# Integrated Monetary and Financial Policies for Small Open Economies*

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**Abstract**

Many small open economies deploy foreign exchange intervention, capital controls, and macroprudential policies to cope with shocks. We develop a micro-founded model that characterizes the optimal joint use of these policies. Our framework incorporates nominal rigidities with producer and dominant currency pricing, pecuniary externalities due to domestic and external borrowing constraints, and shallow foreign exchange markets. We find that: (1) Prudential capital controls to address pecuniary externalities are larger under DCP than PCP; (2) Capital controls and FX intervention enhance monetary autonomy after foreign appetite shocks if FX markets are shallow; (3) Countries with shallow FX markets and currency mismatches face a policy conundrum: while a ban on open FX positions reduces the need for prudential capital controls to address pecuniary externalities, it increases the vulnerability to foreign appetite shocks and can make the economy dependent on FX intervention. 4) Exchange rate flexibility helps relax domestic currency borrowing constraints but may tighten borrowing constraints in FX.

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# 1 Introduction

Despite the widespread adoption of inflation targeting frameworks around the world, many small open economies follow more eclectic approaches. Capital controls and foreign exchange (FX) intervention have a long tradition of being in the policy toolkit of emerging markets, and they have also been deployed by several advanced economies in the post-global-financial-crisis era. At the same time, as figure 1 shows, this era has witnessed a surge in the use of macroprudential tools, as a growing number of countries have implemented or tightened such regulations.

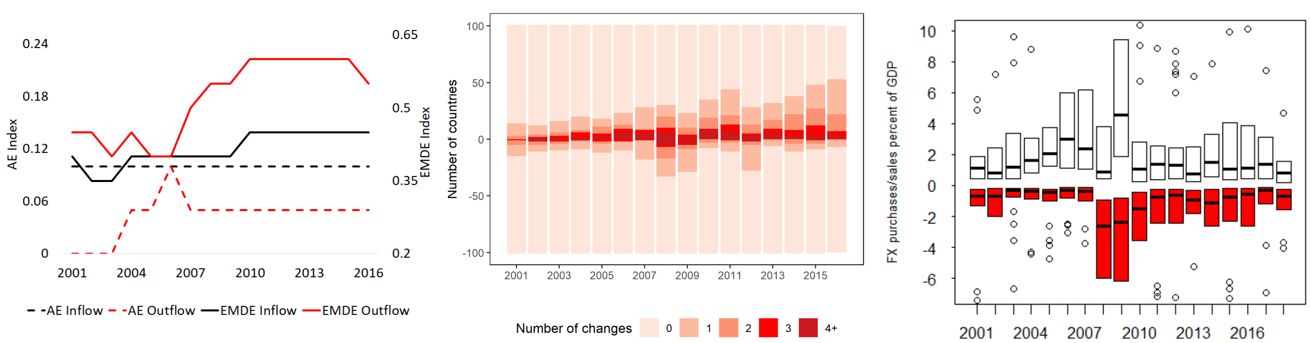
The tradeoff between exchange stability, monetary independence, and capital account openness for open economies goes back at least to the monetary policy trilemma, but the complexities of the world today pose challenges that require a more active use of monetary and financial policies contingent on shocks and frictions. To understand these complex interactions, we characterize the optimal joint configuration of monetary policy, capital controls, foreign exchange intervention, and macroprudential tools in a small open economy framework. We borrow ingredients from a recent theoretical literature in international macroeconomics that provides a careful treatment of the tradeoffs associated with each policy and the externalities they aim to address. While the literature rarely considers more than one policy and friction at a time, our integrated framework makes it possible to analyze not only the costs and benefits of individual policies, but also how the entire range of policies and externalities interact with each other, and how the tradeoffs change when policies are used in combination.

Our framework yields several novel results that underscore the importance of integrating policies and externalities.[1] (1) The weaker impact of exchange rate flexibility on export substitution under dominant currency pricing (DCP) leads to more rather than less volatile exchange rates than under producer currency pricing (PCP) if external FX debt constraints do not bind; while if those constraints do bind, prudential capital controls to address pecuniary aggregate demand externalities

[1] In Basu et al. (our forthcoming IMF Working Paper), we have collected a more comprehensive set of optimal policy results contingent on shocks and country characteristics.

tend to be larger under DCP than under PCP. (2) When financial intermediaries have limited ability to take on the country's currency exposure—in other words, FX markets are shallow—the premia on domestic currency debt are destabilized by shocks to foreigners' willingness to hold that debt, and capital controls can enhance monetary autonomy by facilitating macroeconomic adjustment to those premia while allowing the policy rate to focus on domestic sources of price pressures. (3) While banning domestic residents from taking on open FX exposures can reduce the need for prudential capital controls to address pecuniary externalities, the ban can reduce FX market depth, increasing the economy's vulnerability to foreign appetite shocks, and making it more dependent on FX intervention. (4) Exchange rate depreciations are useful in relaxing domestic housing constraints but may tighten external FX debt constraints, and in an environment where both constraints may bind together, housing macroprudential taxes may be lower under DCP than PCP.

**Figure 1:** Use of Capital Controls, Macroprudential Regulation and FX Intervention




(a) Capital Controls (b) Macroprudential Regulation (c) FX Intervention

Sources: Fernandez et al. (2017); Alam et al. (2019); IMF database on FX interventions
Notes: (a) Median capital inflow and outflow restrictiveness indices for advanced economies (AE) and emerging market and developing economies (EMDE). Higher values are associated with higher barriers. (b) Number of countries that tightened (positive values) or loosened (negative values) macroprudential regulations, broken down by the number of policy actions. (c) Distribution of FX purchases (positive values) and sales (negative values) as percent of GDP based on estimates that strip out valuation effects. Lines in the middle of the boxes represent the medians, box edges show the 25th and 75th percentiles.

Our model features three sectors: the commodity sector, the non-tradable housing sector and the differentiated tradable goods sector. The country is a price taker in commodity markets. Housing services are produced by firms using land as the input to production. The differentiated tradable goods are produced by firms with pricing power. Their price may be sticky in the producer's currency (PCP) or in the dominant currency (DCP). Households do not internalize the impact of their consumption decisions on aggregate demand, paving the way to the well-known Keynesian *aggregate demand externality*. As is standard in open economy models of monetary policy, our assumption that firms have pricing power and face downward-sloping export demand schedule gives rise to a *terms of trade externality*, where individual firms do not take into account that their production decisions impact the position of the aggregate economy on the export demand schedule.[2]

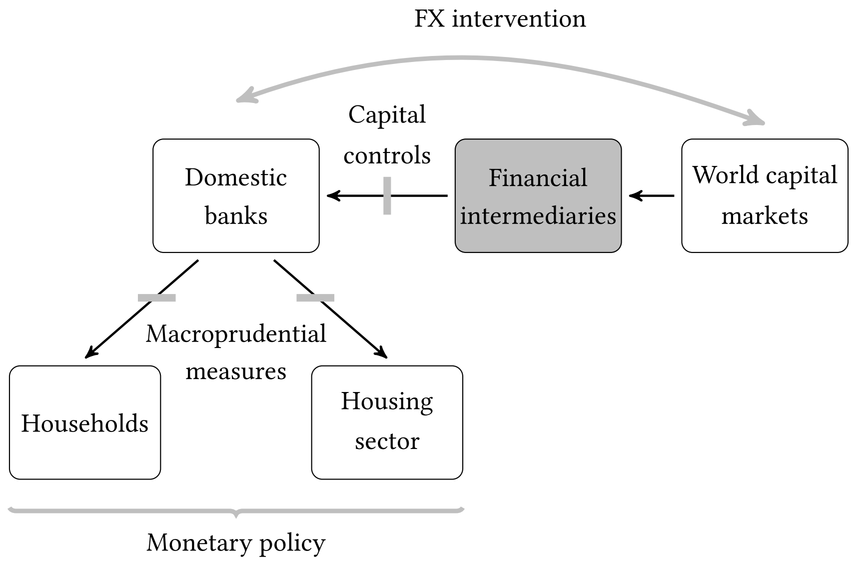
Credit markets feature two kinds of pecuniary externalities. As displayed in Figure 2, domestic banks borrow from financial intermediaries and lend to the domestic households and the housing sector firms. An occasionally-binding borrowing constraint limits domestic banks' external borrowing to a fraction of the domestic price of the differentiated tradable good. This constraint combined with the households' FX exposure generates a *pecuniary aggregate demand externality* as households do not internalize the effects of their individual actions on aggregate demand, the exchange rate, and the tightness of the constraint ex post. Another occasionally-binding borrowing constraint requires the housing sector firms to post a fraction of the value of their land holdings as collateral, leading to a *pecuniary production externality* since these firms do not internalize the effects of their production decisions on land prices.

Financial intermediaries borrow in foreign currency in the world market and satisfy the domestic banks' domestic currency borrowing needs. They are partly foreign and partly owned by the households; this domestic ownership is the source of the economy's FX exposure. We assume that the intermediaries are constrained in their ability to bear the country's currency exposure, which leads to deviations from the uncovered interest parity condition. We refer to this inefficiency as the FX markets being shallow.[3] What we call a *financial terms of trade externality* arises in this case because individual banks do not take into account that their borrowing decisions impact the premium that

[2]Even though this externality arises naturally in our setting, it is not clear that it is relevant for policymaking in the real world, so we focus on results that do not hinge on it.

[3]The uncovered interest parity condition holds in the absence of constraints on the intermediaries ability to bear the country's currency exposure, which we label as the case of deep FX markets.

**Figure 2:** Structure of the Financial Market




the economy as a whole needs to pay to the intermediaries.

While each externality is conceptually tied to a particular friction, in equilibrium, the externalities are connected to each other, and policies typically influence several of them at the same time. Monetary policy, working through changes in the policy rate, affects the interest rate faced by domestic agents when they make consumption, production, and borrowing decisions, as well as the rate that the domestic banks offer to the financial intermediaries. In our integrated framework, it can influence most externalities, one of which is the aggregate demand externality. As in standard open economy models of monetary policy, flexible exchange rates have expenditure-switching benefits whereby an exchange rate depreciation makes imports become more expensive relative to home-produced goods. Households therefore switch away from imports towards home goods. Under PCP, expenditure switching is also operational through exports: an exchange rate depreciation boosts demand by making exports more competitive. Under DCP, exchange rate adjustment becomes a weaker tool because while it continues to affect import consumption, it no longer affects the competitiveness of exports on world markets, as the dollar price remains unchanged.

By reallocating consumption, production, and borrowing intertemporally, capital controls can address a number of externalities. Following the vast literature on emerging market capital flows and sudden stops, we model capital controls as state contingent taxes on capital inflows. When used prudentially, capital controls can contain pecuniary aggregate demand externalities by curbing external debt and consumption ex-ante, which is desirable when a depreciation can make the borrowing constraint bind by worsening the country's balance sheet through FX exposures. When FX markets are shallow, another incentive to curb debt through capital controls arises because doing so reduces the losses incurred due to the inefficiency in the intermediation of debt and mitigates the financial terms of trade externality.

Macroprudential tools are conceptually similar to capital controls in that they also curb overborrowing by affecting debt flows. We model them as taxes on domestic banks' lending to the domestic agents that can differ across consumers and the housing sector. Macroprudential consumer taxes can work as substitutes for prudential capital controls in the absence of housing sector frictions as they affect external debt through affecting households' demand for domestic debt from the banks. As with capital controls, they can affect external debt through impacting domestic debt and address pecuniary aggregate demand externalities or prevent the reduction in available aggregate resources due to the inefficient premia paid to the intermediaries as well as smooth the financial terms of trade externality. Macroprudential housing taxes are an important tool for reducing the risk of fire sales in housing markets.

Sterilized FX intervention primarily circumvents the inefficiency of the financial intermediaries in countries with shallow FX markets. It achieves this by changing the amount of external debt that needs to be absorbed by the financial intermediaries and therefore the associated premium. By doing so, it can allow monetary policy to be aimed at stabilizing households' borrowing and the domestic banks' external borrowing. However, reserve accumulation involves buying low-return foreign currency bonds and selling high-return domestic currency bonds, therefore, a carry cost.

At the constrained efficient allocations, how these policy instruments should tackle the externalities we have described depends on the kind of shock and the obstacles to the simultaneous mitigation of different externalities. We highlight a few key results which emerge from our integrated framework.

While pricing in the dominant currency reduces the benefits of exchange rate flexibility and generally features under- or over-exporting, flexible exchange rates are optimal in the absence of other frictions. In the case of deep FX markets without borrowing constraints, policies such as capital controls do not improve efficiency beyond the terms of trade externality, since they do not address the stickiness of export prices in the dominant currency. Indeed, under most shocks, the DCP economy is characterized by more volatile exchange rates than the PCP economy: under DCP, achieving the same benefits from exchange rate flexibility as PCP requires *larger* exchange rate movements. In other words, when there is no welfare cost associated with exchange rate movements, exchange rates should be more volatile under DCP than PCP so as to stabilize price pressures.

When the pecuniary aggregate demand externality is present, optimal prudential capital controls are larger under DCP than PCP. When future shocks can lead to a binding constraint, the resulting pecuniary externality alters the tradeoffs for monetary policy during the period of the binding constraint: the planner sets the exchange rate to balance price pressures and binding borrowing constraints. The consideration of relaxing the borrowing constraint leads to a more appreciated exchange rate relative to the case without such considerations. Such *relative* appreciation is larger under DCP than PCP. This is because the exchange rate is optimally used more to relax the borrowing constraint ex post rather than to stabilize demand under DCP, since the benefit of exchange rate flexibility for demand stabilization is smaller. It then follows that prudential capital controls are larger under DCP because they are needed to shift demand over time from normal times to the period of distress, by curbing demand before the shock and stimulating it afterwards.

Under shallow FX markets, capital controls and FX intervention improve monetary autonomy after foreign appetite shocks, i.e., shocks to the foreigners' willingness to hold domestic currency debt. After adverse foreign appetite shocks, the country needs to offer higher external premia to foreigners. When the only available tool is monetary policy, this shock leads to a depreciation on impact to reduce imports and generate an expected appreciation. However, monetary policy alone cannot balance domestic price pressures and also target premia. Instead, households excessively deleverage their debt. If FX intervention is also used, FX sales cushion the shock, allowing the policy rate and domestic macro outcomes to move less by partly addressing the financial terms of trade externality. This externality also generates a rationale for capital controls, which pushes capital controls up when UIP premia are inefficiently high and pushes capital controls down when UIP premia are inefficiently low. Capital controls are then appropriate to increase monetary autonomy, in the sense that they address unresolved financial terms of trade externalities while allowing the policy rate to better focus on domestic sources of price pressures.[4] Finally, if FX intervention and capital controls (or macroprudential policies) are used together, monetary policy rate is more de-linked from the shock than when the policies are used individually, enhancing monetary autonomy further.

Countries with shallow FX markets and external borrowing constraints face a policy conundrum: while banning open FX positions can reduce the need for prudential capital controls to address pecuniary aggregate demand externalities, the ban increases the economy's vulnerability to foreign appetite shocks, and can make the economy more dependent on FX intervention. Since the households' FX exposures are due to their ownership of intermediaries in our model, we consider the impact of a ban on open FX positions for those intermediaries which are domestically owned so that domestic currency debt is absorbed by foreign-owned intermediaries instead. Under deep FX markets, such a ban on FX exposures is optimal, because it eliminates currency mismatches and the associated pecuniary aggregate demand externalities with no side effects. However, under shallow FX markets, the ban makes it more expensive to finance external debt and makes FX markets even shallower. As a result, it increases the economy's vulnerability to foreign appetite shocks and may make the economy more dependent on FX intervention by increasing the marginal value of FX intervention after these shocks.

In the presence of housing frictions, a new role for exchange rate flexibility arises. The exist-

[4] While they have similar macro effects in response to this shock, FX intervention and capital controls work through different channels. FX sales reduce the total effective outflow that the private sector has to absorb, reducing the necessary external premia. Capital controls de-link the external premia from the policy rate, so that a loosening of controls can provide higher returns to foreigners without altering the policy rate.

ing literature shows that closed economies with high housing debt should impose macroprudential housing debt taxes in normal times, and relax them (together with monetary policy) to support land prices when the housing constraints bind. Our framework features this standard mechanism. In addition, a new channel arises for open economies: an exchange rate depreciation generates expenditure switching not only towards the domestically produced traded good but also towards housing services. This increased demand for housing services bolsters rents and land prices, and relaxes housing borrowing constraints that are set in domestic currency. This channel is most apparent when we remove all of the policy instruments but let the exchange rate move flexibly in the face of a shock to the housing debt limit. Comparing that case with the one where the exchange rate is fixed reveals that the exchange rate depreciates and relaxes the housing sector constraint, even possibly to the extent of making macroprudential housing debt taxes unnecessary. The desired depreciation required to relax the housing constraint may not match with the desired size of the policy rate reduction to ease the constraint. If the policy rate reduction depreciates the exchange rate beyond the desired size of depreciation, capital inflow subsidies (or reductions in inflow taxes) or FX sales can contain the depreciation associated with the policy rate cut and avoid excessive expenditure switching.

While the larger exchange rate volatility in DCP aggravates FX borrowing constraints, it eases domestic currency borrowing constraints. Recall that, everything else equal, there is higher exchange rate volatility under DCP. This feature of DCP is beneficial in the case of borrowing constraints in domestic currency. The exchange rate depreciation can boost housing sector consumption and relax the housing constraint. The aggregate-demand-destabilizing effects of the depreciation are smaller under DCP because of weaker expenditure switching. Thus, the DCP economy faces an easier tradeoff between relaxing the housing constraint and demand stabilization. This property translates into smaller ex ante macroprudential housing taxes under DCP, since the exchange rate can be used more forcefully ex post.

Considering the borrowing constraint of the housing sector and the banks at the same time, we find that a housing constraint can trigger an external constraint and vice versa. For example, when there is an adverse shock to the housing sector's debt limit, as discussed above, the exchange rate depreciates to help relax the constraint, which is in domestic currency. But when the banks' borrowing constraint is also relevant, the depreciation lowers their debt limit in FX terms and tightens their constraint. In this scenario, we find that interest rates and capital controls are used ex ante to reduce the interest burden on inherited debt for the housing sector and to limit external FX debt.

Symmetrically, the banks' debt limit shock may cause the housing constraint to bind. The external constraint is associated with a large cut in the policy rate and an exchange rate depreciation which tends to increase the domestic currency value of rents and the land price. However, it is also associated with an increase in the borrowing rate for domestic households and the housing sector, and a decrease in household consumption. These latter factors tend to reduce rents and the land price. If the latter effects are larger than the former ones, the housing constraint may bind. Similar to above, in this situation, ex ante policy actions such as ex ante housing macroprudential taxes become optimal. Since the exchange rate depreciation is larger and the increase in the domestic borrowing rate is smaller under DCP after external debt limit shocks, the housing market is better insulated under DCP.

Our paper contributes to four strands of literature. First, we build on the insights developed by Gopinath (2015), Casas et al. (2016) and Gopinath et al. (2019) on the dominant currency paradigm. Following Casas et al. (2016), we compare and contrast the monetary policy implications of producer and dominant currency pricing for a small open economy. But unlike Casas et al. (2016), we consider a smaller scale model that can be solved nonlinearly and develop a rich financial market structure that allows us to analyze policies other than interest rate policy. In this vein, our work is also related to Egorov and Mukhin (2019) who look at optimal monetary policy and the use of capital controls under DCP in a set up without pecuniary externalities or shallow foreign exchange markets.

Second, our paper contributes to the literature on aggregate demand and pecuniary externalities, and the joint analysis of monetary and macroprudential policies. Similar to Farhi and Werning (2016), we build a small scale model with nominal rigidities and monetary policy to form the backbone of our model environment. Also similar to Farhi and Werning (2016), we benefit from the findings of the vast literature on pecuniary externalities and the use of macroprudential policies, exemplified by Mendoza (2010) and Bianchi (2011). We build on these papers by considering alternative forms of nominal rigidities as well as further financial frictions and policy instruments.

Third, our paper borrows elements from the literature on exchange rate determination and foreign exchange intervention in the presence of inefficient financial intermediation. The intermediation inefficiency we consider follows that developed by Gabaix and Maggiori (2015). Similar to Fanelli and Straub (2019) and Cavallino (2019), this inefficiency provides a rationale for FX intervention and at the same time determines its effectiveness. Unlike, these papers, we nest the intermediation inefficiency into a larger setting that features other frictions and policies.

Finally, our modeling of the frictions in the housing sector borrows elements from Kiyotaki and Moore (1997). Specifically, we assume a similar externality to Kiyotaki and Moore (1997), where the use of land depends on a constraint that includes the land price, but we consider an occasionally-binding rather than an always-binding constraint, allowing us to explore the shocks that might cause the constraint to bind. Our work is related to Korinek and Sandri (2016) who also aim to capture the differences between capital controls and macroprudential tools using models that have either the exchange rate or the land price in an occasionally-binding borrowing constraint. Differently from them, we build a unified model nesting both types of constraints and rationales for capital controls and macroprudential policies, while allowing for a broader set of general equilibrium interactions across sectors and policies.

The rest of this paper proceeds as follows. First, section 2 lays out the model environment. Section 3 and 4 respectively describe our results for deep and shallow foreign exchange markets. Section 5 describes our results in the presence of frictions in the housing sector. Finally, section 6 concludes.

## 2 A Three-Period Small Open Economy

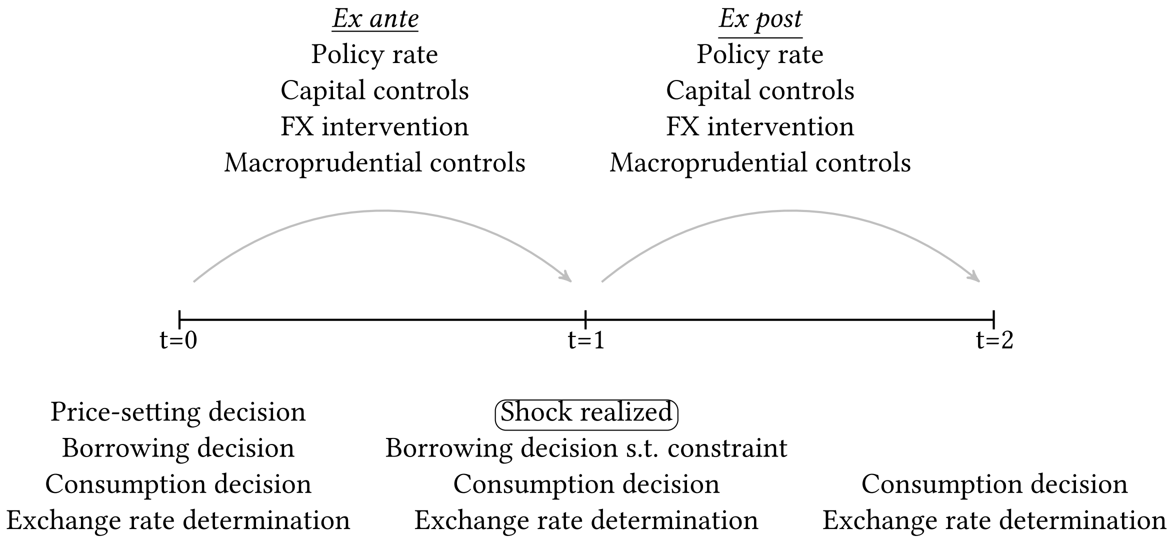
We construct a three-period model of a small open economy composed of households, a government, tradable sector firms, housing sector firms, domestic banks, and international financial intermediaries, a fraction of which is owned by domestic households. Tradable sector firms use labor to produce tradable goods. Housing sector firms use land to produce nontradable housing services; a subset of these firms operates a linear technology and another subset uses a concave technology. The economy receives an endowment of commodities that are exported. Tradable good prices are sticky, and following Gopinath (2015), export prices of home-produced tradable goods may follow producer currency pricing (PCP, i.e., exports are invoiced in domestic currency) or dominant currency pricing (DCP, i.e., exports are invoiced in dollars). Under both PCP and DCP, import and commodity prices are denominated in dollars.

An occasionally-binding borrowing constraint limits domestic banks' debt to a fraction of the domestic price of the tradable good, in the spirit of Mendoza (2010), Bianchi (2011) and Farhi and Werning (2016). Another occasionally-binding borrowing constraint limits the debt of linear firms in the housing sector to a fraction of the value of their landholdings, following Kiyotaki and Moore (1997). There are two noncontingent assets—a domestic currency bond and a dollar bond—and asset market segmentation: domestic agents can only trade the domestic currency bond, while international financial intermediaries can trade in both bonds. This segmentation in international financial markets follows Gabaix and Maggiori (2015). The constrained social planner maximizes households' welfare while taking as given the decisions of private agents.

The financial structure of our model is shown in figure 2 and a stylized timeline of events is shown in figure 3. A variety of shocks strike in period 1, after which all uncertainty is resolved. The planner can implement policies either in period 0 in anticipation of possible shocks (i.e., prudential or ex-ante policy) or in period 1 after the shock has been realized (i.e., ex-post policy):

- Monetary policy. The planner sets the policy rate, which is equal to the interest rate on domestic

**Figure 3:** Timeline of Events




currency bonds, between periods 0 and 1 and between periods 1 and 2.

- Capital inflow controls. The planner can set taxes/subsidies on inflows which generate a spread between the policy rate and the interest rate earned by international financial intermediaries. Such controls can be used in a prudential fashion, between periods 0 and 1, or in an ex-post fashion, between periods 1 and 2. Prudential capital controls are similar to those studied by Bianchi (2011), Korinek (2011), and Farhi and Werning (2016).

- FX intervention. The planner can intermediate between the domestic currency bond and the dollar bond, circumventing the financial intermediaries and their inefficiency, between periods 0 and 1 and between periods 1 and 2. Such intervention is similar to that in Gabaix and Maggiori (2015), Cavallino (2019), and Fanelli and Straub (2019).

- Macroprudential controls. The planner can set taxes/subsidies separately on the borrowing of households and of the linear housing sector firms. Such controls can be used in a prudential fashion, between periods 0 and 1, or in an ex-post fashion, between periods 1 and 2.

Next, we lay out the environment for the private sector agents and derive their optimal decisions before turning to the constrained social planner problem.

*Households*

Households maximize a welfare function which follows the Cole and Obstfeld (1991) formulation over consumption, and the Gali and Monacelli (2005) special case of linear disutility of labor:

$$\mathbb{E}_0\left[\sum{}_{t=0}^{2}\beta^t U\left(C_{Ht}, C_{Ft}, C_{Rt}, N_t\right)\right]$$

where $U\left(C_{Ht}, C_{Ft}, C_{Rt}, N_t\right) = \alpha_H \log C_{Ht} + \alpha_F \log C_{Ft} + \left(1-\alpha_H-\alpha_F\right)\log C_{Rt} - N_t$

Their maximization is subject to a budget constraint:

$$\begin{aligned} & W_t N_t + \Pi_{Tt} + \Pi_{Bt} + \lambda\Pi_{FIt} + T_{FXIt} - T_{Gt} + T_{Rt} + E_t P_{Zt}^{*} Z_t + D_{HHt+1} \\ & \geq P_{Ht}C_{Ht} + E_t P_{Ft}^{*} C_{Ft} + P_{Rt}C_{Rt} + \left(1+\theta_{HHt-1}\right)\left(1+\rho_{t-1}\right)D_{HHt}. \end{aligned} \tag{1}$$

Starting with the right hand side of the budget constraint, $P_{Ht}$ and $C_{Ht}$ are the domestic price and consumption of the tradable good, $E_t$ is the exchange rate in units of domestic currency per dollar, $E_t P_{Ft}^{*}$ (i.e., the exchange rate multiplied by the dollar price of imports) is the domestic currency price of imports, $C_{Ft}$ is the consumption of imports, $P_{Rt}$ and $C_{Rt}$ are the rental price and consumption of nontradable housing services, $D_{HHt+1}$ is the domestic-currency debt at the end of period $t$, $\theta_{HHt}$ is the consumer macroprudential tax, and $\rho_t$ is the interest rate offered by domestic banks on domestic currency debt in period $t$, which applies between periods $t$ and $t+1$.

On the left hand side of the budget constraint, $N_t$ is labor supply, $W_t$ is the wage, $\Pi_{Tt}$ is the profit from tradable sector firms, $\Pi_{Bt}$ is the profit from domestic banks, $\lambda$ is the fraction of international financial intermediaries owned by domestic households while $\Pi_{FIt}$ is the profit of each of them, $T_{FXIt}$ is the profit of the planner from FX operations, $T_{Gt}$ is the lump-sum tax levied by the planner, $T_{Rt}$ is the transfer from housing firms (made only in period 2), $E_t P_{Zt}^{*}$ (i.e., the exchange rate multiplied by the dollar price of commodities) is the domestic price of commodity exports, and $Z_t$ is the endowment of commodities, which are entirely exported.

The households' first order conditions (FOCs) lead to the following intratemporal conditions:

$$\frac{\alpha_H}{C_{Ht}P_{Ht}} = \frac{\alpha_F}{E_t P^*_{Ft} C_{Ft}} = \frac{\alpha_R}{C_{Rt}P_{Rt}} = \frac{1}{W_t} \tag{2}$$

$$\Rightarrow C_{Rt} = \frac{\alpha_H}{\alpha_F} p_{Rt} C_{Ft} \text{ where } p_{Rt} = \frac{E_t P^*_{Ft}}{P_{Rt}}$$

$$\text{and } C_{Ht} = \frac{\alpha_H}{\alpha_F} p_{Ht} C_{Ft} \text{ where } p_{Ht} = \frac{E_t P^*_{Ft}}{P_{Ht}},$$

where $p_{Rt}$ is the price of foreign goods relative to domestic rents, and $p_{Ht}$ is the price of foreign goods relative to home tradable goods. The FOCs also yield the Euler conditions:

$$\frac{\alpha_F}{P^*_{F0}C_{F0}} = \beta\left(1+\theta_{HH0}\right)\left(1+\rho_0\right)\mathbb{E}_0\left[\frac{E_0}{E_1}\frac{\alpha_F}{P^*_{F1}C_{F1}}\right] \text{ and } \frac{\alpha_F}{P^*_{F1}C_{F1}} = \beta\left(1+\theta_{HH1}\right)\left(1+\rho_1\right)\frac{E_1}{E_2}\frac{\alpha_F}{P^*_{F2}C_{F2}}. \tag{3}$$

*Tradable sector firms*

Tradable sector firms are monopolistically competitive and set prices at the beginning of period $t = 0$, after which prices are fully rigid (so we can remove the time subscripts on tradable good prices).[5] Following the New-Keynesian tradition, we assume that they produce a variety $j \in [0,1]$ of tradable goods, $Y_{Tt}(j)$, using labor, $N_t(j)$. The varieties may be consumed domestically, $Y_{Ht}(j)$, or exported, $Y_{Xt}(j)$:

$$Y_{Tt}(j) = Y_{Ht}(j) + Y_{Xt}(j) = A_t N_t(j). \tag{4}$$

Firms face downward-sloping demands for their output from domestic consumption and from export demand. Domestic consumption involves combining the tradable varieties into an aggregate tradable good:

$$Y_{Ht} = \left(\int_0^1 Y_{Ht}(j)^{(\varepsilon-1)/\varepsilon}\, dj\right)^{\varepsilon/(\varepsilon-1)}.$$

[5]This extreme price-setting assumption keeps the model tractable. Under this assumption, we can interpret the optimal exchange rate policy as being related to the planner's desire to mitigate static price pressures, i.e., to ensure that the domestic price level is at an appropriate level relative to the price level of foreign goods. However, this assumption prevents us from considering the welfare costs of price dispersion and inflation dynamics that may damage credibility.

The corresponding domestic demand curve is:

$$Y_{Ht}(j) = \left(\frac{P_H(j)}{P_H}\right)^{-\varepsilon} Y_{Ht},$$

where $P_H(j)$ is the sticky domestic-currency price of each variety and $P_H$ is the price index for the aggregate tradable good. We assume that the export demand curve follows the same form over each traded variety and a unit-elastic expression for the aggregate traded good:

$$Y_{Xt}(j) = \left(\frac{P_X(j)}{P_X}\right)^{-\varepsilon} Y_{Xt} \text{ and } Y_{Xt} = \omega p_{Xt},$$

where $P_X(j)$ is the price fixed by firms for each exported variety, $P_X$ is the corresponding price index for the exported tradable good, and $p_{Xt}$ is the relative price of foreign goods to exports. The denomination of $P_X(j)$ and $P_X$, and the formula for $p_{Xt}$ depend on the pricing paradigm.

Under PCP, firms set identical domestic-currency prices for all of their output, regardless of whether the good is consumed domestically or exported, i.e., $P_X(j) = P_H(j)$. In other words, the law of one price holds, as in Gali and Monacelli (2005). Under DCP, firms set a domestic-currency price, $P_H(j)$, for the domestically-consumed portion of the tradable good, and a separate dollar price, $P_X(j)$, for the exported portion of the good. As a result, the relative price of foreign goods to exports, i.e., the terms of trade, follows separate formulae under PCP and DCP:

$$p_{Xt}^{PCP} = \frac{E_t P_{Ft}^*}{P_H} \text{ and } p_{Xt}^{DCP} = \frac{P_{Ft}^*}{P_X}.$$

Profit maximization for firms *under PCP* is given by:

$$\max \Pi_{Tt}(j) = \Pi_{Ht}(j) + \Pi_{Xt}(j)$$

$$= \max \mathbb{E}_0 \left[\sum_{t=0}^{1} \Lambda_t \left[P_H(j)\left(Y_{Ht}(j) + Y_{Xt}(j)\right) - (1+\phi) W_t N_t(j)\right]\right]$$
$$= \max \mathbb{E}_0 \left[\sum_{t=0}^{1} \Lambda_t \left[P_H(j) - (1+\phi)\frac{W_t}{A_t}\right]\left(Y_{Ht} + Y_{Xt}\right)\left(\frac{P_H(j)}{P_H}\right)^{-\varepsilon}\right],$$

where $\phi$ is a constant labor tax applied on all home production of tradable goods. We assume that firms have perfect access to dollar debt markets, so we set their discount factors as follows: $\Lambda_0 = 1$, $\Lambda_1 = \frac{1}{(1+i_0^*)} \frac{E_0}{E_1}$, and $\Lambda_2 = \frac{1}{(1+i_0^*)(1+i_1^*)} \frac{E_0}{E_2}$.[6] The FOC of the above expression produces a formula for $P_H(j)$—and, since all varieties are identical, for $P_H$—as a function of home demand, export demand, and the labor tax:

$$P_H = P_H(j) = (1+\phi) \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_0 \left[ \sum_{t=0}^{2} \Lambda_t \frac{W_t}{A_t} (Y_{Ht} + Y_{Xt}) \right]}{\mathbb{E}_0 \left[ \sum_{t=0}^{2} \Lambda_t (Y_{Ht} + Y_{Xt}) \right]} \tag{5}$$

The optimal price trades off the profit-maximizing positions the firm wants to target on the two separate home and export demand schedules. By changing the labor tax, $\phi$, the planner can control the domestic price level and the export price level, both given by $P_H$.

Profit maximization for firms *under DCP* follows:

$$\max \Pi_{Tt}(j) = \Pi_{Ht}(j) + \Pi_{Xt}(j)$$

where

$$\Pi_{Ht}(j) = \mathbb{E}_0 \left[ \sum_{t=0}^{1} \Lambda_t \left[ P_H(j) - (1+\phi) \frac{W_t}{A_t} \right] Y_{Ht} \left( \frac{P_H(j)}{P_H} \right)^{-\varepsilon} \right]$$
$$\Pi_{Xt}(j) = \mathbb{E}_0 \left[ \sum_{t=0}^{2} \Lambda_t \left[ E_t P_X(j) - (1+\phi) \frac{W_t}{A_t} \right] Y_{Xt} \left( \frac{P_X(j)}{P_X} \right)^{-\varepsilon} \right].$$

The fact that the labor tax is commonly applied on all home production of tradable goods, and not differentiated across goods according to their final destination, imposes a connection between the domestic price, $P_H$, and the export price, $P_X$, in equilibrium. Taking FOCs of the above expressions

[6] This assumption means that while households own tradable sector firms, the discount factor of tradable sector firms differs from those of the representative household. The reason for this assumption is our goal to de-emphasize the terms of trade externality. If the tradable sector firms have the same discount factors as households, the terms of trade externality would produce a motivation under shallow FX markets for the planner to distort exchange rates in order to alter the firms' discount factors and thereby influence the production of tradable goods.

and rearranging:

$$P_H = P_H(j) = (1+\phi)\frac{\varepsilon}{\varepsilon-1}\frac{\mathbb{E}_0\left[\sum\limits_{t=0}^{2}\Lambda_t \frac{W_t}{A_t} Y_{Ht}\right]}{\mathbb{E}_0\left[\sum\limits_{t=0}^{2}\Lambda_t Y_{Ht}\right]},\ P_X = P_X(j) = (1+\phi)\frac{\varepsilon}{\varepsilon-1}\frac{\mathbb{E}_0\left[\sum\limits_{t=0}^{2}\Lambda_t \frac{W_t}{A_t} Y_{Xt}\right]}{\mathbb{E}_0\left[\sum\limits_{t=0}^{2}\Lambda_t E_t Y_{Xt}\right]}$$

$$\begin{aligned}\Rightarrow P_X &= P_H \frac{\mathbb{E}_0\left[\sum\limits_{t=0}^{2}\Lambda_t \frac{W_t}{A_t} Y_{Xt}\right]}{\mathbb{E}_0\left[\sum\limits_{t=0}^{2}\Lambda_t E_t Y_{Xt}\right]}\frac{\mathbb{E}_0\left[\sum\limits_{t=0}^{2}\Lambda_t Y_{Ht}\right]}{\mathbb{E}_0\left[\sum\limits_{t=0}^{2}\Lambda_t \frac{W_t}{A_t} Y_{Ht}\right]} \\ &= P_X\left(P_H, C_{F0}, \{C_{F1}\}, \{C_{F2}\}, E_0, \{E_1\}, \{E_2\}\right). \end{aligned} \tag{6}$$

The planner needs to take into account that the expression for the export price, $P_X$, is not an independent choice variable, but rather a function of the domestic price, $P_H$, and the levels of tradable consumption and exchange rates in all periods and states. The formula for $P_X$ is provided in Appendix A.1.

*Housing sector firms*

Housing sector firms are perfectly competitive and take rental prices as given, which are flexible in every period. Following Kiyotaki and Moore (1997), there are two housing subsectors, one with a linear production function and another with a concave production function. Firms in subsector $k \in \{Linear, Concave\}$ purchase land, $L_t^k$, in period $t$ in order to produce housing services, $Y_{Rt+1}^k$, in period $t+1$:

$$Y_{Rt+1}^k = \left\{ \begin{array}{cc} L_t^k & \text{for } k = Linear \\ G\left(L_t^k\right) & \text{for } k = Concave \end{array} \right\}$$

where $G' > 0$, $G'' < 0$, and $G'(0) = 1$. They maximize expected profits given by:

$$\mathbb{E}_t \Pi_{Rt+1}^k = \mathbb{E}_t\left[P_{Rt+1} Y_{Rt+1}^k + q_{t+1} L_t^k\right] - \left(1+\theta_{Rt}^k\right)(1+\rho_t) q_t L_t^k,$$

where $P_{Rt+1}$ is the rental price of housing, $q_t$ is the price of land, $\theta_{Rt}^k$ is the housing macroprudential tax applied to each subsector, and $\rho_t$ is the interest rate offered by domestic banks.

Housing sector firms finance their operations by borrowing from domestic banks and remit their final asset position to households in period 2. The domestic currency debt of subsector $k$ evolves as follows:

$$D_{Rt+1}^{k} = \left(1+\theta_{Rt-1}^{k}\right)\left(1+\rho_{t-1}\right) D_{Rt}^{k} + q_t L_t^k - \left[P_{Rt} Y_{Rt}^k + q_t L_{t-1}^k\right] - T_{MPt}^k + T_{Rt}^k. \tag{7}$$

The first term on the right hand side is accumulated debt including interest payments, the second term is the financing of land purchases via additional debt, the third term in square brackets is the repayment of debt using rental income and the resale value of land purchased in the previous period, the fourth term, $T_{MPt}^k$, is a lump-sum transfer to each subsector,[7] and the final term, $T_{Rt}^k$, is a lump-sum transfer made to the households in period 2.

The linear subsector is subject to a borrowing constraint between periods 1 and 2:

$$D_{R2}^{Linear} \leq \kappa_{L1} q_1 L_1^{Linear},$$

where $\kappa_{L1}$ is a parameter governing the pledgability of land value between periods 1 and 2. The right hand side of the constraint becomes tighter when the land price declines.[8] The linear subsector's optimality conditions are:

$$\frac{\mathbb{E}_0\left[P_{R1}+q_1\right]}{\left(1+\theta_{R0}^{Linear}\right)\left(1+\rho_0\right)} = q_0 \text{ and } \frac{P_{R2}+q_2}{\left(1+\theta_{R1}^{Linear}\right)\left(1+\rho_1\right)} \left\{ \begin{array}{ll} = q_1 & \text{if } D_{R2}^{Linear} < \kappa_{L1} q_1 L_1^{Linear} \\ \geq q_1 & \text{if } D_{R2}^{Linear} = \kappa_{L1} q_1 L_1^{Linear} \end{array} \right\} \tag{8}$$

The concave subsector does not face a borrowing constraint. It satisfies the FOCs:

$$\frac{G'\left(L_0^{Concave}\right)\mathbb{E}_0\left[P_{R1}\right]+\mathbb{E}_0\left[q_1\right]}{\left(1+\theta_{R0}^{Concave}\right)\left(1+\rho_0\right)} = q_0 \text{ and } \frac{G'\left(L_1^{Concave}\right)P_{R2}+q_2}{\left(1+\theta_{R1}^{Concave}\right)\left(1+\rho_1\right)} = q_1. \tag{9}$$

[7] We allow the planner to rebate the proceeds from macroprudential taxes back to the agents that have been taxed to begin with. This assumption allows us to abstract from the income effects and to focus only on the substitution effects of the taxes, in the spirit of the earlier literature that studies macroprudential taxes in representative agent models.

[8] We assume that the current price of land enters the constraint, rather than its future price as in Kiyotaki and Moore (1997).

Market clearing in the land market requires:

$$L_t^{Linear} + L_t^{Concave} = 1. \tag{10}$$

Market clearing in the market for nontradable housing services requires:

$$C_{Rt} = Y_{Rt+1}^{Linear} + Y_{Rt+1}^{Concave}. \tag{11}$$

The planner's proceeds from macroprudential taxes on each subsector are rebated back to the same subsector via a lump-sum transfer:

$$T_{MPt}^k = \theta_{Rt-1}^k \left(1 + \rho_t\right) D_{Rt}^k. \tag{12}$$

In Kiyotaki and Moore (1997), the two subsectors are not regulated. If the planner can impose separate macroprudential taxes on both subsectors, it can neutralize the linear subsector's borrowing constraint. In this paper, we will mainly focus on the more interesting case where macroprudential taxes are allowed on the linear subsector, i.e., $\theta_{Rt}^{Linear} \in \mathbb{R}$, while the concave subsector is unregulated, i.e., $\theta_{Rt}^{Concave} \equiv 0$.

*Domestic banks*

The total debt position of the economy sums over household and housing sector debts:

$$D_{t+1} = D_{HHt+1} + D_{Rt+1}^{Linear} + D_{Rt+1}^{Concave}.$$

Domestic banks lend to households and the housing sector by transferring funds in domestic currency from international financial intermediaries. They maximize profits:

$$\Pi_{Bt+1} = \left(\rho_t - i_t\right) D_{t+1}$$

subject to the borrowing constraint between periods 1 and 2:

$$D_2 \leq \kappa_{H1} P_{H1}. \tag{13}$$

This constraint takes a simple form: $\kappa_{H1}$ is a parameter governing the pledgability of domestic tradable goods between periods 1 and 2, and it multiplies the domestic-currency price $P_{H1}$, which means that in dollar terms, the constraint becomes tighter when the exchange rate depreciates. This formulation brings our model closer to practical concerns of policymakers around the world, and it also echoes the constraint in Farhi and Werning (2016).[9] If banks' constraints do not bind, competition between banks ensures that households and the housing sector can borrow and lend at the policy rate: $\rho_t = i_t$. If banks' constraints do bind, the borrowing rate $\rho_t$ rises above the policy rate $i_t$ in order to clear the domestic debt market.

*International financial intermediaries*

International financial intermediaries take positions of $q_{t+1}$ in domestic currency bonds and $-\frac{q_{t+1}}{E_t}$ in dollar bonds in period $t$ in order to maximize their dollar profits subject to a balance sheet friction echoing the one considered by Gabaix and Maggiori (2015):[10]

$$\max_{q_{t+1}} \frac{1}{(1+i_t^*)} \frac{q_{t+1}}{E_t} \mathbb{E}_t \left[ (1-\varphi_t)(1+i_t) \frac{E_t}{E_{t+1}} - (1+i_t^*) \right]$$

$$\text{subject to } \frac{1}{(1+i_t^*)} \frac{q_{t+1}}{E_t} \mathbb{E}_t \left[ (1-\varphi_t)(1+i_t) \frac{E_t}{E_{t+1}} - (1+i_t^*) \right] \geq \frac{1}{(1+i_t^*)} \Gamma_t \left( \frac{q_{t+1}}{E_t} \right)^2 ,$$

where $\Gamma_t \geq 0$ captures the severity of the balance sheet friction, and $\varphi_t$ is the capital inflow tax announced in period $t$ and applies to the repayments made to the financial intermediaries in period $t+1$. A fraction $\lambda$ of the intermediaries are owned by domestic households and the remaining fraction $(1-\lambda)$ are owned by foreigners. Capital controls distort the decisions of all intermediaries, but since the planner rebates all tax revenues to households, only the foreign-owned fraction of the

[9] In subsection 5.3 of Farhi and Werning (2016), the nontradable good has a sticky price, and households can borrow up to a specific fraction of the value of nontradable output. Instead, in our model, we assume that households borrow from banks, and those banks can borrow up to a specific fraction of the sticky price of the home-produced tradable good. Both constraints become tighter when the exchange rate depreciates.

[10] We assume that intermediaries maximize the dollar value of profits, not the domestic currency value of profits. This assumption means that in the absence of balance sheet frictions (i.e., if $\Gamma_t = 0$), the intermediaries' uncovered interest parity (UIP) condition can be written in a form that clearly parallels the households' Euler condition. As a result, when combining the UIP and Euler conditions (as in equation (24), for example), there is no case for prudential capital controls if households' consumption levels across period-1 states are identical and unaffected by period-1 shocks.

intermediaries ends up paying taxes in net terms.

The constraint for financial intermediaries always binds. We can derive the intermediaries' demand for domestic currency bonds:

$$\frac{Q_{t+1}}{E_t} = \frac{1}{\Gamma_t}\mathbb{E}_t\left[(1-\varphi_t)(1+i_t)\frac{E_t}{E_{t+1}} - (1+i_t^*)\right].$$

The intermediaries' realized profit in domestic currency in period $t+1$ is:

$$\Pi_{FIt+1} = Q_{t+1}\left[(1-\varphi_t)(1+i_t) - (1+i_t^*)\frac{E_{t+1}}{E_t}\right].$$

We assume that there is a separate group of non-optimizing foreigners who have exogenous and stochastic demands for domestic-currency debt. Their exogenous debt holdings are $L_{t+1}$ in domestic currency bonds, amounting to $S_t = \frac{L_{t+1}}{E_t}$ in dollar value. They are not subject to the balance sheet friction described above, and their decisions to purchase domestic-currency debt do not depend on the expected returns.

In our model, FX intervention involves the planner taking a position of $O_{t+1}$ in local currency bonds and $FXI_t = -\frac{O_{t+1}}{E_t}$ in dollar bonds. The realized profit for the planner from this transaction is:

$$T_{FXIt+1} = O_{t+1}\left[(1+i_t) - (1+i_t^*)\frac{E_{t+1}}{E_t}\right]. \tag{14}$$

Market clearing in the domestic-currency debt market requires:

$$Q_{t+1} = D_{t+1} - O_{t+1} - L_{t+1},$$

which produces the "Gamma equations" that relate expected excess premia to capital inflows:

$$\Gamma\left(\frac{D_1}{E_0} + FXI_0 - S_0\right) = \mathbb{E}_0\left[\eta_1 - (1+i_0^*)\right] \tag{15}$$

$$\Gamma\left(\frac{D_2}{E_1} + FXI_1 - S_1\right) = \eta_2 - (1+i_1^*), \tag{16}$$

where we define the gross return on domestic assets in dollar terms:

$$\eta_{t+1} = (1 - \varphi_t)(1 + i_t)\frac{E_t}{E_{t+1}} > 0.$$

Since the gross external return is a combination of the ex-ante policy rate, the ex-ante capital controls, and the ex-ante and ex-post exchange rates, it must inherit the contingency properties of its constituent components. Using $H$ and $L$ superscripts for the values of variables after the period-1 realizations of high and low shocks respectively, we derive the following "contingency constraint:"

$$\frac{\eta_1^H}{\eta_1^L} = \frac{E_1^L}{E_1^H} \Rightarrow E_1^H \eta_1^H = E_1^L \eta_1^L. \tag{17}$$

The planner's proceeds from labor taxes, capital inflow taxes, and consumer macroprudential taxes are distributed to households via a lump-sum transfer:

$$T_{Gt+1} + \phi W_{t+1} N_{t+1} + \varphi_t (1 + i_t)(Q_{t+1} + L_{t+1}) + \theta_{HHt}(1 + \rho_t) D_{HHt+1} = 0. \tag{18}$$

*Competitive equilibrium*

**Definition** A competitive equilibrium for this economy is a set of quantities $\{C_{Ht}, C_{Ft}, C_{Rt}, N_t, L_t^{Linear}, L_t^{Concave}, Y_{Ht}, Y_{Xt}, Y_{Rt}^{Linear}, Y_{Rt}^{Concave}\}_{t=0}^{2}$ and prices $\left\{P_H, P_X, \{\rho_t\}_{t=0}^{1}, \{W_t, E_t, P_{Rt}, q_t\}_{t=0}^{2}\right\}$ that satisfy the households' constraints and FOCs (1)-(3), the tradable sector firms' production and price-setting decisions (4) and either (5) or (6), the housing sector firms' production decisions (8) and (9), the land and housing services market clearing conditions (10) and (11), the banks' borrowing constraint (13), the domestic currency bond market clearing conditions (15)-(16), the contingency constraint for gross external returns (17), and the lump sum transfer constraints (12), (14), and (18), taking as given the planner's choice of the policy instruments $\{i_t, \varphi_t, \theta_{HHt}, \theta_{Rt}^{Linear}, FXI_t\}_{t=0}^{1}$.

Substituting the competitive equilibrium equations into the households' budget constraints, we obtain the economy-wide resource constraint for tradable goods:

$$D_{t+1} \geq -E_t P_{Ft}^* \left[\omega C_t^* - C_{Ft}\right] - E_t P_{Zt}^* Z_t - (1-\lambda) O_t \left[\left(1+\widehat{i}_{t-1}\right) - \left(1+i_{t-1}^*\right)\frac{E_t}{E_{t-1}}\right] + \lambda\left(1+i_{t-1}^*\right)\frac{E_t}{E_{t-1}} D_t + (1-\lambda)\left(1+\widehat{i}_{t-1}\right) D_t, \quad (19)$$

where $\left(1+\widehat{i}_t\right) = (1-\varphi_t)(1+i_t)$. Combining capital controls and the policy rate into an "effective foreigners' interest rate" is useful for analytical simplicity. Once the effective rate is pinned down, the planner can decompose it into the two separate policy instruments using the households' Euler condition and the information on whether the banks' borrowing constraint is binding or not.

The resource constraint highlights the importance of the parameter $\lambda$. Households own a fraction $\lambda$ of the intermediaries, and those intermediaries borrow in dollars to purchase the domestic currency debt that is issued by households and the housing sector. Therefore, when considering the economy-wide external debt position, the fraction $\lambda$ of the domestic currency debt position nets out to generate a net dollar exposure. If $\lambda > 0$, households' income moves as if households and the housing sector have issued some dollar bonds themselves: there is a currency mismatch, and a depreciation in the exchange rate increases the domestic currency value of the households' external debt repayments, which may tighten the banks' borrowing constraints. This connection of the banks' constraint to the exchange rate becomes more evident when the constraint is written in dollar terms:

$$\frac{D_2}{E_1} \leq \kappa_{H1}\frac{P_H}{E_1}. \quad (20)$$

The remaining fraction $(1-\lambda)$ represents the domestic currency portion of the external debt position.

Substituting the competitive equilibrium equations and the housing sector conditions into the linear housing subsector's borrowing constraint, we obtain a single equation which summarizes the contribution of the housing sector to the competitive equilibrium:

$$D_{R2}^{Linear} = (1+\rho_0)\left[(1+\rho_{-1}) D_{R0}^{Linear} - P_{R0} L_{-1}^{Linear}\right]$$

$$+\left\{\frac{G'\left(1-L_0^{Linear}\right)\mathbb{E}_0\left[P_{R1}\right]}{\left(1+\theta_{R0}^{Concave}\right)}+\mathbb{E}_0\left[\frac{G'\left(1-L_1^{Linear}\right)P_{R2}+q_2}{\left(1+\theta_{R0}^{Concave}\right)\left(1+\theta_{R1}^{Concave}\right)\left(1+\rho_1\right)}\right]\right\}\left(L_0^{Linear}-L_{-1}^{Linear}\right)$$

$$-P_{R1}L_0^{Linear}+\frac{G'\left(1-L_1^{Linear}\right)P_{R2}+q_2}{\left(1+\theta_{R1}^{Concave}\right)\left(1+\rho_1\right)}\left(\left(1-\kappa_{L1}\right)L_1^{Linear}-L_0^{Linear}\right)\leq 0 \tag{21}$$

$$\text{where } P_{Rt}=\frac{\alpha_R P_H C_{Ht}}{\alpha_H\left[L_{t-1}^{Linear}+G\left(1-L_{t-1}^{Linear}\right)\right]}.$$

This inequality condition is slack if the planner can regulate both housing subsectors, i.e., if $\theta_{R0}^{Concave}\neq 0$ is allowed, or if $\kappa_{L1}$ is high enough such that the flexible adjustment of rents and land prices after shocks poses no financing problems for the linear housing subsector. If so, the non-housing-sector quantities and prices are not affected by the existence of the housing sector. If the constraint does bind in equilibrium, owing to rents and house prices becoming excessively depressed after specific shocks, then the housing sector does distort the competitive equilibrium. Rents and housing prices become excessively depressed after shocks which decrease the pledgability of land, $\kappa_{L1}$, or which decrease domestic aggregate demand, causing a reduction in the consumption of home tradable goods, $C_{Ht}$.

*Constrained Efficient Allocations*

We can write the indirect utility function in period $t$ as follows:

$$V\left(C_{Ft},p_{Ht},p_{Xt},L_{t-1}\right)=U\left(\frac{\alpha_H}{\alpha_F}p_{Ht}C_{Ft},C_{Ft},L_{t-1}^{Linear}+G\left(1-L_{t-1}^{Linear}\right),\frac{\frac{\alpha_H}{\alpha_F}p_{Ht}}{A_t}C_{Ft}+\frac{\omega p_{Xt}}{A_t}C_t^*\right)$$

$$\begin{aligned}\text{where } V_{Ft} &= \frac{\alpha_F}{C_{Ft}}\left[1+\frac{\alpha_H}{\alpha_F}\left(1-\frac{1}{A_t}\frac{C_{Ht}}{\alpha_H}\right)\right], V_{p_Ht}=\frac{\alpha_H}{p_{Ht}}\left(1-\frac{1}{A_t}\frac{C_{Ht}}{\alpha_H}\right),\\ V_{p_Xt} &= -\frac{\omega}{A_t}C_t^*,\text{ and } V_{Lt}=\alpha_R\frac{1-G'\left(1-L_{t-1}^{Linear}\right)}{L_{t-1}^{Linear}+G\left(1-L_{t-1}^{Linear}\right)}.\end{aligned}$$

Next, we define four wedges which summarize the distance of the allocation from the efficient frontier. We identify the key externalities related to each wedge, with the proviso that in our integrated framework, the wedges are jointly determined as a result of all the externalities.

The first wedge is for home consumption, as in Farhi and Werning (2016), and arises from the stickiness of the tradable-good price when sold for domestic consumption:

$$\tau_{Ht} = 1 + \frac{1}{A_t}\frac{U_{Nt}}{U_{Ht}} = 1 - \frac{1}{A_t}\frac{C_{Ht}}{\alpha_H}.$$

This "aggregate demand (AD) wedge" is positive if $P_H$ is too high relative to domestic aggregate demand. There are *aggregate demand externalities* because households do not internalize the impact of their consumption decisions on the time path of aggregate demand, which determines the appropriateness of the pre-set domestic price, $P_H$. There are also *pecuniary aggregate demand externalities* because they do not internalize the impact of their decisions on the level of the exchange rate $E_1$ which enters the banks' borrowing constraint.

The second wedge relates to export production and varies depending on the price-setting paradigm:

$$\tau_{Xt}^{PCP} = \left(1 - \frac{\omega p_{Xt}^{PCP}}{p_{Xt}^{PCP}\frac{d}{dp_{Xt}^{PCP}}\left(\omega p_{Xt}^{PCP}\right)}\right) + p_{Xt}^{PCP}\frac{1}{A_t}\frac{U_{Nt}}{U_{Ft}} = -p_{Xt}^{PCP}\frac{1}{A_t}\frac{C_{Ft}}{\alpha_F}$$

$$\tau_{Xt}^{DCP} = \left(1 - \frac{\omega p_{Xt}^{DCP}}{p_{Xt}^{DCP}\frac{d}{dp_{Xt}^{DCP}}\left(\omega p_{Xt}^{DCP}\right)}\right) + p_{Xt}^{DCP}\frac{1}{A_t}\frac{U_{Nt}}{U_{Ft}} = -p_{Xt}^{DCP}\frac{1}{A_t}\frac{C_{Ft}}{\alpha_F}.$$

This "TOT wedge" highlights that there is a *TOT externality* because while firms do take into account that the demand curve for their own export variety is downward-sloping, they do not internalize that the demand curve for the aggregate export good is also downward-sloping. Under the unit elastic demand assumption for export demand, the first term in the above expressions is zero. These wedges are always negative because firms set the export price, $P_X$, lower than the level that maximizes the economy-wide TOT. In other words, the planner wishes to push the economy to an allocation with a higher export price, $P_X$, and a lower export volume, $Y_{Xt}$, while earning the same dollar value of export revenues.

Together, the first and second wedges capture what we call "static AD and price pressures": whether excess/insufficient domestic and external demand for home-produced traded goods would tend to push the prices of these goods up or down relative to the pre-set rigid level. In sections 3-5, we show that constrained welfare maximization produces a motivation for the planner to minimize overall price pressures, i.e., to minimize a weighted sum of the above wedges, unless other wedges need to be addressed at the same time. Therefore, while we assume rigid prices for analytical tractability, optimal policies from our framework will achieve the major price stabilization motive of the traditional New Keynesian framework. However, we do not capture inefficiencies related to price dispersion or credibility-damaging inflation dynamics.

The third wedge captures the deviation of the gross external return from the level that would prevail if all households could borrow using dollar bonds:

$$\tau_{\Gamma t} = \eta_t - \left(1 + i^*_{t-1}\right).$$

This "UIP wedge" enters the economy-wide resource and borrowing constraints. If the wedge is positive in a particular state and some of the intermediaries are foreign-owned, i.e., $\lambda \in [0, 1)$, then there is a net loss of resources from the domestic economy to those foreign-owned intermediaries in that state. There is what we call a *financial TOT externality* because households do not internalize the impact of their borrowing decisions on the external returns that other households must pay.

The fourth wedge captures the deviation of housing services production from its maximum level:

$$\tau_{Rt} = 1 - G'\left(1 - L^{Linear}_{t-1}\right)$$

This "housing wedge" is positive if land usage is shifted from the linear to the concave subsector of the housing market. The production of housing services is maximized when the linear subsector uses all of the land in the economy for its production. Production is reduced in the presence of borrowing constraints and/or macroprudential taxes for the linear subsector. There is a *pecuniary production externality* because housing sector firms do not internalize the impact of their land usage decisions on the land price $q_1$ which enters their borrowing constraint.

In this economy, the first-best allocation is not feasible: all the wedges $\{\tau_{Ht}, \tau_{Xt}, \tau_{\Gamma t}, \tau_{Rt}\}$ cannot be equal to zero in every state. In a deterministic closed economy, the planner can set the labor tax, $\phi$, to manipulate the domestic price level, $P_H$, such that the distortion owing to monopolistic competition is perfectly eliminated and the AD wedge is zero, i.e., $\tau_{Ht} = 0$. In a deterministic open economy under PCP, the planner cannot perfectly eliminate this distortion. Instead, it has to set the labor tax to balance the AD and TOT wedges, and, as in Gali and Monacelli (2005), some of both distortions remain in equilibrium, i.e., $\tau_{Ht} \neq 0$ and $\tau_{Xt} \neq 0$. The same principle applies under DCP because of the fact that the export price, $P_X$, is still tied to the domestic price, $P_H$. The addition of shocks to the economy, as well as the introduction of the UIP wedge, $\tau_{\Gamma t}$, and housing wedge, $\tau_{Rt}$, reinforces the result that all distortions cannot be entirely eliminated.

This observation leads us to focus on deriving constrained efficient allocations.

**Definition** A constrained efficient allocation is a set of quantities $\left\{C_{Ht}, C_{Ft}, C_{Rt}, N_t, L_t^{Linear}, L_t^{Concave},\right.$ $\left.Y_{Ht}, Y_{Xt}, Y_{Rt}^{Linear}, Y_{Rt}^{Concave}\right\}_{t=0}^{2}$, prices $\left\{P_H, P_X, \{\rho_t\}_{t=0}^{1}, \{W_t, E_t, P_{Rt}, q_t\}_{t=0}^{2}\right\}$, and policy instruments $\left\{i_t, \varphi_t, FXI_t, \theta_{HHt}, \theta_{Rt}^{Linear}\right\}_{t=0}^{1}$ which solve under full commitment:

$$\max_{\left\{C_{Ft}, P_H, E_t, \eta_{t+1}, FXI_t, L_{t-1}^{Linear}\right\}} \left\{ \begin{array}{lr} \mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t V\left(C_{Ft}, \frac{E_t P_{Ft}^*}{P_H}, \frac{E_t P_{Ft}^*}{P_H}, L_{t-1}^{Linear}\right)\right] & \text{if PCP} \\ \mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t V\left(C_{Ft}, \frac{E_t P_{Ft}^*}{P_H}, \frac{P_{Ft}^*}{P_X}, L_{t-1}^{Linear}\right)\right] & \\ \text{with } P_X = P_X\left(C_{F0}, \{C_{F1}\}, \{C_{F2}\}, E_0, \{E_1\}, \{E_2\}, P_H\right) & \text{if DCP,} \end{array} \right\}$$

subject to the restriction that the allocation constitutes a competitive equilibrium. The full set of equations is listed in Appendix A.2, which uses the dollar forms of all the constraints, fixes the dollar values of all initial debt stocks and the period-2 land price, and sets $\theta_{Rt}^{Concave} \equiv 0$.

The joint consideration of the above policy instruments and wedges nests many important results from the literature and also allows us to establish several results that are novel relative to the literature. We describe these results in the following sections, adding one set of frictions at a time to gradually build towards a bigger model. Our framework allows us to determine whether policies which have been highlighted in the literature as being useful to minimize specific wedges after certain shocks can also be used to address other wedges. We are also able to analyze whether policies which have been recommended to reduce specific wedges in the literature may in fact exacerbate other wedges when economies suffer from multiple frictions.[11]

To assess the complementarity and substitutability of instruments, and to facilitate the use of the model for practical policy advice, we can derive optimal policies when different sets of instruments are available in the planner's policy toolkit. Every time an instrument is removed from the toolkit, additional constraints need to be added onto the planner problem:

- If FX intervention is not permitted, we set $FXI_0 = FXI_1 = 0$ and remove the FOCs with respect to FX intervention, $FXI_t$.
- If neither capital controls nor consumer macroprudential controls are permitted, we add the household Euler conditions (3) as constraints with the capital control and macroprudential control terms set to zero.
- If housing macroprudential controls are not permitted, we set $L_0 = 1$.
- If the domestic policy rate cannot be used, we set it equal to the foreign interest rate, i.e., $i_t = i_t^*$, in the formula for the gross external premium, $\eta_{t+1}$.
- If the exchange rate is pegged, we set the exchange rate in all periods and states to the initial value $E_0$.

Our solution approach is as follows. We assume that the constrained planner problem is convex in the region of interest, and correspondingly, we derive the FOCs for the problem in Appendix A.2. In the next two sections, we summarize the salient properties of these FOCs, indexing our results by the pricing paradigm:

$$\mathbb{I}^{PCP} = \left\{ \begin{array}{ll} 1 & \text{if PCP} \\ 0 & \text{if DCP} \end{array} \right\} \text{ and } \mathbb{I}^{DCP} = \left\{ \begin{array}{ll} 0 & \text{if PCP} \\ 1 & \text{if DCP} \end{array} \right\}.$$

[11] In Basu et al. (our forthcoming IMF Working Paper), we catalogue a comprehensive characterization of the optimal integrated use of policy instruments as a function of all the shocks and structural characteristics described in this section.

Then in each section, we explain our results using a mix of analytical and numerical results to qualitatively characterize the optimal integrated use of policies.

# 3 Deep Foreign Exchange Markets

To present our results most clearly, we start with the smallest integrated model and gradually add new frictions one at a time. In this section, we abstract from frictions in FX markets and the housing sector, and focus on the optimal integrated use of the policy rate and capital controls under different pricing paradigms when borrowing constraints are present. Most advanced economies and a few emerging markets have deep FX markets, with their currencies being traded by a substantial number of financial intermediaries, except possibly during episodes of severe global financial stress such as the ongoing COVID-19 crisis.

As we explain below, we study the case with deep FX markets by setting $\Gamma = 0$ (no intermediary frictions), $\lambda = 1$ (households own all intermediaries), and either perfect housing sector regulation or $\kappa_{L1} \to \infty$ (no binding housing frictions) such that housing frictions are irrelevant, and by removing FX intervention from the planner's toolkit.

## 3.1 Policy Instruments and Wedges

The deep FX markets case formally corresponds to setting $\Gamma = 0$ in the constraints and FOCs summarized in Appendix A.2. The value of zero for $\Gamma$ means that financial intermediaries face no balance sheet constraints, so their capacity to hold domestic currency debt is unlimited, and the country's external debt position does not matter for the country's gross external return, $\eta_t$. The Gamma equations therefore reduce to the UIP conditions:

$$\mathbb{E}_0 \left[ \tau_{\Gamma 1} \right] = 0 \text{ and } \tau_{\Gamma 2} = 0. \tag{22}$$

UIP wedges, $\tau_{\Gamma t}$, paid by the domestic economy to intermediaries generate welfare losses if a fraction of the intermediaries are foreign-owned, i.e., $\lambda \in [0, 1)$. However, since the UIP wedges average out to zero, the average external premium is zero. Therefore, to simplify the algebra, we ignore foreign ownership of the intermediaries in this section and set $\lambda = 1$. Note that if domestic households own all financial intermediaries, the economy's liabilities are effectively entirely in dollars. As a further simplification, we assume that housing frictions do not bind. As described in section 2, this result follows either from regulation of both housing subsectors, i.e., both $\theta_{Rt}^{Linear} \in \mathbb{R}$ and $\theta_{Rt}^{Concave} \in \mathbb{R}$ are allowed, or from high housing sector debt capacity, i.e., $\kappa_{L1} \to \infty$.

We define $B_t \equiv \frac{D_t}{E_{t-1}}$ as the total domestic currency debt stock at the beginning of period $t$ converted into a dollar value. Since this debt is entirely sold to international financial intermediaries, and domestic households own all financial intermediaries, $B_t$ also represents the representative household's effective exposure to external dollar-denominated debt.

The absence of binding housing sector frictions means that one of the two instruments of capital controls and consumer macroprudential taxes becomes redundant, because both affect the economy via altering external debt.[12] Their optimal use follows the following expression, in which either of them can be used by the planner while the other one can be set to zero:

$$\frac{(1-\varphi_t)}{(1+\theta_{HHt})} = \begin{cases} \frac{\eta_{t+1} E_{t+1} \beta \mathbb{E}_t \left\{ \frac{1}{E_{t+1}} \frac{\alpha_F}{P^*_{Ft+1} C_{Ft+1}} \right\}}{\frac{\alpha_F}{P^*_{Ft} C_{Ft}}} & \text{if } \Psi_{Bt} = 0 \\ 1 & \text{if } \Psi_{Bt} > 0. \end{cases} \tag{23}$$

Throughout this section, we choose to focus on implementation via capital controls, i.e., set $\theta_{HHt} \equiv 0$ and allow $\varphi_t \in \mathbb{R}$. Equation (22) establishes that FX intervention does not affect the exchange rate, so we defer further consideration of FX intervention to section 4. In addition, we defer further consideration of housing macroprudential taxes to section 5.

The households' Euler conditions can be rewritten as:

$$\frac{\alpha_F}{P^*_{F0} C_{F0}} = \beta \frac{(1+i_0^*)}{(1-\varphi_0)} \frac{1}{\mathbb{E}_0 \left[ \frac{E_0}{E_1} \right]} \mathbb{E}_0 \left[ \frac{E_0}{E_1} \frac{\alpha_F}{P^*_{F1} C_{F1}} \right] \text{ and } \frac{\alpha_F}{P^*_{F1} C_{F1}} \geq \beta \frac{(1+i_1^*)}{(1-\varphi_1)} \frac{\alpha_F}{P^*_{F2} C_{F2}}. \tag{24}$$

[12] In Basu et al. (our forthcoming IMF Working Paper), we show that the two instruments become complements instead of substitutes, and they need to be used together, when there are unregulated sectors and/or circumvention of policy instruments.

These Euler conditions demonstrate that capital controls are effective instruments, as they raise the domestic policy rate above the foreign interest rate and thereby reduce domestic borrowing. Exchange rates $E_0$ and $E_1$ enter the Euler condition between periods 0 and 1 because households have access to domestic-currency bonds only, and not dollar bonds, and the extent of possible risk-sharing depends on the contingency of the available bonds.[13] They do not enter the Euler condition between periods 1 and 2 because there is no uncertainty between those periods.

Next, we turn to the conditions characterizing the constrained efficient allocation, to understand which externalities arise under deep FX markets and how policies should be used to alleviate them. The planner's Euler conditions for $t \in \{0, 1\}$ are:

$$\begin{aligned}
&\frac{\alpha_F}{P_{Ft}^* C_{Ft}} \left[1 + \frac{\alpha_H}{\alpha_F} \tau_{Ht}\right] - \mathbb{I}^{DCP} \cdot \left\{ \mathbb{E}_0 \left[ \sum_{t=0}^{2} \beta^t \omega C_t^* \frac{\alpha_F}{C_{Ft}} \tau_{Xt}^{DCP} \right] \frac{1}{\beta^t \pi_t} \frac{1}{P_{Ft}^*} \frac{1}{P_X} \frac{\partial P_X}{\partial C_{Ft}} \right\} \\
&= \beta \left(1 + i_t^*\right) \mathbb{E}_t \left\{ \frac{\alpha_F}{P_{Ft+1}^* C_{Ft+1}} \left[1 + \frac{\alpha_H}{\alpha_F} \tau_{Ht+1}\right] \right\} + \Psi_{Bt} \frac{1}{\beta \left(1 + i_{t-1}^*\right)} \\
&- \mathbb{I}^{DCP} \cdot \left\{ \mathbb{E}_0 \left[ \sum_{t=0}^{2} \beta^t \omega C_t^* \frac{\alpha_F}{C_{Ft}} \tau_{Xt}^{DCP} \right] \mathbb{E}_t \left[ \frac{\left(1 + i_t^*\right)}{\beta^t \pi_{t+1}} \frac{1}{P_{Ft+1}^*} \frac{1}{P_X} \frac{\partial P_X}{\partial C_{Ft+1}} \right] \right\}, \qquad (25)
\end{aligned}$$

where $\Psi_{Bt}$ is the multiplier on the banks' borrowing constraint. The first term on the left hand side represents the marginal utility of consumption in period $t$, taking into account the AD wedge. The second term on the left hand side captures an effect which only arises under DCP: the impact of the period-$t$ consumption decision on welfare via the period-0 export-price-setting decision. The first term on the right hand side represents the marginal utility of consumption in period $t + 1$, taking into account the AD wedge. The second term on the right hand side captures the distortion in the Euler conditions if the borrowing constraint binds. The third term on the right hand side captures the impact of the period-$t+1$ consumption decision on welfare via the period-0 export-price-setting decision.

Comparing the household and planner Euler conditions, we can see that two of the wedges, $\tau_{Ht}$

[13] If the shock is such that period-1 imports, $C_{F1}$, are perfectly stabilized across high and low realizations of the shock, or if the period-1 exchange rate, $E_1$, is perfectly stabilized across realizations, then the state-contingency of the bond is no longer important, and the exchange rates do not appear in the Euler conditions. The Euler conditions become identical to the Euler conditions of households who are able to participate directly in the dollar bond market subject to capital controls.

and $\tau_{Xt}$, may provide a rationale for policy intervention in the deep FX markets case.

We have mentioned in section 2 that it is not feasible for these two wedges to be equal to zero in every state. Nevertheless, if we set all wedges to zero for illustrative purposes, we observe that both the households' and the planner's Euler conditions would reduce to:

$$\frac{\alpha_F}{P_{Ft}^* C_{Ft}} = \beta \left(1 + i_t^*\right) \mathbb{E}_t \left[\frac{\alpha_F}{P_{Ft+1}^* C_{Ft+1}}\right] \text{ for } t \in \{0, 1\},$$

which is identical to the Euler condition of households who are able to participate without restriction in the dollar bond market. In this case, the planner would set a domestic policy rate consistent with zero capital controls, and would allow full flexibility of the exchange rate.

When the wedges are not zero, there may be a case for the planner to move the domestic policy rate in a different manner, and also to add capital controls into the toolkit in order to stabilize the wedges over time.

The FOCs for exchange rates in each state are:

$$\underbrace{\alpha_H \tau_{Ht}}_{\text{Stabilize demand for home goods}} = \underbrace{\begin{array}{c} -\mathbb{I}^{PCP} \cdot \left\{\omega C_t^* \frac{\alpha_F}{C_{Ft}} \tau_{Xt}^{PCP}\right\} \\ -\mathbb{I}^{DCP} \cdot \left\{\mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t \omega C_t^* \frac{\alpha_F}{C_{Ft}} \tau_{Xt}^{DCP}\right] \frac{1}{\beta^t \pi_t} \frac{E_t}{P_X} \left(-\frac{\partial P_X}{\partial E_t}\right)\right\} \end{array}}_{\text{Optimize TOT on export goods}} + \underbrace{\frac{\Psi_{Bt}}{\beta^t \left(1 + i_{t-1}^*\right)} \kappa_{H1} \frac{P_H}{E_t}}_{\text{Relax bank constraint}}. \tag{26}$$

The planner sets the exchange rate to balance price pressures and binding borrowing constraints within each state. The first term, which includes the AD wedge $\tau_{Ht}$, represents the benefit of moving the exchange rate to generate import substitution and stabilize domestic demand for the home-produced tradable good. The second term, which includes the TOT wedges, $\tau_{Xt}$, represents the benefit of moving the exchange rate to optimize the TOT on export goods by altering the export volume—either just within a specific state (under PCP), or on average across all states via the impact on the period-0 export-price-setting decision (under DCP). Price pressures are balanced if, for example, the first term is positive because prices are too high for consumption purposes, but the second term (including the minus sign in front) is also positive because prices are too low for export purposes. The third term represents the effect of exchange rate movements on the tightness of the borrowing constraint. In line with condition (22), the domestic policy rate moves inversely to the expected exchange rate depreciation.

The expression for capital controls is:

$$\varphi_t = \begin{cases} 1 - \dfrac{\frac{1}{\mathbb{E}_t\left[\frac{E_t}{E_{t+1}}\right]}\mathbb{E}_t\left[\frac{E_t}{E_{t+1}}\frac{\alpha_F}{P^*_{Ft+1}C_{Ft+1}}\right]\left[\begin{array}{c}\frac{\alpha_F}{P^*_{Ft}C_{Ft}}\left[1+\frac{\alpha_H}{\alpha_F}\tau_{Ht}\right] \\ -\mathbb{I}^{DCP}\cdot\left\{\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\frac{1}{\beta^t\pi_t}\frac{1}{P^*_{Ft}}\frac{1}{P_X}\frac{\partial P_X}{\partial C_{Ft}}\right\}\end{array}\right]}{\frac{\alpha_F}{P^*_{Ft}C_{Ft}}\left[\begin{array}{c}\mathbb{E}_t\left\{\frac{\alpha_F}{P^*_{Ft+1}C_{Ft+1}}\left[1+\frac{\alpha_H}{\alpha_F}\tau_{Ht+1}\right]\right\} \\ -\mathbb{I}^{DCP}\cdot\left\{\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\mathbb{E}_t\left[\frac{1}{\beta^{t+1}\pi_{t+1}}\frac{1}{P^*_{Ft+1}}\frac{1}{P_X}\frac{\partial P_X}{\partial C_{Ft+1}}\right]\right\}\end{array}\right]} \\ \text{if } \Psi_{Bt} = 0 \\ \\ 0 \text{ if } \Psi_{Bt} > 0, \end{cases} \tag{27}$$

where capital controls are ineffective, and therefore set to zero, when the banks' borrowing constraint binds.

Capital controls are non-zero if and only if the numerator and denominator of the fraction in equation (27) are unbalanced. The expressions for the numerator and denominator are obtained by substituting for the AD wedge, $\tau_{Ht}$, using equation (26). Therefore, there are two possible rationales for capital controls in this version of our model.

The first potential rationale for capital controls arises if there is a pecuniary AD externality from an occasionally-binding borrowing constraint, i.e., $\Psi_{Bt} > 0$, which captures the concerns of many emerging-market policymakers. Households do not internalize that their borrowing in period $t$ may generate lower aggregate demand and a more depreciated exchange rate in period $t+1$, making the banks' borrowing constraint binding. When the borrowing constraint binds, equation (26) indicates that the AD wedge, $\tau_{Ht}$, is optimally kept higher than the TOT wedges, $\tau_{Xt}$, would justify, i.e., the exchange rate is more appreciated and the domestic policy rate is kept higher in order to address the pecuniary externality and relax the constraint. Therefore, monetary policy and exchange rate flexibility no longer fully address the AD externality. As a result, prudential capital controls become optimal. This finding captures Farhi and Werning's (2016) insights regarding the case for capital controls with occasionally-binding borrowing constraints. Our model additionally allows for the size of the externality to be related to the pricing paradigm (PCP versus DCP).

The second potential rationale for capital controls arises from the TOT externality. Capital controls are non-zero if the weighted TOT wedges, $\tau_{Xt}$, are not balanced over time (as in Costinot, Lorenzoni, and Werning, 2014). Our model nests the results of Farhi and Werning (2014) and extends them to the DCP case. This rationale naturally arises in any open-economy framework with price-setting, but policymakers do not emphasize this channel, so we focus on insights that do not hinge on this motive.

Equations (25)-(27) demonstrate that an integrated model is necessary to characterize the optimal use of multiple instruments: the use of each policy instrument affects several wedges and, as a result, the optimal use of other policy instruments. Specifically, we can see that the level of the domestic policy rate affects exchange rates and thereby the optimal use of capital controls, and vice versa.

## 3.2 Capital Controls and the Pricing Paradigm

In this subsection, we explain the connection between optimal capital controls and the pricing paradigm. Our first lemma relates to the case when pecuniary AD externalities are not relevant because the banks' external debt limit does not bind in any period-1 state.

**Lemma 1.** *Iff $\Psi_{B1} = 0$ for every period-1 state, the sole rationale for capital controls arises from TOT externalities. The size of capital controls differs between PCP and DCP.*

Equation (26) indicates that if the external debt limit never binds, the exchange rate is adjusted to balance price pressures in every period and state. Specifically, the AD wedge, $\tau_{Ht}$, which reflects AD externalities as in Farhi and Werning (2016), is balanced against the TOT wedges, $\tau_{Xt}$, which capture the TOT externalities introduced by Costinot, Lorenzoni, and Werning (2014).

In the expression (27), the AD wedge terms can be substituted out, and the optimal capital controls are related solely to the time path of TOT wedges:

$$\varphi_t = \begin{cases} 1 - \dfrac{\frac{1}{\mathbb{E}_t\left[\frac{E_t}{E_{t+1}}\right]}\mathbb{E}_t\left[\frac{E_t}{E_{t+1}}\frac{\alpha_F}{P^*_{Ft+1}C_{Ft+1}}\right]\left\{1-\frac{\omega C^*_t}{C_{Ft}}\tau^{PCP}_{Xt}\right\}}{\mathbb{E}_t\left[\frac{\alpha_F}{P^*_{Ft+1}C_{Ft+1}}\left\{1-\frac{\omega C^*_{t+1}}{C_{Ft+1}}\tau^{PCP}_{Xt+1}\right\}\right]} & \text{if PCP} \\ 1 - \dfrac{\frac{1}{\mathbb{E}_t\left[\frac{E_t}{E_{t+1}}\right]}\mathbb{E}_t\left[\frac{E_t}{E_{t+1}}\frac{\alpha_F}{P^*_{Ft+1}C_{Ft+1}}\right]\left\{\frac{\alpha_F}{P^*_{Ft}C_{Ft}}-\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\frac{1}{\beta^t\pi_t P^*_{Ft}P_X}\left\{\frac{E_t}{C_{Ft}}\left(-\frac{\partial P_X}{\partial E_t}\right)+\frac{\partial P_X}{\partial C_{Ft}}\right\}\right\}}{\frac{\alpha_F}{P^*_{Ft}C_{Ft}}\left\{\mathbb{E}_t\left[\frac{\alpha_F}{P^*_{Ft+1}C_{Ft+1}}\right]-\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\mathbb{E}_t\left[\frac{1}{\beta^{t+1}\pi_{t+1}P^*_{Ft+1}P_X}\left\{\frac{E_{t+1}}{C_{Ft+1}}\left(-\frac{\partial P_X}{\partial E_{t+1}}\right)+\frac{\partial P_X}{\partial C_{Ft+1}}\right\}\right]\right\}} & \text{if DCP.} \end{cases} \quad (28)$$

The formulae for optimal capital controls are different under PCP and DCP. Under PCP, they follow the results for the rigid-price case in Farhi and Werning (2014). In particular, capital controls are zero for permanent productivity shocks, i.e., changes in the value of $A_1 = A_2$, and they are imposed in the countervailing direction to shocks to the world interest rate, i.e., changes in the value of $i^*_1$. By contrast, under DCP, our simulations show that the results are qualitatively reversed: capital controls are non-zero for permanent productivity shocks and zero for world interest rate shocks.

The reason for the difference between PCP and DCP is that under DCP, export volumes cannot be adjusted via exchange rate movements. Under PCP, the planner adjusts the exchange rate in order to manage export volumes and thereby the balance between the AD and TOT wedges in each period and state. Under DCP, the planner instead influences TOT wedges by using two indirect methods to change the period-0 export-price-setting decision, which in turn pins down the entire schedule of TOT wedges across all periods and states. First, as shown in equation (26), the planner adjusts the exchange rate to balance the AD wedge in each period and state against the indirect impact on the $P_X$-setting decision (via the $\frac{\partial P_X}{\partial E_t}$ terms). Second, as shown in equation (25), the planner adjusts consumption levels in each period and state to influence the $P_X$-setting decision (via the $\frac{\partial P_X}{\partial C_{Ft}}$ terms).[14] In general, the time paths of both the AD and TOT wedges diverge between PCP and DCP. As a result, the optimal capital controls change.

[14] Appendix A.1 records the formulae for these partial derivatives.

Our second lemma records the optimal exchange rate volatility as a function of the pricing paradigm.

**Lemma 2.** *Iff $\Psi_{B1} = 0$ for every period-1 state, shocks which alter the time path of imports optimally produce higher exchange rate volatility under DCP than PCP.*

After a period-1 shock, equation (26) can be rewritten as follows:

$$\alpha_H \left(1 - \frac{1}{A_1}\frac{\alpha_H}{\alpha_F}\frac{P^*_{F1}}{P_H}E_1 C_{F1}\right)$$
$$= \mathbb{I}^{PCP} \cdot \left\{\omega C^*_1 \frac{E_1 P^*_{F1}}{P_H}\frac{1}{A_1}\right\} + \mathbb{I}^{DCP} \cdot \left\{\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t \omega C^*_t \frac{P^*_{Ft}}{P_X}\frac{1}{A_t}\right]\frac{1}{\beta\pi_1}\frac{E_1}{P_X}\left(-\frac{\partial P_X}{\partial E_1}\right)\right\}. \quad (29)$$

Consider a shock which is optimally associated with a reduction in $C_{F1}$, e.g., a permanent negative shock to commodity prices $P^*_{Zt}$ or an increase in the world interest rate $i^*_1$. For a given level of $E_1$, a decrease in $C_{F1}$ makes the left hand side of the above equation higher than the right hand side. Under PCP, equality is restored by depreciating the exchange rate to achieve both import and export substitution: the consumption of home tradable goods, $C_{H1} = \frac{\alpha_H}{\alpha_F}\frac{P^*_{F1}}{P_H}E_1 C_{F1}$, increases while the export volume, $Y_{X1} = \omega C^*_1 \frac{E_1 P^*_{F1}}{P_H}$, also increases.

Under DCP, the import substitution term on the left hand side remains equally operational, but the export substitution term is replaced. The right hand side term has $E_1$ in the numerator, but the exchange rate also appears in the denominator of the (positive) term $\left(-\frac{\partial P_X}{\partial E_1}\right)$. The DCP term captures the small effect of the exchange rate decision on period-0 export-price-setting, so the marginal impact of a change in $E_1$ has a smaller effect on the DCP term than on the PCP term. Therefore, for equality in the above equation to be restored, a larger increase in $E_1$ is necessary under DCP than PCP.

Intuitively, although it may appear at first glance that exchange rate movements should be less useful under DCP than PCP because the traditional export substitution channel disappears under DCP, that logic fails because so far, there is no welfare cost to moving the exchange rate. Even if the marginal benefit is lower, the absence of a marginal cost means that exchange rates are moved until the marginal benefit falls to zero, and that condition is satisfied for a larger depreciation under DCP than PCP.

Next, we introduce welfare costs from moving the exchange rate. Specifically, we consider an environment where the banks' external debt constraint can become binding, and the debt limit is tightened by an exchange rate depreciation.

**Proposition 1.** *Iff $\Psi_{B1} > 0$ in at least one period-1 state, prudential capital controls are higher under DCP than PCP.*

Consider a shock which causes the external debt limit to bind. Figures 4 and 5 illustrate such a shock—a decrease in the value of $\kappa_{H1}$—under PCP and DCP respectively. Equation (29) is altered to the following expression:

$$\begin{aligned} & \alpha_H \left(1 - \frac{1}{A_1}\frac{\alpha_H}{\alpha_F}\frac{P^*_{F1}}{P_H}E_1 C_{F1}\right) \\ = \; & \mathbb{I}^{PCP} \cdot \left\{\omega C^*_1 \frac{E_1 P^*_{F1}}{P_H}\frac{1}{A_1}\right\} + \mathbb{I}^{DCP} \cdot \left\{\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t \omega C^*_t \frac{P^*_{Ft}}{P_X}\frac{1}{A_t}\right]\frac{1}{\beta\pi_1}\frac{E_1}{P_X}\left(-\frac{\partial P_X}{\partial E_1}\right)\right\} \\ & + \frac{\Psi_{B1}}{\beta\left(1+i^*_0\right)}\kappa_{H1}\frac{P_H}{E_1} \end{aligned} \tag{30}$$

A binding debt limit causes a reduction in $C_{F1}$. Let us start from the values of the exchange rate which satisfy equation (29) for PCP and DCP, i.e., for each pricing paradigm, the exchange rate is set to the level that balances AD wedges against TOT wedges and thereby balances domestic and foreign sources of price pressures. From lemma 2, we know that the exchange rate is more depreciated under DCP than PCP to implement the same reduction in $C_{F1}$, and from lemma 1, we also know that at this level of the exchange rate, capital controls are only rationalized by TOT externalities.

The addition of the positive term on the right hand side of the above expression makes the left hand side lower than the right hand side. Equality is restored under both PCP and DCP by appreciating the exchange rate relative to the level which stabilizes price pressures. The necessary appreciation is small under PCP because the appreciation affects both the import and export substitution terms, but it is large under DCP because appreciation affects import substitution, not export substitution, and has only a small effect on the period-0 export-price-setting decision. After the appreciation, the AD wedge is larger than the price-pressure-stabilizing level, and the distance is larger under DCP than PCP.

This argument does not pin down the direction of the difference in the level of the exchange rate or AD wedge between PCP and DCP, but it does establish that the downward deviation of the exchange rate from the level that balances AD wedges against TOT wedges is larger under DCP than PCP, and the upward deviation of the AD wedge above the price-pressure-stabilizing level is larger under DCP than PCP.

The optimal prudential capital control tax in period 0 is now calculated as follows:

$$\varphi_0 = \begin{cases} 1 - \dfrac{\frac{1}{\mathbb{E}_0\left[\frac{E_0}{E_1}\right]}\mathbb{E}_0\left[\frac{E_0}{E_1}\frac{\alpha_F}{P^*_{F1}C_{F1}}\right]\left\{1-\frac{\omega C^*_t}{C_{Ft}}\tau^{PCP}_{Xt}\right\}}{\mathbb{E}_t\left[\frac{\alpha_F}{P^*_{Ft+1}C_{Ft+1}}\left\{1+\frac{\Psi_{B1}}{\beta(1+i^*_0)\alpha_F}\kappa_{H1}\frac{P_H}{E_1}-\frac{\omega C^*_{t+1}}{C_{Ft+1}}\tau^{PCP}_{Xt+1}\right\}\right]} & \text{if PCP} \\ 1 - \dfrac{\frac{1}{\mathbb{E}_0\left[\frac{E_0}{E_1}\right]}\mathbb{E}_0\left[\frac{E_0}{E_1}\frac{\alpha_F}{P^*_{F1}C_{F1}}\right]\left\{\frac{\alpha_F}{P^*_{F0}C_{F0}}-\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\frac{1}{P^*_{F0}P_X}\left\{\frac{E_0}{C_{F0}}\left(-\frac{\partial P_X}{\partial E_0}\right)+\frac{\partial P_X}{\partial C_{F0}}\right\}\right\}}{\frac{\alpha_F}{P^*_{F0}C_{F0}}\left\{\begin{array}{c}\mathbb{E}_0\left[\frac{\alpha_F}{P^*_{F1}C_{F1}}\left\{1+\frac{\Psi_{B1}}{\beta(1+i^*_0)\alpha_F}\kappa_{H1}\frac{P_H}{E_1}\right\}\right] \\ -\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\mathbb{E}_0\left[\frac{1}{\beta\pi_1P^*_{F1}P_X}\left\{\frac{E_1}{C_{F1}}\left(-\frac{\partial P_X}{\partial E_1}\right)+\frac{\partial P_X}{\partial C_{F1}}\right\}\right]\end{array}\right\}} & \text{if DCP.} \end{cases} \tag{31}$$

There is now a rationale for prudential capital controls arising from pecuniary AD externalities. Since the upward deviation of the AD wedge above the price-pressure-stabilizing level is larger under DCP than PCP, the denominator of the fraction in the formula is larger, and prudential capital controls are larger under DCP than PCP. Intuitively, since the planner recognizes that aggregate demand will be excessively low after the shock, the planner imposes larger capital controls to redistribute aggregate demand from period 0 to the period of the shock.

The comparison of figures 4 and 5 confirms this result. We can see that the final exchange rate depreciation after the shock is in fact larger under DCP than PCP, but owing to the larger prudential capital controls and the associated redistribution of aggregate demand, the multiplier on the external constraint is lower under DCP than PCP.

This result concerns the intensive margin: for a given shock to banks' external debt limits in period 1, the prudential capital controls in period 0 are positive under both PCP and DCP, but larger under DCP than PCP. In Basu et al. (our forthcoming IMF Working Paper), we confirm that this result may be true on the extensive margin for other shocks. Specifically, we show that after a permanent negative commodity price shock, the exchange rate depreciation is larger under DCP than PCP, and therefore, the external debt limit may become binding under DCP with levels of initial FX debt for which it would not be binding under PCP. Therefore, prudential capital controls are optimal for a larger set of initial debt levels under DCP than PCP.

Our results in this section nest and correct some intuitions proposed in policy discussions. Although some have proposed that exchange rate movements should be less useful under DCP than PCP because the traditional export substitution channel disappears under DCP, that logic fails when there is no welfare cost to moving the exchange rate, and exchange rates should actually be more volatile under DCP than PCP in order to stabilize price pressures. However, if external debt limits bind, then while the marginal benefit of exchange rate movement remain smaller under DCP than PCP, the marginal costs become positive and indeed similar under PCP and DCP. In this case, it is optimal for the planner to appreciate the exchange rate to relax the constraint more under DCP than PCP, deviating more under DCP from the price-pressure-stabilizing level. And in prior periods, this distortion should be remedied via higher capital controls.

In terms of related literature on capital controls and the pricing paradigm, Egorov and Mukhin (2019) find that capital controls are optimally zero under DCP after foreign monetary policy shocks. Their result appears similar to our result after interest rate shocks in the absence of external debt limits, even though their result on exchange rate volatility under DCP differs from ours: they find that it is optimal to have a "partial peg" of each economy's exchange rate with the dollar. The reason for the difference stems from the differences in the model settings: they assume the use of imported intermediate inputs in production in the context of a continuum of identical small open economies including the U.S. This difference in setting has two implications: first, the dollar price index of intermediate inputs moves according to the dollar depreciation; and second, simultaneous depreciations do not achieve import or export substitution in their global setting. We consider only one small open economy and therefore implicitly allow the composition of economies to be heterogeneous across the world, so that it may be optimal for some countries to depreciate relative to others.

# 4 Shallow Foreign Exchange Markets

Next, we consider the additional friction of shallow FX markets and focus on the optimal integrated use of the policy rate, capital controls, and FX intervention. The shallow FX markets case is relevant for most emerging markets, as their currencies tend to be traded by a limited set of financial intermediaries. Even in normal times in the absence of shocks, these countries may only be able to finance their external debt by offering a premium to foreign investors. Additionally, these countries are vulnerable to risk-on/risk-off phases of the global financial cycle, as the willingness of foreigners to participate in the domestic-currency debt market exhibit boom-bust dynamics.

## 4.1 Policy Instruments and Wedges

The case with shallow FX markets corresponds to setting $\Gamma > 0$ in the constraints and FOCs summarized in Appendix A.2. The relevant household Euler conditions and Gamma equations are equations (3) and (15)-(16). As in section 3, we define $B_t \equiv \frac{D_t}{E_{t-1}}$. In the shallow FX markets case, the representative household's effective exposure to dollar-denominated debt at the beginning of period 0 is given by $\lambda B_0$. At the beginning of any other period $t$, the effective dollar-denominated-debt exposure is given by $\lambda B_t - (1-\lambda)\, FXI_{t-1}$ while the effective domestic-currency-denominated debt exposure is given by $(1-\lambda)\,(B_t + FXI_{t-1})$. As in section 3, we continue to assume that housing frictions do not bind.[15]

Under shallow FX markets, the UIP conditions in equation (22) are violated, with the level of gross external returns depending on the quantity of domestic currency debt that financial intermedi-

[15] This assumption ensures that the substitutability/complementarity of capital controls and macroprudential taxes follows the same logic as in section 3. With shallow FX markets, however, there may be a new rationale for ex post capital controls for macro stabilization after shocks, which would be labeled as CFMs rather than CFM/MPMs in the IMF's taxonomy.

aries must be induced to hold on their balance sheets. These gross external returns must be provided through a combination of the domestic policy rate (which also sets the returns available to households), capital controls, and the expected exchange rate movements (the latter two of which create a gap between households' and intermediaries' returns). For the shallowness of the FX market to matter in welfare terms, we impose that domestic households do not own all the intermediaries, i.e., $\lambda \in [0, 1)$.

Moving from the deep to the shallow FX markets case, the instrument of FX intervention becomes effective through the portfolio balance channel as in Gabaix and Maggiori (2015), Cavallino (2019), and Fanelli and Straub (2019). Under shallow FX markets, the planner can use FX intervention to absorb some of the debt inflows and outflows, thereby altering the equilibrium exposure of financial intermediaries to domestic currency debt. In this manner, FX intervention changes the necessary level of the gross external returns on this debt, which in turn alters exchange rates and allocations.

Specifically, the Gamma equations (15)-(16) establish that the planner should set $(B_{t+1} + FXI_t - S_t) = 0$ if it intends to reduce the expected external premia, $\mathbb{E}_t \tau_{\Gamma t+1}$, to zero for any given level of debt, $B_{t+1}$, and foreign appetite shock, $S_t$. By contrast, the planner should set $(B_{t+1} + FXI_t - S_t) \neq 0$ if some expected external premia are optimal between periods $t$ and $t+1$.

Next we turn to the FOCs for the constrained efficient allocation to understand which additional externalities emerge as we move from deep to shallow FX markets. The planner's Euler conditions for $t \in \{0, 1\}$ are now:

$$
\begin{aligned}
&\frac{\alpha_F}{P^*_{Ft} C_{Ft}} \left[1 + \frac{\alpha_H}{\alpha_F} \tau_{Ht}\right] - \mathbb{I}^{DCP} \cdot \left\{ \mathbb{E}_0 \left[ \sum_{t=0}^{2} \beta^t \omega C^*_t \frac{\alpha_F}{C_{Ft}} \tau^{DCP}_{Xt} \right] \frac{1}{\beta^t \pi_t} \frac{1}{P^*_{Ft}} \frac{1}{P_X} \frac{\partial P_X}{\partial C_{Ft}} \right\} \\
&= \beta \mathbb{E}_t \left\{ \frac{\alpha_F \left[ (1 + i^*_t) + (1 - \lambda) \tau_{\Gamma t+1} \right]}{P^*_{Ft+1} C_{Ft+1}} \left[ 1 + \frac{\alpha_H}{\alpha_F} \tau_{Ht+1} \right] \right\} + \frac{\Psi_{Bt}}{\beta I_{t-1}} + \left( \frac{1}{\beta} \right)^t \Gamma \Omega_t \\
&- \mathbb{I}^{DCP} \cdot \left\{ \mathbb{E}_0 \left[ \sum_{t=0}^{2} \beta^t \omega C^*_t \frac{\alpha_F}{C_{Ft}} \tau^{DCP}_{Xt} \right] \mathbb{E}_t \left[ \frac{(1 + i^*_t) + (1 - \lambda) \tau_{\Gamma t+1}}{\beta^t \pi_{t+1}} \frac{1}{P^*_{Ft+1}} \frac{1}{P_X} \frac{\partial P_X}{\partial C_{Ft+1}} \right] \right\}. \qquad (32)
\end{aligned}
$$

Three wedges, $\{\tau_{Ht}, \tau_{Xt}, \tau_{\Gamma t}\}$, now enter the planner's Euler conditions and generate divergences from the households' Euler conditions. Relative to the deep FX markets Euler condition (25), there are two main additions: the term $\left(\frac{1}{\beta}\right)^t \Gamma\Omega_t$, where $\Omega_t$ is the multiplier on the Gamma equation; and the dependence of the discount factor in period $t+1$ on the UIP wedges in the same period, $\tau_{\Gamma t+1}$.

Focusing first on the $\left(\frac{1}{\beta}\right)^t \Gamma\Omega_t$ term, we observe that the multiplier on the Gamma equation is positive when the external debt level in period $t$ is forcing the UIP wedge, $\tau_{\Gamma t+1}$, to be higher than the planner would otherwise like it to be. We refer to this term as the financial TOT externality, which arises owing to the following channel. When deciding on their level of borrowing, each household takes returns as given, without internalizing the impact of its borrowing decision on the returns facing all households. It does not internalize that since the economy as a whole is the sole supplier of domestic currency bonds to the financial intermediaries, the level of debt determines the UIP wedge in equilibrium. High UIP wedges lower welfare because they constitute excessive premia paid by domestic households to the foreign-owned fraction of the financial intermediaries.

Turning next to the UIP wedges, $\tau_{\Gamma t+1}$, we observe that if households do not own all of the financial intermediaries, i.e., $\lambda \in [0, 1)$, then their external liabilities are effectively partially in domestic currency. The planner can improve welfare by redistributing resources across states using the exchange rate: specifically, the planner should depreciate away the dollar value of repayments on external liabilities in states when economy-wide dollar resources are reduced by shocks, and increase the dollar value of repayments when economy-wide dollar resources are enhanced by shocks.

The expression for capital controls changes relative to the deep FX markets case as follows:

$$\varphi_t = \begin{cases} 1 - \dfrac{\beta \frac{(1+i_t^*)+\mathbb{E}_t\tau_{\Gamma t+1}}{\mathbb{E}_t\left[\frac{E_t}{E_{t+1}}\right]}\mathbb{E}_0\left[\frac{E_t}{E_{t+1}}\frac{\alpha_F}{P^*_{Ft+1}C_{Ft+1}}\right]\left[\begin{array}{c}\frac{\alpha_F}{P^*_{Ft}C_{Ft}}\left[1+\frac{\alpha_H}{\alpha_F}\tau_{Ht}\right] \\ -\mathbb{I}^{DCP}\cdot\left\{\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C_t^*\frac{\alpha_F}{C_{Ft}}\tau_{Xt}^{DCP}\right]\frac{1}{\beta^t\pi_t}\frac{1}{P^*_{Ft}}\frac{1}{P_X}\frac{\partial P_X}{\partial C_{Ft}}\right\}\end{array}\right]}{\frac{\alpha_F}{P^*_{Ft}C_{Ft}}\left[\begin{array}{c}\beta\mathbb{E}_t\left\{\frac{\alpha_F[(1+i_t^*)+(1-\lambda)\tau_{\Gamma t+1}]}{P^*_{Ft+1}C_{Ft+1}}\left[1+\frac{\alpha_H}{\alpha_F}\tau_{Ht+1}\right]\right\}+\left(\frac{1}{\beta}\right)^t\Gamma\Omega_t \\ -\mathbb{I}^{DCP}\cdot\left\{\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C_t^*\frac{\alpha_F}{C_{Ft}}\tau_{Xt}^{DCP}\right]\mathbb{E}_t\left[\frac{(1+i_t^*)+(1-\lambda)\tau_{\Gamma t+1}}{\beta^t\pi_{t+1}}\frac{1}{P^*_{Ft+1}}\frac{1}{P_X}\frac{\partial P_X}{\partial C_{Ft+1}}\right]\right\}\end{array}\right]} \\ \text{if } \Psi_{Bt} = 0 \\ \\ 0 \text{ if } \Psi_{Bt} > 0, \end{cases} \tag{33}$$

This expression captures both the financial TOT externality and the UIP wedge arguments from above. On the financial TOT externality, if $\Omega_t$ is positive, the level of capital controls tends to be larger, as the planner discourages households from borrowing in order to reduce the UIP wedge. On the UIP wedges, the terms $\tau_{\Gamma t+1}$ are multiplied by $(1-\lambda)$ in the denominator but not in the numerator, showing that there is a role for the planner to use capital controls to redistribute resources across states, because the representative household takes its effective borrowing rate (a combination of the foreign and domestic policy rates) as given, while the planner internalizes that it depends on the endogenous UIP wedges.

Exchange rate determination now follows the below expression:

$$\underbrace{\alpha_H\tau_{Ht}}_{\text{Stabilize demand for home goods}} = \underbrace{\begin{array}{c}-\mathbb{I}^{PCP}\cdot\left\{\omega C_t^*\frac{\alpha_F}{C_{Ft}}\tau_{Xt}^{PCP}\right\} \\ -\mathbb{I}^{DCP}\cdot\left\{\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C_t^*\frac{\alpha_F}{C_{Ft}}\tau_{Xt}^{DCP}\right]\frac{1}{\beta^t\pi_t}\left(-\frac{E_t}{P_X}\frac{\partial P_X}{\partial E_t}\right)\right\}\end{array}}_{\text{Optimize TOT on export goods}}$$
$$+\underbrace{\frac{\Psi_{Bt}}{\beta^t\left[\left(1+i_{t-1}^*\right)+(1-\lambda)\,\tau_{\Gamma t}\right]}\kappa_{H1}\frac{P_H}{E_t}}_{\text{Relax bank constraint}} \pm \underbrace{\frac{1}{\beta^t\pi_t}\Lambda E_t\left[\left(1+i_{t-1}^*\right)+\tau_{\Gamma t}\right]}_{\text{Prevent excessive contingency of exchange rate}}. \tag{34}$$

This expression is similar to the expression under deep FX markets, equation (26). Relative to that equation, the discounting of the bank constraint is altered depending on the size of the UIP wedge, and there is an additional final term which ensures that the optimizations by the planner over exchange rates and UIP wedges are connected, i.e., the distribution of exchange rates and UIP wedges must respect the non-contingency of the domestic policy rate.

The following trade-off determines the optimal UIP wedge, $\tau_{\Gamma t+1}$:

$$\underbrace{\Omega_t}_{\text{Ability to borrow more today}} = \underbrace{(1-\lambda)\frac{\Phi_{t+1}+\Psi_{Bt+1}}{\Pi_{s=0}^{t} I_s}\left(B_{t+1}+FXI_t\right)}_{\text{Higher repayments tomorrow}} \pm \underbrace{\frac{1}{\pi_{t+1}}\Lambda E_{t+1}}_{\text{Prevent excessive contingency of premium}}$$
$$+ \underbrace{(1-\lambda)\,\Omega_{t+1}\Gamma\left(B_{t+1}+FXI_t\right)}_{\text{Higher premium tomorrow owing to rollover needs}}, \tag{35}$$

where $B_{t+1} \equiv \frac{D_{t+1}}{E_t}$ is the dollar value of debt, and $\Phi_t$, $\Psi_{HHt}$, and $\Lambda_t$ are the multipliers on the resource constraint, household borrowing constraint, and contingency-check equation in period $t$.

The planner understands that under shallow FX markets, increasing the UIP wedge in order to allow a higher level of debt in period $t$ worsens consumption in period $t+1$ owing to higher external debt repayments, and also requires a higher UIP wedge in period $t+1$ if the debt is rolled over. In addition, the possibility of FX intervention alters the expression for the gross external debt position: when the planner borrows in the domestic currency debt market in order to accumulate dollar assets abroad, a fraction $(1-\lambda)$ of the debt ends up on the balance sheet of the foreign-owned intermediaries, so it constitutes external debt, and the $FXI_t$ terms enter the equation above.

The following trade-off determines the optimal level of FX intervention:

$$\underbrace{\Gamma\Omega_t}_{\text{Lower premium today}} \underbrace{+(1-\lambda)\,\mathbb{E}_t\left[\frac{\Phi_{t+1}+\Psi_{Bt+1}}{\Pi_{s=0}^{t} I_s}\tau_{\Gamma t+1}\right]}_{\text{Change in carry cost}} + \underbrace{(1-\lambda)\,\Gamma\mathbb{E}_t\left[\Omega_{t+1}\tau_{\Gamma t+1}\right]}_{\text{Change in premium tomorrow owing to change in carry cost}} = 0. \tag{36}$$

By absorbing some of the capital inflow or outflow, FX intervention can reduce the external debt that foreign-owned intermediaries have to absorb, and it can thereby reduce the UIP wedge. This benefit should be combined with any carry costs incurred by the FX intervention, taking into account that higher carry costs incurred between periods $t$ and $t+1$ may result in higher external debt, which needs to be rolled over in period $t + 1$. The planner should intervene until the net marginal benefit of intervention is pushed down to zero.

Moving from the deep FX markets case to the shallow FX markets case, one would expect to find a greater role for capital controls and a case for FX intervention. The equations above reveal several interactions between FX intervention and capital controls. FX intervention has two effects on the optimal capital control expression: (i) intervention affects the UIP wedge, $\tau_{\Gamma t}$, which alters the time path of consumption and the AD wedge, $\tau_{Ht}$; and (ii) intervention alters the multiplier on the Gamma equation, $\Omega_t$. Capital controls have two effects on the optimal level of FX intervention: (i) capital controls reduce gross external returns, $\eta_t$, which affect the carry cost of intervention; and (ii) they reduce households' debt, which alters the incentive to absorb the debt via FX intervention.

## 4.2 Capital Controls and Monetary Autonomy

When FX markets are shallow, i.e., foreigners require non-zero UIP premia to hold domestic currency debt because they face limits to arbitrage, the planner's macro stabilization problem may be more difficult than when FX markets are deep. Under deep FX markets, shocks destabilize the AD and (export-volume-based) TOT wedges that the planner is trying to balance over time. Under shallow FX markets, the same shocks additionally destabilize UIP wedges, and the planner needs to stabilize those wedges as well, which may compromise the stabilization of the AD and TOT wedges.

In this subsection, we explain the connection between optimal capital controls and the financial TOT externalities captured by the UIP wedges. We are specifically concerned with ex-post period-1 capital controls rather than prudential period-0 capital controls. Our main result is as follows.

**Proposition 2.** *The financial TOT externality generates a rationale for capital controls to vary across period-1 states in the same direction as the sign of* $\Gamma\Omega_1$*, i.e., capital controls tend to be higher when UIP premia are inefficiently high and lower when UIP premia are inefficiently low.*

Equation (33) shows that there may be many rationales for capital controls, including the AD, TOT, and pecuniary AD externalities from the deep FX markets section. Zooming in on the new financial TOT externality component, which is distinct from the export-volume-based TOT wedge $\tau_{Xt}$, the equation shows that capital controls indeed move in the direction of the sign of $\Gamma\Omega_1$. This result can be seen in its simplest form by writing down the constrained efficient FOCs for $C_{F1}$ and $C_{F2}$ for the case when the banks' external constraint is not binding:

$$\beta I_0 \frac{\alpha_F}{P^*_{F1} C_{F1}} \left[1 + \frac{\alpha_H}{\alpha_F}\tau_{H1}\right] - \mathbb{I}^{DCP} \cdot \left\{ \mathbb{E}_0 \left[ \sum_{t=0}^{2} \beta^t \omega C^*_t \frac{\alpha_F}{C_{Ft}} \tau^{DCP}_{Xt} \right] \frac{I_0}{\pi_1} \frac{1}{P^*_{F1}} \frac{1}{P_X} \frac{\partial P_X}{\partial C_{F1}} \right\} = \Phi_1 + I_0 \Gamma \Omega_1 \tag{37}$$

$$\beta^2 I_0 I_1 \frac{\alpha_F}{P^*_{F2} C_{F2}} \left[1 + \frac{\alpha_H}{\alpha_F}\tau_{H2}\right] - \mathbb{I}^{DCP} \cdot \left\{ \mathbb{E}_0 \left[ \sum_{t=0}^{2} \beta^t \omega C^*_t \frac{\alpha_F}{C_{Ft}} \tau^{DCP}_{Xt} \right] \frac{I_0 I_1}{\pi_1} \frac{1}{P^*_{F2}} \frac{1}{P_X} \frac{\partial P_X}{\partial C_{F2}} \right\} = \Phi_1. \tag{38}$$

The AD wedges, $\tau_{Ht}$, can be replaced by TOT wedges, $\tau_{Xt}$, using equation (34), but that equation does not have $\Omega_1$ in it. Therefore, $\Gamma\Omega_1$ is in the FOC for $C_{F1}$ but not in the FOC for $C_{F2}$, and as a result, the financial TOT externality generates a rationale for the planner to distort the households' borrowing decision between periods 1 and 2.

The above argument establishes that when the planner optimizes the consumption schedule while taking the economy-wide resource constraint as given, there is a financial-TOT-rationale for capital controls to move in the direction of the sign of $\Gamma\Omega_1$. To interpret further what this result means, we need to characterize the formula for $\Gamma\Omega_1$ that emerges from the separate decisions of the planner as it optimizes the resource constraint.

First, consider allocations when the policy rate and capital controls are in the planner's toolkit but FX intervention is not. In period 1, the FOCs for the UIP wedges yield the following expression for $\Omega_1$:

$$\Gamma\Omega_1 = \Gamma(1-\lambda)\Phi \frac{B_2}{I_0 I_1}. \tag{39}$$

Therefore, any shock that is associated with an increase in debt, $B_2$, justifies higher capital controls owing to the financial TOT motive, provided that $\Gamma > 0$ and $\lambda \in [0, 1)$. If $\Gamma > 0$ in the Gamma equation, the higher debt increases the UIP wedge, which means that the economy as a whole pays excessively high gross returns to financial intermediaries. If $\lambda \in [0, 1)$, some of these returns go to foreigners instead of to other domestic agents, so the economy-wide resource constraint shrinks. Increasing capital controls to reduce consumption reduces the increase in debt and thereby reduces these losses. Conversely, any shock that is associated with a decrease in debt calls for lower capital controls, because households are undertaking an excessive deleveraging from the economy-wide perspective.

Figure 6 illustrates the role of capital controls in a DCP economy following shocks to the foreign appetite for domestic currency assets, $S_1$.[16] The dashed lines show the allocations achievable by monetary policy alone. A positive shock causes a decline in the external premium $\eta_2$ between periods 1 and 2. Since the planner cannot eliminate this shock at its source, it is optimal to make the most of it and increase imports. However, monetary policy alone cannot both balance the AD and TOT wedges and optimize the UIP wedge: allowing an exchange rate appreciation so that households expand imports also causes an increase in debt which pushes up the interest rate for all other households. Excessive debt pushes down the absolute size of the UIP wedge, which reduces the ability of the economy to exploit the shock. To mitigate this problem, the planner raises the interest rate, but at the cost of reducing home tradable consumption and employment, i.e., $\tau_{H1} > 0$ and $\tau_{X1} < 0$. Conversely, after a negative shock, the reduction in imports coincides with excessive deleveraging which the planner cannot fully avoid.

The solid lines show the allocations achievable when capital controls are added to the toolkit. After a positive shock, the planner allows some increase in imports but mitigates the increase in debt and the associated financial TOT externalities by increasing capital inflow taxes; while after a negative shock, the planner offers capital inflow subsidies. As a result, the UIP wedge is kept large in absolute size, allowing the economy to exploit the shock, with smaller adverse effects on the destabilization of the AD and TOT wedges.

Next, consider allocations when the policy rate, capital controls, and FX intervention are all available to the planner, and the FX intervention is not subject to any limits beyond carry costs. In

[16] Since the comparison between PCP and DCP is not our primary emphasis in this section, we illustrate our results using the DCP assumption. Our main result on monetary autonomy is robust to the pricing paradigm.

period 1, the FOCs for the UIP wedges and FX intervention can be combined to yield the following expression for $\Omega_1$:

$$\Gamma\Omega_1 = \Gamma(1-\lambda)\frac{\Phi}{I_0 I_1}\frac{S_1}{2}. \tag{40}$$

This expression again establishes that the conditions $\Gamma > 0$ and $\lambda \in [0,1)$ are required for the financial TOT motive to be relevant in the capital controls formula. The expression additionally establishes that the financial TOT motive does not always appear in the capital controls formula if unrestricted FX intervention is available: it is possible that in the process of the planner optimizing the resource constraint, it sets financial TOT externalities to zero.

Let us explain this latter point in more detail. Equation (40) actually holds for all the shocks we consider in this paper. After a shock to the world interest rate, for example, such that $(1+i_2^*) \gtrless (1+i_1^*)$ but $S_1 = 0$, we find that $\Gamma\Omega_1 = 0$. The use of FX intervention to absorb part of the shock at its source, and the possible additional use of capital controls to address AD and TOT wedges as well, does not leave any residual financial TOT externality for capital controls to tackle. Even though debt $B_2$ does actually move across states, if FX intervention is unrestricted, it is set to absorb the financial TOT externalities associated with that debt: $B_2 + FXI_1 = 0$.

However, after shocks to the foreign appetite for domestic currency debt, it is optimal for the planner to absorb only part of the inflow or outflow, because the planner wishes to earn carry profits on the flow: $B_2 + FXI_1 = \frac{S_1}{2}$. This result of partial absorption is consistent with the findings of Cavallino (2019) and Fanelli and Straub (2019), who consider a planner who uses only monetary policy and FX intervention. In our context with the addition of capital controls into the toolkit, partial absorption means that even after the planner optimizes the resource constraint, there remain unaddressed financial TOT externalities which affect the use of capital controls.

Figure 7 illustrates the joint use of monetary policy, FX intervention, and capital controls in this economy following shocks to the foreign appetite for domestic currency assets, $S_1$. The dashed lines show the allocations achievable by monetary policy and FX intervention, excluding capital controls. There is partial absorption of the shock via FX intervention, and imperfect macro stabilization. The solid lines illustrate the joint use of all three instruments. The addition of capital controls does not eliminate the financial TOT externality, as $\Gamma\Omega_1$ continues to be non-zero, but it does fully stabilize macro allocations.

Capital controls and FX intervention are substitutes for the policy rate after foreign appetite shocks. Their joint use allows the policy rate to remain unaffected by the external shock, so that monetary policy can focus instead solely on the domestic sources of price pressures. In terms of the wedges, the destabilization of the UIP wedges can be de-linked from the fully stabilized AD and TOT wedges. The same result holds under PCP.

Our result sheds light on the modern discussions related to the monetary policy trilemma. From a feasibility perspective, if a small open economy has the policy option of exchange rate flexibility, as we allow in our model, the policy rate can feasibly diverge from the world interest rate, and the economy does not find itself in one of the corners of the traditional trilemma. Moving beyond feasibility, our model suggests that we should view the modern discussions about the trilemma through the lens of optimality.

Rey (2013) and Miranda-Agrippino and Rey (2019) have argued that U.S. monetary policy shocks cause downturns in the "global financial cycle" and thereby generate reductions in global asset prices and cross-border flows. This evidence has been used to propose that small open economies do not possess monetary autonomy unless they impose capital controls. In our framework, the issue is reframed as a question of whether in the face of external financial shocks, small open economies can use monetary policy in their traditional sense: to stabilize price pressures, which in our model arise from the combination of AD and export-volume-based TOT wedges.

Our proposed answer is that under deep FX markets and no binding external constraints, countries have an easier task, because no new wedges are introduced, and monetary policy is an appropriate instrument to stabilize the existing wedges in the face of external shocks. Capital controls may also be used to address TOT externalities. Under shallow FX markets, a new UIP wedge arises in addition to the existing AD and TOT wedges, and external shocks destabilize the UIP wedge as well, so the stabilization of the AD and TOT wedges is compromised. Capital controls are then appropriate to increase monetary autonomy, in the sense that they address unresolved financial TOT externalities while allowing the policy rate to better focus on domestic sources of price pressures.

## 4.3 Emerging Market Conundrum

In the previous section, we showed that currency mismatches generate vulnerability to shocks to the ability to issue external FX-denominated debt. In the current section, we have shown that shallow FX markets generate vulnerability to shocks to the foreign appetite for domestic-currency-denominated debt. In practice, many emerging markets may suffer from both currency mismatches and shallow FX markets and may therefore be vulnerable to both shocks. In this case, it is important that policy actions to address one kind of shock do not inadvertently increase the vulnerability of the country to the other kind of shock.

We illustrate this conundrum by considering the impact of a particular macroprudential regulation: a ban on open FX positions for those intermediaries which are domestically owned. Since the representative household acquires currency mismatch through its ownership of intermediaries who have dollar liabilities, such a ban may be seen as a way to reduce the economy's vulnerability to debt limit shocks.

The following lemma establishes that banning open FX positions is appropriate if FX markets are deep.

**Lemma 3.** *If FX markets are deep, i.e., $\Gamma = 0$, a ban on open FX positions by domestically-owned intermediaries is optimal and removes the need for prudential capital controls to address pecuniary AD externalities.*

If FX markets are deep, the economy-wide resource constraint (19) is replaced by the following:

$$D_{t+1} \geq -E_t P_{Ft}^* \left[\omega C_t^* - C_{Ft}\right] - E_t P_{Zt}^* Z_t + \left(1 + \widehat{i}_{t-1}\right) D_t. \tag{41}$$

Relative to equation (19), there is no fraction $\lambda$ of debt with currency mismatch, so an exchange rate depreciation does not increase the domestic currency value of debt repayments, and the pecuniary AD externality disappears. The planner is free to depreciate the exchange rate after all shocks, including debt limit shocks, in order to balance price pressures and stabilize domestic activity. Therefore, prudential capital controls are no longer optimal to address pecuniary AD externalities. There is no side effect because the UIP conditions (22) continue to hold.

However, in a model which integrates both frictions of external debt limits and shallow FX markets, a ban on open FX positions may also generate costs.

**Proposition 3.** *If FX markets are shallow, i.e., $\Gamma > 0$, a ban on open FX positions by domestically-owned intermediaries has the same benefit as under deep FX markets, but also had two side effects: (i) external debt is more expensive in steady state; and (ii) the economy is more vulnerable to foreign appetite shocks and may become more dependent on FX intervention.*

If FX markets are shallow, then the economy-wide resource constraint (19) still changes as in the deep markets case, but includes a term for the carry costs of FX intervention:

$$D_{t+1} \geq -E_t P^*_{Ft} \left[\omega C^*_t - C_{Ft}\right] - E_t P^*_{Zt} Z_t + \left(1 + \widehat{i}_{t-1}\right) D_t - O_t \left[\left(1 + \widehat{i}_{t-1}\right) - \left(1 + i^*_{t-1}\right) \frac{E_t}{E_{t-1}}\right]. \tag{42}$$

Again, the pecuniary AD externality disappears, so prudential capital controls to address this externality go to zero.

Unlike the deep FX markets case, the fact that domestic currency debt can only be absorbed by foreign-owned intermediaries means that the Gamma equations (15) and (16) are altered as follows:

$$\frac{\Gamma}{(1-\lambda)} \left(B_1 + FXI_0 - S_0\right) = \mathbb{E}_0 \left[\eta_1 - (1 + i^*_0)\right] \tag{43}$$

$$\frac{\Gamma}{(1-\lambda)} \left(B_2 + FXI_1 - S_1\right) = \eta_2 - (1 + i^*_1), \tag{44}$$

and the amended system of constraints and FOCs is summarized in Appendix A.4. FX markets effectively become shallower, i.e., the effective Gamma becomes $\frac{\Gamma}{(1-\lambda)}$, which is higher than $\Gamma$. There are two main side effects, which we discuss next with reference to two figures: figure 8, which shows the impact of the regulation on the allocations after external debt limit shocks; and figure 9, which shows the impact of the regulation on the allocations after foreign appetite shocks. In both figures, the dashed lines represent allocations without the regulation and the solid lines represent allocations with the regulation. For illustrative purposes, FX intervention is not allowed. We impose the following constraint for $t \in \{0,1\}$:

$$FXI_t = 0. \tag{45}$$

The first side effect is that external debt is more expensive in steady state. Figure 8, which illustrates debt limit shocks, shows that the regulation generates significant deleveraging when FX intervention is set to zero. Open FX positions only arise in equilibrium for countries who need intermediaries to finance domestic currency debt. If markets are shallow, this financing generates positive UIP wedges. If the FX market becomes shallower, UIP wedges increase and debt decreases in equilibrium. Even though ex ante capital controls are not needed for pecuniary AD externalities (indeed, the external debt limit no longer binds after the regulation), there are large steady-state capital controls to mitigate financial TOT externalities.

The second side effect is that the regulation increases the vulnerability of the economy to foreign appetite shocks and increases the marginal value of ex post FX intervention. Figure 9, which illustrates foreign appetite shocks, plots the allocations with monetary policy (MP) and capital controls (CC) only, and then we can infer the marginal value of ex post FX intervention. Since FX markets are shallower owing to the ban, allocations become more volatile in response to foreign appetite shocks. Both the ex post policy rate and ex post capital controls also become more volatile.

The marginal value of ex post FX intervention can be assessed from the values of the period-1 multipliers, $\left\{\Theta_1^H, \Theta_1^L\right\}$, on constraint (45). Moving from the allocations without the ban to the allocations with the ban, the values of the multipliers increase from $\left\{\Theta_1^H = -0.03, \Theta_1^L = 0.02\right\}$ to $\left\{\Theta_1^H = -0.04, \Theta_1^L = 0.12\right\}$, establishing that the value of ex post FX intervention increases. The country becomes more reliant on FX intervention after foreign appetite shocks.

We believe this conundrum is a fairly general problem for emerging markets whose residents issue both FX-denominated and domestic-currency-denominated debt to foreigners. These countries typically face both kinds of frictions (currency mismatch and shallow FX markets) and both kinds of shocks (binding external FX debt limits and fluctuations in foreign appetite for domestic currency debt).

The conundrum we have identified does not depend on the specific functional forms we have chosen above. The necessary ingredients are fairly simple: some domestic agents should have the option to issue FX debt, their decision to issue FX debt should be endogenous to the UIP premia on domestic currency debt, and the government has the ability to regulate them. If domestic residents borrow in FX in response to an increase in the UIP premia, in order to either reduce their own issuance of domestic currency debt or in order to lend to other domestic agents in domestic currency, their actions mitigate the change in UIP premia and reduce the dependence of the economy on foreign intermediation. However, their new FX debt obligations increase currency mismatch. And if they are forced to reduce currency mismatch to reduce the vulnerability of the economy to external FX debt constraints, they are not available to bypass the foreign intermediaries after negative foreign appetite shocks.

# 5 Housing Sector

Finally, we consider the additional friction of housing sector borrowing constraints and focus on the optimal integrated use of the policy rate, capital controls, FX intervention, and domestic macroprudential taxes. Whether a country as a whole has high or low external debt, there may be substantial stocks of debt contracted between different domestic agents, and domestic borrowing constraints are likely to be related to the domestic currency value of nontraded assets. For the housing sector, an appropriate collateral would be land. Leveraged domestic borrowing is relevant for most advanced economies and a growing number of emerging markets which have gradually developed domestic credit markets over time.

Housing frictions have usually been analyzed in closed economy models where fire sales of land are triggered by domestic shocks, and the possibility of crisis-time fire sales rationalizes taxes or quantity restrictions on domestic housing sector debt in normal times. In this section, we nest such housing sector frictions in an open economy model where fire sales of land may be triggered by both domestic and external shocks, and may rationalize a combination of domestic (policy rate and macroprudential debt taxes) and external adjustment tools (capital controls and FX intervention).

## 5.1 Policy Instruments and Wedges

The case with housing frictions draws on the full set of constraints and FOCs summarized in Appendix A.2. For housing frictions to matter for the equilibrium allocations, we require two conditions: firstly, that the planner can only impose macroprudential taxes on the linear housing subsector, i.e., $\theta_{Rt}^{Linear} \in \mathbb{R}$, while the concave subsector is unregulated, i.e., $\theta_{Rt}^{Concave} \equiv 0$; and secondly, that housing sector borrowing capacity is limited, i.e., $\kappa_{L1}$ is sufficiently low. Housing frictions may rationalize a combination of domestic and external adjustment tools. In this subsection, we first explain the rationale for macroprudential taxes on housing debt. Then we turn to additional policy instruments, including those only available in open economies.

Relative to the previous sections, there is now a meaningful decision for the planner to make regarding the quantity of land held by the linear and concave housing subsectors. Let us begin with describing period-1 crisis-time outcomes and then derive the optimal prudential policies in period 0. The following trade-off determines the constrained-efficient quantity of land held by the linear subsector in the period-1 state $s$:[17]

$$\underbrace{\frac{\beta\alpha_R\tau_{R2}}{Y_{R2}}}_{\text{Minimize housing distortion}} = \underbrace{\Psi_{R1}\widehat{q}_1\left(1-\kappa_{L1}\right)}_{\text{Housing constraint}} + \underbrace{\Psi_{R1}\frac{\partial\widehat{q}_1}{\partial L_1^{Linear}}\left(\left(1-\kappa_{L1}\right)L_1^{Linear}-L_0^{Linear}\right)}_{\text{Tightening of constraint}}$$

[17]In our notation, a derivative of the form $\mathbb{E}_0\left[\Psi_{R1}\frac{\partial X_1}{\partial Y_1^s}\right]$ indicates the marginal impact of changing the variable $Y_1$ in a particular period-1 state $s$ on the expected value of the variable $X_1$ across states, weighted by the housing multiplier $\Psi_{R1}$ in each state. All derivatives depend on whether the planner has access to capital controls or consumer macroprudential taxes, and they are documented in Appendix A.2.

$$+\frac{1}{\pi_1}\mathbb{E}_0\left[\Psi_{R1}\underbrace{\frac{\partial\left(\chi_1\widehat{q}_0\right)}{\partial L_1^{Linear,s}}\left(L_0^{Linear}-L_{-1}^{Linear}\right)}_{\text{Effect on period-0 land price}}\right], \tag{46}$$

where $\Psi_{R1}$ is the multiplier on the housing constraint in period $1$, a hat over a variable indicates the FX value of that variable, and the FX gross return related to domestic interest repayments between periods $t$ and $t+1$ is given by:

$$\chi_{t+1}=\left(1+i_t\right)\frac{E_t}{E_{t+1}}>0.$$

The FX gross return covers both the borrowing rate and the exchange rate movements between the two periods. Unlike the gross return $\eta_{t+1}$ of international financial intermediaries, the gross return $\chi_{t+1}$ is not subject to capital controls because it covers purely domestic transactions.

If the housing constraint is not binding, i.e., $\Psi_{R1}=0$, then there is no distortion to housing output, i.e., $\tau_{R2}=0$, and all land is held by the linear subsector, i.e., $L_1^{Linear}=1$. If the housing constraint is binding, i.e., $\Psi_{R1}>0$, the first term on the right hand side shows that the housing wedge $\tau_{R2}$ exceeds 0, indicating that $L_1^{Linear}$ decreases below 1. The decrease in $L_1^{Linear}$ reduces the period-1 land price $\widehat{q}_1$, i.e., $\frac{\partial\widehat{q}_1}{\partial L_1^{Linear}}>0$, which tightens the constraint via the second term. The associated decrease in the period-0 land price affects the inherited debt of the linear subsector, represented by the third term.

The following trade-off determines the constrained-efficient quantity of land held by the linear subsector in period 0:

$$\underbrace{\frac{\beta\alpha_R\tau_{R1}}{Y_{R1}}}_{\text{Minimize housing distortion}}=\mathbb{E}_0\left[\Psi_{R1}\left\{\begin{array}{c}\underbrace{\left(\chi_1\widehat{q}_0-\widehat{P}_{R1}-\widehat{q}_1\right)}_{\text{Hedging motive}}\\ +\underbrace{\frac{\partial\left(\chi_1\widehat{q}_0\right)}{\partial L_0^{Linear}}\left(L_0^{Linear}-L_{-1}^{Linear}\right)}_{\text{Effect on period-0 land price}}-\underbrace{\frac{\partial\widehat{P}_{R1}}{\partial L_0^{Linear}}L_0^{Linear}}_{\text{Effect on rent}}\end{array}\right\}\right]. \tag{47}$$

If the housing constraint is not binding in any of the period-1 states, there is no distortion to land usage by the linear subsector in period 0. Let us now consider an allocation when the housing constraint is binding in one of the period-1 states.

The first term on the right hand side is the hedging motive. The term is positive if the linear subsector's net profit from land (i.e., the value of rents $\widehat{P}_{R1}$ and the land price $\widehat{q}_1$ minus interest payments $\chi_1\widehat{q}_0$) is negative in the period-1 state where the housing constraint is binding. If so, the linear subsector's constraint in that state could be relaxed if the subsector were holding less land and less inherited debt from the previous period. It is indeed optimal for the planner to relax the constraint in this manner because there is a pecuniary production externality: individual firms in the linear housing subsector do not internalize that their period-0 debt decisions affect the period-1 land price and thereby the tightness of the period-1 constraint. The planner relaxes the constraint by reducing $L_0^{Linear}$ below 1. The second and third terms on the right hand side capture the side-effects of the reduction in $L_0^{Linear}$ on the period-0 land price and on period-1 rents.

The planner can reduce $L_0^{Linear}$ below 1 using a period-0 domestic macroprudential tax on the debt of the linear housing subsector. The housing macroprudential tax in period $t$ follows the expression:

$$\left(1+\theta_{Rt}^{Linear}\right) = \begin{cases} \dfrac{\mathbb{E}_t\left[\frac{1}{L_t^{Linear}+G\left(1-L_t^{Linear}\right)}\frac{\alpha_R}{\alpha_F}\frac{E_{t+1}}{E_t}P_{Ft+1}^*C_{Ft+1}\right]+\mathbb{E}_t\left[\frac{E_{t+1}}{E_t}\widehat{q}_{t+1}\right]}{(1+i_t)\widehat{q}_t} & \text{if } \Psi_{Rt}=0 \\ 0 & \text{if } \Psi_{Rt}>0 \end{cases}, \tag{48}$$

$$\text{where } \widehat{q}_t = \begin{cases} \frac{1}{(1+i_0)}\frac{G'\left(1-L_0^{Linear}\right)}{L_0^{Linear}+G\left(1-L_0^{Linear}\right)}\frac{\alpha_R}{\alpha_F}\mathbb{E}_0\left[\frac{E_1}{E_0}P_{F1}^*C_{F1}\right] \\ \quad +\frac{1}{(1+i_0)}\mathbb{E}_0\left[\frac{1}{\chi_2}\frac{E_1}{E_0}\left(\frac{G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)}\frac{\alpha_R}{\alpha_F}P_{F2}^*C_{F2}+\widehat{q}_2\right)\right] & \text{if } t=0 \\ \frac{1}{\chi_2}\left(\frac{G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)}\frac{\alpha_R}{\alpha_F}P_{F2}^*C_{F2}+\widehat{q}_2\right) & \text{if } t=1 \\ \widehat{q}_2 & \text{if } t=2, \end{cases}$$

where $\widehat{q}_2$ is exogenously fixed.[18] The desired reduction in $L_0^{Linear}$ causes a decrease in the period-0 land price, $\widehat{q}_0$, as the concave subsector is forced to hold some land. To prevent the linear subsector purchasing all the land at this lower price, the planner imposes a positive ex ante tax on that

[18]Fixing $\widehat{q}_2$, the FX value of the land price in period 2, captures the assumption that short-term policy actions cannot alter the long-term relative price of land to foreign tradable goods. The corollary of this assumption is that depreciations increase the period-2 land price in domestic currency.

subsector, i.e., $\theta_{R0}^{Linear} > 0$.

In the absence of other instruments, the hedging motive is positive, because fire sales decrease the net payoffs from land. Therefore, the housing macroprudential tax imposed in period 0 is also positive.

However, the planner possesses additional policy tools to help relax the housing constraint (21), and these tools may alter the hedging motive and thereby the rationale for the housing macroprudential tax. We turn next to the use of these other instruments, both domestic and external. The planner can relax the housing constraint by reducing the policy rate and domestic borrowing rate, which raises the land price via the no-arbitrage condition of the concave housing subsector. The planner can also use a combination of policy tools to depreciate the exchange rate, which relaxes the housing constraint via two channels: firstly, it generates substitution in consumption from imports to home goods including housing, thus boosting rents and house prices; and secondly, it increases the domestic currency price of land in period 2, which filters back to a higher domestic currency price in period 1 as well.

The use of these additional instruments to stabilize the housing sector generates distortions for the non-housing sectors of the economy, which must be balanced against the relaxation of the housing constraint. Exactly which distortions are generated in the rest of the economy depends on the set of available instruments, and in particular whether the planner has access to capital controls or consumer macroprudential taxes. The formula for the FX value of the housing sector's gross borrowing rate depends on which of the two instruments is available:

$$\chi_{t+1} = \begin{cases} \dfrac{\frac{\alpha_F}{P_{Ft}^* C_{Ft}}}{\beta E_{t+1} \mathbb{E}_t \left\{ \frac{1}{E_{t+1}} \frac{\alpha_F}{P_{Ft+1}^* C_{Ft+1}} \right\}} & \text{if } \varphi_t \in \mathbb{R} \text{ but } \theta_{HHt} \equiv 0 \\ \eta_{t+1} & \text{if } \theta_{HHt} \in \mathbb{R} \text{ but } \varphi_t \equiv 0. \end{cases}$$

If capital controls are allowed but consumer macroprudential taxes are not, i.e., $\varphi_t \in \mathbb{R}$ but $\theta_{HHt} \equiv 0$, the housing sector's FX borrowing rate is identical to the borrowing rate of domestic households, and altering the borrowing rate must be balanced against distorting the domestic consumption path. If consumer macroprudential taxes are allowed but capital controls are not, i.e., $\theta_{HHt} \in \mathbb{R}$ but $\varphi_t \equiv 0$, the housing sector's FX borrowing rate is identical to the return received by international intermediaries, and altering the borrowing rate rate must be balanced against altering the UIP wedges paid by the domestic economy to foreigners.

In other words, the occasionally-binding constraint of the housing sector breaks the result of substitutability between capital controls and consumer macroprudential taxes (assuming perfect coverage of both instruments) from sections 3 and 4. The two instruments are in principle substitutable in period 0, but they are not substitutable in period-1 states when the housing constraint binds, and the divergence in allocations in those period-1 states causes a divergence in the optimal period-0 levels of the instruments as well.

We can catalogue the constrained efficient FOCs depending on whether the planner has access to capital controls or consumer macroprudential taxes. First, if capital controls are allowed as in section 4, the expressions (35) and (36) remain unchanged, as do the exchange rate FOCs in periods 0 and 2 represented by equation (34), but the FOC for the exchange rate in period 1 changes to the following:

$$\underbrace{\alpha_H \tau_{H1}}_{\text{Stabilize demand for home goods}} = \underbrace{\begin{matrix} -\mathbb{I}^{PCP} \cdot \left\{ \omega C_1^* \frac{\alpha_F}{C_{F1}} \tau_{X1}^{PCP} \right\} \\ -\mathbb{I}^{DCP} \cdot \left\{ \mathbb{E}_0 \left[ \sum_{t=0}^{2} \beta^t \omega C_t^* \frac{\alpha_F}{C_{Ft}} \tau_{Xt}^{DCP} \right] \frac{1}{\beta \pi_1} \left( -\frac{E_1}{P_X} \frac{\partial P_X}{\partial E_1} \right) \right\} \end{matrix}}_{\text{Optimize TOT on export goods}}$$
$$+ \underbrace{\frac{\Psi_{B1}}{\beta \left[ (1+i_0^*) + (1-\lambda) \tau_{\Gamma 1} \right]} \kappa_{H1} \frac{P_H}{E_1}}_{\text{Relax bank constraint}} \pm \underbrace{\frac{1}{\beta \pi_1} \Lambda E_1 \left[ (1+i_0^*) + \tau_{\Gamma 1} \right]}_{\text{Prevent excessive contingency of exchange rate}}$$
$$+ \mathbb{E}_0 \left[ \Psi_{R1} \left\{ \underbrace{E_1^s \frac{\partial \chi_1}{\partial E_1^s} \left[ \left(1+i_{-1}^*\right) B_{R0}^{Linear} - \widehat{P}_{R0} L_{-1} \right]}_{\text{Inherited housing debt relative to rent and land price}} + \underbrace{E_1^s \frac{\partial \left( \chi_1 \widehat{q}_0 \right)}{\partial E_1^s} \left( L_0 - L_{-1} \right)}_{\text{Effect on period-0 land price}} \right\} \right]. \tag{49}$$

The last term is new and, since $\frac{\partial \chi_1^s}{\partial E_1^s} < 0$ and $\frac{\partial \chi_1^s}{\partial E_1^{-s}} > 0$, indicates that the planner finds it optimal to depreciate the exchange rate in period-1 states in which the housing constraint binds, relative to those period-1 states in which it does not bind. We explained above that such a depreciation raises rents and the land price. Another way to view the same mechanism is that there is a reduction in the ratio of inherited debt to period-1 rents and the land price, and this view is captured in the above expression. The distortion from using the exchange rate to support the housing sector is a reduction in $\tau_{H1}$, indicating positive price and AD pressures.

The new Euler condition between periods 0 and 1 is:

$$\begin{aligned}
&\frac{\alpha_F}{P^*_{F0}C_{F0}}\left[1+\frac{\alpha_H}{\alpha_F}\tau_{H0}\right]-\mathbb{I}^{DCP}\cdot\left\{\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\frac{1}{P^*_{F0}}\frac{1}{P_X}\frac{\partial P_X}{\partial C_{F0}}\right\}\\
&=\beta\mathbb{E}_0\left[\frac{\alpha_F\left[(1+i^*_0)+(1-\lambda)\,\tau_{\Gamma 1}\right]}{P^*_{F1}C_{F1}}\left[1+\frac{\alpha_H}{\alpha_F}\tau_{H1}\right]\right]\\
&-\mathbb{I}^{DCP}\cdot\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\mathbb{E}_0\left[\frac{(1+i^*_0)+(1-\lambda)\,\tau_{\Gamma 1}}{\pi_1 P^*_{F1}}\frac{1}{P_X}\frac{\partial P_X}{\partial C_{F1}}\right]\\
&+\Gamma_0\Omega_0+\mathbb{E}_0\left[\Psi_{R1}\left\{\begin{array}{c}
\left(\frac{1}{P^*_{F0}}\frac{\partial\chi_1}{\partial C_{F0}}-\mathbb{E}_0\left[\frac{(1+i^*_0)+(1-\lambda)\tau_{\Gamma 1}}{P^*_{F1}\pi_1}\right]\frac{\partial\chi_1}{\partial C^s_{F1}}\right)\left[\left(1+i^*_{-1}\right)B^{Linear}_{R0}-\widehat{P}_{R0}L_{-1}\right]\\
-\mathbb{E}_0\left[\frac{(1+i^*_0)+(1-\lambda)\tau_{\Gamma 1}}{P^*_{F1}\pi_1}\right]\frac{\partial(\chi_1\widehat{q}_0)}{\partial C^s_{F1}}\left(L_0-L_{-1}\right)-\chi_1\frac{1}{P^*_{F0}}\frac{\partial\widehat{P}_{R0}}{\partial C_{F0}}L_{-1}\\
-\frac{(1+i^*_0)+(1-\lambda)\tau_{\Gamma 1}}{P^*_{F1}\pi_1}\left\{-\frac{\partial\widehat{P}_{R1}}{\partial C_{F1}}L_0+\frac{\partial\widehat{q}_1}{\partial C_{F1}}\left(\left(1-\kappa_{L1}\right)L_1-L_0\right)\right\}
\end{array}\right\}\right],
\end{aligned}\tag{50}$$

and the new formula for the ex ante capital control tax, $\varphi_0$, is:

$$\varphi_0=1-\frac{\frac{(1+i^*_0)+\mathbb{E}_0\tau_{\Gamma 1}}{\mathbb{E}_0\left[\frac{E_0}{E_1}\right]}\mathbb{E}_0\left[\frac{E_0}{E_1}\frac{\alpha_F}{P^*_{F1}C_{F1}}\right]\left\{\begin{array}{c}\frac{\alpha_F}{P^*_{F0}C_{F0}}\left[1+\frac{\alpha_H}{\alpha_F}\tau_{H0}\right]\\
-\mathbb{I}^{DCP}\cdot\left\{\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\frac{1}{P^*_{F0}}\frac{1}{P_X}\frac{\partial P_X}{\partial C_{F0}}\right\}\end{array}\right\}}{\frac{\alpha_F}{P^*_{F0}C_{F0}}\left\{\begin{array}{c}
\mathbb{E}_0\left\{\frac{(1+i^*_0)+(1-\lambda)\tau_{\Gamma 1}}{(1+i^*_0)}\frac{\alpha_F}{P^*_{F1}C_{F1}}\left[1+\frac{\alpha_H}{\alpha_F}\tau_{H1}\right]\right\}+\Gamma\Omega_0\\
-\mathbb{I}^{DCP}\cdot\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\mathbb{E}_0\left[\frac{(1+i^*_0)+(1-\lambda)\tau_{\Gamma 1}}{\pi_1 P^*_{F1}}\frac{1}{P_X}\frac{\partial P_X}{\partial C_{F1}}\right]\\
+\mathbb{E}_0\left[\Psi_{R1}\left\{\begin{array}{c}
\left(\frac{1}{P^*_{F0}}\frac{\partial\chi_1}{\partial C_{F0}}-\mathbb{E}_0\left[\frac{(1+i^*_0)+(1-\lambda)\tau_{\Gamma 1}}{P^*_{F1}\pi_1}\right]\frac{\partial\chi_1}{\partial C^s_{F1}}\right)\\
\times\left[\left(1+i^*_{-1}\right)B^{Linear}_{R0}-\widehat{P}_{R0}L_{-1}\right]\\
-\mathbb{E}_0\left[\frac{(1+i^*_0)+(1-\lambda)\tau_{\Gamma 1}}{P^*_{F1}\pi_1}\right]\frac{\partial(\chi_1\widehat{q}_0)}{\partial C^s_{F1}}\left(L_0-L_{-1}\right)-\chi_1\frac{1}{P^*_{F0}}\frac{\partial\widehat{P}_{R0}}{\partial C_{F0}}L_{-1}\\
-\frac{(1+i^*_0)+(1-\lambda)\tau_{\Gamma 1}}{P^*_{F1}\pi_1}\left\{-\frac{\partial\widehat{P}_{R1}}{\partial C_{F1}}L_0+\frac{\partial\widehat{q}_1}{\partial C_{F1}}\left(\left(1-\kappa_{L1}\right)L_1-L_0\right)\right\}
\end{array}\right\}\right]
\end{array}\right\}}.\tag{51}$$

This expression captures how the inclusion of the housing sector alters the optimal capital controls. First, the formula reveals how capital controls interact with the policy rate. We know that reducing the policy rate in period 0 (via increasing $C_{F0}$, with $\frac{\partial \chi_1}{\partial C_{F0}} < 0$) and in the period-1 state when the constraint binds (via increasing $C_{F1}$ in that state, with $\frac{\partial \chi_1}{\partial C_{F1}^s} > 0$) help relax the housing constraint. The formula shows that for given UIP wedges, such policy rate reductions reduce the necessary ex ante capital controls. Second, the formula includes terms related to $\frac{\partial \widehat{P}_{Rt}}{\partial C_{Ft}}$ and $\frac{\partial \widehat{q}_1}{\partial C_{F1}}$, which establish that capital controls should shift consumption intertemporally in order to bolster rents and house prices in period-1 states when the housing constraint is binding.

Next, we consider the allocations if consumer macroprudential taxes are allowed but capital controls are not. The FOCs for the exchange rate and FX intervention, given by equations (34) and (36), remain unchanged relative to section 4. However, the expression (35) summarizing the trade-off for the optimal UIP wedge between periods 0 and 1, $\tau_{\Gamma 1}$, is altered to the following:

$$\underbrace{\Omega_0}_{\text{Ability to borrow more today}} = \underbrace{(1-\lambda)\left(\Phi_1 + \Psi_{B1}\right)\frac{B_1 + FXI_0}{I_0}}_{\text{Higher repayments tomorrow}} \pm \underbrace{\frac{1}{\pi_1}\Lambda E_1}_{\text{Prevent excessive contingency of premium}}$$
$$+ \underbrace{(1-\lambda)\,\Omega_1 \Gamma\left(B_1 + FXI_0\right)}_{\text{Higher premium tomorrow owing to rollover needs}} + \underbrace{\Psi_{R1}\left[\left(1+i_{-1}^*\right)B_{R0}^{Linear} - \widehat{P}_{R0}L_{-1}\right]}_{\text{Higher inherited housing debt for linear subsector}}. \tag{52}$$

The last term is new and indicates that the planner finds it optimal to reduce the UIP wedge, i.e., depreciate the exchange rate, in period-1 states in which the housing constraint binds. The distortion from using the exchange rate to support the housing sector is an increase in $\Omega_0$, indicating that international financial intermediaries are less willing to finance domestic currency debt in period 0. To restore the attractiveness of the debt, the planner can commit to appreciate the exchange rate in period-1 states in which the housing constraint does not bind, distorting allocations in those states as well.

The trade-off for the optimal UIP wedge between periods 1 and 2, $\tau_{\Gamma 2}$, is altered to the following:

$$\underbrace{\Omega_1}_{\text{Ability to borrow more today}} = \underbrace{(1-\lambda)\,\Phi_1 \frac{B_2 + FXI_1}{I_0 I_1}}_{\text{Higher repayments tomorrow}} + \underbrace{\frac{1}{\pi_1}\mathbb{E}_0\left[\Psi_{R1}\frac{\partial\left(\chi_1 \widehat{q}_0\right)}{\partial \eta_2^s}\left(L_0 - L_{-1}\right)\right]}_{\text{Reduction in period-0 land price}} + \underbrace{\Psi_{R1}\frac{\partial \widehat{q}_1}{\partial \eta_2}\left(\left(1-\kappa_{L1}\right)L_1 - L_0\right)}_{\text{Lower period-1 land price, tighter constraint}}. \quad (53)$$

The last two terms are again new. The terms capture the impact of the UIP wedge on the constraint via the period-0 and period-1 land prices.

Constrained efficient allocations with consumer macroprudential taxes instead of capital controls follow the Euler condition (51), but some of the derivatives inside the expression take different values.[19] In particular, the borrowing rate for households and the housing sector are no longer connected, so we need to impose that $\frac{\partial \chi_1}{\partial C_{F0}} = \frac{\partial \chi_1}{\partial C_{F1}^s} = 0$. Nevertheless, we preserve the result that consumption levels can be altered (now via consumer macroprudential taxes instead of capital controls) in order to bolster rents and house prices in period-1 states when the housing constraint is binding. The value of the ex ante consumer macroprudential tax is obtained by setting $\varphi_0 = 0$ in the period-0 version of equation (23).

## 5.2 Spillover of Housing and External Constraints

In this subsection, we explain the connection between housing constraints, external constraints, and the pricing paradigm in an open-economy context. Many countries, especially emerging markets, may find themselves vulnerable to two kinds of occasionally-binding borrowing constraints: domestic and external. This topic has been the subject of both theoretical and empirical work (e.g., Caballero and Krishnamurthy, 2001, Kaminsky and Reinhart, 1999). Domestic borrowing constraints typically feature domestic-currency-denominated debt which may be collateralized using domestic nontradable assets such as housing, so the constraints become relaxed as the policy rate is reduced and the exchange rate depreciates. External borrowing constraints typically feature dollar-denominated debt

[19] For more details on the derivatives, please see Appendix A.2.

which may be collateralized using some element of domestic production, so the constraints become tighter as the exchange rate depreciates and the dollar value of domestic collateral declines.

Our first lemma establishes that ignoring the external constraint, it is optimal to depreciate the exchange rate to help loosen the housing constraint.

**Lemma 4.** *If $\Psi_{B1} = 0$ for every period-1 state, it is optimal to depreciate the exchange rate in period-1 states in which the housing constraint binds, i.e., $\Psi_{R1} > 0$, relative to those period-1 states in which it does not bind, i.e., $\Psi_{R1} = 0$. Ex post exchange rate flexibility reduces ex ante housing macroprudential taxes and may make them unnecessary.*

The importance of exchange rate depreciation was described above in our discussion of equation (49). In domestic currency terms, a depreciation relaxes the constraint in two ways. First, it stimulates substitution of consumption from imports to home tradable and nontradable goods, the latter of which includes housing services. This substitution effect increases period-1 rents, which reduces the tightness of the borrowing constraint of the linear housing subsector. Second, it increases the period-1 land price. The reason is that if we fix the long-term relative price of housing to imports (which is implied by monetary neutrality), a depreciation in the exchange rate increases the domestic currency price of land in period 2, which filters back to a higher domestic currency price in period 1 as well. The higher price increases the domestic debt limit of the linear housing subsector.

In FX terms, these two effects can be summarized in a simple manner: there is a reduction in the ratio of inherited debt to period-1 rents and the land price, which loosens the housing constraint. This is the view reflected in the derivatives $\frac{\partial \chi_1^s}{\partial E_1^s} < 0$ and $\frac{\partial \chi_1^s}{\partial E_1^{-s}} > 0$ in equation (49), which establishes that the planner finds it optimal to depreciate the exchange rate in period-1 states in which the housing constraint binds, relative to those period-1 states in which it does not bind. The equation warns that the distortion from using the exchange rate to support the housing sector is a reduction in $\tau_{H1}$, indicating positive price and AD pressures.

To understand the impact of depreciation on the ex ante housing macroprudential tax, consider a shock which causes the housing constraint to bind. Figure 10 shows how the importance of exchange rate flexibility in a DCP economy after such a shock: a decrease in the value of $\kappa_{L1}$, the pledgability parameter in the linear housing subsector's borrowing constraint when they borrow from domestic banks.[20]

The dashed lines show the allocations when the exchange rate is pegged and no policy instruments are available beyond the housing macroprudential tax. In this case, there is a positive hedging motive after the low realization of the debt limit shock, because the shock reduces the land price as it tightens the constraint. As expected, the planner optimally imposes an ex ante housing macroprudential tax to address the pecuniary production externality, reflected in the positive housing wedge.

The solid line in the figure shows that exchange rate flexibility can be strikingly effective. In this case, the planner allows the exchange rate to depreciate after the low realization of the shock in order to relax the constraint: the decline in $\kappa_{L1}$ is partially offset by an increase in the domestic currency value of rents and the land price. This ex post exchange rate depreciation may result in the domestic currency rents and land price being similar across period-1 states, or even being *higher* after the low shock. If so, the hedging motive disappears, and we may obtain the counterintuitive result that an ex ante housing macroprudential tax is not necessary to address a domestic housing sector shock. In the simulation plotted, the optimal value of this tax actually hits its lower bound of zero.[21]

How capital controls and FX intervention should be used depends on how the desired exchange rate depreciation compares to the planner's desired policy rate reduction to support the housing sector. As described in the previous subsection regarding equations (50)-(51), the planner can relax the housing constraint by reducing the policy rate and domestic borrowing rate, which raises the land price via the no-arbitrage condition of the concave housing subsector. This channel is distinct from the exchange rate channel. If the desired policy rate reduction is so large that the associated depreciation exceeds the desired level, then the Gamma equations (15)-(16) indicate that capital inflow subsidies and/or FX sales should be used to contain the depreciation. If the desired policy rate

[20]Our main results on the housing constraint are robust to the pricing paradigm. We explain below how the pricing paradigm affects the transmission channel between external and housing constraints.

[21]The upper bound on land use in the linear subsector, i.e., $L_0^{Linear} \leq 1$, corresponds to a lower bound on the ex ante housing macroprudential tax, i.e., $\theta_{R0}^{Linear} \geq 0$.

reduction is so small that the associated depreciation falls short of the desired level, then capital inflow taxes and/or FX accumulation are appropriate instead.

Figure 11 provides the allocations when more policy instruments are available. The solid lines in the figure show the allocations when all instruments are available except the consumer macroprudential tax. In addition to the exchange rate depreciation, the policy rate is reduced after the debt limit shock, while the ex ante housing macroprudential tax continues to be zero. The monetary loosening comes at the cost of a lower AD wedge, indicating positive price and AD pressures. The policy rate reduction is large, so capital inflow subsidies are optimally used. $(B_2 + FXI_1)$ is kept at zero in both period-1 states, establishing that FX intervention is used solely to minimize external premia and not otherwise to influence the exchange rate.

The dashed lines show the allocations if consumer macroprudential taxes are allowed but capital controls are not. Consumer macroprudential taxes allow a lower ex ante policy rate, which reduces the interest burden and relaxes the housing constraint so much that the ex post policy rate increases substantially. As a result, for the specific parameterization in the simulation, there is no longer an excessive depreciation in the period-1 state when the constraint binds. Instead, the depreciation is insufficient relative to the level which optimally balances the import/export substitution margin against housing sector support. Therefore, the planner accumulates FX, i.e., $(B_2 + FXI_1) > 0$ to further depreciate the exchange rate. The more limited ex post support for the housing market means that the ex ante housing macroprudential tax does actually rise above zero.

Having established that exchange rate depreciation is part of the optimal policy response to the domestic housing constraint, we turn next to the possibility that it could adversely affect the external FX debt constraint.

**Proposition 4.** *If initial external FX debt $B_0 > 0$, exchange rate depreciation in a period-1 state when the housing constraint binds, i.e., $\Psi_{R1} > 0$, can make the external constraint bind in that state, i.e., $\Psi_{B1} > 0$. If so, the ex post exchange rate should be set to the level that balances the tightness of domestic and external constraints. Prudential capital controls are optimal, while it may or may not be* *necessary to impose ex ante housing macroprudential taxes.*

Depreciations pose no problems for countries with no initial unhedged external FX debt, i.e., $B_0 = 0$, but they may cause external constraints to bind for countries with high initial unhedged external FX debt, i.e., $B_0 > 0$.

Figure 12 shows the allocations in a DCP economy with high initial FX debt, where the depreciation to loosen the housing constraint causes the external constraint to bind. In this case, the planner finds it optimal to relax the banks' external debt limit by limiting the depreciation in that state, even at the expense of tightening the housing constraint. Ex post capital controls do not work in the binding state. Ex post FX intervention should be used to absorb external premia but not otherwise to defend the exchange rate: the planner sets $(B_2 + FXI_1)$ to zero. Once the banks' debt limit binds, the interest rate $\rho_1$ in the households' Euler condition and the housing sector's no-arbitrage condition becomes disconnected from the policy rate $i_1$, so the latter can be used to manage the exchange rate.

The limited room for manuever ex post enhances the case for ex ante policy actions. The planner sets a low policy rate in period 0 to reduce the interest burden on inherited debt for the housing sector before the shock hits, thereby mitigating the pecuniary production externality and the ex post housing wedge, and sets positive ex ante capital controls to limit external FX debt, thereby mitigating the pecuniary AD externality. Under PCP, the main results from DCP continue to hold, but the ex ante capital controls are lower for the same reason as in subsection 3.2.

As our final experiment, we consider the possibility of the reverse: a transmission from external to housing constraints.

**Proposition 5.** *If initial housing subsector debt $D_{R0}^{Linear} > 0$, an adverse shock to banks' external debt limits, i.e., to the value of $\kappa_{H1}$, may cause domestic housing constraints to bind. Prudential capital controls are higher under DCP than PCP, but ex ante housing macroprudential taxes may be lower under DCP than PCP.*

This time we begin with PCP and then proceed to DCP to highlight how the difference in exchange rate volatility under PCP and DCP that we identified in subsection 3.2 affects the transmission channel from the external to the domestic constraint.

Under PCP, the banks' debt limit shock may indeed cause the domestic housing constraint to bind. Figure 14 illustrates the mechanism. The binding external constraint is associated with a large decrease in the policy rate $i_1$ and an exchange rate depreciation which tends to increase the domestic currency value of rents and the land price. However, it is also associated with an increase in the borrowing rate $\rho_1$ for domestic households and the housing sector, and a decrease in household consumption. These factors tend to reduce rents and the land price. If the latter effects outweigh the former ones, as they do in our simulations, the housing constraint may bind.

The planner relaxes the housing constraint by reducing the policy rate and allowing more depreciation in that state, even at the expense of tightening the banks' external constraint. Ex post capital controls do not work in the binding state. Ex post FX intervention should be used to absorb external premia but not otherwise to defend the exchange rate. The limited room for manuever ex post enhances the case for ex ante policy actions, and the planner imposes an ex ante housing macroprudential tax.

Under DCP, we know from subsection 3.2 that after the banks' debt limit shock, the exchange rate is more depreciated and yet the external constraint is also more relaxed. Figure 13 illustrates that both of these factors alter the likelihood that the domestic housing constraint binds. The larger depreciation means that there is a larger boost to the domestic currency value of rents and the land price. The more relaxed external constraint means that the borrowing rate $\rho_1$ for domestic households and the housing sector is lower under DCP than PCP, which also supports rents and the land price.

As a result, it is less likely under DCP than PCP that the tightening of external constraints causes the domestic housing constraint to bind. Moreover, even when the housing constraint does bind, it is less severe. Therefore, while ex ante capital controls are larger under DCP than PCP, ex ante housing macroprudential taxes may be lower under DCP than PCP.

# 6 Conclusion

In this paper we built a model of a small open economy that features a range of real and nominal frictions highlighted in the literature. We used this framework to characterize the optimal use of monetary policy, capital controls, and FX intervention for several shocks as a function of frictions such as the currency of trade invoicing, initial debt levels, the severity of currency mismatches and borrowing constraints, and the depth of FX markets. We also showed preliminary results on the effects of external financial shocks on the domestic housing market and the use of domestic macro-prudential tools.

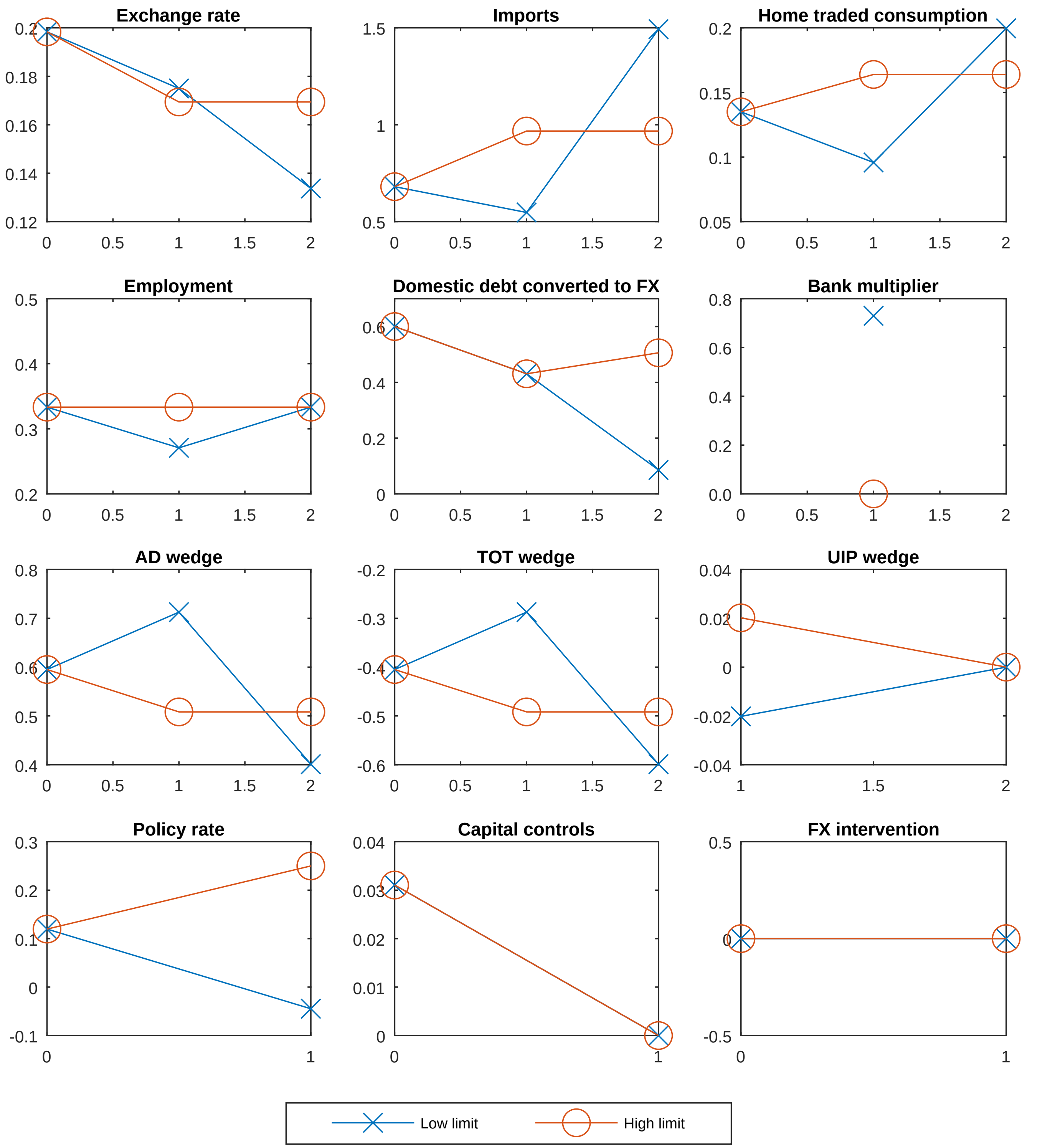
The joint consideration of policy instruments and externalities allows us to establish several results that are novel relative to the literature. Our framework helps determine whether policies which have been highlighted in the literature as being useful to minimize specific externalities after certain shocks can also be used to address other externalities. One result we have highlighted along this dimension is that capital controls may be beneficial in complementing FX intervention to address foreign appetite shocks to domestic currency debt.

We are also able to analyze whether policies which have been recommended to address specific externalities in the literature may exacerbate other externalities when economies suffer from multiple frictions. One result in that vein is that while higher exchange rate volatility under DCP than PCP may help generate import substitution effects to compensate for the lack of export substitution, the associated greater distortion of the exchange rate during crisis times causes higher prudential capital controls under DCP than PCP. Another result in this spirit is that while bans on open FX positions may be beneficial in reducing the vulnerability to debt limit shocks when FX markets are deep, they may generate side-effects in terms of steady-state interest rates and vulnerability to foreign appetite shocks when FX markets are shallow.

Finally, our integrated framework provides valuable guidance in real world policy making. The breadth of our framework allows it to be useful for a wide range of small open economies, including advanced economies and emerging markets. In addition, for complex phenomena such as the

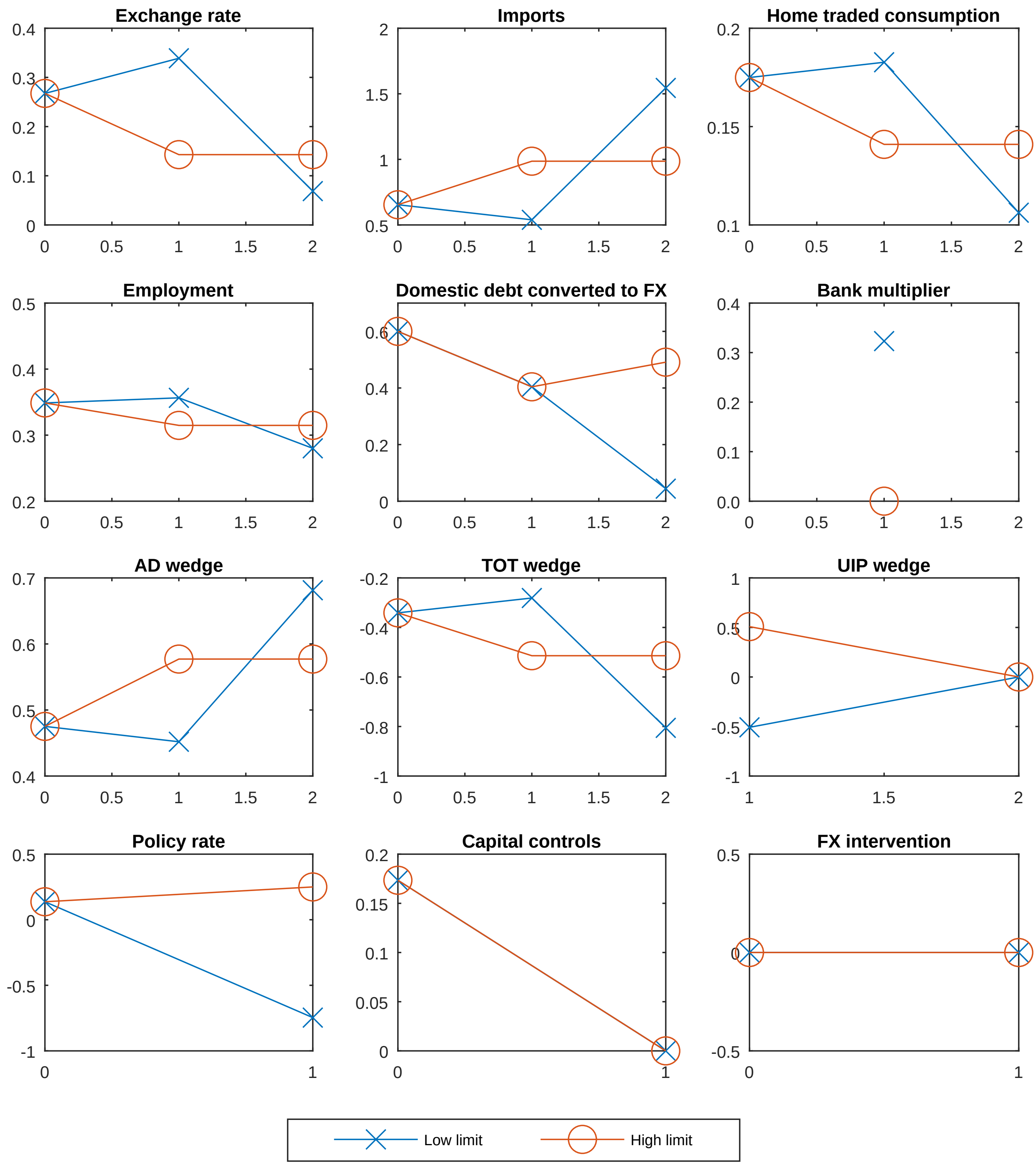
COVID-19 crisis that tend to have multiple underlying economic shocks—including to the world interest rate, commodity prices, FX debt limits, and foreign appetite for domestic currency debt—our framework can help pin down optimal monetary and financial policies for each underlying component, while also pointing out whether policies to address salient shocks today may ameliorate or exacerbate the vulnerabilities to other potential shocks in the future.

**Figure 4:** Debt Limit Shock under PCP with Deep FX Markets



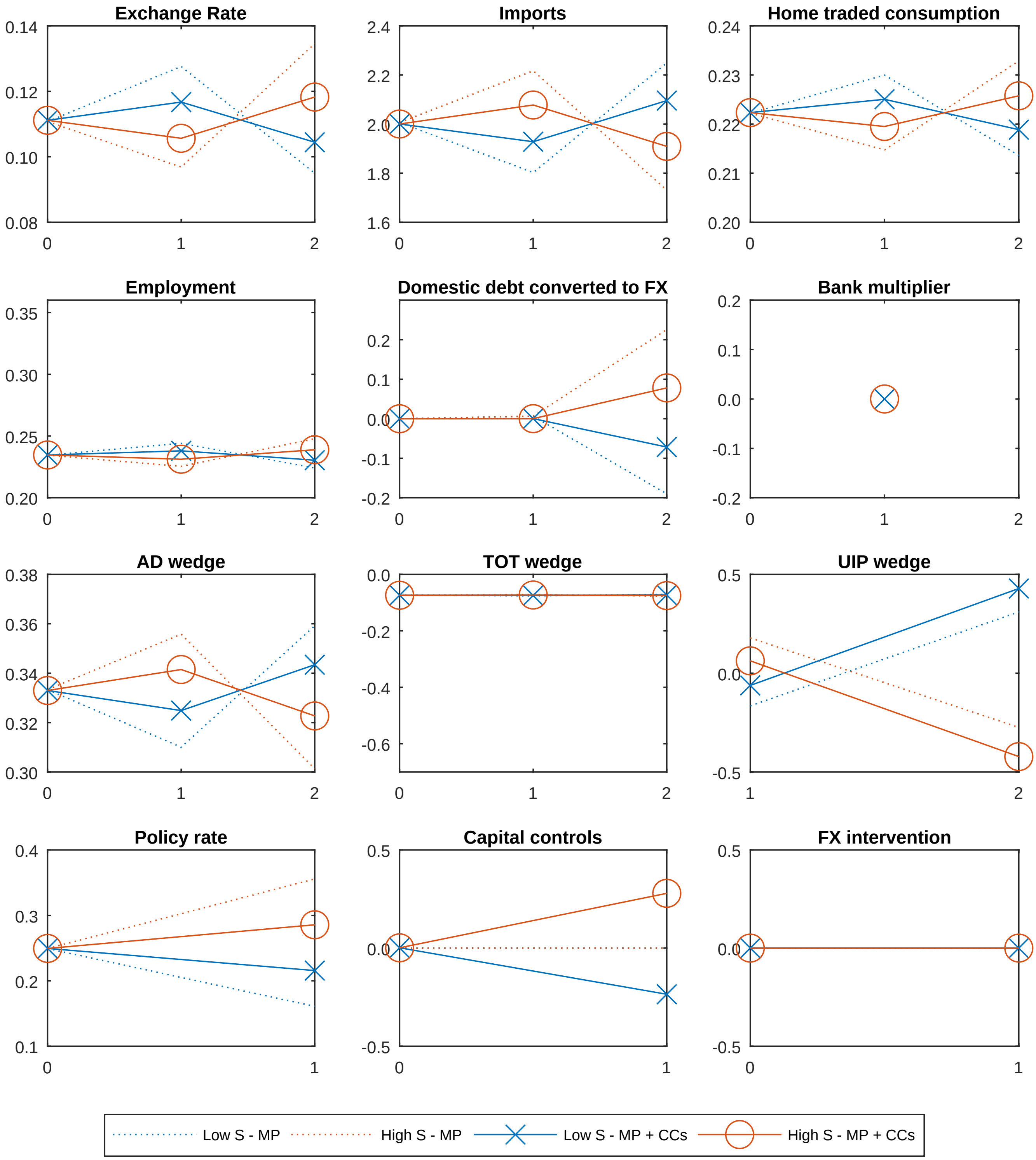

Notes: This figure plots the responses of key variables to a debt limit shock under under PCP with deep FX markets. The shock hits at date-1 and is calibrated as $\kappa \in [0.025, 10]$ such that the constraint binds in the case of a bad realization of the shock but not after a good realization.

**Figure 5:** Debt Limit Shock under DCP with Deep FX Markets



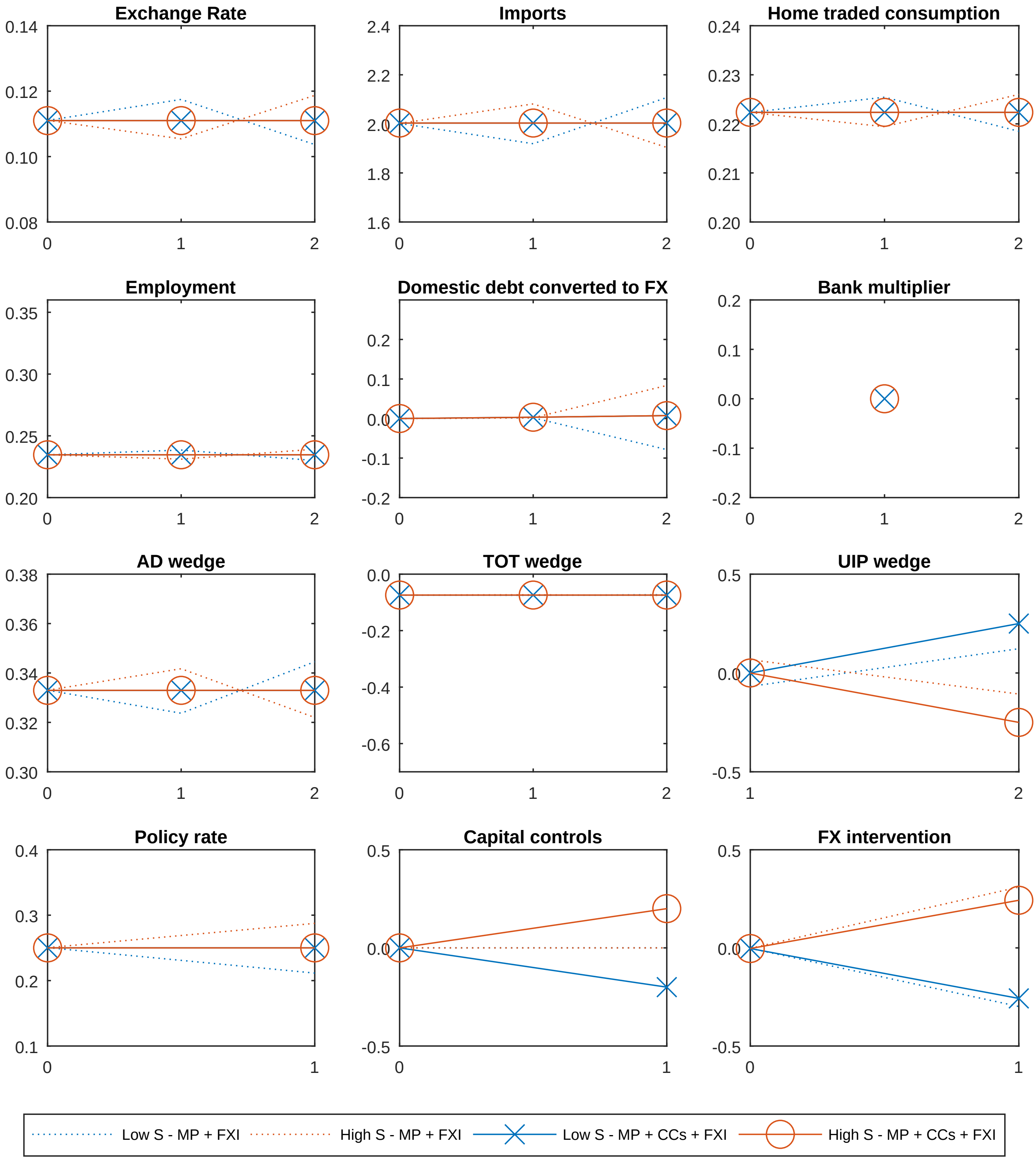

Notes: This figure plots the responses of key variables to a debt limit shock under under DCP with deep FX markets. The shock hits at date-1 and is calibrated as $\kappa \in [0.025, 10]$ such that the constraint binds in the case of a bad realization of the shock but not after a good realization.

**Figure 6:** Foreign Risk Appetite Shock under DCP with MP and CC



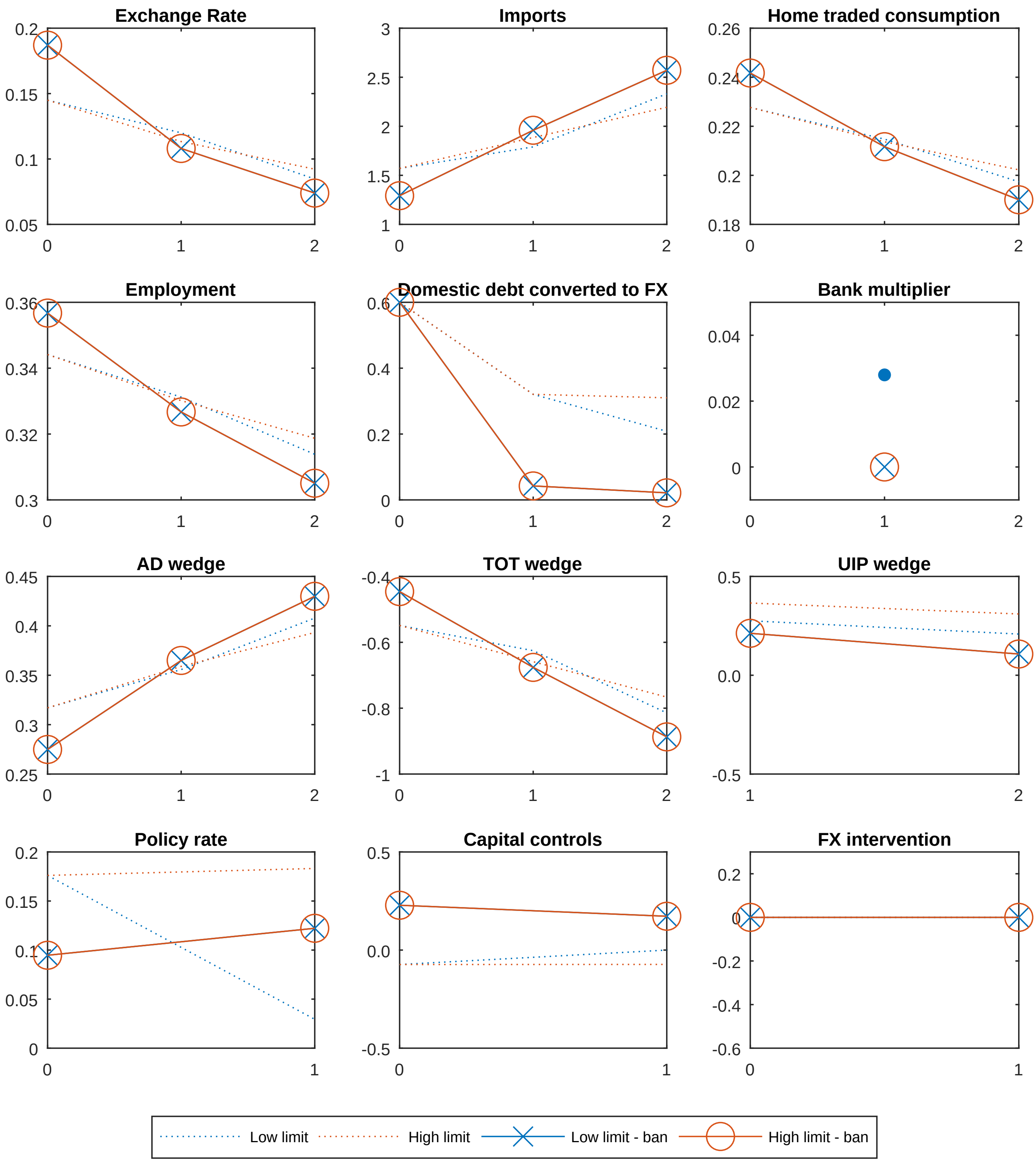

Notes: This figure plots the responses of key variables to a risk foreign appetite shock under DCP with shallow FX markets, monetary policy (MP) and capital controls (CC). The shock hits at date-1 and is calibrated as $S_1 \in [-0.5, 0.5]$.

**Figure 7:** Foreign Risk Appetite Shock under DCP with MP, CC, and FXI



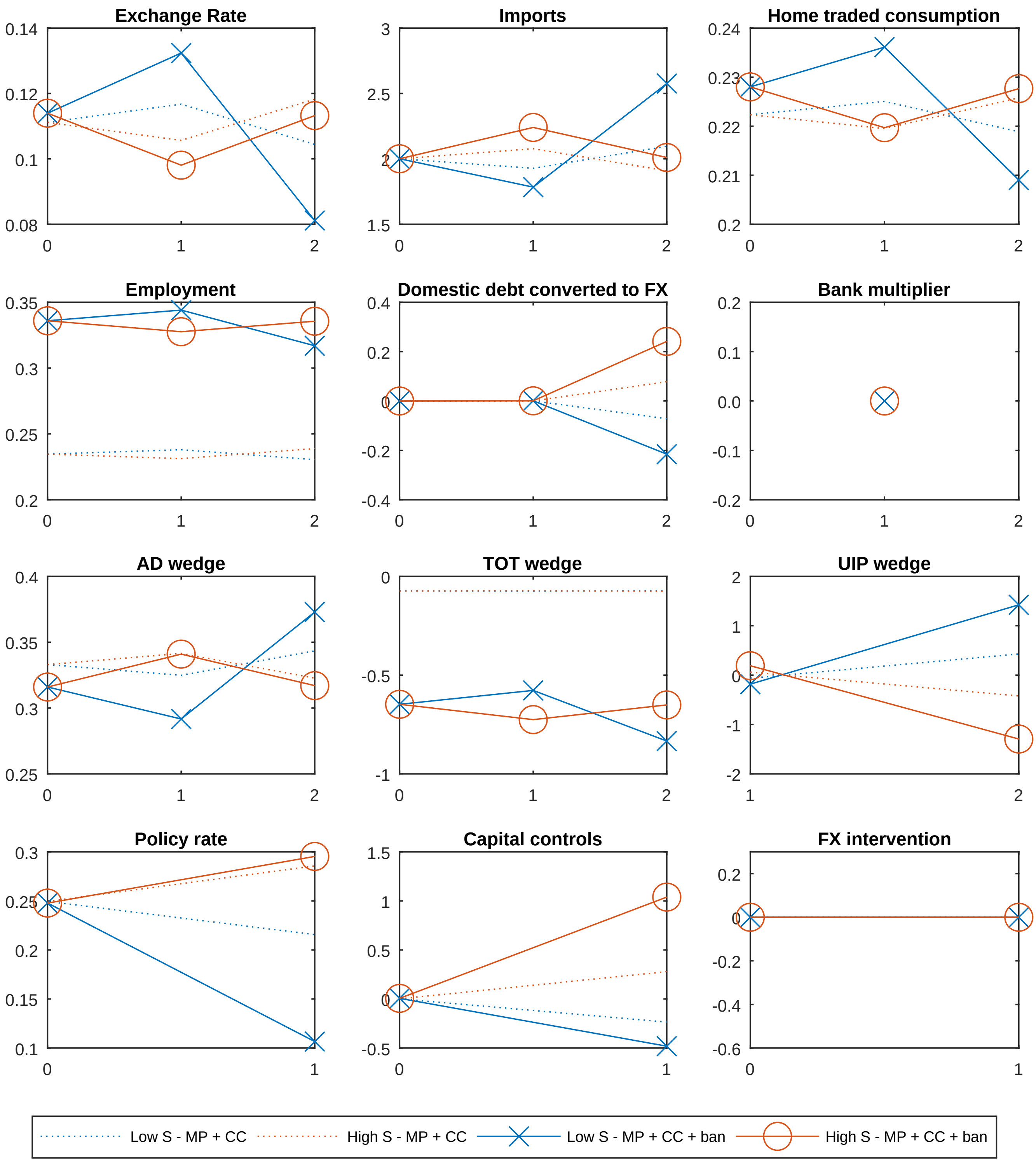

Notes: This figure plots the responses of key variables to a risk foreign appetite shock under DCP with shallow FX markets, monetary policy (MP), capital controls (CC), and FX intervention (FXI). The shock hits at date-1 and is calibrated as $S_1 \in [-0.5, 0.5]$.

**Figure 8:** Debt Limit Shock under DCP and Banning FX Exposures



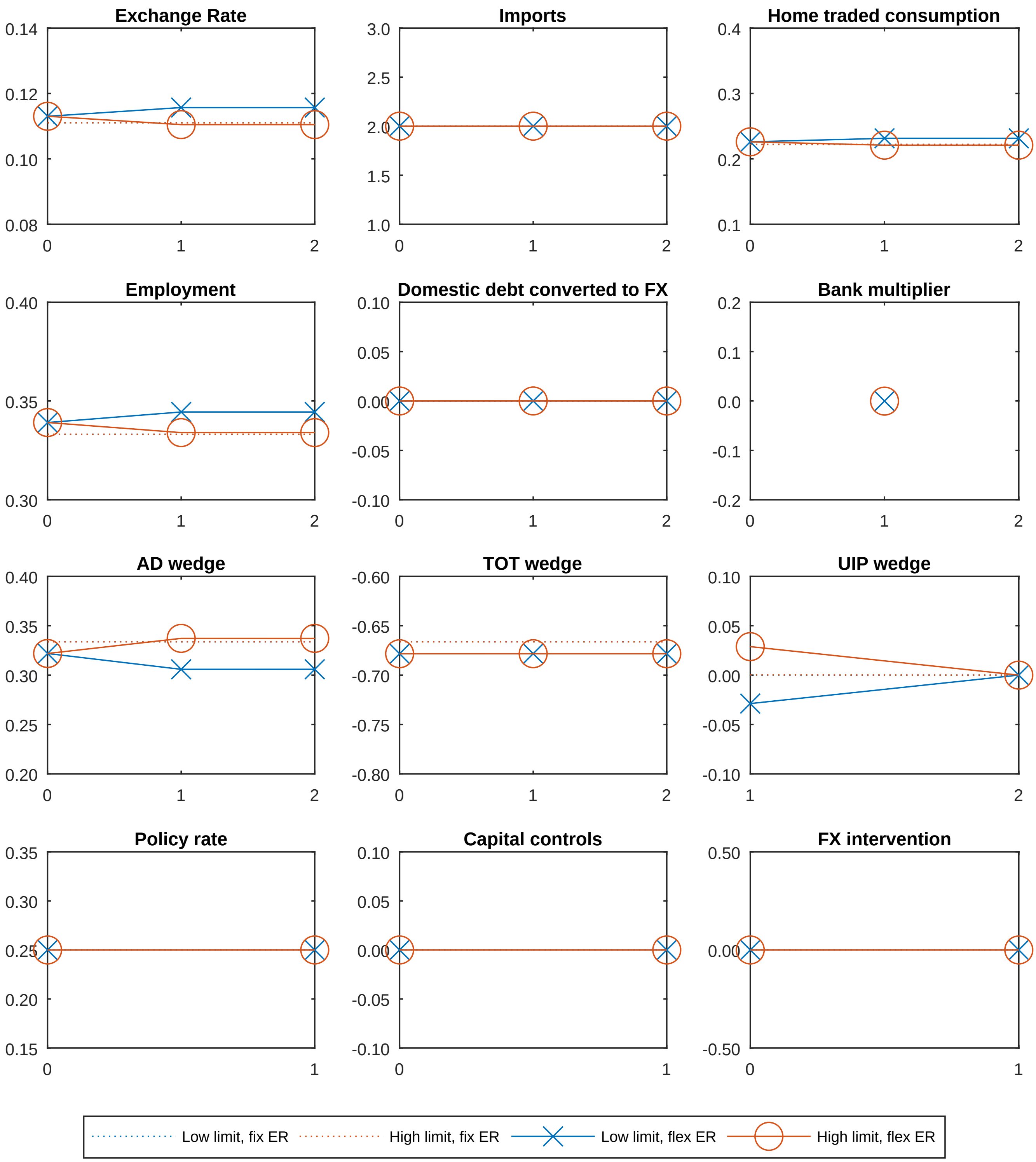

Notes: This figure plots the responses of key variables to a debt limit shock under DCP with shallow FX markets, banning of open FX positions. The shock hits at date-1 and is calibrated as $\kappa \in [0.025, 10]$ such that the constraint binds in the case of a bad realization of the shock but not after a good realization.

**Figure 9:** Foreign Risk Appetite Shock under DCP with MP, CC, and Banning FX Exposures



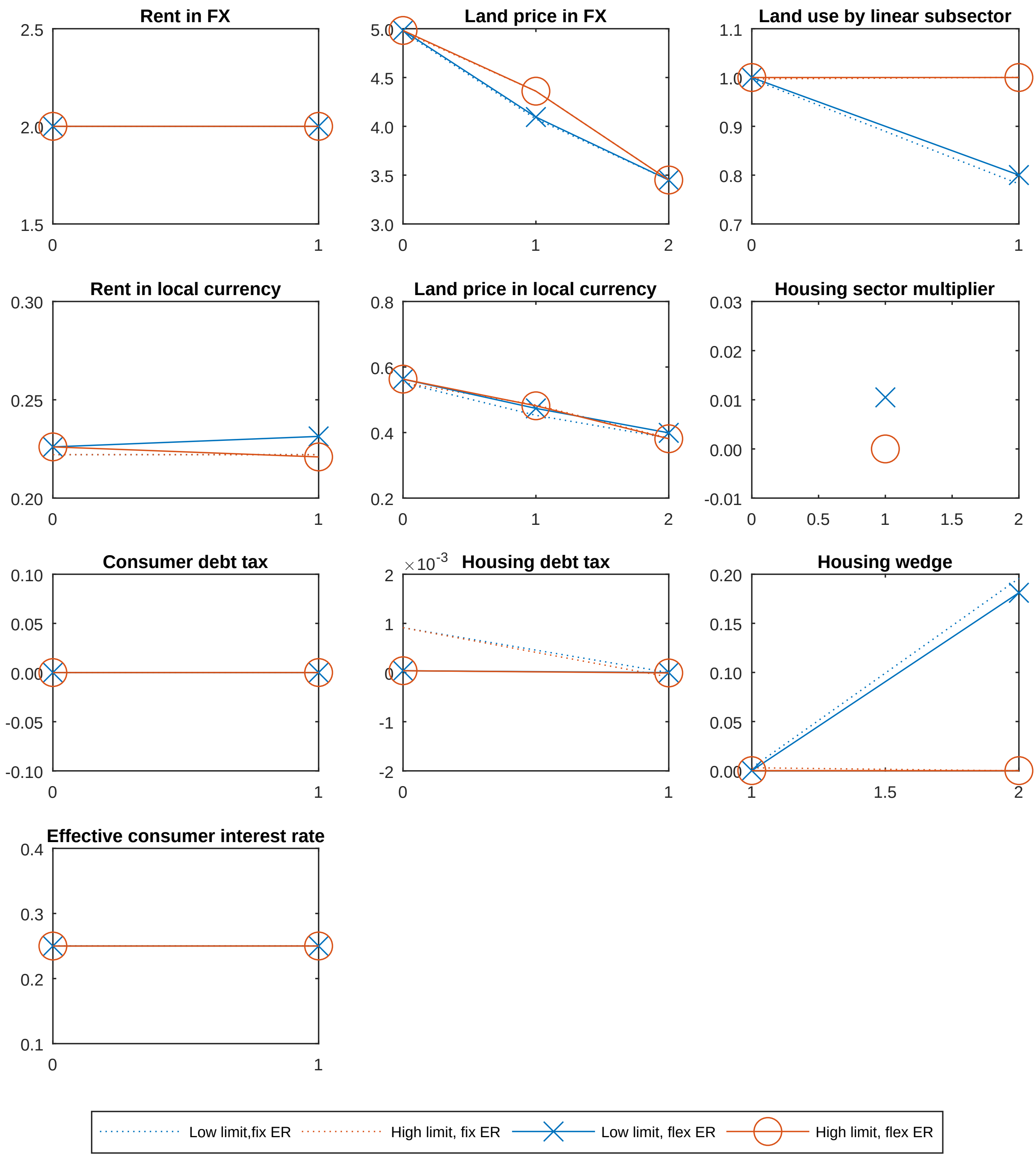

Notes: This figure plots the responses of key variables to a risk foreign appetite shock under DCP with shallow FX markets, monetary policy (MP), capital controls (CC) and banning of open FX positions. The shock hits at date-1 and is calibrated as $S_1 \in [-0.5, 0.5]$.

**Figure 10:** Housing Debt Limit Shock under DCP with Housing Sector Frictions



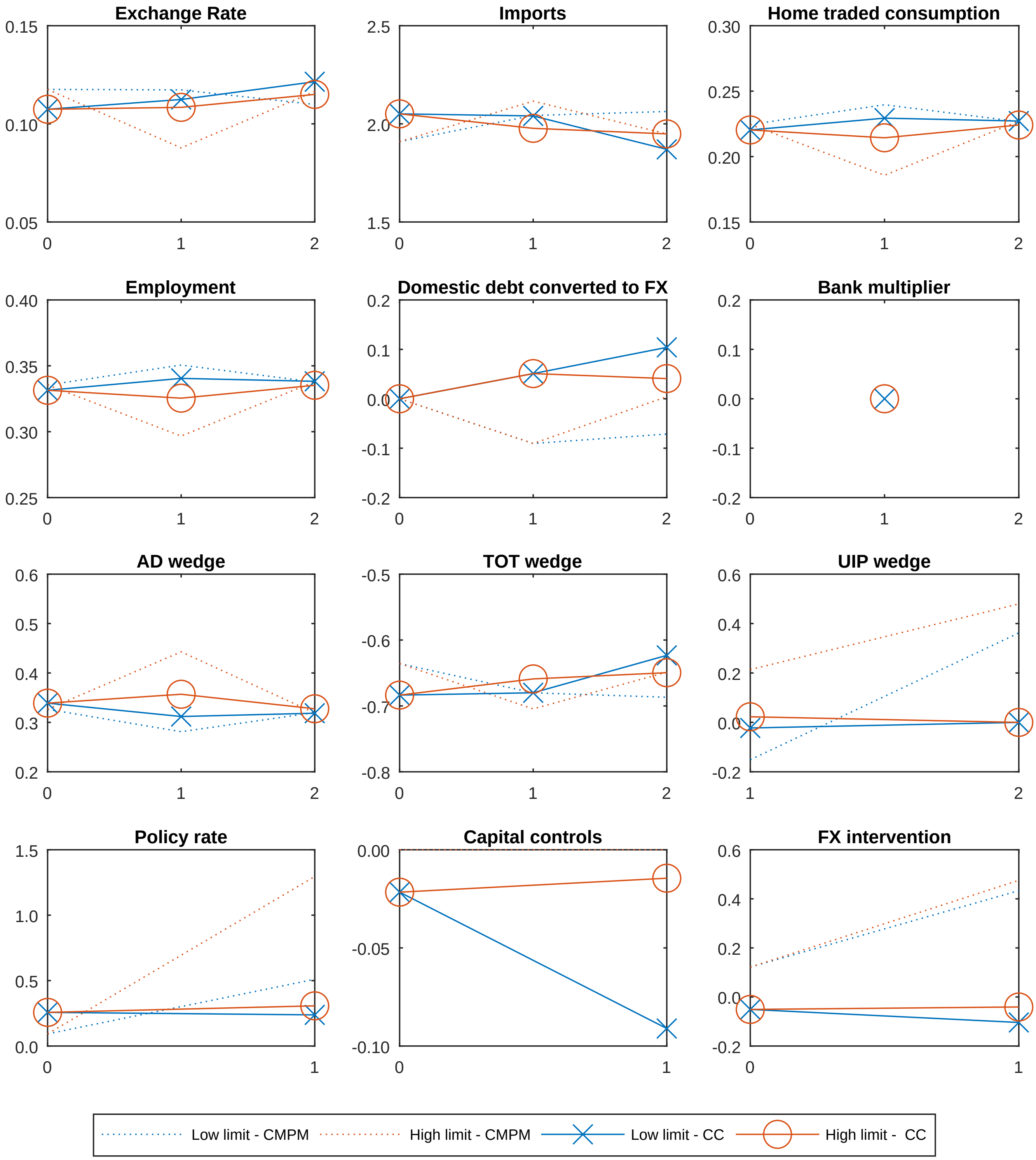

Notes: This figure plots the responses of key variables to a housing debt limit shock under DCP with shallow FX markets and housing sector frictions. Solid lines: housing debt taxes and flexible exchange rates; dashed lines: housing debt taxes and fixed exchange rates. The shock hits at date-1 and is calibrated as $\kappa_{L1} \in [0.025, 10]$.

**Figure 10:** Housing Debt Limit Shock under DCP with Housing Sector Frictions cont.



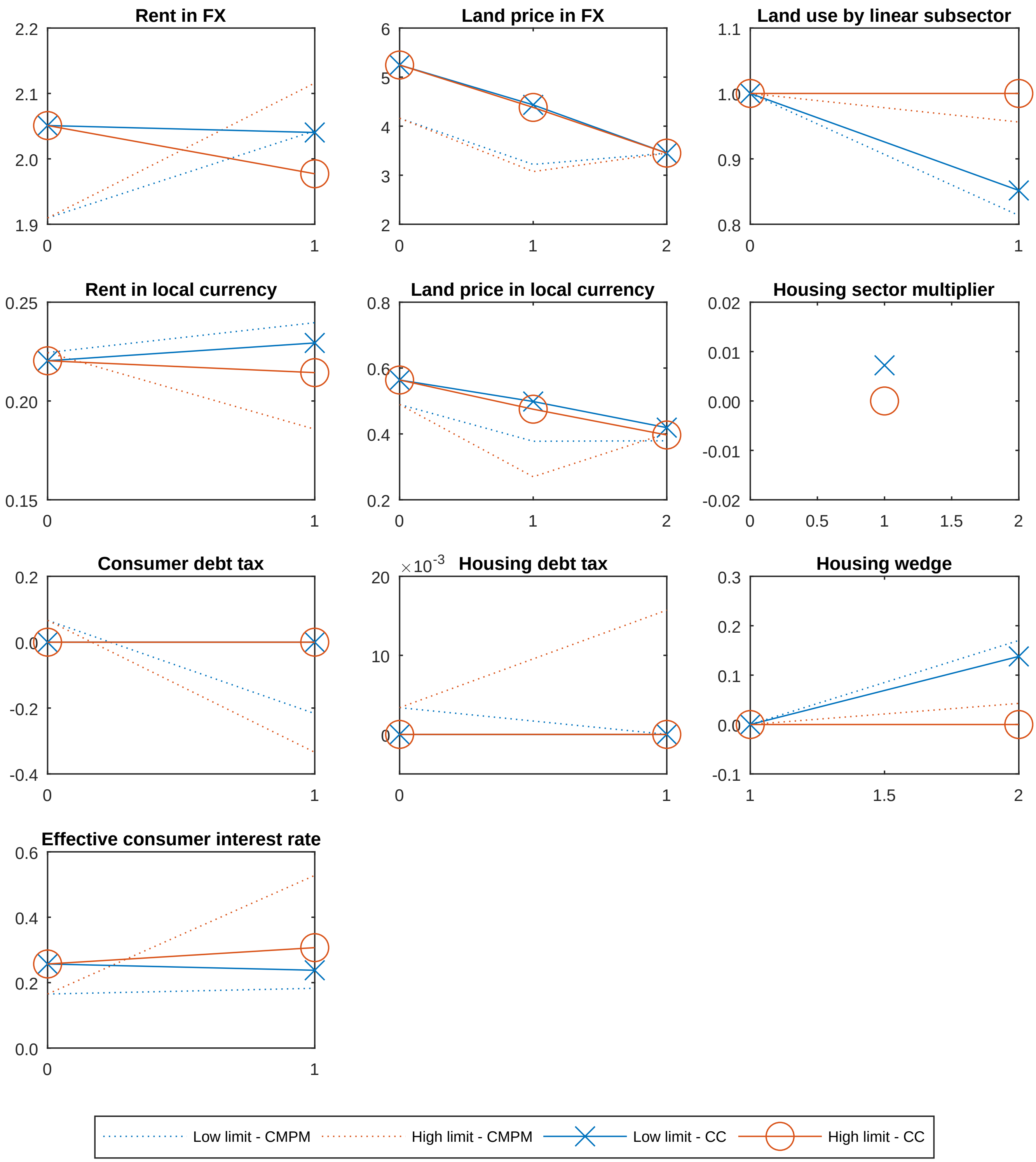

Notes: This figure plots the responses of key variables to a housing debt limit shock under DCP with shallow FX markets and housing sector frictions. Solid lines: housing debt taxes and flexible exchange rates; dashed lines: housing debt taxes and fixed exchange rates. The shock hits at date-1 and is calibrated as $\kappa_{L1} \in [0.025, 10]$.

**Figure 11:** Housing Debt Limit Shock under DCP with Housing Sector Frictions



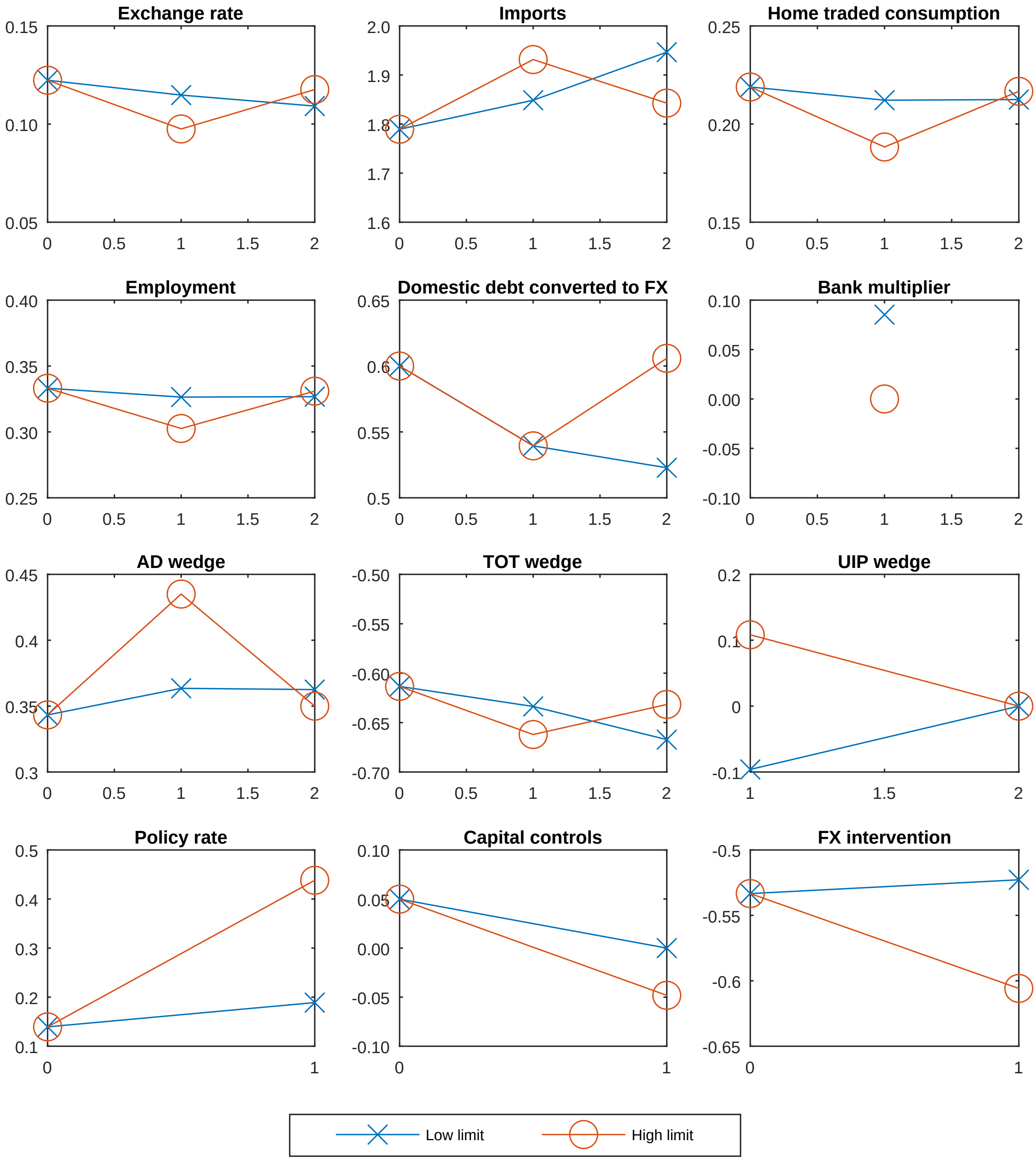

Notes: This figure plots the responses of key variables to a housing debt limit shock under DCP with shallow FX markets and housing sector frictions. Solid lines: capital controls; dashed lines: consumer debt taxes. The shock hits at date-1 and is calibrated as $\kappa_{L1} \in [0.025, 10]$.

**Figure 11:** Housing Debt Limit Shock under DCP with Housing Sector Frictions cont.



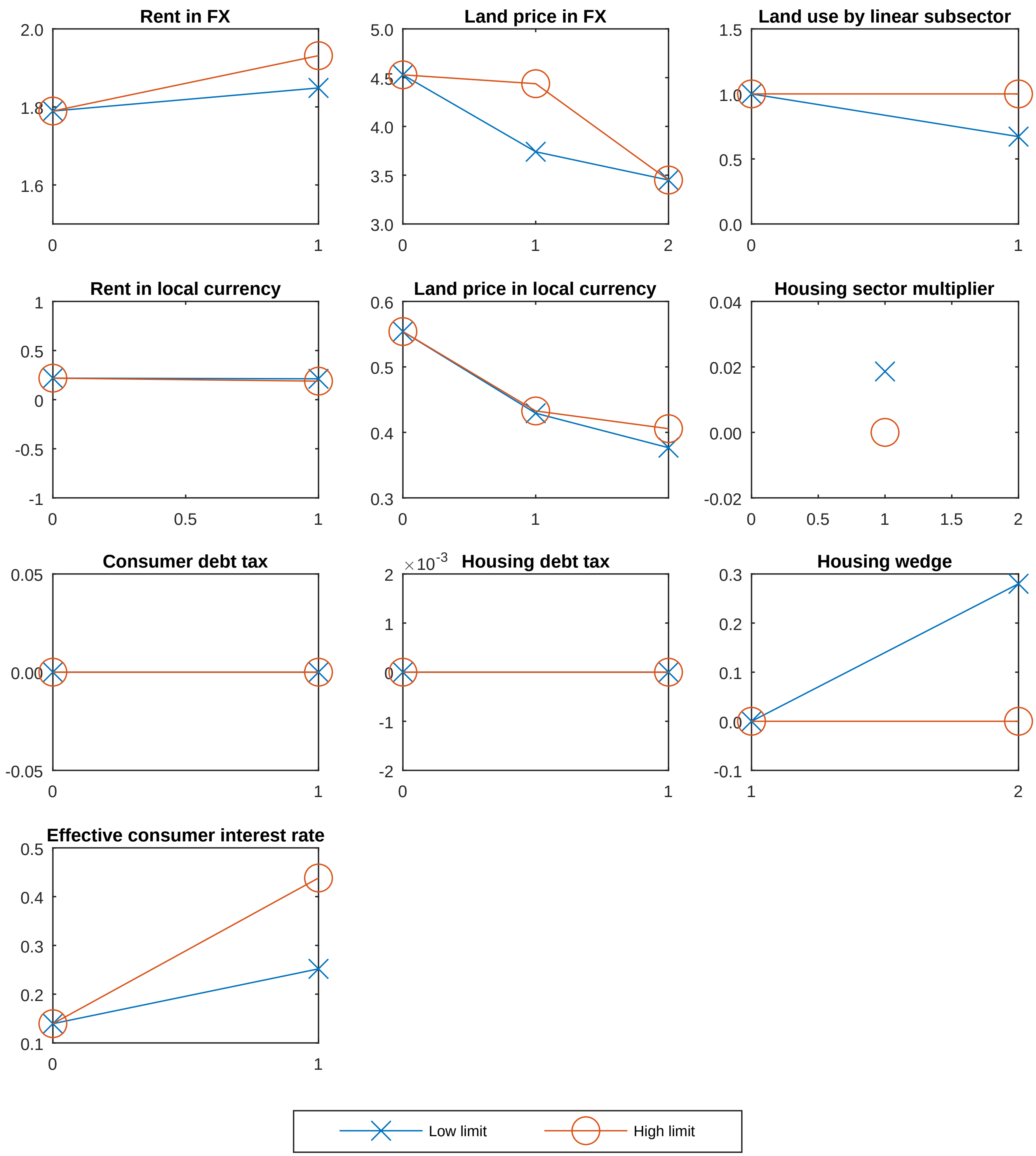

Notes: This figure plots the responses of key variables to a housing debt limit shock under DCP with shallow FX markets and housing sector frictions. Solid lines: capital controls; dashed lines: consumer debt taxes. The shock hits at date-1 and is calibrated as $\kappa_{L1} \in [0.025, 10]$.

**Figure 12:** Housing Constraint Spilling Over to Banks' Constraint under DCP



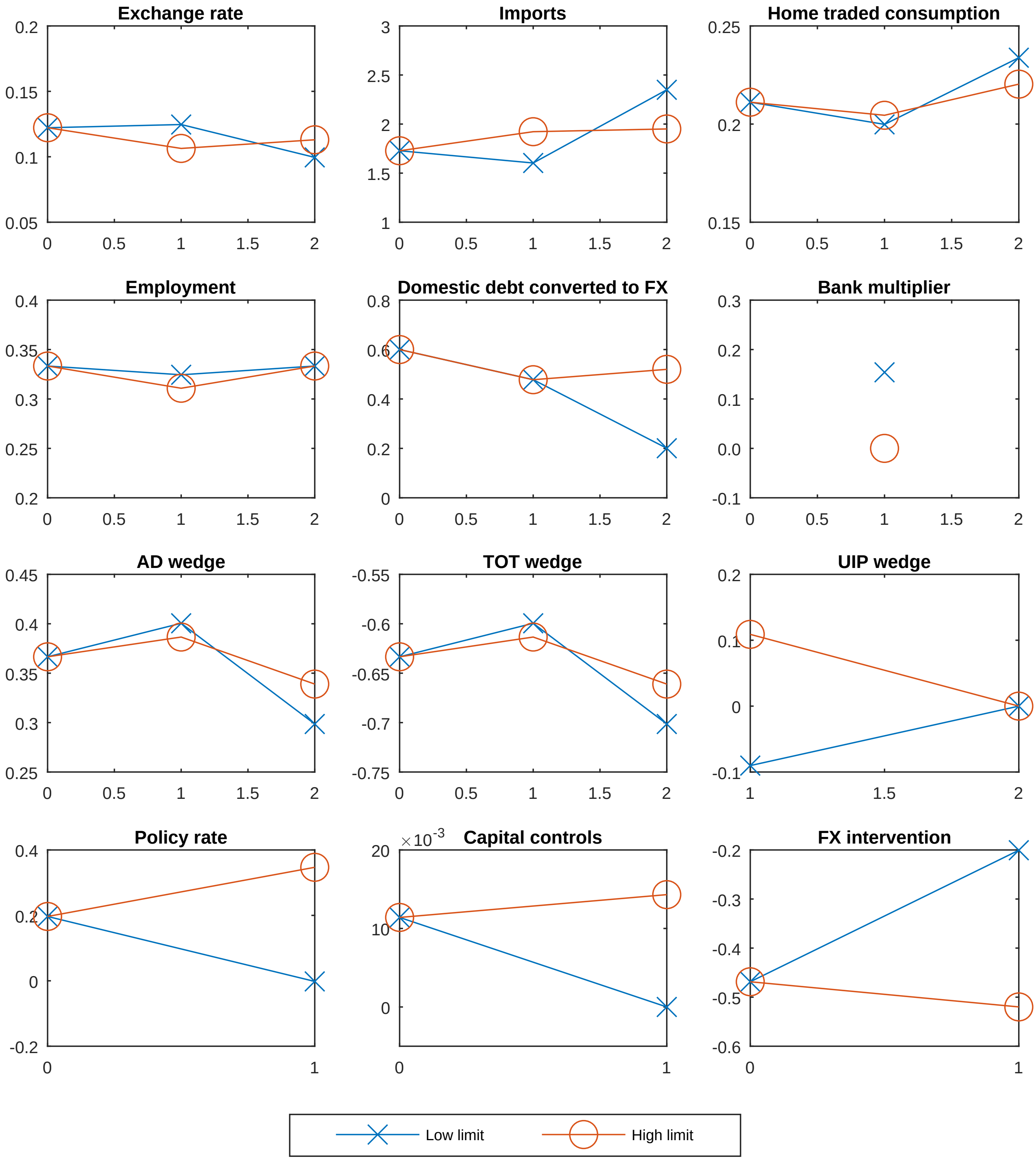

Notes: This figure plots the responses of key variables to a housing debt limit shock under DCP with shallow FX markets and housing sector frictions. The shock hits at date-1 and is calibrated as $\kappa_{L1} \in [0.025, 10]$.

**Figure 12:** Housing Constraint Spilling Over to Banks' Constraint under DCP cont.



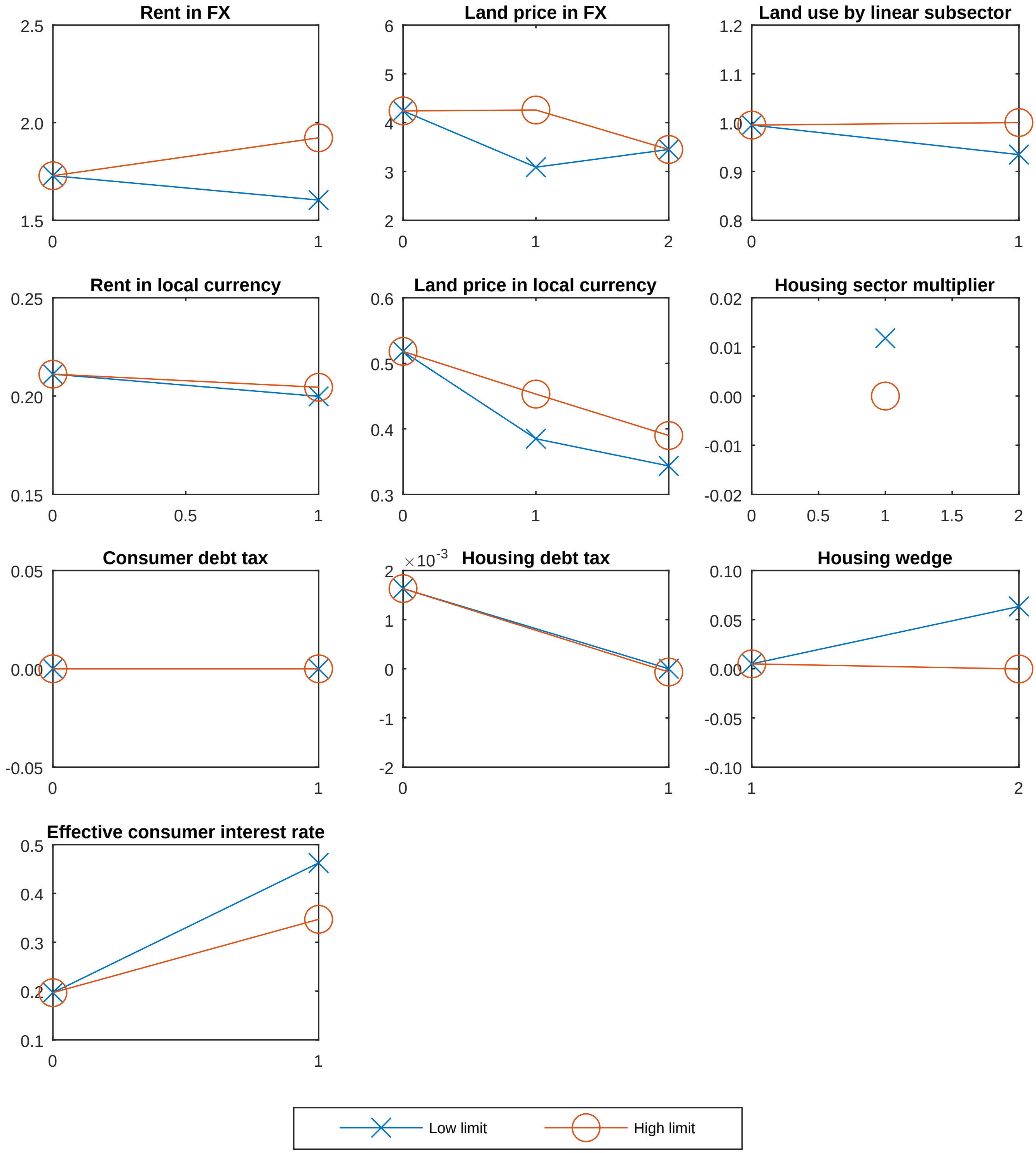

Notes: This figure plots the responses of key variables to a productivity shock under DCP with shallow FX markets and and housing sector frictions. The shock hits at date-1 and is calibrated as $\kappa_{L1} \in [0.025, 10]$.

**Figure 13:** Banks' Constraint Spilling Over to Housing Constraint under PCP



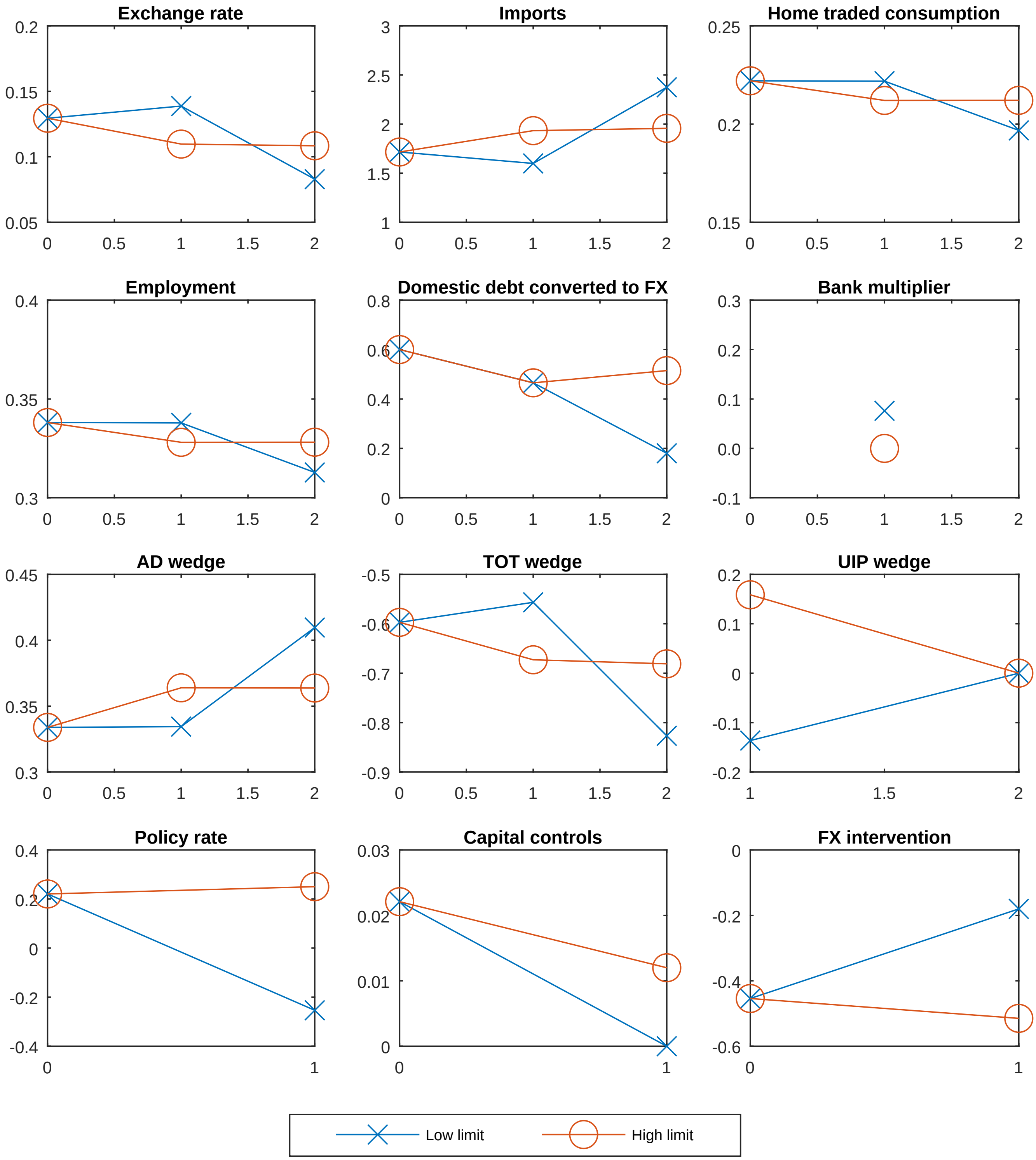

Notes: This figure plots the responses of key variables to a bank debt limit shock under PCP with shallow FX markets and housing sector frictions. The shock hits at date-1 and is calibrated as $\kappa_{H1} \in [0.025, 10]$.

**Figure 13:** Banks' Constraint Spilling Over to Housing Constraint under PCP cont.



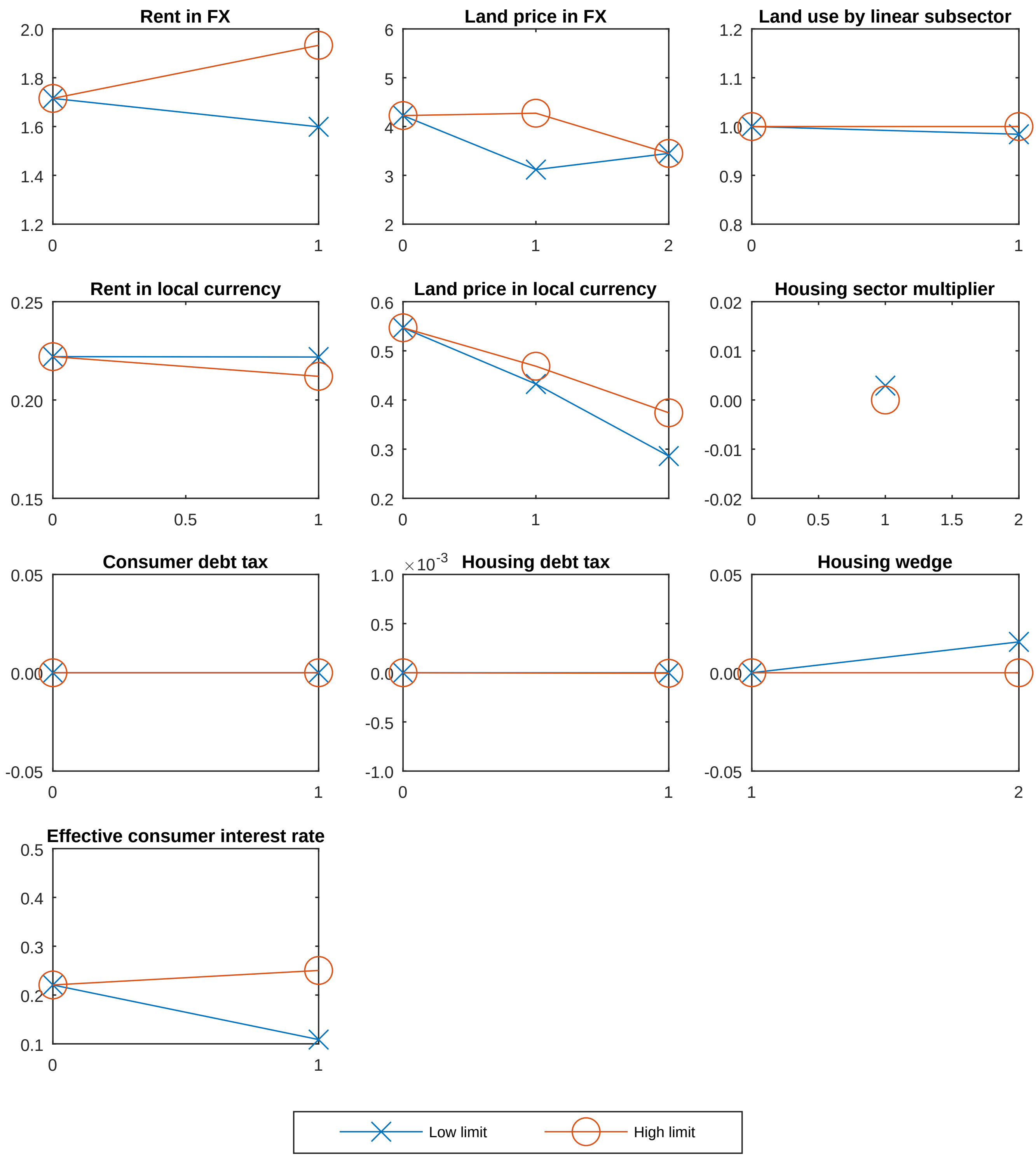

Notes: This figure plots the responses of key variables to a bank debt limit shock under PCP with shallow FX markets and and housing sector frictions. The shock hits at date-1 and is calibrated as $\kappa_{H1} \in [0.025, 10]$.

**Figure 14:** Banks' Constraint Spilling Over to Housing Constraint under DCP




Notes: This figure plots the responses of key variables to a bank debt limit shock under DCP with shallow FX markets and housing sector frictions. The shock hits at date-1 and is calibrated as $\kappa_{H1} \in [0.025, 10]$.

**Figure 14:** Banks' Constraint Spilling Over to Housing Constraint under DCP cont.




Notes: This figure plots the responses of key variables to a bank debt limit shock under DCP with shallow FX markets and and housing sector frictions. The shock hits at date-1 and is calibrated as $\kappa_{H1} \in [0.025, 10]$.

**Table 1:** Parameter Values

| Parameter | Description | Value | | |
|---|---|---|---|---|
| | | Deep FX | Shallow FX | Shallow FX + housing |
| $\alpha_H$ | Expenditure share of tradable goods | 1/3 | 1/3 | 1/3 |
| $\alpha_F$ | Expenditure share of imports | 1/3 | 1/3 | 1/3 |
| $\alpha_R$ | Expenditure share of housing services | 1/3 | 1/3 | 1/3 |
| $\beta$ | Discount factor | 0.8 | 0.8 | 0.8 |
| $\omega$ | Elasticity of export demand | 1 | 1 | 1 |
| $P_F^*$ | Dollar price of imports | 1 | 1 | 1 |
| $C^*$ | World demand level | 1 | 1 | 1 |
| $Y_{NT}$ | Endowment of nontradable goods | 1 | 1 | 1 |
| $Z$ | Endowment of commodities | 1 | 1 | 1 |
| $P_{Z0}^*$ | Initial dollar price of commodity exports | 1 | 1 | 1 |
| $i_0^*$ | Initial world interest rate | $1/\beta$-1 | $1/\beta$-1 | $1/\beta$-1 |
| $A_0$ | Initial level of productivity | 1 | 1 | 1 |
| $B_0$ | Initial debt level | [0, 0.6] | [0, 0.6] | [0, 0.6] |
| $B_{R0}$ | Initial housing sector debt level | NA | NA | 3.5 |
| $L_0$ | Initial land | NA | NA | 1 |
| $\lambda$ | Domestic share of intermediaries | 1 | 0.8 | 0.8 |
| $\Gamma$ | Balance sheet friction | 0 | 1 | 1 |
| Shocks | Description | Value | | |
| $\pi$ | Probability of good/bad shock | 0.5 | 0.5 | 0.5 |
| $\kappa_{H1}$ | Bank Debt limit | [0.025, 10] | [0.025, 10] | [0.025, 10] |
| $\kappa_{L1}$ | Housing Sector Debt limit | NA | NA | [0.025, 10] |
| $S_1$ | Foreign risk appetite | NA | [-0.5, 0.5] | [-0.5, 0.5] |

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# A APPENDIX

## A.1 Price Setting Condition under DCP

The functional form for $P_X = P_X\left(P_H, C_{F0}, \{C_{F1}\}, \{C_{F2}\}, E_0, \{E_1\}, \{E_2\}\right)$ is as follows:

$$
\begin{aligned}
P_X &= P_H \frac{X_1}{X_2}\frac{X_3}{X_4} \\
\text{where } X_1 &= \frac{1}{A_0}\left(P^*_{F0}\right)^2 C^*_0 C_{F0} + \frac{1}{\left(1+i^*_0\right)}\mathbb{E}_0\left[\frac{1}{A_1}\left(P^*_{F1}\right)^2 C^*_1 C_{F1}\right] + \frac{1}{\left(1+i^*_0\right)}\mathbb{E}_0\left[\frac{1}{\left(1+i^*_1\right)}\frac{1}{A_2}\left(P^*_{F2}\right)^2 C^*_2 C_{F2}\right] \\
X_2 &= P^*_{F0}C^*_0 + \frac{1}{\left(1+i^*_0\right)}\mathbb{E}_0\left[P^*_{F1}C^*_1\right] + \frac{1}{\left(1+i^*_0\right)}\mathbb{E}_0\left[\frac{1}{\left(1+i^*_1\right)}P^*_{F2}C^*_2\right] \\
X_3 &= P^*_{F0}C_{F0} + \frac{1}{\left(1+i^*_0\right)}\mathbb{E}_0\left[P^*_{F1}C_{F1}\right] + \frac{1}{\left(1+i^*_0\right)}\mathbb{E}_0\left[\frac{1}{\left(1+i^*_1\right)}P^*_{F2}C_{F2}\right] \\
X_4 &= \frac{1}{A_0}E_0\left(P^*_{F0}C_{F0}\right)^2 + \frac{1}{\left(1+i^*_0\right)}\mathbb{E}_0\left[\frac{1}{A_1}E_1\left(P^*_{F1}C_{F1}\right)^2\right] + \frac{1}{\left(1+i^*_0\right)}\mathbb{E}_0\left[\frac{1}{\left(1+i^*_1\right)}\frac{1}{A_2}E_2\left(P^*_{F2}C_{F2}\right)^2\right]
\end{aligned}
$$

This price-setting constraint on the planner captures the fact that when setting the export price at the beginning of period 0, firms take into account the planner's anticipated actions in all future periods. The solution of the constrained efficient allocation will require the following derivatives:

$$
\frac{\partial P_X}{\partial P_H} = \frac{P_X}{P_H}
$$

$$
\begin{aligned}
\frac{\partial P_X}{\partial C_{F0}} &= P_H \frac{\frac{1}{A_0}\left(P^*_{F0}\right)^2 C^*_0}{X_2}\frac{X_3}{X_4} + P_H\frac{X_1}{X_2}\frac{X_4 P^*_{F0} - X_3\frac{2}{A_0}E_0\left(P^*_{F0}\right)^2 C_{F0}}{\left(X_4\right)^2} \\
&= P_X\left[\frac{\frac{1}{A_0}\left(P^*_{F0}\right)^2 C^*_0}{X_1} + \frac{X_4 P^*_{F0} - X_3\frac{2}{A_0}E_0\left(P^*_{F0}\right)^2 C_{F0}}{X_3X_4}\right], \text{ a single equation}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial P_X}{\partial C_{F1}} &= \pi_1 P_H \frac{\frac{1}{\left(1+i^*_0\right)}\frac{1}{A_1}\left(P^*_{F1}\right)^2 C^*_1}{X_2}\frac{X_3}{X_4} \\
&\quad + \pi_1 P_H \frac{X_1}{X_2}\frac{X_4\frac{1}{\left(1+i^*_0\right)}P^*_{F1} - X_3\frac{1}{\left(1+i^*_0\right)}\frac{2}{A_1}E_1\left(P^*_{F1}\right)^2 C_{F1}}{\left(X_4\right)^2} \\
&= \frac{\pi_1}{\left(1+i^*_0\right)}P_X\left[\frac{\frac{1}{A_1}\left(P^*_{F1}\right)^2 C^*_1}{X_1} + \frac{X_4P^*_{F1} - X_3\frac{2}{A_1}E_1\left(P^*_{F1}\right)^2 C_{F1}}{X_3X_4}\right], \\
&\quad \text{one equation per period-1 state } s_1
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial P_X}{\partial C_{F2}} &= \pi_1 P_H \frac{\frac{1}{\left(1+i^*_0\right)\left(1+i^*_1\right)}\frac{1}{A_2}\left(P^*_{F2}\right)^2 C^*_2}{X_2}\frac{X_3}{X_4} \\
&\quad + \pi_1 P_H\frac{X_1}{X_2}\frac{X_4\frac{1}{\left(1+i^*_0\right)\left(1+i^*_1\right)}P^*_{F2} - X_3\frac{1}{\left(1+i^*_0\right)\left(1+i^*_1\right)}\frac{2}{A_2}E_2\left(P^*_{F2}\right)^2 C_{F2}}{\left(X_4\right)^2}
\end{aligned}
$$

$$= \frac{\pi_1}{(1+i_0^*)(1+i_1^*)} P_X \left[ \frac{\frac{1}{A_2}(P_{F2}^*)^2 C_2^*}{X_1} + \frac{X_4 P_{F2}^* - X_3 \frac{2}{A_2} E_2 (P_{F2}^*)^2 C_{F2}}{X_3 X_4} \right],$$

one equation per period-1 state $s_1$

$$\frac{\partial P_X}{\partial E_0} = -P_H \frac{X_1}{X_2} \frac{X_3}{(X_4)^2} \frac{1}{A_0} (P_{F0}^* C_{F0})^2 = -P_X \frac{\frac{1}{A_0}(P_{F0}^* C_{F0})^2}{X_4}, \text{ a single equation}$$

$$\frac{\partial P_X}{\partial E_1} = -\pi_1 P_H \frac{X_1}{X_2} \frac{X_3}{(X_4)^2} \frac{1}{(1+i_0^*)} \frac{1}{A_1} (P_{F1}^* C_{F1})^2$$
$$= -\frac{\pi_1}{(1+i_0^*)} P_X \frac{\frac{1}{A_1}(P_{F1}^* C_{F1})^2}{X_4},$$

one equation per period-1 state $s_1$

$$\frac{\partial P_X}{\partial E_2} = -\pi_1 P_H \frac{X_1}{X_2} \frac{X_3}{(X_4)^2} \frac{1}{(1+i_0^*)(1+i_1^*)} \frac{1}{A_2} (P_{F2}^* C_{F2})^2$$
$$= -\frac{\pi_1}{(1+i_0^*)(1+i_1^*)} P_X \frac{\frac{1}{A_2}(P_{F2}^* C_{F2})^2}{X_4},$$

one equation per period-1 state $s_1$.

## A.2 FOCs for Constrained Efficient Allocations

The constrained efficient allocation under full commitment is:

$$\max_{\left\{C_{Ft}, P_H, E_t, \eta_{t+1}, FXI_t, L_{t-1}^{Linear}\right\}} \left\{ \begin{array}{cc} \mathbb{E}_0 \left[ \sum_{t=0}^{2} \beta^t V\left(C_{Ft}, \frac{E_t P_{Ft}^*}{P_H}, \frac{E_t P_{Ft}^*}{P_H}, L_{t-1}^{Linear}\right)\right] & \text{if PCP} \\ \mathbb{E}_0 \left[ \sum_{t=0}^{2} \beta^t V\left(C_{Ft}, \frac{E_t P_{Ft}^*}{P_H}, \frac{P_{Ft}^*}{P_X}, L_{t-1}^{Linear}\right)\right] & \text{if DCP,} \\ \text{with } P_X = P_X\left(C_{F0}, \{C_{F1}\}, \{C_{F2}\}, E_0, \{E_1\}, \{E_2\}, P_H\right) & \end{array} \right\}$$

subject to the following constraints:

$$\begin{aligned}(1+i_{-1}^*) B_0 \le\ & P_{F0}^* [\omega C_0^* - C_{F0}] + P_{Z0}^* Z_0 \\ & + \frac{P_{F1}^* [\omega C_1^* - C_{F1}] + P_{Z1}^* Z_1 - (1-\lambda) FXI_0 [\eta_1 - (1+i_0^*)]}{\lambda(1+i_0^*) + (1-\lambda)\eta_1} \\ & + \frac{P_{F2}^* [\omega C_2^* - C_{F2}] + P_{Z2}^* Z_2 - (1-\lambda) FXI_1 [\eta_2 - (1+i_1^*)] + B_3}{[\lambda(1+i_0^*) + (1-\lambda)\eta_1][\lambda(1+i_1^*) + (1-\lambda)\eta_2]},\end{aligned}$$

one equation per period-1 state $s_1$ $[\Phi]$

$$\begin{aligned}(1+i_{-1}^*) B_0 \le\ & P_{F0}^* [\omega C_0^* - C_{F0}] + P_{Z0}^* Z_0 \\ & + \frac{P_{F1}^* [\omega C_1^* - C_{F1}] + P_{Z1}^* Z_1 - (1-\lambda) FXI_0 [\eta_1 - (1+i_0^*)]}{\lambda(1+i_0^*) + (1-\lambda)\eta_1} \\ & + \frac{\kappa_{H1} \frac{P_H}{E_1}}{\lambda(1+i_0^*) + (1-\lambda)\eta_1},\end{aligned}$$

one equation per period-1 state $s_1$ $[\Psi_B]$

$$\Gamma\left(\begin{array}{c}\left(1+i^*_{-1}\right)B_0+FXI_0-S_0\\-P^*_{F0}\left[\omega C^*_0-C_{F0}\right]-P^*_{Z0}Z_0\end{array}\right)=\mathbb{E}_0\left[\eta_1-\left(1+i^*_0\right)\right],\text{ a single equation }[\Omega_0]$$

$$\Gamma\left(\begin{array}{c}\left[\begin{array}{c}\left(1+i^*_{-1}\right)B_0\\-P^*_{F0}\left[\omega C^*_0-C_{F0}\right]-P^*_{Z0}Z_0\end{array}\right]\left[\lambda\left(1+i^*_0\right)+\left(1-\lambda\right)\eta_1\right]+FXI_1-S_1\\-P^*_{F1}\left[\omega C^*_1-C_{F1}\right]-P^*_{Z1}Z_1+\left(1-\lambda\right)FXI_0\left[\eta_1-\left(1+i^*_0\right)\right]\end{array}\right)$$
$$=\eta_2-\left(1+i^*_1\right),\text{ one equation per period-1 state } s_1\ [\Omega_1]$$

$$E^H_1\eta^H_1=E^L_1\eta^L_1,\text{ a single equation }[\Lambda]$$

$$0\geq B^{Linear,s}_{R2}=\chi^s_1\left[\left(1+i^*_{-1}\right)B^{Linear}_{R0}-\frac{\alpha_R}{\alpha_F}\frac{P^*_{F0}C_{F0}}{L^{Linear}_{-1}+G\left(1-L^{Linear}_{-1}\right)}L^{Linear}_{-1}\right]$$
$$+\left\{\begin{array}{c}\frac{G'\left(1-L^{Linear}_0\right)}{L^{Linear}_0+G\left(1-L^{Linear}_0\right)}\frac{\alpha_R}{\alpha_F}\mathbb{E}_0\left[\frac{E_1}{E^s_1}P^*_{F1}C_{F1}\right]\\+\mathbb{E}_0\left[\frac{1}{\chi_2}\frac{E_1}{E^s_1}\left(\frac{G'\left(1-L^{Linear}_1\right)}{L^{Linear}_1+G\left(1-L^{Linear}_1\right)}\frac{\alpha_R}{\alpha_F}P^*_{F2}C_{F2}+\widehat{q}_2\right)\right]\end{array}\right\}\left(L^{Linear}_0-L^{Linear}_{-1}\right)$$
$$-\frac{P^*_{F1}C_{F1}}{L^{Linear}_0+G\left(1-L^{Linear}_0\right)}\frac{\alpha_R}{\alpha_F}L^{Linear}_0$$
$$+\frac{1}{\chi_2}\left(\frac{G'\left(1-L^{Linear}_1\right)}{L^{Linear}_1+G\left(1-L^{Linear}_1\right)}\frac{\alpha_R}{\alpha_F}P^*_{F2}C_{F2}+\widehat{q}_2\right)\left(\left(1-\kappa_{L1}\right)L^{Linear}_1-L^{Linear}_0\right),$$
one equation per period-1 state $s_1$ $[\Psi_R]$

$$\chi^s_{t+1}=\left\{\begin{array}{cc}\eta^s_{t+1} & \text{if capital controls are not permitted}\\ \frac{\frac{\alpha_F}{P^*_{Ft}C_{Ft}}}{\beta\mathbb{E}_t\left\{\frac{E^s_{t+1}}{E_{t+1}}\frac{\alpha_F}{P^*_{Ft+1}C_{Ft+1}}\right\}} & \begin{array}{c}\text{if consumer macroprudential controls}\\\text{are not permitted}\end{array}\end{array}\right.$$

where we define all the constraints in dollar terms, we use the superscript $s$ to refer to the state of nature, and we indicate the multipliers in capital Greek letters in square brackets after each constraint. We fix the dollar value of initial debt repayments for the economy as a whole at $\left(1+i^*_{-1}\right)B_0$ in order to avoid the artefact depreciating away domestic currency debt repayments at time 0, and we fix the dollar value of final debt at $B_3=B_0$ in order to normalize the debt path. We fix the dollar value of initial debt repayments for linear subsector housing firms at $\left(1+i^*_{-1}\right)B^{Linear}_{R0}=-\left(1+i^*_{-1}\right)B^{Concave}_{R0}$, and the dollar value of the final house price at $\widehat{q}_2$, in order to avoid the artefact depreciating away the value of domestic currency in all periods as a means of circumventing this subsector's borrowing constraint.

The above planner problem assumes that all instruments (i.e., the policy rate, capital controls, and FX intervention) are available. For determinacy of the instruments, we need to assume that only one of capital controls and consumer macroprudential taxes are available. The optimal allocations from the problem can be used to produce the implied optimal domestic policy rates and capital controls:

$$\left(1+\widehat{i}_t\right)=\left(1-\varphi_t\right)\left(1+i_t\right)=\eta_{t+1}\frac{E_{t+1}}{E_t}$$

$$\frac{\left(1-\varphi_0\right)}{\left(1+\theta_{HH0}\right)}=\frac{\eta_1\frac{E_1}{E_0}\beta\mathbb{E}_0\left[\frac{E_0}{E_1}\frac{\alpha_F}{P^*_{F1}C_{F1}}\right]}{\frac{\alpha_F}{P^*_{F0}C_{F0}}}\text{ and }\left(1+i_0\right)=\frac{\eta_1\frac{E_1}{E_0}}{\left(1-\varphi_0\right)}$$

$$\begin{array}{ll} \frac{(1-\varphi_1)}{(1+\theta_{HH1})} = \frac{\eta_2 \beta P^*_{F1} C_{F1}}{P^*_{F2} C_{F2}} \text{ and } (1+i_1) = \frac{\eta_2 \frac{E_2}{E_1}}{(1-\varphi_1)} & \text{if } \Psi_{Bt} = 0 \\ (1+i_1) = \left(1+\widehat{i}_1\right) \text{ and } \varphi_1 = \theta_{HH1} = 0 & \text{if } \Psi_{Bt} > 0 \end{array}$$

$$\chi_{t+1} = \frac{\eta_{t+1}}{(1-\varphi_t)}$$

$$(1+\theta_{Rt}) = \begin{cases} \frac{\mathbb{E}_t\left[\frac{1}{L_t^{Linear}+G\left(1-L_t^{Linear}\right)} \frac{\alpha_R}{\alpha_F} \frac{E_{t+1}}{E_t} P^*_{Ft+1} C_{Ft+1}\right] + \mathbb{E}_t\left[\frac{E_{t+1}}{E_t} \widehat{q}_{t+1}\right]}{(1+i_t)\widehat{q}_t} & \text{if } \Psi_{Rt} = 0 \\ 0 & \text{if } \Psi_{Rt} > 0 \end{cases},$$

$$\text{where } \widehat{q}_t = \begin{cases} \frac{1}{(1+i_0)} \frac{G'\left(1-L_0^{Linear}\right)}{L_0^{Linear}+G\left(1-L_0^{Linear}\right)} \frac{\alpha_R}{\alpha_F} \mathbb{E}_0\left[\frac{E_1}{E_0} P^*_{F1} C_{F1}\right] & \\ + \frac{1}{(1+i_0)} \mathbb{E}_0\left[\frac{1}{\chi_2} \frac{E_1}{E_0}\left(\frac{G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)} \frac{\alpha_R}{\alpha_F} P^*_{F2} C_{F2} + \widehat{q}_2\right)\right] & \text{if } t = 0 \\ \frac{1}{\chi_2}\left(\frac{G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)} \frac{\alpha_R}{\alpha_F} P^*_{F2} C_{F2} + \widehat{q}_2\right) & \text{if } t = 1 \\ \widehat{q}_2 & \text{if } t = 2, \end{cases}$$

If FX intervention is not permitted, then we need to set:

$$FXI_0 = FXI_1 = 0,$$

and remove the FOCs with respect to $FXI_t$.

If both capital controls and consumer macroprudential controls are not permitted, then the household Euler conditions need to be added as constraints:

$$\frac{\alpha_F}{P^*_{F0} C_{F0}} = \beta E_1^H \eta_1^H \mathbb{E}_0\left[\frac{1}{E_1} \frac{\alpha_F}{P^*_{F1} C_{F1}}\right], \text{ a single equation } [\Upsilon_0]$$

$$\frac{\alpha_F}{P^*_{F1} C_{F1}} \geq \beta \eta_2 \frac{\alpha_F}{P^*_{F2} C_{F2}} = \beta \chi_2 \frac{\alpha_F}{P^*_{F2} C_{F2}}, \text{ one equation per period-1 state } s_1 \ [\Upsilon_1]$$

If mortgage MPMs are set to zero, i.e., $\theta_{Rt} \equiv 0$, then we need to add the following period-0 constraint:

$$L_0 = 1, \text{ a single equation } [\Delta_0]$$

If capital controls are not permitted and the domestic policy rate cannot be used, then the additional constraints are:

$$E_1^H \eta_1^H = \frac{1}{\beta} E_0, \text{ a single equation } [\Xi_0]$$

$$\eta_2 E_2 = \frac{1}{\beta} E_1, \text{ one equation per period-1 state } s_1 \ [\Xi_1]$$

If consumer macroprudential controls are not permitted and the domestic policy rate cannot be used, then the additional constraints are:

$$\frac{\alpha_F}{P^*_{F0} C_{F0}} = \mathbb{E}_0\left[\frac{E_0}{E_1} \frac{\alpha_F}{P^*_{F1} C_{F1}}\right], \text{ a single equation } [\Sigma_0]$$

$$\frac{\alpha_F}{P^*_{F1} C_{F1}} = \frac{E_1}{E_2} \frac{\alpha_F}{P^*_{F2} C_{F2}}, \text{ one equation per period-1 state } s_1 \ [\Sigma_1]$$

Finally, if the exchange rate regime is a peg, the four additional constraints are:

$$\left\{\begin{array}{l} E_0 = E_1^s \\ E_0 = E_2^s \end{array}\right\}, \text{ one equation per state } s_1 \text{ in each of periods 1 and 2 } [\Pi_1^s \text{ and } \Pi_2^s]$$

The FOCs for the constrained efficient allocation are:

$$\begin{aligned}
&C_{F0}: \frac{\alpha_F}{P_{F0}^* C_{F0}}\left[1+\frac{\alpha_H}{\alpha_F}\left(1-\frac{1}{A_0}\frac{C_{H0}}{\alpha_H}\right)\right] + \mathbb{I}^{DCP}\cdot\left\{\frac{1}{P_{F0}^*}\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\frac{\omega}{A_t}C_t^*\frac{P_{Ft}^*}{(P_X)^2}\right]\frac{\partial P_X}{\partial C_{F0}}\right\} \\
&= \mathbb{E}_0\left[\Phi+\Psi_B\right] + \Gamma\Omega_0 + \Gamma\mathbb{E}_0\left[I_0\Omega_1\right] + \frac{\alpha_F}{\left(P_{F0}^* C_{F0}\right)^2}\left(\Upsilon_0+\Sigma_0\right) \\
&+ \mathbb{E}_0\left[\Psi_R\left\{\frac{1}{P_{F0}^*}\frac{\partial\chi_1}{\partial C_{F0}}\left[\left(1+i_{-1}^*\right)B_{R0}^{Linear} - \widehat{P}_{R0}L_{-1}^{Linear}\right] - \chi_1\frac{1}{P_{F0}^*}\frac{\partial\widehat{P}_{R0}}{\partial C_{F0}}L_{-1}^{Linear}\right\}\right], \text{ a single equation}
\end{aligned}$$

$$\begin{aligned}
&C_{F1}: \beta I_0\frac{\alpha_F}{P_{F1}^* C_{F1}}\left[1+\frac{\alpha_H}{\alpha_F}\left(1-\frac{1}{A_1}\frac{C_{H1}}{\alpha_H}\right)\right] + \mathbb{I}^{DCP}\cdot\left\{\frac{I_0}{\pi_1 P_{F1}^*}\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\frac{\omega}{A_t}C_t^*\frac{P_{Ft}^*}{(P_X)^2}\right]\frac{\partial P_X}{\partial C_{F1}}\right\} \\
&= \Phi+\Psi_B+\Gamma I_0\Omega_1 \\
&+ \frac{I_0}{P_{F1}^*\pi_1}\mathbb{E}_0\left[\Psi_R\left\{\frac{\partial\chi_1}{\partial C_{F1}^s}\left[\left(1+i_{-1}^*\right)B_{R0}^{Linear} - \widehat{P}_{R0}L_{-1}^{Linear}\right] + \frac{\partial\left(\chi_1\widehat{q}_0\right)}{\partial C_{F1}^s}\left(L_0^{Linear}-L_{-1}^{Linear}\right)\right\}\right] \\
&+ \frac{I_0}{P_{F1}^*\pi_1}\Psi_R\left\{-\frac{\partial\widehat{P}_{R1}}{\partial C_{F1}}L_0^{Linear} + \frac{\partial\widehat{q}_1}{\partial C_{F1}}\left(\left(1-\kappa_{L1}\right)L_1^{Linear}-L_0^{Linear}\right)\right\} \\
&+ I_0\frac{\alpha_F}{\left(P_{F1}^* C_{F1}\right)^2}\left[\Upsilon_1-\Upsilon_0\beta\eta_1\right] + I_0\Upsilon_1\beta\frac{\alpha_F}{P_{F2}^* C_{F2}}\frac{1}{P_{F1}^*}\frac{\partial\chi_2}{\partial C_{F1}} + I_0\frac{\alpha_F}{\left(P_{F1}^* C_{F1}\right)^2}\left[\Sigma_1-\frac{E_0}{E_1}\Sigma_0\right], \\
&\text{one equation per period-1 state } s_1
\end{aligned}$$

$$\begin{aligned}
&C_{F2}: \beta^2 I_0 I_1\frac{\alpha_F}{P_{F2}^* C_{F2}}\left[1+\frac{\alpha_H}{\alpha_F}\left(1-\frac{1}{A_2}\frac{C_{H2}}{\alpha_H}\right)\right] + \mathbb{I}^{DCP}\cdot\left\{\frac{I_0 I_1}{\pi_1 P_{F2}^*}\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\frac{\omega}{A_t}C_t^*\frac{P_{Ft}^*}{(P_X)^2}\right]\frac{\partial P_X}{\partial C_{F2}}\right\} \\
&= \Phi + \frac{I_0 I_1}{\pi_1 P_{F2}^*}\mathbb{E}_0\left[\Psi_R\frac{\partial\left(\chi_1\widehat{q}_0\right)}{\partial C_{F2}^s}\left(L_0^{Linear}-L_{-1}^{Linear}\right)\right] + \frac{I_0 I_1}{P_{F2}^*}\Psi_R\frac{\partial\widehat{q}_1}{\partial C_{F2}}\left(\left(1-\kappa_{L1}\right)L_1^{Linear}-L_0^{Linear}\right) \\
&+ \Upsilon_1 I_0 I_1\left\{\beta\chi_2\frac{\alpha_F}{P_{F2}^* C_{F2}}\frac{1}{P_{F2}^*}\frac{\partial\chi_2}{\partial C_{F2}} - \beta\chi_2\frac{\alpha_F}{\left(P_{F2}^* C_{F2}\right)^2}\right\} - \Sigma_1 I_0 I_1\frac{E_1}{E_2}\frac{\alpha_F}{\left(P_{F2}^* C_{F2}\right)^2}, \\
&\text{one equation per period-1 state } s_1
\end{aligned}$$

$$\begin{aligned}
&E_0: \alpha_H\left(1-\frac{1}{A_0}\frac{C_{H0}}{\alpha_H}\right) = \mathbb{I}^{PCP}\cdot\left\{\frac{E_0 P_{F0}^*}{P_H}\frac{\omega}{A_0}C_0^*\right\} + \mathbb{I}^{DCP}\cdot\left\{E_0\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\frac{\omega}{A_t}C_t^*\frac{P_{Ft}^*}{(P_X)^2}\right]\left(-\frac{\partial P_X}{\partial E_0}\right)\right\} \\
&+ \Xi_0\frac{E_0}{\beta} + \Sigma_0\mathbb{E}_0\left[\frac{E_0}{E_1}\frac{\alpha_F}{P_{F1}^* C_{F1}}\right] - E_0\left(\Pi_1^H+\Pi_1^L+\Pi_2^H+\Pi_2^L\right), \text{ a single equation}
\end{aligned}$$

$$E_1^H : \beta\alpha_H\left(1-\frac{1}{A_1}\frac{C_{H1}}{\alpha_H}\right)\pi_1-\frac{\Psi_B}{I_0}\kappa_{H1}\frac{P_H}{E_1}\pi_1-E_1\Pi_1$$
$$+\Lambda E_1\eta_1-\Upsilon_0\beta\eta_1^L\frac{\alpha_F}{P_{F1}^*C_{F1}^L}\pi_1^L+\Xi_0E_1\eta_1-\Xi_1\frac{E_1}{\beta}\pi_1+\Sigma_0\frac{E_0}{E_1}\frac{\alpha_F}{P_{F1}^*C_{F1}}\pi_1-\Sigma_1\frac{E_1}{E_2}\frac{\alpha_F}{P_{F2}^*C_{F2}}\pi_1$$
$$=\mathbb{I}^{PCP}\cdot\left\{\beta\frac{E_1P_{F1}^*}{P_H}\frac{\omega}{A_1}C_1^*\pi_1\right\}+\mathbb{I}^{DCP}\cdot\left\{E_1\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\frac{\omega}{A_t}C_t^*\frac{P_{Ft}^*}{(P_X)^2}\right]\left(-\frac{\partial P_X}{\partial E_1}\right)\right\}$$
$$+\mathbb{E}_0\left[\Psi_R\left\{E_1^H\frac{\partial\chi_1}{\partial E_1^H}\left[\left(1+i_{-1}^*\right)B_{R0}^{Linear}-\widehat{P}_{R0}L_{-1}^{Linear}\right]+E_1^H\frac{\partial\left(\chi_1\widehat{q}_0\right)}{\partial E_1^H}\left(L_0^{Linear}-L_{-1}^{Linear}\right)\right\}\right],$$

a single equation for the $H$-state

$$E_1^L : \beta\alpha_H\left(1-\frac{1}{A_1}\frac{C_{H1}}{\alpha_H}\right)\pi_1-\frac{\Psi_B}{I_0}\kappa_{H1}\frac{P_H}{E_1}\pi_1-E_1\Pi_1$$
$$-\Lambda E_1\eta_1+\Upsilon_0\beta\eta_1\frac{\alpha_F}{P_{F1}^*C_{F1}}\pi_1-\Xi_1\frac{E_1}{\beta}\pi_1+\Sigma_0\frac{E_0}{E_1}\frac{\alpha_F}{P_{F1}^*C_{F1}}\pi_1-\Sigma_1\frac{E_1}{E_2}\frac{\alpha_F}{P_{F2}^*C_{F2}}\pi_1$$
$$=\mathbb{I}^{PCP}\cdot\left\{\beta\frac{E_1P_{F1}^*}{P_H}\frac{\omega}{A_1}C_1^*\pi_1\right\}+\mathbb{I}^{DCP}\cdot\left\{E_1\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\frac{\omega}{A_t}C_t^*\frac{P_{Ft}^*}{(P_X)^2}\right]\left(-\frac{\partial P_X}{\partial E_1}\right)\right\}$$
$$+\mathbb{E}_0\left[\Psi_R\left\{E_1^L\frac{\partial\chi_1}{\partial E_1^L}\left[\left(1+i_{-1}^*\right)B_{R0}^{Linear}-\widehat{P}_{R0}L_{-1}^{Linear}\right]+E_1^L\frac{\partial\left(\chi_1\widehat{q}_0\right)}{\partial E_1^L}\left(L_0^{Linear}-L_{-1}^{Linear}\right)\right\}\right],$$

a single equation for the $L$-state

$$E_2 : \beta^2\alpha_H\left(1-\frac{1}{A_2}\frac{C_{H2}}{\alpha_H}\right)\pi_1-E_2\Upsilon_1\beta\frac{\alpha_F}{P_{F2}^*C_{F2}}\frac{\partial\chi_2}{\partial E_2}\pi_1+\Xi_1E_2\eta_2\pi_1+\Sigma_1\frac{E_1}{E_2}\frac{\alpha_F}{P_{F2}^*C_{F2}}\pi_1-E_2\Pi_2$$
$$+\mathbb{E}_0\left[\Psi_RE_2^s\frac{\partial\left(\chi_1\widehat{q}_0\right)}{\partial E_2^s}\left(L_0^{Linear}-L_{-1}^{Linear}\right)\right]+\Psi_RE_2\frac{\partial\widehat{q}_1}{\partial E_2}\left(\left(1-\kappa_{L1}\right)L_1^{Linear}-L_0^{Linear}\right)\pi_1$$
$$=\mathbb{I}^{PCP}\cdot\left\{\beta^2\frac{E_2P_{F2}^*}{P_H}\frac{\omega}{A_2}C_2^*\pi_1\right\}+\mathbb{I}^{DCP}\cdot\left\{E_2\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\frac{\omega}{A_t}C_t^*\frac{P_{Ft}^*}{(P_X)^2}\right]\left(-\frac{\partial P_X}{\partial E_2}\right)\right\},$$

one equation per period-1 state $s_1$

$$\eta_1^H : \left(\Phi+\Psi_B\right)\left(1-\lambda\right)\frac{B_1+FXI_0}{I_0}+\Psi_B\left(1-\lambda\right)\frac{\kappa_{H1}\frac{P_H}{E_1}-B_2}{\left(I_0\right)^2}$$
$$+\Psi_R\frac{\partial\chi_1}{\partial\eta_1}\left[\left(1+i_{-1}^*\right)B_{R0}^{Linear}-\widehat{P}_{R0}L_{-1}^{Linear}\right]$$
$$=\Omega_0-\Omega_1\Gamma\left(1-\lambda\right)\left(B_1+FXI_0\right)+\frac{1}{\pi_1}\Lambda E_1-\frac{1}{\pi_1}\Upsilon_0\beta E_1\mathbb{E}_0\left\{\frac{1}{E_1}\frac{\alpha_F}{P_{F1}^*C_{F1}}\right\}+\frac{1}{\pi_1}\Xi_0E_1,$$

a single equation for the $H$-state

$$\eta_1^L : \left(\Phi+\Psi_B\right)\left(1-\lambda\right)\frac{B_1+FXI_0}{I_0}+\Psi_B\left(1-\lambda\right)\frac{\kappa_{H1}\frac{P_H}{E_1}-B_2}{\left(I_0\right)^2}$$

$$
\begin{aligned}
&+\Psi_R \frac{\partial \chi_1}{\partial \eta_1}\left[\left(1+i_{-1}^*\right) B_{R0}^{Linear}-\widehat{P}_{R0} L_{-1}^{Linear}\right] \\
&=\Omega_0-\Omega_1 \Gamma(1-\lambda)\left(B_1+FXI_0\right)-\frac{1}{\pi_1} \Lambda E_1, \text{ a single equation for the } L\text{-state}
\end{aligned}
$$

$$
\begin{aligned}
\eta_2 \quad : \quad & \Phi(1-\lambda) \frac{B_2+FXI_1}{I_0 I_1}+\frac{1}{\pi_1} \mathbb{E}_0\left[\Psi_R \frac{\partial\left(\chi_1 \widehat{q}_0\right)}{\partial \eta_2^s}\left(L_0^{Linear}-L_{-1}^{Linear}\right)\right] \\
& +\Psi_R \frac{\partial \widehat{q}_1}{\partial \eta_2}\left(\left(1-\kappa_{L1}\right) L_1^{Linear}-L_0^{Linear}\right) \\
= \quad & \Omega_1-\Upsilon_1 \beta \frac{\alpha_F}{P_{F2}^* C_{F2}}+\Xi_1 E_2, \text{ one equation per period-1 state } s_1
\end{aligned}
$$

$$
\begin{aligned}
FXI_0: 0= & -(1-\lambda) \mathbb{E}_0\left\{\left(\Phi+\Psi_B\right) \frac{\left[\eta_1-\left(1+i_0^*\right)\right]}{I_0}\right\} \\
& -\Gamma \Omega_0-(1-\lambda) \Gamma \mathbb{E}_0\left\{\Omega_1\left[\eta_1-\left(1+i_0^*\right)\right]\right\}, \text{ a single equation}
\end{aligned}
$$

$$
FXI_1:-\Phi \frac{(1-\lambda)\left[\eta_2-\left(1+i_1^*\right)\right]}{I_0 I_1}-\Gamma \Omega_1=0, \text{ one equation per period-1 state } s_1,
$$

$$
\begin{aligned}
& L_0: \beta \alpha_R \frac{1-G'\left(1-L_0^{Linear}\right)}{L_0^{Linear}+G\left(1-L_0^{Linear}\right)} \\
& =\mathbb{E}_0\left[\Psi_R\left\{\left(\chi_1 \widehat{q}_0-\widehat{P}_{R1}-\widehat{q}_1\right)+\frac{\partial\left(\chi_1 \widehat{q}_0\right)}{\partial L_0}\left(L_0^{Linear}-L_{-1}^{Linear}\right)-\frac{\partial \widehat{P}_{R1}}{\partial L_0} L_0\right\}\right], \\
& \text{a single equation}
\end{aligned}
$$

$$
\begin{aligned}
& L_1: \beta^2 \alpha_R \frac{1-G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)}=\frac{1}{\pi_1} \mathbb{E}_0\left[\Psi_R \frac{\partial\left(\chi_1 \widehat{q}_0\right)}{\partial L_1^s}\left(L_0^{Linear}-L_{-1}^{Linear}\right)\right] \\
& +\Psi_R\left\{\widehat{q}_1\left(1-\kappa_{L1}\right)+\frac{\partial \widehat{q}_1}{\partial L_1}\left(\left(1-\kappa_{L1}\right) L_1^{Linear}-L_0^{Linear}\right)\right\}, \text{ one equation per period-1 state } s_1
\end{aligned}
$$

where $\Theta_t \geq 0$ and $\sum_{s=0}^t FXI_t \geq 0$ with complementary slackness, and the FOCs with respect to $P_H$ are redundant, so we normalize $P_H=1$. We define:

$$
I_t=\lambda\left(1+i_t^*\right)+(1-\lambda) \eta_{t+1}
$$

$$
B_1 \equiv \frac{D_1}{E_0}=\left(1+i_{-1}^*\right) B_0-\left(P_{F0}^*\left[\omega C_0^*-C_{F0}\right]+P_{Z0}^* Z_0\right)
$$

$$
B_2 \equiv \frac{D_2}{E_1}=B_1 I_0-\left(P_{F1}^*\left[\omega C_1^*-C_{F1}\right]-(1-\lambda) FXI_0\left[\eta_1-\left(1+i_0^*\right)\right]+P_{Z1}^* Z_1\right)
$$

$$
B_3 \equiv \frac{D_3}{E_2}=B_2 I_1-\left(P_{F2}^*\left[\omega C_2^*-C_{F2}\right]-(1-\lambda) FXI_1\left[\eta_2-\left(1+i_1^*\right)\right]+P_{Z2}^* Z_2\right),
$$

We define the following derivatives for the case when capital controls are not permitted:

$$\frac{\partial \chi_1^s}{\partial C_{F0}} = \frac{\partial \chi_1^s}{\partial C_{F1}^s} = \frac{\partial \chi_1^s}{\partial C_{F1}^{-s}} = \frac{\partial \chi_1^s}{\partial E_1^s} = \frac{\partial \chi_1^s}{\partial E_1^{-s}} = \frac{\partial \chi_2}{\partial C_{F1}} = \frac{\partial \chi_2}{\partial C_{F2}} = \frac{\partial \chi_2}{\partial E_2} = 0$$

$$\frac{\partial \chi_1^s}{\partial \eta_1^s} = \frac{\partial \chi_2}{\partial \eta_2} = 1,$$

and the following derivatives for the case when consumer macroprudential controls are not permitted:

$$\frac{\partial \chi_1^s}{\partial C_{F0}} = -\frac{\frac{\alpha_F}{P_{F0}^*(C_{F0})^2}}{\beta \mathbb{E}_0 \left\{ \frac{E_1^s}{E_1} \frac{\alpha_F}{P_{F1}^* C_{F1}} \right\}}, \frac{\partial \chi_1^s}{\partial C_{F1}^s} = \frac{\frac{\alpha_F}{P_{F0}^* C_{F0}} \beta \frac{\alpha_F}{P_{F1}^* \left(C_{F1}^s\right)^2} \pi_1^s}{\left[\beta \mathbb{E}_0 \left\{ \frac{E_1^s}{E_1} \frac{\alpha_F}{P_{F1}^* C_{F1}} \right\}\right]^2}, \frac{\partial \chi_1^s}{\partial C_{F1}^{-s}} = \frac{\frac{\alpha_F}{P_{F0}^* C_{F0}} \beta \frac{E_1^s}{E_1^{-s}} \frac{\alpha_F}{P_{F1}^* \left(C_{F1}^{-s}\right)^2} \pi_1^{-s}}{\left[\beta \mathbb{E}_0 \left\{ \frac{E_1^s}{E_1} \frac{\alpha_F}{P_{F1}^* C_{F1}} \right\}\right]^2}$$

$$\frac{\partial \chi_1^s}{\partial E_1^s} = -\frac{\frac{\alpha_F}{P_{F0}^* C_{F0}} \beta \frac{1}{E_1^{-s}} \frac{\alpha_F}{P_{F1}^* C_{F1}^{-s}} \pi_1^{-s}}{\left[\beta \mathbb{E}_0 \left\{ \frac{E_1^s}{E_1} \frac{\alpha_F}{P_{F1}^* C_{F1}} \right\}\right]^2}, \frac{\partial \chi_1^s}{\partial E_1^{-s}} = \frac{\frac{\alpha_F}{P_{F0}^* C_{F0}} \beta \frac{E_1^s}{\left(E_1^{-s}\right)^2} \frac{\alpha_F}{P_{F1}^* C_{F1}^{-s}} \pi_1^{-s}}{\left[\beta \mathbb{E}_0 \left\{ \frac{E_1^s}{E_1} \frac{\alpha_F}{P_{F1}^* C_{F1}} \right\}\right]^2}$$

$$\frac{\partial \chi_2}{\partial C_{F1}} = -\frac{P_{F2}^* C_{F2}}{\beta P_{F1}^* \left(C_{F1}\right)^2}, \frac{\partial \chi_2}{\partial C_{F2}} = \frac{P_{F2}^*}{\beta P_{F1}^* C_{F1}}, \frac{\partial \chi_2}{\partial E_2} = 0 \text{ and } \frac{\partial \chi_1^s}{\partial \eta_1^s} = \frac{\partial \chi_2}{\partial \eta_2} = 0$$

We define the following derivatives:

$$\frac{\partial(\chi_1^s \widehat{q}_0)}{\partial C_{F1}^s} = \frac{G'\left(1-L_0^{Linear}\right)}{L_0^{Linear}+G\left(1-L_0^{Linear}\right)} \frac{\alpha_R}{\alpha_F} P_{F1}^* \pi_1^s + \frac{\partial \widehat{q}_1^s}{\partial C_{F1}^s} \pi_1^s \qquad \frac{\partial(\chi_1^s \widehat{q}_0)}{\partial C_{F2}^s} = \frac{\partial \widehat{q}_1^s}{\partial C_{F2}^s} \pi_1^s$$

$$\frac{\partial(\chi_1^s \widehat{q}_0)}{\partial C_{F1}^{-s}} = \frac{G'\left(1-L_0^{Linear}\right)}{L_0^{Linear}+G\left(1-L_0^{Linear}\right)} \frac{\alpha_R}{\alpha_F} \frac{E_1^{-s}}{E_1^s} P_{F1}^* \pi_1^{-s} + \frac{E_1^{-s}}{E_1^s} \frac{\partial \widehat{q}_1^{-s}}{\partial C_{F1}^{-s}} \pi_1^{-s} \qquad \frac{\partial(\chi_1^s \widehat{q}_0)}{\partial C_{F2}^{-s}} = \frac{E_1^{-s}}{E_1^s} \frac{\partial \widehat{q}_1^{-s}}{\partial C_{F2}^{-s}} \pi_1^{-s}$$

$$\frac{\partial(\chi_1^s \widehat{q}_0)}{\partial E_1^s} = -\frac{G'\left(1-L_0^{Linear}\right)}{L_0^{Linear}+G\left(1-L_0^{Linear}\right)} \frac{\alpha_R}{\alpha_F} \frac{E_1^{-s}}{(E_1^s)^2} P_{F1}^* C_{F1}^{-s} \pi_1^{-s} - \frac{E_1^{-s}}{(E_1^s)^2} \widehat{q}_1^{-s} \pi_1^{-s} \qquad \frac{\partial(\chi_1^s \widehat{q}_0)}{\partial E_2^s} = \frac{\partial \widehat{q}_1^s}{\partial E_2^s} \pi_1^s$$

$$\frac{\partial(\chi_1^s \widehat{q}_0)}{\partial E_1^{-s}} = \frac{G'\left(1-L_0^{Linear}\right)}{L_0^{Linear}+G\left(1-L_0^{Linear}\right)} \frac{\alpha_R}{\alpha_F} \frac{1}{E_1^s} P_{F1}^* C_{F1}^{-s} \pi_1^{-s} + \frac{1}{E_1^s} \widehat{q}_1^{-s} \pi_1^{-s} \qquad \frac{\partial(\chi_1^s \widehat{q}_0)}{\partial E_2^{-s}} = \frac{E_1^{-s}}{E_1^s} \frac{\partial \widehat{q}_1^{-s}}{\partial E_2^{-s}} \pi_1^{-s}$$

$$\frac{\partial(\chi_1^s \widehat{q}_0)}{\partial L_0} = \left\{ \frac{\left[-G''\left(1-L_0^{Linear}\right)\right]}{L_0^{Linear}+G\left(1-L_0^{Linear}\right)} - \frac{G'\left(1-L_0^{Linear}\right)\left[1-G'\left(1-L_0^{Linear}\right)\right]}{\left[L_0^{Linear}+G\left(1-L_0^{Linear}\right)\right]^2} \right\} \frac{\alpha_R}{\alpha_F} \mathbb{E}_0 \left[ \frac{E_1}{E_1^s} P_{F1}^* C_{F1} \right] \qquad \frac{\partial(\chi_1^s \widehat{q}_0)}{\partial \eta_2^s} = \frac{\partial \widehat{q}_1^s}{\partial \eta_2^s} \pi_1^s$$

$$\frac{\partial(\chi_1^s \widehat{q}_0)}{\partial L_1^{Linear,s}} = \frac{\partial \widehat{q}_1^s}{\partial L_1^{Linear,s}} \pi_1^s, \frac{\partial(\chi_1^s \widehat{q}_0)}{\partial L_1^{Linear,-s}} = \frac{E_1^{-s}}{E_1^s} \frac{\partial \widehat{q}_1^{-s}}{\partial L_1^{Linear,-s}} \pi_1^{-s} \qquad \frac{\partial(\chi_1^s \widehat{q}_0)}{\partial \eta_2^{-s}} = \frac{E_1^{-s}}{E_1^s} \frac{\partial \widehat{q}_1^{-s}}{\partial \eta_2^{-s}} \pi_1^{-s}$$

$$\frac{\partial \widehat{q}_1}{\partial C_{F1}} = -\frac{1}{\left(\chi_2\right)^2} \left( \frac{G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)} \frac{\alpha_R}{\alpha_F} P_{F2}^* C_{F2} + \widehat{q}_2 \right) \frac{\partial \chi_2}{\partial C_{F1}}$$

$$\frac{\partial \widehat{q}_1}{\partial C_{F2}} = -\frac{1}{\left(\chi_2\right)^2} \left( \frac{G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)} \frac{\alpha_R}{\alpha_F} P_{F2}^* C_{F2} + \widehat{q}_2 \right) \frac{\partial \chi_2}{\partial C_{F2}} + \frac{1}{\chi_2} \frac{G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)} \frac{\alpha_R}{\alpha_F} P_{F2}^*$$

$$\frac{\partial \widehat{q}_1}{\partial E_2} = -\frac{1}{\left(\chi_2\right)^2} \left( \frac{G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)} \frac{\alpha_R}{\alpha_F} P_{F2}^* C_{F2} + \widehat{q}_2 \right) \frac{\partial \chi_2}{\partial E_2}$$

$$\frac{\partial \widehat{q}_1}{\partial \eta_2} = -\frac{1}{\left(\chi_2\right)^2} \left( \frac{G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)} \frac{\alpha_R}{\alpha_F} P_{F2}^* C_{F2} + \widehat{q}_2 \right) \frac{\partial \chi_2}{\partial \eta_2}$$

$$\frac{\partial \widehat{q}_1^s}{\partial L_1^s} = \frac{1}{\chi_2} \left\{ \frac{\left[-G''\left(1-L_1^{Linear}\right)\right]}{L_1^{Linear} + G\left(1-L_1^{Linear}\right)} - \frac{G'\left(1-L_1^{Linear}\right)\left[1-G'\left(1-L_1^{Linear}\right)\right]}{\left[L_1^{Linear} + G\left(1-L_1^{Linear}\right)\right]^2} \right\} \frac{\alpha_R}{\alpha_F} P_{F2}^* C_{F2}$$

$$\frac{\partial \widehat{P}_{R0}}{\partial C_{F0}} = \frac{\alpha_R}{\alpha_F} \frac{P_{F0}^*}{L_{-1}^{Linear} + G\left(1-L_{-1}^{Linear}\right)} \quad \frac{\partial \widehat{P}_{R1}}{\partial C_{F1}} = \frac{\alpha_R}{\alpha_F} \frac{P_{F1}^*}{L_0^{Linear} + G\left(1-L_0^{Linear}\right)} \quad \frac{\partial \widehat{P}_{R1}}{\partial L_0} = -\frac{\alpha_R}{\alpha_F} \frac{P_{F1}^* C_{F1}\left[1-G'\left(1-L_0^{Linear}\right)\right]}{\left[L_0^{Linear} + G\left(1-L_0^{Linear}\right)\right]^2}$$

## A.3 Numerical Solution

We start from the relevant set of planner constraints and FOCs from the preceding subsections. First, we select the set of policy instruments available to the planner:

- If all policy instruments (i.e., the policy rate, capital controls, FX intervention, and macroprudential controls) are available to the planner, then use all the FOCs above but set $\Upsilon_0 = \Upsilon_1 = \Xi_0 = \Xi_1 = \Sigma_0 = \Sigma_1 = 0$.
- If FX intervention is not permitted, then set $FXI_0 = FXI_1 = 0$, and remove the FOCs with respect to $FXI_t$.
- If capital controls and consumer macroprudential controls are not permitted, then use all the FOCs above but set $\Delta_0 = \Xi_0 = \Xi_1 = \Sigma_0 = \Sigma_1 = \Pi_1 = \Pi_2 = 0$.
- If housing sector macroprudential controls are not permitted, then use all the FOCs above but set $\Upsilon_0 = \Upsilon_1 = \Xi_0 = \Xi_1 = \Sigma_0 = \Sigma_1 = \Pi_1 = \Pi_2 = 0$.
- If capital controls and the domestic policy rate are not permitted, then use all the FOCs above but set $\Sigma_0 = \Sigma_1 = \Pi_1 = \Pi_2 = 0$.
- If consumer macroprudential controls and the domestic policy rate are not permitted, then use all the FOCs above but set $\Xi_0 = \Xi_1 = \Pi_1 = \Pi_2 = 0$.
- If the exchange rate is pegged, then use all the FOCs above but set $\Delta_0 = \Xi_0 = \Xi_1 = \Sigma_0 = \Sigma_1 = 0$.

Next, we characterize the solution numerically by running the following iterative process to convergence.

1. Fix guess on whether the banks' external borrowing constraint (20) is slack or binding in every period-1 state. For states where the constraint is slack, fix $\Psi_B = 0$ and remove the borrowing constraint. For states where the constraint is binding, set the borrowing constraint to be satisfied with equality and allow $\Psi_B \neq 0$. Run the following iterative process to convergence.
   - Fix guess on whether the housing sector borrowing constraint (21) is slack or binding in every period-1 state. For states where the constraint is slack, fix $\Psi_R = 0$ and remove the borrowing constraint. For states where the constraint is binding, set the borrowing constraint to be satisfied with equality and allow $\Psi_R \neq 0$. Run the following iterative process to convergence.
   - Verify that the housing sector borrowing constraint is slack for states where the constraint was guessed to be slack; otherwise, change the guess. Verify that $\Psi_R \geq 0$ for states where the constraint was guessed to be binding; otherwise, change the guess.
2. Verify that the household borrowing constraint is slack for states where the constraint was guessed to be slack; otherwise, change the guess. Verify that $\Psi_B \geq 0$ for states where the constraint was guessed to be binding; otherwise, change the guess.

## A.4 Ban on FX Positions

If domestically-owned intermediaries are prohibited from taking open FX positions, then the constrained planner problem changes:

$$\max_{\left\{C_{Ft},P_H,E_t,\eta_{t+1},FXI_t,L^{Linear}_{t-1}\right\}}\left\{\begin{array}{cc}\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t V\left(C_{Ft},\frac{E_tP^*_{Ft}}{P_H},\frac{E_tP^*_{Ft}}{P_H},L^{Linear}_{t-1}\right)\right] & \text{if PCP}\\ \mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t V\left(C_{Ft},\frac{E_tP^*_{Ft}}{P_H},\frac{P^*_{Ft}}{P_X},L^{Linear}_{t-1}\right)\right] & \text{if DCP,}\\ \text{with } P_X=P_X\left(C_{F0},\{C_{F1}\},\{C_{F2}\},E_0,\{E_1\},\{E_2\},P_H\right) & \end{array}\right\}$$

$$\begin{aligned}\left(1+i^*_{-1}\right)B_0 \le\ & P^*_{F0}\left[\omega C^*_0-C_{F0}\right]+P^*_{Z0}Z_0\\ &+\frac{P^*_{F1}\left[\omega C^*_1-C_{F1}\right]+P^*_{Z1}Z_1-FXI_0\left[\eta_1-(1+i^*_0)\right]}{\eta_1}\\ &+\frac{P^*_{F2}\left[\omega C^*_2-C_{F2}\right]+P^*_{Z2}Z_2-FXI_1\left[\eta_2-(1+i^*_1)\right]+B_3}{\eta_1\eta_2},\\ &\text{one equation per period-1 state } s_1\ \ [\Phi]\end{aligned}$$

$$\begin{aligned}\left(1+i^*_{-1}\right)B_0 \le\ & P^*_{F0}\left[\omega C^*_0-C_{F0}\right]+P^*_{Z0}Z_0\\ &+\frac{P^*_{F1}\left[\omega C^*_1-C_{F1}\right]+P^*_{Z1}Z_1-FXI_0\left[\eta_1-(1+i^*_0)\right]}{\eta_1}+\frac{\kappa_{H1}\frac{P_H}{E_1}}{\eta_1},\\ &\text{one equation per period-1 state } s_1\ \ [\Psi_B]\end{aligned}$$

$$\frac{\Gamma}{(1-\lambda)}\left(\begin{array}{c}\left(1+i^*_{-1}\right)B_0+FXI_0-S_0\\ -P^*_{F0}\left[\omega C^*_0-C_{F0}\right]-P^*_{Z0}Z_0\end{array}\right)=\mathbb{E}_0\left[\eta_1-(1+i^*_0)\right]\text{, a single equation }\ [\Omega_0]$$

$$\begin{aligned}&\frac{\Gamma}{(1-\lambda)}\left(\begin{array}{c}\left[\begin{array}{c}\left(1+i^*_{-1}\right)B_0\\ -P^*_{F0}\left[\omega C^*_0-C_{F0}\right]-P^*_{Z0}Z_0\end{array}\right]\eta_1+FXI_1-S_1\\ -P^*_{F1}\left[\omega C^*_1-C_{F1}\right]-P^*_{Z1}Z_1+FXI_0\left[\eta_1-(1+i^*_0)\right]\end{array}\right)\\ &=\eta_2-(1+i^*_1)\text{, one equation per period-1 state } s_1\ \ [\Omega_1]\end{aligned}$$

$$E^H_1\eta^H_1=E^L_1\eta^L_1\text{, a single equation }\ [\Lambda]$$

$$\begin{aligned}0\ge B^{Linear,s}_{R2}=\ &\chi^s_1\left[\left(1+i^*_{-1}\right)B^{Linear}_{R0}-\frac{\alpha_R}{\alpha_F}\frac{P^*_{F0}C_{F0}}{L^{Linear}_{-1}+G\left(1-L^{Linear}_{-1}\right)}L^{Linear}_{-1}\right]\\ &+\left\{\begin{array}{c}\frac{G'\left(1-L^{Linear}_0\right)}{L^{Linear}_0+G\left(1-L^{Linear}_0\right)}\frac{\alpha_R}{\alpha_F}\mathbb{E}_0\left[\frac{E_1}{E^s_1}P^*_{F1}C_{F1}\right]\\ +\mathbb{E}_0\left[\frac{1}{\chi_2}\frac{E_1}{E^s_1}\left(\frac{G'\left(1-L^{Linear}_1\right)}{L^{Linear}_1+G\left(1-L^{Linear}_1\right)}\frac{\alpha_R}{\alpha_F}P^*_{F2}C_{F2}+\widehat{q}_2\right)\right]\end{array}\right\}\left(L^{Linear}_0-L^{Linear}_{-1}\right)\\ &-\frac{P^*_{F1}C_{F1}}{L^{Linear}_0+G\left(1-L^{Linear}_0\right)}\frac{\alpha_R}{\alpha_F}L^{Linear}_0\\ &+\frac{1}{\chi_2}\left(\frac{G'\left(1-L^{Linear}_1\right)}{L^{Linear}_1+G\left(1-L^{Linear}_1\right)}\frac{\alpha_R}{\alpha_F}P^*_{F2}C_{F2}+\widehat{q}_2\right)\left(\left(1-\kappa_{L1}\right)L^{Linear}_1-L^{Linear}_0\right),\end{aligned}$$

one equation per period-1 state $s_1$ $[\Psi_R]$

$$\chi^s_{t+1} = \begin{cases} \eta^s_{t+1} & \text{if capital controls are not permitted} \\ \dfrac{\frac{\alpha_F}{P^*_{Ft} C_{Ft}}}{\beta \mathbb{E}_t \left\{ \frac{E^s_{t+1}}{E_{t+1}} \frac{\alpha_F}{P^*_{Ft+1} C_{Ft+1}} \right\}} & \text{if household macroprudential controls are not permitted} \end{cases}$$

where we define all the constraints in dollar terms, we use the superscript $s$ to refer to the state of nature, we fix the dollar value of initial debt repayments at $\left(1 + i^*_{-1}\right) B_0$, and we set $B_3 = B_0$.