The U.S. Public Debt Valuation Puzzle

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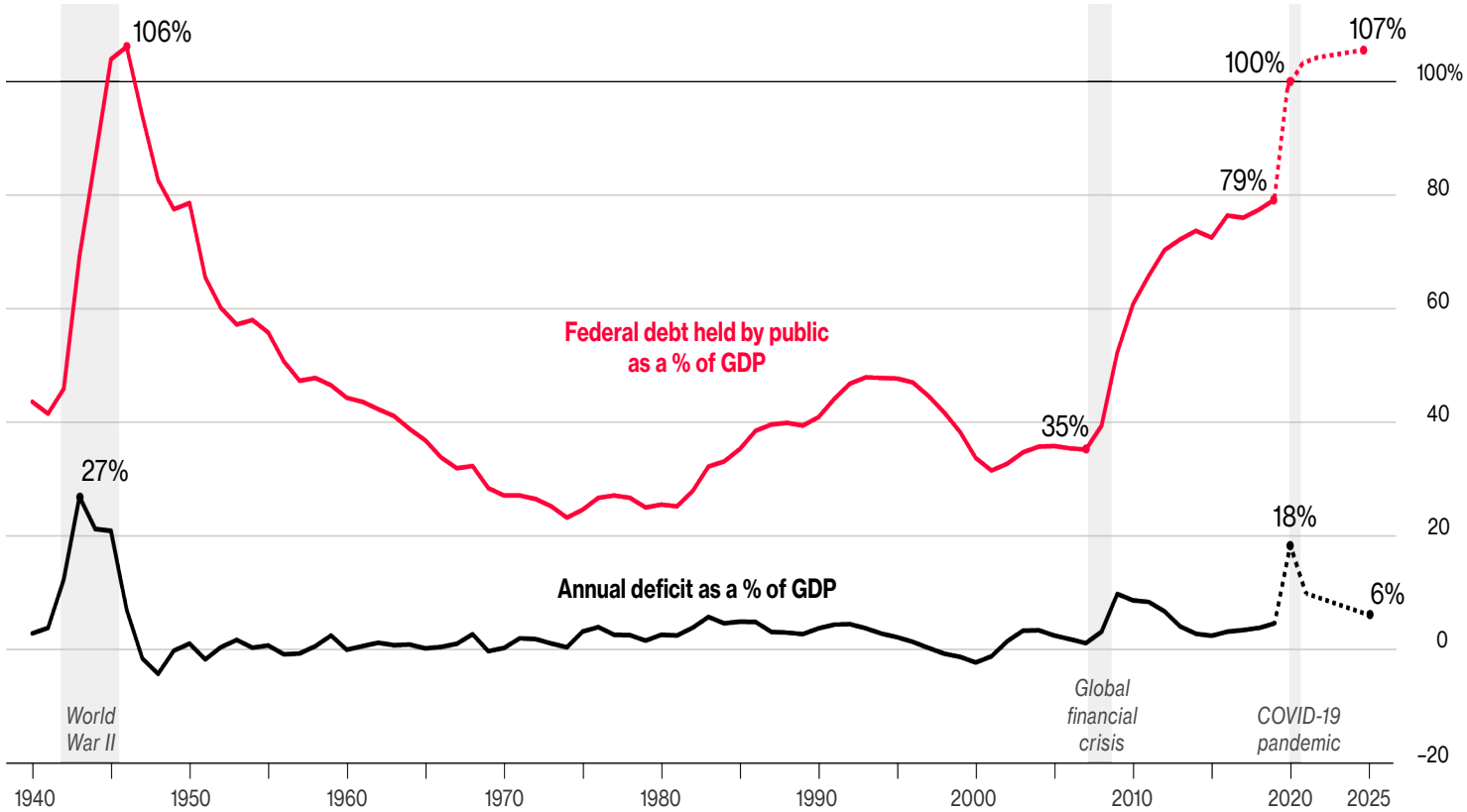
Deteriorating U.S. Fiscal Position



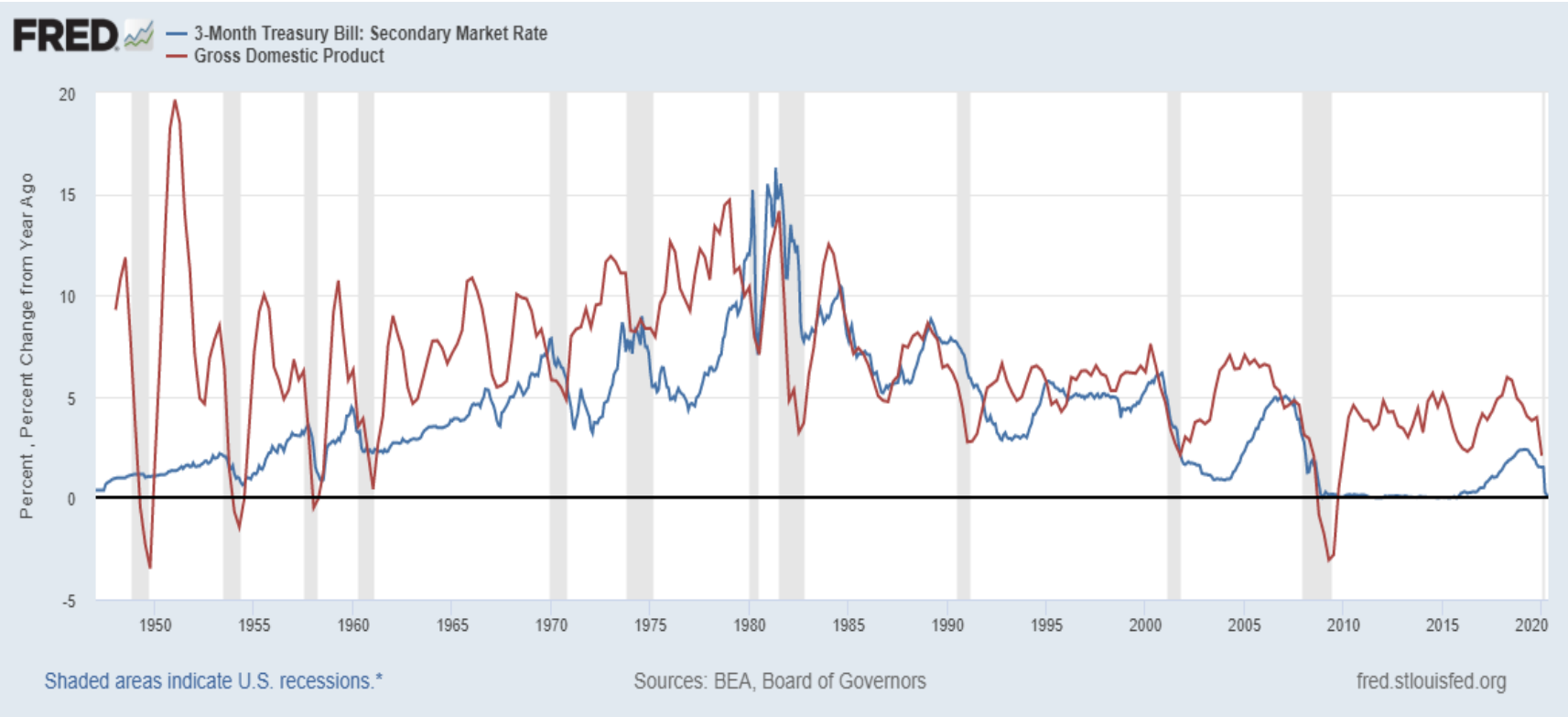
- Source: Committee for a Responsible Federal Budget, Congressional Budget Office, Bloomberg data.

What is U.S.' Debt-Bearing Capacity?

- U.S. federal government is the largest borrower in the world. The outstanding debt held by the public was $17.67 trillion at end of 2019.
- Doubled from 35% of GDP before the Great Recession to 79% of GDP in 2019.
- With covid-19 crisis, U.S. federal government has borrowed trillions more
  - to fund the private sector's payroll
  - to bail out states
  - to lend to banks
- Can the U.S. government continue to borrow trillions more?
- Or should it reduce the deficit to avoid a debt market crash?

"... public debt may have no fiscal cost."

$$r^f < g$$



- Olivier Blanchard's AEA presidential address (2019)

Government Bond Portfolio

- Revisit this question bringing in considerations of **risk**
- Government debt is backed by current and future primary surpluses.
  - Iterate forward on the government budget constraint:

$$G_t + Q_{t-1}^1 = \sum_{h=1}^{H} \left(Q_t^h - Q_{t-1}^{h+1} \right) P_t^h + T_t,$$

  - Impose no-arbitrage: $P_t^h = \mathbb{E}_t \left[M_{t,t+1} P_{t+1}^{h-1} \right], \forall h \leq H$

$$D_t = \sum_{h=0}^{H} Q_{t-1}^{h+1} P_t^h = \mathbb{E}_t \left[\sum_{j=0}^{T} M_{t,t+j} (T_{t+j} - G_{t+j}) \right] + E_t \left[M_{t,t+T} D_{t+T} \right]$$

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  - Impose a TVC: $E_t \left[M_{t,t+T} D_{t+T} \right] \to 0$ as $T \to \infty$
    - TVC can hold even if $r^f < g$

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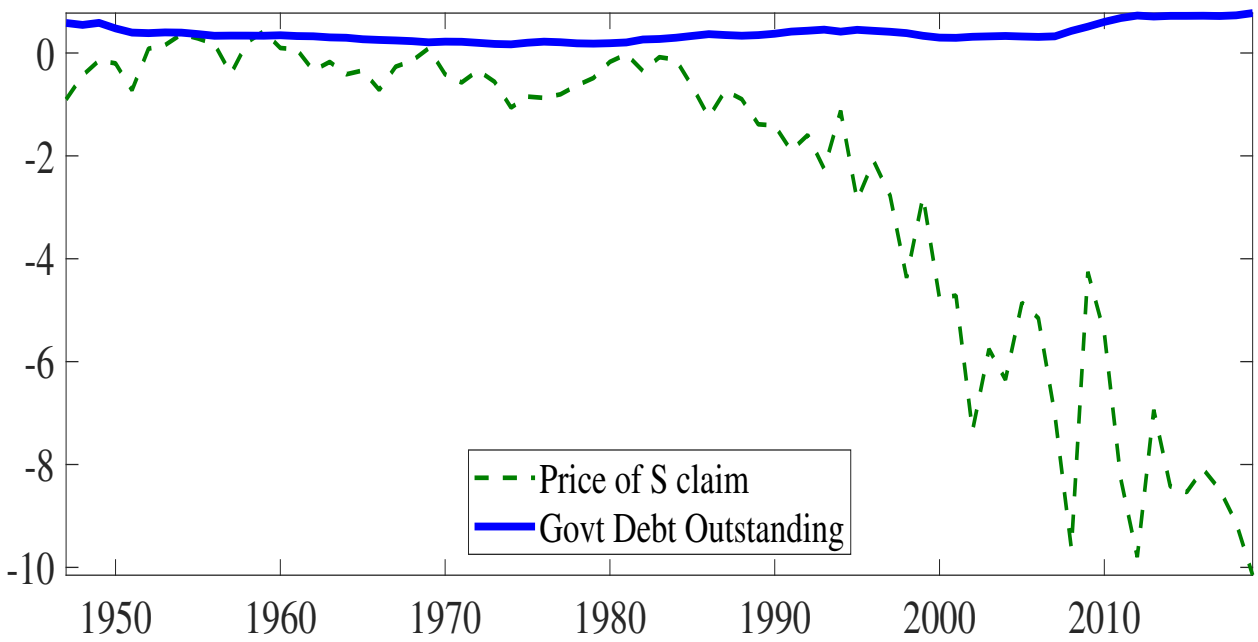
$$\underbrace{D_t = \sum_{h=0}^{H} Q_{t-1}^{h+1} P_t^h}_{\text{the market value of government debt}} = \underbrace{\mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j} (T_{t+j} - G_{t+j}) \right]}_{\text{the expected risk-adjusted PDV of future primary surpluses}}$$

    - Holds ex ante both in real and nominal terms
    - Holds when we allow for sovereign default (extension)

Government Bond Valuation Puzzle

$$D_t = \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j} (T_{t+j} - G_{t+j}) \right]$$



- The wedge between the MV of outstanding debt and the risk-adjusted PDV of future surpluses is 3x GDP; has grown
- For realistic SDF M and realistic cash flow processes $\{T, G\}$

Government Bond Valuation Puzzle

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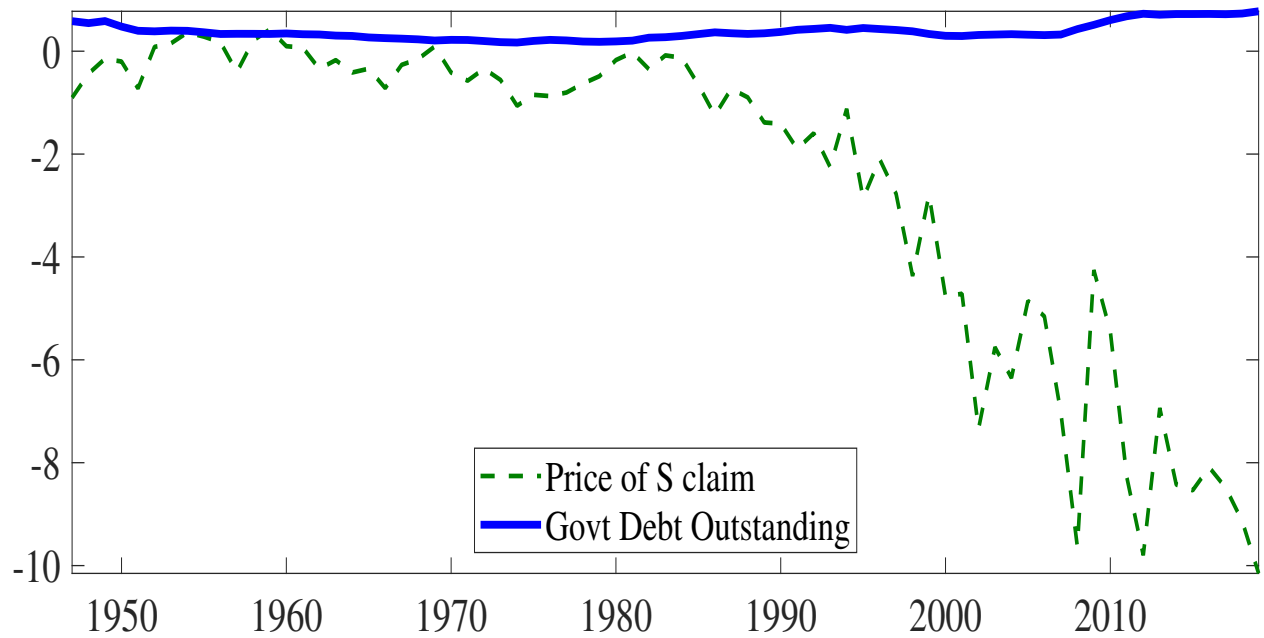
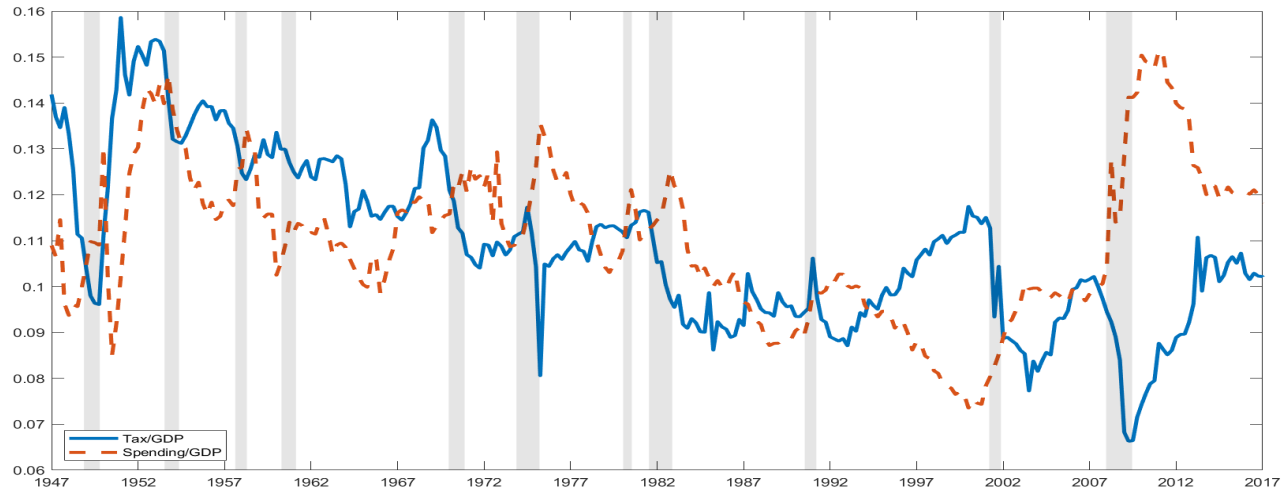


- Investors fail to impose this important restriction on the U.S. government debt **portfolio**

Key Ingredients

1. Cash flow risk in $\{T, G\}$
   - 1.1 Business cycle-frequency risk
     - Tax revenues and revenues/GDP strongly pro-cyclical
     - Government spending and spending/GDP are strongly counter-cyclical



Key Ingredients

1. Cash flow risk in $\{T, G\}$
   1.1 Business cycle-frequency risk
   - $\Rightarrow$ Primary surplus is strongly pro-cyclical
   - Primary surplus is the cash flow of an investment strategy that buys all Treasury debt (net) issuance
   - In recessions, Treasury is net issuer of debt = investor has negative cash flows
   - Cash flow has wrong-way business cycle risk $\Rightarrow$ surplus claim carries business-cycle risk premium

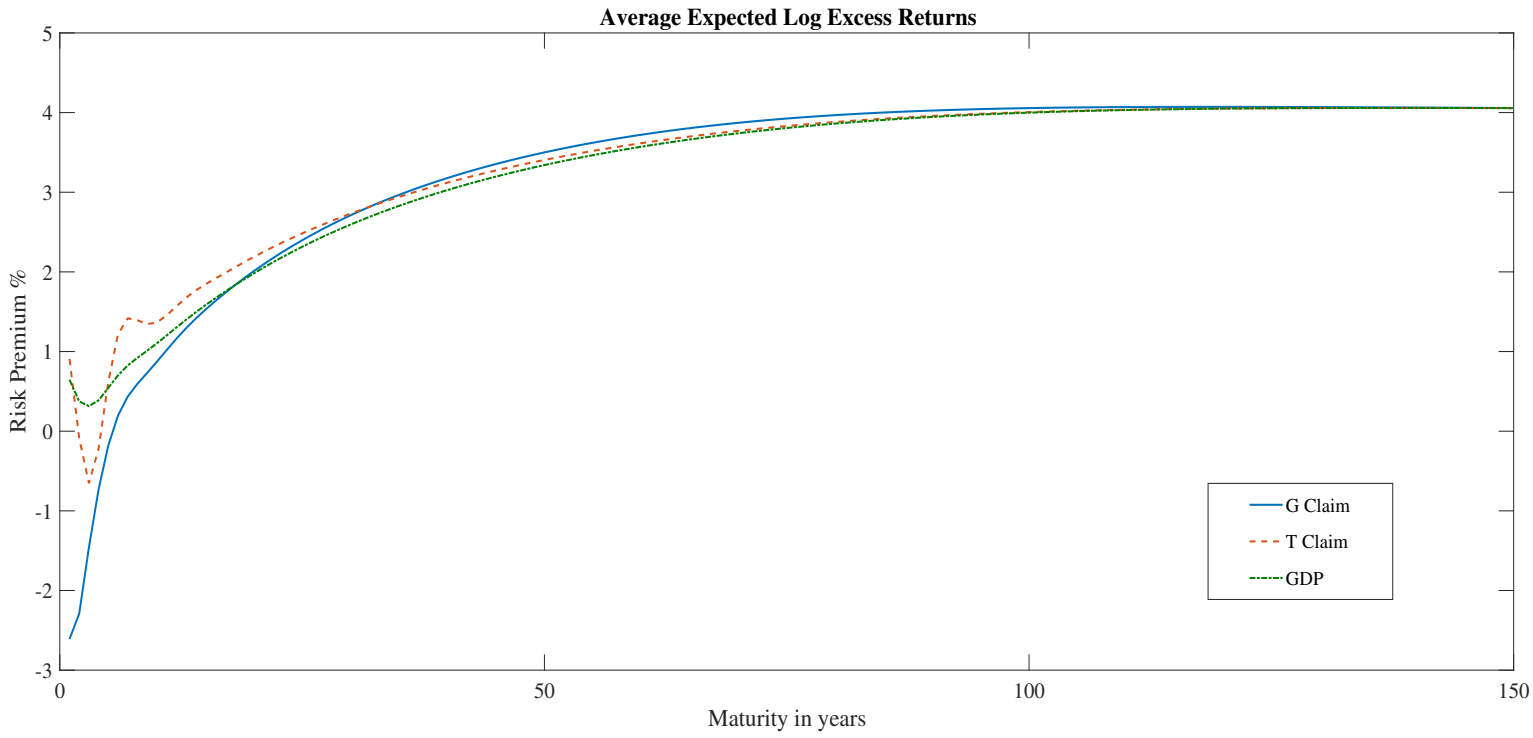
Key Ingredients

1. Cash flow risk in $\{T, G\}$
   - 1.1 Business cycle-frequency risk
   - 1.2 Long-run risk
     - Tax revenue and government spending are cointegrated with GDP $\Rightarrow$ same long-run risk
     - The expected return on a long-dated revenue or spending strip = expected return on long-dated GDP strip
     - Investor who is net long govt debt portfolio faces substantial long-run risk

Key Ingredients

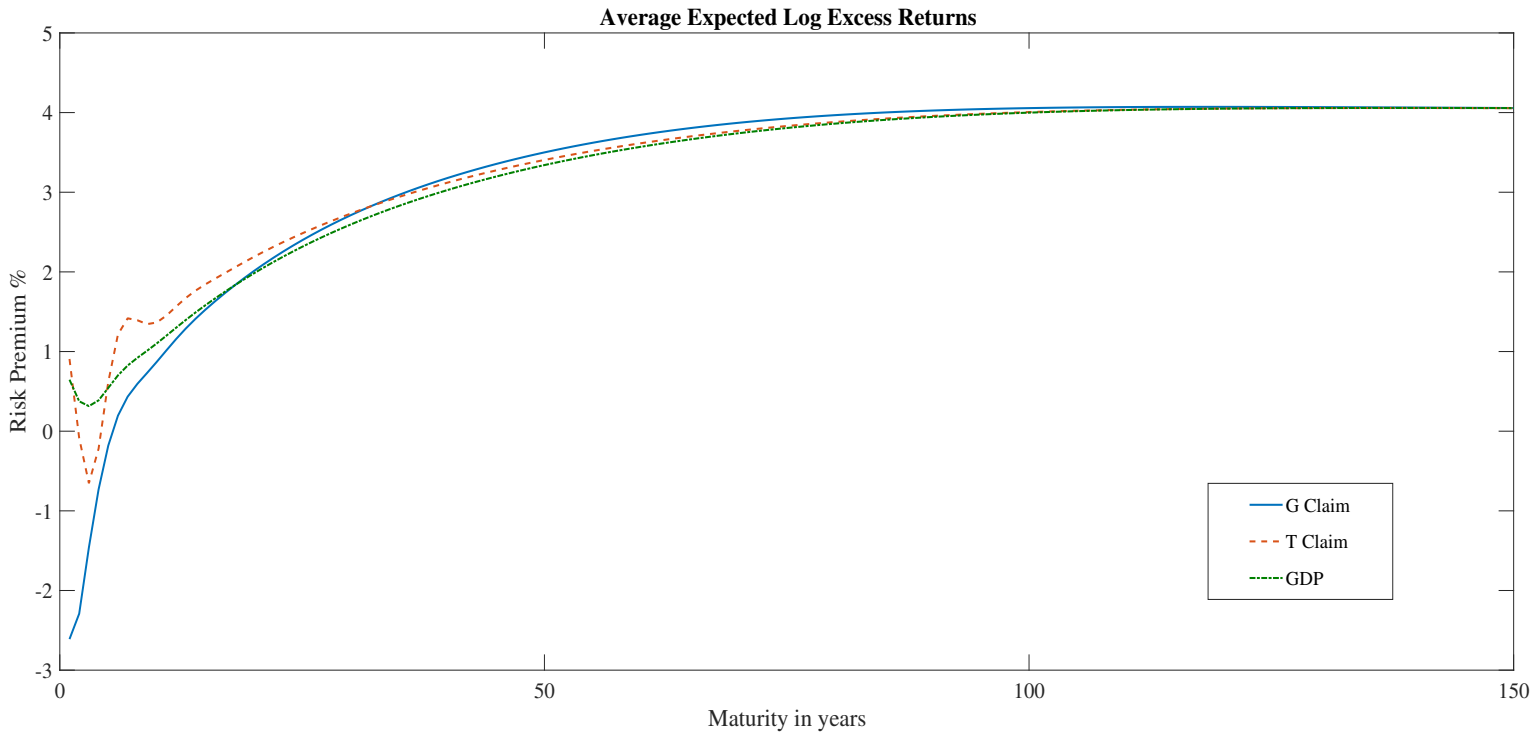
1. Cash flow risk in $\{T, G\}$
2. Realistic SDF M
   - Fits individual bond yields, nominal and real, of various maturities
   - Prices stocks (price levels, and risk premia)
   - Has a sufficiently large permanent component (Alvarez and Jermann, Borovicka, Hansen, Scheinkman)
   - Long-dated GDP claim (unlevered equity claim) has high risk premium $>$ long bond yield
   - Surplus claim has substantial long-run risk premium

Government Debt Risk Premium Puzzle



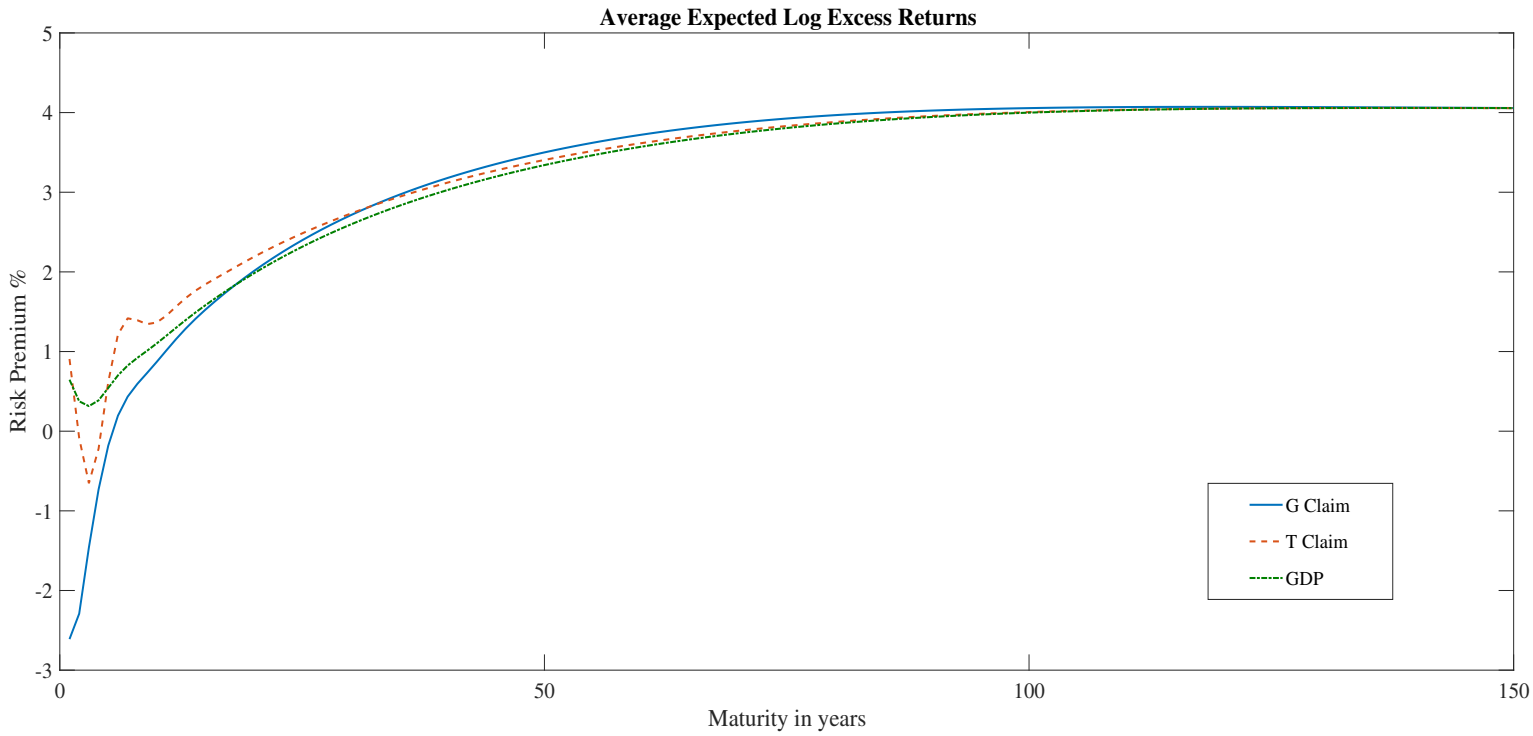
- Short-run: G claim is recession hedge, T claim is exposed

Government Debt Risk Premium Puzzle



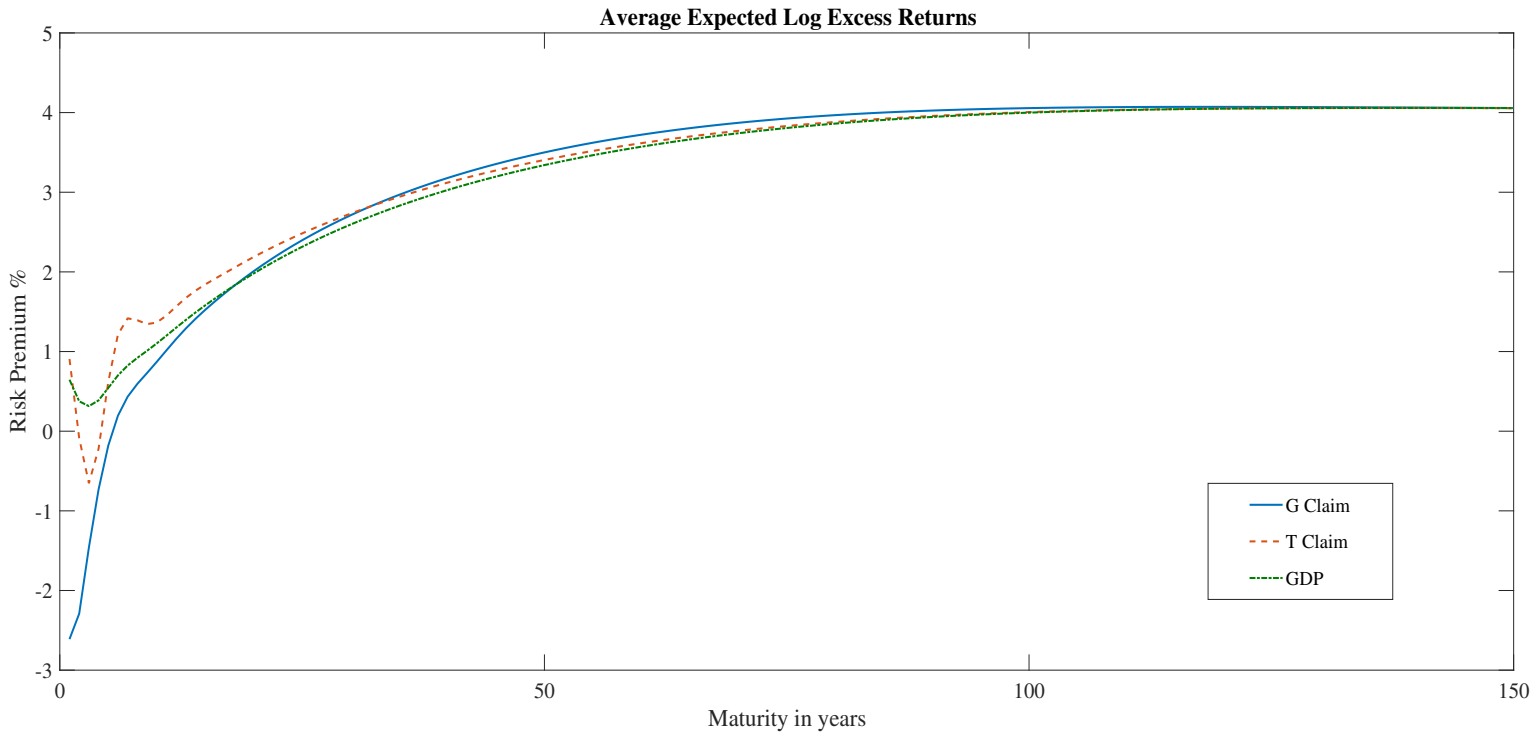
- With cointegration, long-run expected return on T- and on G-claim equals long-run expected return on GDP claim
- High long-run expected return on GDP strip, b/c permanent component in SDF

Government Debt Risk Premium Puzzle



- Short- and long-run risk premia imply that correct discount rate for surplus claim = debt portfolio is **not** the risk-free bond yield
- Expected excess return on surplus claim much higher than average observed excess return on Treasury portfolio of 1.1%

Government Debt Risk Premium Puzzle



- For surplus claim to be risk-free, the T-claim would need to be safer than the G-claim (Jiang et al. 2020b)

$$\mathbb{E}_t\left[R^T_{t+1} - R^f_t\right] = \underbrace{\frac{P^G_t - G_t}{D_t + P^G_t - G_t}}_{\text{less than 1}} \mathbb{E}_t\left[R^G_{t+1} - R^f_t\right]$$

Some Details on Cash Flow Dynamics

- Define $\tau_t = \log(T_t/GDP_t)$, and $g_t = \log(G_t/GDP_t)$
- We let $\Delta\tau_{t+1}$ and Δg_{t+1} depend on lagged macro variables in VAR
  - Real GDP growth, inflation, short interest rate, slope of YC, price-dividend ratio on stock market, aggregate dividend growth, $\Delta\tau_{t+1}$, and Δg_{t+1}
  - Annual data 1947-2019, estimated by OLS
- Tax revenue and spending are cointegrated with GDP
- Model delivers reasonable impulse-responses of fiscal variables
- Results robust to

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- Tax revenue and spending are cointegrated with GDP
  - $\Delta\tau_{t+1}$ and Δg_{t+1} depend on lagged cointegration variables τ_t and g_t.
  - Cointegration indicates (long-run) automatic stabilizers (Bohn, 98)
  - Fiscal shocks temporarily affect the level of τ_t and g_t
- Model delivers reasonable impulse-responses of fiscal variables
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  - Zeroing out insignificant elements in VAR companion matrix
  - Using quarterly instead of annual VAR
  - Starting sample in 1970
  - **Adding debt/gdp as a predictor in the VAR** (see appendix G)

Some Details on Asset Pricing Model

- Takes a stance on the priced sources of aggregate risk in the economy
  - Level & slope factor in the bond term structure
  - Dividend growth on the stock market
- Affine log SDF with market prices of risk Λ_t (Ang and Piazzesi, 2003)

$$\begin{aligned} m^{\$}_{t+1} &= -y^{\$}_t(1) - \frac{1}{2}\boldsymbol{\Lambda}_t'\boldsymbol{\Lambda}_t - \boldsymbol{\Lambda}_t'\boldsymbol{\varepsilon}_{t+1} \\ \boldsymbol{\Lambda}_t &= \boldsymbol{\Lambda}_0 + \boldsymbol{\Lambda}_1 z_t \end{aligned}$$

- Bond yields, price-dividend ratios on stock strips, expected (excess) returns on bonds and stocks are all affine in z_t
- Estimate $(\boldsymbol{\Lambda}_0, \boldsymbol{\Lambda}_1)$ to closely match: nominal and real bond yields of various maturities, nominal bond risk premia, stock price-dividend ratios, equity risk premia

Pricing Claims to Revenue T and Spending G

- With VAR dynamics and the SDF in hand, we can value T and G claims

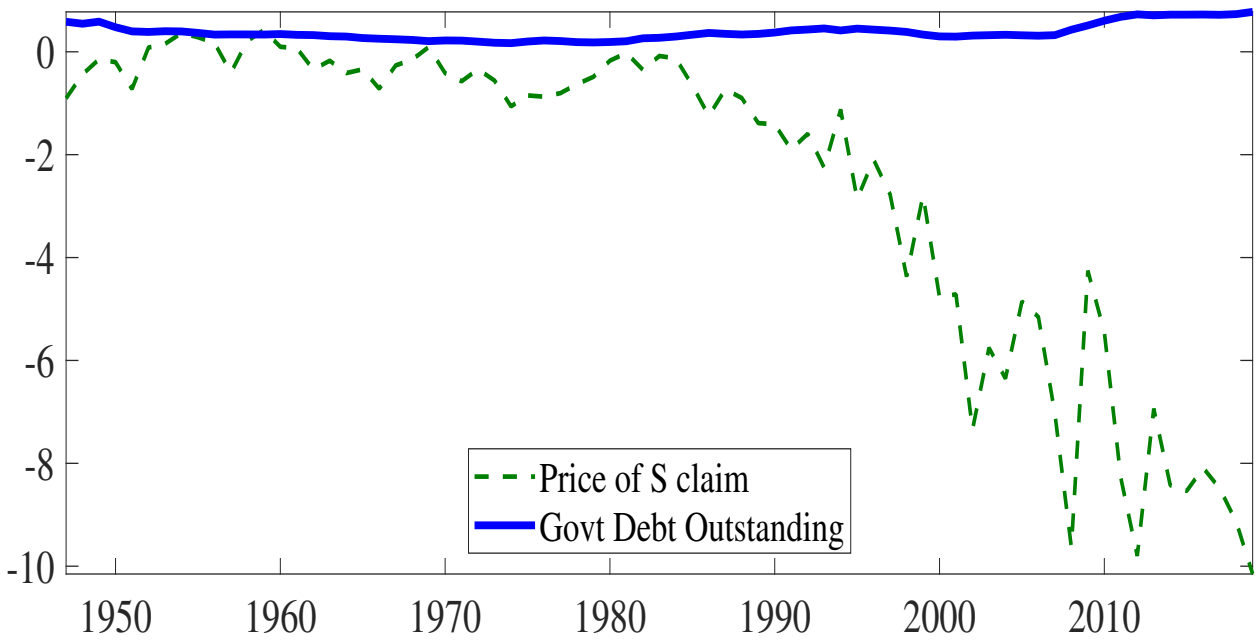
$$\begin{aligned} P_t^T &= \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j} T_{t+j} \right] \\ P_t^G &= \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j} G_{t+j} \right]. \end{aligned}$$

- The price-dividend ratios $PD_t^T = P_t^T / T_t$ and $PD_t^G = P_t^G / G_t$ are affine in the state z_t.
- Value of the surplus claim is $P_t^T - P_t^G = T_t PD_t^T - G_t PD_t^G$
- Scale by GDP for easier comparison to debt/GDP

$$\frac{T_t}{GDP_t} PD_t^T - \frac{G_t}{GDP} PD_t^G$$

And we get the Government Bond Valuation Puzzle

$$D_t = \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j} (T_{t+j} - G_{t+j}) \right]$$



Potential Resolution 1: Convenience Yield

- Convenience yield $\lambda_t \Leftrightarrow$ Treasury bonds paying lower yields than implied from SDF:

$$\begin{aligned} E_t[M_{t+1}] &= P_t^1 e^{-\lambda_t}, \\ E_t[M_{t+1} P_{t+1}^1] &= P_t^2 e^{-\lambda_t}, \\ E_t[M_{t+1} P_{t+1}^K] &= P_t^{K+1} e^{-\lambda_t}. \end{aligned}$$

- Debt now also backed by convenience services that Treasuries offers investors:

$$D_t = E_t \left[\sum_{j=0}^{\infty} M_{t,t+j} \left(T_{t+j} - G_{t+j} + (1 - e^{-\lambda_{t+j}}) D_{t+j} \right) \right]$$

Can Convenience Yields Close the Gap?

- Measure λ_t as the weighted average of CP–T-bill spread and AAA–T-bond spread (Krishnamurthy and Vissing-Jorgensen, 2012).
  - Avg. λ_t is 60 bps p.a.; Avg. conv. revenue is 0.2% of GDP
  - Lines up with measure of Binsbergen et al. (19) ▸ BDG
  - Is strongly counter-cyclical
- Reduces puzzle but does not resolve it
- Leaves open possibility that convenience yields are much larger and counter-cyclical than conventionally thought

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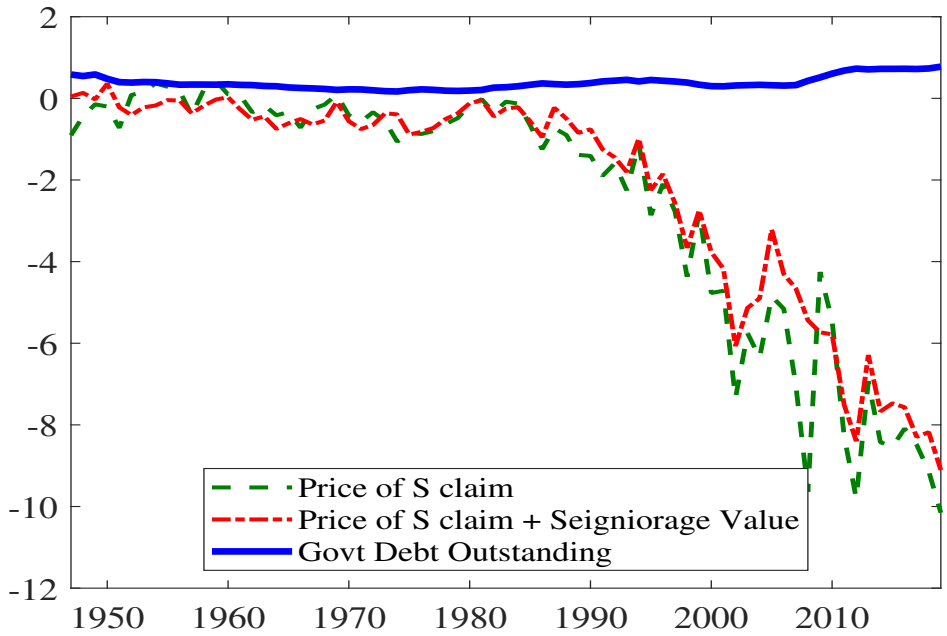
  - PDV of convenience services averages 15.5% of GDP
  - Higher convenience revenue offset by higher discounting because true risk-free rate higher with convenience



- Leaves open possibility that convenience yields are much larger and

Can Convenience Yields Close the Gap?

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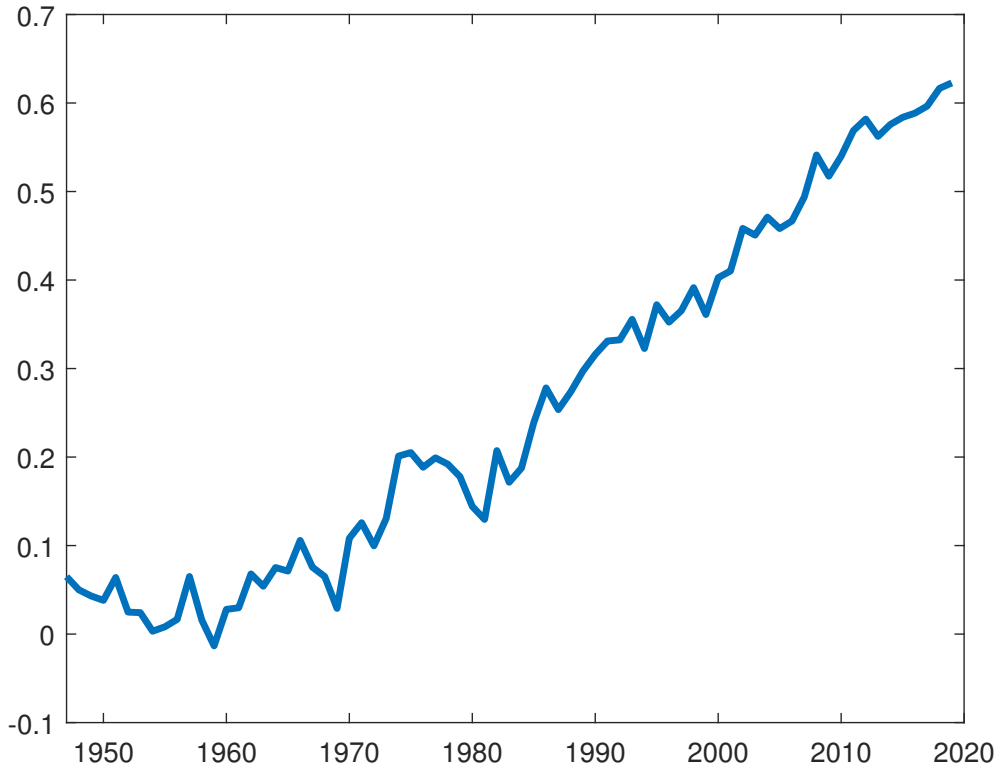
- Reduces puzzle but does not resolve it
- Leaves open possibility that convenience yields are much larger and counter-cyclical than conventionally thought
  - Other dollar-denominated assets also earn convenience yield
  - Krishnamurthy, Jiang, and Lustig (2019) find convenience yields for foreigners between 2 and 3%; Koijen and Yogo (2020) find 2.15% for U.S. long-term bonds
  - U.S. is world's designated supplier of dollar-denominated safe assets, *but that could change*; see Farhi and Maggiori (18)

Potential Resolution 2: Peso Problem

- Hypothesize that probability ϕ_t of a significant, **permanent** spending cut is priced in the surplus claim
- Such a spending cut "disaster" never realizes in post-war U.S. era, a peso event
- Spending cut of 8% of U.S. GDP = 2$\times$stdev of spending shock. Average spending is 11.5% of GDP in sample.
- How large should this spending cut probability ϕ_t be in order to equate the market value of the government debt to the present value of surpluses, period-by-period?

Potential Resolution 2: Peso Problem

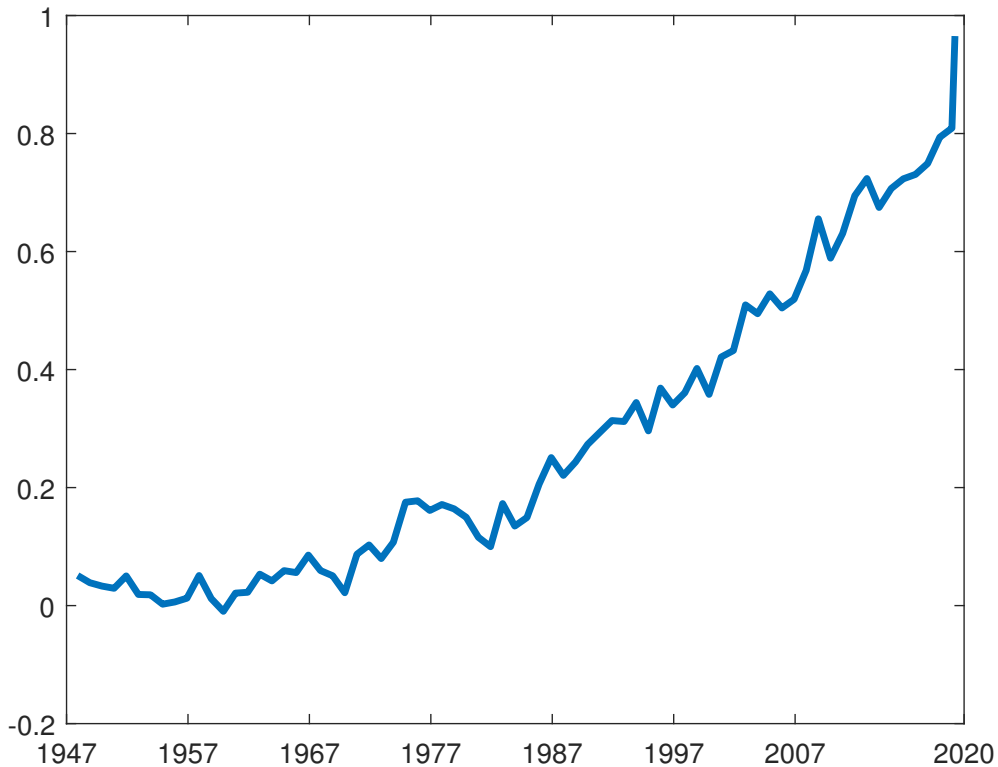
- Large!



- Implied probability ϕ_t at odds with notion of peso event
- Suggests a restatement of the puzzle

Potential Resolution 2: Peso Problem

- Find similar results for probability of major increase in **tax revenues**



- Covid-19 update: implied probability of future tax ↑ from 81% in 2019.Q4 to 96% in 2020.Q1

Potential Resolution 3: Bubble in Treasuries

- Bond markets are not enforcing TVC
  - Bubble = value of outstanding debt − value of surplus claim
  - We quantify the size of the bubble at 287% of GDP unconditionally

- But, TVC may very well hold given large risk premium on debt; $r^f < g$ is not the relevant condition (even if debt is risk-free); $r^f + rp > g$

- TVC violations are hard to sustain in the presence of long-lived investors (Santos and Woodford, 97)

- If Treasury can run Ponzi scheme, why not AAA-rated corporates?

- Rise in sovereign CDS spread after GFC (Chernov et al. 16) seems inconsistent with rational bubble in Treasuries

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Potential Resolutions 4: Pure Fiscal Risk is Priced

- Model assumes that fiscal shocks that are orthogonal to macro-economic and financial sources of risk are not priced
- Mechanically, one can close the wedge by changing this assumption. Allow for non-zero mpr on tax shock and let it depend on the debt/gdp ratio.
- Would need orthogonal tax revenue shocks to have a very large negative risk price to close the wedge
  - That would make the tax claim safer and increase its value, and hence the value of the surplus claim
  - Violates Cochrane and Saa-Requejo (2000) good-deal bound: adds 6.3 to the model's maximum Sharpe ratio.
  - Implausible that positive (orthogonal) tax revenues/GDP shocks occur in bad times
- Similarly, would need very large positive risk price to orthogonalized govt spending/gdp shock

Potential Resolutions 5: Government Assets

- Assets lower **net** government debt held by the public from 77.8% to 69.1% of the GDP; makes little difference for the puzzle
  - Outstanding student loans and other credit transactions, cash balances, and various financial instruments
  - Based on CBO data, total value of these government assets is 8.8% of GDP as of 2018.
- Other assets (national park land, defense assets, critical infrastructure, etc.) arguably off limits for political and military-strategic reasons
- If anything, massive off-balance sheet **liabilities** (Medicare, Social Security) will further **deepen** the puzzle in the future

Conclusion

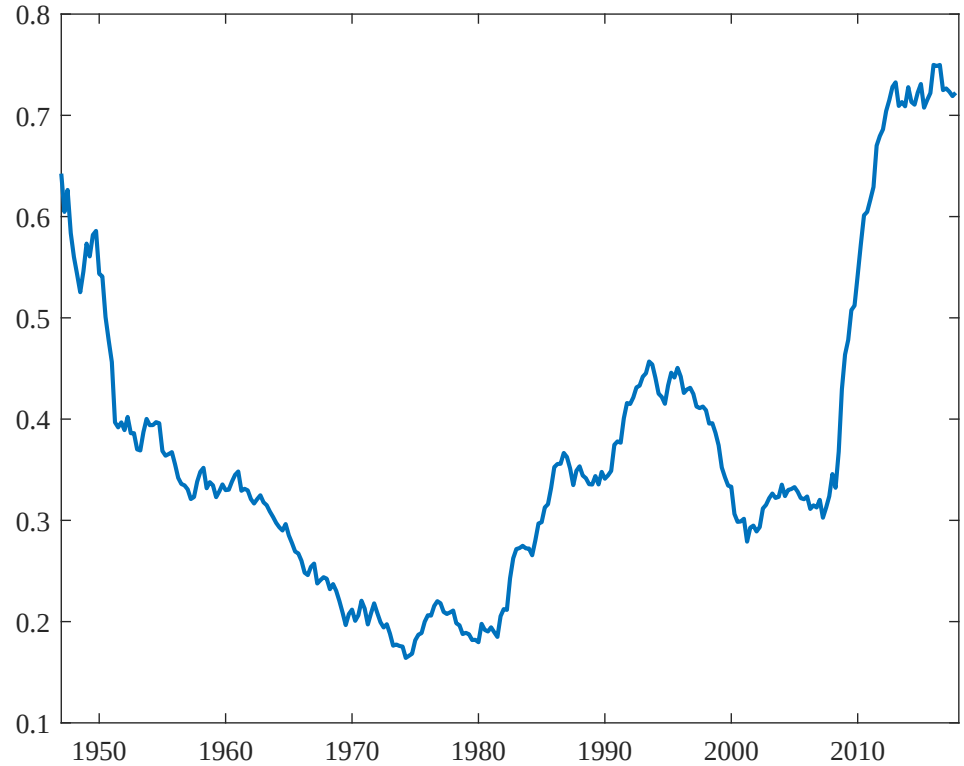
- A portfolio strategy that buys all outstanding Treasuries produces risky cash flows.
- When sources of aggregate risk reflected in bond and stock prices are adequately quantified, substantial risk premium on debt portfolio results.
- Implies that bond yields are puzzlingly low, especially recently.
- Interpretations:
  1. Bond market investors fail to enforce the TVC.
  2. Convenience yields may be much larger than we think.
  3. Investors hold optimistic beliefs about future fiscal rectitude.

Where have all the bond market vigilantes gone?

Related Literature

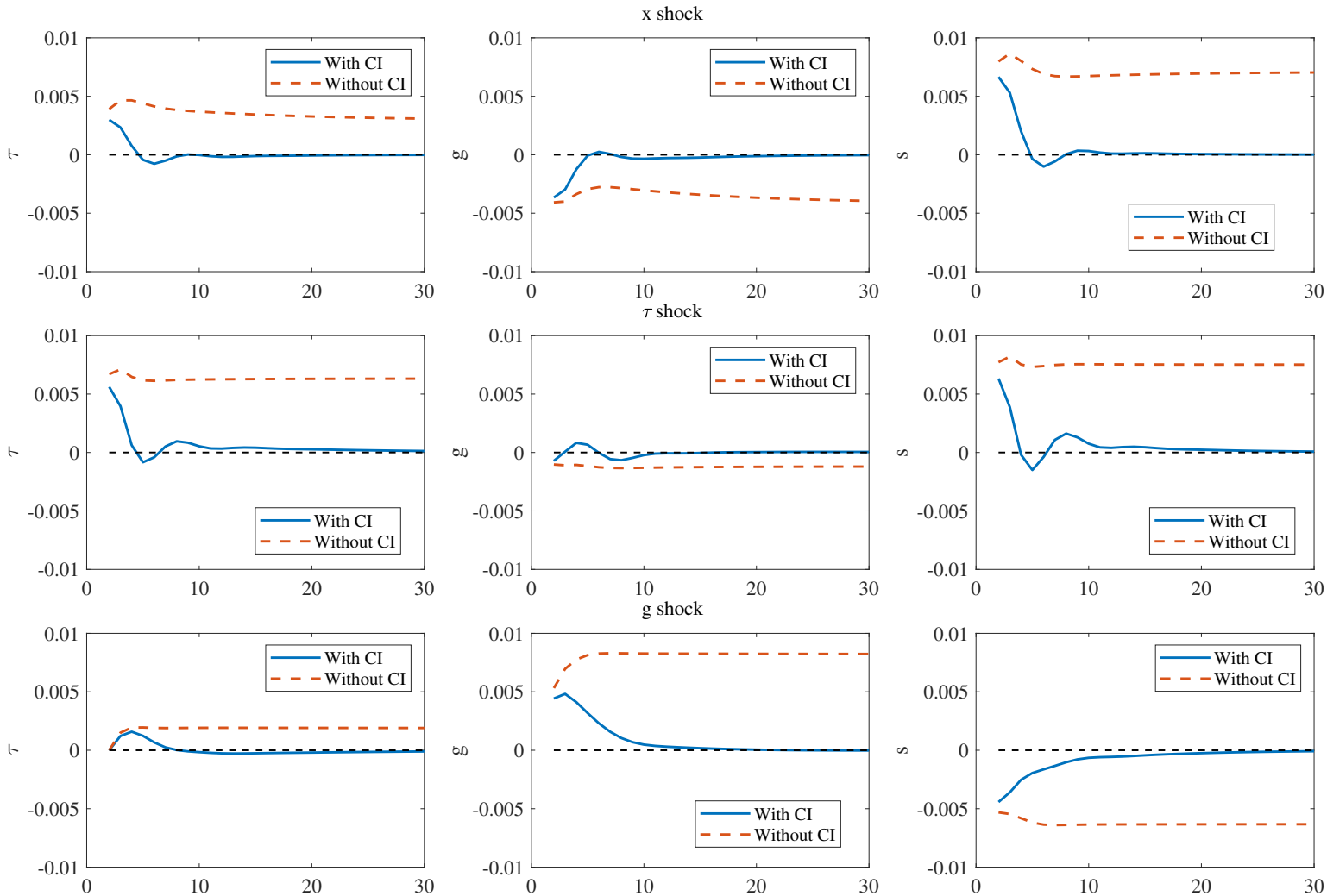
- **Affine no-arbitrage asset pricing models**: Campbell (91, 93, 96); Duffie and Kan (96); Dai and Singleton (00); Ang and Piazzesi (03); Lustig, Van Nieuwerburgh, and Verdelhan (13)
- **Fiscal policy** literature in macro: Hansen and Sargent (80); Lucas and Stokey (83); Hansen, Sargent, and Roberds (91); Angeletos (02); Buera and Nicolini (04); Hall and Sargent (11); Sargent (12); Karantounias (18); Bandhari, Golosov, Evans, and Sargent (17, 19); Blanchard (19), Cochrane (19, 20)
- **Specialness of U.S. bonds**: Longstaff (04); Krishnamurthy and Vissing-Jorgensen (12, 15); Greenwood, Hanson, and Stein (15); Nagel (16); Farhi and Maggiori (18) Du, Im, and Schreger (18); Binsbergen, Diamon, Grotteria (19); Jiang, Krishnamurthy, and Lustig (19)
- **Fiscal policy risk**: Croce, Nguyen, Schmid (12), Croce, Kung, Nguyen, and Schmid (19), Chernov, Schmid, and Schneider (19), Liu, Schmid, and Yaron (20)

The Market Value of Outstanding Debt to GDP



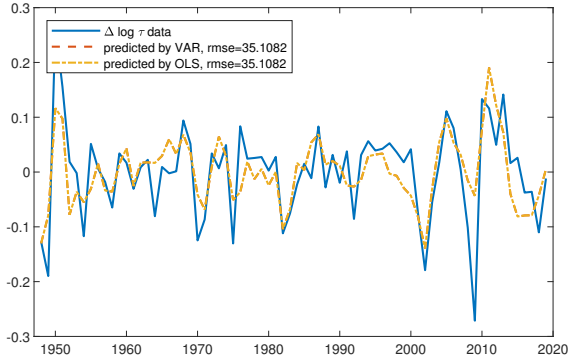
- Build up market value of government debt, cusip by cusip, stripped across horizons
- Follows Hall and Sargent (2011), extended to end of 2019
- Portfolio has low excess return over the T-bill rate: 1.11% per year

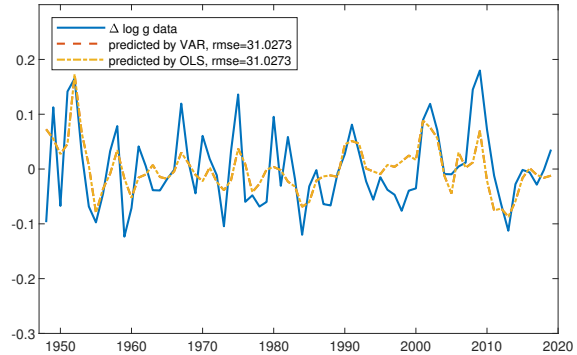
Responses of Tax and Spending



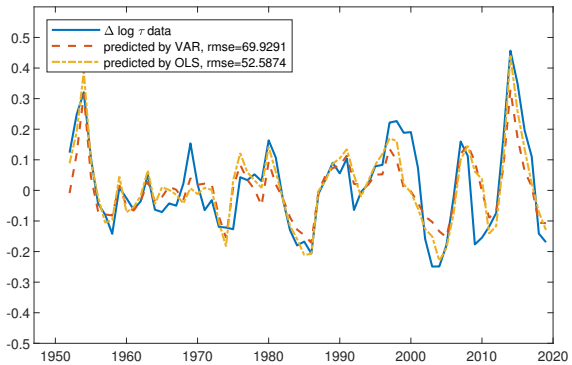
Forecasts of Revenue and Spending Growth

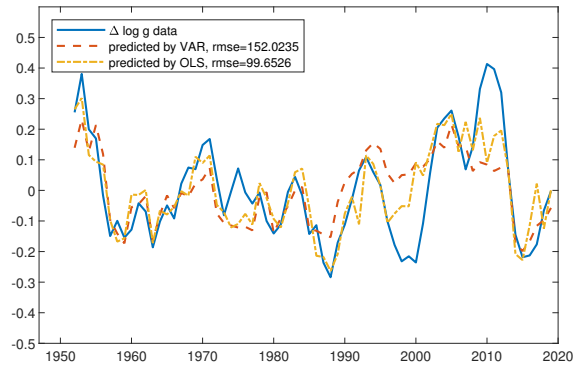
1 Yr Forecast



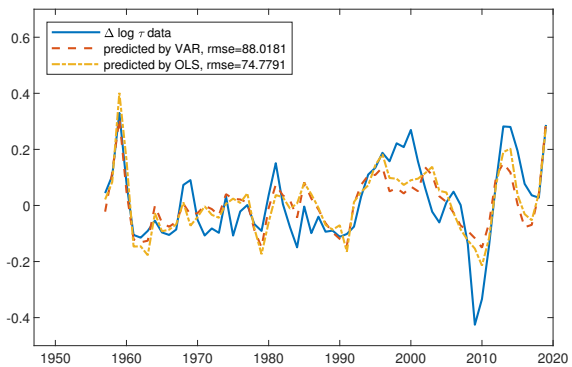


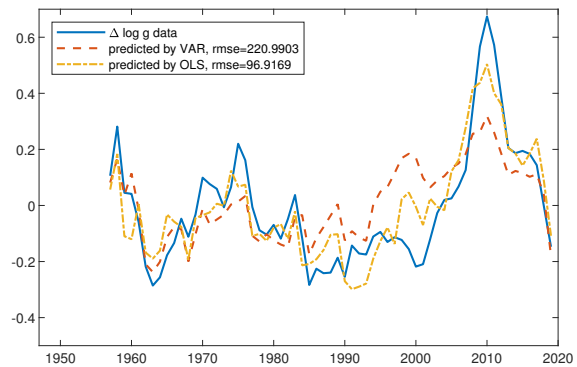
5 Yr Forecast





10 Yr Forecast





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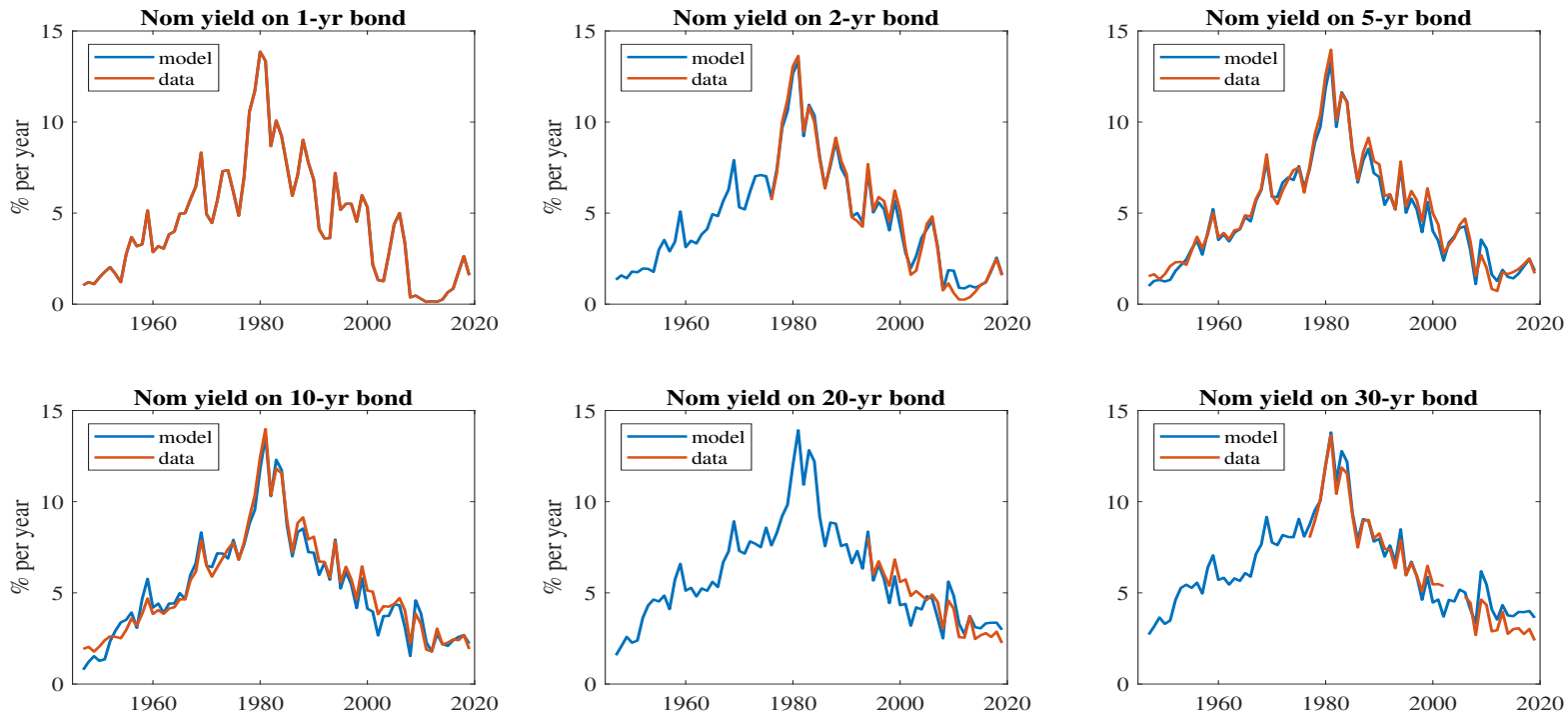

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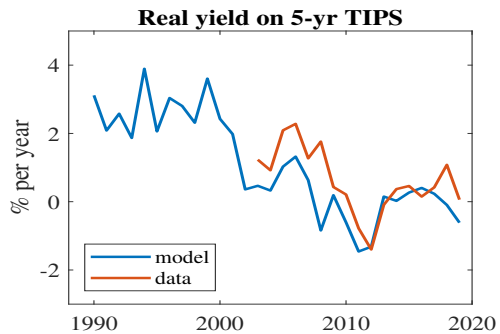

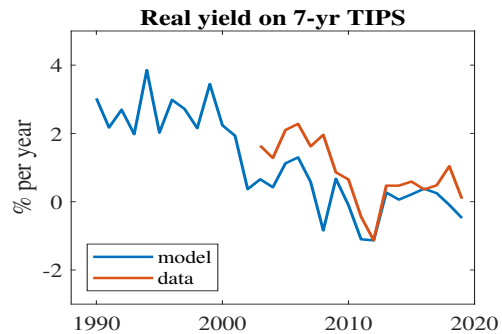

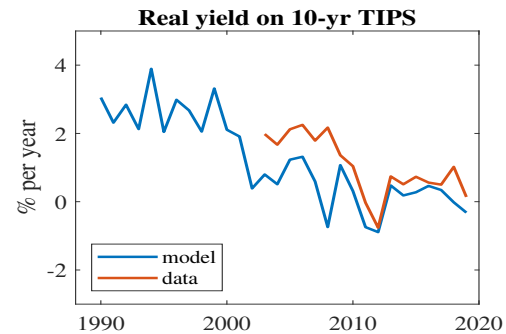

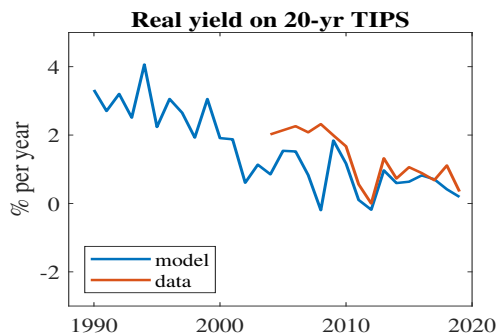

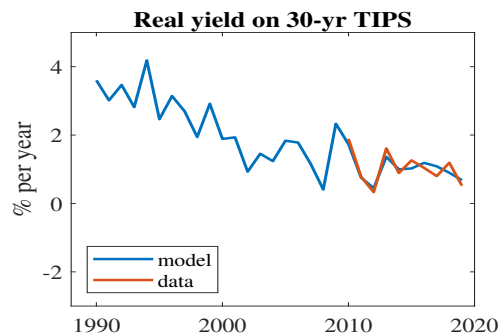

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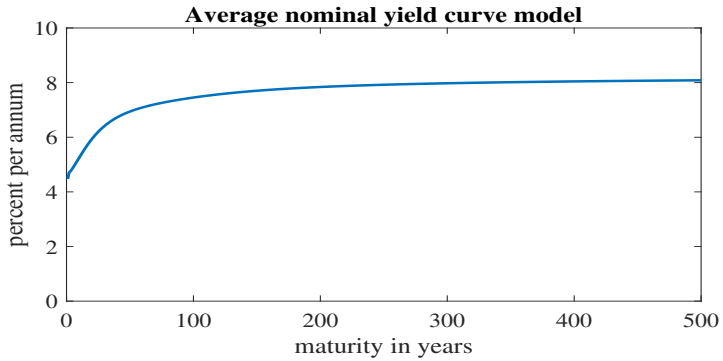

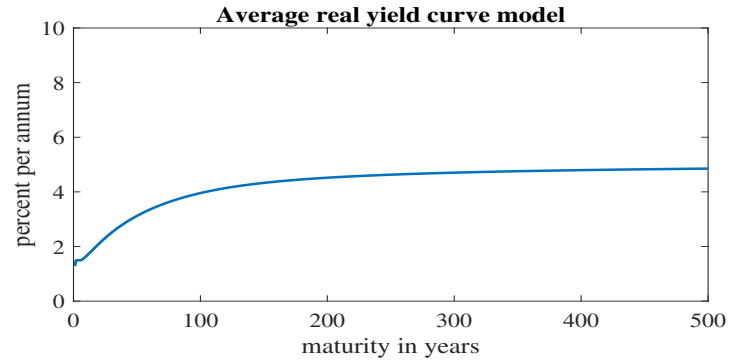

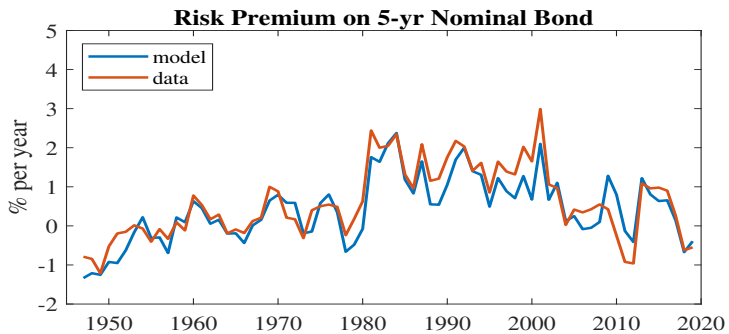

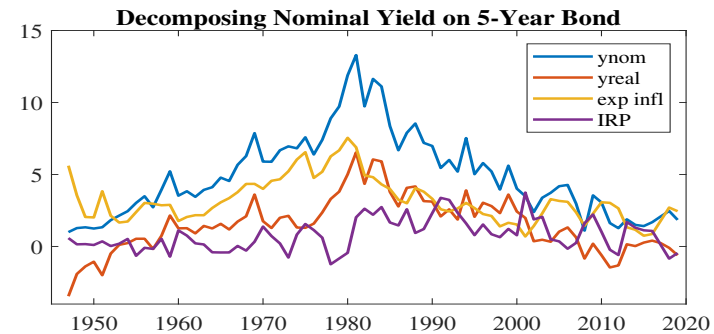

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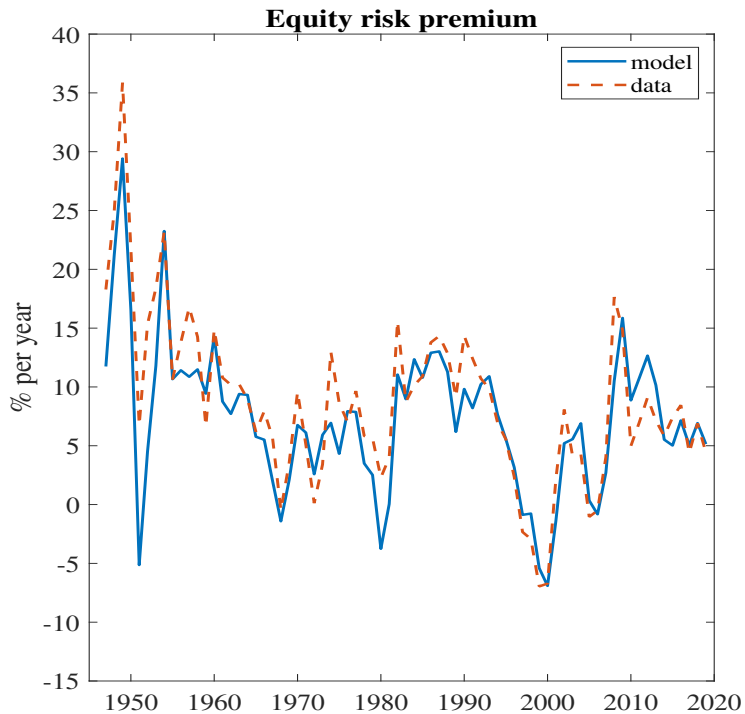
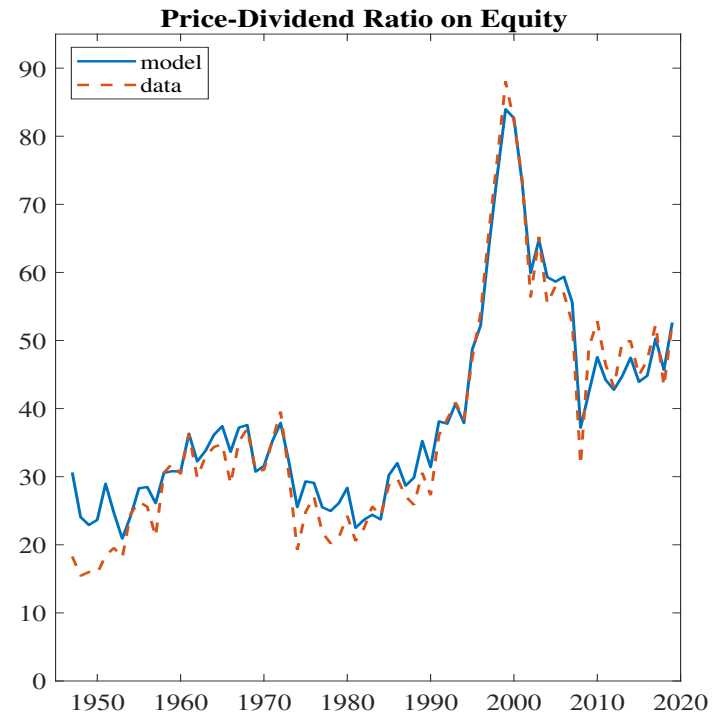
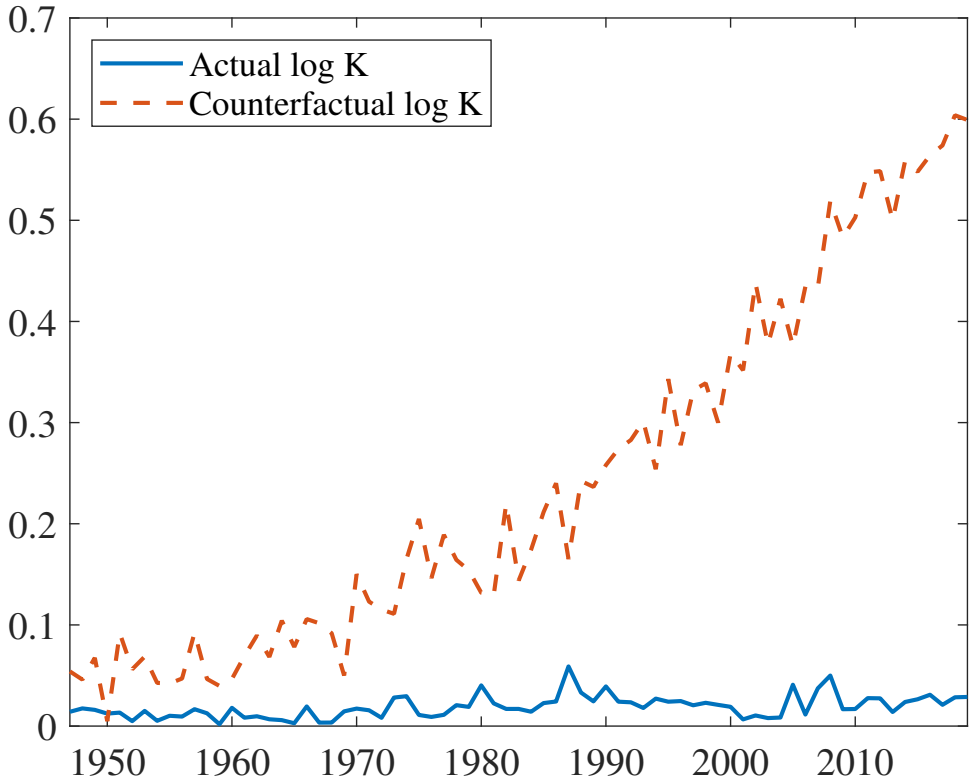


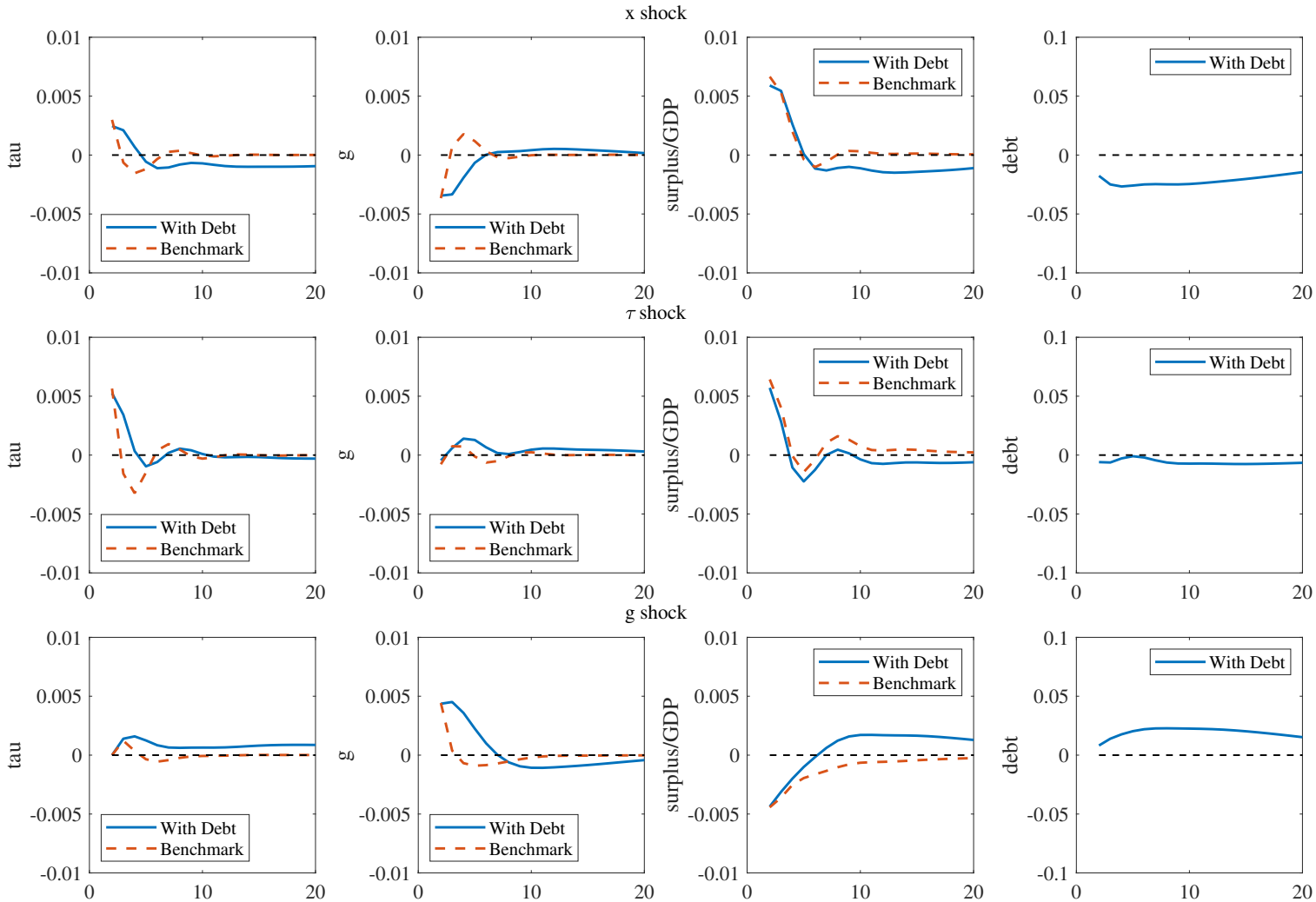


How Large a Convenience Yield to Close the Gap?

- Convenience services would need to be 24.2% of tax revenue
- They are only 1.9% in the data.
- Would need to be 48% of tax revenue in the last 20 years of sample

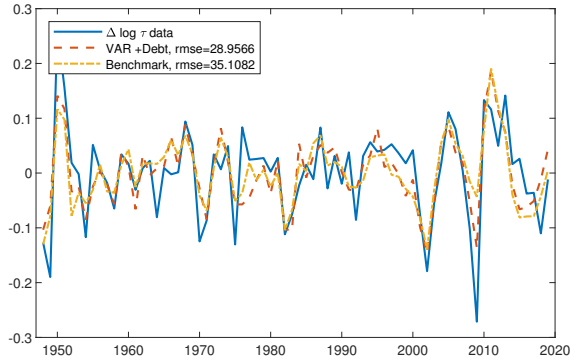


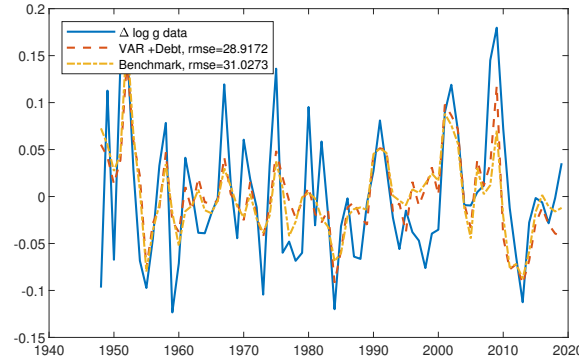
Debt in VAR: IRFs of Tax and Spending



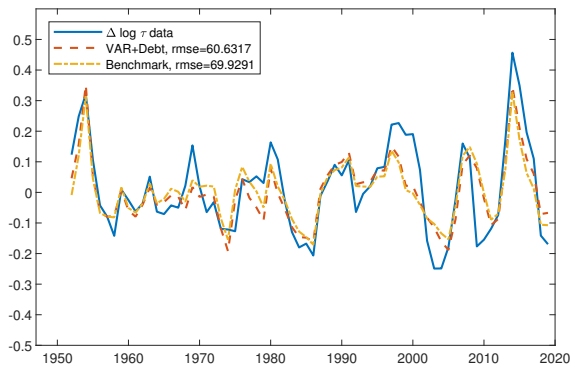
Debt in VAR: Forecasts of Revenue and Spending

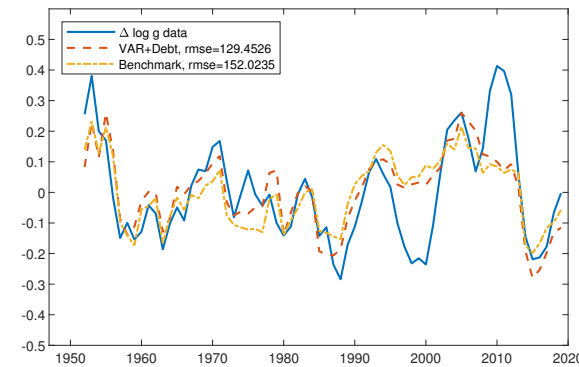
1 Yr Forecast



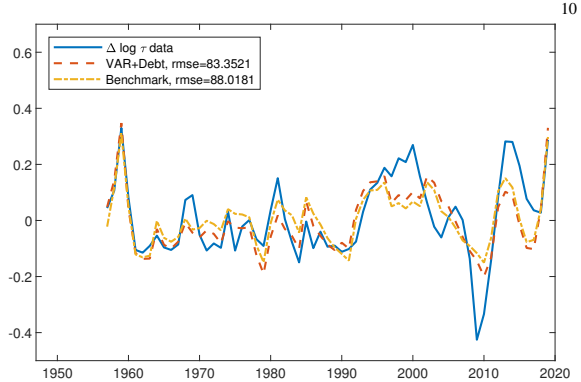


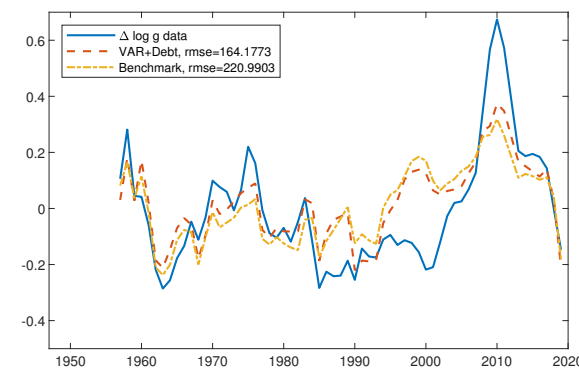
5 Yr Forecast



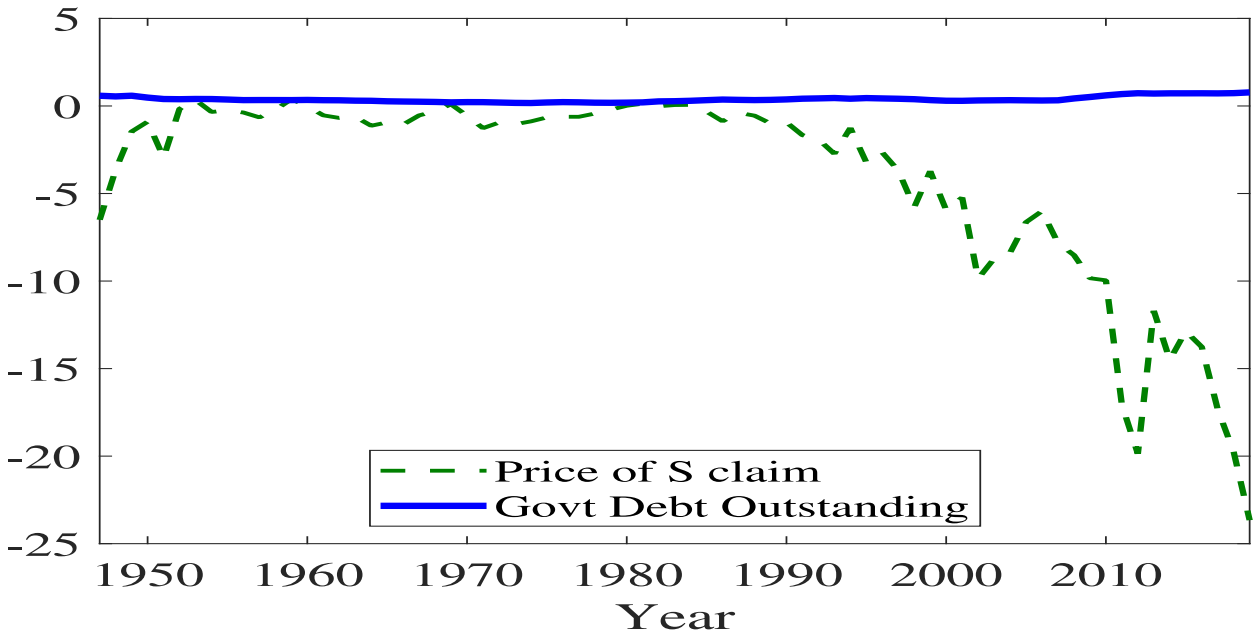


10 Yr Forecast





Debt/GDP in VAR: Valuation Puzzle is Deeper



Debt/GDP Does Not Predict Future Surpluses

$$\frac{S_{t+k}}{GDP_{t+k}} = c_k + b_k \frac{D_t}{GDP_t} + e_{t+k}$$

Horizon k	1	2	3	4	5
b_k	-0.040	-0.023	-0.006	0.001	0.003
$[t-stat]$	[-2.43]	[-1.30]	[-0.30]	[0.04]	[0.13]
R^2	7.81%	2.40%	0.13%	0.00%	0.02%

- If government debt were risk-free, the debt/gdp ratio should be the best forecaster of higher future surpluses
- If anything, coefficients go the wrong way

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- Debt/gdp is a non-stationary variable; need to include it in changes in VAR
- Can allow for mean-reverting debt levels by adding debt/gdp ratio in levels and imposing cointegration

Fiscal Measurability Constraint

- The value of the surplus claim responds in the same way as the bond portfolio to changes in every state variable

$$\frac{\partial D_t}{\partial z_t} = \sum_{h=0}^{H} Q^{\$}_{t-1,h+1} \frac{\partial P^{\$}_t(h)}{\partial z_t} = \frac{\partial P^T_t}{\partial z_t} - \frac{\partial P^G_t}{\partial z_t}$$

- If there is only one-period government debt
- Condition is severely violated in the data

Fiscal Measurability Constraint

- The value of the surplus claim responds in the same way as the bond portfolio to changes in every state variable
- If there is only one-period government debt

$$0 = \frac{\partial P_t^T}{\partial z_t} - \frac{\partial P_t^G}{\partial z_t} = 0$$

- Condition is severely violated in the data

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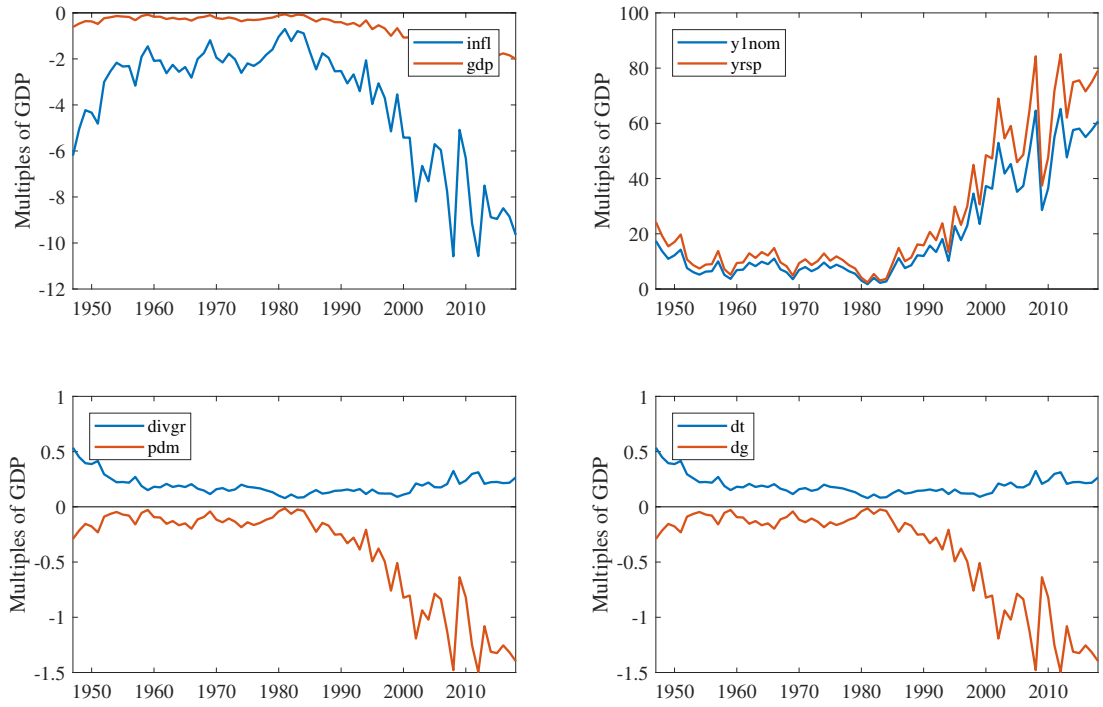
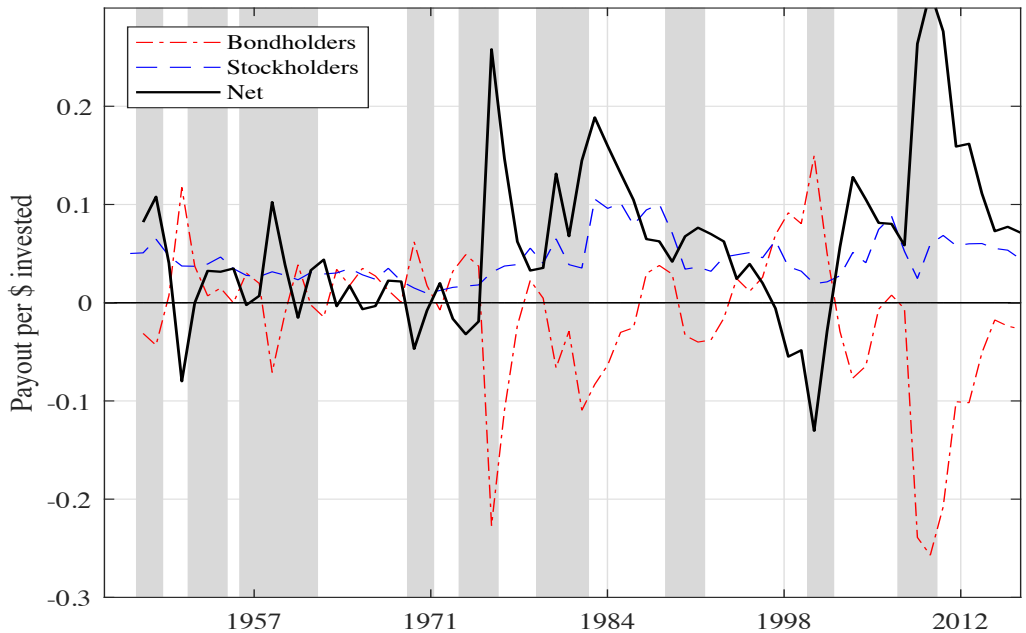


Model-Free Exercise: Betting Against Treasury

- Consider a zero-cost investment strategy: each year, short \$1 of the entire Treasury bond portfolio (overpriced) and invest 1\$ in non-financial equities.
- Cash flows on the long leg: stock dividends plus repurchases minus equity issuance
- Cash flows on the short leg: coupon payments plus principal payments minus Treasury issuance
- Net cash flows on the strategy: cash flows on long leg – cash flows on short leg

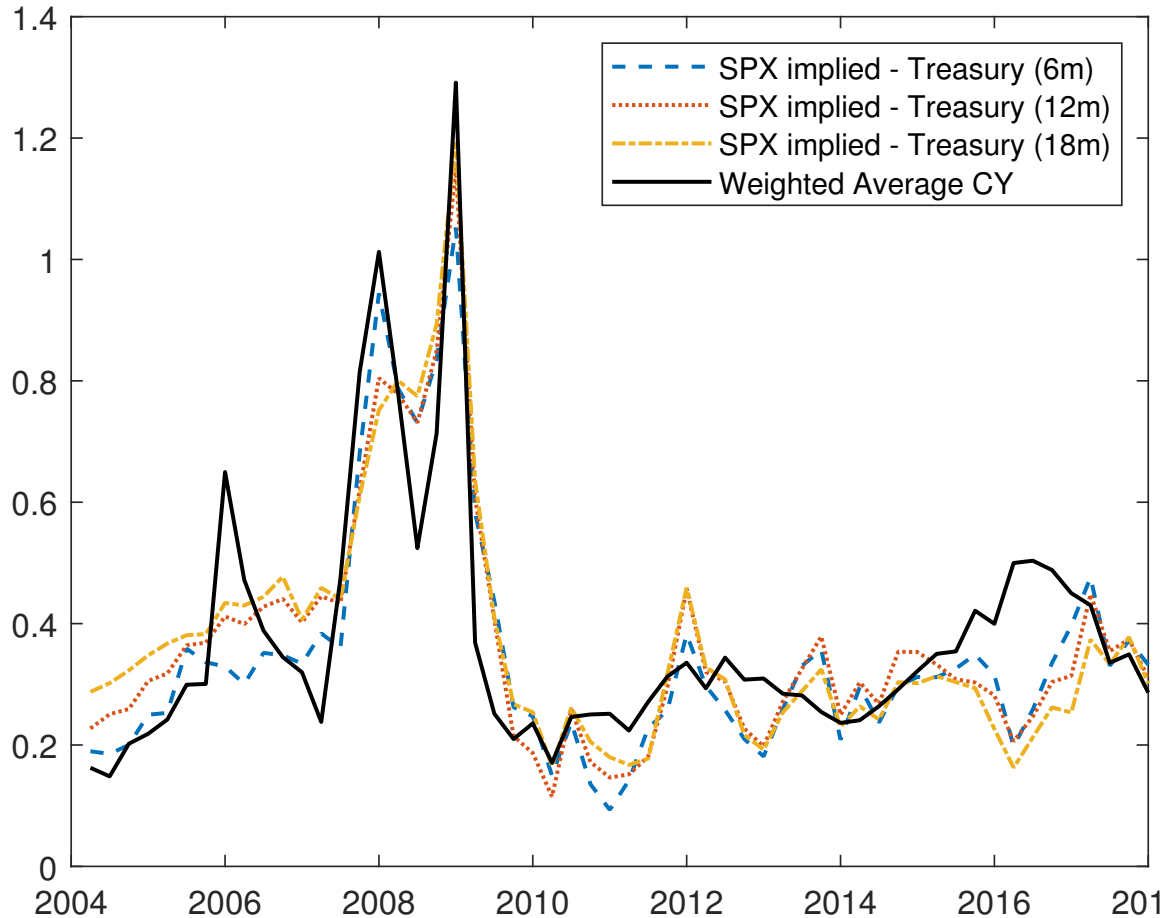
Betting Against Treasury: Net cash flows

- The Treasury cash flows of the short leg are strongly pro-cyclical and hence hedge the equity cash flows of the long leg.
- Annualized excess return is 8.85% and annualized Sharpe ratio is 0.58.
- If the equity risk premium is already a puzzle, here we have a portfolio with *counter-cyclical* cash flows despite its high expected return.



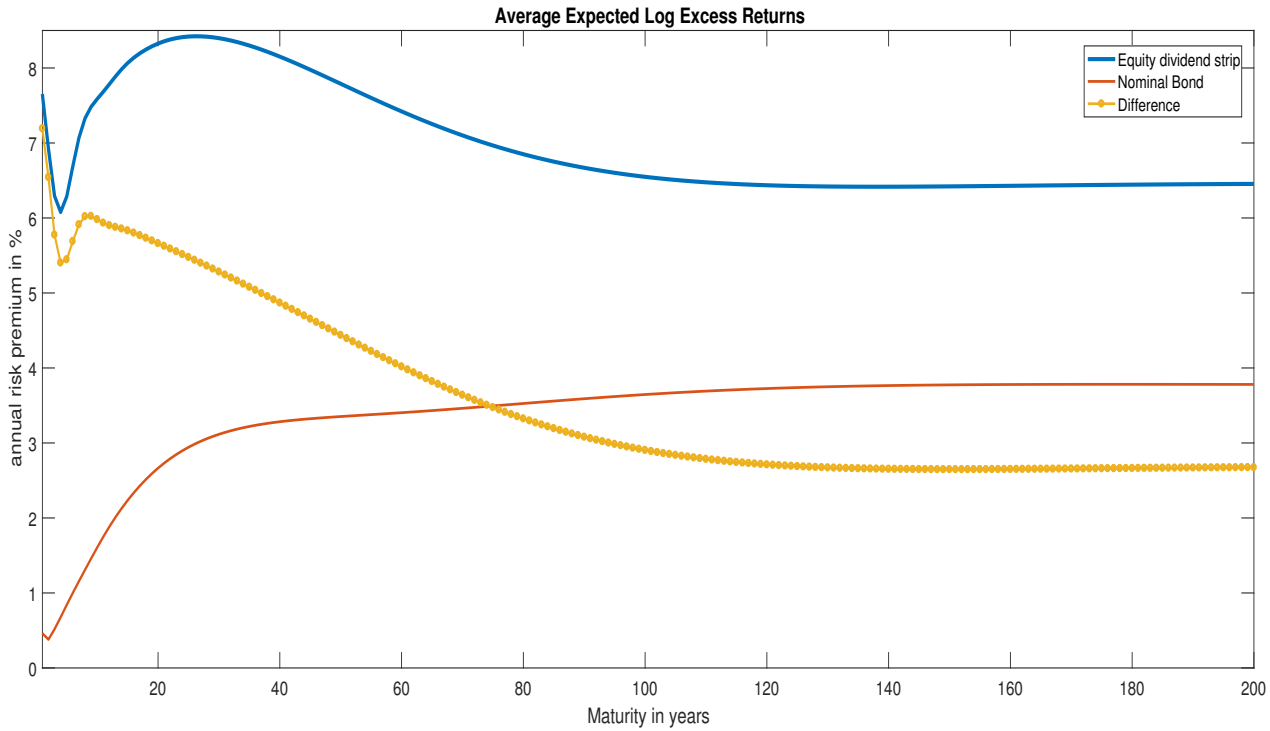
Comparing convenience yields

- Compare our convenience yield, based on Krishnamurty and Vissing-Jorgensen (2012) to that in Binsbergen, Diamond, and Grotteria (2019)



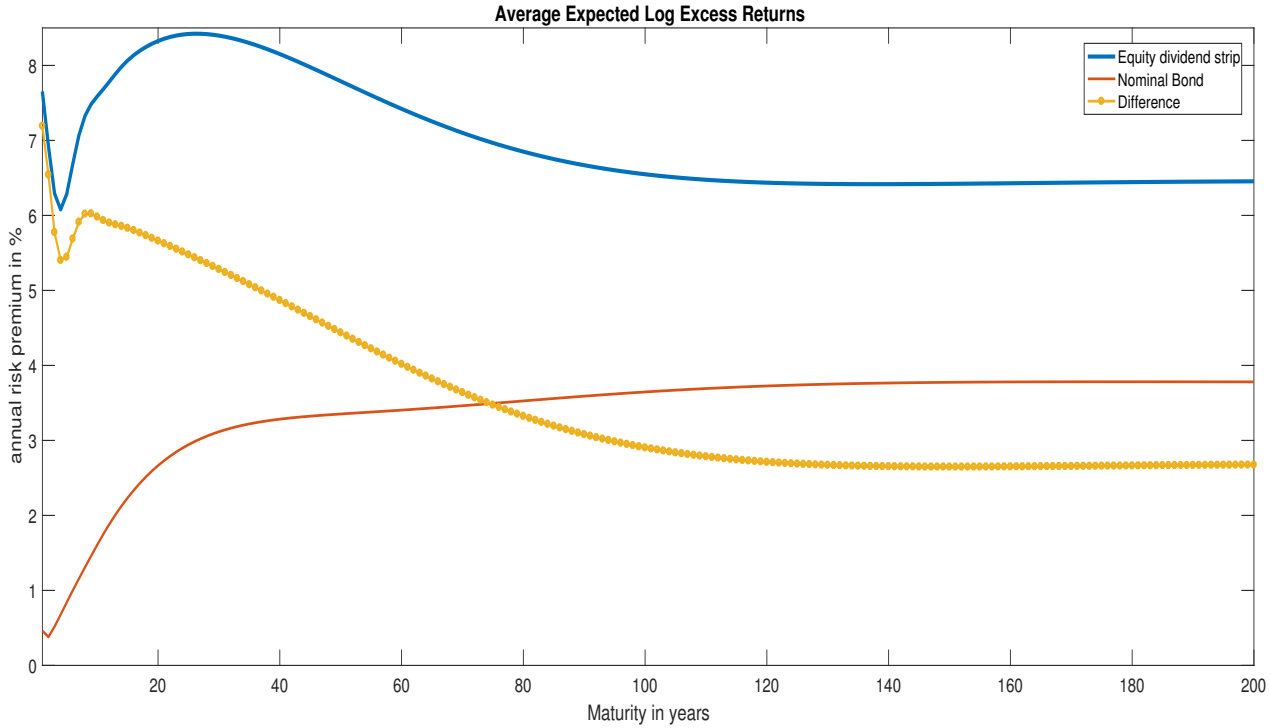
back

Stock and Bond Risk



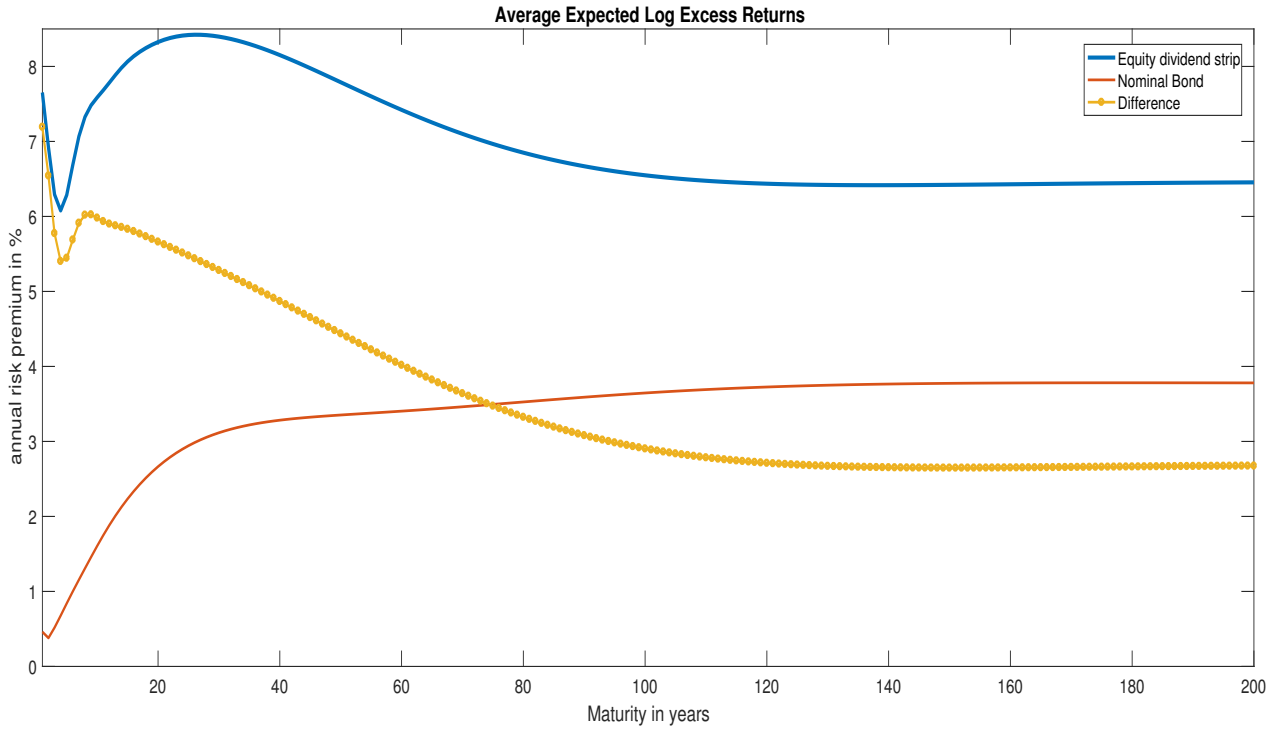
- SDF model implies downward sloping dividend strip-minus-nominal bond risk premium curve
- Weighted-average (div strip weights) difference is 1.75% p.a.
- Matching both price levels and average excess returns on stocks and bonds implies non-trivial market price of risk estimate for dividend risk. Still true in post-1970 sample.

Stock and Bond Risk



- Realized bond returns were indeed very high in last 30 years
- This deepens our puzzle since that subsample pulled up the average excess Treasury portfolio return over the full sample. The (unusually high) full-sample mean is only 1.1%.

Stock and Bond Risk



- If investors expect high future Treasury returns, there is no puzzle
- But, requires large deviations of expected and realized returns

Risk-free Debt in DAPM: Output Risk Matters.

- Risk-free debt valuation as a function of surplus/output ratio $s_t = S_t/Y_t$.
- Output risk does not disappear even when debt is risk-free:

$$\frac{D_t}{Y_t} = \sum_{h=0}^{H} Q_{t-1}^{h+1} P_t^h = \mathbb{E}_t\left[\sum_{j=0}^{T} M_{t,t+j}\frac{Y_{t+j}}{Y_t}s_{t+j}\right] + \mathbb{E}_t\left[M_{t,t+T}\frac{D_{t+T}}{Y_{t+T}}\frac{Y_{t+T}}{Y_t}\right]$$

$$\neq \mathbb{E}_t\left[\sum_{j=0}^{T} M_{t,t+j}s_{t+j}\right] + \mathbb{E}_t\left[M_{t,t+T}\frac{D_{t+T}}{Y_{t+T}}\right]$$

- We cannot just assume a stationary process for s_t and forget about output risk

$$(\mathbb{E}_t - \mathbb{E}_{t-1})\left[\sum_{j=0}^{T} M_{t,t+j}s_{t+j}\right] = 0 \nRightarrow (\mathbb{E}_t - \mathbb{E}_{t-1})\left[\sum_{j=0}^{T} M_{t,t+j}\frac{Y_{t+j}}{Y_t}s_{t+j}\right] = 0$$

- Risk-free debt imposes tight restrictions on s_t. See Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2020b).

Risk-free Debt in DAPM: Risk Premia Matter.

- Risk-free debt valuation as a function of surplus/output ratio:

$$D_t = \sum_{h=0}^{H} Q_{t-1}^{h+1} P_t^h = \mathbb{E}_t \left[\sum_{j=0}^{T} M_{t,t+j} Y_{t+j} s_{t+j} \right] + \mathbb{E}_t \left[M_{t,t+T} Y_{t+T} \frac{D_{t+T}}{Y_{t+T}} \right]$$

- Example: assume constant debt/output ratio d and risk-free debt:

$$D_t = \sum_{h=0}^{H} Q_{t-1}^{h+1} P_t^h = \mathbb{E}_t \left[\sum_{j=0}^{T} M_{t,t+j} Y_{t+j} s_{t+j} \right] + d \times \mathbb{E}_t \left[M_{t,t+T} Y_{t+T} \right]$$

- Impose a TVC: $\mathbb{E}_t \left[M_{t,t+T} Y_{t+T} \right] \to 0$ as $T \to \infty$
  - As long as output strip price $\to 0$, TVC holds, even if $R_{t,t+j}^{rf} < (1+g)^j$

$$D_t = \neq \mathbb{E}_t \left[\sum_{j=0}^{T} (R_{t,t+j}^{rf})^{-1} Y_{t+j} s_{t+j} \right] + d \mathbb{E}_t \left[(R_{t,t+T}^{rf})^{-1} Y_{t+T} \right]$$

- Risk premia matter even when debt is risk-free; $R_{t,t+1}^{rf} < (1+g)$ is irrelevant for TVC