# Expectations of Fundamentals and Stock Market Puzzles

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## Abstract

We revisit several leading puzzles about the aggregate stock market by incorporating into a standard dividend discount model survey expectations of earnings of S&P 500 firms. Using survey expectations, while keeping discount rates constant, explains a significant part of "excess" stock price volatility, price-earnings ratio variation, and return predictability. The evidence is consistent with a mechanism in which good news about fundamentals leads to excessively optimistic forecasts of earnings, especially at long horizons, which inflate stock prices and lead to subsequent low returns. Relaxing rational expectations of fundamentals in a standard asset pricing model accounts for stock market anomalies in a parsimonious way.

## I. Introduction

In the dividend discount model, the price of a stock at time $t$ is given by:

$$P_t = \sum_{s=t+1}^{\infty} \frac{\mathbb{E}_t(D_s)}{R^s},$$

where $R$ is the constant required return and $\mathbb{E}_t(D_s)$ is the rational expectation of the dividend per share at time $s$. Research over the last few decades has shown that this model is a poor description of stock market movements. There are three main problems. First, as originally shown by Shiller (1981) and Leroy and Porter (1981), stock prices are much more volatile than dividends or earnings. In the dividend discount model, all price volatility should be due to news about these fundamentals. Second, the price dividend ratio has a low correlation with future growth in dividends or earnings (Campbell and Shiller 1988). This is also inconsistent with the dividend discount model, in which the price dividend ratio reflects rational forecasts of future growth. Third, stock returns are predictable: a high price dividend ratio today predicts low stock returns over a three to five year horizon (Campbell and Shiller 1988). This is inconsistent with another key assumption of the dividend discount model: constant required returns.

The Campbell-Shiller decomposition shows that these puzzles are related, in the sense that they can be reconciled under rational expectations if the required return is time-varying. Several models of time varying required returns have been proposed, based on disaster risk, recursive utility, and habit formation (Rietz 1988, Barro 2006, Gabaix 2012, Bansal and Yaron 2004, Campbell and Cochrane 1999). This approach is not without problems. It relies on changes in risk attitudes, which are hard to measure directly. It also predicts that investors should expect low returns when stocks are expensive. In survey data, however, the opposite is true: in good times investors expect high, not low, returns (Greenwood and Shleifer 2014). Contrary to rational expectations, such optimism is systematically disappointed in the future.

In this paper we address stock market puzzles by taking an orthogonal route: we hold required returns constant and assess how far we can go by relaxing the rationality of beliefs. We

discipline departures from rationality by using measured expectations of future growth of fundamentals. Recent work shows the promise of using such data. Bordalo, Gennaioli, La Porta, and Shleifer (BGLS 2019) find that analyst forecasts of firms' long-term earnings growth over-react to news about firm-level performance, and that such over-reaction helps explain the cross section of returns. De la O and Myers (2019) show that analyst short-term earnings forecasts for S&P 500 firms have strong explanatory power for the price earnings ratio.

These findings suggest that beliefs about growth in fundamentals may shape stock prices, and raise three questions. First, do measured beliefs about aggregate earnings growth depart from rationality, and if so how? Second, can these beliefs account for the stock market pricing puzzles? Third, can we document the mechanism linking non-rational beliefs to prices and to the predictability of returns?  In this paper, we show that analyst beliefs have remarkable explanatory power for all of the pricing puzzles, that beliefs about long term growth over-react, and that this over-reaction creates systematic forecast errors that help predict returns.

To organize the analysis, Section 2 offers a formulation of beliefs about dividend growth that nests several forms of non-rationality: noise, over-reaction, and under-reaction to fundamentals. We show that – under constant required returns – the puzzles are reconciled if beliefs at long horizons over-react to news. With over-reaction, beliefs are too volatile and systematically revert, so that price booms are followed by busts, consistent with the data.

The remainder of the paper empirically assesses this mechanism. Section 3 studies analysts' expectations.  We collect forecasts of short- and long-term earnings growth of S&P 500 firms over the period 1981-2018 and aggregate these forecasts into an index of market-level expected earnings growth. [2]  We begin by showing that errors in forecasted growth are predictable from forecast revisions, following the method of Coibion and Gorodnichenko (2015), suggesting systematic departures from rationality. To better understand these patterns,

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[2] Most of our analysis focuses on expectations of earnings, where data is available for a much longer period, and crucially includes expectations of long term growth.  We also examine dividend expectations where available.

we examine expectations in light of the process followed by actual earnings growth. We document three facts. First, earnings growth displays sharp short term reversals: after good times, earnings growth declines in the short run and resumes in the long run. Second, expectations of short and long run growth are forward looking, in the sense that they reflect these dynamics. Third, both short and long term expectations are too optimistic after high earnings growth. Together, these facts imply that short term beliefs under-react, while long term beliefs over-react, to news. In light of the model, long term beliefs may be especially helpful to explain stock market puzzles.

To assess whether measured beliefs can account for the puzzles, in Section 4 we compute an expectations based stock price index by using survey expectations of short and long term earnings growth and assuming a constant required return. The volatility of yearly changes in this index is remarkably close to that of actual price changes, suggesting that analyst beliefs can explain the excess volatility puzzle. On its own, this finding however does not yet imply that expectations of fundamentals help account for the other puzzles as well.

We next check whether analyst beliefs co-move with prices. First, we show that our index fits the stock price path remarkably well. Analyst beliefs do indeed appear to be a good proxy for market beliefs. To support this interpretation, we offer evidence that expectations of earnings are not backed out of market prices, but are formed independently by analysts.

Second, we assess whether measured expectations can account for variation in the price earnings ratio. We also assess the explanatory power for the price dividend ratio, in the period where dividend expectations are available. Our analysis confirms the findings of De la O and Myers (2019) that analyst expectations have strong explanatory power. Our expectations based indices account for 62% of price earnings ratio variation and 75% of price dividend variation in our sample. Importantly, relative to previous work we show that expectations about long term earnings growth play a key role in increasing explanatory power.

Finally, we ask whether measured beliefs help explain return predictability. As a preliminary exercise, Section 5 shows that expectations of long term earnings growth negatively predict future returns, while expectations of short term earnings growth do not. This is consistent with over-reaction of beliefs about long term growth: after strong fundamentals, long term beliefs become too optimistic, which inflates stock prices, but also leads to future disappointment and low returns. To directly assess this mechanism, we present several findings: i) sustained periods of high GDP growth and positive earnings surprises predict upward revisions of long term earnings growth, but ii) the revised forecasts are systematically disappointed, and iii) the entailed forecast errors predict low returns. This link from fundamental news to returns confirms the mechanism of our model. As we show in BGLS (2019), the same mechanism operates for individual firms. Over-reaction of beliefs about long term growth can account for return predictability both in the aggregate market and in the cross section of stocks.

Compare our analysis with models of time varying returns. Models of habit formation (Abel 1990, Constatinides 1990, Campbell and Cochrane 1999) generate volatility in required returns by linking risk aversion to recent consumption levels. Models of long run risk (Bansal and Yaron 2004, Bansal, Kiku and Yaron 2010) obtain it through variation in consumption risk. Time varying disaster risk (Gabaix 2012, Wachter 2013) generates volatility in the price to dividend ratio, and in some models also in required returns (Gabaix 2012). These approaches account neither for forecast errors nor for their explanatory power for returns.[3] These models are also inconsistent with survey evidence showing that expected returns are high in good times and when perceived disaster risk is low (Greenwood and Shleifer 2014, Giglio et al. 2019).

We are not the first to explore the link between non-rationality and stock market puzzles. Our approach is closest to models featuring misspecified beliefs about the data generating process (DeLong et al. 1990a, Barberis, Shleifer, and Vishny 1998, Daniel, Hirshleifer, and

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[3] Models of time varying disaster risk may entail forecast errors in the short samples available because disasters are rare events. However, such errors should materialize in the short term, not for long term growth, because in the long run the probability of disaster reverts closer to the baseline. This is contrary to what we find in the data.

Subrahmanyam 1998, 2001, Odean 1998). A growing literature has been bringing measured beliefs into the analysis of these questions (e.g., La Porta 1996, Frankel and Lee 1998, Lee, Myers, and Swaminathan 1999, Lee and Swaminathan 2000, Bachetta et al 2009, Koijen and Nieuwerburgh 2011). Relative to this work, we provide a unified framework for studying the joint evolution of fundamentals and beliefs, and apply it to explaining multiple stock market puzzles simultaneously, using direct evidence on over-reaction of beliefs to fundamental news.

An earlier literature looks at the extrapolation of fundamentals as a source of excess volatility, including the most directly related Barsky and DeLong (1993), but also Lakonishok et al. (1994) or Greenwood and Hanson (2015). An important advance in this literature is Nagel and Xu (2019), who show that a weighted average of past stock payout growth is negatively correlated with future returns. They explain this fact with a model in which, due to recency effects, investors adaptively forecast future dividends. Unlike this work, we show that expectations are forward looking but over-react to shocks to fundamentals, leading to systematic errors that help predict returns. All the main stock market puzzles, as well as the evidence on expectations, are captured by our approach.

Another approach to non-rationality is price extrapolation (e.g., DeBondt and Thaler 1985, DeLong et al. 1990b, Cutler, Poterba, and Summers 1990), which can generate significant price volatility and long run departures from rational prices (Barberis et al. 2015, 2018, Jin and Sui 2019). It is an open question whether extrapolation of prices or fundamental growth matters more for security prices (e.g., Daniel and Titman 2006). We show how far one can go with beliefs about fundamentals: expectations of earnings growth account for prices and predict returns, even when instrumented by news about fundamentals.

## 2. Non-Rational Beliefs and Stock Market Puzzles

Following Campbell and Shiller (1987), the log return $r_{t+1}$ obtained at $t+1$ is given by the log linearized expression:

$$r_{t+1} = \alpha p_{t+1} + (1-\alpha)d_{t+1} - p_t + k, \tag{1}$$

where $p_t$ and $p_{t+1}$ are the log stock prices at $t$ and $t+1$, $d_{t+1}$ is the log dividend at $t+1$, $k$ is a constant, and $\alpha = e^{pd}/(1+e^{pd}) < 1$ depends on the average log price dividend ratio $pd$.

By iterating Equation (1) forward and imposing the transversality condition, one obtains the Campbell-Shiller decomposition:

$$p_t - d_t = \frac{k}{1-\alpha} + \sum_{s\geq 0} \alpha^s g_{t+1+s} - \sum_{s\geq 0} \alpha^s r_{t+1+s}, \tag{2}$$

where $g_{t+s+1} \equiv d_{t+s+1} - d_{t+s}$ is the dividend growth between $t+s$ and $t+s+1$. Here the variation in the price to dividend ratio is due to expected variation in future fundamental growth, in future required returns, or in both. In the rational dividend-discount model, the required return is constant, ruling out variation in expected returns. Price movements then only reflect expectations of future growth $g_{t+1+s}$, and realized excess returns cannot be predicted. In contrast, allowing for time varying expected returns can address the puzzles: changes in expected future returns $r_{t+1+s}$ move prices, accounting not only for excess volatility and price dividend ratio variation, but also implying that current prices predict future realized returns.

However, even if required returns are constant, non-rational beliefs may help address the puzzles. To see this, denote (possibly non-rational) market expectations by $\mathbb{E}_t^M(.)$. Taking the expectation of Equation (2) yields the stock price:

$$p_t^M = d_t + \frac{k-r}{1-\alpha} + \sum_{s\geq 0} \alpha^s \mathbb{E}_t^M(g_{t+s+1}). \tag{3}$$

where $r$ is the constant required return. Suppose market beliefs are given by:

$$\mathbb{E}_t^M(g_{t+s+1}) = \mathbb{E}_t(g_{t+s+1}) + \epsilon_{M,t}, \tag{4}$$

where $\mathbb{E}_t(.)$ denotes rational expectations. In this convenient reduced form expression, departures from rationality are due to the expectations shock $\epsilon_{M,t}$. It follows an AR(1) process $\epsilon_{M,t} = \rho\epsilon_{M,t-1} + u_{M,t}$, where $\rho \in [0,1]$ captures the persistence of mistakes and $u_{M,t}$ is i.i.d.

normal with mean zero and variance $\sigma_M^2$. Beliefs have a forward-looking component, and are rational when $\sigma_M^2 = 0$, but in general are contaminated by a persistent mistake $\epsilon_{M,t}$.

Equation (4) nests several well-known departures from rationality as a function of how the expectational shock $\epsilon_{M,t}$ correlates with fundamental news about dividend growth. To see this, assume that dividend growth follows a covariance stationary process with moving average representation $g_t = \sum_{j \geq 0} \eta_j \epsilon_{t-j}$, where $\epsilon_t$ is the fundamental shock (Gaussian with mean zero and variance $\sigma^2$ and independent over time) and $\eta_j$ is the impulse response for $j$ periods ahead, satisfying square summability $\sum_{j \geq 0} |\eta_j|^2 < \infty$ and $\eta_0 = 1$. We use this highly flexible specification because it allows for short term reversals in earnings growth, which is an important feature of the data documented below.

Denote by $\sigma_{M,F}$ the covariance between the expectations shock $\epsilon_{M,t}$ and the fundamental shock $\epsilon_t$. If the expectations shock is independent of fundamentals, i.e., if $\sigma_M^2 > 0$ but $\sigma_{M,F} = 0$, forecasts at all horizons are distorted by persistent noise unrelated to fundamentals. Some early noise trading models have this feature (Black 1986, DeLong et al. 1990a).

If instead $\sigma_{M,F} > 0$ the belief distortion is positively correlated with the fundamental shock, yielding over-optimism at all horizons after high growth $\epsilon_t > 0$. This can produce the over-reaction to news present in diagnostic expectations (Bordalo et al. 2018, BGLS 2019),[4] but also in different formats in earlier models (e.g., DeLong et al. 1990b, Barberis et al. 1998).

Finally, if $\sigma_{M,F} < 0$, the belief distortion is negatively correlated with the current news $\epsilon_t$. This would temper the forward-looking component of expectations and, if $\sigma_{M,F}$ is not too

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[4] We prove our main results under the more general assumption that the distortion differs across different horizons, namely it is equal to $\delta_{s+1} \epsilon_{M,t}$, at $s+1$. The basic diagnostic formulation falls in this case, as it assumes that $\delta_{s+1}$ equals the true impulse response function. Diagnostic expectations further assume that: i) the belief shock is collinear with fundamentals, $u_{M,t} = \theta_M \epsilon_t$ with $\theta_M > 0$, and ii) the belief distortion is transient, $\rho = 0$. Recent work shows that an empirically more valid formulation of diagnostic expectations allows for persistent distortions $\rho > 0$, and for stronger over-reaction in the long run (see BGLS 2019, and D'Arienzo 2019). This is confirmed by our analysis here.

negative, and yield a muted response to news such as that arising from rational inattention (Sims 2003, Huang and Liu 2007, Bouchaud et al. 2019).[5]

How does the model in Equations (3) and (4) speak to the three pricing puzzles? Excess volatility refers to the fact that the variance of annual prices changes is too high relative to that entailed by rational expectations and constant returns (Shiller 1981, LeRoy and Porter 1981, Campbell and Shiller 1987). Define excess volatility by $\Delta V_{RE} = Var(p_t^M - p_{t-1}^M) - Var(p_t^{RE} - p_{t-1}^{RE})$, where $p_t^{RE}$ is the price prevailing under rational beliefs in Equation (4). Shiller's finding is the statement that $\Delta V_{RE} > 0$.

Consider next the price dividend ratio puzzle. In the dividend discount model, regressing future discounted growth $\sum_{s \geq 0} \alpha^s g_{t+s+1}$ on $p_t - d_t$ should yield a coefficient of 1. This is strongly rejected in the data, where the coefficient is well below 1 (Campbell and Shiller 1987, Cochrane 2011). In our model, errors in beliefs can account for this finding if they lead to prices $p_t^M$ such that the implied regression coefficient $\hat{\beta}_{M,P} = \frac{cov(\sum_{s \geq 0} \alpha^s g_{t+s+1}, p_t^M - d_t)}{var(p_t^M - d_t)}$ is below 1.

Finally, return predictability refers to the fact that when stocks are expensive ($p_t - d_t$ is high) future discounted returns measured by $\sum_{s=0}^{T-1} a^s r_{t+s+1}$ are low (Campbell and Shiller 1987, 1988). This cannot happen in a rational dividend discount model with constant returns. In our model, errors in beliefs help explain this finding if the regression coefficient $\hat{\beta}_{M,R} = \frac{cov(\sum_{s \geq 0} \alpha^s r_{t+s+1}, p_t^M - d_t)}{var(p_t^M - d_t)}$ is negative.

Under what conditions on beliefs do the three puzzles arise in the data?

**Proposition 1** *The non-rational dividend discount model in Equations (3) and (4) yields:*

a) *Excess volatility $\Delta V_{RE} > 0$ when $\sigma_M^2 + \mu(1 - \alpha)\sigma_{M,F} > 0$.*

b) *The price dividend ratio puzzle $\hat{\beta}_{M,P} \in (0,1)$, and the return predictability puzzle $\hat{\beta}_{M,R} < 0$ when $\sigma_M^2 + \omega(1 - \alpha)\sigma_{MF} > 0$.*

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[5] The forward looking nature of Equation (4) is however not consistent with models of backward looking beliefs.

*The model unifies stock market puzzles when departures from belief rationality $\sigma_M^2$ are large . If $\frac{\partial p_t^{RE}}{\partial \epsilon_t}$ is positive and large enough, then $\mu, \omega > 0$, so that higher $\sigma_{M,F}$ helps explain the puzzles.*

All proofs are collected in Appendix A. Departures from rationality in Equation (4) can account for excess volatility of prices, for price dividend ratio variation, as well as for predictability of returns, if beliefs move sufficiently in excess of actual dividends (i.e., if $\sigma_M^2$ is high enough). In principle, the puzzles can obtain if beliefs at all horizons are swayed by pure noise, namely $\sigma_M^2 > 0$ and $\sigma_{M,F} = 0$. But Proposition 1 also highlights the conditions under which the puzzles can be unified if belief distortions are correlated with fundamentals: if the rational price increases sufficiently with positive growth shocks – a realistic assumption – a positive correlation $\sigma_{M,F} > 0$ helps account for the puzzles. Importantly, to the extent that shocks to fundamentals are observable, this is a condition testable using expectations data.

The condition that $\sigma_M^2$ and $\sigma_{M,F}$ are large can be restated as "expectations overreact to news." Good news cause rational beliefs about future payouts to rise on average, increasing the rational price $p_t^{RE}$. If $\sigma_{M,F} > 0$, market beliefs about the future become even more optimistic, causing an even larger increase in $p_t^M$. The price change is excessive, the price to dividend ratio is inflated, which leads to low returns when beliefs revert in the future. All puzzles can then be reconciled if the market over-reacts, i.e., exaggerates the underlying rational patterns.[6]

Proposition 1 specifies conditions on market beliefs, which we do not observe. We instead observe analyst beliefs. In the next section we ask if and how these beliefs depart from rationality. Later we spell out the conditions under which analyst beliefs can shed light on pricing puzzles even if they imperfectly proxy for market beliefs.

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[6] The price to dividend ratio puzzle and the return predictability puzzle rely on the same condition, which follows from the Campbell-Shiller decomposition. Excess volatility relies on a related but distinct condition. Intuitively, excess volatility relies on the volatility of expectational shocks, while the price dividend ratio puzzle and return predictability puzzles also depend on their persistence.

## 3. Data and Evidence on Expectations

### 3.1 Data

*Forecasts of Dividends and Earnings.* We gather monthly data on stock market analyst forecasts for S&P500 firms from the IBES Unadjusted US Summary Statistics file, which surveys analysts during the third Wednesday of each month. We focus on (median) annual forecasts of dividends per share (*DPS*), earnings per share (*EPS*), and long-term earnings growth (*LTG*). IBES data on earnings is more extensive than on dividends, *i.e.* coverage starts on 3/1976 for *EPS*, 12/1981 for *LTG*, and on 1/2002 for *DPS*. Furthermore, forecasts for *EPS* are available at longer horizons than for *DPS*. In principle, IBES tracks annual forecasts for fiscal years one (typically, 4 months into the future) through five (typically 52 months into the future). In practice, *EPS (DPS)* forecasts beyond the third (second) fiscal year are often missing. We fill in for missing *EPS* forecasts by assuming that analysts expect *EPS* to grow at the rate *LTG* starting with the last non-missing *EPS* forecast. This is a sensible assumption since IBES defines *LTG* as the "…expected annual increase in operating earnings over the company's next full business cycle. In general, these forecasts refer to a period of between three to five years." We do not fill in missing values of *DPS* since forecasts for long-term growth in dividends are rare.

We aggregate *DPS* and *EPS* forecasts for firms in the S&P500 index to compute analogue measures for the index. In order to aggregate *DPS* (*EPS*) forecasts at the index level, we begin by linearly interpolating *DPS* (*EPS*) forecasts for each firm *i* and focus on forecasts at horizons ranging from one to five years (in one-year increments). Next, for each firm *i* and month *t*, we compute forecasts for the *level* of dividends (earnings) by multiplying the *DPS* (*EPS*) forecast by the number of shares outstanding at time *t* and we then sum these forecasts across all firms in the index.[7] Finally, we divide these forecasts for the level of dividends and earnings by the total numbers of shares in the S&P500 index.

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[7] We set to missing observations if the market cap of the firms for which we have forecasts at a given horizon is less than 90% of the market cap of the index. We otherwise compute expectations using all available firms.

Analysts may distort their forecasts due to agency conflicts. As we showed in previous work (BGLS 2019), this is unlikely to affect the time series variation in forecasts, which is key here. Furthermore, all brokerage houses typically cover S&P500 firms, so investment banking relationships and analyst sentiment are less likely to play a role in the decision to cover firms in the S&P500.[8] To further alleviate the concern about agency conflicts, and in particular to reduce the impact of outliers, we focus on median forecasts across analysts.

Bordalo et al (2020) show that consensus beliefs such as the median forecast are not ideal to study departures from rationality because informational frictions bias the consensus forecast toward under-reaction even if individual analysts over-react to their individual information. We do not address this issue here, but stress that – if anything – it makes our results stronger.

*Earnings surprise/returns data.* From the CRSP/COMPUSTAT merged file data, we collect data on earnings (income before extraordinary items) and dates when the *Wall Street Journal* published quarterly earnings releases (*rdq*). We aggregate earnings for the S&P500 in the same way as EPS forecasts. Following La Porta et al. (1997), we define the stock return that accrues over earnings' announcement dates as the compounded three-day stock return centered on *rdq*. We then aggregate all event returns for S&P500 firms in a calendar quarter by value weighting each event return by the firm's market capitalization at the beginning of the quarter. From CRSP, we get stock returns around *Wall Street Journal* dates, shares outstanding, stock prices. We also gather from CRSP data on S&P500 index membership and returns.

We obtain monthly data on price dividend and price earnings ratios and dividends for the S&P500 from Shiller's website and seasonally-adjusted GDP from the Saint Louis Fed. Data on expected returns for the S&P500 come from the quarterly survey of CFOs administered by John Graham and Campbell Harvey. Starting in October 2010, the survey tracks, among other things, the returns that CFOs expect for the S&P500 over the following 12 months and ten years.

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[8] For example, in December of 2018, nineteen analysts followed the median S&P500 firm, while four analysts followed the median firm not in index. Analysts are also less likely to rate as "buy" firms in the S&P500 index.

From the St. Louis Fed we gather data on the term spread and the credit spread. The term spread is the log difference between the gross yield of 10-year and 1-year US government bonds. The credit spread is the log difference between the gross yield of BAA and AAA bonds. We also use two standard proxies for risk: the surplus consumption ratio (Campbell and Cochrane 1999) and the consumption-wealth ratio (cay) (Lettau and Ludvigson 2001).[9]

## 3.2 Evidence on Measured Expectations

We first analyze measured expectations of earnings growth, which we use as a proxy for market expectations of earnings growth. In the next section, we rewrite the dividend discount model in terms of expectations of earnings growth, and assess whether the measured expectations analyzed here account for the market price.

We think about analysts' forecasts, denoted $\mathbb{E}_t^O$ (superscript $O$ means observed), through the lens of Equation (4):

$$\mathbb{E}_t^O(g_{t+s+1}) \equiv \mathbb{E}_t(g_{t+s+1}) + \epsilon_{O,t}, \tag{5}$$

Here, $\epsilon_{O,t}$ captures the expectational departures from rationality. Just like for market beliefs, we assume $\epsilon_{O,t}$ has persistence $\rho$ and its innovation is i.i.d. Gaussian, with mean zero, variance $\sigma_O^2$, and covariance $\sigma_{O,F}$ with $\epsilon_t$. However, distortions to analysts' beliefs may be distinct from those of market beliefs, and we denote the covariance between the analysts' $\epsilon_{O,t}$ and the market's $\epsilon_{M,t}$ $\sigma_{O,M}$. This is a key parameter: the validity of using analysts' beliefs as a proxy for market beliefs relies on a high covariance $\sigma_{O,M}$ between them.

Motivated by Proposition 1, which links departures from rationality to stock market puzzles, we test the rationality of measured expectations. A test introduced by Coibion and Gorodnichenko (CG 2015) assesses the predictability of forecast errors, defined as realized minus expected growth, from the current forecast revision. A positive regression coefficient

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[9] The data are available at https://faculty.chicagobooth.edu/john.cochrane/research/Data_and_Programs/index.htm and at sites.google.com/view/martinlettau/data respectively.

indicates under-reaction to the news that prompted the revision: forecasts move in the correct direction but not enough. An insufficient increase in optimism after good news predicts future positive surprise (a positive forecast error). In contrast, a negative coefficient implies over-reaction: forecasts move too much. An excessive increase in optimism after good news predicts future disappointment (a negative forecast error).[10] Proposition 1 highlights that over-reaction to growth shocks can help reconcile all three stock market puzzles.

Table 1 presents regressions of forecast errors on forecast revisions for earnings growth at short and long horizons. For short horizons, we consider forecasts about one and two year ahead earnings growth. For longer horizons, we use *LTG* forecasts. In line with its description, we view *LTG* as the forecast of average yearly earnings growth in the next 3, 4, and 5 years.

### Table 1.
### Forecast Errors and CG Revisions

The dependent variables are the forecast errors in year *t+1*, *t+2*, *t+3*, *t+4*, and *t+5*. Forecast errors beyond year *t+2* are defined relative to forecast for earnings growth in the long run ($LTG_t$). $E^O_t(e_{t+1}-e_t)-E^O_{t-1}(e_{t+1}-e_t)$ is the revision between year t and year *t-1* in the forecast for the one-year earnings growth rate in year *t+1*. $E^O_t(e_{t+2}-e_{t+1})-E^O_{t-1}(e_{t+2}-e_{t+1})$ is the revision between year *t* and year *t-1* in the forecast for the one-year earnings growth rate in year *t+2*. Finally, $\Delta LTG_t$ is the change in LTG between year *t* and *t-1*. We use monthly expectations data starting on December of 1982 (the first period with $\Delta LTG_t$) and data on realized earnings through December of 2018. Newey-West standard errors are reported in parentheses (the number of lags ranges from 12 in the first column to 60 in the last column). Superscripts: [a] significant at the 1% level, [b] significant at the 5% level, [c] significant at the 10% level.

| | $(e_{t+1}-e_t)$ - $E^O_t(e_{t+1}-e_t)$ | $(e_{t+2}-e_{t+1})$ - $E^O_t(e_{t+2}-e_{t+1})$ | $(e_{t+3}-e_t)/3$ - $LTG_t$ | $(e_{t+4}-e_t)/4$ - $LTG_t$ | $(e_{t+5}-e_t)/5$ - $LTG_t$ |
|---|---|---|---|---|---|
| $E^O_t(e_{t+1}-e_t)-E^O_{t-1}(e_{t+1}-e_t)$ | 0.0132 (0.1454) | | | | |
| $E^O_t(e_{t+2}-e_{t+1})-E^O_{t-1}(e_{t+2}-e_{t+1})$ | | 3.2361 (1.9961) | | | |
| $\Delta LTG_t$ | | | -10.0734[a] (2.6286) | -6.8184[a] (1.8928) | -5.1003[a] (1.8928) |
| Observations | 412 | 400 | 397 | 385 | 373 |
| Adjusted $R^2$ | 0% | 3% | 25% | 21% | 19% |

In the case of short term expectations – one or two years ahead – the regression coefficient is positive. Revisions at short term horizons are if anything insufficient rather than

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[10] Here we are considering consensus (i.e. median) beliefs. A positive consensus coefficient is compatible with rationality of individual forecasts when forecasters' information is noisy (CG 2015). Instead, a negative consensus coefficient is unambiguously indicative of over-reaction (BGLS 2019).

excessive, and hence supportive of under-reaction. At longer horizons, instead, there is strong evidence of excessive updating and hence of over-reaction: upward revisions of $LTG$ predict future disappointment, while downward revisions predict positive surprises. Table 1 confirms, at the level of the S&P index, the over-reaction of firm-level $LTG$ forecasts originally documented by BGLS (2019).

What does Table 1 tell us about belief distortions? In Appendix A we show that the CG coefficient entailed by the beliefs in Equation (5) at horizon $s + 1$ is negative, indicating over-reaction if:

$$(1 + \rho)\eta_{s+1}\sigma_{O,F} + \sigma_O^2 > 0, \tag{6}$$

while it is positive when the sign is reversed. If distortions were due to noise alone, $\sigma_{O,F} = 0$, beliefs over-react at all horizons. This cannot deliver the short term under-reaction documented in Table 1. Instead, if $\eta_{s+1}\sigma_{O,F} \neq 0$, the model can account for both under- and over-reaction, depending on the correlation between distortions and fundamentals as well as on the data generating process for earnings growth.

We know that at short horizons earnings growth displays reversals, namely $\eta_{s+1} < 0$ and then gradual convergence to the long run mean (Kothari, Lewellen and Warner 2006). These reversals are confirmed in our data. Table 2 reports aggregate earnings growth in the year after periods in the top and bottom 30% of cyclically adjusted earnings growth, $e_t - cae_{t-1}$.

**Table 2.**
**Short Term Earnings Growth Reversals**

In December of each year $t$ between 1981 and 2014, we rank observations into deciles based on $e_t$-$cae_{t-1}$ and report the average one-year growth rate of earnings in $t$+1, $t$+2, …, $t$+5 for observations in the top 30% and bottom 30%.

|  |  | $e_{t+1}$-$e_t$ | $e_{t+2}$-$e_{t+1}$ | $e_{t+3}$-$e_{t+2}$ | $e_{t+4}$-$e_{t+3}$ | $e_{t+5}$-$e_{t+4}$ |
|---|---|---|---|---|---|---|
| $e_t$-$cae_{t-1}$ | Low | 0.31 | 0.21 | 0.08 | 0.05 | 0.05 |
|  | High | -0.24 | 0.02 | 0.09 | 0.17 | 0.04 |
|  | High-Low | -0.55 | -0.19 | 0.01 | 0.12 | -0.01 |

The evidence of short term reversals is clear. After strong fundamentals, short term growth is very negative and then gradually recovers. After weak fundamentals, short term

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growth is high and then returns to normal. The same finding emerges as we estimate the MA model of Section 2 (Table C.5, Appendix C). This exercise shows, in line with Table 2, that there is reversal in the first two years $\eta_1, \eta_2 < 0$, and then recovery, $\eta_s \geq 0, \ s \geq 3$.

**Proposition 2.** *For an earnings growth process featuring short term reversals ($\eta_1, \eta_2 < 0$) and then recovery ($\eta_s \geq 0, \ s \geq 3$), the patterns of Tables 1 can obtain if belief distortions are positively correlated with fundamentals $\sigma_{O,F} > 0$. In this case, after strong fundamentals: i) short term growth is revised downward while long term growth is revised upward, and ii) these revisions predict positive short term errors and negative long term errors.*

Our model shows that same analysts can under-react when forecasting short term growth and over-react when forecasting long term growth. When $\sigma_{O,F} > 0$, beliefs are forward-looking, but too optimistic after good news. Downward revisions of short term growth are insufficient, and upward revisions of long term growth are excessive. By Proposition 1, this suggests that beliefs about long run growth play a key role in accounting for the stock market puzzles.

To assess the predictions of Proposition 2, we perform a two-stage estimation exercise. In the first stage, we regress forecast revisions on proxies for fundamental news (prediction *i*). In the second stage, we regress the forecast errors on the predicted revisions (prediction *ii*).

We consider three proxies for fundamental news: earnings surprise relative to cyclically adjusted earnings ($eps_t - caeps_{t-1}$), yearly changes in five years GDP growth, and the average return on Wall Street Journal announcements, also averaged over years $t - 5$ to $t$. Table 3 reports the estimation results using expectations of short and long term growth.

### Table 3.

The table presents results of IV regressions. The dependent variable in the first-stage regressions are revisions in growth forecasts. Specifically, $E^O_t(e_{t+1}-e_t)-E^O_{t-1}(e_{t+1}-e_t)$ is the revision between year $t$ and year *t-1* in the forecast for the one-year earnings growth rate in year *t+1*. $E^O_t(e_{t+2}-e_{t+1})-E^O_{t-1}(e_{t+2}-e_{t+1})$ is the revision between year $t$ and year *t-1* in the forecast for the one-year earnings growth rate in year *t+2*. Finally, $\Delta LTG_t$ is the change in the forecast for earnings growth in the long run ($LTG_t$) between year $t$ and *t-1*. The independent variables include: (a) the log of earnings in year $t$ relative to the cyclically-adjusted earnings in year *t-1*, (b) the change between year $t$ and *t-1* in the 5-year growth rate of GDP per capita, and (c) the weighted average of the cumulative

return earned by firms in the S&P500 during earnings announcement days in the preceding 20 quarters.  Please see text for details. The dependent variables in the second-stage regressions in Panel B are forecast errors in year $t+1$, $t+2$, $t+3$, $t+4$, and $t+5$. Forecast errors beyond year $t+2$ are defined relative to $LTG_t$.   The independent variables in the second-stage regressions are the instrumented values of $E^O_t(e_{t+1}-e_t)-E^O_{t-1}(e_{t+1}-e_t)$ in column [1], $E^O_t(e_{t+2}-e_{t+1})-E^O_{t-1}(e_{t+2}-e_{t+1})$ in column [2], and $\Delta LTG_t$ in columns [3]-[5]. We report results using quarterly data starting on December of 1982 (the first period with $\Delta LTG_t$) through December of 2018.  Newey-West standard errors are reported in parentheses (with 4 lags).  We report Newey-West standard errors in parentheses (with 4 lags in columns 1 and 2, 12 in column 3, 16 in column 4, and 20 in column 5).  Superscripts: [a] significant at the 1% level, [b] significant at the 5% level, [c] significant at the 10% level.

### Panel A:  Predicting Changes in Growth Expectations

| | (1) $\mathbb{E}^O_t[e_{t+1}-e_t] - \mathbb{E}^O_{t-1}[e_{t+1}-e_t]$ | (2) $\mathbb{E}^O_t[e_{t+2}-e_{t+1}] - \mathbb{E}^O_{t-1}[e_{t+2}-e_{t-1}]$ | (3) $\Delta LTG_t$ |
|---|---|---|---|
| $e_t - cae_{t-1}$ | -0.6807[a] | -0.0205[a] | 0.4373[a] |
| | (0.1200) | (0.0025) | (0.0936) |
| 1-year Change in GDP growth | 0.0539 | 0.0041 | 0.1634 |
| | (0.0906) | (0.0029) | (0.1025) |
| 5-year event returns | 0.0629 | 0.0061[a] | 0.0998 |
| | (0.0763) | (0.0017) | (0.1000) |
| Constant | 0.0075 | 0.0157[a] | -0.0000 |
| | (0.1059) | (0.0025) | (0.1475) |
| Observations | 143 | 143 | 145 |
| Adjusted R² | 39% | 49% | 33% |

### Panel B:  IV Regressions for forecast errors

| | (1) $e_{t+1}-e_t$ $- \mathbb{E}^O_t[e_{t+1}-e_t]$ | (2) $e_{t+2}-e_{t+1}$ $- \mathbb{E}^O_t[e_{t+2}-e_{t-1}]$ | (3) $\frac{[e_{t+3}-e_t]}{3} - LTG_t$ | (4) $\frac{[e_{t+4}-e_t]}{4} - LTG_t$ | (5) $\frac{[e_{t+5}-e_t]}{5} - LTG_t$ |
|---|---|---|---|---|---|
| $\mathbb{E}_t[FR_1|All]$ | 0.0916 | | | | |
| | (0.1298) | | | | |
| $\mathbb{E}_t[FR_2|All]$ | | 0.2746[c] | | | |
| | | (0.1516) | | | |
| $\mathbb{E}_t[\Delta LTG_t|All]$ | | | -1.2881[a] | -1.2631[a] | -1.3316[a] |
| | | | (0.3165) | (0.2910) | (0.3246) |
| Constant | 0.0061 | -0.0221 | -0.1807 | -0.1472 | -0.1695 |
| | (0.1610) | (0.1429) | (0.1894) | (0.1898) | (0.2114) |
| Observations | 139 | 135 | | | |
| Modified F-stat | 20.458 | 30.737 | 133 | 129 | 125 |
| Sargan overid. Stat | 3.9699 | 4.9264[c] | 18.495 | 18.766 | 13.562 |
| AR Confidence Interval | [-.13, .35] | [-0.13,0.47] | 1.0754 | 1.8006 | 1.5891 |
| Reduced form Adjusted R² | 2% | 11% | 53% | 55% | 59% |

Panel A shows that short term beliefs are revised downward after good fundamental shocks, while long term beliefs are revised upward, consistent with prediction *i*, suggesting that forecasts are forward looking, in line with the rational component of beliefs in Equation (5).[11] Panel B then shows that after strong fundamentals the *downward* revision of short-term growth is insufficient: predicted growth is above subsequent realizations (Columns 1 and 2). In contrast, the contemporaneous *upward* revisions of long-term beliefs is excessive (Columns 3-5), as illustrated in Figure 1. This suggests that analysts become excessively optimistic after good news at all horizons.
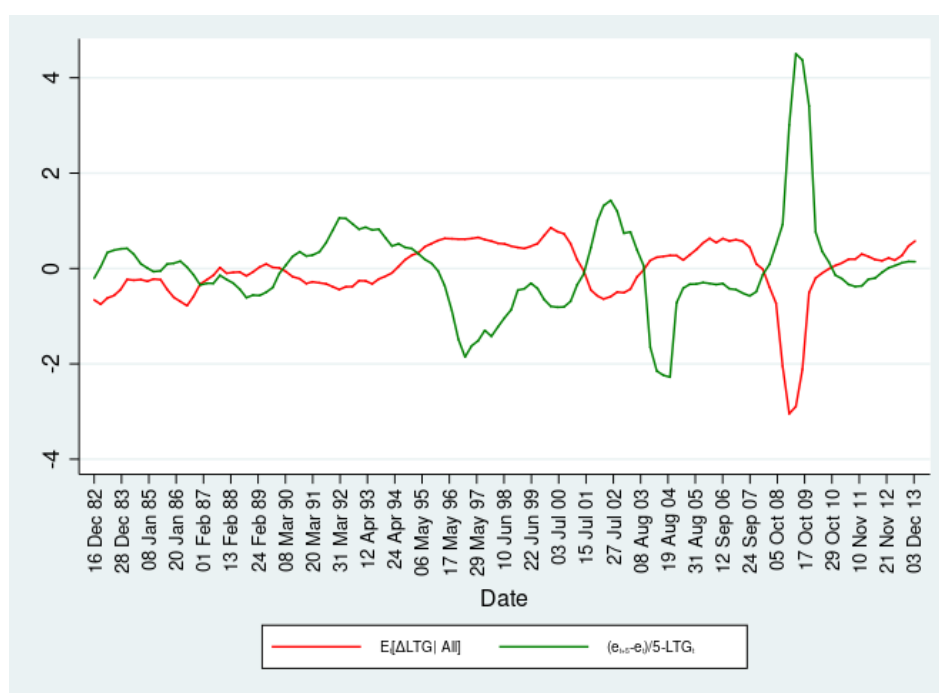


**Figure 1.** We plot the 5-year forecast error (green line) and the predicted one-year change in long term growth in earnings using the specification in Column (3) of Table 3, Panel A (using the log ratio of earnings to cyclically-adjusted earnings in year t-1, the one year change in the 5-year growth rate in GDP, and the 5-year cumulative returns around earnings announcement days).

To summarize, Tables 1 and 3 show that analysts' expectations systematically depart from rationality. Beliefs are forward looking, but are characterized by excess optimism after

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[11] The fact that the coefficients in Table 1, and also in Table 2 Panel B below, have magnitudes above one reflects the fact that movements in LTG are on average followed by movements in growth rates in the opposite direction. This is a non-linear phenomenon concentrated in cases of strong recoveries after poor performance and drops in *LTG*.

positive growth shocks, $\sigma_{O,F} > 0$ in Equation (5). This leads to over-reaction for long run expectations which, according to Proposition 1, plays a key role in accounting for the stock market puzzles. In contrast, under-reaction of short term expectations makes it unlikely to explain those findings.[12]

To conclude this Section, we note that while a cognitive account of Equation (5) is beyond the scope of this paper, the entailed excess optimism after strong fundamentals is consistent with the diagnostic expectations mechanism proposed by BGLS (2019). In that model, a firm's strong earnings growth causes analysts to drastically revise up the probability that it is a "Google", which entails excess optimism about earnings growth at all horizons. We return to this connection in the Conclusions.

## 4. Excess Volatility and Price Dividend Variation

We use analyst forecast data to construct the synthetic price:

$$p_t^O = d_t + \frac{k-r}{1-\alpha} + \sum_{s \geq 0} \alpha^s \mathbb{E}_t^O(g_{t+s+1}), \tag{7}$$

obtained by plugging analyst forecasts of earnings growth in the dividend discount model. We then assess whether $p_t^O$: $i$) displays time series volatility comparable to that of market prices, and $ii$) is strongly correlated with variation in stock prices. We also address the concern that the correlation between $p_t^O$ and the market price may be spuriously due to the fact that analysts mechanically infer one particular expectations measure, $LTG$, from market prices.

In our analysis, we focus on earnings based price measures because expectations for earnings growth are available for a longer sample period and also at longer horizons (i.e. $LTG$). We also perform robustness checks by looking at price dividend proxies constructed using expectations about future dividends.

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[12] The idea that expectations of short term growth display the reversals of Table 3, Panel A features in De la O and Myers' (2019) theoretical analysis.

To build a synthetic (log) price $p_t^O$ based on measured expectations, we first rewrite Equation (7) in terms of expectations of future earnings growth:

$$p_t^O \approx e_t + \frac{\tilde{k} - r}{1 - \alpha} + \sum_{s \geq 0} \alpha^s \mathbb{E}_t^O \left( \Delta e_{t+j+1} \right)$$

where $\tilde{k} = k + (1 - \alpha)\, de$ where $de$ is the average log payout ratio. This expression holds in the limit where $\alpha$ is close to 1. We next implement this expression using the formula:

$$p_t^O = e_t + \frac{\tilde{k} - r}{1 - \alpha} + \ln\left( \frac{\mathbb{E}_t^O EPS_{t+1}}{EPS_t} \right) + \sum_{j=1}^{10} \alpha^{j-1}\, \mathbb{E}_t^O \Delta e_{t+j+1} + \frac{\alpha^{10}}{1 - \alpha}\, g \qquad (8)$$

where we set $r$ to 8.48% (the sample mean), $\alpha$ to 0.9774 (i.e., $\frac{1}{1 + \exp\left( -pd \right)}$), and $\tilde{k}$ to 0.0927. We measure expected growth between $t$ and $t + 1$ and between $t + 1$ and $t + 2$ using forecasted earnings. For longer horizons we use $LTG$, which is analysts' expected growth for the next three to five years. Forecasts for the longer term are not available but a reasonable hypothesis is that growth expectations gradually revert from $LTG$ toward an average long-run level $g$. Accordingly, we use $LTG$ to proxy for the expected growth rate between $t + 3$ and $t + 10$, and set growth expectations beyond $t + 11$ to a level consistent with the average observed stock price.[13,14] In the main analysis we use nominal values, but in Appendix B (Figure B.1) we show that our results are robust when we account for inflation.

We compare of our expectations-based index with a rational (log) price benchmark $p_t^{RE}$ computed following Shiller's (2014) methodology. Starting from the terminal price $p_T^* = \ln\left( \frac{D_T}{r-g} \right)$ at $T$, $p_t^{RE}$ is computed backwards, using the actual earnings over time, and setting $g = 5.81\%$ and $r = 8.48\%$ to reflect sample averages. Setting $T = 2019$, we obtain:

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[13] We use $LTG$ to capture growth until $t + 10$ because in our sample year the average duration of a business cycle is about 10 years. We obtain virtually identical results if we infer growth expectations beyond $t + 5$ by applying the observed decay of observed cyclically adjusted earnings to $LTG_t$. The results are in Appendix B.

[14] Specifically, the long-term growth rate $g$ is the average of the growth rate $g_t$ which solves $p_t = e_t + \frac{\tilde{k} - r}{1 - \alpha} + \ln\left( \frac{\mathbb{E}_t^O EPS_{t+1}}{EPS_t} \right) + \sum_{j=1}^{10} \alpha^{j-1} \mathbb{E}_t^O \Delta e_{t+j+1} + \frac{\alpha^{10}}{1 - \alpha} g_t$.

$$p_t^{RE} = e_t + \sum_{s=t}^{T-1} a^{s-t}(e_{s+1} - e_s) + \alpha^{T-t} * (p_{2019}^* - d_{2019}) + \sum_{s=t}^{T-1} a^{s-t}(\tilde{k} - r).$$

## 4.1 Excess Volatility Puzzle

Proposition 1 identifies a condition under which market expectations generate excess volatility of asset prices. Corollary 1 shows that the price index $p_t^O$ displays excess volatility if *measured* beliefs satisfy the same condition.

**Corollary 1.** *Define* $\Delta V_O = Var(p_t^M - p_{t-1}^M) - Var(p_t^O - p_{t-1}^O)$. *Analyst beliefs reduce excess volatility relative to rational beliefs, namely* $\Delta V_O < \Delta V_{RE}$ *if and only if* $\sigma_O^2 + \mu(1-a)\sigma_{O,F} > 0$, *where* $\mu$ *is as in Proposition 1.a.*

Recall from Tables 1 and 2 that analyst beliefs are too optimistic after good fundamentals, suggesting $\sigma_{O,F}$ is positive and the condition of Corollary 1.a holds. To assess this conjecture, we compare the standard deviation of price changes computed using $p_t^{RE}$ and $p_t^O$ to the standard deviation of actual price changes. Table 4 below reports the results.

**Table 4.**
**Volatility of log price changes**

The standard deviation of one-year change in: (1) the log of the price of the S&P500 index ($\Delta p$), (2) the rational benchmark index ($\Delta p_t^{RE}$), (3) the long-term index ($\Delta p_t^O$) as defined by equation 8 in the text, and (4) the short-term index that incorporates one-year ahead expectations and assumes constant growth after that ($\Delta p_t^{OS}$). The sample period ranges from 12-1982 to 12-2018 and has 424 monthly observations.

| | $\Delta p$ | $\Delta p^{RE}$ | $\Delta p^O$ | $\Delta p^{OS}$ |
|---|---|---|---|---|
| $\sigma$ | 15.2% | 0.7% | 14.6% | 12.0% |

The large gap between volatility of actual prices (Column 1) and the volatility of rational prices (Column 2) is Shiller's puzzle. Our expectations-based index $p_t^O$ dramatically improves the prediction for price volatility (Column 3), explaining nearly all of the observed volatility.

To what extent is year on year price volatility driven by short versus long term expectations? We compute a short-term index, denoted $p_t^{OS}$, that incorporates one year ahead

expectations in Equation (6) and assumes constant growth after that. This index accounts for a price volatility of 12% (Column 4), which is sizable. Short term expectations are indeed quite volatile. Still, adding long term expectations significantly increases price volatility, by another 20%. Table B.2 in Appendix B shows that results are similar if we use a dividends-based price index (in which we continue to use $LTG$ to proxy for long term dividend growth).

One concern on Table 4 is that the price process is non-stationary (Marsh and Merton 1986). To address this issue, we compute the variance of price changes following Shiller (2014). Campbell and Shiller (1987) offer a more systematic approach. They note that under plausible assumptions, dividends and prices are co-integrated. Thus, $P_t - \frac{D_t}{R}$ is stationary, for:

$$P_t - \frac{D_t}{R} = \frac{1}{r} \sum_{s \geq 0} \left(\frac{1}{1+r}\right)^s \mathbb{E}_t(g_{t+s+1}).$$

The logic of Proposition 1 can now be used to compare the volatility $P_t - \frac{D_t}{R}$ to that obtained using $P_t^O - \frac{D_t}{R}$. Table B.3 in Appendix B presents the results for the different specifications above. Similarly to Table 4, incorporating expectations of long-term earnings growth captures a significant share of observed variation in $P_t - \frac{D_t}{R}$.

The finding that analyst beliefs are sufficiently volatile to account for Shiller's puzzle does not on its own imply that such beliefs are highly correlated with market beliefs. We next assess whether this is the case, comparing our expectations-based index $p_t^O$ to actual price levels $p_t^M$. Figure 2 plots the market price (green line) and the synthetic price $p_t^O$ (red line) against the rational price $p_t^{RE}$ (blue line).
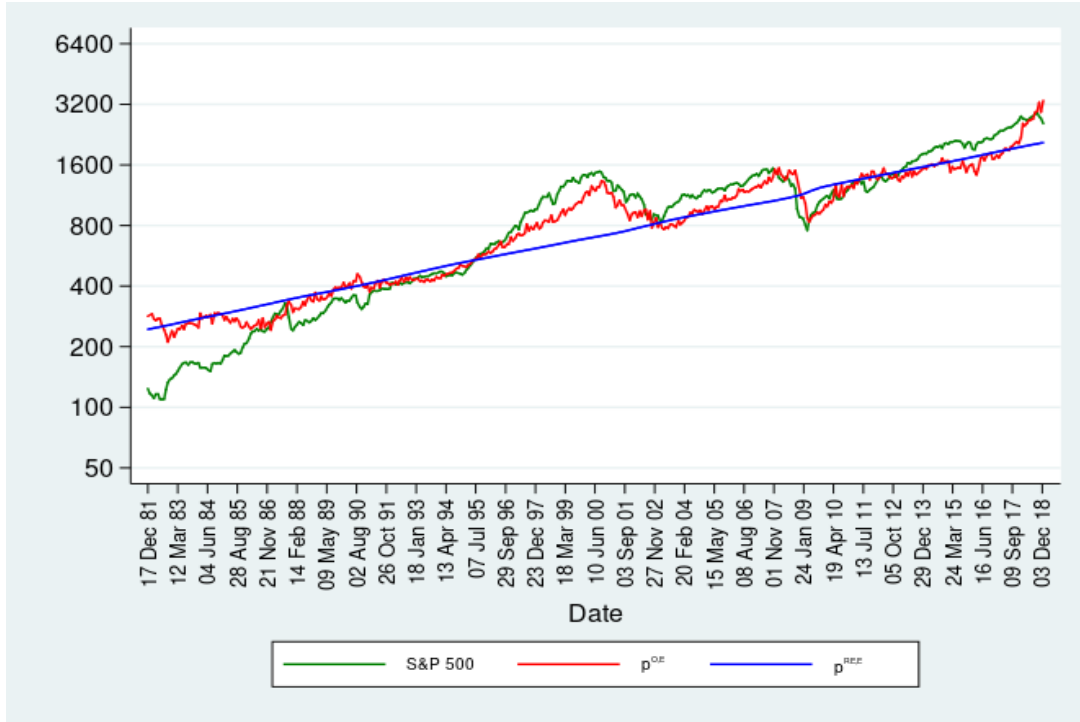
**Figure 2.** We plot the S&P500 index (green line), the rational benchmark index ($p_t^{RE}$, blue line) and the long-term index ($p_t^O$, red line) as defined by equation 8 in the text.

The actual and the synthetic prices are remarkably well aligned. The high frequency volatility of the synthetic index and of the actual price is comparable. Crucially, they also move in tandem relative to the rational price: when the price is above the rational benchmark, so is the synthetic price; and vice versa when the price is below the rational benchmark. This is an indication that observed beliefs may proxy well for market beliefs. Formally:

**Corollary 2.** $cov(p_t^M - p_t^{RE}, p_t^O - p_t^{RE}) > 0$ *if and only if* $\sigma_{OM} > 0$.

With constant discount rates, $p_t^O$ and $p_t^M$ display correlated departures from the rational price $p_t^{RE}$ only if measured beliefs correlate with market beliefs. The fact that these departures are persistent further suggests that expectations errors are persistent, $\rho > 0$.

One concern here is that analyst beliefs are indirectly capturing time varying discount rates rather than market beliefs. While financial analysts are unlikely to confuse earnings growth with rates of return, a connection between estimated earnings growth and discount rates may spuriously arise if analysts use stock prices to *infer* market expectations of fundamentals. This

concern does not apply to short term forecasts, which track well documented short term reversals, but it may be more relevant for the $LTG$ forecast. We consider this concern next.

## 4.2 Are Analysts Inferring $LTG$ from Stock Prices?

Inferring expectations from prices requires information on, or assumptions about, required returns. We consider two cases. In the first, analysts know the market expected return $\mathbb{E}_t^M[\sum_{s\geq0} \alpha^s r_{t+s+1}]$, and can recover the true discounted dividend growth the market expects:

$$\mathbb{E}_t^O\left[\sum_{s\geq0} \alpha^s g_{t+s+1}\right] = p_t^M - d_t - \frac{k-r}{1-\alpha} + \mathbb{E}_t^M\left[\sum_{s\geq0} \alpha^s r_{t+s+1}\right].$$

In this case, analyst beliefs coincide with market expectations, $\epsilon_{O,t} = \epsilon_{M,t}$, and $LTG$ captures exactly the market expectations about long term growth.

In the second, more problematic case, analysts fit growth expectations to justify market prices assuming that the rate of return is constant at $r$, but that assumption is erroneous. In this case, the analyst discounted expected dividend growth is given by:

$$\mathbb{E}_t^O\left[\sum_{s\geq0} \alpha^s g_{t+s+1}\right] = p_t^M - d_t - \frac{k}{1-\alpha}. \tag{9}$$

The key implication here is that *any* price movement causes a revision of analysts' earnings growth, regardless of whether the price movement reflect changes in beliefs or in required returns. To assess this possibility, we perform two exercises.

First, we consider time variation in the aggregate $LTG$ for S&P500 firms. We exploit the finding in Table 3 that $LTG$ is partly driven by fundamentals such as GDP growth, earnings above their cyclically adjusted value, and cumulative announcement returns as covered by the WSJ. Does $LTG$ also respond to stock price movements that are not predicted by fundamentals? If the answer is no, there is little evidence that $LTG$ mechanically responds to changes in the aggregate stock index driven by changes in required rates of return.

To assess this possibility, we perform a two stage exercise. In the first stage we regress the market log price dividend ratio on the same measures of fundamentals used in Table 3, i.e. changes in GDP growth and cumulative returns on earnings announcements over the previous five years. We then check whether the first stage residuals help explain $LTG$.

**Table 5**

The dependent variable in Panel A is the log price-to-dividend ratio. The independent variables are the log of earnings in year $t$ relative to the cyclically-adjusted earnings in year $t-1$, the change between year $t$ and $t-1$ in the 5-year growth rate of GDP per capita, and the weighted average of the cumulative return earned by firms in the S&P500 during earnings announcement days in the preceding 20 quarters. The dependent variable in Panel B is the forecast for long-term growth in earnings (LTG$_t$). The independent variables are the forecast errors generated by the matching regression in Panel A. We report results using quarterly data starting on December of 1981 through December of 2018. We adjust standard errors for serial correlation using the Newey-West procedure (with 12 lags). Superscripts: [a] significant at the 1% level, [b] significant at the 5% level, [c] significant at the 10% level.

**Panel A:  Dependent Variable is Price Dividend Ratio**

|  | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| $e_t$-cae$_{t-1}$ | 0.3945[b] | | | 0.0702 |
| | (0.1894) | | | (0.1421) |
| 5-year WSJ return | | 0.4529[a] | | 0.3148[b] |
| | | (0.1413) | | (0.1314) |
| $\Delta$ 5-year GDP pc growth | | | 0.5726[a] | 0.4963[a] |
| | | | (0.1710) | (0.1571) |
| Observations | 149 | 149 | 149 | 149 |
| Adjusted R$^2$ | 15% | 20% | 32% | 44% |

**Panel B:  Dependent Variable is LTG; Independent variable is prediction error from Panel A**

|  | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| Forecast error $e_t$-cae$_{t-1}$ | 0.4002 | | | |
| | (0.2951) | | | |
| Forecast error 5-year WSJ return | | 0.4610 | | |
| | | (0.2892) | | |
| Forecast error $\Delta$ 5-year GDP pc growth | | | 0.1810 | |
| | | | (0.2321) | |
| Forecast Error Using All Vars | | | | 0.1567 |
| | | | | (0.2686) |
| Observations | 149 | 149 | 149 | 149 |
| Adjusted R$^2$ | 13% | 5% | 2% | -1% |

Note: [a] significant at the 1% level, [b] significant at the 5% level, [c] significant at the 10% level.  Standard errors are corrected for serial correlation using Newey and West (1987).

Panel A shows that fundamentals predict about half of the variation in the dividend price ratio. Panel B shows that price residuals, which still exhibit substantial variation, cannot predict contemporaneous changes in $LTG$, contrary to the hypothesis that analyst mechanically fit $LTG$ to match prices.[15]

The second exercise uses survey measures of expectations of returns, which can be taken as an observable proxy for discount rates. If analysts mistakenly attribute changes in required returns to changes in market expectations, $LTG$ would be negatively correlated with contemporaneous expectations of returns elicited from market participants. Table 6 presents the correlations between $LTG$ and the measures of expectations of returns discussed in Section 2.1.

**Table 6.**

Partial correlation between the forecast for long-term growth in earnings ($LTG_t$) and expected returns from the survey of CFOs using 73 quarterly observations between 10/2001 and 12/2018. $\mathbb{E}_t^O[r_{t+1}]$ is the log of one plus the one-year expected return. $\mathbb{E}_t^O[r_{t+10}]$ is the log of one plus the one-year expected return. Superscripts: [a] significant at the 1% level, [b] significant at the 5% level, [c] significant at the 10% level. Significance levels have the Bonferroni adjustment.

|  | $LTG_t$ | $\mathbb{E}_t^O[r_{t+1}]$ |
|---|---|---|
| $\mathbb{E}_t^O[r_{t+1}]$ | 0.3692[b] | |
| $\mathbb{E}_t^O[r_{t+10}]$ | 0.3087[a] | 0.3901[b] |

Market expectations of returns are *positively*, not negatively, correlated with expectations about long term earnings growth. This is reminiscent of Greenwood and Shleifer's (2014) finding that expectations of returns are positively correlated with past returns.[16] Moreover, after controlling for fundamentals (listed in Table 5), expectations of long term growth and of returns are uncorrelated (Table B.4 in Appendix B).

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[15] One concern with this exercise is that the price dividend ratio may display large high frequency movements while $LTG$ is slower moving, reducing the explanatory power of extrinsic price movements for $LTG$. We thus perform a version of this same exercise using as explanatory variable a variable that is slower moving than the price dividend ratio, namely stock returns in the past five years. This exercise addresses the possibility that analysts fit high $LTG$ after a sustained increase in stock prices. The results, which are reported in Appendix C, Table C2, show that $LTG$ is uncorrelated with return residuals from the first stage. This finding confirms that $LTG$ is unlikely to be mechanically fitted by analysts using price variables.

[16] Cochrane (2011) offers an alternative interpretation of Greenwood and Shleifer's (2014) results in which analysts report risk neutral expectations. Greenwood and Shleifer (2014) argue that such an interpretation is inconsistent with the data, not only because the surveys ask explicitly about returns, but also because under a risk neutral interpretation respondents would answer the risk free rate, which does not match the data.

In sum, *LTG* moves with fundamentals, and it does not seem to be mechanically inferred from prices. This occurs both at the firm level (see BGLS 2019) and at the aggregate level. This finding strengthens the confidence that analyst beliefs are a good proxy for market beliefs.

## 4.3 Variation in the Price Dividend Ratio

The fact that the synthetic price $p_t^O$ closely tracks the market price (Figure 2) and in particular matches its volatility (Table 3) suggests that it may also help account for the variation in valuation ratios such as price dividend or price earnings ratios.  Formally:

**Corollary 3.** *Regressing $p_t^O - d_t$ on $p_t^M - d_t$ has a higher coefficient than regressing $p_t^{RE} - d_t$ on $p_t^M - d_t$ if and only if $\sigma_{O,M} + \omega(1-a)\sigma_{OF} > 0$, where $\omega$ is as in Proposition 2.*

Analyst beliefs help account for the price dividend ratio provided they are positively correlated with market beliefs, $\sigma_{O,M} > 0$, or they over-react sufficiently strongly, $\omega\sigma_{OF} > 0$, or both.

To assess the prediction in Corollary 3, we use the earnings-based synthetic prices of the previous section to build valuation ratios $p_t^{RE} - e_t$, $p_t^O - e_t$ as well as $p_t^{OS} - e_t$, which are expressed in terms of earnings growth expectations alone.[17]  We then regress these ratios on the contemporaneous price earnings ratio.  Similarly, we build synthetic price dividend ratios using expectations of *dividends*, available starting in 2003, according to Equation (7), e.g.

$$p_t^{O,D} - d_t = \frac{k-r}{1-\alpha} + \ln\left(\frac{\mathbb{E}_t^O DPS_{t+1}}{DPS_t}\right) + \sum_{j=1}^{10} \alpha^{j-1}\,\mathbb{E}_t^O \Delta d_{t+j+1} + \frac{\alpha^{10}}{1-\alpha}\,g$$

where we assume that expectations of long run dividend growth are also described by *LTG*, and regress these ratios on the contemporaneous price dividend ratio.  Table 7 reports the results:

**Table 7.**

In Panel A, the dependent variable are: (1) the log of the ratio of the rational benchmark index ($p_t^{RE}$) to earnings ($e_t$), (2) the log of the ratio of the short-term index to earnings, (3) the log of the ratio that incorporates one-year ahead expectations ($p_t^{Os}$ ) to earnings, (3( the log of the ratio of the long-term

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[17] Using earnings-based prices to match the price dividend ratio would require adding a dividend term, which would generate a mechanical correlation between the resulting ratios.

index ($p_t^O$) to earnings, (4) the log of one plus the one-year expected return ($\mathbb{E}_t^O[r_{t+1}]$), and (5) the discounted value of future expected discounts based on an AR(1) model for $\mathbb{E}_t^O[r_{t+1}]$ and $\mathbb{E}_t^O[r_{t+10}]$. The dependent variables in Panel B are the same as in Panel B except that the variables in columns [1]-[3] are scaled by dividends rather earnings. The independent variable is the log price-to-earnings ratio in Panel A and the log price-to-dividend ration in Panel B. Forecasts for earnings and dividends are available monthly while data on expected returns is quarterly. Each regression uses as many observations as possible. In each panel, the last row reports the sample period for each regression. Standard errors are not adjusted for serial correlation. Superscripts: [a] significant at the 1% level, [b] significant at the 5% level, [c] significant at the 10% level.

### Panel A: Price earnings ratio

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| | $p_t^{RE}$-$e_t$ | $p_t^{OS}$-$e_t$ | $p_t^{O}$-$e_t$ | $\mathbb{E}_t^O r_{t+1}$ | $\sum \mathbb{E}_t^O \alpha^j r_{t+j}$ |
| $pe_t$ | 0.5446[a] | 0.5232[a] | 0.6257[a] | 0.0025 | 0.0069 |
| | (0.0771) | (0.0605) | (0.0511) | (0.0054) | (0.0146) |
| Observations | 445 | 440 | 440 | 73 | 73 |
| Adjusted $R^2$ | 0.392 | 0.548 | 0.645 | -0.008 | -0.008 |
| Sample period | 1981:12-2018:12 | 1981:12-2018:12 | 1981:12-2018:12 | 2010:10-2018:12 | 2010:10-2018:12 |

### Panel B: Price dividend ratio

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| | $p_t^{RE,D}$-$d_t$ | $p_t^{Os,D}$-$d_t$ | $p_t^{O,D}$-$d_t$ | $\mathbb{E}_t^O r_{t+1}$ | $\sum \mathbb{E}_t^O \alpha^j r_{t+j}$ |
| $pd_t$ | 0.1534[a] | 0.3454[a] | 0.7178[a] | 0.0516[a] | 0.1411[a] |
| | (0.0106) | (0.0485) | (0.0872) | (0.0087) | (0.0237) |
| Observations | 445 | 178 | 134 | 73 | 73 |
| Adjusted $R^2$ | 23% | 42% | 42% | 46% | 46% |
| Sample period | 1981:12-2018:12 | 2002:10-2018:12 | 2005:11-2018:11 | 2010:10-2018:12 | 2010:10-2018:12 |

Measured expectations of earnings growth account for a large part of the variation in the price earnings and price dividends ratios. In panel A, the synthetic index $p_t^O - e_t$ constructed using long term growth expectations captures 63% of price dividend ratio variation. Nearly one fifth of this comes from expectations of long-term growth. Figure 3 illustrates these results. In Panel B, the index $p_t^O - d_t$ explains roughly 72% of price dividend ratio variation, about half of which comes from *LTG*.
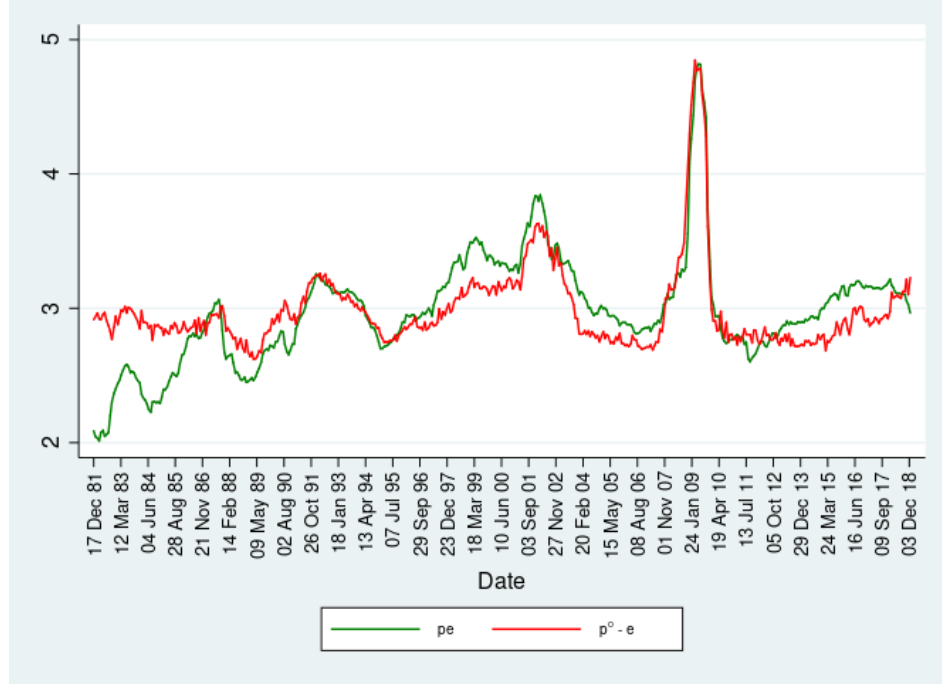
**Figure 3.** We plot the log price-to-earnings ratio (green line) and the difference between the long-term index ($p_t^O$, red line) as defined by Equation (8) in the text and log earnings, i.e. $p_t^O - e_t = \frac{\bar{k}-r}{1-\alpha} + \ln\left(\frac{\mathbb{E}_t^O EPS_{t+1}}{EPS_t}\right) + \sum_{j=1}^{10} \alpha^{j-1} \mathbb{E}_t^O \Delta e_{t+j+1} + \frac{\alpha^{10}}{1-\alpha} g$.

We also assess the explanatory power of contemporaneous measures of expectations of future stock returns. Expectations of returns do not co-vary with the price earnings ratio, and display positive co-movement with the price dividend ratio, the opposite of what one should expect based on rational models of time varying return (this may suggest a role for price extrapolation).[18] The strong explanatory power of expectations of fundamentals in Table 7 lines up with the results of De la O and Myers (2019), but crucially shows that expectations of long term earnings growth greatly increase the explanatory power relative to short term beliefs.

In sum, analyst expectations have strong explanatory power for stock prices and help reconcile Shiller's excess volatility puzzle and the price dividend ratio puzzle. Beliefs about both short and long term growth contribute to the explanatory power. However, only long term beliefs over-react. We next show that this feature is key to obtaining return predictability.

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[18] Overall, measured expectations of fundamentals and of returns fall short of accounting for 100% of price variation. This could be in part due to measurement error in expectations, and in part to genuine variation in market attitudes toward risk not captured by measures of expectations of returns.

## 5. Predictability of Returns

By the Campbell Shiller decomposition, the condition of Corollary 3 ensures that measures of expectations should help predict future returns. In particular, high values of our price indices should signal excess market optimism about future fundamentals, inflated stock prices, and hence low future returns. Table 8 below assesses this explanatory power for both the price earnings ratio and the price dividend ratio.

We separately assess the ability of short and long term expectations to predict future returns. We use raw returns but the results are similar if we use excess returns (Table B.5 Appendix B).

### Table 8.
### Return Predictability

The dependent variable is the log return between year $t$ and $t+1$ in column [1] and the discounted value of the cumulative return between year $t$ and $t+h$ in columns $h=2,\ldots,5$. The independent variables are the: (a) the log of the ratio the long-term index based on dividend forecasts to dividends ($p^{O,D}$-$d$), (b) the log of the ratio the long-term index based on earnings forecasts to earnings ($p^{O,E}$-$e$), (c) the forecast for earnings growth in the long run ($LTG_t$), (d) the time-$t$ forecast for one-year growth in earnings in year $t+1$ ($E_t[e_{t+1}-e_t]$), (e) the time-$t$ forecast for one-year for growth in earnings in year $t+2$ ($E_t[e_{t+2}-e_{t+1}]$). We report results using monthly expectations data for the period 1981:12-2018:12. The last period with stock return data ranges from December of 2017 in column [1] to December 2013 in column [5]. We adjust standard errors for serial correlation using the Newey-West correction (the number of lags ranges from 12 in the first column to 60 in the last one).

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| | $r_{t+1}$ | $\sum_{j=1}^{2}\alpha^{j-1}r_{t+j}$ | $\sum_{j=1}^{3}\alpha^{j-1}r_{t+j}$ | $\sum_{j=1}^{4}\alpha^{j-1}r_{t+j}$ | $\sum_{j=1}^{5}\alpha^{j-1}r_{t+j}$ |
| *Panel A: Returns and $p^{O,D}$-$d$* | | | | | |
| $p_t^{O,D}$-$d_t$ | -0.6497[b] | -0.9688[c] | -0.9561[c] | -1.1931[b] | -1.3618[a] |
| | (0.3238) | (0.5145) | (0.5642) | (0.4786) | (0.4210) |
| Adjusted $R^2$ | 17% | 19% | 19% | 29% | 37% |
| Observations | 80 | 80 | 80 | 80 | 80 |
| *Panel B: Returns and $p^{O}$-$e$* | | | | | |
| $p_t^{O,E}$-$e_t$ | -0.0692 | -0.0436 | -0.0621 | -0.0157 | 0.0624 |
| | (0.0860) | (0.1191) | (0.1521) | (0.1714) | (0.1689) |
| Adjusted $R^2$ | 2% | 0% | 0% | 0% | 0% |
| Observations | 380 | 380 | 380 | 380 | 380 |
| *Panel C: Returns and LTG* | | | | | |

| | | | | | |
|---|---|---|---|---|---|
| LTG$_t$ | -3.3411[a] | -6.8269[a] | -8.7214[a] | -10.0120[a] | -10.8570[a] |
| | (1.1390) | (1.8476) | (1.9357) | (1.9183) | (2.1247) |
| Adjusted R$^2$ | 11% | 22% | 25% | 26% | 26% |
| Observations | 385 | 385 | 385 | 385 | 385 |

**Panel D:  Returns and Short-term earnings growth I**

| | | | | | |
|---|---|---|---|---|---|
| E$_t$[e$_{t+1}$-e$_t$] | -0.0273 | 0.0500 | 0.0545 | 0.1178 | 0.2072[b] |
| | (0.0731) | (0.0702) | (0.0878) | (0.0879) | (0.0842) |
| Adjusted R$^2$ | 380 | 380 | 380 | 380 | 380 |
| Observations | 0% | 0% | 0% | 1% | 3% |

**Panel E:  Returns and Short-term earnings growth II**

| | | | | | |
|---|---|---|---|---|---|
| E$_t$[e$_{t+2}$-e$_{t+1}$] | -0.3605 | 0.1649 | 0.5896 | 1.7040 | 2.9468 |
| | (0.6106) | (1.1681) | (1.7653) | (2.1739) | (2.1259) |
| Adjusted R$^2$ | 0% | 0% | 0% | 3% | 7% |
| Observations | 380 | 380 | 380 | 380 | 380 |

Note: [a] significant at the 1% level, [b] significant at the 5% level, [c] significant at the 10% level. Standard errors are corrected for serial correlation using Newey and West (1987).

Panel A shows that the synthetic price dividend ratio $p_t^O - d_t$ negatively predicts realized returns.  A high $p_t^O - d_t$ today means disappointing returns in the future, suggesting that overly optimistic expectations about future earnings growth may indeed cause overpricing of stocks that subsequently reverses.  Predictability is especially strong at long horizons.  Panel B performs the corresponding analysis for the price earnings ratio.  There is no evidence of predictability here, in line with the fact that price earnings ratios do not predict returns in the same sample.

The most interesting results are in Panels C, D and E. Panel C shows that high current expectations of long run earnings growth $LTG$ strongly predict low future returns. $LTG$ can account for 26% of variation in realized returns over the next five years, or roughly two thirds of the return variation accounted for by $p_t^O - d_t$ at the same horizon.

Panels D and E show that, in contrast, expectations of short-term earnings growth do not predict returns. This is consistent with the fact that short term beliefs capture high frequency variation and, if anything, under-react to news (Table 2). The mispricing associated with short term beliefs may be small, and swamped by the persistent over-reaction induced by $LTG$.

To better understand return predictability, we assess empirically the over-reaction mechanism of Proposition 1. This mechanism implies that after strong fundamentals, upward revisions in *LTG* are associated with excess optimism, inflated prices, and subsequent lower returns. But revisions of short term growth forecasts do not over-react and should therefore not predict returns. Table 9 uses the predicted forecast revisions of short and long term growth from Table 2 to predict future returns.

### Table 9.
### Predicted Expectation Revisions and Returns

We report second-stage results for IV regressions using two- and five-year stock returns as the dependent variable. The independent variables are the instrumented values of: (1) revision between year t and year $t-1$ in the forecast for the one-year earnings growth rate in year $t+1$ ($E^O_t(e_{t+1}-e_t)-E^O_{t-1}(e_{t+1}-e_t)$), (2) the revision between year $t$ and year $t-1$ in the forecast for the one-year earnings growth rate in year $t+2$ ($E^O_t(e_{t+2}-e_{t+1})-E^O_{t-1}(e_{t+2}-e_{t+1})$), and (3) is the change in the long-term growth forecast between year $t$ and $t-1$ ($\Delta LTG_t$). The instruments are: (1) the log of earnings in year $t$ relative to the cyclically-adjusted earnings in year $t-1$, (2) the change between year $t$ and $t-1$ in the 5-year growth rate of GDP per capita, and (3) the weighted average of the cumulative return earned by firms in the S&P500 during earnings announcement days in the preceding 20 quarters. See Table 3:A for first-stage estimates. We report results using quarterly expectations data for the period 1982:4-2018:4. Data on returns between t and t+2 ends on December of 2017 while data on returns between t and t+5 ends on December of 2013. We adjust standard errors for serial correlation using the Newey-West correction (with 8 lags in the first two columns and 20 in the last one). Superscripts: [a] significant at the 1% level, [b] significant at the 5% level, [c] significant at the 10% level.

| | Dep Variable: Log return between years t and: | | |
|---|---|---|---|
| | t+2 | t+2 | t+5 |
| $E[E_t[e_{t+1}-e_t] - E_{t-1}[e_{t+1}-e_t] \mid$ All instruments] | 0.2238 | | |
| | (0.1961) | | |
| $E[E_t[e_{t+2}-e_{t+1}] - E_{t-1}[e_{t+2}-e_{t+1}] \mid$ All instruments] | | 0.1494 | |
| | | (0.1600) | |
| $E[\Delta LTG_t \mid$ All instruments] | | | -0.7857[a] |
| | | | (0.1900) |
| Constant | 0.0022 | -0.0065 | -0.1000 |
| | (0.2143) | (0.2112) | (0.3441) |
| Obs | 135 | 135 | 125 |
| Modified F-stat | 18.73 | 29.68 | 13.56 |
| Sargan overidentification stat | 0.88 | 1.91 | 2.43 |
| AR Confidence Interval | [-.13, .54] | [-.13, .41] | [-1.2,-.41] |
| Reduced form Adj $R^2$ | 3% | 3% | 23% |

Revisions of short run expectations predicted from fundamental shocks do not predict future returns (Column 1). In contrast, predicted revisions of long run earnings growth account

for a significant share of return predictability. An increase in the predicted value of $\Delta LTG_t$ by one standard deviation entails a reduction in 5-year log returns of 0.28 (=.7857 x 0.35; the standard deviation of 5-year log returns is 0.35). Given the average yearly log return of 8.1% this corresponds to losing roughly 42 months' worth of returns over the five years. Expectations of long-term growth thus take the center stage in explaining stock market puzzles.

We can perform a finer three-stage decomposition, and check whether the forecast errors predicted from the excessive forecast revisions in response to fundamental shocks (Table 2, Panel B), in turn predict future realized returns. Table10 reports the results.

### Table 10.
### Predicted Forecast Errors and Returns

We report results for regressions using two- and five-year stock returns as the dependent variable. The independent variables are the predicted values of the: (1) revision between year t and year *t-1* in the forecast for the one-year earnings growth rate in year *t+1* ($E^O_t(e_{t+1}-e_t)-E^O_{t-1}(e_{t+1}-e_t)$), (2) revision between year *t* and year *t-1* in the forecast for the one-year earnings growth rate in year *t+2* ($E^O_t(e_{t+2}-e_{t+1})-E^O_{t-1}(e_{t+2}-e_{t+1})$), and (3) change in the long-term growth forecast between year *t* and *t-1* ($\Delta LTG_t$). In turn, we predict revisions in earnings forecasts and changes in LTG using the following three variables: (1) the log of earnings in year *t* relative to the cyclically-adjusted earnings in year *t-1*, (2) the change between year *t* and *t-1* in the 5-year growth rate of GDP per capita, and (3) the weighted average of the cumulative return earned by firms in the S&P500 during earnings announcement days in the preceding 20 quarters. We report results using quarterly expectations data for the period 1982:4-2018:4. Data on stock returns between t and t+2 ends on December of 2017 while data on stock returns between t and t+5 ends on December of 2013. We adjust standard errors for serial correlation using the Newey-West correction (with 8 lags in the first two columns and 20 in the remaining ones). Superscripts: [a] significant at the 1% level, [b] significant at the 5% level, [c] significant at the 10% level.

| | Dependent Variable: Log return between years t and: | | | | |
|---|---|---|---|---|---|
| | t+2 | t+2 | t+5 | t+5 | t+5 |
| Predicted $(e_{t+1}-e_t)-E^O_t[e_{t+1}-e_t]$ | 0.3128 | | | | |
| | (0.4270) | | | | |
| Predicted $(e_{t+2}-e_{t+1})-E^O_t[e_{t+2}-e_{t+1}]$ | | 0.5170 | | | |
| | | (0.3556) | | | |
| Predicted $(e_{t+5}-e_t)/3-LTG_t$ | | | 4.7231 | | |
| | | | (9.2433) | | |
| Predicted $(e_{t+5}-e_{t+1})/4-LTG_t$ | | | | 1.1421[a] | |
| | | | | (0.3675) | |
| Predicted $(e_{t+5}-e_{t+2})/5-LTG_t$ | | | | | 0.5288[a] |
| | | | | | (0.1847) |
| Constant | 0.0054 | 0.0053 | -0.0000 | 0.0000 | 0.0000 |
| | (0.1446) | (0.1656) | (0.9710) | (0.2897) | (0.2518) |
| Obs | 135 | 135 | 125 | 125 | 125 |
| Modified F-stat | 1.11 | 2.83 | 0.35 | 4.47 | 30.68 |
| Sargan overidentification stat | 1.13 | 1.07 | 0.84 | 0.07 | 2.28 |

| AR Confidence Interval | [entire grid] | [entire grid] | [.,-3.8]U[1.2,.] | [ .48, 2.15] | [0.09, 0.49] |
|---|---|---|---|---|---|
| Reduced form Adj R² | 3% | 3% | 23% | 23% | 23% |

Short term beliefs display modest under-reaction to fundamentals (Table 3 Panel B) and naturally the entailed forecast errors do not account for future returns (Column 1). In contrast, there is a positive and significant association between predicted long-term forecast errors and subsequent returns (columns 4 and 5), further validating the over-reaction mechanism. According to Table 10, this mechanism explains nearly all the predictability of 5 year ahead returns from *LTG* (23% vs 26% in Panel C of Table 8) and over half of that predicted from the price dividend ratio.

The results in this section close the loop of the argument laid out in the Introduction: expectations of long term growth over-react to fundamental shocks, and corresponding forecast errors predict returns. Belief dynamics drive the puzzling price movements.

## 6. Conclusion.

We showed that measured expectations of fundamentals help explain in a parsimonious way leading stock market puzzles even with constant discount rates. Expectations of short and long term earnings growth both contribute to generating excessively volatile prices and realistic time variation in the price dividend ratio. Expectations of long term growth over-react to news, and thus account for persistent boom bust patterns in stock prices and return predictability.

A sceptic may question our measurement of beliefs, arguing that it surreptitiously embodies variation in discount rates. We consider this possibility, but do not find support for it. This is in line with many other studies that have validated the use of survey expectations. At a minimum, beliefs data can help advance our understanding of asset prices. Theories based on discount rate variation may also benefit from external measurement of changing risk attitudes.

The analysis of this paper raises many questions about the role of beliefs. An important next step is to assess how the logic of over-reaction to news can unify cross-sectional and

aggregate puzzles based on a single belief distortion, by introducing a psychologically-founded model of non-rational beliefs into an economy with heterogeneous firms or sectors, in the spirit of BGLS (2019). Expansion of high growth sectors or periods of outstanding aggregate performance may create excess optimism for many firms, leading to an excessive compression in their cost of capital, but also to a reduction in the equity premium for the market as a whole.

The major open area here is psychological foundations of beliefs. Diagnostic expectations (BGLS 2019) provide a micro-foundation for the forward looking over-reaction to news that is central here. They do not yet incorporate the short term persistence of beliefs, which is related to their short term under-reaction. Models in which updating is sluggish due to limited attention or persistent memory signals, but in which the accumulation of signals eventually creates over-reaction and then reversals, offer promising avenues to develop a realistic yet manageable model of beliefs that can help asset pricing research make progress.

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# Appendix A. Proofs.

**Proof of Proposition 1.** The MA representation of the data generating process implies that:

$$g_{t+s+1} = \sum_{j \geq 0} \eta_j \epsilon_{t+s+1-j}.$$

which in turn implies:

$$\mathbb{E}_t(g_{t+s+1}) = \sum_{j \geq s+1} \eta_j \epsilon_{t+s+1-j} = \sum_{j \geq 0} \eta_{j+s+1} \epsilon_{t-j}.$$

Likewise, the expectations shock admits a moving average representation:

$$\epsilon_{M,t} = \sum_{j \geq 0} \rho^j u_{M,t-j},$$

Using the more general formulation of expectational errors:

$$\mathbb{E}_t^M(g_{t+s+1}) = \mathbb{E}_t(g_{t+s+1}) + \delta_{s+1} \epsilon_{M,t}$$

allowing differential impact $\delta_{s+1} \epsilon_{M,t}$ at different horizons $s$, we obtain:

$$\mathbb{E}_t^M(g_{t+s+1}) = \sum_{j \geq 0} \eta_{j+s+1} \epsilon_{t-j} + \rho^j \delta_{s+1} u_{M,t-j}.$$

Expectations have two components: a rational part which responds to current and past shocks $\epsilon_{t-j}$, with a propagation coefficient $\eta_{j+s+1}$, and a distortion part which responds to current and past expectational shocks $u_{M,t-j}$ with propagation coefficient $\rho^j \delta_{s+1}$. This means that an expectational shock has an initial term structure given by $\delta_{s+1}$ which is persistent over time $t$ but decays at a rate $\rho$.

By Equation (5), then, the log stock price at time $t$ is equal to:

$$p_t^M = d_t + \frac{k-r}{1-a} + \sum_{s \geq 0} \sum_{j \geq s+1} a^s \eta_j \epsilon_{t+s+1-j} + \sum_{s \geq 0} \sum_{j \geq 0} a^s \delta_{s+1} \rho^j u_{M,t-j},$$

which can be written as:

$$p_t^M = d_t + \frac{k-r}{1-a} + \sum_{j \geq 0} \epsilon_{t-j} \sum_{s \geq 0} a^s \eta_{j+s+1} + \sum_{j \geq 0} \rho^j u_{M,t-j} \sum_{s \geq 0} a^s \delta_{s+1}$$

$$= d_t + \frac{k-r}{1-a} + \sum_{j \geq 0} H_j \epsilon_{t-j} + \Delta \sum_{j \geq 0} \rho^j u_{M,t-j}$$

where we have defined $H_j \equiv \sum_{s\geq0} a^s \eta_{j+s+1}$ and $\Delta \equiv \sum_{s\geq0} a^s \delta_{s+1}$ as the "average" fundamental impulse response for time j, and the "average" impulse response for expectational distortions. In particular, rationality corresponds to the case $\Delta = 0$.

Consider now the Propositions part i). The log price change is then equal to:

$$p_t^M - p_{t-1}^M = d_t - d_{t-1} + \sum_{j\geq0} \epsilon_{t-j} H_j - \sum_{j\geq0} \epsilon_{t-1-j} H_j + \Delta\left(\sum_{j\geq0} \rho^j u_{M,t-j} - \sum_{j\geq0} \rho^j u_{M,t-1-j}\right)$$

$$= \sum_{j\geq0} \eta_j \epsilon_{t-j} + \epsilon_t H_0 + \sum_{j\geq1}\left((1-a)H_j - \eta_j\right)\epsilon_{t-j} + \Delta u_{M,t}$$

$$+ (\rho-1)\Delta\left(\sum_{j\geq1} \rho^{j-1} u_{M,t-j}\right)$$

$$= \epsilon_t(1+H_0) + (1-a)\sum_{j\geq1} H_j \epsilon_{t-j} + \Delta u_{M,t} + (\rho-1)\Delta\left(\sum_{j\geq1} \rho^{j-1} u_{M,t-j}\right)$$

where we used $d_t - d_{t-1} = \sum_{j\geq0} \eta_j \epsilon_{t-j}$ as well as $\eta_0 = 1$ and

$$H_j - H_{j-1} = \sum_{s\geq0} a^s \eta_{j+s+1} - \sum_{s\geq0} a^s \eta_{j+s} = (1-a)H_j - \eta_j$$

We then have:

$$var(p_t^M - p_{t-1}^M)$$

$$= \left[(1+H_0)^2 \sigma^2 + (1-a)^2 \sum_{j\geq1} H_j^2\right]\sigma^2 + \Delta^2\left[1 + (1-\rho)^2 \sum_{j\geq0} \rho^{2j}\right]\sigma_M^2$$

$$+ 2\Delta\left[(1+H_0) + (\rho-1)(1-a)\sum_{j\geq1} \rho^{j-1} H_j\right]\sigma_{MF}$$

$$= var(p_t^{RE} - p_{t-1}^{RE})\sigma^2 + \frac{2}{1+\rho}\Delta^2 \sigma_M^2$$

$$+ 2\Delta\left[(1+H_0) - (1-\rho)(1-a)\sum_{j\geq1} \rho^{j-1} H_j\right]\sigma_{MF}$$

So there is excess volatility if:

$$\sigma_M^2 + \frac{1+\rho}{\Delta}\left[\sum_{j\geq0} \rho^j H_j + \left(1 - (1-a(1-\rho))\sum_{j\geq1} \rho^{j-1} H_j\right)\right]\sigma_{MF} > 0$$

In the benchmark case where $\delta_{s+1} = 1$ for all $s$, we have $\Delta = \frac{1}{1-a}$ and the condition above can be rewritten:

$$\sigma_M^2 + (1-a)\mu\sigma_{MF} > 0$$

where $\mu = (1+\rho)\left[\sum_{j\geq0}\rho^j H_j + \left(1 - \left(1 - a(1-\rho)\right)\sum_{j\geq1}\rho^{j-1}H_j\right)\right]$. The condition $\mu > 0$ is then equivalent to:

$$1 + H_0 > \sum_{j\geq1}\rho^j H_j\left(\frac{1}{\rho} - 1\right)(1-a),$$

which is fulfilled provided the rational price response to a fundamental shock $\frac{\partial p_t^{RE}}{\partial \epsilon_t} = 1 + H_0$ is large enough. Because $a \approx 0$ and because the long run impulse response converges to zero, so that $\sum_{j\geq1}\rho^j H_j$ is low, the condition $\mu > 0$ is satisfied provided $\frac{\partial p_t^{RE}}{\partial \epsilon_t}$ is not much above zero.

Consider now the Proposition's part ii). The log price dividend ratio is equal to:

$$p_t^M - d_t = \frac{k-r}{1-a} + \sum_{j\geq0}H_j\epsilon_{t-j} + \Delta\sum_{j\geq0}\rho^j u_{M,t-j},$$

which in conventional tests is used as an explanatory variable for future realized dividend growth rates:

$$\sum_{s\geq0}a^s g_{t+s+1} = \sum_{j\geq0}\sum_{s\geq0}a^s\eta_j\epsilon_{t+s+1-j} = \sum_{j\geq0}H_j\epsilon_{t-j} + v',$$

where $v'$ is a combination of future shocks. Then:

$$cov\left[p_t^M - d_t, \sum_{s\geq0}a^s g_{t+s+1}\right] = cov\left[\sum_{j\geq0}H_j\epsilon_{t-j} + \Delta\sum_{j\geq0}\rho^j u_{M,t-j}, \sum_{j\geq0}H_j\epsilon_{t-j}\right]$$

$$= \sum_{j\geq0}H_j^2\sigma^2 + \Delta\sum_{j\geq0}\rho^j H_j\sigma_{MF}$$

while

$$var(p_t^M - d_t) = \sum_{j\geq0}H_j^2\sigma^2 + \left(\frac{\Delta^2}{1-\rho^2}\right)\sigma_M^2 + 2\Delta\sum_{j\geq0}\rho^j H_j\sigma_{MF}$$

So the coefficient from regressing the future discounted dividend growth on the log price dividend is:

$$\beta = \frac{\sum_{j\geq0}H_j^2\sigma^2 + \Delta\sum_{j\geq0}\rho^j H_j\sigma_{MF}}{\sum_{j\geq0}H_j^2\sigma^2 + \left(\frac{\Delta^2}{1-\rho^2}\right)\sigma_M^2 + 2\Delta\sum_{j\geq0}\rho^j H_j\sigma_{MF}}$$

Note that under rational expectations, $\Delta = 0$, we have $\beta = 1$. Instead, the coefficient is smaller than 1 if

$$\sigma_M^2 + (1-\rho^2)\frac{\sum_{j\geq0}\rho^j H_j}{\Delta}\sigma_{MF} > 0$$

Again, in the benchmark case where $\delta_{s+1} = 1$ for all $s$, we have:

$$\sigma_M^2 + (1-a)\omega\sigma_{MF} > 0$$

with $\omega = (1-\rho^2)\sum_{j\geq0}\rho^j H_j$.

Finally, consider the predictability of returns. By Equation (1), the one period stock return is equal to:

$$r_{t+1} = a(p_{t+1} - d_{t+1}) + g_{t+1} - (p_t - d_t),$$

where we have set $k = 0$ for convenience. By iterating the equation forward until $t + T$ we obtain:

$$\sum_{s=0}^{T-1} a^s r_{t+s+1} = a^T(p_{t+T} - d_{t+T}) + \sum_{s=0}^{T-1} a^s g_{t+s+1} - (p_t - d_t).$$

By using the price rule (where for convenience we have also set $r = 0$), it is immediate to obtain:

$$\sum_{s=0}^{T-1} a^s r_{t+s+1} = \sum_{s\geq0} a^{s+T}\mathbb{E}_{t+T}^M(g_{t+T+s+1}) + \sum_{s=0}^{T-1} a^s g_{t+s+1} - \sum_{s\geq0} a^s\mathbb{E}_t^M(g_{t+s+1}),$$

which can be written as:

$$\sum_{s=0}^{T-1} a^s r_{t+s+1} = \sum_{s\geq T} a^s[\mathbb{E}_{t+T}^M(g_{t+s+1}) - \mathbb{E}_t^M(g_{t+s+1})] + \sum_{s=0}^{T-1} a^s[g_{t+s+1} - \mathbb{E}_t^M(g_{t+s+1})],$$

so that $T$-period ahead returns combine the forecast revisions up until $T$ as well as the term structure of forecast errors made at time $t$. Note that:

$$\mathbb{E}_{t+T}^M(g_{t+s+1}) - \mathbb{E}_t^M(g_{t+s+1}) = \sum_{j\geq T}\rho^j\delta_{s+1-T}u_{M,t+T-j} - \sum_{j\geq0}\rho^j\delta_{s+1}u_{M,t-j} + \nu$$

$$= \sum_{j\geq0}(\rho^{j+T}\delta_{s+1-T} - \rho^j\delta_{s+1})u_{M,t-j} + \nu$$

where $\nu$ captures shocks that occur after time $t$. From the perspective of time $t$, realized future returns are:

$$\mathbb{E}_t\left[\sum_{s=0}^{T-1} a^s r_{t+s+1}\right]$$

$$= \sum_{s\geq T} a^s \sum_{j\geq 0}\left(\rho^{j+T}\delta_{s+1-T} - \rho^j \delta_{s+1}\right)u_{M,t-j}$$

$$- \left(\sum_{s=0}^{T-1} a^s \delta_{s+1}\right)\left(\sum_{j\geq 0}\rho^j u_{M,t-j}\right) = \left[\rho^T \sum_{s\geq T} a^s \delta_{s+1-T} - \Delta\right]\left(\sum_{j\geq 0}\rho^j u_{M,t-j}\right)$$

$$= (\rho^T a^T - 1)\Delta\left(\sum_{j\geq 0}\rho^j u_{M,t-j}\right)$$

This implies that regressing the $T$-period return on the current price dividend ratio $p_t^M - d_t$ yields a coefficient $cov(\sum_{s=0}^{T-1} a^s r_{t+s+1}, p_t^M - d_t)/var(p_t^M - d_t)$, where

$$cov\left(\sum_{s=0}^{T-1} a^s r_{t+s+1}, p_t^M - d_t\right)$$

$$= (\rho^T a^T - 1)\Delta cov\left(\sum_{j\geq 0}\rho^j u_{M,t-j}, \sum_{j\geq 0} H_j \epsilon_{t-j} + \Delta\sum_{j\geq 0}\rho^j u_{M,t-j}\right)$$

$$= -\frac{1-\rho^T a^T}{1-\rho^2}\Delta^2\left((1-\rho^2)\frac{\sum_{j\geq 0}\rho^j H_j}{\Delta}\sigma_{M,F} + \sigma_M^2\right)$$

The coefficient is negative provided

$$(1-\rho^2)\frac{\sum_{j\geq 0}\rho^j H_j}{\Delta}\sigma_{M,F} + \sigma_M^2 > 0$$

Again, in the benchmark case where $\delta_{s+1} = 1$ for all $s$, we have:

$$\sigma_M^2 + (1-a)\omega\sigma_{MF} > 0$$

with $\omega = (1-\rho^2)\sum_{j\geq 0}\rho^j H_j$. ∎


**Proof of Proposition 2.**

We begin by deriving the Coibion Gorodnichenko coefficient that links forecast errors to forecast revisions. From Equation (4), the expected forecast error at time $t$ is:

$$\mathbb{E}_t[g_{t+s+1} - \mathbb{E}_t^O(g_{t+s+1})] = -\delta_{s+1}\left(\sum_{j\geq 0}\rho^j u_{O,t-j}\right) = -\delta_{s+1} u_{O,t} - \delta_{s+1}\left(\sum_{j\geq 1}\rho^j u_{O,t-j}\right)$$

while the revision at $t$ is has two components, one driven by the shocks at t (both fundamental and to expectations) and another driven by the change in the impact of past expectation shock on the forecast:

$$\mathbb{E}_t^O(g_{t+s+1}) - \mathbb{E}_{t-1}^O(g_{t+s+1}) = \eta_{s+1}\epsilon_t + \delta_{s+1}u_{O,t} + \frac{\delta_{s+1}\rho - \delta_{s+2}}{\rho}\left(\sum_{j\geq 1}\rho^j u_{O,t-j}\right).$$

so that:

$$cov[g_{t+s+1} - \mathbb{E}_t^O(g_{t+s+1}), \mathbb{E}_t^O(g_{t+s+1}) - \mathbb{E}_{t-1}^O(g_{t+s+1})]$$

$$= -\delta_{s+1}(\eta_{s+1}\sigma_{OF} + \delta_{s+1}\sigma_O^2) - \delta_{s+1}(\delta_{s+1}\rho - \delta_{s+2})\frac{\rho}{1-\rho^2}\sigma_O^2$$

$$var[\mathbb{E}_t^O(g_{t+s+1}) - \mathbb{E}_{t-1}^O(g_{t+s+1})]$$

$$= \eta_{s+1}^2\sigma^2 + \delta_{s+1}^2\sigma_O^2 + \eta_{s+1}\delta_{s+1}\sigma_{OF} + (\delta_{s+1}\rho - \delta_{s+2})\frac{\rho}{1-\rho^2}\sigma_O^2.$$

The CG coefficient is negative provided:

$$\delta_{s+1}[(\eta_{s+1}\sigma_{OF} + \delta_{s+1}\sigma_O^2)(1-\rho^2) + (\delta_{s+1}\rho - \delta_{s+2})\rho\sigma_O^2] > 0$$

which is equivalent to:

$$\delta_{s+1}[\eta_{s+1}\sigma_{OF}(1-\rho^2) + (\delta_{s+1} - \delta_{s+2}\rho)\sigma_O^2] > 0.$$

In the benchmark case where $\delta_{s+1} = 1$ for all $s$, this becomes:

$$(1+\rho)\eta_{s+1}\sigma_{OF} + \sigma_O^2 > 0.$$

which is reminiscent of the conditions in Propositions 1 to 3. Belief updating at horizon $s + 1$ is excessive when the distortion co-moves with rational updating ($\sigma_{OF} > 0$ and $\eta_{s+1} > 0$) or when beliefs are noisy (large $\sigma_O^2$). In the first case analysts over-react to fundamental news, in the second they over-react to noise.

We can now interpret the results of Table 1. Predictability of forecast errors depends on the combination of analyst optimism in reaction to a good shock, $\sigma_{OF}$, with the impact of that shock on subsequent growth, $\eta_{s+1}$. The results in Table 1 can be reconciled if a positive shock to earnings growth displays short term reversal, namely $\eta_1, \eta_2 < 0$, and long-term higher growth, namely $\eta_{s+1} > 0$ for $s > 2$. In short, the patterns for short and long term forecasts are

reconciled if analysts become too optimistic after a good growth shock, $\sigma_{OF} > 0$. According to Equation (5), after a positive shock analysts revise their forecasts downward, anticipating mean reversion, but because $\sigma_{OF} > 0$ they do not revise enough. Insufficient reversion of beliefs creates short term under-reaction. On the other hand, after the same positive growth shock, analysts revise up their long run beliefs due to both the rational and irrational components in Equation (5), causing over-reaction of long term forecasts.

We now turn to Table 2, where $\sigma_{OF}$ is assessed following a two-stage approach. In a first stage we regress the revision of growth forecasts on our news proxies (Table 2, Panel A). In a second stage, we regress the forecast error on the revision predicted from the first stage (Table 2, Panel B).

We now show that a positive first stage coefficient $\varphi_s > 0$ at horizon $s + 1$ means that $\eta_{s+1}\sigma^2 + \sigma_{O,F} > 0$: expectations move in the direction of the shock provided the shock is persistent ($\eta_{s+1} > 0$) and expectational distortions correlate with the shock itself ($\sigma_{O,F} > 0$). Recall that the forecast revision is:

$$\mathbb{E}_t^O(g_{t+s+1}) - \mathbb{E}_{t-1}^O(g_{t+s+1}) = \eta_{s+1}\epsilon_t + \delta_{s+1}u_{O,t} + \frac{\delta_{s+1}\rho - \delta_{s+2}}{\rho}\left(\sum_{j \geq 1}\rho^j u_{O,t-j}\right).$$

It follows that:

$$\varphi_{fs} = \frac{cov[\mathbb{E}_t^O(g_{t+s+1}) - \mathbb{E}_{t-1}^O(g_{t+s+1}), \epsilon_t]}{var[\epsilon_t]} = \frac{\eta_{s+1}\sigma^2 + \delta_{s+1}\sigma_{O,F}}{\sigma^2}.$$

The predicted forecast revision is then equal to:

$$\widehat{\mathbb{E}}_t^O(g_{t+s+1}) - \widehat{\mathbb{E}}_{t-1}^O(g_{t+s+1}) = \varphi_{fs}\epsilon_t$$

If news proxies indeed predict forecast revisions, the second stage coefficient is given by:

$$\frac{cov[g_{t+s+1} - \mathbb{E}_t^O(g_{t+s+1}), \widehat{\mathbb{E}}_t^O(g_{t+s+1}) - \widehat{\mathbb{E}}_{t-1}^O(g_{t+s+1})]}{var[\widehat{\mathbb{E}}_t^O(g_{t+s+1}) - \widehat{\mathbb{E}}_{t-1}^O(g_{t+s+1})]} = -\frac{\delta_{s+1}\sigma_{OF}}{\varphi_{fs}\sigma^2}.$$

To derive this equation, we use the forecast error:

$$\mathbb{E}_t[g_{t+s+1} - \mathbb{E}_t^O(g_{t+s+1})] = -\delta_{s+1}\left(\sum_{j \geq 0}\rho^j u_{O,t-j}\right) = -\delta_{s+1}u_{O,t} - \delta_{s+1}\left(\sum_{j \geq 1}\rho^j u_{O,t-j}\right)$$

The analysis yields $\sigma_{OF} > 0$ if, for horizons $s$ such that news positively predict revisions, $\varphi_{fs} > 0$, the second stage is negative. Intuitively, in this case analysts become too optimistic after good shocks, as suggested by Table 1 for beliefs about long run growth. But $\sigma_{OF} > 0$ also holds if the first stage coefficient is negative, $\varphi_{fs} < 0$ and the second stage coefficient is positive. In this case, after a positive shock beliefs get revised downwards, as in the case of mean reversion, but insufficiently so. Insufficient mean reversion after good news is also a sign of excess optimism, which entails $\sigma_{OF} > 0$.

∎

**Proof or Corollary 1.** From Proposition 1 we have:

$$\Delta V_O - \Delta V_{RE} = Var(p_t^{RE} - p_{t-1}^{RE}) - Var(p_t^O - p_{t-1}^O)$$

$$= -2\frac{\Delta^2}{1+\rho}\sigma_O^2 - 2\Delta\left[\sum_{j\geq0}\rho^j H_j + \left(1 - (1 - a(1-\rho))\sum_{j\geq1}\rho^{j-1}H_j\right)\right]\sigma_{OF}$$

This holds provided there is excess volatility under measured expectations, namely:

$$\sigma_O^2 + \frac{1+\rho}{\Delta}\left[\sum_{j\geq0}\rho^j H_j + \left(1 - (1 - a(1-\rho))\sum_{j\geq1}\rho^{j-1}H_j\right)\right]\sigma_{OF} > 0.$$

∎

**Proof or Corollary 2.** From Proposition 1 we know that:

$$p_t^M - p_t^{RE} = \Delta\sum_{j\geq0}\rho^j u_{M,t-j}$$

and similarly for $p_t^O - p_t^{RE}$. It follows that:

$$cov(p_t^M - p_t^{RE}, p_t^O - p_t^{RE}) = \frac{\Delta^2}{1-\rho^2}\sigma_{OM}$$

∎

**Proof of Corollary 3.** From Proposition 1, we have

$$p_t^M - d_t = \frac{k-r}{1-a} + \sum_{j\geq 0} H_j \epsilon_{t-j} + \Delta \sum_{j\geq 0} \rho^j u_{M,t-j}.$$

and analogously for $p_t^O - d_t$ and $p_t^{RE} - d_t$ (where the last term drops out). If follows that

$$cov(p_t^O - d_t, p_t^M - d_t) = \sum_{j\geq 0} H_j^2 \sigma^2 + \frac{\Delta^2}{1-\rho^2}\sigma_{O,M} + \Delta \sum_{j\geq 0} \rho^j H_j \left(\sigma_{M,F} + \sigma_{O,F}\right)$$

while

$$cov(p_t^{RE} - d_t, p_t^M - d_t) = \sum_{j\geq 0} H_j^2 \sigma^2 + \Delta \sum_{j\geq 0} \rho^j H_j \, \sigma_{M,F}$$

Thus, $cov(p_t^O - d_t, p_t^M - d_t) > cov(p_t^{RE} - d_t, p_t^M - d_t)$ if and only if

$$\sigma_{O,M} + \frac{1-\rho^2}{\Delta}\sum_{j\geq 0} \rho^j H_j \, \sigma_{O,F} > 0$$

since the denominator $var(p_t^M - d_t)$ is the same in both cases. In the benchmark case where $\delta_{s+1} = 1$ for all $s$, the condition becomes:

$$\sigma_{O,M} + (1-a)\omega\sigma_{O,F} > 0$$

with $\omega = (1-\rho^2)\sum_{j\geq 0} \rho^j H_j$.

∎

# Appendix B. Robustness and Further Results on Price Indices

In this Appendix, we collect several results that complement the analysis of the synthetic price indices in the text.

*Correlations across indices.*

|  | p-d | p$^{RE,D}$-d | p$^{O,D}$-d | p$^{OS,D}$-d | p-e | p$^{RE,E}$-e | p$^{O,E}$-e |
|---|---|---|---|---|---|---|---|
| p$^{RE,D}$-d | 0.4850 | | | | | | |
| p$^{O,D}$-d | 0.6489 | -0.3119 | | | | | |
| p$^{OS,D}$-d | 0.6500 | -0.0392 | 0.7367 | | | | |
| p-e | 0.5765 | 0.2308 | -0.5664 | -0.6455 | | | |
| p$^{RE,E}$-e | -0.2248 | 0.0972 | -0.6947 | -0.7084 | 0.6275 | | |
| p$^{O,E}$-e | 0.0874 | 0.0717 | -0.4927 | -0.6257 | 0.8034 | 0.8603 | |
| p$^{OS,E}$-e | -0.0402 | 0.1341 | -0.6076 | -0.6316 | 0.7412 | 0.9446 | 0.9392 |

*Alternative definitions and excess volatility.* Here we consider an alternative definition of price where expectations at time $t$ of growth beyond year $t + 5$ is inferred by applying the observed decay of observed cyclically adjusted earnings to $LTG_t$. Regressing $caeps_t - caeps_{t-t}$ on $caeps_{t-5} - caeps_{t-10}$ yields a slope coefficient of roughly 0.4. Thus, for a ten-year forecasting horizon we set:

$$p_t^{O,E10} = e_t + \frac{\tilde{k} - r}{1 - \alpha} + \ln\left(\frac{E_t^O \, EPS_{t+1}}{EPS_t}\right) + \sum_{j=1}^{5} \alpha^{j-1} E_t^O \Delta e_{t+j+1}$$

$$+ \sum_{j=6}^{10} \alpha^{j-1} \ln\left(1 + 0.4 * E_t^O \Delta e_{t+5}\right) + \frac{\alpha^{10}}{1 - \alpha} \, g_{10}.$$

and similarly for a 15 and 20-year forecasting horizon, as well as for an alternative dividend based index $p_t^{O,D10}$ (where long term growth is assumed to be described by LTG). Table B.2 shows the results.

**Table B.2**
**Panel A: Dividend based synthetic price**

| Δp | Δp$^{O,D}$ | Δp$^{O,D10}$ | Δp$^{O,D15}$ | Δp$^{O,D20}$ |
|---|---|---|---|---|

| σ | 15.2% | 18.2% | 15.9% | 16.5% | 16.7% |
|---|---|---|---|---|---|

<div align="center">**Panel B: Earnings based synthetic price**</div>

| | $\Delta p$ | $\Delta p^{O,E}$ | $\Delta p^{O,E10}$ | $\Delta p^{O,E15}$ | $\Delta p^{O,E20}$ |
|---|---|---|---|---|---|
| σ | 15.2% | 14.6% | 12.7% | 13.2% | 13.4% |

*Volatility of cointegrated series.* Finally, following Campbell and Shiller (1987) we assess the volatility of the cointegrated series $P_t - \frac{D_t}{r}$ for different measures of prices. The first column in Table B.3 reproduces the volatility of changes in log prices from Table 3. The remaining columns assess the volatility of the co-integrated series $P_t - \frac{D_t}{r}$ for different measures of prices.

<div align="center">**Table B.3**</div>

| | $\frac{P_t}{D_t}$ | $\frac{P_t^{RE,D}}{D_t}$ | $\frac{P_t^{O,D}}{D_t}$ | $\frac{P_t^{Os,D}}{D_t}$ |
|---|---|---|---|---|
| σ | 427 | 211 | 561 | 390 |
| N | 440 | 440 | 440 | 440 |

| | $\frac{P_t}{D_t}$ | $\frac{P_t^{RE,E}}{D_t}$ | $\frac{P_t^{O,E}}{D_t}$ | $\frac{P_t^{Os,E}}{D_t}$ |
|---|---|---|---|---|
| σ | 550 | 378 | 446 | 437 |
| N | 440 | 440 | 440 | 440 |

| | $\frac{P_t}{E_t}$ | $\frac{P_t^{RE,E}}{E_t}$ | $\frac{P_t^{O,E}}{E_t}$ | $\frac{P_t^{Os,E}}{E_t}$ |
|---|---|---|---|---|
| σ | 363 | 223 | 268 | 234 |
| N | 440 | 440 | 440 | 440 |

Next, we expand on the link between LTG and expectations of returns of Table 6, introducing further measures of expected returns, and showing that controlling for fundamentals, LTG is uncorrelated with expectations of returns.

<div align="center">**Table B.4**</div>

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| | Dependent Variable: LTG | | | |

| | | | | |
|---|---|---|---|---|
| $e_t$-$cae_{t-1}$ | 0.0075[a] | 0.0078[a] | 0.0086[c] | -0.0026 |
| | (0.0028) | (0.0028) | (0.0044) | (0.0041) |
| 1-year Change in GDP growth ($\Delta gGDP_t$) | 0.2207 | 0.2219 | 0.3289 | 0.5470 |
| | (0.3991) | (0.3906) | (0.3749) | (0.4751) |
| 5-year event returns (5-yr WJS) | 0.4432[a] | 0.4309[a] | 0.2815[a] | 0.3559[a] |
| | (0.1473) | (0.1070) | (0.0809) | (0.0695) |
| $E_t^{Duke}[R_{t+1}]$ | -0.0002 | | | |
| | (0.1737) | | | |
| $E_t^{Duke}[R_{t+10}]$ | | 0.0605 | | |
| | | (0.2754) | | |
| $E_t^{SPF}[R_{t+1}]$ | | | 0.0559 | |
| | | | (0.0401) | |
| $E_t^{SPF}[R_{t+10}]$ | | | | -0.1938 |
| | | | | (0.1803) |
| Constant | 0.1034[a] | 0.0994[a] | 0.1031[a] | 0.1182[a] |
| | (0.0092) | (0.0226) | (0.0059) | (0.0130) |
| Obs | 69 | 69 | 57 | 27 |
| Adj $R^2$ | 31% | 31% | 40% | 38% |

We next reproduce Table 8, which examines the predictability of returns on the basis of expectations, using excess (as opposed to raw) returns.

**Table B.5**

| | $r_{t+1}$ | $\sum_{j=1}^{2} \alpha^{j-1} r_{t+j}$ | $\sum_{j=1}^{3} \alpha^{j-1} r_{t+j}$ | $\sum_{j=1}^{4} \alpha^{j-1} r_{t+j}$ | $\sum_{j=1}^{5} \alpha^{j-1} r_{t+j}$ |
|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) |
| **Panel A: Returns and $p^{O,D}$-d** | | | | | |
| $p_t^{O,D}$-$d_t$ | -0.7056[b] | -1.0571[c] | -1.0671 | -1.3133[b] | -1.4787[a] |
| | (0.3406) | (0.5734) | (0.6531) | (0.5867) | (0.5373) |
| Adj $R^2$ | 19% | 18% | 17% | 25% | 30% |
| **Panel B: Returns and $p^{O,E}$-d** | | | | | |
| $p_t^{O,E}$-$e_t$ | -0.0446 | 0.0012 | 0.0002 | 0.0607 | 0.1499 |
| | (0.0866) | (0.1226) | (0.1534) | (0.1702) | (0.1654) |
| Adj $R^2$ | 1% | 0% | 0% | 0% | 2% |

|  | | | | | |
|---|---|---|---|---|---|
| | | | *Panel C: Returns and LTG* | | |
| LTG$_t$ | -3.7290[a] | -7.5329[a] | -9.6691[a] | -11.1317[a] | -12.0825[a] |
| | (1.0799) | (1.6616) | (1.6631) | (1.9254) | (2.5059) |
| Adj R$^2$ | 14% | 27% | 31% | 33% | 34% |

|  | | | | | |
|---|---|---|---|---|---|
| | | | *Panel D: Returns and LTG net of expected inflation* | | |
| $LTG_t$ | -2.2707[b] | -4.4494[b] | -6.0472[b] | -7.0270[a] | -7.6013[a] |
| | (1.1006) | (2.0635) | (2.3795) | (2.5105) | (2.7102) |
| Adj R$^2$ | 8% | 15% | 19% | 21% | 21% |

|  | | | | | |
|---|---|---|---|---|---|
| | | | **Panel E: Returns and Short-term growth** | | |
| $E_t^O[e_{t+2} - e_{t+1}$ | -0.3245 | 0.1585 | 0.5243 | 1.5456 | 2.6627 |
| | (0.6524) | (1.2784) | (1.8276) | (2.1985) | (2.1747) |
| Adj R$^2$ | 0% | 0% | 0% | 2% | 6% |

|  | | | | | |
|---|---|---|---|---|---|
| | | | **Panel F: Returns and Short-term growth net of inflation** | | |
| $E_t^O[e_{t+2}$ | -0.2627 | 0.2204 | 0.4249 | 1.2858 | 2.2613 |
| | (0.6289) | (1.1049) | (1.5969) | (2.0282) | (2.0845) |
| Adj R$^2$ | 0% | 0% | 0% | 2% | 5% |

**Figure B.1**
**Prices adjusted for inflation.**

We plot the S&P500 index (green line), the rational benchmark index ($p^{RE}$, blue line) and our benchmark expectations based price index ($p^O$, red line). All values are adjusted for inflation using the CPI index.
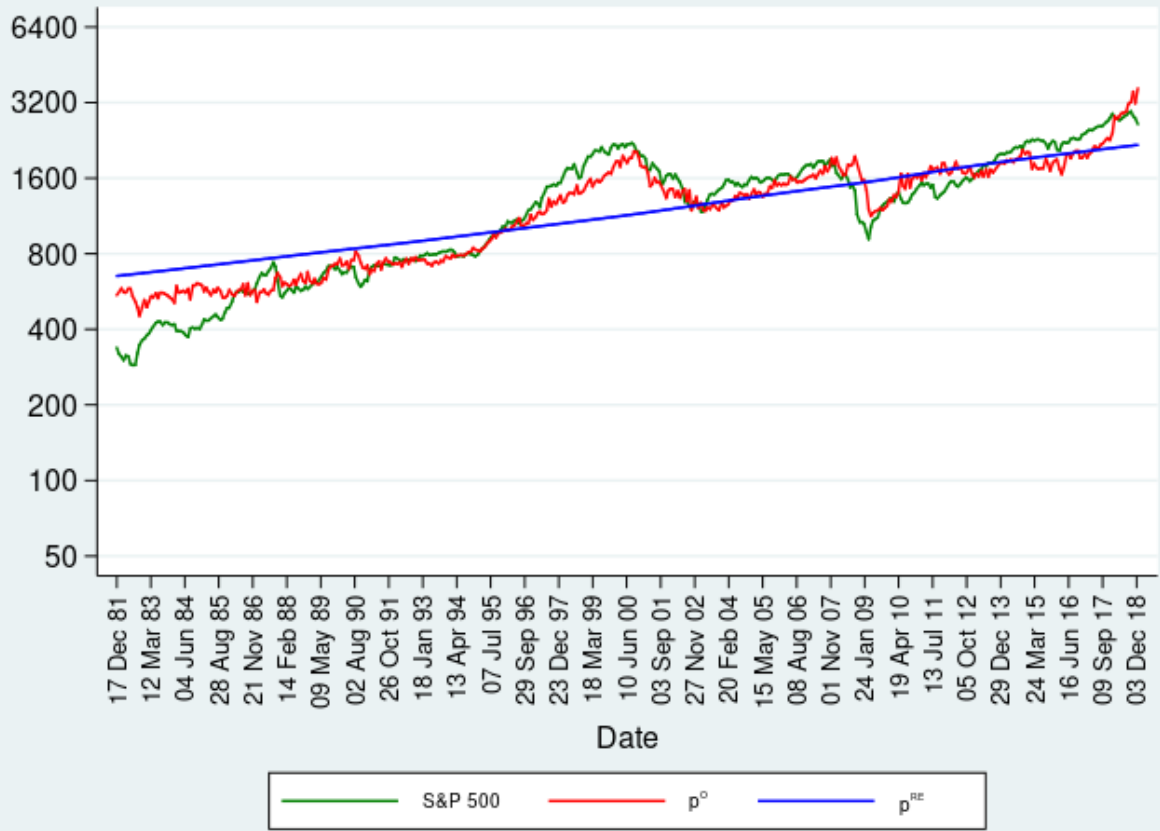
# Appendix C. Other robustness checks

In this Appendix, we collect two sets of robustness checks: i) on the analysis of whether analyst expectations are inferred from prices, and ii) on the predictability of returns from expectation-based synthetic prices.

*Prices and analyst expectations.* Table C.1 generalizes Table 5 in the text by including several other proxies for fundamental news.

## Table C.1
## Panel A: Dependent variable is dividend price ratio

|  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| Term Spread | 2.0114 |  |  |  |  |  |  |  | 0.2461 |
|  | (6.0444) |  |  |  |  |  |  |  | (3.6722) |
| Credit Spread |  | 47.1636[a] |  |  |  |  |  |  | 4.6921 |
|  |  | (14.7006) |  |  |  |  |  |  | (13.0758) |
| spc (Cochrane & Campbell) |  |  | -2.9513 |  |  |  |  |  | 8.0017[b] |
|  |  |  | (6.4096) |  |  |  |  |  | (3.2712) |
| cay |  |  |  | 7.2134[b] |  |  |  |  | 12.4786[a] |
|  |  |  |  | (2.9067) |  |  |  |  | (1.7969) |
| 5-year Div. growth |  |  |  |  | -0.3209 |  |  |  | 0.9857[b] |
|  |  |  |  |  | (0.2713) |  |  |  | (0.4831) |
| 5-year EPS growth |  |  |  |  |  | -0.2176[a] |  |  | -0.0519 |
|  |  |  |  |  |  | (0.0773) |  |  | (0.0603) |
| 5-year WSJ return |  |  |  |  |  |  | -5.4363[a] |  | -5.5172[a] |
|  |  |  |  |  |  |  | (1.6753) |  | (1.3792) |
| 5-year growth in GDP pc |  |  |  |  |  |  |  | -13.3910[b] | -6.9125 |
|  |  |  |  |  |  |  |  | (5.1701) | (5.9754) |
| Observations | 445 | 445 | 445 | 144 | 445 | 445 | 145 | 149 | 140 |
| Adjusted $R^2$ | 0.0% | 23.9% | 0.4% | 10.7% | 1.9% | 10.1% | 31.0% | 22.0% | 73.0% |

## Panel B: Dependent variable is LTG

|  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| Error Term Spread | -0.0194[b] |  |  |  |  |  |  |  |  |
|  | (0.0085) |  |  |  |  |  |  |  |  |
| Error Credit Spread |  | -0.0212[b] |  |  |  |  |  |  |  |
|  |  | (0.0087) |  |  |  |  |  |  |  |  |
| Error spc (Cochrane |  |  | -0.0193[b] |  |  |  |  |  |  |

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| Campbell) | | | (0.0085) | | | | | | |
| Error cay | | | | -0.0219 | | | | | |
| | | | | (0.0143) | | | | | |
| Error 5yr Div growth | | | | | $-0.0213^b$ | | | | |
| | | | | | (0.0085) | | | | |
| Error 5yr GDP | | | | | | $-0.0205^b$ | | | |
| | | | | | | (0.0090) | | | |
| 5-year WSJ return | | | | | | | -0.0136 | | |
| | | | | | | | (0.0112) | | |
| 5yr growth in GDP pc | | | | | | | | -0.0123 | |
| | | | | | | | | (0.0136) | |
| Error Using All Vars | | | | | | | | | -0.0136 |
| | | | | | | | | | (0.0143) |
| Observations | 445 | 445 | 445 | 144 | 445 | 445 | 145 | 149 | 140 |
| Adjusted $R^2$ | 19% | 17% | 18% | 25% | 22% | 19% | 5% | 5% | 2% |

Note: Standard errors are corrected for serial correlation using Newey and West (1987).

*Return Predictability.* Here we separately assess the predictability of returns from the components of the dividend price ratio that are predicted from, or orthogonal to, LTG.

Table C.2 repeats the exercise in Table 5, showing that LTG is not explained by movements in 5-year cumulative returns that are not explained by fundamentals.

### Table C.2

Panel A:  Dependent Variable is Cumulative Return between t-5 and t

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| $e_t\text{-}cae_{t-1}$ | 0.1872 | | | -0.0537 |
| | (0.1519) | | | (0.1718) |
| 5-year WSJ return | | $0.2447^c$ | | 0.1875 |
| | | (0.1358) | | (0.1331) |
| $\Delta$ 5-year GDP pc growth | | | $0.5115^a$ | $0.4896^a$ |
| | | | (0.1429) | (0.1409) |
| Observations | 149 | 149 | 149 | 149 |
| Adjusted $R^2$ | 3% | 5% | 26% | 27% |

Panel B:  Dependent Variable is LTG; Independent variable is prediction error from Panel A

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| Forecast error $e_t\text{-}cae_{t-1}$ | 0.3707 | | | |
| | (0.2412) | | | |

| | | | | |
|---|---|---|---|---|
| Forecast error 5-year WSJ return | | 0.3889 | | |
| | | (0.2473) | | |
| Forecast error Δ 5-year GDP pc growth | | | 0.1577 | |
| | | | (0.2234) | |
| | | | | 0.1528 |
| Forecast Error Using All Vars | | | | (0.2235) |
| Observations | 149 | 149 | 149 | 149 |
| Adjusted R² | 13% | 14% | 1% | 1% |

## Table C.3  Return predictability from LTG and price to dividend ratio

## Panel A:  Returns and the component of dp that is orthogonal to LTG

| | Dependent Variable: Log return between years t and: | | | | |
|---|---|---|---|---|---|
| | t+1 | t+2 | t+3 | t+4 | t+5 |
| $dp_t - E[dp_t \mid LTG_t]$ | -0.1255[b] | -0.2049[b] | -0.3283[a] | -0.4567[a] | -0.5700[a] |
| | (0.0499) | (0.0882) | (0.1116) | (0.1363) | (0.1695) |
| Observations | 385 | 385 | 385 | 385 | 385 |
| Adjusted R2 | 7% | 9% | 16% | 24% | 31% |

Note: Standard errors are corrected for serial correlation using Newey and West (1987).

## Panel B:  Returns and the predicted and orthogonal components of dp

| | Dependent Variable: Log return between years t and: | | | | |
|---|---|---|---|---|---|
| | t+1 | t+2 | t+3 | t+4 | t+5 |
| $dp_t - E[dp_t \mid LTG_t]$ | -0.1113[b] | -0.1750[b] | -0.2905[a] | -0.4138[a] | -0.5239[a] |
| | (0.0491) | (0.0774) | (0.0971) | (0.1157) | (0.1336) |
| $E[dp_t \mid LTG_t]$ | -0.3024[a] | -0.6365[a] | -0.8057[a] | -0.9148[a] | -0.9821[a] |
| | (0.1084) | (0.1718) | (0.1616) | (0.1721) | (0.2361) |
| Observations | 385 | 385 | 385 | 385 | 385 |
| Adjusted R2 | 16% | 29% | 37% | 45% | 52% |

Note: Standard errors are corrected for serial correlation using Newey and West (1987).

## Table C.4

## Revisions about short term growth expectations do not predict returns

## Panel A:  Returns and growth forecast for year 1

| | Dependent Variable: Log return between years t and: | | | | |
|---|---|---|---|---|---|
| | t+1 | t+2 | t+3 | t+4 | t+5 |
| $E_t e_{t+1} - e_t$ | -0.0084 | 0.0519 | 0.0734 | 0.1599[c] | 0.2286[b] |
| | (0.0444) | (0.0702) | (0.0837) | (0.0905) | (0.0904) |

| Observations | 380 | 380 | 380 | 380 | 380 |

Note: Standard errors are corrected for serial correlation using Newey and West (1987).

### Panel B:  Returns and growth forecast for year 2

`

| | Dependent Variable: Log return between years t and: | | | | |
|---|---|---|---|---|---|
| | t+1 | t+2 | t+3 | t+4 | t+5 |
| $E_t[e_{t+2}-e_{t+1}]$ | -0.3605 | 0.1771 | 0.6216 | 1.8153 | 3.1773 |
| | (0.6106) | (1.1817) | (1.8082) | (2.2459) | (2.1957) |
| Observations | 380 | 380 | 380 | 380 | 380 |

Note: Standard errors are corrected for serial correlation using Newey and West (1987).

**Table C.5**
**Moving average representation of earnings growth**

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| $ge_{t-1}$ | -1.0000[a] | -0.5794[a] | -0.6069[a] | -0.8893[a] | -0.8421[a] |
| | (0.0819) | (0.1583) | (0.2189) | (0.2182) | (0.2290) |
| $ge_{t-2}$ | | -0.4206[a] | -0.4321[a] | -0.6381[a] | -0.6519[a] |
| | | (0.1436) | (0.1558) | (0.1833) | (0.1894) |
| $ge_{t-3}$ | | | 0.0390 | -0.0287 | -0.0469 |
| | | | (0.2161) | (0.2478) | (0.2425) |
| $ge_{t-4}$ | | | | 0.5687[a] | 0.4664[c] |
| | | | | (0.1834) | (0.2686) |
| $ge_{t-5}$ | | | | | 0.0904 |
| | | | | | (0.1674) |
| Observations | 38 | 38 | 38 | 38 | 38 |