Firm Wages in a Frictional Labor Market*

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Abstract

This paper studies a directed search model of multi-worker firms facing within-firm equity constraints on wages. I show that introducing such constraints leads to lower wages, as the firms' incentive to profit from their existing workers via low wages depresses also those of new hires, increasing the profitability of hiring. In a dynamic model, these elements further give rise to a time-inconsistency in the firm problem affecting allocations, as the firm effectively faces a less elastic labor supply in the short than the long run. To consider outcomes when firms reoptimize wages each period in the face of this time-inconsistency, I consider Markov perfect equilibria, also proposing a tractable solution approach to the problem. In two applications, I show that the constraints dampen wage variation over the business cycle and can amplify that in unemployment in a significant way. Second, firms may find it profitable to fix wages for a period of time, and an equilibrium with fixed wages be good for worker welfare, as well as resource allocation, despite added volatility in the labor market.
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1 Introduction

Large firms play an important role in the labor market.[1] Motivated by literature in personnel economics characterizing worker compensation in large organizations as governed by administrative rules reflecting horizontal equity concerns, this paper thinks about firm wage setting in an environment where firms face within-firm equity constraints on wages. I show, in the context of a directed search model of multi-worker firms, that introducing such constraints leads to lower wages, as the firms' incentive to profit from their existing workers via low wages depresses also those of new hires, increasing the profitability of hiring. In a dynamic model, these elements further give rise to a time-inconsistency in the firm problem affecting allocations, as the firm effectively faces a less elastic labor supply in the short than the long run. To analyze outcomes when firms reoptimize wages each period in the face of this time-inconsistency, I consider Markov perfect equilibria, proposing a tractable solution approach that allows studying the implications of the model for the dynamics of wages and hiring in response to shocks. Two applications show that such cross-worker constraints can give rise to rigidity/stickiness in wages.

I study a labor market with search frictions and competitive search (Moen 1997), where firms employ a measure of workers and must pay all their (equally productive) workers the same. I refer to such constraints as firm wage constraints. I begin by showing, in the context of a static model, that introducing such constraints alters the tradeoffs firms face in choosing a wage to offer. In competitive search, firms set wages to resolve a tradeoff between the wage and vacancy costs of hiring: offering a higher wage increases hires per vacancy, but at the cost of having to pay those hires more. With firm wage constraints, this decision is influenced by the firm's incentive to profit from its existing workers via low wages, causing the firm to set a lower wage instead. With all firms affected, the equilibrium shifts toward lower wages in a way that hurts workers and benefits firms, encouraging vacancy creation and leading to overhiring in equilibrium. As equilibrium allocations absent constraints are socially optimal, the constraints also imply a departure from efficiency.[2]

I then show, in the context of a dynamic infinite horizon model, that the firm's wage-setting problem involves a time-inconsistency affecting allocations. In the initial period, the firm's incentive to profit from its existing workers again leads the firm to set lower wages

[1] Seventy percent of US private employment is in firms with 50 employees or more (Moscarini and Postel-Vinay 2012).

[2] The efficiency of the competitive search equilibrium is discussed, e.g., by Rogerson, Shimer, and Wright (2005). The overhiring happens for a different reason than in the literature on multi-worker firms with random search and bargaining, where it arises due to decreasing returns in technology.

than an unconstrained firm would. The firm optimally plans on higher wages in future periods, however, if it can commit to such wages when making plans. To understand the firm's differing incentives in setting wages over time, note that the firm effectively faces a less elastic labor supply in the short run because it inherits a set of existing workers that are (to a degree) locked in due to the frictions and taken as given by the firm. Meanwhile, in making plans for future periods the firm does not treat its future workforce as exogenous, leaving future labor supply more elastic.[3] Of course, commitment to future wages is necessary to implement the plan, as the firm would otherwise again choose lower wages ex post.

While an unconstrained firm also faces a similar incentive to profit from its existing workers via low wages, it is due to the within-firm constraints that these incentives affect allocations here. An unconstrained firm is able to circumvent the incentive to cut the wages of existing workers ex post by front-loading compensation into the hiring period.[4] As a result, the unconstrained firm does not value commitment to future wages, nor does such commitment affect allocations.[5] It is the firm wage constraints that – in preventing the firm from paying new hires more than existing workers – simultaneously make commitment to future wages valuable, as well as something that matters for allocations.

To consider outcomes when firms cannot commit to future wages, I study Markov perfect equilibria, also offering a tractable solution approach to the problem. Analyzing Markov perfect equilibria in an environment with a time-inconsistency can be challenging because the decision-maker's objective does not coincide with maximizing his/her value function, which means that standard dynamic programming arguments cannot be directly applied. An approach that has been developed for characterizing differentiable Markov perfect equilibria involves deriving a *generalized* Euler equation, which spells out the tradeoffs faced by the decision-maker, as well as serves as a basis for solving the problem numerically. Analyzing the generalized Euler equation remains more involved, however, due to the dependence of this functional equation on the derivative of choice variables with respect to the state.[6] To

[3]The time-inconsistency of the dynamic firm problem is reminiscent of that in optimal capital taxation (Chamley 1986, Judd 1985) in that the firm prefers to tax labor (via low wages) more in the short run, where labor supply is less elastic.

[4]Note that the competitive search equilibrium always requires some degree of commitment, to allow for a tradeoff between what is posted and the ensuing market tightness. In line with this, I assume throughout that the firm has commitment to the current period wage.

[5]Commitment to future wages does matter for the timing of wage payments for the unconstrained firm as well: without commitment to future wages the firm would optimally pay as little as possible after the hiring period (the worker's opportunity cost), front-loading compensation into the hiring period. With linear preferences this need not have allocative effects, however.

[6]Time-inconsistencies appear in multiple contexts, due to either preferences directly or the economic environment, such as in problems of optimal fiscal or monetary policy. See Klein, Krusell, and Rios-Rull (2008) for a discussion on characterizing Markov perfect equilibria in problems with time-inconsistency, in

avoid this complication, I consider equilibria that are consistent with the size-independence of the firm problem, which implies that the firm's decisions are independent of the relevant endogenous state – firm size – and thus a standard Euler equation approach can be used. In addition to simplifying solving the model, this approach allows incorporating stochastic shocks without difficulty.

I then use the model in two applications. The first studies how such within-firm constraints alter the cyclical behavior of wages and unemployment relative to the unconstrained case. Intuitively, one might expect such constraints to dampen wage increases in expansions, if the downward pressure on wages associated with the constraints strengthens as the share of already matched workers in the labor market rises (and vice versa for recessions). The second application considers whether firms in this environment would prefer to fix wages – adopting a simple fixed wage rule over discretion – as well as the equilibrium implications of all firms doing so.

To study the impact of firm wage constraints on business cycle variation in wages and unemployment, I compare shock propagation in the firm wage model (without commitment) with the unconstrained case. Wages in the firm wage model are less responsive to shocks, leading to amplification in the vacancy-unemployment ratio. Parameterizing the constrained and unconstrained models to the same steady state, the amplification in labor market flows associated with firm wages can be substantial, with a tenfold increase in the response of the vacancy-unemployment ratio to the shock relative to the unconstrained case. Overall, this allows the model mechanism to explain roughly a third of the observed variation in the vacancy-unemployment ratio.

To study the profitability and equilibrium implications of infrequent wage adjustment, I extend the model to allow firms to commit to a simple wage rule of a fixed wage for a probabilistic period of time. In the context of this extended firm wage model, I show that a single firm deviating to a fixed wage when other firms reoptimize each period chooses a higher wage and grows faster, due to being more forward-looking in its wage setting. In particular, firm value increases as a result of the commitment involved, something that holds also in the presence of shocks, even though the fixed wage limits the firm's ability to respond to them.

Concluding that fixing the wage is profitable for firms, I then consider equilibrium outcomes when all firms fix wages for a probabilistic period of time, in a staggered way. I show that longer wage durations work to undo the equilibrium effects of firm wages: By

the context of a study of optimal government spending.

making firms more forward-looking, they raise the level of wages, shifting the labor market equilibrium to make workers better off, while reducing overhiring, thus improving the efficiency of resource allocation. Moreover, these effects hold also in the presence of aggregate shocks, despite the added volatility in the labor market associated with longer wage durations. Thus, in an environment characterized by firm wage constraints, fixed wages may be welfare-improving, despite the seeming "rigidities" in the labor market.

The firm wage model also accommodates firm-specific idiosyncratic shocks. In a stationary equilibrium with firm heterogeneity, more productive firms offer higher wages and grow faster than less productive ones, giving rise to cross-sectional dispersion in wages and a large firm wage premium (Brown and Medoff 1989, Mortensen 2003). I show that firm wages dampen the responses of wages to firm shocks also, amplifying those of firm growth, and that the effects of fixed wages carry over to a setting with non-trivial firm risk as well.

Related Literature This paper is related to several strands of existing literature. The within-firm constraints relate to literature characterizing worker compensation within firms as governed by internal pay structures. Such structures involve a hierarchy of positions within the firm, with horizontal equity concerns typically viewed as limiting wage differences within levels of the hierarchy.[7] Bewley (1999) finds that worker compensation in firms with 50 employees or more is generally governed by formal structures of this nature, largely motivated by managers by internal equity concerns.[8] Such structures have also been argued to be important for explaining worker compensation within firms: Baker, Gibbs, and Holmstrom (1994) offer an early case study, arguing that the hierarchy explains the bulk of wage differences within their firm. More recently, Lazear and Oyer (2004) and Bayer and Kuhn (2018) provide evidence that incorporating information on hierarchies allows explaining as much 80 percent of the cross-sectional variation in wages – a substantial increase over cross-sectional wage regressions that typically explain only about a third of this variation with observables.

More than the specific form of the structure (which could involve explicit tenure premia

[7]See, e.g., Baker, Jensen, and Murphy (1988). More recently, Lazear and Shaw (2007) discuss compression in firm pay structures, arguing that managers and human resource professionals within firms generally view compensation as more compressed than output, with the objective of making pay more equitable, while noting that measurement remains difficult for lack of systematic data on individual output. Lazear and Oyer (2012) state that a look inside firms shows that actual wage dynamics are driven largely by the jobs people hold, while individual jobs have a fairly narrow band of possible wages.

[8]Of course, equity concerns may be at play in small organizations as well. A growing body of evidence emphasizes that workers are concerned with how their wages compare with peers (Card, Mas, Moretti, and Saez 2012, Bracha, Gneezy, and Loewenstein 2015, Breza, Kaur, and Shamdasani 2018, Dube, Giuliano, and Leonard 2019). According to these authors, workers appear to prefer equal treatment with peers, and wage differentials to reduce effort and output, as well as lead to quits and withholding participation.

for example), what is relevant here is the notion of a policy that systematically connects the wages of different workers within the firm, influencing how the firm sets them. If new hires must be brought in on similar terms as comparable existing workers – in line with the structure – hiring wages will be influenced by the firm's incentive to profit from their existing workers.[9] The question then becomes: what are the implications for equilibrium outcomes in the labor market?

This question is becoming increasingly timely as many US states are recently reinforcing pay equity laws seeking to prohibit employers from discriminating among employees – formally requiring employers to pay the same wage to employees performing substantially similar work.[10] Such measures force employers to be increasingly transparent in their wage setting practices, encouraging making compensation decisions more systematic and documented across the board. In terms of the implications, the present study cautions that imposing stringent regulation on firms in this regard may come with unexpected costs for workers, via reduced pay as well as added volatility in the labor market.

The paper is related to an existing literature studying models of multi-worker firms in frictional labor markets. Some of these models feature random search, such as the bargaining models of Acemoglu and Hawkins (2014) and Elsby and Michaels (2013), as well as the canonical model of wage dispersion of Burdett and Mortensen (1998) and its dynamic extensions by Moscarini and Postel-Vinay (2013, 2016). Others feature directed search, as in Rudanko (2011), Kaas and Kircher (2015) and Schaal (2017). While the models with random search mentioned also feature firm wages in practice, the models with directed search do not – making it interesting to study the implications of imposing such constraints relative to the unconstrained benchmark.

Note that despite similarities between modeling frameworks, the existing work on models of multi-worker firms with random search and bargaining has not emphasized a related time-inconsistency affecting the firm problem.[11] Although firms in both random and directed search have an incentive to profit from their existing workers through low wages ex post, commitment plays more of a role in directed search in the following sense: In directed search, the firm's wage setting problem centers around the tradeoff it faces between its offered wages and hiring rate, with commitment to those wages playing a key role as the firm has an

[9] Note that this view aligns with literature arguing that the wages of new hires are equally cyclical as those of existing workers (Gertler and Trigari 2009, Hagedorn and Manovskii 2013, Gertler, Huckfeldt, and Trigari 2019), conditional on match quality.

[10] See, e.g., Biro (2018) and Martinez (2019).

[11] The literature on the Burdett-Mortensen framework does recognize the role of commitment for outcomes: Moscarini and Postel-Vinay (2013, 2016) assume full commitment to future wages, while Coles (2001) considers the implications of relaxing it.

incentive to promise high wages ex ante but not pay them ex post. In random search, on the other hand, an individual firm has an incentive to pay little both ex ante as well as ex post, because its wages have no impact on its hiring rate (as long as they exceed the worker's opportunity cost).[12]

Given the central role played by commitment in directed search, it becomes interesting to consider the implications of relaxing the assumption of full commitment to future wages made in the literature. The present paper thus extends the existing work on multi-worker firms with directed search in two ways: it incorporates within-firm constraints on wages and relaxes the assumption that firms have full commitment to future wages. And as discussed, relaxing commitment becomes particularly interesting in the presence of constraints, as they prevent the firm from side-stepping the issue by front-loading pay.

The first application is motivated by the long-standing puzzle facing macroeconomists of why wages vary so little while unemployment varies so much over the business cycle, and a question of whether within-firm constraints on wages could play a role in generating rigidity in wages over time. In the context of search models it is related to the literature on the unemployment volatility puzzle discussed by Hall (2005) and Shimer (2005) and a literature that followed. This literature has sought mechanisms generating amplification in the responses of unemployment and vacancy creation to shocks, typically via rigidity in wages.[13] In an early contribution in this vein, Menzio (2005) also sought to think about the implications of firm wages for labor market dynamics. His work considers a random search model with on-the-job search where firms have private information about their productivity. The present paper, on the other hand, abstracts from on-the-job search and asymmetric information, highlighting a commitment problem arising in the context of competitive search, and its implications.[14]

The second application is motivated by the observation that wages adjust relatively infrequently relative to labor market flows, and a related modeling tradition imposing fixed wages with staggered adjustment (Taylor 1999, 2016).[15] In the spirit of work studying the

[12] Of course, equilibrium outcomes do depend on how wages are set across firms in random search as well. If firms in random search can freely set wages ex post, they will be driven to the workers' opportunity cost. There is no reason for an individual firm to depart from doing so, or plan to depart from doing so ex ante.

[13] See, e.g., Rogerson and Shimer (2011) for a discussion.

[14] Snell and Thomas (2010) also consider the implications of equity concerns for the cyclical behavior of wages, in a (non-search) framework where equity concerns combine with the motive of risk-neutral firms to insure risk-averse workers, resulting in wage rigidity.

[15] Barattieri, Basu, and Gottschalk (2014) and Grigsby, Hurst, and Yildirmaz (2019) document average durations of wages between 4 and 8 quarters. For European countries, Lamo and Smets (2009) report an average duration of wages of 15 months. Clearly, wage adjustment is less frequent than the monthly, or even weekly, frequencies labor market flows vary at.

tradeoffs between rules and discretion in settings with time-inconsistencies (Athey, Atkeson, and Kehoe 2005, Amador, Werning, and Angeletos 2006), I consider whether the time-inconsistency in the firm problem could be viewed as motivating the adoption of fixed wage rules.

In the labor market context the application relates to the work of Gertler and Trigari (2009), who study the impact of sticky wages on business cycles in unemployment and vacancy creation in a random search model of multi-worker firms that rebargain wages only when a Calvo draw allows it. While they argue that firms in their context may find the added volatility associated with sticky wages profitable due to convexities in profits, they largely refrain from relating equilibrium outcomes to socially optimal ones.[16] The present paper argues that in the context of directed search, where a firm's offered wages also influence job seeker behavior, there may be a stronger argument for fixed wages due to the time-inconsistency in the firm objective: not only can longer wage durations be profitable for firms, they may also be desirable from both a worker and planner perspective (in a second best sense).

The paper is organized as follows. Section 2 begins with a one-period model to illustrate the static tradeoffs involved with firm wages, while Section 3 turns to a dynamic infinite horizon model to illustrate the time-inconsistency. Section 4 extends the baseline model to allow longer wage commitments/fixed wages. Section 5 considers the implications for business cycles in wages and unemployment, as well as the impact of infrequent wage adjustment, in a quantitative setting. Appendixes A-G contain proofs, a two-period model demonstrating the time-inconsistency in a simpler environment, details on the parametrization and solution methods, an extension to firm-level shocks, as well as additional figures.

2 Static Model

This section begins by considering the impact of firm wages in the context of a static, one-period model, before proceeding to the dynamic model in the next section.

Within a single period, consider a labor market with measure one workers, and a large number I firms. Each firm begins the period with n_i existing workers, for all $i \in I$. The total measure of matched workers in the beginning of the period is thus $N = \sum_{i \in I} n_i$, leaving $1 - N$ unmatched workers looking for jobs. All firms have access to a linear production technology with output z per worker, while workers who do not find jobs have access to a

[16] They state that in their model efficiency requires wages being driven to workers' opportunity cost.

home production technology with output b ($< z$) per worker.

In addition to their existing workers, firms can hire new workers in a frictional labor market. Firms seeking to hire must post vacancies, where posting v vacancies is subject to a convex cost $\kappa(v, n) = \hat{\kappa}(v/n)n$, where n is the firm's existing workforce and $\hat{\kappa}' > 0, \hat{\kappa}'' > 0$.[17] The search frictions in bringing these vacancies and unmatched workers together are formalized with a matching function, with constant returns to scale. I denote the probability a worker finds a job in a market with tightness (vacancies per job seeker) θ as $\mu(\theta)$, with $\mu' > 0, \mu'' < 0$, and the probability a vacancy is filled as $q(\theta)$, where $\mu(\theta) = \theta q(\theta)$.

In posting vacancies firms also specify the wage that will apply to those jobs and take into account that the offered wage will affect their ability to fill vacancies. Specifically, they expect the measure of job seekers they attract per vacancy to be such that job seekers are left indifferent between applying to this firm versus elsewhere, the latter yielding the equilibrium value of search U. Formally, given wage w_i, the market tightness θ_i they expect to face is such that

$$U = \mu(\theta_i)w_i + (1 - \mu(\theta_i))b, \tag{1}$$

where the firm takes the value of search U as given (because the firm is small relative to the market). Here a worker applying to the firm finds a job with probability $\mu(\theta_i)$, attaining the wage w_i, and remains unmatched with probability $1 - \mu(\theta_i)$, attaining b. Per equation (1), the firm anticipates that offering a higher wage attracts more job seekers per vacancy, which increases the probability these vacancies are filled, $q(\theta_i)$.

Each firm chooses a wage and a measure of vacancies to maximize its profits:

$$\max_{w_i, \theta_i, v_i} (n_i + q(\theta_i)v_i)(z - w_i) - \kappa(v_i, n_i), \tag{2}$$

taking as given n_i and constraint (1) characterizing the beliefs of the firm regarding the market tightness to prevail in response to its chosen wage. The profits reflect the firm's n_i existing workers and $q(\theta_i)v_i$ new hires all producing z units of output at the firm wage w_i, with vacancies subject to the vacancy cost $\kappa(v_i, n_i)$.

Note that the firm problem is effectively independent of the firm's initial size. Defining the firm's rate of vacancy creation as $x_i := v_i/n_i$, one can scale and rewrite the problem as:

$$\max_{w_i, \theta_i, x_i} (1 + q(\theta_i)x_i)(z - w_i) - \hat{\kappa}(x_i), \tag{3}$$

[17] The convexity in the vacancy cost is introduced to help ensure that first order conditions characterize optimizing behavior, and the homothetic form plays a role in allowing solving the dynamic model in a tractable way. Note that the derivatives $\kappa_v(v, n)$ and $\kappa_n(v, n)$ are functions of the ratio v/n only, and for expositional reasons I hence denote them as $\kappa_v(v/n)$ and $\kappa_n(v/n)$ in what follows.

taking as given constraint (1) characterizing beliefs. This means that heterogeneity in initial sizes across firms does not translate into differences in wages or vacancy rates, as well as that firm growth is independent of size (Gibrat's law holds). While larger firms do hire more, they do so only to the extent that initial differences in size are preserved. Assuming firms are equally productive, I thus drop the firm indexes on w_i, θ_i, x_i in what follows.

The firm's first order condition for vacancy creation,

$$\kappa_v(x) = q(\theta)(z - w), \tag{4}$$

states that the firm creates vacancies to a point where the marginal cost of an additional vacancy, on the left, equals the expected profits from the additional workers hired, on the right.

The firm's first order condition for the wage reads

$$1 + q(\theta)x = q'(\theta)x(z - w)g_w(w; U), \tag{5}$$

where I denote the beliefs of the firm regarding the market tightness implied by wage w (defined via (1)) by $g(w; U)$, with g_w representing the corresponding derivative. The firm raises the wage to a point where the marginal increase in wage costs, on the left, equals the marginal increase in profits from greater vacancy filling rates, on the right. Note that the firm's existing workers make raising the wage more costly to the firm, as any wage increase must be paid to new and existing workers alike.[18]

The firm wage policy is embodied in the single wage appearing in problem (2). In the absence of such constraints, the firm problem would instead read:

$$\max_{w_i, \theta_i, v_i} n_i(z - w_i^e) + q(\theta_i)v_i(z - w_i) - \kappa(v_i, n_i), \tag{6}$$

subject to constraint (1) characterizing beliefs in response to the hiring wage w_i, and where the average wage of existing workers is denoted by w_i^e.

The first order conditions for this unconstrained firm problem include the same condition for optimal vacancy creation as for the constrained firm (4), but with a different optimality condition for the hiring wage:

$$q(\theta)x = q'(\theta)x(z - w)g_w(w; U). \tag{7}$$

[18] The firm could in some circumstances also prefer the corner solution of opting out of hiring altogether, while paying its existing workers the minimum to keep them, with $v_i = 0, w_i = b$. I focus on interior solutions characterised by first order conditions in what follows, checking in the quantitative exercises that the firm values dominate deviating to such a corner.

The firm again raises the wage to a point where the marginal increase in wage costs equals the increase in profits from greater vacancy filling rates. Raising the wage is less costly for the unconstrained firm, however, as any wage increase now applies to new hires only.

For the existing workers, outcomes depend on whether the firm has some pre-commitment to their wages or not. If not, the firm would optimally pay these workers as little as possible: their outside option b, retaining them at minimum compensation. Note that doing so would mean paying new hires more than the firm's existing workers, something the latter might object to – bringing us to the original firm problem imposing equity.

Definition 1. *A competitive search equilibrium with firm wages is an allocation* $\{w, \theta, x\}$ *and value of search* U *such that the allocation and value solve the problem* (3) *with each job seeker applying to one firm:* $1 - N = xN/\theta$.

The level effects of firm wages on equilibrium outcomes are summarized in the following proposition:

Proposition 1. *Assuming the vacancy cost is strictly increasing and convex and the matching function elasticity weakly decreasing, the competitive search equilibrium with firm wages satisfying* (1), (4), (5), *and* $1-N = xN/\theta$ *is unique, with a strictly lower wage* w *and higher market tightness* θ*, as well as strictly greater vacancy creation and employment, than without the firm wage policy.*

Intuitively, firm wage constraints lead to downward pressure on wages, due to the constrained firms' incentive to profit from their existing workers via low wages. With all firms affected, this causes the equilibrium to shift toward lower wages in a way that encourages vacancy creation and hiring. As the competitive search equilibrium absent constraints is known to be efficient, it follows that the firm wage equilibrium is inefficient, featuring overhiring.[19]

Similar effects emerge in the dynamic model, where the measure of existing matches is endogenous. I turn to this dynamic model next.

3 Dynamic Model

This section extends the firm wage model to a dynamic infinite-horizon setting, demonstrating a related time-inconsistency in the firm problem. I begin by assuming firms have commitment to future wages, before turning to the case where they reoptimize each period,

[19]Rogerson, Shimer, and Wright (2005) discuss the efficiency of the competitive search equilibrium.

considering Markov perfect equilibria. I also relate outcomes to the unconstrained case, as well as what a benevolent planner would choose.[20]

3.1 Firm Wages

Time is discrete and the horizon infinite. All agents are rational and discount the future at rate β. Each period a large number I firms inherit a measure of existing workers n_{it} from the previous period and hire new ones in a frictional labor market. Employment relationships are long term and end at the end of each period with probability δ. Labor productivity z_t is stochastic, and follows a Markovian process.

In posting vacancies, firms now specify a fully state-contingent wage contract that will apply to those jobs. Given a contract $\{w_{it+k}\}_{k=0}^{\infty}$ offered in period t, the market tightness θ_{it} firms expect to face is such that

$$U_t = \mu(\theta_{it})E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (w_{it+k} + \beta\delta U_{t+1+k}) + (1-\mu(\theta_{it}))(b + \beta E_t U_{t+1}), \tag{8}$$

where the firm takes the value(s) of search $\{U_{t+k}\}_{k=0}^{\infty}$ as given. Here a worker applying to the firm in period t finds a job with probability $\mu(\theta_{it})$, subsequently receiving the specified wages until a separation returns him to job search, and remains unmatched with probability $1-\mu(\theta_{it})$, receiving b and continuing to search in the following period. Per equation (8), the firm anticipates that offering a better contract attracts more job seekers per vacancy, which increases the probability these vacancies are filled.

Note that as far as wages are concerned, what workers care about is the expected present value $E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k w_{it+k}$ rather than the details of how these wages are paid out. Given this, it is convenient to rewrite equation (8) using the shorthand

$$X_t = \mu(\theta_{it})(W_{it} + Y_t), \tag{9}$$

where W_{it} represents the present value of wages and I define the variables $X_t := U_t - b - \beta E_t U_{t+1}$ and $Y_t := E_t \beta\delta \sum_{k=0}^{\infty} \beta^k (1-\delta)^k U_{t+1+k} - b - \beta E_t U_{t+1}$. By way of interpretation, X_t represents the option value of search and Y_t the value of forgone home production and search during employment.[21] Firms take these values $\{X_t, Y_t\}_{t=0}^{\infty}$ as given, as they do $\{U_t\}_{t=0}^{\infty}$.

[20] For a two-period version of the model that illustrates the time-inconsistency in a simpler setting, see Appendix B.

[21] To make the interpretations more evident, the equations can be rewritten as: $U_t = b + \beta E_t U_{t+1} + X_t$ and $Y_t = -b - E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k (b + X_{t+k})$.

Assuming commitment to future choices, each firm then solves the sequence problem

$$\max_{\{w_{it},\theta_{it},v_{it}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t[(n_{it} + q(\theta_{it})v_{it})(z_t - w_{it}) - \kappa(v_{it}, n_{it})] \tag{10}$$

$$\text{s.t. } n_{it+1} = (1-\delta)(n_{it} + q(\theta_{it})v_{it}), \ \forall t \geq 0, \tag{11}$$

$$X_t = \mu(\theta_{it})E_t(\sum_{k=0}^{\infty} \beta^k (1-\delta)^k w_{it+k} + Y_t), \ \forall t \geq 0, \tag{12}$$

taking as given n_{i0} and $\{X_t, Y_t\}_{t=0}^{\infty}$. Firms maximize the expected present value of profits, where in each period t the firm's existing and new workers produce z_t units of output at the firm wage w_{it}, with vacancies subject to the vacancy cost $\kappa(v_{it}, n_{it})$. In doing so, they take as given the law of motion for their workforce (11), as well as the constraints (12) characterizing the beliefs of the firm regarding the market tightnesses to prevail in response to its offered wages.[22]

Note that in choosing a sequence of per-period wages $\{w_{it}\}_{t=0}^{\infty}$ the firm is effectively choosing a sequence of present values of (firm) wages $\{W_{it}\}_{t=0}^{\infty}$. In line with the fact that the allocative wage variable in these models typically is the present value rather than per-period wages, one can rewrite the firm problem in terms of these values directly. Or, going a step further, in terms of allocations directly, by using the constraints (12) to substitute out the present values. I adopt the latter approach to allow a transparent comparison of the firm wage model with the unconstrained case, as well as planner problem.

Proceeding along these lines, the firm problem becomes:

$$\max_{\{\theta_{it},v_{it}\}_{t=0}^{\infty}} -\frac{n_{i0}X_0}{\mu(\theta_{i0})} + E_0 \sum_{t=0}^{\infty} \beta^t[(n_{it} + q(\theta_{it})v_{it})(z_t - b) - \kappa(v_{it}, n_{it}) - X_t(\frac{v_{it}}{\theta_{it}} + n_{it})] \tag{13}$$

$$\text{s.t. } n_{it+1} = (1-\delta)(n_{it} + q(\theta_{it})v_{it}), \ \forall t \geq 0,$$

taking as given n_{i0} and $\{X_t\}_{t=0}^{\infty}$.

Formally, we have the result:

Proposition 2. *Problem* (10) *is equivalent to problem* (13) *if the firm hires each period.*

Rewriting the firm problem this way makes it clear that the initial period is different from later periods: the objective consists of a present value repeating the same terms over time,

[22] As noted in the context of the static model, I focus on equilibria where firms hire each period and (12) thus always holds. There are circumstances in which firms could prefer to opt out of hiring completely, paying their existing workers such low wages as to make them indifferent between remaining employed and quitting to look for a new job (or lower, if possible). I discuss this possibility in Appendix A and provide checks in the quantitative exercises to make sure that such a deviation would not appear profitable for firms.

together with an added term where the initial market tightness appears separately. This asymmetry makes sense if one thinks of the firm as choosing a sequence of present values of wages to offer new hires in a setting where the same value applies to the firm's existing workers at each point in time as well. In choosing the initial period value, the firm has an incentive to profit from its existing workers via a low value (as in the static model). In making plans for future hiring, on the other hand, the firm does not view its future workforce as exogenous, implying that the tradeoffs the firm faces in the initial period and later periods differ. Intuitively, the firm effectively faces a less elastic labor supply in the initial period than later on.

In what follows I characterize outcomes using an Euler equation approach that relates equilibrium allocations to those attained absent constraints, as well as socially optimal allocations, directly. The unconstrained firm problem and corresponding planner problem are provided explicitly in Section 3.2. For now, it is useful to note that the unconstrained firm objective, as well as the planner objective, coincide with maximizing the present value on the right in (13) (reinterpreting X_t as a corresponding planner shadow value). The three problems are thus related in a simple way that makes it clear that the firm wage constraints lead to a departure from outcomes in the unconstrained case and what is efficient.

The firm's first order condition for vacancy creation, taking into account the influence of vacancies on subsequent employment, yields for $t \geq 0$:[23]

$$\kappa_v(x_{it}) + \frac{X_t}{\theta_{it}} = q(\theta_{it})[z_t - b + E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k (z_{t+k} - b - X_{t+k} - \kappa_n(x_{it+k}))], \tag{14}$$

with x_{it} again defined as v_{it}/n_{it}. With wages substituted out, the interpretation of this equation reads closer to a planner optimality condition.[24] The firm creates vacancies to a point where the marginal costs of additional vacancies, $\kappa_v(x_{it})$, together with the value of search the firm must deliver to attract the additional job seekers for these vacancies, X_t/θ_{it}, equal the marginal surpluses – vacancies are filled with probability $q(\theta_t)$, resulting in the new hires producing at the market productivity z_{t+k} instead of at home at b and continuing to search (which yields value X_{t+k}) while the employment relationship lasts. The hires also reduce the costs of vacancy creation going forward, as reflected in the term $-\kappa_n(x_{it+k})$.

[23]For example for v_0, taking into account its influence on n_t, $\forall t > 0$, differentiating yields $-\kappa_v(x_{i0}) - \frac{X_0}{\theta_{i0}} + q(\theta_{i0})(z_0 - b) + q(\theta_{i0})\beta(1-\delta)E_0(z_1 - b - X_1 - \kappa_n(x_{i1})) + q(\theta_{i0})\beta^2(1-\delta)^2 E_0(z_2 - b - X_2 - \kappa_n(x_{i2})) + \ldots$

[24]In fact, equation (14) corresponds to the familiar condition where the firm creates vacancies to a point where the marginal costs of additional vacancies equals the present value of profits from those vacancies.

The first order condition for the market tightness in future periods, $t > 0$, reads[25]

$$\frac{X_t}{\theta_{it}^2} = -q'(\theta_{it})[z_t - b + E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k (z_{t+k} - b - X_{t+k} - \kappa_n(x_{it+k}))]. \tag{15}$$

The firm targets a tightness equating the marginal surpluses from attracting more job seekers per vacancy – the increase in the probability vacancies are filled and the marginal surpluses associated with the resulting new hires – with the marginal cost – the increase in job seeker value the firm must deliver to attract the additional job seekers per vacancy, X_t/θ_{it}^2.

The first order condition for the market tightness in the initial period reads

$$\frac{X_0}{\theta_0^2} + \frac{X_0 \mu'(\theta_{i0})}{x_{i0}\mu(\theta_{i0})^2} = -q'(\theta_{i0})[z_0 - b + E_0 \sum_{k=1}^{\infty} \beta^k (1-\delta)^k (z_k - b - X_k - \kappa_n(x_{ik}))]. \tag{16}$$

The firm again targets a tightness that equates the marginal surpluses from attracting more job seekers per vacancy with the marginal costs, with the difference that the marginal costs are greater here. In addition to the increase in job seeker value the firm must deliver to attract the additional job seekers per vacancy, doing so also reduces firm profits on its existing workers (as wages rise), with the marginal loss per vacancy $X_0\mu'(\theta_{i0})/x_{i0}\mu(\theta_{i0})^2$.

Note that conditions (14)-(15) are again identical across firms, because also the dynamic firm problem is independent of the firm's initial size. I hence drop the firm indexes in what follows.

Proposition 3. *The firm wage firm problem* (10) *is independent of the firm's initial size.*

The optimality conditions yield inter-temporal Euler equations characterizing the evolution of allocations. For future periods $t > 0$, we have:[26]

$$\frac{\kappa_v(x_t)}{\mu'(\theta_t)} = z_t - b + \beta(1-\delta)E_t[(1 - \mu(\theta_{t+1}) + \mu'(\theta_{t+1})\theta_{t+1})\frac{\kappa_v(x_{t+1})}{\mu'(\theta_{t+1})} - \kappa_n(x_{t+1})]. \tag{17}$$

Here the marginal costs of new matches today, $\kappa_v(x_t)/\mu'(\theta_t)$, are equated to the flow surpluses from those matches today together with the expected present value of the matches tomorrow, $\kappa_v(x_{t+1})/\mu'(\theta_{t+1})$, as well as the resulting decrease in vacancy costs. The expectations take into account the probability of a separation, as well as that an increase in hires today reduces hires per vacancy tomorrow, by reducing the measure of unmatched workers.

[25] For example for θ_1, taking into account its influence on n_t, $\forall t > 1$, differentiating yields $\frac{X_1 v_{i1}}{\theta_{i1}^2} + q'(\theta_{i1})v_{i1}(z_1 - b) + q'(\theta_{i1})v_{i1}\beta(1-\delta)E_1(z_2 - b - X_2 - \kappa_n(x_{i2})) + q'(\theta_{i1})v_{i1}\beta^2(1-\delta)^2 E_1(z_3 - b - X_3 - \kappa_n(x_{i3})) + \ldots$

[26] From (14), $\frac{\kappa_v(x_t) + X_t/\theta_t}{q(\theta_t)} = z_t - b + \beta(1-\delta)E_t[\frac{\kappa_v(x_{t+1}) + X_{t+1}/\theta_{t+1}}{q(\theta_{t+1})} - X_{t+1} - \kappa_n(x_{t+1})]$ holds for $t \geq 0$. To arrive at (17), note that the equilibrium values X_t can be substituted out using that (14) and (15) imply $X_t = \kappa_v(x_t)\frac{\mu(\theta_t) - \mu'(\theta_t)\theta_t}{\mu'(\theta_t)}$ for $t > 0$. For (18), similarly, (14) and (16) imply that $X_0 = \kappa_v(x_0)\frac{\mu(\theta_0) - \mu'(\theta_0)\theta_0}{\mu'(\theta_0)}\frac{q(\theta_0)x_0}{1 + q(\theta_0)x_0}$.

For the initial period, the influence of the firm's existing workers introduces a wedge on the left hand side of the equation:

$$\frac{\kappa_v(x_0)}{\mu'(\theta_0)}[1 - \frac{1-\mu'(\theta_0)\theta_0/\mu(\theta_0)}{1+q(\theta_0)x_0}] = z_0 - b + \beta(1-\delta)E_0\big[\big(1-\mu(\theta_1)+\mu'(\theta_1)\theta_1\big)\frac{\kappa_v(x_1)}{\mu'(\theta_1)} - \kappa_n(x_1)\big], \tag{18}$$

indicating that the effective cost of creating matches is lower in the initial period than in later periods (consistent with lower equilibrium wages in the initial period).

Finally, to relate the firm wage Euler equations those of unconstrained firms or (equivalently) a benevolent planner, it is useful to note that equations (17)-(18) differ from the latter only in the initial period wedge appearing in (18). The corresponding equations for unconstrained firms, as well as planner, coincide with (17) for all periods.[27] Even though the constraints thus influence firm behavior (in an allocative way), with commitment to future wages this influence is limited to the initial period.

Reoptimizing firms Firms must have commitment to future wages to implement the above plans. But what if they instead expect to reoptimize each period? Ultimately this seems the more natural specification to consider. To think about this case without commitment, I consider Markov perfect equilibria, where the firms' choices each period depend on the set of payoff-relevant state variables in the firm problem. In particular, I focus on equilibria that are consistent with the size-independence of the firm problem, an approach that simplifies analyzing the problem and permits the extensions considered.

Suppose the current aggregate state is denoted $S := (N, z)$. Based on (13), the firm problem can then be written recursively as:

$$\max_{\theta,v} -\frac{nX(S)}{\mu(\theta)} + (n+q(\theta)v)(z-b) - \kappa(v,n) - X(S)(\frac{v}{\theta}+n) + \beta E_S V(n';S') \tag{19}$$
$$\text{s.t. } n' = (1-\delta)(n+q(\theta)v),$$

together with the accounting equation

$$V(n;S) = (n+q(\theta)v)(z-b) - \kappa(v,n) - X(S)(\frac{v}{\theta}+n) + \beta E_S V(n';S'). \tag{20}$$

That the problem consists of a separate objective and accounting equation reflects the time-inconsistency of the problem. Relative to the sequence problem (13), the firm objective (19) now features the added term $-nX/\mu(\theta)$ reflecting the influence of existing workers on

[27] To see this, compare the firm objective in (13) with those in (28) and (31).

firm behavior each period. In line with the sequence problem, this term does not enter the accounting equation (20) keeping track of continuation values, however.

It is convenient to note that the recursive firm problem also scales by size. Defining $\hat{V}(S) := V(n; S)/n$ and using the law of motion for the firm's workforce, scaling by n yields the firm problem

$$\max_{\theta,x} -\frac{X(S)}{\mu(\theta)} + (1+q(\theta)x)(z-b+\beta(1-\delta)E_S\hat{V}(S')) - \hat{\kappa}(x) - X(S)(\frac{x}{\theta}+1) \tag{21}$$

with the accounting equation

$$\hat{V}(S) = (1+q(\theta)x)(z-b+\beta(1-\delta)E_S\hat{V}(S')) - \hat{\kappa}(x) - X(S)(\frac{x}{\theta}+1). \tag{22}$$

The first order conditions for vacancy creation and market tightness now read:

$$\kappa_v(x) + \frac{X(S)}{\theta} = q(\theta)(z-b+\beta(1-\delta)E_S\hat{V}(S')), \text{ and} \tag{23}$$

$$\frac{X(S)}{\theta^2} + \frac{X(S)\mu'(\theta)}{x\mu(\theta)^2} = -q'(\theta)(z-b+\beta(1-\delta)E_S\hat{V}(S')), \tag{24}$$

respectively. Note that the optimality condition for the tightness (24) now reflects the influence of the firm's existing workers each period.

The optimality conditions and accounting equation yield the following intertemporal Euler equation characterizing the evolution of allocations, for all $t \geq 0$:[28]

$$\frac{\kappa_v(x_t)}{\mu'(\theta_t)}[1 - \frac{1-\frac{\mu'(\theta_t)\theta_t}{\mu(\theta_t)}}{1+q(\theta_t)x_t}] = z_t - b \tag{25}$$
$$+\beta(1-\delta)E_t\left\{\frac{\kappa_v(x_{t+1})}{\mu'(\theta_{t+1})}[1-\mu(\theta_{t+1})+\mu'(\theta_{t+1})\theta_{t+1} - (1-\mu(\theta_{t+1}))\frac{1-\frac{\mu'(\theta_{t+1})\theta_{t+1}}{\mu(\theta_{t+1})}}{1+q(\theta_{t+1})x_{t+1}}] - \kappa_n(x_{t+1})\right\}.$$

Because the firm's existing workers influence firm behavior each period, the reoptimizing firm's Euler equation now features wedges each period, and on both the left and right sides of the equation (as the firm also anticipates its incentive to cut wages in the future). These wedges effectively reduce the cost of creating matches (consistent with lower equilibrium wages) relative to the case of unconstrained firms and planner allocations.[29]

[28]Equation (22) can be rewritten, using (23) and $\kappa_n(x) = \hat{\kappa}(x) - \kappa_v(x)x$, as $\hat{V}_t = z_t - b + \beta(1-\delta)E_t\hat{V}_{t+1} - X_t - \kappa_n(x_t)$. Combining this with (23) yields, again, $\frac{\kappa_v(x_t)+X_t/\theta_t}{q(\theta_t)} = z_t - b + \beta(1-\delta)E_t[\frac{\kappa_v(x_{t+1})+X_{t+1}/\theta_{t+1}}{q(\theta_{t+1})} - X_{t+1} - \kappa_n(x_{t+1})]$, where the equilibrium values of X_t can be substituted out using that (23) and (24) imply $X_t = \kappa_v(x_t)\frac{\mu(\theta_t)-\mu'(\theta_t)\theta_t}{\mu'(\theta_t)}\frac{q(\theta_t)x_t}{1+q(\theta_t)x_t}$.

[29]The Euler equations for the latter two coincide with (17), but for all $t \geq 0$.

From a practical point of view it is convenient to note that equation (25) takes the form of a standard Euler equation, instead of the *generalized* Euler equations that typically appear in problems with time-inconsistencies (see, e.g., Klein, Krusell, and Rios-Rull (2008)). Such generalized Euler equations generally involve derivatives of choice variables with respect to an endogenous state variable – reflecting the decision-maker taking into account the effects of his/her choices on the magnitude of future biases – something that makes the generalized Euler equation a more complicated object to analyze than standard Euler equations.[30] Here the focus on size-independent behavior eliminates such a dependence.

Finally, defining an equilibrium where firm wage firms reoptimize each period, we have:

Definition 2. *A competitive search equilibrium with firm wages is an allocation* $\{w_t, \theta_t, x_t\}_{t=0}^{\infty}$ *and job seeker values* $\{X_t\}_{t=0}^{\infty}$ *such that the allocation and values solve the problem* (21)*, with each job seeker applying to one firm:* $1 - N_t = x_t N_t / \theta_t$, $\forall t$.

Ultimately, the core of the firm wage equilibrium reduces to a simple three-equation dynamic system in endogenous variables θ_t, x_t, N_t given by the Euler equation (25), law of motion $N_{t+1} = (1-\delta)(N_t + \mu(\theta_t)(1 - N_t))$, and adding up constraint $1 - N_t = x_t N_t/\theta_t$, together with an initial value for N_0 and process for shocks.

3.2 The Unconstrained Case and Planner Problem

This section considers the benchmark case of unconstrained firms, as well as efficient allocations.

In the absence of constraints, the problem of a firm deciding on hiring in any period $t \geq 0$ can be written (nearly) independently of hiring at other points in time as:

$$\max_{\{w_{it+k}^t\}_{k=0}^{\infty}, \theta_{it}, v_{it}} E_t[q(\theta_{it}) v_{it} \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (z_{t+k} - w_{it+k}^t) - \sum_{k=0}^{\infty} \beta^k \kappa(v_{it+k}, n_{it+k})] \tag{26}$$

$$\text{s.t. } X_t = \mu(\theta_{it}) E_t(\sum_{k=0}^{\infty} \beta^k (1-\delta)^k w_{it+k}^t + Y_t), \tag{27}$$

where n_{it} is given. The first term in the objective (26) represents the present value of output net of wages associated with workers hired in period t, while the second the costs of creating the vacancies together with the influence of the hiring on vacancy costs in the future, assuming the workforce follows the corresponding law of motion over time.

[30]Even solving for a steady state is non-trivial: One cannot simply evaluate the Euler equation in steady state and solve, because the derivative introduces an additional unknown.

This problem can be rewritten in terms of allocations by substituting out wages using (27). To then arrive at a full firm problem involving decisions regarding hiring over time, aggregating across cohorts hired at different points in time yields the problem:[31]

$$\begin{aligned} \max_{\{\theta_{it}, v_{it}\}_{t=0}^{\infty}} \; & E_0 \sum_{t=0}^{\infty} \beta^t [(n_{it} + q(\theta_{it}) v_{it})(z_t - b) - \kappa(v_{it}, n_{it}) - X_t(\frac{v_{it}}{\theta_{it}} + n_{it})] \\ \text{s.t. } \; & n_{it+1} = (1-\delta)(n_{it} + q(\theta_{it}) v_{it}), \; \forall t \geq 0. \end{aligned} \tag{28}$$

Comparing this problem with the constrained firm's problem in (13) reveals the two to be identical except for the initial period, with the unconstrained firm's first order conditions coinciding with (14) and (15) for all $t \geq 0$ – absent the initial period wedge. Similarly, the unconstrained firm's Euler equation coincides with that in (17) for all $t \geq 0$.

A key difference between the constrained and unconstrained firm problems is that while the unconstrained firm sets wages cohort by cohort solely with the hiring margin in mind, the constrained firm's decisions are influenced by the incentive to profit from its existing workforce in any optimization period. If the constrained firm has commitment to future wages, it is able to avoid this influence in future periods, but if it does not, the influence is there each period. Commitment to future wages thus has allocative effects when firms are constrained. When firms are not constrained, on the other hand, commitment to future wages does not matter for allocations. This reflects in the form of problem (28), with no special role for the initial period. It is also intuitive, as an unconstrained firm is able to front-load compensation into the hiring period and thus does not require commitment to future wages to deliver hires their ex-ante preferred present value of wages.[32]

How do the two firm problems compare with planner objectives? A planner maximizing the expected present value of output would allocate resources according to:

$$\max_{\{\theta_{it}, v_{it}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \sum_{i \in I} [(n_{it} + q(\theta_{it}) v_{it}) z_t - \kappa(v_{it}, n_{it})] + [1 - \sum_{i \in I} (n_{it} + q(\theta_{it}) v_{it})] b \right\} \tag{29}$$

$$\text{s.t. } \; n_{it+1} = (1-\delta)(n_{it} + q(\theta_{it}) v_{it}), \; \forall i \in I, t \geq 0,$$

$$\sum_{i \in I} v_{it}/\theta_{it} = 1 - \sum_{i \in I} n_{it}, \; \forall t \geq 0, \tag{30}$$

[31] See Appendix A.

[32] For the unconstrained firm: Commitment does matter for the timing of wage payments over the course of an employment relationship, but without affecting the present value of wages the firm can offer new hires, and hence without affecting allocations. If the firm has commitment, it is left indifferent across a variety of alternative ways of paying out any desired present value of wages. But even if it does not, the firm should still be able to implement the same present value with an appropriate wage payment in the hiring period, even if subsequent wages are low enough to make the worker indifferent between remaining employed and quitting to search for a new job. Having commitment within the hiring period is thus enough.

with n_{i0} given for all producers i. The planner maximizes the present value of producer and home output net of the costs of vacancy creation, taking into account the law of motion for the workforce of each producer. In addition, the planner's choices of θ_{it}, v_{it} must be consistent with the measure of unmatched workers each period, as the job seekers allocated to each producer, v_{it}/θ_{it}, must add up to the latter.

Reorganizing terms and including the adding up constraints (30) with corresponding Lagrange multipliers λ_t, the planner objective becomes

$$E_0 \sum_{i \in I} \sum_{t=0}^{\infty} \beta^t [(n_{it} + q(\theta_{it}) v_{it})(z_t - b) - \kappa(v_{it}, n_{it}) - \lambda_t (\frac{v_{it}}{\theta_{it}} + n_{it})]. \tag{31}$$

Comparing this objective with that of the unconstrained firm reveals the two to be closely related, as the producer-level objective the planner faces is essentially the same as the firm's, with the equilibrium value of search replaced by the shadow value of job seekers. This is consistent with the conclusion of Kaas and Kircher (2015) that equilibrium allocations absent constraints are efficient.

Note that it follows that the firm wage equilibrium is inefficient (with or without commitment) due to the influence of the firms' existing workers on allocations.

4 Infrequent Wage Adjustment

The commitment problem suggests that firms may find it profitable to adopt rules governing their wage setting, instead of reoptimizing continually. A simple rule firms could consider would be to fix the wage for a period of time, reoptimizing it only from time to time. To consider the profitability and equilibrium implications of such rules, this section extends the model to allow firms to fix wages for a probabilistic period of time. I begin by considering the profitability of such a rule from an individual firm's perspective, before turning to an equilibrium where all firms adopt them.

4.1 Single Firm Deviation to Fixed Wage

Consider an equilibrium with firm wages where firms reoptimize each period in the face of aggregate shocks to productivity, and where a single firm contemplates a deviation to a fixed wage for a probabilistic period of time.

Recall that the competitive search equilibrium involves firms offering contracts that dominate alternative contracts they could consider, whether the latter are offered in equilibrium or not. In evaluating such alternative contracts, the firm's beliefs regarding the market tightness are assumed to be consistent with the job seeker constraints, as they are in problem (10) with (12). To arrive at a corresponding constraint for a firm deviating to a fixed wage, note that for a firm that deviates to fixed wage w, expecting to revert to equilibrium behavior with probability α each period, the expected present value of wages is given by

$$\phi(w, S) = \frac{w}{1 - \beta(1-\delta)(1-\alpha)} + \Lambda(S), \tag{32}$$

where $\Lambda(S) = E_S \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (1-\alpha)^k \beta(1-\delta)\alpha W(S^{k+1})$ represents the expected present value of reverting to equilibrium wages (whose present values are here denoted by $W(S)$). The tightness(es) prevailing during the deviation are then determined by the constraints $X(S) = \mu(\theta)(\phi(w, S) + Y(S))$ each period.

The problem of the deviating firm is, then, to choose a wage w and vacancy creation v to maximize

$$- \frac{nX(S)}{\mu(\theta)} + (n + q(\theta)v)(z - b) - \kappa(v, n) - X(S)(\frac{v}{\theta} + n) + \beta E_S(\alpha V(n', S') + (1-\alpha)V^f(n', w, S'))$$
$$\text{s.t. } n' = (1-\delta)(n + q(\theta)v),$$
$$X(S) = \mu(\theta)(\phi(w, S) + Y(S)),$$

given n, S. The objective is the same as in (19) except for the continuation values, where the firm now attains the equilibrium value $V(n', S')$ only if it reverts to equilibrium wages immediately, and a value of holding the wage fixed $V^f(n', w, S')$ otherwise.

While the deviation lasts, the wage is fixed and the firm only chooses vacancies v to maximize:

$$(n + q(\theta)v)(z - b) - \kappa(v, n) - X(S)(\frac{v}{\theta} + n) + \beta E_S(\alpha V(n', S') + (1-\alpha)V^f(n', w, S'))$$
$$\text{s.t. } n' = (1-\delta)(n + q(\theta)v),$$
$$X(S) = \mu(\theta)(\phi(w, S) + Y(S)),$$

given n, w, S – where the maximized value further determines $V^f(n, w, S)$.

These firm problems, again, scale with size, and I maintain the focus on size-independent behavior in what follows. The first order condition for the deviation wage reads

$$\begin{aligned} \frac{X(S)}{\theta^2} + \frac{X(S)\mu'(\theta)}{x\mu(\theta)^2} =& - q'(\theta)[z - b + \beta(1-\delta)E_S[\alpha\hat{V}(S') + (1-\alpha)\hat{V}^f(w, S')]] \\ & - \beta(1-\delta)(1-\alpha)(1 + q(\theta)x)/x E_S \hat{V}_w^f(w, S')/\theta_w, \end{aligned} \tag{33}$$

where $\hat{V}^f(w,S) := V^f(n,w,S)/n$ and the final term reflects the impact of the wage on future profits.[33] As the fixed wage will prevail for longer, the deviating firm is more forward-looking in setting it than firms that reoptimize continually.

The first order condition for vacancy creation remains the same throughout the deviation:

$$\kappa_v(x) + \frac{X(S)}{\theta} = q(\theta)(z - b + \beta(1-\delta)E_S(\alpha\hat{V}(S') + (1-\alpha)\hat{V}^f(w,S'))). \tag{34}$$

The differences between the optimality conditions of the deviating firm (with $\alpha < 1$) and equilibrium firms (with $\alpha = 1$) suggest that the deviating firm will choose a different wage and grow at a different rate from equilibrium firms. Section 5 demonstrates these differences, and how they depend on the duration of the wage, in the context of a parameterized model. In particular, I show that deviating raises firm value, making it interesting to consider the equilibrium implications of all firms adopting such rules. I turn to this problem next.

4.2 Equilibrium with Infrequent Adjustment

Consider an equilibrium where firm wage firms reoptimize their wage with probability α each period, and otherwise hold it fixed, in the face of aggregate shocks to productivity. Reoptimization occurs independently across firms, implying that wage adjustment is staggered. In this context the equilibrium generally features a distribution of wages (reflecting wages set during past aggregate states), and this distribution becomes part of the aggregate state S.

The problem of a reoptimizing firm is to choose a wage w and vacancy creation v to maximize

$$-\frac{nX(S)}{\mu(\theta)} + (n + q(\theta)v)(z-b) - \kappa(v,n) - X(S)(\frac{v}{\theta} + n) + \beta E_S(\alpha V^r(n',S') + (1-\alpha)V^f(n',w,S'))$$

s.t. $n' = (1-\delta)(n + q(\theta)v)$,

$X(S) = \mu(\theta)(\phi(w,S) + Y(S))$,

given n, S, and where the implied continuation value satisfies the accounting equation

$$V^r(n,S) = (n + q(\theta)v)(z-b) - \kappa(v,n) - X(S)(\frac{v}{\theta} + n) + \beta E_S(\alpha V^r(n',S') + (1-\alpha)V^f(n',w,S')).$$

[33] From the job seeker constraint, the change in tightness associated with a change in the wage is given by $\theta_w = -\mu(\theta)^2/(\mu'(\theta)X(1-\beta(1-\delta)(1-\alpha)))$, whereas the corresponding change in the continuation value satisfies

$$\begin{aligned}\hat{V}^f_w(w,S) =& xq'(\theta)[z - b + \beta(1-\delta)E_S[\alpha\hat{V}(S') + (1-\alpha)\hat{V}^f(w,S')]]\theta_w \\ &+ \frac{xX(S)}{\theta^2}\theta_w + \beta(1-\delta)(1-\alpha)(1 + q(\theta)x)E_S\hat{V}^f_w(w,S').\end{aligned}$$

Note that the firm objective is the same as for the deviating firm except for the continuation values, where the firm now attains an equilibrium value of a new fixed wage $V^r(n', S')$ when it reoptimizes.[34]

While the wage remains fixed, the firm only chooses vacancies v to maximize

$$
\begin{aligned}
&(n + q(\theta)v)(z - b) - \kappa(v, n) - X(S)(\frac{v}{\theta} + n) + \beta E_S(\alpha V^r(n', S') + (1 - \alpha)V^f(n', w, S')) \\
&\text{s.t. } n' = (1 - \delta)(n + q(\theta)v), \\
&\qquad X(S) = \mu(\theta)(\phi(w, S) + Y(S)),
\end{aligned}
$$

given n, w, S – where the maximized value again determines $V^f(n, w, S)$.

The firm problems continue to scale with size, and yield first order conditions that coincide with those of the deviating firm (33)-(34), but with continuation values $\hat{V}^r(S) := V^r(n, S)/n$ and $\hat{V}^f(w, S) := V^f(n, w, S)/n$ corresponding to an equilibrium with fixed wages. That the optimality conditions differ from those of firms reoptimizing each period suggests differences in firm behavior – something that should also lead to equilibrium effects in this case with all firms affected. I illustrate the impact of fixed wages on equilibrium outcomes and welfare in the next section.

5 Quantitative Illustration

This section uses the model to study the implications of firm wage constraints for labor market outcomes: first, how they affect the responses of wages and hiring to shocks, and then, the profitability and equilibrium implications of infrequent wage adjustment. I begin with an environment where firms face aggregate shocks to labor productivity, but also discuss firm-level shocks toward the end of the section.

5.1 Parameterizing and Solving the Model

I begin with a parametrization and discussion of the solution approach, before turning to results. Appendixes C-G provide details.

Parametrization I adopt a monthly frequency, set the discount rate to $\beta = 1.05^{-1/12}$, and normalize steady-state labor productivity to $z = 1$. To be consistent with an average duration

[34] Similarly, the function ϕ is as defined in (32), but with $\Lambda(S) = E_S \sum_{k=0}^{\infty} \beta^k (1 - \delta)^k (1 - \alpha)^k \beta(1 - \delta)\alpha W^r(S^{k+1})$ where $W^r(S)$ now corresponds to equilibrium values of new fixed wages.

of employment of 2.5 years, I set the separation rate to $\delta = 0.033$. To then be consistent with an average unemployment rate of 5 percent, when steady-state unemployment in the model is $\delta(1-\mu(\theta))/(\mu(\theta)+\delta(1-\mu(\theta)))$, requires a steady-state job-finding probability of $\mu(\theta) = 0.388$. I adopt the matching function $m(v,u) = vu/(v^\ell + u^\ell)^{1/\ell}$ for this discrete time model, as in den Haan, Ramey, and Watson (2000), and target a steady-state level of θ of 0.43, as in Kaas and Kircher (2015). With this, fitting the above job-finding probability requires $\ell = 1.85$. Finally, I follow Kaas and Kircher (2015) in adopting the vacancy cost $\kappa(v,n) = \frac{\kappa_0}{1+\gamma}(v/n)^\gamma v$ with $\gamma = 2$. This leaves two remaining free parameters, κ_0 and b.

To then arrive at comparable parametrizations for the constrained and unconstrained models, I begin with a benchmark parametrization for the latter: Following Shimer (2005), I adopt the value $b = 0.4$ and set κ_0 such that the corresponding Euler equation (17) holds in steady state.[35] I then seek alternative values of κ_0, b for the firm wage model that hold the level of the wage and hence firm profit rate unchanged across models, while ensuring the corresponding Euler equation (25) holds. This approach allows comparing firm responses to shocks in a setting where the shock is similarly sized relative to the profitability of hiring across models.[36] It turns out that doing so requires holding the value of κ_0 unchanged across models, while raising b to bring wages in the constrained model to their levels in the unconstrained model (see Appendix C for details). The implied value of b for the constrained model is here 0.89. In addition to this baseline parametrization, I consider robustness to alternatives.

Solution Approach The baseline firm wage model where firms reoptimize each period is relatively straightforward to solve, as the equilibrium conditions reduce to a set of nonlinear difference equations that can be solved with standard methods, such as Dynare.[37] The complete system of equations is provided in Appendix D. In solving the model, I also check that the solution characterized by the first order conditions dominates the corner solution of opting out of hiring for a period: zero vacancies and a low wage making existing workers indifferent between remaining employed and quitting to search for a new job (see Appendix

[35] To connect the unconstrained model to the model in Shimer (2005) – a random search model with bargaining – note that calibrations of the latter models typically assume a bargaining power parameter ensuring that equilibrium allocations coincide with planner allocations (as in Shimer (2005)). One can thus view the dynamics of the unconstrained model as representing those of random search models calibrated in this way.

[36] This approach to comparing models coincides with that in e.g. Hall and Milgrom (2008), Hagedorn and Manovskii (2008), and Elsby and Michaels (2013), who adopt an explicit target for the vacancy cost (and hence firm profit rate) to hold across models.

[37] The tractability is due to the structure of the problem together with the focus on equilibria consistent with the size-independence of the firm problem. Klein, Krusell, and Rios-Rull (2008) discuss solving the more general case.

A for a discussion). These checks can be found in Appendix G.

The extension to infrequent wage adjustment has two parts: the single deviating firm fixing its wage and the equilibrium with fixed wages. Solving the first involves simply adding the deviating firm's first order conditions to the baseline system and solving as before. The second requires an adjustment, however, because the distribution of wages becomes a state variable: Individual firms' choices of θ_{it}, x_{it} depend on their wage and in equilibrium these must satisfy the adding up constraint across firms $\sum_i x_{it} n_{it}/\theta_{it} + \sum_i n_{it} = 1$ each period. I solve this extended model using the approach of Gertler and Trigari (2009), by first linearizing the model equations and then aggregating across firms, arriving at a system where the average wage across firms becomes a sufficient statistic for the distribution of wages. The resulting linear system is provided in Appendix E and can again be solved with standard approaches.

In addition to aggregate shocks, I consider an environment where firms face firm-specific idiosyncratic shocks, discussed in more detail in Appendix F. To solve the baseline model with firm shocks, one can either use Dynare with higher-order approximations or solve the non-linear firm problem on a grid for productivity directly, as in this case the only state variable in the firm problem is the firm's current productivity. For the baseline model, the latter approach involves solving a nonlinear system of equations in the equilibrium firm choices of $\{\theta, x\}$ for each possible productivity realization, and simulating the model to find the job seeker value X consistent with the equilibrium adding up constraint. For the equilibrium with fixed wages, the set of unknowns is larger but a similar approach can be used.

The next sections describe the results.

5.2 Firm Wages over the Business Cycle

How do firm wage constraints affect the responses of wages and labor market flows to shocks? Can they help explain why wages vary so little while unemployment varies so much over the business cycle?

A side-by-side comparison of the two models shows that the firm wage model features clearly more rigid wages in response to aggregate shocks than the unconstrained model. To illustrate, Figure 1 plots impulse responses to a one percent positive shock to labor productivity across the two models, parameterized to maintain the steady-state levels of unemployment, wages and profits the same across models as described. As the figure shows, the wage increase in the firm wage model is only about a quarter of that in the unconstrained model, where the wage increase is almost identical to that of productivity. The profitability of

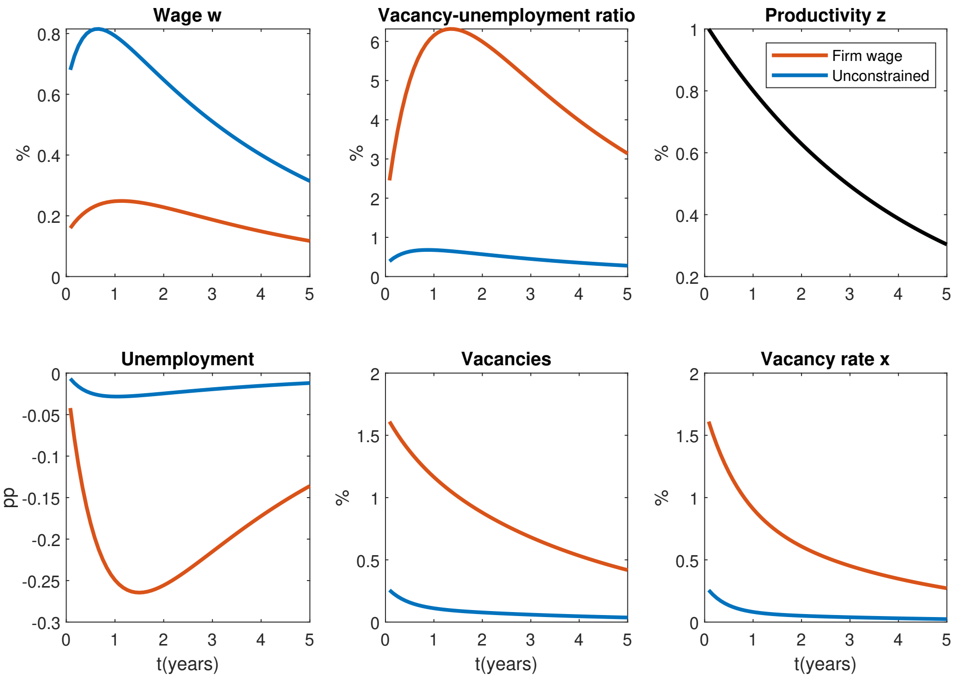

Figure 1: Impulse Responses in Firm Wage vs Unconstrained Model

Notes: The figure plots the percentage responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model and the unconstrained model. Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$. The two models have the same steady-state levels of wage, market tightness, unemployment, as described in the section on parametrization.

hiring thus rises more in the firm wage model, whereas in the unconstrained model the wage increase absorbs the bulk of that in productivity, leaving limited room for the profitability of hiring to rise. This results in an increase in the vacancy-unemployment ratio that is an order of magnitude greater in the firm wage model than the unconstrained model, with equally significant amplification in the increase in vacancy creation and drop in unemployment.[38]

A literature has studied the cyclical behavior of unemployment and vacancy creation in the search and matching framework specifically, arguing that the model produces little variation in these variables over the business cycle relative to the data. The impact of firm wages is significant relative to the gap between model and data emphasized in this literature: According to Shimer (2005), the unconstrained model would require a tenfold increase in the volatility of the vacancy-unemployment ratio to be consistent with measured volatility in the

[38] In stating that the firm wage model generates rigidity in wages I mean allocative rigidity, i.e. rigidity in the present value of wages. Strictly speaking, the unconstrained model does not pin down per-period wages without making additional assumptions, so to arrive at a series for per-period wages for the unconstrained model that speaks to allocative rigidity I make the symmetric assumption that the unconstrained firm pays all its workers the same at each point in time.

Figure 2: Impulse Responses in Firm Wage vs Unconstrained Model across Parametrizations
Notes: The figure plots the percentage responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model parameterized with alternative combinations of κ_0, b and the unconstrained model. Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$.

vacancy-unemployment ratio. Taking into account the impact of the convex adjustment costs introduced as part of the firm wage model, firm wages explain roughly a third of the observed volatility: In the model the vacancy-unemployment ratio rises by 6.5 percent in response to a one percent shock to labor productivity, while in the data the relative standard deviation of the vacancy-unemployment ratio to that of labor productivity is $38/2 \approx 19$ (Shimer 2005).

For robustness, one can also consider parameterizations of the firm wage model involving a more modest increase in the value of home production, where κ_0 instead adjusts to ensure the firm wage Euler equation continues to hold. Note that the less b increases the more κ_0 must increase, however, to counteract the increased tendency for hiring in the firm wage model. The corresponding model comparison is shown in Figure 2, which plots a band of impulse responses under alternative combinations of κ_0, b, from only b adjusting across models (as in Figure 1) to only κ_0 adjusting. The latter case corresponds to the upper envelope of the wage responses, and lower envelope of the flow responses.

The figure shows that the firm wage model features less variable wages and more variable labor market flows than the unconstrained model throughout. The magnitude of the effect ranges from significant to moderate as one moves from the original parametrization toward

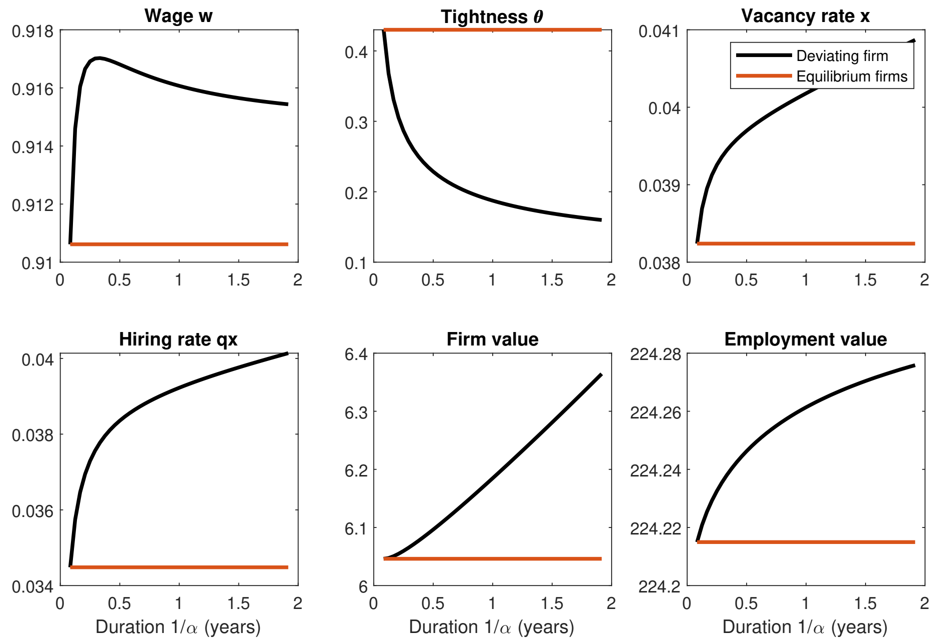

Figure 3: Single Firm Deviating to Longer Wage Commitment

Notes: The figure displays steady-state values in a firm wage equilibrium without shocks, together with corresponding values for an individual firm that deviates by setting a fixed wage for a probabilistic period of time. The latter are plotted as a function of the expected duration of the wage $1/\alpha$. The firm value plotted is the scaled firm value per initial size.

a more modest increase in b and greater increase in κ_0 instead. Note that in the case where b is unchanged across models, the vacancy cost is six times greater in the firm wage model than in the unconstrained model, dampening labor market flows significantly. Even then unemployment and vacancy creation remain more variable in the firm wage model than in the unconstrained model – reflecting allocative rigidity in wages.[39]

5.3 Infrequent Wage Adjustment

Would firms in this environment find it profitable to fix wages? And how would equilibrium outcomes change if they did?

The Firm Wage Equilibrium: Levels and Welfare I begin by describing the level effects of firm wages on equilibrium outcomes, including those on welfare. To that end, consider the firm wage equilibrium where firms reoptimize wages each period, parameterized

[39]Figure G.1 in Appendix G provides impulse responses holding parameters unchanged across models, showing that the qualitative features continue to hold. In this context the steady state levels of unemployment, market tightness and job-finding probability differ across models however.

as described, together with the corresponding planner allocation/unconstrained equilibrium. I compare the steady states of the two below (the effects can be confirmed in Figure 5).

As discussed, constrained firms tend to offer lower wages, and hence attract fewer job seekers per vacancy, than unconstrained firms. With all firms affected, this downward pressure on wages shifts the labor market equilibrium toward lower wages in a way that makes workers worse off – both the value of employment and unemployment fall – despite an increase in the job-finding probability. Meanwhile firm value increases, as does the profitability of vacancy creation, leading to overhiring relative to the unconstrained case/what is efficient.

Perhaps surprisingly, the constraints thus appear to make firms better off in equilibrium, rather than being purely costly, as one would expect in partial equilibrium. Relatedly, the equity would appear to come at a cost for workers, in addition to distorting allocations from what a benevolent planner would choose.

The Profitability of a Fixed Wage Consider then an individual firm deviating to a fixed wage in this setting. Beginning with a version of the model without shocks, to isolate the benefit of the commitment provided by the wage, Figure 3 shows how the firm compares to equilibrium firms in terms of its choices of wage, corresponding market tightness, vacancy rate, hiring rate, as well as firm and worker value. The deviating firm offers a higher wage than equilibrium firms, thus attracting more job seekers and hiring more workers per vacancy – consistent with the deviating firm being more forward-looking in its wage setting (as it avoids the incentive to cut wages ex post). It also creates more vacancies than equilibrium firms – consistent with the ability to commit raising the profitability of vacancy creation. And for both reasons, the deviating firm grows faster than equilibrium firms while the deviation lasts.[40]

These effects also appear to generally be monotonic in the duration of the wage – with the exception of the per-period wage. Even though the present value of wages rises in the duration of wages, the implied per-period wage need not, because the probability of reverting to the (low) equilibrium wage also falls in duration.

Most importantly, note that the deviation is profitable for the firm, raising firm value relative to equilibrium firms. This is due to the time-inconsistency in the firm problem, which makes commitment to future wages valuable. As discussed, the constraints play a role

[40] I have checked that the deviating firm remains small relative to the market in Figure 3. If the deviating firm grows at rate g during the deviation (with $1+g=(1+qx)(1-\delta)>1$), then expected initial firm size $t>1$ periods after the deviation started is $[\alpha\sum_{k=0}^{t-2}(1-\alpha)^k(1+g)^k+(1-\alpha)^{t-1}(1+g)^{t-1}]n_1$, where $n_1=(1+g)n_0$ is initial size after one period of deviation. It follows that firm size remains bounded as t grows if and only if $(1-\alpha)(1+g)<1$.

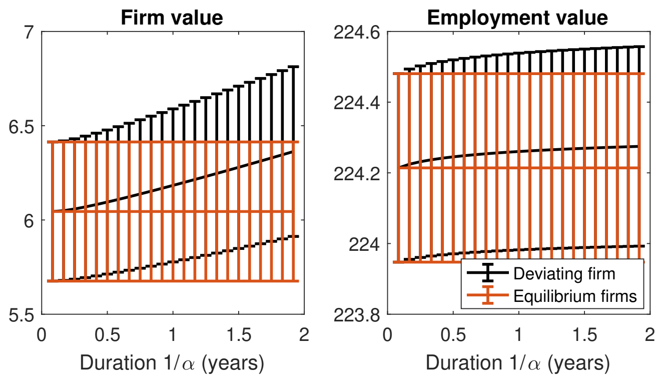

Figure 4: Deviation to Longer Wage Commitment with Aggregate Shocks

Notes: The figure displays simulation means together with standard deviation bounds in a firm wage equilibrium with aggregate shocks, together with corresponding values for an individual firm that deviates by setting a fixed wage for a probabilistic period of time. The latter are plotted as a function of the expected duration of wages $1/\alpha$. Labor productivity follows an AR(1) with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$. The firm value plotted is the scaled firm value per initial size.

in making commitment valuable, and correspondingly, a similar deviation in the absence of firm wage constraints would have no effect on firm behavior or value (as illustrated in Figure G.4 in Appendix G). Note also that in addition to the firm itself being better off, also the workers employed at the firm are better off, due to the higher wages while the deviation lasts.

When firms face shocks, a fixed wage involves costs as well as benefits. To consider the profitability of deviating to a fixed wage when firms face aggregate shocks, Figure 4 compares firm valuations between the deviating firm and equilibrium firms in this context. The figure plots simulation means together with corresponding standard deviation bounds, showing that even though aggregate shocks clearly affect valuations, deviating continues to be associated with greater overall profitability. Similarly, while the welfare of workers employed at the deviating firm clearly varies in response to aggregate shocks, the deviation remains welfare-improving for them as well.

To illustrate the costs involved with fixed wages, Figure G.5 in Appendix G compares the impulse responses of the deviating firm to those of equilibrium firms. As shown, the fixed wage hampers the firm's ability to respond to shocks by shutting down one of the two instruments it would normally use for doing so. Instead of raising wages together with other firms when productivity rises, the firm holds its wage fixed, which makes it less attractive to job seekers. The firm still increases vacancy creation in response to the shock, but the profitability of vacancy creation suffers from not being able to offer more attractive terms,

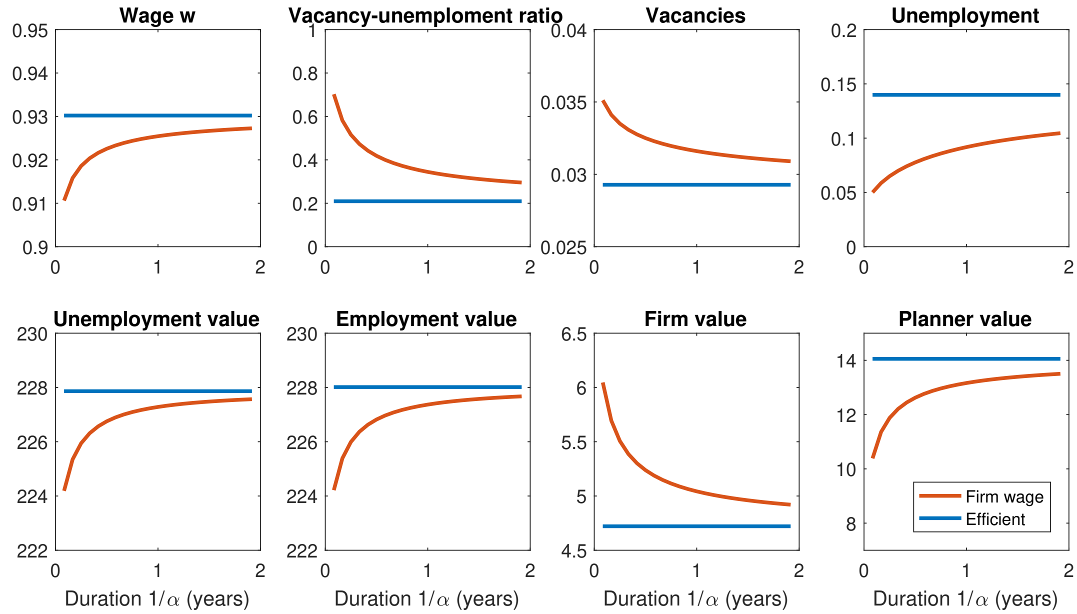

Figure 5: Equilibrium with Longer Wage Commitments

Notes: The figure displays steady-state values in a firm wage equilibrium with infrequent adjustment, as a function of the expected duration of wages $1/\alpha$, together with the corresponding efficient allocation/unconstrained equilibrium. The firm value plotted is the scaled firm value per initial size, but also the unscaled firm value declines in wage duration. Correspondingly, the planner value plotted is the scaled value per initial size, but also the unscaled value increases in wage duration. The figure also displays the corresponding values in the efficient allocation.

dampening the increase. On net, the deviating firm's hiring barely increases with the shock. Overall, the value of the commitment provided by the fixed wage appears to dominate these limitations in responding to shocks here, however.

Equilibrium with Fixed Wages Given that fixed wages thus appear profitable for firms, I then turn to the question of how they affect equilibrium outcomes.

Beginning again with a version of the model without shocks, Figure 5 shows how the firm wage equilibrium compares with the unconstrained case/efficient allocations (as discussed earlier), as well as showing how longer wage durations affect outcomes relative to this starting point. As in the case of the deviating firm, longer wage durations tend to raise wages, as firms become more forward-looking in their wage setting. With all firms affected, this upward pressure on wages shifts the equilibrium toward higher wages. In a sense this works to reverse the effects of firm wages on equilibrium outcomes: As equilibrium wages rise, both employed and unemployed workers become better off, despite a decline in the job finding probability. Meanwhile firm values fall, as does the profitability of vacancy creation, which also declines. The decline in the job finding probability implies an increase in unemployment, as the overhiring associated with the constraints subsides. Overall, longer wage durations thus make workers better off, as well as improving the efficiency of resource allocation.

When firms face aggregate shocks, fixed wages also influence labor market volatility. To consider the impact of longer wage durations in the context of shocks, Figure 6 again plots simulation means together with standard deviation bounds, both for the firm wage model (as a function of the duration of the wage) and the efficient allocation. The figure confirms that – as one might expect with sticky wages – longer wage durations lead to added volatility in vacancy creation and unemployment. Note that there are two competing effects at play here, however. One the one hand, longer wage durations should – by making wages sticky – reduce wage volatility and increase that of labor market flows. On the other, they should also – by reversing the effects of firm wages on volatility – increase wage volatility and reduce that of labor market flows. Between the two effects, the net effect on, for example, wage volatility is non-monotonic, despite the volatility of vacancy creation and unemployment rising in the duration of wages.[41]

Finally, note that despite making worker and firm values more variable, longer wage durations continue to raise worker value and decrease firm value, while bringing allocations closer to socially optimal ones in terms of levels.

5.4 Firm-Level Shocks

In practice firms face non-trivial firm-level risk as well. The firm wage model extends to accommodate a stationary equilibrium with idiosyncratic firm-level shocks in a natural way, and this section illustrates the impact of firm wages, as well as fixed wages, on outcomes in that context. I relegate the equations to Appendix F, proceeding directly to discuss the results below.

Firm Wage Equilibrium with Firm Shocks In a stationary equilibrium with idiosyncratic firm-level shocks to productivity, individual firms grow and shrink over time in response to the shocks they face. Figure F.1 illustrates this churn by plotting impulse responses.[42] An

[41]The effect is also not clearly monotonic for the vacancy-unemployment ratio. As longer durations raise the volatility of vacancy creation and unemployment they also reduce the correlation between the two, as longer wage durations lead to lagged unemployment responses. Figure G.7 provides corresponding impulse responses, showing that *in percentage terms* the volatility of the vacancy-unemployment ratio, unemployment and vacancy creation rise, while that of the aggregate wage falls, as wage durations increase.

[42]I adjust the model calibration for the case of firm shocks, increasing the size of the adjustment cost in order to curb firm responses to large and persistent firm-level shocks in the context of linear production

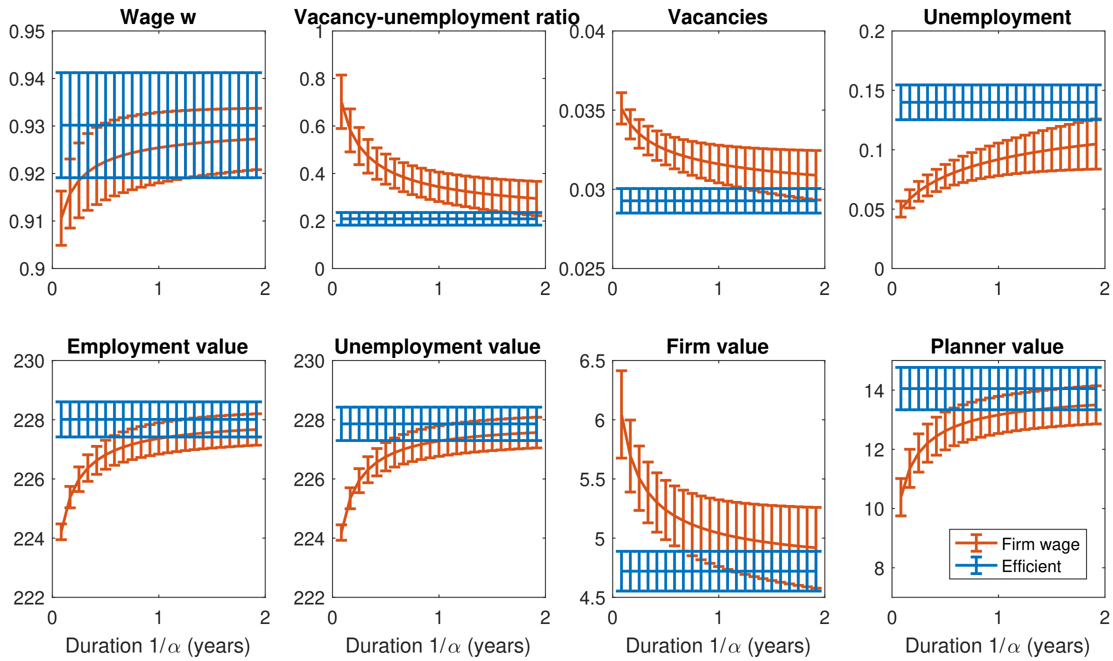

Figure 6: Equilibrium with Longer Wage Commitments with Aggregate Shocks

Notes: The figure displays simulation means together with standard deviation bounds in a firm wage equilibrium with infrequent adjustment, as a function of the expected duration of wages $1/\alpha$, together with the corresponding efficient allocation/unconstrained equilibrium. Labor productivity follows an AR(1) with autocorrelation $\rho_z = 0.96$ and standard deviation $\sigma_z = 0.02$. The firm value plotted is the scaled firm value per initial size, but also the unscaled firm value declines in wage duration. Correspondingly, the planner value plotted is the scaled value per initial size, but also the unscaled value increases in wage duration. The figure also shows the corresponding values in the efficient allocation.

increase in firm productivity causes the firm to raise its offered wage, thus attracting more job seekers per vacancy, as well as to increase its vacancy creation. As a result firm growth accelerates, with employment expanding over time relative to other firms. Relative to the unconstrained case, firm wages again work to dampen the responses of wages to shocks, and amplify those of hiring and employment.

Note that despite the size-independence of the firm problem, which carries over to the case of firm-level shocks, the above indicates that the model is consistent with a large firm wage premium in the cross section (Brown and Medoff 1989) in the sense that if productivity is persistent, more productive firms will both offer higher wages and become larger.

technologies. Specifically, I lower the target tightness to 0.4, which implies $\ell = 2.67$, $b = 0.81$ and an average cost of vacancies of 3.8 in the firm wage model.

Fixed Wages with Firm Shocks Can fixing wages remain profitable in the face of significant firm-level shocks? To shed light on this, Figure F.2 again considers a firm deviating to a fixed wage when other firms reoptimize each period. Firm productivity is discretized to take on one of three values: low, intermediate or high, with the deviating firm depicted in the intermediate productivity state. The figure shows that, in line with the impulse responses, among the equilibrium firms more productive firms pay higher wages, attract more job seekers per vacancy, and grow faster, as well as having greater firm value. Despite the differences driven by productivity, the effects of the deviation remain very similar, however, with the deviating firm offering a higher wage and creating more vacancies, and thus growing faster than equilibrium firms with similar productivity. In particular, the deviation remains profitable despite productivity being an important determinant of firm value when shocks are large.

Finally, I also revisit the equilibrium effects of fixed wages in this setting with firm shocks. Figure F.3 plots the results, confirming that while prevailing productivity is important for firms, longer wage durations continue to have similar effects as before. As firms become more forward-looking, the equilibrium shifts toward higher wages, making workers better off and reducing overhiring.

6 Conclusions

Motivated by literature in personnel economics characterizing worker compensation in large organizations as governed by administrative rules reflecting horizontal equity concerns, this paper thinks about firm wage setting in an environment where firms face within-firm equity constraints on wages. I show that introducing such constraints leads to lower wages, as the firms' incentive to profit from their existing workers via low wages depresses also those of new hires, increasing the profitability of hiring. In a dynamic model, these elements further give rise to a time-inconsistency in the firm problem affecting allocations. To consider outcomes when firms reoptimize wages each period in the face of this time-inconsistency, I consider Markov perfect equilibria, also proposing a tractable solution approach to the problem. In two applications, I show that the constraints dampen wage variation over the business cycle and can amplify that in unemployment in a significant way. Second, firms may find it profitable to fix wages for a period of time, and an equilibrium with fixed wages be good for worker welfare, as well as resource allocation, despite added volatility in the labor market.

The model environment could naturally be enriched in various ways. Instead of imposing constraints on firms, one could think more carefully about the underlying problem between firm owners, managers and workers giving rise to them. Instead of imposing equal treatment among equally productive workers within the firm, one could seek to explicitly incorporate tenure premia, or wage compression among heterogenous workers within the firm. And further, one could attempt to formalize the internal wage structures more fully, including transitions from one position to another within the firm. At the same time, the present approach offers a simpler setting for demonstrating the time-inconsistency arising in the context of constraints, as well as allowing a tractable approach to solving for equilibrium outcomes in the presence of aggregate and firm-level shocks. The results suggest that the institutional constraints affecting firms can play an important role for understanding important features of the labor market.

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Appendix

A Proofs and Details

Proof of Proposition 1 Equation (1) yields the derivative $g_w(w;U) = -\mu(\theta)/(\mu'(\theta)(w-b))$, equation (4) the wage $w = z - \kappa_v(x)/q(\theta)$ and the equilibrium condition the vacancy rate $x = \theta(1-N)/N$. Using these in (5) yields an equation determining equilibrium θ:

$$1 + q(\theta)\theta\frac{1-N}{N} = -q'(\theta)\theta\frac{1-N}{N}\frac{\kappa_v(\theta\frac{1-N}{N})}{q(\theta)}\frac{\mu(\theta)}{\mu'(\theta)(z-b-\frac{\kappa_v(\theta\frac{1-N}{N})}{q(\theta)})},$$

or dividing by θ,

$$\frac{1}{\theta} + q(\theta)\frac{1-N}{N} = \frac{1-\varepsilon_\mu(\theta)}{\varepsilon_\mu(\theta)}\frac{1-N}{N}\frac{\kappa_v(\theta\frac{1-N}{N})}{z-b-\frac{\kappa_v(\theta\frac{1-N}{N})}{q(\theta)}},$$

where I denote the matching function elasticity by $\varepsilon_\mu(\theta) := \mu'(\theta)\theta/\mu(\theta)$.

The left hand side is strictly decreasing and the right hand side strictly increasing in θ, given the assumptions on the vacancy cost and matching function. Hence, the equation pins down a unique equilibrium θ.

For the unconstrained model one simply leaves out the $1/\theta$ term on the left hand side, which implies that the tightness in the firm wage model is strictly greater, $\theta_{FW} > \theta_{SD}$, and hence employment, $N + \mu(\theta)(1-N)$, is strictly higher in the firm wage model. From $x = \theta(1-N)/N$, the hiring rate in the firm wage model is then strictly greater, $x_{FW} > x_{SD}$, as is total vacancy creation xN. Finally the wage, from $w = b + \kappa_v(x)/q(\theta)$, is strictly lower in the firm wage model, $w_{FW} < w_{SD}$.

Proof of Proposition 2 For convenience, let $y_t := Y_t - \beta(1-\delta)E_t Y_{t+1}$. We have $y_t = E_t[\beta\delta U_{t+1} - b - \beta U_{t+1} + \beta(1-\delta)(b + \beta U_{t+2})] = E_t[-b - \beta(1-\delta)(U_{t+1} - b - \beta U_{t+2})]$, meaning that $y_t = -b - \beta(1-\delta)E_t X_{t+1}$.

First, the firm objective in (10) can be rewritten as

$$E_0[n_0 \sum_{t=0}^{\infty} \beta^t (1-\delta)^t (z - w_t) + \sum_{t=0}^{\infty} \beta^t \sum_{k=0}^{t-1} (1-\delta)^{t-k} q(\theta_k) v_k (z - w_t) - \sum_{t=0}^{\infty} \beta^t \kappa(v_t, n_t)], \quad (35)$$

using that $n_t + q(\theta_t)v_t = (1-\delta)^t n_0 + \sum_{k=0}^{t-1} (1-\delta)^{t-k} q(\theta_k) v_k$.

The first term in (35) can then be rewritten as

$$E_0 n_0 \sum_{t=0}^{\infty} \beta^t (1-\delta)^t (z - w_t) = n_0[Z_0 + Y_0 - \frac{X_0}{\mu(\theta_0)}], \quad (36)$$

using that the job seeker value constraint (12) implies $W_0 = X_0/\mu(\theta_0) - Y_0$.

The second term in (35) can be rewritten as

$$\begin{aligned} & E_0 \sum_{k=0}^{\infty} \beta^k q(\theta_k) v_k \sum_{t=k}^{\infty} \beta^{t-k} (1-\delta)^{t-k} (z_t - w_t) \\ &= E_0 \sum_{k=0}^{\infty} \beta^k [q(\theta_k) v_k \sum_{t=k}^{\infty} \beta^{t-k} (1-\delta)^{t-k} (z_t + y_t) - \frac{v_k}{\theta_k} \sum_{t=k}^{\infty} \beta^{t-k} (1-\delta)^{t-k} X_t] \\ &= E_0 \sum_{t=0}^{\infty} \beta^t \sum_{k=0}^{t-1} (1-\delta)^{t-k} [q(\theta_k) v_k (z_t + y_t) - \frac{v_k}{\theta_k} X_t], \quad (37) \end{aligned}$$

where the first equality follows from rearranging terms, and the second uses the job seeker value constraint to substitute out the present value of wages.

Combining the terms in (36) and (37) and rearranging, the firm objective becomes

$$
\begin{aligned}
& E_0 n_0 [\sum_{t=0}^{\infty} \beta^t (1-\delta)^t (z_t + y_t) - \frac{X_0}{\mu(\theta_0)}] \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \sum_{k=0}^{t-1} (1-\delta)^{t-k} [q(\theta_k) v_k (z_t + y_t) - \frac{v_k}{\theta_k} X_t] - E_0 \sum_{t=0}^{\infty} \beta^t \kappa(v_t, n_t) \\
& = -\frac{n_0 X_0}{\mu(\theta_0)} + E_0 \sum_{t=0}^{\infty} \beta^t [(n_t + q(\theta_t) v_t)(z_t + y_t) - \frac{v_t X_t}{\theta_t} - \kappa(v_t, n_t)]. \qquad (38)
\end{aligned}
$$

Using that $y_t = -b - \beta(1-\delta) E_t X_{t+1}$, and rearranging, the firm objective can be written as

$$
-\frac{n_{i0} X_0}{\mu(\theta_{i0})} + n_{i0} X_0 + E_t \sum_{t=0}^{\infty} \beta^t [(n_{it} + q(\theta_{it}) v_{it})(z_t - b) - \kappa(v_{it}, n_{it}) - X_t (\frac{v_{it}}{\theta_{it}} + n_{it})]. \qquad (39)
$$

Note that the term $n_{i0} X_0$ is independent of the firm's actions, and has been omitted for brevity in writing the firm problem as in (13) in the text. In calculating the actual firm value, it must be added back.

Opting Out of Hiring Note that because the firm begins with a stock of existing workers, it could potentially find it optimal to, instead of following the interior solution characterized by the first order conditions, not hire at all in the first period and instead set a wage that is so low as to make those existing workers indifferent between remaining with the firm and unemployment. The latter would mean that $W_0 + Y_0 = 0$ and no hiring that $v_0 = 0$. How would this change firm value?

In the derivation above, it would mean that the expression in (36) would reduce to $n_{i0}[Z_0 + Y_0]$, and the expression in (37) would have $v_{i0} = 0$, such that θ_{i0} no longer appears. Firm value, as in (39), would then become

$$
n_{i0} X_0 + E_t \sum_{t=0}^{\infty} \beta^t [(n_{it} + q(\theta_{it}) v_{it})(z_t - b) - \kappa(v_{it}, n_{it}) - X_t (\frac{v_{it}}{\theta_{it}} + n_{it})]
$$

with $v_{i0} = 0$. With commitment, after this initial period the firm problem becomes equivalent to the planner problem, and hence hiring should be consistent with efficient allocations and interior as long as standard conditions are met (z sufficiently above b). In the initial period, one would want to check that this value does not dominate the equilibrium value. Note that due to the size-independence of the firm problem, if one firm prefers to deviate, all firms will.

In the context of no commitment, if a firm in any period were to deviate to this non-hiring option, its value would be

$$
\begin{aligned}
& nX(S) + n(z-b) - nX(S) + \beta E_S V(n'; S') \\
& \text{s.t. } n' = (1-\delta)n,
\end{aligned}
$$

where the continuation value $V(n;S)$ follows (20). In solving the model using first order conditions, one would want to make sure this deviation value does not exceed equilibrium values, something that can restrict parameter values. In practice high aggregate levels of existing matches tend to make deviating more attractive, so one would choose parameters such that the desired steady-state measure of matches is sufficiently below this range, keeping the economy below a range where deviating becomes attractive.

Second Order Conditions For the sequence problem, denoting the firm objective as g, second order conditions read, for $t > 0$: $g_{x_t x_t} = -\hat{\kappa}''(x_t) < 0$, $g_{\theta_t\theta_t} = q''(\theta_t)x_t(z_t - b + \beta(1-\delta)E_t\hat{V}_{t+1}) - \frac{2X_t x_t}{\theta_t^3} < 0$, and $det = g_{x_t x_t}g_{\theta_t\theta_t} - g_{x_t\theta_t}^2 > 0$, where $g_{x_t\theta_t} = q'(\theta_t)(z_t - b + \beta(1-\delta)E_t\hat{V}_{t+1}) + \frac{X_t}{\theta_t^2} = 0$. For the initial period: $g_{x_0x_0} = -\hat{\kappa}''(x_0)$, $g_{\theta_0\theta_0} = \frac{X_0\mu''(\theta_0)}{\mu(\theta_0)^2} - \frac{2X_0\mu'(\theta_0)^2}{\mu(\theta_0)^3} + q''(\theta_0)x_0(z_0 - b + \beta(1-\delta)E_0\hat{V}_1) - \frac{2X_0x_0}{\theta_0^3}$ and $det = g_{x_0x_0}g_{\theta_0\theta_0} - g_{x_0\theta_0}^2 > 0$, where $g_{x_0\theta_0} = q'(\theta_0)(z_0 - b + \beta(1-\delta)E_0\hat{V}_1) + \frac{X_0}{\theta_0^2}$. The periods separate when calculating second order conditions.

For the no commitment case, again denoting the firm objective as g, second order conditions read: $g_{xx} = -\hat{\kappa}''(x) < 0$, $g_{\theta\theta} = \frac{X\mu''(\theta)}{\mu(\theta)^2} - \frac{2X\mu'(\theta)^2}{\mu(\theta)^3} + q''(\theta)x(z-b+\beta(1-\delta)E\hat{V}) - \frac{2Xx}{\theta^3} < 0$, and $det = g_{xx}g_{\theta\theta} - g_{x\theta}^2 > 0$, where $g_{x\theta} = q'(\theta)(z - b + \beta(1-\delta)E\hat{V}) + \frac{X}{\theta^2}$.

Proof of Proposition 3 The firm problem (10) is equivalent to the problem

$$
\begin{aligned}
& \max_{\{w_{it},\theta_{it},x_{it}\}} E_0 \sum_{t=0}^{\infty} \beta^t (1-\delta)^t \prod_{k=0}^{t-1}(1 + q(\theta_{ik})x_{ik})[(1 + q(\theta_{it})x_{it})(z_t - w_{it}) - \hat{\kappa}(x_{it})] \\
& \text{s.t. } X_t = \mu(\theta_{it})(E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k w_{it+k} + Y_t), \ \forall t \geq 0,
\end{aligned}
$$

which does not depend on n_{i0}. This can be seen by expressing the profits in problem (10) in each period t scaled by size n_{it} and using the law of motion to adjust the discounting for this scaling. Finally, normalizing the firm problem with initial size n_{i0} yields the expression above.

Unconstrained Firm Problem Starting from the firm problem for hiring in period t in (26), one can substitute wages out using (27) to arrive at

$$
\max_{\theta_{it},v_{it}} E_t \left[q(\theta_{it})v_{it}[z_t - b + \sum_{k=1}^{\infty} \beta^k (1-\delta)^k (z_{t+k} - b - X_{t+k})] - \frac{X_t v_{it}}{\theta_{it}} - \sum_{k=0}^{\infty} \beta^k \kappa(v_{it+k}, n_{it+k}) \right],
$$

using the relationship $Y_t = -b + \beta(1-\delta)E_t(Y_{t+1} - X_{t+1})$, or $Y_t = -b - E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k (b + X_{t+k})$.

Adding up over cohorts of workers hired at different points in time (with discounting) yields

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[q(\theta_{it}) v_{it} [z_t - b + \sum_{k=1}^{\infty} \beta^k (1-\delta)^k (z_{t+k} - b - X_{t+k})] - \frac{X_t v_{it}}{\theta_{it}} - \kappa(v_{it}, n_{it}) \right]$$
$$= E_0 \sum_{t=0}^{\infty} \beta^t \left[(n_{it}^n + q(\theta_{it}) v_{it})(z_t - b) - X_t (n_{it}^n + \frac{v_{it}}{\theta_{it}}) - \kappa(v_{it}, n_{it}) \right] + X_0 n_{i0}^n,$$

where n_{it}^n denotes the workforce hired on or after period zero, with $n_{i0}^n = 0$ and $n_{it+1}^n = (1-\delta)(n_{it}^n + q(\theta_{it}) v_{it})$ for all $t \geq 0$.

Total firm value in period zero can then be written, adding the present value associated with existing workers at time zero, $n_{i0}[E_0 \sum_{t=0}^{\infty} \beta^t (1-\delta)^t z_t - W_{i0}^e]$, where W_{i0}^e represents the average present value of wages among the initial workforce, as

$$E_0 \sum_{t=0}^{\infty} \beta^t [(n_{it}^n + q(\theta_{it}) v_{it})(z_t - b) - X_t (n_{it}^n + \frac{v_{it}}{\theta_{it}}) - \kappa(v_{it}, n_{it})] + X_0 n_{i0}^n + n_{i0}[E_0 \sum_{t=0}^{\infty} \beta^t (1-\delta)^t z_t - W_{i0}^e]$$
$$= E_0 \sum_{t=0}^{\infty} \beta^t [(n_{it} + q(\theta_{it}) v_{it})(z_t - b) - X_t (n_{it} + \frac{v_{it}}{\theta_{it}}) - \kappa(v_{it}, n_{it})]$$
$$- n_{i0} E_0 \sum_{t=0}^{\infty} \beta^t (1-\delta)^t (z_t - b - X_t) + X_0 n_{i0}^n + n_{i0}[E_0 \sum_{t=0}^{\infty} \beta^t (1-\delta)^t z_t - W_{i0}^e],$$

where the equality uses that $n_{it} = n_{it}^n + (1-\delta)^t n_{i0}$.

Note that the terms on the last line are either given or independent of the firm's choice variables $\{\theta_{it}, v_{it}\}_{t=0}^{\infty}$ which appear only on the previous line, and thus the firm problem effectively coincides with maximizing the former.

B Two Period Model of Firm Wages

Consider a deterministic, two-period version of the dynamic model in Section 3.

The value of entering period $t = 0, 1$ as an unemployed worker satisfies

$$U_1 = \mu(\theta_{i1}) w_{i1} + (1 - \mu(\theta_{i1})) b,$$
$$U_0 = \mu(\theta_{i0})(w_{i0} + \beta(1-\delta) w_{i1} + \beta\delta U_1) + (1 - \mu(\theta_{i0}))(b + \beta U_1).$$

If we define $X_0 := U_0 - b - \beta U_1$, $X_1 := U_1 - b$, $Y_0 := -b - \beta(1-\delta)U_1$, and $Y_1 := -b$, the above can be written as

$$X_t = \mu(\theta_{it})(W_{it} + Y_t),\ t = 0, 1,$$

where $W_{i1} = w_{i1}$ and $W_{i0} = w_{i0} + \beta(1-\delta)w_{i1}$.

Commitment Assuming firms have commitment, the firm problem reads

$$\max_{\{w_{it},\theta_{it},v_{it}\}_{t=0}^{1}} \sum_{t=0}^{1} \beta^t[(n_{it} + q(\theta_{it})v_{it})(z_t - w_{it}) - \kappa(v_{it}, n_{it})],$$

$$\text{s.t. } n_{i1} = (1-\delta)(n_{i1} + q(\theta_{i1})v_{i1}), \tag{40}$$

$$X_t = \mu(\theta_{it})(\sum_{k=0}^{1} \beta^k(1-\delta)^k w_{it+k} + Y_t),\ \text{for } t = 0, 1, \tag{41}$$

with n_{i0} given for all i. The firm maximizes the present discounted value of profits, taking into account the law of motion for employment relationships, as well as the constraint reflecting job seeker behavior each period, where the firm takes the market-determined values of X_t, Y_t as given.

Using the job seeker constraints (41) to substitute out wages, the law of motion (40), and dividing by initial size, the firm problem can be rewritten as

$$\begin{aligned} \max -\frac{X_0}{\mu(\theta_{i0})} &+ X_0 + (1 + q(\theta_{i0})x_{i0})(z_0 - b) - \hat{\kappa}(x_{i0}) - X_0(\frac{x_{i0}}{\theta_{i0}} + 1) \\ &+ \beta(1-\delta)(1 + q(\theta_{i1})x_{i1})[(1 + q(\theta_{i1})x_{i1})(z_1 - b) - \hat{\kappa}(x_{i1}) - X_1(\frac{x_{i1}}{\theta_{i1}} + 1)]. \end{aligned}$$

Note that this problem is independent of firm size, and hence in what follows the firm-level indicators are dropped.

The first order conditions in period one read[43]

$$\kappa_v(x_1) + \frac{X_1}{\theta_1} = q(\theta_1)(z_1 - b),$$

$$\frac{X_1}{\theta_1} = -q'(\theta_1)(z_1 - b).$$

[43]Denoting the firm objective as g, second order conditions read: $g_{x_1x_1} = -\hat{\kappa}''(x_1) < 0$, $g_{\theta_1\theta_1} = q''(\theta_1)x_1(z_1 - b) - \frac{2X_1x_1}{\theta_1^3} < 0$, and $det = g_{x_1x_1}g_{\theta_1\theta_1} - g_{x_1\theta_1}^2 > 0$, where $g_{x_1\theta_1} = q'(\theta_1)(z_1 - b) + \frac{X_1}{\theta_1^2} = 0$, and $g_{x_0x_0} = -\hat{\kappa}''(x_0)$, $g_{\theta_0\theta_0} = \frac{X_0\mu''(\theta_0)}{\mu(\theta_0)^2} - \frac{2X_0\mu'(\theta_0)^2}{\mu(\theta_0)^3} + q''(\theta_0)x_0(z_0 - b + \beta(1-\delta)E_0\hat{V}_1) - \frac{2X_0x_0}{\theta_0^3}$ and $det = g_{x_0x_0}g_{\theta_0\theta_0} - g_{x_0\theta_0}^2 > 0$, where $g_{x_0\theta_0} = q'(\theta_0)(z_0 - b + \beta(1-\delta)E_0\hat{V}_1) + \frac{X_0}{\theta_0^2} < 0$. The periods separate when calculating second order conditions.

Taken together, these imply that $\frac{\kappa_v(x_1)}{\mu'(\theta_1)} = z_1 - b$, where $X_1 = \kappa_v(x_1)\frac{\mu(\theta_1)-\mu'(\theta_1)\theta_1}{\mu'(\theta_1)}$.

Given an allocation θ_1, x_1, period one firm value (normalized by size) is

$$\hat{V}_1^o = -\frac{X_1}{\mu(\theta_1)} + X_1 + (1+q(\theta_1)x_1)(z_1-b) - \hat{\kappa}(x_1) - X_1(\frac{x_1}{\theta_1}+1),$$

while the continuation value is

$$\hat{V}_1 = (1+q(\theta_1)x_1)(z_1-b) - \hat{\kappa}(x_1) - X_1(\frac{x_1}{\theta_1}+1).$$

Using the optimality conditions, these can be written as

$$\hat{V}_1^o = -\frac{X_1}{\mu(\theta_1)} + z_1 - b - \kappa_n(x_1),$$
$$\hat{V}_1 = z_1 - b - \kappa_n(x_1) - X_1.$$

The first order conditions in the initial period read

$$\kappa_v(x_0) + \frac{X_0}{\theta_0} = q(\theta_0)[z_0 - b + \beta(1-\delta)\hat{V}_1],$$
$$\frac{X_0}{\theta_0^2}[1 + \frac{\mu'(\theta_0)\theta_0^2}{x_0\mu(\theta_0)^2}] = -q'(\theta_0)[z_0 - b + \beta(1-\delta)\hat{V}_1],$$

and taken together, imply that

$$\frac{\kappa_v(x_0)}{\mu'(\theta_0)}[1 - \frac{(1-\mu'(\theta_0)\theta_0/\mu(\theta_0))}{1+q(\theta_0)x_0}] = z_0 - b + \beta(1-\delta)\big[\frac{\kappa_v(x_1)}{\mu'(\theta_1)}\big(1-\mu(\theta_1)+\mu'(\theta_1)\theta_1\big) - \kappa_n(x_1)\big],$$

with

$$X_0 = \kappa_v(x_0)\frac{\mu(\theta_0)-\mu'(\theta_0)\theta_0}{\mu'(\theta_0)}\frac{q(\theta_0)x_0}{1+q(\theta_0)x_0}.$$

Given allocations and the continuation value, and using the optimality conditions, the normalized firm value in the initial period can be written as

$$\hat{V}_0^o = -\frac{X_0}{\mu(\theta_0)} + z_0 - b + \beta(1-\delta)\hat{V}_1 - \kappa_n(x_0).$$

Limited Commitment If firms cannot commit, the firm problem is solved backward.

In period one, firms maximize the firm value

$$\max -\frac{X_1}{\mu(\theta_1)} + X_1 + (1+q(\theta_1)x_1)(z_1-b) - \hat{\kappa}(x_1) - X_1(\frac{x_1}{\theta_1}+1).$$

The first order conditions for optimality read

$$\kappa_v(x_1) + \frac{X_1}{\theta_1} = q(\theta_1)(z_1 - b),$$
$$\frac{X_1}{\theta_1^2}[1 + \frac{\mu'(\theta_1)\theta_1^2}{x_1\mu(\theta_1)^2}] = -q'(\theta_1)(z_1 - b),$$

and imply

$$\frac{\kappa_v(x_1)}{\mu'(\theta_1)}[1 - \frac{(1 - \mu'(\theta_1)\theta_1/\mu(\theta_1))}{1 + q(\theta_1)x_1}] = z_1 - b,$$

with

$$X_1 = \kappa_v(x_1)\frac{\mu(\theta_1) - \mu'(\theta_1)\theta_1}{\mu'(\theta_1)}\frac{q(\theta_1)x_1}{1 + q(\theta_1)x_1}.$$

Given allocations, the normalized firm value then satisfies

$$\hat{V}_1^o = -\frac{X_1}{\mu(\theta_1)} + (1 + q(\theta_1)x_1)(z_1 - b) - \frac{X_1 x_1}{\theta_1} - \hat{\kappa}(x_1) = -\frac{X_1}{\mu(\theta_1)} + z_1 - b - \kappa_n(x_1)$$

and the corresponding continuation value

$$\hat{V}_1 = z_1 - b - \kappa_n(x_1) - X_1.$$

The first order conditions in the initial period read[44]

$$\kappa_v(x_0) + \frac{X_0}{\theta_0} = q(\theta_0)[z_0 - b + \beta(1-\delta)\hat{V}_1],$$
$$\frac{X_0}{\theta_0^2}[1 + \frac{\mu'(\theta_0)\theta_0^2}{x_0\mu(\theta_0)^2}] = -q'(\theta_0)[z_0 - b + \beta(1-\delta)\hat{V}_1],$$

and imply

$$\frac{\kappa_v(x_0)}{\mu'(\theta_0)}[1 - \frac{(1 - \mu'(\theta_0)\theta_0/\mu(\theta_0))}{1 + q(\theta_0)x_0}]$$
$$= z_0 - b + \beta(1-\delta)\big[\frac{\kappa_v(x_1)}{\mu'(\theta_1)}\big(1 - \mu(\theta_1) + \mu'(\theta_1)\theta_1 - (1 - \mu(\theta_1))\frac{(1 - \mu'(\theta_1)\theta_1/\mu(\theta_1))}{1 + q(\theta_1)x_1}\big) - \kappa_n(x_1)\big],$$

with

$$X_0 = \kappa_v(x_0)\frac{\mu(\theta_0) - \mu'(\theta_0)\theta_0}{\mu'(\theta_0)}\frac{q(\theta_0)x_0}{1 + q(\theta_0)x_0}.$$

[44]Denoting the firm objective as g, second order conditions read: $g_{x_1x_1} = -\hat{\kappa}''(x_1) < 0$, $g_{\theta_1\theta_1} = \frac{X_1\mu''(\theta_1)}{\mu(\theta_1)^2} - \frac{2X_1\mu'(\theta_1)^2}{\mu(\theta_1)^3} + q''(\theta_1)x_1(z_1 - b) - \frac{2X_1x_1}{\theta_1^3} < 0$, and $det = g_{x_1x_1}g_{\theta_1\theta_1} - g_{x_1\theta_1}^2 > 0$, where $g_{x_1\theta_1} = q'(\theta_1)(z_1 - b) + \frac{X_1}{\theta_1^2} < 0$, and $g_{x_2x_2} = -\hat{\kappa}''(x_2)$, $g_{\theta_0\theta_0} = \frac{X_0\mu''(\theta_0)}{\mu(\theta_0)^2} - \frac{2X_0\mu'(\theta_0)^2}{\mu(\theta_0)^3} + q''(\theta_0)x_0(z_0 - b + \beta(1-\delta)E_0\hat{V}_1) - \frac{2X_0x_0}{\theta_0^3}$ and $det = g_{x_0x_0}g_{\theta_0\theta_0} - g_{x_0\theta_0}^2 > 0$, where $g_{x_0\theta_0} = q'(\theta_0)(z_0 - b + \beta(1-\delta)E_0\hat{V}_1) + \frac{X_0}{\theta_0^2} < 0$. The periods separate when calculating second order conditions.

Given allocations and continuation values, normalized firm value in the initial period equals

$$\hat{V}_0^o = -\frac{X_0}{\mu(\theta_0)} + z_0 - b + \beta(1-\delta)\hat{V}_1 - \kappa_n(x_0).$$

Whether or not firms have commitment, equilibrium requires allocations to be optimal for firms, as well as the total measure of job seekers allocated to firms to be consistent with the measure of job seekers in the market: $x_t = \theta_t(1-N_t)/N_t$ for $t = 0, 1$.

Planner Problem The planner problem reads:

$$\begin{aligned}
\max_{\{\theta_{it}, v_{it}\}_{t=0}^1} & \sum_{t=0}^{1} \beta^t [\sum_i [(n_{it} + q(\theta_{it})v_{it})z_t - \kappa(v_{it}, n_{it})] + (1 - \sum_i (n_{it} + q(\theta_{it})v_{it})b)] \\
\text{s.t. } & n_{i1} = (1-\delta)(n_{i0} + q(\theta_{i0})v_{i0}), \\
& \sum_i v_{it}/\theta_{it} = 1 - \sum_i n_{it}, \text{ for } t = 0, 1,
\end{aligned} \tag{42}$$

with n_{i0} given for all i. The planner maximizes the present discounted value of output produced by employed workers with the market technology and by unemployed workers with the home technology, net of the costs of vacancy creation. The planner takes as given the law of motion for employment relationships, as well as a constraint (42) that imposes that the planner's choices of vacancies and market tightness across markets must be consistent with the total measure of job seekers in each period. In what follows, the latter constraint is associated with a Lagrange multiplier λ_t for $t = 0, 1$, reflecting the planner's shadow value of job seekers.

The first order conditions for the planner's choice of v_{it}, θ_{it} for $t = 0, 1$, read

$$\begin{aligned}
\kappa_v(x_{i1}) + \frac{\lambda_1}{\theta_{i1}} &= q(\theta_{i1})(z_1 - b), \\
\frac{\lambda_1}{\theta_{i1}^2} &= -q'(\theta_{i1})(z_1 - b), \\
\kappa_v(x_{i0}) + \frac{\lambda_0}{\theta_{i0}} &= q(\theta_{i0})[z_0 - b + \beta(1-\delta)(z_1 - b - \kappa_n(x_{i1}) - \lambda_1)], \\
\frac{\lambda_0}{\theta_{i0}^2} &= -q'(\theta_{i0})[z_0 - b + \beta(1-\delta)(z_1 - b - \kappa_n(x_{i1}) - \lambda_1)].
\end{aligned}$$

Note that these are independent of producer size, and in what follows I hence drop the producer index i to consider symmetric allocations.

Taken together, the optimality conditions imply that

$$\frac{\kappa_v(x_1)}{\mu'(\theta_1)} = z_1 - b,$$
$$\frac{\kappa_v(x_0)}{\mu'(\theta_0)} = z_0 - b + \beta(1-\delta)(\frac{\kappa_v(x_1)}{\mu'(\theta_1)}(1 - \mu(\theta_1) + \theta_1\mu'(\theta_1)) - \kappa_n(x_1)),$$

with the Lagrange multipliers satisfying

$$\lambda_t = \kappa_v(x_t)\frac{\mu(\theta_t) - \mu'(\theta_t)\theta_t}{\mu'(\theta_t)}$$

for $t = 0, 1$.

In addition, the planner's allocation must also satisfy the constraint (42), $x_t = \theta_t(1 - N_t)/N_t$ for $t = 0, 1$, where the total measure of existing relationships satisfies the law of motion $N_1 = (1-\delta)(1 + q(\theta_0)x_0)N_0$.

C Calibration Details

The law of motion for matches implies steady-state unemployment:

$$u = 1 - N - \mu(\theta)(1-N) = \frac{\delta(1-\mu(\theta))}{\delta(1-\mu(\theta)) + \mu(\theta)},$$

and if δ is given, a target for steady-state u determines $\mu(\theta)$.

Given a target for the tightness θ, the matching function parameter γ is then pinned down (uniquely) from $\mu(\theta) = \theta/(1+\theta^\ell)^{1/\ell}$. This also determines steady-state values of $x = \theta(1-N)/N = \delta\theta/((1-\delta)\mu(\theta))$ and $\mu'(\theta)$.

These labor market flows must also be consistent with the model Euler equation: in the case of the firm wage model, equation (25), and the case of the unconstrained model, equation (17). The Euler equation pins down a unique value of $(z-b)/\kappa_0$ that allows the Euler equation to hold with the flows chosen. This still allows alternative combinations of b, κ_0 consistent with any such value, however.

To consider the implications for wages and profits, note that in either case, the firm's first order conditions for vacancy creation together with the dynamic equation for the continuation value of the firm imply that in steady state:

$$\frac{\kappa_v(x) + \frac{X}{\theta}}{q(\theta)} = z - b + \beta(1-\delta)[\frac{\kappa_v(x) + \frac{X}{\theta}}{q(\theta)} - \kappa_n(x) - X].$$

To connect this to wages, note that the present value of wages satisfies $W = X/\mu(\theta) - Y$, where $Y = -(b + \beta(1-\delta)X)/(1-\beta(1-\delta))$.[45] Using this above, we have

$$\frac{\kappa_v(x) + \frac{X}{\theta}}{q(\theta)} = z + \beta(1-\delta)[\frac{\kappa_v(x) + \frac{X}{\theta}}{q(\theta)} - \kappa_n(x)] + (1-\beta(1-\delta))Y,$$

or

$$\frac{\kappa_v(x)}{q(\theta)} + W = z + \beta(1-\delta)[\frac{\kappa_v(x)}{q(\theta)} + W - \kappa_n(x)],$$

which implies that the steady-state per-period wage $w = W(1-\beta(1-\delta))$ satisfies

$$w = z - \frac{\kappa_v(x)}{q(\theta)} + \beta(1-\delta)[\frac{\kappa_v(x)}{q(\theta)} - \kappa_n(x)].$$

For both models to have the same steady-state wage, conditional on having the same steady-state flows, they must have the same κ_0. If this is the case, it follows that firm profits are also the same across models, as firm profit per worker equals

$$\frac{(n + q(\theta)v)(z-w) - \kappa(v,n)}{n + q(\theta)v} = \frac{(1 + q(\theta)x)(z-w) - \kappa(x)}{1 + q(\theta)x}.$$

The calibration approach used first adopts a baseline parametrization for the unconstrained model, involving targets for steady-state flows and a choice of the parameter b, with κ_0 set to satisfy the corresponding Euler equation. For a comparable parametrization of the firm wage model then, the targets for the steady-state flows are held unchanged, as is the value of κ_0, to keep the steady-state wage and profit rate unchanged across models. The value of b is then determined by the Euler equation for that model.

D Solving: Firm Wages with Aggregate Shocks

The full non-linear dynamic system to solve for the firm wage equilibrium with aggregate shocks is given below. The last five equations define some variables of interest based on the solution (employment, unemployment, the vacancy-unemployment ratio, firm value, and

[45] Appendix A shows that $y_t = -b - \beta(1-\delta)X_{t+1}$, and by definition $Y = y/(1-\beta(1-\delta))$.

firm value if the firm did not hire in the current period at all).

$$\begin{aligned}
&\hat{\kappa}'(x_t) + \frac{X_t}{\theta_t} = q(\theta_t)(z_t - b + \beta(1-\delta)V_{t+1}) \\
&\frac{X_t}{\theta_t^2}(1 + \frac{\mu'(\theta_t)\theta_t^2}{x_t\mu(\theta_t)^2}) = -q'(\theta_t)(z_t - b + \beta(1-\delta)V_{t+1}) \\
&V_t = z_t - b - X_t + \beta(1-\delta)V_{t+1} - \hat{\kappa}(x) + x\hat{\kappa}'(x) \\
&N_{t+1} = (1-\delta)(N_t + \mu(\theta_t)(1-N_t)) \\
&x_t = v_t/N_t \\
&\theta_t(1-N_t) = v_t \\
&X_t = \mu(\theta_t)(W_t + Y_t) \\
&W_t = w_t + \beta(1-\delta)W_{t+1} \\
&y_t = -b - \beta(1-\delta)X_{t+1} \\
&Y_t = y_t + \beta(1-\delta)Y_{t+1} \\
&z_{t+1} - 1 = \rho_z(z_t - 1) + \epsilon_{zt+1} \\
&e_t = N_t + \mu(\theta_t)(1-N_t) \\
&u_t = 1 - e_t \\
&vuratio_t = v_t/u_t \\
&V_{obj,t} = -X_t/\mu(\theta_t) + z_t - b + \beta(1-\delta)V_{t+1} - \hat{\kappa}(x) + x\hat{\kappa}'(x) \\
&V_{objnh,t} = z_t - b + \beta(1-\delta)V_{t+1}
\end{aligned}$$

This uses that $\kappa_v(x) = \hat{\kappa}'(x)$ and $\kappa_n(x) = \hat{\kappa}(x) - x\hat{\kappa}'(x)$.

E Solving: Infrequent Adjustment and Aggregate Shocks

This section considers the solution approach adopted for the equilibrium with infrequent adjustment and aggregate shocks. The challenge is that in principle the distribution of wages is a state variable, with individual firm behavior affected by the firm's prevailing wage, and feeding into the equilibrium adding up condition. The model is solved by linearization, following the approach of Gertler and Trigari (2009). Once the equations are linearized, only the average wage appears in the system characterizing equilibrium.

Given a wage w, we have the present value of wages:

$$W(w) = \frac{w}{1-\beta(1-\delta)(1-\alpha)} + \beta(1-\delta)\alpha\sum_{k=0}^{\infty}\beta^k(1-\delta)^k(1-\alpha)^k W_{t+k+1}.$$

For short, let $\Lambda_t = \beta(1-\delta)\alpha \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (1-\alpha)^k W_{t+k+1}$, which satisfies the dynamic equation

$$\frac{\Lambda_t}{\beta(1-\delta)\alpha} = W_{t+1} + \beta(1-\delta)(1-\alpha)\frac{\Lambda_{t+1}}{\beta(1-\delta)\alpha}.$$

First, I solve for a linear approximation to the firm continuation value when the wage is fixed: $V_t^f(w) - \bar{V} = V_t^0 + V_t^1(w-\bar{w})$.

While a firm's wage w is fixed, the present value of wages at the firm follows:

$$W_t(w) - \bar{W} = \frac{w-\bar{w}}{1-\beta(1-\delta)(1-\alpha)} + \Lambda_t - \bar{\Lambda},$$

where the equilibrium contracting wages (not the wage held fixed w) determine Λ_t according to

$$\frac{\Lambda_t - \bar{\Lambda}}{\beta(1-\delta)\alpha} = W_{t+1} - \bar{W} + \beta(1-\delta)(1-\alpha)\frac{\Lambda_{t+1}-\bar{\Lambda}}{\beta(1-\delta)\alpha}.$$

The present value of wages $W_t(w)$ determines the tightness according to:

$$X_t - \bar{X} = \mu'(\bar{\theta})(\bar{W}+\bar{Y})(\theta_t(w)-\bar{\theta}) + \mu(\bar{\theta})(W_t(w) - \bar{W} + Y_t - \bar{Y}),$$

as a linear function $\theta(w,S) - \bar{\theta} = A_t + B(w-\bar{w})$ with

$$B = -\frac{\mu(\bar{\theta})}{\mu'(\bar{\theta})(\bar{W}+\bar{Y})(1-\beta(1-\delta)(1-\alpha))},$$

$$A_t = \frac{1}{\mu'(\bar{\theta})(\bar{W}+\bar{Y})}(X_t - \bar{X} - \mu(\bar{\theta})(\Lambda_t - \bar{\Lambda} + Y_t - \bar{Y})).$$

The firm's choice of x follows:

$$\begin{aligned}\hat{\kappa}''(\bar{x})(x_t-\bar{x}) + \frac{X_t-\bar{X}}{\bar{\theta}} - \frac{\bar{X}}{\bar{\theta}^2}(\theta_t-\bar{\theta}) &= q'(\bar{\theta})(\bar{z}-b+\beta(1-\delta)\bar{V})(\theta_t-\bar{\theta}) \\ &+ q(\bar{\theta})(z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1}-\bar{V}) + (1-\alpha)(V_{t+1}^0 + V_{t+1}^1(w-\bar{w})))).\end{aligned}$$

Substituting in for $\theta_t(w)$, this gives the hiring rate x as a linear function $x_t(w) - \bar{x} = \hat{A}_t + \hat{B}_t(w-\bar{w})$, where

$$\hat{B}_t = \frac{B\bar{X}}{\hat{\kappa}''(\bar{x})\bar{\theta}^2} + \frac{Bq'(\bar{\theta})(\bar{z}-b+\beta(1-\delta)\bar{V})}{\hat{\kappa}''(\bar{x})} + \frac{q(\bar{\theta})}{\hat{\kappa}''(\bar{x})}\beta(1-\delta)(1-\alpha)V_{t+1}^1,$$

$$\begin{aligned}\hat{A}_t = &-\frac{1}{\hat{\kappa}''(\bar{x})\bar{\theta}}(X_t-\bar{X}) + \frac{\bar{X}}{\hat{\kappa}''(\bar{x})\bar{\theta}^2}A_t + \frac{q'(\bar{\theta})}{\hat{\kappa}''(\bar{x})}(\bar{z}-b+\beta(1-\delta)\bar{V})A_t \\ &+ \frac{q(\bar{\theta})}{\hat{\kappa}''(\bar{x})}(z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1}-\bar{V}) + (1-\alpha)V_{t+1}^0)).\end{aligned}$$

Finally, the dynamic equation for the value $V^f(w,S)$ implies that for all such w we have:

$$\begin{aligned}
&V_t^0 + V_t^1(w-\bar{w})\\
&= z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1}-\bar{V}) + (1-\alpha)(V_{t+1}^0 + V_{t+1}^1(w-\bar{w}))) + \bar{x}\hat{\kappa}''(\bar{x})(x_t(w)-\bar{x}) - (X_t - \bar{X}).
\end{aligned}$$

Using the expression for $x_t(w)$, the expression yields equations for the constant and coefficient on w for this equation to hold.

The coefficient on w thus satisfies:

$$V_t^1 = \beta(1-\delta)(1-\alpha)V_{t+1}^1 + \frac{\bar{x}\bar{X}B}{\bar{\theta}^2} + \bar{x}q'(\bar{\theta})(\bar{z} - b + \beta(1-\delta)\bar{V})B + \bar{x}q(\bar{\theta})\beta(1-\delta)(1-\alpha)V_{t+1}^1.$$

Note that this is an unstable equation with constant coefficients, implying the coefficient V_t^1 is a constant. Further, $\hat{B}_t$ is also then a constant.

The constant satisfies:

$$\begin{aligned}
V_t^0 &= z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1}-\bar{V}) + (1-\alpha)V_{t+1}^0) - (X_t - \bar{X}) - \frac{\bar{x}}{\bar{\theta}}(X_t - \bar{X}) + \frac{\bar{x}\bar{X}}{\bar{\theta}^2}A_t\\
&\quad + \bar{x}q'(\bar{\theta})(\bar{z} - b + \beta(1-\delta)\bar{V})A_t + \bar{x}q(\bar{\theta})(z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1}-\bar{V}) + (1-\alpha)V_{t+1}^0)).
\end{aligned}$$

This is a dynamic equation that is also unstable, but with coefficients that can vary over time. Add this equation into the model system, to determine the coefficients (they enter into the system).

Second, proceed to solve for equilibrium.

Firms that are optimizing this period choose a wage according to:

$$\begin{aligned}
&\frac{X_t - \bar{X}}{\bar{\theta}^2} - 2\frac{\bar{X}}{\bar{\theta}^3}(\theta_t - \bar{\theta}) + \frac{\mu'(\bar{\theta})}{\bar{x}\mu(\bar{\theta})^2}(X_t - \bar{X}) - \frac{\mu'(\bar{\theta})\bar{X}}{\bar{x}^2\mu(\bar{\theta})^2}(x_t - \bar{x}) + \frac{\bar{X}}{\bar{x}}\frac{\mu(\bar{\theta})^2\mu''(\bar{\theta}) - 2\mu(\bar{\theta})\mu'(\bar{\theta})^2}{\mu(\bar{\theta})^4}(\theta_t - \bar{\theta})\\
&= -q''(\bar{\theta})(\bar{z} - b + \beta(1-\delta)\bar{V})(\theta_t - \bar{\theta})\\
&- q'(\bar{\theta})(z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1}-\bar{V}) + (1-\alpha)(V_{t+1}^0 + V_{t+1}^1(w_t - \bar{w}))))\\
&- \beta(1-\delta)(1-\alpha)[\frac{q'(\bar{\theta})\bar{V}^1}{\bar{\theta}_w}(\theta_t - \bar{\theta}) - \frac{\bar{V}^1}{\bar{\theta}_w x^2}(x_t - \bar{x}) + \frac{(1+q(\bar{\theta})\bar{x})}{\bar{\theta}_w\bar{x}}(V_{t+1}^1 - \bar{V}^1) - \frac{(1+q(\bar{\theta})\bar{x})\bar{V}^1}{\bar{\theta}_w^2\bar{x}}(\theta_{wt} - \bar{\theta}_w)]
\end{aligned}$$

with $\theta_t(w) = A_t + B(w-\bar{w})$, $x_t(w) = \hat{A}_t + \hat{B}_t(w-\bar{w})$ from above and

$$\frac{\mu'(\bar{\theta})}{\mu(\bar{\theta})^2}\bar{\theta}_w(X_t - \bar{X}) + \frac{\mu'(\bar{\theta})}{\mu(\bar{\theta})^2}\bar{X}(\theta_{wt} - \bar{\theta}_w) + \frac{\mu(\bar{\theta})^2\mu''(\bar{\theta}) - 2\mu(\bar{\theta})\mu'(\bar{\theta})^2}{\mu(\bar{\theta})^4}\bar{X}\bar{\theta}_w(\theta_t - \bar{\theta}) = 0.$$

The rest of firms apply a previously set wage, and the cross-firm average wage follows: $\hat{w}_t = \alpha w_t + (1-\alpha)\hat{w}_{t-1}$.

The cross-firm average tightness and vacancy rate are: $\hat{\theta}_t = A_t + B(\hat{w}_t - \bar{w})$, $\hat{x}_t = \hat{A}_t + \hat{B}_t(\hat{w}_t - \bar{w})$.

The average firm size follows the law of motion:

$$\hat{n}_{t+1} - \bar{n} = (1-\delta)((1 + q(\bar{\theta})\bar{x})(\hat{n}_t - \bar{n}) + \bar{n}q(\bar{\theta})(\hat{x}_t - \bar{x}) + \bar{n}q'(\bar{\theta})\bar{x}(\hat{\theta}_t - \bar{\theta})).$$

Finally, the equilibrium adding up constraint reads:

$$\frac{\bar{n}}{\bar{\theta}}(\hat{x}_t - \bar{x}) + \frac{\bar{x}}{\bar{\theta}}(\hat{n}_t - \bar{n}) - \frac{\bar{x}\bar{n}}{\bar{\theta}^2}(\hat{\theta}_t - \bar{\theta}) = -(\hat{n}_t - \bar{n}).$$

Steady state: Guess θ. This implies values for $N = \mu(\theta)(1-\delta)/(1-(1-\delta)(1-\mu(\theta)))$ and $x = \theta(1-N)/N$. Firm continuation value satisfies $V = (z - b - \hat{\kappa}(x) + x\hat{\kappa}'(x) - X)/(1 - \beta(1-\delta))$. Plugging this into the optimality condition for vacancies allows solving for X:

$$\hat{\kappa}'(x) + \frac{X}{\theta} = q(\theta)[z - b + \beta(1-\delta)\frac{z - b - \hat{\kappa}(x) + x\hat{\kappa}'(x) - X}{1-\beta(1-\delta)}],$$

$$X = \frac{-\hat{\kappa}'(x) + q(\theta)[z - b + \beta(1-\delta)\frac{z-b-\hat{\kappa}(x)+x\hat{\kappa}'(x)}{1-\beta(1-\delta)}]}{\frac{1}{\theta} + \frac{q(\theta)\beta(1-\delta)}{1-\beta(1-\delta)}}.$$

One can then solve for θ_w and V_w. Finally, the optimality condition for wage/tightness gives an equation determining θ. Then, $W = X/\mu(\theta) - Y$ and $\Lambda = \beta(1-\delta)\alpha W/(1 - \beta(1-\delta)(1-\alpha))$.

Linearized system:

$$\frac{\Lambda_t}{\beta(1-\delta)\alpha} = W_{t+1} + \beta(1-\delta)(1-\alpha)\frac{\Lambda_{t+1}}{\beta(1-\delta)\alpha}$$

$$W_t - \bar{W} = \frac{w_t - \bar{w}}{1-\beta(1-\delta)(1-\alpha)} + \Lambda_t - \bar{\Lambda}$$

$$\frac{\bar{\mu}'}{\bar{\mu}^2}\bar{\theta}_w(X_t - \bar{X}) + \frac{\bar{\mu}'}{\bar{\mu}^2}\bar{X}(\theta_{wt} - \bar{\theta}_w) + \frac{\bar{\mu}^2\bar{\mu}'' - 2\bar{\mu}(\bar{\mu}')^2}{\bar{\mu}^4}\bar{X}\bar{\theta}_w(\theta_t - \bar{\theta}) = 0$$

$$\frac{X_t - \bar{X}}{\bar{\theta}^2} - 2\frac{\bar{X}}{\bar{\theta}^3}(\theta_t - \bar{\theta}) + \frac{\bar{\mu}'}{\bar{x}\bar{\mu}^2}(X_t - \bar{X}) - \frac{\bar{\mu}'\bar{X}}{\bar{x}^2\bar{\mu}^2}(x_t - \bar{x}) + \frac{\bar{X}}{\bar{x}}\frac{\bar{\mu}^2\bar{\mu}'' - 2\bar{\mu}(\bar{\mu}')^2}{\bar{\mu}^4}(\theta_t - \bar{\theta})$$

$$= -\bar{q}''[\bar{z} - b + \beta(1-\delta)\bar{V}](\theta_t - \bar{\theta})$$

$$- \bar{q}'[z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)(V^0_{t+1} + V^1_{t+1}(w_t - \bar{w})))]$$

$$- \beta(1-\delta)(1-\alpha)[\frac{\bar{q}'\bar{V}^1}{\bar{\theta_w}}(\theta_t - \bar{\theta}) - \frac{\bar{q}\bar{V}^1}{\bar{\theta}_w\bar{x}^2}(x_t - \bar{x}) + \frac{1+\bar{q}\bar{x}}{\bar{\theta}_w\bar{x}}(V^1_{t+1} - \bar{V}^1) - \frac{(1+\bar{q}\bar{x})\bar{V}^1}{\bar{\theta}_w^2\bar{x}}(\theta_{wt} - \bar{\theta}_w)]$$

$$V^0_t = z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)V^0_{t+1}) - (X_t - \bar{X}) - \frac{\bar{x}}{\bar{\theta}}(X_t - \bar{X}) + \frac{\bar{x}\bar{X}}{\bar{\theta}^2}A_t$$

$$+ \bar{x}\bar{q}'[\bar{z} - b + \beta(1-\delta)\bar{V}]A_t + \bar{x}\bar{q}[z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)V^0_{t+1})]$$

$$\hat{A}_t = -\frac{X_t - \bar{X}}{\bar{\kappa}''\bar{\theta}} + \frac{\bar{X}A_t}{\bar{\kappa}''\bar{\theta}^2} + \frac{\bar{q}'}{\bar{\kappa}''}[\bar{z} - b + \beta(1-\delta)\bar{V}]A_t$$

$$+ \frac{\bar{q}}{\bar{\kappa}''}[z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)V^0_{t+1})]$$

$$A_t = \frac{1}{\bar{\mu}'(\bar{W} + \bar{Y})}(X_t - \bar{X} - \bar{\mu}(\Lambda_t - \bar{\Lambda} + Y_t - \bar{Y}))$$

$$\hat{n}_{t+1} - \bar{n} = (1-\delta)((1+\bar{q}\bar{x})(\hat{n}_t - \bar{n}) + \bar{n}\bar{q}(\hat{x}_t - \bar{x}) + \bar{n}\bar{q}'\bar{x}(\hat{\theta}_t - \bar{\theta}))$$

$$\frac{\bar{n}}{\bar{\theta}}(\hat{x}_t - \bar{x}) + \frac{\bar{x}}{\bar{\theta}}(\hat{n}_t - \bar{n}) - \frac{\bar{x}\bar{n}}{\bar{\theta}^2}(\hat{\theta}_t - \bar{\theta}) = -(\hat{n}_t - \bar{n})$$

$$\hat{w}_t = \alpha w_t + (1-\alpha)\hat{w}_{t-1}$$

$$\theta_t - \bar{\theta} = A_t + B(w_t - \bar{w})$$

$$x_t - \bar{x} = \hat{A}_t + \hat{B}(w_t - \bar{w})$$

$$\hat{\theta}_t - \bar{\theta} = A_t + B(\hat{w}_t - \bar{w})$$

$$\hat{x}_t - \bar{x} = \hat{A}_t + \hat{B}(\hat{w}_t - \bar{w})$$

$$V_t - \bar{V} = V^0_t + V^1(w_t - \bar{w})$$

F Model with Firm-Level Shocks

Consider an environment where firms face idiosyncratic shocks to their productivity. In a stationary equilibrium with firm heterogeneity, the aggregate measure of matches N and the value of job seekers X remain constant, while firm shocks lead to reallocation of labor across firms over time.[46]

The firm problem in this context reads:

$$\max_{\theta,v} -\frac{nX}{\mu(\theta)} + (n+q(\theta)v)(z-b) - \kappa(v,n) - X(\frac{v}{\theta}+n) + \beta E_z V(n',z')$$
$$\text{s.t. } n' = (1-\delta)(n+q(\theta)v),$$

where the continuation value satisfies

$$V(n,z) = (n+q(\theta)v)(z-b) - \kappa(v,n) - X(\frac{v}{\theta}+n) + \beta E_z V(n',z').$$

Scaling by size, these equations again yield the size-independent problem:

$$\max_{\theta,x} -\frac{X}{\mu(\theta)} + (1+q(\theta)x)(z-b+\beta(1-\delta)E_z\hat{V}(z')) - \hat{\kappa}(x) - X(\frac{x}{\theta}+1) \tag{43}$$

where

$$\hat{V}(z) = (1+q(\theta)x)(z-b+\beta(1-\delta)E_z\hat{V}(z')) - \hat{\kappa}(x) - X(\frac{x}{\theta}+1). \tag{44}$$

To arrive at an intertemporal Euler equation, one can combine the firm first order conditions to arrive at:

$$\frac{\kappa_v(x_t)+\frac{X}{\theta_t}}{q(\theta_t)} = z_t - b + \beta(1-\delta)[\frac{\kappa_v(x_{t+1})+\frac{X}{\theta_{t+1}}}{q(\theta_{t+1})} - \kappa_n(x_{t+1}) - X],$$

where the value of job seekers is now constant in this stationary setting, satisfying

$$X = \kappa_v(x_t)\frac{\mu(\theta_t)-\mu'(\theta_t)\theta_t}{\mu'(\theta_t)}\frac{q(\theta_t)x_t}{1+q(\theta_t)x_t}.$$

Definition 3. *A stationary competitive search equilibrium with firm wages is an allocation* $\{w_{it},\theta_{it},x_{it}\}_{t=0}^{\infty}$ $\forall i$ *and job seeker value* X *such that the allocation and value solve the problem (43-44), and that each job seeker applies to one firm:* $1-\sum_i n_{it} = \sum_i x_{it}n_{it}/\theta_{it}$, $\forall t$.

Single Firm Deviation to Longer Wage Commitment Consider introducing into the above equilibrium an individual firm, small relative to the size of the market, that today makes a wage commitment for a probabilistic period of time, returning to equilibrium behavior once the commitment expires.

[46] I abstract from firm entry and exit here, but one could incorporate such a margin by adding exit shocks into the firm problem, with new firms replacing exiting ones. The behavior of new and existing firms is identical if new firms are assumed to enter with at least one worker, due to the size-independence.

The deviating firm chooses a wage w, expecting each period going forward to revert to equilibrium behavior with probability α and to maintain the wage with probability $1-\alpha$. To connect the per-period wage to the market tightness, note that the equilibrium firms' market tightnesses imply these firms offer their workers specific present values of wages for each z, due to the job seeker constraint. Taking these equilibrium values as given, one can solve for the present value of wages for the deviating firm as a function of the wage w and productivity z, denoted below as $\phi(w, z)$.[47] Finally, the job seeker constraint gives the implied tightness: $X = \mu(\theta)(\phi(w, z) + Y)$.

In the period the firm deviates to the longer wage commitment, it chooses a wage w and vacancy creation v, to solve the problem

$$\max_{w,v} -\frac{nX}{\mu(\theta)} + (n + q(\theta)v)(z - b) - \kappa(v, n) - X(\frac{v}{\theta} + n) + \beta E_z(\alpha V(n', z') + (1-\alpha)V^f(n', w, z'))$$
$$\text{s.t. } n' = (1-\delta)(n + q(\theta)v),$$
$$X = \mu(\theta)(\phi(w, z) + Y),$$

given n, z. Here the firm expects to revert to equilibrium behavior in the following period with probability α, implying the continuation value $V(n', z')$, and to maintain the wage commitment otherwise, implying the continuation value $V^f(n', w, z')$, discussed below.

In periods when the firm maintains the wage commitment, it only chooses vacancies, to solve the problem:

$$\max_{v}(n + q(\theta)v)(z - b) - \kappa(v, n) - X(\frac{v}{\theta} + n) + \beta E_z(\alpha V(n', z') + (1-\alpha)V^f(n', w, z'))$$
$$\text{s.t. } n' = (1-\delta)(n + q(\theta)v),$$

where the tightness θ is determined by the job seeker constraint $X = \mu(\theta)(\phi(w, z) + Y)$. The continuation value $V^f(n', w, z')$ satisfies

$$V^f(n, w, z) = (n + q(\theta)v)(z - b) - \kappa(v, n) - X(\frac{v}{\theta} + n) + \beta E_z(\alpha V(n', z') + (1-\alpha)V^f(n', w, z')).$$

The deviating firm's problems can also be scaled to arrive at size-independent problems. Defining $\hat{V}^f(w, z) := V^f(n, w, z)/n$, the deviating firm chooses w, x to solve

$$\max_{w,x} -\frac{X}{\mu(\theta)} + (1 + q(\theta)x)(z - b + \beta(1-\delta)E_z(\alpha\hat{V}(z') + (1-\alpha)\hat{V}^f(w, z'))) - \hat{\kappa}(x) - X(\frac{x}{\theta} + 1)$$
$$\text{s.t. } X = \mu(\theta)(\phi(w, z) + Y).$$

[47] Denote the vector of equilibrium present values of wages across z as $\mathbf{W}$ and that of the deviating firm as $\mathbf{W}^f(w)$. We have that $\mathbf{W}^f(w) = w\mathbf{i} + \beta(1-\delta)[\alpha\Pi\mathbf{W} + (1-\alpha)\Pi\mathbf{W}^f(w)]$, where Π is the transition matrix for the productivity process and $\mathbf{i}$ a vector of ones. This gives the deviating firm's present values as $\mathbf{W}^f(w) = (I - \beta(1-\delta)(1-\alpha)\Pi)^{-1}(w\mathbf{i} + \beta(1-\delta)\alpha\Pi\mathbf{W})$. I denote the components of this vector in the text by $\phi(w, z)$. Note that the derivative of the value satisfies $\phi_w(w, z) = (1 - \beta(1-\delta)(1-\alpha))^{-1}$.

In periods when the firm maintains the commitment to w, it chooses x to solve

$$\max_x (1+q(\theta)x)(z-b+\beta(1-\delta)E_z(\alpha\hat{V}(z')+(1-\alpha)\hat{V}^f(w,z')))-\hat{\kappa}(x)-X(\frac{x}{\theta}+1),$$

where the tightness θ is determined by the job seeker constraint $X=\mu(\theta)(\phi(w,z)+Y)$. The continuation value satisfies

$$\hat{V}^f(w,z)=(1+q(\theta)x)(z-b+\beta(1-\delta)E_z(\alpha\hat{V}(z')+(1-\alpha)\hat{V}^f(w,z')))-\hat{\kappa}(x)-X(\frac{x}{\theta}+1).$$

The two problems yield the same first order condition for vacancy creation

$$\hat{\kappa}'(x)+\frac{X}{\theta}=q(\theta)(z-b+\beta(1-\delta)E_z(\alpha\hat{V}(z')+(1-\alpha)\hat{V}^f(w,z'))),$$

for the deviation period and periods when the commitment is maintained. Meanwhile, the deviating firm's first order condition characterizing the wage-tightness tradeoff reads

$$\begin{aligned}\frac{X}{\theta^2}[1+\frac{\mu'(\theta)\theta^2}{x\mu(\theta)^2}]=&-q'(\theta)[z-b+\beta(1-\delta)E_z[\alpha\hat{V}(z')+(1-\alpha)\hat{V}^f(w,z')]]\\&-\beta(1-\delta)(1-\alpha)(1+q(\theta)x)/xE_z\hat{V}^f_w(w,z')/\theta_w,\end{aligned}$$

where the derivative of θ with respect to w is $\theta_w=-\mu(\theta)^2/(\mu'(\theta)X(1-\beta(1-\delta)(1-\alpha)))$, while the derivative of the continuation value satisfies

$$\begin{aligned}\hat{V}^f_w(w,z)=&xq'(\theta)[z-b+\beta(1-\delta)E_z[\alpha\hat{V}(z')+(1-\alpha)\hat{V}^f(w,z')]]\theta_w\\&+\frac{xX}{\theta^2}\theta_w+\beta(1-\delta)(1-\alpha)(1+q(\theta)x)E_z\hat{V}^f_w(w,z').\end{aligned}$$

Equilibrium with Infrequent Adjustment If a longer wage commitment is profitable for the deviating firm, it becomes interesting to consider an equilibrium where all firms follow a strategy of infrequent adjustment.

To think about these questions, suppose all firms reoptimize their wage w each period with probability α and maintain their existing wage commitment with probability $1-\alpha$. To connect the per-period wage to the corresponding market tightness, one can again solve for the present value of wages as a function of the wage w and productivity z, denoted $\phi(w,z)$.[48]

[48]Denote the vector of equilibrium present values of wages for a reoptimizing firm across z as $\mathbf{W}^r$ and that of a firm maintaining wage commitment w as $\mathbf{W}^f(w)$. We have that $\mathbf{W}^f(w)=w\mathbf{i}+\beta(1-\delta)[\alpha\Pi\mathbf{W}^r+(1-\alpha)\Pi\mathbf{W}^f(w)]$, where Π is the transition matrix for the productivity process and $\mathbf{i}$ a vector of ones. This gives the deviating firm's present values as $\mathbf{W}^f(w)=(I-\beta(1-\delta)(1-\alpha)\Pi)^{-1}(w\mathbf{i}+\beta(1-\delta)\alpha\Pi\mathbf{W}^r)$. I denote the components of this vector in the text by $\phi(w,z)$.

In this case, firms reoptimizing wages solve

$$\max_{w,v} -\frac{nX}{\mu(\theta)} + (n+q(\theta)v)(z-b) - \kappa(v,n) - X(\frac{v}{\theta}+n) + \beta E_z(\alpha V^r(n',z') + (1-\alpha)V^f(n',w,z'))$$
$$\text{s.t. } n' = (1-\delta)(n+q(\theta)v),$$
$$X = \mu(\theta)(\phi(w,z)+Y),$$

where the implied continuation value satisfies

$$V^r(n,z) = (n+q(\theta)v)(z-b) - \kappa(v,n) - X(\frac{v}{\theta}+n) + \beta E_z(\alpha V^r(n',z') + (1-\alpha)V^f(n',w,z')),$$

and firms holding the wage commitment fixed solve

$$\max_{v}(n+q(\theta)v)(z-b) - \kappa(v,n) - X(\frac{v}{\theta}+n) + \beta E_z(\alpha V^r(n',z') + (1-\alpha)V^f(n',w,z'))$$
$$\text{s.t. } n' = (1-\delta)(n+q(\theta)v),$$

where the tightness θ is determined by the job seeker constraint $X = \mu(\theta)(\phi(w,z)+Y)$ and the continuation value $V^f(n',w,z')$ satisfies

$$V^f(n,w,z) = (n+q(\theta)v)(z-b) - \kappa(v,n) - X(\frac{v}{\theta}+n) + \beta E_z(\alpha V^r(n',z') + (1-\alpha)V^f(n',w,z')).$$

Once again, the problems can be scaled. Thus, firms reoptimizing wages solve

$$\max_{w,x} -\frac{X}{\mu(\theta)} + (1+q(\theta)x)(z-b+\beta E_z(\alpha\hat{V}^r(z') + (1-\alpha)\hat{V}^f(w,z'))) - \hat{\kappa}(x) - X(\frac{x}{\theta}+1)$$

where the tightness θ is determined by the job seeker constraint $X = \mu(\theta)(\phi(w,z)+Y)$, and the implied continuation value satisfies

$$\hat{V}^r(z) = (1+q(\theta)x)(z-b+\beta E_z(\alpha\hat{V}^r(z') + (1-\alpha)\hat{V}^f(w,z'))) - \hat{\kappa}(x) - X(\frac{x}{\theta}+1),$$

and firms holding the wage commitment fixed solve

$$\max_{x}(1+q(\theta)x)(z-b+\beta E_z(\alpha\hat{V}^r(z') + (1-\alpha)\hat{V}^f(w,z'))) - \hat{\kappa}(x) - X(\frac{x}{\theta}+1),$$

where the tightness θ is determined by the job seeker constraint $X = \mu(\theta)(\phi(w,z)+Y)$ and the continuation value $V^f(w,z')$ satisfies

$$\hat{V}^f(w,z) = (1+q(\theta)x)(z-b+\beta E_z(\alpha\hat{V}(z') + (1-\alpha)\hat{V}^f(w,z'))) - \hat{\kappa}(x) - X(\frac{x}{\theta}+1).$$

The first order conditions for the firms' choice of wage and vacancy creation rate coincide with those for the deviating firm, with the continuation values $\hat{V}^r(z)$ and $\hat{V}^f(w,z)$ as characterized above.

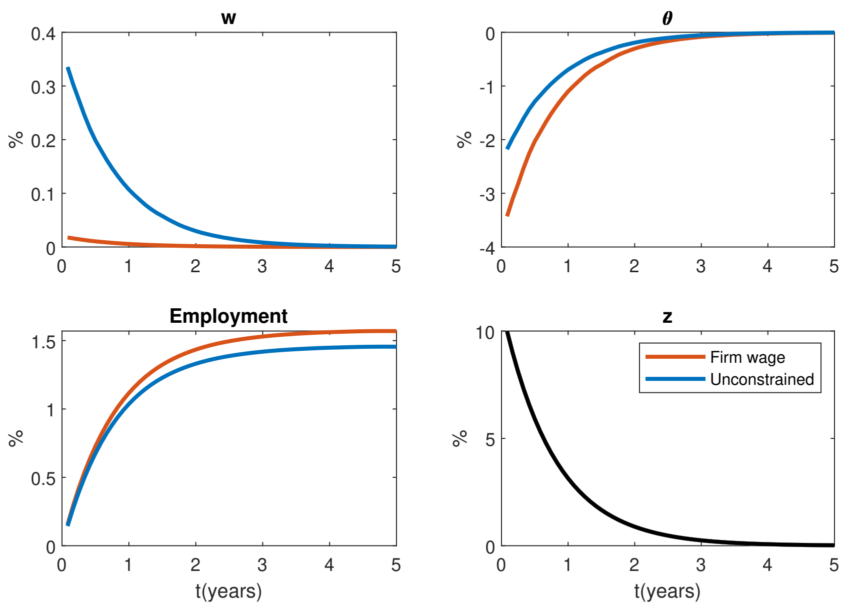

Figure F.1: Impulse Responses in Firm Wage vs Unconstrained Model

Notes: The figure plots the percentage responses of model variables to a ten percent increase in firm-level labor productivity in the firm wage model and the unconstrained competitive search model without firm wages. Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.9$ and standard deviation $\sigma_z = 0.1$. The response is based on a quadratic approximation, produced with Dynare. The two models compared have the same steady-state levels of wage, tightness, unemployment.

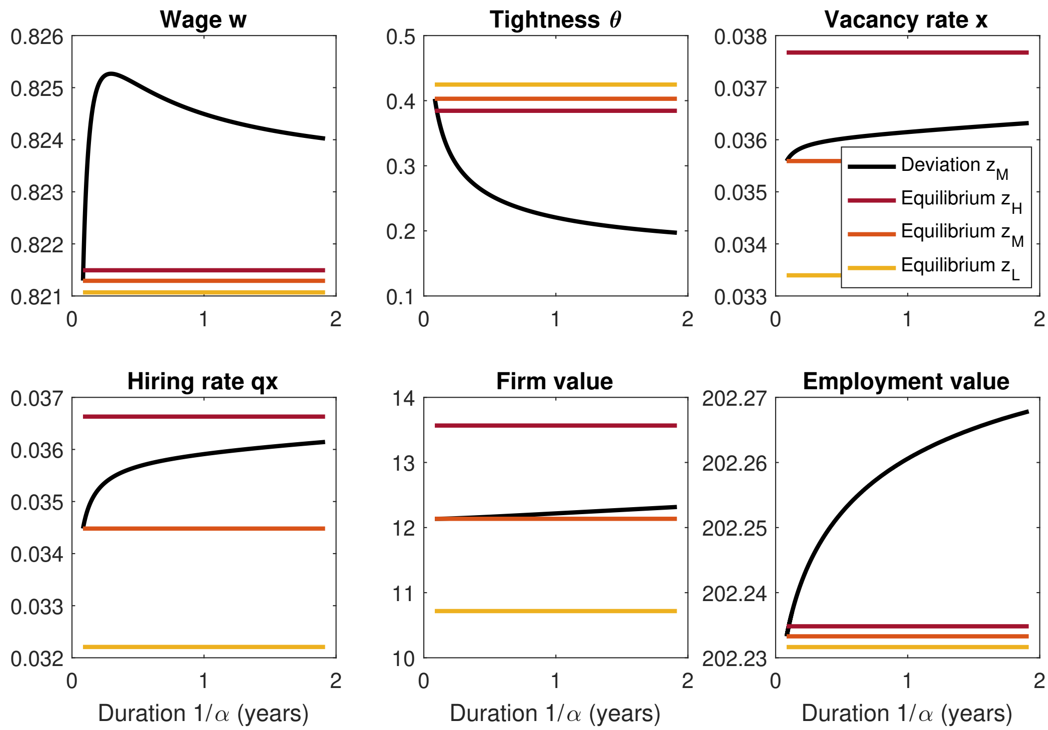

Figure F.2: Single Firm Deviating to Longer Wage Commitment with Firm-Level Shocks

Notes: The figure displays the equilibrium values of a number of variables in the stationary equilibrium with firm wages, along with the corresponding values for an individual firm in that equilibrium that is able to set a wage commitment for a probabilistic period of time. The model is solved on three state grid for productivity, approximating an $AR(1)$ with autocorrelation $\rho_z = 0.9$ and standard deviation $\sigma_z = 0.1$ based on the Rouwenhorst method. The deviating firm is in the intermediate productivity state and its choices are plotted as a function of $1/\alpha$, the expected duration of the wage commitment. The firm value plotted is the scaled value per initial size.

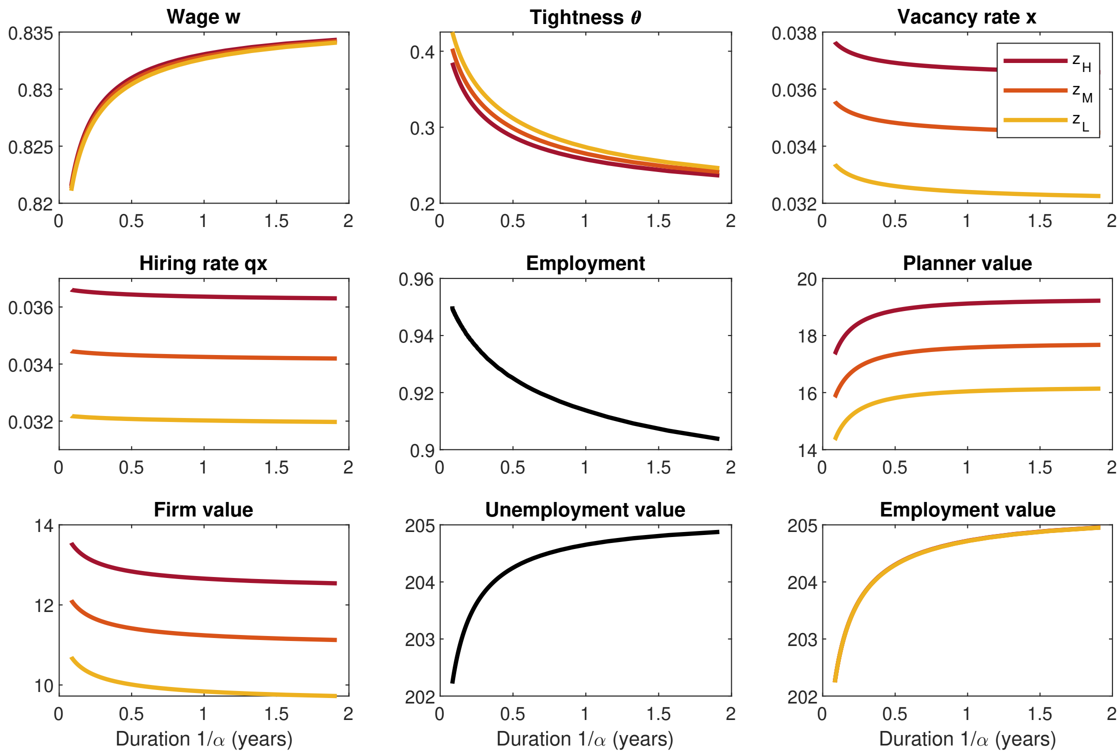

Figure F.3: Equilibrium with Longer Wage Commitments

Notes: The figure displays the values of a number of variables in the stationary equilibrium with firm wages and infrequent adjustment, as a function of $1/\alpha$, the expected duration of wages. The firm value plotted is the scaled value per initial size, but also the unscaled firm value declines in wage duration. Correspondingly, the planner value plotted is the scaled value per initial size. The figure also shows the corresponding values in the efficient allocation.

G Additional Figures

Figure G.1: Impulse Responses with Identical Parameters

Notes: The figure plots the responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model versus the unconstrained model. Productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$. The two models are parameterized identically, with $b = 0.89$. The firm wage model attains the 5 percent steady-state unemployment target with a steady-state wage of 0.91, while the unconstrained model has a higher steady-state wage, at 0.93, with 14 percent steady-state unemployment.

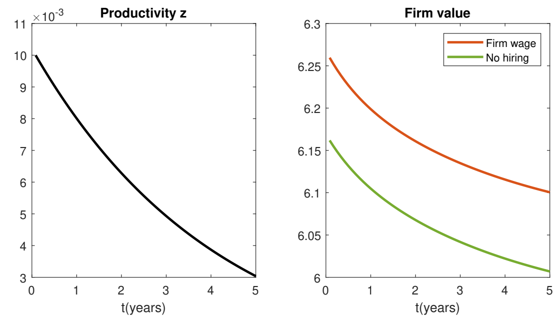

Figure G.2: Impulse Response of Firm Value vs No Hiring Value

Notes: The figure refers to the impulse response in Figure 1. It shows that the firm value attained by following the first order conditions dominates opting out of hiring for a period, throughout the impulse response.

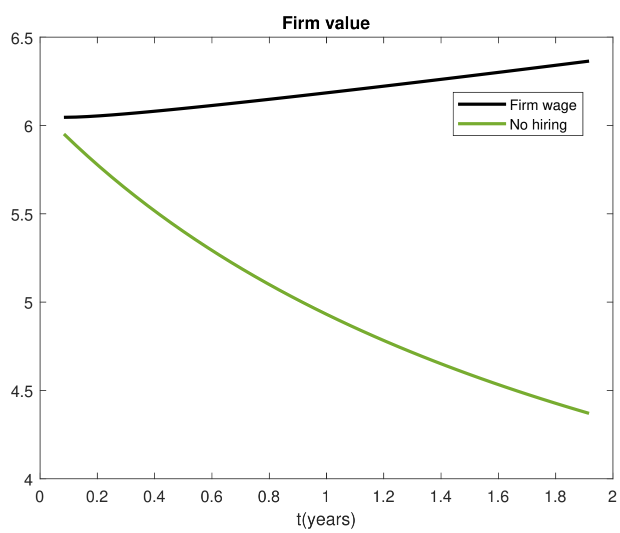

Figure G.3: Single Firm Deviation Value vs No Hiring Value

Notes: The figure refers to the deviating firm in Figure 3. It shows that the firm value attained by following the first order conditions dominates opting out of hiring for the deviation period, across wage durations.

Figure G.4: Single Firm Deviating in the Unconstrained Competitive Search Model

Notes: The figure displays the steady-state values of a number of variables in a stationary equilibrium with competitive search, along with the corresponding values for an individual firm in that equilibrium that is able to set a wage commitment for a probabilistic period of time. The latter are plotted as a function of $1/\alpha$, the expected duration of the wage commitment. The firm value plotted is the scaled firm value per initial size.

Figure G.5: Impulse Response of Fixed Wage vs Equilibrium Firms

Notes: The figure plots the percentage responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model and for a single firm deviating to a longer wage commitment. Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$.

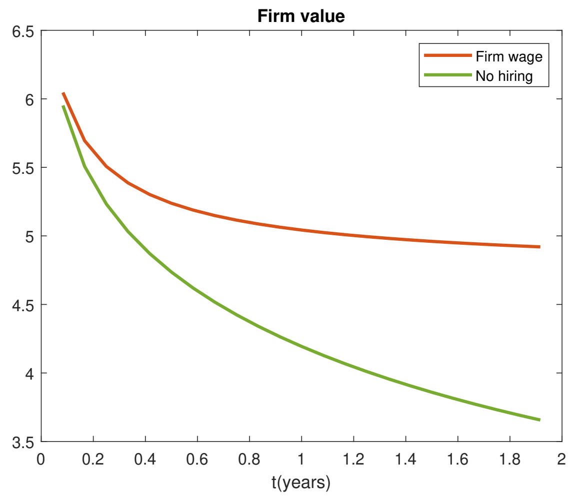

Figure G.6: Equilibrium Firm Value vs No Hiring Value

Notes: The figure refers to the deviating firm in Figure 5. It shows that the firm value attained by following the first order conditions dominates opting out of hiring for the duration of a fixed wage, across wage durations.

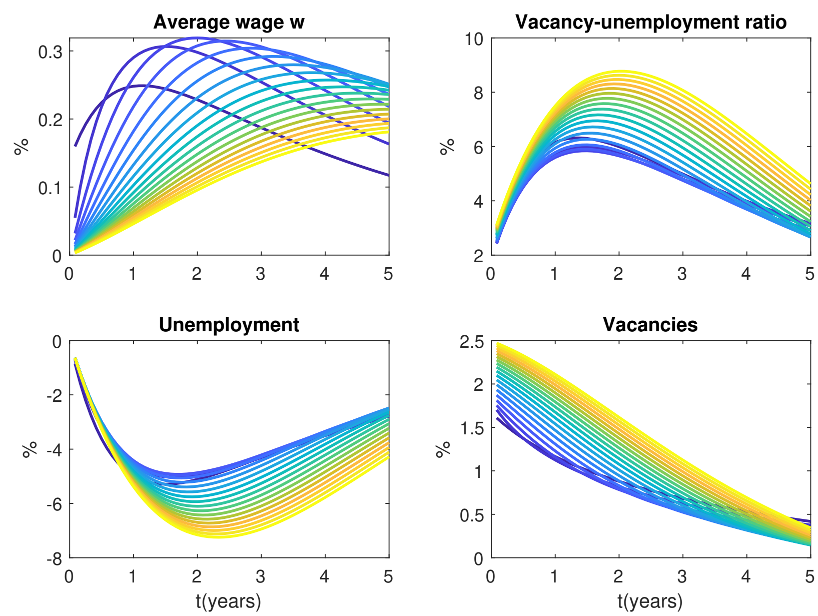

Figure G.7: Impact of Duration on Labor Market Volatility

Notes: The figure displays impulse responses to an aggregate shock to productivity, as a function of $1/\alpha$, the expected duration of wages rises from one month to 1.5 years. Labor productivity follows an AR(1) with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$.