The Job Ladder: Inflation vs. Reallocation*

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Abstract

We introduce on-the-job search frictions in an otherwise standard monetary DSGE New-Keynesian model. Heterogeneity in productivity across jobs generates a job ladder. Firms Bertrand-compete for employed workers using the Sequential Auctions protocol of Postel-Vinay and Robin (2002). Outside job offers to employed workers, when accepted, reallocate employment up the productivity ladder; when declined, because matched by the current employer, they raise production costs and, due to nominal price rigidities, compress mark-ups, building inflationary pressure. When employment is concentrated at the bottom of the job ladder, typically after recessions, the reallocation effect prevails, aggregate supply expands, moderating marginal costs and inflation. As workers climb the job ladder, reducing slack in the employment pool, the inflation effect takes over. The model generates endogenous cyclical movements in the Neo Classical labor wedge and in the New Keynesian wage mark-up. The economy takes time to absorb cyclical misallocation and features propagation in the response of job creation, unemployment and wage inflation to aggregate shocks. The ratio between job finding probabilities from other jobs and from unemployment, a measure of the "acceptance rate" of job offers to employed workers, predicts negatively future inflation, independently of the unemployment rate, both in the model and in reduced-form empirical evidence that we provide.

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1 Introduction

The Phillips curve, an inverse short-run relationship between the rates of unemployment and inflation, has long been a guiding principle of monetary policy. Microfoundations of this relationship build on price-setting frictions, due either to explicit costs of price adjustment or to incomplete information about the nature of demand shocks faced by producers. In this body of work, the labor market is typically modeled as competitive, and features no unemployment; the relevant measure of slack is an output gap. Nominal wage rigidity — which, in the presence of pricing frictions, implies real wage rigidity — can generate classical unemployment associated with such a gap (Erceg, Henderson and Levine 2000). But the canonical model of unemployment, supported by a vast arsenal of empirical evidence on labor market flows, builds on search frictions, a primitive feature of the trading environment, rather than on exogenously imposed sources of wage rigidity. In the so-called DMP framework, wages are set by Nash Bargaining, the value of unemployment being the worker's outside option. When the economy is expanding and firms post many vacancies, the unemployed have an easy time finding a new job, hence unemployment declines, while employed workers have a strong threat and bargaining power, and real wages rise (although this may happen gradually due to infrequent bargaining — Gertler and Trigari 2009). This view seems to capture well the original idea behind the Phillips curve: low unemployment signals scarcity of labor, hence pressure on its price.[1].

In this paper, we advocate shifting emphasis away from unemployment, as the relevant indicator of slack to predict inflation, and towards the (mis)allocation of employment on a "job ladder". In Moscarini and Postel-Vinay (2017), using microdata from the Survey of Income and Program Participation to control for composition effects in employment, we provide empirical evidence that neither the unemployment rate nor the job-finding rate from

[1]Krause, Lopez-Salido and Lubik (2008) show that modeling the labor market according to this DMP tradition does not have much additional explanatory power for inflation dynamics in an otherwise standard monetary DSGE model, but Christiano, Eichenbaum and Trabandt (2016) find that it significantly improves the overall empirical fit the model.

unemployment have any significant comovement over time with nominal wage inflation.[2] In contrast, the rate at which workers move from job to job (or employer to employer, EE) has a significant positive relationship with contemporaneous nominal wage inflation, even for stayers who do not switch jobs. While the latter comovement is certainly driven, to some extent, by unobservable shocks to labor demand, the missing comovement between wage growth and unemployment is natural under an alternative view of labor markets characterized by search frictions. In this view, wages are not subject to bargaining, but are offered unilaterally by firms. Workers' bargaining power derives not from their outside option of unemployment, which is irrelevant as long as it does not bind, but from their ability to receive outside offers.[3] Outside offers can either be accepted, moving the worker up a job ladder, or matched and declined, pushing wages closer to marginal product and representing, for the employer, a cost-push shock. The latter outcome is more likely after a sufficiently long aggregate expansion, when workers have been moving up the ladder for a while and have therefore become difficult to poach away. In that case, cost pressure builds and, with a lag due to price rigidities, eventually manifests itself as price inflation.[4]

Our claim is that competition for employed, not unemployed, workers transmits aggregate shocks to wages, and that the distinction is important, for two reasons. First and

[2]In a similar vein, Jager *et al.* (2018) find in administrative Austrian data that sudden and large changes in Unemployment Insurance benefit size and duration by age have no discernible effect on the continuing wages of UI eligible workers.

[3]Hall and Milgrom (2008) replace Nash Bargaining with another bargaining protocol that also insulates wages from outside options at high frequency. They do not feature on the job search, which we emphasize as key to the transmission of aggregate shocks.

[4]Ashley and Verbrugge (2018) show that, in a forecasting, reduced-form sense, the statistical relationship between the rates of inflation and unemployment is highly non linear, and characterized by two distinct measures of slack or unemployment gap, "bust" and "boom", and three distinct phases. The first ("bust") relationship is the one highlighted by Stock and Watson (2010): there is a sharp reduction in inflation that occurs as the unemployment rate is rising rapidly. The second ("null") relationship occurs as the unemployment rate subsequently begins to fall; during this phase, inflation is unrelated to any conventional unemployment gap. The final ("overheating") relationship begins once the unemployment rate drops below its natural rate. In our view, the transmission channel of aggregate demand to inflation is employment misallocation, which is, in the last phase, small and highly correlated with the "boom" unemployment gap. Crump et al. (2019) combine microdata on labor market transition rates, to control for the effects of demographics on the unemployment rate without on the job search, with data on inflation expectations, in a standard New Keynesian Phillips Curve framework. They estimate the time series of the natural rate of unemployment, where inflation remains stable. They find that the unemployment gap from their estimated natural rate declined very slowly after the Great Recession.

foremost, these two types of competition point to two different observable proxies of slack to guide monetary policy. Unemployment signals poor job creation. Because of the same frictions, however, employed workers are misallocated on a job ladder. The more severe this misallocation, the easier to poach workers away from competitors, the stronger the incentives to job creation, independently of unemployment. Conversely, with less misallocation most outside job offers will be either rejected or matched, causing little employment gains and large wage gains, which are cost-push shocks from the point of view of sticky-price firms. Unemployment can thus be seen as the bottom rung of a high job ladder. As such, it is at best an incomplete measure of slack, which must be supplemented by measures of employment misallocation, or symptoms thereof.

Second, unemployment and misallocation of employment have different cyclical patterns. As Shimer (2012) showed, cyclical movements in the unemployment rate are driven to a large extent by those in the job-finding rate from unemployment, which in turn reflect closely the vacancy/unemployment rate, thus job creation. The latter is a very volatile variable, but the unemployment rate tracks it very closely, because the job finding probability in the US is high, over 25% per month, ruling out substantial propagation. In contrast, the EE transition probability is low, about 2% per month, so the reallocation of employment up the job ladder unfolds slowly, and the propagation of aggregate shocks through the poaching/outside offers channel can be strong. Because firms cannot perfectly target their pool of job applicants, they create more jobs and post more vacancies either when many job applicants are unemployed or when the employed are mismatched and easy to poach (or both). Thus, independently of the state of unemployment, the distribution of employment on the job ladder, a very slow-moving state variable, determines job creation and, ultimately, also the pace of job finding from unemployment. Wages do not respond, despite falling unemployment, for quite some time, until few workers are left at the bottom of the ladder, and competition for employed workers intensifies. In terms of observables, monetary authorities should pay attention to the *lagged* EE transition probability.

To formalize and quantitatively investigate this hypothesis, we introduce search in the labor market, both on- and off-the job, and endogenous job creation into an otherwise standard monetary DSGE model with complete financial markets, a representative risk-averse household, and Calvo pricing. Wages are set by Postel-Vinay and Robin (2002)'s Sequential Auctions protocol: firms make unilateral offers that can be renegotiated only by mutual consent, when outside offers arrive. We are interested in business cycles and monetary shocks, hence we must move from the steady state analysis that is common in search models to allow for aggregate uncertainty. Accordingly, we allow firms to offer and commit to contracts that are state-contingent wages, and to Bertrand-compete in such contracts for already employed workers. In Moscarini and Postel-Vinay (2018) we analyze in great detail, also quantitatively, the risk-neutral real version of this model, a business cycle search model with Sequential Auctions. Here we introduce on-the-job search and ex post competition in a DSGE model with risk-averse agents and nominal price rigidities. Essentially, we replace the household neoclassical labor supply of the standard DSGE models, where the labor market is perfectly competitive, with a search model of the labor market, where both (un)employment and the distribution of job quality within employment are state variables, the dynamics of which are determined by labor demand, through endogenous job creation.

Our model features an endogenous Neoclassical labor wedge, which maps into an endogenous New-Keynesian wage mark-up, both measuring the deviation from the efficient benchmark where the marginal rate of substitution of consumption for leisure equals the marginal product of labor. This object matters to our firms because it captures the ex post profits from hiring an unemployed worker, necessary to cover upfront recruitment posting costs. In our model, however, the impact of aggregate shocks on employment or wages also depends on a new object, that we dub the *productivity wedge*, which is a measure of the shortfall in average labor productivity due to employment misallocation on the job ladder. Because workers can search on the job, they can upgrade their position, shifting the distribution up the ladder, and closing the productivity wedge.

To evaluate the quantitative properties of the model, we calibrate it in steady state, log-linearize the dynamic equations, and simulate time series subject to various combinations of demand and supply side aggregate shocks. We find that the model provides significant propagation of Average Labor Productivity (ALP), due to employment movement up and down the job ladder, and some amplification of aggregate shocks on the job finding probability and the unemployment rate. Nominal price stickiness dampens the response of ALP, but amplifies the response of unemployment. Most notably, we study the lead/lag relationship of various labor market variables with the inflation rate. Besides the conventional unemployment rate, we also study the UE job finding probability from unemployment, as well as the "acceptance probability" of outside offers to employed workers, a measure of the degree of misallocation of the employed workforce. In the model, the acceptance probability can be directly measured by taking the ratio between EE and UE probabilities, because the latter measures the frequency of contacts with open vacancies, thus the two probabilities diverge only because employed workers do not accept all job offers, to an extent that varies with the misallocation of employment on the job ladder. We find that, in model-simulated data, the lagged acceptance probability has a strong and negative correlation with inflation: when employment is misallocated, and employed workers are more likely to accept outside offer, the economy features more slack, and output can expand more easily, without putting pressure on marginal cost and inflation. Importantly, the acceptance probability swamps the unemployment rate and the UE probability as a predictor of future inflation. We relate this prediction to the scarce available aggregate data.

Section 2 illustrates motivating empirical evidence, Section 3 describes the model, Section 4 its equilibrium, Section 5 quantitative results. The Appendix contains additional details.

2 Conceptual Framework and Motivating Evidence

The main argument of this paper is simple. If employed workers are willing to entertain outside offers, then by revealed preferences, they must be mismatched, so they can be poached relatively easily and any wage raise they receive will likely reflect an improved allocation. If instead these workers are already well matched, outside offers will often be countered by current employers, causing wages to rise with no reallocation — a classic wage-push shock. Hence, mismatch and willingness to accept outside offers are negative predictors of inflation.

How to make this insight operational and practically useful? We propose a simple but intuitive empirical proxy for mismatch and willingness to accept outside offers: the ratio between the observed transition probabilities employer-to-employer (EE) and unemployment-to-employment (UE). The idea is that the UE probability accurately measures job availability, as the unemployed are unlikely to decline many offers. In fact, the empirical UE probability from the monthly Current Population Survey (CPS) is almost exactly log-linear in the ratio between vacancy counts from the Job Openings and Labor Turnover Survey (JOLTS) and unemployed counts from the CPS. In standard random job search models, the employed receive offers at a rate that is roughly proportional to that of the unemployed, but are choosier. The EE/UE ratio isolates how choosy the employed are: we refer to it as the "acceptance rate"" (of outside offers), and we explore its predictive power for price and wage inflation. In other words, we explore the role of the EE/EU ratio as a measure of slack, alternative to the unemployment rate, in the Phillips curve.

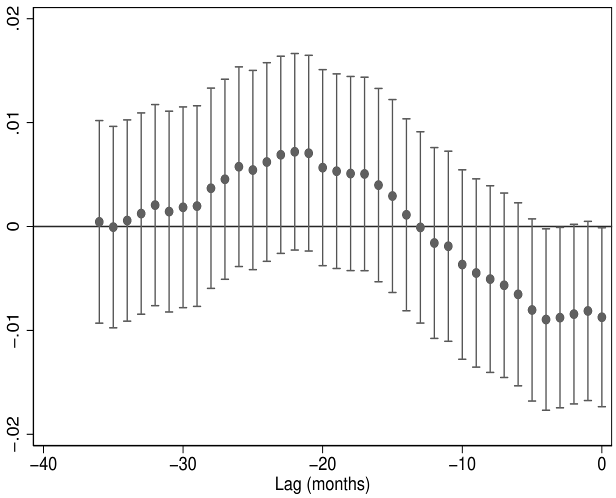
A key piece of empirical information, and the binding data constraint, is the EE transition probability. A time series for the US is available at monthly frequency since 1995 from the CPS, and since 2000 at quarterly frequency from "Job-to-job Flows" series of the Census's Longitudinal Employer Household Dynamics (LEHD) data set. The CPS is monthly and longer, and allows to relate labor market transitions to the monthly inflation rate. The LEHD is quarterly and shorter, but has comprehensive coverage and huge sample size, which allow to precisely estimate the EE probability also at the state level and exploit geographical variation in the variables of interest. In order to interpret our coefficient estimates as elasticities, we take logs and HP filter labor market rates. We explore both price inflation and wage inflation in the data.

Because the UE transition probability is meant to approximate the meeting rate, it co-varies highly with labor demand, as measured by job market tightness, hence potentially with wage growth due to labor demand shocks. This comovement could generate a spurious negative relationship between acceptance rate, whose denominator is this transition probability itself, and wage growth. For this reason, we also control directly for the UE transition probability in the various regressions shown in this section.

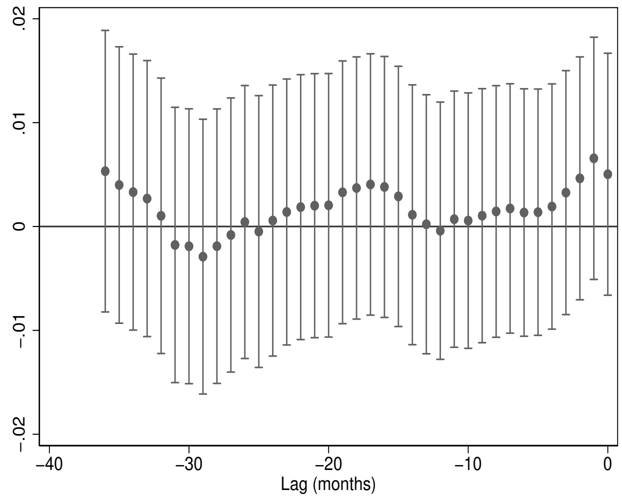
2.1 Aggregate evidence: price Phillips curve

We estimate EE and UE transition probabilities from the CPS matched files following the methodology in Fallick and Fleischman (2004). For the EE probability after 2007 we use the series corrected by Fujita, Moscarini and Postel-Vinay (2019) to compensate for an important source of measurement error and bias. We then consider two measures of price inflation: 12-month log changes in the CPI and in the Personal Consumption Expenditure deflator. We HP filter all logged variables with a parameter of 8.1×10^6 (as advocated by Shimer, 2012), and study their lead-lag relationship, using OLS regressions. Specifically, we regress the inflation rate on the rates of unemployment, UE transition, and acceptance lagged τ months, *separately for each* $\tau = 0, 1, 2 \cdots 36$ months. Figure 1 illustrates the results for CPI inflation; it plots the regression coefficient and 95% error bands for each lag τ running from right to left (hence, the time of the regressors $t - \tau$ runs from left to right relative to the timing of inflation t). Each estimate is from a separate regression.

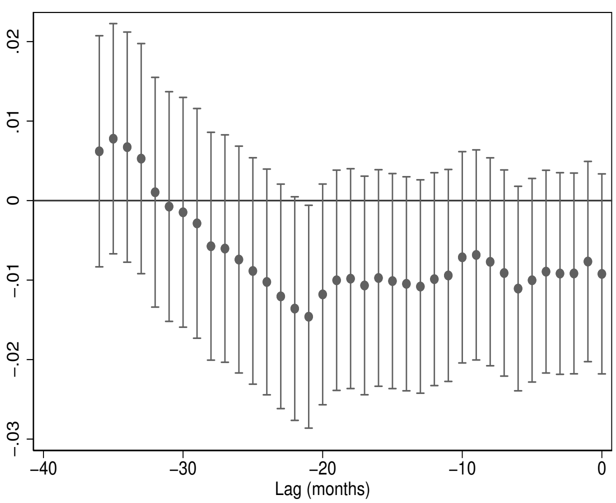
The estimated coefficients show a significant negative relationship between inflation and contemporaneous and lagged unemployment, not with the UE hiring rate or acceptance rate of outside offers. The latter tends to be negative, in line with our hypothesis, but lacks significance. Results are analogous when predicting inflation in the Personal Consumption Expenditure index. All estimates are small in magnitude. The small sample size is clearly a hurdle to accuracy of estimates. If we omit hiring and acceptance rate, we can run the regression of inflation on the unemployment rate on a much longer monthly time series,



(a) unemployment rate



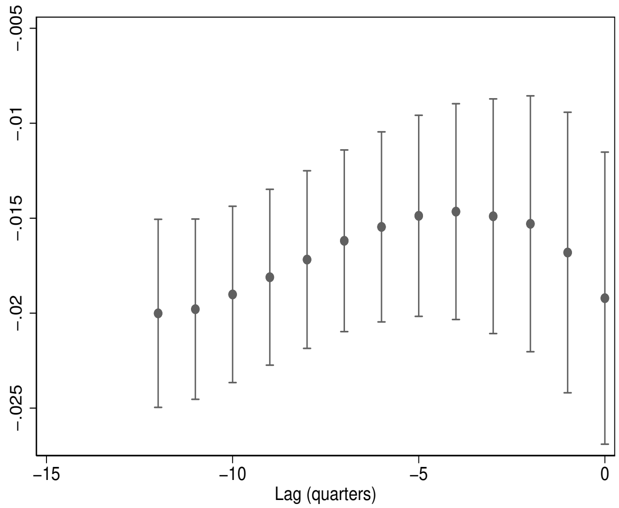
(b) hiring rate from non-employment



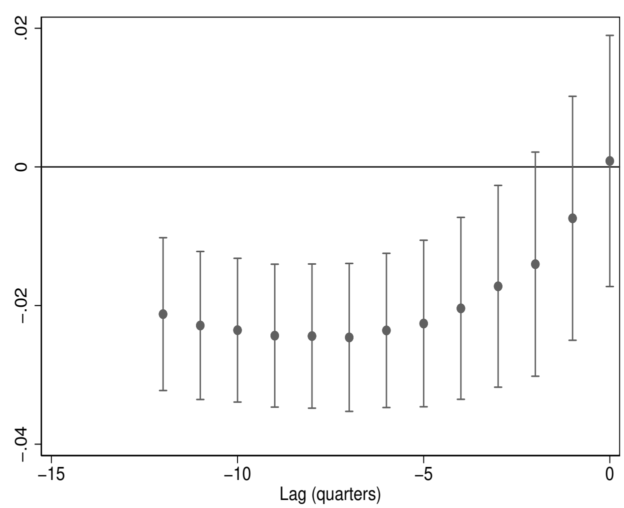
(c) acceptance rate of outside offers

Fig. 1: Multivariate regression coefficients of CPI inflation on lagged labor market indicators.

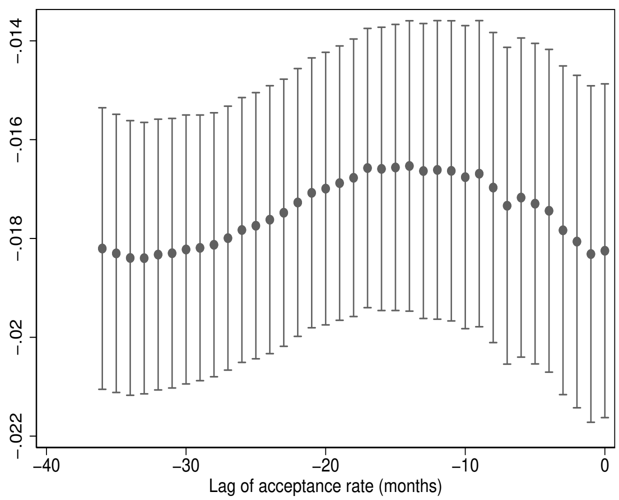
starting in 1948. The contemporaneous negative relationship disappears, and resurfaces only when the unemployment rate is lagged three or more months.



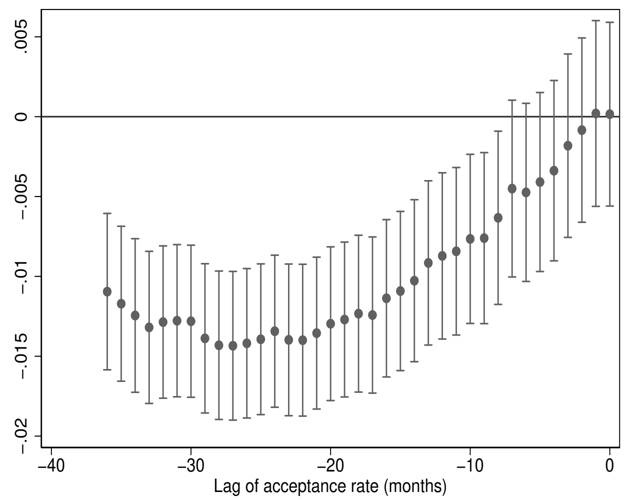
(a) unemployment rate



(b) acceptance rate of outside offers



(c) unemployment rate



(d) acceptance rate of outside offers

Fig. 2: Slope of wage Phillips curve augmented with lags of acceptance rate.

2.2 Aggregate evidence: wage Phillips curve

In a series of articles, Galì (2011) and coauthors (see Galì and Gambetti 2019 for a recent summary) find it difficult to detect a stable price Phillips curve in the US, possibly due to changing monetary policy regimes, but do find in post-war US quarterly data robust evidence of a wage Phillips curve, an inverse contemporaneous comovement between the quarterly rates of nominal wage growth and unemployment, similar to the original Phillips

(1958) finding for the United Kingdom. They use monthly data on wages from the Current Employment Statistics, Average Hourly Earnings of Production and Nonsupervisory workers, but aggregate data to quarterly frequency, starting in 1964. The results of reduced-form regressions are corroborated by those of estimated VARs, identified through a combination of ordering and sign restrictions. Critically, the wage Phillips curve is most visible when regressing nominal wage growth not only on the contemporaneous unemployment rate but also on the one-quarter lagged rate of inflation, as measured by the GDP deflator, to capture any indexation, whether automatic or implicit.

We revisit Galì and Gambetti (2019)'s exercise, using their sample, starting in 1964:Q1. The estimated coefficients of wage inflation on unemployment and lagged price inflation are (resp.) $-.012$ (stderr .002) and .493 (.041), in line with their results. If we focus on the post 1995:Q3 period, when our acceptance rate measure will be available, the same coefficients change to $-.019$ (.002) and $-.103$ (.09). So, the wage Phillips curve is robust in the last quarter century, while the effect of lagged inflation is not.

Next, we add our acceptance rate measure, at lags $\tau = 0, 1, 2 \cdots 12$ to the covariates of that baseline regression of wage inflation Δw_t on contemporaneous unemployment and one-quarter lagged price inflation. This requires restricting attention to the post-1995 period, when the acceptance rate series is available. Figure 2 plots the estimated regression coefficients of contemporaneous unemployment u_t and acceptance rate at time $t - \tau$ lagged $\tau = 0, 1, 2 \cdots 12$ quarters, each value of τ a separate regression, against this lag τ. Note that, in the upper left panel, the estimate always concerns the contemporaneous unemployment rate, and the lag refers to the other regressor, the acceptance rate. The relationship of wage growth remains negative with both the unemployment rate and the lagged acceptance rate, although the latter only after several lags (upper right). Standard errors are sizable, given the short time series. Since the time series for nominal wage growth, unemployment rate and acceptance rate are available monthly since 1995, we repeat the exercise for this later period with monthly data. For the lagged inflation rate, we use the (chained) price index for Personal Consumption Expenditure Excluding Food and Energy. The bottom panels of Figure 2 show that the results are similar, but more precisely estimated.[5]

2.3 State-level evidence: wage Phillips curve

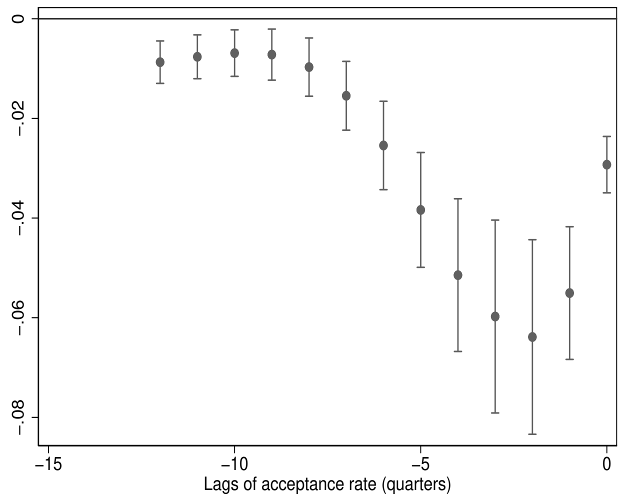
Finally, we complement the time series evidence shown thus far with a longitudinal study exploiting geographic variation across states in addition to time variation. Since price series are not available at the state level, in this longitudinal study we focus on wage inflation. We draw evidence on the variables of interest from a combination of three data sources, for fifty states plus the District of Columbia. First, quarterly state-level measures of wages are from the Quarterly Census of Employment and Wages (QCEW). Specifically, our measure of inflation is the QCEW's *over-the-year average weekly wage percentage change*, the percentage change in state-level average weekly wages between quarters t and $t-4$, a measure of wage inflation which we will denote by Δw_{st} for state s in quarter t. Second, we obtain quarterly state-level measures of the hiring rate from non-employment and the rate of employer-to-employer transitions from the dedicated LEHD series. Third, we get quarterly state-level measures of the labor force and unemployment rate from the BLS.[6] Note that in this longitudinal study, our measure of the hiring rate, which substitutes the UE rate we used in previous sections, is the hiring rate from non-employment (including non-participation) as the UE rate is not directly available from the LEHD data.

The combination of those three data sets covers the period 2000:Q2 to 2018:Q1. To match the fact that our measure of inflation is based on over-the-year average wage growth, we consider four-quarter trailing moving averages of the (logged) hiring, acceptance, and unemployment rates.[7] Finally, we take out state-level trends in all variables (including

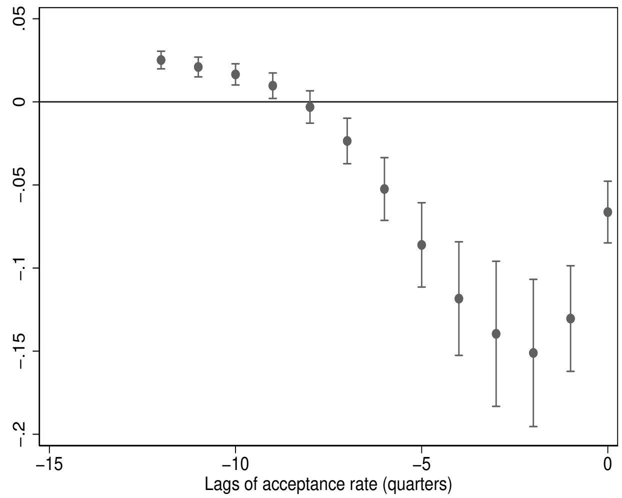
[5]We also repeat the baseline specification without acceptance rate using monthly data since 1964; the estimated coefficient on the unemployment rate is $-.013$ (.001), in line with quarterly evidence.

[6]Full details on the QCEW, as well as data, are available from the BLS at `https://www.bls.gov/cew/downloadable-data-files.htm`. Details and data relating to the LEHD are available from the Census Bureau at `https://lehd.ces.census.gov/data/j2j_beta.html`. BLS labor force data are available to download from `https://www.bls.gov/lau/`.

[7]Results are qualitatively robust to not smoothing.



(a) unemployment rate



(b) acceptance rate of outside offers

Fig. 3: Slope of state-level wage Phillips curve augmented with lags of acceptance rate.

inflation) by HP-filtering all series with a smoothing parameter of 10^5. Armed with those logged, averaged and filtered series, we run a set of OLS regressions which parallel the ones we ran on aggregate data in the previous subsection. In each case, observations are weighted by average state-level labor force size over the observation period. Lagged inflation is national, from the GDP deflator.

The baseline specification relating wage inflation to contemporaneous unemployment yields a negative and significant but small estimate, which further shrinks to zero when including lagged inflation as in Galì's work. When we include the acceptance rate at various lags, and repeat in Figure 3 the upper panels of Figure 2 with this state panel, both the coefficient on the contemporaneous unemployment rate and that on the lagged acceptance rate turn negative, but the latter is larger in magnitude.

We conclude that the ratio between the employer-to-employer transition probability and the non-employment-to-employment transition probability, our measure of the propensity of employed workers to accept outside offers conditional on receiving them (thus, of the quality of their current matches), is a leading negative predictor of wage growth. Overall, the data show a robust negative relationship between our empirical "acceptance rate" and subsequent wage and, to a lesser extent, price inflation. We now propose a theoretical model that makes sense, qualitatively and quantitatively, of this evidence.

3 The Economy

Agents, goods, endowments and technology. Time $t = 0, 1, 2, \cdots$ is discrete. Two vertically integrated sectors produce each a different kind of non-storable output: an intermediate input, that we call Service, and differentiated varieties of a final good.

Firms in the Service sector produce with linear technology using only labor. Each unit of labor ("job match") produces y units of the Service, which is then sold on a competitive market at price ω_t. Productivity y is specific to each match and is drawn, once and for all, when the match forms, in a i.i.d. manner from a cdf Γ with $\Gamma(\underline{y}) = 0, \Gamma(\overline{y}) = 1$ for some $\overline{y} > \underline{y} > 0$, and mean $\mathbb{E}_\Gamma y := \mu$.

Each variety $i \in [0, 1]$ of the Final good is produced by a single firm, also indexed by i, with a linear technology that turns y units of the homogeneous Service into $z_t y$ units of variety i, which are then sold in a monopolistic competitive market. TFP z_t follows a first-order Markov process.

A representative household is a collection of agents $j \in [0, 1]$. Each household member has an indivisible unit endowment of time per period, and the household is collectively endowed with ownership shares of all firms in both sectors. We indicate whether household member j is employed at time t by $e_t(j) \in \{0, 1\}$.

A Government issues numéraire liabilities, that we will refer to as "bonds", and taxes households, to purchase and consume final good varieties.

Preferences. The household has preferences

$$U(C_t) - b \int_0^1 e_t(j)\, dj$$

over work and consumption of Final good varieties, where, for $\eta > 1$

$$C_t = \left(\int_0^1 c_t(i)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}} \tag{1}$$

and leisure, with $U' > 0 > U''$, $b \geq 0$. For simplicity, we assume that b is low enough that all matches are preferable to unemployment at all points in time, and separations into unemployment are only exogenous.[8]

The household maximizes the present value of expected utility discounted with factor $\beta_t \in (0,1)$. The subjective discount factor β_t follows an exogenous stationary first-order Markov process with support in $(0,1)$. We denote the total discount factor between times t and $t+\tau$ by $\mathcal{B}_t^{t+\tau} = \prod_{\tau'=0}^{\tau-1} \beta_{t+\tau'}$, with the convention $\mathcal{B}_t^t = 1$.

Search frictions in the labor market. Service sector producers can advertise vacancies which randomly meet jobseekers. Advertising a vacancy entails a flow utility cost of κ_v in units of the aggregator C_t. Once a vacancy and a jobseeker meet, before making any offer the employer has to pay an additional one-off utility cost equal to κ_s, which we interpret as an administrative/screening cost, to process the job application, learn the employment status of the applicant, observe the productivity of own match y and of the current match of the job applicant, if employed.[9] If the match is also acceptable, production can begin in the following period.

Previously unemployed workers search for open job vacancies. Previously employed workers are separated from their jobs with probability $\delta \in (0,1]$ and become unemployed, in which case they have to wait until next period to search; if not, they also receive this period, with

[8]In Moscarini and Postel-Vinay (2018) we allow for endogenous separations in the flexible price, risk-neutral version of this economy.

[9]Note that we specify hiring costs — i.e. advertising and screening costs — as utility (or "effort", or "psychic") costs, rather than costs to be paid in units or Services or Final goods. That is, adjustment costs do not appear in resource constraints and market-clearing equations. While this is not essential, we explain the reason, and alternatives, in Section 4.4.

probability $s \in (0,1]$, an opportunity to search for a vacant job (a new match). Let

$$\theta = \frac{v}{u + s(1-\delta)(1-u)}$$

be effective job market tightness, the ratio between vacancies and total search effort by (previously) unemployed and (remaining) employed. A homothetic meeting function determines the probability $\phi(\theta) \in [0,1]$, increasing in θ, that a searching worker locates an open vacancy, thus the probability $\phi(\theta)/\theta$, decreasing in θ, that an open vacancy meets a worker. Therefore, the output of the Service sector can be thought of as a bundle of efficiency units of labor, assembled by Service sector firms in a frictional labor market, and leased to Final good producers in a competitive market at unit price ω_t. Service sector firms are essentially labor market intermediaries, solving the hiring problems of Final good producers.

Price determination. Each Final good variety producer $i \in [0,1]$ is a monopolistic competitor for its variety i, and draws every period with probability $\nu \in (0,1)$ in an i.i.d. fashion an opportunity to revise its price $p_t(i)$, which must be expressed in units of the numéraire controlled by the Government. Given the price, either newly revised or not, the firm serves all the resulting demand $q_t(i)$ by buying the required quantity $q_t(i)/z_t$ of Service in a competitive market at nominal unit price ω_t.

Wage setting. A Service-producing firm can commit to guarantee each worker a state-contingent expected present value of payoffs in utility terms (a "contract"), including wages paid directly to the worker, wages paid by future employers, and value of leisure during unemployment spells. This contract can be implemented by state-contingent wage payments while the relationship lasts. The contract can be renegotiated by mutual consent only. The firm's commitment is limited, in that it can always unilaterally separate, so firms' profits cannot be negative (in expected PDV). Same for the worker: if the utility value from staying in the contract falls below the value of unemployment, the worker will quit. When an employed worker contacts an open vacancy, the recruiting firms and the worker's incumbent employer observe each other's match qualities with the worker and engage in Bertrand competition over contracts. The worker chooses the contract that delivers the larger value. When indifferent, the worker stays.

Monetary and fiscal policy. The government sells to households B_{t+1} bonds at price $(1 + R_t)^{-1}$ units of numéraire in exchange for one unit of it for sure one period later. The monetary authority is the monopolist of the unit of account and controls the nominal interest rate R_t on newly issue bonds following some (typically Taylor) rule.

On the fiscal side, the Government spends units of account to purchase final goods, each variety i in a quantity equal to a multiple G_t/C_t of the household's current consumption $c_t(i)$ of each variety i. Government spending in real terms G_t follows a policy rule to be defined later. The Government levies on households a lump-sum tax, or rebates a lump-sum subsidy if negative, equal to T_t in nominal terms, to balance its budget every period.

Financial assets. Besides buying and redeeming nominal bonds, households trade ownership shares of all producers, of Final good (F) and Service (S), in competitive financial markets. Producers of Final good varieties earn, due to product differentiation, pure profits (or losses), which change randomly with infrequent Calvo pricing. Service producers also earn pure profits ex post, to cover hiring costs due to search frictions in the labor market. These profits also fluctuate randomly depending on the outcome of job posting. Both flows of profits are rebated to shareholders as net dividends. To eliminate idiosyncratic risk in these dividends, the household combines these shares in mutual funds that own a representative cross-section of all firms.

Timing of events within a period.

1. all agents observe random innovations to subjective discount factor, TFP in the Final good sector, monetary policy and fiscal policy;

2. the monetary authority chooses the nominal interest rate, and the fiscal authority chooses the size and allocation of its expenditure;
3. each producer of a Final good variety receives with some probability an opportunity to change its price, independent over time and across varieties;
4. some existing job matches break up exogenously, and those workers become unemployed;
5. firms in the Service sector post vacancies;
6. previously unemployed and (still) employed workers search for those vacancies;
7. upon meeting a job applicant, a vacancy-posting firm pays the screening cost and then makes the worker a new offer; if the worker is already employed, his current employer makes a counteroffer;
8. if the worker is employed, receives and accepts an outside offer, he becomes employed in the new match, otherwise he remains in his current state, either unemployed or employed in the current match;
9. firms and employed workers produce; firms and households trade Final good and Service; firms in the Service sector pay their workers wages to fulfill the current contracts they are committed to; firms in all sectors pay dividends to mutual fund owners; unemployed workers receive utility from leisure;
10. households trade nominal bonds with the monetary authority and mutual fund shares with each other, and pay taxes.

4 Equilibrium

4.1 Household optimization

The household chooses stochastic processes for Final good consumption varieties $c_t(i)$, holdings of bonds B_t^d and ownership shares of (mutual funds of) firms in both sectors (h_t^F, h_t^S), given their prices, resp. $p_t(i), R_t, \vartheta_t^F, \vartheta_t^S$. The household does not freely choose its member j's labor supply $e_t(j)$, because of search frictions: rather, the household chooses the probability $a_t(j)$ that member j accept any new job offer they might receive at the end of period t. The household then solves:

$$\max_{\left\{c_t(i), B_t^d, h_t^F, h_t^S, a_t(j)\right\}} \mathbb{E}_0 \left[\sum_{t=0}^{+\infty} \mathcal{B}_0^t \left(U\left(C_t\right) - b \int_0^1 e_t(j)\, dj \right) \right]$$

subject to:

- the utility aggregator (1);
- the budget constraint (in nominal terms)

$$\int_0^1 p_t(i) c_t(i) di + \frac{B_{t+1}^d}{1+R_t} + \sum_{\mathcal{I}=S,F} h_{t+1}^{\mathcal{I}} \vartheta_t^{\mathcal{I}} \leq \sum_{\mathcal{I}=S,F} h_t^{\mathcal{I}} \left(\Pi_t^{\mathcal{I}} + \vartheta_t^{\mathcal{I}} \right) + \int_0^1 e_t(j) w_t(j) dj + B_t^d - T_t$$

  where $\Pi_t^F = \int_0^1 \Pi_t^F(i) di$ are the total nominal profit flows earned by all Final good producers, $\Pi_t^F(i)$ by the only firm producing Final good variety i, and Π_t^S by each Service producer (after paying any hiring costs ex ante), while $\int_0^1 e_t(j) w_t(j) dj$ are the household's nominal earnings, the sum of wages $w_t(j)$ paid to those workers $j \in [0,1]$ within the household who are currently employed by Service producers; because of search frictions, different workers receive different wages;

- the stochastic processes for each member j's employment status...

$$e_{t+1}(j) = \begin{cases} e_t(j) & \text{with prob. } e_t(j)(1-\delta) + [1 - e_t(j)]\, \phi\left(\theta_t\right) a_t(j) \\ 1 - e_t(j) & \text{otherwise} \end{cases} \tag{2}$$

- ...and wage $w_t(j)$, to be determined;

- the No Ponzi Game condition

$$\Pr\left(\lim_{t\to\infty} B_t^d \prod_{\tau=0}^{t-1} (1+R_\tau)^{-1} = 0\right) = 1$$

We solve the household's maximization problem in steps: consumption and financial portfolio allocation first, then labor market turnover decisions.

The demand for each Final good variety is standard

$$c_t(i) = C_t \left(\frac{p_t(i)}{P_t}\right)^{-\eta} \tag{3}$$

where the ideal price index is

$$P_t = \left(\int_0^1 p_t(i)^{1-\eta} di\right)^{\frac{1}{1-\eta}} \tag{4}$$

and minimum expenditure equals $P_t C_t = \int_0^1 p_t(i) c_t(i) di$.

Divide the time-t budget constraint by P_t and attach to it a Lagrange multiplier λ_t, which then equals $U'(C_t)$, so it converts units of the consumption aggregator C_t into utils. The demand for bonds gives rise to the standard Euler equation

$$(1+R_t)\,\beta_t \mathbb{E}_t\left[\frac{U'(C_{t+1})}{U'(C_t)}\frac{P_t}{P_{t+1}}\right] = 1 \tag{5}$$

which discounts the real interest rate $(1+R_t)\,\mathbb{E}_t[P_t/P_{t+1}]$ with the pricing kernel $\mathcal{D}_t^{t+1}$, where we define the pricing kernel τ periods forward:

$$\mathcal{D}_t^{t+\tau} = \prod_{s=t}^{t+\tau-1}\left(\beta_s \frac{\lambda_{s+1}}{\lambda_s}\right) = \left(\prod_{s=t}^{t+\tau-1}\beta_s\right)\left(\prod_{s=t}^{t+\tau-1}\frac{\lambda_{s+1}}{\lambda_s}\right) = \left(\prod_{s=t}^{t+\tau-1}\beta_s\right)\frac{\lambda_{t+\tau}}{\lambda_t} = \mathcal{B}_t^{t+\tau}\frac{U'(C_{t+\tau})}{U'(C_t)}$$

For each sector $\mathcal{I} = S, F$, optimal portfolio allocations and market clearing, ruling asset price bubbles out, imply a standard asset pricing formula:

$$\frac{\vartheta_t^{\mathcal{I}}}{P_t} = \mathbb{E}_t\left[\sum_{\tau=0}^{+\infty}\mathcal{D}_t^{t+\tau}\frac{\Pi_{t+\tau}^{\mathcal{I}}}{P_{t+\tau}}\right]$$

Firms maximize the value to their owners, or utility value of the share price of each mutual fund, which is the present value of real profits, discounted by the pricing kernel, the representative household's stochastic discount factor.

We now turn to labor market turnover decisions $a_t(j)$. The only objects in the household's maximization problem that depend on those decisions are the total disutility of work $b\int_0^1 e_t(j)\,dj$ and nominal labor income $\int_0^1 e_t(j)w_t(j)dj$ through the stochastic laws of motion of each member's employment status $e_t(j)$, namely (2), and nominal wage $w_t(j)$. Thus, when deciding upon $a_t(j)$, the household solves the sub-problem:

$$\max_{\{a_t(j)\}} \mathbb{E}_0 \left[\sum_{t=0}^{+\infty} \mathcal{B}_0^t \left(-b \int_0^1 e_t(j)\,dj + \lambda_t \int_0^1 e_t(j) \frac{w_t(j)}{P_t} dj \right) \right] \tag{6}$$

subject to (2) and the stochastic process for equilibrium wages $w_t(j)$.

Job acceptance decisions are taken independently by different household members, because they do not affect each other's employment prospects: the household is one of many, and does not internalize congestion externalities in the search market, not even the externalities that its own members create on each other. The only interaction between household members is through income pooling, which explains the common weight λ_t on earnings, independent of each member's identity j and employment status $e_t(j)$. Therefore, the household maximizes the integrand of (6) separately for each member j, and the household's labor turnover problem (6) separates into two types: one for each currently unemployed member ($e_t(j) = 0$), yielding an optimal real value of unemployment $V_{u,t}^j$, and one for each employed member j ($e_t(j) = 1$), with $V_{e,t}^j\left(w^t(j), y_t(j)\right)$ the value of working in a match of quality $y = y_t(j)$ and holding a contract $w^t(j)$ specifying a continuation stream of promised state-contingent wages, starting with a current wage $w_t(j)$. These can be written in recursive form as follows. The Bellman equations for values in utility terms read:

$$
\begin{aligned}
\lambda_t V_{u,t}^j &= b + \max_{\{a_\tau(j)\}} \mathbb{E}_t \left[\sum_{\tau=1}^{+\infty} \mathcal{B}_t^{t+\tau} \left(-b + \lambda_{t+\tau} \frac{w_{t+\tau}(j)}{P_{t+\tau}} \right) e_{t+\tau}(j) \mid e_t(j) = 0 \right] \\
&= b + \beta_t \max_{\{a_t(j)\}} \mathbb{E}_t \big[(1 - \phi(\theta_t) a_t(j)) \lambda_{t+1} V_{u,t+1}^j \\
&\qquad + \phi(\theta_t) a_t(j) \lambda_{t+1} V_{e,t+1}^j \left(w^{t+1}(j), y_{t+1}(j) \right) \mid e_t(j) = 0 \big]
\end{aligned}
$$

$$
\begin{aligned}
&\lambda_t V_{e,t}^j \left(w^t(j), y_t(j) \right) \\
&\quad = \lambda_t \frac{w_t(j)}{P_t} + \max_{\{a_\tau(j)\}} \mathbb{E}_t \left[\sum_{\tau=1}^{+\infty} \mathcal{B}_t^{t+\tau} \left(-b + \lambda_{t+\tau} \frac{w_{t+\tau}(j)}{P_{t+\tau}} \right) e_{t+\tau}(j) \mid e_t(j) = 1, w^t(j), y_t(j) \right] \\
&\quad = \lambda_t \frac{w_t(j)}{P_t} + \beta_t \max_{\{a_t(j)\}} \mathbb{E}_t \left[\delta \lambda_{t+1} V_{u,t+1}^j + (1-\delta) \lambda_{t+1} V_{e,t+1}^j \left(w^{t+1}(j), y_{t+1}(j) \right) \mid e_t(j) = 1, w^t(j), y_t(j) \right]
\end{aligned}
$$

The time index t subsumes the dependence of the value on payoff-relevant variables that are exogenous to the employment relationship but endogenous to the economy, such as job market tightness θ_t and price level P_t. Dividing throughout by λ_t and using our notation for the pricing kernel, we can now represent those two problems in real terms and in the recursive form that is common in equilibrium models with on-the-job search:

$$
\begin{aligned}
V_{u,t}^j &= \frac{b}{\lambda_t} + \mathbb{E}_t \left\langle \mathcal{D}_t^{t+1} \left[\phi(\theta_t) a_t(j) V_{e,t+1}^j \left(w^{t+1}(j), y_{t+1}(j) \right) + (1 - \phi(\theta_t) a_t(j)) V_{u,t+1}^j \right] \right\rangle \\
V_{e,t}^j \left(w^t(j), y_t(j) \right) &= \frac{w_t(j)}{P_t} \\
&\quad + \mathbb{E}_t \left\langle \mathcal{D}_t^{t+1} \left[\delta V_{u,t+1}^j + (1-\delta) V_{e,t+1}^j \left(w^{t+1}(j), y_{t+1}(j) \right) \right] \mid e_t(j) = 1, w^t(j), y_t(j) \right\rangle
\end{aligned}
$$

4.2 Final good producers' optimization

The producer of Final good variety i chooses its price $p_t(i)$ and produces quantity $q_t(i)$ to serve the resulting demand $c_t(i)$ from the consumers' isoelastic demand function (3) and the additional proportional demand from the Government, and maximizes profits, given the technology that turns one unit of the homogeneous Service, purchased at given unit price ω_t, into z_t units of Final good variety i. Serving the demand $c_t(i)$ thus requires producing

$q_t(i) = \left(1+\frac{G_t}{C_t}\right)c_t(i)$ and paying a nominal input cost $\omega_t q_t(i)/z_t = \left(1+\frac{G_t}{C_t}\right)\omega_t c_t(i)/z_t$. Dropping the variety index and using (3), this producer quotes price p and earns

$$\widetilde{\Pi}_t^F(p) = (C_t + G_t)\left(\frac{p}{P_t}\right)^{-\eta}\left(p - \frac{\omega_t}{z_t}\right)$$

nominal profits, scaled by total demand from the household private sector and the Government. Each producer i is allowed to revise its price with probability ν each period. When this opportunity arises at time t, firm i chooses a price, that will be in effect until the future random time $t+\tau > t$ of the next opportunity, to maximize the expected PDV of real profits:

$$\max_{p(i)} \mathbb{E}_t\left[\sum_{\tau=0}^{+\infty}(1-\nu)^\tau \mathcal{D}_t^{t+\tau}\frac{\widetilde{\Pi}_{t+\tau}^F(p(i))}{P_{t+\tau}}\right].$$

The optimal reset price, p_t^*, is the same for all firms i:

$$p_t^* = \frac{\eta}{\eta-1}\frac{\mathbb{E}_t\left[\sum_{\tau=0}^{+\infty}(1-\nu)^\tau \mathcal{D}_t^{t+\tau}\left(C_{t+\tau}+G_{t+\tau}\right)P_{t+\tau}^{\eta-1}\frac{\omega_{t+\tau}}{z_{t+\tau}}\right]}{\mathbb{E}_t\left[\sum_{\tau=0}^{+\infty}(1-\nu)^\tau \mathcal{D}_t^{t+\tau}\left(C_{t+\tau}+G_{t+\tau}\right)P_{t+\tau}^{\eta-1}\right]} \tag{7}$$

Because the selection of firms that get to reset their prices is random, using (4) the Final good price index P_t then solves:

$$P_t^{1-\eta} = \nu\left(p_t^*\right)^{1-\eta} + (1-\nu)P_{t-1}^{1-\eta} \tag{8}$$

This price adjustment technology causes dispersion in the prices of Final good varieties. Specifically, in each period t prices are geometrically distributed across inputs, with a fraction $\nu(1-\nu)^\tau$ of the varieties priced at $p_{t-\tau}^*$, for $\tau \in \mathbb{N}$. Total demand for the Service by households and Government is then:

$$\frac{1}{z_t}\int_0^1\left(1+\frac{G_t}{C_t}\right)c_t(i)di = \frac{G_t+C_t}{z_t}\nu\sum_{\tau=0}^{+\infty}(1-\nu)^\tau\left(\frac{p_{t-\tau}^*}{P_t}\right)^{-\eta}.$$

The total profits that this firm rebates to its shareholders at time t equal $\Pi_t^F(i) = \widetilde{\Pi}_t^F(p^*_{t-\tau(i)})$ where $\tau(i)$ is the age of firm i's price.

4.3 Service producer's optimization and labor market equilibrium

Match values. In the Service sector, firms hire workers in a frictional labor market to assemble a (labor) Service that they sell in a competitive market to downstream Final good producers. Service sector firms can commit to pay their workers streams of wages, and can only renegotiate the deal by mutual consent, which only occurs if either the worker receives a better outside offer or if an aggregate shock makes one of the two parties' participation constraint bind, causing that party to want to walk away from the relationship, while the other disagrees. Because we assumed that the value of leisure b is small enough to rule out any endogenous separations under any history of aggregate shocks, only outside offers can trigger renegotiation.

We now drop the individual-member superscript j from labor market values and investigate said values further. Because employers extract the full match rent from unemployed workers, the value they offer them as of the beginning of period t, after aggregate shocks are observed, is $V_{e,t}\left(w^t(j), y_t\right) = V_{u,t}$, which solves

$$V_{u,t} = \frac{b}{\lambda_t} + \mathbb{E}_t\left[\mathcal{D}_t^{t+1} V_{u,t+1}\right] = \frac{b}{\lambda_t} + \beta_t \mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t} V_{u,t+1}\right] = \frac{b}{\lambda_t}\sum_{\tau=0}^{\infty}\mathbb{E}_t \mathcal{B}_t^{t+\tau},$$

so that $V_{u,t}$ is a known multiple of b. Hence, if b is small enough, for example equal to zero, no match will ever break up endogenously, so all separations will be exogenous, with probability δ.

Next, let $\overline{V}_{e,t}(y)$ denote the maximum value that a firm is willing to pay at the time offers are made (i.e. at stage 8 of the within-period timing outlined in Section 3) to a worker with whom it can produce a flow y of Service, without violating its participation constraint (remember, the firm has a zero outside option by free entry). In auction theory parlance, this is the firm's willingness to pay for a match y.

When a worker who is currently employed in a match of quality y and is promised an expected continuation value $V_{e,t}(w^t, y)$, namely a wage w_t/P_t today and then a continuation contract, meets an open vacancy and draws a new match quality y' in period t, Bertrand competition produces one of three possible outcomes: (i) $V_{e,t}(w^t, y) \geq \overline{V}_{e,t}(y')$, in which case the incumbent employer needs to do nothing to retain the worker, and the offer is irrelevant as the poacher cannot profitably match the worker's current value; (ii) $V_{e,t}(w^t, y) < \overline{V}_{e,t}(y') \leq \overline{V}_{e,t}(y)$, in which case the incumbent employer profitably retains the worker by raising its offer from $V_{e,t}(w^t, y)$ to $\overline{V}_{e,t}(y')$; (iii) and finally $V_{e,t}(w^t, y) \leq \overline{V}_{e,t}(y) < \overline{V}_{e,t}(y')$, in which case the worker is poached with an offer worth $\overline{V}_{e,t}(y)$. In any case, the worker moves if and only if $\overline{V}_{e,t}(y) < \overline{V}_{e,t}(y')$, and turnover decisions depend solely on the full-rent extraction value function $\overline{V}_{e,t}(y)$. Thus, in period t, the maximum value $\overline{V}_{e,t}(y)$ that the worker can receive in a type-y match includes a wage, as well as a continuation value which equals the discounted expected value of unemployment $\mathbb{E}_t\left[\mathcal{D}_t^{t+1} V_{u,t+1}\right]$ in case the worker is laid off at stage 6 of period t (probability δ), and otherwise equals the (expected future) willingness to pay $\mathbb{E}_t\left[\mathcal{D}_t^{t+1} \overline{V}_{e,t+1}(y)\right]$ of the current employer, received either from the incumbent employer itself, as part of the current contract, or from a poacher. This is because the incumbent firm is already promising the maximum it can in period $t+1$, so it will not match any outside offers: the worker either stays at the same value or leaves and receives the same value from a more productive poacher. Hence, the only remaining choice is the flow wage, and the maximum the firm can pay without making a loss is full revenues. Therefore:

$$\overline{V}_{e,t}(y) = \frac{\omega_t}{P_t} y + \delta \mathbb{E}_t\left[\mathcal{D}_t^{t+1} V_{u,t+1}\right] + (1-\delta)\mathbb{E}_t\left[\mathcal{D}_t^{t+1} \overline{V}_{e,t+1}(y)\right]$$

Since households value a marginal dollar of profit as much as a dollar of labor income (namely, $\lambda_t P_t$), value is perfectly transferable between individual workers and firms. Therefore, a worker's value $\overline{V}_{e,t}(y)$ of extracting full rents from a type-y job is also the value of said job to the firm-worker pair under any sharing rule, and we can define a type-y job *surplus* $S_t(y) = \overline{V}_{e,t}(y) - V_{u,t}$ at the offer-making stage of period t. Subtracting (**??**) from both sides of the last equation and solving forward:

$$
\begin{aligned}
S_t(y) = \overline{V}_{e,t}(y) - V_{u,t} &= \frac{\omega_t}{P_t} y - \frac{b}{\lambda_t} + (1-\delta)\mathbb{E}_t \left[\mathcal{D}_t^{t+1} S_{t+1}(y) \right] \\
&= \mathbb{E}_t \left[\sum_{\tau=0}^{+\infty} (1-\delta)^\tau \mathcal{D}_t^{t+\tau} \left(\frac{\omega_{t+\tau}}{P_{t+\tau}} y - \frac{b}{\lambda_{t+\tau}} \right) \right] \\
&= W_t y - \frac{b}{U'(C_t)} \sum_{\tau=0}^{\infty} (1-\delta)^\tau \mathbb{E}_t \mathcal{B}_t^{t+\tau}
\end{aligned}
$$

where we define recursively the expected PDV of a unit flow of Service ω_t/P_t — an object that can be interpreted as the "average real wage rate" — until match separation, in units of the consumption aggregator C_t:

$$
W_t = \frac{\omega_t}{P_t} + (1-\delta)\mathbb{E}_t \left[\mathcal{D}_t^{t+1} W_{t+1} \right] \tag{9}
$$

Crucially, the surplus $S_t(y)$ is affine increasing in y. Because the willingness to pay in the auction can be written as $\overline{V}_{e,t}(y) = V_{u,t} + S_t(y)$, this too is affine increasing in y, with intercept and slope that vary stochastically over time. Therefore, the firm with the higher match quality y wins the auction, and we draw the main conclusion of this subsection: the equilibrium is Rank Preserving (RP), and the direction of reallocation is efficient, always from less to more productive matches.

Note that worker compensation depends on the expected PDV W_t of the real price ω_t/P_t of the Service he produces, while the pricing decision (7) of the Final good producers who buy that Service and can reset their prices today depend on the expected PDV of the marginal cost, which is $\omega_t/(P_t z_t)$, adjusted for TFP.

Evolution of worker stocks: employment distribution on the job ladder and unemployment. Let $\ell_{t+1}(y)$ denote the population density of employment at match quality y, at the end of time t, after separations and hiring, when production takes place. (Equivalently, $\ell_t(y)$ is the density of employment at the beginning of period t.) Due to the RP property of equilibrium, this evolves according to:

$$\ell_{t+1}(y) = (1-\delta)\left\{\left[1 - s\phi(\theta_t)\,\overline{\Gamma}(y)\right]\ell_t(y) + s\phi(\theta_t)\,\gamma(y)\int_{\underline{y}}^{y}\ell_t(y')dy'\right\} + \phi(\theta_t)\,\gamma(y)u_t \quad (10)$$

Combining with the definition of the employment rate, $\int_{\underline{y}}^{\overline{y}}\ell_t(y')dy' = 1 - u_t$, yields the familiar law of motion of unemployment:

$$u_{t+1} = [1 - \phi(\theta_t)]\,u_t + \delta\,(1 - u_t) \quad (11)$$

Free entry and labor demand. By the time a firm and a worker who have met on the search market must decide whether or not to consummate the match, they know the quality of the potential match, y', which yields surplus $S_t(y')$. The surplus in the worker's previous situation is known, too: it is zero if the worker was unemployed, and $S_t(y)$ if the worker was employed in a type-y match.

The free entry condition equates the flow cost of vacancy posting to the vacancy contact probability times the expected return from a successful contact, net of screening costs. The firm appropriates the entire surplus $S_t(y)$ from unemployed job applicants, and the difference in surplus between own match y and existing match y', namely $S_t(y)-S_t(y') = \overline{V}_t(y)-\overline{V}_t(y')$, from employed job applicants. When meeting an unemployed worker, the firm cannot make a "blind offers", but has to pay the hiring cost κ_s and learn match quality y in order to calculate the outcome of future auctions for this worker, and thus the initial wage that implements the value $V_{u,t}$. When meeting an employed, the firm must learn new and current match quality to bid in the auction. We assume, and later verify, that parameter values are such that, in equilibrium, firms are willing to post vacancies and, ex post, pay κ_s.

Using the expression for the match surplus previously derived, the free entry condition then writes as:

$$\kappa_v \frac{\theta_t}{\phi(\theta_t)} + \kappa_s = \frac{u_t}{u_t + (1-\delta)s(1-u_t)} \left\{ W_t \mu - \frac{b}{U'(C_t)} \sum_{\tau=0}^{\infty} (1-\delta)^{\tau} \mathbb{E}_t \mathcal{B}_t^{t+\tau} \right\} + \frac{(1-\delta)s(1-u_t)}{u_t + (1-\delta)s(1-u_t)} W_t \int_{\underline{y}}^{\overline{y}} \gamma(y) \int_{\underline{y}}^{y} (y - y') \frac{\ell_t(y')}{1-u_t} dy' dy \quad (12)$$

On the LHS are vacancy posting costs times the expected duration of a vacancy, plus the screening cost, on the RHS are the expected profits earned by Service sector producers, namely the expected PDV of Π^S_{t+s}/P_{t+s}, all in units of the aggregator of Final goods. This is the average of the expected profits from hiring an unemployed and an employed job applicant, weighted by the respective shares of the two types of job applicants in the pool of job searchers. Unemployed hires are homogeneous, while employed hires are distributed, at the time of vacancy posting, according to the probability density $\ell_t(y')/(1-u_t)$ of match quality y' in their current jobs, which gives them bargaining power in wage negotiations.

4.4 Market-clearing

Financial markets. The representative household holds all shares of all firms, $h^F_t = h^S_t = 1$, and purchases (redeems) all bonds (previously) issued by the Government, so bond demand B^d_t equal supply B_t. The government balances its budget every period: $T_t = P_t G_t + B_t - B_{t+1}/(1+R_t)$, so the households pay back to the Government in taxes all the net surplus of bonds redemptions (including interest) minus new bond purchases, neither borrow nor save, but spend all their income on the Final good.

Good markets. Hiring costs are in units of the utility aggregator C_t and do not absorb any physical resources, thus enter no market-clearing conditions.[10] Market-clearing in the Service market requires the supply by its producers to equal its demand by Final good

[10] An alternative specification of hiring costs is in units of the Service, essentially using workers to hire and recruit other workers. Since the nominal Service price ω_t is flexible, so is the real price ω_t/P_t, which determines the incentives to create jobs. As aggregate recruiting activity rises in response to a shock, and the demand for Service with it, its price ω_t/P_t rises, raising real hiring costs and curbing the job creation response.

producers:

$$\int_{\underline{y}}^{\overline{y}} y\ell_{t+1}(y)dy = \int_0^1 \frac{q_t(i)}{z_t}di.$$

Market-clearing in each Final good variety i requires the supply $q_t(i)$ to equal the isoelastic demand $c_t(i)$ by households in (3) plus the demand from the Government, which scales up $c_t(i)$ by a factor G_t/C_t:

$$\left(1 + \frac{G_t}{C_t}\right) C_t P_t^\eta \int_0^1 p_t(i)^{-\eta} di = \int_0^1 q_t(i)\, di$$

Denoting $\tilde{P}_t = \left(\int_0^1 p_t(i)^{-\eta} di\right)^{-1/\eta}$ and combining the last two identities, we obtain a consolidated market-clearing condition:

$$\int_{\underline{y}}^{\overline{y}} y\ell_{t+1}(y)dy = \frac{C_t + G_t}{z_t} \left(\frac{P_t}{\tilde{P}_t}\right)^\eta. \tag{13}$$

4.5 General Equilibrium

The economy enters period t with a set of pre-determined aggregate objects: the employment distribution $\ell_t(\cdot)$, hence unemployment $u_t = 1 - \int_{\underline{y}}^{\overline{y}} \ell_t(y)dy$, the distribution of Final good variety prices $p_{t-1}(\cdot)$ and, at the beginning of the period, the new realizations of TFP z_t, discount factor β_t, and interest rate R_t. The first two are endogenous, infinitely-dimensional state variables. TFP and the discount factor have exogenous laws of motion. Monetary policy is assumed to follow a rule that makes R_t a stochastic function of the other four.

A key observation is that the price distribution $p_{t-1}(\cdot)$ enters equilibrium conditions only through the two price indexes $P_{t-1}, \tilde{P}_{t-1}$, which have known laws of motion: P_t follows (8) and, by the same reasoning, $\tilde{P}_t$ follows

$$\tilde{P}_t^{-\eta} = \nu\,(\,p_t^*)^{-\eta} + (1-\nu)\tilde{P}_{t-1}^{-\eta} \tag{14}$$

where we note that the reset price p_t^* that updates these two price indexes only depends on the processes of C_t, P_t and ω_t through (7).

Definition 1 *A* ***Recursive Rational Expectations Equilibrium*** *is a collection of measurable functions* $\{C, \theta, \omega\}$ *of the state vector* $\left\langle P_{-1}, \tilde{P}_{-1}, \ell(\cdot), z, \beta \right\rangle$*, a nominal interest rate rule* R*, a real government spending rule* G *and a tax rule* T*, given functions of the same state vector, that solve the consumption Euler equation* (5)*, the optimal reset price equation* (7)*, the free entry condition* (12)*, Government budget balance, and market-clearing in financial markets and in goods markets* (13)*, and which determine a first-order Markov process for each endogenous component of the state vector:* (8) *for* P*,* (14) *for* $\tilde{P}$*,* (10) *for* $\ell(\cdot)$*. The exogenous components* z *and* β *follow exogenous first-order Markov processes.*

4.6 Discussion: Two Wedges

The Neoclassical labor wedge and the New Keynesian wage mark-up. From the free entry condition (12), vacancy creation θ_t depends on the average of two expected returns, from unemployed and employed hires, weighted by the shares of these two groups in the job searching pool. The expected returns from an unemployed hire equal the expected PDV

$$W_t \mu - \frac{b}{U'(C_t)} \sum_{\tau=0}^{\infty} (1-\delta)^{\tau} \mathbb{E}_t \mathcal{B}_t^{t+\tau} = \mathbb{E}_t \left[\sum_{\tau=0}^{+\infty} (1-\delta)^{\tau} \mathcal{D}_t^{t+\tau} \left(MPL_{t+\tau} - MRS_{t+\tau} \right) \right] \tag{15}$$

of the difference between the Marginal Product of Labor in units of the Final good

$$MPL_{t+\tau} = \frac{\omega_{t+\tau}}{P_{t+\tau}} \mu$$

and the Marginal Rate of Substitution between consumption of the Final good and leisure:

$$MRS_{t+\tau} = \frac{b}{U'(C_{t+\tau})}$$

Indeed, the term labeled MPL_t is the average flow Service output of an extra unit of work μ, converted into consumption goods by the relative price ω_t / P_t, an expected Marginal Product of Labor. The term labeled MRS_t is the ratio between the additional utility b from one less unit of work and the marginal utility of consumption of the Final good, namely the MRS between consumption and leisure.

Chari, Kehoe and McGrattan (2007) define the "labor wedge" as the ratio between the MRS and the MPL. Measured in the data through the lens of a neoclassical growth model with balanced growth preferences, this labor wedge is procyclical, that is, the implicit "tax" rate on labor income is countercyclical, and plays the central role in amplifying business cycle fluctuations. In our model, the expected returns to hiring an unemployed worker in (15) equal the expected present value of the MPL times one minus the labor wedge. A procyclical labor wedge makes the returns to hiring unemployed workers procyclical. The MRS, however, contributes a countercyclical component to the labor wedge: in recessions, when consumption is low, workers value income more, so they are willing to work for less. So the procyclical movement in the labor wedge required to account for business cycle must originate from a strongly procyclical MPL, or relative price $\omega_{t+\tau}/P_{t+\tau}$.

An alternative interpretation of the "Service" in our model is a composite quantity of labor, with Service producers acting as labor market intermediaries, or temp-agencies, that hire workers in a frictional labor market and sell their services to good producers in a competitive market. Therefore, $\omega_{t+\tau}/P_{t+\tau}$ is the average cost of efficiency units of labor to good producers, and the firm discounts the difference between this real wage index (scaled by average efficiency units μ) and the MRS between consumption and leisure.

Estimated New-Keynesian models (Smets and Wouters 2007) define the "wage markup" as the ratio between the real wage and the MRS, and find that changes in this mark-up are key to explain inflation and output dynamics. Lacking a mechanism to generate endogenous changes in the wage mark-up, they attribute them to shocks, that they estimate to be procyclical. Erceg, Henderson and Levine (2000) generate wage mark-ups by assuming sticky nominal wages. Galí (2011) calls for a theory of an endogenous wage mark-up. Our model delivers just that. The expected returns to hiring an unemployed worker in (15) equal the expected present value of the MRS times the wage mark-up minus one. Thus, in our model the labor wedge is the reciprocal of the wage mark up. If the markets for both input (labor) and output (Service) were competitive, both the labor wedge and the wage mark-up would be identically equal to one, with workers on their labor supply curve and firms on their labor demand curve. If the labor market was competitive but the output (Service) market was monopolistically competitive, with Service providers charging a constant mark-up over the marginal cost of labor, the labor wedge would be less than one (i.e., the implicit tax rate on labor earnings would be positive) and the wage mark-up larger than one, but both would be constant over time. With our frictional labor market, the labor wedge is smaller than one and the wage mark-up is larger than one, to compensate for hiring costs, and, crucially, both are endogenous and time-varying.

The Productivity wedge: A new source of propagation and inflation. Our model contains an additional, novel transmission mechanism of aggregate shocks to job creation, absent in either of those two strands of the literature. Service providers, when posting vacancies, also mind the expected return from an *employed* hire, the double integral in (12). This is independent of the MRS, and depends entirely on the distribution of employment $\ell_t(\cdot)$, which is a slow-moving aggregate state variable. We call this object the "productivity wedge", because it is zero in the frictionless limit, where every worker is always in the best possible match, and it is larger the more misallocated is employment on the ladder.[11]

This term introduces an additional component to labor demand, with a complex cyclical pattern. At a cyclical peak, workers have had time and opportunities to climb the ladder, so it is both difficult and expensive to poach employees from other firms, and the expected returns from hiring employed workers are weak. After a recession, as the unemployed regain employment, they restart from random rungs on the match quality ladder, which are worse than the employment distribution at the cyclical peak. Hence, early in a recovery, many recent hires are easily "poachable". The transition of cheap unemployed job applicants into

[11]Note that our "productivity wedge" is somewhat reminiscent — although clearly not identical to — the "OP covariance" term first introduced by Olley and Pakes (1996) in their study of productivity in the telecommunications equipment industry, and taken up as a measure of mismatch in Bartelsman et al. (2013). In the context of our model, the OP covariance would be defined as the covariance between match productivity and match employment share, formally $\omega_t \int_{\underline{y}}^{\overline{y}} (\ln y - \mu) \left(\frac{\ell_t(y)}{(1-u_t)\gamma(y)} - 1 \right) d\Gamma(y)$.

low-quality jobs makes these workers only slightly more expensive, and still quite profitable, to hire. As time goes by, and unemployment declines, employment reallocation up the ladder through job-to-job quits picks up, employed workers grow more and more expensive to hire, ultimately putting pressure on wages, until we are back to a cyclical peak. The productivity wedge implies a procyclical wage mark-up, or countercylical labor wedge, as long as employment is still misallocated and "poachable".

In the US economy, the transition probability from job to job is fairly small, of similar magnitude to the separation rate into unemployment, and both are an order of magnitude smaller than the transition probability from unemployment to employment. Therefore, movements in the employment distribution up the job ladder are slow. An important implication is that, in our model, job market-tightness, thus the unemployment rate, have sluggish transitional dynamics. This stands in contrast to the canonical model with only search from unemployment, where tightness is a jump variable, with no transitional dynamics, and the unemployment rate converges very quickly to its new steady state. This is important, because the slow but prolonged decline in the U.S. unemployment rate after 2009 can only be explained in the canonical model by a long (and implausible) sequence of small, consecutive, positive aggregate shocks. A slowly mean-reverting process for the aggregate driver of business cycles will not do, because the free entry condition is forward-looking and would incorporate the expected recovery. In contrast, our model has a built-in, slow-moving, endogenous propagation mechanism of temporary aggregate shocks. Even more notably, the propagation is also transmitted to real wages, thus, ultimately, to inflation, amplifying the classic propagation that derives from staggered price-setting.

DSGE models with search frictions (Andolfatto 1996, Merz 1996, Krause and Lubik 2007, Gertler and Trigari 2009, Christiano, Eichenbaum and Trabandt 2016) typically focus on unemployment and abstract from on-the-job search. Within the linear-utility labor market search tradition, Robin (2011) adopts the Sequential Auction model of a labor market with on-the-job search, but stresses permanent worker heterogeneity. Firms are identical, thus the job ladder has only two steps. Only unemployed hires generate profits for firms. An employed job searcher extracts all rents from both incumbent and prospective employer. Therefore, in Robin's model no productivity wedge appears in the returns to job creation. The full stochastic job ladder mechanism, which gives rise to a positive and time-varying productivity wedge, appears in two recent business cycle models with on the job search: Moscarini and Postel-Vinay (2013) assume wage-contract posting without renegotiation, but cannot easily accommodate nominal price stickiness; Lise and Robin (2017) allow for ex ante worker and firm heterogeneity and sorting within the more tractable renegotiation framework. The latter model, although still cast in a linear utility framework, is the closest comparison. We assume a much simpler model of the job ladder, based on ex post match quality draws rather than ex ante two-sided heterogeneity, in order to flesh out the propagation mechanism of aggregate shocks that poaching introduces, and to be embed it in a full-fledged general equilibrium framework, with sticky prices and savings, where we can study monetary policy.

4.7 Special cases

Our model features three important "frictions": risk aversion in consumer preferences for Final goods, nominal price rigidity in Final good markets, and search frictions in the labor market. Barring search frictions in the labor market (e.g., the distribution of match qualities is a mass point at the upper bound $\overline{y}$, all workers receive offers every period, and $\delta = 0$ so no worker falls off the job ladder), employment concentrates at $\overline{y}$, the Service price ω_t is the nominal wage, and the model reduces to a standard New Keynesian model.

To gain understanding about the response of the economy to aggregate shocks, we can also shut down price rigidity and then risk aversion. In the next section, we will compute numerically equilibrium in a calibrated version of the full model. The flex price, risk-neutral benchmark is especially important to this exercise, because much of the steady state calibration strategy does not depend on price rigidity and risk aversion (both irrelevant in steady state with complete financial markets). Therefore, we follow the calibration strategy in Moscarini and Postel-Vinay (2018), who analyze that simpler case in detail.

Barring nominal rigidities, namely assuming $\nu = 1$, we obtain the most interesting benchmark, the flexible price economy. It is easy to show that Final good producers that face no pricing frictions all choose the same optimal static mark-up price, so that

$$p_t = P_t = \frac{\eta}{\eta - 1} \frac{\omega_t}{z_t} \tag{16}$$

and supply the same quantity, which then equals also the consumption aggregator plus Government absorption, the natural rate of output.[12] It is easy to show that money is neutral and nominal variables are determined by monetary policy.

Further removing consumer risk aversion, the model boils down to the standard case analyzed in search model of the labor market, linear utility and competitive output market, a business cycle version of the Sequential Auction model of Postel-Vinay and Robin (2002) as analyzed in Moscarini and Postel-Vinay (2018). Equilibrium computation in that case is very simple. Accounting for OJS yields a much higher estimated elasticity of the matching function with respect to vacancies, in US data from about .31 to .5. The reason is that employment is procyclical, hence so is the congestion created by the employed on the unemployed job searchers. In order to match the same observed cyclical volatility of the job finding probability from unemployment, vacancies have to be much more "important", and workers unimportant, in generating meetings. This effect amplifies aggregate shocks. A composition effect in the search pool works in the opposite direction: employed job searchers are more expensive to hire and less profitable than unemployed ones, and are relatively more prevalent in good economic times, thus they tame the response of job creation through the free entry condition. Finally, *within* employment, procyclical movements in the contact rate

[12]In the canonical New Keynesian model, this level of economic activity is not first-best only because of the monopoly distortion, which can be and usually is undone with an appropriate tax and subsidy scheme. In our model, there is a second, unavoidable distortion due to search frictions, and no presumption that the natural rate is constrained efficient. Nonetheless, we maintain the nomenclature for ease of comparison. Furthermore, since optimal mark-ups are constant, monopoly power turns out to have no impact on the business cycle properties of the model. Therefore, any conclusions that we reach in this particular case will extend, with minor modifications, to the economy with both flexible prices and perfectly competitive input producers.

generate countercyclical misallocation of employment on the job ladder, which moves slowly, but affects job creation through free entry, hence propagates aggregate shocks.

5 Quantitative analysis

In any version of this model with labor market frictions, independently of price rigidities, monopoly power, or risk aversion, the distribution of employment $\ell_t(\cdot)$ is a state variable, which makes equilibrium computation difficult in the presence of aggregate shocks. To compute stochastic equilibrium paths, we log linearize the system around its deterministic steady state, and assume that match quality has finite support, so we treat employment on each of the K rungs as a scalar variable in the linearization. We describe the equations, including steady state, in Appendix B. We calibrate the model at a monthly frequency. Unless otherwise noted, the calibration strategy follows Moscarini and Postel-Vinay (2018), to which the reader is referred for details. We then simulate the model, and report implied aggregate statistics after aggregating them at the quarterly frequency, which is the only one available in the data for many relevant variables.

5.1 Functional forms and Steady State calibration

We begin with production technology. Log TFP is described by the following AR(1) process:

$$\ln(z_t) = (1 - \varpi_z)\mu_z + \varpi_z \ln(z_{t-1}) + \varepsilon_t^z, \qquad \varepsilon_t^z \overset{\text{i.i.d.}}{\sim} \mathcal{N}\left(0, \sigma_z^2\right)$$

Because Average Labor Productivity is endogenous and depends on employment allocation on the ladder, we treat TFP as a latent variable that drives ALP. To calibrate $\varpi_z = .95$ and $\sigma_z = .0067$, in Moscarini and Postel-Vinay (2018) we targeted a quarterly standard deviation of filtered log ALP equal to 1.9% in the stochastic simulation of a risk-neutral, flex price version of this model.

We specify the distribution Γ of match quality draws as a Pareto with slope coefficient equal to 1.1 and mean normalized to 1. We approximate it over a 500-point discrete grid.

We assume a Calvo parameter $\nu = 1/10$, implying a 10-month average duration of prices. While this is higher than the corresponding empirical duration of about 7 months, heterogeneity across sectors in ν that we ignore is sizable and tends to raise price stickiness.

Next, we move to preferences. We set the value of leisure $b = 0$ so that no existing job is ever destroyed endogenously. Importantly, once we allow for OJS, the amplification properties of the model are much less dependent on the value of b. This value determines the returns to hire unemployed job applicants, while the returns from hiring employed job applicants depend on their current wages, which may have been renegotiated multiple times and thus no longer retain any memory of the opportunity cost b. So OJS allows to sidestep the debate on the opportunity cost of time that originated from Hagedorn and Manovskii (2008). We choose a value $\eta = 6$ for the elasticity of substitution between final good varieties, implying a 20% optimal net mark-up of price over marginal cost in steady state. The utility function over the CES consumption aggregator of final good varieties is CRRA: $U(C) = \sigma C^{1-1/\sigma}/(\sigma - 1)$, and we choose IES $\sigma = .5$.

For the discount factor, we assume $\beta_t = 1/(1+\varrho_t)$ where $\varrho_t > 0$ follows an AR(1) process in logs: $\ln(\varrho_{t+1}) = (1-\varpi_\beta)\ln\left(\frac{1-\beta}{\beta}\right) + \varpi_\beta \ln(\varrho_t) - \frac{1}{1-\beta}\varepsilon^\beta_{t+1}$ for some $\beta \in (0,1)$, $\varpi_\beta \in (0,1)$ and ε^β_{t+1} i.i.d. In the Appendix, we show that this implies, up to a log-linear approximation:

$$\ln(\beta_{t+1}) = (1-\varpi_\beta)\ln(\beta) + \varpi_\beta \ln(\beta_t) + \varepsilon^\beta_{t+1}$$

To calibrate the discount factor process, we downloaded from FRED monthly time series for the yield on 1-year Treasury bonds and CPI, and quarterly time series for chain-weighted aggregate private consumption and population size. We use 1-year bonds, rather than 3-month Bills, because the latter series is available only since 1982. We compute one-year ahead CPI inflation, every month, and average the inflation rate and the yield on Treasuries over quarters. Subtracting the former from the latter we obtain at quarterly frequency a time series of the ex post annual real interest rate, that we convert to quarterly real interest rate by taking its fourth root. We then compute the quarterly growth rate of per capita real consumption growth from NIPA. We end up with quarterly time series of per capita consumption growth and ex post real interest rate from 1962:Q1 to 2018:Q3. We multiply consumption growth by the calibrated value of σ, the inverse Intertemporal Elasticity of Substitution, and subtract the real interest rate. This residual variation in consumption growth unexplained by real interest rate movements is an estimate of changes in the discount factor. We take the log of that residual and subtract one, to obtain an empirical counterpart of ϱ_t. To eliminate volatility in this latter series, due for example to expectation errors that contaminate actual vs expected inflation, we MA-smooth it with a two-sided window of one quarter on each side. To this smoothed series we fit an AR(1). The estimated persistence and standard deviation of innovations imply monthly values of $\varpi_\beta = .947$ and standard deviation of innovations $\sigma_\beta = .00284$. Finally, we calibrate the mean discount factor to $\beta = .9957$ per month, corresponding to .95 per year.

We assume that log Government spending in real terms follows an AR(1)

$$\ln(G_t) = (1 - \varpi_G)\ln(G) + \varpi_G \ln(G_{t-1}) + \varepsilon_t^G \tag{17}$$

for some $G \in (0, Q)$, $\varpi_G \in (0, 1)$, and ε_t^G is a random i.i.d. innovation. To calibrate the parameters, we take chain-weighted total Government expenditures for consumption and gross investment from NIPA, quarterly, starting after the Korean war, we normalize it by chain-weighted GDP, we fit a linear trend and then an AR(1). We obtain a high estimated persistence and significant volatility of innovations, although the latter declined dramatically after the 1980s. For the long-run mean of government expenditure G, we calibrate it at 30% of private consumption C, roughly the post-Korean war average.

We specify monetary policy as a Taylor rule with contemporaneous timing:[13]

$$\ln(1+R_t) = \varpi_R \ln(1+R_{t-1}) + (1-\varpi_R)\left[\psi_\pi \ln\left(\frac{P_t}{P_{t-1}}\right) + \psi_Q \ln\left(\frac{C_t+G_t}{Q}\right) - \ln(\beta)\right] + \varepsilon_t^R \tag{18}$$

where $Q = C + G$ is steady state output of the final good, which is also total value added, and ε_t^R is a random i.i.d. innovation. We ignore the Zero Lower Bound. To calibrate this Taylor rule, we assume high interest rate smoothing $\varpi_R = .87$ and standard values for the Taylor rule coefficients, $\psi_\pi = 1.5$, $\psi_Q = .5$ and small shocks $\sigma_R = .0024$.

Finally, we specify matching frictions. Since for now all separations into unemployment are exogenous, we set δ equal to the average monthly transition probability from employment into unemployment (EU). Since all new matches are acceptable to the unemployed, we set the job contact probability in steady state equilibrium $\phi(\theta)$ equal to the average monthly transition probability from unemployment into employment (UE). We estimate these two probabilities from unemployment duration stocks (Shimer 2012) in the monthly CPS, respectively the number of workers who report being unemployed for 5 weeks or less divided by employment a month before (EU), which averages 2.4%, and one minus the ratio between the number of workers who report being unemployed for more than 5 weeks and unemployment a month before (UE), which averages 41%. The implied steady-state unemployment rate is $u = .024/(.024 + .41) = .055$.

Given these parameter values, we identify the relative efficiency of OJS s from the pace of EE reallocation. Because of the Rank-Preserving property of equilibrium, in steady state this is independent of the specific match quality distribution Γ: when given the opportunity, workers move up the job ladder, no matter how steep it is, at a speed that depends only on $s\phi(\theta)$. Given values of $\phi(\theta)$ and δ, hence u, we solve for the value of s that equates the the model-implied steady state EE probability to the average monthly transition probability from job to job, which is about 2% in the monthly CPS after its 1994 survey re-design. This

[13] A potentially interesting alternative rule to look at would be an interest rate peg, where the Central Bank simply keeps the interest rate constant. However, as it generically the case in the simple NK model, the steady state of our model is locally indeterminate under such a monetary policy rule.

yields $s = .176$, in line with existing estimates.

For the job finding probability we use the unemployment-duration based measure described above, which has standard deviation (in log deviations from HP trend) equal to .147 over the post-war period. For vacancies we use the monthly Composite Help-Wanted Index of Barnichon (2010), updated by the author to cover 1955-2016, and very close to JOLTS vacancies since its 2001 inception. For u_t we use the civilian unemployment rate from the monthly CPS, 1948-2018. We filter the log of each series separately using the longest time span available for each. We then run regressions of the job-finding probability on vacancies and unemployment rate for the time period where the series overlap, 1955-2016. We cannot reject empirically the hypothesis of constant returns to scale in matching. Hence, we assume a Cobb-Douglas matching function, with elasticity α, so that the job finding probability is $\phi(\theta) = \phi_0 \theta^\alpha$. We estimate the value of α at $\hat{\alpha} = .5$.[14]

We calibrate the values of vacancy posting cost κ_v and screening cost κ_s so that κ_s equals 60% of the total hiring cost per hire $\kappa_v \theta / \phi(\theta) + \kappa_s$ in steady state equilibrium. In turn, the average hiring cost per hire equals, by free entry, the expected return from a random hire, whether unemployed or employed, which can be calculated based entirely on parameter values set above. The resulting values yield screening cost of about five months, and therefore an average advertising cost of about a month, of average output, per hire.

The linearized system of equations solved by a Rational Expectations Equilibrium can be reduced to two jump variables and three predetermined variables. Details are in the appendix. The calibration generates a determinate equilibrium: the resulting matrix has five real eigenvalues, of which two are outside the unit sphere. We study the quantitative

[14] As argued in Moscarini and Postel-Vinay (2018), the scale ϕ_0 of the matching function only reflects the units in which vacancies are measured and has no impact on the model's dynamic properties. The only constraint on ϕ_0 in a discrete-time model like ours is that it must be such that both the job finding probability $\phi(\theta)$ and the vacancy filling probability $\phi(\theta)/\theta$ are less than one at all dates. But for the results that follow, we only need to specify the value of the matching function elasticity α. When we estimate a standard matching function ignoring OJS, i.e. when we identify the search pool with just unemployment u_t, we obtain a lower elasticity $\hat{\alpha} = .32$. The reason is simple: this standard method incorporates a term equal to α times the log of relative search effort by the unemployed vs the employed $u_t/[u_t + (1-\delta)s\,(1-u_t)]$, into the residual, which is then negatively correlated with $\ln(v_t/u_t)$, creating a downward bias in the estimated elasticity $\hat{\alpha}$.

Technology		
aggregate TFP (log):		
mean	μ_z	0
persistence	ϖ_z	0.95
volatility	σ_z	0.067
Pareto distribution of match quality Γ*:*		
mean	μ	1
shape parameter		1.1
Preferences		
flow value of leisure	b	0
elasticity of substitution btw varieties	η	6
intertemporal elasticity of substitution	σ	0.5
discount factor:		
mean	β	0.9957
persistence (log)	ϖ_β	0.947
volatility (log)	σ_β	0.00284
Search frictions		
matching function elasticity	α	0.5
on the job search efficiency	s	0.176
prob. of exogenous job destruction	δ	0.024
Pricing frictions		
Calvo probability	ν	0.1
Monetary policy rule, nominal interest rate (log)		
interest rate smoothing	ϖ_R	0.87
volatility	σ_R	0.0024
inflation parameter	ψ_π	1.5
output parameter	ψ_Q	0.5
Government spending		
persistence (log)	ϖ_G	0.966
volatility (log)	σ_G	0.018
share of private consumption	G/C	0.3

Table 1: Parameter values

properties of this unique equilibrium.

5.2 Results: aggregate volatility and comovement

To understand the average stochastic properties of our model economy, we simulate the model's equilibrium monthly time series over a period of fifty years, after an initial longer burn-in period. We then study the unconditional moments of various macroeconomic variables of interest.

A key new variable is the ratio between EE and UE probabilities. In the model, this ratio equals the average "acceptance probability" of outside offers, namely

$$\text{Acceptance Probability} := \int_{\underline{y}}^{\overline{y}} \overline{\Gamma}(y) \frac{\ell_t(y)}{1-u_t}.$$

Unlike UE and EE in isolation, which reflect demand and productivity shocks, their ratio is a direct measure of "misallocation" of employment on the job ladder, and of "poachability" of employed workers. In simpler words, it is a measure of aggregate labor supply elasticity.

Main results. Table 2 reports standard deviations and correlations of aggregate outcomes. For comparability with widely available data sets, all model-generated series are aggregated from months to quarters, logged and HP-filtered with parameter 1,600. The last column reports results from simulations that activate all four independent aggregate shocks, to TFP in the final good sector, to Government spending, to the subjective discount factor, and to monetary policy. To understand the mechanism, the previous columns activate only one or two shocks at a time. We begin by discussing the results in the last column.

Average Labor Productivity in the Final good sector, defined as

$$\text{ALP} = \frac{\omega_t}{P_t} \int_{\underline{y}}^{\overline{y}} y \frac{\ell_t(y)}{1-u_t}$$

has a standard deviation of about 1.7%, still slightly lower than in the data (about 2%). Unemployment and job finding rate from unemployment vary about five time as much, a

	Shocks to				
	TFP z_t	Gov't G_t	z_t & G_t	β_t & G_t	All
Standard Deviations					
Average Labor Productivity	.010	.003	.011	.013	.017
U to E transition prob.	.057	.018	.059	.101	.119
U rate	.038	.014	.041	.080	.090
Consumption	.013	.011	.017	.021	.025
Inflation rate	.014	.004	.014	.021	.025
Correlation					
U rate and Vacancies	−.398	−.693	−.421	−.659	−.581
Estimated elasticities of inflation rate to					
one-quarter lagged:					
U rate	−.012	.036	−.007	.048	.037
	(.004)	(.004)	(.004)	(.004)	(.004)
U to E transition prob.	.045	−.051	.029	−.008	−.020
	(.003)	(.003)	(.003)	(.003)	(.003)
Acceptance prob. outside offers	−1.35	.219	−.850	.071	.063
	(.046)	(.021)	(.042)	(.022)	(.026)
four-quarter lagged:					
U rate	−.016	.038	−.011	.033	.025
	(.005)	(.005)	(.005)	(.004)	(.004)
U to E transition prob.	.022	−.017	−.016	−.008	−.006
	(.003)	(.003)	(.003)	(.003)	(.003)
Acceptance prob. outside offers	−.555	−.243	−.375	−.035	−.308
	(.049)	(.021)	(.044)	(.023)	(.026)

Table 2: Results. Standard errors in parentheses.

significant degree of amplification, albeit still short of the factor of ten observed in the data. Aggregate consumption is too volatile relative to nondurable consumption from post-war NIPA data in the US (1.2%). Inflation exhibits significant quarterly variability. The Beveridge-curve negative correlation between unemployment and vacancies is a bit understated compared to that observed in the data ($-.85$).

Our main focus is on the predictive power of labor market indicators on inflation, in a reduced-form sense, from these model-generated data, to mirror the empirical evidence we presented in Section 2. In Table 2 we also report the estimated elasticities from a regression of (log) inflation on a constant and various combinations of lagged (log) unemployment rate, UE job finding probability from unemployment, EE probability, and the EE/UE probability ratio.

If employment is misallocated, many workers are unhappy about their jobs, and are easy to poach in new matches that are likely to beat their existing ones. Then, an increase in the demand for the intermediate input (Service), thus in the returns to hiring, is met by an easy expansion in employment, because many job applications result in a hire, and generate a large surplus. Hence, quantity responds much more than the marginal production cost of the Final good. Conversely, if most employment has already climbed the job ladder, most outside offers will be either ignored or matched, creating a bottleneck that limits the expansion of Service input and amplifies the rise of its price, the marginal cost.

Consistently with this view, the results show that the (lagged) acceptance probability is a much stronger (in a statistical and economic sense) predictor of inflation than the more traditional unemployment rate and the UE probability. The regression coefficient on the acceptance probability is systematically large, negative and significant at a four quarter lag. The other coefficients are all much smaller and unstable. The results at a one quarter lag are much less clear cut, presumably because it takes time for misallocated employment to be poached and to expand the supply of Service input, and moderate marginal cost.

The first four columns study various partial combinations of aggregate supply and de-

	Shocks to				
	TFP z_t	Gov't G_t	z_t & G_t	β_t & G_t	All
Average Labor Productivity	.013	.001	.013	.002	.013
U to E transition prob.	.067	.025	.072	.142	.160
U rate	.049	.022	.053	.122	.133
Consumption	.017	.012	.020	.015	.022
Inflation rate	.016	.003	.016	.021	.024
Correlation					
U rate and Vacancies	−.225	−.668	−.268	−.629	−.517
Estimated elasticities of inflation rate to					
one-quarter lagged:					
U rate	−.060	−.051	−.050	−.042	−.025
	(.004)	(.003)	(.004)	(.003)	(.003)
U to E transition prob.	.030	−.056	.024	−.041	−.014
	(.003)	(.003)	(.003)	(.003)	(.003)
four-quarter lagged:					
U rate	−.034	−.034	−.031	−.036	−.028
	(.004)	(.003)	(.004)	(.003)	(.003)
U to E transition prob.	.016	−.007	.015	−.003	.002
	(.003)	(.003)	(.003)	(.003)	(.003)

Table 3: Results, model with no on the job search. Standard errors in parentheses.

mand shocks. Fewer shocks of the same magnitude mean, of course, less aggregate volatility. The slope of the Beveridge curve is robustly negative, independently of the nature of the shocks, despite on the job search. The results using four-quarter lags of labor market indicators are qualitatively similar across shock configurations: both unemployment and the exit probability from it into employment have weak and unstable predictive power for inflation, while the acceptance rate systematically predicts inflation inversely. At one quarter lag, the results are different. TFP shock generate a strong negative relationship between acceptance of outside offers and subsequent inflation. Demand shocks generate a positive one. At such a short lag, it is difficult to disentangle the direct effect of a favorable shock on inflation (negative for TFP, positive for demand) from the dynamic effect of expanding output supply through accepted outside offers and hires from employment.

The role of on-the-job search. Table 3 reports results from the same model where we shut down on-the-job search, by setting $s = 0$. For comparison, we keep the rest of the calibration unchanged. In fact, this is almost without loss in generality, because the values of all other calibrated parameters are independent of the value of s. The only exception is that matching function elasticity α, which is estimated by regressing the UE probability on the ratio between vacancies and total search, measured as the unemployment rate plus s times the employment rate. The resulting value $\alpha = .5$ is higher than the .37 that we would obtain if we set $s = 0$ and defined job market tightness as the usual vacancy/unemployment ratio. A value of .5, however, is fairly common in the literature that studies models without on the job search, hence we maintain for comparability. The equilibrium is still determinate. The regression results omit the acceptance probability, because without OJS that is no longer relevant: the unemployed accept all offers, and the employed cannot search.

A comparison with Table 2 reveals that shutting down OJS has three main effects: it reduces the volatility of average labor productivity, in the absence of slow employment reallocation on the productivity ladder; it reduces the correlation between vacancies and unemployment; and it generates a robust negative correlation between unemployment and inflation at one quarter lag. The slope of this Phillips curve, however, is small, and the elasticity of inflation is negative, and small, also with respect to the previous quarter's UE job finding probability, which is counterintuitive. Since unemployment rate and UE probability are themselves strongly negatively correlated in the data, in practice these labor market variables provide little predictive power for inflation. Nothing interesting emerges at four quarter lag. This is in contrast to the acceptance probability of outside job offers in Table 2.

Additional results. Faccini and Melosi (2018) study a version of our model with two possible match qualities, bad and good jobs, and time-varying search effort by the employed. They calibrate the model in steady state, and then use empirical time series of EE and UE transition probabilities and the calibrated model's equilibrium equations to estimate, by Maximum Likelihood, the implied time series for aggregate demand (preference) shocks and for on-the-job search effort. The real marginal cost $\omega_t/(P_t z_t)$ series predicted by the model is much more in line with observed post-2008 inflation than estimates of marginal costs derived from either the labor share, as in standard New Keynesian models with competitive labor market, or from the UE transition rate, as in versions of the New Keynesian model that introduce only unemployed job search. Simply put, both the EE transition probability and the inflation rate in the US exhibited a profound decline and lack of recovery following the 2008-2009 recession, much slower than even the slow recovery of employment. But the EE probability fell by much less than UE, as workers mismatched and were eager to upgrade, slowly expanding supply and keeping inflation in check. What caused the persistently low propensity to search on the job remains to be investigated. In our structural model, where on the job search effort is fixed by assumption, the action is all loaded on the the acceptance probability, which in turn originates from cyclical mismatch.

6 Conclusion

TBC

References

[1] Andolfatto, D. (1996), Business Cycles and Labor Market Search. *American Economic Review* 86(1): 112-32.

[2] Ashley, R. and Verbrugge, R. (2018), The Phillips Curve Includes Two Unemployment Gaps, Not One. Mimeo, Cleveland FED.

[3] Bartelsman, E., Haltiwanger, J. and Scarpetta, S. (2013), Cross-Country Differences in Productivity: The Role of Allocation and Selection. *American Economic Review* 103(1): 305-334.

[4] Chari, V. V., Kehoe, P. J. and McGrattan, E. R. (2007), Business Cycle Accounting. *Econometrica*, 75: 781-836.

[5] Christiano, L. J., Eichenbaum, M. S. and Trabandt, M. (2016), Unemployment and Business Cycles. *Econometrica*, 84: 1523-1569.

[6] Crump, R.K., Eusepi, S., Giannoni, M., and Sahin, A. (2019), A Unified Approach to Measuring u^*. Prepared for *Brookings Papers on Economic Activity.*

[7] Erceg, C., Henderson, D. and Levin, A. (2000). Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics* 46: 281-313

[8] Faccini, R., and Melosi, L. (2018). Bad Jobs and Low Inflation. Mimeo, Queen Mary University London and Chicago FED.

[9] Fujita, S., Moscarini, G. and Postel-Vinay, F. (2019). Measuring Employer-to-Employer Reallocation. Mimeo.

[10] Galì, J. (2011), The Return of the Wage Phillips Curve. *Journal of the European Economic Association*, 9: 436-461.

[11] Galì, J. and Gambetti, M. (2019), Has the U.S. Wage Phillips Curve Flattened? A Semi-Structural Exploration, (with L. Gambetti), forthcoming in: J. Gali and D. Saravia (eds.) Changing Inflation Dynamics, Evolving Monetary Policy, Central Bank of Chile.

[12] Gertler, M. and Trigari, A. (2009), Unemployment Fluctuations with Staggered Nash Wage Bargaining. *Journal of Political Economy* 117(1): 38-86.

[13] Hall, R.E., and Milgrom, P. (2008). The Limited Influence of Unemployment on the Wage Bargain. *The American Economic Review* 98(4):1653-1674.

[14] Jager, S., Schoefer, B., Young, S. and Zweimuller, J. (2018), "Wages and the Value of Nonemployment", mimeo.

[15] Krause, M. and Lubik T.A. (2007). The (ir)relevance of real wage rigidity in the New Keynesian model with search frictions. *Journal of Monetary Economics*, 54(3), 706-727.

[16] Krause, M., Lopez-Salido, D.J. and Lubik, T.A. (2008). Do search frictions matter for inflation dynamics? *European Economic Review*, 52(8), 1464-1479.

[17] Lise, J. and Robin, J-M. (2017), The Macrodynamics of Sorting between Workers and Firms. *American Economic Review*, 107(4): 1104-1135.

[18] Merz, M. (1996), Search in the Labor Market and the Real Business Cycle. *Journal of Monetary Economics* 36(2), 269-300.

[19] Moscarini, G. and Postel-Vinay, F. (2013), Stochastic Search Equilibrium. *Review of Economic Studies*, 80,4: 1545-1581

[20] Moscarini, G. and Postel-Vinay, F. (2017), The Relative Power of Employment-to-Employment Reallocation and Unemployment Exits in Predicting Wage Growth. *American Economic Review, Papers and Proceedings*, 107(5):364-68

[21] Moscarini, G. and Postel-Vinay, F. (2018), On the Job Search and Business Cycles. Mimeo Yale University and University College London.

[22] Olley, G. S. and Pakes, A. (1996), The Dynamics of Productivity in the Telecommunications Industry. *Econometrica*, 64(6): 1263-1297.

[23] Phillips, A.W., (1958), The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957. *Economica*, 25(100): 283-299.

[24] Postel-Vinay, F. and Robin, J.-M. (2002), Equilibrium Wage Dispersion with Worker and Employer Heterogeneity. *Econometrica*, 70(6): 2295-350.

[25] Robin, J.-M. (2011), On the Dynamics of Unemployment and Wage Distributions. *Econometrica*, 79(5): 1327-1355.

[26] Shimer R. (2012). Reassessing the Ins and Outs of Unemployment. *Review of Economic Dynamics*, 15:127-148

[27] Sims, C. A. (2001). Solving Linear Rational Expectations Models'. *Computational Economics*, 20(1), 1-20.

[28] Smets, F. and Wouters, R. (2007), Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, 97(3): 586-606.

[29] Stock, J. and Watson, M. (2010), Modeling inflation after the crisis.*Proceedings - Economic Policy Symposium - Jackson Hole*, Federal Reserve Bank of Kansas City, 173-220.

APPENDIX

A Steady state

An important benchmark for stochastic equilibrium computation is the steady state equilibrium. Absent aggregate shocks to the discount factor β_t, TFP z_t and nominal interest rate R_t, price rigidity is irrelevant, because prices never need to change. Therefore, the steady state of the full, frictional economy closely resembles the stochastic equilibrium of the flex price benchmark.

Let $L_t(y) = \int_{\underline{y}}^{y} \ell_t(y')dy'$ denote the population c.d.f of employment on the job ladder. Integrating (10) over $(\underline{y}, y)$,

$$L_{t+1}(y) = (1-\delta)\left[1 - s\phi\left(\theta_t\right)\overline{\Gamma}(y)\right] L_t(y) + \phi\left(\theta_t\right) u_t \Gamma(y)$$

The stationary employment distribution solves the ordinary linear differential equation:

$$L'(y) = (1-\delta)\left[1 - s\phi(\theta)\overline{\Gamma}(y)\right] L'(y) + s\phi(\theta)\gamma(y)L(y) + \phi(\theta)\gamma(y)u$$

The solution can be found in closed form:

$$L(y) = \frac{\phi(\theta)\Gamma(y)u}{\delta + (1-\delta)s\phi(\theta)\overline{\Gamma}(y)}$$

Using this expression and integrating by parts, total Service output equals:

$$\int_{\underline{y}}^{\overline{y}} y dL(y) = \overline{y}(1-u) - \int_{\underline{y}}^{\overline{y}} \frac{\phi(\theta)\Gamma(y)u}{\delta + (1-\delta)s\phi(\theta)\overline{\Gamma}(y)} dy.$$

Normalizing to one the steady state levels of prices and TFP, steady state equilibrium solves:

$$
\begin{aligned}
P &= \widetilde{P} = p^* = 1 \\
R &= \frac{1-\beta}{\beta} \\
z &= 1 \\
u &= \frac{\delta}{\delta + \phi(\theta)} \\
C + G &= \overline{y}(1-u) - \int_{\underline{y}}^{\overline{y}} \frac{\phi(\theta)\Gamma(y)u}{\delta + (1-\delta)s\phi(\theta)\overline{\Gamma}(y)} dy \\
\frac{\omega}{Pz} &= \frac{\eta - 1}{\eta} \\
W &= \frac{1}{1-\beta(1-\delta)} \frac{\omega}{P}
\end{aligned}
$$

$$
\kappa_v \frac{\theta}{\phi(\theta)} + \kappa_s = \frac{u\left[W\mu - \frac{b}{U'(C)\left[1-\beta(1-\delta)\right]}\right] + W(1-\delta)s\int_{\underline{y}}^{\overline{y}} \overline{\Gamma}(y) \frac{\phi(\theta)\Gamma(y)u}{\delta + (1-\delta)s\phi(\theta)\overline{\Gamma}(y)} dy}{u + (1-\delta)s(1-u)}
$$

B Log linearization

B.1 Notation

We will carry out our linearization and numerical exercises under the assumption that the support of match quality y is discrete, i.e. that the support of y consists of the finite set of values $\underline{y} = y_1 < y_2 < \cdots < y_K = \overline{y}$, with $K \geq 2$, and with corresponding probability masses $\gamma(y_1), \cdots, \gamma(y_K)$ in γ (and likewise in ℓ).

Then, CDFs and survivor functions are:

$$
\Gamma(y_k) = \sum_{i=1}^{k} \gamma(y_i) \qquad \text{and} \qquad \overline{\Gamma}(y_k) = 1 - \Gamma(y_k) = \sum_{i=k+1}^{K} \gamma(y_i)
$$

and similarly for ℓ. This implies $\Gamma(y_1) = \gamma(y_1) > 0$ and $L(y_1) = \ell(y_1) > 0$. In what follows, we expand the notation by introducing a "dummy" y_0 such that $\Gamma(y_0) = L(y_0) = 0$. We also define, for every $n = 0, 1, 2\cdots$

$$\mathbf{I}_{n,t} := \sum_{k=1}^{K-1} \left[\overline{\Gamma}(y_k)\right]^n L_t(y_k)\,(y_{k+1} - y_k)\,.$$

With this notation, total Service output equals:

$$\begin{aligned}\sum_{k=1}^{K} y_k \ell_t(y_k) &= \sum_{k=1}^{K} y_k \left[L_t(y_k) - L_t(y_{k-1})\right] \\ &= y_K L_t(y_K) - \sum_{k=1}^{K-1} L_t(y_k)\,(y_{k+1} - y_k) = \overline{y}(1-u_t) - \mathbf{I}_{0,t},\end{aligned}$$

and the expected returns from an employed hire equal:

$$\begin{aligned}\sum_{k=1}^{K} \gamma(y_k) \sum_{i=1}^{k} \ell_t(y_i)\,(y_k - y_i) &= \sum_{k=1}^{K} \left[\overline{\Gamma}(y_{k-1}) - \overline{\Gamma}(y_k)\right] \sum_{i=1}^{k} \ell_t(y_i)\,(y_k - y_i) \\ &= \sum_{k=1}^{K-1} \overline{\Gamma}(y_k) \left(\sum_{i=1}^{k+1} \ell_t(y_i)\,(y_{k+1} - y_i) - \sum_{i=1}^{k} \ell_t(y_i)\,(y_k - y_i)\right) \\ &= \sum_{k=1}^{K-1} \overline{\Gamma}(y_k) L_t(y_k)\,(y_{k+1} - y_k) \\ &= \mathbf{I}_{1,t}\end{aligned}$$

Finally, we introduce the notation x_t for the real marginal cost of production of any variety of intermediate inputs:

$$x_t = \frac{\omega_t}{P_t z_t}$$

B.2 Recap of equilibrium conditions

Pre-determined variables.

Discount factor

$\beta_{t+1} = 1/(1 + \varrho_{t+1})$ where

$$\ln(\varrho_{t+1}) = (1 - \varpi_\beta) \ln\left(\frac{1-\beta}{\beta}\right) + \varpi_\beta \ln(\varrho_t) - \frac{1}{1-\beta}\varepsilon^{\beta}_{t+1}$$

TFP

$$\ln(z_{t+1}) = \varpi_z \ln(z_t) + \varepsilon^z_{t+1}$$

Nominal interest rate: Monetary Policy Rule

$$\ln(1+R_{t+1}) = \varpi_R \ln(1+R_t) + (1-\varpi_R)\left[\psi_\pi \ln(1+\pi_{t+1}) + \psi_Q \ln\left(\frac{C_{t+1}+G_{t+1}}{C+G}\right) - \ln(\beta)\right] + \varepsilon^R_{t+1}$$

Government spending.

$$\ln(G_{t+1}) = (1-\varpi_G)\ln(G) + \varpi_G \ln(G_t) + \varepsilon^G_{t+1}$$

Unemployment

$$u_{t+1} = [1-\phi(\theta_t)]\, u_t + \delta(1-u_t)$$

Employment distribution dynamics

For each $k \in \{1, \cdots, K-1\}$ (note that $L_t(y_K) = 1 - u_t$ duplicates the unemployment equation above):

$$L_{t+1}(y_k) = (1-\delta)\left[1 - s\phi(\theta_t)\,\overline{\Gamma}(y_k)\right] L_t(y_k) + \phi(\theta_t)\, u_t \Gamma(y_k)$$

"Static" equations, where no $(t+1)$-dated variables appear, either directly or in expectation.

Market-Clearing

$$\frac{G_t + C_t}{z_t}\left(\frac{P_t}{\tilde{P}_t}\right)^\eta = \overline{y}(1-u_{t+1}) - \mathbf{I}_{0,t+1}$$

Free-Entry Condition

$$\kappa_v \frac{\theta_t}{\phi(\theta_t)} + \kappa_s = \frac{u_t\left(W_t\mu - \frac{b}{U'(C_t)}\sum_{\tau=0}^{\infty}(1-\delta)^\tau\, \mathbb{E}_t \mathcal{B}_t^{t+\tau}\right) + W_t(1-\delta)s\mathbf{I}_{1,t}}{u_t + (1-\delta)\, s\,(1-u_t)}$$

where $\mathcal{B}_t^{t+\tau} = \prod_{\tau'=0}^{\tau-1} \beta_{t+\tau'}$.

Price indices

$$P_t^{1-\eta} = \nu\; p_t^{*1-\eta} + (1-\nu) P_{t-1}^{1-\eta}$$

$$\left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} = \nu\left(\frac{p_t^*}{P_t}\right)^{-\eta} + (1-\nu)\left(\frac{P_t}{P_{t-1}}\right)^{\eta}\left(\frac{\tilde{P}_{t-1}}{P_{t-1}}\right)^{-\eta}$$

Dynamics of non-predetermined, forward-looking variables.

Consumption: Euler Equation

$$\beta_t (1+R_t)\,\mathbb{E}_t\left[\frac{U'(C_{t+1})}{U'(C_t)}\frac{1}{1+\pi_{t+1}}\right] = 1$$

Present value of Service relative price

$$W_t = x_t z_t + (1-\delta)\mathbb{E}_t\left[\mathcal{D}_t^{t+1} W_{t+1}\right]$$

Optimal reset price

$$p_t^* = \frac{\eta}{\eta-1}\frac{\mathbb{E}_t\left[\sum_{\tau=0}^{+\infty}(1-\nu)^{\tau}\mathcal{D}_t^{t+\tau}\left(C_{t+\tau}+G_{t+\tau}\right)P_{t+\tau}^{\eta}x_{t+\tau}\right]}{\mathbb{E}_t\left[\sum_{\tau=0}^{+\infty}(1-\nu)^{\tau}\mathcal{D}_t^{t+\tau}\left(C_{t+\tau}+G_{t+\tau}\right)P_{t+\tau}^{\eta-1}\right]}$$

B.3 Log-linearizing the equilibrium conditions

We use hats to denote log deviations from steady state, such as $\widehat{\theta}_t = \ln(\theta_t) - \ln(\theta)$. For inflation, since we cannot take logs of $\pi = 0$, we use a linearization in levels: $\widehat{\pi}_t = \pi_t - \pi = \pi_t$. Moreover, in steady state, from the Euler equation $R = -\ln(\beta)$ and we define $\widehat{R}_t = R_t + \ln(\beta)$.

Discount factor.

$$\boxed{\widehat{\beta}_{t+1} = \varpi_{\beta}\widehat{\beta}_t + \varepsilon_{t+1}^{\beta}}$$

TFP. This is already linear in logs:

$$\boxed{\widehat{z}_{t+1} = \varpi_z \widehat{z}_t + \varepsilon_{t+1}^{z}}$$

Monetary Policy Rule. This is log-linearized with respect to C and G, but linearized with respect to the levels of R and π as discussed earlier:

$$\boxed{\widehat{R}_{t+1} = \varpi_R \widehat{R}_t + (1-\varpi_R)\left[\psi_\pi \pi_{t+1} + \psi_Q \frac{C\widehat{C}_{t+1} + G\widehat{G}_{t+1}}{C+G}\right] + \varepsilon^R_{t+1}}$$

Government spending. This is already linear in logs:

$$\widehat{G}_{t+1} = \varpi_G \widehat{G}_t + \varepsilon^G_{t+1}$$

Employment distribution. With a finite support of match quality $\{y_k\}_{k=1}^K$, the employment distribution $L_t(\cdot)$ is a finitely-dimensional vector which is part of the state variable. In log-linear form, for each y_k in the support:

$$\boxed{\widehat{L}_{t+1}(y_k) = (1-\delta)\left[1 - s\phi(\theta)\overline{\Gamma}(y_k)\right]\widehat{L}_t(y_k) + \frac{\phi(\theta)u\Gamma(y_k)}{L(y_k)}\widehat{u}_t + \alpha(\theta)\phi(\theta)\left[\frac{u\Gamma(y_k)}{L(y_k)} - (1-\delta)s\overline{\Gamma}(y_k)\right]\widehat{\theta}_t}$$

where $\alpha(\theta)$ is the elasticity of the matching function w.r. to vacancies.

Unemployment. The log-linearized version of the law of motion of unemployment can be obtained either by direct log-linearization of (11), or by noticing that $\widehat{L}_t\left(\overline{y}\right) = -\frac{u}{1-u}\widehat{u}_t$ and applying the derivation above. Either way:

$$\boxed{\widehat{u}_{t+1} = [1-\delta-\phi(\theta)]\widehat{u}_t - \phi(\theta)\alpha(\theta)\widehat{\theta}_t}$$

Consumption Euler Equation. Standard derivations produce:

$$\boxed{\mathbb{E}_t\left[\widehat{C}_{t+1}\right] - \widehat{C}_t = \sigma\left(\widehat{R}_t - \mathbb{E}_t\left[\pi_{t+1}\right] + \widehat{\beta}_t\right)}$$

Free-entry condition.

$$\boxed{\frac{(1-\alpha(\theta))\kappa\theta}{\kappa\theta+\kappa_s\phi(\theta)}\widehat{\theta}_t = \left[\frac{\mu-m}{u\,(\mu-m)+(1-\delta)s\mathbf{I}_1} - \frac{1-s(1-\delta)}{u+(1-\delta)\,s\,(1-u)}\right] u\cdot\widehat{u}_t}$$

$$\boxed{+\frac{u\mu+(1-\delta)s\mathbf{I}_1}{u\,(\mu-m)+(1-\delta)s\mathbf{I}_1}\widehat{W}_t + \frac{(1-\delta)s}{u\,(\mu-m)+(1-\delta)s\mathbf{I}_1}\sum_{j=1}^{K-1}(y_{j+1}-y_j)\left[1-\Gamma(y_j)\right]L(y_j)\widehat{L}_t(y_j)}$$

$$\boxed{-\frac{1}{\sigma}\frac{um}{u\,(\mu-m)+(1-\delta)s\mathbf{I}_1}\widehat{C}_t - \frac{um}{u\,(\mu-m)+(1-\delta)s\mathbf{I}_1}\frac{\beta(1-\delta)}{1-\varpi_\beta\beta(1-\delta)}\widehat{\beta}_t}$$

Present value of Service relative price.

$$\boxed{\widehat{W}_t = \left[1-\beta(1-\delta)\right](\widehat{x}_t+\widehat{z}_t) + \beta(1-\delta)\mathbb{E}_t\left[\widehat{\beta}_t - \frac{1}{\sigma}\widehat{C}_{t+1} + \frac{1}{\sigma}\widehat{C}_t + \widehat{W}_{t+1}\right]}$$

Prices.

$$\widehat{P}_t = \nu\widehat{p^*}_t + (1-\nu)\widehat{P}_{t-1}$$

where we used the fact that in steady state, $P_t = p_t^* = P_{t-1} = P$. Similarly,

$$\widehat{\tilde{P}}_t = \nu\widehat{p}_t^* + (1-\nu)\widehat{\tilde{P}_{t-1}}$$

Combining those two log-linear equations $\widehat{\tilde{P}}_t = (1-\nu)\widehat{P}_t$. Thus $\widehat{\tilde{P}}_t - \widehat{P}_t$ converges to zero deterministically. Near steady state, prices are close to their steady-state benchmark, there is little price dispersion. This implies that $\widehat{\tilde{P}}_t$ and $\widehat{P}_t$ are approximately the same:

$$\boxed{\widehat{\tilde{P}}_t \simeq \widehat{P}_t}$$

Market-Clearing. Using the laws of motion of $\widehat{u}_t$ and $\widehat{L}_t(y_k)$

$$\boxed{C\widehat{C}_t = -G\widehat{G}_t + (C+G)\widehat{z}_t - \left[(1-\delta)\overline{y} - \phi(\theta)\mu\right]u\widehat{u}_t + \alpha(\theta)\phi(\theta)\left[u\mu+(1-\delta)s\mathbf{I}_1\right]\widehat{\theta}_t}$$

$$\boxed{-(1-\delta)\sum_{k=1}^{K-1}\left[1-s\phi(\theta)\overline{\Gamma}(y_k)\right](y_{k+1}-y_k)\,L(y_k)\widehat{L}_t(y_k)}$$

Optimal reset price. The log-linearization yields a standard New-Keynesian Phillips curve

$$\boxed{\pi_t = \nu \frac{1-\beta(1-\nu)}{1-\nu} \hat{x}_t + \beta \mathbb{E}_t \left[\pi_{t+1} \right]}$$

B.4 Recap of log-linearized equations

This system comprises $10 + (K-1)$, linear stochastic difference equations, previously displayed in "boxes", in the 9 variables $\left(\widehat{C}_t, \widehat{\theta}_t, \pi_t, \widehat{x}_t, \widehat{W}_t, \widehat{R}_t, \widehat{z}_t, \widehat{\beta}_t, \widehat{G}_t, \widehat{u}_t\right)$ and the $K-1$ variables $L_t(y_k)$ for $k = 1, 2, \cdots, K-1$. Price indices P_t and $\tilde{P}_t$ no longer appear, only their growth rate π_t is relevant to equilibrium.

The first two equations, market-clearing and free entry, are contemporaneous, i.e. contain only variables dated at t. The next 3 equations are forward-looking, i.e. also contain time$-t$ expectations of variables dated at $t+1$. The last $5+(K-1)$ equations are backward-looking, i.e. only contain variables dated at $t+1$ as function of variables dated t, but without any expectation. The system only contains variables dated t, $t+1$, and expectations thereof conditional on information available at time t. Here the "contemporaneous" variables $\widehat{\theta}_t, \widehat{x}_t$ only appear at time t, the "predetermined" variables $\widehat{R}_t, \widehat{z}_t, \widehat{\beta}_t, \widehat{u}_t, \widehat{G}_t, L_t(y_k)$ can be solved at $t+1$ as a function of variables at time t and exogenous innovations, and the "jump" variables $\widehat{C}_t, \pi_t, \widehat{W}_t$ appear both at time t and as time$-t$ conditional expectations of their values at time $t+1$.