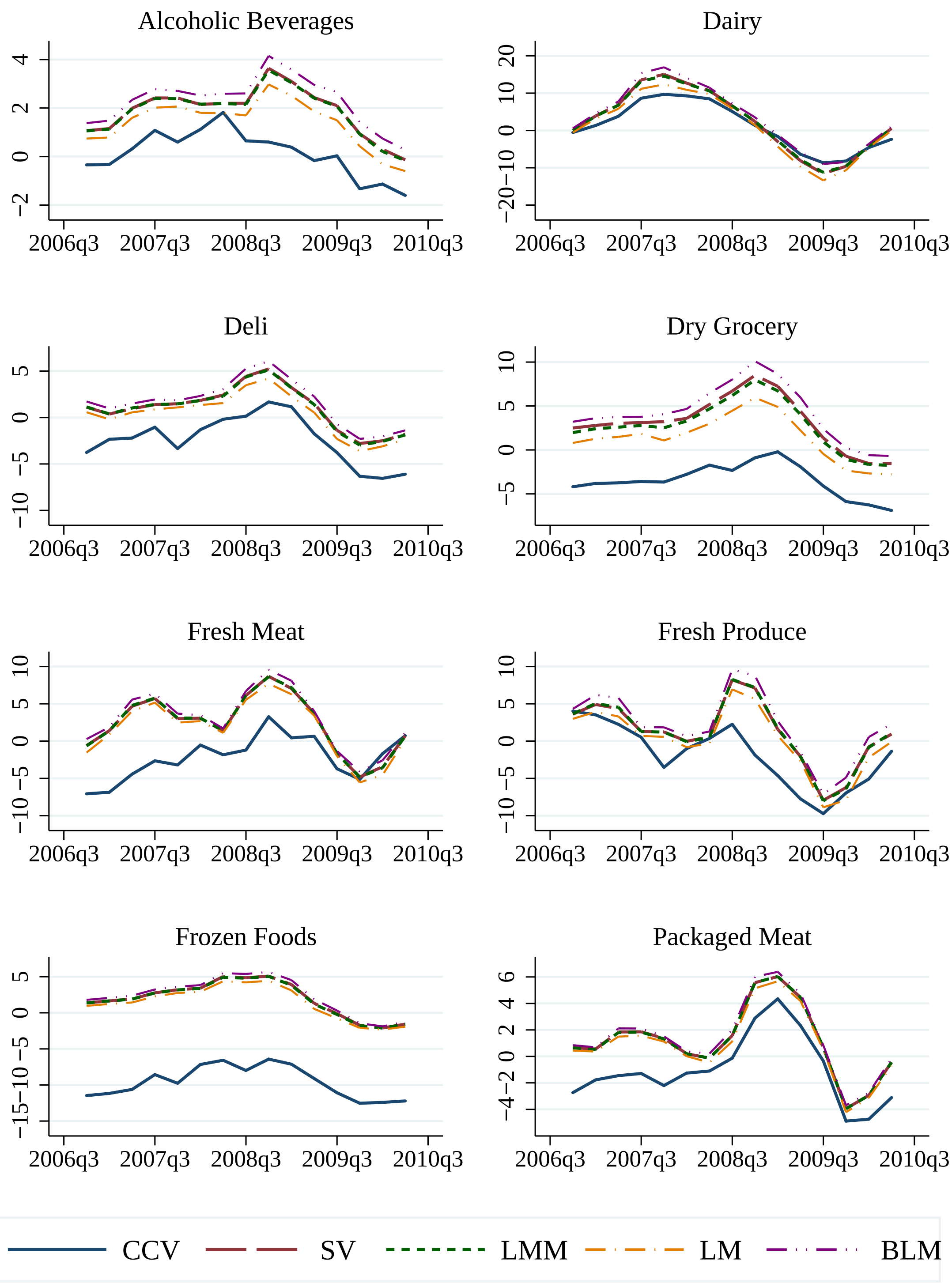
Figures

Figure 1: Scantrak CES Price Index Averages (% change versus year ago)



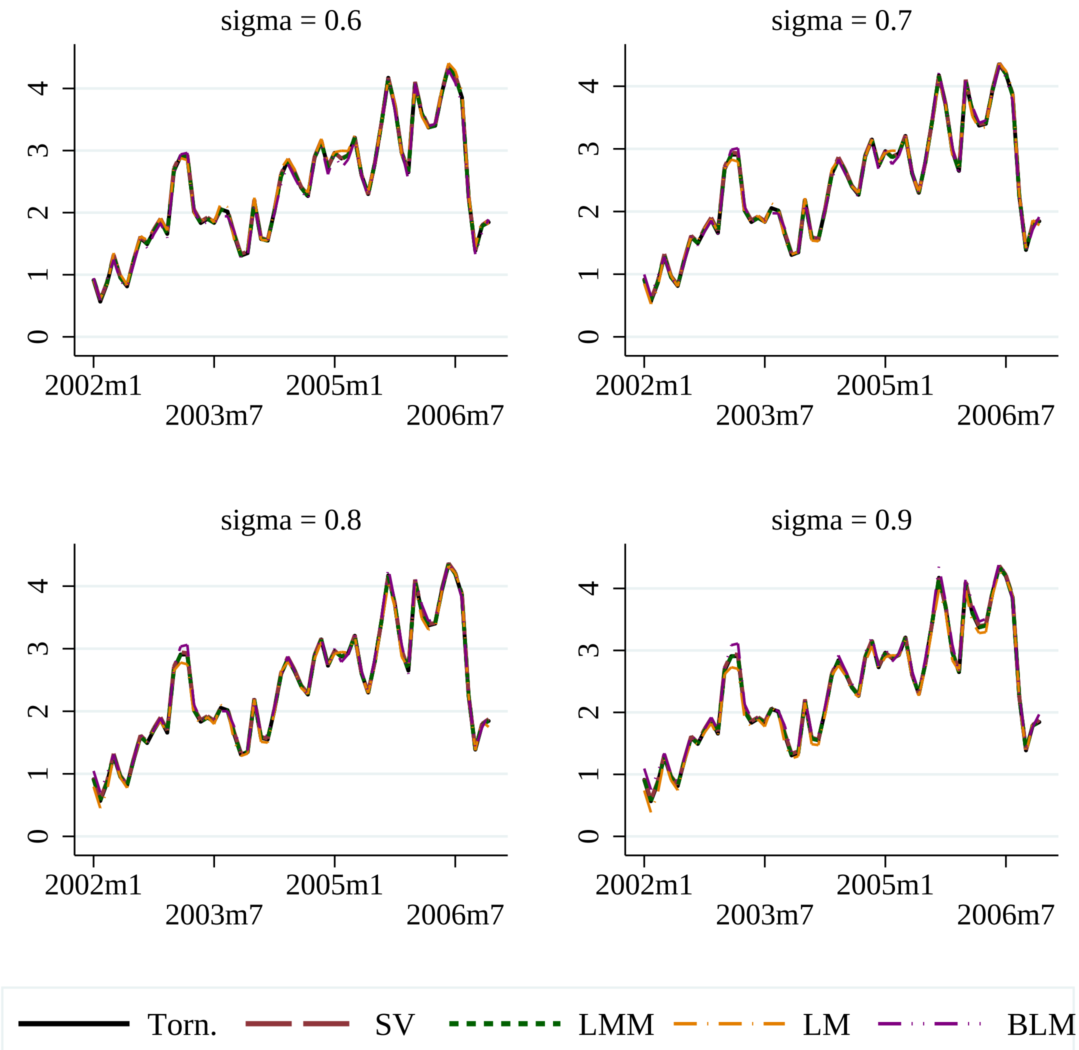
Note: Based on data provided by The Nielsen Company (U.S.), LLC. Plots are comparison period expenditure-weighted averages of the four-quarter proportional changes implied by product group-level indexes for food and beverage products. CCV refers to RW's CES Common Varieties Index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM. All but the SV indexes require estimated elasticities of substitution.

Figure 2: Scantrak CES Price Index Averages By Dept. (% change versus year ago)



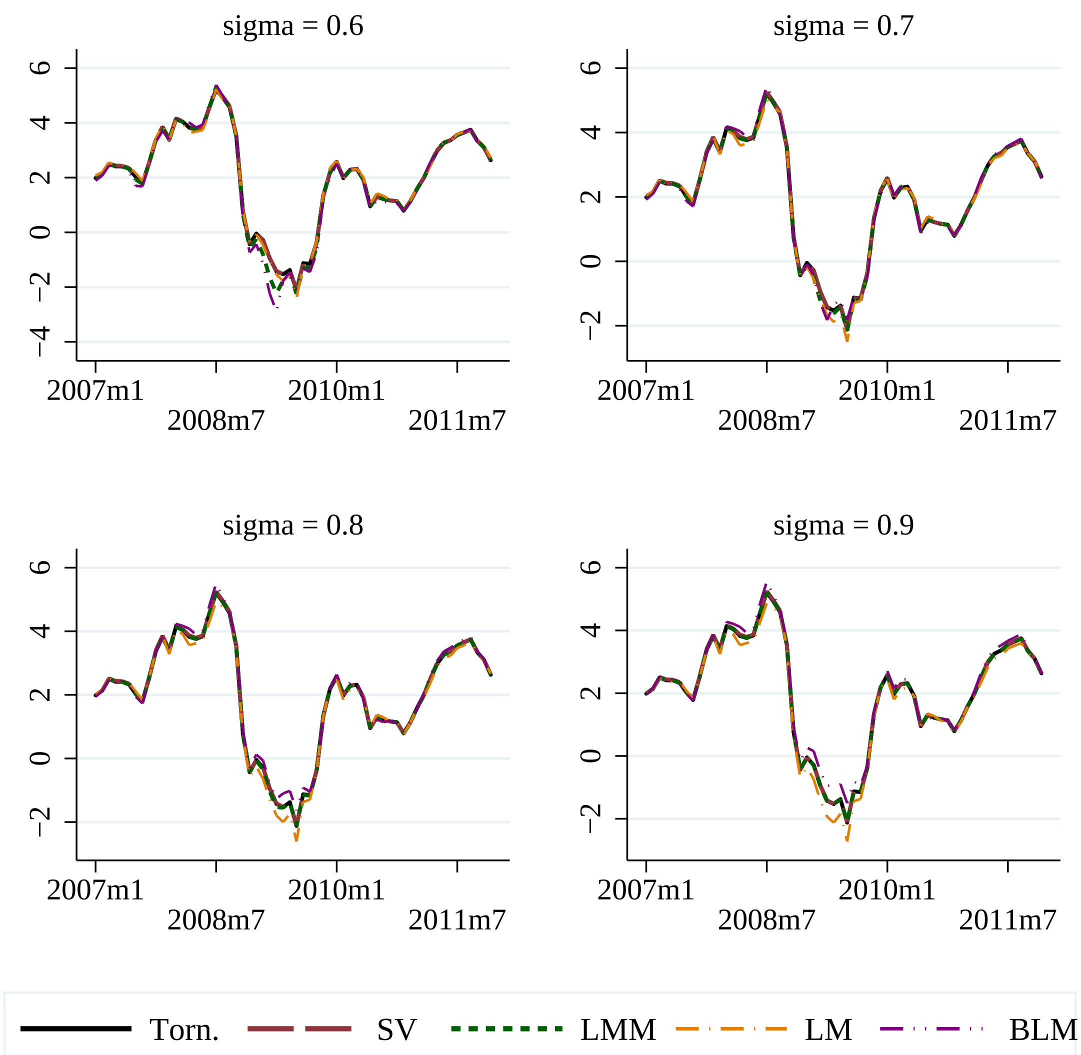
Note: Based on data provided by The Nielsen Company (U.S.), LLC. Plots are comparison expenditure-weighted averages of the four-quarter proportional changes implied by product group-level indexes for food and beverage products. CCV refers to RW's CES Common Varieties Index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM. All but the SV indexes require estimated elasticities of substitution.

Figure 3: Comparison of 12-mo. CPI Aggregates, 2002m1-2006m12



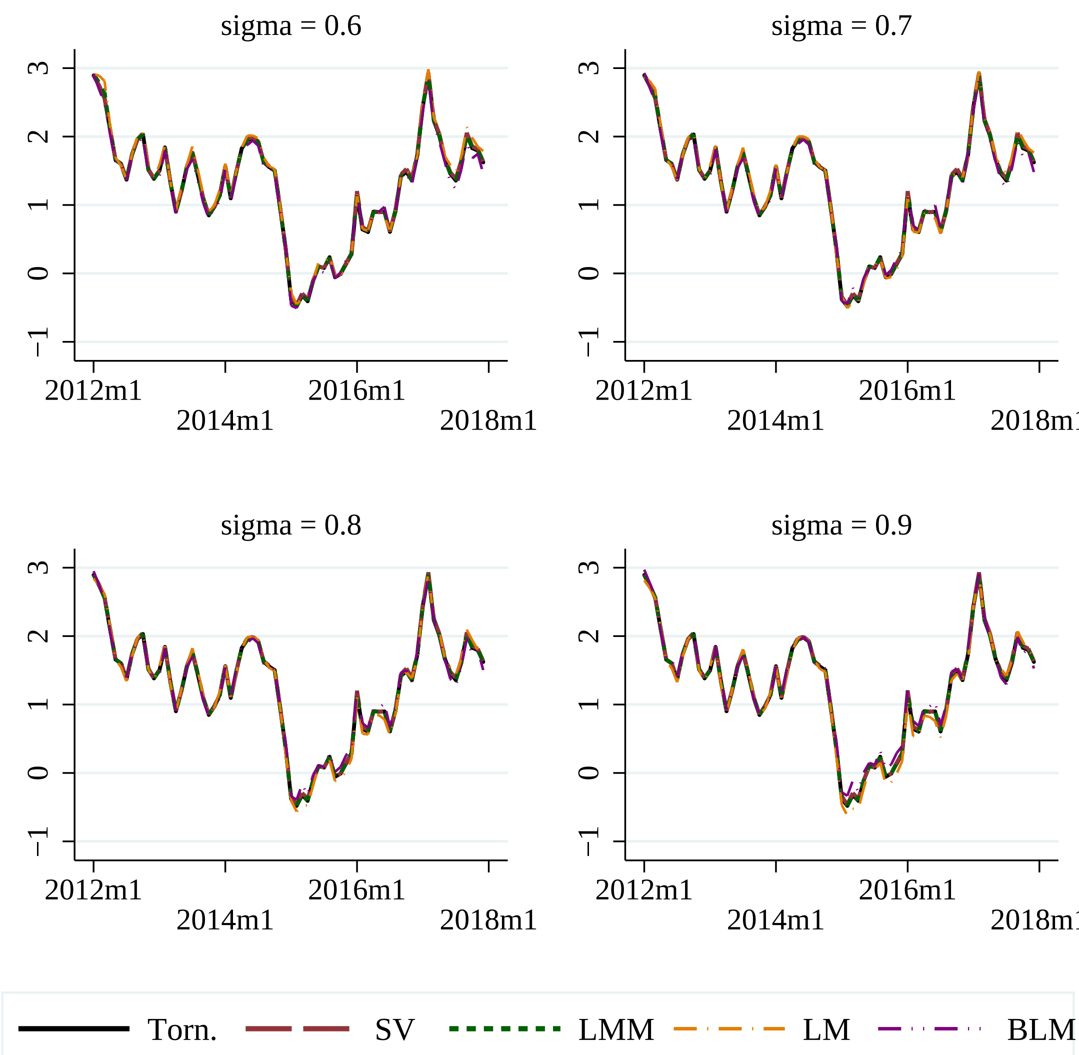
Note: Plots are twelve-month percent changes. Torn refers to the Tornqvist index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM. The LMM, LM, and BLM indexes are calculated using indicated elasticity of substitution (sigma).

Figure 4: Comparison of 12-mo. CPI Aggregates, 2007m1-2011m12



Note: Plots are twelve-month percent changes. Torn refers to the Tornqvist index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM. The LMM, LM, and BLM indexes are calculated using indicated elasticity of substitution (sigma).

Figure 5: Comparison of 12-mo. CPI Aggregates, 2012m1-2017m12



Note: Plots are twelve-month percent changes. Torn refers to the Tornqvist index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM. The LMM, LM, and BLM indexes are calculated using indicated elasticity of substitution (sigma).

A Traditional price indexes with retail scanner data

As in Section 5, I calculate four-quarter price indexes for each food and beverage product group in the Scantrack data using the Fisher, Tornqvist, Sato-Vartia, and Laspeyres formulas. All but the Laspeyres are variable-weight indexes, meaning they reflect consumer substitutions over time. While the Sato-Vartia reflects substitutions according to the CES expenditure function, the Fisher and Tornqvist are superlative, meaning they approximate an arbitrary homothetic expenditure function (Diewert, 1976). Figure A1 plots kernel density estimates of the differences between each index and the Fisher, while Table A1 lists mean differences and mean absolute differences within Scantrack department.

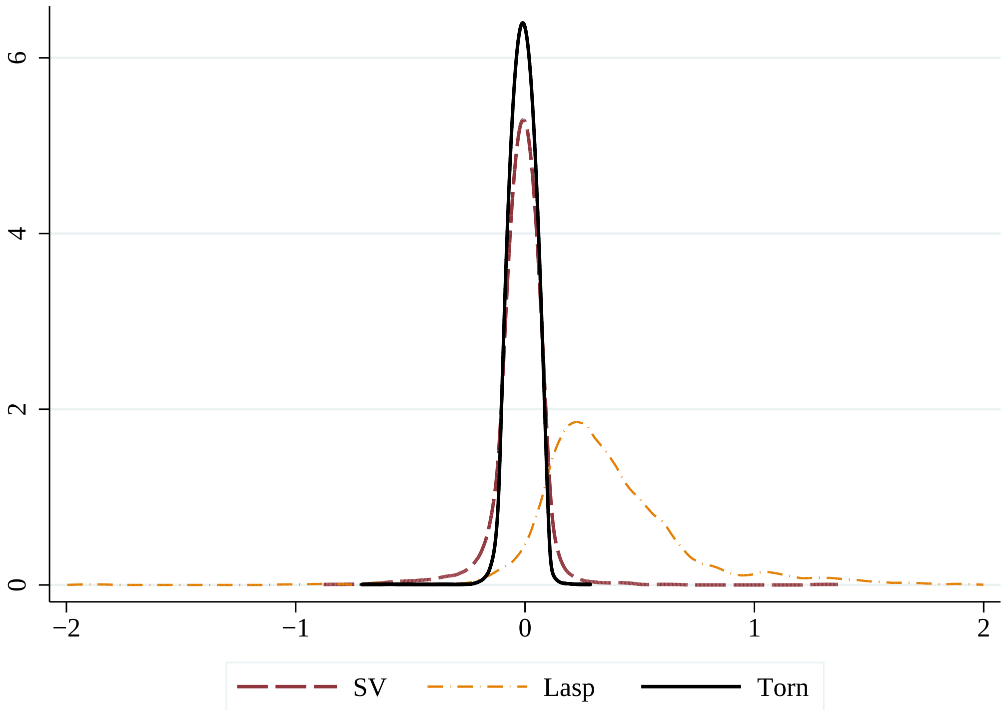
As many have found previously, there tends to be substantial agreement between the three traditional variable-weight indexes, with mean differences and mean absolute differences all less than one tenth of one percentage point in magnitude. In contrast, the Laspeyres indexes exceeds the Fisher index by about 0.4 percentage points, on average, which is on the order of estimates from Boskin, et. al. (1996) and others for lower-level substitution bias.

Table A1: Differences from a Fisher Index by Department (percentage points)

	Mean Difference			Mean Absolute Difference		
	Torn.	SV	Lasp.	Torn.	SV	Lasp.
Alcoholic Beverages	0.0005	0.0013	0.2093	0.0041	0.0101	0.2102
Dairy	−0.0019	−0.0021	0.2903	0.0071	0.0251	0.2916
Deli	−0.0029	0.0093	0.3257	0.0046	0.0252	0.3257
Dry Grocery	−0.0170	−0.0454	0.4043	0.0213	0.0683	0.4168
Fresh Meat	0.0031	0.0027	0.6404	0.0154	0.0276	0.6404
Fresh Produce	0.0097	0.0169	0.6203	0.0182	0.0428	0.6203
Frozen Foods	−0.0166	−0.0186	0.4255	0.0199	0.0416	0.4350
Packaged Meat	−0.0022	0.0012	0.4027	0.0046	0.0075	0.4027
All	*−0.0109*	*−.0257*	*0.3858*	*0.0165*	*0.0495*	*0.3940*

Note: Based on data provided by The Nielsen Company (U.S.), LLC. Statistics are average differences between index indicated and a Fisher index, weighted by the product group's share of expenditure in the comparison period.

Figure A1: Scanner Data: Differences from a Fisher Index



Note: Based on data provided by The Nielsen Company (U.S.), LLC. Data are differences between index indicated and a Fisher index at the product group level, expressed as percent changes over four quarters. Density estimates use Epanechnikov kernel with bandwidth of 0.05 percentage points.

B Product turnover with CES preferences

Dating back to Feenstra (1994), the CES function is convenient for modeling the cost-of-living effects of entering and exiting varieties. Following this framework, I consider that some items may be unavailable in one or more periods. Denote the set of varieties available in each period as $\mathcal{I}_0$ and $\mathcal{I}_1$, such that $\mathcal{I}_0 \cup \mathcal{I}_1 \subseteq \bar{\mathcal{I}}$. We assume the consumer has CES preferences over the superset of varieties $\bar{\mathcal{I}}$. Denote the sets of common varieties as $\mathcal{I}^C = \mathcal{I}_0 \cap \mathcal{I}_1$, exiting varieties as $\mathcal{I}^E = \mathcal{I}_0 \setminus \mathcal{I}^C$, and new varieties as $\mathcal{I}^N = \mathcal{I}_1 \setminus \mathcal{I}^C$.

An important feature of CES preferences is that optimal expenditure on a subset of varieties depends only on prices and taste parameters for varieties in that subset. Adjusting notation to account for an arbitrary set $\mathcal{I}$, Eq. 8 becomes (taking $\bar{u}$ to be one):

$$C(\boldsymbol{p};\boldsymbol{\varphi},\mathcal{I}) = \left[\sum_{i\in\mathcal{I}}\left(\frac{p_i}{\varphi_i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{18}$$

Eq. 9 becomes

$$s_i(\boldsymbol{p};\boldsymbol{\varphi},\mathcal{I}) = \frac{(p_i/\varphi_i)^{1-\sigma}}{\sum_{j\in\mathcal{I}}(p_j/\varphi_j)^{1-\sigma}} = \frac{(p_i/\varphi_i)^{1-\sigma}}{[C(\boldsymbol{p};\boldsymbol{\varphi},\mathcal{I})]^{1-\sigma}} \tag{19}$$

If a variety is unavailable, the COLI framework uses the reservation price implied by the model of preferences. Assuming $\sigma > 1$, the CES model implies infinite reservation prices.[19] Together, these properties imply $C(\boldsymbol{p};\boldsymbol{\varphi},\bar{\mathcal{I}}) = C(\boldsymbol{p};\boldsymbol{\varphi}_t,\mathcal{I}_t)$..

The class of conditional COLI is therefore given by

Definition B.1 *Conditional Cost-of-living Index with product turnover*

$$\Phi(\boldsymbol{p}_0,\boldsymbol{p}_1;\boldsymbol{\varphi},\bar{\mathcal{I}}) = \frac{C(\boldsymbol{p}_1;\boldsymbol{\varphi},\mathcal{I}_1)}{C(\boldsymbol{p}_0;\boldsymbol{\varphi},\mathcal{I}_0)} \tag{20}$$

Let $\mathcal{I}^* \subseteq \mathcal{I}^C$. Similar to Feenstra (1994), we can rewrite Eq. 20 as

$$\begin{aligned}\Phi(\boldsymbol{p}_0,\boldsymbol{p}_1;\boldsymbol{\varphi},\bar{\mathcal{I}}) &= \frac{C(\boldsymbol{p}_1;\boldsymbol{\varphi},\mathcal{I}^*)}{C(\boldsymbol{p}_0;\boldsymbol{\varphi},\mathcal{I}^*)}\frac{C(\boldsymbol{p}_0;\boldsymbol{\varphi},\mathcal{I}^*)}{C(\boldsymbol{p}_0;\boldsymbol{\varphi},\mathcal{I}_0)}\frac{C(\boldsymbol{p}_1;\boldsymbol{\varphi},\mathcal{I}_1)}{C(\boldsymbol{p}_1;\boldsymbol{\varphi},\mathcal{I}^*)} \\ &\equiv \Phi(\boldsymbol{p}_0,\boldsymbol{p}_1;\boldsymbol{\varphi},\mathcal{I}^*)\lambda_0(\boldsymbol{\varphi})^{\frac{1}{1-\sigma}}\lambda_1(\boldsymbol{\varphi})^{\frac{1}{\sigma-1}}, \end{aligned} \tag{21}$$

where $\Phi(\boldsymbol{p}_0,\boldsymbol{p}_1;\boldsymbol{\varphi},\mathcal{I}^*)$ is the conditional COLI over the common set $\mathcal{I}^*$, and

$$\lambda_t(\boldsymbol{\varphi}) = \frac{\sum_{i\in\mathcal{I}^*}\left(\frac{p_{it}}{\varphi_i}\right)^{1-\sigma}}{\sum_{i\in\mathcal{I}_t}\left(\frac{p_{it}}{\varphi_i}\right)^{1-\sigma}},\ t=0,1. \tag{22}$$

The term $\lambda_0(\boldsymbol{\varphi})^{\frac{1}{1-\sigma}}$ adjusts the COLI for the welfare loss from exiting products, while $\lambda_1(\boldsymbol{\varphi})^{\frac{1}{\sigma-1}}$ adjusts it for the welfare gain from new products.

[19] When $\sigma < 1$, consumption of all commodities is necessary for positive utility.

As before, $\boldsymbol{\varphi}_0$ and $\boldsymbol{\varphi}_1$ are interesting choices. From Feenstra (1994), $\lambda_t(\boldsymbol{\varphi}_t) = \frac{\sum_{i \in \mathcal{I}^*} p_{it} q_{it}}{\sum_{i \in \mathcal{I}_t} p_{it} q_{it}} \equiv \lambda_t$, which is the share of common varieties expenditure out of total expenditure occurring in period t. A clear challenge arises, however, with the terms $\lambda_s(\boldsymbol{\varphi}_t), s \neq t$. Intuitively, Eq. 19 implies the taste parameters for absent varieties are not identified—given an infinite price, expenditure shares are zero for any finite value of φ_{it}.

The situation is helped somewhat by the fact that $\lambda_t(\boldsymbol{\varphi}) \in [0, 1]$, and so $\lambda_0(\boldsymbol{\varphi})^{\frac{1}{1-\sigma}} \geq 1$ and $\lambda_1(\boldsymbol{\varphi})^{\frac{1}{\sigma-1}} \leq 1$.[20] This implies the following bounds:

$$\bar{P}_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*) \equiv P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)\lambda_0^{\frac{1}{1-\sigma}} \geq \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \bar{\mathcal{I}}) \tag{23}$$

$$\bar{P}_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*) \equiv P_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)\lambda_1^{\frac{1}{\sigma-1}} \leq \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1, \bar{\mathcal{I}}), \tag{24}$$

where $\bar{P}_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)$ and $\bar{P}_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)$ are Lloyd-Moulton style indexes which include only adjustments for either exit or entry, not both.

Given these bounds, it is possible to apply the method of proof in Konüs (1924) and Diewert (2001) (Proposition 8) to show that there exists an intermediate taste vector $\check{\boldsymbol{\varphi}}$ such that either $\bar{P}_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*) \leq \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \check{\boldsymbol{\varphi}}, \bar{\mathcal{I}}) \leq \bar{P}_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)$ or $\bar{P}_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*) \leq \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \check{\boldsymbol{\varphi}}, \bar{\mathcal{I}}) \leq \bar{P}_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)$. Of course, these bounds may not be particularly tight, and a symmetric average (e.g., a geometric mean) might not be attractive because the missing factors $\lambda_1(\boldsymbol{\varphi}_0)^{\frac{1}{\sigma-1}}$ and $\lambda_0(\boldsymbol{\varphi}_1)^{\frac{1}{1-\sigma}}$ are not likely to be of comparable magnitudes. For instance, Feenstra (1994), Broda and Weinstein (2010), and others have found that $\lambda_1^{\frac{1}{\sigma-1}}$ dominates $\lambda_0^{\frac{1}{1-\sigma}}$, resulting in a net downward adjustment to the COLI. Consequently, $\bar{P}_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)$ might be a lot closer to its target than $\bar{P}_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)$.

As market entry and exit decisions may themselves be tied to tastes, it seems reasonable that the welfare loss from exiting varieties is larger if conditioning on reference period preferences, and that the welfare gain from new varieties is larger if conditioning on comparison

[20]The term $\lambda_1(\boldsymbol{\varphi})^{\frac{1}{\sigma-1}} \leq 1$ is also greater than zero.

period preferences. This motivates the following assumption:

Assumption B.1 *Non-continuing varieties are valued more in the period in which they are available.*

$$\lambda_t(\boldsymbol{\varphi}_t) \leq \lambda_t(\boldsymbol{\varphi}_s), t = 0, 1;\ s \neq t$$

The implies $\lambda_0^{\frac{1}{1-\sigma}} \geq \lambda_0(\boldsymbol{\varphi}_1)^{\frac{1}{1-\sigma}}$ and $\lambda_1^{\frac{1}{\sigma-1}} \leq \lambda_1(\boldsymbol{\varphi}_0)^{\frac{1}{\sigma-1}}$. Assumption B.1 would be true, for example, if taste parameters for common varieties were constant while taste parameters were lower for varieties when they were absent from the market.

Suppose we use Feenstra's adjustment $\lambda_0^{\frac{1}{1-\sigma}}\lambda_1^{\frac{1}{\sigma-1}}$ on both P_{LM} and P_{BLM}. Define

$$P_{FLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \bar{\mathcal{I}}) = P_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)\lambda_0^{\frac{1}{1-\sigma}}\lambda_1^{\frac{1}{\sigma-1}} \tag{25}$$

and

$$P_{FBLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \bar{\mathcal{I}}) = P_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*)\lambda_0^{\frac{1}{1-\sigma}}\lambda_1^{\frac{1}{\sigma-1}}. \tag{26}$$

Under assumption B.1, we have:

$$P_{FLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \bar{\mathcal{I}}) \leq \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \bar{\mathcal{I}}) \leq \bar{P}_{LM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*) \tag{27}$$

and

$$\bar{P}_{BLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \mathcal{I}^*) \leq \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1, \bar{\mathcal{I}}) \leq P_{FBLM}(\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{q}_0, \boldsymbol{q}_1, \sigma, \bar{\mathcal{I}}). \tag{28}$$

A priori, neither of these bounds must be tight enough to be useful, but previous research has found that the effect of new varieties tends to dominate that of disappearing varieties in Feenstra-style CES indexes (Broda and Weinstein, 2010). Indeed, using the Nielsen Retail Scanner data (Table B1), I find the adjustment for exiting varieties is relatively small on average for many departments, ranging from 0.03 percentage points for Alcoholic Beverages to 1.43 percentage points for Deli. The adjustments for new varieties is between two and twelve times larger in magnitude, ranging from -0.38 percentage points for Alcoholic Beverages to -4.12 percentage points for Deli.[21] As a result, the bounds on $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1, \bar{\mathcal{I}})$ appear tighter than the bounds on $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \bar{\mathcal{I}})$. Figure B1 plots the average P_{BLM} index over common varieties versus the upper and lower bounds for $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1, \bar{\mathcal{I}})$. Overall, the average difference in bounds is 0.34 percentage points for food products and 0.26 percentage points for non-food products. As the graph indicates, this margin is small relative to the net adjustment for product turnover. Consequently, a geometric mean of the observable bounds seems reasonable to estimate the COLI conditional on comparison period tastes.[22] Note, such an index will imply a smaller welfare effect from exiting varieties, leading to a larger net adjustment or "new goods bias" for the common varieties index.

Table B1: Mean Product Turnover Adjustments and Bounds by Department

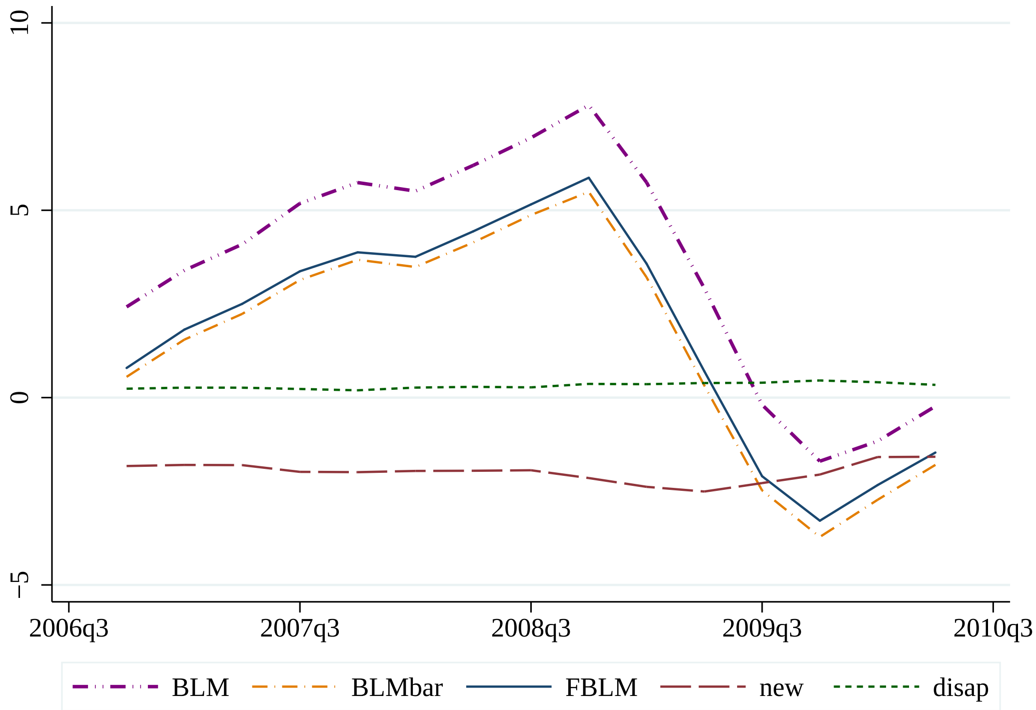
	Adj. New	Adj. Disapp.	LMbar – FLM	FBLM – BLMbar
Alcoholic Beverages	−0.3824	0.0332	0.3858	0.0336
Dairy	−1.4611	0.3869	1.4921	0.3797
Deli	−4.1236	1.4308	4.1879	1.3852
Dry Grocery	−1.9752	0.2197	2.0076	0.2235
Fresh Meat	−2.2178	1.0607	2.2682	1.0673
Fresh Produce	−2.0288	0.5090	2.0362	0.5073
Frozen Foods	−3.4847	0.4003	3.5433	0.3886
Packaged Meat	−1.8188	0.6936	1.8458	0.6886
All	*−1.9961*	*0.3169*	*2.0284*	*0.3153*

Notes: Based on data provided by The Nielsen Company (U.S.), LLC. Product group differences are weighted by comparison period expenditure share. BLM is the Backwards Lloyd-Moulton index over common varieties, BLMbar also includes the Feenstra adjustment for new varieties only, and FBLM uses the Feenstra adjustments for new and exiting varieties.

[21] These adjustment magnitudes are significantly larger than what was reported in Broda and Weinstein (2010) using consumer scanner data over 1994-2003. As my estimates of σ using the retail data are lower, the larger adjustments are to be expected.

[22] In the style of Konüs (1924) and Diewert (2001), one can show P_{FBLM} and P_{FLM} bound a COLI evaluated at an intermediate taste level, though with retail scanner data, I found these bounds to also be wide for most departments.

Figure B1: Scanner Data BLM Indexes with Product Turnover (% change versus year ago)



Note: Based on data provided by The Nielsen Company (U.S.), LLC. Plots are averages of the four-quarter proportional changes implied by product group-level indexes, weighted by comparison period expenditure shares. BLM is the Backwards Lloyd-Moulton index over common varieties, BLMbar also includes the Feenstra adjustment for new varieties only, and FBLM uses the Feenstra adjustments for new and exiting varieties.

C Scale and normalization of tastes

This appendix discusses how the unconditional CES COLI depends on changes in the scale of tastes over time, as well as how alternative normalizations of tastes in the RW framework affect the index. The results I derive are similar to Kurtzon (2019).

For simplicity, suppose that σ is known. Re-arranging the CES expenditure share equation (Eq. 9) evaluated at $\boldsymbol{p}_t, \boldsymbol{\varphi}_t$, we see the φ_{it} are identified only up to a common scale factor.

$$\varphi_{it} C(\boldsymbol{p}_t, \boldsymbol{\varphi}_t) = p_{it} s_{it}^{\frac{1}{\sigma-1}} \; i \in \mathcal{I}. \tag{29}$$

To eliminate the unknown constant (across items) factor $C(\boldsymbol{p}_t, \boldsymbol{\varphi}_t)$, choose a normalization

$\tilde{x} = F(x_1, \ldots, x_N)$ such that $F(a, \ldots, a) = a$ and $F(ay_1, \ldots, ay_N) = aF(y_1, \ldots y_N)$. Leading examples include a geometric mean, $F(x_1, \ldots, x_N) = \prod_{i=1}^{N} x_i^{w_i}$, where $\sum_{i=1}^{N} w_i = 1$, or a reference variety, $F(x_1, \ldots, x_N) = x_j$. Apply $F()$ to each side of Eq. 29 and divide Eq. 29 by the result. This yields Eq. 30 below.

$$\begin{aligned}\frac{\varphi_{it}}{\tilde{\varphi}_t} &= \frac{r_{it}}{\tilde{r}_t} \\ &\equiv \ddot{\varphi}_{it},\ i \in \mathcal{I}, \end{aligned} \tag{30}$$

where $r_{it} = p_{it} s_{it}^{\frac{1}{\sigma-1}}$. Note we can plug in $\ddot{\varphi}_{it}$ (based on any normalization) into the conditional COLIs $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1)$ and $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0)$ and get exactly P_{BLM} and P_{LM}, respectively, since the scale factors cancel. In addition, it is straightforward to show that if we use the unweighted geometric mean as the normalization, set $\tilde{\varphi}_0 = \tilde{\varphi}_1 = \varphi$, plug $\ddot{\varphi}_{it}$ into the CES expenditure function Eq. 8, and then take the ratio $C(\boldsymbol{p}_1, \ddot{\boldsymbol{\varphi}}_1)/C(\boldsymbol{p}_0, \ddot{\boldsymbol{\varphi}}_0)$, we get Eq. 14 for P_{CCV} exactly.

While the conditional COLI estimates are invariant to the normalization, the unconditional estimate is not. Because the CES expenditure function, Eq. 8, is homogeneous of degree -1 in $\boldsymbol{\varphi}$, we have the following relationship:

$$\begin{aligned}\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1) &= \frac{\tilde{\varphi}_0}{\tilde{\varphi}_1} \frac{\left[\sum_{i \in \mathcal{I}} \left(\frac{p_{i1}}{\ddot{\varphi}_{i1}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\left[\sum_{i \in \mathcal{I}} \left(\frac{p_{i0}}{\ddot{\varphi}_{i0}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}} \\ &= \frac{\tilde{\varphi}_0}{\tilde{\varphi}_1} \Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \ddot{\boldsymbol{\varphi}}_0, \ddot{\boldsymbol{\varphi}}_1). \end{aligned} \tag{31}$$

The unconditional COLI based on normalized taste parameters therefore differs from the true unconditional COLI $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1)$ by the factor $\tilde{\varphi}_0/\tilde{\varphi}_1$, which is unidentified (RW assume it to be equal to one by setting $\tilde{\varphi}_0 = \tilde{\varphi}_1$). Therefore, the normalization imposed by the P_{CCV} is not really "free", because it implicitly defines the estimand $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \ddot{\boldsymbol{\varphi}}_0, \ddot{\boldsymbol{\varphi}}_1)$. In reality, we do not even know if $\Phi_U(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1)$ is greater than or less than one. In fact, Kurtzon (2019) shows that the taste-shock bias is measured to be identically zero if using the geometric mean of tastes weighted by the w_i from Eq. 5.

C.1 Alternative normalizations

It is clear from Eq. 31 that an unconditional COLI that uses normalized taste parameters is not invariant to the normalization chosen, of which there are infinitely many. Another aspect of the issue, relevant to Redding and Weinstein (2020), occurs when we assume the constant elasticity model (e.g., Eq. 8) for a commodity set $\mathcal{I}$, but estimate an unconditional sub-COLI over a smaller set. As a component of their CES Universal Price Index (CUPI), Redding and Weinstein (2020) actually calculate P_{CCV} over a subset of the varieties that are available in both the reference and comparison periods. By assumption, however, the CES expenditure share equation (Eq. 9) holds for all $i \in \mathcal{I}_t$, $t = 0, 1$. If normalizing using a geometric mean, then a natural question is whether this mean should be over all varieties or just the subset.

RW's CUPI consists of a P_{CCV} and product turnover adjustments from Feenstra (1994). Restricting the set over which the P_{CCV} is calculated is possible because of the following decomposition of the CES unconditional COLI with product turnover. We can write

$$\begin{aligned}
\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1, \bar{\mathcal{I}}) &= \frac{C(\boldsymbol{p}_1; \boldsymbol{\varphi}_1, \mathcal{I}_1)}{C(\boldsymbol{p}_0; \boldsymbol{\varphi}_0, \mathcal{I}_0)} \\
&= \frac{C(\boldsymbol{p}_1; \boldsymbol{\varphi}_1, \mathcal{I}^*)}{C(\boldsymbol{p}_0; \boldsymbol{\varphi}_0, \mathcal{I}^*)} \frac{C(\boldsymbol{p}_0; \boldsymbol{\varphi}_0, \mathcal{I}^*)}{C(\boldsymbol{p}_0; \boldsymbol{\varphi}_0, \mathcal{I}_0)} \frac{C(\boldsymbol{p}_1; \boldsymbol{\varphi}_1, \mathcal{I}_1)}{C(\boldsymbol{p}_1; \boldsymbol{\varphi}_1, \mathcal{I}^*)} \\
&\equiv \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1, \mathcal{I}^*) \lambda_0^{\frac{1}{1-\sigma}} \lambda_1^{\frac{1}{\sigma-1}},
\end{aligned} \tag{32}$$

where the $\lambda_t = \frac{\sum_{i \in \mathcal{I}^*} p_{it} q_{it}}{\sum_{i \in \mathcal{I}_t} p_{it} q_{it}}$ as in Appendix B and $\Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1, \mathcal{I}^*)$ is the unconditional COLI for some common set $\mathcal{I}^*$. This decomposition holds for any $\mathcal{I}^* \subseteq \mathcal{I}^C = \mathcal{I}_0 \cap \mathcal{I}_1$, and RW use this fact to restrict $\mathcal{I}^*$ to include only products that had a lifespan of six years and were not within three quarters of birth or death in periods 0 or 1. In contrast, Redding and Weinstein (2018) uses the full set $\mathcal{I}^C$. Using the smaller set, the taste-shock bias estimate in RW is much lower in magnitude (around 0.4 percentage points per year), than that reported in Redding and Weinstein (2018) (around 2-4 percentage points per year).

I attempt to replicate RW's use of a more restricted set of varieties. Since I have only about five years of data, my restricted set includes only those products appearing in every quarter from 2005Q3 to 2010Q2. To avoid products that may have been within three quarters of birth or death, I focus only on the period from 2007Q3 to 2009Q3. The restricted sets end up having about half as many UPC's as the full sets of common varieties. Using the more restricted sets of common varieties and the CCV formula in Eq. 14 (i.e., normalizing based on the geometric mean across the narrower set of varieties), I also find lower estimates of taste-shock bias, as shown in Table C1 (compare to Table 7). For food products, the average taste-shock bias ranges from -0.17 percentage points per year for Deli, to 3.24 percentage points per year for Fresh Meat. Across all food and beverage products, the average taste-shock bias is 1.2 percentage points per year, which is higher than RW, but less than one fourth of what it was using the full set of common UPCs. Figure C1 plots the index averages. The patterns of comparisons between LM and BLM indexes for food and nonfood products largely follow what was observed in Figure 1 using the full set of common varieties.

This could be because the taste shock bias associated with the more restrictive set of common varieties is smaller than that of the full set. However, this interpretation appears to be closely dependent on the normalization employed. As discussed in the previous subsection, computing P_{CCV} over the set $\mathcal{I}^*$ using Eq. 14 is equivalent to using the normalized taste parameters in Eq. 33 below, which impose $\tilde{\varphi}_0^* = \tilde{\varphi}_1^* = 1$.

$$\ddot{\varphi}_{it} = \left(\frac{p_{it}}{\tilde{p}_t^*}\right)\left(\frac{s_{it}^*}{\tilde{s}_t^*}\right)^{\frac{1}{\sigma-1}}, i \in \mathcal{I}^*, \tag{33}$$

where $s_{it}^* = \frac{p_{it}q_{it}}{\sum_{jin\mathcal{I}^*} p_{jt}q_{jt}}$, and $\tilde{x}_t^* = \prod_{j\in\mathcal{I}^*} x_{jt}^{1/N^*}$ for $x = p, q$.

When using the restricted subset $\mathcal{I}^*$, however, there are additional options for normalizing the φ_{it}. The starting point for the CUPI is to assume the CES model over all products available in either the reference or comparison periods. As in Eq. 19, expenditure shares out of an arbitrary subset of goods $\mathcal{I}$ depend only on expenditures in that subset. This means that we could normalize the φ_{it} using larger sets of items up to and including the full common goods set $\mathcal{I}^C$, or even $\mathcal{I}_t$. Using $\mathcal{I}^C$, this would mean computing

$$\bar{\varphi}_{it} = \left(\frac{p_{it}}{\tilde{p}_t^C}\right)\left(\frac{s_{it}^C}{\tilde{s}_t^C}\right)^{\frac{1}{\sigma-1}}, i \in \mathcal{I}^*, \tag{34}$$

where $s_{it}^C = \frac{p_{it}q_{it}}{\sum_{jin\mathcal{I}^C} p_{jt}q_{jt}}$, and $\tilde{x}_t^C = \prod_{j\in\mathcal{I}^C} x_{jt}^{1/N^C}$ for $x = p, q$.

I also estimate a version of P_{CCV} over the restricted set of common varieties, but using the normalization reflected in Eq. 34. Since this is the entire set of common varieties, this is same normalization used for the CCV indexes presented in Section 5, even though the price index is for the restricted set only. In Figure C2, I compare the averages for of all these possible CCV indexes, along with the SV indexes for both sets of common varieties. As in Section 5, SV and CCV cover all common varieties and are computed using Eq.'s 5 and 14, respectively. SV(R), CCV(R, 1) and CCV(R, 2) cover the restricted set of varieties. CCV and CCV(R,2) normalize using Eq. 34 (e.g., geometric means over all common varieties), while CCV(R, 1) normalizes using Eq. 33 (e.g., geometric means over the restricted set of varieties). The results indicate a large influence of the choice of normalization on the CCV index. Comparison of SV and SV(R), or CCV and CCV(R, 2) suggest that there is little difference in the average price changes across the two sets of varieties. However, the wide gap between CCV(R, 2) and CCV(R, 1) suggests that the choice of normalization is playing a large role in the SV to CCV comparison.

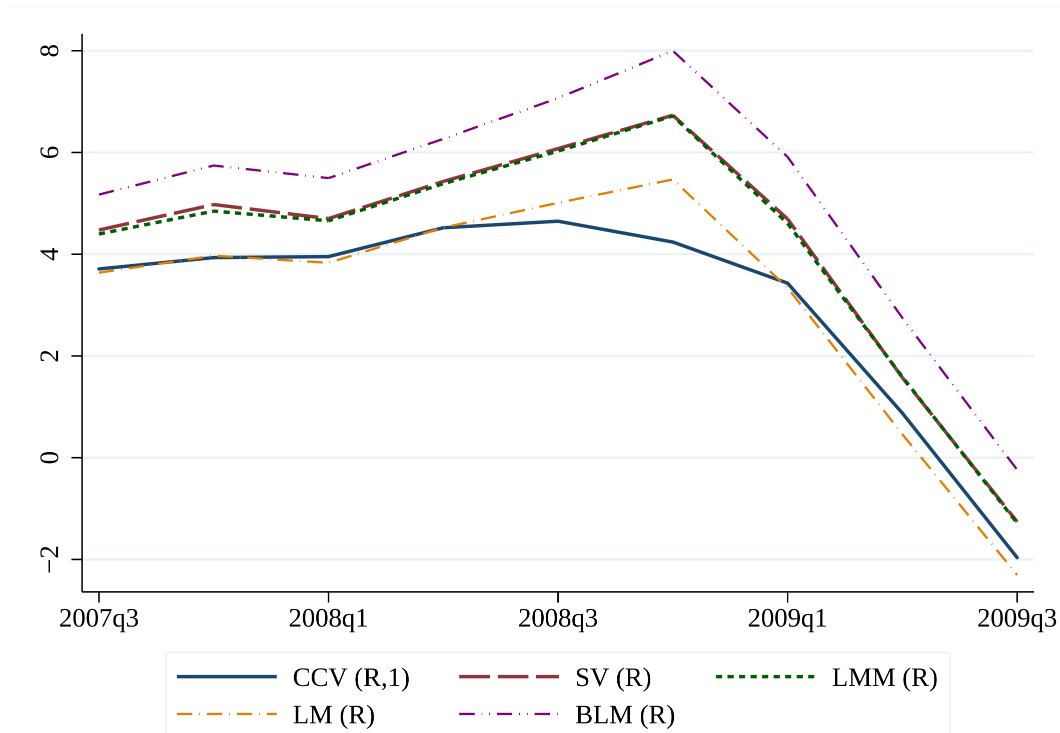
When proposing their restricted common goods set, RW argue that expenditure patterns near the beginning and end of a products life may "make it appear as if consumer tastes for a common variety are changing rapidly when in fact they are not." This would seem to suggest that the CES model fits the data poorly for varieties in the set $\mathcal{I}^C \setminus \mathcal{I}^*$, and so CCV and CCV(R, 2) are unreliable because they include (the latter through the normalization only) expenditure information for these varieties. But if this is true, it should also call into question the validity of the product turnover adjustment terms λ_t, as these also depend on expenditure information for the questionable items. If relatively new, relatively old, or relatively short-lived products follow different expenditure patterns, then it would seem a richer model might be needed for the entire consumption basket, rather than a shifting of items out of the CCV component of the CUPI.

Table C1: Mean Differences of CES Indexes Over More Restrictive Common Goods Sets (percentage points)

	SV − CCV	SV − LMM	SV − BLM	BLM − LM
Alcoholic Beverages	0.4516	0.0014	−0.3923	0.7859
Dairy	0.1392	0.0207	−1.4827	2.9646
Deli	−0.1797	0.0449	−0.6407	1.3662
Dry Grocery	1.7494	0.0846	−1.1414	2.4202
Fresh Meat	3.2415	−0.0351	−0.6033	1.1329
Fresh Produce	0.8757	0.0072	−0.8923	1.7901
Frozen Foods	1.3658	0.0299	−0.4703	0.9961
Packaged Meat	−0.4275	−0.0018	−0.3004	0.5962
All	*1.1965*	*0.0532*	*−1.0035*	*2.0886*

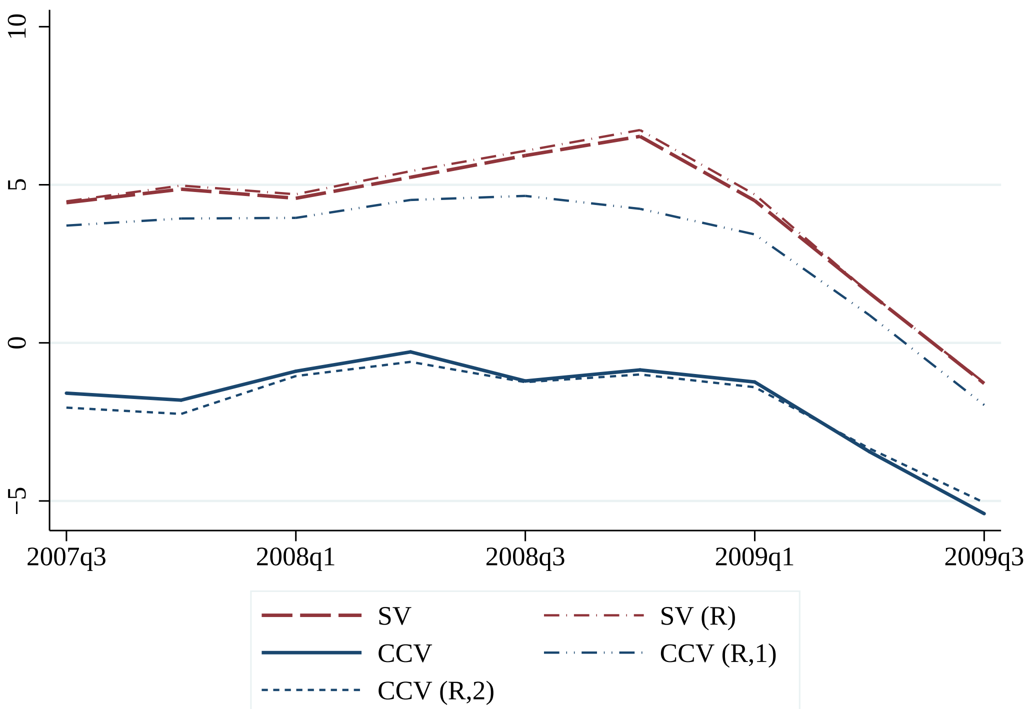
Notes: Based on data provided by The Nielsen Company (U.S.), LLC. Product group index differences are weighted by comparison-period expenditure share. Indexes include varieties observed continuously from 2005Q3 to 2010Q2. CCV refers to RW's CES Common Varieties Index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM.

Figure C1: Scanner Data CES Price Indexes Over More Restrictive Common Goods Sets (% change versus year ago)



Note: Based on data provided by The Nielsen Company (U.S.), LLC. Plots are averages of the four-quarter proportional changes implied by product group-level indexes, weighted by comparison period expenditure shares. CCV refers to RW's CES Common Varieties Index, SV refers to Sato-Vartia, LM refers to Lloyd-Moulton, BLM refers to Backwards Lloyd-Moulton, and LMM refers to the geometric mean of LM and BLM. Indexes include varieties observed in all quarters between 2005Q3 and 2010Q2.

Figure C2: Scanner Data SV and CCV Index Comparison (% change versus year ago)



Note: Based on data provided by The Nielsen Company (U.S.), LLC. Plots are averages of the four-quarter proportional changes implied by product group-level indexes, weighted by comparison period expenditure shares. SV and CCV cover all common varieties. SV (R), CCV (R, 1) and CCV (R, 2) cover a restricted set of varieties with a lifespan of 2005Q3-2010Q2. CCV and CCV (R,2) assume that the taste parameter geomean across all common varieties is constant across periods. CCV (R, 1) assumes the taste parameter geomean across the restricted set of varieties is constant across periods.

D Homothetic Translog

This section shows how to derive conditional COLI for the homothetic translog model. To match RW's parameterization of tastes, the representative agent's minimized unit expenditure function is given in the following definition.

Assumption D.1 *Homothetic translog expenditure function*

$$\ln C(\boldsymbol{p};\boldsymbol{\varphi}) = \ln \alpha_0 + \sum_{i\in\mathcal{I}} \alpha_i \ln\left(\frac{p_i}{\varphi_i}\right) + \frac{1}{2}\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}} \gamma_{ij} \ln\left(\frac{p_i}{\varphi_i}\right)\ln\left(\frac{p_j}{\varphi_j}\right),\ t = 0,1. \tag{35}$$

where the restriction $\gamma_{ij} = \gamma_{ji}$ is made without loss of generality.

After some algebra, we can rewrite Eq. 35 as

$$\ln C(\boldsymbol{p};\boldsymbol{\varphi}) = \ln\left[a_0(\boldsymbol{\varphi})\right] + \sum_{i\in\mathcal{I}} a_i(\boldsymbol{\varphi}) \ln p_i + \frac{1}{2}\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}} \gamma_{ij} \ln p_i \ln p_j, \tag{36}$$

where $\ln\left[a_0(\boldsymbol{\varphi})\right] = \ln \alpha_0 - \sum_{i\in\mathcal{I}} \alpha_i \ln \varphi_i + \frac{1}{2}\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}} \ln \varphi_i \ln \varphi_j$ and $a_i(\boldsymbol{\varphi}) = \alpha_i - \sum_{j\in\mathcal{I}} \gamma_{ij} \ln \varphi_j$. From Diewert (1976), homogeneity and symmetry then imply the restrictions $\sum_{i\in\mathcal{I}} a_i(\boldsymbol{\varphi}) = 1$ and $\sum_{j\in\mathcal{I}} \gamma_{ij} = 0$.

Eq. 36 reveals two salient points. First, the time variation in $\boldsymbol{\varphi}$ affects the parameter on the first order $\ln p$ terms only, and so the Caves, Christensen, and Diewert (1982) result on the Tornqvist index applies. Second, the $\ln\left[a_0(\boldsymbol{\varphi})\right]$ term captures the pure effect of tastes on unit expenditure, but cancels from the ordinal index that holds tastes fixed.

Under Assumption D.1, the $a_i(\boldsymbol{\varphi}_0)$ and $a_i(\boldsymbol{\varphi}_1)$ are recoverable up to estimates of the γ_{ij}. To see this, the expenditure share equation for variety i is given by:

$$\begin{aligned} s_i(\boldsymbol{p};\boldsymbol{\varphi}) =& \alpha_i + \sum_{j\in\mathcal{I}} \gamma_{ij} \ln\left(\frac{p_j}{\varphi_j}\right) \\ =& a_i(\boldsymbol{\varphi}) + \sum_{j\in\mathcal{I}} \gamma_{ij} \ln p_j. \end{aligned} \tag{37}$$

This implies the following counterfactual expenditure shares do not depend on the α_i.

$$s_i(\boldsymbol{p_1}; \boldsymbol{\varphi}_0) = s_{i0} + \sum_{j \in \mathcal{I}} \gamma_{ij} \ln\left(\frac{p_{j1}}{p_{j0}}\right), \tag{38}$$

$$s_i(\boldsymbol{p_0}; \boldsymbol{\varphi}_1) = s_{i1} - \sum_{j \in \mathcal{I}} \gamma_{ij} \ln\left(\frac{p_{j1}}{p_{j0}}\right). \tag{39}$$

Denote $s_{it} = s_i(\boldsymbol{p_t}; \boldsymbol{\varphi}_t)$ the observed expenditure share, $t = 0, 1$, $s_{i1}^* = s_i(\boldsymbol{p_1}; \boldsymbol{\varphi}_0)$ and $s_{i0}^* = s_i(\boldsymbol{p_0}; \boldsymbol{\varphi}_1)$. Define the following Tornqvist style price indexes.

Definition D.1 *Tornqvist Price Index*

$$\ln P_T = \sum_{i \in \mathcal{I}} \frac{1}{2}\left(s_{i0} + s_{i1}\right) \ln\left(\frac{p_{i1}}{p_{i0}}\right) \tag{40}$$

Definition D.2 *Reference taste Tornqvist index*

$$\ln P_{T0} = \sum_{i \in \mathcal{I}} \frac{1}{2}\left(s_{i0} + s_{i1}^*\right) \ln\left(\frac{p_{i1}}{p_{i0}}\right) \tag{41}$$

Definition D.3 *Comparison period taste Tornqvist index*

$$\ln P_{T1} = \sum_{i \in \mathcal{I}} \frac{1}{2}\left(s_{i0}^* + s_{i1}\right) \ln\left(\frac{p_{i1}}{p_{i0}}\right) \tag{42}$$

Proposition 3 *Under Assumption D.1,* $P_{T0} = \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_0)$ *and* $P_{T1} = \Phi(\boldsymbol{p}_0, \boldsymbol{p}_1; \boldsymbol{\varphi}_1)$.

The proof follows from substitution of Eq. 37 into Eq. 36.