Rational Sentiments and Economic Cycles*

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Abstract

We propose a rational model of endogenous credit cycles generated by the two-way interaction of credit market sentiments and real outcomes. Sentiments are high when most lenders optimally choose lax lending standards. This leads to low interest rates and high output growth, but also to the deterioration of future credit application quality. When the quality is sufficiently low, lenders endogenously switch to tight standards, i.e. sentiments become low. This implies high credit spreads, low quantity of issued credit and a gradual improvement in the quality of applications, which eventually triggers a shift to lax lending standards. The equilibrium cycle might feature a long boom or a lengthy, possibly double-dip recession. It is generically different from the optimal cycle as atomistic lenders ignore their aggregate effect on the composition of borrowers. Carefully chosen risk-weighted capital requirements or monetary policy can often improve the decentralized equilibrium cycle.

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1 Introduction

A growing body of empirical evidence suggests that periods of overheating in credit markets forecast low excess bond returns and recessions. These periods are characterized by an increased total quantity of credit, low interest rates, and, importantly, deteriorating quality of newly issued credit. In the subsequent recessions, credit turns scarce and expensive even for ex-post high-value projects. (Greenwood and Hanson, 2013; López-Salido et al., 2017)[1]

A major conundrum for policy makers and academics alike is how economic policy should respond to periods of overheating and subsequent recessions. For this, one needs to understand what triggers these periods, what determines the lengths of the different stages, and how they respond to policy.

To answer these questions we build a rational model to analyze the two-way interaction between credit market sentiments and real economic outcomes. In our model the dynamic interactions between lenders endogenous choice of lending standards and the resulting loan performance leads to alternating market sentiments, which in turn creates cycles.

We capture sentiments as lenders choice of tolerance to missing out on good investment versus extending bad loans when granting credit. We show that the credit market exhibits the symptoms of overheating or high sentiments whenever lenders optimally choose lax lending standards. In these periods, a mixed quality of credit is issued at a low interest rate inducing high credit growth, high economic output, and a deteriorating quality of projects. When the pool of applications is of sufficiently low quality, lenders optimally switch to tight standards implying high credit spreads, high quality and low quantity of issued credit. This leads to an improving pool of credit applications, eventually triggering a shift to lax lending standards. We characterize when the implied endogenous economic cycles feature long booms, and when the economy suffers lengthy, possibly double-dip recessions. A constrained planner often prefers a cycling economy to one with persistently high or persistently low sentiments. A macro prudential policy that implements carefully chosen risk-weighted capital requirements or a monetary policy can often improve the welfare in the decentralized economy. Our predictions match stylized facts on the co-movement of credit composition

[1]See also Morais et al. (2019) for US and international evidence on lax bank lending standards in booms, and Baron and Xiong (2017) on the negative relationship between banks credit expansion and banks' equity returns. More generally, there is ample evidence of pro-cyclical volume and countercyclical value of investment in a wide range of contexts. For instance, Eisfeldt and Rampini (2006) demonstrates this for sales of property, plant and equipment, while Kaplan and Stromberg (2009) shows similar evidence on venture capital deals.

and spreads, lending standards, market fragmentation, rebalancing and realized returns of credit portfolios, and real outcomes.

In our stochastic OLG model, entrepreneurs run projects and obtain credit from investors to scale up their operation. Some entrepreneurs are running high-return projects and are paying back their loans. Others run low-return projects and default on their credit. The majority of investors are not sufficiently skilled to distinguish between good and bad entrepreneurs. These investors, whom we refer to as unskilled, can run one of two types of imperfect tests to decide which entrepreneurs to grant credit to. A *bold test* represents lax lending standards. This test passes the credit application of all good entrepreneurs along with some bad ones. A *cautious test* represents tight lending standards as it rejects all applications from bad entrepreneurs along with some of the good ones. Unskilled investors make a rational choice over the test to run based on the fundamentals of the economy.

We assume that credit is essential for survival. That is, at the end of each period, entrepreneurs exit either because of natural death or because they did not get credit. In each case, they are replaced by newborns drawn from an exogenous pool of types. The resulting type distribution serves as the evolving state of the economy.

Typically, our economy features cycles in equilibrium. These cycles are an outcome of the two-way interaction between credit sentiment and the fundamentals of the economy.

An economy with a small fraction of bad projects exhibits the symptoms of overheating or high credit market sentiment. In this case, unskilled investors choose lax lending standards, and credit to a wide range of entrepreneurs is issued at the same, relatively low interest rate. Because the bold tests implies false positive mistakes, a fraction of bad projects are also financed implying a mixed quality of issued credit. Given the low interest rates and lax lending standards, most entrepreneurs scale up their projects implying high output. Also, a large fraction of bad entrepreneurs survive, deteriorating the quality of loan applications.

At some point, when the quality of potential borrowers is sufficiently low, unskilled investors tighten their lending standards, providing credit only to a fraction of good projects. These projects are financed at a relatively low interest rate, because unskilled capital is abundant and the obtained loan quality is high due to the tight lending standards. However, the remaining group of good entrepreneurs can only obtain limited credit at high interest rates from the few skilled investors. Bad entrepreneurs cannot obtain credit at all. That is, the economy is in a recession, with low output and consumption, and high credit spreads across different borrower groups indicating a fragmented credit market. At the same time, the issued credit is of high quality and only a small fraction of bad entrepreneurs survive, improving the quality of loan applications over time. However, at the point where the sentiments switch, the real economy suffers a large crash even if the change in the fundamental quality of available projects is small. Put together, booms with overheated credit markets eventually turn to low-sentiment busts, while downturns eventually turn to booms.

We show that the model can generate a rich pattern of deterministic cyclical behavior. Cycles might feature long booms and short recessions, or vice-versa. They might also feature a double-dip recession. We characterize how the properties of cycles change in response to changing parameters. We choose not to incorporate aggregate shocks in our model to demonstrate our mechanism as clearly as possible. In the presence of aggregate shocks, our economy would feature more realistic, stochastic cycles.

Once we have a theory of the origin and properties of economic cycles induced by credit market sentiment, we turn to welfare and economic policy. First, we explore the rational for policy makers to intervene in this economy. We study a planner's problem who can choose lending standards conditional on the state. We argue that the planner can often improve on the decentralized outcome, because investors do not internalize how their individual choice of lending standards affects the aggregate dynamics of the state space. In particular, the planner often prefers a cycling economy, where low sentiment stages keep the fraction of bad projects in the economy at bay which makes the high sentiment stages more beneficial.

Then, we connect the constrained planner's solution to realistic monetary and macro-prudential policies. We show that both changing the risk-free rate and specifying capital requirements can be used to influence investors' lending standards. Therefore, each of these policies affects the dynamics of the state distribution, and, consequently, welfare. However, the policy maker can improve the quality of loan applications only at the expense of increasing the average cost of capital. This trade-off determines the ranking across policies. Under our representation, we show thata non-state contingent risk-weighted capital requirements dominate a non-state contingent risk-free rate policy. However, a sophisticated regulator might be able to implement a countercyclical monetary policy which can push the economy even closer to the constrained optimal cycle.

Finally, we contrast our results with a wide range of stylized facts on market segmentation, the fluctuation of credit market sentiment, output, the heterogeneity of returns and portfolios of investors and international spillovers of monetary policy.

Literature. To the best of our knowledge our paper is the first to provide a mechanism where economic cycles are endogenously generated by the interaction of lending standards and average borrower quality.

Our paper is closest the growing literature on dynamic lending standards (Martin, 2005; Hu, 2017; Asriyan et al., 2018; Figueroa and Leukhina, 2018; Fishman et al., 2019). Just as in Hu (2017) and in the contemporaneous work of Fishman et al. (2019), we capture lending standards as lenders' informational choice which affects the borrowers' average quality in the future, which feeds back to the future choice of lending standards. We emphasize two deviations. First, these papers feature economies which converge to either a high or a low steady state. That is, these models do not provide an endogenous force turning a boom into a recession, or vice-versa.[2] This is why our model is more suited to study the effect of economic policies on the characteristics of economic cycles. Second, as our mechanism is different, our model captures some features of credit cycles which the previous literature does not. Notably, as sentiment shifts and booms turns into recessions, credit conditions across fundamentally similar firms drastically diverge, both in price and in quantity, implying a fragmented credit market and a discontinuous drop in output and consumption.

There is also an earlier literature on endogenous investment cycles (Azariadis and Smith, 1998; Matsuyama, 2007) and credit constraints which builds on the interaction of entrepreneurs net worth, the implied tightness of their credit constraint and the implied choice among available projects with different productivity. Instead, we focus on a mechanism building on the endogenous fluctuation of lending standards.

Our paper is also connected to the literature on credit cycles (Kiyotaki and Moore, 1997; Lorenzoni, 2008; Mendoza, 2010; Gorton and Ordonez, 2014; Gorton and Ordoñez, 2016). While our topic is also booms and busts induced by changing availability of credit, these papers focus on how exogenous shocks are amplified by the effect through the price of the collateral. In our model the price of collateral or exogenous shocks play no role.

There is a long tradition in economics starting with Keynes' metaphor of animal spirits to associate boom-bust cycles with fluctuating investors' sentiment.[3] As opposed to models

[2] Notable exceptions are Martin (2005) and Figueroa and Leukhina (2018) where changing lending standards create fluctuations without exogenous shocks, just as in our model. However, these models capture sentiments as contracting choice and specify the state of the economy as the level of capital or net worth implying a different mechanism and less rich set of potential cycles.

[3] Angeletos and La'O (2013) provides a conceptually distinct approach to capture sentiment in a rational framework as rationally over weighted public information.

based on extrapolative expectations (Bordalo et al., 2018; Greenwood et al., 2019), we capture credit market sentiment as a rational choice of lending standards. Our model generates some of the leading facts of the empirical side of this literature; for instance the deterioration of credit quality in booms, or the strong correlation between high credit growth and low subsequent returns. However, as a rational model, our mechanism does not generate an exploitable anomaly under the least informed agent's information set. That is, regarding evidence that points to such anomalies to exist, our approach can only play a complementary role to behavioral models in explaining those facts.

Finally, from a methodological perspective, the structure of our credit market builds on Kurlat (2016) which we further develop in the companion paper of Farboodi and Kondor (2018). None of the these papers focus on endogenous economic cycles.

2 Set Up. Rational Sentiments and Economic Cycles

There is one type of consumption good in the model and two types of agents, entrepreneurs and investors. All agents are risk neutral. The type distribution of entrepreneurs, to be explained later, serves as the state variable of the economy.

The dynamic economy consists of an infinite number of periods and the following stage game describes the sequence of events within each period.

Stage Game

Each period is divided into two parts: morning and evening. There are two types of agents: entrepreneurs who produce and investors who provide funding for entrepreneurs. Each agent is endowed with one unit of the good in the morning. Any agent can invest in a safe technology in the morning and get $1 + r_f$ return in the evening.[4]

Here we explain the optimization problem of each type of agent. The formal optimization problems are stated in Appendix A.

[4]The return on the safe technology, r_f, can represent a physical return or a policy rate. In sections 3 and 4, we think of it as the rate of return on the storage technology, which can be normalized to zero. In section 5 we reintroduce r_f as the return on a risk-free asset provided by the policy maker.

Entrepreneurs. There is a unit mass of entrepreneurs each day. Each entrepreneur is endowed with one unit of wealth, and a single project with a two dimensional type distribution, in the morning. The project is good or bad, $\tau = g, b$, and opaque or transparent, $\omega = 0, 1$. We refer to (τ, ω) as the type of the project or the entrepreneur, interchangeably. Entrepreneurs know their own type. We denote the fraction of opaque and transparent bad entrepreneurs by μ_0 and μ_1, respectively. As it will be clear later, the type distribution evolves endogenously and these fractions serve as state variables in each period.

Production. Entrepreneurs invest in the morning and produce in the evening. Each entrepreneur (τ, ω) chooses investment $i(\tau, \omega)$ in the morning. Each unit of investment costs one unit of consumption and can be covered by entrepreneur's initial endowment or by borrowing from investors in the credit market. Each unit of investment returns ρ_τ depending on entrepreneur's type τ. We assume that $\rho_g > 1 + r_f$ and

$$\left(1 - \frac{1}{\rho_g}\right)(1 + r_f) < \rho_b < 1. \tag{2.1}$$

The first inequality in (2.1) ensures that bad entrepreneurs want to borrow even if the return on their project is smaller than its cost.

Investors. Each investor lives for one period. A small, w_1, mass of investors are skilled, while a large, w_0 mass of investors are unskilled. Skill is privately observable. All investors are born in the morning, provide loans in the afternoon, and consume and die in the evening. Let $h \in [0, w_0 + w_1]$ denote an individual investor.

Skilled investors can observe the type of each project. Unskilled investors instead can observe imperfect signals for the project sample they receive. These signals are generated by a test of investor choice: each investor can opt for a *bold test* or a *cautious test*. The tests differ in the signal they generate for opaque projects. The bold test pools all opaque projects, good or bad, with good transparent ones (a false positive error). The cautious test pools all opaque projects with bad transparent ones (a false negative error). Intuitively, we can imagine the bold test to reject bad transparent projects only and green pass all other ones, while the cautious test passes only for good transparent projects.[5] When an investor

[5]For simplicity we restrict investor's choice set to these two tests. In appendix B we show that this restriction is not essential for our results. In particular, we enrich the investor choice set so that they are able to choose between the continuum of test lying between the bold and cautious extreme tests, and we

is indifferent between the two tests, we break the tie by assuming that she chooses the bold test.

The size of the sample an investor tests is limited by the investor's unit endowment; she cannot test more applications than the quantity she could finance if all pass her test. The cost of the test on this sample is $0c$, $c \in (0,1)$, and each unskilled investor runs exactly one test.

Credit Market. Credit market operates in the morning. An entrepreneur can get a loan from investors collateralized by her investment. Let ℓ denote the amount of credit (loans) the entrepreneur raises on the credit market.

Each investor can seize ξ fraction of the collateral to cover the interest and principal of the loan from good projects only. For the rest of the paper we normalize $\xi = 1$. Therefore, the collateral constraint implies $\ell(1+r) \leq i$. That is, an entrepreneur facing positive interest rate cannot borrow the full amount she needs for investing i. At least $r\ell$ has to come from her endowment. The budget constraint of the entrepreneur is $i \leq \ell + 1$.

In the credit market, after choosing the type of the test, each investor advertises an interest rate, $\tilde{r}(h)$, at which he is willing to give loans to applications passing his test. While each entrepreneur chooses the measure of loan applications $\sigma(r;\tau,\omega)$ she wishes to submit at each interest rate r with the maximum of $\sigma(r;\tau,\omega) \leq \frac{1}{r}$.[6] The credit market clears starting from the lowest interest rate and unskilled investors sample first. We provide further detail on the market clearing protocol in Appendix A.

Dynamic Evolution

The economy runs for infinitely many periods and the stage game describes the sequence of events within each period. Each generation of investors lives for one period only, and is replaced by a new identical generation the following period. Entrepreneurs live for a random number of periods, implying a stochastic overlapping generation model for entrepreneurs. The fractions of different type of entrepreneurs serve as the state variables for agents' opti-

show that the dominant choice is always one of the extreme.

[6] Note that the maximum credit any entrepreneur can apply for at a given rate corresponds to interest rate payment equal to her endowment. This corresponds to the maximum enforceable repayment as a binding collateral constraint, $\ell(1+r) = i$ and a binding budget constraint $\ell + 1 = i$ implies $\ell = \frac{1}{r}$.

mization problem.

There is a unit mass of entrepreneurs alive every period. At the end of each period, random fraction δ of the entrepreneurs die and are replaced by an inflow of new entrepreneurs, of the same measure. Furthermore, we assume that credit is essential for survival. Therefore, any entrepreneur who is not able to raise financing dies and is also replaced by a new entrant entrepreneur. Since the credit allocation is an equilibrium object in each period, we postpone the mathematical formulation of the transition rule to section 3.1, where we characterize the credit market equilibrium . The next assumption formally states the replacement rule.

Assumption 2.1 *Randomly chosen δ fraction of the living entrepreneurs, as well as any entrepreneur not financed by investors is replaced by a randomly chosen new entrant from the outside pool of entrepreneurs in the next period. The type distribution of the new entrants has λ fraction of bad entrepreneurs, and an independent $\frac{1}{2}$ fraction of opaque ones.*

For simplicity, we assume that there is no credit history recorded for entrepreneurs. That is, investors cannot learn from the past. Also, there is no saving technology available across periods. Therefore, entrepreneurs consume their wealth at the end of each period, and if survived, they start the new period with their unit endowment received in the morning.

The decentralized equilibrium is defined as follows.

Definition 2.1 (Equilibrium) *A decentralized dynamic equilibrium is a sequence of stage game equilibria. A stage game equilibrium is a set of entrepreneurs' investment , $i_t(\tau, \omega)$, credit demand schedules, $\sigma_t(r, \tau, \omega)$, along with investors' advertised interest rate schedule $\tilde{r}_t(h)$, unskilled investors' choice of test, an equilibrium interest rate schedule $r_t(\tau, \omega)$, and credit allocation schedule $\ell_t(\tau, \omega)$ for each entrepreneur, and allocation of applications to investors such that*

(i) each agent's choice is optimal given the strategy profile of all other agents;

(ii) the implied interest rate schedule $r_t(\tau, \omega)$, and credit allocation schedule $\ell_t(\tau, \omega)$ for each entrepreneur, and allocation of applications to investors are consistent with agents' choices and the market clearing process.

In each period, the stage game equilibrium is consistent with the realized type distribution, while the dynamics of the type distribution is consistent with Assumption 2.1.

Thus the relationship among periods is through the law of motion for the fractions of different types of entrepreneurs. These fractions are the state variables of the economy each period and each equilibrium objects defined for the stage game should be conditioned on them. For simplicity, we will suppress this dependence whenever it does not cause any confusion.

We focus on the case where there are many unskilled investors, but few skilled investors in the following sense.

Assumption 2.2 *The mass of skilled and unskilled investors, w_1, w_0, satisfies the following criteria.*

(i) Skilled investors capital is not sufficient to cover the credit demand of all opaque good entrepreneurs at any interest rate that any good entrepreneur is willing to borrow at.

(ii) Unskilled investors capital, w_0, is abundant. In particular, it is sufficiently large that it covers the credit demand of all entrepreneurs that unskilled investors are willing to lend to at any equilibrium interest rate.

Our structure allows for solving for the full equilibrium in steps. In section 3 we first solve for the credit market equilibrium in the stage game. In doing so, we take the entrepreneur type distribution as given. Then we characterize the credit market dynamics. In Section 4, we describe the real economy outcomes in the stage game and its dynamics.

3 Credit Market Equilibrium

State Variables. (μ_0, μ_1), the fraction of bad opaque and transparent entrepreneurs in the economy, are sufficient state variables in each period. Following proposition 3.2 the measures of good opaque and good transparent entrepreneurs are both equal to $\frac{1-\mu_0-\mu_1}{2}$, thus (μ_0, μ_1) fully characterizes the distribution of entrepreneurs each period.

Stage Game

We start the derivation of the equilibrium with a few basic properties of entrepreneurs' credit demand.

Lemma 3.1 *In any equilibrium entrepreneurs' credit demand schedule, $\sigma(r, \tau, \omega)$, simplifies as follows:*

(i) Each type chooses a reservation interest rate $r^{max}(\tau, \omega)$ and submits maximum demand to all weekly lower interest rates and 0 otherwise.

(ii) There is a maximum interest rate at which good firms are willing to borrow: $r^{max}(g, \omega) \leq \bar{r} \equiv \rho_g - 1$.

(iii) Under inequality (2.1), bad entrepreneurs never choose a lower reservation rate than $\bar{r}$: $r^{max}(b, \omega) \geq \bar{r}$.

This Lemma simplifies the analysis considerably. It shows that it is sufficient to find the equilibrium reservation interest rate of entrepreneurs instead of working out a full credit demand schedule. It also clarifies the role of inequality (2.1). It ensures that the return of bad entrepreneurs' project is sufficiently high that, despite the negative returns on each invested unit, they are willing to take it on if they can borrow against it, as they do not plan to pay back the loan. This is true even if interest rate on the loan reaches the maximum good entrepreneurs are willing to pay, $\bar{r}$.

We next show that the unique equilibrium in the credit market is one of three distinct types, depending on the parameters. In order to do so, it is useful to first define three interest rate functions.

Definition 3.1 (Dynamic Interest Rates)

$$r_B(\mu_0, \mu_1, c, r_f) \equiv \frac{\mu_0}{1-\mu_1-\mu_0} + \frac{1-\mu_1}{1-\mu_1-\mu_0} r_f + \frac{1}{1-\mu_1-\mu_0} c \tag{3.1}$$

$$r_C(\mu_0, \mu_1, c, r_f) \equiv r_f + \frac{2}{1-\mu_1-\mu_0} c \tag{3.2}$$

$$r_I(\mu_0, \mu_1, c, r_f) \equiv \frac{2\mu_0}{1-\mu_1-\mu_0} + \frac{1+\mu_0-\mu_1}{1-\mu_1-\mu_0} r_f + \frac{1+\mu_1+\mu_0}{1-\mu_1-\mu_0} c. \tag{3.3}$$

Let $\tilde{\mu}_0(\mu_1) \equiv \frac{\bar{r}-r_f-c-\mu_1(\bar{r}+c-r_f)}{2+c+\bar{r}+r_f}$. The next proposition characterizes the three type of equilibria depending on $\tilde{\mu}_0(\mu_1)$, and shows that in each of them, the entrepreneurs who can obtain credit face exactly one of the above three interest rates or the maximum interest rate $\bar{r}$.

Proposition 3.1 *When* $\min\{r_B(\mu_0, \mu_1, c, r_f), r_C(\mu_0, \mu_1, c, r_f)\} < \bar{r}$,

(i) $\mu_0 \in [0, \frac{c}{1+r_f}]$ *is associated with a* bold stage. *In a bold stage the credit market has a pooling equilibrium where all entrepreneurs who obtain credit (all good and some bad), do so at interest rate* $r_B(\mu_0, \mu_1, c, r_f)$. *Every unskilled investor chooses the bold test.*

(ii) $\mu_0 \in (\max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)\}, 1]$ *is associated with a* cautious stage. *In a cautious stage the credit market has a separating equilibrium, where opaque good entrepreneurs obtain credit at interest rate* $\bar{r}$, *transparent good entrepreneurs obtain credit at* $r_C(\mu_0, \mu_1, c, r_f)$ *and bad entrepreneurs don't obtain any credit. Every unskilled investor chooses the cautious test.*

(iii) $\mu_0 \in (\frac{c}{1+r_f}, \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)\}]$ *is associated with a* mix stage. *In a mix stage the credit market has a semi-separating equilibrium, where opaque good and bad entrepreneurs obtain credit at interest rate* r_I. *Good transparent entrepreneurs obtain credit at interest rate* $r_C(\mu_0, \mu_1, c, r_f)$. *Some unskilled investors choose the bold test while others choose the cautious test.*

Otherwise the economy is in autarky, where unskilled investors do not lend, bad entrepreneurs do not borrow, and good firms obtain credit at interest rate $\bar{r}$ *from skilled investors only.*

When there are not too many bad projects around, investors are more concerned about losing out on good project by applying too harsh lending standards. Thus lending standards are lax, and many projects including bad ones are able to raise financing at the same relatively low rate. On the other hand, if there are many bad projects, lending standards are tightened and credit market becomes segmented. Not only bad projects are unable to raise financing, even some good projects are able to do so only at extremely high rates. Lastly, if the fraction of bad projects are at some intermediate level, then some investors apply lax and some tight lending standards. Markets are still fragmented but still some bad projects are able to raise financing.

The intuition relies on the fact that abundant supply of unskilled capital implies a zero profit condition for unskilled investors. In fact, interest rates $r_B(\mu_0, \mu_1, c, r_f)$, $r_C(\mu_0, \mu_1, c, r_f)$ and $r_I(\mu_0, \mu_1, c, r_f)$ are the rates at which an unskilled investor is indifferent between lending to entrepreneurs and earning the risk free rate r_f without running a test in the corresponding equilibrium. The indifference holds as long as all types apply for credit at that rate. The break-even interest rates, r_B, r_C and r_I depends on investors' choice of test due to the following trade-off. The cautious test results in a loan portfolio of higher quality as unlike the bold test, only good entrepreneurs pass it. Thus investors are always paid back when they run a cautious test and end up extending a loan. However, their rejection rate is higher than with a bold test as a cautious test fails all the opaque good entrepreneurs. As running the test has a fixed cost, not lending to tested applications is costly.

The bold stage arises when the break-even rate for bold investors is smaller than that for cautious investors, $r_B(\mu_0, \mu_1, c, r_f) \leq r_C(\mu_0, \mu_1, c, r_f)$. This is the case when $\mu_0 \leq \frac{c}{1+r_f}$. This is the left region on the left panel of Figure 1, on the left of the vertical line. Here there are few bad opaque entrepreneurs and thus the rejection rate of cautious test is relatively high. Thus cautious investors cannot compete with bold investors and all active unskilled investors are bold. As the bold test passes all the good projects, opaque or transparent, skilled investors offering higher rate than r_B would end up with no applications. Therefore, there is a single prevailing market interest rate at which all good projects and some bad ones raise funding from both skilled and unskilled investors. Skilled investors obtain a rent as they finance only good projects.

The cautious stage arises if cautious investors are willing to enter at a lower interest rate than bold investors as long as all types apply for credit at that interest rate, $r_B(\mu_0, \mu_1, c, r_f) > r_C(\mu_0, \mu_1, c, r_f)$.[7] In this case, all unskilled investors are cautious in equilibrium. This is the right region in the left panel of Figure 1 where different good projects raise financing at different interest rates. Bad projects cannot raise any financing. However, as cautious investors reject opaque good projects, in this stage, skilled investors can advertise a higher interest rate and attract applications from all opaque good entrepreneurs. Indeed, as skilled capital is in short supply, they are advertising the highest possible rate a good entrepreneur is willing to accept, $\bar{r}$.

A bold stage exhibits several features of an overheated, high sentiment credit market. Interest rates are uniformly low and most projects including some bad ones are financed. Thus the overall quality of initiated credit contracts is low with a significant share eventually defaulting. This is in contrast with the cautious stage which exhibits feature of a low sentiment credit market. Most importantly, this market is fragmented. Some good entrepreneurs (transparent ones) enjoy a lot of funding at low interest rates. However, aside

[7] An additional requirement for a cautious stage is that the break-even interest rate for bold investors under the condition that transparent good entrepreneurs are not applying is not feasible. That is, $r_I(\mu_0, \mu_1, c, r_f) > \bar{r}$, or, equivalently, $\tilde{\mu}_0(\mu_1) < \frac{c}{1+r_f}$. Otherwise we are in the mixed stage. We return to this distinction in Section 6.

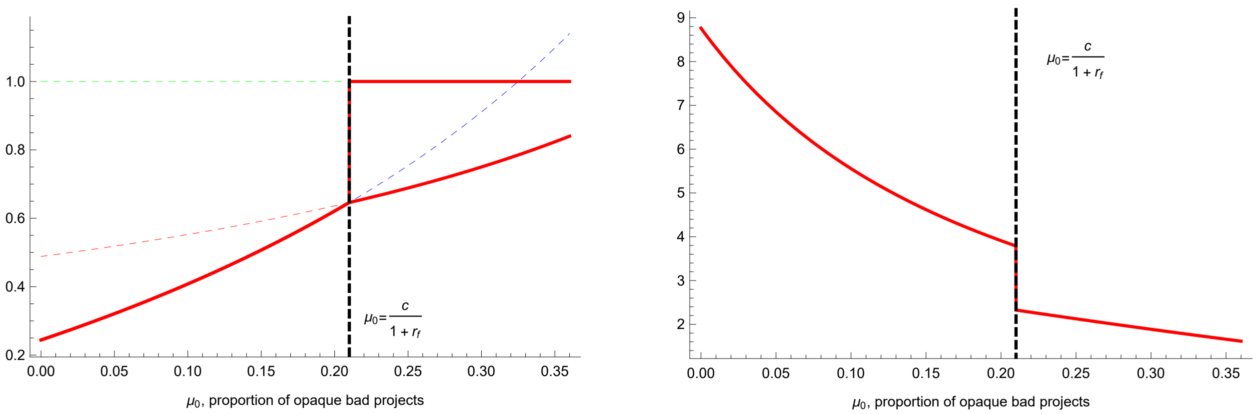


Figure 1: Interest rates and output in the two-stage equilibrium as μ_0 changes in a two-stage economy. The left panel displays the reservation interest rates r_B, r_C, (dashed curves), the maximum feasible rate $\bar{r}$ (dashed horizontal line), and the equilibrium interest rates (solid curves). The right panel displays output. The vertical line in both figures corresponds to $\mu_0 = \frac{c}{1+r_f}$, the threshold where the equilibrium changes between cautious and bold.

from bad projects not being financed at all, some good entrepreneurs (opaque ones) can get only limited funding at very high rates. Therefore, the total loan quantity is relatively low, but its quality is high, which leads to high subsequent realized returns.

In economies where μ_1 is such that $\tilde{\mu}_0(\mu_1) < \frac{c}{1+r_f}$, the equilibrium fluctuates only between the bold and the cautious stage. We refer this case as a two-stage economy. The left panel of Figure 1 displays the relevant zero profit interest rates along with $\bar{r}$, as a function of the proportion of opaque bad projects μ_0.

The third part of the proposition shows that there might be an intermediate case which we refer to as the mix stage. When $\tilde{\mu}_0(\mu_1) > \frac{c}{1+r_f}$, the mix stage arises for the intermediate range of μ_0. In this equilibrium the credit market is segmented, but unskilled investors are some bold and some cautious so both bad and good projects get financing. We postpone the discussion of this equilibrium to section 6.

3.1 Dynamics and Endogenous Cycles

The key to the dynamics of the model is the determination of endogenous response of the state variables to credit market sentiment. The quality of the pool of credit applications deteriorates in the bold equilibrium when the credit market is overheated, and improves in a cautious equilibrium when credit market sentiment is low. At the same time, the changing type distribution induces rational shifts in investors information test choice and implies fluctuations in sentiment. This endogenous interaction leads to deterministic economic cycles without exogenous aggregate shocks to the economy.

In this part, we start with the general description of the evolution of the state variable. Then, we restrict our attention to the two-stage economy. Later in section 6, we extend our discussion to a three-stage economy which might experience a double-dip recession before eventually recovering.

Evolution of State Variable. Let μ_0' and μ_1' denote the state variables next period.

When at least some lenders run the bold test, only bad transparent projects cannot raise financing. However, when lenders are all cautious, opaque bad projects are not financed either. In either stage who cannot raise financing exit the pool of projects and are replaced by a new draw from the outside pool. The next proposition summarizes the law of motion for measure of opaque and transparent bad entrepreneurs.

Proposition 3.2 *Assume* $\min\{r_B(\mu_0, \mu_1, c, r_f), r_C(\mu_0, \mu_1, c, r_f)\} < \bar{r}\,(\rho_g, \xi, \phi)$ *so that the economy is not in autarky.*

(i) If $\mu_0 \in \left[0, \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)\}\right]$, *then law of motion for* μ_0 *and* μ_1 *follows*

$$\mu_{0B}(\delta, \lambda, \mu_0, \mu_1) = (1-\delta)\mu_0 + \big(\delta + (1-\delta)\mu_1\big)\frac{\lambda}{2}, \tag{3.4}$$

$$\mu_{1B}(\delta, \lambda, \mu_0, \mu_1) = \big(\delta + (1-\delta)\mu_1\big)\frac{\lambda}{2}. \tag{3.5}$$

(ii) If $\mu_0 \in (\max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)\}, 1]$, *then law of motion for* μ_0 *and* μ_1 *follows*

$$\mu_{0C}(\delta, \lambda, \mu_0, \mu_1) = \big(\delta + (1-\delta)(\mu_0 + \mu_1)\big)\frac{\lambda}{2}, \tag{3.6}$$

$$\mu_{1C}(\delta, \lambda, \mu_0, \mu_1) = \big(\delta + (1-\delta)(\mu_0 + \mu_1)\big)\frac{\lambda}{2}. \tag{3.7}$$

These laws of motion are quite intuitive. For instance, consider the mass of opaque bad types μ_0. When the economy is not in a cautious stage, function $\mu_{0B}(\delta, \lambda, \phi, \mu_0, \mu_1)$ defines the evolution of μ_0. It consists of survivals from this period, plus the replacements from the outside pool. From the existing bad opaque entrepreneurs, fraction $(1-\delta)$ survives.

The replacements consists of two parts itself: δ measure of all entrepreneurs are exogenously replaced. Furthermore, $(1-\delta)\mu_1$ is the measure of the transparent bad types who have survived endogenously cannot raise funding and this are replaced. From the replacements, a fraction $\lambda/2$ enter as opaque bad. All the other cases follow a similar intuition.

Following the same logic, the opaque and transparent good types are subject to the same law of motion in both cases, given that their measures in the outside pool is the same. Thus in the long run both measures will be equal to $\frac{1-\mu_0-\mu_1}{2}$. This validates that our (μ_0, μ_1) are sufficient state variables for the economy despite four types of entrepreneurs.

We next characterize the endogenous cycle in a two-stage economy where $\tilde{\mu}_0(\mu_1) < \frac{c}{1+r_f}$ along the equilibrium path. We will provide a characterization of the more general case in section 6.

Two-Stage Economy To ensure $\tilde{\mu}_0(\mu_1) < \frac{c}{1+r_f}$ in the long run, for the rest of this section we maintain the following assumption to ensure the investors are all bold or cautious.

Assumption 3.3

$$\frac{\bar{r} - r_f - c - \frac{2c}{1+r_f}}{\bar{r} + c} - \frac{c}{1+r_f} \leq \frac{\delta\lambda}{2-\lambda(1-\delta)}. \tag{3.8}$$

In the two-stage economy, the dynamic economy is characterized by a single state variable, μ_0. The following four constant levels of μ_0 are very useful in our credit cycle characterization

- $\bar{\mu}_{0B}(\delta, \lambda)$: value of μ_0 in a steady state where every lender is bold and remains bold forever,
- $\bar{\mu}_{0C}(\delta, \lambda)$: value of μ_0 in a steady state where every lender is cautious and remains cautious forever.
- $\mu^*_{0B}(\delta, \lambda)$ and $\mu^*_{0C}(\delta, \lambda)$: values of μ if the economy fluctuates between two states in the long run, one in which every one is bold ($\mu^*_{0B}(\delta, \lambda)$), and one in which everyone is cautious ($\mu^*_{0C}(\delta, \lambda)$).

The first two values, $\bar{\mu}_{0B}$ and $\bar{\mu}_{0C}$, correspond to steady states that are not cycles. μ^*_{0B} and μ^*_{0C} on the other hand correspond to a cycle of length 2. The appendix provides detail for derivation of these levels. It also shows that $\bar{\mu}_{0B}(\delta,\lambda) > \mu^*_{0C}(\delta,\lambda) > \mu^*_{0B}(\delta,\lambda) > \bar{\mu}_{0C}(\delta,\lambda)$. Using these values, the following proposition characterizes the steady state dynamic cycles of the economy.

Proposition 3.3 *Given $\bar{\mu}_{0B}(\delta,\lambda) > \mu^*_{0C}(\delta,\lambda) > \mu^*_{0B}(\delta,\lambda) > \bar{\mu}_{0C}(\delta,\lambda)$ above, the ergodic distribution of the economy is characterized as follows.*

(i) $\frac{c}{1+r_f} \geq \bar{\mu}_{0B}$*: the ergodic distribution is degenerate,* $\mu_0 \to \bar{\mu}_{0B}$*. The economy converges to a permanent bold stage.*

(ii) $\frac{c}{1+r_f} < \bar{\mu}_{0C}$*: the ergodic distribution is degenerate,* $\mu_0 \to \bar{\mu}_{0C}$*. The economy converges to a permanent cautious stage.*

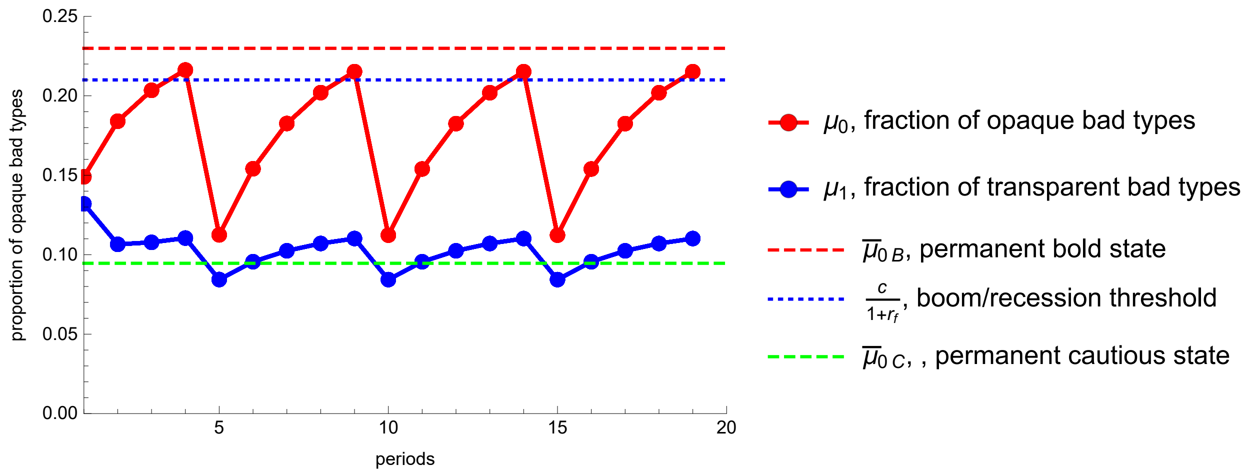
(iii) $\mu^*_{0B} \leq \frac{c}{1+r_f} \leq \mu^*_{0C}$*: the ergodic distribution has a two-point support,* μ^*_{0C}, μ^*_{0B}*. The economy oscillates between a one-period bold stage and a one-period cautious stage for ever. Thus the economy has a credit cycle of length* 2.

(iv) $\mu^*_{0C} < \frac{c}{1+r_f} < \bar{\mu}_{0B}$*: the ergodic distribution has more than two points of support. The credit cycle consists of a multi-period bold stage (while* μ_0 *increases), followed by a one-period cautious stage when* μ_0 *declines to that of the bold stage with the lowest* μ_0.

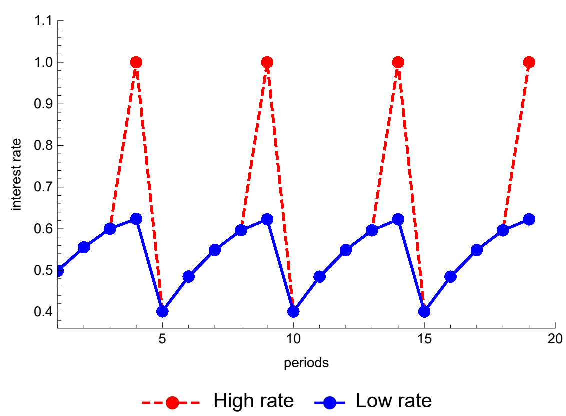
(v) $\bar{\mu}_{0C} \leq \frac{c}{1+r_f} < \mu^*_{0B}$*: the ergodic distribution has more than two points of support. The credit cycle consists of a multi-period cautious stage (while* μ_0 *decreases), followed by a one-period bold stage when* μ_0 *rises to that of the cautious stage with the highest* μ_0.

One can think of $\frac{c}{1+r_f}$ as investor opportunity cost of time, or opportunity cost of giving up on good investment. When it is large, investors have a tendency to use the bold test over cautious test since the opportunity cost of giving up on good investment is high. When this value is very large, then the economy is doomed to end in a permanent overheated bold stage. Basically the fraction of opaque bad projects never reaches a high enough value where it would be wise to be cautious. As a mirror image when $\frac{c}{1+r_f}$ is very low, there is a permanent low-sentiment cautious stage.

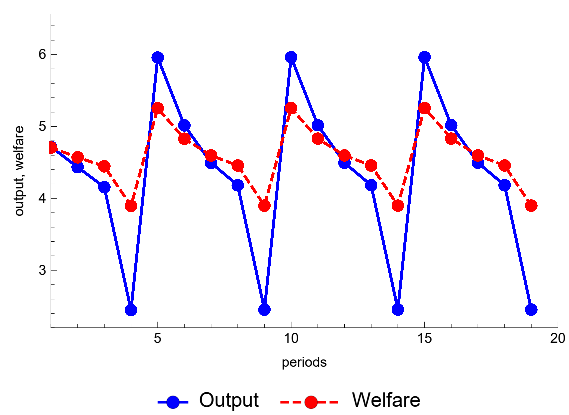
In contrast when investors has an intermediate opportunity cost $\frac{c}{1+r_f}$, the economy features endogenous, deterministic cycles of various types. We refer to this set of parameters as the cyclicality region. Within the cyclicality region, when investor opportunity cost of giving up good investment is relatively high, the cycle features multi periods of boom and



(a) Ergodic distribution of the state space



(b) Interest rates



(c) Output and Welfare

Figure 2: Panel (a) depicts the law of motions of state variables. Panel (b) shows the interest rates, and Panel (c) depicts the total gross output and welfare in a two-stage economy with a multi-period boom and a one period recession cycle.

a one period recession. In this case, a short recession is enough to improve the quality of loan applications sufficiently such that investors are happy to be bold again so they do not risk losing good investment, at the cost of financing some bad investment. A lower $\frac{c}{1+r_f}$ implies a symmetric cycle, where one-period booms are followed by one period recessions. An even lower investor opportunity cost of time implies multi-period recessions followed by a one period booms.

The top panel of Figure 2 helps clarify he intuition behind deterministic cycles. Given the evolution of entrepreneurs' type distribution, under a fixed information choice of investors, the proportion of bad opaque types would converge to a single steady state. The upper (lower) dashed horizontal line denotes this steady state when investors are bold (cautious). The fraction of opaque bad entrepreneurs (μ_0) in the bold steady state has to be higher that of the cautious one, as the exit rate of opaque bad entrepreneurs is higher when investors are cautious. If investors test switching threshold lies between these two steady states, the economy must exhibit deterministic cycles of one type or another. For instance, consider starting at a low μ_0, below the threshold $\frac{c}{1+r_f}$. When fraction of opaque bad firms is low, investors are bold and hence μ_0 moves up, towards the higher μ_0, bold steady state. Therefore, there must be a point when μ_0 surpasses the threshold $\frac{c}{1+r_f}$ and triggers a switch to being cautious. But then, μ_0 immediately moves towards the lower μ_0, cautious steady state. Intuitively, the length of booms and recessions depend on how many steps the system needs in any of these stages to cross the threshold. The Figure depicts the case when booms are long and recessions are short.[8]

The next panel of Figure 2 plot the corresponding ergodic distribution of interest rates. Consistently with Proposition 3.1, we see that that there is no credit spread across different entrepreneurs in the bold stage. However, the credit market is fragmented in the cautious stage. As unskilled investors stop lending to opaque good firms, their interest rate, and the observed credit spread spikes.

Note that cycles are an outcome of the two-way interaction between investor sentiment and the fundamentals of the economy. In booms investors are bold because the opportunity

[8]Is it possible to construct an equilibrium where the economy is permanently at the threshold $\mu_0 = \frac{c}{1+r_f}$ by disposing the assumption that indifferent investors choose to be bold? At that point investors are indifferent, hence, one could require any given fraction of them to be bold. However, under our market clearing protocol, as long as any positive measure of investors choose to be bold, the law of motion is given by (3.4)-(3.5). The reason is that bold investors capital is distributed pro rata among passed applicants, and Assumption 2.1 implies that even minimal credit is enough for survival. This implies that the economy cannot be stuck at $\mu_0 = \frac{c}{1+r_f}$ even if investors mix between bold and cautious.

cost of losing a good project is high for them, since the fraction of opaque and bad applicants are relatively low. Thus lending standards are lax and there is a lot of credit. However, as a result the quality of the credit pool starts to deteriorate. At some point, the fraction of opaque bad applicants becomes so high that investors prefer to turn cautious. Being cautious implies tight lending standards, high interest rate and little credit for opaque projects, which stops opaque bad entrepreneurs from raising funding. Hence, they are replaced by newborns which improves the quality of the credit pool. Therefore, the cycle continues.

The last panel plots the corresponding ergodic distribution output and welfare which we study in the next sections.

4 Investment and Output

In the previous sections we described the credit market equilibrium for any given state μ_0, μ_1. We emphasized that using the limited properties the credit market schedule of entrepreneurs derived in Lemma 3.1, we can establish the interest rates, $r(\tau, \omega)$ each entrepreneur faces in equilibrium. Given the structure described in Proposition 3.1, we foresee that some types (τ, ω) might face a credit quantity constraint, $\ell \leq \bar{\ell}(\tau, \omega)$. Namely, any group whom unskilled investors are not willingly lend to will be constrained given Assumption 2.2.

Therefore, given the credit market outcome and the collateral and budget constraints of entrepreneurs, we can formalize the choice of investment, i, and borrowing, ℓ for any entrepreneur (τ, ω) as follows.[9]

$$\max_{\ell,i} (1 + \ell - i)(1 + r_f) + \rho_\tau i - 1_{\tau=g} \ell (1 + r(\tau, \omega)) \tag{4.1}$$

$$s.t.$$

$$\ell \leq \min\left(\frac{i}{1+r}, \bar{\ell}(\tau, \omega)\right)$$

$$i \leq \ell + 1$$

where the entrepreneur takes $\bar{\ell}(\tau, \omega)$ and $r(\tau, \omega)$ as given. She maximizes her consumption at the end of the period given, by the sum of her return from the risk-free technology and from her project minus the repayment in case she is a good type.

[9]In Appendix A.1, we spell out the entrepreneur's problem in its most general form. In Appendix A.2, we define $\ell \leq \bar{\ell}(\tau, \omega)$ and show that problem 4.1 is a compact version of the general problem.

Good entrepreneurs. As $\rho_g > 1+r_f$, a good entrepreneur prefers to invest in the project rather than risk free rate, $i = 1 + \ell$. Furthermore, as long as $r(g,\omega) \leq \bar{r} \equiv \rho_g - 1$ her objective function is increasing in ℓ, therefore she prefers to borrow as much as possible. This is consistent with the definition of $\bar{r}$. Therefore, if ℓ is sufficiently large, the constraints of problems 4.1 imply $\ell = \frac{1}{r(\tau,\omega)}$ and $i = \ell + 1 = \frac{1}{r(\tau,\omega)} + 1$. If $\bar{\ell} < \frac{1}{r(\tau,\omega)}$, she is rationed in the credit market and borrows $\ell = \bar{\ell}$ and invests $i = \bar{\ell} + 1$.

Bad entrepreneurs. In the maximization problem 4.1 for bad entrepreneurs, constraint $\ell \leq \min\left(\frac{i}{1+r}, \bar{\ell}\right)$ always binds with equality since more loan is always better for a bad agent, who never pays back. At the same time, her maximand is decreasing in i, because $\rho_b < 1 + r_f$. In fact, she is investing only because more collateral can relax her borrowing constraint. Given the structure of the credit market equilibrium, we conjecture and later verify that bad entrepreneurs are always constrained in the sense that $\bar{\ell}(b,\omega) < \frac{1}{r(b,\omega)}$. Then, $\ell = \bar{\ell}(b,\omega)$ and the bad entrepreneur invests just enough to ensure that she can borrow up to this limit, $i = \bar{\ell}(1 + r(b,\omega))$. The entrepreneur understands that to implement that investment, she has to invest $r\bar{\ell}$ from her own endowment. It is easy to check that she is willing to do that, as long as $\rho_b > (1 + r_f)\frac{r(b,\omega)}{1+r(b,\omega)}$, which always holds if $r(b,\omega) \leq \bar{r}$ under (2.1). However, unlike good entrepreneurs, she puts rest of her endowment in the risk-free storage technology.

The last step is to work out $\bar{\ell}(\tau,\omega)$ for each type by market clearing in all market segments which unskilled investors are not willingly serve. The summarize the result in the next proposition in each type of equilibria. We spell out the derivation in the Appendix.

Proposition 4.1

(i) In any equilibrium transparent bad entrepreneurs are not financed by any investors, hence $i(b,1) = 0$.

(ii) In the bold stage, all entrepreneurs face interest rate r_B. All good entrepreneurs invest $i(g,\omega) = \frac{1}{r_B} + 1$, while bad opaque entrepreneurs' investment plan is limited by unskilled investors' mistakes at interest rate r_B, implying $\ell(b,0) = \frac{1}{r_B} - \frac{w_1}{1-\mu_0-\mu_1}$ and $i(b,0) = \ell(b,0)(1 + r_B)$.

(iii) In a cautious stage, all transparent good entrepreneurs face interest rate r_C thus invest at $i(g,1) = \frac{1}{r_C} + 1$, while good opaque ones face $\bar{r}$ and their investment $i(g,0)$ is

limited by the capital of skilled investors, implying $i(g,0) = \frac{2w_1}{1-\mu_0-\mu_1} + 1$. Bad opaque entrepreneurs are not financed by investor, hence $i(b,0) = 0$.

(iv) *In a mix stage, all transparent good entrepreneurs face r_C while opaque good face r_I, and they both implement the maximum scale, $i(g,1) = \frac{1}{r_C} + 1$ and $i(g,0) = \frac{1}{r_I} + 1$. Opaque bad investment $i(b,0)$ is limited by unskilled investor's mistake at r_I, $\ell(b,0) = \frac{1}{2r_I} - \frac{w_1}{1-\mu_0-\mu_1}$ and $i(b,0) = \ell(b,0)(1+r_I)$.*

Note that the output of entrepreneur (τ,ω) is given by $y(\tau,\omega) \equiv \rho_\tau i(\tau,\omega)$.

In a bold stage all entrepreneurs apply for loans at interest rate r_B. All unskilled investors use the bold test and they have abundant capital, thus every good entrepreneurs can obtain all the credit they are willing to absorb at interest rate r_B. Among the bad entrepreneurs, transparent ones cannot obtain credit as they are rejected by the bold test. On the other hand, opaque bad entrepreneurs can obtain some credit because unskilled investors cannot distinguish them from good entrepreneurs using the bold test. However, their credit and thus investment is limited by the mistakes made by unskilled investors who choose to participate in the credit market. Since all good entrepreneurs and even some bad ones invest, the output is high in a bold stage, thus it corresponds to a "boom".

In a cautious stage, good transparent entrepreneurs obtain credit from unskilled investors using the cautious test at interest rate r_C. Unskilled capital is in large supply, therefore good transparent entrepreneurs can implement $i = \frac{1}{r_C} + 1$. Good opaque entrepreneurs instead are obtain credit only from skilled investors at the maximum feasible interest rate $\bar{r}$. As the capital of skilled investors is in short supply, their capital limits the credit of these entrepreneurs implying low low credit quantities. None of the bad entrepreneurs can raise any financing from investors. Thus investment is low in a cautious stage and it corresponds to a "recession".

We discuss the mix stage (iv) in section 6 in detail. Here we just note that is in between the other two regimes of equilibria.

The aggregate output in state (μ_0,μ_1) is given by

$$Y(\mu_0,\mu_1) \equiv \rho_g \frac{1-\mu_0-\mu_1}{2}\left(y\left(g,1\right)+y\left(g,0\right)\right) + \rho_b\left(\mu_1 y\left(b,1\right)+\mu_0 y\left(b,0\right)\right). \tag{4.2}$$

The right panel of Figure 1 illustrates aggregate output conditional on state state μ_0

(for a fixed state μ_1) in a two-state economy. A natural observation is that aggregate output is smoothly monotonically decreasing in the fraction of bad opaque entrepreneurs within any range of parameters where the type of the equilibrium is not changing. This is so because of two reasons. First, the average productivity of entrepreneurs is smaller when μ_0 as bad projects are less productive. Second, Figure 1 illustrates that within the range of a given equilibrium interest rates are (weakly) increasing in μ_0. A larger proportion of bad entrepreneurs increases the equilibrium interest rates, because of adverse selection This increases the cost of capital for production, which decreases investment and total output according to Proposition 4.1.

More interestingly, the change in total output is not smooth when the economy switches between the bold and cautious stage. Continuous changes in μ_0 can lead to discontinuous jumps in $Y(\mu_0, \mu_1)$. In this sense, the economy crashes around the thresholds where agents switch from bold to cautious strategy.

Proposition 4.2 *Consider a set of parameters for which* $r_B\left(\frac{c}{1+r_f}, \mu_1, c, r_f\right) < \bar{r}$. *Total output,* $Y(\mu_0, \mu_1)$, *jumps downward at* $\mu_0 = \frac{c}{1+r_f}$, *when the economy switches from the bold stage to the cautious stage in a two-stage economy.*

The crash at the switching point is intuitive. For a marginal increase in μ_0 around the thresholds bad entrepreneurs stop getting credit and interest rate jumps and credit quantity drops for opaque good projects. All these effects leads to less investment and discontinuously smaller output.

The bottom panel on Figure 2 illustrate the cyclicality of output, $Y(\mu_0, \mu_1)$, the crash when sentiment switches, and its co-movement with the spread between opaque and transparent rates in the corresponding two-stage economy. Comparison with the top panel shows the co-movement with the fraction of opaque bad entrepreneurs μ_0. Unsurprisingly, larger fraction of bad opaque entrepreneurs implies smaller output. However, the effect of a change from an overheated credit market to a low-sentiment credit market is very pronounced. The top panel of Figure 2 shows that this switch occurs in periods 4, 9 , 14, and 19 in our example. While μ_0 increases only slightly in those periods, the bottom panel of Figure 2 shows a sizable drop in output. This is the result of the switch in sentiment. In these periods, the deterioration of the pool of credit applications triggers investors to become cautious. Therefore all bad projects lose financing, and opaque good projects are significantly squeezed. As the middle panel on Figure 2 shows, the fragmentation in the credit market means the opaque good entrepreneurs face a significantly larger interest rate than before. On the bright side though, this crash has a "purification effect" on the economy. Bad entrepreneurs exit the economy and are replaced by an average entrepreneur from the outside pool. This leads to a sufficient improvement in the credit application quality for the next period that increases the opportunity cost of giving up on good investments, and triggers investors to switch to bold test. Over the next couple of periods, the credit market becomes overheated again.

5 Welfare, Economic Cycles and Economic Policy

In the previous sections, we demonstrated how fluctuations of sentiment and that of fundamentals feed onto each other, creating endogenous cycles. As we explicitly model the mechanism which turns booms to busts and vice-versa, our framework is well suited to explore how certain economic policies could and should influence these economic cycles.

We first explore the rational for policy makers to intervene in this economy. In particular, we study a constrained planner's problem where the planner can choose which test investors should use in each aggregate state. We argue that the planner can often improve on the decentralized outcome, because investors do not internalize how their individual choice of lending standards affects the long run dynamics evolution of the state. In particular, the planner often prefers a cycling economy, where low sentiment stages keep the fraction of bad projects in the economy at bay, which in turn makes the high sentiment stages more beneficial.

Then, we connect the constrained planner's solution to realistic monetary and macro-prudential policies. We show that both changing the risk-free rate and specifying capital requirements can be used to influence investors' lending standards. Therefore, each of these policies affects the long run cyclical behavior of the economy, i.e. the dynamic evolution of the aggregate state, and consequently, the welfare. In general, the policy maker can improve the quality of loan applications only at the expense of increasing the average cost of capital. The choice between these policies depend on a trade-off. In our particular specification, we show that capital requirements with carefully chosen risk weights tends to be a more efficient tool than monetary policy in pushing the economy towards constrained efficient cycles.

5.1 Welfare

A natural welfare measure is the aggregate consumption of all entrepreneurs and investors. This is equivalent with all production from the risky and riskless technologies minus the cost of testing:

$$
\begin{aligned}
W(\mu_0,\mu_1) \equiv & \frac{1-\mu_0-\mu_1}{2}\left(y\left(g,1\right)+y\left(g,0\right)\right)+\mu_0 y\left(b,0\right)+\mu_1 y\left(b,1\right) \\
& +\left(1+r_f\right)\left(\mu_1+\mu_0-\mu_1 r(b,1)\ell\left(b,1\right)-\mu_0 r(b,0)\ell\left(b,0\right)\right) \\
& +\left(w_0+w_1-\frac{1-\mu_0-\mu_1}{2}\left(\ell\left(g,1\right)+\ell\left(g,0\right)\right)-\mu_1\ell\left(b,1\right)-\mu_0\ell\left(b,0\right)\right)\left(1+r_f\right) \\
& -c\left(1-k(\mu_0,\mu_1)\right)w_0
\end{aligned}
$$

The terms in each line corresponds to total production by the risky technology, production by entrepreneurs by the risk-free technology, production by investors by the risk-free technology, and the cost of testing, respectively. As we derive in the proof of Proposition 4.1, $k(\mu_0,\mu_1)$ is the fraction of unskilled investors not lending to any entrepreneurs.

The next proposition states that welfare decreases in the fraction of bad entrepreneurs, μ_0, within any segment of the state space where the type of the equilibrium is not changing. Also welfare discontinuously drops when the economy switches to the cautious stage from a bold stage.

Proposition 5.1 *Consider a two-stage economy. Then welfare is decreasing in the fraction of bad projects, μ_0. There is a discontinuous drop in $W(\mu_0,\mu_1)$ around $\mu_0=\frac{c}{1+r_f}$.*

Similarly to its effect on total output, an increase in the fraction of bad entrepreneurs decreases welfare because this group is less productive and imposes higher cost of capital for all firms through adverse selection. While the higher cost of capital can increase skilled investors consumption, this effect is always dominated by the smaller consumption of entrepreneurs. When the economy switches from bold to cautious, bad opaque entrepreneurs loose all their financing, while good opaque entrepreneurs experience both a jump in their interest rate from r_B to $\bar{r}$ and a drop in their loans as unskilled investors stop financing them. Again the resulting upward jump in the consumption for good investors is dominated by those adverse effects.

In the dynamic economy the state distribution is endogenous as we described in 3.1. In a cycling economy, just as output, welfare is higher in the bold stage and lower in the cautious stage, enforcing our interpretation of these stages as booms and busts. Figure 2c depicts the dynamics of welfare and output under our baseline parameterization.

5.2 Optimal Cycles

As the focus of our analysis is the relationship between the choice of investors' lending standards and that of fundamentals, it is instructive to study the following constrained planner's problem.

Definition 5.1 *The constrained planner's solution is a dynamic equilibrium where the planner chooses which test is available in any given state* (μ_0, μ_1) *for all investors to maximize the average welfare along the ergodic state distribution path.*

As the definition shows, we give the planner limited tools to influence economic outcomes. The planner can force all agents to choose the bold test in some states and choose the cautious test in other states. Given the implied information structure, the equilibrium interest rates, quantities and the law of motion of the state distribution are determined as before. For instance, if the planner chooses all agents to use the bold test in all states, the economy will feature only bold stages with no cycles and a degenerate ergodic distribution of $(\mu_0 = \bar{\mu}_{0B}, \mu_1 = \bar{\mu}_{1B})$. Similarly, the planner can implement an only-cautious economy with a degenerate ergodic distribution of $(\bar{\mu}_{0C}, \bar{\mu}_{1C})$. Following the intuition in Proposition 3.3, the planner can also implement various cycling economies, for instance, by forcing agents to be cautious if and only if $\mu_0 < \hat{\mu}_0^P$ for some $\hat{\mu}_0^P \in [\bar{\mu}_{0C}, \bar{\mu}_{0B}]$.

In the next proposition states sufficient conditions that the constrained planner's solution is a cycling economy.

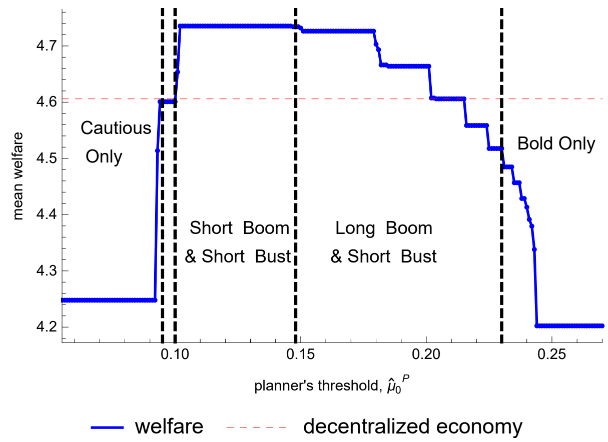
Proposition 5.2 *Suppose that* $\rho_g - 1 > \rho_b$ *and* $r_B(0, 0, c, r_f) < \bar{r}$. *Then, there is an interval* $[\lambda^{\min}, \lambda^{\max}]$ *and a* $\bar{\delta}$ *that if* $\lambda \in [\lambda_{\min}, \lambda^{\max}]$ *and* $\delta < \bar{\delta}$ *then the constrained planner's solution features endogenous cycles.*

The proposition states that rational sentiment driven cycles can be the choice of a welfare maximizing planner. Intuitively, the planner's main motivation for choosing the test is to influence the ergodic state distribution. Tight lending standards keep the fraction of bad firms at bay. However, if the planner forces agents to be always cautious, opaque good firms are always squeezed. Therefore, to maximize average welfare, the planner might decide to periodically allow investors to be bold when the fraction of entrepreneurs running negative NPV projects is sufficiently low.

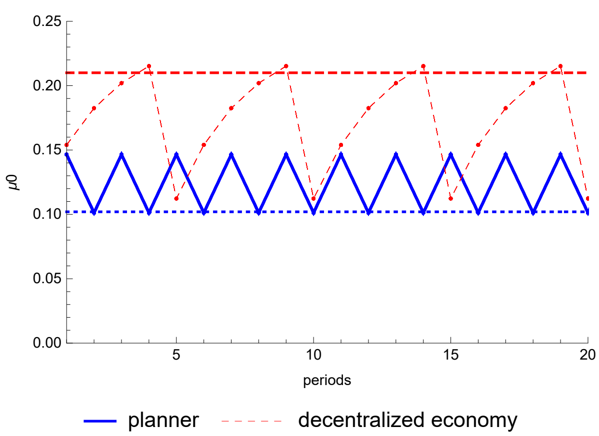
Interestingly, type of cyclicality in the constrained planner's solution determined very differently from the decentralized outcome. This is so, because investors do not internalize that their individual choice of test affects determines the ergodic state distribution. Note that this would be the case even if investors were long lived. The reason is that each investor is atomistic; a unilateral deviation to another test would not affect the ergodic distribution.

To illustrate the planner's problem, Panel (a) on Figure 3 shows the mean welfare over the cycle corresponding to different levels of planner choice of threshold $\hat{\mu}_0^P$. The dashed horizontal line represents welfare in the decentralized economy. The vertical dashed lines partition the figures according to the cyclicality of the implied economy in line with Proposition 3.3. From left to right, the first region corresponds to a cautious-only economy, while the rest of the regions corresponds to a short boom/long recession, a short boom/short recession, a long boom/short recession and a bold-only economy, in order. Welfare changes discontinuously wherever the choice of the planner changes the cyclicality of the economy and it is flat otherwise. Within the long boom and short recession region the drops correspond to the points where the long boom is becoming a period longer.

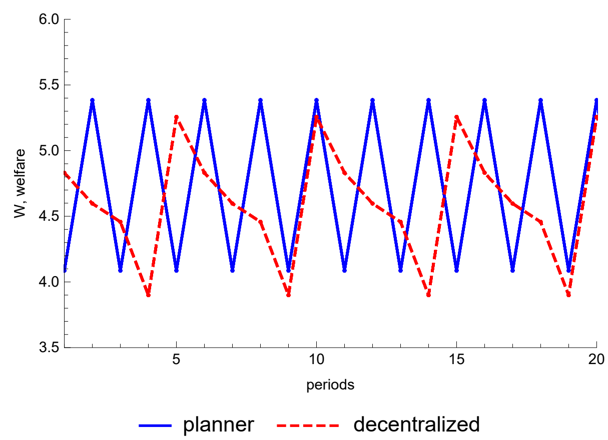
It is apparent that instead of the decentralized choice implying a long boom and a short recession, the planner prefers to push the economy to short booms and short recessions. Panel (b) and (c) illustrates the intuition. Panel (c) compares the path of the state variable μ_0 between the planner optimal choice and the decentralized solution. The dashed horizontal line is the threshold between bold and cautious stages under the decentralized economy, $\frac{c}{1+r_f}$. The dotted horizontal line is the same object, $\hat{\mu}_0^P$, under the planner's optimal choice. The planner, by choosing a lower threshold, forces the economy to a purifying cautious stage more often. This keeps the fraction of bad types in the applicant pool low in average. Panel (d) compares the welfare paths between the planner's choice and the decentralized economy. Because of the lower fraction of bad types, both the booms and the recessions are leading to higher welfare than the corresponding states in decentralized economy. As panel (a) shows, the mean welfare along the planner's path is higher as a consequence.



(a) Mean welfare



(b) Fraction of bad opaque projects



(c) Welfare paths

Figure 3: Mean welfare for different levels of planner choice of threshold $\hat{\mu}_0^P$, as well as the comparison between the implied paths for the fraction of bad opaque entrepreneurs μ_0, and welfare, along the optimal versus the decentralized cycle.

5.3 Economic Policy

In the previous section we established that intervention by a constrained planner generally can push the economy to a higher welfare economic cycle. In this section, we connect the constrained planner's solution to realistic monetary and macro-prudential policies. We show that both changing the risk-free rate and specifying capital requirements can be used to influence investors' lending standards. Therefore, each of these policies affects the dynamics of the state distribution, and, consequently, welfare. However, the policy maker can improve the quality of loan applications only on the expense of increasing the average cost of capital. This trade-off determines the ranking of these policies.

In this section, we compare a non-state-contingent capital requirement policy and risk-free rate policy. We show that capital requirements with carefully chosen risk weights which depends only on individual choices increases the cost of capital less than the risk-free policy, often making the earlier a more efficient tool in pushing the economy towards constrained efficient cycles.

At the end of the section, we consider a sophisticated policy maker who can compute the resulting aggregate state from individual choices, and set a higher risk-free rate only when the corresponding outcome equilibrium state will be a boom and keep the interest rate at zero otherwise, a countercyclical monetary policy. We show that a ountercyclical monetary policy dominates the macro-prudential policy we consider.

For the rest of this section, we normalize the physical return to storage technology to zero. The monetary policy rate, r_f, is the net return of a risk-free asset introduced by the policy maker. This asset is available in perfectly elastic supply for entrepreneurs and investors alike. The monetary policy rate is permanent and non-state-contingent. To ensure that the budget constraint of the policy maker is satisfied, assume that a lump-sum tax on all agents is introduced which exactly covers the aggregate expenditure of providing return r_f. We adjust welfare accordingly.

As a macro-prudential tool, we model risk-weighted capital requirements as follows. Let v_g be the capital a bold investor invests in loans which have passed the bold test. Suppose that the regulator imposes a risk weight $x \geq 1$ on those investments as they are risky. Importantly, this policy is only a function of individual investor choice and is independent of the resulting aggregate state. Put differently, the macro-prudential policy is also permanent and non-contingent.[10] Let v_r her investment in the risk-free asset. Recall that a bold investor, after financing all the loans producing a green-light invests all of her remaining capital in risk-free assets. As this is safe investment, its risk-weight is 1. That is, the investor has to respect the capital requirement $v_g x + v_r = 1$. If $x = 1$, this reduces to the budget constraint of the investor in our unregulated economy. When $x > 1$ the capital constraint forces an investor running the bold to test to not to invest $v_g(x-1)$ units of her endowment. We assume that the investor can put this capital in storage, – or the risk-free asset if available – and consume it in the last period.

In the next proposition, we show that both of these policy tools affects the decentralized equilibrium through changing both the cost of capital for entrepreneurs and investors' choice of lending standards.

Proposition 5.3 *When risk-weight $x > 1$, and r_f is the return on the risk-free asset, the equilibrium characterized by Propositions 3.1-4.1 with the modification that the interest rate functions (3.1) and (3.3) are replaced by*

$$
\begin{aligned}
r_B^{x,r_f}(\mu_0,\mu_1,c,r_f,x) &\equiv r_B(\mu_0,\mu_1,c,r_f) + \frac{(x-1)(1-\mu_1)}{1-\mu_1-\mu_0}c \\
r_C^{x,r_f}(\mu_0,\mu_1,c,r_f,x) &\equiv r_C(\mu_0,\mu_1,c,r_f) \\
r_I^{x,r_f}(\mu_0,\mu_1,c,r_f,x) &\equiv x r_I(\mu_0,\mu_1,c,r_f) + (x-1)(1-\frac{2\mu_1}{1-\mu_0-\mu_1}c)
\end{aligned}
$$

respectively, and thresholds $\frac{c}{1+r_f}$ and $\tilde{\mu}_0(\mu_1,c,r_f,\rho_g)$ on μ_0 are replaced by

$$
\begin{aligned}
\hat{\mu}_0(\mu_1,c,r_f,x) &\equiv \frac{c}{1+r_f}(1-(1-\mu_1)(x-1)) \\
\tilde{\mu}_0(\mu_1,c,r_f,\rho_g,x) &\equiv \frac{(1-\mu_1)(\rho_g-(1+r_f)-(x-1)(c+r_f+1))-(1+\mu_1)c}{\rho_g+x(1+c+r_f)},
\end{aligned}
$$

respectively.

Using our results, the next corollary summarizes the effect of tighter monetary or tighter macro-prudential policy on equilibrium cycles.

Corollary 5.1 *Consider a two-stage economy. Larger risk-free rate, r_f, or larger risk-weight, x, implies (weakly) higher cost of capital for any fixed state (μ_0,μ_1). A sufficiently*

[10] Introducing a different risk-weight for cautious investors' loans would be straightforward. For simplicity, and because they are investing only in loans which certainly pay back, we omit that treatment.

large, one-off increase in r_f or x pushes a high credit sentiment boom to a low sentiment recession. Furthermore it results in the economy spending more time in low credit market sentiment stages where lending standards are tight, and less time in high sentiment stages, where lending standards are lax.

The corollary is a direct implication of Propositions 3.1, 3.3, 4.1 and 5.3. The idea is that while both policies change the ergodic distribution at different expenses in terms of the cost of capital. In fact, as the next proposition demonstrates, achieving the same tightening of lending standards is always accompanied by higher lending rates if monetary policy is used instead of macro-prudential policy.

Definition 5.2 (Equivalent Policies) *Consider a two-stage economy currently at r_f^0, x^0, implying $\hat{\mu}_0^0 = \hat{\mu}_0(x^0, r_f^0, \cdots)$. We call the monetary policy r_f^N and macro-prudential policy x^N equivalent if keeping every other parameter constant, $\hat{\mu}_0^N = \hat{\mu}_0(x^0, r_f^N, \cdots) = \hat{\mu}_0(x^N, r_f^0, \cdots)$. That is, implementing either new policy leads to the same new sequence of state variables along the corresponding equilibrium cycles.*

Proposition 5.4 *Consider a two stage economy and two equivalent policies $r_f^N > r_f^0$ and $x^N > x^0$. In every state of the corresponding equilibrium cycle, the equilibrium interest rate that any entrepreneur faces is weakly higher under the monetary policy.*

The higher interest rate implied by the risk-free rate policy tend to translate to lower welfare as the next lemma suggests.

Lemma 5.1 *Consider a two stage economy in state (μ_0, μ_1) currently at r_f^0, x^0, a marginally higher monetary policy $dr_f > 0$ and a marginally stricter macro-prudential policy $dx > 0$. Keeping the state constant, if $c \leq \bar{c}$, the welfare is higher under the marginally more strict macro prudential policy, compared to the monetary policy.*

Note that the comparison depends on the size of the cost of testing, c. It is so, because increasing the risk-weight, x, implies that each bold unskilled investor lends a smaller amount of capital to entrepreneurs. This implies that more unskilled investors have to enter to satisfy the same demand by capital. This is costly, if c is high.

5.3.1 State-Contingent Monetary Policy

que In the previous section we have found that among non-state-contingent policies, carefully chosen risk-weights might be a more efficient tool to push the economy to higher-welfare cycles than monetary policy. One reason is that although the risk-weight x applies to investor testing and risky lending choices the same way in both booms and recessions, it endogenously only limits lending in the bold stage. In a cautious stage investors lend only to safe entrepreneurs, therefore they are not subject to the higher capital requirement. This is in contrast with the non-state-contingent monetary policy tool. A higher risk-free rate implies higher opportunity cost of lending in all states.

To assess the role of this asymmetry, we examine the effect of a state-contingent, countercyclical monetary policy. That is, we consider a policy maker who can solve for equilibrium outcome resulting from his policy tool, and sets the interest rate at $r_f > 0$ only when the aggregate equilibrium outcome will be a bold stage, while keeping it at 0 whenever the resulting economy will be in the cautious stage. The next proposition states that a countercyclical monetary policy always dominates an equivalent macro-prudential policy.

Proposition 5.5 *Consider a two stage economy and two equivalent policies. The state-contingent countercyclical monetary policy increases the risk-free rate to r_f^N from the initial level of r_f^0 when the resulting economy is in bold stage, and keeps it at zero if it is in cautious stage. The non-state-contingent macroprudential policy sets the risk weight to x^N from the initial level of x^0.*

(i) In every state of the corresponding equilibrium cycle, the equilibrium interest rate is the same under the two policies.

(ii) As long as the implied equilibrium path has a bold stage, the implied expected welfare is strictly higher under the state-contingent countercyclical monetary policy.

The result states that a countercyclical monetary policy has the same effect on the cost of capital as an equivalent macro-prudential policy. However, as the macro-prudential policy implies a higher testing cost, the countercyclical monetary policy is a more efficient tool to push the economy towards higher-welfare cycles. An important caveat is that the countercyclical monetary policy assumes a relatively high level of sophistication of the policy maker. She essentially has to solve a fixed point problem. When increasing the interest rate, she has to be able to foresee whether that interest rate keeps the economy in a boom, or shifts it towards a downturn.

6 Three-Stage Economy and Double-dip Recessions

When $\tilde{\mu}_0(\mu_1) < \frac{c}{1+r_f}$ does not hold everywhere along the equilibrium path in the long run, the economy features more elaborate dynamics. Importantly, the economy not only can be in bold and cautious stages, but also in a mix stage. In this case, we need two state variables to characterize the dynamic economy, (μ_0, μ_1).

We first explain the mix stage in credit market and real economy, and then proceed to considering the dynamics associated with a three-stage economy.

Mix stage in credit market. If the maximum feasible interest rate $\bar{r}$ is sufficiently high, the interest rate $r_I(\mu_0, \mu_1, c, r_f)$ in proposition 3.1.(iii) becomes feasible. In this case, some unskilled investors choose to be bold and some cautious. A semi-separating equilibrium arises where opaque good entrepreneurs are financed by bold unskilled investors and skilled investors at a relatively high interest rate, $r_I(\mu_0, \mu_1, c, r_f)$. Furthermore, the bold unskilled investors mistake opaque bad entrepreneurs with good opaque ones and finance them at the same interest rate. Alternatively, cautious unskilled investors finance all transparent good entrepreneurs at the lower interest rate $r_C(\mu_0, \mu_1, c, r_f)$. Interest rates $r_I(\mu_0, \mu_1, c, r_f)$ and $r_C(\mu_0, \mu_1, c, r_f)$ are such that unskilled investors are indifferent whether to be bold and finance a worse quality portfolio for a higher interest rate, or to be cautious and finance a high quality portfolio for a lower interest rate.

Real economy. In a mix stage, the masses of entering cautious and bold unskilled investors are such that the first group can satisfy the credit demand of transparent good investors at the low interest rate r_C while the second group, together with skilled capital, can satisfy the credit demand at the higher interest rate r_I. Therefore, the investment of both of good entrepreneurs are given by $i = \frac{1}{r} + 1$ with the relevant different interest rates. Similar to the bold stage, the credit to opaque bad entrepreneurs is given by the share of capital of bold unskilled investors who cannot identify their loan applications from good opaque ones. The credit of of bad opaque entrepreneurs is lower in mix relative to bold stage since they face a higher interest rate $r_I > r_B$ in the mix stage.

When $\tilde{\mu}_0(\mu_1) < \frac{c}{1+r_f})$ does not always hold in the long run equilibrium the economy can cycle through all three stages, going from bold to mix and then to cautions, and then jumping back to bold stages. Such a three-stage economy features two different type of downturns. The mix stage has signs of low credit market sentiment as it leads to a similar fragmentation of the market as the cautious stage. However, the mixed stage corresponds to a recession which is not sufficiently deep to trigger the "purification effect" on the entrepreneur pool we observed in the cautious stage. This is so, because even a small mass of bold investors is sufficient to mistakenly provide credit to bad opaque entrepreneurs and prevent them from exiting. Therefore, in a mix stage the fraction of bad entrepreneurs keeps increasing to the point when the cautious stage is triggered. In a cautious stage the sufficiently tight credit conditions reverses the direction of the economy.

The following assumption ensures a three-stage cycle for the dynamic economy.

Assumption 6.4 *Assume*

(i) $\frac{c}{1+r_f} < \frac{\bar{r}-c-r_f-\frac{\delta\lambda(c+\bar{r})}{2((1-\delta)(1-\lambda)+\delta)}}{c+\bar{r}+2} < \frac{\lambda}{2-\lambda(1-\delta)}$

(ii) $\frac{c}{1+r_f} > \frac{\delta\lambda(2-\delta)(2+\lambda(1-\delta))}{2(2(2-\delta)^2\delta^2-(1-\delta)^4(2-\lambda)(1-\lambda)+3\delta(2-\delta)(1-\delta)^2(2-\lambda)+(4(1-\lambda)-(2-\delta)\delta(\lambda^2+6(1-\lambda)))(1-\delta)^2)}$.

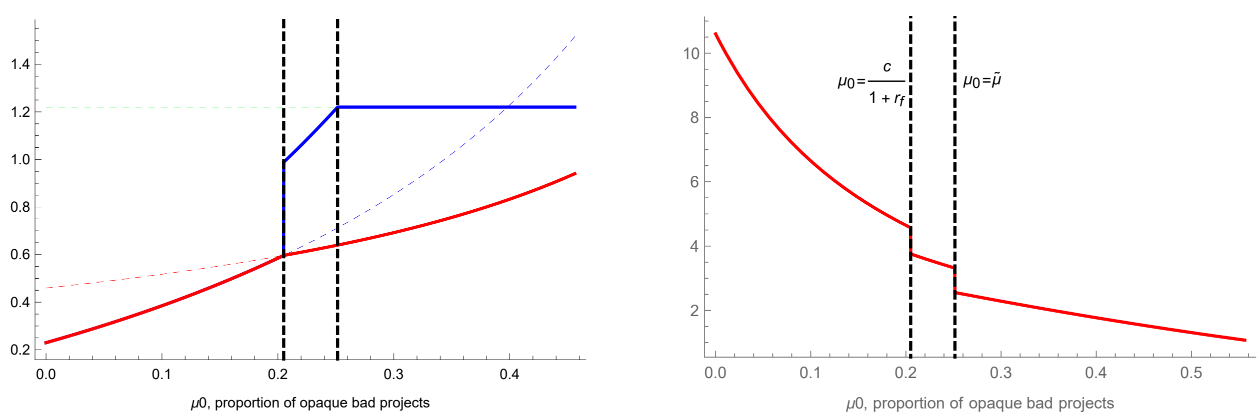


Figure 4: Interest rates (left panel) and output (right panel) in a three-stage economy as the fraction of bad opaque entrepreneurs μ_0 changes.

Assumption 6.4 ensures that $\mu^*_{0C} < \frac{c}{1+r_f} < \tilde{\mu}_0(\mu_1) < \bar{\mu}_{0B}$. This is sufficient to make sure that in the long run, the economy goes through all the three stages, as formalized in the following proposition.

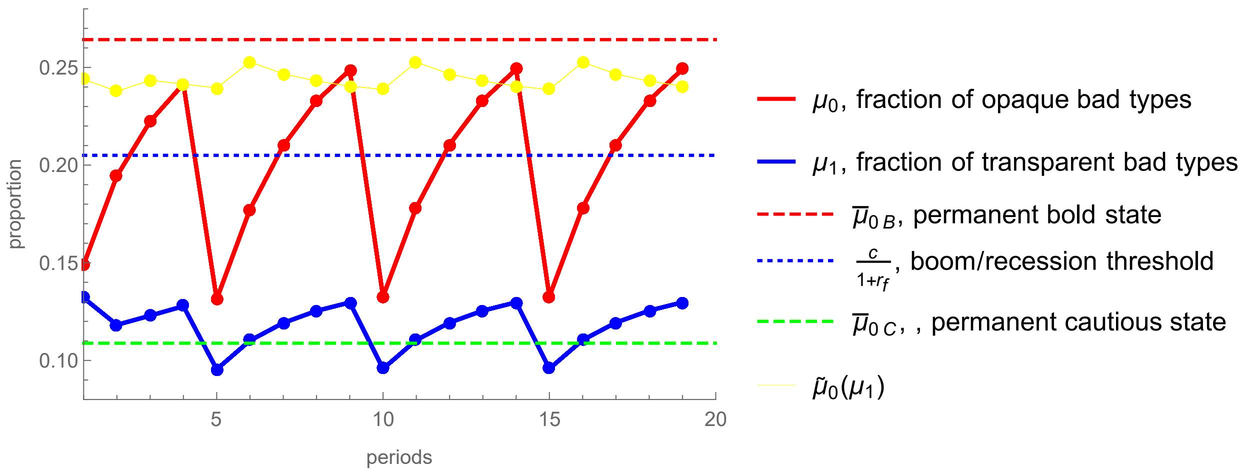
Proposition 6.6 *Consider $\bar{\mu}_{0B}(\delta,\lambda) > \mu^*_{0C}(\delta,\lambda) > \mu^*_{0B}(\delta,\lambda) > \bar{\mu}_{0C}(\delta,\lambda)$, and assume assumption 6.4 holds. Then the ergodic distribution has more than two points of support. The credit cycle consists of a multi-period bold stage when μ_0 increases, followed by a multi-period mix stage when μ_0 still increases, followed by a one-period cautious stage when μ_0 declines to that of the bold stage with the lowest μ_0.*

Figure 5 portraits an example of the dynamic cycle of the economy. The top panel illustrates the dynamics of the state variables. In the beginning of each upward segment, μ_0 is sufficiently small and the composition of borrowers sufficiently good, $\mu_0 < \frac{c}{1+r_f}$. In this region (below the dotted blue line) all investors are bold. Between the dotted blue line and the yellow curve, some unskilled investors turn cautious but some are still bold (semi-separating equilibrium). In this region, opaque bad entrepreneurs still get funded (although at worse interest rates), so μ_0 is still growing. When μ_0 becomes sufficiently large, i.e. it crosses the yellow threshold, then the composition of borrowers become sufficiently bad that no investor chooses to be bold. Thus all investors turn cautious and a one period crash happens.

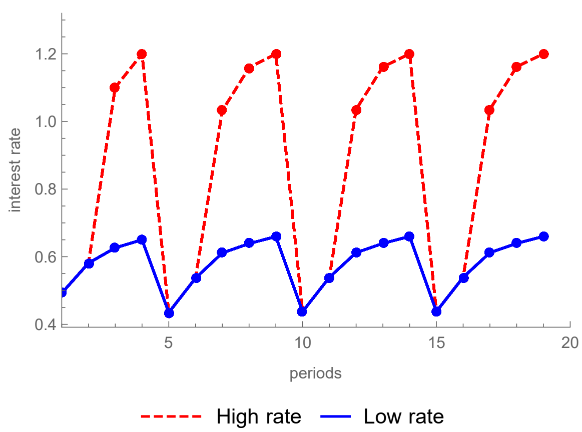
Observe that although the dynamics of the state variables are qualitatively similar to the two-stage economy depicted in Figure 2, the implied outcomes are quite different. The second panel of Figure 5 depicts the interest rates in the three-stage economy. inn the mix stage there is already a considerable spread between the interest rates faced by opaque and transparent entrepreneurs although μ_0 is still increasing. This spread drops to zero again, only when the economy reverses to the bold stage. Output dynamics in the bottom panel shows that in the three-stage economy output crashes twice in each cycle, creating a double dip recession.

Further comparison of Figure 2 and Figure 5 shows that although the interest rate in the mix stage and in the cautious stage are at similar levels, the output effect of switching to the cautious stage is significant in the three-stage economy. In fact, the output dynamics shows a double-dip recession. Despite the significant drop in output around period 3 , the recession is not deep enough for the economy to experience the "purification effects" of a cautious equilibrium. Therefore, output drops further until a second drop in output occurs in period 6 . In this period finally the fraction of opaque bad entrepreneurs drops, and corresponds to a deep downturn which then triggers a boom in the following period.

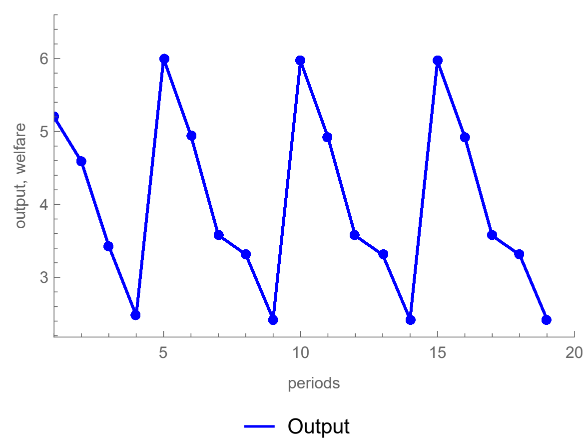
A few observations are worth mentioning. First, if $\mu^*_{0B} < \frac{c}{1+r_f} \leq \tilde{\mu}_0(\mu_1) < \mu^*_{0C}$ (assumption 3.3 and 6.4 both violated), the ergodic set consists of the same two point distribution as



(a) Ergodic distribution of the state space



(b) Interest rates



(c) Output

Figure 5: Panel (a) depicts the law of motions of state variables. Panel (b) shows the interest rates, and Panel (c) depicts the total gross output in a three-stage economy with a multi-period bold, a multi-period mix, and a one period cautious stage cycle.

described in Proposition 3.3.(iii), i.e. a two-stage economy with a cycle of length two which fluctuates between all lenders being bold or cautious.

Second, it is not possible to have a three-stage economy with a long downturn. In other words, whenever there is a cycle which consists of multiple consecutive cautious stages, it ends either by a single bold stage (if $\tilde{\mu}_0(\mu_1) < \frac{c}{1+r_f}$), or by a single mix stage (if $\tilde{\mu}_0(\mu_1) > \frac{c}{1+r_f}$). The key to this observation is that whenever μ_0 falls below $\max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)\}$, the dynamics is dictated by $\mu_{0B}(\delta, \lambda, \mu_0, \mu_1)$ function and is upward sloping, so it cannot enter a third (lower) stage.

Finally, note that the threshold $\tilde{\mu}_0(\mu_1)$ depends on one of the state variable μ_1. This implies that the dynamic economy might even fluctuate between a two-stage and a three-stage economy. This can lead to cycles with varying length.

7 Model and Facts

Despite its simple structure, our model generates a rich set of empirical predictions.

When mapping the model outcomes to the data, a critical question is how to think about the distinction of credit to firms with different types. The most conservative mapping is to think of types as heterogeneity uncorrelated with publicly available real-time information. This is the point of view of an unskilled investor who does not run any test. As we will discuss, under this approach, the model can be applied to correlation across aggregate credit quantity, quality or average returns. Also, the model can give predictions on the relationship between ex-ante choices of different investor groups and ex-post outcomes.

A less conservative approach is to allow for the possibility that there is publicly available information which partitions firms similarly to the bold or the cautious tests. Then, the interpretation of choosing the test is that the investor has to decide on which piece of public information to include in her due diligence process. For instance, we can think of all borrowers who would be rejected by a cautious test as the group of the risky firms which can issue only low-grade bonds. In the context of the subprime crises, they might be identified as low-documentation borrowers. Similarly, in the context of the European crises, they might be identified as borrowers from (South of) Europe. From the point of view of unskilled investors, lending to these firms is risky as a fraction of them is not paying back. While these groups are identifiable by information which available to the lender in real time, only cautious investors reject all applications from this group. Bold investor decides to apply less tight lending standards and reject only a subgroup of these applicants. Under this approach, our model can give predictions on the level of these borrower groups.

This section focuses on some of these empirical predictions and contrasts it with empirical evidence when available under each of these approaches.

Tightness of credit, interest rates and economic cycles Under the conservative mapping, our model predicts that within any group of risky borrowers formed by observables, larger aggregate quantity of credit is granted at less dispersed interest rates in booms compared to recessions. Also, any proxy for ex-ante lending standards or the quality of the issued loans should show less tight and deteriorating lending standards in booms.

As a simple sign for cyclicality of lending standards we plot on Figure 6 the percentage of senior loan officers claiming to tighten C&I loan conditions in the given quarter as the blue curve. The shaded areas are NBER recessions. As a more systematic treatment Morais et al. (2019) finds both US and international evidence for lax lending standards in booms in the bank loan market. In a different context, Demyanyk and Van Hemert (2009) documents that the quality of subprime loans deteriorated for six consecutive years before the 2007 crisis.

Credit composition, the quality spread and credit market sentiment Following the less conservative mapping, in this part we identify junk bond issuance as credit to firms a cautious test would reject, i.e., credit to opaque firms. Then loans to transparent good firms map to high-grade bond issuance.

A basic prediction of our model is that the quality spread should be counter cyclical. The red curve in Figure 6, the Moody's seasoned AAA-BAA spread, demonstrates that this is indeed the case.

Under this approach, we can also contrast our finding with the growing body of evidence suggesting that periods of overheating in credit markets forecasts low excess bond or loan returns. This is not a tautology if credit market overheating is measured ex-ante by the quantity or composition of credit. As an influential example, Greenwood and Hanson (2013) show that the share of junk bond issuance out of total issuance inversely predicts the excess

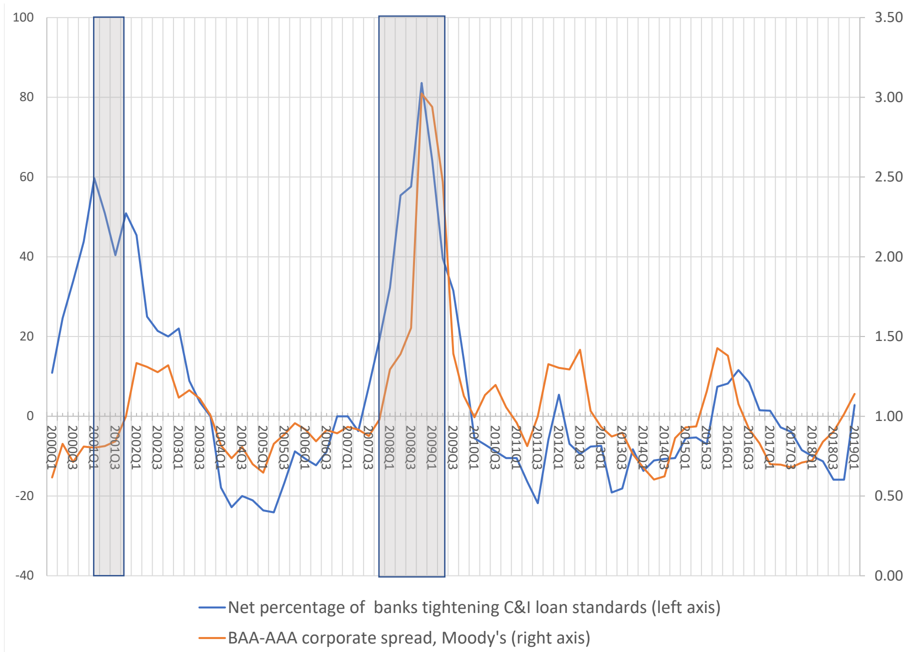


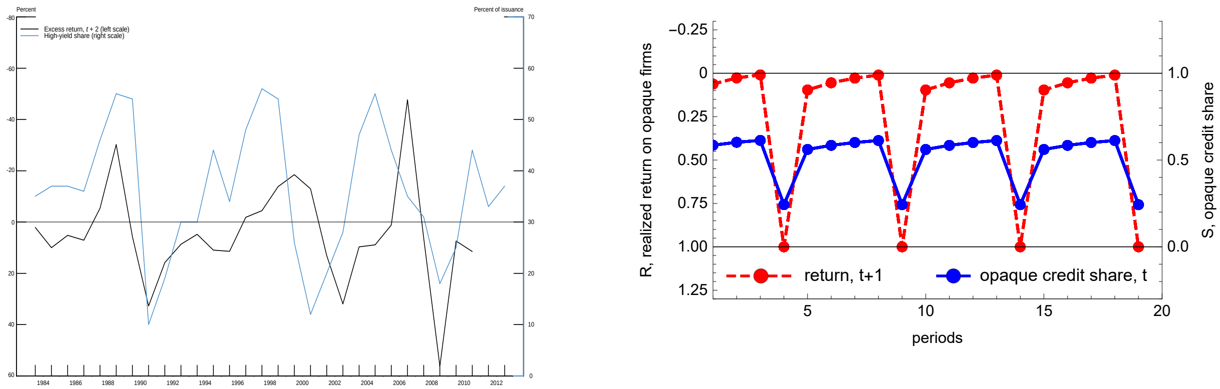
Figure 6: The percentage of senior loan officers claiming to tighten C&I loan conditions in the given quarter (Survey of Senior Loan Officers, Federal Reserve, solid curve, left axis), and the Moody's seasoned BAA-AAA spread (FRED, St. Louis FED, dashed curve, right axis). The shaded areas are NBER recessions.

return on these bonds.[11] Panel (a) on Figure 7 is the illustration of this fact using the reproduction of Stein (2013). The blue curve is the share of junk bonds issued in a given period as a fraction of all issuance, measured on the right axis. The black curve is the excess return on those low-grade bonds in the subsequent two years, measured on the reverse scale on the left axis. For instance, years when both curves are high correspond to overheated periods with low subsequent returns. Periods where both curves are low tend to correspond to recessions: low sentiment credit markets with high subsequent returns.

Panel (b) of Figure 7 plots the model-equivalent time-series for the two-stage economy simulated on Figure 2. In the model, the share of transparent credit to all credit is

$$S(\mu_0, \mu_1) \equiv \frac{\mu_0 \ell(b,0) + \frac{1-\mu_0-\mu_1}{2}\ell(g,0)}{\mu_0 \ell(b,0) + \frac{1-\mu_0-\mu_1}{2}\left(\ell(g,0) + \ell(g,1)\right)}.$$

[11]The inverse relationship between credit expansion and subsequent returns is remarkably widespread across various financial markets. For instance, Baron and Xiong (2017) documents the negative relationship between bank's credit expansion and banks' equity returns, Kaplan and Stromberg (2009) finds a similar inverse relationship between venture capitalists aggregate flow to new investments and their subsequent returns. A related early work is Eisfeldt and Rampini (2006), who shows that volume of transactions are procyclical while return on transactions is counter-cyclical in the sales of property, plant and equipment.



(a) High-yield share and excess bond return

(b) Opaque credit share and realized return

Figure 7: Opaque credit share and its realized excess return. Panel (a) is reproduced from Stein (2013). High-yield (HY) share is the ratio of speculative-grade issuance to total rated corporate bond issuance. Excess returns are calculated as the difference between the log return on a HY index and a corresponding treasury index. Excess returns are calculated between t and $t+2$ and shown alongside the HY share at time on an inverted scale, so that the negative correlation appears positive. Panel (b) shows the share of issued credit to opaque projects relative to all credit in a given period (solid, right scale), and the realized return on opaque credit one period later (dashed, left inverse scale).

Note that $\ell(\tau, \omega)$ depends on the state variables as we described in Section 4. The net excess return on opaque credit is

$$R(\mu_0, \mu_1) \equiv \frac{\frac{1-\mu_0-\mu_1}{2}\ell(g,0)(1+r(g,0))}{\mu_0 \ell(b,0) + \frac{1-\mu_0-\mu_1}{2}\ell(g,0)} - (1+r_f)$$

where the denominator is the total credit opaque projects receive in a given period, while the numerator is the total repayment on that credit in the subsequent period. Note the strong co-movement between $S(\mu_0, \mu_1)$ and $R(\mu_0, \mu_1)$ on a reverse scale, both within the bold stage and across periods. To show the intuition, we calculate these expressions in the limit when skilled capital is negligible:

$$\lim_{w_1 \to 0} S(\mu_0, \mu_1) = \frac{1+\mu_0-\mu_1}{2(1-\mu_1)}, \lim_{w_1 \to 0} R(\mu_0, \mu_1) = \frac{c - \mu_0(1+r_f)}{1+\mu_0-\mu_1}$$

in the bold stage and

$$\lim_{w_1 \to 0} S(\mu_0, \mu_1) = \frac{2c + r_f(1 - \mu_0 - \mu_1)}{2c + (1 - \mu_0 - \mu_1)(r_f + \rho_g)}, \lim_{w_1 \to 0} R(\mu_0, \mu_1) = \bar{r} - r_f$$

in the cautious stage. It is apparent that the (inverse) co-movement within the bold stage is driven by both of our state variables. The co-movement across the bold and cautious stages are driven by the fact that in the cautious stage, $S(\mu_0, \mu_1)$ is converges to its minimal value, while $R(\mu_0, \mu_1)$ converges to its maximal value for $w_1 \to 0$.

It is important to note that even if our model generates the strong correlation across the quality composition of issued credit and subsequent realized returns, this correlation does not amounts to an exploitable anomaly based on the information set of unskilled investors. That is, regarding evidence that points to such anomalies to exist, our rational model can only play a complementary role to behavioral models in explaining those facts.

The critical assumption why the quality-return correlation does not translate to an anomaly is that unskilled investors cannot mimic skilled investors. In booms, the low realized returns on junk bonds make unskilled investors exactly brake even. They could choose to reject all applications from risk firms by a cautious test, but that would be sub-optimal as the fraction of bad credit in the applicant pool is low. They would prefer to lend only to those firms which skilled lends to, but they cannot condition on the skilled loan portfolio. In recessions, they understand that the issued opaque credit generate high returns for skilled investors. But, again they cannot mimic their investment decisions. In particular, if an unskilled were to issue opaque credit they would end up financing all bad opaque firms as well given their bold test.

Market fragmentation and heterogeneous portfolio rebalancing We argue that through the unique structure of the stage game, our model provides crucial insights on the market fragmentation in recessions. As a bold stage turns into a cautious stage, skilled and unskilled investors rebalance their portfolio very differently. Skilled investors rebalance from high-quality bonds (to transparent entrepreneurs) to low-quality ones (to opaque entrepreneurs), while unskilled investors do the opposite. This implies that good entrepreneurs face heterogeneous experience. Transparent good entrepreneurs enjoy abundant credit supply, while opaque good entrepreneurs are squeezed, although in the bold stage they faced the same market conditions. This market fragmentation and the implied heterogeneous effect of a downturn is a unique feature of our model.

This type of market fragmentation was especially salient in context of the Euro zone crisis. Indeed, Ivashina et al. (2015) and Gallagher et al. (2018) find that in 2011 a group of US money market funds stopped lending only to European banks but not to other banks which had similar fundamentals. In particular, Gallagher et al. (2018) finds that when these money market funds stopped financing entrepreneurs in a European country, they did so irrespective of a entrepreneur's implied risk of default. These predictions are consistent with our mechanism when considering these funds as low skilled investors. Moreover, Ivashina et al. (2015) also find evidence that this process led to a significant disruption in the syndicated loan market, a possible channel for the real effects that our model predicts.[12]

International spillovers of US monetary policy Our model is applicable beyond a developed economy such as the US. As an example, consider our set up as a description of an emerging market economy where the banking sector is sensitive to external risk-free rate, as this determines the opportunity cost of lending domestically. In particular, we can think of the return on risk-free storage in our model as the US policy rate, controlled by the FED. Just as in our model, this rate is effectively exogenous for the emerging market banking sector, but clearly influences their activity. Recall from Corollary 5.1 that a small increase in the risk-free rate can have a significant adverse effect both in the short run and in the longer run. Even a small increase in the risk-free rate can push a bold, overheated economy to a low-sentiment, recessionary one. A permanent change in the rate can change the nature of the cycle in the economy. In general, higher opportunity cost of lending leads to longer recessions and shorter booms. This is in line with several emerging market officials including Raghu Rajan's warning, who was the Chair of India's central bank around 2013-14, when the emerging markets were expected to be adversely affected when US starts to raise rates.[13]

8 Conclusion

We present a model of rational sentiments and economic cycles. We capture sentiment by the credit market conditions which follow investors choice of test that they use to grant credit. Their choice leads either tight or lax lending standards. Tight credit standards forces low quality entrepreneurs to exit at a higher rate implying an improving quality of the borrowing

[12]See also our companion paper Farboodi and Kondor (2018) providing a substantially richer picture on market fragmentation by treating sentiment switches as exogenous.

[13]See Financial Times, September 3, 2013 and January 31, 2014.

pool. This change then influences investors lending standards in the future. We show that the two-way interaction between sentiment and fundamentals generates endogenous economic cycles. We show that the planner, with carefully chosen capital requirements for investors, prefers to change the cyclicality of the economy, using recessions to keep the borrower pool quality at bay. We further demonstrate that the predictions of the model matches numerous stylized facts related to credit cycles.

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A Appendix: Agent Optimization Problem and the Market Clearing Protocol

In this Appendix, we define the problem of each agent along with the market clearing protocol. We also introduce a robustness criterium. The structure of our credit market is a modified version of Kurlat (2016). The entrepreneur and investor problems are simplified versions of those in Farboodi and Kondor (2018).

A.1 Entrepreneur and Investor Problem

Let R be the a set of trading posts, each of them identified by an interest rate r. Then, the formal problem for an entrepreneur (τ, ω) is

$$\begin{aligned}
&\max_{\{\sigma(r;\tau,\omega)\}_{r\in R}, i(\tau,\omega)} (1+\ell(\tau,\omega)-i(\tau,\omega))(1+r_f)+\rho_\tau i(\tau,\omega)-1_{\tau=g}\ell(\tau,\omega)(1+r(\tau,\omega)) \\
&s.t. \\
&0 \leq \sigma(r;\tau,\omega) \leq \frac{1}{r} \qquad \forall r \in R \\
\ell(\tau,\omega) &= \int_R \sigma(r;\tau,\omega) d\eta(r;\tau,\omega) \\
r(\tau,\omega) &= \frac{\int_R r\sigma(r;\tau,\omega) d\eta(r;\tau,\omega)}{\int_R \sigma(r;\tau,\omega) d\eta(r;\tau,\omega)} \\
\ell(\tau,\omega) &\leq \frac{i}{1+r(\tau,\omega)} \\
i(\tau,\omega) &\leq \ell(\tau,\omega)+1.
\end{aligned}$$

where η is a rationing function assigning $\eta(R_0;\tau,\omega)$ measure of credit, per unit application, for an entrepreneur of type (τ,ω) submitting applications to the subset of trading posts $R_0 \in R$. η is an equilibrium object, determined by the choices of the agents and the market clearing protocol as we explain below. It is taken by entrepreneurs as given.

The problem of an unskilled investor $h \in [0, w_0]$ is

$$\max_{\chi(h),\tilde{r}(h)} (1+\tilde{r}(h))(S_u(r;g,1)+\mathbb{1}_{\chi(h)=B}S_u(r;g,0))+(1+r_f)\left(S_u(r;b,1)+\mathbb{1}_{\chi(h)=C}\left(S_u(r;b,0)+S_u(r;g,0)\right)\right)$$

and that of a skilled investor $h \in (w_0, w_0+w_1]$ is

$$\max_{\tilde{r}(h)} \left((1+\tilde{r}(h))(S_s(r;g,1)+S_s(r;g,0))\right).$$

$\chi(h)$ is the unskilled agent's choice of test, while S_u and S_s are the sampling functions for unskilled and skilled investors respectively. A sampling function, S_u, assigns a measure $S_u(r;\tau,\omega)$ to any interest rate advertised by an unskilled investor. This describes the measure of credit applications submitted by entrepreneurs of type (τ,ω) in the sample of the unskilled investor advertising that rate. $S_s(r;\tau,\omega)$ is the analogous object for skilled investors. Each investor takes the sampling functions as given. Just as the rationing function, the sampling functions are endogenous objects determined by the market clearing protocol and the choices of agents as described next.

A.2 Market Clearing Protocol

Let $r^{\max}(\tau, \omega)$ be the maximum interest rate at which $\sigma(r, \tau, \omega)$ is positive for some given type of entrepreneur. If $\max_{(\tau,\omega)} r^{\max}(\tau, \omega) \leq \min_h \tilde{r}(h)$, then no applications are financed. Otherwise the market clearing process starts from the smallest advertised interest rate $\tilde{r}(h)$, say, r'. First, each entrepreneurs submitting applications at that rate has to post r' per unit of application from her endowment which we refer to as down-payment. Applications without a down-payment are automatically discarded. Then, each unskilled investor advertising rate r' runs his chosen test and grant credit for investment for all applications which pass the testin the sample he receives. He obtains a sample of the (non-discarded) applications submitted at that rate. The applications are allocated pro rata. That is, the proportion of each type (τ, ω) in each sample is identical to their proportion in the (non-discarded) application pool at that interest rate. If there are not enough applications to fill up every present investor's capacity limit, then all applications have been sampled and the sampling process stops. Otherwise, all unskilled investors received a sample up to their sampling capacity. Their remaining endowment is invested in the risk-free asset. Entrepreneurs have to invest the credit amount along with the down-payment and the invested units are posted as collateral for the loan. These invested units enter into a public registry, so they cannot serve as collateral to other loan applications. Applications which enter into a sample, but do not pass the test are discarded, and the down-payment is returned to the entrepreneur who can invest it in the risk free asset.

If all unskilled investors reached their sampling capacity and there are remaining good projects, then they are distributed pro rata across skilled investors up to their capacity given by their endowment. Skilled investors grant credit to these projects and the investment and colletarilization process are as before. Any remaining applications are discarded and the down-payment is returned. Then, the process is repeated at the next lowest advertised interest rate, etc. If there are no more advertised rates, or there are no more applications for which $r^{\max}(\tau, \omega)$ is larger than a remaining advertised rate, the process stops. This process pins down the objects η, S_u, S_s.

Note that (4.1) is a compact version of the entrepreneur's problem. Instead of considering the choice over the optimal credit application strategy, $\{\sigma(r; \tau, \omega)\}_{r \in R}$ for a given rationing function, we define the upper limit of obtainable credit as

$$\bar{\ell}(\tau, \omega) \equiv \max_{\{\sigma(r;\tau,\omega)\}_{r\in R}} \int_R \sigma(r; \tau, \omega) d\eta\,(r; \tau, \omega)\,.$$

Clearly, if an entrepreneur can obtain a given level of credit, she can also obtain less. Therefore, the choice of ℓ and i in (4.1), will be the same as ℓ and i obtained in the entrepreneur's full problem.

A.3 Robustness

Following Kurlat (2016), we make a robustness assumption implying tie-braking rules and avoiding multiple equilibria.

Assumption A.1 *Suppose that ε fraction of applications submitted at an advertised interest rate are granted unconditionally. We require that the equilibrium strategy of each entrepreneur is the limit of equilibrium strategies as ε goes to 0.*

Apart from avoiding multiplicity of equilibria, as we will see, an additional consequence of this assumption is that each type who chooses to submit loan applications at a given interest rate submits the same, maximal amount That is, $\sigma(r;\tau,\omega) > 0$ implies $\sigma(r;\tau,\omega) = \frac{1}{r}$. This feature has the convenient consequence that the application pool at any given interest rate is independent of cross-sectional distribution of choices $i(\tau,\omega)$, therefore, we can solve the credit market equilibrium independently of choices $i(\tau,\omega)$. This simplifies the analysis considerably.

B Appendix: Continuum of Tests

Assume there is a continuum of tests, indexed by $s \in [0,1]$. Every test s passes all $\frac{1-\mu_0-\mu_1}{2}$ transparent good projects and rejects all μ_1 transparent bad projects. Furthermore, test s passes s fraction of the opaque projects, i.e. $s\frac{1-\mu_0-\mu_1}{2}$ good projects and $s\mu_0$ bad opaque projects. Thus, $s=0$ corresponds to the cautious test, and $s=1$ corresponds to the bold test. Tests with $s \in (0,1)$ cover everything in between. We follow the logic as in proof of Proposition 3.1 to show that both the bold and the cautious equilibrium are robust to this modification. In particular, investors strictly prefer to choose the bold test when $\mu_0 < \frac{c}{1+r_f}$ and the cautious test when $\mu_0 > \frac{c}{1+r_f}$ even if the intermediate choices are also available.

Recall that the unskilled investors choose a test which allows them to advertise the lowest break even interest rate under the conjecture that at that interest rate all types will submit an application. If that were not true, unskilled investors not entering in equilibrium could choose a test and advertise an interest rate which leads to higher profit than staying outside. (We rely here on Lemma 3.1 (i) ensuring that if an entrepreneurs applies for a given rate in equilibrium, he also applies for all lower rates, advertised or not.) The break even interest rate for any test characterized by s is

$$\left(\frac{1-\mu_0-\mu_1}{2}+s\frac{1-\mu_0-\mu_1}{2}\right)(1+r(s)) \\ +\left(\mu_1+(1-s)\mu_0+(1-s)\frac{1-\mu_0-\mu_1}{2}\right)(1+r_f)-c=1+r_f,$$

which in turn implies

$$\frac{(1+r_f)\left(1-\left(\mu_1+(1-s)\,\mu_0+(1-s)\,\frac{1-\mu_0-\mu_1}{2}\right)\right)+c}{\left(\frac{1-\mu_0-\mu_1}{2}+s\frac{1-\mu_0-\mu_1}{2}\right)}-1=r\,(s)\,.$$

Note that

$$\frac{\partial r(s)}{\partial s}=-2\frac{c-\mu_0-\mu_0 r_f}{(s+1)^2\,(1-\mu_0-\mu_1)},$$

implying that whenever $\mu_0<\frac{c}{(1+r_f)}$, the smallest interest rate is implied by the test $s=1$, while in the opposite case it is $s=0$. Thus, by the same argument as in the main text, if $\mu_0<\frac{c}{(1+r_f)}$, the equilibrium advertised interest rate by unskilled investors corresponds to the test $s=1$ (bold test), and in the opposite case they choose $s=0$ (cautious test). In this sense, the continuum of intermediate tests are always dominated by either the bold or the cautious test, and restricting investor choice to these two tests is without loss of generality.

C Appendix: Proofs

Proof of Lemma 3.1

The market clearing mechanism and Assumption A.1 implies that if any agent would like to raise credit at an interest rate $r^{\max}$, she would want to submit a maximum measure of applications, $\sigma\,(r;\tau,\omega)=\frac{1}{r}$ at every interest rate smaller than $r^{\max}$ too. The reason is that it makes it possible that they are receiving a fraction of their credit at a lower rate (as markets clear from the lowest interest rate), and potentially even without the requirement to invest the received amount (Assumption A.1). This latter possibility is attractive for bad entrepreneurs. Because applications with no down-payment are discarded, there is no possibility of having more credit granted as intended. Agents also want to submit the maximum measure of applications at $r^{\max}$. Given the linear structure, if, at a given interest rate an agent would like borrow to invest, she also would like to borrow up to the limit $\frac{1}{r}$ and invest at that rate. This concludes the first part of the Lemma.

For the second part, recognize that good entrepreneurs would want to invest the maximum as long as their post repayment return is above what they would get from investing their own endowment. Using the collateral constraint this requires $\rho_g<(\rho_g-1)\frac{1+r}{r}$, which implies $r<\bar{r}=\rho_g-1$. For the last part, suppose that bad entrepreneurs can obtain a maximum of $\bar{\ell}$ loan at an interest rate not higher than their chosen reservation rate. Given the collateral constraint, to obtain ℓ loan, they have to invest a minimum of $i=\ell(1+r)$ into their project. Given $\rho_b<1$, they invest their remaining endowment into the risk-free rate.

That is, for any choice of $\ell < \bar{\ell}$, they can consume

$$(1+\ell-\ell(1+r))(1+r_f)+\rho_b\ell(1+r)$$

which is increasing in ℓ if $r \leq \bar{r}$ and assumption 2.1 holds.

Proof of Proposition 3.1

The main steps of the proof are explained in the text. Here, we just have to specify the details.

First, we show that if all entrepreneurs submit the maximum demand to an advertised rate r_B^p than bold, unskilled investors are indifferent to stay out or enter. The superscript refers to the fact that it is a pooled market where all entrepreneurs submit. In fact, r_B^p is defined by the indifference condition

$$(1-\mu_1-\mu_0)\left(1+r_B^p\right)+\mu_1\left(1+r_f\right)-c=1+r_f \tag{A.1}$$

Note that $((1-\mu_1-\mu_0)+\mu_0)\frac{(1-\mu_1-\mu_0)}{((1-\mu_1-\mu_0)+\mu_0)}=(1-\mu_1-\mu_0)$ is the probability of ending up financing a good project with a bold test (Pr(green signal) $\times$ Pr(good project|green signal)), while μ_1 is the probability that a entrepreneur in the sample will not pass the bold test, hence the investor invests in the risk-free asset instead. Therefore, the left hand side is the expected utility of running the bold test on a proportional sample of applications. Note that we are using the assumption that unskilled investors sample first.

Similarly, a cautious investor is indifferent to enter to a pooled market at interest rate r_C^p, which is defined as:

$$\frac{(1-\mu_1-\mu_0)}{2}\left(1+r_C^p\right)+\left(\frac{(1-\mu_1-\mu_0)}{2}+(\mu_1+\mu_0)\right)\left(1+r_f\right)-c=1+r_f \tag{A.2}$$

We claim that iff $r_B^p \leq r_C^p$ holds, r_B^p supports a bold equilibrium where the entering mass of unskilled investors is determined by the following market clearing condition. Given the fraction of bold investors' capital financing good projects, together with the capital of skilled investors (which all finance good projects) all good projects, opaque or transparent, have all their credit demand satisfied. (This market clearing condition is spelled out in the proof of Proposition 4.1). Then, following the intuition in the text, it is easy to check that no one has a profitable deviation: skilled or unskilled investors do not want to change their interest rate from r_B^p, and none of the entrepreneurs want to demand less than $\bar{L}$ at that rate. While, if the condition above did not hold, investors would be motivated to choose to be cautious advertising a rate $\tilde{r} \in (r_C^p, r_B^p)$.

Now consider a cautious equilibrium where all unskilled are cautious and advertise r_C^s. This implies that opaque good projects can be financed only by skilled investors. As skilled capital is scarce, they will advertise the maximum feasible rate $\bar{r}$. As unskilled capital is abundant, therefore r_C^s has to make cautious unskilled indifferent whether to enter. As all entrepreneurs demand credit at all advertised rate which is lower than their reservation rate, the pool of applicants in that low interest rate post is identical to the one in the pooled equilibrium at r_P^p. That is, r_C^s solves (A.2) and $r_C^s = r_C^p = r_C$ holds. If an unskilled investors is to deviate to a bold test, she has two options. She can advertise an interest rate $\tilde{r} \leq r_C^s$ attracting the pool of all type of entrepreneurs or it can advertise a high rate $\tilde{r} \in (r_C^s, \bar{r}]$ attracting all, but the transparent good ones. The earlier is a profitable deviation if and only if $r_B^s \leq r_C^s$ where r_B^s solves (A.1). That is, a necessary condition for a cautious equilibrium is $r_B = r_B^p > r_C$. The latter option is a profitable deviation if and only if $r_I \leq \bar{r}$ where r^I is determined by the indifference condition

$$\frac{\frac{(1-\mu_1-\mu_0)}{2}}{\frac{(1-\mu_1-\mu_0)}{2} + (\mu_1 + \mu_0)} (1 + r_I) + \frac{\mu_1}{\frac{(1-\mu_1-\mu_0)}{2} + (\mu_1 + \mu_0)} (1 + r_f) - c = (1 + r_f) .$$

Note that $r_I > r_B$ because it refers to an adversely selected pool of applicants. Checking that neither skilled investors nor any type of entrepreneurs want to deviate from the assigned strategies concludes the construction of the cautious equilibrium.

Finally, if $r_I < \bar{r}$ and $r_B > r_C$, then there is a mix equilibrium. In this case, skilled investors cannot offer $\bar{r}$ as they would be undercut by bold unskilled ones. Instead, skilled and bold unskilled investors advertise r_I. This high interest rate post is cleared similarly to the one at the bold equilibrium: the fraction of entering bold unskilled investors have to be sufficient to satisfy, together with skilled investors, all the credit demand of good opaque projects. At the same time, a group of unskilled investors choose to be cautious and advertise r_C to serve good transparent projects. Note that the two groups of unskilled investors make the same expected profit of $1 + r_f$ by the definition of r_I and r_C. Again, we can check that none of the agents prefer to deviate from the assigned strategies. Given that the conditions for each type of equilibria are mutually exclusive, we have uniqueness.

Observe that the static reasoning can be applied in each period of the dynamic set up, and express the equilibrium criteria in terms of μ_0.

Proof of Propositions 3.2 and 3.3

Since the switch between the two regimes is only a function of μ_0 in a two-stage economy, it is sufficient to compare μ_0 across different regimes. In other words, what determines whether the economy is in a boom or a recession is measure of bad entrepreneurs who are opaque.

Step 1. The first step is to find the single point steady states in the dynamic model, i.e. the measures that correspond to being in a cautious market, and end up in the same cautious market, and similarly for bold market. One can use the system of equations (3.7-3.4) to get these fixed points.

$$\bar{\mu}_{0B}(\delta,\lambda) = \frac{\lambda}{2-(1-\delta)\lambda} \tag{A.3}$$

$$\bar{\mu}_{1B}(\delta,\lambda) = \frac{\delta\lambda}{2-(1-\delta)\lambda} \tag{A.4}$$

$$\bar{\mu}_{0C}(\delta,\lambda) = \frac{\delta\lambda}{2((1-\delta)(1-\lambda)+\delta)} \tag{A.5}$$

$$\bar{\mu}_{1C}(\delta,\lambda) = \frac{\delta\lambda}{2((1-\delta)(1-\lambda)+\delta)} \tag{A.6}$$

Note that $\bar{\mu}_{0C} = \bar{\mu}_{1C}$, and $\bar{\mu}_{0B} > \bar{\mu}_{0C}$. Since $\bar{\mu}_0$ is exogenous to these steady states (using c and r_t, we will focus on the case where $\bar{\mu}_0$ is in the middle, i.e. $\bar{\mu}_{0B} > m\bar{u}_0 > \bar{\mu}_{0C}$.

Step 2. Two point oscillating distribution. Using the law of motions (3.7-3.4) we get the two point oscillating distribution by conjecturing that if the lower of those two points, μ^*_{0B} corresponds to a boom, then the implied next period value is μ^*_{0C} and it corresponds to a recession. Two two points has to be such that given the recession, the implied next period value is μ^*_{0B} again. The implied points are

$$\mu^*_{0B}(\delta,\lambda) = \frac{\delta\lambda((\delta-1)\lambda-1)}{(\delta-1)^2\lambda(\lambda+1)-2} \tag{A.7}$$

$$\mu^*_{1B}(\delta,\lambda) = \frac{\delta\lambda((\delta-1)\lambda-1)}{(\delta-1)^2\lambda(\lambda+1)-2} \tag{A.8}$$

$$\mu^*_{0C}(\delta,\lambda) = -\frac{(\delta-2)\delta\lambda((\delta-1)\lambda-2)}{2(\delta-1)^2\lambda(\lambda+1)-4} \tag{A.9}$$

$$\mu^*_{1C}(\delta,\lambda) = \frac{\delta\lambda((\delta-1)\delta\lambda-2)}{2(\delta-1)^2\lambda(\lambda+1)-4}. \tag{A.10}$$

It is clear that $\mu^*_{0C} > \mu^*_{0B}$. Thus, the statement follows.

Proof of Proposition 4.1

We described in the main text how entrepreneurs' decide on investment i and borrowing ℓ taking the interest rate $r(\tau,\omega)$ and the borrowing limit $\bar{\ell}(\tau,\omega)$ as given. Then, expressions in Proposition 4.1 follow from the determination of $r(\tau,\omega)$ in Proposition 3.1 and the borrowing limits $\bar{\ell}(\tau,\omega)$ which we derive here. We also derive here $k(\mu_0,\mu_1)$, the equilibrium fraction of unskilled investors who decide to not to enter the credit market in a given state. Consider the bold stage first. The market clearing condition for credit to good transparent and opaque entrepreneurs is

$$w_1 + (1 - b_P)\, w_0\, (1 - \mu_0 - \mu_1) = (1 - \mu_0 - \mu_1)\, \frac{1}{r_B}$$

where $k(\mu_0, \mu_1) = k_B$ in a bold stage. Then, $\bar{\ell}(b, 0)$ is determined by the endowment of unskilled investors which is allocated to bad, opaque credit by mistake:

$$\mu_0 \bar{\ell}\,(b, 0) = (1 - k_B)\, w_0 \mu_0$$

implying

$$\bar{\ell}\,(b, 0) = \frac{1}{r_B} - \frac{w_1}{(1 - \mu_0 - \mu_1)}$$

and

$$i\,(b, 0) = \bar{\ell}\,(b, 0)\,(1 + r_B) = \frac{(1 + r_B)}{r_B} - \frac{(1 + r_B)\, w_1}{(1 - \mu_0 - \mu_1)}.$$

Assumption 2.2 requires $\frac{w_1}{(1-\mu_0-\mu_1)} < \frac{1}{r_B}$, validating that bold entrepreneurs are indeed constrained.

In the cautious stage market clearing for opaque good firms gives

$$\frac{(1 - \mu_0 - \mu_1)}{2} \bar{\ell}(g, 0) = w_1$$

implying

$$\bar{\ell}(g, 0) = \frac{2 w_1}{(1 - \mu_0 - \mu_1)}$$

and investment

$$i\,(g, 0) = 1 + \frac{2 w_1}{(1 - \mu_0 - \mu_1)}.$$

Assumption 2.2 requires $\frac{w_1}{(1-\mu_0-\mu_1)} < \frac{1}{2\bar{r}}$ implying that good opaque entrepreneurs are indeed constrained in this stage. The fraction of entering unskilled investors in a cautious stage, $(1 - k_C)$, is determined by the market clearing condition for the low interest rate market,

$$\frac{(1 - \mu_0 - \mu_1)}{2} \frac{1}{r_C} = (1 - k_C) w_0 \frac{(1 - \mu_0 - \mu_1)}{2}.$$

Turning to the mix stage recall from the proof of Proposition 3.1 that $\frac{1-\mu_0-\mu_1}{\mu_0+\mu_1+1}$ fraction of invested unskilled capital finances good, opaque projects at the high interest rate market, $2\frac{\mu_0}{\mu_0+\mu_1+1}$ finances bad opaque projects and $2\frac{\mu_1}{\mu_0+\mu_1+1}$ ends up at risk-free storage. Then market clearing for good opaque firms then is

$$\frac{(1-\mu_1-\mu_0)}{2}\ell(g,0) = (1-k_I)\,w_0\frac{(1-\mu_1-\mu_0)}{1+(\mu_1+\mu_0)} + w_1$$

as good opaque entrepreneurs are not constrained, this implies

$$\frac{1}{2}\frac{1}{r_I} - \frac{w_1}{1-\mu_0-\mu_1} = (1-k_I)\,w_0\frac{1}{1+(\mu_1+\mu_0)}$$

Then market clearing for bad, opaque entrepreneurs gives

$$\mu_0\bar{\ell}(b,0) = (1-b_I)\,w_0 2\frac{\mu_0}{\mu_0+\mu_1+1}.$$

Substituting back $(1-b_I)$ implies

$$\ell(\bar{b},0) = \left(\frac{1}{2}\frac{1}{r_I} - \frac{w_1}{1-\mu_0-\mu_1}\right)$$

and

$$i\,(b,0) = (1+r_I)\left(\frac{1}{2}\frac{1}{r_I} - \frac{w_1}{1-\mu_0-\mu_1}\right).$$

Assumption 2.2 requires $\frac{w_1}{(1-\mu_0-\mu_1)} < \frac{1}{2r_I}$. Also, w_0 has to be sufficiently large that $k_I, k_B, k_C \in [0,1]$. We can summarize the requirements on w_1 for later use as:

$$\frac{w_1}{(1-\mu_0-\mu_1)} < \min\left(\frac{1}{2\bar{r}}, \frac{1}{2r_I}, \frac{1}{r_B}\right) = \frac{1}{2\bar{r}}. \tag{A.11}$$

Proof of Proposition 5.3

Clearly, a risk weight of $x > 1$ does not influence the interest rate in a cautious stage as investors are lending to projects which they all pay back.

In a bold stage, we require

$$v_g x + v_r = 1$$

but still assume that the technology of a bold test did not change implying

$$\frac{v_g}{v_g+v_r} = (1-\mu_1)\,.$$

Therefore,

$$v_g = \frac{1-\mu_1}{x(1-\mu_1)+\mu_1}, v_r = \frac{\mu_1}{x(1-\mu_1)+\mu_1}$$

which modifies the indifference condition determining the zero profit rate r_B^x as follows

$$\frac{1-\mu_1}{x(1-\mu_1)+\mu_1}\left(1+r_B^x\right)\frac{(1-\mu_1-\mu_0)}{1-\mu_1} + \frac{\mu_1}{x(1-\mu_1)+\mu_1}\left(1+r_f\right) - c = 1+r_f$$

implying the expression for r_B^x in the proposition.

In the mix stage, the bold test on the high interest rate market (at which transparent good entrepreneurs do not apply for credit) implies

$$\frac{v_g}{v_g+v_r} = \frac{\frac{(1-\mu_1-\mu_0)}{2}+\mu_0}{\frac{(1-\mu_1-\mu_0)}{2}+(\mu_1+\mu_0)}.$$

Therefore

$$v_g = \frac{\mu_0-\mu_1+1}{x+2\mu_1+x\mu_0-x\mu_1}, v_r = 2\frac{\mu_1}{x+2\mu_1+x\mu_0-x\mu_1}$$

in the mix stage. This implies that the indifference condition determining the zero profit rate r_I^x is modified as follows:

$$\frac{1-\mu_0-\mu_1}{x+2\mu_1+x\mu_0-x\mu_1}\left(1+r_I^x\right) + 2\frac{\mu_1}{x+2\mu_1+x\mu_0-x\mu_1}\left(1+r_f\right) - c = 1+r_f$$

which gives the expression of r_I^x in the proposition. Finally, by analogous arguments to the baseline case, the threshold between the bold and cautiuos stages is given by identity

$$r_B^x\left(\bar{\mu}_0^x\left(\mu_1,c,r_f\right),\mu_1,c,r_f,x\right) \equiv r_C\left(\bar{\mu}_0^x\left(\mu_1,c,r_f\right),\mu_1,c,r_f\right)$$

while the threshold $\tilde{\mu}^x(\cdot)$ is given by identity

$$r_I^x\left(\tilde{\mu}_0^x\left(\mu_1,\rho_g,c,r_f\right),\mu_1,c,r_f,x\right) \equiv \rho_g - 1.$$

Proof of Propositions 4.2 and 5.1

Proposition 4.2 follows as described in the text. Proposition 5.1 follows from the following three Lemmas.

Lemma C.1 *Within the pooling region, welfare is decreasing in* μ_0. $\frac{\partial W_B(\mu_0,\mu_1)}{\partial \mu_0} < 0$

Proof. Total consumption is

$$\begin{aligned}
W_B(\mu_0,\mu_1) \equiv{} & (1-\mu_0-\mu_1)(\rho_g-1)\left(\frac{1}{r_B}+1\right)+ \\
& +\mu_0\left((1+r_f)+(\rho_b(1+r_B)-r_B((1+r_f)))\left(\frac{1}{r_B}-\frac{w_1}{(1-\mu_0-\mu_1)}\right)\right) \\
& +\mu_1(1+r_f)+(1+r_f)w_0+w_1(1+r_B)
\end{aligned}$$

where the first line corresponds to good entrepreneurs, the second line corresponds to bad, opaque entrepreneurs, while the last line corresponds to investors and bad, transparent entrepreneurs. We rewrite this expression as

$$\begin{aligned}
W_B(\mu_0,\mu_1) ={} & w_1\rho_g+w_0(1+r_f)+\rho_g-\mu_1(\rho_g-(1+r_f)) \\
& +X\left(Y-(1+r_f)-\frac{c}{1-\mu_1}\right) \\
& +\mu_0\left(w_1 r_B\frac{(1+r_f-\rho_b)}{1-\mu_0-\mu_1}-(\rho_g-\rho_b)\right)
\end{aligned} \tag{A.12}$$

where

$$X\equiv\mu_0\left(\frac{1}{r_B}-\frac{w_1}{1-\mu_0-\mu_1}\right)+(1-\mu_0-\mu_1)\frac{1}{r_B}-w_1=(1-\mu_1)\left(\frac{1}{r_B}-\frac{w_1}{1-\mu_0-\mu_1}\right)$$

is the total investment of unskilled capital to risky projects, while

$$Y\equiv\frac{\rho_g((1-\mu_0-\mu_1)\frac{1}{r_B}-w_1)+\rho_b\mu_0(\frac{1}{r_B}-\frac{w_1}{1-\mu_0-\mu_1})}{(1-\mu_1)(\frac{1}{r_B}-\frac{w_1}{1-\mu_0-\mu_1})}=\frac{\rho_g(1-\mu_0-\mu_1)+\rho_b\mu_0}{(1-\mu_1)}$$

is the per unit return on this investment. Note that $X>0$, and

$$Y>(1+r_f)+\frac{c}{1-\mu_1}$$

because it is equivalent to

$$\rho_g(1-\mu_0-\mu_1)+\rho_b\mu_0>(1+r_B)(1-\mu_0-\mu_1)$$

and

$$(1+r_B)\leq 1+\bar{r}=\rho_g.$$

Therefore, as $\frac{\partial X}{\partial\mu_0}$ and $\frac{\partial Y}{\partial\mu_0}$ are clearly negative, and the first line of (A.12) is independent of

μ_0, we only need to show that the derivative of the term in the third line of (A.12),

$$\frac{\partial\left(-\mu_0\left(\rho_g-\rho_b-w_1 r_B\frac{(1+r_f-\rho_b)}{1-\mu_0-\mu_1}\right)\right)}{\partial\mu_0}=-\left(\rho_g-\rho_b\right)+w_1 r_B\frac{(1+r_f-\rho_b)}{1-\mu_0-\mu_1}\frac{1-\mu_1}{1-\mu_0-\mu_1}$$

is negative. But this is always true in a pooling region as

$$\begin{aligned}
&-\left(\rho_g-\rho_b\right)+w_1 r_B\frac{(1+r_f-\rho_b)}{1-\mu_0-\mu_1}\frac{1-\mu_1}{1-\mu_0-\mu_1}\leq-\left(\rho_g-\rho_b\right)+\frac{(1+r_f-\rho_b)}{1-\mu_0-\mu_1}\left(1-\mu_1\right)\\
&\leq\rho_g\left(-\left(\frac{\rho_g-\rho_b}{\rho_g}\right)+\frac{(1+r_f-\rho_b)}{\left(\left(1-\mu_1\right)\left(1+r_f\right)+c\right)}\left(1-\mu_1\right)\right)<\\
&<\rho_g\left(-\left(\frac{1+r_f-\rho_b}{1+r_f}\right)+\frac{(1+r_f-\rho_b)}{\left(\left(1-\mu_1\right)\left(1+r_f\right)+c\right)}\left(1-\mu_1\right)\right)=\\
&\qquad=\rho_g(1+r_f-\rho_b)\left(-\frac{1}{1+r_f}+\frac{\left(1-\mu_1\right)}{\left(\left(1-\mu_1\right)\left(1+r_f\right)+c\right)}\right)<0,
\end{aligned}$$

where the first inequality comes from (A.11), the second one comes form

$$\left(1+r_B\right)=\frac{\left(\left(1-\mu_1\right)\left(1+r_f\right)+c\right)}{1-\mu_0-\mu_1}\leq\rho_g$$

while the third one comes from $\rho_g>1+r_f$. ■

Lemma C.2 *Within the separating region, welfare is decreasing in μ_0, .$\frac{\partial W_C(\mu_0,\mu_1)}{\partial\mu_0}<0$*

Proof. Total consumption is

$$\begin{aligned}
W_C\left(\mu_0,\mu_1\right)\equiv\frac{\left(1-\mu_0-\mu_1\right)}{2}\left(\rho_g-1\right)\left(\frac{1}{r_C}+1\right)+\frac{\left(1-\mu_0-\mu_1\right)}{2}\rho_g+\\
+\left(\mu_0+\mu_1\right)\left(1+r_f\right)+\left(1+r_f\right)w_0+w_1\left(1+\bar{r}\right)
\end{aligned}$$

where,. for the second term, we used that facing $\bar{r}$, opaque good firms obtain the same consumption with any choice of ℓ. The implied slope in μ_0 of

$$\frac{\partial W_C\left(\mu_0,\mu_1\right)}{\partial\mu_0}=-\left(\left(\rho_g-1\right)\frac{\left(1-\mu_0-\mu_1\right)}{2}\frac{2c+\frac{(1-\mu_0-\mu_1)}{2}r_f}{\left(c+\frac{(1-\mu_0-\mu_1)}{2}r_f\right)^2}+2\rho_g-1\right)\frac{1}{2}+\left(1+r_f\right) \tag{A.13}$$

The following inequalities proof the statement:

$$
\begin{aligned}
&-\left(\left(\rho_g-1\right)\frac{\left(1-\mu_0-\mu_1\right)}{2}\frac{2c+\frac{(1-\mu_0-\mu_1)}{2}r_f}{\left(c+\frac{(1-\mu_0-\mu_1)}{2}r_f\right)^2}+2\rho_g-1\right)\frac{1}{2}+\left(1+r_f\right)\leq\\
&\leq-\left(\left(\rho_g-1\right)\frac{c}{\rho_g-1-r_f}\frac{2c+\frac{c}{\rho_g-1-r_f}r_f}{\left(c+\frac{c}{\rho_g-1-r_f}r_f\right)^2}+2\rho_g-1\right)\frac{1}{2}+\left(1+r_f\right)=\\
&\quad=-\left(\frac{2\rho_g-r_f-2}{\rho_g-1}+2\rho_g-1\right)\frac{1}{2}+\left(1+r_f\right)=-\frac{1}{2}\left(2\rho_g-1\right)\frac{\rho_g-\left(r_f+1\right)}{\rho_g-1}<0.
\end{aligned}
$$

where, for the first inequality, we used that (A.13) is decreasing in $\frac{(1-\mu_0-\mu_1)}{2}$ and that

$$
\frac{2}{1-\mu_1-\mu_0}c=r_C-r_f\leq\bar{r}-r_f=\rho_g-\left(1+r_f\right).
$$

■

Lemma C.3 *Let us fix μ_1 and μ_0 at any level $\mu_0\leq\frac{c}{1+r_f}$. Under the pooling equilibrium welfare is strictly larger than under the separating equilibrium, $W_B\left(\mu_0,\mu_1\right)-W_C\left(\mu_0,\mu_1\right)$ as long as $\mu_0\leq\frac{c}{1+r_f}$.*

Proof. The difference in transparent good consumption is

$$
\frac{\left(1-\mu_0-\mu_1\right)}{2}\left(\rho_g-1\right)\left(\frac{1}{r_B}+1\right)-\frac{\left(1-\mu_0-\mu_1\right)}{2}\left(\rho_g-1\right)\left(\frac{1}{r_C}+1\right)
$$

which is nonnegative in any point when $r_B\leq r_C$, that is, in the pooling region. The difference in opaque good plus skilled consumption is

$$
\left[\frac{\left(1-\mu_0-\mu_1\right)}{2}\left(\rho_g-1\right)\left(\frac{1}{r_B}+1\right)+w_1\left(1+r_B\right)\right]-\left[\frac{\left(1-\mu_0-\mu_1\right)}{2}\rho_g+w_1\left(1+\bar{r}\right)\right] \tag{A.14}
$$

note that the term in the first squared bracket is decreasing in r_B as

$$
\begin{aligned}
&\frac{\partial\left(\frac{(1-\mu_0-\mu_1)}{2}\left(\rho_g-1\right)\left(\frac{1}{r_B}+1\right)+w_1\left(1+r_B\right)\right)}{\partial r_B}=\\
&=-\frac{1}{r_B^2}\frac{\left(1-\mu_0-\mu_1\right)}{2}\left(\rho_g-1\right)+w_1\leq-\frac{1}{r_B^2}\frac{\left(1-\mu_0-\mu_1\right)}{2}\left(\rho_g-1\right)+\frac{1-\mu_0-\mu_1}{r_B}=\\
&\qquad\qquad=\frac{\left(1-\mu_0-\mu_1\right)}{r_B}\left(1-\frac{\rho_g-1}{r_B}\right)<0
\end{aligned}
$$

where we used (A.11), and equals to the term in the second left bracket when $r_B = \bar{r}$. That is, (A.14) is non-negative at any point as long as $r_B \leq \bar{r}$. Unskilled consumption is equal under the two regimes, while the difference in bad consumption is

$$\mu_0 \left(\left(\rho_b \left(1 + r_B\right) - r_B \left(\left(1 + r_f\right)\right)\right) \left(\frac{1}{r_B} - \frac{w_1}{\left(1 - \mu_0 - \mu_1\right)} \right) \right)$$

which is positive by (2.1). ■

Proof of Proposition 5.2

We will show that under the conditions of the proposition, there is at least one cyclical economy (the one with short-booms and short recessions) which is preferred by the planner compared to both the always bold and always cautious economies. We will argue that for this conclusion, it is sufficient to show that $\rho_g - 1 > \rho_b$ and $\lambda \in [\lambda_{\min}, 1 - \frac{r_f + c + 1}{2\rho_g - \left(c + r_f + 1\right)}]$ implies

$$\max(\lim_{\delta \to 0} W_C\left(\bar{\mu}_{0C}, \bar{\mu}_{1C}\right), \lim_{\delta \to 0} W_B\left(\bar{\mu}_{0B}, \bar{\mu}_{1B}\right)) < \lim_{\delta \to 0} \frac{W_B\left(\mu^*_{0B}, \mu^*_{1B}\right) + W_C\left(\mu^*_{0C}, \mu^*_{1C}\right)}{2}.$$

Note that $\lim_{\delta \to 0} \bar{\mu}_{0B} = \frac{\lambda}{2-\lambda}$ and

$$\lim_{\delta \to 0} \bar{\mu}_{1B}, \bar{\mu}_{1C}, \mu^*_{1C}, \mu^*_{1B}, \bar{\mu}_{0C}, \mu^*_{0C}, \mu^*_{0B} = 0.$$

Note that $\lambda^{\max} \equiv 1 - \frac{r_f + c + 1}{2\rho_g - \left(c + r_f + 1\right)}$ is the solution of

$$r_B \left(\frac{\lambda^{\max}}{2 - \lambda^{\max}}, 0, c, r_f \right) = \bar{r}.$$

In an economy where investors are always bold or always cautious, welfare converges to $W_B\left(\mu^*_{0B}, \mu^*_{1B}\right)$ and $W_C\left(\bar{\mu}_{0C}, \bar{\mu}^*_{1C}\right)$ by definition. First, note that

$$\lim_{\delta \to 0} W_C\left(\bar{\mu}_{0C}, \bar{\mu}_{1C}\right) = W_C\left(0, 0\right) < \lim_{\delta \to 0} \frac{W_B\left(\mu^*_{0B}, \mu^*_{1B}\right) + W_C\left(\mu^*_{0C}, \mu^*_{1C}\right)}{2} = \frac{W_B\left(0, 0\right) + W_C\left(0, 0\right)}{2}.$$

This is implied by Lemma C.3. Then, we show that $\rho_g - 1 > \rho_b$ is a sufficient condition that if $\lambda = \lambda^{\max}$ then

$$\lim_{\delta \to 0} W_C\left(\mu^*_{0C}, \mu^*_{1C}\right) > \lim_{\delta \to 0} W_B\left(\bar{\mu}_{0B}, \bar{\mu}_{1B}\right). \tag{A.15}$$

At that point $r_B = \bar{r}$ implying that (A.15) is equivalent to

$$W_C(0,0) > W_B\left(\frac{\lambda^{\max}}{2-\lambda^{\max}}, 0\right)$$

which we can rewrite as

$$\frac{1}{2}(\rho_g - 1)\left(\frac{1}{r_C}+1\right) + \frac{1}{2}\rho_g >$$
$$> (1-\mu_0)\rho_g + \mu_0\left((1+r_f) + (\rho_b(1+\bar{r}) - \bar{r}((1+r_f)))\left(\frac{1}{\bar{r}} - \frac{w_1}{(1-\mu_0-\mu_1)}\right)\right).$$

Note that the left hand side is the weighted average of ρ_g and $(\rho_g - 1)\left(\frac{1}{r_f+2c}+1\right)$ and also

$$(\rho_g - 1)\left(\frac{1}{r_C}+1\right) > (\rho_g - 1)\left(\frac{1}{\rho_g - 1}+1\right) = \rho_g.$$

Therefore it is sufficient to show that

$$\rho_g > \frac{\rho_b(1+\bar{r})}{\bar{r}} - \frac{(\rho_b(1+\bar{r}) - (1+r_f)\bar{r})w_1}{(1-\mu_0-\mu_1)}$$

which, given (2.1), holds as long as $\rho_g - 1 > \rho_b$. Therefore, we can conclude that

$$\lim_{\delta\to 0}\frac{W_B(\mu_{0B}^*, \mu_{1B}^*) + W_C(\mu_{0C}^*, \mu_{1C}^*)}{2} > \lim_{\delta\to 0} W_C(\bar{\mu}_{0C}, \bar{\mu}_{1C}) > \lim_{\delta\to 0} W_B(\bar{\mu}_{0B}, \bar{\mu}_{1B})$$

at $\lambda = \lambda^{\max}$ and $\rho_g - 1 > \rho_b$. As all inequalities are strict and all relevant functions are continuous from the left in (μ_0, μ_1), for any $\lambda < \lambda^{\max}$ sufficiently close to $\lambda^{\max}$ we can pick a $\bar{\delta}(\lambda)$ that if $\delta < \bar{\delta}(\lambda)$ then our statement holds. Picking the smallest such λ defines $\lambda_{\min}$ of the proposition and picking

$$\bar{\delta} = \max_{\lambda\in[\lambda_{\min},\lambda_{\max}]} \bar{\delta}(\lambda)$$

defines the threshold for δ.

Proof of Proposition 5.4

Bold States. Consider large integer $N \to \infty$. Let x^N defined by $\hat{\mu}_0\left(x^N, r_f^0, \cdot\right) = \hat{\mu}_0^N$. Also define

$$\Delta x = \frac{x^N - x^0}{N}$$

and the series

$$x^n = x^0 + n\Delta x, \forall n < N.$$

We also define a corresponding series $r^1, r^2, \cdots, r^n$ by

$$\left(x^n - x^{n-1}\right) \frac{\partial \hat{\mu}_0\left(x, r_f, \cdot\right)}{\partial x}|_{x=x^{n-1}, r_f=r_f^0} \equiv \left(r^n - r^{n-1}\right) \frac{\partial \hat{\mu}_0\left(x, r_f, \cdot\right)}{\partial r_f}|_{x=x^0, r_f=r_f^{n-1}}. \quad \text{(A.16)}$$

Rewrite (A.16) as

$$\left(r^n - r^{n-1}\right) = \left(x^n - x^{n-1}\right) \frac{\frac{\partial \hat{\mu}_0\left(x, r_f, \cdot\right)}{\partial x}|_{x=x^{n-1}, r_f=r_f^0}}{\frac{\partial \hat{\mu}_0\left(x, r_f, \cdot\right)}{\partial r_f}|_{x=x^0, r_f=r_f^{n-1}}} = \left(x^n - x^{n-1}\right) \frac{c\frac{1-\mu_1}{r_f^0+1}}{c\frac{1-(1-\mu_1)(x^0-1)}{\left(r_f^{n-1}+1\right)^2}}. \quad \text{(A.17)}$$

We will work under the assumption that $1-(1-\mu_1)(x^0-1) > 1$, thus the ratio on the right hand side of the above equation is a constant bounded away form infinity. As a result, as $N \to \infty$ and $x^n - x^{n-1} \to 0$, $r^n - r^{n-1} \to 0$ as well. As such, using Reimann summation we have:

$$\hat{\mu}_0^N \approx \hat{\mu}_0^0 + \sum_{i=1}^{N} \left(x^n - x^{n-1}\right) \frac{\partial \hat{\mu}_0\left(x, r_f, \cdot\right)}{\partial x}|_{x=x^{n-1}, r_f=r_f^0} = \hat{\mu}_0^0 + \sum_{i=1}^{N} \left(r^n - r^{n-1}\right) \frac{\partial \hat{\mu}_0\left(x, r_f, \cdot\right)}{\partial r_f}|_{x=x^0, r_f=r_f^{n-1}}$$

We want to show that everything else equal, with the same state variable μ_0, the bold interest rate under the macro prudential policy, (x^N, r_f^0), is smaller than the bold interest rate under the monetary policy, (x^0, r_f^N). As

$$r_B\left(x^N, r_f^0, \cdot\right) \approx r_B\left(x^0, r_f^0, \cdot\right) + \sum_{i=1}^{N} \left(x^n - x^{n-1}\right) \frac{\partial r_B\left(x, r_f, \cdot\right)}{\partial x}|_{x=x^{n-1}, r_f=r_f^0}$$

and

$$r_B\left(x^0, r_f^N, \cdot\right) \approx r_B\left(x^0, r_f^0, \cdot\right) + \sum_{i=1}^{N} \left(r^n - r^{n-1}\right) \frac{\partial r_B\left(x, r_f, \cdot\right)}{\partial r_f}|_{x=x^0, r_f=r_f^{n-1}}$$

and both approximations are arbitrarily precise for large N, it is sufficient to show that

$$\sum_{i=1}^{N} \left(x^n - x^{n-1}\right) \frac{\partial r_B\left(x, r_f, \cdot\right)}{\partial x}|_{x=x^{n-1}, r_f=r_f^0} < \sum_{i=1}^{N} \left(r^n - r^{n-1}\right) \frac{\partial r_B\left(x, r_f, \cdot\right)}{\partial r_f}|_{x=x^0, r_f=r_f^{n-1}}.$$

Using (A.17), this is equivalent with the following inequalities:

$$\sum_{i=1}^{N}\left(x^{n}-x^{n-1}\right)\frac{\partial r_{B}\left(x,r_{f},\cdot\right)}{\partial x}|_{x=x^{n-1},r_{f}=r_{f}^{0}}<\sum_{i=1}^{N}\left(x^{n}-x^{n-1}\right)\frac{c\frac{1-\mu_{1}}{r_{f}^{0}+1}}{c\frac{1-(1-\mu_{1})(x^{0}-1)}{\left(r_{f}^{n-1}+1\right)^{2}}}\frac{\partial r_{B}\left(x,r_{f},\cdot\right)}{\partial r_{f}}|_{x=x^{0},r_{f}=r_{f}^{n-1}}$$

$$\Delta x\sum_{i=1}^{N}c\frac{1-\mu_{1}}{1-\mu_{0}-\mu_{1}}<\sum_{i=1}^{N}\Delta x\frac{c\frac{1-\mu_{1}}{r_{f}^{0}+1}}{c\frac{1-(1-\mu_{1})(x^{0}-1)}{\left(r_{f}^{n-1}+1\right)^{2}}}\frac{1-\mu_{1}}{1-\mu_{0}-\mu_{1}}$$

$$\Delta xNc<\Delta x\frac{(1-\mu_{1})}{1-(1-\mu_{1})\left(x^{0}-1\right)}\sum_{i=1}^{N}\frac{\left(r_{f}^{n-1}+1\right)^{2}}{r_{f}^{0}+1}$$

$$\Delta xN<\Delta x\frac{(1-\mu_{1})}{\frac{c}{1+r_{f}^{0}}\left(1-(1-\mu_{1})\left(x^{0}-1\right)\right)}\sum_{i=1}^{N}\frac{\left(r_{f}^{n-1}+1\right)^{2}}{\left(r_{f}^{0}+1\right)^{2}}$$

$$\Delta xN<\Delta x\frac{1-\mu_{1}}{\hat{\mu}_{0}\left(x^{0},r_{f}^{0-}\right)}\sum_{i=1}^{N}\frac{\left(r_{f}^{n-1}+1\right)^{2}}{\left(r_{f}^{0}+1\right)^{2}}.$$

But this should hold as long as $x^{N}>x^{0}$, so $x^{1},..x^{n}$ and $r_{f}^{1},...r_{f}^{n}$ are both increasing series as $1-\mu_{1}>\bar{\mu}_{0}\left(x^{0},r_{f}^{0}\right)$ in a two stage economy.

Cautious States. In cautious states, there is risk weight in macro prudential policy, thus $x=1$. So the interest rates are both with $x=1$, and with monetary policy $r_{f}>0$ while $r_{f}=0$ under macro prudential policy. Direct comparison of interest rate function 3.2 shows that it is higher when monetary policy is implemented, which completes the proof.

Proof of Lemma 5.1

With the return on storage technology normalized to zero, the welfare function incorporating macro prudential policy ($x>1$ risk weights), and monetary policy (interest rate r_{f}, finance by lump sum taxes) is given by

$$\begin{aligned}
W_{B}^{x,r_{f}} &= w_{1}\rho_{g}+w_{0}+\rho_{g}-\mu_{1}\left(\rho_{g}-1\right)\\
&+(1-\mu_{1})(\frac{1}{r_{B}}-\frac{w_{1}}{1-\mu_{0}-\mu_{1}})\left(\rho_{g}-1-\frac{\mu_{0}}{1-\mu_{1}}(\rho_{g}-\rho_{b})-\frac{c}{1-\mu_{1}}\right)\\
&-\mu_{0}\left(\rho_{g}-\rho_{b}-w_{1}r_{B}\frac{1-\rho_{b})}{1-\mu_{0}-\mu_{1}}\right)-(x-1)\,c\,(1-\mu_{1})\left(\frac{1}{r_{B}}-\frac{1}{(1-\mu_{1}-\mu_{0})}w_{1}\right)
\end{aligned}$$

Change in the welfare keeping the state variable (μ_0) constant is given by

$$\frac{dW}{dy} = \frac{\partial W}{\partial y} + \frac{\partial W}{\partial r_B}\frac{dr_B}{dy}$$

where $y = x, r_f$ depend on the policy.

We can further simplify the welfare function to

$$\begin{aligned} W_B^{x,r_f} =& w_1\rho_g + w_0 + \rho_g - \mu_1\left(\rho_g - 1\right) \\ &+ (1-\mu_1)(\frac{1}{r_B} - \frac{w_1}{1-\mu_0-\mu_1})\left(\rho_g - 1 - \frac{\mu_0}{1-\mu_1}(\rho_g - \rho_b) - \left(x + \frac{\mu_1}{1-\mu_1}\right)c\right) \\ &- \mu_0\left(\rho_g - \rho_b - w_1 r_B\frac{(1-\rho_b)}{1-\mu_0-\mu_1}\right) \end{aligned}$$

Also for notational convenience, let

$$AIRU = \rho_g - \frac{\mu_0}{1-\mu_1}(\rho_g - \rho_b).$$

Bold Equilibrium

(1) Marginal change in welfare due to the macro-prudential policy:

$$\begin{aligned} \frac{dW}{dx} &= \frac{\partial W}{\partial x} + \frac{\partial W}{\partial r_B}\frac{dr_B}{dx} \\ &= -c(1-\mu_1)(\frac{1}{r_B} - \frac{w_1}{1-\mu_0-\mu_1}) \\ &+ \frac{dr_B}{dx}\left(-\frac{1-\mu_1}{r_B^2}\left(AIRU - (1+r_f) - \frac{c}{1-\mu_1}\right) + w_1(1+r_f-\rho_b)\frac{\mu_0}{1-\mu_0-\mu_1}\right). \end{aligned}$$

The first line is the direct effect and the second line is the indirect effect through the change in interest rate. The direct effect is negative for the following reason: the demand for loans is only affected through the interest rates, so it is unaffected by the macro prudential policy directly. As such, to satisfy the same demand, more unskilled investors has to enter because each of them can now finance fewer loans due to the capital requirements. As such, a higher cost of testing is paid which is the direct effect.

(2) Marginal change in welfare due to the monetary policy

$$\begin{aligned} \frac{dW}{dr_f} =& \frac{\partial W}{\partial r_f} + \frac{\partial W}{\partial r_B}\frac{dr_B}{dr_f} \\ =& \frac{dr_B}{dr_f}\left(-\frac{1-\mu_1}{r_B^2}\left(AIRU - (1+r_f) - \frac{c}{1-\mu_1}\right) + w_1(1+r_f-\rho_b)\frac{\mu_0}{1-\mu_0-\mu_1}\right). \end{aligned}$$

The marginal change in the monetary policy has no direct effect on welfare since the interest rate payments are financed by lump sum taxes.

$$\frac{dW}{dx} - \frac{dW}{dr_f} =$$
$$- c(1-\mu_1)(\frac{1}{r_B} - \frac{w_1}{1-\mu_0-\mu_1})$$
$$+ (\frac{dr_B}{dx} - \frac{dr_B}{dr_f}) \left(-\frac{1-\mu_1}{r_B^2} \left(AIRU - (1+r_f) - \frac{c}{1-\mu_1} \right) + w_1(1+r_f-\rho_b)\frac{\mu_0}{1-\mu_0-\mu_1} \right)$$

Consider the indirect effect through interest rates. The first term is negative since

$$\frac{\frac{dW}{dx}}{\frac{dW}{dr_f}} = \frac{c\frac{1-\mu_1}{1-\mu_0-\mu_1}}{\frac{1-\mu_1}{1-\mu_0-\mu_1}} = c,$$

and $c < 1$ implies that $\frac{dW}{dx} < \frac{dW}{dr_f}$.

We continue by finding sufficient conditions which insures the term in parenthesis is negative:

$$- \frac{1-\mu_1}{r_B^2} \left(AIRU - (1+r_f) - \frac{c}{1-\mu_1} \right) + w_1(1+r_f-\rho_b)\frac{\mu_0}{1-\mu_0-\mu_1} < 0$$
$$\frac{(1-\mu_0-\mu_1)\rho_g + \mu_0\rho_b - (1-\mu_1)(1+r_f) - c}{r_B^2} > w_1(1+r_f-\rho_b)\frac{\mu_0}{1-\mu_0-\mu_1}$$

Substitute in for the bound on w_1

$$\frac{(1-\mu_0-\mu_1)\rho_g + \mu_0\rho_b - (1-\mu_1)(1+r_f) - c}{r_B} > (1+r_f-\rho_b)\mu_0$$

Since we are in bold equilibrium, $\mu_0 < \bar{\mu}_0^x$.

Also, from definition of r_B^{x,r_f} (i.e. bold interest rate in the presence of policy tools), we have

$$(1-\mu_1)(1+r_f) + c = (1-\mu_0-\mu_1)(1+r_B^{x,r_f}) - (x-1)(1-\mu_1).$$

Substitute in the last equation and rearrange to get the following sufficient condition

$$(1-\mu_0-\mu_1)(\rho_g - 1 - r_B^{x,r_f}) + \mu_0\rho_b(1+r_B^{x,r_f}) + (x-1)(1-\mu_1) > \mu_0(1+r_f)r_B^{x,r_f}$$

A sufficient condition is

$$\rho_b(1+r_B^{x,r_f}) > (1+r_f)r_B^{x,r_f},$$
$$\rho_b > \frac{r_B^{x,r_f}}{1+r_B^{x,r_f}}(1+r_f),$$

which is the same condition as (2.1).

Thus if

$$-(1-\mu_1)(\frac{1}{r_B} - \frac{w_1}{1-\mu_0-\mu_1})c+$$
$$\left(-\frac{1-\mu_1}{r_B^2}(AIRU-(1+r_f)-\frac{c}{1-\mu_1}) + w_1(1+r_f-\rho_b)\frac{\mu_0}{1-\mu_0-\mu_1}\right)(c-1)\frac{1-\mu_1}{1-\mu_0-\mu_1} > 0$$

Then the indirect effect through the interest rate dominates the direct negative effect of macro prudential policy, and a marginal macro prudential capital requirement dominates a marginal monetary policy.

To simplify notation, let

$$A = \frac{1-\mu_1}{r_B^2}(AIRU-(1+r_f)-\frac{c}{1-\mu_1}) - w_1(1+r_f-\rho_b)\frac{\mu_0}{1-\mu_0-\mu_1} > 0.$$

Thus if

$$c\frac{1-\mu_1}{1-\mu_0-\mu_1}(A+\frac{1-\mu_0-\mu_1}{r_B}-w_1) < \frac{1-\mu_1}{1-\mu_0-\mu_1}A,$$
$$c < \bar{c} = \frac{A}{A+\frac{1-\mu_0-\mu_1}{r_B}-w_1},$$

then the static effect of monetary policy is more adverse than the macro prudential policy in the bold states.

Cautious Equilibrium. Here there is no effect for macro prudential policy since in the cautious states $x=1$. As $\frac{dr_C}{dr_f} > 0$, we only need to compute the sign of $\frac{\partial W}{\partial r_C}$.

$$\frac{\partial W}{\partial r_C} = -\frac{1-\mu_0-\mu_1}{2}(\rho_g-1)\frac{1}{r_C^2} < 0$$

Thus the static effect of monetary policy is more adverse than the macro prudential policy in the cautious states as well.

Proof of Proposition 5.5

When the policy maker can choose a risk-free rate r_f for the bold stage, keeping the risk-free rate at 0 in the cautious then the implied threshold $\hat{\mu}_0$ is determined by the equation

$$r_B^{x,r_f}(\hat{\mu}_0, \mu_1, c, r_f, x) = r_C^{x,r_f}(\hat{\mu}_0, \mu_1, c, 0, x)$$

implying

$$\hat{\mu}_0 = c(1 - (1 - \mu_1)(x - 1)) - (1 - \mu_1)r_f.$$

By observing that $\frac{\partial \hat{\mu}_0}{\partial x} = c\frac{\partial \hat{\mu}_0}{\partial r_f}$ and applying the definition, if x^N is equivalent with r_f^N then

$$x^N - x^0 = \frac{1}{c}\left(r_f^N - r_f^0\right).$$

This implies that

$$\begin{aligned}
&r_B^{x,r_f}(\mu_0, \mu_1, c, r_f^0, x^N) - r_B^{x,r_f}(\mu_0, \mu_1, c, r_f^N, x^0) = \\
&= \frac{1 - \mu_1}{1 - \mu_1 - \mu_0}\left(r_f^0 - r_f^N\right) + \left(x^N - x^0\right)\frac{(1 - \mu_1)}{1 - \mu_1 - \mu_0}c = \\
&\qquad = \frac{1 - \mu_1}{1 - \mu_1 - \mu_0}\left(r_f^0 - r_f^N\right) + \frac{1}{c}\left(r_f^N - r_f^0\right)\frac{(1 - \mu_1)}{1 - \mu_1 - \mu_0}c = 0
\end{aligned}$$

None of the policies affect the interest rate in a cautious stage. This concludes the proof of the first statement. Also, the macro-prudential policy still has the adverse effect through the cost of testing derived in the proof of Lemma 5.1. This implies the second statement.