Behavioral & Experimental Macroeconomics and Policy Analysis: a Complex Systems Approach

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NBER Behavioral Macroeconomics Research Boot Camp
Boston, 12 March 2020

[1] **Disclaimer:** the views expressed in this talk are solely those of the author and may differ from official Bank of Canada views

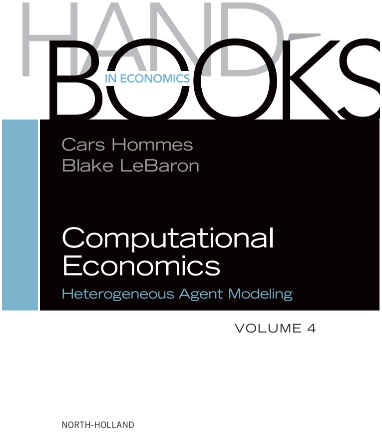
Examples of Complex Systems

Some Characteristics of Complex Systems

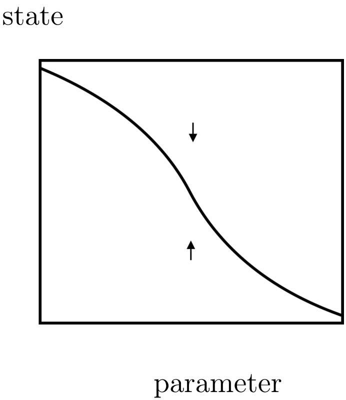
- **interactions** of particles/**heterogeneous agents** at **micro** level create patterns and structure at aggregate level (**emergent macro behaviour**); **More is different**
- **nonlinear** and **critical transitions**:
  small changes at micro-level may lead to **large and irreversible** changes at macro level
- complex economic systems: “the **particles can think**”
  agents learn and adapt their behavior, thus changing the laws of motion of the system
  How to model **(ir)rationality**?
  How to model **expectations in a complex environment?**
  Behavioural Theory in this talk: learning of simple, optimal heuristics in a complex, unknown environment

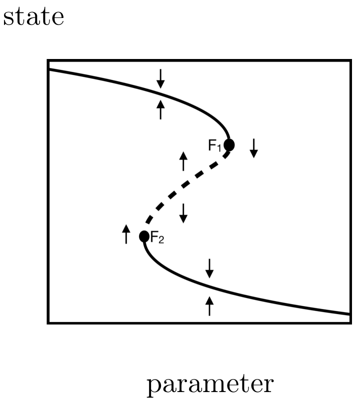
This talk focuses on stylized ‘few types’ models

But large literature on detailed **Agent-Based Models (ABMs)**



Key Feature Complex Systems: Critical Transitions between Multiple Equilibria; Tipping points





Plan of the Talk

Hommes, JEL 2020, forthcoming

Focus of the survey: boundedly rational expectations in stylized complex systems.

Five behavioural take-aways

- Complex/nonlinear systems exhibit **critical transitions** and **tipping points**
- Simple forecasting heuristics that make us smart
  - learning optimal homogeneous **AR(1)** rule
  - switching between heterogeneous **anchor and adjustment** rules
- Empirical validation of expectations through laboratory macro experiments
- Policy insight: how to **manage** complex economic systems?

Outline

Behavioral Learning Equilibrium (BLE)

Hommes and Zhu, JET 2014

- **simplest/parsimonious misspecification equilibrium**
- for each endogenous variable in the economy
  perceived law of motion (PLM) $\equiv$ AR1 process
  $\neq$ actual law of motion (ALM)
- **consistency requirements**: fixed point **observable statistics**
  - **unconditional mean** + **autocorrelation** of **PLM** $\equiv$
    unconditional mean + autocorrelation of **ALM**
- simple learning mechanism for parameters through
  sample autocorrelation learning to learn
  the **optimal** AR(1) heuristic

Simplest example: asset pricing model with AR(1) driving dividends

1-D linear model driven by autocorrelated shocks/fundamentals

p_t: price
y_t: driving dividends

$$\begin{cases} p_t = \frac{1}{R}\left[p^e_{t+1} + a + \rho y_t\right] + \delta_t \\ y_t = a + \rho y_{t-1} + \varepsilon_t, \end{cases} \quad (1)$$

δ_t, ε_t: i.i.d. noise

no noise case: $\delta_t \equiv 0$

Asset pricing model with linear AR(1) forecasts

- **Perceived law of motion (PLM)** of agents:
  AR(1) process
  $$p_t = \alpha + \beta(p_{t-1} - \alpha) + v_t$$
- α is the **mean**; β is **first-order autocorrelation**
- **Actual law of motion (ALM)**:

$$\begin{cases} p_t = \dfrac{1}{R}\left[\alpha + \beta^2(p_{t-1} - \alpha) + a + \rho y_t\right] + \delta_t \\ y_t = a + \rho y_{t-1} + \varepsilon_t \end{cases}$$

Rational Expectations Equilibrium

$$p_t^* = \frac{aR}{(R-1)(R-\rho)} + \frac{\rho}{R-\rho} y_t. \tag{3}$$

In **special case** when $\{y_t\}$ is i.i.d., i.e. $a = \bar{y}$ and $\rho = 0$, then

$$p_t^* = \frac{a}{R-1} = \frac{\bar{y}}{R-1}$$

first order ACF under rational expectations:

$$\text{Corr}(p_t^*, p_{t-1}^*) = \rho$$

Behavioral Learning Equilibrium (BLE)

Consistency requirements:
Mean and **first order autocorrelation** of price must satisfy

$$\bar{p} := \frac{\alpha(1-\beta^2)+\bar{y}}{R-\beta^2} = \alpha,$$
$$F(\beta) := \frac{\beta^2+R\rho}{\rho\beta^2+R} = \beta.$$

If $0 < \rho < 1$ and no noise ($\delta_t \equiv 0$) then there exists a **unique** behavioural learning equilibrium (BLE) (α^*, β^*), given by

$$\alpha^* = \frac{\bar{y}}{R-1} = \overline{p^*} \quad \textbf{(unbiased)}$$
$$\beta^* > \rho \quad \textbf{(persistence \& volatility amplification)}$$

no free parameters; optimal AR(1) rule

Unique BLE in asset pricing model; near unit root

no noise case

$$(\alpha^*, \beta^*) \;=\; (1.0, 0.997)$$

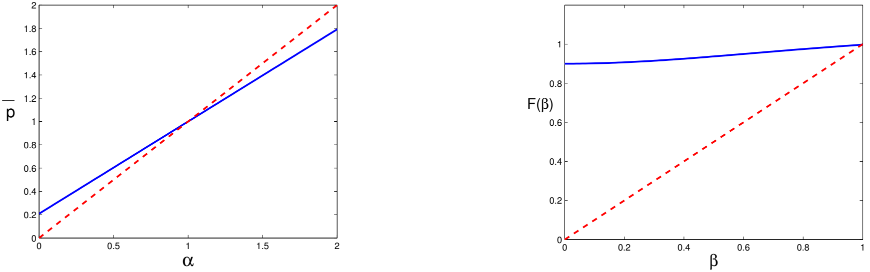


Figure: (a) α^* where mean $\bar{p} = \frac{\alpha(1-\beta^2)+\bar{y}}{R-\beta^2}$ intersects red diagonal α;
(b). β^*, where blue $F(\beta) = \frac{\beta^2+R\rho}{\rho\beta^2+R}$ intersects red diagonal;
parameters $R = 1.05$, $\rho = 0.9$, $a = 0.015$.

Sample Autocorrelation Learning (SAC-learning)

- SAC-learning: Hommes and Sorger (1998)

$$\alpha_t = \frac{1}{t+1}\sum_{i=0}^{t} p_i, \quad \beta_t = \frac{\sum_{i=0}^{t-1}(p_i - \alpha_t)(p_{i+1} - \alpha_t)}{\sum_{i=0}^{t}(p_i - \alpha_t)^2}$$

- PLM under SAC-learning:

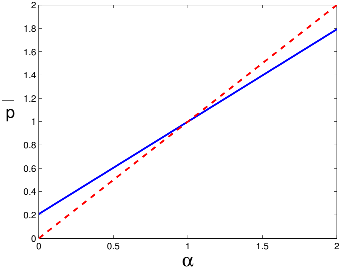
$$p_t = \alpha_{t-1} + \beta_{t-1}(p_{t-1} - \alpha_{t-1}) + v_t$$

- ALM under SAC-learning

$$\begin{cases} p_t = \frac{1}{R}\left[\alpha_{t-1} + \beta_{t-1}^2(p_{t-1} - \alpha_{t-1}) + a + \rho y_t\right], \\ y_t = a + \rho y_{t-1} + \varepsilon_t. \end{cases}$$

unique BLE stable under SAC-learning

$$(\alpha^*, \beta^*) \ = \ (1.0, 0.997)$$



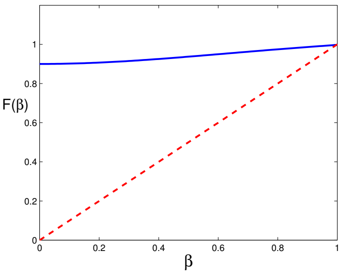


Figure: (a) α^* where mean $\bar{p} = \frac{\alpha(1-\beta^2)+\bar{y}}{R-\beta^2}$ intersects red diagonal α;
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parameters $R = 1.05$, $\rho = 0.9$, $a = 0.015$.

Simulation of SAC-learning

$\rho = 0.9;, \beta^* = 0.997$

Learning to believe in near-unit root

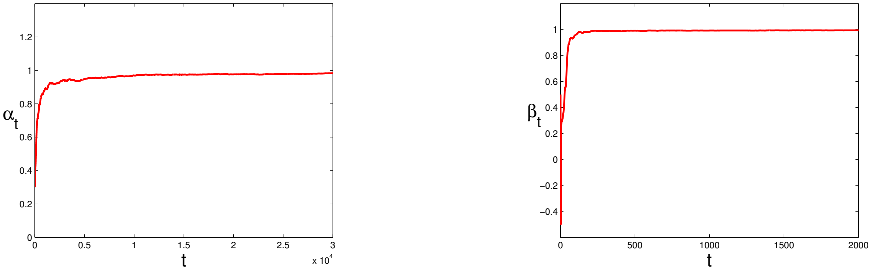


Figure: Time series of α_t and β_t under SAC learning.

- **Converging slowly to (unique) stable SCEE** $(\alpha^*, \beta^*) = (1.0, 0.997)$

Behavioral Learning Equilibrium

$\rho = 0.9;, \beta^* = 0.997$ (no noise case)

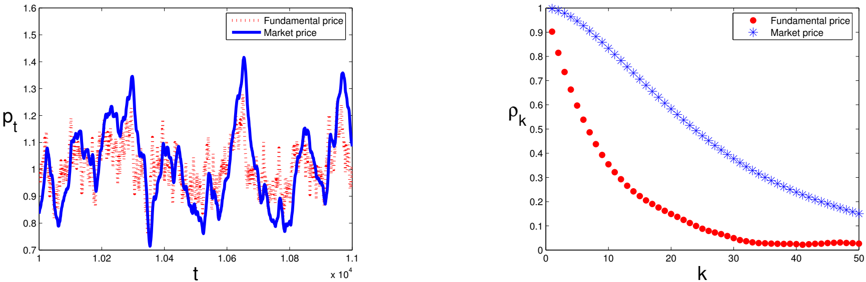
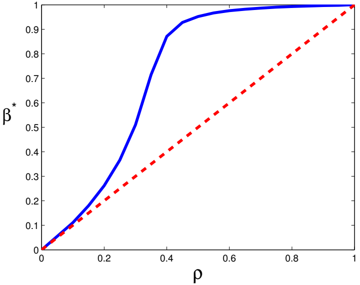


Figure: Time series of fundamental prices (red) and market prices (blue).

- **Market prices fluctuate around fundamental prices**
- **Persistence & volatility amplification**

Persistence & Volatility Amplification in Behavioral Learning Equilibrium

$\rho = 0.9;$, $\beta^* = 0.997$ (no noise case)



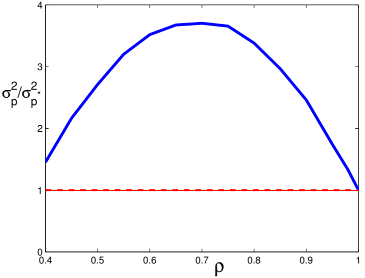
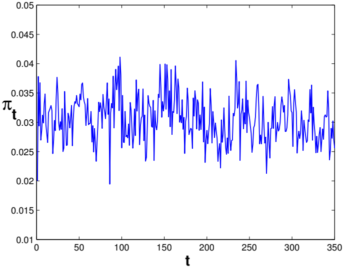


Figure: (a) SCEE β^* as a function of ρ;
(b) ratio of variance of market prices and variance of RE fundamental prices as a function of ρ.

Behavioral Equilibria with low and high persistence

co-existence of stable equilibria $\beta^* = 0.3066$ and $\beta^* = 0.9961$ (with noise δ_t)



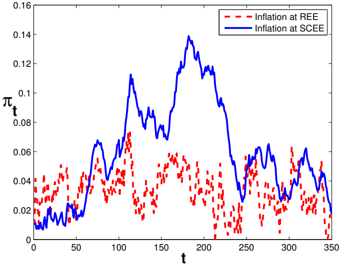


Figure: Convergence to low or high persistence equilibria β^* depending on initial states

Critical Transitions of Equilibria β^* depending on ρ

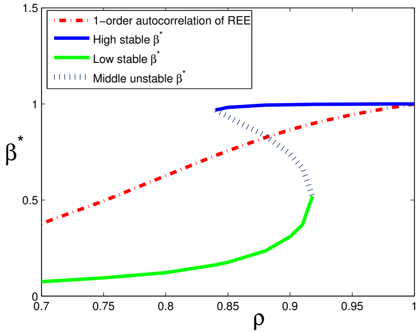
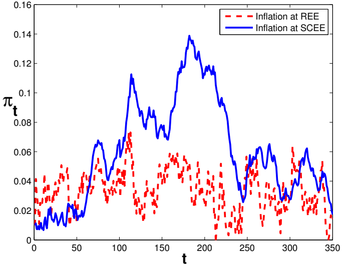


Figure: $\beta^*(1 \to 2 \to 3 \to 2 \to 1)$ as $\rho \uparrow$, where $\delta = 0.99, \gamma = 0.075, \frac{\sigma_u^2}{\sigma_\epsilon^2} = 0.1$.

High persistence BLE matches US inflation



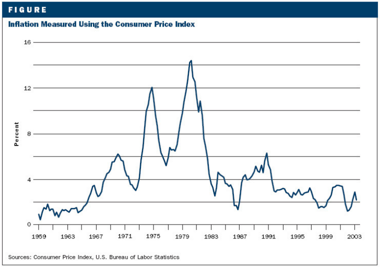


Figure: (a). Time series of inflation at stable SCEE $(\alpha^*, \beta^*) = (0.02, 0.995)$; (b). Empirical time series of inflation: Tallman (Federal Reserve Bank of Atlanta, ECONOMIC REVIEW, Third Quarter 2003).

Recent and ongoing work

Hommes, Mavromatis, Özden and Zhu, (2020)

- application and estimation of BLE in 3-Eq. NK-model
  optimal AR(1) rules for both inflation and output
- estimation of BLE in Smets-Wouters DSGE model
- **Relevance**: **simple behavioral learning equilibria** are important, because **coordination of expectations** may be more likely;
  (**different propagation mechanism** of shocks than under RE)
- **future extensions:** optimal AR(2) rule

$$p_t^e = \alpha + \beta_1 p_{t-1} + \beta_2 (p_{t-1} - p_{t-2})$$

Is there **trend-extrapolation** and **mean-reversion**?

Why Macro Experiments?

- If a theory does not work in the lab, why would it work in reality?
- A **macro experiment** studies **group behaviour** in a (simple) **complex system** in the lab, where aggregate behaviour depends on all individual **interactions** and decisions
- A **learning-to-forecast** experiment studies individual expectations and aggregate macro behaviour in simple **expectations feedback systems**
- **Main question:** do agents coordinate on **RE equilibrium** or on **behavioural learning** outcome?

Lucas, JPE, 1986 on Learning and Experiments

“Recent theoretical work is making it increasingly clear that the **multiplicity of equilibria** ... can arise in a wide variety of situations involving sequential trading, in competitive as well as finite agent games. All but a few of these equilibria are, I believe, behaviorally uninteresting: They do not describe **behavior that collections of adaptively behaving people** would ever hit on. I think an appropriate **stability theory** can be useful in weeding out these uninteresting equilibria ... But to be useful, stability theory must be more than simply a fancy way of saying that one does not want to think about certain equilibria. I prefer to view it as an **experimentally testable hypothesis**, as a special instance of the adaptive laws that we believe govern all human behavior.”

Positive versus Negative Feedback Experiments

Heemeijer et al. (JEDC 2009); Bao et al. (JEDC 2012)

- **negative feedback** (strategic substitute environment)

$$p_t = 60 - \frac{20}{21}[\sum_{h=1}^{6} \frac{1}{6} p^e_{ht}] - 60] + \epsilon_t$$

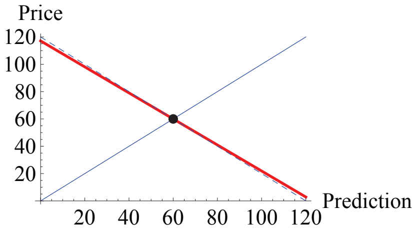
- **positive feedback** (strategic complementarity environment)

$$p_t = 60 + \frac{20}{21}[\sum_{h=1}^{6} \frac{1}{6} p^e_{ht} - 60] + \epsilon_t$$

- **common feature**: same RE equilibrium 60
- **only difference**: sign in the slope of linear map $+0.95$ vs -0.95

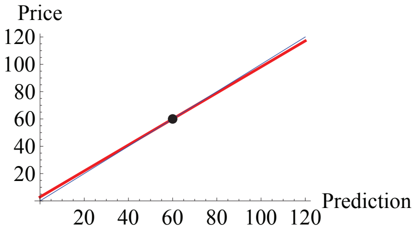
Feedback Mappings in LtFE

negative feedback



$$p_t = 60 - \frac{20}{21}\left(\overline{p_t^e} - 60\right) + \varepsilon_t$$

positive feedback



$$p_t = 60 + \frac{20}{21}\left(\overline{p_t^e} - 60\right) + \varepsilon_t$$

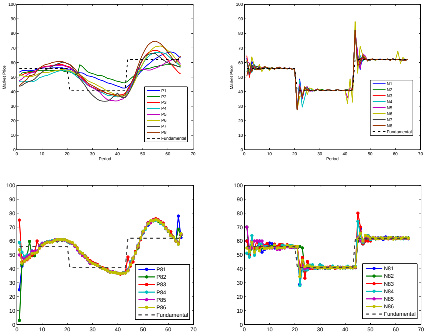
Concern with macroeconomic theory:
full information rational expectations ignores **almost self-fulfilling equilibria** in (strong) **positive feedback** systems

Positive vs Negative Feedback; Large Shocks

Bao, Hommes, Sonnemans, Tuinstra, JEDC 2012

positive FB (8 groups) **coordination failures**

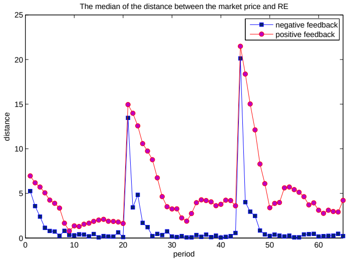
negative FB (8 groups) **coordination on RE**



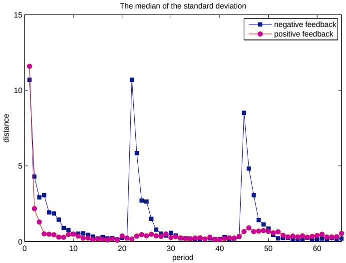
group 8, 6 individuals

Positive/Negative Feedback; Large Shocks

distance to RE price



degree of heterogeneity



positive feedback: quick coordination on 'wrong' price

negative feedback: slower coordination on correct RE price

Outline

Heuristics Switching Model

Brock and Hommes, ECMA 1997; Anufriev and Hommes, AEJ:Micro 2012

- agents choose from a number of simple **forecasting heuristics**
- **performance based reinforcement learning:**
  agents evaluate the **performances** of all heuristics, and tend to **switch** to more successful rules;

fractions of belief types are gradually updated in each period:
(discrete choice model with asynchronous updating)

$$n_{ht} = \delta n_{h,t-1} + (1-\delta)\frac{e^{\beta U_{h,t-1}}}{Z_{t-1}}$$

where Z_{t-1} is normalization factor.

- U_{ht} **fitness measure** (e.g. utility, forecasting errors, etc.)
- β is **intensity of choice**.
- δ **asynchronous** updating

Heuristic Switching Model: four forecasting heuristics

Anufriev and Hommes, AEJ:Micro 2012

- **adaptive expectations** rule, $[w = 0.65]$

  ADA $\quad p^e_{1,t+1} = 0.65\, p_{t-1} + 0.35\, p^e_{1,t}$

- **weak trend-following** rule, $[\gamma = 0.4]$

  WTR $\quad p^e_{2,t+1} = p_{t-1} + 0.4\,(p_{t-1} - p_{t-2})$

- **strong trend-following** rule, $[\gamma = 1.3]$

  STR $\quad p^e_{3,t+1} = p_{t-1} + 1.3\,(p_{t-1} - p_{t-2})$

- **anchoring and adjustment heuristic** with learnable anchor

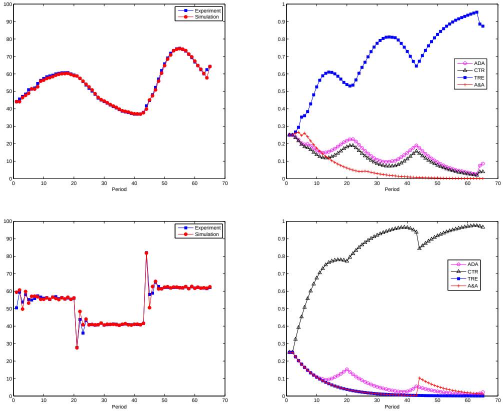
  LAA $\quad p^e_{4,t+1} = \frac{1}{2}\left(p^{av}_{t-1} + p_{t-1}\right) + (p_{t-1} - p_{t-2})$

Problem: but where do these 4 rules and their coefficients come from?

Positive vs Negative Feedback; Large Shocks

Heuristics Switching Model Simulations

prices strategy frequencies



positive feedback: trend-followers amplify fluctuations

Outline

Learning First Order Forecasting Heuristic

Simple heuristics that make us smart (Anufriev et al., 2019)

Agents learn two **parameters** of linear heuristic?

- Agent i uses a **first order forecasting heuristic** h to predict p_t:

  anchor and adjustment rule

$$p^e_{i,h,t} = \alpha_{i,h,t} p_{t-1} + (1 - \alpha_{i,h,t}) p^e_{i,t-1} + \beta_{i,h,t}(p_{t-1} - p_{t-2}).$$

- The rule h requires **two** parameters: an **anchor** $\alpha_{i,h,t}$ and a **trend** $\beta_{i,h,t}$
- General constraint: $\alpha \in [0, 1]$, $\beta \in [-1.1, 1.1]$.
- The rule generalizes popular HSM heuristics: naive, adaptive expectations and trend extrapolation.
- **RE**: $\alpha = 0$, $\beta = 0$, $p^e_{i,t-1} = p^f$.

Learning by GA's through simple heuristics

Simple heuristics that make us smart (Anufriev et al., 2019)

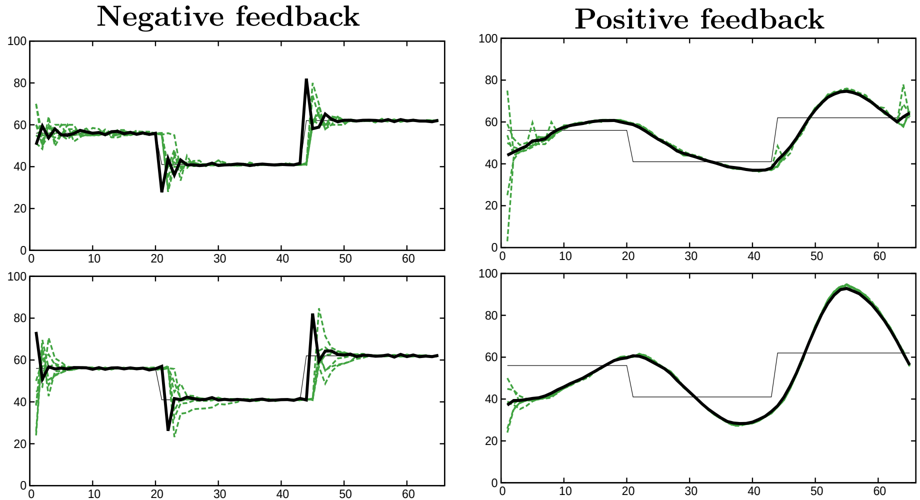
- Every agent has a list of $H = 20$ different heuristics (α, β).
- When agent i learns the last realized price p_{t-1}, she tries to re-optimize the rules with GA evolutionary operators:
    1. sample (with replacement) 20 new heuristics from the old depending on their *hypothetical forecasting performance* (**reproduction**); **(survival of the fittest)**
    2. **mutation:** with some small probability "mutate" them (modify (α, β) of each heuristic);
    3. **election:** compare the new and the old heuristics in terms of their hypothetical forecasting performance – pick the better ones.

Process mimics natural selection: worse forecasting heuristics are likely to be *replaced* by better; *inefficient experimentation* screened out.

Remark: agents learn *independently*.

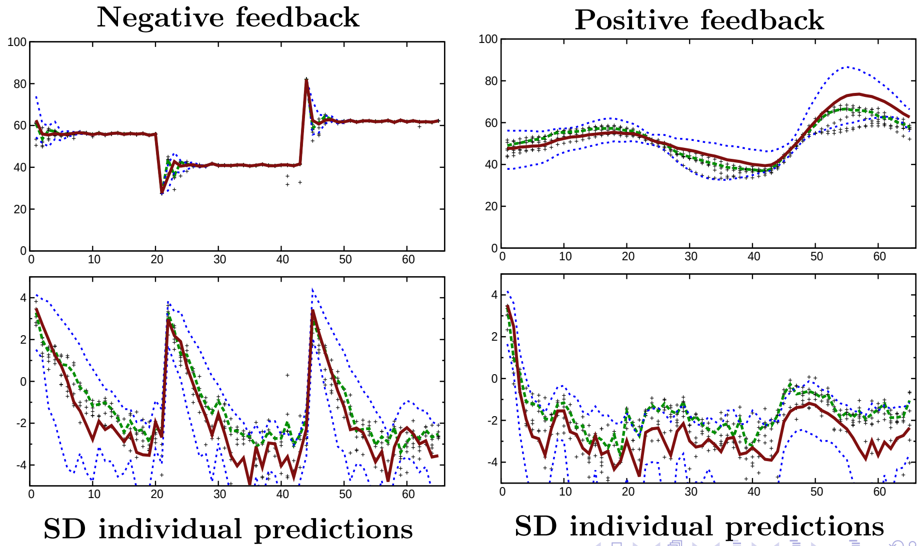
Lab experiment (top) and 65-period simulations (bottom)

experimental data Bao et al. (2012)



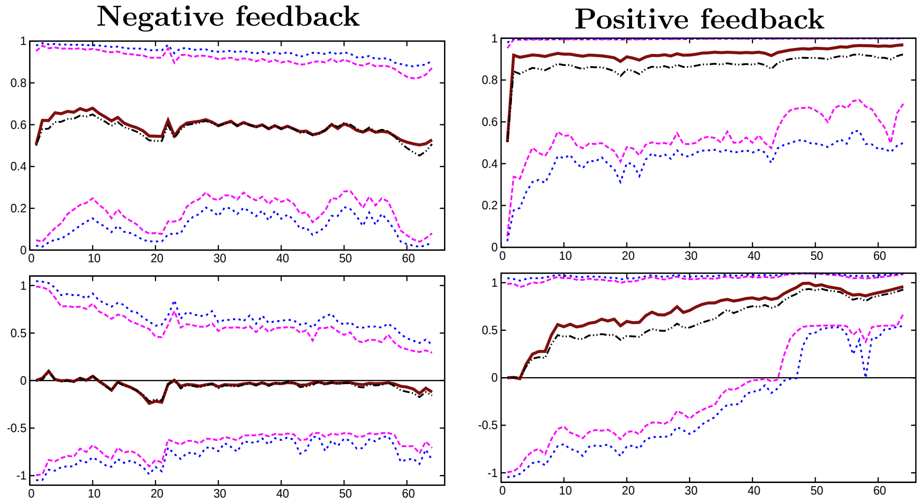
65-period ahead Monte Carlo simulations (1000)

experimental data Bao et al. (2012)



Anchor α_t (top) and trend β_t (bottom) parameters

experimental data Bao et al. (2012)



Average Heuristics

Under **negative feedback** agents learn to use **adaptive expectations**:

$$p^e_{i,t} \approx 0.5p_{t-1} + 0.5p^e_{i,t-1}$$

Under **positive feedback** agents learn to become **trend-follower**:

$$p^e_{i,t} \approx 0.95p_{t-1} + 0.05p^e_{i,t-1} + 0.9(p_{t-1} - p_{t-2})$$

Outline

Housing market experiment with **positive** versus **negative feedback**

Bao and Hommes, 2019

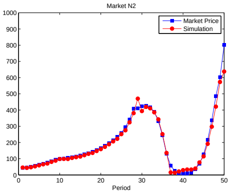
$$p_t = \frac{1}{1+r}[(1-c)\overline{p}^e_{t+1} + \overline{y}] + \nu_t, \qquad \lambda = \frac{1-c}{1+r}$$

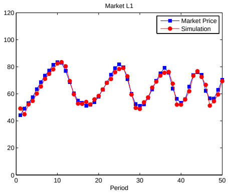
Behavioural intuition: an increase of housing supply (parameter c) adds **negative feedback** to the system, **weakening the overall positive feedback** (through speculators) making the system more stable

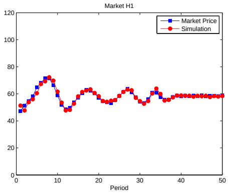
Managing Positive Feedback through Negative FB Policy

Housing Market Experiments, Bao and Hommes, 2019

no FB policy	weak negative FB policy	strong negative FB policy
large bubble	**oscillations**	**stable**
$(\lambda = 0.95;\ r = 5\%)$	$(\lambda = 0.85;\ r = 18\%)$	$(\lambda = 0.71;\ r = 40\%)$





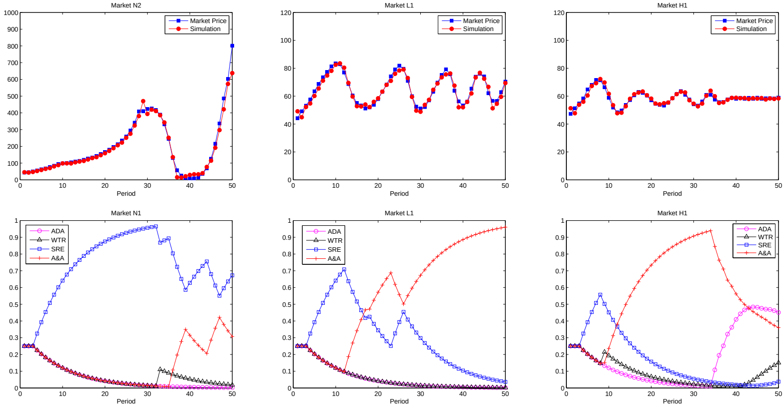


adding negative FB **stabilizes** complex positive FB system

Note: policy under RE: **do not interfere**

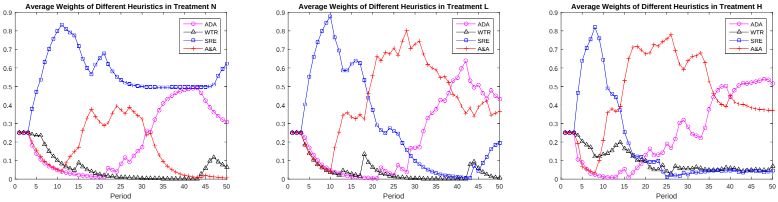
Simulated 1-period ahead forecasts HSM

strong positive FB	medium positive FB	weak positive FB
$\lambda = 0.95$	$\lambda = 0.85$	$\lambda = 0.7$



Average simulated 1-period ahead forecasts HSM

strong positive FB $\lambda = 0.95$ | medium positive FB $\lambda = 0.85$ | weak positive FB $\lambda = 0.7$



Policy Implication: negative FB policies that weaken the overall positive feedback may **stabilize** markets by preventing coordination on trend-following behaviour

Outline

Five Behavioral Take-aways

- complex systems (non-linearity, heterogeneity, etc.) exhibit **critical transitions** between **multiple equilibria**
- adaptive learning of optimal AR(1) rule generates **near-unit root** and **excess volatility** and **persistence amplification**
- parsimonious **heterogeneous expectations** switching model based on relative performance
- heuristics switching between anchor and adjustment rules **fits experimental** & **empirical** data well
- 'negative feedback' policies can affect the self-organisation process, **prevent coordination on trend-following behaviour** and stabilize complex markets

Open Questions

- optimal AR(1) versus optimal **AR(2)**
  short-run **trend-extrapolation** versus average **mean-reversion**
  Which data are better explained by learning AR(2)?
- **homogeneous** versus **heterogeneous** expectations
  homogeneous AR(2) versus heterogeneous switching
  between mean-reverting rule and trend-extrapolating rule
  Are **bubbles and crashes** better explained by heterogeneous agents model?
- **optimal policy** under simple behavioral **forecasting heuristics**

Thank you very much!
Good luck with your thesis on behavioral macro

Open Questions

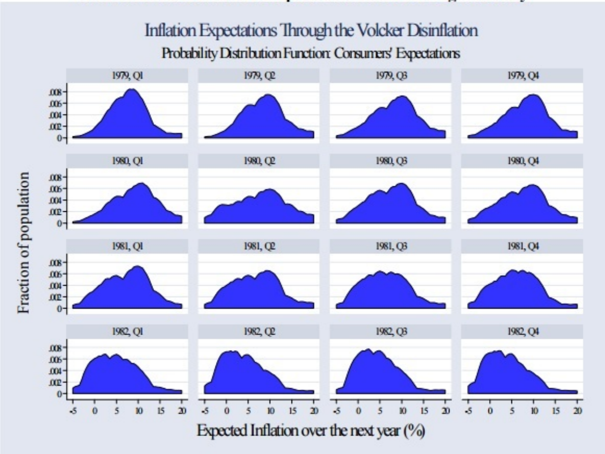
- optimal AR(1) versus optimal **AR(2)**
  short-run **trend-extrapolation** versus average **mean-reversion**
  Which data are better explained by learning AR(2)?
- **homogeneous** versus **heterogeneous** expectations
  homogeneous AR(2) versus heterogeneous switching
  between mean-reverting rule and trend-extrapolating rule
  Are **bubbles and crashes** better explained by heterogeneous agents model?
- **optimal policy** under simple behavioral **forecasting heuristics**

Thank you very much!
Good luck with your thesis on behavioral macro

Survey of Professional Forecasters: bimodal distribution

Mankiw, Reis and Wolfers, 2003

Figure 12: The Volcker Disinflation:
The Evolution of Inflation Expectations in the Michigan Survey



NK model with fundamentalists versus naive

Cornea-Madeira, Hommes and Massaro, JBES 2017

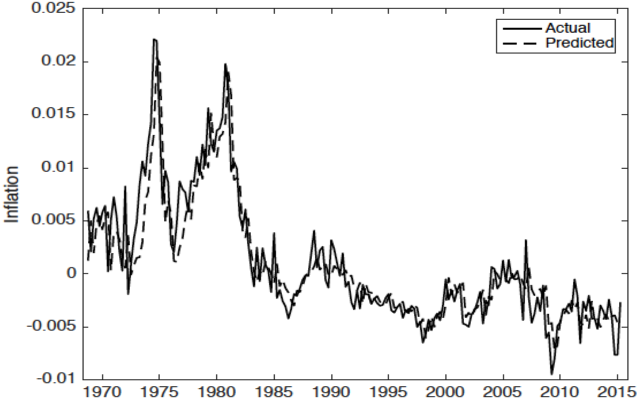
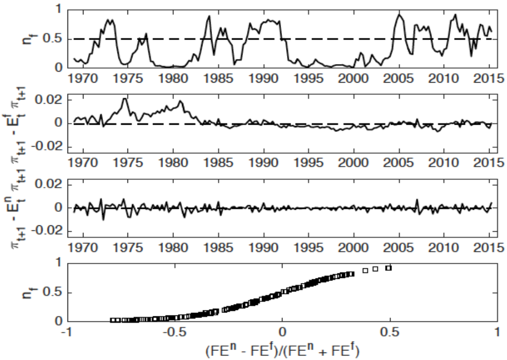


Figure: Actual vs. predicted inflation

NK model with fundamentalists versus naive

Cornea-Madeira, Hommes and Massaro, JBES 2017

Evolution of weight of fundamentalists $n_{f,t}$



on average more **backward looking** agents

Mean	**0.353**
Median	0.276
Maximum	0.924
Minimum	0.019
Std. Dev.	0.282
Skewness	0.418
Kurtosis	1.720
Auto-corr. Q(-1)	0.887

Top panel: Time series of the fraction of fundamentalists $n_{f,t}$
Second panel: Distance between actual and fundamental inflation
Third panel: Distance between inflation and naive forecast
Bottom: Scatter plot $n_{f,t}$ vs relative forecast error naive rule

NK model with fundamentalists versus naive expectations estimated on survey data professional forecasters

Cornea-Madeira, Hommes and Massaro, JBES 2017

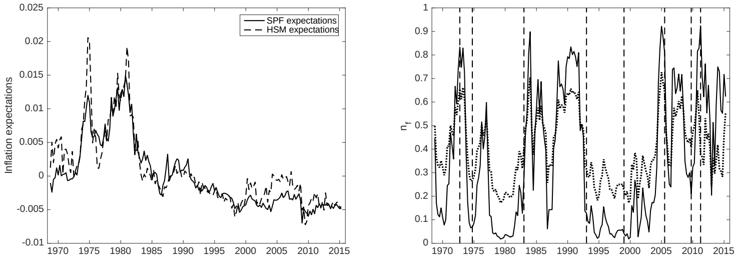
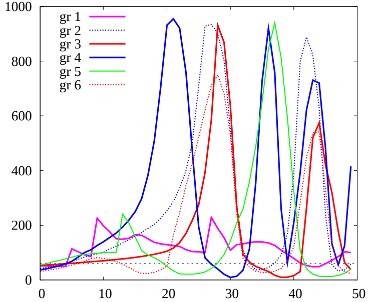


Figure: SPF forecasts vs. HSM expectations and estimated structural breaks with fractions of fundamentalists for inflation and SPF. **SPF switch slower than inflation expectations**

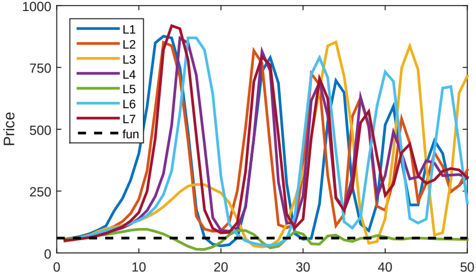
What happens without fundamental robot traders?

Hommes, Sonnemans, Tuinstra, vd Velden, JEBO 2008; Bao et al, 2016

groups of 6 subjects



groups of 25-30 subjects



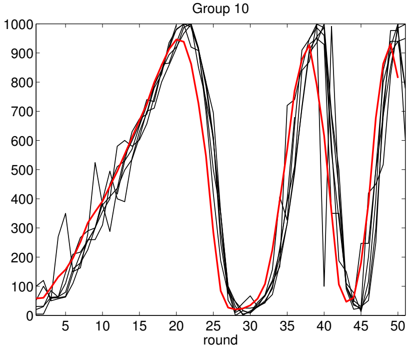
Does **positive feedback** cause **instability**?

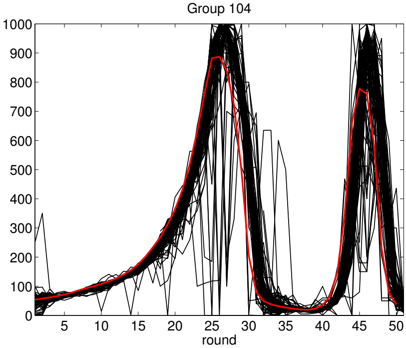
Coordination on bubbles in even larger groups

IBSEN Horizon 2020; Hommes, Kopanyi-Peuker, Sonnemans, 2018

group of 6 subjects

group of 100 subjects





Are bubbles caused by (strong) positive feedback?

Switching model estimated on housing markets

Bolt et al., JEDC 2019

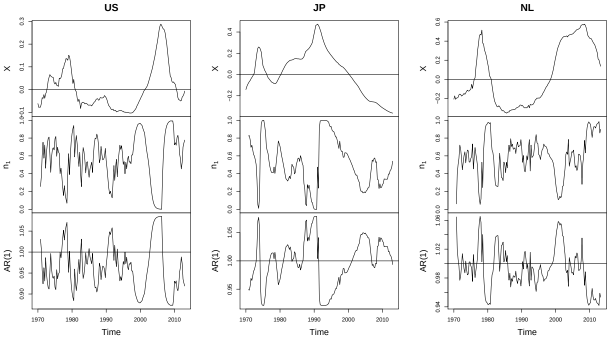


Figure: **Top panels**: relative house price deviations X_t from fundamentals; **Middle panels**: time-varying fractions of mean-reverting fundamentalists; **Bottom**: estimated market sentiment as time-varying AR(1) coefficient.