Uncovered Interest Parity, Forward Guidance and the Exchange Rate

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Abstract

I analyze the effectiveness of forward guidance policies in open economies, focusing on the role played by the exchange rate in their transmission. Under uncovered interest parity (UIP), the effect on the real exchange rate of an anticipated change in the real interest rate does not decline with the horizon. Empirical evidence using US, euro area and UK data points to a substantial deviation from that invariance prediction: expectations of interest rate differentials in the near (distant) future are shown to have much larger (smaller) effects on the real exchange rate than is implied by UIP. Some possible explanations are discussed.

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1 Introduction

The challenges posed by the global financial crisis to central bankers and the latter’s increasing reliance on unconventional monetary policies has triggered an explosion of theoretical and empirical research on the effectiveness of such policies, i.e. policies that seek to substitute for changes in the short-term nominal rate –the instrument of monetary policy in normal times– when the latter attains its zero lower bound (ZLB). A prominent example of an unconventional policy adopted by several central banks in recent years is given by forward guidance, i.e. the attempt to influence current macroeconomic outcomes by managing expectations about the future path of the policy rate once the ZLB is no longer binding.

In the present paper I analyze the effectiveness of forward guidance policies in an open economy, focusing on the role played by the exchange rate in their transmission. When doing so, I take as a benchmark the implications of uncovered interest parity (UIP, henceforth) on the impact of anticipated interest rates on the current exchange rate. This is of particular interest since most open economy macroeconomic models generally assume UIP. Yet, and to the best of my knowledge, neither the implications of UIP for the effectiveness forward guidance policies nor the role of the exchange rate in the transmission of those policies have been analyzed before.

As discussed below, UIP makes the current exchange rate depend, to a first-order approximation, on the undiscounted sum of expected future interest rate differentials. Importantly, that relation relies only a relatively weak assumption: the existence at each point in time of some deep pocket investors with unconstrained access to both domestic and foreign bonds.

In the first part of the paper I analyze the effects of forward guidance on the exchange rate, under the assumption of constant prices (or, equivalently, when the induced effects of interest rates and the exchange rate on output and prices are ignored). In that environment, the combination of UIP with the long run neutrality of monetary policy yields a strong implication: the impact on the current exchange rate of an announcement of a future adjustment of the nominal rate is invariant to the timing of that adjustment.

Next I turn to the analysis of forward guidance policies when allowing for feedback effects on output and prices, using a simple New Keynesian model of a small open economy. I show how, in that environment, the effect of a given anticipated change in the short-term nominal rate on the current exchange rate is larger the longer is the horizon of implementation of a given adjustment in the nominal interest rate. A similar prediction applies to the effect on output and inflation. As discussed below, both results are closely connected to the so called forward guidance puzzle uncovered in the recent literature, though that literature has invariably focused on closed economy models and has thus ignored the real exchange rate channel.\footnote{See Carlstrom et al. (2015), Del Negro et al. (2015), and McKay et al. (2016, 2017), among others,}

In the second part of the paper, I turn to the data, and provide some empirical evidence on the actual effects of anticipated future interest rate differentials at different horizons on the exchange rate. This is of special interest since, as is well known, the UIP condition is generally rejected in the data.\footnote{See, e.g., Bacchetta (2013) and Engel (2014) for a survey of the empirical literature on UIP.} An open question, which is the focus of the present inquiry, is what the empirical failure of UIP implies with regard to the relation between the current real exchange rate and anticipated real interest rate differentials at different horizons. In other words, the objective of the empirical analysis below is to characterize the potential deviations observed in
the data from the horizon-invariance property implied by the UIP.

Using data for the US, UK and euro area on bilateral real exchange rates and market-based proxies for anticipated real interest rate differentials at different horizons, I test the horizon-invariance property linking those variables, as implied by the UIP. The evidence points to a strong rejection of that property. Perhaps more interestingly, it suggests a simple characterization of the empirical deviations from horizon-invariance: expectations of interest rate differentials in the near (distant) future have much larger (smaller) effects than is implied by UIP. I refer to this particular dimension of the empirical failure of UIP as the forward guidance exchange rate puzzle.

The third part of the paper discusses possible interpretations of the previous findings. In particular, I argue that some of the solutions to the forward guidance puzzle proposed in the closed economy literature are unlikely to apply to the exchange rate channel emphasized in the present paper. On the other hand, deviations from UIP involving departures from rational expectations and/or portfolio adjustment costs have a better chance to capture the evidence reported in the present paper.

The remainder of the paper is organized as follows. Section 2 briefly describes the forward guidance puzzle in a closed economy setting. Section 3 discusses the effects of forward guidance on the exchange rate in a partial equilibrium framework. Section 4 revisits that analysis in general equilibrium, using a small open economy New Keynesian model as a reference framework. Section 5 presents the empirical evidence. Section 6 discusses possible interpretations of the evidence. Section 7 summarizes and concludes.

2 Background: The Forward Guidance Puzzle

In the present section I briefly review the literature on the forward guidance puzzle. The analysis in that literature has been invariably conducted using a closed economy framework.

The effectiveness of forward guidance and its role in the design of the optimal monetary policy under a binding ZLB was analyzed in Eggertsson and Woodford (2003) and Jung et al. (2005), using a standard New Keynesian model. Those papers emphasized the high effectiveness of forward guidance as a stabilizing instrument, as implied by the theory, at least under the maintained assumption of credible commitment.

More recently, the contributions of Carlstrom et al. (2015), Del Negro et al. (2015), and McKay et al. (2016, 2017), among others, have traced the strong theoretical effectiveness of forward guidance to a "questionable" property of one of the key blocks of the New Keynesian model, the Euler equation, which in its conventional form implies that future interest rates are not "discounted" when determining current consumption. Formally, the standard dynamic IS equation (DIS) of the New Keynesian model can be solved forward and written as:^{3}

\[ \hat{y}_t = -\frac{1}{\sigma} \sum_{k=0}^{\infty} \mathbb{E}_t \{ \hat{r}_{t+k} \} \]

where \( y_t \) is (log) output and \( r_t \equiv i_t - \mathbb{E}_t \{ \pi_{t+1} \} \) is the real interest rate. The \( \hat{\cdot} \) denotes deviations from steady state. Two predictions of the model stand out. Firstly, the effect on output of a

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^{3} I am implicitly assuming the most basic version of the model, with consumption as the only aggregate demand component. See, e.g., chapter 3 in Galí (2015).
given anticipated change in the real interest rate is *invariant to the horizon of implementation* of that change. Secondly, when combined with a forward-looking New Keynesian Phillips curve, the previous property implies that the announcement of a future *nominal* rate adjustment of a given size and persistence is predicted to have a *stronger* effect on current output and inflation *the longer the horizon of implementation*, given the positive relation between that horizon and the size of the inflation response. The previous two predictions stand at odds with conventional wisdom, and as such they have been (jointly) labeled the *forward guidance puzzle*.

Several potential "solutions" to the forward guidance puzzle have been proposed in the literature, in the form of modifications of the benchmark model that may generate some kind of discounting in the Euler equation. Those modifications include the introduction of finite lives (Del Negro et al. (2015)), incomplete markets (McKay et al. (2016, 2017), Werning (2015), Farhi and Werning (2017)), lack of common knowledge (Angeletos and Lian (2017)), and behavioral discounting (Gabaix (2017)). The proposed solutions typically generate an approximate "discounted" DIS equation of the form

\[
\hat{y}_t = \alpha \mathbb{E}_t \{ \hat{y}_{t+1} \} - \frac{1}{\sigma} \mathbb{E}_t \{ \hat{r}_t \}
\]

where \( \alpha \in (0, 1) \), leading to the forward-looking representation

\[
\hat{y}_t = -\frac{1}{\sigma} \sum_{k=0}^{\infty} \alpha^k \mathbb{E}_t \{ \hat{r}_{t+k} \}
\]

which implies that the effect of future interest rate changes on current output is more muted the longer is the horizon of their implementation.

Interestingly, and as discussed in section 6 below, several of those solutions would not seem to be relevant to the exchange rate channel emphasized in the present paper.

Next I show that, under the assumption of uncovered interest parity, a phenomenon analogous to the forward guidance puzzle applies to the real exchange rate in an open economy.

### 3 Forward Guidance and the Exchange Rate in Partial Equilibrium

Consider the asset pricing equations

\[
1 = (1 + i_t) \mathbb{E}_t \{ \Lambda_{t,t+1}(P_t/P_{t+1}) \}
\]

\[
1 = (1 + i^*_t) \mathbb{E}_t \{ \Lambda_{t,t+1}(E_{t+1}/E_t)(P_t/P_{t+1}) \}
\]

for all \( t \), where \( i_t \) denotes the yield on a nominally riskless one-period bond denominated in domestic currency purchased in period \( t \) (and maturing in period \( t+1 \)). \( i^*_t \) is the corresponding yield on an analogous bond denominated in foreign currency. \( E_t \) is the nominal exchange rate, expressed as the price of foreign currency in terms of domestic currency. \( \Lambda_{t,t+1} \) is the (real) stochastic discount factor for a (domestic) investor with unconstrained access to the two bonds in period \( t \).
Combining (1) and (2) we have
\[ \mathbb{E}_t \{ A_{t,t+1} (P_t/P_{t+1}) [(1 + i_t) - (1 + i^*_t)(\mathcal{E}_{t+1}/\mathcal{E}_t)] \} = 0 \] (3)

In a neighborhood of a perfect foresight steady state, and to a first-order approximation, we can rewrite the previous equation as:
\[ i_t = i^*_t + \mathbb{E}_t \{ \Delta e_{t+1} \} \] (4)

for all \( t \), where \( e_t \equiv \log \mathcal{E}_t \). This is the familiar uncovered interest parity (UIP) condition.

Letting \( q_t \equiv p^*_t + e_t - p_t \) denote the (log) real exchange rate, one can write the "real" version of UIP as:
\[ q_t = r^*_t - r_t + \mathbb{E}_t \{ q_{t+1} \} \] (5)

where \( r_t \equiv i_t - \mathbb{E}_t \{ \pi_{t+1} \} \) is the real interest rate and \( \pi_t \equiv p_t - p_{t-1} \) denotes (CPI) inflation, both referring to the home economy. \( r^*_t \) and with \( \pi^*_t \) are defined analogously for the foreign economy. Assume for simplicity that \( \lim_{T \to \infty} \mathbb{E}_t \{ q_T \} \) is well defined and bounded.\(^4\) In that case, (5) can be solved forward and, after taking the limit as \( T \to \infty \), rewritten as:
\[ q_t = \sum_{k=0}^{\infty} \mathbb{E}_t \{ r^*_{t+k} - r_{t+k} \} + \lim_{T \to \infty} \mathbb{E}_t \{ q_T \} \] (6)

Equation (6) is a straightforward implication of uncovered interest parity, combined with the assumptions of rational expectations and a bounded long run real exchange rate. It determines the real exchange rate as a function of (i) current and expected real interest rate differentials and (ii) the long run expectation of the real exchange rate. Forward-looking real exchange rate equations similar to (21) have often been used in the empirical exchange rate literature, though not in connection to forward guidance.\(^5\) For the purposes of the present paper a key property of (6) must be highlighted, namely, the lack of discounting of expected future real interest rate differentials. This property is analogous to the one featured by the dynamic IS equation of the New Keynesian model and which is at the root of the forward guidance puzzle, as discussed above.

In what follows I discuss some of the implications of that property for the real exchange rate and its connection to forward guidance policies, and explore its empirical support.

### 3.1 A Forward Guidance Experiment

Assume that at time \( t \) the central bank of a small open economy credibly announces an increase of the nominal interest rate of size \( \delta \), starting \( T \) periods from now and of duration \( D \) (i.e., from period \( t + T \) to \( t + T + D - 1 \)). Interest rates and prices in the rest of the world are assumed to remain unchanged in response to that announcement and its subsequent implementation. Furthermore, assume that the path of domestic prices also remains unchanged (this assumption is relaxed below). Under the assumption of long run neutrality of monetary

\(^4\)Note that the previous assumption is weaker than a weak version of purchasing power parity. In the empirical section below, when taking the model to the data, I relax that assumption by allowing for a time trend, possibly resulting from long term productivity growth rate differentials.

\(^5\)See, e.g., Engel and West (2005) and Engel (2016), among many others.
policy, \( \lim_{T \to \infty} \mathbb{E}_t \{ q_T \} \) should not change in response to the previous announcement. It follows from (6) that the real exchange rate will vary in response to the announcement by an amount given by

\[
\tilde{q}_t = -D \delta
\]

i.e. the exchange rate appreciation at the time of the announcement is proportional to the duration and the size of the announced interest rate increase, but is independent of its planned timing \((T)\). Thus, a \(D\)-period increase of the real interest rate 10 years from now is predicted to have the same effect on today’s real exchange rate as an increase of equal size and duration to be implemented immediately.

Once the interest rate increase is effectively implemented in period \(t + T\), the exchange rate depreciates at a constant rate \(\delta\) per period, i.e.

\[
\Delta q_{t+T+k} = \delta \quad \text{for} \quad k = 1, 2, \ldots D
\]

and stabilizes at its initial level once the intervention concludes, i.e. \(q_{t+T+k} = q_t\) for \(k = D + 1, D + 2, \ldots\)

Figure 1 illustrates the implied path of the interest rate and the exchange rate when an interest rate rise of 1% (in annual terms) is announced at \(t = 0\), to be implemented at \(T = 4\) and lasting for \(D = 4\) periods.

## 4 Forward Guidance and the Exchange Rate in General Equilibrium

Consider the (log-linearized) equilibrium conditions of a standard small open economy model with Calvo staggered price-setting, law of one price (producer pricing), and complete markets.\(^6\)

\[
\pi_{H,t} = \beta \mathbb{E}_t \{ \pi_{H,t+1} \} + \kappa y_t - \omega q_t \tag{7}
\]

\[
y_t = (1 - \nu) c_t + \vartheta q_t \tag{8}
\]

\[
c_t = \mathbb{E}_t \{ c_{t+1} \} - \frac{1}{\sigma} (i_t - \mathbb{E}_t \{ \pi_{t+1} \}) \tag{9}
\]

\[
c_t = \frac{1}{\sigma} q_t \tag{10}
\]

where \(\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}\) denotes domestic inflation, \(y_t\) is (log) output and \(c_t\) is (log) consumption. Equation (7) is a New Keynesian Phillips curve for the small open economy. Coefficients \(\kappa\) and \(\omega\) are defined as \(\kappa \equiv \lambda (\sigma + \varphi)\) and \(\omega \equiv \frac{\lambda (\sigma - 1) (1 - \nu)}{1 - \nu}\) where \(\nu \in [0, 1]\) is an index of openness (equal the share of imported goods in domestic consumption in the steady state), \(\sigma > 0\) is the (inverse) elasticity of intertemporal substitution, \(\eta > 0\) is the elasticity of substitution between domestic and foreign goods, and \(\lambda \equiv \frac{(1 - \theta) (1 - \beta \theta)}{\theta} > 0\) is inversely related to the Calvo price stickiness parameter \(\theta\). (8) is the goods market clearing condition, with \(\vartheta \equiv \eta \nu \left(1 + \frac{1}{1 - \nu}\right) > 0\).

(9) is the consumption Euler equation, with \(\pi_t \equiv p_t - p_{t-1}\) denoting CPI inflation. (10) is the international risk sharing condition, derived under the assumption of complete markets. The above specification of the equilibrium conditions assumes constant output, prices and real interest rates in the rest of the world, normalized to zero for notational ease (i.e. \(r^*_t = y^*_t = p^*_t = 0\),

\(^6\)Detailed derivations of the equilibrium conditions can be found in Galí and Monacelli (2005) and Galí (2015, chapter 8). With little loss of generality I assume an underlying technology that is linear in labor input.
all $t$). Also for simplicity I abstract from any non-policy shocks, with the analysis focusing exclusively on the effects of exogenous monetary policy changes.

Note that (9) and (10) imply the real version of UIP introduced in the previous section:

$$q_t = \mathbb{E}_t \{q_{t+1}\} - (i_t - \mathbb{E}_t \{\pi_{t+1}\})$$

(11)

Furthermore, under the maintained assumption of full pass through, CPI inflation and domestic inflation are linked by

$$\pi_t \equiv (1 - v)\pi_{H,t} + v \Delta e_t$$

$$= \pi_{H,t} + \frac{v}{1 - v} \Delta q_t$$

(12)

As shown in Galí and Monacelli (2005) the previous equilibrium conditions can be combined to obtain a system of two difference equations for domestic inflation $\pi_{H,t}$ and output $y_t$ that is isomorphic to that of the closed economy, namely:

$$\pi_{H,t} = \beta \mathbb{E}_t \{\pi_{H,t+1}\} + \kappa_v y_t$$

(13)

$$y_t = \mathbb{E}_t \{y_{t+1}\} - \frac{1}{\sigma_v} (i_t - \mathbb{E}_t \{\pi_{H,t+1}\})$$

(14)

where $\sigma_v \equiv \frac{\sigma}{1 + (\sigma - 1)\eta(2 - \nu)} > 0$ and $\kappa_v \equiv \lambda (\sigma_v + \varphi) > 0$ are now both functions of the open economy parameters $(\nu, \eta)$. In addition, by combining (8) and (10) one can derive the following simple relation between the real exchange rate and output:

$$q_t = \sigma_v (1 - v) y_t$$

(15)

In order to close the model, a description of monetary policy is required. I assume the simple rule

$$i_t = \phi_\pi \pi_{H,t}$$

(16)

where $\phi_\pi > 1$. It can be easily checked that in the absence of exogenous shocks the equilibrium in the above economy is (locally) unique and given by $\pi_{H,t} = y_t = q_t = i_t = 0$ for all $t$.

Consider next a forward guidance experiment analogous to the one analyzed in the previous section, but allowing for an endogenous response of inflation to the anticipated change in the interest rate. More specifically, assume that at time 0, the home central bank credibly announces a one-period increase in the nominal interest rate of 0.25 (i.e. one percentage point in annualized terms), to be implemented in period $T$. Furthermore, the central bank commits to keeping the nominal interest rate at its initial level (normalized to zero in the impulse responses) between

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7 The assumption of complete markets at the international level is sufficient (though not necessary) to derive the uncovered interest parity equation. As discussed in section 2 above that equation can be derived as long as there are some investors each period with unconstrained access to both domestic and foreign one-period bonds.

8 All of the results below carry over unaltered if we assume that the central bank responds to other variables (e.g. output or the exchange rate) in addition to domestic inflation. The reason is twofold: (i) the rule is assumed to be "suspended" between the announcement and the end of the implementation, and (ii) that once the intervention comes to an end (and in the absence of other shocks) the economy immediately jumps to the steady state, independently of the precise form of the rule (as long as equilibrium uniqueness is guaranteed).
periods 0 and $T - 1$, independently of the evolution of inflation. At time $T + 1$ it restores the interest rate rule (16) and, with it, the initial equilibrium. I use (13), (14) and (15) to determine the response of output, domestic inflation and the real exchange rate to the previous forward guidance experiment. Given the response of $\pi_{t,t}$ and $q_t$, (12) can be used to back out the response of CPI inflation, $\pi_t$. The latter can then be used to derive the response of the (consumption) price level, which combined with the relation $e_t = q_t + p_t$ allows one to derive the response of the nominal exchange rate.

Figure 2 displays the response of interest rates, the exchange rate, output, and inflation, to the above experiment under three alternative time horizons for implementation: $T = \{1, 2, 4\}$. The parameters of the model are calibrated as follows: $\beta = 0.99$, $\nu = 0.4$, $\sigma = \varphi = 1$, $\eta = 2$, and $\theta = 0.75$. Note that a version of the "forward guidance puzzle" for the open economy emerges: the longer is the horizon of implementation, the larger is the impact of the announcement on the real and nominal exchange rates as well as on output and inflation. As emphasized by McKay et al. (2016), the reason for the amplification has to do with the fact that inflation depends on current and expected future output, combined with the property that the longer is the implementation of a given interest rise the more persistent the output response. It follows that the longer is the implementation horizon of a given change in the nominal rate the larger will be the response of the real rate—and hence of output and the real exchange rate—between the time of the announcement and that of policy implementation.

Figure 3 illustrates more explicitly the forward guidance puzzle as applies to the nominal and real exchange rates. It displays the percent response of those two variables on impact when a one-period increase in the nominal rate is announced, to be implemented at alternative horizons represented by the horizontal axis. As the Figure makes clear the percent appreciation of the home currency, both in real and nominal terms, increases exponentially with the horizon of implementation. Note also that the appreciation of the nominal exchange rate is substantially larger than that of the real exchange rate, with the gap between the two increasing with the horizon of implementation. That gap, which corresponds to the percent decrease in the CPI in response to the forward guidance announcement, is also increasing in the horizon due to the forward-lookingness of the New Keynesian Phillips curve. The fall in inflation, in turn, leads to a further rise in current and future real interest rates (given an unchanged path for the nominal rate), thus generating an additional appreciation of the real exchange rate.

An alternative perspective on the previous experiment can be obtained by focusing on the determination of the nominal exchange rate. Consider an announcement of an interest rate increase of size $\delta$ and duration $D$, to be implemented $T$ periods ahead. Iterating forward equation (4) we can express the nominal exchange rate at the time of the policy announcement as:

$$ e_t = \delta D + \mathbb{E}_t\{e_{t+T+D}\} $$

The first term on the right hand side of (17) captures the dependence of the nominal exchange rate on anticipated changes in nominal interest rate differentials. As discussed in section 2 that effect is a function of the size ($\delta$) and duration ($D$) of the anticipated policy intervention, but not of its timing. This captures the partial equilibrium dimension of the forward guidance exchange rate puzzle. The second term, $\mathbb{E}_t\{p_{t+T+D}\}$, which reinforces the
The effect of the first term, is the result of general equilibrium effects working through (i) the impact on aggregate demand and output of the changes in consumption and the real exchange rate induced by the anticipation of higher future nominal interest rates (given prices), and (ii) their subsequent effects on inflation and the price level, which depend on the duration of the output effects (as implied by (13)) and, hence, on the timing of the policy implementation.

The strength of some the general equilibrium effects at work in the previous simulations is, from an empirical perspective, a controversial subject. This is true, in particular, with regard to the degree of forward-lookingness of inflation, i.e. that variable’s sensitivity to expected future output developments. An empirical analysis of the role played by the response of inflation (and, hence, of real interest rates) to anticipated changes in nominal interest rates in the determination of the exchange rate is clearly beyond the scope of the present paper.\(^9\) Instead, in the remainder of the paper I turn to an empirical exploration of the (partial equilibrium) link between the real exchange rate and current future real interest rate differentials, with a focus on the role played by the horizon of anticipated interest rate changes, and having as a benchmark the relationship between those variables implied by the real version of UIP and shown in (6).

5 Expected Interest Rate Differentials and the Exchange Rate: Does the Horizon Matter?

Next I provide some evidence on the extent to which fluctuations in the real exchange rate can be accounted for by variations in expected interest rate differentials at different horizons. The starting point of my empirical analysis is the relation between the real exchange rate and expected future real interest rate differentials implied by the UIP condition. The exact form of the equation to be taken to the data depends on the maintained assumption regarding the long run properties of the real exchange rate. Next I discuss three alternative assumptions regarding those properties.

In the first case considered, a (weak) version of purchasing power parity (PPP) is assumed to hold, so that \(\{q_t\}\) is stationary around a constant mean \(q\), and \(\lim_{T \to \infty} E_t\{q_T\} = q\). In that case one can rewrite (6) as

\[
q_t = q + \sum_{k=0}^{\infty} E_t \{ r^{*}_{t+k} - r_{t+k} \}
\]

(18)

I assume that for a sufficiently long horizon \(m\) the following approximation is valid:

\[
\sum_{k=m}^{\infty} E_t \{ r^{*}_{t+k} - r_{t+k} \} \approx 0
\]

In the empirical implementation below, I assume \(m = 360\), which corresponds to 30 years. This seems a conservative assumption.

In that case the infinite sum on the right hand side of (18) can be decomposed as the sum of two terms:

\(^9\)See, e.g. Mavroeidis et al. (2014), Rudd and Whelan (2005) and Galí et al. (2005), as well as other contributions to the special issue of the Journal of Monetary Economics (vol. 52, issue 6) on the empirics of the New Keynesian Phillips curve for a discussion of some the issues in that controversy.
\[ q_t \simeq q + \sum_{k=0}^{n-1} \mathbb{E}_t \{ r_{t+k}^* - r_{t+k} \} + \sum_{k=n}^{m-1} \mathbb{E}_t \{ r_{t+k}^* - r_{t+k} \} \]

(19)

for any horizon \( n \in \{1, 2, 3, \ldots, m-1\} \). Note that \( D^S_t(n) \) and \( D^L_t(n) \) capture the anticipated real interest rate differentials at "short" and "long" horizons, respectively, with \( n \) representing the number of lead periods that define the boundary between the two. The empirical strategy pursued below consists of using measures of the (log) real exchange rate, together with empirical proxies for \( D^S_t(n) \) and \( D^L_t(n) \) to estimate equation

\[ q_t = \alpha + \gamma_S D^S_t(n) + \gamma_L D^L_t(n) \]

(20)

In what follows I refer to (20) as the baseline specification. Note that the joint null of UIP and (weak) PPP implies \( \gamma_S = \gamma_L = 1 \), which can be tested. Most interestingly, one may want to examine the size and sign of the estimated deviations from that null, as well as its dependence on the horizon. Before discussing the details of the empirical implementation, I briefly describe two alternative representations of the exchange rate equation that relax the assumption of (weak) PPP underlying the above specification.

Consider the case of a (log) real exchange rate that is stationary around a deterministic trend \( \alpha + \delta t \), possibly as a result of different trend productivity growth rates in the tradable sectors of the two economies considered. Note that in that case \( \lim_{T \to \infty} \mathbb{E}_t \{ q_T \} \) is unbounded, so that representation (6) is not well defined (and (18) is invalid). Nevertheless, combining the previous assumptions with the arbitrage condition (5) (which remains valid, independently of the long run properties of \( q_t \)) one can derive:

\[ \hat{q}_t = \sum_{k=0}^{\infty} \mathbb{E}_t \{ r_{t+k}^* - r_{t+k} - \delta \} + \lim_{T \to \infty} \mathbb{E}_t \{ \hat{q}_T \} \]

(21)

where \( \hat{q}_t \equiv q_t - (\alpha + \delta t) \) is the detrended (log) real exchange rate and where \( \delta \) becomes the unconditional mean of the real interest rate differential. Under the trend stationarity assumption made here, \( \lim_{T \to \infty} \mathbb{E}_t \{ \hat{q}_T \} = 0 \). Accordingly, (21) can be rewritten as

\[ q_t = \alpha + \delta t + \sum_{k=0}^{\infty} \mathbb{E}_t \{ r_{t+k}^* - r_{t+k} - \delta \} \]

Again, I assume that for a sufficiently long horizon \( m \) the following approximation holds:

\[ \sum_{k=m}^{\infty} \mathbb{E}_t \{ r_{t+k}^* - r_{t+k} - \delta \} \simeq 0 \]

(22)

i.e., real interest rate differentials are expected to return to their unconditional mean \( \delta \) within a horizon of \( m \) months. It follows from (22), that

\[ q_t \simeq \alpha + \delta(t - m) + \sum_{k=0}^{m-1} \mathbb{E}_t \{ r_{t+k}^* - r_{t+k} \} \]

\[ \simeq \alpha + \delta(t - m) + D^S_t(n) + D^L_t(n) \]

(23)
which motivates the estimation of the empirical equation

$$q_t = \alpha_0 + \delta t + \gamma_S D_t^S(n) + \gamma_L D_t^L(n) \quad (24)$$

given empirical proxies for $D_t^S(n)$ and $D_t^L(n)$. Below I refer to (24) as the time trend specification.

Finally, I consider the case in which the (log) real exchange rate is an I(1) stochastic process, possibly with a deterministic component (i.e., $\Delta q_t$ is stationary with mean $\delta$). This would be a likely implication of (log) productivity differentials in the tradable sector being themselves I(1) processes. In that case equation (21) remains valid, and can be rewritten under assumption (22) as

$$q_t \simeq \alpha + \delta(t - m) + \sum_{k=0}^{m-1} \mathbb{E}_t \{ r^*_{t+k} - r_{t+k} \} + \lim_{T \to \infty} \mathbb{E}_t \{ \hat{q}_T \}$$

Taking first differences, we can write

$$\Delta q_t \simeq \delta + \Delta D_t^S(n) + \Delta D_t^L(n) + \varepsilon_t \quad (25)$$

where $\varepsilon_t \equiv \lim_{T \to \infty} (\mathbb{E}_t - \mathbb{E}_{t-1}) \{ q_T \}$, i.e. the period $t$ innovation in the expected long-run real exchange rate. Motivated by the previous considerations, below I also report estimates of the empirical equation

$$\Delta q_t = \delta + \gamma_S \Delta D_t^S(n) + \gamma_L \Delta D_t^L(n) + \varepsilon_t \quad (26)$$

henceforth referred to as the first difference specification.

As in the baseline representation, the absence of discounting in (6) implies that, ceteris paribus, a change in $D_t^S(n)$ (given $D_t^L(n)$) should have the same effect on the real exchange rate as a commensurate change in $D_t^L(n)$ (given $D_t^S(n)$). Furthermore, under UIP that effect should be "one-for-one" in both cases, i.e. $\gamma_S = \gamma_L = 1$, which can be tested. Also, estimates of $\gamma_S$ and $\gamma_L$ can help characterize the deviations from that null, with $\gamma_S$ measuring the sensitivity of the real exchange rate with respect to changes in expected interest rate differentials at shorter run horizons (i.e. over the next $n$ periods), while $\gamma_L$ captures the corresponding effect at longer horizons (i.e. beyond $n$ periods).

### 5.1 Empirical Strategy

In order to estimate (20), (24) and (26) I need to construct empirical counterparts to $D_t^S(n)$ and $D_t^L(n)$. This requires some assumptions, which I take as reasonable approximations, given the purpose at hand. First, I use (zero coupon) yields on government debt at different maturities to approximate expectations of future short term rates. Taking the time unit to be a month (so that $i_t$, $\pi_t$ and $r_t$ have the interpretation of monthly rates), and letting $i_t(n)$ denote the (annualized) zero coupon yield on a government bond maturing in $n$ months, I assume

$$\sum_{k=0}^{n-1} \mathbb{E}_t \{ i_{t+k} \} \simeq \frac{n}{12} i_t(n) \quad (27)$$

with an analogous relation holding for the foreign economy.

Secondly, I use (average) monthly data on inflation swaps at different maturities to approximate inflation expectations at different horizons. Letting $\pi_t(n)$ denote (annualized) expected
inflation between month \( t \) and month \( t + n \), the previous assumptions call for the use of the following approximation

\[
\sum_{k=0}^{n-1} \mathbb{E}_t \{ r_{t+k} \} \simeq \frac{n}{12} [(i_t(n) - \pi_t^e(n)]
\]

(28)

with an analogous relation holding for the foreign economy. The previous result is used below in order to compute, for each month, an empirical counterpart to the sum of expected real rate differentials over the subsequent \( n \) months, given data on government bond yields and inflation expectations of equivalent horizons for the home and foreign economies. More specifically,

\[
D^S_t(n) \equiv \sum_{k=0}^{n-1} \mathbb{E}_t \{ r^*_{t+k} - r_{t+k} \} \\
\simeq \frac{n}{12} [(i^*_t(n) - \pi^e_t(n)) - (i_t(n) - \pi^e_t(n))] 
\]

(29)

Finally, and given the assumptions above, one can obtain an approximate measure of \( D^L_t(n) \), the anticipated real interest rate differentials at "long" horizons, using the approximation

\[
D^L_t(n) \equiv \sum_{k=n}^{m-1} \mathbb{E}_t \{ r^*_{t+k} - r_{t+k} \} = D^S_t(m) - D^S_t(n) 
\]

(30)

given available measures of \( D^S_t(m) \) and \( D^S_t(n) \).

5.2 Data

I use monthly data on (zero coupon) yields for German, US and UK government bonds with 1, 2, 5, 10 and 30 year maturities, combined with monthly measures of expected inflation over the same five horizons derived from inflation swap contracts. I construct monthly time series for the real exchange rate, using data on the euro-dollar, pound-dollar, and pound-euro nominal exchange rates, and the CPI indexes for the US, euro area and UK economies. Data are monthly averages. Constraints on data availability on inflation swap contracts force me to start the sample period in 2004:8. As noted above I assume a "long horizon" \( m \) equal to 360 (corresponding to a 30 year horizon). I construct time series for \( D^S_t(n) \) and \( D^L_t(n) \) using the approximate relations (29) and (30) above, for \( n = 12, 24, 60, 120, 360 \).

5.3 Empirical Findings

Tables 1-3 report the main empirical findings of the paper, based on data for the US and the euro area (Table 1), US and the UK (Table 2) and euro area and UK (Table 3). Each table contains three panels, displaying respectively the OLS estimates of \( \gamma_S \) and \( \gamma_L \) for each of the three specifications introduced above. In each case, estimates are reported for horizons \( n \in \{12, 24, 60, 120\} \). In the case of \( n = 360 \) I only report the estimate for \( \gamma_S \) since \( D^L_t(360) \simeq 0 \) under the assumptions made. The sample period is 2004:8-2018:12. Standard errors, reported in brackets, were computed using the Newey-West adjustment for serial correlation, with a 12 lag window.
I start by describing the evidence for the euro-dollar exchange rate, based on US and euro area data, and summarized in Table 1. Note that most of the estimated coefficients are positive and highly significant. Thus, the evidence confirms the link between the real exchange rate and current and expected real interest rate differentials, with the sign of the relation consistent with the theory. The associated $R^2$ is very high for the baseline and time trend specifications, regardless of the horizon; perhaps not surprisingly it is lower for the first difference specification, given the amount of exchange rate "noise" at high frequencies. On the other hand, the null $\gamma_S = \gamma_L = 1$ is easily rejected for all specifications ($p$ values are extremely low and not reported). Most interestingly, the estimates for $\gamma_S$ are in all cases much larger than those of $\gamma_L$, by an order of magnitude. In words: changes in expected real interest rate differentials in the near future are associated with much larger variations in the real exchange rate than commensurate changes anticipated to take place in the more distant future. Furthermore, and consistent with that interpretation, a look at the pattern of $\gamma_S$ estimates across different values of $n$ suggests that the exchange rate elasticity with respect to expected interest rate differentials diminishes monotonically with the horizon. In particular, for all the specifications, $\gamma_S$ is larger than one – the value implied by the UIP theoretical benchmark – for horizons up to two years. In general the point estimates for both $\gamma_S$ and $\gamma_L$ are smaller in the first difference specification. For the baseline and time trend specifications the $\gamma_S$ estimate is also significantly above one for $n = 60$, corresponding to a horizon of five years, and for shorter horizons it is more than twice the size implied by the benchmark model. On the other hand, the elasticity of the real exchange rate with respect to expected real interest rate differentials at long horizons, given by $\gamma_L$, is systematically less than one, and significantly so. The previous findings imply that, relative to the UIP benchmark, exchange rates tend to overreact to changes in expected interest rate differentials at short horizons, while they tend to underreact to similar expected changes at long horizons. I refer to this apparent disconnect between theory and empirics as the forward guidance exchange rate puzzle.

The evidence for the pound-dollar exchange rate, based on US and UK data, is summarized in Table 2. That evidence is qualitatively very similar to that for the euro-dollar exchange rate. In particular, the estimates for $\gamma_S$ systematically decrease with the horizon, and are significantly larger than one at short horizons. Similarly, the estimates for $\gamma_L$ are smaller than one uniformly and, with one exception, insignificantly different from zero at long horizons in some cases (with some point estimates being slightly negative).

Table 3 reports the evidence for the pound-euro exchange rate. Again, most of the main qualitative findings emphasized above also obtain when data for the UK and the euro area are used. The only discrepancy in that regard are given by the estimates corresponding to the first difference specification, which are mostly insignificant in the case of the pound-euro evidence.

6 Discussion and Possible Explanations

The empirical analysis of the dynamic relation between the exchange rate and anticipated interest rate differentials described in the previous section has taken as a benchmark the UIP condition (5). It is that UIP condition which, combined with rational expectations and alternative assumptions on the long run properties of the real exchange rate, yields the forward-looking "undiscounted" exchange rate representations (19), (23) and (25) that have been taken to the
From that perspective, the rejection of the undiscounted model reported above should not be much of a surprise, given the well known empirical failure of UIP. The question remains as to what alternative model can account for the relationship between the exchange rate and expected interest rate differentials at different horizons uncovered in the previous section, and which I have labeled the forward guidance exchange rate puzzle.

Let \( \zeta_t \equiv r_t^* - r_t + \mathbb{E}_t\{q_{t+1}\} \) define the deviation from the UIP condition. Under rational expectations, \( \zeta_t \) has a natural interpretation as the (foreign exchange) risk premium, i.e. the expected excess return on foreign bonds relative to home bonds required by investors. Thus, we can generalize (5) and write it as:

\[
q_t = r_t^* - r_t + \mathbb{E}_t\{q_{t+1}\} - \zeta_t \quad (31)
\]

Note that, under the maintained assumption of rational expectations, (31) holds by construction, since it involves no more than the definition of the risk premium. With little loss of generality, assume that the risk premium \( \{\zeta_t\} \) is stationary with a zero unconditional mean. Moreover, suppose that for a sufficiently long horizon \( m \) the following approximation is valid:

\[
\sum_{k=m}^{\infty} \mathbb{E}_t\{\zeta_{t+k}\} \simeq 0
\]

Accordingly, and assuming PPP for expositional convenience, the real exchange rate equation (6) can now be written as

\[
q_t \simeq q + D^S_t(n) + D^L_t(n) - v_t
\]

where \( v_t \equiv \sum_{k=0}^{m-1} \mathbb{E}_t\{\zeta_{t+k}\} \) is the expected cumulative risk premium. Under the assumption that the risk premium is orthogonal to interest rate differentials at all leads and lags, OLS estimates of \( \gamma_S \) and \( \gamma_L \) in (??) and (??) should be consistent and thus converge to one asymptotically, even though UIP no longer holds (in a strict sense) due the presence of a time-varying risk premium. The evidence reported above is thus in conflict with the joint hypothesis of rational expectations and uncorrelated fluctuations in the risk premium.

When thinking about possible explanations for the above evidence it is worth noting that some of the solutions to the closed economy forward guidance puzzle found in the literature are unlikely to apply to the case at hand. Those solutions involve a "downward adjustment" in the elasticity of individual expected future marginal utility with respect to aggregate consumption as a consequence of a variety of assumptions, including the risk of death (Del Negro et al. (2015)) or the risk of lower future consumption in the presence of idiosyncratic shocks, incomplete markets, and borrowing constraints (e.g. McKay et al. (2016)). As a result, aggregate consumption in those models often becomes less sensitive to interest rates, especially future ones. In some simple examples of those models (e.g. McKay et al. (2017), the aggregate consumption Euler equation can be written, up to a first order approximation, as

\[
c_t = \alpha \mathbb{E}_t\{c_{t+1}\} - \frac{1}{\sigma} \mathbb{E}_t\{r_t\}
\]

\[\text{See, e.g. Bacchetta (2013) and Engel (2014) for surveys of the literature on UIP.}\]

\[\text{The generalization to a risk premium with a nonzero unconditional mean is straightforward.}\]

\[\text{As argued by Werning (2015) that "discounting" of future interest rates is not a general consequence of the presence of incomplete markets, depending critically on the cyclicality of household income risk.}\]
where \( \alpha \in (0, 1) \), thus implying the geometric discounting of anticipated future interest rates.

The interest parity condition (5), on the other hand, should hold to a first order approximation, independently of the properties of the discount factor, or the presence of incomplete markets, as long as each period there are some investors that have (frictionless) access to both home and foreign riskless bonds (or deposits). Intuitively, the reason is that (5) involves a "contemporaneous arbitrage" between two assets (whose payoffs are subject to the same discounting), as opposed to the "intertemporal arbitrage" associated with the consumer’s Euler equation.

Next I briefly discuss other candidate explanations, starting with some that maintain the assumption of rational expectations.

As emphasized in the literature, a possible reason for the observed deviations from UIP is that (5) may be an inadequate approximation to the arbitrage condition (3), since it ignores potentially important higher-order terms that may account for the joint comovement of the risk premium and interest rate differentials needed to explain the empirical findings above. I view as a challenge for future research to come up with a model of risk premium determination which can reconcile the frictionless arbitrage condition (3) with the evidence reported in the previous section, while preserving the assumption of rational expectations.

In a recent paper, Bacchetta and van Wincoop (2018) make some inroads at accounting for the forward guidance exchange rate puzzle by introducing a friction in the form of convex portfolio adjustment costs in a two-country overlapping generations model.\(^{13}\) The presence of those costs pose a limit to arbitrage which makes room for the emergence in equilibrium of a time-varying risk premium in response to changes in interest rate differentials. In particular, the authors show that in the equilibrium of their model the following representation for the (log) real exchange rate holds:

\[
q_t = \varphi q_{t-1} + \sum_{k=0}^{\infty} \alpha^k \mathbb{E}_t \{ r_{t+k}^* - r_{t+k} \} \tag{32}
\]

where coefficients \( \alpha \in [0, 1) \) and \( \varphi \in [0, 1) \) are, respectively, increasing and decreasing in the parameter that indexes the importance of portfolio adjustment costs. The Bacchetta-van Wincoop model displays two features that could potentially render it consistent with the empirical evidence above. Firstly, the dependence on anticipated interest rate differentials declines with the horizon, as (32) clearly implies. Secondly, the presence of the lagged exchange rate in (32) implies that lagged interest rate differentials (in addition to current and anticipated) are a determinant of the current exchange rate. To the extent that, as seems plausible, lagged interest rates are more strongly correlated with anticipated interest rates in the near future than with their counterparts at a more distant future, the estimated coefficient \( \gamma_S \) for relatively short horizons (small \( n \)) would likely be biased upwards and could be above one, especially if \( \alpha \) is not too small. A full-fledged quantitative analysis of the Bachetta-van Wincoop model and its ability to account for the forward guidance exchange rate puzzle seems an interesting avenue for further research, but one beyond the scope of the present paper.

\[^{13}\]In addition to the forward guidance exchange rate puzzle, the authors show that their model can potentially account for other five exchange rate puzzles: delayed overshooting, forward discount puzzle, predictability reversal, the Engel puzzle, and the Lustig-Stathopoulos-Verdelhan puzzle.
Alternatively, one may seek to account for the forward guidance exchange rate puzzle by allowing for some deviation from rational expectations. Consider the alternative UIP condition:

\[ q_t = r_t^* - r_t + \tilde{E}_t \{ q_{t+1} \} \]  

(33)

where \( \tilde{E}_t \{ \cdot \} \) denotes the subjective expectations operator. Assume that, as in the behavioral model of Gabaix (2018), subjective expectations involve some discounting relative to rational expectations, in particular, when applied to future deviations of the real exchange rate from its long run value, i.e. \( \tilde{E}_t \{ \tilde{q}_{t+1} \} = \alpha \tilde{E}_t \{ q_{t+1} \} \), for \( \alpha \in [0, 1) \). Under the assumption of long run PPP (for convenience), we can thus rewrite

\[ \tilde{q}_t = \sum_{k=0}^{\infty} \alpha^k \tilde{E}_t \{ r_{t+k}^* - r_{t+k} \} \]

with anticipated changes in real interest rate differentials in the distant future predicted to have a more muted effect on the real exchange rate than those anticipated over a shorter horizon. Note, however, that such an assumption would not be able to account for the seeming overreaction of the real exchange rate to anticipated changes in interest rate differentials in the near future, as implied by the estimates reported above. That shortcoming could in principle be overcome by a simple variation on the previous behavioral model, which I briefly describe next. Suppose that the relevant no arbitrage condition is given by

\[ r_t = r_t^* + \gamma \tilde{E}_t \{ \Delta q_{t+1} \} \]

where \( \gamma \geq 0 \) is the weight that investors attach to exchange rate changes when forming expectations about returns on the foreign asset. Note that \( \gamma = 1 \) under UIP, as consistent with rational behavior. Under the assumption made above that \( \tilde{E}_t \{ \tilde{q}_{t+1} \} = \alpha \tilde{E}_t \{ q_{t+1} \} \) with \( \alpha \in [0, 1) \), we have

\[ \tilde{q}_t = \frac{1}{\gamma} \sum_{k=0}^{\infty} \alpha^k \tilde{E}_t \{ r_{t+k}^* - r_{t+k} \} \]  

(34)

Thus, if \( \gamma \) is smaller than one, i.e. if investors *downweight* the role of exchange rate changes when forming expectations about future returns on foreign assets, (34) can potentially account for the evidence reported above: relative to the UIP benchmark, exchange rates tend to overreact to changes in expected interest rate differentials at short horizons, while they tend to underreact to similar expected changes at long horizons. Providing possible micro-foundations for the above reduced form model, based on either information frictions or behavioral assumptions is, however, beyond the scope of the present paper.

7 Concluding Comments

The present paper has explored the implications of uncovered interest parity (UIP) for the effectiveness of forward guidance policies in open economies, focusing on the role played by the exchange rate in the transmission of those policies. UIP implies that the current exchange rate is determined by current and expected future interest rate differentials, *undiscounted*. Accordingly, in partial equilibrium (i.e. ignoring the feedback effects on inflation) the effect on the current exchange rate of a given anticipated change in the interest rate does not decline
with the horizon of its implementation. Using a New Keynesian model of a small open economy as a reference framework, I show that when prices are allowed to respond endogenously, the size of the effect of anticipated changes in the nominal rate on the current exchange rate, as well as on output and inflation, is larger the longer is the horizon of implementation of the announced policies.

Using data on real exchange rates and market-based forecasts of real interest rate differentials for the US, UK and the euro area, I have provided evidence that conflicts with the prediction of undiscounted effects of anticipated real interest rate differentials. In particular, my findings suggest that expectations of interest rate differentials in the near (distant) future appear to have much larger (smaller) effects than is implied by the theory, an observation which I refer to as the forward guidance exchange rate puzzle. Several candidate deviations from the joint assumption of UIP and rational expectations are discussed that could potentially provide a theoretical explanation to that puzzle.
REFERENCES


### Table 1
**U.S. - Euro Area Evidence**

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<tr>
<th></th>
<th>$\hat{\gamma}_S$</th>
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Note: The table reports the OLS estimates of $\gamma_S$ and $\gamma_L$ in the regression equations (1), (2) and (3) in the main text, respectively. Sample period 2004:8-2018:12. Standard errors reported in brackets, computed using the Newey-West adjustment with 12 lags. One and two asterisks indicate significance at 10 and 5 percent level, respectively.
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Note: The table reports the OLS estimates of $\gamma_S$ and $\gamma_L$ in the regression equations (), (), and () in the main text, respectively. Sample period 2004:8-2018:12. Standard errors reported in brackets, computed using the Newey-West adjustment with 12 lags. One and two asterisks indicate significance at 10 and 5 percent level, respectively.
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<td></td>
<td>(0.45)</td>
<td>(0.06)</td>
<td></td>
</tr>
<tr>
<td>$n = 120$</td>
<td>1.74**</td>
<td>-0.00</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>(0.30)</td>
<td>(0.07)</td>
<td></td>
</tr>
<tr>
<td>$n = 360$</td>
<td>0.14</td>
<td>-</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>First differences</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n = 12$</td>
<td>-0.09</td>
<td>0.08*</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(0.67)</td>
<td>(0.04)</td>
<td></td>
</tr>
<tr>
<td>$n = 24$</td>
<td>0.39</td>
<td>-0.06*</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(0.70)</td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td>$n = 60$</td>
<td>0.41</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(0.42)</td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td>$n = 120$</td>
<td>0.23</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td>$n = 360$</td>
<td>0.07</td>
<td>-</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The table reports the OLS estimates of $\gamma_S$ and $\gamma_L$ in the regression equations (1), (2) and (3) in the main text, respectively. Sample period 2004:8-2018:12. Standard errors reported in brackets, computed using the Newey-West adjustment with 12 lags. One and two asterisks indicate significance at 10 and 5 percent level, respectively.
Figure 1. Forward Guidance and the Exchange Rate: Partial Equilibrium
Figure 2. Forward Guidance in the Open Economy: 
The Role of the Horizon
Figure 3. Forward Guidance: Exchange Rate Response and Implementation Horizon