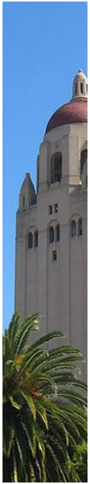

The End of Economic Growth?
Unintended Consequences of a Declining Population

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NBER Growth Meeting
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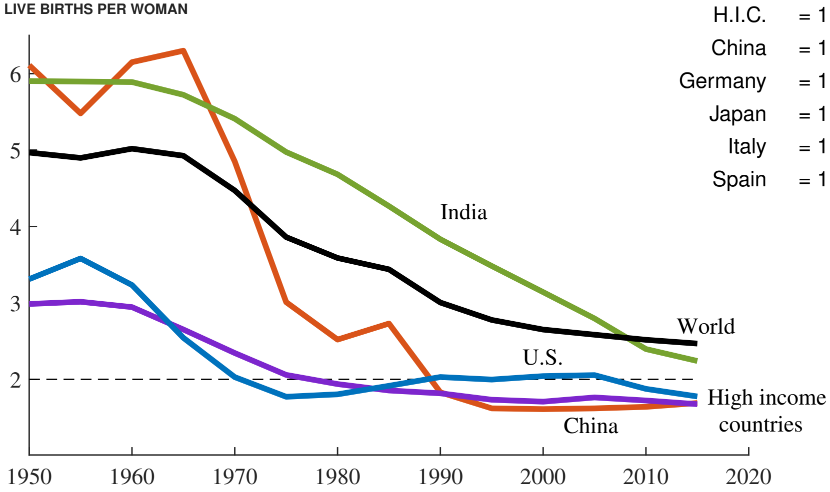
Key Role of Population

- People $\Rightarrow$ ideas $\Rightarrow$ economic growth
  - Romer (1990), Aghion-Howitt (1992), Grossman-Helpman
  - Jones (1995), Kortum (1997), Segerstrom (1998)
  - And most idea-driven growth models
- The future of global population?
  - Conventional view: stabilize at 8 or 10 billion
- Bricker and Ibbotson's *Empty Planet* (2019)
  - Maybe the future is **negative population growth**
  - High income countries already have fertility below replacement!

The Total Fertility Rate (Live Births per Woman)



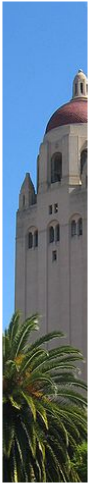
U.S. = 1.8
H.I.C. = 1.7
China = 1.7
Germany = 1.6
Japan = 1.4
Italy = 1.3
Spain = 1.3

What happens to economic growth if population growth is negative?

- Exogenous population decline
  - **Empty Planet Result**: Living standards stagnate as population vanishes!
  - Contrast with standard **Expanding Cosmos** result: exponential growth for an exponentially growing population
- Endogenous fertility
  - Parameterize so that the equilibrium features negative population growth
  - A planner who prefers **Expanding Cosmos** can get trapped in an **Empty Planet** – if society delays implementing the optimal allocation

Literature Review

- Many models of fertility and growth (but not $n < 0$)
  - Too many papers to fit on this slide!
- Falling population growth and declining dynamism
  - Krugman (1979) and Melitz (2003) are semi-endogenous growth models
  - Karahan-Pugsley-Sahin (2019), Hopenhayn-Neira-Singhania (2019), Engbom (2019), Peters-Walsh (2019)
- Negative population growth
  - Feyrer-Sacerdote-Stern (2008) and changing status of women
  - Christians (2011), Sasaki-Hoshida (2017), Sasaki (2019a,b) consider capital, land, and CES
  - Detroit? Or world in 25,000 BCE?



The Empty Planet Result

A Simplified Romer/AH/GH Model

Production of goods (IRS)	$Y_t = A_t^\sigma N_t$
Production of ideas	$\frac{\dot{A}_t}{A_t} = \alpha N_t$
Constant population	$N_t = N$

- Income per person: levels and growth

$$y_t \equiv Y_t / N_t = A_t^\sigma$$

$$\frac{\dot{y}_t}{y_t} = \sigma \frac{\dot{A}_t}{A_t} = \sigma \alpha N$$

- Exponential growth with a constant population
  - But population growth means exploding growth? (Semi-endogenous fix)

Negative Population Growth in Romer/AH/GH

Production of goods (IRS)	$Y_t = A_t^\sigma N_t$
Production of ideas	$\frac{\dot{A}_t}{A_t} = \alpha N_t$
Exogenous population decline	$N_t = N_0 e^{-\eta t}$

- Combining the 2nd and 3rd equations (note $\eta > 0$)

$$\frac{\dot{A}_t}{A_t} = \alpha N_0 e^{-\eta t}$$

- This equation is easily integrated...

The Empty Planet Result in Romer/GH/AH

- The stock of knowledge A_t is given by

$$\log A_t = \log A_0 + \frac{g_{A0}}{\eta}\left(1 - e^{-\eta t}\right)$$

  where g_{A0} is the initial growth rate of A

- A_t and $y_t \equiv Y_t/N_t$ converge to constant values A^* and y^*:

$$A^* = A_0 \exp\left(\frac{g_{A0}}{\eta}\right)$$

$$y^* = y_0 \exp\left(\frac{g_{y0}}{\eta}\right)$$

- **Empty Planet Result**: Living standards stagnate as the population vanishes!

Semi-Endogenous Growth

Production of goods (IRS)	$Y_t = A_t^\sigma N_t$
Production of ideas	$\frac{\dot{A}_t}{A_t} = \alpha N_t^\lambda A_t^{-\beta}$
Exogenous population growth	$N_t = N_0 e^{nt}, \quad n > 0$

- Income per person: levels and growth

$$y_t = A_t^\sigma \quad \text{and} \quad A_t^* \propto N_t^{\lambda/\beta}$$

$$g_y^* = \gamma n, \quad \text{where} \quad \gamma \equiv \lambda\sigma/\beta$$

- **Expanding Cosmos**: Exponential income growth for growing population

Negative Population Growth in the Semi-Endogenous Setting

Production of goods (IRS)	$Y_t = A_t^{\sigma} N_t$
Production of ideas	$\frac{\dot{A}_t}{A_t} = \alpha N_t^{\lambda} A_t^{-\beta}$
Exogenous population decline	$N_t = N_0 e^{-\eta t}$

- Combining the 2nd and 3rd equations:

$$\frac{\dot{A}_t}{A_t} = \alpha N_0^{\lambda} e^{-\lambda \eta t} A_t^{-\beta}$$

- Also easily integrated...

The Empty Planet in a Semi-Endogenous Framework

- The stock of knowledge A_t is given by

$$A_t = A_0 \left(1 + \frac{\beta g_{A0}}{\lambda\eta}\left(1 - e^{-\lambda\eta t}\right)\right)^{1/\beta}$$

- Let $\gamma \equiv \lambda\sigma/\beta$ = overall degree of increasing returns to scale.

- Both A_t and income per person $y_t \equiv Y_t/N_t$ converge to constant values A^* and y^*:

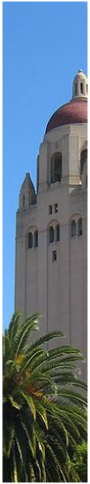
$$A^* = A_0 \left(1 + \frac{\beta g_{A0}}{\lambda\eta}\right)^{1/\beta}$$

$$y^* = y_0 \left(1 + \frac{g_{y0}}{\gamma\eta}\right)^{\gamma/\lambda}$$

First Key Result: The Empty Planet

- Fertility has trended down: 5, 4, 3, 2, and less in rich countries
  - For a family, nothing special about “above 2” vs “below 2”

- But macroeconomics makes this distinction critical!
  - Negative population growth may condemn us to stagnation on an **Empty Planet**
    - Stagnating living standards for a population that vanishes
  - Vs. the exponential growth in income and population of an **Expanding Cosmos**



Endogenous Fertility

The Economic Environment

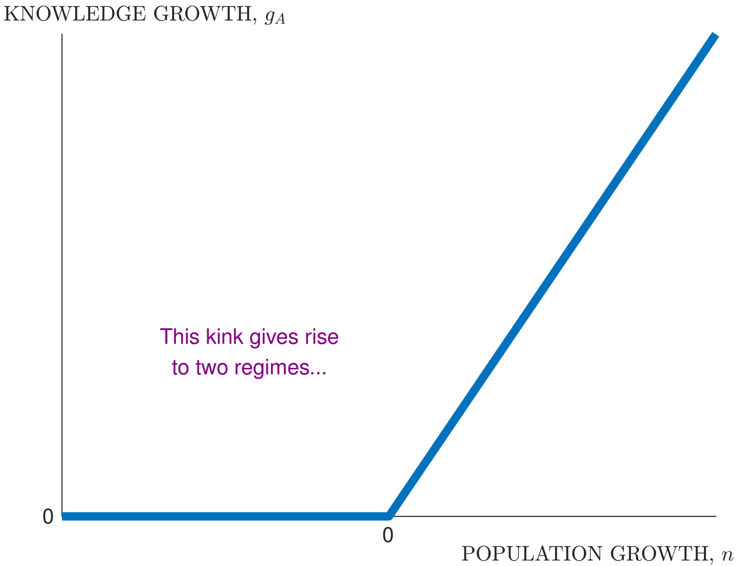
ℓ = time having kids instead of producing goods

Final output	$Y_t = A_t^\sigma (1 - \ell_t) N_t$
Population growth	$\frac{\dot{N}_t}{N_t} = n_t = b(\ell_t) - \delta$
Fertility	$b(\ell_t) = \bar{b}\ell_t$
Ideas	$\frac{\dot{A}_t}{A_t} = N_t^\lambda A_t^{-\beta}$
Generation 0 utility	$U_0 = \int_0^\infty e^{-\rho t} u(c_t, \tilde{N}_t) dt, \quad \tilde{N}_t \equiv N_t / N_0$
Flow utility	$u(c_t, \tilde{N}_t) = \log c_t + \epsilon \log \tilde{N}_t$
Consumption	$c_t = Y_t / N_t$

Overview of Endogenous Fertility Setup

- All people generate ideas here
  - Learning by doing vs separate R&D
- Equilibrium: ideas are an externality (simple)
  - We have kids because we like them
  - We ignore that they might create ideas that benefit everyone
  - Planner will desire higher fertility
- This is a modeling choice — other results are possible
- Abstract from the demographic transition. Focus on where it settles

Steady State Knowledge Growth



Key Features of the Equilibrium and Optimal Allocations

- Fertility in both

$$n = \bar{b}\ell - \delta$$

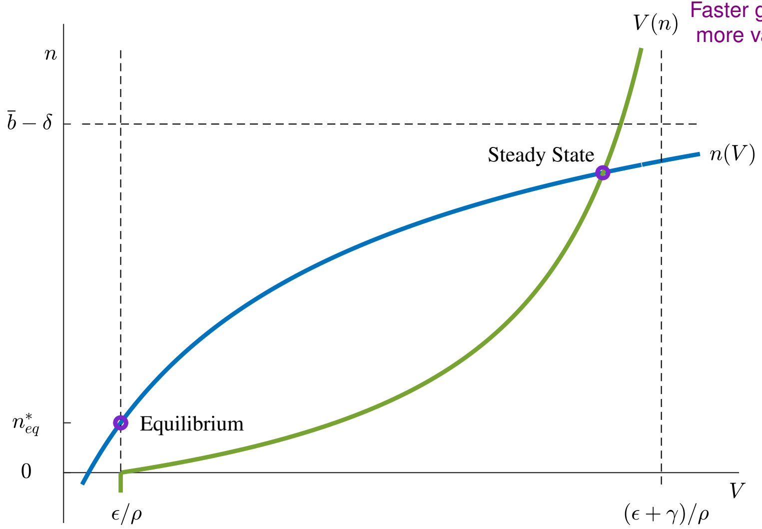
$$\ell = 1 - \frac{1}{\bar{b}V}$$

where V is the "utility value of people" (eqm vs optimal). Therefore

$$n(V) = \bar{b} - \delta - \frac{1}{V}$$

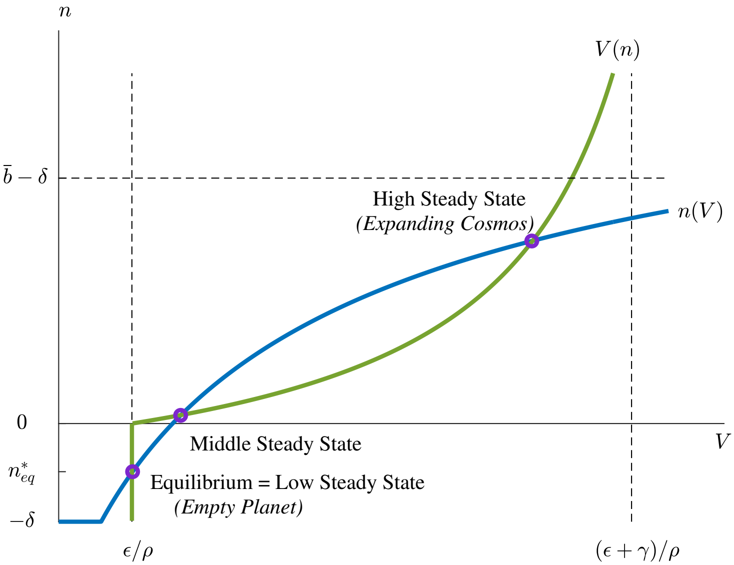
- Equilibrium: value kids because we love them (only): $V^{eqm} = \frac{\epsilon}{\rho}$
  - We can support $n < 0$ as an equilibrium for some parameter values
- Planner also values the ideas our kids will produce: $V^{sp} = \frac{\epsilon + \mu \dot{A}}{\rho} \Rightarrow V(n)$

A Unique Steady State for the Optimal Allocation when $n_{eq}^* > 0$



Faster growth makes people more valuable – more ideas

Multiple Steady State Solutions when $n_{eq}^* < 0$



Transition Dynamics

- State variables: N_t and A_t
- Redefine "state-like" variables for transition dynamics solution: N_t and

  $x_t \equiv A_t^{\beta}/N_t^{\lambda}$ = "Knowledge per person"

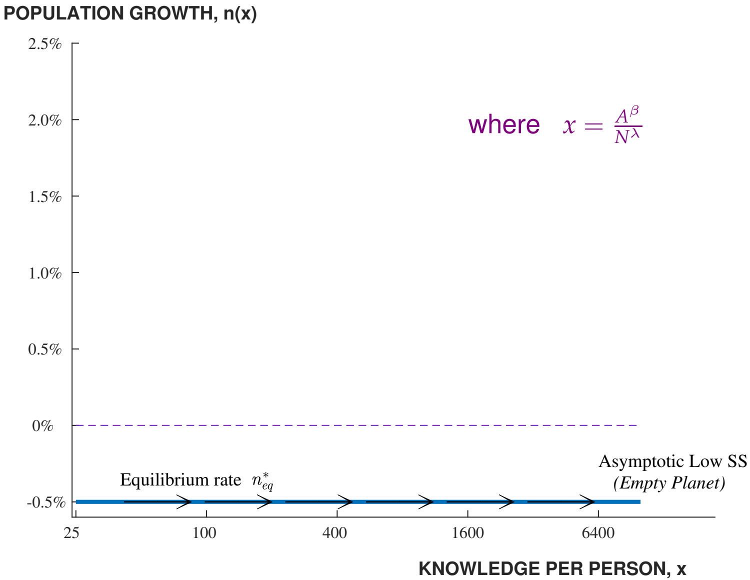
- Why?

$$\frac{\dot{A}_t}{A_t} = \frac{N_t^{\lambda}}{A_t^{\beta}} = \frac{1}{x_t}$$

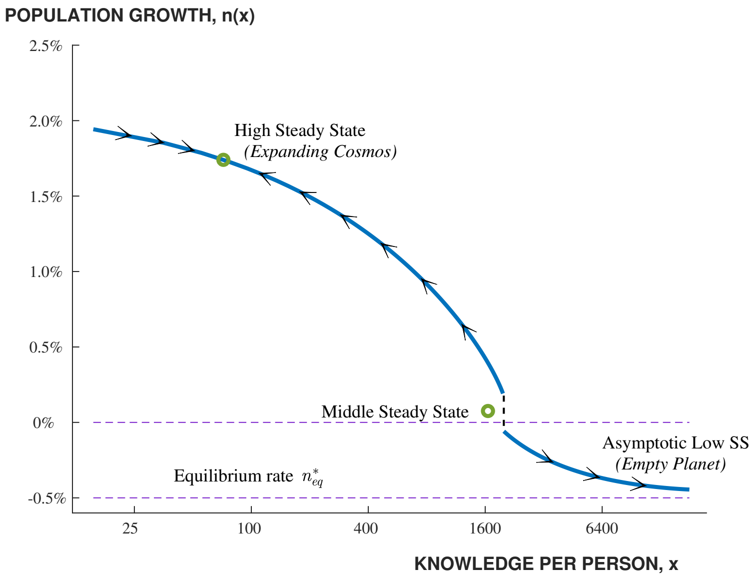
Key insight: optimal fertility only depends on x_t

- Note: x is the ratio of A and N, two stocks that are each good for welfare.
  - So a bigger x is not necessarily welfare improving.

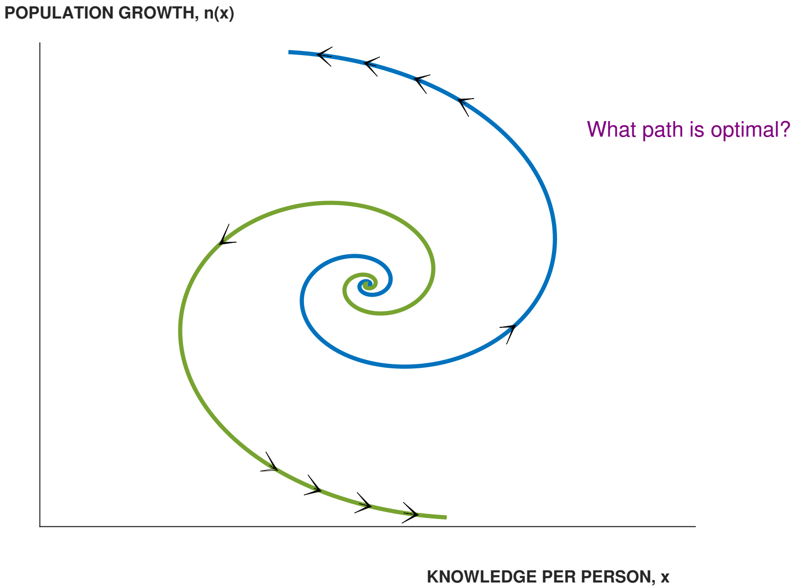
Equilibrium Transition Dynamics



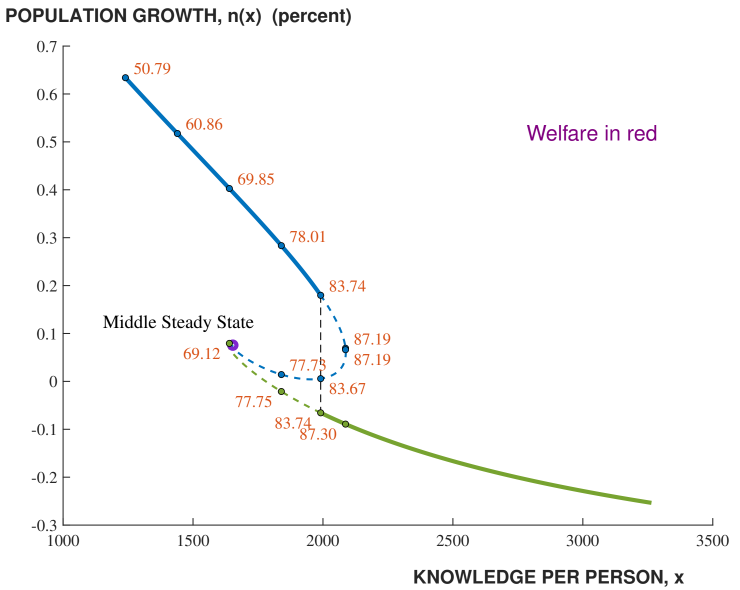
Optimal Population Growth



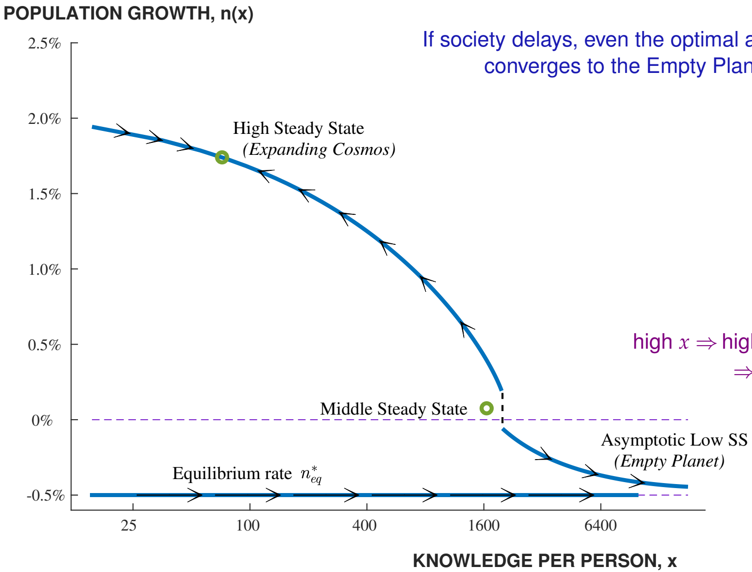
The Middle Steady State: Unstable Spiral Dynamics



Population Growth Near the Middle Steady State



Even the optimal allocation can get trapped



If society delays, even the optimal allocation converges to the Empty Planet

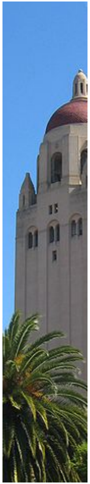
high $x \Rightarrow$ high ideas per person

$\Rightarrow$ low $\mu \dot{A}$

Conclusion

- Fertility has trended down: 5, 4, 3, 2, and less in rich countries
  - For a family, nothing special about “above 2” vs “below 2”
- But macroeconomics makes this distinction critical:
  - Negative population growth may condemn us to stagnation on an **Empty Planet**
  - Vs. the exponential growth in income and population of an **Expanding Cosmos**
- Surprise: Even the optimum can get trapped in the Empty Planet if society delays.

Fertility considerations may be more important than we thought!



Extra Slides

Numerical Example

- Parameter values
  - $g_{y0} = 2\%, \quad \eta = 1\%$
  - $\beta = 3 \quad \Rightarrow \quad \gamma = 1/3$ (from BJVW)

- How far away is the long-run stagnation level of income?

	y^*/y_0
Romer/AH/GH	7.4
Semi-endog	1.9

- The Empty Planet result occurs in both, but quantitative difference

A Competitive Equilibrium with Externalities

- Representative generation takes w_t as given and solves

$$\max_{\{\ell_t\}} \int_0^\infty e^{-\rho t} u(c_t, \tilde{N}_t) dt$$

subject to

$$\dot{N}_t = (b(\ell_t) - \delta) N_t$$

$$c_t = w_t (1 - \ell_t)$$

- Equilibrium wage $w_t = \text{MP}_L = A_t^\sigma$
- Rest of economic environment closes the equilibrium

Solving for the equilibrium

- The Hamiltonian for this problem is

$$\mathcal{H} = u(c_t, \tilde{N}_t) + v_t[b(\ell_t) - \delta]N_t$$

where v_t is the shadow value of another person.

- Let $V_t \equiv v_t N_t$ = shadow value of the population
- Equilibrium features constant fertility along transition path

$$V_t = \frac{\epsilon}{\rho} \equiv V^*_{eq}$$

$$\ell_t = 1 - \frac{1}{\bar{b}V_t} = 1 - \frac{1}{\bar{b}V^*_{eq}} = 1 - \frac{\rho}{\bar{b}\epsilon} \equiv \ell_{eq}$$

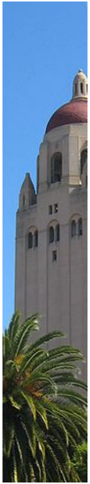
Discussion of the Equilibrium Allocation

$$n^{eq} = \bar{b} - \delta - \frac{\rho}{\epsilon}$$

- We can choose parameter values so that $n^{eq} < 0$
  - Constant, negative population growth in equilibrium

- Remaining solution replicates the exogenous fertility analysis

The Empty Planet result can arise in equilibrium



The Optimal Allocation

The Optimal Allocation

- Choose fertility to maximize the welfare of a representative generation
- Problem:

$$\max_{\{\ell_t\}} \int_0^\infty e^{-\rho t} u(c_t, \tilde{N}_t) dt$$

  subject to

$$\dot{N}_t = (b(\ell_t) - \delta) N_t$$

$$\frac{\dot{A}_t}{A_t} = N_t^\lambda A_t^{-\beta}$$

$$c_t = Y_t / N_t$$

- Optimal allocation recognizes that offspring produce ideas

Solution

- Hamiltonian:

$$\mathcal{H} = u(c_t, \tilde{N}_t) + \mu_t N_t^{\lambda} A_t^{1-\beta} + v_t(b(\ell_t) - \delta)N_t$$

μ_t is the shadow value of an idea

v_t is the shadow value of another person

- First order conditions

$$\ell_t = 1 - \frac{1}{\bar{b}V_t}, \quad \text{where} \quad V_t \equiv v_t N_t$$

$$\rho = \frac{\dot{\mu}_t}{\mu_t} + \frac{1}{\mu_t}\left(u_c \sigma \frac{y_t}{A_t} + \mu_t(1-\beta)\frac{\dot{A}_t}{A_t}\right)$$

$$\rho = \frac{\dot{v}_t}{v_t} + \frac{1}{v_t}\left(\frac{\epsilon}{N_t} + \mu_t \lambda \frac{\dot{A}_t}{N_t} + v_t n_t\right)$$

Steady State Conditions

- The social value of people in steady state is

$$V_{sp}^* = v_t^* N_t^* = \frac{\epsilon + \lambda z^*}{\rho}$$

where z denotes the social value of new ideas:

$$z^* \equiv \mu_t^* \dot{A}_t^* = \frac{\sigma g_A^*}{\rho + \beta g_A^*}$$

- If $n_{sp}^* > 0$, then we have an **Expanding Cosmos** steady state

$$g_A^* = \frac{\lambda n_{sp}^*}{\beta}$$

$$g_y^* = \gamma n_{sp}^*, \quad \text{where} \quad \gamma \equiv \frac{\lambda \sigma}{\beta}$$

Optimal Steady State(s)

- Two equations in two unknowns (V, n)

$$V(n) = \begin{cases} \frac{1}{\rho}\left(\epsilon + \frac{\gamma}{1+\frac{\rho}{\lambda n}}\right) & \text{if } n > 0 \\ \frac{\epsilon}{\rho} & \text{if } n \leq 0 \end{cases}$$

$$n(V) = \bar{b}\ell(V) - \delta = \bar{b} - \delta + \frac{1}{V}$$

- We show the solution graphically

Parameter Values for Numerical Solution

Parameter/Moment	Value	Comment
σ	1	Normalization
λ	1	Duplication effect of ideas
β	1.25	BJVW
ρ	.01	Standard value
δ	1%	Death rate
n^{eq}	-0.5%	Suggested by Europe, Japan, U.S.
ℓ^{eq}	1/8	Time spent raising children

Implied Parameter Values and "Expanding Cosmos" Steady-State Results

Result	Value	Comment
$\bar{b}$	.040	$n^{eq} = \bar{b}\ell^{eq} - \delta = -0.5\%$
ϵ	.286	From equation for ℓ^{eq}
n^{sp}	1.74%	From equations for ℓ^{sp} and n^{sp}
ℓ^{sp}	0.68	From equations for ℓ^{sp} and n^{sp}
$g_y^{sp} = g_A^{sp}$	1.39%	Equals γn^{sp} with $\sigma = 1$

The Economics of Multiple SS's and Transition Dynamics

- **The High SS is saddle path stable as usual**
  - Equilbrium fertility depends on utility value of kids
  - Planner also values the ideas the kids will produce $\Rightarrow n_t^{sp} > n_t^{eq}$
- **Why is there a low SS?**
  - Diminishing returns to each input, including ideas
  - As knowledge per person, x, goes to ∞, the "idea value" of an extra kid falls to zero $\Rightarrow n_{sp}(x) \to n_{eq}$
- **Why is the low SS stable?**
  - Since $n_{eq} < 0$, we also have $n_{sp}(x) < 0$ for x sufficiently high
  - With $n_{sp}(x) < 0$, $x = A^{\beta}/N^{\lambda}$ *rises* over time

What about the middle candidate steady state?

- Linearize the FOCs. Dynamic system has
  - imaginary eigenvalues
  - with positive real parts

- So the middle SS is an unstable spiral — a “Skiba point” (Skiba 1978)

- Numerical solution reveals what is going on...