Are Markups Too High? Competition, Strategic Innovation, and Industry Dynamics *

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Abstract

Since the 1970s, there have been significant changes in firm dynamics within and across industries in the US. Industries are increasingly dominated by a small number of large firms ("superstars"). Markups, market concentration, profits, and R&D spending are increasing, whereas business dynamism, productivity growth, and the labor share are in decline. We develop a unified framework to explore the underlying economic mechanisms driving these changes, and the implications for economic growth and social welfare. Our theoretical framework combines a detailed oligopolistic competition structure featuring endogenous entry and exit with a new Schumpeterian growth model. Within each industry, there are an endogenously determined number of superstars that compete *à la* Cournot and a continuum of small firms which collectively constitute a competitive fringe. Firms dynamically choose their innovation strategies, cognizant of other firms' choices. The model is consistent with the changes in the macroeconomic aggregates, and it replicates the observed hump-shaped relationship between innovation and competition within and across industries. We estimate the model to disentangle the effects of separate mechanisms on the structural transition, which yields striking results: (1) While the increase in the average markup causes a significant static welfare loss, this loss is overshadowed by the dynamic welfare gains from increased innovation in response to higher profit opportunities. (2) The decline in productivity growth is largely driven by the increasing costs of innovation, i.e., ideas are getting harder to find.

Keywords: innovation, markups, growth, strategic investment, industry dynamics, business dynamism.

JEL Classification: E20, L10, O30, O40.

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1 Introduction

Since the 1970s, there have been significant changes in firm dynamics within and across industries in the US, with quantitatively important implications for the macroeconomy at large. Industries are increasingly dominated by a small number of large firms ("superstars," "megafirms") which compete against each other in the product market, as well as through dynamic strategic innovation decisions. At the aggregate level, average markup, market concentration, profits, and R&D spending are increasing, whereas business dynamism, productivity growth, and the labor share are in decline.[1] These sweeping changes have drawn considerable attention from academics, policymakers, and the public alike. Discovering and understanding the economic mechanisms underlying this transformation are key to assess the implications for efficiency, economic growth, and social welfare, which can then be used to formulate optimal policy responses. In this paper, we offer a unified framework to address this need.

What are the desirable properties such a framework should satisfy? At the aggregate level, it should be consistent with the overarching trends in the listed macroeconomic variables. At the industry level, it should be able to explain the advent of superstar firms versus small firms as well as the increase in market concentration, and offer realistic industry dynamics with firm heterogeneity and endogenous entry and exit. At the firm level, it should capture the strategic interactions between firms: (1) product market competition which determines relative prices, market shares, markups, profits, labor demand, and labor share endogenously; (2) dynamic innovation decisions to improve firm productivity over time, optimally chosen in response to innovation policies of competing firms. Finally, the results from the framework should be consistent with the observed non-linear relationship between innovation and competition: (1) a hump-shaped relationship between industry innovation and market concentration across industries,[2] and (2) a hump-shaped relationship between firm innovation and relative sales within industries.[3] Consistency in these last two dimensions is crucial, since welfare implications of the transition do not only depend on static inefficiencies caused by markups, but also on the dynamic inefficiencies in innovation and aggregate productivity growth.

We develop a new quantitative model with all these desired properties that remains highly tractable despite its rich dynamics and heterogeneity, the quantification of which yields some striking insights. It can be described as a combination of a detailed oligopolistic competition model with endogenous entry and exit, and a new Schumpeterian growth model that features step-by-step innovation. Within each industry and at any given time, there are an endogenously determined number of large firms – the superstars – and a continuum of small firms which collectively constitute a competitive fringe. Entrepreneurs choose the entry rate of new small businesses. Industry output is a CES aggregate of the production of superstar firms and the competitive fringe, and the superstars compete *à la* Cournot. This specification yields non-degenerate distributions of sales, employment, and markups in

[1] See Akcigit and Ates (2019a) for an extensive summary and discussion of these trends.
[2] See Aghion, Bloom, Blundell, Griffith, and Howitt (2005), and our own findings in Section 3.
[3] See Section 3.

each industry, a feature missing in Schumpeterian growth models with Bertrand competition and homogeneous goods. The distributions depend on the number of superstar firms and the distribution of their productivities, as well as the relative productivity of the competitive fringe. These industry characteristics endogenously change over time according to the strategic innovation decisions of the superstars and the small firms. Motivated by higher profits, the superstars undertake costly R&D in order to improve their productivity relative to their competitors. Small firms also spend resources on R&D, and conditional on success, they join the ranks of the superstars. At the same time, a large firm that lags behind can lose its status as a superstar if its productivity relative to the leader falls to a sufficiently low level. Combined, these dynamics generate transitions between industry states, resulting in a stationary distribution along a balanced growth path equilibrium. The aggregate productivity growth in the economy is determined by the innovation decisions in each industry, weighted by this invariant distribution across industry states in a stationary economy. The model can generate the described hump-shaped relationship between innovation and competition within and across industries.

We estimate our model via indirect inference using macro and micro level moments related to growth, R&D investment, markup distribution, profitability, productivity, and the relationship between market shares and innovation from three different samples. First, we estimate the model using all available data between 1976 and 2004, and use the estimated equilibrium to clarify the workings of the mechanisms in the model, with a special emphasis on the relationship between competition and innovation, and how it differs from other models in the literature that study similar questions. We also split our data into two sub-samples – an early sub-sample from 1976 to 1990, and a late sub-sample from 1991 to 2004 – and re-estimate. We use these two estimates to disentangle the mechanisms underlying the structural transition in the US over these three decades. We achieve this by starting from the late period economy, and considering counterfactual economies where parameters governing individual mechanisms are reset back to their early period values. This helps us quantify what channels contribute the most to – or work against – the observed changes in the macroeconomic aggregates.

Perhaps one of the most important questions to ask is whether the observed increase in markups is detrimental to welfare and economic growth. While the static losses from increased markups are well-known and unambiguous, the dynamic effects can go in either direction, as evidenced by the hump-shaped relationship between innovation and competition. Increased competition can boost innovation as firms try to improve their relative productivity ("escape competition"), or lower it if they get discouraged by lower expected profits. Which channel dominates is a quantitative question. First, we show that most of the rise in markups is driven by a decrease in competition from small firms instead of a decrease in competition among large firms. Next, we conduct a counterfactual exercise in which we reset the competition from small firms to its early period level, and compute the consumption-equivalent welfare change. The results are striking: the growth rate goes down by 17.53% of its value, and instead of a net gain, social welfare is reduced by 4.52%. Decomposing the change in welfare into its individual components reveals that although the static efficiency gains would improve welfare by 3.73%, the fall in profitability discourages innovation, and the dynamic losses from the decline in endogenous productivity growth more than offset the static gain. In other words, if markups had stayed the same across the period, the observed slowdown in economic growth would be even further magnified. Our results suggest that the dynamic effects of increasing market concentration on innovation and productivity growth should not be ignored when trying to understand the transformation in the US in the last four decades, and the significant increase in markups is not necessarily detrimental to welfare.

If the increase in markups and market concentration are not the culprits behind the productivity slowdown, what is? Our model's answer is the increase in the costs of innovation. If the costs of innovation of small firms which are trying to become superstars were set back to their earlier level, it would account for roughly half of the decline in productivity growth. A similar experiment for the cost of innovation of superstars yields another significant effect that can explain more than 100% of the observed decline. These results point towards "the ideas are getting harder to find" hypothesis studied in Gordon (2012) and Bloom, Jones, Van Reenen, and Webb (2020).

We also investigate the distributional implications of the structural transition. Due to wealth inequality, the gains from growth are unequally distributed in the society. Individuals who rely more on labor income instead of capital income would benefit less from the dynamic welfare gains from higher markups. In fact, we find that workers with no capital income would be slightly worse off. This implies that more redistribution might be needed to counteract the uneven distribution of welfare gains.

Our estimated model also delivers several predictions that can be tested, especially related to trends in productivity, market concentration, and the labor share. Although they are not targeted, we show that our calibration correctly predicts the increase in productivity dispersion documented by Barth, Bryson, Davis, and Freeman (2016) and the negative correlation between productivity dispersion and the labor share across industries highlighted in Gouin-Bonenfant (2018). In addition, our model is in line with several facts related to changes in the labor share documented in Autor, Dorn, Katz, Patterson, and Van Reenen (2017). Finally, our model also predicts a U-shape relationship between entry into superstar firms and market concentration that is in line with empirical evidence for US-listed firms. These tests further confirm the external validity of the model.

Our quantitative results are found to be robust to attributing only half of the observed growth to innovation, introducing endogenous capital accumulation, as well as using different values for the elasticity of intertemporal substitution. Neither are they sensitive to using different targets to discipline markups. Our baseline estimation targets the sales-weighted markup moments from De Loecker, Eeckhout, and Unger (2019). Targeting cost-weighted markups from Edmond, Midrigan, and Xu (2018) does not lead to sizable differences. Motivated by the recent criticism in Bond, Hashemi, Kaplan, and Zoch (2020) regarding the consistency of the markup estimation methodology developed in De Loecker and Warzynski (2012), we also conduct an estimation which does not rely on any markup targets. If anything, this amplifies our results.

Solving for the non-stationary equilibria of our model is significantly more challenging than in standard macroeconomic models, given that it requires calculating the complete time paths of 86 continuous state variables given our choices.[4] Despite the complexity, the tractability of our framework and our choice to use a continuous-time setting render the computation of non-stationary equilibria feasible. We find that our welfare results remain robust to taking the transitional dynamics into account.

For further elucidation, we also solve the (unconstrained as well as constrained) social planner's problem dynamically, which is once again complex, yet feasible.[5] Consistent with our results regarding the welfare effect of increasing markups, the solution to the planner's problem reveals that the dynamic inefficiencies due to under-investment in innovation are much more severe than the static inefficiencies due to market power, even though the latter are quite significant on their own. This result suggests that the optimal design of corporate taxation and R&D subsidies, studied in Akcigit, Hanley, and Stantcheva (2019), is first-order for social welfare and economic growth.

This paper is related to the literature on the welfare cost of markups and on a recent body of literature that investigates the increase in market concentration and the associated increase in markups and profit shares over the last few decades in the US as highlighted in Barkai (2020), Gutiérrez and Philippon (2017a), Eggertsson, Robbins, and Wold (2018), Hall (2018), De Loecker, Eeckhout, and Unger (2019), and Grullon, Larkin, and Michaely (2019).[6] Gutiérrez and Philippon (2017b) and Gutiérrez, Jones, and Philippon (2019) show that the increase in market concentration and the decline in competition can explain underinvestment by US firms. Baqaee and Farhi (2020) estimate that misallocation due to large and dispersed markups results in a TFP loss as large as 15%. Recent work by Edmond, Midrigan, and Xu (2018) is closely related to our paper. Using a dynamic model with size-dependent markups, they find sizeable welfare losses from markups. Weiss (2019) studies the role played by intangible capital on markup trends and welfare in a model with oligopolistic competition and a one-time invesment in intangible capital by firms. Compared to those papers, our model has continuous and strategic investment in productivity growth, and oligopolistic competition between an endogenous number of heterogeneous innovative firms. Our Schumpeterian structure delivers significant differences regarding the dynamics, growth, and welfare. In ongoing work, De Loecker, Eeckhout, and Mongey (2019) also study the causes and consequences of market power in general equilibrium. Our model differs from their approach, as we endogenize productivity growth and model strategic interactions between firms in innovation. This difference is significant, since allowing for endogenous productivity growth can completely overturn the welfare implications of higher markups as we discussed earlier.

[4]For comparison, computing non-stationary equilibria in the canonical neoclassical growth model requires finding the complete time path of a single continuous state variable, the capital stock K_t.

[5]Even though we simplify the full dynamic problem significantly through our derivations, this still leaves 253 positive scalars to be simultaneously solved for through global optimization methods.

[6]De Loecker and Eeckhout (2018) also document similar trends for markups in other regions of the world.

Our paper also adds to the endogenous growth literature studying the relationship between competition and innovation. In early Schumpeterian models of growth through innovation (e.g., Aghion and Howitt (1992)), more product market competition reduces rents and hence the incentives to invest in R&D and innovation. Aghion, Harris, Howitt, and Vickers (2001) propose a model of step-by-step innovation by superstar firms. By introducing an additional incentive to escape competition, this model can generate an inverted-U shape relationship between the degree of market competition (measured by the elasticity of substitution between products) and innovation. Using a similar model, Aghion, Bloom, Blundell, Griffith, and Howitt (2005) also generate an inverted-U shape relationship between competition and innovation.[7] They further provide empirical evidence for such an inverted-U shape between innovation activity and industry competition using data from the UK. We also undertake an empirical analysis in which we look at the relationship between competition and innovation using data from the US. We verify that the findings in Aghion, Bloom, Blundell, Griffith, and Howitt (2005) also hold in the US using several measures of innovation. Furthermore, we also document a robust inverted-U relationship between the relative sales of a firm and its innovation. Our model is able to replicate the two hump-shaped relationships within and across industries without relying on exogenous heterogeneity,[8] and in the presence of endogenous firm entry.[9]

Within the class of endogenous growth models, the closest papers to ours are Peters and Walsh (2019), Akcigit and Ates (2019a,b), and Aghion, Bergeaud, Boppart, Klenow, and Li (2019).[10] Peters and Walsh (2019) propose an extension of Peters (2013) to study the role of the decline in the growth rate of the labor force on the observed decline in business dynamism, increased market power, and slower productivity growth. Building on Acemoglu and Akcigit (2012), Akcigit and Ates (2019a,b) propose a model of endogenous growth and firm dynamics with heterogenous markups and show that declining knowledge diffusion played a significant role in declining business dynamism. Aghion, Bergeaud, Boppart, Klenow, and Li (2019) explain the rise in concentration and profits through falling firm-level costs of spanning multiple markets due to accelerating IT advances. All of the listed papers assume Bertrand competition with homogeneous goods in each industry. Markups depend directly on the productivity gap between the leader and the follower. Unless firms are neck-and-neck, the leader takes over the whole industry. Our paper differs from these four papers in its rigorous treatment of within-industry dynamics as discussed earlier. Our model features non-degenerate distributions of sales, employment, markups, and profits within each industry. These advances upon the previous literature allow us to hit the hump-shaped relationship between competition and innovation within

[7]See Gilbert (2006) for an extensive review of the literature on competition and innovation.

[8]For instance, in Aghion, Bloom, Blundell, Griffith, and Howitt (2005), exogenous heterogeneity in collusion across industries is assumed to generate the inverted-U result.

[9]Etro (2007) argues that the industry-level inverted-U relationship breaks down when endogenous entry is introduced to Aghion, Harris, Howitt, and Vickers (2001). Our model provides a counterexample. Other counterexamples can be found in Bento (2014, 2019)

[10]In other related work, Liu, Mian, and Sufi (2019) propose a model of endogenous growth with Bertrand competition with non-homogeneous goods between two firms within each product market. They show that a decline in the interest rate can lead to a rise in market concentration and a slowdown in productivity growth. Corhay, Kung, and Schmid (2019) use a Romer-type endogenous growth model to study the asset pricing implications of increasing markups.

and across industries, which is crucial for estimating the growth and welfare implications of higher market concentration and markups. The new competitive fringe feature of our model allows for more realistic firm life-cycles where entrants do not immediately become industry leaders, and we can distinguish new business entry from emergence of new superstars. Apart from the technical differences, our study also differs in its quantitative approach. Instead of searching for a single mechanism which can jointly explain all the changes in the macroeconomic aggregates to some degree, our estimation tightly hits all the changes, and we use the model to disentangle the contribution of each channel. This provides some guidance on which mechanisms should be further investigated to better understand the underlying sources of the structural transition, such as the widening productivity gap between the superstars and the rest, and the increasing costs of R&D.

Our paper is related to the broader literature on the decline in business dynamism. A large body of that literature shows that the entry rate of new firms has significantly decreased in the US since the early 1980s (see for instance, Hathaway and Litan (2014), Decker, Haltiwanger, Jarmin, and Miranda (2016), Pugsley and Sahin (2018)). The decline in firm entry can further affect productivity growth. For instance, Lentz and Mortensen (2008), Garcia-Macia, Hsieh, and Klenow (2019) and Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018) find that a significant share of aggregate TFP growth and job creation is due to firm entry and young firms. Our quantitative exercise reveals that not only the share of output by small firms decreases but also that these firms are less likely to become superstar firms due to higher R&D costs.

Our results also add to the recent literature on the slowdown of TFP growth in the US. Gordon (2012, 2014) argues that the significant slowdown in the rate of productivity growth in the US since the 1970s is due to diminishing returns from R&D. Bloom, Jones, Van Reenen, and Webb (2020) show empirical evidence for a steady decline in R&D productivity over the course of the 20th century. As ideas are getting harder to find, sustaining a constant economic growth rate has required a simultaneous increase in research inputs. Our quantitative results suggest that increasing R&D costs are the primary drivers of the productivity slowdown over this time period.

Another important trend over the last decades in the US relates to the evolution of factor shares. In particular, the share of income paid to workers has steadily decreased over the last decades as highlighted in, among others, Karabarbounis and Neiman (2013) and Elsby, Hobijn, and Şahin (2013). At the same time, the profit share has followed an opposite trajectory (Barkai (2020)). A large literature has highlighted several potential sources for the decrease in the labor share such as technological change and automation (see, for instance, Zeira (1998), Acemoglu and Restrepo (2018), Martinez (2018)), globalization (Elsby, Hobijn, and Şahin (2013)), the decline in the relative price of capital (Karabarbounis and Neiman (2013)), increased cost of housing (Rognlie (2015)) or a rise in productivity dispersion (Gouin-Bonenfant (2018)). Barkai (2020) and De Loecker, Eeckhout, and Unger (2019) relate the decrease in the labor share to the rise in profitability and markups. Autor, Dorn, Katz, Patterson, and Van Reenen (2017) and Kehrig and Vincent (2017) attribute this trend to the rise of superstar firms which tend to display high markups and low labor shares. Our model is also able to generate a decline in the labor share over the same time period. In our model, the decrease in the labor share is largely driven by the reallocation of production towards superstars with high markups, which is consistent with Autor, Dorn, Katz, Patterson, and Van Reenen (2017) and Kehrig and Vincent (2017).

The paper is organized as follows. In Section 2, we introduce our new model. In Section 3, we describe the data sources and variables construction that we use to investigate the inverted-U relationship between innovation and competition both *within* and *across* industries. In Section 4, we estimate the model and use it to conduct several quantitative exercises. We further validate the model in Section 5 and check its robustness in Section 6. Section 7 concludes.

2 Model

2.1 Environment

Preferences Time is continuous and indexed by $t \in \mathbb{R}_+$. The economy is populated by an infinitely-lived representative consumer who discounts the future at rate $\rho > 0$. The representative consumer maximizes lifetime utility:

$$U = \int_0^\infty e^{-\rho t} \ln(C_t) \; dt \tag{1}$$

where C_t is consumption of the final good at time t, the price of which is normalized to one.

The household inelastically supplies one unit of labor in exchange for an endogenously determined wage rate w_t. Households own all the assets in the economy and face the following budget constraint:

$$\dot{A}_t = r_t A_t + w_t - C_t \tag{2}$$

where A_t is household wealth and r_t is the rate of return on assets.

Final Good Production The final good Y_t is produced competitively using inputs from a measure one of industries:

$$\ln\left(Y_t\right) = \int_0^1 \ln\left(y_{jt}\right) \; dj \tag{3}$$

where y_{jt} is production of industry j at time t.

Industry Production Each industry is populated by an endogenous number of superstar firms ($N_{jt} \in \{1, ..., \bar{N}\}$), each producing a differentiated variety, as well as by a competitive fringe composed of a mass m_{jt} of small firms producing a homogeneous good (small firm k in the fringe of industry j at time t produces y_{ckjt}). Given that there is a continuum of small firms and their products are homogeneous, each small firm in the competitive fringe is a price taker. Superstars compete in quantities. In particular, we allow for strategic interaction among superstars as variety production is the result of a static Cournot game. Total production of industry j at time t is given by:

$$y_{jt} = \left(\sum_{i=1}^{N_{jt}} y_{ijt}^{\frac{\eta-1}{\eta}} + \tilde{y}_{cjt}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{4}$$

where y_{ijt} is the production of superstar firm i in industry j at time t, $\tilde{y}_{cjt} = \int_{F_{jt}} y_{ckjt}\, dk$ is the production of the competitive fringe in industry j at time t, F_{jt} is the set of small firms in the fringe in industry j at time t and $\eta > 1$ is the elasticity of substitution between varieties.

Variety Production Superstar firms in each industry produce their own variety using a linear production technology in labor:

$$y_{ijt} = q_{ijt} l_{ijt} \tag{5}$$

where q_{ijt} is the productivity of superstar firm i in industry j at time t and l_{ijt} is labor.

Similarly for every small firm k in the competitive fringe:

$$y_{ckjt} = q_{cjt} l_{ckjt} \tag{6}$$

We assume that each small firm in the fringe in a given industry has the same productivity q_{cjt}. Superstar firms within each industry differ according to their level of productivity which can be built over time through R&D and innovation.

R&D and Innovation Each superstar can perform R&D to improve the productivity of its variety. To generate a Poisson rate z_{ijt} of success in R&D, firm i must pay a cost in units of the final good equal to:

$$R_{ijt} = \chi z_{ijt}^{\phi} Y_t. \tag{7}$$

If a superstar successfully innovates at time t, its productivity is multiplied by $(1+\lambda)$. We assume that the maximum number of productivity steps between any two superstar firms within an industry is $\bar{n} \geq 1$. For the competitive fringe, we assume that the relative productivity of small firms with respect to the leader is a constant, denoted by $\zeta = \frac{q_{cjt}}{q_{jt}^{leader}}$.

Entry and Exit of Superstar Firms At any time t, each small firm k in the competitive fringe can generate a Poisson arrival density X_{kjt} of entry into superstar firms when $N_{jt} < \bar{N}$. The associated R&D cost is given by

$$R_{kjt}^{e} = \nu X_{kjt}^{\epsilon} Y_t. \tag{8}$$

Because all small firms are homogeneous within an industry, they all perform the same level of innovation in equilibrium. We can rewrite the industry level Poisson rate of innovation $X_{jt} = \int X_{kjt} dk = m_{jt} X_{kjt}$ and the industry level R&D expenditures of small firms $R^e_{jt} = m_{jt} R^e_{kjt}$.

When a small firm successfully innovates, it becomes a superstar, and the number of superstar firms in the industry is increased by one unless the number of firms in the industry is already equal to $\bar{N}$, in which case entry is not allowed. The new entrant is assumed to enter as the smallest superstar firm in the industry, i.e., it is assumed to have a productivity level $\bar{n}$ steps below the leader. In this sense, entry into superstars in our model should be interpreted as a small firm becoming large enough to strategically interact with other superstars. In particular, our assumptions imply that firms more than $\bar{n}$ steps below the leader in any industry are not large enough to be considered as having any strategic interaction with other firms. Consistent with this interpretation of our model, a firm endogenously loses its superstar firm status when it is $\bar{n}$ steps below the industry leader and the leader innovates. In that case, the number of superstar firms in the industry decreases by one.

Entry and Exit of Small Firms We also introduce entry into and exit from the competitive fringe, which allows for a realistic mapping between small firm entry in the model and new business creation in the data. This is different from the existing endogenous growth literature where entrants can immediately become the leader in their industry, or large enough to interact strategically with the current leader. Therefore, our model features a more realistic firm life-cycle. We assume an exogenous exit rate of small firms equal to τ. Regarding entry, there is a mass one of entrepreneurs who can pay a cost $\psi e_t^2 Y_t$ to generate a Poisson rate e_t of starting a new small firm. The new firms are randomly allocated to the competitive fringe of an industry, which implies $m_{jt} = m_t$ for all industries j. In order to keep the mass of entrepreneurs unchanged, we assume that they sell their firm on a competitive market at its full value and remain in the set of entrepreneurs.

2.2 Equilibrium

Consumer's problem Household lifetime utility maximization delivers the standard Euler equation:

$$\frac{\dot{C}_t}{C_t} = r_t - \rho. \tag{9}$$

Final Good Producers The final good is produced competitively. The representative final good producer chooses the quantity of each variety in each industry which maximizes profit:

$$\max_{[\{y_{ijt}\}_{i=1}^{N_{jt}}, \tilde{y}_{cjt}]_{j=0}^{1}} \exp\left(\frac{\eta}{\eta-1}\int_0^1 \ln\left[\sum_{i=1}^{N_{jt}} y_{ijt}^{\frac{\eta-1}{\eta}} + \tilde{y}_{cjt}^{\frac{\eta-1}{\eta}}\right] dj\right) - \int_0^1 \left(\sum_{i=1}^{N_{jt}} p_{ijt} y_{ijt} + p_{cjt}\tilde{y}_{cjt}\right) dj. \tag{10}$$

where p_{ijt} (p_{cjt}) is the price of variety i (the competitive fringe variety) in industry j at time t. This leads to the following inverse demand functions:

$$p_{ijt} = \frac{y_{ijt}^{-\frac{1}{\eta}} Y_t}{\sum_{k=1}^{N_{jt}} y_{kjt}^{\frac{\eta-1}{\eta}} + \tilde{y}_{cjt}^{\frac{\eta-1}{\eta}}} \tag{11}$$

and

$$\frac{y_{ijt}}{y_{kjt}} = \left(\frac{p_{kjt}}{p_{ijt}}\right)^{\eta} \tag{12}$$

where y_{ijt} should be replaced by $\tilde{y}_{cjt}$ for the competitive fringe.

Variety Producers We assume that superstar firms within the same industry compete *à la* Cournot. Each firm maximizes profit:

$$\max_{y_{ijt}} \; p_{ijt} y_{ijt} - w_t l_{ijt} = \max_{y_{ijt}} \; \frac{y_{ijt}^{\frac{\eta-1}{\eta}} Y_t}{\sum_{k=1}^{N_{jt}} y_{kjt}^{\frac{\eta-1}{\eta}} + \tilde{y}_{cjt}^{\frac{\eta-1}{\eta}}} - \frac{w_t y_{ijt}}{q_{ijt}}. \tag{13}$$

This delivers the following best response functions for superstar firms:

$$y_{ijt} = \left[\frac{\eta-1}{\eta} q_{ijt} \frac{\sum_{k\neq i} y_{kjt}^{\frac{\eta-1}{\eta}} + \tilde{y}_{cjt}^{\frac{\eta-1}{\eta}}}{\left[\sum_{k=1}^{N_{jt}} y_{kjt}^{\frac{\eta-1}{\eta}} + \tilde{y}_{cjt}^{\frac{\eta-1}{\eta}}\right]^2} \frac{Y_t}{w_t}\right]^{\eta} \tag{14}$$

$$= \frac{\eta-1}{\eta} q_{ijt} \frac{\sum_{k\neq i} \left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}{\left[\sum_{k=1}^{N_{jt}} \left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}\right]^2} \frac{Y_t}{w_t} \tag{15}$$

The total production of the competitive fringe is given by:

$$\tilde{y}_{cjt} = q_{cjt} \frac{\frac{Y_t}{w_t}}{\sum_{k=1}^{N_{jt}} \frac{y_{kjt}}{\tilde{y}_{cjt}}^{\frac{\eta-1}{\eta}} + 1} \tag{16}$$

Relative production between each superstar variety and the competitive fringe within the industry can then be written as:

$$\left(\frac{y_{ijt}}{y_{kjt}}\right)^{\frac{1}{\eta}} = \frac{q_{ijt}}{q_{kjt}} \frac{\sum_{l\neq i} \left(\frac{y_{ljt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}{\sum_{l\neq k} \left(\frac{y_{ljt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}} \tag{17}$$

$$\left(\frac{y_{ijt}}{\tilde{y}_{cjt}}\right)^{\frac{1}{\eta}} = \frac{\eta-1}{\eta} \frac{q_{ijt}}{q_{cjt}} \frac{\sum_{l\neq i} \left(\frac{y_{ljt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}{\sum_{l=1}^{N_{jt}} \left(\frac{y_{ljt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}. \tag{18}$$

For each industry j, this is a system of N_{jt} equations and N_{jt} unknown production ratios which can be solved given relative productivities within the industry.

We can further derive variety prices (p_{ijt}) and profits before R&D expenditures (π_{ijt}) which only depend on relative productivities within the industry:

$$p_{ijt} = \frac{\eta}{\eta-1}\frac{\sum_{k=1}^{N_{jt}}\left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}+\left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}{\sum_{k\neq i}\left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}+\left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}\frac{w_t}{q_{ijt}} \tag{19}$$

$$\pi_{ijt} = \frac{Y_t}{\left[\sum_{k=1}^{N_{jt}}\left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}+\left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}\right]^2}\frac{\eta+\sum_{k\neq i}\left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}+\left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}{\eta} \tag{20}$$

Firms charge varying markups (M_{ijt}) over marginal cost that depends on the number of competitors as well as the relative productivity of the firm compared to them:

$$M_{ijt} = \frac{\eta}{\eta-1}\frac{\sum_{k=1}^{N_{jt}}\left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}+\left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}{\sum_{k\neq i}\left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}+\left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}} \tag{21}$$

Superstar Value Function and R&D Decision Static production decision, markups, and profits within each industry only depend on the number of superstars, and the distribution of their relative productivities. In other words, the relevant state variables for a firm i in industry j at time t can be summarized by the vector of the number of productivity steps between superstar firm i and every other superstar firm $k \in \{(1,2,...,N_{jt})\backslash\{i\}\}$ in the industry. Letting $n_{ijt}^k \in \{-\bar{n}, -\bar{n}+1, ..., \bar{n}-1, \bar{n}\}$ be the number of steps by which firm i in industry j leads firm k at time t, the relevant state variables for firm i in industry j at time t is given by the vector $\mathbf{n}_{ijt} = \{n_{ijt}^k\}_{k\neq i}$ and $N_{jt} = |\mathbf{n}_{ijt}| + 1$.[11]

Henceforth, we drop the time subscripts unless otherwise needed. A superstar firm i chooses an innovation rate (z_{ij}) to maximize the value of the firm given by:

$$\begin{aligned} rV(\mathbf{n}_i, N) = \quad & \max_{z_i} \pi(\mathbf{n}_i, N) - \chi z_i^{\phi} Y \\ & + z_i \left[V(\mathbf{n}_i\backslash\{n_i^k = \bar{n}\} + \mathbf{1}, N - |\{n_i^k = \bar{n}\}|) - V(\mathbf{n}_i, N)\right] \\ & + \sum_{k:n_i^k=-\bar{n}} z_{kj}(0 - V(\mathbf{n}_i, N)) \\ & + \sum_{k:n_i^k\neq-\bar{n}} z_{kj}\left[V(\mathbf{n}_i\backslash\{n_i^k\}\cup\{n_i^k-1\}\backslash\{n_i^l = \bar{n}+n_i^k\}, N - |\{n_i^l = \bar{n}+n_i^k\}|) - V(\mathbf{n}_i, N)\right] \\ & + X_j\left[V(\mathbf{n}_i \cup \{\min\{\bar{n}, \bar{n}+\min(\mathbf{n}_i)\}\}, \min(N+1, \bar{N})) - V(\mathbf{n}_i, N)\right] + \dot{V}(\mathbf{n}_i, N) \end{aligned} \tag{22}$$

The first line is the flow profit minus the cost of R&D. The second line is the change in firm value

[11] We can rewrite the relative productivity of firm i and k as $\frac{q_{ijt}}{q_{kjt}} = (1+\lambda)^{n_{ijt}^k}$.

due to a successful innovation by firm i which happens with Poisson rate z_i. If firm i innovates, it increases its lead to any other firm by one. Any firm $\bar{n}$ productivity steps below firm i exits the set of superstars. The third line corresponds to the change in value due to endogenous exit which arises if one of the industry leaders is $\bar{n}$ steps ahead of firm i and innovates.

The fourth line comes from any other firm (not leading i by $\bar{n}$) innovating. In that case, the lead of firm i with respect to the innovating firm decreases by one. In addition, if the innovating firm k is also leading any other firm l by $\bar{n}$ (which happens if $n_i^l - n_i^k = \bar{n}$), firm l exits. The first term in the fifth line is the effect of firm entry on the value of firm i. In that case, the entrant starts $\bar{n}$ productivity steps below the industry leader. The second term on the same line is the growth in firm value.

We can guess and verify that, in a balanced growth path (BGP), $V(\mathbf{n}_i, N) = v(\mathbf{n}_i, N)Y$. In that case, $\dot{V}(\mathbf{n}_i, N) = gv(\mathbf{n}_i, N)Y$ (where g is the growth rate of Y). Using equation (9), we can write:

$$\begin{aligned}
\rho v(\mathbf{n}_i, N) = \quad & \max_{z_i} \frac{\pi(\mathbf{n}_i, N)}{Y} - \chi z_i^{\phi} \\
& + z_i \left[v(\mathbf{n}_i \backslash \{n_i^k = \bar{n}\} + \mathbf{1}, N - |\{n_i^k = \bar{n}\}|) - v(\mathbf{n}_i, N) \right] \\
& + \sum_{k: n_i^k = -\bar{n}} z_{kj}(0 - v(\mathbf{n}_i, N)) \\
& + \sum_{k: n_i^k \neq -\bar{n}} z_{kj} \left[v(\mathbf{n}_i \backslash \{n_i^k\} \cup \{n_i^k - 1\} \backslash \{n_i^l = \bar{n} + n_i^k\}, N - |\{n_i^l = \bar{n} + n_i^k\}|) - v(\mathbf{n}_i, N) \right] \\
& + X_j \left[v(\mathbf{n}_i \cup \{\min\{\bar{n}, \bar{n} + \min(\mathbf{n}_i)\}\}, \min(N+1, \bar{N})) - v(\mathbf{n}_i, N) \right]. \qquad (23)
\end{aligned}$$

The optimal level of innovation is given by:

$$z_i = \left\{ \frac{\left[v(\mathbf{n}_i \backslash \{n_i^k = \bar{n}\} + 1, \max(1, N - |\{n_i^k = \bar{n}\}|) - v(\mathbf{n}_i, N) \right]}{\chi \phi} \right\}^{\frac{1}{\phi - 1}}. \qquad (24)$$

Small Firm Innovation and Entry into Superstar Firms Since relative sales and innovation policies only depend on pairwise productivity differences and the number of firms in the industry, we can define $\Theta = (N, \vec{n})$ as the state of the industry with $N \in \{1, ..., \bar{N}\}$ being the number of superstars in the industry and $\vec{n} \in \{0, ..., \bar{n}\}^{N-1}$ denoting the number of steps followers are behind the leader (in ascending order). We let $f(\Theta) = \frac{1}{\eta-1} \ln \left(\sum_{i=1}^{N_j} \left(\frac{y_{ij}}{y_{cj}}(\Theta) \right)^{\frac{\eta-1}{\eta}} + 1 \right)$ and define $p_{li}(\Theta)$ as the arrival rate of a leader innovation and $p(\Theta, \Theta')$ as the instantaneous flows from state Θ to Θ'. In each industry j (with $N_j < \bar{N}$), each small firm in the competitive fringe can perform R&D. All small firms within an industry are symmetric and choose R&D investment to maximize:

$$\begin{aligned}
rV^e(\Theta_j) \quad = \quad & \max_{X_{kj}} X_{kj} V(\{-\tilde{\mathbf{n}}_j + \bar{n}\} \cup \{-\bar{n}\}, N_j + 1) - \tau V^e(\Theta_j) - \nu X_{kj}^{\epsilon} Y \\
& + \sum_{\Theta'} p(\Theta_j, \Theta')(V^e(\Theta') - V^e(\Theta_j)) + \dot{V}^e(\Theta_j) \qquad (25)
\end{aligned}$$

where $V^e(\Theta_j)$ is the value of a small firm in industry j and $\tilde{\mathbf{n}}_j = \mathbf{n}_{kj}$ where k denotes a productivity leader in industry j.[12]

Guessing and verifying that, in a BGP, $V^e(\Theta_j) = v^e(\Theta_j)Y$, we can rewrite:

$$
\begin{aligned}
(\rho+\tau)v^e(\Theta_j) &= \max_{X_{kj}} X_{kj} v(\{-\tilde{\mathbf{n}}_j + \bar{n}\} \cup \{-\bar{n}\}, N_j + 1) - \nu X_{kj}^{\epsilon} && (26)\\
&\quad + \sum_{\Theta'} p(\Theta_j, \Theta')(v^e(\Theta') - v^e(\Theta_j)) && (27)
\end{aligned}
$$

The optimal innovation intensity by a small firm in industry j is then:

$$
X_{kj} = \left(\frac{v(\{-\tilde{\mathbf{n}}_j + \bar{n}\} \cup \{-\bar{n}\}, N_j + 1)}{\nu\epsilon} \right)^{\frac{1}{\epsilon-1}} \tag{28}
$$

Plugging in the optimal solution, the normalized value of a small firm is calculated as

$$
v^e(\Theta_j) = \frac{1}{\rho+\tau}\left(1 - \frac{1}{\epsilon}\right) \frac{v(\{-\tilde{\mathbf{n}}_j + \bar{n}\} \cup \{-\bar{n}\}, N_j + 1)^{\frac{\epsilon}{\epsilon-1}}}{(\nu\epsilon)^{\frac{1}{\epsilon-1}}} \tag{29}
$$

Entrepreneurs and Entry into the Competitive Fringe There is a mass one of entrepreneurs in the economy who can pay a cost $\psi e^2 Y$ to create a Poisson rate e of becoming a small firm in a randomly selected industry. We assume that a successful entrepreneur immediately sells the small firm in a competitive market. The expected selling price of the new small firm of a successful entrepreneur is equal to: $W = \sum_{\Theta} V^e(\Theta)\mu(\Theta)$, where $\mu(\Theta)$ is the mass of industries of type Θ.[13] The value of being an entrepreneur (S) can be written as:

$$
\rho S = \max_{e} -\psi e^2 Y + eW \tag{30}
$$

Guessing and verifying that, in a BGP, $S = sY$, we obtain that:

$$
e = \frac{W}{2\psi Y} = \frac{\sum_{\Theta} v^e(\Theta)\mu(\Theta)}{2\psi} \tag{31}
$$

which implies:

$$
s = \frac{\left[\sum_{\Theta} v^e(\Theta)\mu(\Theta)\right]^2}{4\psi\rho} \tag{32}
$$

In a stationary equilibrium, entry into the competitive fringe equals exit from the competitive

[12] Note that we use $\int_{k=i} V_k^e(\Theta_j)dk = 0$ in the first term, i.e., the value of the small firm is insignificant compared to the value of the superstar firm it becomes, since it is of mass zero in the competitive fringe.

[13] We can show that the expected value of $\sum_{\Theta'} p(\Theta, \Theta')(V^e(\Theta') - V^e(\Theta))$ in a stationary equilibrium is equal to zero (see Proposition 1 in Appendix A.2). W is thus equal to $\frac{1-\frac{1}{\epsilon}}{\rho+\tau} \frac{\int_0^1 V(\{-\tilde{\mathbf{n}}_j + \bar{n}\} \cup \{-\bar{n}\}, N_j+1)^{\frac{\epsilon}{\epsilon-1}} \, dj}{(\nu\epsilon)^{\frac{1}{\epsilon-1}}}$.

fringe which means:

$$e = \tau m \tag{33}$$

Combining equations (31) and (33), we get an equation that pins down the value of m as:

$$m = \frac{\sum_{\Theta} v^e(\Theta)\mu(\Theta)}{2\psi\tau} \tag{34}$$

Equilibrium Definition An equilibrium is defined by a set of allocations $\{C_t, Y_t, y_{ijt}, y_{ckjt}\}$, policies $\{l_{ijt}, l_{ckjt}, z_{ijt}, X_{kjt}, e_t\}$, prices $\{p_{ijt}, p_{cjt}, w_t, r_t\}$, the number of superstars in each industry N_{jt}, a mass of small firms m_t, a set of vectors $\{\mathbf{n}_{ijt}\}$ that denote the relative productivity distance between firm i and every other firm in the same industry j at time t, such that, $\forall t \geq 0$, $j \in [0,1]$, $i \in \{1, ..., N_{jt}\}$:

(i) Given prices, final good producers maximize profit.

(ii) Given $\mathbf{n}_{ij}$ and N_{jt}, superstars choose y_{ijt} to maximize profit.

(iii) Given prices, small firms in the competitive fringe choose y_{ckjt} to maximize profit.

(iv) Superstar firms choose innovation policy z_{ijt} to maximize firm value.

(v) Small firms choose innovation policy X_{kjt} to maximize firm value.

(vi) Entrepreneurs choose e_t to maximize profit.

(vii) The real wage rate w_t clears the labor market.

(viii) Aggregate consumption C_t grows at rate $r_t - \rho$.

(ix) Resource constraint is satisfied: $Y_t = C_t + \int_0^1 \sum_{i=1}^{N_{jt}} \chi z_{ijt}^{\phi} Y_t \, dj + \int_0^1 m_t \nu X_{kjt}^{\epsilon} Y_t \, dj + \psi e_t^2 Y_t$.

Growth Rate and Balanced Growth Path We can derive the growth rate of the economy at time t (g_t) as:[14]

$$\begin{aligned} g_t &= -g_{\omega,t} + \sum \left[p_{lit}(\Theta) \ln(1+\lambda)\right] \mu_t(\Theta) \\ &\quad + \sum_{\Theta} \sum_{\Theta'} \left[f_t(\Theta') - f_t(\Theta)\right] p_t(\Theta, \Theta') \mu_t(\Theta) \end{aligned} \tag{35}$$

where $g_{\omega,t}$ is the growth rate of the relative wage, $\omega_t = \frac{w_t}{Y_t}$, the second term comes from the growth rate of the industry leaders, and the third term accounts for production reallocation as industries move between states. In a balanced growth path with time-invariant distribution over Θ, $g_{\omega,t} = 0$, $\mu_t(\Theta) = \mu(\Theta)$ and:

$$g = \sum \left[p_{li}(\Theta) \ln(1+\lambda)\right] \mu(\Theta) \tag{36}$$

[14] See Appendix A.1 for the full derivation.

3 Relationship Between Competition and Innovation

Unlike other recent models that study the effects of rising markups, we require our model to be consistent with the observed empirical relationship between competition and innovation, which allows us to discipline the counterfactual model implications for innovation, economic growth, and welfare. In this section, we present the empirical regularities between competition and innovation across and within industries.

3.1 Data Sources

USPTO Utility Patents Grant Data: The patent grant data are obtained from the NBER Patent Database Project and contain information for all 3,279,509 utility patents the USPTO granted between 1976 and 2006. This dataset includes extensive information on each granted patent, including the unique patent number, a unique identifier for the assignee, the technology class, and backward and forward citations in the sample up to 2006. Following a dynamic assignment procedure, we link this dataset to the Compustat dataset.

Compustat North American Fundamentals: We draw our main sample from Compustat for publicly traded firms in North America. This dataset contains balance sheets reported annually by companies between 1974 and 2006. It comprises 29,378 different companies and 390,467 firm×year observations.

3.2 Variable Construction

Patent Citations: Our first measure of innovation is the number of citations a patent received as of 2006. We use the truncation correction weights devised by Hall, Jaffe, and Trajtenberg (2001) to correct for systematic citation differences across different technology classes and for the fact that earlier patents have more years during which they can receive citations (truncation bias).

Tail Innovations: In order to distinguish disruptive patents from ordinary ones, we declare a patented innovation as a tail innovation if it is among the top 10% patents according to citations received among all patents applied for in the same year. The tail innovation index is constructed as the fraction of tail innovations among all granted patents of the firm in a given year, similar to Acemoglu, Akcigit, and Celik (2018). Tail count is likewise defined as the total number of tail innovations a firm receives in a given year. The first variable is scale-free, whereas the second one is scale-dependent.

Originality: We use the originality index devised by Hall, Jaffe, and Trajtenberg (2001). Let $i \in I$ denote a technology class and $s_{ij} \in [0, 1]$ denote the share of citations that patent j makes to patents in technology class i (with $\sum_{i \in I} s_{ij} = 1$). Then for a patent j that makes positive citations, we define: $\text{Originality}_j = 1 - \sum_{i \in I} s_{ij}^2$. This index thus measures the dispersion of the citations made by a patent in terms of the technology classes of cited patents. Greater dispersion of citations is interpreted as a sign of greater originality, since the patented innovation combines information from a diverse range of technological fields. The patent classes used in the baseline analysis are the 36 two-digit technological subcategories defined in Hall, Jaffe, and Trajtenberg (2001).[15] The average originality of a firm's innovation in a given year is the average originality of all the patents for which the firm applied in that year. Originality count is constructed by summing the originality scores of all patents the firm applied for. The first variable is scale-free, whereas the second one is scale-dependent.

Generality: Similar to originality, we use the generality index devised by Hall, Jaffe, and Trajtenberg (2001). Let $i \in I$ denote a technology class and $s_{ij} \in [0, 1]$ denote the share of citations that patent j receives from patents in technology class i (with $\sum_{i \in I} s_{ij} = 1$). Then for a patent j that receives positive citations, we define: $\text{Generality}_j = 1 - \sum_{i \in I} s_{ij}^2$. This index measures the dispersion of the citations made to a patent in terms of the technology classes of citing patents. Greater dispersion of citations is interpreted as a sign of greater generality, since the patented innovation contributes to the creation of patents in a diverse range of technological fields. Average generality and generality count are constructed in the same way as average originality and originality count.

R&D Spending: Patent based measures capture a successful innovation outcome. However, it might also be worthwhile to look at the amount of resources spent by a firm to conduct innovation regardless of success, as this captures the firm's intent. To this purpose, we use the R&D spending reported in Compustat. We have two variables, *log R&D spending* and *log R&D spending 2*. The first one excludes firms when the variable value is missing, whereas the second one replaces missing values with zeroes.

Other Variables: While many inventions are patented, firms can choose not to patent some inventions and keep them as trade secrets. Alternatively, a firm might improve its productivity through methods that are not considered novel enough to warrant a patent by the patent authorities. In such cases, it might be better to look at other firm outcomes that are likely to be correlated with productivity improvements. To do so, we consider investment in advertising and physical capital on the cost side. In addition, we directly look at the measured growth rates of firms' sales, employment, and total assets.[16]

3.3 Industry Innovation and Market Concentration

As documented in Aghion, Bloom, Blundell, Griffith, and Howitt (2005), innovation and market concentration have a non-linear relationship. While higher competition provides incentives for firms to improve their relative productivity compared to their peers ("escape competition") to improve their market share and profits, increased competition can also reduce the incentives to innovate as it pushes down profits in the whole market, which makes R&D investment less worthwhile. According

[15]For robustness, the same measures are recalculated using the three-digit International Patent Classification categories, and the US Patent Class categories assigned internally by the USPTO.

[16]The growth rates are defined as in Davis, Haltiwanger, and Schuh (1996). This bounds the growth rates in the interval $[-2, +2]$, addressing concerns regarding outliers.

Table I: Industry Innovation and Market Concentration (HHI) – Baseline Specification

Panel A: Total Innovation by Industry

	patent count	total citations	tail count	original count	general count
HHI	256.792	57.539	52.281	43.730	28.863
	(73.620)***	(13.716)***	(11.841)***	(16.929)***	(13.850)**
HHI sq.	-184.904	-39.240	-34.843	-29.048	-20.688
	(58.133)***	(10.692)***	(9.399)***	(13.384)**	(10.914)*
number of firms	5.303	0.943	0.921	1.185	0.812
	(0.619)***	(0.104)***	(0.095)***	(0.145)***	(0.101)***
R^2	0.22	0.22	0.23	0.22	0.22
N	11,305	11,305	11,305	11,305	11,305

Panel B: Industry Average of Total Innovation by Firms

	patent count	total citations	tail count	original count	general count
HHI	10.632	191.412	195.286	196.976	274.055
	(5.295)**	(67.088)***	(65.003)***	(128.902)	(94.877)***
HHI sq.	-10.377	-180.346	-181.419	-171.365	-247.240
	(4.395)**	(54.975)***	(54.239)***	(111.115)	(81.058)***
number of firms	-0.076	-0.694	-0.543	-1.714	-1.675
	(0.015)***	(0.179)***	(0.151)***	(0.338)***	(0.295)***
R^2	0.07	0.06	0.06	0.06	0.10
N	11,305	11,305	11,305	11,305	11,305

Panel C: Industry Average of Average Innovation Quality by Firms

	avg. citations	tail innov	avg. originality	avg. generality
HHI	2.015	2.333	3.248	3.046
	(0.483)***	(0.725)***	(0.972)***	(0.965)***
HHI sq.	-1.791	-1.962	-2.275	-2.907
	(0.464)***	(0.746)***	(1.057)**	(1.007)***
number of firms	0.008	0.010	-0.003	-0.006
	(0.001)***	(0.001)***	(0.001)**	(0.002)***
R^2	0.32	0.15	0.34	0.35
N	11,305	11,305	11,305	11,305

Notes: Robust asymptotic standard errors are reported in parentheses. The sample period is from 1976 to 2004 at the annual frequency. All regressions control for year dummies, and a full set of two-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

to which one of the two effects dominates in a particular market, increased competition can reduce or increase overall innovation.

We document the same empirical regularity using our own sample of firms from the US. We are interested in the relationship between our innovation variables and market concentration. Table I contains the results of this exercise. In Panel A, we regress total patent count, total citations, tail innovation count, and originality- and generality-weighted patent counts on market concentration as captured by the Herfindahl-Hirschman Index (HHI) of SIC4 industries. All columns control for the number of firms in the industry, as well as year and SIC2 industry fixed effects. As expected, the linear term has a strong positive coefficient, whereas the quadratic term has a strong negative coefficient, replicating the inverted-U relationship.[17] Panel B repeats the same regressions where the innovation variables are constructed as the average values for all firms in an SIC4 industry instead of the total across the industry. The results are quite similar with the exception of originality-weighted patent count, which has the same signs, but the coefficients are not statistically significant. Panel C regresses the average of firm-level average innovation quality metrics for each industry. The results with these scale-independent innovation quality measures are identical to the previous specifications.

3.4 Firm Innovation and Relative Sales

Different from earlier studies, we also consider the relationship between the relative market share of a firm and its innovation. In this specification, we have *firm* $\times$ *year* level observations. Table II documents our main findings. In Panel A, we regress average citations, tail innovations, average originality, and average generality on relative sales of the firm in its SIC4 industry and its square. The control variables include profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, and a full set of year and SIC4 industry fixed effects. Robust standard errors are clustered at the firm level. In all four columns, we observe a strong inverted-U relationship between a firm's relative sales and its innovation output measures.[18] As a firm's market share increases, it invests more resources into innovation. However, there are diminishing returns. As the firm becomes more dominant in its industry and enjoys a larger market share, it begins to lower its investment in innovation.

Panel B repeats the same regressions where the scale-free innovation measures are replaced with scale-dependent ones such as log total patents, log total citations, log R&D spending where missing values are dropped, and log R&D spending 2 which replaces missing R&D values with zeroes. The inverted-U relationship is still present when we look at the totals instead of the average innovation quality. Panel C replaces the independent variable with other firm moments that are expected to be positively related to productivity improvements. The same non-linear relationship holds if we consider

[17]We should stress that we do not claim that a causal relationship exists. This exercise documents correlation patterns that we would like our model to be able to reproduce. Market concentration and innovation are both endogenous variables in our model.

[18]To our knowledge, we are the first to document this regularity.

Table II: Firm Innovation and Relative Sales – Baseline Specification

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative sales	7.919	6.861	8.396	17.845
	(1.192)***	(1.374)***	(1.789)***	(1.903)***
relative sales sq.	-7.851	-6.793	-6.271	-15.182
	(1.435)***	(1.803)***	(2.208)***	(2.363)***
R^2	0.15	0.10	0.26	0.25
N	104,911	104,911	104,911	104,911

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative sales	2.144	3.582	1.331	0.966
	(0.197)***	(0.307)***	(0.093)***	(0.080)***
relative sales sq.	-1.462	-2.691	-1.169	-0.896
	(0.269)***	(0.402)***	(0.119)***	(0.104)***
R^2	0.57	0.50	0.96	0.94
N	104,911	104,911	61,186	104,911

Panel C

	log(xad)	log(capx)	sales growth	employment growth	asset growth
relative sales	10.702	12.054	0.254	0.194	0.269
	(0.333)***	(0.228)***	(0.020)***	(0.016)***	(0.021)***
relative sales sq.	-10.034	-11.145	-0.236	-0.183	-0.248
	(0.436)***	(0.297)***	(0.025)***	(0.020)***	(0.025)***
R^2	0.73	0.68	0.12	0.12	0.13
N	37,779	103,558	102,726	96,718	103,598

Notes: Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample period is from 1976 to 2004 at the annual frequency. All regressions control for profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, year dummies, and a full set of four-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

log advertising spending, log physical capital expenditures, firm sales growth, firm employment growth, or firm asset growth. Since these firm moments are not reliant on the patent data to calculate, these results reassure us that the regularities we document are not driven by the properties of patent-based innovation metrics.

Our results do not depend on our choice to look at the relative market share of a firm. Tables B1 and B2 in the Appendix replicate the results in Table II where relative sales is replaced with relative employment and relative total assets respectively. The same inverted-U relationship is present in all specifications.

We further test the robustness of our results by replacing the SIC4 fixed effects with SIC3, SIC2, and firm fixed effects. The results can be found in Tables B3, B4, and B5, respectively. The results with SIC2 and SIC3 fixed effects are quite similar to the baseline specification. Although the last specification with firm fixed effects is much more demanding, the inverted-U relationship is still present in all seven regressions, with some loss of significance in columns 2 and 3 in Panel A.

Finally, we consider how the relationship changes across time. Tables B6 and B7 split the sample into two periods: early (1976-1990) and late (1991-2004). Although the quantitative magnitudes are quite different, the inverted-U relationship is present in both samples.

3.5 Robustness of the Inverted-U Relationship

Thus far, we have sought to identify an inverted-U relationship between competition and innovation across and within industries using the standard approach found in numerous economic studies investigating such non-linear relationships. This involves running regressions with linear and quadratic terms, establishing the significance of their coefficients, and showing that the extremum lies within the data range. However, the sufficiency of this methodology to establish the existence of a genuinely U or inverted-U shape relationship has been the subject of some debate. In light of these concerns, Lind and Mehlum (2010) develop a hypothesis test for the existence of U- and inverted-U-shape relationships.[19] To further establish the robustness of our results, we conduct the hypothesis test proposed in Lind and Mehlum (2010) for all specifications in Tables I and II, where the null hypothesis is the lack of an inverted-U relationship. This involves testing whether or not the slope of the curve is positive at the start and negative at the end of the interval of the variable of interest. Correspondingly, Tables B8 and B9 report the t- and p-values at the lower and upper bounds of the interval of the explanatory variable. The null hypothesis is firmly rejected in all 27 specifications but one. The inverted-U relationships that we have identified pass the formal test of existence, with p-values below 1% in the vast majority of cases.[20]

[19] Arcand, Berkes, and Panizza (2015), Rodrik (2016), Bazzi, Gaduh, Rothenberg, and Wong (2016), Kesavan, Staats, and Gilland (2014), Tan and Netessine (2014), and Batt and Terwiesch (2017) among others use the test proposed in Lind and Mehlum (2010) to establish the existence of U- and inverted-U-shape relationships.

[20] Out of the 27 specifications, the null hypothesis is rejected at 1%-level at both bounds in 19 cases, 5%-level in 4 cases, and 10%-level in 3 cases. The p-value is 13.8% in the only setting without significance.

4 Quantitative Analysis

4.1 Estimation

Ten parameter values must be determined: $\rho, \lambda, \eta, \chi, \nu, \zeta, \phi, \epsilon, \psi, \tau$. The consumer discount rate ρ is set to 0.04, which implies a real interest rate of 6% when the growth rate is 2%.[21] The rest are structurally estimated following a simulated method of moments approach.

The success of SMM estimation depends on model identification, which requires that we choose moments that are sensitive to variations in the structural parameters. Nine parameters remain to be determined: innovation step size λ, elasticity of substitution within an industry η, superstar innovation cost scale parameter χ, small firm innovation cost scale parameter ν, relative quality of the competitive fringe ζ, superstar innovation convexity parameter ϕ, small firm innovation convexity parameter ϵ, entry cost scale parameter ψ, and exogenous exit rate τ. All these parameters are jointly estimated to match the following targeted data moments: the growth rate of real GDP per capita, aggregate R&D intensity, labor share, firm entry rate, (sales-weighted) average markup,[22] within-year standard deviation of markups, the linear term and the top point obtained from the intra-industry regression of a firm's innovation on its relative sales, average profitability, average relative quality of the leader, and its standard deviation across industries. A detailed description of the data moments and the estimation procedure is provided in Appendix A.3.

Panel A of Table III reports the values of the parameters, whereas Panel B provides an overview of the values of the targeted moments in the data and the estimated model. The model tightly matches the eleven data moments. The Jacobian matrix of the model moments with respect to the model parameters in percentage terms is displayed in Table B10.

4.2 Policy Functions

Figure I displays the optimal innovation policy functions followed by the firms in the estimated equilibrium obtained by targeting data moments for the whole sample. The left panel of Figure I depicts the innovation policy of a firm in an industry with two superstar firms. The relevant state variable is how many steps the current firm is ahead of its competitor, where negative numbers indicate that the current firm is lagging behind the competing firm. We see that the incentive to innovate is increasing from -5 to -1, and it is decreasing from -1 to 5. This means that a two-firm industry experiences the highest amount of superstar innovation when the two competing firms are

[21]We target a relatively high real interest rate to remain conservative. For instance, a lower real interest rate of 4% would halve the implied discount rate to $\rho = 0.02$. This would double the welfare contribution of the output growth rate relative to that from the initial consumption level, significantly amplify the dynamic welfare gains, and further strengthen our findings.

[22]In Section 6.4, we re-estimate the model using cost-weighted markups from Edmond, Midrigan, and Xu (2018), and the results are found to be similar. Motivated by Bond, Hashemi, Kaplan, and Zoch (2020), we also conduct another re-estimation that does not rely on any markup-based moments obtained through the De Loecker and Warzynski (2012) methodology. This strengthens our results.

Table III: Baseline Model Parameters and Target Moments

A. Parameter estimates

Parameter	*Description*	*Whole sample*	*Early sub-sample*	*Late sub-sample*
λ	innovation step size	0.3126	0.3369	0.3261
η	elasticity within industry	6.6800	16.5759	6.9717
χ	superstar cost scale	120.5659	198.7544	73.1135
ν	small firm cost scale	3.4046	1.3209	2.5502
ζ	competitive fringe ratio	0.5912	0.616	0.5454
ϕ	superstar cost convexity	3.8711	4.1409	3.3975
ϵ	small firm cost convexity	2.6594	2.8913	2.5583
τ	exit rate	0.1151	0.1257	0.1052
ψ	entry cost scale	0.0149	0.0079	0.0115

B. Moments

	Whole sample		Early sub-sample		Late sub-sample	
Target moments	Data	Model	Data	Model	Data	Model
growth rate	2.20%	2.20%	2.42%	2.42%	1.98%	1.98%
R&D intensity	2.43%	2.02%	2.38%	1.83%	2.50%	2.33%
average markup	1.3498	1.3462	1.2805	1.2801	1.4242	1.4195
std. dev. markup	0.346	0.387	0.299	0.319	0.396	0.428
labor share	0.652	0.628	0.656	0.630	0.648	0.611
firm entry rate	0.115	0.115	0.126	0.126	0.105	0.105
β(innovation, relative sales)	0.629	0.726	0.435	0.682	0.699	0.756
top point (intra-industry)	0.505	0.448	0.447	0.470	0.507	0.453
average profitability	0.144	0.176	0.137	0.153	0.147	0.204
average leader relative quality	0.749	0.642	0.750	0.621	0.747	0.643
std. dev. leader relative quality	0.223	0.161	0.224	0.145	0.222	0.149

Notes: The estimation is done with the simulated method of moments. Panel A reports the estimated parameters. Panel B reports the simulated and actual moments.

very close to each other in terms of productivity (neck-and-neck and one step difference.)

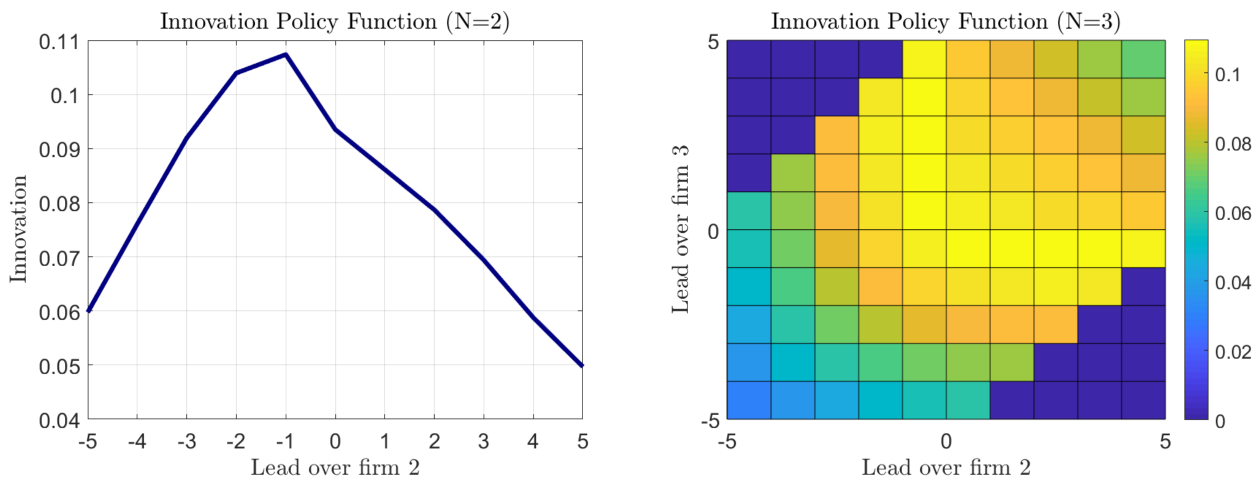

Figure I: Innovation Policy Function

The right panel of Figure I does the same for a firm in an industry with three superstar firms. This time the state variable is two-dimensional, and the innovation decision of the firm is a function of how many steps the current firm is ahead of both of its competitors. Since competitors that have the same relative distance in terms of quality are identical, the two-dimensional surface is symmetric along the antidiagonal. There are also some illegal states which cannot happen as we assume the maximum number of steps between two firms cannot exceed $\bar{n}$. These correspond to the two blue triangles at the top left and bottom right corner, and should be ignored. Overall, the policy function shares some properties with the case for a two firm industry. In particular, along the antidiagonal where the two competitors have the same quality, the innovation policy is once again increasing until the firms are close to being neck-and-neck, and then decreasing.

Figure B1 in the Appendix is the case for a firm in an industry with four superstars. In this case, the state variable is three-dimensional. Therefore, we split the innovation policy function into ten separate subfigures constructed similar to the right panel of Figure I, where each subfigure corresponds to the fourth competitor being a certain number of steps behind the current firm. Note that there are more illegal cases that arise whenever $n_4 \neq 0$ which are once again colored blue. Similar properties along the antidiagonal are observed.

4.3 Model-Implied Relationship Between Innovation and Competition

One key feature of our model compared to other models with endogenous markups is its ability to deliver a hump-shaped relationship between innovation and competition at the industry level, which is shown in the left panel of Figure II.[23] Each marker on the plot corresponds to the innovation choice of a firm given an industry state. The horizontal axis depicts the relative sales of the firm compared

[23]It is worth noting that the model is flexible enough to generate the opposite relationship, i.e., a U-shape. The inverted-U relationship is an estimation result.

to the total sales of all superstars in the same industry. The legend for the figure clarifies the number of superstar firms in the industry the observation is coming from. The blue curve is the quadratic fit to the observations. We see the observations constitute an inverted-U shape both when they are all considered at the same time, and also separately conditioning on a certain number of superstar firms. The innovation choices are normalized by demeaning and dividing by the standard deviation. Each observation is assigned weights based on their occurrence dictated by the time-invariant distribution of industry types $\mu(\Theta)$. Likewise, the right panel of Figure II depicts the same for R&D spending on relative sales. The overall shape looks similar, but the differences are magnified as the innovation level increases, owing to the convexity of the superstar innovation cost function.

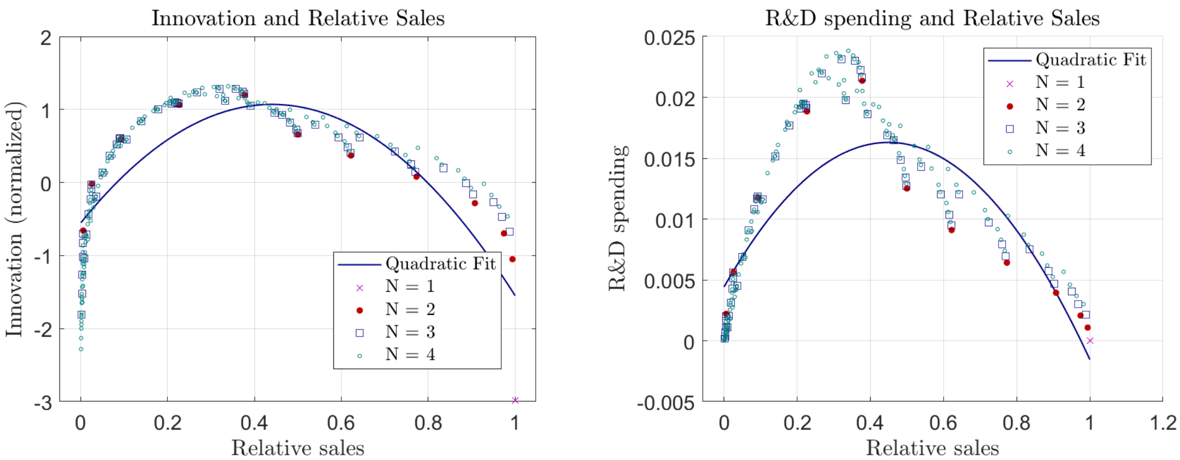

Figure II: Innovation, R&D Expenses, and Firm Relative Sales

Figure III shows the model-implied relationship between competition and innovation across industries. We use the Herfindahl-Hirschman Index to measure market concentration.[24] The figure shows that there is an inverted-U relationship between an industry's market concentration and its total innovation. The vertical blue line corresponds to the top point of the quadratic polynomial fitted to the model-generated data, whereas the vertical red line corresponds to the average HHI of industries in the economy, weighted by their shares in the time-invariant industry state distribution $\mu(\Theta)$. Note that this average is lower than the top point. This foreshadows our finding that a change that increases market concentration would increase innovation overall. We would expect the opposite to be true if the red line was to the right of the blue line.

It is worth mentioning that our model is able to generate this second relationship without explicitly targeting the quadratic relationship in the estimation. Furthermore, our model generates the inverted-U without introducing any exogenous heterogeneity.[25] This is different from recent models that endogenize markups.[26] In Schumpeterian models with a single active firm, all observations would be

[24] Note that the market share percentage of each individual small firm in the competitive fringe is zero.

[25] The original inverted-U paper by Aghion, Bloom, Blundell, Griffith, and Howitt (2005) achieves this through the introduction of exogenous collusion heterogeneity across industries.

[26] See Akcigit and Ates (2019a,b), Aghion, Bergeaud, Boppart, Klenow, and Li (2019), Peters and Walsh (2019), De Loecker, Eeckhout, and Mongey (2019), and Edmond, Midrigan, and Xu (2018) among others.

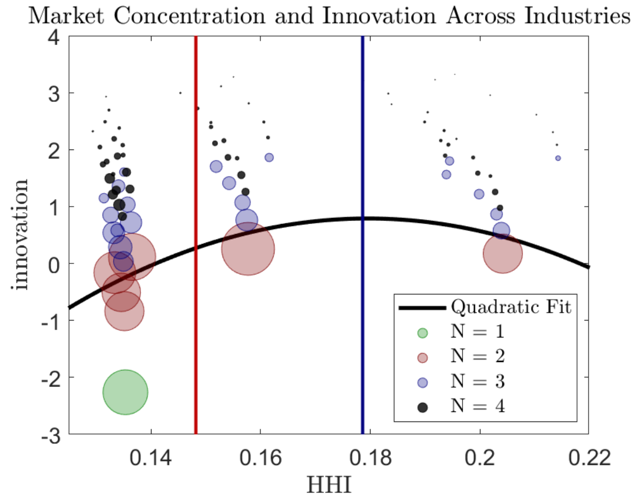

FIGURE III: INNOVATION AND HHI

clustered at 1 for relative sales and HHI. In step-by-step innovation models with two firms, there would be two points at 0.5 and 1 for relative sales, and 0.5 and 1 for HHI. In models with exogenous productivity evolution, we would get a flat line. Finally, in models with innovation only at entry, there would be a monotone relationship between competition and innovation, the shape of which depends on that of the cost function. Our model's ability to match this relationship aids in disciplining the counterfactual behavior of aggregate productivity growth in response to endogenous changes in market concentration, which is crucial for calculating the social welfare implications of increasing markups.

4.4 Computing Social Welfare

We would like to compute social welfare in counterfactual economies and compare them against the estimated equilibrium. To calculate welfare, we need to compute the consumption stream of the representative household. In a non-stationary equilibrium, the time path of consumption is directly calculated.[27] In a BGP equilibrium, two components are sufficient: the growth rate of consumption g, and the initial consumption level C_0. This, in turn, requires us to compute initial output Y_0 and aggregate spending on R&D and new business creation. The level of initial output Y_0 is given by:

$$\ln(Y_0) = \int_0^1 \ln q_{j0}^{\text{leader}} \, dj + \ln \zeta - \ln \omega + \sum f(\Theta)\mu(\Theta) \qquad (37)$$

All terms are time-invariant except for the average log productivity level of the industry leaders at time 0, given by $Q_0 = \int_0^1 \ln q_{j0}^{\text{leader}} dj$. When comparing welfare across economies, we fix this term to

[27]See Section A.6.1 in the Appendix for details.

be equal to zero in all economies without loss of generality.[28] Next, initial consumption C_0 is given by

$$C_0 = Y_0 \frac{C_0}{Y_0} = Y_0 \left(1 - \int_0^1 \sum_{i=1}^{N_{j0}} \chi z_{ij0}^{\phi} \; dj - \int_0^1 m_0 \nu X_{kj0}^{\epsilon} \; dj - \psi e_0^2 \right) \tag{38}$$

where the second factor is the share of output left for consumption after R&D and entrepreneur entry costs are subtracted. Then, the welfare of the representative household in a BGP equilibrium can be calculated as:

$$W = \int_0^{\infty} e^{-\rho t} ln(C_t) dt = \frac{ln(C_0)}{\rho} + \frac{g}{\rho^2} \tag{39}$$

The model allows for a closed-form decomposition of changes in welfare across two economies as follows:

$$\Delta W = \frac{1}{\rho} \left[\Delta \ln \zeta - \Delta \ln \omega + \Delta \sum f(\Theta) \mu(\Theta) + \Delta \ln \left(\frac{C}{Y} \right) \right] + \frac{1}{\rho^2} \Delta g \tag{40}$$

For two economies A and B, we can define a consumption equivalent welfare measure (ϖ) which corresponds to the percentage increase in lifetime consumption that an agent in economy A would need to be indifferent between being in economy A and B:

$$W_B = \frac{\ln(C_0^A (1 + \varpi))}{\rho} + \frac{g^A}{\rho^2} \tag{41}$$

Solving for ϖ, we get:

$$\varpi = \exp\left(\left(W_B - \frac{g^A}{\rho^2} \right) \rho - \ln(C_0^A) \right) - 1 \tag{42}$$

4.5 Comparison of the Early and Late Period Economies

In Section 4.1, we estimate the model by targeting data moments from the early and late periods of our sample (1976-1990 and 1991-2004). The estimated parameter values in the two sub-samples are quite different, which captures the structural changes in the US economy throughout this time period. These estimates are reported in Table III.

The elasticity of substitution within an industry, η, decreases from 16.6 to 6.97. This parameter primarily governs the degree of static product market competition between superstar firms, and the significant decrease in its value indicates that superstar firms enjoy higher monopoly power in the late sub-sample. At the same time, the ratio of the competitive fringe's productivity relative to the industry leader, ζ, decreases from 0.616 to 0.545. This parameter captures the product market competition from non-superstar firms, and a decrease in its value indicates that superstar firms can

[28] In other words, we keep the initial frontier technology level the same across counterfactual economies.

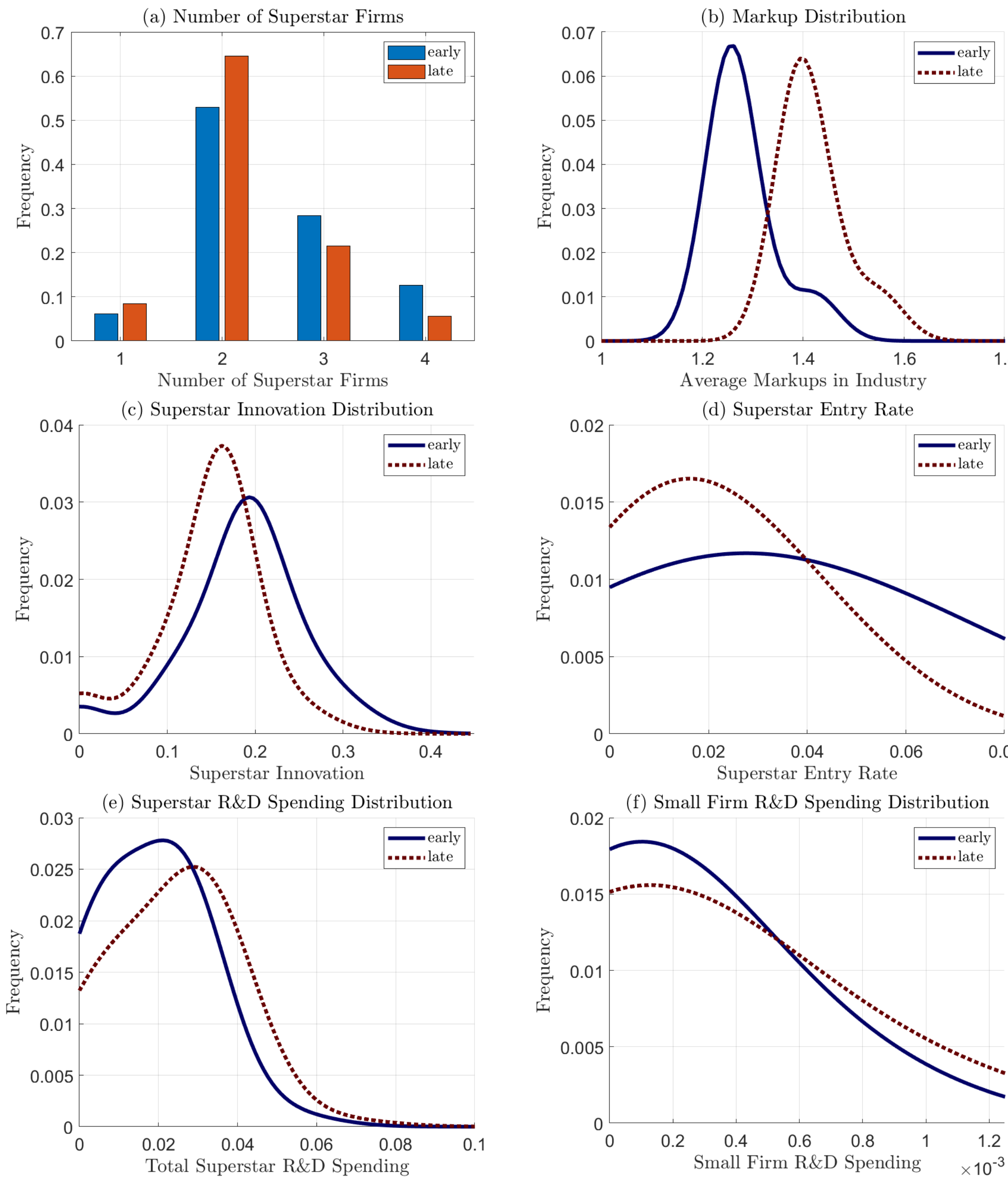

Figure IV: Changes in Distribution: Early vs. Late charge higher markups thanks to reduced competition.

If we turn to parameters that govern the innovation cost function of the superstars, we observe a decrease in the scale parameter χ from 199 to 73.1, and the convexity ϕ from 4.14 to 3.40. The first change reduces the cost of superstar innovation, whereas the second change allows innovation to be more concentrated across firms, as it reduces the diminishing returns to innovation within a firm. Given other parameter values, the decrease in ϕ also increases innovation costs on average. When we turn to small firms, the picture changes. The innovation scale parameter for small firm innovation, ν, increases from 1.32 to 2.55, and the convexity ϵ decreases from 2.89 to 2.56. Both changes increase the cost of innovation for small firms, which reduces the entry rate of new superstar firms. Finally, the cost of new business entry increases as ψ increases from 0.79% to 1.15%.

Figure IV shows that there are considerable changes in the industrial structure between the two periods. Panel (a) of Figure IV shows the distribution of industries over states with a different number of superstar firms. There is an increase in the share of industries with one or two superstar firms, whereas the share of industries with more than two superstar firms decreases. There is an increase in market concentration. This is also seen in Panel (b) of Figure IV: the distribution of average markup within industries shifts strongly to the right. Panel (c) of Figure IV depicts superstar innovation at the industry level, and the distribution moves to the left towards less innovation. The distribution of the superstar entry rate – or, equivalently, small firm innovation – across industries is seen in Panel (d) of Figure IV. A large overall decrease is observed, and the heterogeneity of superstar entry rates across industries also goes down. Despite the decrease in innovation by both small and large firms, the distribution of R&D costs remains largely the same with slight increases, as seen in Panels (e) and (f) of Figure IV. This owes to the overall increase in the costs to innovate.

4.6 Disentangling the Structural Transition

In order to better understand and disentangle the effects of the transition implied by the estimated parameter values in the two sub-samples, we conduct counterfactual exercises where we investigate the effects of each change separately. To do so, we compare the estimated late sub-sample economy against counterfactual economies in which we set the values of selected parameters to their early sub-sample estimates. These exercises illustrate how the economy would look like if there were no structural change in a particular mechanism. This, in turn, allows us to understand which mechanisms are the primary drivers of the time trends in important quantities such as the average markup, the labor share, output growth, and so on, as well as their implications for social welfare. In other words, we use our model as a tool to uncover how the structural parameters change over time, and establish changes in which mechanisms induce the observed macroeconomic changes, in what magnitude, and in which direction.

Table IV: Disentangling the Structural Transition

	Benchmark	Early η	% change	Early ζ	% change	Early ν, ϵ	% change
growth rate	1.98%	1.79%	-9.38%	1.63%	-17.53%	2.19%	10.64%
R&D intensity	2.33%	2.20%	-5.79%	1.54%	-34.04%	2.79%	19.61%
average markup	1.420	1.415	-0.29%	1.307	-7.91%	1.423	0.28%
std. dev. markup	0.428	0.404	-5.68%	0.363	-15.23%	0.420	-1.91%
labor share	0.611	0.601	-1.76%	0.642	5.02%	0.600	-1.90%
entry rate	0.105	0.105	0.00%	0.105	0.00%	0.105	0.00%
β(innov, relative sales)	0.756	0.699	-7.62%	0.750	-0.81%	0.833	10.18%
top point (intra-industry)	0.453	0.440	-2.92%	0.446	-1.63%	0.469	3.42%
avg profitability	0.204	0.209	2.23%	0.162	-20.53%	0.204	-0.05%
avg leader relative quality	0.643	0.687	6.82%	0.684	6.34%	0.569	-11.53%
std. dev. leader relative quality	0.148	0.163	9.63%	0.165	10.88%	0.123	-17.45%
superstar innovation	0.147	0.127	-13.61%	0.117	-20.30%	0.179	21.89%
small firm innovation	0.019	0.010	-46.92%	0.011	-42.78%	0.042	120.72%
output share of superstars	0.515	0.537	4.26%	0.433	-15.88%	0.535	3.73%
avg superstars per industry	2.244	1.988	-11.38%	2.028	-9.61%	2.791	24.42%
mass of small firms	1.000	0.674	-32.56%	0.653	-34.69%	1.281	28.12%
initial output	0.801	0.748	-6.63%	0.826	3.16%	0.811	1.31%
CE Welfare change		-10.64%		-4.52%		6.38%	

	Benchmark	Early χ, ϕ	% change	Early ψ, τ	% change	All	% change
growth rate	1.98%	2.63%	33.06%	1.99%	0.52%	2.42%	22.58%
R&D intensity	2.33%	2.61%	11.74%	2.35%	0.76%	1.83%	-21.55%
average markup	1.420	1.418	-0.09%	1.420	0.01%	1.280	-9.82%
std. dev. markup	0.428	0.425	-0.78%	0.428	-0.08%	0.319	-25.53%
labor share	0.611	0.603	-1.38%	0.603	-1.36%	0.647	5.88%
entry rate	0.105	0.105	0.00%	0.126	19.44%	0.126	19.44%
β(innov, relative sales)	0.756	0.700	-7.48%	0.757	0.07%	0.682	-9.79%
top point (intra-industry)	0.453	0.456	0.53%	0.454	0.13%	0.470	3.61%
avg profitability	0.204	0.202	-1.01%	0.204	0.01%	0.153	-25.20%
avg leader relative quality	0.643	0.609	-5.32%	0.640	-0.43%	0.621	-3.48%
std. dev. leader relative quality	0.148	0.149	0.42%	0.147	-0.80%	0.145	-2.08%
superstar innovation	0.147	0.212	44.44%	0.148	0.79%	0.184	25.27%
small firm innovation	0.019	0.036	90.19%	0.019	2.79%	0.032	66.57%
output share of superstars	0.515	0.522	1.23%	0.516	0.15%	0.454	-11.98%
avg superstars per industry	2.244	2.527	12.64%	2.260	0.71%	2.475	10.32%
mass of small firms	1.000	1.581	58.09%	1.041	4.08%	1.000	0.00%
initial output	0.801	0.803	0.32%	0.801	0.05%	0.774	-3.35%
CE Welfare change		17.88%		0.39%		8.73%	

Notes: The table reports the change in model moments when setting the parameter of interest back to its estimated level in early sub-sample while keeping other parameters fixed at their estimated value in the late sub-sample.

The results are presented in Table IV, where the first column displays the benchmark values of chosen model moments in the late sub-sample. The last two columns of the bottom panel show the results of changing all the parameters to their early period values to provide context. All the remaining columns show how the moments change in each exercise. Below, we discuss these results in detail.

4.6.1 Competition from Superstars vs. Small Firms

We first look at what happens when we set the elasticity of substitution within an industry, η, to its early period value. The direct effect of the change is a decrease in the monopoly power of the superstar firms. Average markup falls, but only slightly by 0.29% of its value. The reduction in monopoly power hurts superstar firms as a whole, but across superstar firms, it favors the industry leaders who have the highest productivity. Consequently, the average number of superstars per industry falls by 11.4%, and so does the initial output by 6.63%. There is a modest drop of 13.6% in superstar innovation, whereas small firm innovation drops significantly by 46.9% of its value. The consequent decrease in the profitability of small firms reduces their mass by 32.6%. As a combined effect of the changes in innovation, there is a significant decrease in the growth rate by 9.38% of its value, which translates into a welfare loss of 10.64% in consumption-equivalent terms.[29]

This experiment reveals that the increase in the estimated monopoly power of superstar firms results in increased economic growth due to better incentives to innovate and the resultant changes in the industrial structure. If the elasticity of substitution had not fallen, there would be less incentive to innovate to become superstar firms, which implies industries being dominated by an even smaller number of superstar firms. These superstar firms, in turn, would not innovate as much, since the "escape competition" effect is weaker – fewer superstar firms means less peer competition. Therefore, the average markup would stay virtually the same, and the static welfare gains would be limited. The dynamic losses from reduced growth overshadow slightly reduced markups, resulting in lower welfare.

Next, we look at what happens when we set small firms' productivity relative to the industry leader, ζ, to its early period value. Unlike η, changes in this parameter capture the competition from small firms. This time, the average markup falls from 1.42 to 1.31. Given that the average markup in the early sample is 1.28, the change in ζ can explain nearly all of the change in the average markup across time. While the reduction in markups improves static efficiency, increasing initial output by 3.16% and the labor share by 5.02%, average profitability falls by 20.5%. Consequently, firms have less incentive to innovate, reducing small firm innovation by 42.8% and superstar innovation by 20.3%. Combined together, this reduces the output growth rate from 1.98% to 1.63%. Despite the gains in static efficiency, social welfare drops by 4.52%.

[29]Note that this welfare number should be interpreted with caution, since the change in η can be considered a change in consumer preferences rather than a change in production technology. Therefore, the drop in initial consumption might not indicate a welfare loss. However, the significant drop in the growth rate still means that a hypothetical consumer with unchanged preferences would be worse off.

There are two main takeaways from this experiment. First, our model suggests that the overall increase and the polarization in markups which were observed after 1975 owe mostly to a reduction in competition from small firms, rather than a reduction in competition between superstar firms, or the changing costs of innovation. This is consistent with the previous findings in the literature regarding the decline in business dynamism and "winner-takes-most" dynamics as in Decker, Haltiwanger, Jarmin, and Miranda (2016) and Autor, Dorn, Katz, Patterson, and Van Reenen (2017) among others. The output share of superstars increases because the relative productivity of small firms is lower. Second, our model's implications regarding the social costs of increasing markups are quite different from other papers in the literature which focus on static efficiency gains alone. Our model suggests that the increase in markups can be welfare-improving when the dynamic effects on productivity growth are taken into account. Since this is a key difference of our model, we discuss the static vs. dynamic effects of increasing markups in richer detail in Section 4.6.4.

4.6.2 Costs of Superstar vs. Small Firm Innovation

In our third experiment, we set the parameters that govern the innovation cost function of small firms, ν and ϵ, to their early period values. Unsurprisingly, there is a very significant direct effect on small firm innovation which increases by 120.7% of its value. The average number of superstar firms per industry increases by 24.4%, which is roughly double the total difference between the early and late periods. Average profitability is virtually the same, so the Schumpeterian creative destruction effect does not change. On the other hand, the increased number of superstars boosts the "escape competition" effect, and superstar innovation increases by 21.9% of its value. Hence output growth increases from 1.98% to 2.19%, which can explain around half of the growth rate difference between the two sub-samples. Coupled with a slight gain in initial output of 1.31%, social welfare improves by 6.38%.

Recall that in our model we define entry into superstars not as newly established firms. Through successful innovation, small firms become superstar firms. Whereas the decrease in the competitive fringe's productivity ζ captures the declining product market competition from small firms, the increase in the cost of small firm innovation reduces the frequency at which small firms can join the ranks of superstar firms. Therefore, our estimates suggest that the number of small firms with high growth potential ("gazelles") has gone down, which allows industries to be dominated by fewer superstar firms. This finding is consistent with the recent evidence by Pugsley, Sedlacek, and Sterk (2018), who find that both the share of such "gazelles" among all firms, and their average growth rates have fallen over time after 1975. As the experiment suggests, this leads to a weakening of the incentives to innovate by the remaining superstar firms, which hurts productivity growth.

Our fourth experiment reverts the parameters of the superstar innovation cost function, χ and ϕ, to their early period values. Superstar innovation increases by 44.4% in response. However, this increase is more than matched by a simultaneous increase in small firm innovation by 90.2%.

Average profitability and the output share of superstars do not change by much, leading to virtually unchanged average markup and initial output. This means static efficiency remains comparable to the benchmark, and the sizable 33.1% increase in output growth translates to a similarly large 17.9% increase in welfare.

This experiment relates directly to the literature on the increasing costs of innovation ("ideas are getting harder to find"). As noted in Bloom, Jones, Van Reenen, and Webb (2020), the number of researchers employed in research and development of new ideas has been constantly increasing over the time period, despite the observed decline in productivity growth. Our experiment reveals that the increase in the cost of superstar innovation can more than account for the 22.6% drop in output growth by itself. Looking more broadly at the determinants of output growth in our model, the reduced competition from superstars and small firms push for higher growth, whereas increased innovation costs dominate the competition channel, and deliver the observed decline in productivity growth. In this aspect, our findings are not consistent with the popular hypotheses regarding how the decline in competition might have led to the productivity slowdown. Instead, our model's answer is a significant technological decline in research productivity. There can be many reasons behind the decline in research productivity, ranging from the reduced arrival rate of general purpose technologies (Gordon (2012)) to stronger intellectual property protection (Han (2018)), increased patent litigation (Galasso and Schankerman (2014)), protective patenting (Argente, Baslandze, Hanley, and Moreira (2020)), or increasing misallocation of talent in innovation due to increasing wealth inequality (Celik (2017)). Our results suggest future research directed at identifying the reasons behind the decline in research productivity might hold the key to understanding the decline in productivity growth.

4.6.3 Entrepreneurship and Firm Entry

In our final experiment, we consider the effects of the decline in the firm entry rate, and the changes to the costs of founding new businesses. To this purpose, we set the scale parameter of the entrepreneur cost function, ψ, and the exit rate, τ, to their early period values. From the early period to the late period, the exit rate τ goes down, and the scale parameter ψ goes up. The prior increases the survival rate of small firms, therefore increasing the expected value obtained from founding a new business. The latter increases the cost of doing so. Therefore, the two changes push the equilibrium mass of small firms in opposite directions. The total effect seems to be only a slight increase of 4.08%. This increase boosts small firm innovation by 2.79%, which also encourages slightly more innovation by superstars at 0.79%. Initial output is virtually the same, and the growth rate increases by 0.52% of its value. The combined welfare effect is calculated as 0.39%.

These results suggest that the increasing cost of entrepreneurship and the decline in firm exit rate act to counterbalance each other. If we consider their effects separately, reverting the value of ψ by itself leads to a slight increase in growth by 2.85% and welfare by 1.73%, whereas repeating the same exercise for τ reduces growth and welfare by 2.72% and 1.41%, respectively. Compared to the quantitative magnitudes obtained from other experiments, these two mechanisms seem to have muted effects on the economy. For average markups and market concentration, the relative productivity of small firms seems to matter much more than their total number. For productivity growth, the changes in costs of innovation dominate the changes in firm entry and the cost of founding new businesses. It is important to note that these results do not imply that the new business entry margin is unimportant. As seen in Table IV, the mass of small firms changes by a large amount in all of the previous exercises, and this mass boosts the amount of small firm innovation and, consequently, the average number of superstars in an industry. Shutting down this endogenous business entry margin would moderately diminish the growth effects in other quantitative experiments.

4.6.4 Static vs. Dynamic Costs of Higher Markups

The second counterfactual experiment in Section 4.6.1 where the relative productivity of the competitive fringe ζ was reverted to its early period value revealed that increased competition from small firms reduced the average markup to nearly its early period value, yet the dynamic change in welfare was a loss of 4.52%. To better understand why this happens, it is useful to decompose the change in welfare to its constituent parts using equation (40). At the same time, our finding differs significantly from static analyses that focus on the efficiency gains from reduced markups without taking the implications for productivity growth and endogenous industry dynamics into account. This motivates us to also perform the decomposition where we only consider the static changes from reduced markups, and the static changes plus the endogenous change in the distribution of industries, while keeping the innovation policies and the output growth rate the same. The results are presented in Table V.

The first column of Table V calculates and decomposes the change in welfare if we ignored the dynamics completely. The dynamics come into play through three model components: (1) the growth rate, (2) the change in consumption to output ratio due to R&D spending and the cost of new business entry, (3) the changes in the relative wage rate ω and the production by superstar firms $\sum f(\Theta)\mu(\Theta)$ which both depend on the distribution of industries $\mu(\Theta)$. To calculate the static effects, we keep the growth rate, R&D spending, new business creation, and $\mu(\Theta)$ at the late period levels. There is a large increase in output and welfare as a direct effect of increasing ζ. Increased production increases labor demand, which pushes the relative wage ω up, the effect of which is negative. Finally, increased production by small firms results in reduced supply from superstar firms, so the third component is negative as well. The direct effect dominates these endogenous (static) responses, and welfare is increased. Column 2 shows the change in consumption-equivalent welfare by each individual component. The combined effect is a significant 3.73% gain in welfare.

The third column repeats the same exercise as in column one with a single difference: we use the distribution of industries $\mu(\Theta)$ implied by the dynamic long-run change in response to the increase in ζ rather than retaining the late sample values. This changes the effects from the relative wage

Table V: Static vs. Dynamic Costs of Higher Markups

	Static		Static+New Distribution		Dynamic	
	ΔW	CEWC	ΔW	CEWC	ΔW	CEWC
competitive fringe productivity	3.043	12.94%	3.043	12.94%	3.043	12.94%
relative wage	-1.495	-5.81%	-1.561	-6.05%	-1.561	-6.05%
output of superstar firms	-0.632	-2.50%	-0.703	-2.77%	-0.703	-2.77%
consumption/output	0.000	0.00%	0.000	0.00%	0.229	0.92%
output growth	0.000	0.00%	0.000	0.00%	-2.166	-8.30%
total	0.916	3.73%	0.779	3.16%	-1.158	-4.52%

Notes: The table decomposes the change in welfare to its constituent parts using equation (40).

and the output of superstars slightly, and the gain in welfare remains close but lower at 3.16%. This means ignoring the long term effects on industrial structure – i.e., the number and relative qualities of superstars – would result in overestimating the welfare gains by 18%, which is not insignificant.

The fifth column displays the welfare decomposition for the full dynamic response. As alluded to in the second exercise, the difference in the initial output (and hence the static efficiency) is limited. There is a small welfare gain from the consumption to output ratio term due to reduced R&D spending and new business entry. However, there is a very significant decline due to reduced growth that wipes out both the static welfare gains from reduced markups and the gains from higher initial consumption. Therefore, the final tally is a loss of 4.52%, as the dynamic losses from reduced productivity growth valued at 8.30% of consumption completely dominate the 3.16% gain from improved static efficiency. A static model that does not endogenize productivity growth would not be able to obtain this result, and highly overestimate the cost of increased markups in the US between 1976 and 2004.

4.6.5 Distributional Implications of the Structural Transition

Throughout our analysis, we have opted to use a representative household. However, as the empirical findings indicate, the structural transition in the US over the last four decades decreased labor's share of income while driving up profits. Wealth is heavily concentrated in the US: According to the 2013 Survey of Consumer Finances, households in the bottom 50% of the wealth distribution hold only 1% of total wealth, whereas the top 5% hold 63% and top 1% hold 36%. Therefore, the gains from the increase in the profit share accrue to a very small portion of the population, whereas the decline in the labor share hurts most of the population who derive their income primarily from labor, not assets. This means the increase in markups and the decline in the labor share can have significant distributional implications, as studied in Boar and Midrigan (2019).

While adding a complete heterogeneous agent framework on the household side with credit market imperfections remains beyond the scope of our paper, there are less costly ways to uncover the first-order implications for inequality with little alteration to the model. We can separate the representative household into two types of consumers: (1) workers, who derive all of their income

Table VI: Distributional Implications of the Structural Transition

	Early η	Early ζ	Early ν, ϵ	Early χ, ϕ	Early ψ, τ	All
Benchmark CEWC	-10.64%	-4.52%	6.38%	17.88%	0.39%	8.73%
Worker CEWC	-11.24%	0.70%	6.17%	18.07%	0.29%	15.96%
Capitalist CEWC	-8.34%	-24.39%	7.15%	17.18%	0.80%	-18.80%
$\Delta\%$ $C^{\text{capitalist}}/C^{\text{worker}}$	3.27%	-24.92%	0.92%	-0.75%	0.51%	-29.97%

Notes: The table reports the change in model moments when setting the parameter of interest back to its estimated level in early sub-sample while keeping other parameters fixed at their estimated value in the late sub-sample.

from labor, and cannot own any assets, (2) capitalists, who have no labor income, but own all assets in the economy and receive all entrepreneurial income. With this modification, the consumption of workers along the BGP is given by $C_t^{\text{worker}} = \omega Y_t$, and the consumption of capitalists is given by $C_t^{\text{capitalist}} = C_t - C_t^{\text{worker}}$. With these consumption streams, we can compute the consumption equivalent welfare change (CEWC) for both types of consumers. We can also compute how the relative consumption of the two groups, $C^{\text{capitalist}}/C^{\text{worker}}$ changes in each counterfactual exercise.

The results are shown in Table VI. The first row repeats the consumption equivalent welfare change for the representative consumer in Section 4.6, whereas the second and third rows report it for the workers and capitalists, respectively. The last row reports the percentage change in relative consumption. In all exercises except the second one, the welfare changes of the workers and capitalists go in the same direction, and they are very close in magnitude. The change in relative consumption also remains below 3.5% in absolute terms in these exercises. Naturally, the only exception to this is the exercise where markups are reduced. When the relative productivity of the competitive fringe, ζ, is reset to its early period value, the welfare of the capitalists drops by 24.4%, whereas that of the workers increases slightly by 0.7%. The relative consumption of the capitalists goes down by 24.9%, reducing the inequality between the two groups.

Recall that the overall welfare loss was 4.52% for the representative household. This means that while increased competition from small firms would be detrimental for the overall economy, since the gains from innovation are not shared equally, consumers who primarily rely on labor income would be net winners by a slight margin, whereas wealthy consumers would be the obvious losers. One can draw a parallel to the welfare implications of reducing barriers to trade, where free trade is usually found to be efficient from a representative agent perspective, but the gains from trade are distributed unequally, creating winners and losers. In our setting, it means that higher markups are not the problem, but the unequal distribution of the gains from higher growth can be. Policies that aim to directly reduce markups might be detrimental to efficiency and economic growth. Redistributing the gains from innovation in a more equitable way through transfers can, therefore, be a more successful policy than a direct reduction in markups if the aim is to improve the well-being of the average consumer.

4.7 Social Planner's Problem

In the previous sections, we have highlighted the importance of taking dynamic considerations into account when studying the welfare implications of structural changes that occurred over the last few decades and the observed rise in markups. Our model features several inefficiencies that interact, such as markup dispersion, R&D externalities, and business stealing. In Section A.4 of the Appendix, we solve for the constrained and unconstrained social planner's problems dynamically. We derive two main results.[30] First, even though removing all markups from the economy increases welfare significantly, removing dynamic inefficiencies plays a much larger role in the welfare difference between the decentralized and Pareto-optimal allocations.[31] In other words, under-investment in innovation is a more severe problem than market power. This supports our results regarding the effect of increased markups over time on welfare as, in our calibrations, higher markups lead to more innovation overall and dynamic welfare gains. Second, the constrained optimal allocation, where the planner cannot choose positive R&D for large firms in single-superstar industries,[32] delivers a very large share of the welfare difference between the unconstrained optimal and decentralized allocation.

5 Model Validation

Beyond its ability to replicate the (untargeted) inverted-U shape relationship between market concentration and innovation (see Figure III), our estimated model also delivers several predictions that can be tested, especially related to trends in productivity, market concentration, and the labor share. In particular, we show that our calibration correctly predicts the increase in productivity dispersion documented by Barth, Bryson, Davis, and Freeman (2016) and the negative correlation between productivity dispersion and the labor share across industries highlighted in Gouin-Bonenfant (2018). In addition, our model is in line with several facts related to changes in the labor share documented in Autor, Dorn, Katz, Patterson, and Van Reenen (2017). Finally, our model also predicts a U-shape relationship between entry into superstar firms and market concentration that is in line with empirical evidence for US-listed firms (Compustat).

5.1 Increase in Productivity Dispersion

Recent studies emphasize the role played by increased between-firm dispersion in explaining trends in income inequality. For instance, Song, Price, Guvenen, Bloom, and Von Wachter (2018)

[30] A full description of the results for the full sample as well as for both subperiods can be found in Appendix A.4.4.

[31] We find static welfare gains from removing markups comparable to those reported in Baqaee and Farhi (2020) and Edmond, Midrigan, and Xu (2018).

[32] In a decentralized equilibrium, single superstars do not perform R&D as they have no incentive to do so. As a result, implementing the first-best allocation with taxes and subsidies would require a 100% subsidy of R&D by large firms in single-superstar industries. Our constraint is motivated by the severe implementation problems this would entail.

find that two-thirds of the rise in the variance of earnings is due to increased between-firm wage dispersion. In addition, Barth, Bryson, Davis, and Freeman (2016) document a significant rise in the variance of between-firm productivity (log revenue per worker) between 1977 and 2007. In our model, productivity dispersion both within and across industries arises endogenously as the outcome of innovation. In particular, our quantitative exercise allows us to determine how productivity dispersion responds to the structural transition of the economy. Using the same measure as in Barth, Bryson, Davis, and Freeman (2016) (the variance of log revenue per worker), our estimation predicts a rise in productivity dispersion by 33.21% between the early and late subsamples (i.e., from 1976 to 2004) which is very close to the estimated 32.6% increase in Barth, Bryson, Davis, and Freeman (2016) between 1977 and 2007.

5.2 Productivity Dispersion, Value-Added, and the Labor Share

Building on the evidence from Barth, Bryson, Davis, and Freeman (2016), Gouin-Bonenfant (2018) further shows that an exogenous increase in productivity dispersion could explain the observed fall in the labor share in a model with labor market monopsony power. Testing one of the main predictions of his model, he shows, using Canadian data, that the industry-level labor share is negatively correlated with dispersion in productivity. In our model, both labor share and productivity dispersion are endogenously determined. Productivity dispersion within an industry depends on innovation activity whose returns, in turn, shape the productivity distribution. Given the rich structure of our model, we can also compute the correlation between the industry-level labor share and productivity dispersion. In our model, there is a negative association between the industry-level labor share and the dispersion of productivity (TFPQ) in both sub-samples, in line with the findings in Gouin-Bonenfant (2018).

Furthermore, as in Autor, Dorn, Katz, Patterson, and Van Reenen (2017) and Kehrig and Vincent (2017), Gouin-Bonenfant (2018) also documents a negative relationship between (log) firm-level labor share and (log) value-added. We repeat this regression in our model, and obtain a coefficient of -0.0883, which is very close to the value -0.112 that he documents. This means larger firms have a smaller labor share on average.

5.3 Market Concentration and the Labor Share

Using US firm-level data, Autor, Dorn, Katz, Patterson, and Van Reenen (2017) also highlight how the decrease in the labor share has been associated with several dimensions of the rise in market concentration and, in particular, the rise of superstar firms. First, they show that industry sales increasingly concentrate in a small number of superstar firms. In our model, the market share of superstar firms increases by 13.6% between the early and late sub-samples. This result is comparable with those found in Autor, Dorn, Katz, Patterson, and Van Reenen (2017): it is within the range of estimates for the six sectors they report, and closest to the rise in manufacturing. This untargeted change in our model is directly related to the competition between superstars and small firms which we have shown to be decreasing over time and which is the main driver of the observed increase in markups.

Regressing the change in the 4-digit industry labor share on the change in market concentration (measured by the market share of top 4 and top 20 firms as well as the Herfindahl index), they further show that industries in which market concentration rose the most also experienced the sharpest decline in their labor share. Our model delivers the same prediction between our two sub-samples. In particular, focusing on the corresponding two relevant measures of concentration in our model (i.e., top 4 market share and the Herfindahl index), we also find a negative association between the change in market concentration and in the labor share.[33] Interestingly, the estimates from those regressions fall in the ballpark of those reported in Autor, Dorn, Katz, Patterson, and Van Reenen (2017). Our regression based on the top 4 share of sales delivers a coefficient of -0.192 (the estimates in Autor, Dorn, Katz, Patterson, and Van Reenen (2017) range between -0.146 and -0.339). For the Herfindahl index regression, our estimate is -0.696 (between -0.213 and -0.502 in Autor, Dorn, Katz, Patterson, and Van Reenen (2017)).

5.4 Decomposing the Source of the Decrease in the Labor Share

Using our model, we can write the change in the labor share (LS) between the early (subscript e) and late (subscript l) sub-samples as follows:

$$\begin{aligned}\Delta LS &= \sum_{\Theta} \mu_l(\Theta) \left[\sum_{k=1}^{N(\Theta)} l_{k,l}(\Theta) + m_l l_{c,l}(\Theta) \right] \omega_l - \sum_{\Theta} \mu_e(\Theta) \left[\sum_{k=1}^{N(\Theta)} l_{k,e}(\Theta) + m_e l_{c,e}(\Theta) \right] \omega_e \\ &= \sum_{\Theta} \mu_l(\Theta) \left[\sum_{k=1}^{N(\Theta)} LS_{k,l}(\Theta)\Upsilon_{k,l}(\Theta) + LS_{c,l}(\Theta)\Upsilon_{c,l}(\Theta) \right] \\ &\quad - \sum_{\Theta} \mu_e(\Theta) \left[\sum_{k=1}^{N(\Theta)} LS_{k,e}(\Theta)\Upsilon_{k,e}(\Theta) + LS_{c,e}(\Theta)\Upsilon_{c,e}(\Theta) \right] \qquad (43)\end{aligned}$$

where $\Upsilon_{k,t}(\Theta)$ is the market share of firm k in industry Θ at time t and $\Upsilon_{c,t}(\Theta)$ is the market share of the fringe. We can further decompose the change in the labor share as:

$$\begin{aligned}\Delta LS = &\left(\sum_{\Theta} \mu_l(\Theta) - \sum_{\Theta} \mu_e(\Theta) \right) \left[\sum_{k=1}^{N(\Theta)} LS_{k,l}(\Theta)\Upsilon_{k,l}(\Theta) + LS_{c,l}(\Theta)\Upsilon_{c,l}(\Theta) \right] \\ &+ \sum_{\Theta} \mu_e(\Theta) \left\{ \left[\sum_{k=1}^{N(\Theta)} LS_{k,l}(\Theta)\Upsilon_{k,l}(\Theta) + LS_{c,l}(\Theta)\Upsilon_{c,l}(\Theta) \right] - \left[\sum_{k=1}^{N(\Theta)} LS_{k,l}(\Theta)\Upsilon_{k,e}(\Theta) + LS_{c,l}(\Theta)\Upsilon_{c,e}(\Theta) \right] \right\} \\ &+ \sum_{\Theta} \mu_e(\Theta) \left\{ \left[\sum_{k=1}^{N(\Theta)} LS_{k,l}(\Theta)\Upsilon_{k,e}(\Theta) + LS_{c,l}(\Theta)\Upsilon_{c,e}(\Theta) \right] - \left[\sum_{k=1}^{N(\Theta)} LS_{k,e}(\Theta)\Upsilon_{k,e}(\Theta) + LS_{c,e}(\Theta)\Upsilon_{c,e}(\Theta) \right] \right\}\end{aligned}$$

[33] In particular, we compute the change in market concentration and in the labor share for industries with the same number of firms and the same productivity step distribution in the early and late subsamples.

where the first term captures the change in the aggregate labor share due to the change in the distribution of industry states $\mu(\Theta)$, the second term captures the change in the aggregate labor share due to within industry reallocation of sales, and the third term captures the change in the aggregate labor share due to changes in firm-level labor shares.

Our model predicts that almost all of the 5.65% decrease in the aggregate labor share between both sub-samples is due to changes in market share reallocation (2.17%) and changes in firm-level labor shares (3.48%). Even though Autor, Dorn, Katz, Patterson, and Van Reenen (2017) use a different decomposition, our results are comparable, in that they also find within-industry market share reallocation to be responsible for a large part of the decrease in the aggregate labor share.

5.5 Emergence of Superstars and Market Concentration

Besides being able to match the (targeted) rate of entry of new businesses, our model also has some additional predictions regarding the rate at which small firms become superstars. Our model generates an industry-specific entry rate of superstars. In particular, our estimated model implies that entry into superstar firms follows a U-shaped relationship with market concentration. This prediction of the model can be tested using our Compustat sample. If we consider the entry of new firms into the Compustat sample as a proxy for the emergence of new superstars, then we find a U-shaped relationship between industry-level superstar entry rate and market concentration (HHI). Since this is an imperfect proxy, we do not view the empirical relationship that we find as conclusive. However, it would be worthwhile for future research exploring the emergence of superstars to look further into this relationship.

6 Robustness of Quantitative Results

Our model delivers some sharp predictions with sizable growth and welfare implications. It is important to test whether these findings are driven by our modeling assumptions. In this section, we change some of our modeling assumptions, re-estimate the model, and repeat the counterfactual experiments.

6.1 Exogenous vs. Endogenous Productivity Growth

In the benchmark model, productivity growth along a balanced growth path equilibrium is completely driven by the endogenous innovation strategies of the firms in the economy. The implicit assumption is that there are no other mechanisms at play that might change total factor productivity between the two time periods. If there are other sources of TFP growth, such as a reduction in unmodeled sources of inefficiency, this would mean that we would be attributing too much importance to the role of innovation in the determination of economic growth. To see how much our quantitative results would change if we attribute a smaller role to innovation, we introduce exogenous productivity growth to the model, where we replace the production technology given in equation (5) with

$$y_{ijt} = a_t q_{ijt} l_{ijt} \tag{44}$$

with $a_t = a_0 e^{g_a t}, \forall t$. Consequently, the output growth rate along a BGP equilibrium becomes

$$g = g_a + \sum \left[p_{lit}(\Theta) \ln(1+\lambda) \right] \mu_t(\Theta) \tag{45}$$

By picking a higher value for g_a, we can reduce the importance of innovation. To be conservative, we conduct an exercise where only 50% of the observed growth is attributed to endogenous firm effort to improve productivity. To do so, we assign $g_a = g_{data}/2$ and re-estimate the model for the early and late periods. Estimation results can be seen in Table B11. Next, we repeat the counterfactual exercises in Section 4.6 with the new estimated equilibria, and the results are displayed in Table B12. Compared to the baseline model, the counterfactual growth changes are milder. However, the consumption-equivalent welfare changes are diminished only slightly. For instance, the experiment in which we reset the value of the relative productivity of the competitive fringe, ζ, still results in a sizeable welfare loss of 3.25%, as opposed to 4.52% in the baseline. Table B13 repeats the welfare decomposition exercise in Section 4.6.4, which shows that although the dynamic loss from reduced growth is smaller, the static gains from reduced markups are also lower under the new estimation. Hence, the dynamic losses still dominate the static gains, and our results regarding the welfare effect of reduced markups remain robust to attributing a smaller role to innovation in productivity growth.

6.2 Lower Elasticity of Intertemporal Substitution

The period utility function in the baseline model is the natural logarithm, which implies an elasticity of intertemporal substitution (EIS) of 1. This is done for tractability, as it allows an intuitive decomposition of welfare, and the computation of stationary and non-stationary equilibria are simpler since the discount term in the firm value functions, $r - g$, is equal to the discount factor of the representative household ρ. Conducting a survey of 1429 studies, Havranek, Horvath, Irsova, and Rusnak (2015) report that the average estimate of the EIS for the US is 0.594, with a standard error of 0.036. Since this number is quite low in comparison, one might worry that our choice regarding the preferences might exaggerate the dynamic welfare gains from increased growth. To establish the robustness of our results, we change the preferences given in equation (1) to

$$U = \int_0^\infty e^{-\rho t} \frac{C_t^{1-\theta}}{1-\theta} \, dt \tag{46}$$

where θ is the constant relative risk aversion parameter. To remain conservative, we impose $\theta = 2$, which implies an EIS of 0.5; lower than the mean estimate for the US. We repeat the estimation with the new preferences, and the results are displayed in Table B14. To remain consistent with the baseline, we change the value of ρ to 0.02, so that the implied real interest rate under 2% growth is the same across estimations. Using the new estimation results, we repeat the counterfactual exercises in Section 4.6, and report the results in Table B15. In general, the results remain quite comparable, where the growth and welfare implications are higher for three of the five mechanisms, and lower for the remaining two. In particular, the dynamic losses from welfare still dominate the static gains in the experiment where markups are reduced to their early period level. The total welfare impact is still negative at -2.58%. We conclude that our choice of utility function does not drive our results.

6.3 Capital Accumulation

In the baseline model, production only uses labor as an input. This is done for tractability and ease of comparison to other work on endogenous markups discussed earlier. In this section, we show that extending our model to include endogenous physical capital accumulation does not change the results significantly. The details of this extended model can be found in Appendix A.5.

Unlike the previous two robustness checks, we do not re-estimate the model. This is because all targeted moments except R&D intensity remain exactly the same after the introduction of capital accumulation. However, consumption equivalent welfare changes are affected, since output is now also used for investment in physical capital. Table B16 presents the results of repeating the counterfactual exercises in Section 4.6. It is seen that the direction of welfare changes are maintained. In particular, reverting the value of ζ to its early period value still results in a welfare loss of 2.32%. We conclude that abstracting away from capital accumulation does not drive our results.

6.4 Sensitivity to Markup Estimates

In our baseline estimation, we use the sales-weighted average markup estimates from De Loecker, Eeckhout, and Unger (2019) as an estimation target. Concerns have been raised regarding whether the cost- or sales-weighted average markup should be the focus of attention, given that it is the cost-weighted average markup that summarizes the distortions in allocative efficiency in commonly-used theoretical frameworks.[34] Despite our choice to target the sales-weighted average markup, our model delivers lower values for the cost-weighted average markup in all three samples.[35] This is consistent with what is observed in the data, and our numbers for the cost-weighted average markup are found to be close to, but slightly higher than, the cost-weighted markups reported in Edmond, Midrigan, and Xu (2018).

To further establish the robustness of our results, we re-estimate the model using cost-weighted average markup targets obtained from Edmond, Midrigan, and Xu (2018) to hit them precisely, and

[34] See Edmond, Midrigan, and Xu (2018) among others.

[35] The sales- vs. cost-weighted average markup in the estimated economies are 1.3462 vs. 1.2436 in the whole sample, 1.2801 vs. 1.2063 in the early sub-sample, and 1.4195 vs. 1.2945 in the late sub-sample.

repeat our counterfactual experiments. Table B17 displays the results of re-estimation, whereas Tables B18, B19, and B20 repeat the counterfactual experiments in Tables IV, V, and VI. The quantitative magnitudes are found to be similar to those in the baseline analysis. One major difference worth highlighting is the change in inferred distributional consequences: While the rise in markups is still welfare-enhancing for a representative agent, the welfare loss of hypothetical individuals who solely rely on labor income is found to be 1.83%, which is higher than the 0.70% figure found in the baseline estimation.

More recently, Bond, Hashemi, Kaplan, and Zoch (2020) have raised concerns over the consistency of average markup estimates obtained using the De Loecker and Warzynski (2012) methodology, which both De Loecker, Eeckhout, and Unger (2019) and Edmond, Midrigan, and Xu (2018) follow. This raises the question whether our results hinge on markup estimates that might potentially be biased. Fortunately, given the rich structure of our setting that ties several aggregate moments together, we can still estimate our model even if we do not explicitly target any markup-based moments. As seen in the decomposition exercise in Table IV, the relative productivity of small firms ζ which governs the average markup, is also responsible for the majority of the change in the labor share. Therefore, even if we do not explicitly target the average markup, the labor share can be used to identify the value of ζ, which in turn can identify the implied average markup through indirect inference. Following this line of reasoning, we re-estimate the model after dropping the average markup and the standard deviation of markups from the set of targeted moments.[36] Table B21 displays the results of re-estimation, whereas Tables B22, B23, and B24 repeat the counterfactual experiments in Tables IV, V, and VI. The indirect inference suggests that the average markup has increased by 8.69% of its value between the early and late periods, which is similar to the 10.9% increase in the baseline. However, the average markup level in the early period is found to be lower at 1.21, as opposed to the 1.28 estimate in De Loecker, Eeckhout, and Unger (2019). Despite the fall in the estimated level of the average markup, the positive welfare effects associated with the increase in markups are found to be amplified instead of diminished: the welfare impact of keeping the relative productivity of small firms ζ the same as in the early sub-sample would decrease welfare by 8.58% as opposed to 4.52% in the baseline. Not relying on markup estimates obtained through the De Loecker and Warzynski (2012) methodology, if anything, strengthens our results.

6.5 Non-Stationary Dynamics and the Welfare Costs of the Transition

The welfare results in Section 4.6 are obtained by comparing the actual and hypothetical late sub-sample steady-states. One might be concerned that a comparison across steady-states might be insufficient to capture the full differences in welfare across the separate scenarios, since some effects of the changes are instantaneous (e.g., the effects on static product market competition), whereas the rest take more time to manifest in full (e.g., the changes to the industry-state distribution $\mu_t(\Theta)$.) To

[36] This is feasible since our model is already overidentified (9 parameters vs. 11 targets.)

capture the welfare differences from the point of view of an agent at $t = 0$, one needs to take the transitional dynamics in non-stationary equilibria into account.

Computing a non-stationary equilibrium of our proposed model is more complex than in standard macroeconomic models. The relevant state variables to keep track of are (1) the average log productivity level of industry leaders Q_t, (2) the mass of small firms m_t, and (3) the industry-state distribution $\mu_t(\Theta)$. Given our choices for $\bar{n}$ and $\bar{N}$, this means solving for a non-stationary equilibrium requires finding the complete time paths of 86 continuous state variables under rational expectations, which might seem daunting at first.[37] Despite the complexity, the tractability of our framework and our choice to use a continuous-time setting render the computation of non-stationary equilibria feasible. This is accomplished without any deviation from the assumption of rational expectations. For brevity, the full details including the algorithm are relegated to Section A.6 of the Appendix.

In Section A.6.2 of the Appendix, we conduct the equivalents of the decomposition experiments, where we require all economies to start from the early steady-state, and converge over time to the new steady-state. The welfare numbers are recomputed taking the full transition into account, and are presented in Table A3. The welfare difference between the realized transition and the counterfactual of remaining in the early steady-state in perpetuum is now calculated to be 7.32%, as opposed to 8.73% in the baseline analysis. This shows that taking the non-stationary dynamics into account does not significantly change the calculated welfare impact of the structural transition in the US. The structural change that contributes the most to the loss in welfare is once again the decline in the R&D efficiency of large firms, the impact of which is calculated as 15.28%, which is quite close to the 17.88% found in the baseline. Finally, we find that the dynamic gains in welfare associated with higher markups still dominate the static losses in efficiency, but the total welfare gain is smaller. This is because the increase in aggregate productivity growth takes time to fully manifest due to the time it takes for the industry-state distribution $\mu_t(\Theta)$ to converge to its stationary value, whereas the static losses from a less productive competitive fringe are instantaneous.

7 Conclusion

We offer a unified framework to study the economic mechanisms that underlie the structural transformation in the US in the last four decades. To this end, we develop and estimate a dynamic model in which firms strategically compete with other firms and dynamically choose their innovation strategies. This approach departs from much of the previous literature on endogenous growth studying markups, competition, and innovation, in which researchers use models featuring degenerate firm distributions with Bertrand competition in the product market. Our model can account for an arbitrarily high number of firms in an industry, with endogenous entry and exit, and it can generate non-degenerate sale, employment, markup, and innovation distributions within industries. It also

[37]For comparison, computing non-stationary equilibria in the canonical neoclassical growth model requires finding the complete time path of a single continuous state variable, the capital stock K_t.

can generate endogenous industry dynamics in rich detail, and replicate the observed inverted-U relationship between innovation and competition both within and across industries, which is crucial to discipline the aggregate relationship between markups and economic growth, and consequently, the dynamic gains in welfare.

We use the estimated model to gauge whether increasing markups boost or hinder aggregate innovation and economic growth. The findings reveal that while the increase in average markups causes a significant static welfare loss, this loss is overshadowed by the dynamic welfare gains from increased innovation in response to higher profit opportunities. Overall, our results suggest that the dynamic effects of increasing market concentration on innovation and productivity should not be ignored when trying to understand the transformation in the US in the last four decades; and the rise of superstar firms and markups is not necessarily detrimental to welfare.

If the increase in markups and market concentration are not the culprits behind the productivity slowdown, what is? Our model's answer to the question is the increase in the costs of innovation. If the costs of innovation of small firms which are trying to become superstars were set back to their earlier level, it would account for roughly half of the observed decline in productivity growth. A similar experiment for the cost of innovation of superstars yields another significant effect that can explain more than 100% of the observed decline. These results point towards "the ideas are getting harder to find" hypothesis studied in Gordon (2012) and Bloom, Jones, Van Reenen, and Webb (2020).

What are the implications for optimal policy? In our analysis, we find that although higher markups are not detrimental to welfare from an aggregate perspective, it can be so for individuals who rely primarily on labor income. Increasing markups are beneficial for dynamic efficiency, yet they also increase the inequality between groups that rely on labor income vs. capital income. This suggests that the optimal amount of transfer payments must increase if the goal is to improve the economic well-being of the average consumer. This draws a parallel to the welfare implications of reducing barriers to trade: reducing markups is inefficient, but redistribution might be necessary for political support, since the gains from higher growth are not shared equally.

We view our research as a starting point for understanding the aggregate implications of strategic interactions among heterogeneous firms. While our model shows the importance of using a model with non-degenerate firm distribution and realistic product market competition to gauge the welfare impact of rising markups, it can also be used in different settings where heterogeneous firms compete staticly in the product market, and dynamically to improve their relative market shares. The model is highly tractable and could be easily extended to study the implications of various kinds of government policies, such as size-dependent R&D tax credits, subsidizing new business entry, and corporate taxation. We expect future studies along these lines to be both promising and fruitful.

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Online Material for "Are Markups Too High? Competition, Strategic Innovation, and Industry Dynamics" (Not for Publication)

A Appendix

A.1 Growth rate

This section derives the growth rate of the economy.

$$
\begin{aligned}
\ln(Y_t) &= \int_0^1 \frac{\eta}{\eta-1} ln \left[\sum_{i=1}^{N_{jt}} y_{ijt}^{\frac{\eta-1}{\eta}} + y_{cjt}^{\frac{\eta-1}{\eta}} \right] dj \\
&= \int_0^1 \frac{\eta}{\eta-1} \ln \left[y_{cjt}^{\frac{\eta-1}{\eta}} \left(\sum_{i=1}^{N_{jt}} \left(\frac{y_{ijt}}{y_{cjt}} \right)^{\frac{\eta-1}{\eta}} + 1 \right) \right] dj \\
&= \int_0^1 \left[\ln(y_{cjt}) + \frac{\eta}{\eta-1} \ln \left(\sum_{i=1}^{N_{jt}} \left(\frac{y_{ijt}}{y_{cjt}} \right)^{\frac{\eta-1}{\eta}} + 1 \right) \right] dj \\
&= \int_0^1 \left[\ln \left(\frac{1}{\omega_t} \frac{q_{cjt}}{\sum_{i=1}^{N_{jt}} \left(\frac{y_{ijt}}{y_{cjt}} \right)^{\frac{\eta-1}{\eta}} + 1} \right) + \frac{\eta}{\eta-1} \ln \left(\sum_{i=1}^{N_{jt}} \left(\frac{y_{ijt}}{y_{cjt}} \right)^{\frac{\eta-1}{\eta}} + 1 \right) \right] dj \\
&= \int_0^1 \left[\ln \left(\frac{q_{cjt}}{\omega_t} \right) + \frac{1}{\eta-1} \ln \left(\sum_{i=1}^{N_{jt}} \left(\frac{y_{ijt}}{y_{cjt}} \right)^{\frac{\eta-1}{\eta}} + 1 \right) \right] dj \\
&= \int_0^1 \ln \left(\frac{q_{cjt}}{\omega_t} \right) dj + \sum f_t(\Theta)\mu_t(\Theta) \qquad (47) \\
\ln(Y_{t+\Delta t}) - \ln(Y_t) &= -\ln(\omega_{t+\Delta t}) + \ln(\omega_t) + \sum p_{lit}(\Theta)\Delta t \ln(1+\lambda)\mu_t(\Theta) \\
&\quad + \sum_{\Theta} \sum_{\Theta'} \left[f_t(\Theta') - f_t(\Theta) \right] p_t(\Theta, \Theta')\mu_t(\Theta)\Delta t + o(\Delta t) \qquad (48) \\
g_t &= -g_{\omega,t} + \sum p_{lit}(\Theta) \ln(1+\lambda)\mu_t(\Theta) \\
&\quad + \sum_{\Theta} \sum_{\Theta'} \left[f_t(\Theta') - f_t(\Theta) \right] p_t(\Theta, \Theta')\mu_t(\Theta) \qquad (49)
\end{aligned}
$$

A.2 Proposition 1

Let $\hat{\Theta}$ denote the set of all industry-states Θ. Let $h : \hat{\Theta} \to \mathbb{R}$ be a function. Then:

$$
\begin{aligned}
\mathbb{E}\left[\sum_{\Theta'} p(\Theta, \Theta')(h(\Theta') - h(\Theta)) \right] &= \sum_{\Theta} \sum_{\Theta'} p(\Theta, \Theta')(h(\Theta') - h(\Theta))\mu(\Theta) \\
&= \sum_{\Theta} \sum_{\Theta'} p(\Theta, \Theta')h(\Theta')\mu(\Theta) - \sum_{\Theta} \sum_{\Theta'} p(\Theta, \Theta')h(\Theta)\mu(\Theta) \\
&= \sum_{\Theta'} h(\Theta') \sum_{\Theta} p(\Theta, \Theta')\mu(\Theta) - \sum_{\Theta} h(\Theta) \sum_{\Theta'} p(\Theta, \Theta')\mu(\Theta) \\
&= \sum_{\Theta'} h(\Theta')\mu(\Theta') - \sum_{\Theta} h(\Theta)\mu(\Theta) \\
&= \mathbb{E}\left[h(\Theta') \right] - \mathbb{E}\left[h(\Theta) \right] \\
&= 0
\end{aligned}
$$

A.3 Estimation Procedure

We pick the maximum number of superstars in an industry $\bar{N} = 4$ and the maximum number of productivity steps between any two superstar firms $\bar{n} = 5$, which delivers 84 unique industry states Θ.[38] The model has ten parameters to be determined: $\rho, \lambda, \eta, \chi, \nu, \zeta, \phi, \epsilon, \psi, \tau$. The consumer discount rate ρ is set to 0.04, which implies a real interest rate of 6% when the growth rate is 2%.[39] The remaining nine parameters are structurally estimated following a simulated method of moments approach. In this section, we discuss the data moments we use to discipline the parameter values, and provide the relevant data sources for each of these moments. The associated Jacobian matrix is presented in Table B10.

1. **Growth rate:** To discipline output growth in our model, we obtain the annual growth rate of real GDP per capita from the US Bureau of Economic Analysis, and calculate the geometric averages for each sub-sample.

2. **Labor share:** We obtain the labor share estimates from Karabarbounis and Neiman (2013); in particular the time series for corporate labor share (OECD and UN). For capital share, we rely on the data from Barkai (2020). For both time series, we calculate the averages across all years for each sub-sample. In our baseline model, there is no capital. Therefore, the model-generated labor share $\omega L = wL/Y$ corresponds to the share of labor income among labor income plus profits. For comparability, we multiply this number by $(1 - \kappa)$ where κ is the (exogenous) capital share, following Akcigit and Ates (2019b).[40]

3. **R&D intensity:** The data for aggregate R&D intensity is taken from the National Science Foundation, who report total R&D expenditures divided by GDP.

4. **Level and dispersion of markups:** To discipline markups, we target the sales-weighted average markup and the sales-weighted standard deviation of markups found in De Loecker, Eeckhout, and Unger (2019). In Section 6.4, we re-estimate the model using cost-weighted markups from Edmond, Midrigan, and Xu (2018), and the results are found to be similar. Motivated by Bond, Hashemi, Kaplan, and Zoch (2020), we also conduct another re-estimation that does not rely on any markup-based moments obtained through the De Loecker and Warzynski (2012) methodology. This, if anything, strengthens our results.

5. **Relationship between firm innovation and relative sales:** As discussed earlier, replicating the observed inverted-U relationship between competition and innovation is key to discipline the counterfactual implications of the model regarding economic growth and social welfare. In order to achieve this, we require the model-generated relationship between firm innovation and relative sales to

[38]The results do not significantly depend on these choices. The estimated value of λ adjusts to absorb the choice of a different $\bar{n}$. The relative productivity of the competitive fringe ζ adjusts to absorb the changes in $\bar{N}$.

[39]We target a relatively high real interest rate to remain conservative. For instance, a lower real interest rate of 4% would halve the implied discount rate to $\rho = 0.02$. This would double the welfare contribution of the output growth rate relative to that from the initial consumption level, significantly amplify the dynamic welfare gains, and further strengthen our findings.

[40]In Section 6.3, we explicitly add physical capital to the model, and calculate labor share without any correction, and the labor share in the model becomes $(1 - \kappa)\omega L$, justifying this correction.

be the same as in the data. This is done through indirect inference, where we target the coefficients for the linear and quadratic terms of the regression in column 1 of Table II Panel A after normalization. Innovation in the model is measured as the Poisson arrival event of quality improvement, whereas it is measured as average citations in the data. We normalize both by subtracting their means and dividing by the standard deviation.

6. **Average profitability:** In the model, average profitability is calculated as static profit flow minus R&D expenses divided by sales. In the data, it is defined as operating income before depreciation divided by sales (OIBDP/SALE in Compustat.)

7. **Level and dispersion of leader quality:** We target the average relative quality of the leader in an industry, and its standard deviation across all industries. In the model, quality is known. In the data, we proxy quality by calculating the stock of past patent citations. The relative quality of the leader is defined as the quality of the leader divided by the sum of the qualities of the top four firms in an industry (SIC4 in the data.)

8. **Firm entry:** In our model, firm entry rate is defined as the entry rate of new small firms. We obtain the data counterpart – the entry rate of new businesses – from the Business Dynamics Statistics (BDS) database compiled by the US Census.

A.4 Social Planner's Problem

There are several distortions in the decentralized equilibrium of the economy in our model. On the static side, the superstar firms use their market power to increase their profits through charging positive markups. On the dynamic side, the superstars, small firms, as well as entrepreneurs all ignore their effects on the rest of the economy: the positive contribution of their innovation to productivity growth, as well as the negative contribution of their investments that result in business-stealing. Due to these reasons, it is useful to solve the social planner's problem so that we can compare the inefficient decentralized equilibrium allocation to the Pareto-efficient allocation. In this section, we solve the problem in steps, and undertake the comparison.

A.4.1 The Complete Social Planner's Problem

The goal of the social planner is to maximize the lifetime utility of the representative household subject to technological constraints. Given the initial conditions, $\mu_0(\Theta), m_0$, and Q_0, the full problem can be stated as follows:

$$\max_{[[\{l_{ijt},z_{ijt}\}_{i=1}^{N_{jt}},[l_{kcjt},X_{kjt}]_{k=0}^{m_t}]_{j=0}^{1},e_t]_{t=0}^{\infty}} \int_0^\infty e^{-\rho t}\ln(C_t)dt, \text{ such that} \tag{50}$$

$$C_t + R_t Y_t \leq Y_t \tag{51}$$

$$R_t = \int \left(\sum_{i=1}^{N_{jt}} \chi z_{ijt}^{\phi} + \int \nu X_{kjt}^{\epsilon} dk \right) dj + \psi e_t^2 \tag{52}$$

$$\ln(Y_t) = \int_0^1 \ln(y_{jt}) dj \tag{53}$$

$$y_{jt} = \left(\sum_{i=1}^{N_{jt}} y_{ijt}^{\frac{\eta-1}{\eta}} + \tilde{y}_{cjt}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{54}$$

$$\tilde{y}_{cjt} = \int y_{kcjt} dk \tag{55}$$

$$y_{ijt} = q_{ijt} l_{ijt} \tag{56}$$

$$y_{kcjt} = q_{cjt} l_{kcjt} \tag{57}$$

$$\int \left(\sum_{i=1}^{N_{jt}} l_{ijt} + \int l_{kcjt} dk \right) dj \leq L = 1 \tag{58}$$

$$q_{jt}^{leader} = \max\{q_{1jt}, ..., q_{N_{jt}jt}\} \tag{59}$$

$$q_{cjt} = \zeta q_{jt}^{leader} \tag{60}$$

$$\{q_{1jt}, ..., q_{N_{jt}jt}\} = \left\{ q_{jt}^{leader}, \frac{q_{jt}^{leader}}{(1+\lambda)^{\vec{n}_{jt}(1)}}, ..., \frac{q_{jt}^{leader}}{(1+\lambda)^{\vec{n}_{jt}(N_{jt}-1)}} \right\} \tag{61}$$

$$\Theta_{jt} = (N_{jt}, \vec{n}_{jt}) \tag{62}$$

$$Q_t = \int \ln(q_{jt}^{leader}) dj \tag{63}$$

$$\frac{\dot{Q}_t}{Q_t} = \ln(1+\lambda) \sum_{\Theta} p_{lit}(\Theta)\mu_t(\Theta) \tag{64}$$

$$\dot{\mu}_t(\Theta) = \sum_{\Theta'} p_t(\Theta', \Theta)\mu_t(\Theta') - \sum_{\Theta'} p_t(\Theta, \Theta')\mu_t(\Theta) \tag{65}$$

$$\sum_{\Theta} \mu_t(\Theta) = 1 \tag{66}$$

$$\dot{m}_t = e_t - \tau m_t \tag{67}$$

where equation (51) is the resource constraint, and equation (52) is total R&D and business creation investment as a share of GDP. Equations (53) and (54) are respectively the final good and industry output production functions. Equation (55) is the production of the competitive fringe in industry j at time t. Equations (56) and (57) are the production functions of superstars and small firms. Equation (58) is the aggregate labor feasibility constraint. Equation (59) defines the productivity of industry j leader at time t (q_{jt}^{leader}) as the highest firm-level productivity in the industry. Equation (60) imposes that the productivity of each small firm in the competitive fringe in industry j is a fraction ζ of the industry leader's productivity at any time. Equation (61) defines the vector of productivity in industry j at time t for an industry with N_{jt} superstar firms where $\vec{n}_{jt}$ is the vector of productivity steps between each firm in the industry and the leader. Θ_{jt} in equation (62) is the state of industry j at time t, which can be summarized by the number of superstars in the industry (N_{jt}) and the number of productivity steps between each firm and the industry leader $\vec{n}_{jt}$. Q_t is the average (log) productivity of leaders across industries at time t (equation (63)). The growth rate of Q_t is given by equation (64) where $\mu_t(\Theta)$ is the mass of industries in state Θ and $p_{lit}(\Theta)$ is the arrival rate at which one of the industry leaders innovates. This arrival rate in turn depends on the mass of small firms and innovation policies. Equation (65) is the law of motion of the industry distribution. The first term corresponds to inflows into state Θ while the second term represents outflows. $p(\Theta, \Theta')$ is the instantaneous flow from state Θ to Θ' which depends on the innovation policies and the mass of small firms in the economy. Equation (66) states that the mass of industries has to sum to one. Finally, equation (67) is the law of motion of the mass of small firms.

The social planner maximizes welfare by choosing labor allocation to every superstar firms i in industry j at time t (l_{ijt}) and to every small firms k in industry j at time t (l_{kcjt}). The social planner also chooses R&D innovation policies for every superstar firms (z_{ijt}) and small firms (X_{kjt}) as well as the entry policy of entrepreneurs (e_t). Since small firms within the fringe of a given industry are symmetric, we can write the total labor allocation to small firms in industry j at time t as $l_{cjt} = m_t l_{kcjt}$ and the Poisson rate of innovation by small firms as $X_{jt} = m_t X_{kjt}$.

This is a large problem to solve. However, it can be split into a static problem and a dynamic problem. First, note that the final good and labor feasibility constraints (equations (51) and (58)) must bind with equality. This is because the preferences of the representative household are increasing in consumption C_t, and there is no disutility of labor. As a consequence, given the productivity distribution $[\{q_{ijt}\}_{i=1}^{N_{jt}}, q_{cjt}]_{j=0}^{1}$, the social planner's optimal solution must maximize total output Y_t for all t, subject to the production technologies outlined in equations (53) to (57) and the labor feasibility constraint (58). We solve this static output maximization problem in the next subsection.

A.4.2 Static Output Maximization

Given the productivity distribution $[\{q_{ijt}\}_{i=1}^{N_{jt}}, q_{cjt}]_{j=0}^{1}$, the social planner's static (log-)output maximization problem at time t can be stated as follows:

$$\max_{[\{l_{ijt}\}_{i=1}^{N_{jt}}, l_{cjt}]_{j=0}^{1}} \quad \int_0^1 \frac{\eta}{\eta-1} \ln\left(\sum_{i=1}^{N_{jt}} (l_{ijt}q_{ijt})^{\frac{\eta-1}{\eta}} + (l_{cjt}q_{cjt})^{\frac{\eta-1}{\eta}}\right) dj, \text{ such that} \tag{68}$$

$$\int_0^1 \left(\sum_{i=1}^{N_{jt}} l_{ijt} + l_{cjt}\right) dj = 1 \tag{69}$$

This delivers the first order conditions

$$\frac{q_{ijt}^{\frac{\eta-1}{\eta}} l_{ijt}^{-\frac{1}{\eta}}}{\sum_{i=1}^{N_{jt}} (l_{ijt}q_{ijt})^{\frac{\eta-1}{\eta}} + (l_{cjt}q_{cjt})^{\frac{\eta-1}{\eta}}} = \omega_t, \forall i \in \{1, ..., N_{jt}\}, \forall j \in [0,1] \tag{70}$$

$$\frac{q_{cjt}^{\frac{\eta-1}{\eta}} l_{cjt}^{-\frac{1}{\eta}}}{\sum_{i=1}^{N_{jt}} (l_{ijt}q_{ijt})^{\frac{\eta-1}{\eta}} + (l_{cjt}q_{cjt})^{\frac{\eta-1}{\eta}}} = \omega_t, \forall j \in [0,1] \tag{71}$$

where ω_t is the Lagrange multiplier associated with the labor feasibility constraint (69). To find the exact labor allocations, first note the following:

$$\omega_t \left(\sum_{i=1}^{N_{jt}} l_{ijt} + l_{cjt} \right) = \frac{\sum_{i=1}^{N_{jt}} (l_{ijt} q_{ijt})^{\frac{\eta-1}{\eta}} + (l_{cjt} q_{cjt})^{\frac{\eta-1}{\eta}}}{\sum_{i=1}^{N_{jt}} (l_{ijt} q_{ijt})^{\frac{\eta-1}{\eta}} + (l_{cjt} q_{cjt})^{\frac{\eta-1}{\eta}}} = 1 \tag{72}$$

$$\int_0^1 \omega_t \left(\sum_{i=1}^{N_{jt}} l_{ijt} + l_{cjt} \right) dj = \int_0^1 1 dj \tag{73}$$

$$\omega_t = 1 \tag{74}$$

The first equation obtained by using equations (70) and (71). The last step uses the labor feasibility constraint (69). In turn, plugging $\omega_t = 1$ into the first equation delivers:

$$\sum_{i=1}^{N_{jt}} l_{ijt} + l_{cjt} = 1, \forall j \in [0,1] \tag{75}$$

This equation establishes that the total labor allocated to each industry j is always equal. Next, using equations (70) and (71), we establish:

$$\frac{l_{ijt}}{l_{kjt}} = \left(\frac{q_{ijt}}{q_{kjt}} \right)^{\eta-1}, \forall i, k \in \{1, ..., N_{jt}\}, \forall j \in [0,1] \tag{76}$$

$$\frac{l_{ijt}}{l_{cjt}} = \left(\frac{q_{ijt}}{q_{cjt}} \right)^{\eta-1}, \forall i \in \{1, ..., N_{jt}\}, \forall j \in [0,1] \tag{77}$$

Combined with (75), we have:

$$l_{ijt} = \frac{1}{\sum_{k=1}^{N_{jt}} \left(\frac{q_{kjt}}{q_{ijt}} \right)^{\eta-1} + \left(\frac{q_{cjt}}{q_{ijt}} \right)^{\eta-1}}, \forall i \in \{1, ..., N_{jt}\}, \forall j \in [0,1] \tag{78}$$

$$l_{cjt} = \frac{1}{\sum_{k=1}^{N_{jt}} \left(\frac{q_{kjt}}{q_{cjt}} \right)^{\eta-1} + 1}, \forall j \in [0,1] \tag{79}$$

Table A1: Social Planner's Problem: Static Welfare Gains

	Whole sample	Early sub-sample	Late sub-sample
output DE	0.824	0.774	0.801
output SPP	1.033	1.006	1.028
CEWC	25.37%	30.01%	28.36%

Notes: This table reports the static gains from removing all markups in each of our samples. The first row shows the level of initial output in the decentralized equilibrium (DE). The second line reports initial output when all markups are removed (using the decentralized equilibrium industry distribution). The third row displays the static consumption-equivalent welfare gains from removing all markups.

Table A2: Social Planner's Problem: Dynamic Welfare Gains

	Whole sample			Early sub-sample			Late sub-sample		
	DE	SPP (unconstrained)	SPP ($z_1 = 0$)	DE	SPP (unconstrained)	SPP ($z_1 = 0$)	DE	SPP (unconstrained)	SPP ($z_1 = 0$)
growth rate	0.022	0.056	0.052	0.024	0.058	0.055	0.020	0.055	0.051
initial output	0.824	1.009	1.021	0.774	1.000	1.013	0.801	1.004	1.011
CEWC		115.02%	97.61%		129.39%	82.76%		122.67%	103.02%

Notes: This table reports the results of the unconstrained and constrained social planner's problems for the full sample and for both sub-periods. For each sample, the first column reports the growth rate and initial output in the decentralized equilibrium (DE). The second column shows the results for the unconstrained social planner's problem (SPP) for the growth rate, initial output, and the consumption-equivalent welfare gain compared to the decentralized equilibrium. The third column displays the same information for the constrained social planner in which large firms in single-superstar industries perform no R&D.

This concludes finding the optimal labor allocation that maximizes output. We plug in the optimal solution into the production function to calculate the implied log-output:

$$\begin{aligned}
\ln(Y_t) &= \int_0^1 \frac{\eta}{\eta-1} \ln\left[\sum_{k=1}^{N_{jt}} y_{kjt}^{\frac{\eta-1}{\eta}} + y_{cjt}^{\frac{\eta-1}{\eta}}\right] dj \\
&= \int_0^1 \ln(y_{cjt}) + \frac{\eta}{\eta-1} \ln\left[\sum_{k=1}^{N_{jt}} \left(\frac{y_{kjt}}{y_{cjt}}\right)^{\frac{\eta-1}{\eta}} + 1\right] dj \\
&= \int_0^1 \ln\left[\frac{q_{cjt}}{\sum_{k=1}^{N_{jt}} \left(\frac{q_{kjt}}{q_{cjt}}\right)^{\eta-1} + 1}\right] dj + \frac{\eta}{\eta-1} \int_0^1 \ln\left[\sum_{k=1}^{N_{jt}} \left(\frac{q_{kjt}}{q_{cjt}}\right)^{\eta-1} + 1\right] dj \\
&= \int_0^1 \ln(q_{cjt}) dj + \frac{1}{\eta-1} \int_0^1 \ln\left[\sum_{k=1}^{N_{jt}} \left(\frac{q_{kjt}}{q_{cjt}}\right)^{\eta-1} + 1\right] dj \\
&= \ln\zeta + \int_0^1 \ln q_{jt}^{leader} dj + \frac{1}{\eta-1} \sum_{\Theta} \ln\left[\sum_{k=1}^{N(\Theta)} \left(\frac{q_{kjt}}{q_{cjt}}\right)^{\eta-1} + 1\right] \mu_t(\Theta) \\
&= \ln\zeta + \int_0^1 \ln q_{jt}^{leader} dj + \sum_{\Theta} \tilde{f}_t(\Theta)\mu_t(\Theta) \\
&= \ln\zeta + Q_t + \sum_{\Theta} \tilde{f}_t(\Theta)\mu_t(\Theta)
\end{aligned}$$

The last line provides a closed-form solution, which establishes the efficient amount of log-output as a function of the average productivity level of the leaders Q_t, and the industry state distribution $\mu_t(\Theta)$.

Before we move on to the full dynamic problem of the social planner, we can first derive the static welfare gain that would be obtained by removing all markups but keeping the distribution and dynamic policies unchanged. Table A1 shows that the static welfare gains can be substantial (around 25% in consumption equivalent terms). It is interesting to note that our model does not underestimate the static cost of markups. If anything, our estimates are slightly larger than those found in Baqaee and Farhi (2020) and Edmond, Midrigan, and Xu (2018).

A.4.3 Dynamic Welfare Maximization

Given the results of the static output maximization, we can greatly simplify the complete problem of the social planner. As we would like to compare the results to the general equilibrium along a balanced growth path, we also focus on the stationary version of the social planner's problem, where the state variables $\mu_0(\Theta)$ and m_0 are initialized at their stationary values. The average productivity of the leaders, Q_0, is initialized at 0 to remain consistent with Section 4.4. The dynamic welfare maximization problem of the social planner can therefore be re-stated as:

$$\max_{\left\{\left\{\{z_i(\Theta)\}_{i=1}^{N(\Theta)}, X_k(\Theta)\right\}_{\Theta}, e\right\}} \left\{\frac{\ln C_0}{\rho} + \frac{g}{\rho^2}\right\}, \text{such that} \tag{80}$$

$$C_0 = (1-R)Y_0 \tag{81}$$

$$\ln Y_0 = \ln\zeta + Q_0 + \sum_{\Theta} \tilde{f}(\Theta)\mu(\Theta) \tag{82}$$

$$g = \ln(1+\lambda)\sum_{\Theta} p_{li}(\Theta)\mu(\Theta) \tag{83}$$

$$R = \sum_{\Theta}\left(\sum_{i=1}^{N(\Theta)} \chi(z_i(\Theta))^{\phi} + \nu m(X_k(\Theta))^{\epsilon}\right)\mu(\Theta) + \psi e^2 \tag{84}$$

$$\dot{\mu}_t(\Theta) = \sum_{\Theta'} p(\Theta',\Theta)\mu(\Theta') - \sum_{\Theta'} p(\Theta,\Theta')\mu(\Theta) = 0, \forall\Theta \tag{85}$$

$$\sum_{\Theta}\mu_t(\Theta) = 1 \tag{86}$$

$$\dot{m}_t = e - \tau m = 0 \tag{87}$$

where initial consumption is equal to output minus R&D and business creation investment cost (equation 81), equation (82) is (log) initial output consistent with the optimal static allocation chosen by the social planner (see Section A.4.2), g is the growth rate of output in a balanced growth path (equation (83)), R is the share of output allocated to R&D and business creation (equation (84)), equation (85) is the law of motion of the industry distribution, equation (86) imposes that the mass of industries in the economy is equal to one and the dynamics of the mass of small firm is given by equation (87). In addition, those constraints require that g, $\mu_t(\Theta)$, and m_t remain constant in a balanced growth path.

The social planner chooses the innovation policy of superstar firms ($\{z_i(\Theta)\}_{i=1}^{N(\Theta)}$) and small firms ($X_k(\Theta)$) as well as investment in new business creation (e). We can notice that this formulation of the social planner's problem reduces the dimensionality of the maximization problem. Instead of solving for a continuum of continuous functions, we have reduced the problem to solving for a finite number of positive scalars, $\left\{\left\{\{z_i(\Theta)\}_{i=1}^{N(\Theta)}, X_k(\Theta)\right\}_{\Theta}, e\right\}$

The dynamic social planner's problem, while greatly simplified, still requires 90 (=84+6) constraints to be satisfied. However, we can plug in all of the constraints into the objective function, and turn the problem into an unconstrained maximization problem (except for the non-negativity constraints for the choice variables.) It is trivial to see this is the case for equations (81) to (84), and equation (87). The inflow-outflow equations (85) and equation (86) that determine the stationary industry state distribution $\mu(\Theta)$ are less obvious.

First, note that given the choice variables, the values $p(\Theta,\Theta')$ are constants. Therefore, the 84 equations described by equation (85) constitute a system of 84 linear equations in $\mu(\Theta)$. One of these 84 equations is superfluous since the system is closed. Combined together with equation (86), they constitute a linear system of 84 equations in 84 unknowns. Rewrite this system of equations in matrix form as $A\vec{\mu} = b$, where A is an invertible square matrix, b is a column vector, and $\vec{\mu}$ is the industry state distribution written in vector form. Hence, we have $\vec{\mu} = A^{-1}b$. In other words, given the choice variables, the stationary industry state distribution $\mu(\Theta)$ can be obtained using matrix algebra, and the resulting values can be plugged into the objective function.

At the end, we are left with an unconstrained optimization problem where we need to determine the optimal values of 253 positive scalars. We solve this problem using global optimization methods. The results are discussed below.

A.4.4 Results from Social Planner's Optimization

In this section, we report the results of the unconstrained social planner's problem. First, the optimal (static) allocation corresponds to a decentralized equilibrium allocation with no markups. In addition, the Planner chooses an industry distribution which converges to a degenerate distribution with only one superstar firm which does all the R&D. Results comparing welfare and output between the optimal and the decentralized allocation can be found in Table A2 for the full sample as well as for both subperiods. The optimal allocation consistently features higher initial output (static welfare gain) and growth rates (dynamic welfare gain) with large overall welfare gains. Table A2 shows that the static welfare gains from higher initial consumption are substantial (between 20% and 25%). It is interesting to note that our model does not underestimate the static cost of markups. If anything, our estimates are slightly larger than those found in Baqaee and Farhi (2020) and Edmond, Midrigan, and Xu (2018). However, despite the higher estimated static cost of markups, we find that the dynamic benefits from increased markups easily dominates the static gains.

A.4.5 Constrained Planner's Problem

In addition to the full social planner's problem discussed in Section A.4.4, in this section, we report the results from a constrained social planner's problem where we impose that there is no superstar innovation in industries with a single superstar firm. In the decentralized equilibrium, there is no innovation by superstars in single superstar industries since they have no incentive to do R&D. Under the optimal allocation, however, the distribution converges to a degenerate industry distribution with only single-superstar industries. In addition, only the incumbent superstar performs R&D. The objective of the following exercise is to analyze how much worse the social planner would do in terms of welfare if we impose that single superstars cannot perform R&D, i.e., we only allow the planner to allocate R&D resources to firms which perform R&D in the decentralized equilibrium. Results for the full sample and both subperiods can be found in Table A2. Once again, most of the welfare gains are due to removing dynamic inefficiencies (as is the case for the unconstrained optimal allocation). In addition, a large share of the welfare difference between the optimal and decentralized allocations can be achieved under the constrained optimal allocation with no R&D by large firms in single-superstar industries

A.5 Extended Model with Capital Accumulation

In this section, we introduce an extension of our baseline model with endogenous capital accumulation. In particular, we assume the following production function for the final good producer:

$$\ln(Y_t) = \int_0^1 \ln\left(y_{jt}^{1-\kappa} k_{jt}^{\kappa}\right)\, dj \tag{88}$$

where k_{jt} is capital which depreciates at a rate δ. Households own the stock of capital ($K_t = \int_0^1 k_{jt}\, dj$) and rent it to final good producers at a rental rate $R_t = r_t + \delta$. The final good is now used for consumption, investment in R&D, costs of new business entry, and investment in physical capital.

Profit maximization by final good producers implies:

$$k_{jt} = \frac{\kappa Y_t}{R_t} \tag{89}$$

$$p_{ijt} = \frac{(1-\kappa) y_{ijt}^{-\frac{1}{\eta}} Y_t}{\sum_{k=1}^{N_{jt}} y_{kjt}^{\frac{\eta-1}{\eta}} + \tilde{y}_{cjt}^{\frac{\eta-1}{\eta}}} \tag{90}$$

where κ is the capital share of income. This implies that, in a balanced growth path, the stock of capital grows at the same rate as aggregate output, Y_t. This delivers the same system of equations in unknown production ratios as in equation (17). Profits of superstars can be written as:

$$\pi_{ijt} = \frac{(1-\kappa)Y_t}{\left[\sum_{k=1}^{N_{jt}} \left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}\right]^2} \frac{\eta + \sum_{k\neq i} \left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}{\eta} \tag{91}$$

Compared to the baseline model, the dynamic problem of superstar firms and small firms is only affected by the multiplicative constant $(1-\kappa)$ that appears in the profit function of the superstar firm. Final good production is equal to:

$$\ln(Y_t) = \int_0^1 \ln q_{jt}^{\text{leader}}\, dj + \ln \zeta - \ln \omega + \sum f(\Theta)\mu(\Theta) + \ln(1-\kappa) + \frac{\kappa}{1-\kappa}\left[\ln(\kappa) - \ln(R_t)\right] \tag{92}$$

The law of motion for capital is given by $\dot{K}_t = I_t - \delta K_t$, where I_t is investment in physical capital at time t. We can show that the investment-to-GDP ratio is equal to:

$$\frac{I_t}{Y_t} = \frac{\kappa(r_t - \rho + \delta)}{r_t + \delta} \tag{93}$$

which is constant in a balanced growth path. The equation for the growth rate remains the same. The level of initial consumption, C_0, becomes:

$$C_0 = Y_0 \frac{C_0}{Y_0} = Y_0 \left(1 - \int_0^1 \sum_{i=1}^{N_{j0}} \chi z_{ij0}^{\phi}\, dj - \int_0^1 \nu X_{j0}^{\epsilon}\, dj - \psi e_0^2 - \frac{I_0}{Y_0}\right) \tag{94}$$

A.6 Non-Stationary Equilibria

In this section, we describe the algorithm used to compute non-stationary equilibria of our model, and repeat the quantitative experiments in Table IV while taking transitional dynamics into account.

A.6.1 Algorithm for Computing Non-Stationary Equilibria

Define $\varphi_i = [\rho_i, \lambda_i, \eta_i, \zeta_i, \nu_i, \epsilon_i, \chi_i, \phi_i, \psi_i, \tau_i]$ as the vector of structural parameters of economy i. Suppose an economy is initially at its stationary equilibrium implied by the initial parameter values φ_b, where b stands for the beginning. Without loss of generality, normalize the initial average log productivity of the leaders $Q_t = \int_0^1 \ln q_{j0}^{\text{leader}} dj = 0$. At time $t = 0$, the vector of structural parameters of the economy changes from φ_b to φ_e, where e stands for the end. As $t \to \infty$, the economy will converge to the stationary equilibrium implied by the final parameter values φ_e. To compute the described non-stationary equilibrium, we need to characterize the full transition path of all prices and allocations in the economy. Below, we describe the algorithm used to this purpose.

1. Compute the stationary equilibrium implied by the initial parameter vector φ_b. Call the associated time-invariant mass of small firms m_b, and the time-invariant industry-state distribution $\mu_b(\Theta)$.

2. Compute the stationary equilibrium implied by the final parameter vector φ_e. Call the associated time-invariant mass of small firms m_e, and the time-invariant industry-state distribution $\mu_e(\Theta)$.

3. Define $T > 0$ as the amount of time (measured in years) for which the time paths of allocations and prices will be computed. T must be large enough such that the state variables $\mu_t(\Theta)$ and m_t are sufficiently close to their stationary values; i.e., $\|\mu_T(\Theta) - \mu_e(\Theta)\| < \epsilon_{\mu_e(\Theta)}$ and $\|m_T - m_e\| < \epsilon_{m_e}$ for some tolerance values $\epsilon_{\mu_e(\Theta)}, \epsilon_{m_e} > 0$ where $\|.\|$ denotes the sup-norm. Choose a value for T — e.g., $T = 1000$.

4. Divide the time interval $[0, T]$ into a uniform grid with step size $\Delta t > 0$. Denote the resultant (discrete) time grid as $\vec{t} = \{s\Delta t\}_{s=0}^{S}$ with $S = T/\Delta t$. A lower grid step size Δt increases approximation accuracy (discussed below) at the cost of computation time.

5. Create an initial guess for the time path of the mass of small firms over the time grid. A reasonable initial guess is a linear interpolation with boundary values $m_0 = m_b$ and $m_T = m_e$. Call this initial guess $\vec{m}_{old}$.

6. Repeat the following until the time path of the mass of small firms $\vec{m}$ converges; i.e., $\|\vec{m}_{old} - \vec{m}_{new}\| < \epsilon_{\vec{m}}$ for some tolerance value $\epsilon_{\vec{m}} > 0$:

   (a) Calculate the quantities, prices, static profit flows, and markups of all firms in each industry-state $\Theta \in \hat{\Theta}$ that arise as a result of static product market competition. (Solve a system of N non-linear equations for each industry-state, where N is the number of superstar firms.)

   (b) Denote the time path of the normalized superstar firm value function as $\vec{v} = \{v_s(\mathbf{n}_i, N)\}_{s=0}^{S}$. Given the convergence of the state variables, $v_S(\mathbf{n}_i, N) \approx v_e(\mathbf{n}_i, N)$. Set $v_S(\mathbf{n}_i, N) = v_e(\mathbf{n}_i, N)$. We know $\dot{v}_t(\mathbf{n}_i, N) = \lim_{\Delta t \to 0} \frac{v_{t+\Delta t}(\mathbf{n}_i, N) - v_t(\mathbf{n}_i, N)}{\Delta t}$. Then $\dot{v}_t(\mathbf{n}_i, N) \approx \frac{v_{t+\Delta t}(\mathbf{n}_i, N) - v_t(\mathbf{n}_i, N)}{\Delta t}$, where the approximation is more accurate for smaller values of Δt. Plugging this approximate value into equation (22) yields an equation linking $v_s(.)$ to $v_{s+1}(.)$. Starting from $s = S$, use backward iteration to obtain the time path of the normalized superstar firm value function $\vec{v}$, along with the time paths of superstar innovation policy function $\vec{z} = \{z_{is}(\mathbf{n}_i, N)\}_{s=0}^{S}$ and small firm innovation policy function $\vec{X} = \{X_{kjs}(\Theta_j)\}_{s=0}^{S}$.

(c) The initial value of the industry-state distribution at time $t = 0$ is determined by the value in the initial stationary equilibrium, $\mu_0(\Theta) = \mu_b(\Theta), \forall \Theta \in \hat{\Theta}$. Unlike a stationary equilibrium, the inflow and outflow rates for each industry-state Θ are now time-varying. Using the values obtained for $\vec{z}$ and $\vec{X}$, starting from $s = 0$, construct the time paths of inflows to $(\sum_{\Theta'} p_t(\Theta', \Theta)\mu_t(\Theta'))$ and outflows from $(\sum_{\Theta'} p_t(\Theta, \Theta')\mu_t(\Theta))$ each industry-state $\Theta \in \hat{\Theta}$ and the resultant time path of the industry-state distribution $\vec{\mu} = \{\mu_s(\Theta)\}_{s=0}^{S}$ using forward iteration. This yields $\mu_T(\Theta) = \mu_S(\Theta)$.

(d) Using $\vec{v}$ and $\vec{X}$, construct the time path of expected profit flows of small firms in each industry-state Θ, denoted $\vec{\pi^e} = \{\pi_s^e(\Theta)\}_{s=0}^{S}$.

(e) Denote the time path of the normalized small firm value function as $\vec{v^e} = \{v_s^e(\mathbf{n}_i, N)\}_{s=0}^{S}$. Given the convergence of the state variables, $v_S^e(\mathbf{n}_i, N) \approx v_e^e(\mathbf{n}_i, N)$. Set $v_S^e(\mathbf{n}_i, N) = v_e^e(\mathbf{n}_i, N)$. We know $\dot{v}_t^e(\mathbf{n}_i, N) = \lim_{\Delta t \to 0} \frac{v_{t+\Delta t}^e(\mathbf{n}_i,N) - v_t^e(\mathbf{n}_i,N)}{\Delta t}$. Then $\dot{v}_t^e(\mathbf{n}_i, N) \approx \frac{v_{t+\Delta t}^e(\mathbf{n}_i,N) - v_t^e(\mathbf{n}_i,N)}{\Delta t}$, where the approximation is more accurate for smaller values of Δt. Plugging this approximate value into equation (25) yields an equation linking $v_s^e(.)$ to $v_{s+1}^e(.)$. Starting from $s = S$, use backward iteration to obtain the time path of the normalized small firm value function $\vec{v^e}$.

(f) Using the time paths of the industry-state distribution $\vec{\mu}$ and the small firm value function $\vec{v^e}$, solve for the time path of the small business creation rate $\vec{e} = \{e_s\}_{s=0}^{S}$ consistent with entrepreneur profit maximization.

(g) The mass of small firms at time $t = 0$ is determined by the value in the initial stationary equilibrium, $m_0 = m_b$. Starting from $s = 0$, calculate the new time path of the mass of small firms $\vec{m}_{new}$ through forward iteration using $\vec{e}$ and the exogenous small firm exit rate τ.

(h) Calculate the time path of the relative wage rate $\vec{\omega}$ using the static product market competition results and $\vec{\mu}$.

(i) Calculate the time path of the output growth rate $\vec{g}$ using equation (35). Calculate the time path of the average log productivity level of industry leaders $\vec{Q}$ through forward iteration.

(j) Calculate other allocations and statistics of interest as necessary. To calculate lifetime utility of the representative consumer at time $t = 0$: Calculate the initial output Y_0, and the time path of aggregate output $\vec{Y}$ using $\vec{g}$. Calculate the time paths of aggregate R&D intensity and business creation costs to derive the time path of the consumption-to-output ratio. Using all, calculate the time path of aggregate consumption $\vec{C}$. Plug $\vec{C}$ into the utility function of the representative consumer to obtain lifetime utility at time $t = 0$.

(k) Check if the time path of the mass of small firms has converged ($\|\vec{m}_{old} - \vec{m}_{new}\| < \epsilon_{\vec{m}}$.) If true, exit. If false, update $\vec{m}_{old} = \xi\vec{m}_{new} + (1-\xi)\vec{m}_{old}$, and go back to step (a).[41]

7. Check if the state variables have converged ($\|\mu_T(\Theta) - \mu_e(\Theta)\| < \epsilon_{\mu_e(\Theta)}$ and $\|m_T - m_e\| < \epsilon_{m_e}$). If true, the non-stationary equilibrium is found. If false, either increase T or decrease Δt, and go back to step 5.

[41] $\xi \in (0, 1]$ is an update weight. $\xi = 0.5$ works well.

Table A3: Disentangling the Structural Transition with Non-Stationary Dynamics

	Early η	Early ζ	Early ν, ϵ	Early χ, ϕ	Early ψ, τ	All
CEWC	-7.22%	-1.08%	1.35%	15.28%	0.04%	7.32%

A.6.2 Disentangling the Structural Transition with Non-Stationary Dynamics

In this section, we repeat the quantitative experiments in Table IV while taking transitional dynamics into account. Before we conduct the counterfactual experiments, we need to compute the realized non-stationary equilibrium between 1976 and 2004 in the US. To do so, we use the estimated parameter values from the early period sub-sample as φ_b, and those from the late period sub-sample as φ_e, and compute the baseline non-stationary equilibrium following the algorithm described in the preceding subsection. All consumption-equivalent welfare numbers in the following counterfactual experiments use the lifetime utility of the representative consumer at time $t = 0$ in this baseline economy as the yardstick.

As in Table IV, we conduct six separate counterfactual experiments. The initial vector of structural parameters φ_b is always the same, and uses the estimated parameter values from the early period sub-sample. The final vector of structural parameters φ_e is changed in each experiment. These are listed below in sequence:

1. Early η: $\varphi_e = [\rho, \lambda^{late}, \eta^{early}, \zeta^{late}, \nu^{late}, \epsilon^{late}, \chi^{late}, \phi^{late}, \psi^{late}, \tau^{late}]$
2. Early ζ: $\varphi_e = [\rho, \lambda^{late}, \eta^{late}, \zeta^{early}, \nu^{late}, \epsilon^{late}, \chi^{late}, \phi^{late}, \psi^{late}, \tau^{late}]$
3. Early ν, ϵ: $\varphi_e = [\rho, \lambda^{late}, \eta^{late}, \zeta^{late}, \nu^{early}, \epsilon^{early}, \chi^{late}, \phi^{late}, \psi^{late}, \tau^{late}]$
4. Early χ, ϕ: $\varphi_e = [\rho, \lambda^{late}, \eta^{late}, \zeta^{late}, \nu^{late}, \epsilon^{late}, \chi^{early}, \phi^{early}, \psi^{late}, \tau^{late}]$
5. Early ψ, τ: $\varphi_e = [\rho, \lambda^{late}, \eta^{late}, \zeta^{late}, \nu^{late}, \epsilon^{late}, \chi^{late}, \phi^{late}, \psi^{early}, \tau^{early}]$
6. Early all: $\varphi_e = [\rho, \lambda^{early}, \eta^{early}, \zeta^{early}, \nu^{early}, \epsilon^{early}, \chi^{early}, \phi^{early}, \psi^{early}, \tau^{early}]$

Table A3 presents the resultant consumption-equivalent welfare change numbers. First, looking at the final column, the welfare difference between the realized transition and the counterfactual of remaining in the early steady-state in perpetuum is now calculated to be 7.32%, as opposed to 8.73% in the baseline analysis. This shows that taking the non-stationary dynamics into account does not significantly change the calculated welfare impact of the structural transition in the US.

The individual experiments themselves also maintain the same signs as in Table IV, but the quantitative magnitudes change in some cases. In the first experiment, the welfare loss in the counterfactual economy with the early period elasticity of substitution η is now 7.22% instead of 10.64%. In the second experiment where the relative productivity of the competitive fringe ζ is held constant, we find that the dynamic gains in welfare associated with higher markups still dominate the static losses in efficiency, albeit with a smaller magnitude (roughly four thirds of the static losses instead of two times found in the baseline). This is because the increase in aggregate productivity growth takes time to fully manifest due to the time it takes for the industry-state distribution $\mu_t(\Theta)$ to converge to its stationary value, whereas the static losses from a less productive competitive fringe are instantaneous. The third column shows that restoring the R&D efficiency of small firms back to its early-period value is still welfare-enhancing, but the value is now lower at 1.08% as opposed to 6.38%. This, however, is not the case for the experiment in the fourth column. The structural change that contributes the most to the loss in welfare is once again the decline in the R&D efficiency of superstar firms, the impact of which is calculated as 15.28%, which is quite close to the 17.88% found in the baseline. Finally, the total effect of keeping new business creation costs and firm exit rate the same as in the early period sub-sample is still a wash on average, where the welfare barely moves at 0.04%, similar to the small 0.39% value found in the baseline.

To summarize, the overall message remains the same: The primary driver that lies behind the observed increase in the average markup, the fall in the relative productivity of small firms, is still welfare-enhancing since the dynamic gains from improved productivity growth still dominate the static losses from lower static allocative efficiency. The primary driver of the observed decline in productivity growth is still the decline in the R&D efficiency of superstar firms ("ideas are getting harder to find") with a very similar magnitude.

B Additional Tables and Figures

Table B1: Firm Innovation and Relative Employment

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative employment	6.821	5.908	7.094	16.539
	(1.110)***	(1.293)***	(1.718)***	(1.836)***
relative employment sq.	-6.692	-5.728	-5.346	-14.595
	(1.310)***	(1.642)***	(2.085)**	(2.241)***
R^2	0.15	0.10	0.26	0.25
N	101,853	101,853	101,853	101,853

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative employment	2.088	3.445	1.185	0.886
	(0.187)***	(0.294)***	(0.088)***	(0.076)***
relative employment sq.	-1.572	-2.766	-1.031	-0.812
	(0.229)***	(0.353)***	(0.110)***	(0.096)***
R^2	0.57	0.50	0.96	0.94
N	101,853	101,853	59,829	101,853

Notes: This table replicates the results in Table II where relative sales is replaced with relative employment. Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample period is from 1976 to 2004 at the annual frequency. All regressions control for profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, year dummies, and a full set of four-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

Table B2: Firm Innovation and Relative Total Assets

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative total assets	8.693	7.966	9.437	18.534
	(1.123)***	(1.338)***	(1.762)***	(1.854)***
relative total assets sq.	-8.555	-7.718	-7.669	-16.079
	(1.360)***	(1.754)***	(2.231)***	(2.325)***
R^2	0.15	0.10	0.26	0.25
N	104,911	104,911	104,911	104,911

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative total assets	2.149	3.631	1.381	0.967
	(0.192)***	(0.297)***	(0.091)***	(0.080)***
relative total assets sq.	-1.450	-2.717	-1.223	-0.896
	(0.271)***	(0.400)***	(0.117)***	(0.108)***
R^2	0.57	0.50	0.96	0.94
N	104,911	104,911	61,186	104,911

Notes: This table replicates the results in Table II where relative sales is replaced with relative total assets. Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample period is from 1976 to 2004 at the annual frequency. All regressions control for profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, year dummies, and a full set of four-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

Table B3: Firm Innovation and Relative Sales – SIC3 Fixed Effects

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative sales	5.879	4.976	7.035	15.018
	(1.162)***	(1.356)***	(1.788)***	(1.900)***
relative sales sq.	-6.422	-5.499	-5.508	-13.491
	(1.436)***	(1.783)***	(2.239)**	(2.398)***
R^2	0.14	0.10	0.25	0.24
N	104,911	104,911	104,911	104,911

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative sales	1.774	2.932	0.967	0.798
	(0.191)***	(0.299)***	(0.093)***	(0.080)***
relative sales sq.	-1.216	-2.258	-0.862	-0.773
	(0.264)***	(0.397)***	(0.121)***	(0.111)***
R^2	0.55	0.49	0.96	0.94
N	104,911	104,911	61,186	104,911

Notes: Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample period is from 1976 to 2004 at the annual frequency. All regressions control for profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, year dummies, and a full set of three-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

Table B4: Firm Innovation and Relative Sales – SIC2 Fixed Effects

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative sales	3.825	2.677	7.088	13.541
	(1.306)***	(1.513)*	(1.769)***	(1.982)***
relative sales sq.	-3.714	-2.455	-4.576	-11.063
	(1.613)**	(1.976)	(2.424)*	(2.705)***
R^2	0.13	0.08	0.23	0.22
N	104,911	104,911	104,911	104,911

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative sales	1.698	2.702	0.728	0.700
	(0.214)***	(0.332)***	(0.089)***	(0.078)***
relative sales sq.	-1.149	-1.983	-0.662	-0.695
	(0.333)***	(0.494)***	(0.114)***	(0.109)***
R^2	0.53	0.47	0.95	0.94
N	104,911	104,911	61,186	104,911

Notes: Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample period is from 1976 to 2004 at the annual frequency. All regressions control for profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, year dummies, and a full set of two-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

Table B5: Firm Innovation and Relative Sales – Firm Fixed Effects

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative sales	6.181	4.026	6.030	11.929
	(1.817)***	(2.298)*	(2.701)**	(2.735)***
relative sales sq.	-5.310	-2.986	-3.321	-9.253
	(1.913)***	(2.485)	(3.098)	(2.952)***
R^2	0.44	0.34	0.52	0.50
N	104,911	104,911	104,911	104,911

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative sales	1.160	2.261	1.604	0.930
	(0.217)***	(0.357)***	(0.179)***	(0.132)***
relative sales sq.	-0.794	-1.665	-1.195	-0.729
	(0.244)***	(0.388)***	(0.207)***	(0.150)***
R^2	0.86	0.78	0.98	0.97
N	104,911	104,911	61,186	104,911

Panel C

	log(xad)	log(capx)	sale growth	employment growth	asset growth
relative sales	5.058	6.120	0.557	0.229	0.261
	(0.455)***	(0.285)***	(0.044)***	(0.035)***	(0.047)***
relative sales sq.	-3.973	-4.881	-0.475	-0.198	-0.226
	(0.537)***	(0.310)***	(0.047)***	(0.036)***	(0.048)***
R^2	0.95	0.91	0.31	0.28	0.29
N	37,779	103,558	102,726	96,718	103,598

Notes: Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample period is from 1976 to 2004 at the frequency. All regressions control for profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, year dummies, and firm fixed effects. $***p < 0.01, **p < 0.05, *p < 0.1$.

Table B6: Firm Innovation and Relative Sales – Early Sample (1976-1990)

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative sales	4.822	4.814	6.751	15.188
	(1.686)***	(2.151)**	(1.894)***	(2.875)***
relative sales sq.	-5.411	-5.834	-6.173	-14.151
	(1.993)***	(2.751)**	(2.315)***	(3.538)***
R^2	0.18	0.12	0.22	0.30
N	44,130	44,130	44,130	44,130

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative sales	1.635	2.704	1.154	0.696
	(0.254)***	(0.404)***	(0.115)***	(0.098)***
relative sales sq.	-0.924	-1.838	-0.918	-0.600
	(0.356)***	(0.537)***	(0.157)***	(0.139)***
R^2	0.62	0.56	0.97	0.95
N	44,130	44,130	23,939	44,130

Notes: Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample period is from 1976 to 1990 at the annual frequency. All regressions control for profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, year dummies, and a full set of four-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

Table B7: Firm Innovation and Relative Sales – Late Sample (1991-2004)

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative sales	9.473	6.064	12.913	13.466
	(1.475)***	(1.553)***	(2.460)***	(1.828)***
relative sales sq.	-9.349	-6.239	-8.795	-10.047
	(1.772)***	(1.937)***	(3.131)***	(2.321)***
R^2	0.15	0.11	0.28	0.23
N	60,781	60,781	60,781	60,781

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative sales	2.488	3.947	1.730	1.306
	(0.235)***	(0.346)***	(0.127)***	(0.108)***
relative sales sq.	-1.687	-2.922	-1.569	-1.223
	(0.312)***	(0.440)***	(0.162)***	(0.147)***
R^2	0.56	0.48	0.95	0.94
N	60,781	60,781	37,247	60,781

Notes: Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample period is from 1991 to 2004 at the annual frequency. All regressions control for profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, year dummies, and a full set of four-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

Table B8: Industry Innovation and Market Concentration (HHI) – Baseline Specification (Inverted-U Hypothesis Test)

Panel A: Total Innovation by Industry

	patent count	total citations	tail count	original count	general count
lower bound					
t-value	3.488	4.195	4.415	2.583	2.084
P>|t|	0.000	0.000	0.000	0.005	0.019
upper bound					
t-value	-2.472	-2.612	-2.377	-1.352	-1.470
P>|t|	0.007	0.005	0.009	0.088	0.071

Panel B: Industry Average of Total Innovation by Firms

	patent count	total citations	tail count	original count	general count
lower bound					
t-value	2.008	2.853	3.004	1.528	2.889
P>|t|	0.022	0.002	0.001	0.063	0.002
upper bound					
t-value	-2.617	-3.658	-3.605	-1.426	-2.989
P>|t|	0.004	0.000	0.000	0.077	0.001

Panel C: Industry Average of Average Innovation Quality by Firms

	avg. citations	tail innov	avg. orginality	avg. generality
lower bound				
t-value	4.172	3.219	3.340	3.158
P>|t|	0.000	0.001	0.000	0.001
upper bound				
t-value	-3.271	-1.944	-1.088	-2.492
P>|t|	0.001	0.026	0.138	0.006

Notes: To further check the robustness of the inverted-U relationship between industry innovation and market concentration, we test whether or not the slope of the fitted curve is positive at the start and negative at the end of the interval of the market concentration following Lind and Mehlum (2010). This table reports the hypothesis testing results.

Table B9: Firm Innovation and Relative Sales – Baseline Specification (Inverted-U Hypothesis Test)

Panel A

	avg. citations	tail innov	avg. orginality	avg. generality
lower bound				
t-value	6.614	4.952	4.625	9.277
P>|t|	0.000	0.000	0.000	0.000
upper bound				
t-value	-4.246	-2.779	-1.442	-4.020
P>|t|	0.000	0.003	0.075	0.000

Panel B

	log total patents	log total citations	log R\&D Spending	log R]&D spending 2
lower bound				
t-value	10.671	11.494	14.158	12.012
P>|t|	0.000	0.000	0.000	0.000
upper bound				
t-value	-2.048	-3.256	-6.363	-5.808
P>|t|	0.020	0.001	0.000	0.000

Panel C

	log(xad)	log(capx)	sales growth	employment growth	asset growth
lower bound					
t-value	31.841	52.415	12.784	11.965	12.938
P>|t|	0.000	0.000	0.000	0.000	0.000
upper bound					
t-value	-16.054	-25.941	-6.824	-6.877	-7.018
P>|t|	0.000	0.000	0.000	0.000	0.000

Notes: To further check the robustness of the inverted-U relationship between firm innovation and relative sales, we test whether or not the slope of the fitted curve is positive at the start and negative at the end of the interval of relative sales following Lind and Mehlum (2010). This table reports the hypothesis testing results.

Table B10: Identification: Jacobian Matrix

	λ	η	χ	ν	ζ	ϕ	ϵ	ψ	τ
growth rate	0.332	-0.399	-0.333	-0.120	-1.325	3.138	0.935	-0.101	-0.175
R&D intensity	-0.853	-0.658	-0.265	-0.182	-3.371	1.868	1.451	-0.153	-0.265
average markup	-0.033	-0.061	0.002	-0.003	-0.653	-0.011	0.024	-0.003	-0.004
std. dev. markup	0.150	-0.177	-0.004	0.014	-1.343	-0.010	-0.112	0.012	0.020
labor share	0.052	0.041	-0.002	0.005	0.495	0.009	-0.038	0.004	0.007
entry rate	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
β(innovation, rel. sales)	0.431	0.064	-0.155	0.014	-0.044	0.374	0.024	0.012	0.020
turning point (intra)	0.089	0.000	-0.016	-0.025	-0.105	0.091	0.217	-0.021	-0.036
avg profitability	-0.145	-0.117	0.039	-0.002	-1.887	-0.241	0.011	-0.002	-0.003
avg leader rel. quality	0.646	0.318	-0.015	0.099	0.523	-0.164	-0.794	0.083	0.143
std. dev. leader rel. quality	0.433	0.292	-0.027	0.139	0.659	0.183	-0.949	0.117	0.202

Notes: The table shows the Jacobian matrix associated with the estimation of the baseline model. Each entry of the matrix reports the percentage change in each moment given one percent increase in each parameter.

Table B11: Model Parameters and Target Moments — Exogenous Growth (50%)

A. Parameter estimates

Parameter	*Description*	*Early sub-sample*	*Late sub-sample*
λ	innovation step size	0.1389	0.1506
η	elasticity within industry	16.5783	8.4861
χ	superstar cost scale	199.8551	34.2025
ν	small firm cost scale	1.3205	3.3011
ζ	competitive fringe ratio	0.6421	0.5622
ϕ	superstar cost convexity	4.1448	3.0994
ϵ	small firm cost convexity	3.9939	2.0535
τ	exit rate	0.1257	0.1052
ψ	entry cost scale	0.0438	0.0398

B. Moments

	Early sub-sample		Late sub-sample	
Target moments	Data	Model	Data	Model
growth rate	2.42%	2.42%	1.98%	1.98%
R&D intensity	2.38%	2.42%	2.50%	2.47%
average markup	1.280	1.280	1.424	1.421
std. dev. markup	0.299	0.259	0.396	0.353
labor share	0.656	0.619	0.648	0.596
entry rate	0.126	0.126	0.105	0.105
β(innovation, relative sales)	0.435	0.618	0.699	0.559
top point (intra-industry)	0.447	0.381	0.507	0.356
average profitability	0.137	0.162	0.147	0.223
average leader relative quality	0.750	0.378	0.747	0.526
std. dev. leader relative quality	0.224	0.095	0.222	0.138

Notes: The estimation is done with the simulated method of moments. Panel A reports the estimated parameters. Panel B reports the simulated and actual moments.

Table B12: Disentangling the Structural Transition — Exogenous Growth (50%)

	Benchmark	Early η	% change	Early ζ	% change	Early ν, ϵ	% change
growth rate	1.98%	1.91%	-3.75%	1.78%	-10.27%	2.15%	8.15%
R&D intensity	2.47%	2.12%	-14.51%	1.40%	-43.32%	3.79%	53.06%
average markup	1.421	1.435	0.98%	1.291	-9.12%	1.410	-0.74%
std. dev. markup	0.353	0.345	-2.07%	0.306	-13.18%	0.306	-13.31%
labor share	0.596	0.589	-1.17%	0.648	8.72%	0.591	-0.82%
entry rate	0.105	0.105	0.00%	0.105	0.00%	0.105	0.00%
β(innov, relative sales)	0.559	0.307	-45.12%	0.420	-24.90%	1.124	100.95%
top point (intra-industry)	0.356	0.263	-25.96%	0.326	-8.46%	0.334	-5.98%
average profitability	0.223	0.235	5.16%	0.168	-24.99%	0.217	-2.90%
average leader relative quality	0.526	0.579	10.20%	0.591	12.38%	0.353	-32.87%
std. dev. leader relative quality	0.138	0.142	3.56%	0.166	20.44%	0.059	-57.36%
superstar innovation	0.162	0.135	-16.84%	0.118	-27.12%	0.259	59.89%
small firm innovation	0.017	0.005	-70.18%	0.007	-61.38%	0.062	261.68%
output share of superstars	0.625	0.644	3.05%	0.495	-20.76%	0.715	14.48%
avg. superstars per industry	2.377	2.015	-15.26%	2.036	-14.35%	3.850	61.95%
mass of small firms	1.000	0.498	-50.20%	0.523	-47.66%	2.003	100.25%
initial output	0.855	0.810	-5.30%	0.861	0.69%	0.897	4.84%
CE Welfare change		-6.70%		-3.25%		7.53%	

	Benchmark	Early χ, ϕ	% change	Early ψ, τ	% change	All	% change
growth rate	1.98%	2.19%	10.43%	1.96%	-1.42%	2.42%	21.71%
R&D intensity	2.47%	2.77%	12.07%	2.32%	-6.32%	2.42%	-2.34%
average markup	1.421	1.411	-0.68%	1.420	-0.02%	1.280	-9.88%
std. dev. markup	0.353	0.338	-4.15%	0.357	1.21%	0.259	-26.58%
labor share	0.596	0.597	0.16%	0.597	0.17%	0.644	8.13%
entry rate	0.105	0.105	0.00%	0.126	19.44%	0.126	19.44%
β(innov, relative sales)	0.559	0.550	-1.74%	0.513	-8.27%	0.618	10.58%
top point (intra-industry)	0.356	0.356	0.06%	0.346	-2.61%	0.381	7.21%
average profitability	0.223	0.221	-1.04%	0.223	0.06%	0.162	-27.40%
average leader relative quality	0.526	0.483	-8.08%	0.544	3.59%	0.378	-28.12%
std. dev. leader relative quality	0.138	0.153	11.52%	0.145	5.34%	0.095	-31.27%
superstar innovation	0.162	0.236	45.56%	0.153	-5.56%	0.278	71.55%
small firm innovation	0.017	0.033	94.90%	0.014	-18.78%	0.060	248.36%
output share of superstars	0.625	0.649	3.92%	0.616	-1.47%	0.594	-4.99%
avg. superstars per industry	2.377	2.736	15.08%	2.267	-4.64%	3.555	49.53%
mass of small firms	1.000	1.609	60.90%	0.759	-24.14%	1.000	0.00%
initial output	0.855	0.864	1.02%	0.851	-0.51%	0.839	-1.94%
CE Welfare change		5.96%		-1.07%		9.22%	

Notes: The table reports the change in model moments when setting parameter of interest back to its estimated level in early sub-sample while keeping other parameters fixed at their estimated value in the late sub-sample.

Table B13: Static vs. Dynamic Costs of Higher Markups — Exogenous Growth (50%)

	Static		Static+New Distribution		Dynamic	
	ΔW	CEWC	ΔW	CEWC	ΔW	CEWC
competitive fringe productivity	3.322	14.21%	3.322	14.21%	3.322	14.21%
relative wage	-1.973	-7.59%	-2.091	-8.02%	-2.091	-8.02%
output of superstar firms	-0.893	-3.51%	-1.059	-4.15%	-1.059	-4.15%
consumption/output	0.000	0.00%	0.000	0.00%	0.276	1.11%
output growth	0.000	0.00%	0.000	0.00%	-1.274	-4.97%
total	0.456	1.84%	0.172	0.69%	-0.827	-3.25%

Notes: The table decomposes the change in welfare to its constituent parts using equation (40). The first column of the table calculates and decomposes the change in welfare if we ignored the dynamics completely. Column 2 shows the change in consumption-equivalent welfare by each individual component. The third column repeats the same exercise as in column one with a single difference: we use the distribution of industries $\mu(\Theta)$ implied by the dynamic long-run change in response to the increase in ζ rather than retaining the late sample values. The fifth column displays the welfare decomposition for the full dynamic response.

Table B14: Model Parameters and Target Moments — CRRA Utility

A. Parameter estimates

Parameter	*Description*	*Early sub-sample*	*Late sub-sample*
λ	innovation step size	0.3156	0.3159
η	elasticity within industry	16.5924	6.6383
χ	superstar cost scale	195.5605	79.0478
ν	small firm cost scale	1.3196	2.8855
ζ	competitive fringe ratio	0.6184	0.5441
ϕ	superstar cost convexity	4.1450	3.4400
ϵ	small firm cost convexity	4.0872	2.5928
τ	exit rate	0.1257	0.1052
ψ	entry cost scale	0.0114	0.0176

B. Moments

	Early sub-sample		Late sub-sample	
Target moments	Data	Model	Data	Model
growth rate	2.42%	2.42%	1.98%	1.98%
R&D intensity	2.38%	1.93%	2.50%	2.40%
average markup	1.280	1.283	1.424	1.425
std. dev. markup	0.299	0.313	0.396	0.430
labor share	0.656	0.627	0.648	0.609
entry rate	0.126	0.126	0.105	0.105
β(innovation, relative sales)	0.435	0.747	0.699	0.745
top point (intra-industry)	0.447	0.485	0.507	0.451
average profitability	0.137	0.155	0.147	0.206
average leader relative quality	0.750	0.538	0.747	0.630
std. dev. leader relative quality	0.224	0.122	0.222	0.148

Notes: The estimation is done with the simulated method of moments. Panel A reports the estimated parameters. Panel B reports the simulated and actual moments.

Table B15: Disentangling the Structural Transition — CRRA Utility

	Benchmark	Early η	% change	Early ζ	% change	Early ν, ϵ	% change
growth rate	1.98%	1.82%	-8.00%	1.72%	-12.96%	2.22%	12.22%
R&D intensity	2.40%	2.34%	-2.44%	1.74%	-27.50%	3.01%	25.13%
average markup	1.425	1.419	-0.39%	1.309	-8.12%	1.430	0.35%
std. dev. markup	0.430	0.404	-6.01%	0.364	-15.42%	0.414	-3.70%
labor share	0.609	0.607	-0.37%	0.650	6.66%	0.604	-0.92%
entry rate	0.105	0.105	0.00%	0.105	0.00%	0.105	0.00%
β(innov, relative sales)	0.745	0.682	-8.50%	0.735	-1.38%	0.927	24.41%
top point (intra-industry)	0.451	0.440	-2.35%	0.448	-0.68%	0.463	2.78%
average profitability	0.206	0.210	1.59%	0.162	-21.71%	0.207	0.52%
average leader relative quality	0.630	0.679	7.83%	0.663	5.26%	0.496	-21.28%
std. dev. leader relative quality	0.148	0.162	9.31%	0.160	8.21%	0.101	-31.85%
superstar innovation	0.154	0.133	-13.49%	0.129	-15.87%	0.212	37.80%
small firm innovation	0.021	0.011	-47.29%	0.014	-33.14%	0.074	246.34%
output share of superstars	0.521	0.542	4.14%	0.436	-16.20%	0.556	6.66%
avg. superstars per industry	2.304	2.012	-12.69%	2.123	-7.87%	3.476	50.86%
mass of small firms	1.000	0.681	-31.93%	0.737	-26.25%	1.491	49.12%
initial output	0.809	0.748	-7.49%	0.836	3.32%	0.828	2.40%
CE Welfare change		-11.02%		-2.58%		8.02%	

	Benchmark	Early χ, ϕ	% change	Early ψ, τ	% change	All	% change
growth rate	1.98%	2.45%	23.84%	1.99%	0.63%	2.42%	22.25%
R&D intensity	2.40%	2.26%	-5.76%	2.42%	0.82%	1.93%	-19.47%
average markup	1.425	1.423	-0.14%	1.425	0.02%	1.283	-9.99%
std. dev. markup	0.430	0.429	-0.31%	0.430	-0.13%	0.313	-27.27%
labor share	0.609	0.610	0.08%	0.609	-0.04%	0.654	7.26%
entry rate	0.105	0.105	0.00%	0.126	19.44%	0.126	19.44%
β(innov, relative sales)	0.745	0.695	-6.69%	0.746	0.18%	0.747	0.28%
top point (intra-industry)	0.451	0.446	-1.06%	0.452	0.16%	0.485	7.65%
average profitability	0.206	0.207	0.47%	0.206	0.06%	0.155	-24.91%
average leader relative quality	0.630	0.617	-2.03%	0.625	-0.70%	0.538	-14.58%
std. dev. leader relative quality	0.148	0.156	4.96%	0.146	-1.21%	0.122	-17.41%
superstar innovation	0.154	0.200	30.46%	0.155	1.12%	0.217	41.06%
small firm innovation	0.021	0.031	44.70%	0.022	4.07%	0.060	177.36%
output share of superstars	0.521	0.523	0.32%	0.522	0.26%	0.472	-9.43%
avg. superstars per industry	2.304	2.437	5.78%	2.331	1.14%	3.066	33.07%
mass of small firms	1.000	1.281	28.13%	1.063	6.33%	1.000	0.00%
initial output	0.809	0.809	0.02%	0.810	0.09%	0.782	-3.37%
CE Welfare change		12.13%		0.48%		7.93%	

Notes: The table reports the change in model moments when setting parameter of interest back to its estimated level in early sub-sample while keeping other parameters fixed at their estimated value in the late sub-sample.

Table B16: Disentangling the Structural Transition — With Capital

	Benchmark	Early η	% change	Early ζ	% change	Early ν, ϵ	% change
growth rate	1.98%	1.79%	-9.38%	1.63%	-17.53%	2.19%	10.64%
R&D intensity	1.85%	1.72%	-7.05%	1.20%	-34.92%	2.18%	18.01%
average markup	1.420	1.415	-0.29%	1.307	-7.91%	1.423	0.28%
std. dev. markup	0.428	0.404	-5.68%	0.363	-15.23%	0.420	-1.91%
labor share	0.611	0.601	-1.76%	0.642	5.02%	0.600	-1.90%
entry rate	0.105	0.105	0.00%	0.105	0.00%	0.105	0.00%
β(innov, relative sales)	0.756	0.699	-7.62%	0.750	-0.81%	0.833	10.18%
top point (intra-industry)	0.453	0.440	-2.92%	0.446	-1.63%	0.469	3.42%
average profitability	0.204	0.209	2.23%	0.162	-20.53%	0.204	-0.05%
average leader relative quality	0.643	0.687	6.82%	0.684	6.34%	0.569	-11.53%
std. dev. leader relative quality	0.148	0.163	9.63%	0.165	10.88%	0.123	-17.45%
superstar innovation	0.147	0.127	-13.61%	0.117	-20.30%	0.179	21.89%
small firm innovation	0.019	0.010	-46.92%	0.011	-42.78%	0.042	120.72%
output share of superstars	0.515	0.537	4.26%	0.433	-15.88%	0.535	3.73%
avg. superstars per industry	2.244	1.988	-11.38%	2.028	-9.61%	2.791	24.42%
mass of small firms	1.000	0.674	-32.56%	0.653	-34.69%	1.281	28.12%
initial output	0.909	0.872	-4.15%	0.966	6.28%	0.938	3.10%
CE Welfare change		-8.95%		-2.32%		7.24%	

	Benchmark	Early χ, ϕ	% change	Early ψ, τ	% change	All	% change
growth rate	1.98%	2.63%	33.06%	1.99%	0.52%	2.42%	22.58%
R&D intensity	1.85%	2.04%	10.25%	1.84%	-0.58%	1.43%	-22.60%
average markup	1.420	1.418	-0.09%	1.420	0.01%	1.280	-9.82%
std. dev. markup	0.428	0.425	-0.78%	0.428	-0.08%	0.319	-25.53%
labor share	0.611	0.603	-1.38%	0.603	-1.36%	0.647	5.88%
entry rate	0.105	0.105	0.00%	0.126	19.44%	0.126	19.44%
β(innov, relative sales)	0.756	0.700	-7.48%	0.757	0.07%	0.682	-9.79%
top point (intra-industry)	0.453	0.456	0.53%	0.454	0.13%	0.470	3.61%
average profitability	0.204	0.202	-1.01%	0.204	0.01%	0.153	-25.20%
average leader relative quality	0.643	0.609	-5.32%	0.640	-0.43%	0.621	-3.48%
std. dev. leader relative quality	0.148	0.149	0.42%	0.147	-0.80%	0.145	-2.08%
superstar innovation	0.147	0.212	44.44%	0.148	0.79%	0.184	25.27%
small firm innovation	0.019	0.036	90.19%	0.019	2.79%	0.032	66.57%
output share of superstars	0.515	0.522	1.23%	0.516	0.15%	0.454	-11.98%
avg. superstars per industry	2.244	2.527	12.64%	2.260	0.71%	2.475	10.32%
mass of small firms	1.000	1.581	58.09%	1.041	4.08%	1.000	0.00%
initial output	0.909	0.920	1.14%	0.930	2.27%	0.890	-2.13%
CE Welfare change		17.41%		1.74%		8.83%	

Notes: The table reports the change in model moments when setting parameter of interest back to its estimated level in early sub-sample while keeping other parameters fixed at their estimated value in the late sub-sample.

Table B17: Model Parameters and Target Moments — Cost-Weighted Markups

A. Parameter estimates

Parameter	*Description*	*Early sub-sample*	*Late sub-sample*
λ	innovation step size	0.3246	0.3185
η	elasticity within industry	16.7242	7.5737
χ	superstar cost scale	207.7198	72.9946
ν	small firm cost scale	1.3520	2.6121
ζ	competitive fringe ratio	0.6568	0.5972
ϕ	superstar cost convexity	4.1954	3.4056
ϵ	small firm cost convexity	3.8951	3.3837
τ	exit rate	0.1257	0.1052
ψ	entry cost scale	0.0089	0.0116

B. Moments

	Early sub-sample		Late sub-sample	
Target moments	Data	Model	Data	Model
growth rate	2.42%	2.42%	1.98%	1.98%
R&D intensity	2.38%	1.67%	2.50%	2.19%
average markup (cost-weighted)	1.168	1.168	1.240	1.241
std. dev. markup	0.299	0.274	0.396	0.367
labor share	0.656	0.651	0.648	0.638
entry rate	0.126	0.126	0.105	0.105
β(innovation, relative sales)	0.435	0.721	0.699	0.819
top point (intra-industry)	0.447	0.490	0.507	0.471
average profitability	0.137	0.127	0.147	0.173
average leader relative quality	0.750	0.555	0.747	0.576
std. dev. leader relative quality	0.224	0.129	0.222	0.127

Notes: The estimation is done with the simulated method of moments. Panel A reports the estimated parameters. Panel B reports the simulated and actual moments.

Table B18: Disentangling the Structural Transition — Cost-Weighted Markups

	Benchmark	Early η	% change	Early ζ	% change	Early ν, ϵ	% change
growth rate	1.98%	1.84%	-6.88%	1.75%	-11.63%	2.06%	4.26%
R&D intensity	2.19%	2.00%	-8.51%	1.57%	-28.18%	2.39%	9.57%
average markup (cost-weighted)	1.241	1.234	-0.59%	1.185	-4.52%	1.244	0.24%
average markup (sales-weighted)	1.335	1.317	-1.39%	1.256	-5.95%	1.337	0.15%
std. dev. markup	0.367	0.339	-7.68%	0.316	-13.90%	0.364	-0.77%
labor share	0.638	0.633	-0.75%	0.659	3.33%	0.628	-1.58%
entry rate	0.105	0.105	0.00%	0.105	0.00%	0.105	0.00%
β(innov, relative sales)	0.819	0.733	-10.52%	0.781	-4.65%	0.897	9.61%
top point (intra-industry)	0.471	0.464	-1.39%	0.466	-1.10%	0.478	1.48%
avg. profitability	0.173	0.170	-1.72%	0.141	-18.57%	0.172	-0.07%
avg. leader relative quality	0.576	0.618	7.38%	0.602	4.62%	0.529	-8.18%
std. dev. leader relative quality	0.127	0.135	6.66%	0.135	6.45%	0.110	-13.06%
superstar innovation	0.162	0.140	-13.47%	0.138	-15.04%	0.181	11.79%
small firm innovation	0.037	0.021	-42.20%	0.025	-31.38%	0.058	58.23%
output share of superstars	0.479	0.488	1.88%	0.413	-13.82%	0.488	1.90%
avg. superstars per industry	2.713	2.365	-12.84%	2.499	-7.91%	3.158	16.38%
mass of small firms	1.000	0.670	-33.01%	0.710	-28.98%	1.156	15.58%
initial output	0.821	0.771	-6.16%	0.846	2.99%	0.826	0.54%
CE Welfare change		-9.03%		-2.06%		2.56%	

	Benchmark	Early χ, ϕ	% change	Early ψ, τ	% change	All	% change
growth rate	1.98%	2.68%	35.31%	1.98%	-0.20%	2.42%	22.28%
R&D intensity	2.19%	2.49%	13.92%	2.18%	-0.38%	1.67%	-23.69%
average markup (cost-weighted)	1.241	1.241	0.02%	1.241	-0.01%	1.167	-5.93%
average markup (sales-weighted)	1.335	1.334	-0.09%	1.335	-0.01%	1.223	-8.41%
std. dev. markup	0.367	0.364	-0.81%	0.367	0.03%	0.274	-25.28%
labor share	0.638	0.629	-1.36%	0.629	-1.33%	0.669	4.89%
entry rate	0.105	0.105	0.00%	0.126	19.44%	0.126	19.44%
β(innov, relative sales)	0.819	0.741	-9.53%	0.817	-0.27%	0.721	-11.97%
top point (intra-industry)	0.471	0.471	-6.42E-05	0.471	-0.07%	0.490	4.00%
avg. profitability	0.173	0.170	-1.52%	0.173	-1.97E-05	0.127	-26.46%
avg. leader relative quality	0.576	0.536	-6.87%	0.577	0.30%	0.555	-3.52%
std. dev. leader relative quality	0.127	0.128	0.98%	0.127	0.54%	0.129	1.40%
superstar innovation	0.162	0.247	52.39%	0.161	-0.43%	0.208	28.47%
small firm innovation	0.037	0.063	69.97%	0.036	-1.65%	0.055	48.96%
output share of superstars	0.479	0.485	1.32%	0.478	-0.08%	0.415	-13.29%
avg. superstars per industry	2.713	3.102	14.33%	2.699	-0.53%	2.979	9.79%
mass of small firms	1.000	1.652	65.23%	0.973	-2.72%	1.000	0.00%
initial output	0.821	0.823	0.26%	0.821	-0.02%	0.795	-3.24%
CE Welfare change		19.13%		-0.01%		8.71%	

Notes: The table reports the change in model moments when setting the parameter of interest back to its estimated level in early sub-sample while keeping other parameters fixed at their estimated value in the late sub-sample.

Table B19: Static vs. Dynamic Costs of Higher Markups — Cost-Weighted Markups

	Static		Static+New Distribution		Dynamic	
	ΔW	CEWC	ΔW	CEWC	ΔW	CEWC
competitive fringe productivity	2.378	9.98%	2.378	9.98%	2.378	9.98%
relative wage	-1.129	-4.42%	-1.156	-4.52%	-1.156	-4.52%
output of superstar firms	-0.456	-1.81%	-0.487	-1.93%	-0.487	-1.93%
consumption/output	0.000	0.00%	0.000	0.00%	0.184	0.74%
output growth	0.000	0.00%	0.000	0.00%	-1.439	-5.59%
total	0.792	3.22%	0.736	2.99%	-0.519	-2.06%

Notes: The table decomposes the change in welfare to its constituent parts using equation (40). The first column of the table calculates and decomposes the change in welfare if we ignored the dynamics completely. Column 2 shows the change in consumption-equivalent welfare by each individual component. The third column repeats the same exercise as in column one with a single difference: we use the distribution of industries $\mu(\Theta)$ implied by the dynamic long-run change in response to the increase in ζ rather than retaining the late sample values. The fifth column displays the welfare decomposition for the full dynamic response.

Table B20: Distributional Implications of the Structural Transition — Cost-Weighted Markups

	Early η	Early ζ	Early ν, ϵ	Early χ, ϕ	Early ψ, τ	All
Benchmark CEWC	-9.03%	-2.06%	2.56%	19.13%	-0.01%	8.71%
Worker CEWC	-8.76%	1.83%	2.43%	19.38%	-0.11%	14.85%
Capitalist CEWC	-10.27%	-20.32%	3.16%	17.93%	0.45%	-20.14%
$\Delta\%\ C^{\text{capitalist}}/C^{\text{worker}}$	-1.65%	-21.75%	0.71%	-1.22%	0.56%	-30.46%

Notes: The table reports the change in model moments when setting the parameter of interest back to its estimated level in early sub-sample while keeping other parameters fixed at their estimated value in the late sub-sample.

Table B21: Model Parameters and Target Moments — Without Markup-Based Targets

A. Parameter estimates

Parameter	*Description*	*Early sub-sample*	*Late sub-sample*
λ	innovation step size	0.3125	0.3239
η	elasticity within industry	16.1641	8.4312
χ	superstar cost scale	219.815	78.0269
ν	small firm cost scale	1.3979	2.3002
ζ	competitive fringe ratio	0.6649	0.5992
ϕ	superstar cost convexity	4.3875	3.7336
ϵ	small firm cost convexity	2.9949	2.3166
τ	exit rate	0.1257	0.1052
ψ	entry cost scale	0.0080	0.0079

B. Moments

	Early sub-sample		Late sub-sample	
Target moments	Data	Model	Data	Model
growth rate	2.42%	2.42%	1.98%	1.98%
R&D intensity	2.38%	1.46%	2.50%	1.66%
labor share	0.6562	0.6562	0.6475	0.6475
entry rate	0.126	0.126	0.105	0.105
β(innovation, relative sales)	0.435	0.670	0.699	0.795
top point (intra-industry)	0.447	0.470	0.507	0.448
average profitability	0.137	0.122	0.147	0.165
average leader relative quality	0.750	0.611	0.747	0.718
std. dev. leader relative quality	0.224	0.151	0.222	0.176

Notes: The estimation is done with the simulated method of moments. Panel A reports the estimated parameters. Panel B reports the simulated and actual moments.

Table B22: Disentangling the Structural Transition — Without Markup-Based Targets

	Benchmark	Early η	% change	Early ζ	% change	Early ν, ϵ	% change
growth rate	1.98%	1.74%	-12.46%	1.45%	-26.77%	2.60%	31.23%
R&D intensity	1.66%	1.45%	-12.46%	0.96%	-42.17%	2.48%	49.18%
labor share	0.648	0.643	-0.76%	0.672	3.83%	0.633	-2.31%
entry rate	0.105	0.105	0.00%	0.105	0.00%	0.105	0.00%
β(innov, relative sales)	0.795	0.783	-1.53%	0.823	3.44%	0.821	3.25%
top point (intra-industry)	0.448	0.445	-0.78%	0.447	-0.36%	0.478	6.54%
avg. profitability	0.165	0.163	-1.66%	0.129	-21.76%	0.165	-3.88E-05
avg. leader relative quality	0.718	0.754	4.95%	0.778	8.30%	0.574	-20.08%
std. dev. leader relative quality	0.176	0.184	4.40%	0.188	6.55%	0.128	-27.14%
average markup	1.316	1.299	-1.28%	1.227	-6.75%	1.325	0.71%
std. dev. markup	0.367	0.343	-6.67%	0.305	-16.98%	0.359	-2.34%
superstar innovation	0.141	0.121	-14.01%	0.101	-28.29%	0.215	52.70%
small firm innovation	0.013	0.009	-26.40%	0.007	-42.29%	0.049	279.26%
output share of superstars	0.441	0.445	1.04%	0.365	-17.12%	0.474	7.60%
avg. superstars per industry	1.911	1.773	-7.21%	1.692	-11.45%	2.776	45.24%
mass of small firms	1.000	0.800	-19.97%	0.652	-34.81%	1.526	52.56%
initial output	0.794	0.758	-4.49%	0.822	3.53%	0.809	1.92%
CE Welfare change		-9.93%		-8.58%		18.10%	

	Benchmark	Early χ, ϕ	% change	Early ψ, τ	% change	All	% change
growth rate	1.98%	2.44%	23.29%	1.86%	-6.03%	2.42%	21.93%
R&D intensity	1.66%	1.77%	6.41%	1.55%	-6.83%	1.46%	-11.81%
labor share	0.648	0.638	-1.53%	0.640	-1.18%	0.674	4.16%
entry rate	0.105	0.105	0.00%	0.126	19.44%	0.126	19.44%
β(innov, relative sales)	0.795	0.703	-11.64%	0.805	1.18%	0.670	-15.76%
top point (intra-industry)	0.448	0.447	-0.39%	0.447	-0.32%	0.470	4.79%
avg. profitability	0.165	0.166	0.44%	0.165	-0.08%	0.122	-26.37%
avg. leader relative quality	0.718	0.682	-4.99%	0.738	2.69%	0.611	-14.91%
std. dev. leader relative quality	0.176	0.171	-2.92%	0.181	2.97%	0.151	-14.00%
average markup	1.316	1.317	0.13%	1.314	-0.12%	1.211	-7.98%
std. dev. markup	0.367	0.365	-0.59%	0.369	0.32%	0.267	-27.25%
superstar innovation	0.141	0.181	28.75%	0.131	-6.83%	0.199	41.31%
small firm innovation	0.013	0.021	62.30%	0.011	-12.79%	0.035	175.04%
output share of superstars	0.441	0.448	1.77%	0.436	-1.12%	0.397	-9.80%
avg. superstars per industry	1.911	2.094	9.59%	1.837	-3.90%	2.500	30.79%
mass of small firms	1.000	1.380	38.04%	0.811	-18.91%	1.000	0.00%
initial output	0.794	0.797	0.41%	0.792	-0.28%	0.797	0.39%
CE Welfare change		12.68%		-3.01%		12.25%	

Notes: The table reports the change in model moments when setting the parameter of interest back to its estimated level in early sub-sample while keeping other parameters fixed at their estimated value in the late sub-sample.

Table B23: Static vs. Dynamic Costs of Higher Markups — Without Markup-Based Targets

	Static		Static+New Distribution		Dynamic	
	ΔW	CEWC	ΔW	CEWC	ΔW	CEWC
competitive fringe productivity	2.601	10.96%	2.601	10.96%	2.601	10.96%
relative wage	-1.191	-4.65%	-1.277	-4.98%	-1.277	-4.98%
output of superstar firms	-0.385	-1.53%	-0.456	-1.81%	-0.456	-1.81%
consumption/output	0.000	0.00%	0.000	0.00%	0.205	0.82%
output growth	0.000	0.00%	0.000	0.00%	-3.317	-12.43%
total	1.025	4.19%	0.868	3.53%	-2.244	-8.58%

Notes: The table decomposes the change in welfare to its constituent parts using equation (40). The first column of the table calculates and decomposes the change in welfare if we ignored the dynamics completely. Column 2 shows the change in consumption-equivalent welfare by each individual component. The third column repeats the same exercise as in column one with a single difference: we use the distribution of industries $\mu(\Theta)$ implied by the dynamic long-run change in response to the increase in ζ rather than retaining the late sample values. The fifth column displays the welfare decomposition for the full dynamic response.

Table B24: Distributional Implications of the Structural Transition — Without Markup-Based Targets

	Early η	Early ζ	Early ν, ϵ	Early χ, ϕ	Early ψ, τ	All
Benchmark CEWC	-9.93%	-8.58%	18.10%	12.68%	-3.01%	12.25%
Worker CEWC	-9.69%	-4.58%	17.81%	12.47%	-3.07%	18.15%
Capitalist CEWC	-11.10%	-28.56%	19.56%	13.73%	-2.69%	-17.16%
$\Delta\%\ C^{\text{capitalist}}/C^{\text{worker}}$	-1.56%	-25.14%	1.49%	1.12%	0.40%	-29.88%

Notes: The table reports the change in model moments when setting the parameter of interest back to its estimated level in early sub-sample while keeping other parameters fixed at their estimated value in the late sub-sample.

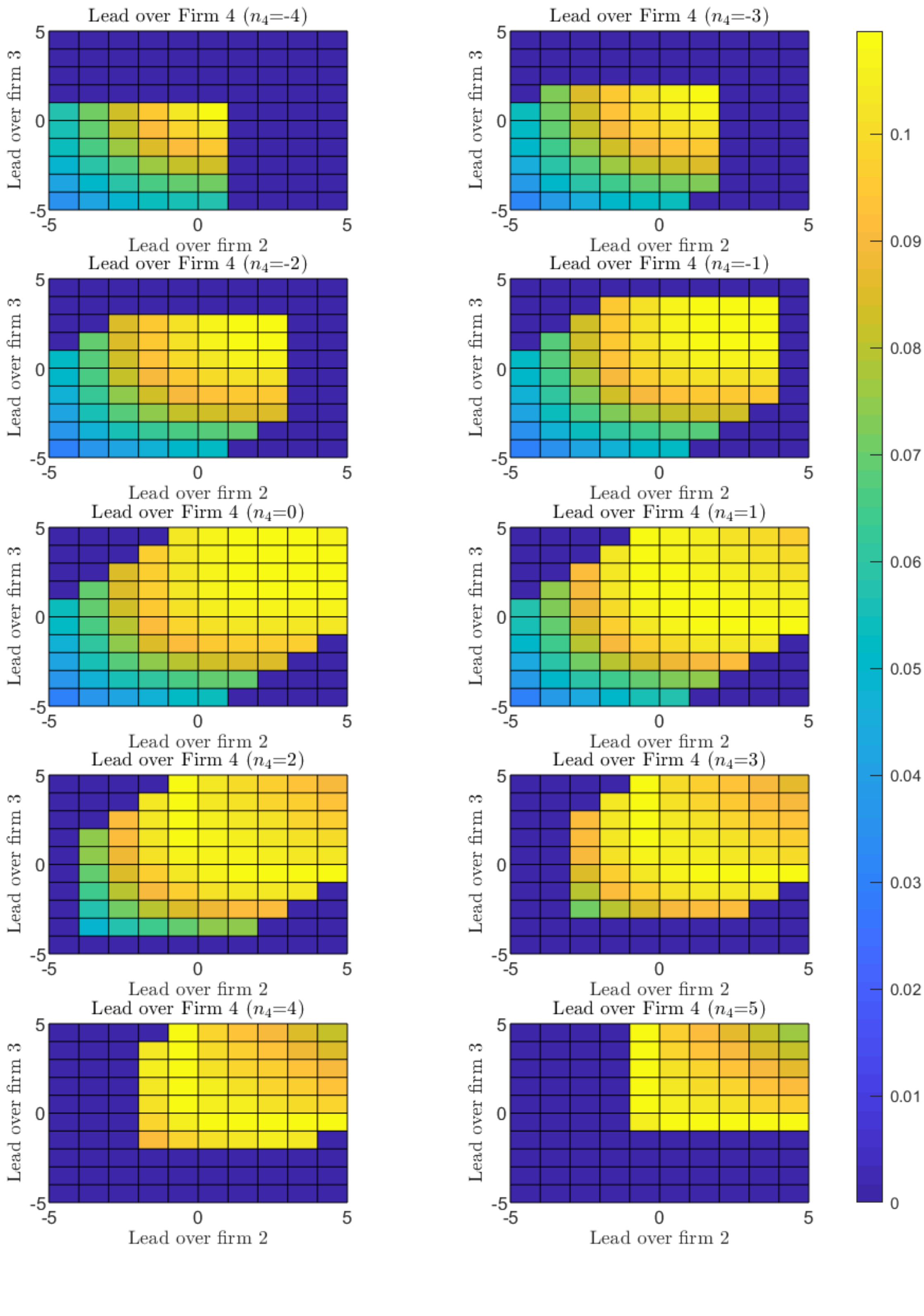

FIGURE B1: INNOVATION POLICY FUNCTION (N=4)

Notes: This figure displays the optimal innovation policy functions followed by the firms in an industry with four superstars. Each subfigure corresponds to the fourth competitor being a certain number of steps behind the current firm.